AIRCRAFT REPAIR DAMAGE TOLERANCE ANALYSIS

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ABSTRACT

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Master of Engineering, 2006

Mechanical Engineering

By Dejan Markovic, Ryerson University

Modern air travel has become a perpetual evolution both from a practical and scientific point of view. However, it is also becoming increasingly common to fly in an airplane with little or no regard for the immense engineering involvement that goes into making air travel as safe and efficient as possible. This report considers the problems of aircraft fatigue and how it translates to inspectability for safety in order to predict problems and solve them before they actually occur. The most common aircraft repair is a crack in a pressurized skin panel. This report evaluates the structural integrity of a particular panel that is assumed to have failed in service and thus been repaired by the addition of a doubler. Damage tolerance analysis is used to evaluate a conservative crack growth scenario for a typical business jet with a structural economic life of 15,000 flight hours. The steps shown follow the guidelines approved by the regulating aviation bodies of both Canada and the United States (Transport Canada and the FAA respectively). Structural inspections are a common practice for aircraft at their half lives; in this case it would be 7,500 flights. The report determines that this particular scenario defines a threshold inspection interval of 8,414 flights and a repeat of 2,944 flights thereafter. In comparison with an actual test aircraft, having experienced an almost identical failure and repair program, the test aircraft experienced failure at 9,963 flights. Therefore, the intervals presented herein provide adequate clearance for the detection and repair of such damage. The purpose of this report is to introduce the underlying principals of damage tolerance analysis to the reader and illustrate the analytical process with a real world example. Such is the job of an aerospace stress engineer.

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TABLE OF CONTENTS

AUTHOR'S DECLARATION	II
ABSTRACT.....	III
ACKNOWLEDGMENTS	IV
TABLE OF CONTENTS	V
LIST OF FIGURES	VII
LIST OF TABLES.....	X
NOMENCLATURE.....	XI
1 CHAPTER ONE – HISTORICAL OVERVIEW.....	1
1.1 INTRODUCTION.....	1
2 CHAPTER TWO - LINEAR ELASTIC FRACTURE MECHANICS (LEFM).....	8
2.1 LEFM - INTRODUCTION.....	8
2.2 SUPERPOSITION AND COMPOUNDING	14
2.3 FRACTURE TOUGHNESS	15
2.4 RESIDUAL STRENGTH	17
2.5 CRACK PROPAGATION UNDER CYCLIC LOADING.....	19
3 CHAPTER THREE - DAMAGE TOLERANCE PRINCIPLES.....	24
3.1 WHAT IS DAMAGE TOLERANCE?.....	24
3.2 DAMAGE TOLERANCE EVALUATION PROCEDURE.....	27
3.3 PSES, ALIS AND SSIS	29
3.3.1 SELECTION OF PSES.....	31
3.4 FAILURE ANALYSIS QUALIFICATION PRINCIPLES	32
3.5 SPECTRA AND DESIGN MISSION ANALYSIS	38
3.6 FLAW SIZE AND CRACK LENGTHS	46
3.6.1 INSPECTABLE SIZE	48
3.6.2 CRITICAL SIZE	48
3.7 RESIDUAL STRENGTH	49
3.7.1 PLASTIC FAILURE	50
3.7.2 ELASTIC FAILURE	52
3.8 FATIGUE CRACK GROWTH	53
3.8.1 CRACK GROWTH MODELS.....	53
3.8.2 WALKER-CHANG MODEL.....	55
3.8.3 LOAD INTERACTION.....	55
3.8.4 NASGRO MODEL	57
3.8.5 STRESS INTENSITY FACTOR MODELS	57

3.8.6 INSPECTION INTERVALS	58
3.8.7 THRESHOLD INSPECTION.....	58
3.8.8 REPEAT INSPECTION	60
3.9 CRACK GROWTH SCENARIOS	61
4 CHAPTER FOUR - DAMAGE TOLERANCE ANALYSIS	63
4.1 INTRODUCTION.....	63
4.1.1 SUBJECT AIRCRAFT & TECHNICAL SPECIFICATIONS	63
4.2 PSE OVERVIEW	66
4.3 DEFINITION AND ANALYSIS	69
4.3.1 DAMAGE OVERVIEW	72
4.3.2 CRACK GROWTH FAILURE SEQUENCE	73
4.3.3 SECTION PROPERTIES	75
4.3.4 MATERIAL PROPERTIES	78
4.4 FINITE ELEMENT MODEL.....	79
4.4.1 ELEMENT FORCE.....	79
4.4.2 ELEMENT SHEAR	80
4.4.3 ENDLOADS.....	81
4.5 FLIGHT CASE SPECTRUM.....	83
4.5.1 APPLIED LOADS	83
4.5.2 STRESS TRANSFER FUNCTIONS	84
4.5.3 PRESSURE CASE.....	88
4.6 LIMIT STRESS	90
4.7 GEOMETRY FACTOR MODEL (BETA).....	91
4.8 RESIDUAL STRENGTH	93
4.9 CRACK GROWTH	94
4.10 INSPECTION INTERVALS.....	95
5 CHAPTER FIVE – AIRCRAFT REPAIR DAMAGE TOLERANCE ANALYSIS CONCLUSION.....	97
5.1 DISCUSSION AND COMPARISON OF RESULTS	97
5.2 ANALYSYS UNCERTAINTY	101
5.3 SOME RECOMMENDATIONS	102
APPENDIX A: NASTRAN INPUT EXTRACTION FILE.....	105
APPENDIX B: GAG STRESS OUTPUT	109
APPENDIX C	114
REFERENCES	118

LIST OF FIGURES

FIGURE 1-1: ALOHA AIRLINES B737 FATIGUE FAILURE (REF 2).....	6
FIGURE 2-1: CRACK LOADING MODES (REF 3).....	9
FIGURE 2-2: INFINITE PLATE WITH A CRACK UNDER MODE I LOADING (REF 3).....	9
FIGURE 2-3: SIZE EFFECT ON CRACK PROPAGATION.	13
FIGURE 2-4: PRINCIPAL OF SUPERPOSITION.....	14
FIGURE 2-5: FRACTURE TOUGHNESS FOR TITANIUM (REF 1).....	16
FIGURE 2-7: RESIDUAL STRENGTH VS. HALF CRACK LENGTH (REF 7)	18
FIGURE 2-8: SCHEMATIC ILLUSTRATION OF THE FATIGUE CRACK GROWTH RATE AS A FUNCTION OF STRESS INTENSITY RANGE (REF 8).	19
FIGURE 2-9: SCHEMATIC ILLUSTRATION OF THE FATIGUE CRACK GROWTH RATE AS A FUNCTION OF STRESS INTENSITY RANGE.....	20
FIGURE 2-10: TEMPERATURE EFFECT ON CRACK PROPAGATION (REF 1,5).....	21
FIGURE 2-11: EFFECT OF OVERLOAD ON CRACK PROPAGATION RATE (REF 6).	22
FIGURE 2-12: STRESS DISTRIBUTION UNDER ZERO APPLIED LOAD (REF 5).	23
FIGURE 3-1: DAMAGE TOLERANCE CERTIFICATION REQUIREMENTS COMPLIANCE (REF 1,10).....	28
FIGURE 3-2: STRUCTURAL ANALYSIS LOGIC (REF 1,2,10).	30
FIGURE 3-3: SAFE LIFE PRINCIPLE	33
FIGURE 3-4: SINGLE LOAD PATH PRINCIPLE.....	35

FIGURE 3-5: MULTIPLE LOAD PATH PRINCIPLE	37
FIGURE 3-6: EXAMPLE OF TYPICAL AIRCRAFT FLIGHT PROFILE.....	39
FIGURE 3-8: FLAW SIZES	47
FIGURE 3-9: EXAMPLE OF A CRACK GROWTH PLOT	48
FIGURE 3-10: EXAMPLE OF A RESIDUAL STRENGTH DIAGRAM.....	49
FIGURE 3-11 CRACK GROWTH MATERIAL DATA (REF 8).	54
FIGURE 4-1: 3 VIEW DRAWING (BOMBARDIER AEROSPACE).....	64
FIGURE 4-2: PSE OF INTEREST	66
FIGURE 4-3: TYPICAL STRUCTURE OF BAGGAGE DOOR CUTOUT (REF 18).	67
FIGURE 4-4: CRACK GROWTH CURVE	68
FIGURE 4-5: TYPICAL FEM REPRESENTATION.....	70
FIGURE 4-6: TYPICAL DETAILED FEM AT EMERGENCY EXIT DOOR CUT- OUTS	71
FIGURE 4-7: DAMAGED SKIN PANEL WITH CUTOUT & REPAIR DOUBLER	72
FIGURE 4-8: FAILURE SEQUENCE	74
FIGURE 4-9: SECTION CUTS	75
FIGURE 4-10: SECTION A-A	76
FIGURE 4-11: SECTION B-B	77
FIGURE 4-12: FEM REPRESENTATION	82
FIGURE 4-13: TYPICAL LINTEL FEM	83
FIGURE 4-14: BETA VS. HALF CRACK LENGTH.....	92
FIGURE 4-15: RESIDUAL STRENGTH CURVES	93

FIGURE 4-16: CRACK GROWTH CURVE	94
FIGURE 5-1: CYCLIC LOADING FATIGUE TEST SETUP 1	99
FIGURE 5-2: CYCLIC LOADING FATIGUE TEST SETUP 2	99
FIGURE 5-3: PANEL FAILURE FOLLOWING CYCLIC TESTS.....	100
FIGURE 5-4: CRACK IDENTIFICATION BY NON-DESTRUCTIVE INSPECTIONS (FPI).....	100

LIST OF TABLES

TABLE 3-1: EXAMPLE OF FLIGHT PERFORMANCE / MISSION BASED TABLE	41
TABLE 3-2: EXAMPLE OF A FLIGHT DISTRIBUTION TABLE	42
TABLE 3-3 WALKER-CHANG CRACK GROWTH PARAMETERS (REF 8)	56
TABLE 3-4: TABLE OF SCATTER FACTORS (REF 2,10)	59
TABLE 3-5: CRACK GROWTH EXAMPLE SCENERIOS (REF 10).	62
TABLE 4-1: CHALLENGER 300 TECHNICAL SPECIFICATIONS	65
TABLE 4-2: STRINGER AND SKIN CROSS-SECTION AREA	76
TABLE 4-3: FRAME X AND SKIN CROSS-SECTION AREA	77
TABLE 4-4: FRAME Y AND SKIN CROSS-SECTION AREA	78
TABLE 4-5: NASTRAN EXTRACTION FILE INPUT SUMMARY	87
TABLE 4-6: MOST CRITICAL GAG STRESSES FROM NASTRAN OUTPUT	89
TABLE 4-7 SCATTER FACTOR DEFINITION	96
TABLE 5-1 INSPECTION INTERVALS	97

NOMENCLATURE

σ_x	Stress along the crack growth direction.
σ_y	Stress in the perpendicular direction of the crack growth.
σ_{xy}	Shearing stress in the region of the crack tip.
K_I	Stress intensity factor. (I for mode I loading).
σ	Reference stress (as a function of K_I).
σ_{Flugge}	Flugge stress for pressure shells.
a	Crack length in inches.
β	Geometrical Factor, commonly referred to as 'Beta Value'.
r	distance to the crack tip.
a_i	Initial crack length.
a_d	Detectable crack length.
a_c	Critical crack length.
P	Load (inches).
P_i	Inner cap load.
P_o	Outer cap load.
K_c	Fracture toughness
K_a	Applied stress intensity factor.
F_{ty}	Material yield strength
W	Width
D	Diameter
C	Intercept of $\log(da/dN)$ vs $\log(\Delta K)$.
N	slope of $\log(da/dN)$ vs $\log(\Delta K)$
ΔK	Stress intensity range.

R	Ratio of minimum stress to maximum stress from σ/σ_K .
PSE	Primary Structural Element
N_i	Flight cycles when crack growth initiates.
N_d	Flight cycles when crack becomes detectable
N_c	Flight cycles when crack becomes critical in length.
da/dN	Crack growth rate.
q	Chang exponent
m	Walker collapsing constant
F_{thr}	Flights to threshold inspection.
F_{crit}	threshold inspection band, which is the number of flights from the initial flaw size to critical crack length.
F_{rep}	flights for repeat inspections intervals
F_{insp}	repeat inspection band, which is the number of flights from the inspectable size to the critical crack length
K_1	scatter factor for source of crack growth data.
K_2	scatter factor for number of load paths
K_3	scatter factor for environmental effects.
K_4	scatter factor to account for uncertainties in the analysis.
A	Area (inches).
y	Offset (inches).
I	Inertia (inches ⁴).
M	Moment (lb-in.)

1 CHAPTER ONE – HISTORICAL OVERVIEW

1.1 INTRODUCTION

The transportation industry as a whole has been plagued by fatigue related failures ever since the industrial revolution vastly increased the use of metals. Although a significant number of fatigue failures had been experienced by the railways during the first part of the 20th century and much of the early research on fatigue was carried out by that industry, relatively little effort was devoted to addressing fatigue concerns on aircraft during the early evolutionary stages of flight. According to T. Swift of the McDonnell Douglas Aircraft Company, the first systematic investigation of fatigue failures was initiated by a German Railway Company, Imperial Railways (1847 to 1889). They investigated experiments related to failures in railway axles (Ref 1).

Metal fatigue has traditionally always been a predicament for the aircraft manufacturers. The Wright brothers themselves have reported that the first flight of their flyer had been delayed because a hollow propeller shaft had developed a fatigue crack and they had to order a solid derivative.

It was not until the advent of larger transport aircraft during the latter part of the 1940's that fatigue avoidance became a significant consideration during the design process. This new emphasis on fatigue was probably driven by the use of progressively stronger materials, which could be operated at higher stress levels, an advantage that usually entailed some sacrifice in fatigue and fracture properties. Another factor may have been the steady improvement in stress analysis techniques that permitted the use of lower static strength margins and higher operating stresses that unfortunately also increased the risk of premature fatigue failure.

A radical design paradigm that was heavily favoured in the United Kingdom was the imposition of a fatigue life limit on all primary structure considered to be at risk from fatigue. Fatigue life for such structure was usually determined by test and then divided by a factor (a value of 5 was typical) to obtain a "safe-life". Structure certified in this manner could remain in service until the safe-life was reached, at which time it had to be scrapped. This criteria was, at the time, considered to be the safest and most cost reliable solution, as it was seen to be a cheaper and more cost effective to replace whole components rather than risk having to replace an entire aircraft in the event of a catastrophe; something that was occurring with frequent regularity.

Conversely, the North American standard, at around this time, was favouring an alternative approach known as "fail-safe" certification. This required the structure be designed with multiple load paths sufficiently strong so that if one load path failed (through fatigue, accidental damage or corrosion), the remaining intact paths could continue to bear all loads until the failure was detected and repaired.

This objective was achieved by the duplication of primary structural members. Mandatory retirement times were not considered necessary for such structure as the aircraft was considered safe for operation indefinitely so long as fatigue cracks, service damage, or corrosion were detected in a timely manner and repaired. To ensure that this would happen, periodic inspection tasks for potentially critical areas had to be incorporated into the scheduled maintenance program for the aircraft. Inspection intervals were determined by a panel of maintenance engineers primarily using judgment based on past experience with similar structure. With these provisions in place, it was considered that fail-safe structure could remain in service until the aircraft became uneconomic to operate. On the other hand, safe-life structure would in most cases have to be withdrawn from service at a time when only a fraction of the structure's fatigue life had been used up, a natural consequence of the imposition of a drastically factored down test life. In all probability the aircraft would at that point still be

perfectly viable economically. The fail-safe approach, as well as providing the potential for enhanced safety through load path duplication therefore also offered an economic advantage over the safe-life approach.

Despite the implementation of these safety measures, further fatigue related accidents took place. In this case, two DeHavilland Comet passenger aircraft broke apart during high altitude cruise flight in 1951 as a result of a pressure cabin fatigue failure. Completely unexpected at the time, this was a severe shock to the aviation industry as a whole, because this aircraft type had a prior fatigue test of the front fuselage section by the manufacturer. The testing had been performed for 18,000 pressure cycles, whereas the fuselages on the two accident aircraft had failed after only 1,290 and 900 pressure cycles. This accident had led to one of the most thorough aircraft accident investigations undertaken up to that time. The final conclusion yielded an understanding of the mechanism that had produced such a misleading test result. The aircraft test specimen that was used in the fatigue test evaluation also happened to be the same aircraft used for static testing of high pressurization loads. The significance of these findings was that the high pressurization loads had created fatigue life enhancing residual stresses that led to an unrealistic extension of fatigue life on test aircraft.

The Comet disasters, probably more than any other single event up to that time, prompted the funding of research during the following two decades that led to major advances in fatigue evaluation and the development of the design features needed to obtain damage tolerant pressure cabins.

The safe-life approach eventually fell out of favour on civil transport aircraft, even in the U.K., no doubt partly for the economic reason cited above and fail-safe design came to be adopted almost universally for most of the structure. Landing gears continued to be safe-life as it was considered impractical to incorporate multiple load paths in these components. The safe-life approach was also

retained for fighter and attack aircraft, possibly because their smaller size and "crowded" structure made fail-safe design more difficult to apply.

Another surprise came in 1969, when an in-flight break-up of a low-time F-111 aircraft occurred due to a wing fatigue failure. Interestingly, as had been the case in the Comet accidents, fatigue testing had indicated a significantly longer life than the 105 flight hours of service attained by the accident aircraft. An investigation established that in this instance premature fatigue failure was caused by an undetected manufacturing flaw in the welded wing box centre section. This wasn't the first time the military had experienced substantial loss. In 1958, two B-47's disintegrated in mid-air after having their lower wing skins fail. The investigations of these aircraft were still underway, when three more in-flight accidents occurred.

This series of catastrophic events had resulted in a complete re-think of the fatigue qualification process by the U.S. Air Force (USAF) and culminated in the replacement of the safe-life approach by a radical and somewhat daring new procedure which came to be known as the "Damage Tolerance" method. This method now required that at the very least, the aircraft manufacturer would have to show that the aircraft possessed the contracted service life despite the presence of manufacturing flaws of specified size at any potentially critical location in the structure. Additionally, the designer was encouraged to employ structural configurations and materials that facilitated the detection of cracks before dangerous structural weakening occurred.

Complimenting this innovative approach was the emerging discipline of "fracture mechanics" which had been evolving over the preceding 20 years to the point where it could be used to provide a means of determining crack growth by calculation. Having had to bear the grunt of criticism and lawsuits, not to mention the general perception that flying is becoming more and more dangerous, aircraft manufacturers couldn't afford to have any more catastrophes

overshadow their progress towards improved safety and reliability. The general consensus was that fracture mechanics methods had not yet matured sufficiently for use in an aircraft design. As a result, the United States government, namely the USAF, took up the onus, and over the next several years, they funded development of the required tools to improve the techniques and capabilities of damage tolerance methodologies.

The U.S. Navy (USN) on the other hand, had decided against the trend and thus retained the safe-life approach, which led to an interesting situation in 1991 that highlighted the philosophical difference in the maintenance of continued structural airworthiness by the USAF and the USN. A situation arose, where cracks were discovered in the fuselages of a number of F-16 aircraft operated by both agencies. Based on their maturing methodologies of damage tolerance principals, the USAF, was able to declare its F-16s safe for further service, whereas the USN, which demanded a crack-free structure, had to immediately ground its aircraft. The USN since has adopted an amalgamation of the two methodologies.

Meanwhile, civilian aviation was also having the share of related issues and deficiencies in fatigue certification and requirements. For example, the 1974 wing failure of a HS-748 over Argentina, and again in 1977 the failure of a Boeing 707 tailplane failure in Lusaka where both aircraft had been certified to fail-safe requirements. As a result of the success the USAF was having at implementing their damage tolerance principals the, United States Federal Aviation Administration (FAA) in 1978, mandated that in the future all newly certified FAR Part 25 transport aircraft would have to satisfy damage tolerance requirements.

FIGURE 1-1: ALOHA AIRLINES B737 FATIGUE FAILURE (REF 2).

The need for the durability objective was demonstrated quite dramatically by the in-flight fuselage pressure cabin failure of a Boeing 737-200 jetliner operated by Aloha Airlines of Hawaii on April 28, 1988. A subsequent investigation reported the cause was by the sudden in-flight link-up of multiple small fatigue cracks that had initiated independently at a large number of adjacent sites. Several passengers were sucked out of the aircraft still strapped in their seats, (Ref 2), see Figure 1-1.

A major enhancement introduced by the damage tolerance requirements is that inspection intervals now have to be substantiated by test and/or analysis, rather than being judgment based. Also, for multi load path structures, residual strength has to be demonstrated with appropriate account taken of fatigue

damage sustained in service prior to failure of the primary load path. Previous fail-safe certification practice had not accounted for this. As can be expected, this change in requirements resulted in a significant increase in the certification workload. Not long after, in 1980 the European Joint Airworthiness Authority followed the FAA's lead by requiring damage tolerance certification of all new designs. It is interesting to note that a Canadian design (the Canadair Challenger) was the first aircraft to be fully certified to the new FAA damage tolerance requirements.

The objective of this study is to thoroughly introduce the basic concepts behind the theoretical applications of fracture mechanics that yield the practical results required for damage tolerance evaluation of aircraft structures and components for the sole purpose of developing inspection criteria on such structures and components. This is done through a brief synopsis of the historical implications of having the ability to pre-determine fatigue critical locations on aircraft structures as well as a succinct abridgment of past and present aviation regulations that govern the development and regulations in this field of engineering. Furthermore, a methodological breakdown of the steps required to perform an exacting analysis is followed by an actual industry example of a repaired aircraft structure requiring damage tolerance analysis. The example simulates exact operating conditions of a typical mid-range business jet aircraft, requiring inspection intervals on the aforementioned structural repair. Finally, the results are compared to an actual service aircraft that has had an identical repair with known cracking at known flight cycles. This comparison measures the level of conservatism in the analysis assumptions as well as providing the reader with an idea of the importance pertaining to structural and damage tolerance analysis in aircraft engineering. The scope of this report is limited to historical research, methodology review and practical analysis execution with a final conclusion outlining the results and providing the insight into analysis uncertainty and recommendations based on the author's professional aerospace engineering experience.

2 PCHAPTER TWO - LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

2.1 LEFM - INTRODUCTION

Fracture Mechanics is that technology concerned with the modeling of cracking phenomena. A material's tolerance to flaws resides in a material's ability to deform locally. Since the source of fractures can be identified with the lack of material tolerance to cracks, it seems only natural that attention should be focused on the crack tip region where the material must resist crack extension.

Damage Tolerance methodology necessitates the evaluation of crack behaviour, as a result fracture mechanics becomes the primary analysis tool. It is therefore appropriate to begin an overview of damage tolerance methods with brief outline of fracture mechanics principles.

The application of a fracture mechanics model to solve crack problems came about through the following realization: component fractures that result from the extension of small crack-like defects are failures that depend on localized phenomena. There are three possible ways in which a crack can be loaded. These are illustrated in Figure 2-1 and are referred to as Modes I, II and III. In most damage tolerance analysis situations, only Mode I loading is of practical interest. Although Mode II behaviour can exist initially under pure shear loading, a propagating crack loaded in this way tends to change direction in a manner that transitions to Mode I loading. Mode III is rarely encountered in practice. The emphasis in the practical application of fracture mechanics methods to aircraft structures therefore tends to be on Mode I loading behaviour.

Crack loading modes

FIGURE 2-1: CRACK LOADING MODES (REF 3)

Arguably, fracture mechanics had become as a practical crack analysis tool following George Irwin's successful analysis of the crack tip region stress field using elasticity theory.

A brief synopsis of his work considers a flat, infinitely large, perfectly elastic plate with a crack of length '2a' that is subjected to a uniform remote stress σ as shown below (i.e. a Mode I loading condition).

FIGURE 2-2: INFINITE PLATE WITH A CRACK UNDER MODE I LOADING (REF 3)

Using elasticity theory and Airy stress functions, Irwin was able to show that for a linear elastic material the magnitude of the stress components σ_x , σ_y and σ_{xy} in the immediate vicinity of the crack tips, expressed in terms of polar co-ordinates r and θ is given by (Ref 4,5,6):

$$\sigma_y = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} f_y(\theta) \quad (1)$$

$$\sigma_x = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - \sigma = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} f_x(\theta) - \sigma \quad (2)$$

$$\sigma_{xy} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} f_{xy}(\theta) \quad (3)$$

$$\sigma_{xz} = \sigma_{yz} = 0$$

$$\sigma_z = 0 \quad \text{..... (Plane Stress)}$$

$$\sigma_z = \mu(\sigma_x + \sigma_y) \quad \text{..... (Plane Strain)}$$

Where $f_y(\theta)$, $f_x(\theta)$ and $f_{xy}(\theta)$ are trigonometric functions of θ .

The following can be noted regarding these results:

1. The right hand side of the above equations for σ_x , σ_y and σ_{xy} is an approximation in that only the first term of an infinite series is included. This is acceptable since the first term dominates at small values of r , which is the primary region of interest in the evaluation of crack behaviour. These equations are therefore restricted to $r \ll a$.
2. The term $-\sigma$ in the equation for σ_x is usually neglected since it is insignificantly small compared to the rest of the terms.
3. Stresses in the crack tip region tend to infinity as $r \rightarrow 0$.

4. Stresses in the crack tip region are equal to the product of a geometrical position parameter $f_{ij}(\theta) / \sqrt{(2\pi r)}$ and the parameter $\sigma\sqrt{(\pi a)}$.

The parameter $\sigma\sqrt{(\pi a)}$ is called the "Stress Intensity Factor" and is denoted by the symbol K_I where the subscript I denotes Mode I loading. With this notation the equations for σ_x , σ_y and σ_{xy} become:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} f_y(\theta) \quad (4)$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} f_x(\theta) \quad (5)$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} f_{xy}(\theta) \quad (6)$$

The crack tip region stress-field equations for σ_x , σ_y and σ_{xy} all contain the geometrical position parameter $f_{ij}(\theta) / \sqrt{(2\pi r)}$. It has been shown that this term is the same for all Mode I loading conditions and structure geometry, which is a very significant finding since that leaves the stress intensity factor K_I as the sole determinant of crack loading severity for different loading & geometry combinations. For other combinations of geometry and loading an adjustment factor β (often referred to as the crack geometry factor) can be derived by various methods, so that K_I is given by (Ref 5):

$$K_I = \beta \sigma \sqrt{(\pi a)} \quad (7)$$

Handbooks are available which provide β values for many commonly encountered configurations and loading conditions, such as the NASGRO Material Database

and Peterson's Stress Intensity Factors. Since $f_i(\theta) / \sqrt{(2\pi r)}$ is the same for all Mode I loading conditions.

The following example demonstrates the ability of the stress intensity factor to account for a size effect that cannot be readily accounted for by traditional fatigue analysis. Consider two flat plates of finite width, each with a centered hole and subjected to constant amplitude cyclic loading with a nominal magnitude of $P_{\max} = P_{\max}$ ksi and $P_{\min} = 0$. Assuming that each plate is made of the same material and that they are identical in every respect except that one is twice as large as the other. Both panels will have the same stress concentration factor (by virtue of their being geometrically similar) and are subjected to identical stress levels. Crack propagation analysis commenced from an industry standard 0.005" through-thickness crack located at the edge of the hole and carried out with appropriate stress intensity factors, yields a significantly longer life for the half-sized panel, as is apparent in the figure 2-3 (Ref 5).

The conclusion is as follows, the shorter crack propagation life obtained for the larger panel can be explained by the fact that the larger panel, the elevated stress field in the region of the hole, although not increased in magnitude, extends over twice the length. The stress intensity factors used in crack propagation analysis account for this longer stress field and a shorter life results.

FIGURE 2-3: SIZE EFFECT ON CRACK PROPAGATION.

2.2 SUPERPOSITION AND COMPOUNDING

Superposition - Superposition of stress intensity factors is suitable when a structure of a particular geometry is subjected to more than one loading condition. This is achieved by adding the crack geometry factors for each of the constituent loading conditions. The only constraint on the summation process is that the stress-intensity factors must be associated with the same structural geometry, including crack geometry. Thus, stress-intensity factors associated with edge crack problems cannot be added to that of a crack growing radially from a hole. The following example in figure 2-4 illustrates the conditions under which one might linearly add stress-intensity factors (Ref 6). Compounding is the process of multiplying stress intensity factors.

$$\beta_{EFF} = \sum_{i=1}^n \beta_i$$

FIGURE 2-4: PRINCIPAL OF SUPERPOSITION

2.3 FRACTURE TOUGHNESS

Fracture toughness properties of a material are essential for reliable crack growth analyses. These include plane strain fracture toughness (K_{Ic}) values, part-through fracture toughness (K_{Ie}) values, and any other available fracture toughness (K_c) values as a function of thickness. K_{Ie} values are especially important because flaws in real structures are often part-through (surface or corner) cracks. The main reason that fracture toughness depends on thickness is that differences in constraint produce changes in the stress state in the material. Thin structures, where the constraint is small, experience a plane stress condition, whereas thicker structures that are in plane strain have more constraint.

Having already established that the stress intensity factor K is principal determinant of crack loading severity, fracture toughness is thus defined as the upper limit of K at which failure occurs in a cracked structure in the same manner that failure occurs in an uncracked structure when the stress exceeds the material ultimate value. This upper limit is referred to as the "fracture toughness" of the material and is denoted by K_{Ic} for mode I loading. It can be determined by test from (Ref 4):

$$K_{Ic} = \beta \sigma_{fail} \sqrt{\pi a} \quad (8)$$

Where,

σ_{fail} = the test determined stress at failure

a = the associated crack length

Plane stress K_{Ic} data, are of significant use in aircraft damage tolerance analysis because the plane stress conditions usually prevail in the thin-skinned shell structures. The following is an example of K_{Ic} variation with thickness in figure 2-5.

FIGURE 2-5: FRACTURE TOUGHNESS FOR TITANIUM (REF 1).

The variation in fracture toughness with respect to grain direction can be quite large, with the highest values occurring when the crack plane is normal to the grain direction (L-T and L-S orientations) and lowest when the crack plane is parallel to the grain direction (S-T and S-L orientations).

2.4 RESIDUAL STRENGTH

The strength of a structure can be significantly affected by the presence of a crack and is usually substantially lower than the strength of the undamaged structure. To prevent catastrophic failure, it is important to evaluate the load carrying capacity that will exist in the potentially cracked structure throughout its expected service life. The load carrying capacity of a cracked structure is the residual strength of that structure and it is a function of material toughness, crack size, crack geometry and structural configuration.

The residual strength capability curve defines the level of load that the structure can withstand without failing in the presence of a growing crack. The change in residual strength capacity is a function of time, and this is accounted for in the crack growth calculation, as the crack size is a function of time. The analysis of residual strength capability for built-up structure requires the estimation of the critical stress level at which the partial failure occurs, as well as an understanding of the capability of the total structure to withstand this partial failure at and subsequent to the time of failure. The required load associated with the time subsequent to failure is based on the inspection category and, the partially failed structure must be able to maintain this load until the time of inspection.

A materials residual strength may be calculated if its fracture toughness and crack geometry factors are known from the following equation:

$$\sigma_c = \frac{K_c}{\beta \sqrt{\pi a}} \quad (9)$$

The residual strength curve in figure 2-7, displays results for a typical panel based on the above residual strength expression. It is important to note that at the ultimate limit small crack lengths will exceed it. It is generally accepted, however, that failure occurs when the average stress in a particular cross-section reaches the material yield stress. Furthermore, for a panel of finite width, another problem

that presents itself when the crack approaches the panel's edge and the residual strength is zero. As a result, (Ref 7) Feddersen proposed that in the short crack region the residual strength curve could be represented by a tangential straight line **DC** as indicated in the diagram and another tangential straight line **EF** could be used to represent residual strength in the long crack region. Residual strength is then defined by curve **DCEF**. This engineering approximation is in fairly good agreement with test results and is widely used.

FIGURE 2-6: RESIDUAL STRENGTH VS. HALF CRACK LENGTH (REF 7)

As an example, for a lug of width W , diameter D and thickness t , and material yield strength σ_y , the residual strength limit imposed by net section yielding is defined by the following straight line,

$$\sigma_c = \sigma_y (W - D - a/(W - D)) \quad (10)$$

2.5 CRACK PROPAGATION UNDER CYCLIC LOADING

Crack propagation under cyclic loading is expressed as the amount of crack growth occurring per load cycle (da/dN), and is primarily a function of the change in stress intensity factor (ΔK), the stress ratio ($R = \sigma_{\min} / \sigma_{\max}$) and thickness. Available data for constant amplitude loading has been generated from tests carried out over many years for all materials likely to be used in aircraft construction.

FIGURE 2-7: SCHEMATIC ILLUSTRATION OF THE FATIGUE CRACK GROWTH RATE AS A FUNCTION OF STRESS INTENSITY RANGE (REF 8).

FIGURE 2-8: SCHEMATIC ILLUSTRATION OF THE FATIGUE CRACK GROWTH RATE AS A FUNCTION OF STRESS INTENSITY RANGE.

Figure 2-9 above depicts the general fatigue cracking behaviour exhibited by most structural materials. This data can often be represented with reasonable accuracy by a straight line over a significant ΔK range. This is indicated in the diagram and leads to the well known "Paris Equation" (Ref 1):

$$da/dN = C (\Delta K)^n \quad (11)$$

where 'C' and 'n' are empirical constants.

The curve implies that at low ΔK there exists a threshold below which no crack growth is observed. At the same time, there exists a threshold in the stress-intensity range where rapid crack propagation occurs when the maximum stress-intensity-factor in the fatigue cycle approaches the fracture toughness of the material.

FIGURE 2-9: TEMPERATURE EFFECT ON CRACK PROPAGATION (REF1,5).

Other factors that contribute to crack propagation rate include the environment that the material is exposed to, the frequency of the applied loading and certainly the service temperature. Cracks have a tendency to grow faster with increasing temperature, as illustrated in Figure 2-10. Corrosive environments such as high humidity air, salt laden sea air and contaminants often present in fuel tanks usually increase crack propagation rate as well, however the effect is subject to a lot of scatter (Ref 1).

Aircraft operations often experience occasional high loads within a cyclic load sequence and these spikes can have a very pronounced effect on crack propagation rate. Figure 2-11 depicts how the propagation of a crack can be lowered by inducing a periodic tensile overload in a constant amplitude load sequence. A compressive overload has the opposite effect and when coupled with a tensile overload will reduce the effect of the tensile load, as also indicated below. (Ref 1).

FIGURE 2-10: EFFECT OF OVERLOAD ON CRACK PROPAGATION RATE (REF 6).

FIGURE 2-11: STRESS DISTRIBUTION UNDER ZERO APPLIED LOAD (REF 5).

This phenomenon is termed "plasticity induced crack closure" and it was introduced by Elber, who hypothesized that a crack subjected to cyclic loading remains tightly closed below a certain load level and that only the portion of the cyclic load sequence which exceeds this threshold has any affect crack propagation. He formulated that the higher this threshold, the lower the crack propagation rate for a given load sequence. It is the existence of this threshold could be explained by the plastically stretched material left behind in the wake of an advancing crack, which results from material yielding in the plastic zone located ahead of the crack tip. This stretched material prevents the crack faces from separating during the load application phase until a certain load level is attained.

3 CHAPTER THREE - DAMAGE TOLERANCE PRINCIPLES

3.1 WHAT IS DAMAGE TOLERANCE?

The Boeing Commercial Airplane Company defines Damage Tolerance in the following way:

"Damage tolerance is the ability of structure to sustain anticipated loads in the presence of fatigue, corrosion or accidental damage until such damage is detected through inspections or malfunctions and repaired".

The Federal Aviation Agency (FAA) defines Damage Tolerance as follows:

"Damage tolerance means that the structure has been evaluated to ensure that should serious fatigue, corrosion, or accidental damage occur within the operational life of the aircraft, the remaining structure can withstand reasonable loads without failure or excessive structural deformation until the damage is detected".

The objectives of the damage tolerance approach could be summarized as follows:

- DAMAGE TOLERANCE OBJECTIVE:
 - 1) To devise an inspection program for the primary structure which ensures that cracking initiated by fatigue, accidental damage or corrosion will not result in a catastrophic failure before being detected and repaired.
 - 2) To show that safe completion of a flight is possible following discrete source damage due to bird impact or uncontained engine rotor burst

- **DURABILITY OBJECTIVE:**

To select structural configurations and materials that will eliminate, or at least minimize the possibility of fatigue cracking at multiple sites during the expected service life of the aircraft.

The damage tolerance evaluation must cover all parts of the primary structure that are at risk from fatigue, accidental and environmental damage and whose failure could jeopardize the safe operation of the aircraft. If damage tolerance evaluation of any part(s) of the structure falling into this category is not feasible, safe-life certification is permitted by the current rules, provided that it can be demonstrated to the satisfaction of the certification authorities that damage tolerance evaluation is impractical. Current certification authority policy is that the landing gear and its attachments are the only components not amenable to damage tolerance certification.

For damage tolerance evaluation purposes, each structural detail, structural element, or structural assembly which is at risk from fatigue, accidental or environmental damage and whose failure would result in a reduction in aircraft residual strength or loss of structural function that could have catastrophic consequences is classified as a Primary Structural Element (PSE). A PSE may consist of only a single structural detail such as a joint, or of a larger structural component such as an entire wing skin panel assembly. In the case of the latter, only the most critical location on the PSE would usually be evaluated and the limitations determined for this location would be applied to the entire structure covered by the PSE. The limitations that flow from the damage tolerance evaluation are the inspection requirements for each PSE and these have to be approved by the certifying authority. Following approval, these inspection requirements are entered in the Airworthiness Limitations Section of the Structural Maintenance Manual. As is the case for all airworthiness limitations, subsequent

changes require Airworthiness Authority approval that is granted only on the basis of further substantiation.

Although damage tolerance qualification could in principle be achieved using only analytical methods, some testing is often needed to validate analysis methods and/or to obtain representative material data. For some structural details analysis may not be sufficiently reliable with currently available analytical techniques, in which case qualification will have to be entirely by test.

In practice, PSE selection can be confined to structure, which is at risk from fatigue, accidental or environmental damage and whose failure is likely to have catastrophic consequences, especially if the structure possesses one or more of the following characteristics:

1. A low strength margin in tension or shear.
2. High stress concentration and/or high fastener load transfer.
3. Poor inspectability.
4. Early cracking during fatigue test.

(Ref 10).

3.2 DAMAGE TOLERANCE EVALUATION PROCEDURE

The first step in the evaluation of Damage Tolerant Structures is to establish a usage profile that defines how the aircraft is expected to be operate in typical service. With this in mind, a "global load spectra" has to be derived to define the loads likely to be encountered during all flight phases of the anticipated usage profile. Frequency-of-occurrence load data is available from various accepted sources and will be consulted for this purpose.

Next, a comprehensive list of PSEs (Primary Structural Elements) will then be prepared using the criteria discussed previously. In all likelihood this list will grow as analysis and fatigue testing progresses. PSEs may even be added after the aircraft has entered service. "Local load spectra" must now be generated for each PSE from the global spectra. These are expressed in terms of stress or load in the critical element(s) of each PSE. This is a prerequisite before any analysis or component detail testing can be carried out.

At this point it will be necessary to determine which PSEs are to be certified as safe-life; the remainder will have to be certified as damage tolerant. Although conservative analysis may be accepted by the certification authorities for establishing interim lives for safe-life parts of the structure, it is generally expected that final safe-lives will be determined by fatigue test of representative structure. Test requirements for these components must therefore be issued as soon as possible. For the damage tolerance evaluation of the remaining PSEs, which will constitute the major portion of the structure, a decision must be made for each PSE whether the evaluation is to be as "single load path" or "multi load path" structure. Multi load path structure may be evaluated by analysis or, as an alternative, entirely by test. Generally however, fracture mechanics analysis is the preferred option since this is likely to be less costly and usually provides early results. Figure 3-1 illustrates an organizational breakdown of the DTA evaluation.

FIGURE 3-1: DAMAGE TOLERANCE CERTIFICATION REQUIREMENTS COMPLIANCE (REF 1,10).

3.3 PSES, ALIS AND SSIS

Principal Structural Elements, PSEs, are those constituent parts of the airframe which contribute significantly to the carriage of flight, ground and pressurization loads, and whose failure could result in catastrophic failure of the aircraft. The Structures Group select PSEs.

The PSEs that require mandatory inspections (damage tolerant) or replacements (safe life) become Airworthiness Limitation Items, ALIs. The operator must inspect damage tolerant ALIs at specific intervals, usually specified in numbers of flight cycles or flight hours. The mandatory inspections and safe life limits are contained in their worthiness Requirements Section of the Maintenance Requirements Manual.

A Structural Significant Item (SSI) is any detail, element or assembly, which contributes significantly to carrying flight, ground, pressurization or control loads. Its failure could adversely affect the structural integrity necessary for the safety of the aircraft (Ref 1, 2, 10).

FIGURE 3-2: STRUCTURAL ANALYSIS LOGIC (REF 1,2,10).

3.3.1 SELECTION OF PSES

PSEs are selected by reviewing the failure consequences of each detail, component or assembly. Only those elements that could result in catastrophic failure are PSEs.

Typically, flaw growth within a PSE starts at load path discontinuities and/or other areas of stress concentration subjected to repeated tensile loading. Stress concentrations could be a consequence of design details, or result from service, manufacturing or environmental damage. Examples of load path discontinuities include stringer run-outs, lugs, skin cutouts and splices. All major structural attachments and openings are selected for analysis.

Scratches and notches caused by in-service damage may initiate cracks, so locations of relatively high tensile stress structure should also be selected for analysis. Locations, which are susceptible to environmental deterioration, should also be considered.

The selection of sites for analysis is initially based on service and test experience of similar aircraft. Reviewing the static stresses and the fatigue test results of actual structures later refines the selection. It is important to control the number of PSEs. This will reduce effort expended on inspection and analysis and lessen the amount of documentation required. To ensure adequate and efficient structural coverage, the full PSE should be inspected, but inspection intervals should be based on the most critical site only (Ref 1, 2, 10).

3.4 FAILURE ANALYSIS QUALIFICATION PRINCIPLES

The consequences of structural failure are divided into three categories:

Catastrophic: Resulting in loss of the aircraft and/or fatalities.

Hazardous: Resulting in a large reduction in safety margins and a potentially overwhelming increase in crew workload; or causing serious injury to, or death of, a small proportion of occupants.

Major: Resulting in a significant reduction in safety margins and a reduction in the effectiveness of the crew, or injury to occupants.

If the total or partial failure of a structural item has catastrophic consequences, it is a PSE. It should be noted that the multiple elements of a fail-safe PSE are essential to the integrity of that structure and each must be damage tolerant, even though failure of any one in isolation will not cause catastrophic failure. For a fatigue failure to be considered improbable, this report will assume that the item must have a mean life of 1 SEL (the Structural Economic Life) or 15,000 flight cycles. This is a common value pertaining to most business jet aircraft. If the failure consequence of a structure is neither catastrophic nor hazardous, but is major, the structure shall have a mean fatigue life of ½ SEL (7,500 flight cycles).

If a PSE is single load path, it must exhibit slow and easily inspectable crack growth. Slow crack growth is when the first inspection is at half-life and the subsequent inspections are not less than a quarter life.

The majority of PSEs have mandatory structural inspections and so constitute ALIs. But there are three cases that are exceptions:

1) Safe Life replacement ALIs

These components are not conducive to damage tolerant design and are replaced once a safe life limit is reached. This option is permitted only if it is

concluded, with certifying authority concurrence, that damage tolerance qualification is impractical. Under damage tolerance rationale, this is the case when, for a single load path structure, $a_d > a_c$ where a_d and a_c are the detectable and critical crack sizes respectively, as indicated in figure 3-3. For such structure a safe-life is declared on the basis of fatigue testing:

$$\text{SAFE LIFE} = \frac{\text{TEST BASED MEAN LIFE}}{\text{SAFE-LIFE FACTOR}}$$

FIGURE 3-3: SAFE LIFE PRINCIPLE

It should be noted that the United States Military Authority allows "slow crack growth non-inspectable" damage tolerance qualification for USAF aircraft in cases where critical crack size is smaller than detectable size, provided that durability equivalent to 2 design lives can be achieved in the presence of a

major manufacturing flaw. This amounts, in effect, to safe-life certification with the added requirement that 2 design lifetimes must be achieved in the presence of a manufacturing flaw. However, civil certification authorities (FAA, JAA, Transport Canada, etc.) believe that civil transport aircraft structure must be inspectable to qualify for damage tolerance certification. The slow crack growth non-inspectable option has therefore not been accepted - structure that can only be treated this way must be classified as safe-life or redesigned to permit damage tolerance qualification with acceptable inspectability.

2) *Single Load Path PSEs*

This option applies to single load path structure when partial member failure is readily detectable well before critical crack size is reached (i.e. $a_d < a_c$) and where total member failure cannot be tolerated.

Damage tolerance evaluation is carried out by assuming that a Major Initial Flaw (MIF) of a size agreed to by the Certification Authority exists at the critical location in the structure. The damage tolerance evaluation will have to determine how long it takes for this initial flaw, treated as a crack, to reach detectable size (a_d) and critical size (a_c). The time taken for the MIF to propagate to a_c provides the basis for the inspection threshold, i.e. the time when the first inspection must be carried out. Time from a_d to a_c provides the basis for the repeat inspection interval. Although this method of damage tolerance qualification is permitted, certification by the multi-load path method (as indicated on the next page in figure 3-4) is often the more preferred method by the certification authorities (Ref 1, 2, 10).

FIGURE 3-4: SINGLE LOAD PATH PRINCIPLE

3) Multiple Load Path PSEs with obvious failure of first load path

Mandatory inspections are not required for multiple load path PSEs where failure of the first load path will be evident to flight or ground crew in the flight cycle in which it occurs. Following this partial failure, there must be a secondary load path that will enable completion of the flight with no or slow flaw growth. This option is available when more than one load path exists; a 'primary' load path which is the first to crack and at least one 'secondary' load path which can continue to carry all required loads after primary path total or partial failure. The following possibilities can arise:

a) Although two or more discrete load paths exist, residual strength capability is inadequate following failure of one load path. Critical crack size is therefore reached before total failure of the first load path occurs. Such structure effectively becomes single load path from the damage tolerance standpoint and should be treated as such.

b) Failure of the primary load path is not readily detectable and cracking becomes detectable only after it has progressed into the secondary load path. The inspection objective is therefore the detection of cracks in the secondary load path before critical size is reached. As an example, this situation may arise in fuselage shell structure if, to reduce the maintenance work load, only external inspection of the skin is specified. Failure of a stringer, which would be considered the primary path, is then not detectable from the outside, so cracking would become detectable only after reaching detectable size in the skin, which in this case would be treated as the secondary load path. Although this option simplifies inspections, this advantage is usually obtained at the price of a shorter repeat inspection interval.

c) Cracking becomes detectable upon total failure of the primary load path. The inspection objective is then the detection of the totally failed primary load path before the secondary load path reaches critical size.

Figure 3-5 illustrates how threshold and repeat inspection times are defined for multi-load path structure where detectable crack size is total failure of the primary load path.

FIGURE 3-5: MULTIPLE LOAD PATH PRINCIPLE

3.5 SPECTRA AND DESIGN MISSION ANALYSIS

Although the topic of fatigue mission profile development is beyond the scope of this report, a general overview and source precedence is introduced that the reader may refer to for further information. The global load spectra and the mission profile details utilized in this report are based on actual working experience with the structure in question and thus is eligible for comparison with a tested static example as is discussed in the discussion chapter.

The last piece of information required to perform a damage tolerance analysis is usage data. In order for a damage tolerance analysis to be accurate, it is critical that the usage, and therefore the loads, that the aircraft experiences in service are well understood. The detailed procedure to develop aircraft usage estimates is presented in this section. Typical flight lengths are defined through customer surveys and statistical analysis. These flight lengths are used as a basis for determining flight profiles. Aircraft usage estimates are usually presented in the form of defined flight profiles. These flight profiles encompass typical usage of the aircraft and are used to develop gust load spectra and stress equations.

A flight profile is defined as a typical flight that represents part or all (if there is only one usage profile) of an aircraft's normal usage. It consists of takeoff, climb, cruise, descent, approach, and landing segments that define the flight. Flight parameters are calculated for each of these segments for use in load spectra and stress calculations. Typically there are up to six flight profiles of varying flight length and altitude that are created to define the typical usage of an aircraft.

The next step in creating a flight profile is to determine how many flight segments make up a profile and to calculate the associated flight parameters for each segment. See Figure 3-6 for a typical flight profile example.

FIGURE 3-6: EXAMPLE OF TYPICAL AIRCRAFT FLIGHT PROFILE

In an attempt to abbreviate the discussion regarding the global load spectra, it is important to note that it must adequately account for all significant flight and ground loads encountered during the expected operation of the aircraft. Data based on records obtained from previous load surveys conducted on other aircraft is available for this purpose from a number of reliable published sources. When utilizing such data, care should be taken to ensure that the published data has been obtained from aircraft of similar design to those about to be evaluated. Typical sources for such data are:

- a) VERTICAL GUST & MANOEUVRE LOADS: ESDU 69023
- b) LATERAL GUST LOADS: ESDU 69023.
- c) VERTICAL TAXI LOADS: MIL -A-8866B.
- d) LATERAL TAXI LOADS: ESDU 75008.
- e) LANDING LOADS: MIL -A-8866B.
- f) BRAKING LOADS: Very little published data is available. Judgment based criteria tempered by past experience are usually used.
- g) CONTROL SURFACE LOADS: Virtually no published data is available, mainly based on test and company experience.
- h) PRESSURISATION: Maximum cabin normal operating differential pressure is usually assumed to occur on each flight, although in principle it should be permissible to generate a less severe pressurization spectrum based on the usage profile of the aircraft.

Flight Load Segment	Start-End Altitude	Total Fuel (lb)	GRWT (lb)	Segment Time-min	KEAS	Mach	ALT (ft)	%CG	Diff. Press (psi)	Thrust (lb)
Taxi/Takeoff Roll		860	6170	N/A	0	0	0	23.21	0	0
Climb	0-5000 ft	834	6144	6.04	120	0.19	2500	23.22	0	874
Climb	5000-8300 ft	816	6126	4.45	120	0.205	6650	23.23	0	801
Cruise	8300-8300 ft	773	6083	26.60	146	0.257	8300	23.24	0	517
Cruise	8300-8300 ft	701	6011	26.60	146	0.257	8300	23.27	0	514
Cruise	8300-8300 ft	629	5939	26.60	146	0.257	8300	23.30	0	514
Descent	8300-2000 ft	574	5884	12.60	170	0.282	5150	23.33	0	451
Approach	2000-0 ft	543	5853	2.97	102	0.158	1000	23.34	0	76
Maneuver - 15° Flap		543	5853	N/A	102	0.158	1000	23.34	0	76
Maneuver - 45° Flap		543	5853	N/A	102	0.158	1000	23.34	0	76
Landing Impact		531	5841	N/A	0	0	0	23.34	0	0
Landing Roll/Taxi to Ramp		516	5826	N/A	0	0	0	23.44	0	0

TABLE 3-1: EXAMPLE OF FLIGHT PERFORMANCE / MISSION BASED TABLE

After determining the utilization of each of the flight (Table 3-1) profiles a mission mix must be determined for crack growth analysis (Table 3-2). A mission mix is defined as the distribution or number of each flight of each profile occurring in a predefined loading block. If the loading block is 1,000 flights, then the 24 flights that make up the six profiles used as an example, would have to occur a total of 1,000 times. This mission mix is used by crack growth programs to define how often and when each flight in the stress spectrum occurs.

Flight Profile	% Total of Flights	Total Flights (1000 ft block)	1/1 Flights (90% Tot. flts)	1/10 Flights (9% Tot. flts)	1/100 Flights (0.9% Tot. flts)	1/1000 Flights (0.09% Tot. flts)
1	11.9%	119	107.1	10.71	1.07	0.107
2	27.3%	274	246.6	24.66	2.47	0.247
3	20.5%	206	185.4	18.54	1.85	0.185
4	19.7%	197	177.3	17.73	1.77	0.177
5	11.6%	116	104.4	10.44	1.04	0.104
6	8.8%	88	79.2	7.92	0.79	0.079
	$\Sigma = 100\%$	$\Sigma = 1000$				

TABLE 3-2: EXAMPLE OF A FLIGHT DISTRIBUTION TABLE

Before such spectra can be used for fatigue analysis, fracture mechanics analysis or detail fatigue test, the spectra must be expressed in terms of stress or local element load for each PSE through the use of appropriate stress or local load transfer functions. These can be determined by analyzing a number of unit fatigue cases. This procedure is beyond the scope of this report; however the reader is encouraged to follow the references for further information pertaining to this topic. All fatigue loads used in this report are typical of a jet of this category and are supplied by Bombardier Aerospace's stress department as an unofficial loading characteristic for the purposes of this report.

Before a damage tolerance analysis can be conducted, the load factor spectrum must be developed. The load spectrum is used to generate a stress spectrum for each critical area of a selected PSE. In turn, the stress spectrum for each PSE is used for analytical fatigue and crack growth calculations.

A design mission is required to establish load spectra for various components and locations on the aircraft. Global load spectra must adequately account for all significant flight and ground loads encountered during the expected operation of the aircraft. Data based on records obtained from load surveys conducted on other aircraft is available for this purpose from a number of reliable published sources. Also, all "load transition cycles" that might arise as the aircraft transitions from one flight phase to another should be accounted for. The diagram in figure 3-7 illustrates how one such transition cycle, the well known Ground-Air-Ground

(G-A-G) cycle arises as the aircraft transitions from the ground borne mode to flight and back to ground borne. Additional transition cycles can arise from other flight phases in some parts of the structure, their significance depending on the location within the structure being considered. These cycles can be readily determined by "rainflow" or "range pair" counting. The following is a series of steps to used to define the mission analysis procedures:

- Step 1 Obtain statistical data on payload missions from potential operators or from operators of similar aircraft.
- Step 2 Reduce the statistical data from Step 1 into one or more representative payload missions.
- Step 3 Define non-payload mission(s) and the frequency of usage. Non-payload missions should include training flights.
- Step 4 Establish a mission mix by combining the payload mission(s) and the non-payload mission(s) into one mission mix.
- Step 5 Using comparative crack growth analysis at various locations on the aircraft, determine an equivalent design mission. The equivalent design mission must be substantiated by test.
- Step 6 A detailed load spectrum is generated based on the design missions established in Step 5. The load spectrum must account for loading from the following sources.

⇒ Air Manoeuvres

⇒ Ground Manoeuvres

⇒ Vertical Gusts

- ⇒ Lateral Gusts
- ⇒ Rotation and Flare
- ⇒ Impact at Landing
- ⇒ Thrust
- ⇒ Reverse Thrust
- ⇒ Brakes
- ⇒ Cabin Pressurization
- ⇒ Control Surface Movements
- ⇒ Pre-flight checks
- ⇒ Dynamic Effects
- ⇒ Component Transportation
- ⇒ Aerodynamic Configuration
- ⇒ Towing and Ground Handling

Step 7 The generated load spectrum is reduced by the following methods:

- ⇒ Stepping-off of exceedances
- ⇒ Aggregation of load occurrences
- ⇒ Clipping of high loads
- ⇒ Truncation of low loads

Step 8 The fatigue loads are arranged into a format which can be used for testing and analysis.

The ground-air-ground (GAG) spectrum is defined as the stress cycle per flight encompassing the maximum flight stress excursion and the minimum ground stress excursion. This cycle represents the effect of transition from the landing gear to the wing, and back to the landing gear again, as the support for the aircraft weight. This large stress change occurs once per flight due to the operational cycle. The airplane has negative or low positive loads while on the ground, encounters variable loads while taxiing, is subjected to variable gust and maneuver loads, fuel burn-off occurs, and finally experiences landing and taxi loads. The GAG cycle is not an additional spectrum (as with gust or maneuver), but is instead a method of accounting for the large stress range that otherwise would not be included. (Ref 1, 2, 10).

FIGURE 3-7: EXAAMPLE OF A TYPICAL G-A-G CYCLE

3.6 FLAW SIZE AND CRACK LENGTHS

If damage tolerance evaluation is to be carried out using fracture mechanics methods, it is a necessary prerequisite that at least one crack must exist in the structure. In the rare case of a single load path component where only the repeat inspection interval had to be determined, the detectable crack size could be used as the initial size. The time taken for this crack to propagate to critical size would then provide all the information needed to define the repeat inspection interval. However, in most instances, the structure to be analyzed is nominally uncracked, such as in a new aircraft at service entry, or a propagating crack may have run into a hole from which it will re-initiate on the as yet uncracked opposite side. It is reasoned that in practice such cracks initiate from small flaws that result from material processing and structural assembly and are present throughout the entire structure.

The flaw size is the crack length at which crack growth is assumed to start. There are two types of flaw which occur:

- *Induced Flaw* (IF), which starts at physical damage incurred during manufacturing, maintenance, or by corrosion;
- *Material Flaw* (MF), which normally exists in the material as an impurity, inclusion or void.

From Figure 3-8, an induced flaw is typically 0.05 inches in length, is larger than a material flaw (typically 0.005 inches), and so becomes the initiation site for the lead crack that could propagate under fatigue loading (Ref 1,2,10).

ASSUMED MAJOR INITIAL FLAW SIZES

AT HOLES AND CUT-OUTS	LOCATIONS OTHER THAN HOLES AND CUT-OUTS
<p>$t < 0.05"$ 0.050"</p> <p>$t > 0.05"$ 0.050"</p>	<p>$t < 0.05"$ 0.100"</p> <p>$t > 0.05"$ 0.050" 0.100"</p>

ASSUMED TYPICAL IMPERFECTION FLAW SIZES

AT HOLES AND CUT-OUTS	LOCATIONS OTHER THAN HOLES AND CUT-OUTS
<p>0.005"</p>	<p>0.010" 0.005"</p>

FIGURE 3-8: FLAW SIZES

3.6.1 INSPECTABLE SIZE

All areas defined by the PSEs must be inspectable either by Visual Techniques (VT) or by Non-Destructive Inspection (NDI) techniques. Cracks originating from an assumed flaw size may grow to an inspectable size or may cause an obvious failure of an element in a fail-safe multiple element structure. For most PSEs, cracks should be detectable by general (surveillance) visual inspections detailed visual detection should be viewed as an exception - and recourse to NDI is considered only in a limited number of PSEs. This is of course to limit the time an aircraft spends on the ground for the inspection, in order to conserve operator expenditures as well, the finer the inspection technique, the more care one must exude when performing the tasks. This care or lack thereof can lead to additional-incurred costs on behalf of the operator.

3.6.2 CRITICAL SIZE

The critical crack size is determined by the residual strength capability of the structural element in which the crack propagates, Figure 3-9. Residual strength is discussed in the following section.

FIGURE 3-9: EXAMPLE OF A CRACK GROWTH PLOT

3.7 RESIDUAL STRENGTH

The residual strength is the capacity of an element to withstand non-failure limit and fail-safe loads in the presence of damage. The residual strength capability of a structure containing a crack is limited by elastic failure (crack instability) or by plastic failure (material yield). Interaction may also occur between the two types of failure. The critical crack size is determined by the most severe failure condition. The residual strength capability can be determined analytically, as explained in the following sections, or by testing representative structure with residual strength cuts introduced equivalent to the critical crack lengths, see Figure 3-10.

FIGURE 3-10: EXAMPLE OF A RESIDUAL STRENGTH DIAGRAM

3.7.1 PLASTIC FAILURE

Plastic failure is failure by net section yield. This occurs when the increased stress caused by the reduction in cross-sectional area exceeds the yield strength of the material. In most cases for ductile materials, such as the metals used on aircraft, the von Mises-Hencky theory should be used when calculating the stress for net section yield (Ref 2).

The general equation, which applies when net section yield occurs, is:

$$S'_n = F_{ty} \quad (12)$$

Where; S'_n = Von Mises stress of the net cross section.

F_{ty} = material yield strength.

For the convenience of residual strength plots the following equation is used:

$$S_p = \Phi(a) \cdot F_{ty} \quad (13)$$

$S' = S_p$ for plastic failure.

where; S_p = maximum allowable stress level for plastic failure in the presence of a crack.

S_p is the variable plotted on a residual strength diagram as a function of crack length.

$\Phi(a)$ = ratio of the stress level without a crack to the stress level with a crack.

Φ is a function of geometry and crack length. It is usually calculated using a ratio of areas or inertia. Note the reference section property must be consistent with that used for the stress intensity factor.

a = one tip crack length. By convention a two tip crack length would be $2a$.

S' = Von Mises stress due to the applied limit or fail-safe load in an unflawed or un-failed section.

As an example the plastic failure for a flat panel in pure uniform tension with a crack is given by:

$$S_p = \frac{(W - n \cdot a)}{W} \cdot F_{ty} \quad (14)$$

where: W = total panel width.

n = number of crack tips (1 or 2).

Care must be taken to base the net section yield model on the appropriate sectional area. For example, in an axially loaded lug with a single through crack growing from the hole, the model should consider only the side of the lug in which the crack is growing. It is unrealistic to presume that the load in cracked leg will be re-distributed effectively to the un-cracked side. The initial load carrying capability, $P/2$, is progressively diminished by the ratio of net areas with crack, $\frac{(W - D)}{2} - a$, and without crack, $\frac{(W - D)}{2}$.

3.7.2 ELASTIC FAILURE

Elastic failure occurs when the crack growth or tearing becomes unstable (rapid crack growth). Crack instability is assumed when the Stress Intensity Factor (K) induced by the crack exceeds the material fracture toughness (K_c). The general equation that applies when elastic failure occurs is (Ref 1,2,10):

$$K_a = K_c \quad (15)$$

where: K_a = applied stress intensity factor.

K_c = fracture toughness of the material which is a function of thickness, panel width and loading direction. K_c can be plane stress, plane strain or transitional. The K_c value should be the apparent value, which includes slow stable tearing, and not the direct crack resistance curve (R-curve) value.

For the convenience of residual strength plots, the following equation is used:

$$S_e = \frac{K_c}{\beta(a) \cdot \sqrt{(a \cdot \pi)}} \quad (16)$$

$S = S_e$ for elastic failure.

where: S_e = maximum allowable stress level for elastic failure in the presence of a crack. S_e is the variable plotted on a residual strength diagram as a function of crack length.

$\beta(a)$ = geometry factor, is a function of geometric discontinuities, stress distribution and crack length. The geometry factor models are given by various methods and references.

S = maximum principal stress due to the applied fail-safe load in an unflawed or un-failed section.

3.8 FATIGUE CRACK GROWTH

The number of flights required for a crack to grow from the flaw size to critical size and from inspectable size to critical size is determined by crack growth analysis. The crack growth is determined analytically by crack growth models substantiated by experimental results.

3.8.1 CRACK GROWTH MODELS

Crack growth is dependent cyclic loads applied to the part. The rate of growth is

$$\frac{da}{dN} = f(\Delta K) \quad (17)$$

related to fluctuations in the stress field at the crack tip, characterized by the stress intensity factor range, ΔK :

where: a = crack length.
 N = number of cycles
 ΔK = stress intensity factor range.

$$\Delta K = \sigma \beta \sqrt{\pi a} \quad (18)$$

The stress intensity factor range is given by:

where: β = geometry factor
 N = number of cycles
 ΔK = stress intensity factor range.

The crack growth relationship is determined from test data and is typically quantified by a curve-fit function or as a data look-up table.

Figure 3-11 is extracted from the NASGRO program and shows the NASGRO curve-fit. Also shown is a linear curve-fit of the type used in the Walker-Chang crack growth equation developed by Rockwell and used on many aircraft programs (Ref 8).

FIGURE 3-11 CRACK GROWTH MATERIAL DATA (REF 8).

3.8.2 WALKER-CHANG MODEL

Aircraft manufacturers traditionally use the "Walker-Chang" curve-fit equation, given below (Ref 1, 8).

$$\begin{aligned} &\text{For } R \geq 0 \\ &\frac{da}{dN} = C \left(\frac{\Delta K}{(1 - R)^{1-m}} \right)^n \\ &\text{For } R < 0 \\ &\frac{da}{dN} = C \left((1 + R^2)^q K_{\max} \right)^n \end{aligned} \quad (19)$$

where:

$$R = \sigma_{\min} / \sigma_{\max}$$

$$\Delta K = K_{\max} - K_{\min}$$

$$C = \text{intercept of } \log(da/dN) \text{ vs } \log(\Delta K)$$

$$n = \text{slope of } \log(da/dN) \text{ vs } \log(\Delta K)$$

$$m = \text{Walker collapsing constant}$$

$$q = \text{Chang exponent}$$

See table 3-3 for a list of crack growth parameters.

3.8.3 LOAD INTERACTION

The simple crack growth relationship, above, does not allow for retardation and acceleration effects resulting from the sequence of loads in the mission model. To account for these sequence effects, the stress intensity factor values are adjusted by the "Modified Generalized Willenborg" load interaction model. This method is beyond the scope of this report and is therefore not considered.

Material	Form	Environmental Conditions	C	n	m	q	Kth	R+cut	R-cut
2024-T3	Sheet	Ambient	5.41E-10	3.68	0.6	1	1.5	0.75	-0.5
2024-T351	Plate	Ambient	1.00E-09	3.53	0.6	1	1.5	0.75	-0.5
2024-T42	Sheet	Ambient	5.41E-10	3.68	0.6	1	1.5	0.75	-0.5
2024-T851	Plate	Low	7.50E-10	3.92	0.6	1	1.5	0.75	-0.5
7050-T73651	Plate	Low	7.09E-10	3.59	0.6	1	1.5	0.75	-0.5
7050-T73652	Forging	High	6.99E-11	5.33	0.6	1	1.5	0.75	-0.5
7050-T73652	Forging	Low	1.59E-10	4.16	0.5	1	1.5	0.75	-0.5
7050-T7651	Plate	High	1.29E-09	3.69	0.6	1	1.5	0.75	-0.5
7075-T6	Sheet	Ambient	2.45E-09	3.43	0.6	1	1.5	0.75	-0.5
7075-T6510	Extrus	Ambient	1.16E-08	2.96	0.6	1	1.5	0.75	-0.5
7075-T76511	Extrus	Low	6.68E-09	2.89	0.6	1	1.5	0.75	-0.5
7075-T7351	Plate	Ambient	3.07E-09	3.28	0.6	1	1.5	0.75	-0.5
7075-T7351	Plate	High	2.61E-09	3.49	0.6	1	1.5	0.75	-0.5
7075-T7351	Plate	Low	1.10E-09	3.59	0.6	1	1.5	0.75	-0.5
7175-T73652	Forging	Low	1.00E-13	8.41	0.6	1	1.5	0.75	-0.5
7175-T73652	Forging	Low	3.70E-10	4.12	0.6	1	1.5	0.75	-0.5
7475-T7351	Plate	High	8.92E-10	3.78	0.6	1	1.5	0.75	-0.5
7475-T7351	Plate	Low	3.19E-10	3.85	0.6	1	1.5	0.75	-0.5
Ti-6AL-2Sn	Bar	Ambient	5.15E-11	4.03	0.6	1	4.5	0.75	-0.5
Ti-6AL-4V	Plate & Sheet	Ambient	1.96E-10	3.71	0.7	1	4.5	0.75	-0.5
4340 Steel	Bar	Ambient	1.05E-09	2.54	0.7	1	5.5	0.75	-0.5
4140 Steel	Bar	Ambient	1.05E-09	2.54	0.7	1	5.5	0.75	-0.5
301 Stainless Steel	Sheet	Ambient	1.70E-08	2.07	0.7	1	5.5	0.75	-0.5

TABLE 3-3 WALKER-CHANG CRACK GROWTH PARAMETERS (REF 8)

3.8.4 NASGRO MODEL

The NASGRO model, below, is a preferred alternative to Walker-Chang. It is included in the NASGRO and CRACKS damage tolerance analysis programs, where (Ref 8):

$$\frac{da}{dN} = C \left[\left(\frac{1-f}{1-R} \right) \Delta K \right]^n \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left(1 - \frac{K_{max}}{K_c} \right)^q} \quad (20)$$

C , n , p , and q are empirically constants and f is the crack opening function
 K_{max} , K_c , K_{th} are, respectively, the maximum, critical and threshold values of stress intensity factor

3.8.5 STRESS INTENSITY FACTOR MODELS

The crack growth rate (da/dN) is a function of the material characteristics and the stress intensity factor. The stress intensity factor (K) is a function of crack length, stress and geometry. The following relationship describes this:

$$\Delta K = \beta(a) \cdot \Delta \sigma \sqrt{(a \cdot \pi)} \quad (21)$$

where:

ΔK = change in stress intensity factor magnitude.

$\Delta \sigma$ = change in applied reference stress level.

$\beta(a)$ = geometry factor

a = crack length.

Geometry factors can be obtained by various methods, such can be found in Ref 3, 5, 8, 9, 11,12.

3.8.6 INSPECTION INTERVALS

For most structures, there are two types of inspection intervals. The first inspection is the threshold inspection that occurs when an assumed crack has reached a nominal size. The inspections that occur at regular intervals after the threshold inspection are the repeat inspections.

3.8.7 THRESHOLD INSPECTION

The threshold inspection is the first mandatory inspection of an ALI. Typically, during the preceding period crack sizes will be less than the inspectable length and so too small for reliable detection.

The threshold inspection is determined by factoring the crack growth interval between the flaw size and the critical crack length. The following equation summarizes the method:

$$F_{thr} = \frac{F_{crit}}{K_1 \cdot K_3 \cdot K_4} \leq \frac{\text{Economic Life}}{2} \quad (22)$$

where:

F_{thr} = flights to threshold inspection.

F_{crit} = threshold inspection band, which is the number of flights from the initial flaw size to critical crack length.

K_1 = scatter factor for source of crack growth data.

K_3 = scatter factor for environmental effects.

K_4 = scatter factor to account for uncertainties in the analysis.

Factors for Threshold and Repeat Inspections		
Factor	Value	Description
K₁	2.0	For analysis based on an induced flaw and substantiated by correlation with testing.
	4.0	For experimental results based on 2 tests or on symmetric structure. Only applicable if the structure has a multiple load path and is fail-safe.
	5.0	For experimental results based on a single test. Only applicable if the structure has a multiple load path and is fail-safe.
K₂	2.0	For multiple load path structure that is fail-safe.
	3.0	For single load path and multiple load path non fail-safe structure.
K₃	1.0	Where the correct or more severe material properties or environments are used.
	1.5	Where less severe material properties have been used.
	2.0	
K₄	1.0	For all analysis based on a conservative load spectrum and a conservative stress analysis.

TABLE 3-4: TABLE OF SCATTER FACTORS (REF 2,10)

3.8.8 REPEAT INSPECTION

The repeat inspection period is determined by applying a factor to the crack growth that occurs between the inspectable crack length to the critical crack length. The following equation summarizes the method:

$$F_{rep} = \frac{F_{insp}}{K_2 \cdot K_3 \cdot K_4} \leq \frac{\text{Economic Life}}{2} \quad (23)$$

where:

F_{rep} = flights for repeat inspections intervals

F_{insp} = repeat inspection band, which is the number of flights from the inspectable size to the critical crack length

K_2 = scatter factor for number of load paths

K_3 = scatter factor for environmental effects

K_4 = scatter factor to account for uncertainties in the analysis

3.9 CRACK GROWTH SCENARIOS

As a general rule, it will be necessary to analyze several locations within a PSE. For each, the crack growth scenario chosen must be the most conservative in terms of inspection intervals. Further analyses could be required to assess the effect of continuing damage.

In most cases, the static stress level, alone, is not a sufficient basis for selection of crack initiation locations. The following should be considered when selecting a location and a crack growth scenario:

- 1 Geometric discontinuities causing high stress intensity factors.
- 2 Interaction of geometric discontinuities along the crack growth path.
- 3 A high load transfer at attachment points.
- 4 A large cross sectional area transferring loads to relatively small area.
- 5 Splices may have hidden elements.
- 6 Some load distributions should not be assumed as uniform.
- 7 Crack growth in 2 directions in 2 dimensions.
- 8 A high magnitude Ground-Air-Ground stress level.
- 9 Material susceptibility to crack growth or low fracture toughness. Material grain direction and sensitivity to the environment should also be considered.

Table 3-5 gives example of two typical crack growth scenarios.

CRACK GROWTH SCENARIO

GEOMETRY FACTOR MODELS

EXAMPLE 1: HOLE TO EDGE

1a) Start in direction of highest stress concentration

1b) Continuing crack growth

1a) **BETA MODEL:**

1 tip through crack from a hole towards an edge.

CRACK SIZES:

Start at 0.05", end at edge.

1b) **BETA MODEL:**

1 tip through crack from an edge.

CRACK SIZES:

Start at 0.005" + radius + edge distance + crack growth during 1a), end has no boundary

OR

BETA MODEL:

1 tip through crack from a semi-elliptical notch.

CRACK SIZES:

Start at 0.005" + crack growth during phase 1a) (semi-ellipse = radius + edge distance), end has no boundary.

EXAMPLE 2: HOLE/HOLE/EDGE

2a) Start with highest stress concentration

2b) Continuing growth with highest stress concentration

2c) Continuing crack growth

2a) **BETA MODEL:**

1 tip through crack from a hole towards another hole away from an edge.

CRACK SIZES:

Start at 0.05", end at hole.

2b) **BETA MODEL:**

1 tip through crack from an elliptical hole towards an edge.

CRACK SIZES:

Start at 0.005" + crack growth during phase 2a), end at edge.

2c) **BETA MODEL:**

1 tip through crack from an edge.

CRACK SIZES:

Start at 0.005" + edge distance + crack growth during phase 2b), end has no boundary.

NOTE: Edge distance is dimension from hole centre to edge of part.

Dimension for minor axis of ellipse is given by:

$a = \sqrt{r(b+r)}$: b = maj axis, r = hole rad

Ref 19, p20.2

TABLE 3-5: CRACK GROWTH EXAMPLE SCENERIOS (REF 10).

4 CHAPTER FOUR - DAMAGE TOLERANCE ANALYSIS

4.1 INTRODUCTION

The subsequent section performs a detailed damage tolerance analysis of a typical primary structural element commonly found in general aviation business jet aircraft. The ensuing analysis is performed in accordance with Transport Canada approved methodology based on information readily available in the public domain. The analysis problem requires that an aircraft fuselage primary structure be analyzed in terms of the damage tolerance principals, as outlined in the foregoing chapters, in order to determine that adequate inspection intervals, for the purposes of efficient maintenance practices, can be obtained such that the aircraft may be flown according to its mission profile via the most economic means.

4.1.1 SUBJECT AIRCRAFT & TECHNICAL SPECIFICATIONS

The Challenger 300, originally known as the Continental Business Jet BD-100, is medium size trans-continental business jet from Bombardier Aerospace of Canada. The aircraft is capable of carrying up to eight passengers in a cabin with stand-up headroom over a non-stop range of 3,100nm.

Launched in 1999, the first flight of the aircraft took place in August 2001 from Bombardier's Learjet plant in Wichita, Kansas. The aircraft received certification from Transport Canada in May 2003, US Federal Aviation Authority in June 2003 and European Joint Aviation Authorities in August 2003. The Challenger 300 entered service with Flexjet in January 2004.

The main performance parameters are the aircraft's non-stop range of 5,741km combined with its ability to take-off from airstrips of 1,509m. The high cruise speed is 0.82 Mach or 870km per hour and the maximum operating altitude is 45,000ft or 13,716m. The Challenger 300 has a conventional all-metal airframe with winglets intended to reduce lift-induced drag. Canadair is responsible for building the

forward section of the fuselage including the cockpit and primary flight controls. Mitsubishi Heavy Industries build the wings. Bombardier Aerospace in Belfast is responsible for construction of the centre fuselage. AIDC of Taiwan builds the rear fuselage and tail. Hawker DeHavilland Australia supplies the tailcone and the auxiliary power installation unit. The component sections are transported to the Bombardier Aerospace facility in Montreal for final assembly.

FIGURE 4-1: 3 VIEW DRAWING (BOMBARDIER AEROSPACE)

Capacity				
Crew: 2				
Typical seating: 8-9 (double-club)				
Dimensions				
External				
Length:	68.3 ft			20.92 m
Wingspan:	63.84 ft			19.46 m
Wing area (basic):	522 ft ²			48.50 m ²
Height overall:	20.33 ft			6.20 m
Internal				
Cabin length:	28.6 ft			8.72 m
(From cockpit divider to end of pressurized compartment)				
Cabin maximum width: (Centerline)	7.17 ft			2.19 m
Cabin width: (Floorline)	5.08 ft			1.55 m
Cabin height:	6.08 ft			1.85 m
Floor area: (Excluding cockpit)	146 ft ²			13.56 m ²
Cabin volume:	860 ft ³			24.35 m ³
(Excluding cockpit and baggage)				
Weights				
A. Maximum ramp weight:	38,650 lb			17,531 kg
B. Maximum takeoff weight:	38,500 lb			17,463 kg
C. Maximum landing weight:	33,750 lb			15,309 kg
D. Maximum zero fuel weight:	26,100 lb			11,839 kg
E. Typical basic operating weight:	23,350 lb			10,591 kg
F. Maximum fuel weight:	14,150 lb			6,418 kg
Maximum payload (D-E):	2,750 lb			1,247 kg
Payload – full fuel (A-E-F):	1,150 lb			522 kg
Fuel with maximum payload:	12,550 lb			5,693 kg
Performance				
Range				
Maximum range:	3,100 NM	3,568 SM		5,741 km
(NBAA IFR Reserves, ISA, with 8 pax/2 crew and maximum fuel)				
Speed				
	Mach	kts	mph	km/h
High-speed cruise	0.82	470	541	870
Typical cruise speed	0.8	459	528	850
Airfield Performance				
Balanced field length:	4,720 ft			1,438 m
(SL, ISA, MGTOW)				
Landing distance				
(SL, ISA, MLW):	2,600 ft			792 m
Ceiling				
Maximum operating altitude:	45,000 ft			13,716 m
Initial cruise altitude (MGTOW):	41,000 ft			12,497 m

TABLE 4-1: CHALLENGER 300 TECHNICAL SPECIFICATIONS

(WWW.AERO.BOMBARDIER.COM)

4.2 PSE OVERVIEW

The skin encompassing the fuselage of an aircraft has historically proven to be an area of frequently recurring cracking scenarios in aircraft design. Not necessarily due to poor design, rather due to production damages and/or in-service damages, repairs and accidental induced flaw.

Typical major manufacturing flaws initially are assumed to exist either at the cut-out edge or in the bore of a fastener hole close to the edge. Concurrent cracking is also considered to exist in the lintel, frame or sill towards which the crack is propagating. The scenario of a major initial flaw in the frame, sill, or lintel must also be considered. Secondary cracking should simultaneously be considered to exist in the attached skin.

FIGURE 4-2: PSE OF INTEREST

FIGURE 4-3: TYPICAL STRUCTURE OF BAGGAGE DOOR CUTOUT (REF 18).

The development of a fatigue crack is analysed by the technique of linear elastic fracture mechanics, LEFM. Crack growth is assumed to start at an initial defect and propagate under in-service loadings. At some stage, the crack length will be sufficiently large to be detectable by inspection; a size that was initially determined by the analyst based on the category of structure, in conjunction with economic and practical considerations. If left unchecked, the crack will continue to grow until it renders the structure incapable of supporting limit load. The preferred inspection technique for skins including those surrounding door cut-outs is a detailed visual inspection capable of detecting a 1.0" exposed crack length. Inspection of the frames, lintels, sills and door fitting back-up structure will normally require removal of internal furnishings to enable access for the preferred detailed visual inspection.

NDI methods should only be requested where inspection intervals are prohibitive for a detailed visual inspection. Door fitting should be inspected for the failure of any single attachment location. An inspection schedule must be implemented so that sufficient inspection tasks are carried out during the period from detectable until this final, critical size to ensure that a crack will be found before catastrophic failure of the airframe occurs. Three key cracks lengths are necessary to develop this inspection schedule:

- Initial flaw size
- Detectable crack size
- Critical crack size.

FIGURE 4-4: CRACK GROWTH CURVE

4.3 DEFINITION AND ANALYSIS

Separate stress surveys are performed for the skin cut-out and for the frames, sills and lintels. The skin cut-out stress spectrum survey should be performed initially using the skin elements around any cut-outs. This will allow determination of the critical location (typically a cut-out corner) and the anticipated orientation of cracking. Having identified the critical area a more refined survey should be completed using the same method with the detailed FEM output for the elements in the critical area. This option is not always exercised due to time and cost constraints.

The resultant map of GAG stresses will allow the crack path to be forecast as it will follow perpendicular to the maximum GAG stress orientation through the critical elements. By reviewing the peak GAG stress in each of the elements along the crack path a stress distribution can be plotted. Where it is found that the detailed model based GAG levels equalise with the coarse elements into which the crack growth is plotted, this coarse element can be used as the basis for the skin analysis stress spectrum and is considered to be the critical remote skin element.

Frame, lintel and sill, inner and outer cap GAG stress surveys should be generated for the extent of these structures also giving particular consideration to where they transition to the basic geometry. Considering the results of this survey with knowledge of where joints and section changes occur should allow the potential number of critical locations to be reduced. However, cracking in metallic structures, particularly in pressure vessel type circular fuselages are readily predictable. It is the authors experience that in general longitudinal panel stress is four times (4x) less severe than it is in the circumferential direction (hoop stress). This is particularly valid for a frame and stringer type fuselage arrangement. As a result, in an attempt to simplify the process of load extraction only the circumferential panel stress is determined and used for crack growth.

FIGURE 4-5: TYPICAL FEM REPRESENTATION

FIGURE 4-6: TYPICAL DETAILED FEM AT EMERGENCY EXIT DOOR CUT-OUTS

4.3.1 DAMAGE OVERVIEW

This report will evaluate a section of aircraft skin, bounded by longitudinally running stringers and circumferentially bounding fuselage frames. The skin panel in question contains a cutout which has cracked.

FIGURE 4-7: DAMAGED SKIN PANEL WITH CUTOUT & REPAIR DOUBLER

4.3.2 CRACK GROWTH FAILURE SEQUENCE

The ligament between the cutout and the adjacent rivet holes on each side are assumed failed. A two-tip through crack grows in the stiffened panel with bulging until failure.

A repair doubler is introduced to maintain a continuous load transfer path (axial and shear). Additionally, the doubler will support the crack arresting capability of the overall panel, as the doubler via the attaching fasteners now mainly takes the tension required for further crack propagation. It is true however, that the fasteners will experience a greater degree of pin loading; however, there are 5 rows (circumferentially) which are more than adequate to distribute the circumferential stress between them.

The forthcoming analysis conservatively ignores the presence of the repair doubler, as is standard practise with many aircraft manufacturers, however, it will assume that the doubler does contribute to the cutout and its physical presence will be assumed to act and support the repair design. Figures 4-8 to 4-11 and Tables 4-2 to 4-4 indicate the failure sequence and section properties.

4.3.2.1 INITIAL AND DETECTABLE CRACK LENGTH

The initial full crack length is 1.56 (cutout length) + 2×0.111 (ligament width) + 2×0.132 (hole diameter) + 0.05 (induced flaw) + 0.005 (material flaw) = 2.101 in. and the half crack length is 1.0505 in. The inspectable crack length is 1.0 in. from a hole (detailed visual inspection). However, the low edge distance for the first rivet and the close pitch leaves very little material between the rivets and makes it difficult to inspect. Therefore, it is assumed that the inspectable crack length is 1.0 in. from the third rivet head, and the inspectable full crack length is 1.56 (cutout length) + 2×0.111 (ligament width) + 0.132 (hole diameter) + 4×0.375 (rivet pitch) +

0.1742 (rivet head diameter) + 2×1.0 (inspectable length) = 5.5882 in. and the half crack length is 2.7941 in

FIGURE 4-8: FAILURE SEQUENCE

4.3.3 SECTION PROPERTIES

FIGURE 4-9: SECTION CUTS

FIGURE 4-10: SECTION A-A

TABLE 4-2: STRINGER AND SKIN CROSS-SECTION AREA

Item	W(in)	t(in)	A(in ²)
Skin Pad up STR A	2.14	0.058	0.1241
Skin Pad up STR B	1.80	0.058	0.1044
½ Skin Chem-mill STR A-C	$6.74/2 - 2.14/2 = 2.30$	0.045	0.1035
½ Skin Chem-mill STR A-B	$5.61/2 - 2.14/2 = 1.735$	0.045	0.0781
½ Skin Chem-mill STR B-A	$5.61/2 - 1.80/2 = 1.905$	0.045	0.0857
½ Skin Chem-mill STR B-D	$5.61/2 - 1.80/2 = 1.905$	0.045	0.0857
Stringer A	S6788T		0.3265
Stringer B	S6807T		0.1439

$$\text{Area } (A_{\text{str17}}) = 0.1241 + 0.1035 + 0.0781 + 0.3265 = 0.6322 \text{ in.}^2$$

$$\text{Area } (A_{\text{str18}}) = 0.1044 + 0.0857 + 0.0857 + 0.1439 = 0.4197 \text{ in.}^2$$

FIGURE 4-11: SECTION B-B

TABLE 4-3: FRAME X AND SKIN CROSS-SECTION AREA

Item	B (in)	H (in)	A (in ²)	Y (in)	AY (in ³)	AY' (in ³)	I (in ⁴)
Skin aft FS X	6.1300	0.0450	0.2759	-0.0355	-0.0098	0.0003	0.0000
Skin forward FS X	5.3700	0.0450	0.2417	-0.0355	-0.0086	0.0003	0.0000
Skin Pad up	3.0200	0.0130	0.0393	-0.0065	-0.0003	0.0000	0.0000
Doubler	2.8200	0.0630	0.1777	0.0315	0.0056	0.0002	0.0001
Frame C Member	0.7500	0.0500	0.0375	0.0880	0.0033	0.0003	0.0000
	0.0500	2.8370	0.1419	1.5315	0.2172	0.3327	0.0951
	0.6500	0.0500	0.0325	2.9750	0.0967	0.2876	0.0000
Sum			0.9463		0.3042	0.6215	0.0953

Area (A_{fr}) = 0.9463 in.²

Offset (Y) = 0.3215 in.

Inertia (I) = 0.6190 in.⁴

TABLE 4-4: FRAME Y AND SKIN CROSS-SECTION AREA

Item	E (in)	H (in)	A (in ²)	Y (in)	AY (in ³)	AY (in ³)	I (in ⁴)
Skin aft FS Y	8.3700	0.0450	0.3767	-0.0355	-0.0134	0.0005	0.0001
Skin forward FS Y	5.3700	0.0450	0.2417	-0.0355	-0.0086	0.0003	0.0000
Skin Pad up	1.0500	0.0130	0.0137	-0.0065	-0.0001	0.0000	0.0000
Frame C Member	0.7500	0.0500	0.0375	0.0250	0.0009	0.0000	0.0000
	0.0500	2.9000	0.1450	1.5000	0.2175	0.3263	0.1016
	1.0000	0.0500	0.0500	2.9750	0.1488	0.4425	0.0000
Frame L Bracket	0.0500	0.7000	0.0350	2.5500	0.0893	0.2276	0.0014
	0.9500	0.0500	0.0475	2.9250	0.1389	0.4064	0.0000
Sum			0.9470		0.5733	1.4036	0.1032

Area (A_{fr}) = 0.9470 in.²
Offset (Y) = 0.6055 in.
Inertia (I) = 1.1596 in.⁴

4.3.4 MATERIAL PROPERTIES

The material used for the skin is 2024-T3 Al Alloy clad in ambient air (Ref 8,15):

$$C = 5.410 \times 10^{-10}$$

$$n = 3.68$$

$$m = 0.60$$

$$K_c = 68 \text{ ksi} \cdot \text{in}^{1/2} \text{ (TL)}$$

$$F_{ty} = 40 \text{ ksi (LT, A-basis)}$$

To account for high humidity air (below floor), a factor $K_3 = 1.5$ will be used on the results.

4.4 FINITE ELEMENT MODEL

The purpose of a complete aircraft finite element model is to provide internal loads to the stress engineers. The main objective is to identify the load path in the aircraft structure when external loads are applied.

MSC. Patran and Nastran are used to extract unofficial loads supplied by the stress department. Linear analysis under ultimate loads is performed on complete aircraft finite element model. Stress engineers have to be aware that under ultimate loads, parts of the aircraft are working in a non-linear regime (material yielding, fastener load distribution, skin buckling. As a result, this situation requires some approximations.

Standard Outputs

For the purposes of this report, the 3 most utilized FEM outputs are:

- Element Force
- Element Shear
- Grid Point Force Balance (Endloads)

4.4.1 ELEMENT FORCE

The element force provides the load passing through each element individually. Since the elements are typically linear, the load can vary linearly from one end to the other within the element, so the location of the results given has to be specified/known. The most common usage is to use the element force at the center of the element, giving an average value of the loading of an element. If more detailed values are needed, the output for element force can be provided at each corner/end of elements.

The element force is expressed in force (lb) and moment (lb.in) for all 1-D elements (CROD, CBAR, CBEAM...), while it is expressed in force flow (lb/in) and moment flow (lb.in/in) for all 2-D elements (CTRIA, CQUAD). Note that there is no such output for a CSHEAR element. The sign convention for element force is Positive for Tension and Negative for compression.

4.4.2 ELEMENT SHEAR

The element shear output represents the shear flow component of the force passing in a 2D element. This value corresponds to the FXY field of the element force output for a shell element. This output is also the only output for CSHEAR element.

In the same minds as Element Force, the element shear can vary from one edge to the other within the same element, usually, the Element Shear is provided as an average value for the element, representing the mean values of all edge shear flow. The Element Shear is expressed in shear flow (lb/in) which is pounds per inch length of the element edge.

4.4.3 ENDLOADS

As its name clearly defines, the Grid Point Force Balance is the balance of all forces acting on a specific node (grid point); this is the free body diagram of a node. All forces acting on a node should sum to zero, giving a state of equilibrium to the node between the external forces and internal forces. Another name for this is endload extraction.

“End Loads” or “Summation of Element Oriented Forces on Adjacent Elements”. This output provides the Total Load passing from one node to the other by doing a summation of all element load connected to both nodes. The next figure shows the general principle around the end loads to be used for typical axial load stringer analysis.

In general, the flanges of each frame are modeled using CROD elements. The webs are modeled with CQUAD4 elements with normal pointing aft for consistency. Nodes 1 and 2 of their web are on the Outer Mold Line (OML) of the fuselage. The stringers are modeled by CROD element. The skin is modeled by CQUAD4 elements with membrane properties only. Their nodes 1 and 2 should be on the same stringer line, node 2 aft of node 1. The normal of these elements must point outside of the aircraft.

Figure 4-12 illustrates the FEM model developed to represent the damage scenario.

4.5 FLIGHT CASE SPECTRUM

The loads for the flight case spectrum were extracted from the fatigue database of a typical business jet aircraft profile with a structural economic life of 15,000 flights, as predicted by bombardier stress department.

4.5.1 APPLIED LOADS

Skin Loads Extraction

The skin load should be determined using the output from the skin quad elements. The three load components, F_x , F_y , and F_{xy} should be extracted for utilisation in the stress equations.

Frame, Sill, Lintel Loads Extraction

The process for the extraction of loads from frames, sills and lintels are the same. The outer cap end-load (L_o) should be extracted from the outer cap node associated with the outer cap rod element. The inner cap end-load (L_i) should be extracted from the inner cap node associated with the inner cap rod element. Typically FEM endload output will include the loads in any skins attached to the cap. The actual modelling convention should be checked for each specific model.

FIGURE 4-13: TYPICAL LINTEL FEM

4.5.2 STRESS TRANSFER FUNCTIONS

The same Stress Transfer Functions are used whether the loads used are from the coarse or detailed FEM output, or if they are to be used for the purpose of a stress survey or for crack growth spectrum generation.

$$\sigma_x = \frac{F_x}{t} \quad \sigma_y = \frac{F_y}{t} \quad \tau_{xy} = \frac{F_{xy}}{t} \quad (24)$$

Where t is the actual skin thickness at the element location

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad (25)$$

4.5.2.1 STRESS TRANSFER FUNCTION FOR FRAMES, SILLS, AND LINTELS

The stress level in the flanges of the frames, sills and lintels are determined by considering bending in the section and direct end-load on it, where:

L_i = Inner Cap End Load, and L_o = Outer Cap Load and Skin Load

This is equivalent to:

Where:

L = Total End load on Section = $L_o + L_i$; and

M = Moment at Centroid = $L_o \bar{y} - L_i(h - \bar{y})$

The Inner and Outer Cap stresses are:

$$\sigma_i = \frac{L}{A_t} - \frac{M(h - \bar{y})}{I} \quad (26)$$

$$\sigma_o = \frac{L}{A_t} + \frac{M\bar{y}}{I} \quad (27)$$

In order to simplify these generic equations and make them compatible with the FEM model for ease of load extraction, the following constants and variables are introduced and used in the NASTRAN extraction input file, see appendix A and table 4-7.

$$\sigma_{simplified} = \frac{\sigma_{avg}}{2} \pm \sqrt{\left(\frac{\sigma_{avg}}{2}\right)^2 + \tau_{XY}^2} \quad (28)$$

$$\text{Where: } \sigma_{avg} = \frac{\sigma_X}{2} + \frac{\sigma_Y}{2} \quad (29)$$

The following table summarizes the nodal points associated with the FEM model illustrated in figure 4-13 and develops the principal fatigue and limit stress equations that determined the residual limits of the crack growth scenarios.

σ_Y	$B_{01} = \text{Outer capload Frame}_Y = \frac{V_{01} + aV_{09} + V_{02}}{2}$ $B_{02} = \text{Inner capload Frame}_Y = \frac{V_{03} + V_{04} + bV_{10}}{2}$ $= \frac{B_{01} + B_{02}}{A_{fY}} + B_{01} \frac{y_Y^2}{I_Y} - B_{02} (h_{FEMY} - y_Y) \frac{y_Y}{I_Y}$	V_{01} : EL (127037,127036) Endload Outer, Str A towards Str B V_{02} : EL (127036,127037) Endload Outer, Str B towards Str A V_{03} : EL (127137,127136) Endload Inner, Str A towards Str B V_{04} : EL (127136,127137) Endload Inner, Str B towards Str A V_{09} : EL (127037,127137) Endload Str A Outer towards Inner V_{10} : EL (127136,127036) Endload Str B Inner towards Outer
	$B_{03} = \text{Outer capload Frame}_X = \frac{V_{05} + aV_{11} + V_{06}}{2}$ $B_{04} = \text{Inner capload Frame}_X = \frac{V_{07} + V_{08}}{2}$ $= \frac{B_{03} + B_{04}}{A_{fX}} + B_{03} \frac{y_X^2}{I_X} - B_{04} (h_{FEMX} - y_X) \frac{y_X}{I_X}$	V_{05} : EL (126037,126036) Endload Outer, Str A towards Str B V_{06} : EL (126036,126037) Endload Outer, Str B towards Str A V_{07} : EL (126137,126136) Endload Inner, Str A towards Str B V_{08} : EL (126136,126137) Endload Inner, Str B towards Str A V_{11} : EL (126037,126137) Endload Str A Outer towards Inner
	$\tau_{XY} = \frac{V_{12}}{t}$	V_{12} : ES (126637) Shear, FS321-333, Str A-B

TABLE 4-5: NASTRAN EXTRACTION FILE INPUT SUMMARY

The variable A and I are defined in Section 4.3.3 and thickness is 0.045". Here y is the offset from the skin OML; $y = Y + 0.058$. Y and height (h_{FEM}) is defined in Section 4.3.3 also. From the FEM 'a' is the cosine of 61.5° and 'b' is the cosine of 52.4°. The NASTRAN input file and the resulting GAG stresses from the combination of σ_{avg} and τ_{XY} shown in Appendix A.

4.5.3 PRESSURE CASE

For this analysis, the stress from pressurization (unit case of 10 psi is standard in fuselage structural design) is evaluated using the Flugge equation. If the resulting "Flugge stress" is higher than the pressure stress from the FEM, the FEM stress is replaced by the "Flugge stress". Flugge equation (Ref 13) accounts for pressure loads carried in the stiffeners to produce an average skin stress across the bay.

$$\sigma_{Flugge} = \frac{t_x P \rho + \left(\frac{1}{2}\right) v (t_\phi - t) P \rho}{(1 - v^2) t_\phi t_x + v^2 t (t_\phi + t_x - t)} \quad (30)$$

Where:

P = Pressure level = 10 psi

ρ = Fuselage radius = 53 in.

ν = 0.33 (Poisson's ratio)

A_{str} = Stringer A area = 0.3265 in²

= Stringer B area = 0.1439 in²

A_{fr} = Frame area (outer cap + part of web) = (2 x 0.75 - 0.05) x 0.05 = 0.0725 in²

s = Stringer spacing = 5.61 in.

B = Frame spacing = 11.5 in.

t = skin thickness = 0.045 in.

$$t_x = t + \frac{A_{str}}{s} = 0.045 + \frac{0.1439}{5.61} = 0.07065 \text{ or } t_x = t + \frac{A_{str}}{s} = 0.045 + \frac{0.3265}{5.61} = 0.103$$

$$t_\phi = t + \frac{A_{fr}}{B} = 0.045 + \frac{0.0725}{11.5} = 0.0513$$

σ_{Flugge} = 10.514 ksi for P = 10 psi (unit case), STR A (Minimum value)

σ_{Flugge} = 10.534 ksi for P = 10 psi (unit case), STR B (Maximum value)

The maximum value of 10.534 ksi is used.

Occurrence	Max (ks)	Min (ks)
900 times per 1000 flights	9.3845	0.1318
90 times per 1000 flights	9.4457	0.0822
9 times per 1000 flights	9.5242	0.0248
1 time per 1000 flights	9.5863	-0.0583

**TABLE 4-6: MOST CRITICAL GAG STRESSES FROM NASTRAN OUTPUT, SEE
APPENDIX B FOR THE OUTPUT.**

4.6 LIMIT STRESS

For a pressurized cabin, the limit stress for residual strength calculations is determined in accordance with FAR 25.571(b)(5) parts (i) and (ii).

Part (i) Normal operating pressure combined with the expected external aerodynamic pressure applied simultaneously with flight loading conditions¹. Note that landing and ground handling cases are not combined with pressure. For the aircraft considered in this report, the design static limit loads are used. Details of calculations are shown in Appendix A.

The highest limit principal stress is:

$$s_{\text{limit}} = 10.300 \text{ ksi} - \text{Case 7819017, PSD Lateral Gust+ 9 psi}$$

Part (ii) Expected external aerodynamic pressure in 1g flight combined with a cabin differential pressure equal to 1.1 times the normal operating differential pressure without any other load.

$$P = 0.2 + (1.1 * 8.8 \text{ psi}) = 9.88 \text{ psi (failsafe pressure)}$$

Using the equation for hoop stress of pressurized cylinder:

$$s_{\text{Hoop-Pressure}} = (P \times r) / (t) = (9.88 \text{ psi} \times 53 \text{ in.}) / 0.045 \text{ in.}) = 11.6364 \text{ ksi}$$

Note that the principal stress calculated using “flugge” hoop stress and FEM shear stress is 10.410 ksi and the principal stress calculated using the FEM pressure stress is 3.832 ksi. The limit hoop stress calculated above is thus conservative.

The limit stress used for the determination of the critical crack length for residual strength analysis is the greater value of parts (i) and (ii), here the hoop stress of part (ii); 11.6364 ksi.

¹ Maximum operating pressure is 8.8 psi and external aerodynamic pressure is 0.2 psi.

4.7 GEOMETRY FACTOR MODEL (BETA)

The ligament between the cutout and the adjacent hole on each side is assumed failed. A two-tip through crack is assumed to grow in the skin panel, with an initial half crack length of 1.0716 in. The beta model used is a two-tip through crack, combined with bulging and crack in a stiffened panel: $\beta = \beta_{2tip} \beta_{bulg} \beta_{stiff}$.

Parameters β_{2tip} :

Model: 2 tip center through crack

Initial half crack length = 1.0505 in.

Final half crack length = 10.0 in.

Panel width (W) = 23 in. (2 frame bays)

Parameters β_{bulg} :

Model: Bulging in a pressurized fuselage

Crack location: Bay centered

Initial half crack length = 1.0505 in.

Final half crack length = 10.0 in.

Panel thickness (t) = 0.045 in.

Fuselage radius (R) = 53 in.

Stiffener spacing (b) = 2*7.11=14.22 in.

Panel modulus (E_1) = 10 500 ksi

Internal pressure (P) = 6.8 psi (Often the most frequent pressure is also the lowest pressure, which, based on frequency combined with a higher beta value yields a more critical scenario, thus it is conservative)

Parameters β_{stiff} :

Model: Crack in a stiffened panel

Crack location: Towards an intact stiffener

Initial half crack length = 1.0505 in.

Final half crack length = 10.0 in.

Panel thickness (t) = 0.045 in.

Stiffener spacing (b) = 14.22 in.

Fastener pitch (h) = 0.78 in.

Panel modulus (E_1) = 10 500 ksi

Stiffener modulus (E_2) = 10 300 ksi

Stiffener area (A) = $(2 \times 0.75 - 0.05) \times 0.05 = 0.0725 \text{ in}^2$

FIGURE 4-14: BETA VS. HALF CRACK LENGTH

4.8 RESIDUAL STRENGTH

For this analysis, the limit stress is 11.6364 ksi as defined previously. The critical crack length is determined from whichever residual strength expression reaches the limit stress first. The expression for residual strength based on fracture toughness is:

$$\sigma_{res} = \frac{K_c}{\beta\sqrt{\pi a}} = \frac{68}{\beta\sqrt{\pi a}} \text{ where } K_c = 68 \text{ ksi}\cdot\text{in}^{1/2}$$

The expression for residual strength based on net section yield is:

$$\sigma_{res} = F_{ty} \frac{(W - 2a)}{W} = 40 \frac{(39 - 2a)}{39} ?$$

where $F_{ty} = 40$ ksi ($W = 3$ frame bays FS (A-1) – (B+1) = 39 in.

The curves in Figure 4-15 present the residual strength vs. crack length.

FIGURE 4-15: RESIDUAL STRENGTH CURVES

4.9 CRACK GROWTH

The fracture analysis in this report adopts the methodology of the Walker-Chang crack growth rate equation with the Willenborg retardation model is used. The crack growth program used for this analysis is *CRPRKG87*. The technical approach employed in the *CRPRKG87* program is applicable to metallic structures containing cracks or crack-like flaws subjected to cyclic loading. This crack growth analysis methodology is based on linear elastic fracture mechanics (LEFM), i.e. the range of the stress intensity factor, ΔK , is the controlling parameter for characterizing the crack growth rates of a crack. Refer to appendix C for the output parameter. The material constants for these equations together with other required material properties are presented in MATERIAL PROPERTIES section 4.3.4. The crack growth curve is shown in Figure 4-16.

FIGURE 4-16: CRACK GROWTH CURVE

4.10 INSPECTION INTERVALS

The threshold and repeat inspection periods are calculated to allow timely detection of structural damage.

Following the methodology from chapter 3, the threshold inspection band is the period of time during which a crack grows from its initial damage size to the critical size, i.e.

$$\text{Threshold interval} = \frac{N_{crit}}{K_1 \times K_3 \times K_4}$$

The repeat detection band is the period of time during which a crack grows from its inspectable size to the defined critical size, i.e.

$$\text{Repeat interval} = \frac{N_{crit} - N_{insp}}{K_2 \times K_3 \times K_4}$$

The variable K_1 to K_4 are analytical variables used to obtain conservative inspection intervals by taking into account data limitations and uncertainties:

K_1 = Factor dependent on source of crack growth data (analysis, single test, or multiple test)

K_2 = Allowance for number of load paths

K_3 = Allowance for environmental effects

K_4 = Allowance for uncertainties in

- i) Stress level prediction, e.g. accuracy of finite element model
- ii) True distribution of load between elements in a multiple load path structure
- iii) Accuracy of basic material properties
- iv) Accuracy of residual strength analysis in the absence of test values
- v) Assumptions/omissions made in generating the flight spectra
- vi) Correction when extrapolating to get threshold inspections

Factor	Value	
K ₁	2.0	for analysis
	4.0	for test results from 2 tests (or single test on a symmetric structure)
	5.0	for test results from a single test
K ₂	2.0	for multiple load path
	3.0	for single load path
K ₃	1.0	where the correct or more severe environmental properties are used
	1.5	analysis uses less severe environmental properties
K ₄	1.0	for analysis based on: i) conservative stress analysis ii) conservative flight spectrum iii) good material property data
	1.5	for threshold inspection time when extrapolating data for a centre holed model

TABLE 4-7 SCATTER FACTOR DEFINITION

For this analysis, with:

Inspection type = Detailed Visual Inspection

Inspectable crack length (a_{insp}) = 2.7941 in. (half length)

Critical crack length (a_{crit}) = 8.5726 in. (half length)

The following factors are used:

$N_{init} = 0$ flights $K_1 = 2.0$

$N_{insp} = 16\,411$ flights $K_2 = 2.0$

$N_{critical} = 25\,243$ flights $K_3 = 1.5$

$K_4 = 1.0$

The following inspection intervals are obtained:

$$\text{Threshold} = \frac{N_{crit}}{K_1 \times K_3 \times K_4} = \frac{25243}{2.0 \times 1.5 \times 1.0} = 8,414 \text{ flights}$$

$$\text{Repeat} = \frac{N_{critical} - N_{insp}}{K_2 \times K_3 \times K_4} = \frac{(25243 - 16411) \text{ flights}}{2.0 \times 1.5 \times 1.0} = 2,944 \text{ flights}$$

5 CHAPTER FIVE – AIRCRAFT REPAIR DAMAGE TOLERANCE ANALYSIS CONCLUSION

Based on fracture mechanics analysis, the intervals and method of required maintenance inspections are determined for the reworked area of a section of fuselage skin between stringers A and B and between Frame Station X and Y of a generic medium Challenger class aircraft. It is therefore concluded that compliance with FAR 25.571 (reference 14) is demonstrated.

The inspection intervals are tabulated below:

TABLE 5-1 INSPECTION INTERVALS

Threshold Inspection	Repeat Inspection	Inspection method
8,414	2,944	Detailed Visual

The damage tolerance analysis presented herein has resulted in inspections intervals of 8,414 hrs and 2,944 hrs for the threshold and repeat respectively. These inspections consist of detailed visual inspection for cracks in the skin between stringer A and B and between Frame Station X and Y.

5.1 DISCUSSION AND COMPARISON OF RESULTS

Most manufacturers of certified commercial aircraft develop a static and fatigue test article that is used for both static and damage tolerance bench evaluations. This type of in-service skin damage is fairly common among frequent usage pressurized aircraft. Therefore, it is common practise to keep observation for such damages when cycling a test article. This exact scenario took place on a regional jet test article during cyclic fatigue testing at a skin panel location identical to the scenario presented in this report. A subsequent repair was immediately implemented and cyclic fatigue testing was resumed. As a result, this location automatically becomes an SSI and is thus required to be monitored periodically for

signs of further failure and / or crack initiation. Having already experienced a failure in that particular section of fuselage, it is probable that the continued application of cyclic fatigue loading would undoubtedly induce a second failure, this time one relative to the repair that is similar to that described in this report. The analysis and report aims to simulate this scenario and predict the accuracy of the damage tolerance methods as they are compared to actual test data. Figures 5-1 and 5-2 depict the test rig set-up containing aircraft test specimen.

Figure 5-3 shows the panel of interest after a routine inspection at intervals of 5000 flights and a subsequent NDI inspection has revealed a panel failure by way of excessive crack propagation. This failure took place at 9,963 flight cycles with an equivalently large crack length. This is in fact a very good result even though the predicted and actual flights have approximately a 13% difference between them. The main positive for this large difference is that the inspection predictions for this location would mandate the operator of this particular aircraft to inspect the panel and luckily find the crack initiation before it reaches its critical length, resulting in failure of the pressurized panel and perhaps even catastrophic loss of the entire aircraft.

In theory, the closer a damage tolerance analysis is to the actual conditions it is simulating it should yield more accurate results, however, when one takes into consideration the fact that there are lives at stake, then the added conservatism and all of its associated benefits and non-benefits are a small price to pay. In this comparison, it could be argued that the repair should in fact be more conservative. Repairing the damage with an even larger and/or thicker doubler could do this. This would add weight to the overall aircraft, however, it would also increase the maintenance inspection period and thus reduce the overall cost of maintaining the aircraft due to the larger repeat inspections associated with the repair analysis.

FIGURE 5-1: CYCLIC LOADING FATIGUE TEST SETUP 1

FIGURE 5-2: CYCLIC LOADING FATIGUE TEST SETUP 2

FIGURE 5-3: PANEL FAILURE FOLLOWING CYCLIC TESTS

FIGURE 5-4: CRACK IDENTIFICATION BY NON-DESTRUCTIVE INSPECTIONS (FPI).

5.2 ANALYSIS UNCERTAINTY

Fatigue and fracture analysis is not a precise science. Analysis uncertainties can arise for various reasons. A few can be listed as follows:

1. For many situations there are as yet no standardized damage tolerance analysis procedures, so different analysts may tackle a given problem in different ways, leading to some variability in the results obtained for a nominally identical problem.
2. Limitations in analysis tool capability may force the analyst to make simplifying assumptions based on personal experience and engineering judgment. Although this is true for traditional static strength analysis as well, it is encountered more frequently with fatigue and fracture analysis.
3. Fracture properties of materials exhibit greater variability than mechanical properties used for static strength analysis.
4. Inspection times are determined for a load spectrum based on anticipated aircraft usage. There is bound to be some variability in how different operators use their aircraft.
5. In the absence of a component test, structural failure may occur in a manner not anticipated by the analyst.

5.3 SOME RECOMMENDATIONS

This report will be concluded with a few recommendations based on past experience with the damage tolerance analysis of conventional metallic structure.

1. The damage tolerance certification process will be facilitated considerably if the principles and criteria to be used in the damage tolerance evaluation are documented and approved by the prime certification authority during the early stages of a new program.
2. It is desirable to prepare a summary report which documents in concise form the results from the damage tolerance evaluation (inspection times, detectable crack sizes, analysis report references etc.) of every Primary Structural Element. This will become an invaluable reference for both the damage tolerance analysts and other users of the damage tolerance evaluation results after the analyses are completed.
3. To state the obvious, excessive stress levels should be avoided. Some approximate guidelines which could be used during the preliminary design stage for transport category aircraft constructed of conventional materials are as follows:

It is highly desirable to adopt a simple screening procedure which can identify excessive stress locations as soon as possible. For example, a subroutine which determines equivalent once/flight stress could be incorporated in the spectrum derivation computer program so that every time a stress spectrum is generated, the equivalent once/flight stress is also determined. This provides advance warning of any problem areas. This is also useful for the identification of non-critical areas that do not require damage tolerance analysis - a valuable time saver.

4. In multi load path structure it is desirable to keep the stress level low enough to ensure that the time taken for a typical imperfection flaw (0.005") to propagate to critical size under intact load path conditions exceeds the design life by an adequate margin. This avoids repeat inspection intervals that diminish with time in service and the need for a damage tolerance imposed life limit.
5. Multi rather than single load path structure is usually preferred because the former is generally considered to be inherently safer. Also, such structure is often easier to inspect since the requirement is usually the detection of total rather than partial failure of a structural member.
6. In multi load path structure consisting of 'back-to-back' duplicated load paths, the paths should be of equal or near equal area since inspection time will be governed by the life of the load path with the lowest area.
7. Material with good fracture properties (low crack propagation rate, high fracture toughness) should be used. In this context the following is worth noting:
 - a) Replacement of 2XXX with 7XXX aluminium alloys, although improving static strength, may result in faster crack propagation and therefore a reduction in inspection intervals.
 - b) Sometimes available handbook data, such as that supplied by the material manufacturer, is not sufficient by itself to guide selection of the best material. Handbook crack propagation data generated under constant amplitude loading may show little difference in propagation rate for the candidate materials being considered, whereas in reality there may actually be a significant difference under spectrum loading. Where this is an important consideration, such as on wing lower skins, some comparative coupon

testing under spectrum loading is desirable if appropriate data is not already available.

c) For high strength steel such as 300M it may not be possible to exploit the high tensile strength properties because fracture toughness is not on a par with these properties.

8. Easy inspectability should be a high priority design goal to minimize aircraft maintenance cost. Despite advances in NDI, visual inspection is generally easiest to perform and is still preferred by many operators.
9. For a new structure it is crucial to confirm that all critical areas are inspectable before any configuration is frozen. Failure to do this can result in very costly modifications, possibly even after aircraft have entered service.
10. Design for ease of repair should also be given adequate priority, bearing in mind that repairs have to comply with the same damage tolerance requirements as the originally certified structure.

APPENDIX A: NASTRAN INPUT EXTRACTION FILE

PARAMETERS AND RESULTING LIMIT STRESS CALCULATIONS

The equations for the fatigue GAG stresses are used outlined in section 4.5.2.1.
The section properties are given in Section 4.3.3.

Project Name

FSX - FSX, STR A-B

Stress Equation

C01=0.9470 #Area frame Y + skin
C02=0.0450 #Thickness skin
C03=0.6055 #Offset frame Y
C04=1.1596 #Inertia frame Y
C05=0.9463 #Area frame X + skin
C06=0.3215 #Offset frame X
C07=0.6190 #Inertia frame X
C08=3 #Frame Height in FEM
C09=0.47716 #Cosinus Angle a
C10=0.61015 #Cosinus Angle b

V01=EL(127037,127036) #End Load Outer Cap at FSX, Str A towards Str B
V02=EL(127036,127037) #End Load Outer Cap at FSX, Str B towards Str A
V03=EL(127137,127136) #End Load Inner Cap at FSX, Str A towards Str B
V04=EL(127136,127137) #End Load Inner Cap at FSX, Str B towards Str A
V05=EL(126037,126036) #End Load Outer Cap at FSX, Str A towards Str B
V06=EL(126036,126037) #End Load Outer Cap at FSX, Str B towards Str A
V07=EL(126137,126136) #End Load Inner Cap at FS, Str A towards Str B
V08=EL(126136,126137) #End Load Inner Cap at FSX, Str B towards Str A
V09=EL(127037,127137) #End Load STR A at FSX, Outer towards Inner
V10=EL(127136,127036) #End Load STR B at FSX, Inner towards Outer
V11=EL(126037,126137) #End Load STR A at FSX, Outer towards Inner
V12=ES(126637) #Shear in skin at FSX-Y, STR A-B

#Load Outer Cap FSX

B01={V01+C09*V09+V02}/2

#Load Inner Cap FSX

B02={V03 +V04+C10*V10}/2

#Load Outer Cap FSX

B03={V05+C09*V11+V06}/2

#Load Inner Cap FSX

B04={V07 +V08}/2

#Stress FSX

Q1={B01+B02}/C01+{B01}*C03*C03/C04-{B02}*{C08-C03}*C03/C04

#Stress FSX

Q2={B03+B04}/C05+{B03}*C06*C06/C07-{B04}*{C08-C06}*C06/C07

#Average Stress FSX and FSX

Q3={Q1+Q2}/2

#Shear Stress

Q4={V12}/C02

#Simplified principal stress

$$QZ1=\{(Q3/2)+\text{sign}\{Q3\}\{0.25*Q3^2+Q4^2\}^{.5}\}$$

Rangpair program parameters

UNIT = KSI

REVERT= 1.0

*OUTPUT=STATIC

USER MODIFIED UNIT CASE

STRESS(Q1,9999)=10534

STRESS(Q2,9999)=10534

Target Directory

/

Output file names

QZ1: Stress_Along_Frame

LIMIT STRESS CALCULATIONS FROM EXTRATED LOADS

Part (i): Normal operating pressure combined with the expected external aerodynamic pressure applied simultaneously with flight loading conditions².

Loadcase	Description	Ultimate Stress no ΔP added	Limit Stress no ΔP added	+ 9 psi (9/10* Flugge)	Limit Stress + 9 psi
Pressure	18.35 psi	7117.76			
Flugge	10 psi	10534			
7139001	2 Point Level Landing	1168.89	779.26		N/A
p7139001	2 Point Level Landing + 6.6 psi	3742.83	2495.22		N/A

² Maximum operating pressure is 8.8 psi and external aerodynamic pressure is 0.2 psi (See FARs).

7139003	2 Point Level Landing	2564.88	1709.92		N/A
p7139003	2 Point Level Landing + 6.6 psi	4277.18	2851.45		N/A
7139008	2 Point Level Landing	1166.68	777.79		N/A
p7139008	2 Point Level Landing + 6.6 psi	3742.24	2494.83		N/A
7139009	2 Point Level Landing	2483.83	1655.89		N/A
p7139009	2 Point Level Landing + 6.6 psi	4577.66	3051.77		N/A
7159005	Tail Down Landing	3384.77	2256.51		N/A
p7159005	Tail Down Landing + 6.6 psi	5033.48	3355.65		N/A
7159006	Tail Down Landing	1295.38	863.59		N/A
p7159006	Tail Down Landing + 6.6 psi	3800.82	2533.88		N/A
7159007	Tail Down Landing	-3020.41	-2013.61		N/A
p7159007	Tail Down Landing + 6.6 psi	4560.39	3040.26		N/A
7159010	Tail Down Landing	4832.19	3221.46		N/A
p7159010	Tail Down Landing + 6.6 psi	6312.42	4208.28		N/A
7159033	Tail Down Landing	3803.88	2535.92		N/A
p7159033	Tail Down Landing + 6.6 psi	5558.09	3705.39		N/A
7189101	Drift Landing	2026.51	1351.01		N/A
p7189101	Drift Landing + 6.6 psi	3690.58	2460.39		N/A
7219001	3 Point Braked Roll	-2058.76	-1372.51		N/A
7219002	3 Point Braked Roll	1487.14	991.43		N/A
7219003	3 Point Braked Roll	1878.42	1252.28		N/A
7259011	Static Nose Wheel Yaw	1245.73	830.49		N/A
7309001	Pitch Checked Maneuver	669.47	446.31	9480.60	9926.91
7309002	Pitch Checked Maneuver	-268.18	-178.79	9480.60	9301.81
7309005	Pitch Checked Maneuver	-275.85	-183.90	9480.60	9296.70
7419001	Negative Formula Vertical Gust	-1108.13	-738.75	9480.60	8741.85
p7419001	Negative Formula Vertical Gust + 10.2 psi	3987.9	2658.60		N/A
7419011	Negative Formula Vertical Gust	-1034.77	-689.85	9480.60	8790.75
p7419011	Negative Formula Vertical Gust + 10.2 psi	3919.81	2613.21		N/A
7439125	Negative PSD Vertical Gust	-625	-416.67	9480.60	9063.93
p7439125	Negative PSD Vertical Gust + 10.2 psi	3723.99	2482.66		N/A
7719169	Inadvertent Thrust Reversal	-1242.99	-828.66	9480.60	8651.94
7819011	PSD Lateral Gust	1217.74	811.83	9480.60	10292.43
7819013	PSD Lateral Gust	1168.89	779.26	9480.60	10259.86

7819015	PSD Lateral Gust	1168.9	779.27	9480.60	10259.87
7819017	PSD Lateral Gust	1229.05	819.37	9480.60	10299.97
7999002		-2305.54	-1537.03	9480.60	7943.57

The highest limit principal stress is:

$$\sigma_{\text{limit}} = 10.300 \text{ ksi} - \text{Case 7819017, PSD Lateral Gust} + 9 \text{ psi}$$

Part (ii) Expected external aerodynamic pressure in 1g flight combined with a cabin differential pressure equal to 1.1 times the normal operating differential pressure without any other load.

$$P = 0.2 + (1.1 * 8.8 \text{ psi}) = 9.88 \text{ psi (failsafe pressure)}$$

Using the equation for hoop stress of pressurized cylinder:

$$\sigma_{\text{Hoop}} = (P \times r) / (t) = (9.88 \text{ psi} \times 53 \text{ in.}) / 0.045 \text{ in.} = 11.6364 \text{ ksi}$$

The principal stress using “flugge” hoop stress ($9.88/10 \times 10.534 = 10.408 \text{ ksi}$) and FEM shear stress ($9.88/18.35 \times -266.7 = -143.6 \text{ psi}$) is:

$$\frac{10.408}{2} \pm \sqrt{\left(\frac{10.408}{2}\right)^2 + 0.1436^2} = 10.410 \text{ ksi}$$

The principal stress calculated using the FEM pressure stress is:

$$9.88/18.35 \times 7.118 = 3.832 \text{ ksi}$$

The limit stress is the greater value of parts (i) and (ii), the hoop stress of part (ii); 11.6364 ksi.

APPENDIX B: GAG STRESS OUTPUT

INPUT FILE NAME :

./AVG_PRINCIPAL_STRESS.rang.in/data/stress/01.AIRFRAME/04.CH

OUTPUT FILE NAME : ./AVG_PRINCIPAL_STRESS.rang.ou./Q7.nrang.out

RANGE PAIRED OUTPUT						
X900	0.3960	-0.4453	1.	1.	5	15
X900	0.3147	-0.2981	1.	1.	9	9
X900	2.5319	2.4943	1.	1.	23	24
X900	7.2495	7.2243	1.	1.	31	32
X900	9.3601	9.2912	1.	1.	49	55
X900	9.3595	9.3580	3.	1.	50	50
X900	9.3593	9.3582	8.	1.	51	51
X900	9.3845	0.1318	1.	1.	55	115 GAG
CYCLE						
X900	9.4010	9.3165	2.	1.	56	56
X900	9.1815	9.1396	1.	1.	63	63
X900	9.1814	9.1468	1.	1.	71	64
X900	7.2831	7.2288	1.	1.	75	75
X900	7.2731	7.2387	2.	1.	76	76
X900	7.2688	7.2430	1.	1.	77	77
X900	7.2890	7.2229	1.	1.	81	81
X900	2.5591	2.4921	1.	1.	85	85
X900	2.5475	2.5034	2.	1.	86	86
X900	2.5418	2.5090	5.	1.	87	87
X900	2.5382	2.5126	3.	1.	88	88
X900	2.5416	2.5093	1.	1.	92	92
X900	2.5358	2.5150	1.	1.	93	93
X900	0.9933	0.8154	1.	1.	97	97
X900	0.9606	0.8478	2.	1.	98	98
X900	0.9454	0.8630	3.	1.	99	99
X900	0.9239	0.8844	1.	1.	103	103
X900	0.3945	-0.3179	1.	1.	122	127
END1	0.3117	-0.2743	1.	1.	126	126
X090	0.4374	-0.4675	1.	1.	4	14
X090	0.3960	-0.3866	1.	1.	5	5
X090	0.3218	-0.2859	1.	1.	8	8
X090	0.3147	-0.2981	1.	1.	9	9
X090	2.5463	2.4556	1.	1.	22	27
X090	2.5319	2.4854	1.	1.	23	23
X090	2.5230	2.4943	1.	1.	24	24
X090	7.2598	7.1657	1.	1.	30	35
X090	7.2495	7.2188	1.	1.	31	31
X090	7.2438	7.2243	1.	1.	32	32
X090	9.1611	9.0835	1.	1.	38	42
X090	9.1510	9.1264	1.	1.	39	39
X090	9.1939	9.1193	1.	1.	42	45
X090	9.3609	9.2406	1.	1.	48	54
X090	9.3601	9.3575	1.	1.	49	49
X090	9.3595	9.3580	3.	1.	50	50
X090	9.3593	9.3582	8.	1.	51	51
X090	9.4457	0.0822	1.	1.	54	114 GAG
CYCLE						
X090	9.4264	9.2912	1.	1.	55	55
X090	9.4010	9.3165	2.	1.	56	56

X090	9.1982	9.1230	1.	1.	62	62
X090	9.1815	9.1396	1.	1.	63	63
X090	9.1743	9.1468	1.	1.	64	64
X090	9.2068	9.1143	1.	1.	67	67
X090	9.1814	9.1356	1.	1.	71	70
X090	7.2990	7.2131	1.	1.	74	74
X090	7.2831	7.2288	1.	1.	75	75
X090	7.2731	7.2387	2.	1.	76	76
X090	7.2688	7.2430	1.	1.	77	77
X090	7.3135	7.1984	1.	1.	80	80
X090	7.2890	7.2229	1.	1.	81	81
X090	2.5757	2.4760	1.	1.	84	84
X090	2.5591	2.4921	1.	1.	85	85
X090	2.5475	2.5034	2.	1.	86	86
X090	2.5418	2.5090	5.	1.	87	87
X090	2.5382	2.5126	3.	1.	88	88
X090	2.5522	2.4991	1.	1.	91	91
X090	2.5416	2.5093	1.	1.	92	92
X090	2.5358	2.5150	1.	1.	93	93
X090	1.0413	0.7680	1.	1.	96	96
X090	0.9933	0.8154	1.	1.	97	97
X090	0.9606	0.8478	2.	1.	98	98
X090	0.9454	0.8630	3.	1.	99	99
X090	0.9375	0.8708	1.	1.	102	102
X090	0.9239	0.8844	1.	1.	103	103
X090	0.4369	-0.3585	1.	1.	121	122
X090	0.3945	-0.3179	1.	1.	122	127
X090	0.3192	-0.2641	1.	1.	125	125
END2	0.3117	-0.2743	1.	1.	126	126
X009	0.4790	-0.4929	1.	1.	3	13
X009	0.4374	-0.4113	1.	1.	4	4
X009	0.3960	-0.3866	1.	1.	5	5
X009	0.3296	-0.2753	1.	1.	7	7
X009	0.3218	-0.2859	1.	1.	8	8
X009	0.3147	-0.2981	1.	1.	9	9
X009	2.5628	2.4279	1.	1.	21	26
X009	2.5463	2.4710	1.	1.	22	22
X009	2.5319	2.4854	1.	1.	23	23
X009	2.5230	2.4943	1.	1.	24	24
X009	2.5898	2.4556	1.	1.	26	27
X009	7.2713	7.1299	1.	1.	29	34
X009	7.2598	7.2088	1.	1.	30	30
X009	7.2495	7.2188	1.	1.	31	31
X009	7.2438	7.2243	1.	1.	32	32
X009	7.3385	7.1657	1.	1.	34	35
X009	9.1756	9.0446	1.	1.	37	41
X009	9.1611	9.1165	1.	1.	38	38
X009	9.1510	9.1264	1.	1.	39	39
X009	9.2329	9.0835	1.	1.	41	42
X009	9.1939	9.1121	1.	1.	42	44
X009	9.1638	9.1193	1.	1.	45	45
X009	9.3619	9.1711	1.	1.	47	53
X009	9.3609	9.3567	1.	1.	48	48
X009	9.3601	9.3575	1.	1.	49	49
X009	9.3595	9.3580	3.	1.	50	50
X009	9.3593	9.3582	8.	1.	51	51

X009	9.5263	0.0248	1.	1.	53	95 GAG
CYCLE						
X009	9.4771	9.2406	1.	1.	54	54
X009	9.4264	9.2912	1.	1.	55	55
X009	9.4010	9.3165	1.	1.	56	56
X009	9.4096	9.3165	1.	1.	58	56
X009	9.3923	9.3161	1.	1.	59	58
X009	9.2214	9.1001	1.	1.	61	61
X009	9.1982	9.1230	1.	1.	62	62
X009	9.1815	9.1396	1.	1.	63	63
X009	9.1743	9.1468	1.	1.	64	64
X009	9.2386	9.0827	1.	1.	66	66
X009	9.2068	9.1143	1.	1.	67	67
X009	9.1918	9.1280	1.	1.	70	69
X009	9.1814	9.1356	1.	1.	71	70
X009	7.3170	7.1954	1.	1.	73	73
X009	7.2990	7.2131	1.	1.	74	74
X009	7.2831	7.2288	1.	1.	75	75
X009	7.2731	7.2387	2.	1.	76	76
X009	7.2688	7.2430	1.	1.	77	77
X009	7.3418	7.1704	1.	1.	79	79
X009	7.3135	7.1984	1.	1.	80	80
X009	7.2890	7.2229	1.	1.	81	81
X009	2.5955	2.4573	1.	1.	83	83
X009	2.5757	2.4760	1.	1.	84	84
X009	2.5591	2.4921	1.	1.	85	85
X009	2.5475	2.5034	2.	1.	86	86
X009	2.5418	2.5090	5.	1.	87	87
X009	2.5382	2.5126	3.	1.	88	88
X009	2.5639	2.4880	1.	1.	90	90
X009	2.5522	2.4991	1.	1.	91	91
X009	2.5416	2.5093	1.	1.	92	92
X009	2.5358	2.5150	1.	1.	93	93
X009	1.0981	-0.6244	1.	1.	95	113
X009	1.0413	0.7680	1.	1.	96	96
X009	0.9933	0.8154	1.	1.	97	97
X009	0.9606	0.8478	2.	1.	98	98
X009	0.9454	0.8630	3.	1.	99	99
X009	0.9538	0.8545	1.	1.	101	101
X009	0.9375	0.8708	1.	1.	102	102
X009	0.9239	0.8844	1.	1.	103	103
X009	0.4795	-0.3833	1.	1.	120	121
X009	0.4369	-0.3585	1.	1.	121	122
X009	0.3945	-0.3179	1.	1.	122	127
X009	0.3275	-0.2558	1.	1.	124	124
X009	0.3192	-0.2641	1.	1.	125	125
END3	0.3117	-0.2743	1.	1.	126	126
X001	0.5139	-0.5152	1.	1.	2	12
X001	0.4790	-0.4365	1.	1.	3	3
X001	0.4374	-0.4113	1.	1.	4	4
X001	0.3960	-0.3866	1.	1.	5	5
X001	0.3368	-0.2681	1.	1.	6	6
X001	0.3296	-0.2753	1.	1.	7	7
X001	0.3218	-0.2859	1.	1.	8	8
X001	0.3147	-0.2981	1.	1.	9	9
X001	2.5819	2.3953	1.	1.	20	25
X001	2.5628	2.4546	1.	1.	21	21

X001	2.5463	2.4710	1.	1.	22	22
X001	2.5319	2.4854	1.	1.	23	23
X001	2.5230	2.4943	1.	1.	24	24
X001	2.6229	2.4279	1.	1.	25	26
X001	2.5898	2.4556	1.	1.	26	27
X001	7.2847	7.0877	1.	1.	28	33
X001	7.2713	7.1979	1.	1.	29	29
X001	7.2598	7.2088	1.	1.	30	30
X001	7.2495	7.2188	1.	1.	31	31
X001	7.2438	7.2243	1.	1.	32	32
X001	7.3810	7.1299	1.	1.	33	34
X001	7.3385	7.1657	1.	1.	34	35
X001	9.1931	8.9912	1.	1.	36	40
X001	9.1756	9.1025	1.	1.	37	37
X001	9.1611	9.1165	1.	1.	38	38
X001	9.1510	9.1264	1.	1.	39	39
X001	9.2866	9.0446	1.	1.	40	41
X001	9.2329	9.0835	1.	1.	41	42
X001	9.1939	9.1061	1.	1.	42	43
X001	9.1749	9.1121	1.	1.	44	44
X001	9.1638	9.1193	1.	1.	45	45
X001	9.3629	9.0996	1.	1.	46	52
X001	9.3619	9.3557	1.	1.	47	47
X001	9.3609	9.3567	1.	1.	48	48
X001	9.3601	9.3575	1.	1.	49	49
X001	9.3595	9.3580	3.	1.	50	50
X001	9.3593	9.3582	8.	1.	51	51
X001	9.5863	-0.0583	1.	52	95	GAG CYCLE
X001	9.5470	9.1711	1.	1.	53	53
X001	9.4771	9.2406	1.	1.	54	54
X001	9.4264	9.3165	1.	1.	55	56
X001	9.4010	9.3165	1.	1.	56	56
X001	9.4339	9.2912	1.	1.	57	55
X001	9.4096	9.3042	1.	1.	58	57
X001	9.3923	9.3161	1.	1.	59	58
X001	9.2469	9.0750	1.	1.	60	60
X001	9.2214	9.1001	1.	1.	61	61
X001	9.1982	9.1230	1.	1.	62	62
X001	9.1815	9.1396	1.	1.	63	63
X001	9.1743	9.1468	1.	1.	64	64
X001	9.2801	9.0415	1.	1.	65	65
X001	9.2386	9.0827	1.	1.	66	66
X001	9.2226	9.1143	1.	1.	68	67
X001	9.2051	9.1218	1.	1.	69	68
X001	9.1918	9.1280	1.	1.	70	69
X001	9.1814	9.1356	1.	1.	71	70
X001	7.3373	7.1756	1.	1.	72	72
X001	7.3170	7.1954	1.	1.	73	73
X001	7.2990	7.2131	1.	1.	74	74
X001	7.2831	7.2288	1.	1.	75	75
X001	7.2731	7.2387	2.	1.	76	76
X001	7.2688	7.2430	1.	1.	77	77
X001	7.3737	7.1387	1.	1.	78	78
X001	7.3418	7.1704	1.	1.	79	79
X001	7.3135	7.1984	1.	1.	80	80
X001	7.2890	7.2229	1.	1.	81	81
X001	2.6161	2.4381	1.	1.	82	82

X001	2.5955	2.4573	1.	1.	83	83
X001	2.5757	2.4760	1.	1.	84	84
X001	2.5591	2.4921	1.	1.	85	85
X001	2.5475	2.5034	2.	1.	86	86
X001	2.5418	2.5090	5.	1.	87	87
X001	2.5382	2.5126	3.	1.	88	88
X001	2.5777	2.4752	1.	1.	89	89
X001	2.5639	2.4880	1.	1.	90	90
X001	2.5522	2.4991	1.	1.	91	91
X001	2.5416	2.5093	1.	1.	92	92
X001	2.5358	2.5150	1.	1.	93	93
X001	1.1594	-0.6892	1.	1.	94	94
X001	1.0981	-0.6454	1.	1.	95	112
X001	1.0413	0.7680	1.	1.	96	96
X001	0.9933	0.8154	1.	1.	97	97
X001	0.9606	0.8478	2.	1.	98	98
X001	0.9454	0.8630	3.	1.	99	99
X001	0.9722	0.8361	1.	1.	100	100
X001	0.9538	0.8545	1.	1.	101	101
X001	0.9375	0.8708	1.	1.	102	102
X001	0.9239	0.8844	1.	1.	103	103
X001	0.5153	-0.4087	1.	1.	119	120
X001	0.4795	-0.3833	1.	1.	120	121
X001	0.4369	-0.3585	1.	1.	121	122
X001	0.3945	-0.3179	1.	1.	122	127
X001	0.3352	-0.2507	1.	1.	123	123
X001	0.3275	-0.2558	1.	1.	124	124
X001	0.3192	-0.2641	1.	1.	125	125
END*	0.3117	-0.2743	1.	1.	126	126

APPENDIX C

This is crack growth output parameter file.

```

MATERIAL PROPERTIES
C= 0.541000E-09    SMALLN= 3.6800    KSUBC= 80.0    YIELD STRESS= 42.0
LIMIT STRESS= 11.64    EM= 0.60    RCP= 0.75    RCN= -0.50    AKTH= 1.50    CHANG= 1.00
INITIAL (SEMI-)CRACK LENGTH = 0.55600E+00, HIDDEN FLAW LENGTH= 0.00000E+00, NLINK= 0
CORRECTION FACTORS USED FOR DELTA K
DELTAK=(DELTA T)*SQRT(PI*A)*BETA(1)*BETA(2)*...BETA(I)
BETA(3)-TABULAR FUNCTION OF A/B
WHERE....B= 0.100000E+01

      A/B      BETA(3)      A/B      BETA(3)      A/B      BETA(3)
0.990200E+00 0.555600E+00 0.111840E+01 0.662200E+00 0.103060E+01 0.768700E+00
0.946900E+00 0.875200E+00 0.967700E+00 0.981700E+00 0.954400E+00 0.108830E+01
0.108490E+01 0.119480E+01 0.943400E+00 0.145760E+01 0.111640E+01 0.156410E+01
0.106930E+01 0.167060E+01 0.107190E+01 0.177720E+01 0.106780E+01 0.188370E+01
0.125650E+01 0.199020E+01 0.107510E+01 0.225300E+01 0.128980E+01 0.235950E+01
0.128360E+01 0.246610E+01 0.125420E+01 0.257260E+01 0.126470E+01 0.267910E+01
0.162610E+01 0.278570E+01 0.130940E+01 0.289220E+01 0.134180E+01 0.315500E+01
0.179540E+01 0.326150E+01 0.165470E+01 0.336800E+01 0.171310E+01 0.347460E+01
0.305370E+01 0.358110E+01 0.190440E+01 0.368760E+01 0.204850E+01 0.395040E+01
0.405690E+01 0.357420E+01 0.416350E+01 0.489240E+01 0.427000E+01

INITIAL CYCLE NUMBER = 0.
REPEAT INPUT SPECTRUM 1000 TIMES
**/**MISSION DESCRIPTION**/**
LOADS INPUT CONSISTS OF 4 BLOCKS,MISSION PROFILE USES 38 BLOCKS,(SOME MAY BE
REPEATED), I.E. ---
1,/ 45 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 45 OF BLOCK# 1,/ 9 OF BLOCK# 2,/ 165 OF BLOCK#
2,/ 1 OF BLOCK# 3,/ 30 OF BLOCK# 1,/ 9 OF BLOCK# 2,/ 60 OF BLOCK# 1,/ 9 OF BLOCK#
1,/ 30 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 30 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 30 OF BLOCK#
1,/ 1 OF BLOCK# 3,/ 9 OF BLOCK# 2,/ 60 OF BLOCK# 1,/ 9 OF BLOCK# 2,/ 15 OF BLOCK#
2,/ 1 OF BLOCK# 4,/ 90 OF BLOCK# 1,/ 9 OF BLOCK# 2,/ 45 OF BLOCK# 1,/ 18 OF BLOCK#
1,/ 15 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 120 OF BLOCK# 1,/ 9 OF BLOCK# 2,/ 15 OF BLOCK#
2,/ 1 OF BLOCK# 3,/ 15 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 30 OF BLOCK# 1,/ 9 OF BLOCK#
30 OF BLOCK# 1,/ 1 OF BLOCK# 3,/ 30 OF BLOCK# 1,/

BLOCK      BLOCK      BLOCK      NUMBER OF      NUMBER OF
NUMBER      START      FINISH      LOADS      CYCLES
1           1           27          27.0        48.0
2           28          76          49.0        70.0
3           77          149         73.0        93.0
4          150          215         96.0       116.0

```

*/** TOTAL CYCLES IN MISSION= 50453. */**

*/**/*THE FOLLOWING CALCULATIONS ARE BEYOND THE RANGE OF "AOVERB" TABLE, MODIFY OR EXTEND
TABLE*/**/*

*****ARGUMENT(AOVERB)= 0.480542E+01*****

CRITICAL (SEMI-)CRACK LENGTH AT MAXIMUM STRESS OF 11.640= 3.6696; (42 ITERATIONS)

"AMF",CALCULATED AUTOMATICALLY= 2.0

1 INTEGRATION INTERVAL (CYCLES)	MINIMUM VALUE OF LOAD STEP	(SEMI-)CRACK LENGTH/ NUMBER OF CYCLES PER LOAD STEP	(SEMI-)CRACK GROWTH IN ANY NUMBER OF CYCLES AT END OF STEP	INTEGRATION=9999.0 DELTA SIGMA	R
X900	1	1.00	1.00	0.594	-0.5000
1.000					
X900	2	1.00	2.00	0.472	-0.5000
1.000					
X900	3	1.00	3.00	0.038	0.7500
1.000					
X900	4	1.00	4.00	0.025	0.7500
1.000					
X900	5	1.00	5.00	0.069	0.7500
1.000					
X900	6	3.00	8.00	0.002	0.7500
1.000					
X900	7	8.00	16.00	0.001	0.7500
1.000					
X900	8	1.00	17.00	10.006	-0.0615
1.000					
X900	9	2.00	19.00	0.085	0.7500
1.000					
X900	10	1.00	20.00	0.042	0.7500
1.000					
X900	11	1.00	21.00	0.035	0.7500
1.000					
X900	12	1.00	22.00	0.054	0.7500
1.000					
X900	13	2.00	24.00	0.034	0.7500
1.000					
X900	14	1.00	25.00	0.026	0.7500
1.000					
X900	15	1.00	26.00	0.066	0.7500
1.000					
X900	16	1.00	27.00	0.067	0.7500
1.000					
X900	17	2.00	29.00	0.044	0.7500
1.000					
X900	18	5.00	34.00	0.033	0.7500
1.000					
X900	19	3.00	37.00	0.026	0.7500
1.000					
X900	20	1.00	38.00	0.032	0.7500
1.000					
X900	21	1.00	39.00	0.021	0.7500
1.000					
X900	22	1.00	40.00	0.178	0.7500
1.000					
X900	23	2.00	42.00	0.113	0.7500
1.000					
X900	24	3.00	45.00	0.082	0.7500
1.000					
X900	25	1.00	46.00	0.039	0.7500
1.000					
X900	26	1.00	47.00	0.592	-0.5000
1.000					
END1	27	1.00	48.00	0.468	-0.5000
1.000					
X090	28	1.00	1.00	0.656	-0.5000
1.000					
X090	29	1.00	2.00	0.594	-0.5000
1.000					
X090	30	1.00	3.00	0.483	-0.5000
1.000					
X090	31	1.00	4.00	0.472	-0.5000
1.000					

X090	32	1.00	5.00	0.091	0.7500
1.000					
X090	33	1.00	6.00	0.046	0.7500
1.000					
X090	34	1.00	7.00	0.029	0.7500
1.000					
X090	35	1.00	8.00	0.094	0.7500
1.000					
X090	36	1.00	9.00	0.031	0.7500
1.000					
X090	37	1.00	10.00	0.020	0.7500
1.000					
X090	38	1.00	11.00	0.078	0.7500
1.000					
X090	39	1.00	12.00	0.025	0.7500
1.000					
X090	40	1.00	13.00	0.075	0.7500
1.000					
X090	41	1.00	14.00	0.120	0.7500
1.000					
X090	42	1.00	15.00	0.003	0.7500
1.000					
X090	43	3.00	18.00	0.002	0.7500
1.000					
X090	44	8.00	26.00	0.001	0.7500
1.000					
X090	45	1.00	27.00	10.078	-0.0634
1.000					
X090	46	1.00	28.00	0.135	0.7500
1.000					
X090	47	2.00	30.00	0.085	0.7500
1.000					
X090	48	1.00	31.00	0.075	0.7500
1.000					
X090	49	1.00	32.00	0.042	0.7500
1.000					
X090	50	1.00	33.00	0.028	0.7500
1.000					
1	LOAD	NUMBER OF	NUMBER OF	DELTA	R
INTEGRATION	STEP	CYCLES PER	CYCLES AT	SIGMA	
INTERVAL		LOAD STEP	END OF STEP		
(CYCLES)					

Crack Growth truncated

Final flight and crack length information:

APLOT	CPLLOT	XPLOT
0	0	1.051
1813.63	1.81363	1.11069
3455.36	3.45536	1.17454
5097.1	5.0971	1.24882
6517.13	6.51713	1.32273
7780.01	7.78001	1.4004
8916.59	8.91659	1.48172
9926.89	9.92689	1.56576
10937.19	10.93719	1.66424
11858.39	11.85839	1.76716
12567	12.567	1.85528
13412.07	13.41207	1.97555
14043.51	14.04351	2.09266
14611.8	14.6118	2.21615
15116.95	15.11695	2.34474
15622.1	15.6221	2.49577

16000.96	16.00096	2.62804
16379.83	16.37983	2.78044
16790.09	16.79009	2.95866
17105.8	17.1058	3.13115
17421.52	17.42152	3.32976
17705.67	17.70567	3.53428
17958.24	17.95824	3.73929
18210.82	18.21082	3.96783
18431.82	18.43182	4.18671
18652.82	18.65282	4.4214
18905.4	18.9054	4.70749
19126.4	19.1264	4.9714
19378.98	19.37898	5.27749
19663.12	19.66312	5.60807
19950.43	19.95043	5.91147
20333.85	20.33385	6.27333
20870.57	20.87057	6.66889
21726.17	21.72617	7.06777
23052.19	23.05219	7.48688
24381.36	24.38136	7.93
25115.23	25.11523	8.39134
25470.24	25.47024	8.89602
25612.18	25.61218	9.38527
25658.52	25.65852	9.97156

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