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CERVICAL SPINE MULTI-BODY MATHEMATICAL MODEL

By
Ali Taha, P.Eng, M.A.Sc
University of Technology, 1993

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In Partial Fulfillment of the Requirement for the Degree of
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ABSTRACT

CERVICAL SPINE MULTI BODY MATHEMATICAL MODEL

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Mechanical Engineering

Ryerson University

The human cervical spine is the most complicated structure in the vertebral column; its seven mobile vertebrae are interconnected in different ways and interact with many other parts of the body, in particularly the neck. Individual variations exist; with time, growth and aging can induce substantial changes in the components of each individual's spine.

Knowledge of the function of the spine, from a mechanical viewpoint, is important to the study of both normal function and pathological processes. However, because of the structural complexity of the spine, only a limited understanding of the subject exists at present.

Mathematical analogies are often used to study complex biological systems, which are difficult to investigate by using conventional experimental techniques. Hence, they are simulated and their behavior may then be studied in great detail.

This project describes a mathematical model of the human cervical vertebral column. The model is comprised of a collection of equations expressing the mechanical relationships that must be obeyed among its elements, each of which represents a segment of a real vertebral column. Some degree of descriptive reality has been sacrificed in an effort to gain simplicity in approximating the behavior of the spine under a variety of conditions.

The mathematical model was solved by preparing a special code as part of this project; determining dimensions and positions of neck components were another task since no such data are available in previous studies. However, most of the physical data of the neck was utilized from existing literature. The solution was determined by analyzing deflections, forces and stresses on each individual ligament and on each individual vertebra. Results of two case studies shown at the end of this project illustrate by figures depicted the projection views of the deflections of ligaments and total magnitude forces exerted on each vertebra.

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NOMENCLATURE

AS	Anterior shear
PS	Posterior shear
LS	Lateral shear
TNS	Tension
CMP	Compression
LB	Lateral bending
FLX	Flexion
EXT	Extension
ALL-L	Left anterior longitudinal ligament
ALL-C	Center anterior longitudinal ligament
ALL-R	Right anterior longitudinal ligament
ISL- L	Left interspinous ligaments
ISL- C	Central interspinous ligaments
ISL- R	Right interspinous ligaments
PLL-L	Left posterior longitudinal ligaments
PLL-OL	Left outer posterior longitudinal ligaments
PLL-C	Central posterior longitudinal ligaments
PLL-OR	Right outer posterior longitudinal ligaments
PLL-R	Right posterior longitudinal ligaments
IL-L	Left intertransverse ligament
IL-R	Right intertransverse ligament
FL-OL	Left outer flavum ligament
FL-IL	Left anterior flavum ligament
FL-OC	Centralouter flavum ligament
FL-IC	Central anterior flavum ligament
FL-IR	Right anterior flavum ligament
FL-OR	Right outer flavum ligament
AL-L	Left Alar ligament
AL-R	Right Alar ligament
TL-L	Left Transverse atlantal ligament
TL-R	Right Transverse atlantal ligament
C1	Cervical vertebra #1 (atlas)
C2	Cervical vertebra #2 (axis)
C3	Cervical vertebra #3
C4	Cervical vertebra #4
C5	Cervical vertebra #5
C6	Cervical vertebra #6
C7	Cervical vertebra #7
G_i	Vertebra local centre of rotation; $i=1:7$
Ψ_i	Vertebra rotational angles
$\rightarrow$	
p_i	Position relative to global coordinate system; $i=1:7$
$\rightarrow$	
r_{Gi}	Position vector of the radius from G_i to P_i ; $i=1:7$
$\rightarrow$	
$\vec{S}_{pi}$	Vertebra linear displacements matrix of point at C_i ,
R_j^i :	Transformation matrix from vertebra C_i to the lower one C_j or C_{i+1}
$\rightarrow$	
p_i	Position relative to global coordinate system
$\rightarrow$	
P_{ii}	Position relative to the local coordinate system (x_i, y_i, z_i
$\rightarrow$	
F_l	Ligament reaction force vector
$\rightarrow$	
F_{Ci}	Resultant force on C_i , with respect to the global coordinate system.
$\rightarrow$	
F	Resultant force the cervical spine, with respect to the global coordinate system.

CHAPTER ONE

INTRODUCTION

1-1 Objective

It has been noted that helicopter pilot's long hours of operation while monitoring navigation instruments, located on the bottom sides of the pilot seat, is the cause of mild pain in the neck. Pilot's neck comes under severe static and dynamic loads, due to head weight, helmet weight and the G forces caused by gravity and inertia loads, as shown in Figure 1-1. This subject was issued to a scientific firm for investigation to decide what could be done. They began by analyzing the cervical spine and verifying the types of loads and how much the neck can withstand before any type of fatigue occurs. The first step in solving this type of problem is to identify the involved criteria, and build a mathematical model accordingly to simulate internal and external cervical spine actions and reactions.

The neck, according to the pathologist perspective, has been considered the most complicated part of the human anatomy. It has the most complicated muscle system and the most complicated vertebra system in the spine, not to mention the other systems, which are not directly related to the biomechanical point of view. Therefore, biomechanical literature has divided the neck into two main systems: the cervical spine components system and the neck muscles components system. In order to begin mechanical analysis on these systems, a mathematical model must be introduced depicting the biological systems. Another step must also be included to introduce components into a system of solvable equations, including all possible components coupled with their physical properties leading to an understanding of the functions and the responses of each individual component. As for this project, the target is to develop a mathematical model and solve it to determine stresses, force and deflection on each ligament, also to find forces applied on each vertebra and on the whole cervical spine, leaving the muscle system to be the next step. However, the model has to be capable of handling further expansions by adding the unconsidered components. Therefore, this project is considered to be the first step in the analysis of the entire neck.

The model used in this project has idealized the cervical spinal column as a collection of rigid bodies connected by deformable elements. The vertebrae are represented by rigid bodies, while the intervertebral discs, ligament and other connective tissues are represented by deformable elements. The model has to be capable of treating any three-dimensional force, moment or motion of the spine. Each element may have axial or torsional bending and/or shear resistance in any combination. Forces and moments exerted by and on the elements can be arbitrary. Material nonlinearity of the biological tissues such as the absence of compressive strength in ligaments, the damping factor and hardening occurs as deformations increase, and can also be considered.

However, these factors will be examined in future work, in addition to any flexible elements other than ligaments.

Figure 1-1

Illustration of the head and cervical spine at an arbitrary position, G_i : vertebra local center of rotation, C_i : vertebra body notation, ψ_i : rotational angles

1-2 Project Overviews

The goal of this study is to use information collected from human neck anatomy literature and previous experimental results on element properties, in a set of equations governing cervical spine kinematics-kinetic behavior forming a numerical mathematical model. The model will then be solved by utilizing a computer program to obtain the mechanical behavior such as stresses, force and deflection on each ligament, also to find forces applied on each vertebra and on the whole cervical spine.

This study consists of five chapters; an introduction to the study stating the problem and a review of the current literature. The second chapter is an overview of the neck, definitions, components, functions and some useful mechanical properties. The third chapter will present the mathematical side of the study demonstrating detailed steps to the final mathematical model. The fourth chapter addresses the solution of the model and uses the model in specific cases as an example of the neck simulation. The final chapter discusses the potential practical application of this study.

In order to achieve the above goals, this project will include the following steps:-

Step “1”:-

Explain and identify the cervical spine from a mechanical engineering view, listing the physical and dimensional properties, which a 3D CAD model has to be developed, which it's useful in finding the position vectors and verifying results later.

Step “2”:-

Propose a mathematical set of equations representing the kinematics of the human cervical spine; it should govern the 3D motions of all points within the cervical spine.

Step “3”

Present and interconnect a new set of equations with step”2” to determine the reaction force vectors and stresses on the individual elements, vertebra and the whole cervical spine.

Step “4”:

Create a MAT-LAB™ code to solve the above mathematical model, and apply it to a case study.

Step “5”:

Design a structure for solving test cases using volunteers; implement a procedure to utilize the project for practical testing.

1-3 Literature Review

The neck is one of the most sophisticated parts of the human body; the anatomy is well described by H. Gray [1] and R.M.H. Mc Minnin [2]. Biomechanic researchers have done a wide range of studies on this part of the human body. Some are interested in studying the details of a specific tissue like a specific bone or muscle in the neck and focus on the material physical properties, such as Vee-Sin Lee Peter and others, and Donald T.R. , Albert H. B. and Victor h. F. [34, 35, 36, 37, 38, 39, 40] Some investigate groups of similar tissues, for example A. B. Schultz and others [30, 31], investigate the stresses on the intervertebral disc under axial load, while Karin B. and Peter L. [32] study the muscle activities on neck response. Others like De Jager, Marike and Schultz focus on the mechanism of injury in the cervical spine,[20, 21, 26], with most of their efforts focusing on simulating whiplash as a result of car accidents. M. Panjabi, [27, 29] has made a major contribution to this field with the development of a new biofidelic model to describe cervical spine curvature and evaluate the potential for injury in the upper and lower cervical spine during simulated whiplash. Finally, the research of C. Helleur [41] looks at solutions to prevent neck injury by studying the use of the helmet.

As for studying and analyzing the cervical spine, biomechanical models of the neck have different degrees of complexity. The simplest model done by John D. [10], consists of a two-pivot lumped mass as model, in which a graphical model was developed to illustrate relevant force vectors acting on the head and neck in upright posture. This model was used to calculate theoretical static loading on the structures of the neck under various loading conditions resulting from specific configurations of passive elements and active supportive muscle effort. On the other side of complexity is the finite element analysis (FEA) models. It is remarkable that more FE, models than multibody models have been published over the last decade. Geometry material property, loading and boundary conditions, followed by validations are the critical issues in FE modeling of cervical spine. N. Yoganandan, S.K and F. Pintar [16] and [14], have extensively studied the sensitivity of FEA on cervical spine, using a cadaver of a 32 years old adult free from spinal diseases for their 3D solid model. In this model, the responses and contributions of the soft tissue structures of the human neck are described with a focus on mathematical modeling. Spinal ligaments, intervertebral discs, zygapophysial joints, and uncovertebral joints of the cervical spine are included. FE modeling approaches have been emphasized. Representative data relevant to the development and execution of the model are discussed. The basis for the multibody (MB) model by De Jager [21] proceeds from a relatively simple multi-body model to a more detailed one utilized by the FEA. It consists of rigid vertebrae and a rigid head, linear viscoelastic intervertebral discs, nonlinear viscoelastic ligaments, contractile muscles and frictionless, nonlinear viscoelastic facet joints. He concludes that although FE codes are better suited than multibody models for detailed descriptions of the cervical spine, the differences in detail of the currently available neck models (FE and MB) are negligible.

This subject has attracted many researchers since 1930, but it was not until the seventies that a three-dimensional model for the nonlinear force analysis of the spine was developed. The development of a model requires the accomplishment of a number of interrelated tasks including the development of the computational techniques. There was no access to the relevant biomechanical data, or available data isn't in the form to be useful for an engineers. A.B. Schoultz and T.B. Belytschko [5, 12] considered to be pioneers of the spine 3D modeling, developed their mathematical model for a three-dimensional force analysis of the human vertebral column. The vertebrae are idealized as rigid bodies, while the discs, ligaments, and connective tissues are represented by deformable elements. An incremental stiffness method which accounts for nonlinearity due to large displacements is used. The use of 3D radiographic technique by Richard H.B, Albert H.B, Clyde L.N and Charles C.S [7] and A.B. Schoultz [8] contribute to determining the models accuracy and reliability. The results of these tests were used to evaluate mechanical models of spine. M. Panjabi also contributed to the multibody 3D mathematical model, using the autopsy specimens to obtain the data he needed, and evaluating both the experimental and mathematical errors. Another method called "part to whole" approach by N. Yoganandan, F.A. Pinter [13] has been developed to find useful data for the mathematical model.

The multibody mathematical models described above are not readily available in a useful detailed form. Since Ryerson University is now addressing this topic there is a need to build our own efficient model and solution. Therefore this project will present a fully detailed cervical spine multibody model analysis, using whatever data and mechanical properties available from the existing literatures, utilizing an independent approach in analyzing the neck movement and finding the stress. Implementing a CAD model depicted from a plastic mannequin, could be used in analyzing the neck by the FE analysis in the future. Finally this project includes a proposal of a conceptual design for the experimental test to verify results.

CHAPTER 2

PHYSICAL MODEL OF THE CERVICAL SPINE

2-1 Introduction

This chapter reviews the biomechanics of the cervical spine, demonstrating the major functions of its components and tabulates the necessary data in order to build the mathematical model. This chapter serves as an interface between the human tissues from a biological view and a mathematical/ engineering prospective. The mechanical properties have been taken from published literatures in this field, while geometrical properties have been obtained from a CAD model implemented as part of this project and depict the geometry of an average human spine using the human mannequin “plastic model” as a reference.

Cervical spine structure consists of two main types of tissue, the vertebra and the soft tissue structures or so called “the motion segment”, which consist of the following components:

- a) The Ligaments,
- b) The Intervertebral disc joint between the adjacent vertebrae,
- c) The Facet joint, another set of joints between the adjacent vertebrae
- d) The Uncovertebral joints

The responses and contributions of the soft tissue structures of the human neck are described in this chapter with a focus on the biomechanical and mathematical modeling properties and behavior, that includes the spinal ligaments, intervertebral discs, facet joints, and uncovertebral joints of the cervical spine, in addition to their functions and mechanical role.

For this and subsequent chapters geometrical characteristics will refer to the properties such as length, position vectors and cross-sectional areas, and the mechanical characteristics refers to the properties such as stresses, spring constants, damping coefficient, force-displacement and stress-strain responses.

2-2 Cervical Spine Load Characteristics

Following a similar load convention adopted by M. Pinjabi [9], D. Jager [21] and B. schoultz [4] as shown in Figure 2-1 and Table 2-1, it is representative of the general experimental set up in determining the load-kinematics of the motion segment set. The lower solid piece (the lower vertebra) is rigidly fixed to a stationary base, while the upper part (upper vertebra) is free to move in its six degrees of freedom upon the applied load.

The original right-handed orthogonal coordinate system is placed at the geometric center of the upper vertebral body. The *geometric center* is assumed to lie in the midsagittal area at the intersection of the body. The x , y , z defines the positive directions for the loads (forces F and moments M) and the corresponding displacements (translation S and/ or rotation ψ). The names assigned to the loads and displacements reflect the mechanical behavior which may differ for positive and negative displacements along the same axis. Assuming midsagittal symmetry, the behavior in the left and right lateral shear, lateral bending, or axial rotation is identical.

Figure 2-1

Load	Disp.	Name	Abbr.
$-F_y$	$-S_y$	Anterior shear	AS
$+F_y$	$+S_y$	Posterior shear	PS
$\pm F_x$	$\pm S_x$	Lateral shear	LS
$+F_z$	$+S_z$	Tension	TNS
$-F_z$	$-S_z$	Compression	CMP
$\pm M_x$	$\pm \psi_x$	Lateral bending	LB
$-M_y$	$-\psi_y$	Flexion	FLX
$+M_y$	$+\psi_y$	Extension	EXT
$\pm M_z$	$\pm \psi_z$	Axial rotation	AR

Table 2-1

2-3 Spinal Vertebral Anatomy

The vertebral column is a flexible column, formed by a series of bones called vertebra, H. Gray [1] and R.M.H. Mc Minnin [2]. Of the thirty three vertebrae, only 24 are mobile vertebrae. They are grouped under the following names: cervical, lumbar, sacral and coccygeal, according to the regions they occupy. There are seven in the cervical region (C1-C7), twelve in the thoracic (T1-T12), five in the lumbar (L1-L5) five fused in the sacral (S1) and four fused in the coccygeal. The thoracic vertebrae connect with the ribs. The vertebral bodies are separated from each other by the intervertebral discs, which account for about one-fourth of the total length of the column. The adult vertebral column exhibits several curvatures in the sagittal plane: an anterior convex cervical curve (cervical lordosis), an posterior convex thoracic curve (thoracic kyphosis), an anterior convex lumbar curve (lumbar lordosis), and a fixed curve in the sacro-coccygeal area. Figure 2-2 [1].

2-3-1 The Cervical Vertebra

The cervical spine consists of seven vertebrae; numbered C1 to C7. The 1st, 2nd and last vertebra, the 7th, are distinctive compared to the rest. However, cervical vertebra, Figure 2-3, H. Gray [1], are the smallest of the true vertebra, and can be readily distinguished from those of the thoracic or lumbar regions by the presence of a foramen in each transverse process. The first, second, and seventh present exceptional features and must be separately described; the following characteristics are common to the remaining four. Each vertebra consists of:

- The body: is small and broader from side to side than from front to back.
- The pedicles: are attached to the body midway
- The lamina: are narrow, and thinner above than below,
- the vertebral foramen is large, and triangular.
- The spinous process: is short and bifid, often of unequal size.
- The superior and inferior articular processes: on either side are fused to form
- the articular pillar,
- The articular facets: are flat and ovalar.
- The transverse process: gives passage to the vertebral artery and vein and a plexus of sympathetic nerves in the upper six vertebra
- The anterior and the posterior tubercle.

Figure 2-2
Vertebral column in the sagittal plane [1]

Figure 2-3
A cervical vertebra [1]

2-3-2 First Cervical Vertebra

The first cervical vertebra, Figure, 2-4 [1], is named the **atlas** because it supports the globe of the head. It has no body due to the fact that the body of the atlas has fused with that of the next vertebra. It is ring-like, and consists of an anterior and a posterior arch.

Figure 2-4
First cervical vertebra the Atlas [1]

2-3-3 Second Cervical Vertebra

The second cervical vertebra, Figure 2-5, [1] and Figure 2-6 [1], is named the **Epistropheus** or **Axis** because it forms the pivot upon which the first vertebra, carrying the head, rotates. The most distinctive characteristic of this bone is the strong odontoid process, which rises perpendicularly from the upper surface of the body.

Figure 2-5
Second cervical vertebra, or axis, from above [1]

Figure 2-6
Second cervical vertebra axis, from the side [1]

2-3-4 The Seventh Cervical Vertebra

Figure 2-7, [1]. The most distinctive characteristic of this vertebra is the existence of a long and prominent spinous process, hence the name **vertebra prominens**.

Figure 2-7
Seventh cervical vertebra [1]

2-4 Soft Tissue (Motion Segment) Anatomy, Role and Properties

As mentioned earlier, this part of the human cervical spine consists of four major parts: ligaments, the intervertebral disc, the facet joint, and the uncovertebral joints. This section deals with the functions and anatomy of each of these soft tissues.

2-4-1 Ligaments

Ligaments are uniaxial structures that resist only tensile or distractive forces. Some ligaments are capable of resisting tensile forces in a range of directions because of their orientation. Since more than one type of ligament is found in adjacent vertebrae, the response is dependent on the nature of the external load vector. The response is dependent on the severity, i.e., the magnitude and direction of the applied load. Internal forces resisted by the various ligaments differ according to ligament location and characteristics. Ligaments are generally divided into five distinctive types:

- The Anterior Longitudinal Ligament (ALL): is firmly united with the anterior side of the vertebra bodies. It is considered to be the most effective under an extension bending moment.
- Interspinous Ligaments (ISL): are relatively weak sheets of fibrous tissue which unite spinous processes. They are effective under a extension moment.
- The Posterior Longitudinal Ligaments (PLL): extends from and connects to the back of the vertebra bodies. These bodies, which lie close to the center of rotation, respond with less force resistance than the anterior longitudinal ligament or the interspinous ligament.
- Flavum Ligament (FL): characterized by a higher proportion of elastin than collagen, therefore it is more elastic than other type of ligaments. This results in a more mechanical elastic behavior,
- The Intertransverse Ligament (IL): relatively weaker sheets of fibrous tissue joining the transverses along their adjacent borders.

Different ways in determining the geometrical specification were investigated by N. Yoganandan and F. Pintar [14], The researchers began the study by collecting data from radiographs and X-rays including the Magnetic Resonance Imaging (MRI). The results of this research are shown in Table 2-2.

	Area (mm ²)	Length (mm)
C2-C5		
ALL	11.1 (1.9)	18.8 (1.0)
PLL	11.3 (2.0)	19.0 (1.0)
LF	46.0 (5.8)	8.5 (0.9)
ISL	13.0 (3.3)	10.4 (0.8)
C5-T1		
ALL	12.1 (2.7)	18.3 (0.5)
PLL	14.7 (6.8)	17.9 (0.5)
LF	48.9 (7.9)	10.6 (0.6)
ISL	13.4 (1.0)	9.9 (0.7)

Where:

ALL: anterior longitudinal ligament;

PLL: posterior longitudinal ligament;

LF: ligamentum flavum;

ISL: interspinous ligament.

Table 2-2
Length and cross-sectional area of cervical spine ligaments,[4]

They also were able to determine the fundamental properties of these ligaments, such as stiffness (structural property) and elastic modulus. Results were obtained by subjecting the ligament to the tensile load test. Experiments conducted with human cadaver cervical spine structures include isolated ligament tests and in situ bone-ligament-bone preparations.

The typical force-displacement (solid line) and stiffness displacement (dotted line) responses of a ligament are shown in Figure 2-8 [21], Non-linearity in the force-displacement behavior is apparent. OA denotes the ambient phase or “toe” region representing a highly non-linear curve corresponding to low stiffness; AB denotes the most linear phase (these two correspond to physiologic loading); BC denotes the traumatic phase (region of gradually decreasing stiffness); C denotes failure. Post-traumatic phases represented by the portion of the curve beyond the traumatic phase (arrow). OA1, A1B1, B1C1 denote the stiffness pattern in these phases. At the point of failure, the stiffness is zero. The physiological, traumatic, and post-traumatic phases were renamed to neutral, elastic, and plastic zones, respectively. Removal of the ligament from

the spinal column for testing often results in damage to the structure and induces fixation failures. In situ preparations have the unique advantage that the ligament under test is not isolated from its surroundings. Stiffness value on each individual ligament have been observed from [13], [14], [20], [21] and [23] and tabulated in Table 2-3.

Figure 2-8
Force-Displacement (solid line) and stiffness- displacement (dash line), [21]

Spinal level	Type	Stiffness (N/mm)
OC-C1	JC	32.6
OC-C1	AA-OM	16.9
OC-C1	PA-OM	5.7
C1-C2	ALL	24.0
C1-C2	JC	32.3
C1-C2	LF	11.6
OC-C2	TM	7.1
OC-C2	Apical	28.6
OC-C2	Alar	21.2
OC-C2	CLV	19.0

Biomechanical data for ligaments in upper cervical spine

Where:

JC: joint capsules;

AA-OM : anterior atlanto-occipital membrane;

PA-OM : posterior atlanto-occipital membrane;

ALL :anterior longitudinal ligament;

LF :ligamentum flavum;

TM : tectorial membrane;

CLV :cruciate ligament, vertical portion.

Spinal level	Type	Stiffness (N/mm)
C2-C5		16.0
	ALL	25.4
	PLL	25.0
	LF	7.74
	ISL	17.9
C5-T1		23.0
	ALL	21.6
	PLL	6.4
	LF	16.0
	ISL	25.4

Table 2-3

Biomechanical data for ligaments in lower cervical spine

The above described force, deflection, energy, stiffness, stress, and strain parameters are applicable to (quasi) static tensile loading. Wherein the rate of load application is slow, sufficient time is allowed for the ligament structure to react (process of gradual reorientation of fibers during loading).

In particular, the failure force, stiffness, and energy have all been reported to increase with rate of loading, Figure 2-9, Narayan Y, A.B. Kumaresan and F A. Pintar [14]. For example, with an increase in loading rate from 1.0 to 250 cm/s, the tensile failure load, stiffness, and energy for the anterior longitudinal ligament and ligamentum flavum of the cervical spine increases by a factor of 2-4.

Figure 2-9
Load rate effect on force-displacement diagram [14]

An important but frequently ignored aspect is the effect of ligament pre-stress on the internal and external biomechanical responses of the spine. From functional and anatomical perspectives, ligaments have varying levels of pre-stress. For example, the ligamentum flavum has more pre-stress than the anterior longitudinal ligament.

2-4-2 Intervertebral Disc

The cervical spine is a segmented structure consisting of mobile vertebrae separated by intervertebral discs. The intervertebral disc consists of three parts:

- The annulus fibrosis in the periphery:
- The cartilage plates above and below.
- The nucleus pulposus in the center.

The annulus fibrosis Figure 2-10, B. Schultz [30] is formed by a series of concentric encircling lamellae. The lamellae consist of collagen fibers, which have two well-defined axes of orientation. The fibers in each lamella enter, alternating from the previous one and aligned at an angle of 30° with the horizontal axis according to B. Schultz [30] or at an angle between 24° and 37° according to other researchers.

Fig. 2-10
The annulus fibrosis [30]

The nucleus pulposus is located close to the center of the disc; it is bound above and below by the superior and inferior cartilagenous end-plates, and on its periphery by the annulus fibrosis. It is composed of non-orientated collagen fibrils enmeshed in a mucoprotein gel. The water content of the nucleus ranges from 88 per cent at birth to approximately 69 per cent by the age of 77.

In contrast to ligaments, which are considered to be uniaxial and bear tension only, intervertebral discs respond to (or experience) multiple load vectors. Under any external loading, with the exception of direct uniaxial tension (e.g., airbag loading), discs carry compressive forces along with other components. During normal physiologic conditions, the weight of the head places the C2-T1 discs in the neck. These discs are placed with respect to the cervical column. Intervertebral discs do not carry or resist pure

loading. In other words, compression is associated with the moment. The off-centered anatomy of the nucleus pulposus also contributes to the dissimilar proportions of anterior and posterior annulus internal load sharing. Thus, during physiologic and traumatic load applications, cervical intervertebral discs respond to a variety of load vectors including compression, bending, and tension.

Several modeling approaches have been used to simulate the disc, B. Schultz [30]. For example, a composite, single-entity definition that incorporates Young's modulus of elasticity in compression and tension has been used. This definition does not differentiate between the properties of the nucleus, annulus ground substance, and fibers. Solid finite elements have already been used in previous research for this purpose.

2-4-3 Facet Joints

Like the intervertebral disc, facet joints respond to multiple load vectors; they stand against compressive forces in the sub-axial cervical spine. The distribution of the compressive load resisted by the two facet joints at any cervical level depends on the orientation of the joint and eccentricity of the external compressive force. From this point of view, facet joints provide a complementary role to the disc. Because of the oblique orientation of the facet processes, the external load is resisted by normal and shear forces in the joint. Thus, antero-posterior shear loading is supported.

E.C. Teo and H.W. Ng [18] Because the facet joints are related with pain-sensitive structures, any deviation from its normal physiologic function due to degenerative disorders or trauma may lead to facet-induced pain in the head, neck, and shoulders depending on the level at which integrity of the joint is compromised. Furthermore, due to posterior placement in the vertebra, it is a major stabilizing structure for other tissues in the region of the neck. Facet joints also act to limit the torsion of the disc. The coupling motion of the lower cervical spine in lateral bending and axial rotation is determined by the oblique orientation of the facet joint.

2-4-4 Uncovertebral Clefts

Located in the vertebral body-disc-vertebral body medium, from C2 to T1, are clefts (formerly referred to as Luschka's joints),[14]. The clefts are not formed at birth, and therefore, do not constitute joints. They arise late in childhood; become more evident in the young adult; and increase in size with advancing age, extending to meet in the midline to produce a transverse fissure across the back of the disc. They appear laterally in the anulus fibrosus between the uncinate processes of the lower vertebral body and medially in the saddle contour of the caudolateral aspect of the upper vertebral body. Along with the facet, they have been considered to be a frictionless deformation articulation.

2-5 Geometrical Model and Analysis

The objective of this section is to develop a simplified graphical 3D model appropriate for the evaluation and illustration of the proposed theoretical mechanical properties as indicated in the previous sections, including the forces transmission mechanisms and the kinematics of the cervical spine.

To create a geometrical model that relates to the positional and functional integrity of the cervical spine, while yielding to the configuration and functional aspects, a researcher must have a source of reliable data, such as frozen sections, x-rays, radiographs, MRIs, or other resources. In this project, the CAD model is based on a “life-size mannequin” also called “a highly detailed Spine plastic Model”, Figure 2-11. It was used in an effort to be as simple as possible, in terms of geometrical compilations such as: approaching complicated sections, contours and shapes to the nearest engineering determined figure. It is the intention that this kind of rounding will not affect the purpose of this study, since the model is to serve as a measurement task as well as to figure out vector directions and vertebra positions and orientations.

Numerical occupant models for simulations of dynamic behavior can be divided into two major groups; Multi Body System (MBS), which is the type of analysis, used in this project, and the Finite Elements model, which lack detailed knowledge about the properties of materials and involves long computer processing times.

The coordinate system and geometry, forces and moments convention based on Figure 2-1 and Figure 2-11, De Jager [21]. The range of angular movement of the head is based on data observed by [21] on volunteers. A general consensus on the span of movements is a range between 115° and 160° ; 40° for flexion and 105° for extension. According to the same reference, the initial position was defined to be as in Figure 2-12, 190 mm radius, and 40.8° sector in the sagittal plane. A detailed range of motion has been presented by [21], and tabulated here in Table 2-4.

Detailed Dimensions and configurations of the seven cervical vertebrae are shown in Figure 2-13: Figure 2-16. For more geometrical details, please check the CAD model. Note that reference dimensions are shown in millimeters.

Motion Deg.	C0- C1	C1- C2	C2- C3	C3- C4	C4- C5	C5- C6	C6- C7	C7- T1
One side lateral bending	5	5	10	11	11	8	7	4
Combined flexion/ extension	25	20	10	15	20	20	17	9
One side axial rotation	5	40	3	7	7	7	6	2

Table 2-4
The range of motion of cervical vertebrae, (angles in degree)

Figure 2-11
Cervical spine assembly, "Iso. view"

Figure 2-12
Cervical spine assembly, "Side view"

Figure 2-13
7th Cervical spine vertebra, Atlas vertebra

Figure 2-14
6th Cervical spine vertebra, Axis vertebra

Figure 2-15
2nd - 6th cervical spine vertebra

Figure 2-16
1st Cervical spine vertebra

2-5-1 Vertebra Positions and Center Of Motion

The center of motion is defined as the rotational center of the vertebrae, relative to the adjacent lower one. It is referred to as the “vertebra reference point”. This point has an important role in defining the relative axis in the next chapter.

Different locations have been given by different researchers in relation to flexion/extension [4], [5], [6] and [21]. However, De-Jager [21] formulated his decision utilizing X-Rays of the cervical spine of humans who held their heads in full extension. These centers of motion reflect the apparent vertebra rotational centers of motion from full extension to full flexion. The above references agree that for the lowest cervical vertebra C7, the center of motion is located in the middle of the upper endplate, even though it is fixed rigidly to the base. As for the rest of the vertebrae (C6-C5), the center of motion moves gradually downward and backward with each higher vertebra level. For example, C3-C2 has a rotational center that is located halfway down the C3 vertebra body and close to the posterior margin of dens near the transverse ligament, Figure 2-12. While the top, C1-C0, centers are off the average, and located above the occipital condyles.

2-6 Segments and Joints Geometric Modeling

The model of the cervical spine, as discussed earlier, consists of seven solid vertebrae and soft tissues represented by the ligaments, inter-vertebral joints & facets. For simplicity, ligaments are modeled as a spring line element (or more than one line), while the joint capsules are described as four single spring line elements around each facet joint. Keep in mind that ligaments resist only tensile forces, while the facet and the intervertebral disc have been modeled to be individual correspondent systems of spring and a dashpot, with six degrees of freedom. Figure 2-17 demonstrates two adjacent vertebrae and the ligaments that bind them.

Figure 2-17

One set of ligaments ;1- ALL: anterior longitudinal ligament; 2- PLL: posterior longitudinal ligament; 3- IL: The intertransverse ligament; 4- LF: ligamentum flavum; 5- ISL: Interspinous ligament.; A- capsule (the facet joint); B- intervertebral disk.

CHAPTER THREE

THE MATHEMATICAL MODEL

3-1 General Assumptions

Building a mathematical model of a musculoskeletal system starts by formulating a set of motion and loads governing equations, to represents the kinematics and mechanical behavior of the model, replicate displacements, internal and external loads. As for this study, the aim is to idealize the whole spinal structure to a vectorial mathematical model, starting by modeling the displacement vectors for the first couple of vertebrae, then expanding this model to the next vertebra, as a farther step, ending by the full number of vertebrae and final displacement vectors model, followed by determining the internal forces and stresses of the vertebra and ligament at any possible 3D position. Where, each vertebra considered having six degrees-of-freedom, allowing general three-dimensional motions. Some assumptions and consideration applied to the features of the model are as follows:

1. The model consists of any number of bodies and springs, connected to each other in any manner required to form an anatomically true model.
2. The bodies are three-dimensional rigid bodies and each may have all of the possible 6 degrees-of-freedom of motion. They could be at any position or orientation. No symmetry of any kind is required, even though in the anatomy of the human, a median plane divides the body into two mirrored parts.
3. Springs (ligaments) are connected in a pre-defined combination to the vertebra, they have to be able to accommodate more elements for more detailed model.
4. The system has an initial configuration, the initial position determined by the model analysis. This position will be taken as a reference where vertebra motions are relative to it.
5. External loads, static and dynamic, consisting of force and torque vectors, may be applied singularly or simultaneously to any number of the bodies.

6. Matrix methods are used throughout which make the method versatile and suitable for the computer.
7. The C7 vertebra is considered to have a zero absolute movements, i.e. it has neither linear nor rotational displacements,
8. Joints are the only guidance for the angular displacements.

3-2 Vertebra Reference Points

The reference point of vertebra C_i is a virtual point outside the body of the vertebra and represents the instantaneous center of rotation of vertebra C_i . The importance of defining reference point of vertebra is due to the fact that vertebra absolute displacements are the result of its relative displacement to previous vertebra plus the previous vertebra absolute displacements. In the next sections, reference points will be used and named as G_i to locate local coordinate systems and use in transformation axis.

3-3 Kinematics of Cervical Spine

3-3-1 Kinematics of Reference Point

Figure 3-1 is an example of the kinematics of reference point, which illustrate a 2D sketch of the 1st and 2nd vertebrae, G_6 is the reference points of vertebra C6, it shows the position of vertebra C6 rotates by $\vec{\psi}_6$, while C7 is rigidly fixed to the ground, follows assumption #7 in the last section. Absolute displacement vector for a point, P_6 , at C6 will be the vector sum due to C_6 rotation with respect to point G_6 , plus the displacement vector of vertebra C7:

$$\vec{S}_{P6} = \vec{S}_7 + f(\vec{\psi}_6, \vec{r}_{G6}) \quad (1)$$

Where

$\vec{S}_{P6}$: Displacement vector of point in P_6 ,
 $\vec{S}_7$: Displacement vector of G_7 ,

$\vec{\psi}_6$: Angular displacement vector of C6,
 $\vec{r}_{G6}$: Position vector from G₆ to point at P6.

Since $S_7=0$, Equation (1) becomes:

$$\vec{S}_{P6} = f(\vec{\psi}_6, \vec{r}_{G6}) \quad (2)$$

Adding next vertebra, C5, on the top of the previous setting, displacement of point P₅ at vertebra C5, is the vector summation of displacements resulted from the rotation around G₆, plus its own rotation around G₅. Figure 3-2 illustrate the above, it separates vertebra movement into two components, first is to disregard the rotation around G₅ while the second is to consider rotation around C5 only, adding these two components result to the final position.

Figure 3-2
Summation of position vectors

$$\vec{S}_{P5} = f(\vec{\psi}_6, \vec{r}_{56}) + f(\vec{\psi}_5, \vec{r}_{G5}) \quad (3)$$

Where

$\vec{S}_{P5}$: Displacement vector of point P5,
 $\vec{r}_{56}$: Position vector from G₆ to point P5,
 $\vec{\psi}_6, \vec{\psi}_5$: Angular displacement vector of C6, C5
 $\vec{r}_{G5}$: Position vector from G₅ to point P5.

Adding more vertebrae to the previous model leads to a general form which describes the displacement vectors at any point of the cervical vertebrae:

$$\vec{S}_{Pi} = f(\vec{\psi}_6, \vec{r}_{i6}) + f(\vec{\psi}_5, \vec{r}_{i5}) + \dots + f(\vec{\psi}_i, \vec{r}_{Gi}) \quad (4)$$

Where

- $\vec{S}_{Pi}$: Displacement vector of point at Ci,
- $\vec{r}_{i6}$: Position vector from G₆ to a point at Ci.
- $\vec{r}_{i5}$: Position vector from G₅ to a point at Ci.
- $\vec{\psi}_6, \vec{\psi}_5$ & $\vec{\psi}_i$: Angular displacement vector of C6, C5 & C_i
- $\vec{r}_{Gi}$: Position vector from G_i to a point at Ci.

Note that $\vec{r}_{56}, \vec{r}_{46}, \vec{r}_{45} \dots \vec{r}_{ij}$ denote the position vectors from the center of rotation of the “j” vertebra to a point on the “i” vertebra, where; “i” denote to the existing vertebra, “j” denotes the lower one which cause the additional portion of displacement.

Repeating the above procedure for the rest of the vertebrae and formulate the results in a matrix format:

$$\begin{bmatrix} \vec{S}_{Pi} \end{bmatrix} = f\left(\begin{bmatrix} \vec{\psi}_i \end{bmatrix}, \begin{bmatrix} \vec{r} \end{bmatrix}\right) \quad (5)$$

Where:

- $\begin{bmatrix} \vec{S}_{Pi} \end{bmatrix}$: Vertebra linear displacements matrix of point at Ci,
- $\begin{bmatrix} \vec{\psi}_i \end{bmatrix}$: Vertebra angular displacement
- $\begin{bmatrix} \vec{r} \end{bmatrix}$: position vectors, includes a combination of $\vec{r}_{ij}$ ’s and $\vec{r}_{Gi}$ ’s

Rewrite Equation (5) in expanded form:

$$\begin{Bmatrix} \vec{S}_{P7} \\ \vec{S}_{P6} \\ \vec{S}_{P5} \\ \vec{S}_{P4} \\ \vec{S}_{P3} \\ \vec{S}_{P2} \\ \vec{S}_{P1} \end{Bmatrix} = f\left(\begin{bmatrix} \vec{\psi}_7 & \vec{\psi}_6 & \vec{\psi}_5 & \vec{\psi}_4 & \vec{\psi}_3 & \vec{\psi}_2 & \vec{\psi}_1 \end{bmatrix}, \begin{bmatrix} \vec{r}_{G7} & 0 & 0 & 0 & 0 & 0 & 0 \\ \vec{r}_{67} & \vec{r}_{G6} & 0 & 0 & 0 & 0 & 0 \\ \vec{r}_{57} & \vec{r}_{56} & \vec{r}_{G5} & 0 & 0 & 0 & 0 \\ \vec{r}_{47} & \vec{r}_{46} & \vec{r}_{45} & \vec{r}_{G4} & 0 & 0 & 0 \\ \vec{r}_{37} & \vec{r}_{36} & \vec{r}_{35} & \vec{r}_{34} & \vec{r}_{G3} & 0 & 0 \\ \vec{r}_{27} & \vec{r}_{26} & \vec{r}_{25} & \vec{r}_{24} & \vec{r}_{23} & \vec{r}_{G2} & 0 \\ \vec{r}_{17} & \vec{r}_{16} & \vec{r}_{15} & \vec{r}_{14} & \vec{r}_{13} & \vec{r}_{12} & \vec{r}_{G1} \end{bmatrix}^T \right) \quad (6)$$

Note that the $\begin{bmatrix} \vec{\psi}_i \end{bmatrix}$ matrix is a square 7 X 7 matrix with seven identical rows. $\vec{\psi}_7$ is equal to 0 as per assumption #7. However, it will be kept in the matrix for future work to include lower vertebrae.

The general solution of Equations (6) is not feasible if it is treated the way shown, since the $\vec{r}_{ij}$ matrix is related and driven from $\begin{bmatrix} \vec{\psi}_i \end{bmatrix}$. In other words, $\vec{r}_{ij}$ matrix doesn't contain constant elements, since vector directions change due to the movements of the spine.

Therefore, $\vec{r}_{ij}$ matrix has to be reconstructed in order to have constant coefficients. A typical solution to this type of problem could be achieved by introducing new sets of local axes, one set for each vertebra, and relate them together, by using a system of transformation matrices which will relate the moving axes together.

3-3-2 Coordinate Systems and Vector Transformation [3]

Vector transformation in kinematics is the process of setting a relationship between the elements of two coordinate systems in which they translate and/or rotate relative to each other. For this study, each mobile vertebra will be connected to an individual coordinate system, called the local coordinate system, in which the elements of the local $\begin{bmatrix} \vec{r} \end{bmatrix}$ matrix for a vertebra relative to its local coordinate system will be

constants. The next step is to transfer the $\begin{bmatrix} \vec{r} \end{bmatrix}$ matrix associated with its projection to the next local coordinate system located in the lower vertebra, this process neither changes

the magnitude of the $\begin{bmatrix} \vec{r} \\ r \end{bmatrix}$ vectors, in fact it will be considered to be independent values from the whole process, nor the direction. However, it will change the way of describing them according to the relative coordinate systems.

The first step in this procedure is determining the Global “fixed” Cartesian system, the (x, y, z) coordinate system. Its origin will be located at the reference point of C7, and it will be considered to be fixed relative to the frame holds C7. The second step is to determine local “mobile” coordinate systems. Each of them is rigidly fixed to a vertebra reference point, translated and rotated relative to the global coordinate system.

The procedure of transformation of the axis in 3D space is well explained and developed in analytical geometry handbooks and the literature; however, a summarized illustration will be considered as an introduction to this mathematical model. For the two systems shown in Figure 3-3, the position vector $\vec{P}$ is defined in the 3D space, associated with two different Cartesian coordinate systems, sharing one common point of origin, 1st system named a “0” system” or the (x₀, y₀, z₀) system, and system “1” or (x₁, y₁, z₁) system. Vector $\vec{P}$ is expressed in term of a unit vector in either frame, i₀, j₀ and k₀ or i₁, j₁ and k₁. Therefore:

$$\vec{P} = P_{x0} \cdot i_0 + P_{y0} \cdot j_0 + P_{z0} \cdot k_0 = P_{x1} \cdot i_1 + P_{y1} \cdot j_1 + P_{z1} \cdot k_1 \quad (7)$$

In order to determine one representation for $\vec{P}$ in terms of other, the following inner product would solve it:

$$\begin{aligned} P_{x0} &= (P_{x1} \cdot i_1 + P_{y1} \cdot j_1 + P_{z1} \cdot k_1) \cdot i_0 \\ P_{y0} &= (P_{x1} \cdot i_1 + P_{y1} \cdot j_1 + P_{z1} \cdot k_1) \cdot j_0 \\ P_{z0} &= (P_{x1} \cdot i_1 + P_{y1} \cdot j_1 + P_{z1} \cdot k_1) \cdot k_0 \end{aligned} \quad (8)$$

Equation (8) could be written in a matrix form as:

$$\begin{bmatrix} P_{x0} \\ P_{y0} \\ P_{z0} \end{bmatrix} = \begin{bmatrix} i_1 \cdot i_0 & j_1 \cdot i_0 & k_1 \cdot i_0 \\ i_1 \cdot j_0 & j_1 \cdot j_0 & k_1 \cdot j_0 \\ i_1 \cdot k_0 & j_1 \cdot k_0 & k_1 \cdot k_0 \end{bmatrix} \begin{bmatrix} P_{x1} \\ P_{y1} \\ P_{z1} \end{bmatrix}$$

Or simply:

$$P_o = R_0^1 P_1 \quad (9)$$

Equation (9) could be re-written as:

$$P_1 = R_1^0 P_0 \quad (10)$$

Where:

$$(R_1^0)^{-1} = R_1^0$$

In the same manner, transformation of axis could be extend to more than one step, i.e. a first coordinate system transfer to a second one then to third and fourth, etc. This is achieved by determining the inner product of the systems transformation matrices.

The general form of the transformation matrix of multi step will look like Equation (11):

$$R_q^k = \begin{bmatrix} i_k \cdot i_q & j_k \cdot i_q & k_k \cdot i_q \\ i_k \cdot j_q & j_k \cdot j_q & k_k \cdot j_q \\ i_k \cdot k_q & j_k \cdot k_q & k_k \cdot k_q \end{bmatrix} = R_q^{q+1} \cdot R_{q+1}^{q+2} \dots R_{k-1}^k \quad (11)$$

In other words, Equation (11) implies a succession of coordinate transformations, each represented by a particular transformation matrix, which can be represented by the matrix product or “composition” of individual transformation matrices. This principle will be very useful in developing a systematic procedure to determine the kinematics relationship associated with specific vertebra motion.

For this project, it will be assume that the heads travels from one point to another causing the cervical vertebras to translate and rotate. In order to build a systematic model, a convention has to be set up for the proceeding work. The project will always consider the rotation of any individual vertebra starts by rotating on its own x_i -axis by an angle of θ_i , to be followed by rotation along the y_i -axis with the amount of β_i and finally the rotation along z_i -axis by the angle γ_i . Hence the transformation matrix between two consecutive vertebras will be:

$$R_j^i = \begin{bmatrix} \cos \theta \cdot \cos \beta \cdot \cos \gamma - \sin \theta \cdot \sin \gamma & -\cos \theta \cdot \cos \beta \cdot \sin \gamma - \sin \theta \cdot \cos \gamma & \cos \theta \cdot \sin \beta \\ \sin \theta \cdot \cos \beta \cdot \cos \gamma + \cos \theta \cdot \sin \gamma & -\sin \theta \cdot \cos \beta \cdot \sin \gamma + \cos \theta \cdot \cos \gamma & \sin \theta \cdot \sin \beta \\ -\sin \beta \cdot \cos \gamma & \sin \beta \cdot \sin \gamma & \cos \beta \end{bmatrix} \quad (12)$$

Where:

R_j^i : is the transformation matrix from vertebra C_i to the lower one C_j or C_{i+1}

3-3-3 Position Vector of a Point at the Cervical Spine

The aim of this section is to develop a systematic method to determine the positions of any point in the cervical vertebra, which includes all possibilities of head locations, i.e. with all possibilities of the translation and rotation of the vertebra. To illustrate this, consider the situation in Figure 3-4, the global coordinate system located at point G6; where G6 is a common point representing the reference point of C6 and it could be considered to be part of C7 which has zero rotation and translation. The first system of the local coordinate systems, (lets call it frame 5), located at the same point, G6, but it's attached to vertebra C6, while frame 5 is fixed to vertebra C5, located at G5.

Figure 3-4

A- first three cervical vertebra in the initial position

B- first three cervical vertebra in an arbitrary position

Figure 3-4 (A) depicts the first three vertebra in the cervical spine in their absolute vertical position, which might never occur in real life. However, for this project it will be assumed as the initial position of the head, and will be considered to be the reference position for all other positions.

The position of an arbitrary point P_5 or P_i , could be defined by a position vector from its vertebra reference point. This position vector will keep same magnitude and direction relative to its vertebra local coordinate system. This will also be true for the position vectors that connect consequent reference points. For example, $\vec{r}_{56}$, will keep the same magnitude and direction relative to frame 6.

From the above observations and the formulas from the previous section the position vector of point P_7 at vertebra C7 (P_7 not shown in Figure 3-4) relative to the global coordinate system will be:

$$\vec{P}_7 = \vec{P}_{7i} = P_{7x} \cdot i + P_{7y} \cdot j + P_{7z} \cdot k \quad (13)$$

Where:

$\vec{P}_7$: Position relative to global coordinate system

$\vec{P}_{7i}$: Position relative to the local coordinate system

This special case is simply represents the absolute position of it, as for the scope of this project, points on C7 will never change its positions. While the position of point P_6 at vertebra C6 (P_6 not shown in Figure 3-4) relative to the global coordinate system will be:

$$\vec{P}_6 = R_7^6 \cdot \vec{P}_{6i} \quad (14)$$

Where:

$\vec{P}_6$: Position relative to global coordinate system

$\vec{P}_{6i}$: Position relative to the local coordinate system (x_6, y_6, z_6)

R_7^6 : Transformation matrix from the local coordinates system (x_6, y_6, z_6) to the global coordinate system

The position of point P_5 at vertebra C5 relative to the global coordinate system will be, as in Figure 3-4-B:

$$\vec{P}_5 = R_7^6 \cdot \vec{r}_{56} + R_7^5 \vec{P}_{5i} \quad (15)$$

Where:

$\vec{P}_5$: Position relative to global coordinate system

$\vec{P}_{5i}$: Position relative vector to the local coordinate system (x_5, y_5, z_5)

$\vec{r}_{56}$: Position vector relative to the local coordinate system (x_6, y_6, z_6), from G_6 to G_5

R_7^6 : Transformation matrix from local coordinates system (x_6, y_6, z_6) to global coordinate system

R_7^5 : Transformation matrix from the local coordinates system (x_5, y_5, z_5) to the global coordinate system

$$\text{Where: } R_7^5 = R_7^6 \cdot R_6^5$$

The position of point P_4 at vertebra C_4 relative to the world coordinate system will be:

$$\vec{P}_4 = R_7^6 \cdot \vec{r}_{56} + R_7^5 \vec{r}_{45} + R_7^4 \vec{P}_{4i} \quad (16)$$

Where:

$\vec{P}_4$: Position relative to global coordinate system

$\vec{P}_{4i}$: Position relative to the local coordinate system (x_4, y_4, z_4)

$\vec{r}_{45}$: Position vector relative to the local coordinate system (x_5, y_5, z_5) from G_5 to G_4

R_7^6 : Transformation matrix from the local coordinates system (x_6, y_6, z_6) to the global coordinate system

R_7^5 : Transformation matrix from the local coordinates system (x_5, y_5, z_5) to the global coordinate system

R_7^4 : Transformation matrix from (x_4, y_4, z_4) coordinates system to global coordinate system

$$\text{Where: } R_7^4 = R_7^6 \cdot R_6^5 \cdot R_5^4$$

The position of point P_i at vertebra C_i relative to the world coordinate system will be:

$$\vec{P}_i = R_7^6 \cdot \vec{r}_{56} + R_7^5 \vec{r}_{45} + \dots + R_7^i \vec{P}_{ii} \quad (17)$$

Where:

$\vec{P}_i$: Position relative to global coordinate system

$\vec{P}_{ii}$: Position relative to the local coordinate system (x_i, y_i, z_i)

R_7^6 : Transformation matrix from the local coordinates system (x_6, y_6, z_6) to the global coordinate system

R_7^5 : Transformation matrix from the local coordinates system (x_5, y_5, z_5) to the global coordinate system

R_7^i : Transformation matrix from (x_i, y_i, z_i) coordinates system to global coordinate system

Where: $R_7^4 = R_7^6 . R_6^5 . \dots . R_{i+1}^i$

Finally, Summarizing position vectors for a point of cervical vertebrae as follow:

$$\begin{aligned}
 \vec{P}_7 &= \vec{P}_{7i} \\
 \vec{P}_6 &= R_7^6 . \vec{P}_{6i} \\
 \vec{P}_5 &= R_7^6 . \vec{r}_{56} + R_7^5 . \vec{P}_{5i} \\
 \vec{P}_4 &= R_7^6 . \vec{r}_{56} + R_7^5 . \vec{r}_{45} + R_7^4 . \vec{P}_{4i} \\
 \vec{P}_3 &= R_7^6 . \vec{r}_{56} + R_7^5 . \vec{r}_{45} + R_7^4 . \vec{r}_{34} + R_7^3 . \vec{P}_{3i} \\
 \vec{P}_2 &= R_7^6 . \vec{r}_{56} + R_7^5 . \vec{r}_{45} + R_7^4 . \vec{r}_{34} + R_7^3 . \vec{r}_{23} + R_7^2 . \vec{P}_{2i} \\
 \vec{P}_1 &= R_7^6 . \vec{r}_{56} + R_7^5 . \vec{r}_{45} + R_7^4 . \vec{r}_{34} + R_7^3 . \vec{r}_{23} + R_7^2 . \vec{r}_{12} + R_7^1 . \vec{P}_{1i}
 \end{aligned} \tag{18}$$

Or in matrix form:

$$\begin{Bmatrix} \vec{P}_7 \\ \vec{P}_6 \\ \vec{P}_5 \\ \vec{P}_4 \\ \vec{P}_3 \\ \vec{P}_2 \\ \vec{P}_1 \end{Bmatrix} = \begin{bmatrix} \vec{P}_{7i} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \vec{P}_{6i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \vec{r}_{56} & \vec{P}_{5i} & 0 & 0 & 0 & 0 \\ 0 & \vec{r}_{56} & \vec{r}_{45} & \vec{P}_{4i} & 0 & 0 & 0 \\ 0 & \vec{r}_{56} & \vec{r}_{45} & \vec{r}_{34} & \vec{P}_{3i} & 0 & 0 \\ 0 & \vec{r}_{56} & \vec{r}_{45} & \vec{r}_{34} & \vec{r}_{23} & \vec{P}_{2i} & 0 \\ 0 & \vec{r}_{56} & \vec{r}_{45} & \vec{r}_{34} & \vec{r}_{23} & \vec{r}_{12} & \vec{P}_{1i} \end{bmatrix} \cdot \begin{Bmatrix} 1 \\ R_7^6 \\ R_7^5 \\ R_7^4 \\ R_7^3 \\ R_7^2 \\ R_7^1 \end{Bmatrix} \tag{19}$$

3-4 External and Internal Cervical Spine Forces

The cervical spine system withstands two types of loads: the *internal forces*, created by the ligaments, intervertebral and joints, and the *external forces*, created by the action of muscles, head weight, G-forces, and other forces created by the external environment of the neck. This study will focus on the internal forces, considered as the reaction to these external ones.

The first step in solving a cervical spine equation system is to break down the system into seven sub-systems and deal with each vertebra individually. Each of these sub-systems is considered to be under static equilibrium to which Newton's First Law is applied. Finding the solution to the above sub-systems will determine the vertebra rotating angles (vectors) around its own reference point. The position vectors of the soft tissues will be obtained by locating the points of interest, tissue connection points with the vertebra body, relative to the global coordinate system by the aid of Equation (19). To illustrate the above a simplified 2D mathematical model has to be solved such as the one shown in Figure 3-5 which depicts the first three cervical vertebrae. This simplified model will be expanded to cover the whole seven cervical vertebrae in three dimensional motion. "F" is the only external force, "k's" represent connection ligament constants. Figure 3-5 (A) shows the initial position while Figure 3-5 (B) illustrates the final position.

Figure 3-5

The first three cervical vertebrae, k_i is ligaments spring constants; C_{ij} is connection points of ligaments to vertebrae, "F" external force

The reaction force on ligament “ l_i ” at vertebra (i) is the product of the ligament constant “ k_i ” and its length difference. The external force “ F ” applied while the system is in its initial position. The direction of ligament reaction will be the vector along the ligament “ l_i ” in its initial direction; however, the magnitude of the length difference will be the magnitude of the final position vector subtracted from the magnitude of the initial position. The product of the ligament constant “ k_i ” by the inner product of the unit vector of the final position and the deflection vector results in the ligament reaction force vector. Therefore, from in Figure 3-6:-

$$\vec{F}_l = k_i \cdot \vec{\delta} \quad (20)$$

Where:

$\vec{F}_l$: Ligament reaction force vector

k : Ligament constant

$\vec{\delta}$: Ligament length difference

$$\vec{\delta} = \left[\left| \vec{C'_i C'_j} \right| - \left| \vec{C_i C_j} \right| \right] \cdot \vec{u_{c'_i c'_j}} \quad (21)$$

Where:

$\left| \vec{C_i C_j} \right|$: Initial magnitude of the ligament length vector, identified by the initial position from the CAD file directly,

$\left| \vec{C'_i C'_j} \right|$: Final magnitude of the ligament vector, need farther calculation,

$\vec{u_{c'_i c'_j}}$: Unit vector of final ligament position vector, calculated by the code.

$$\vec{C'_i C'_j} = \vec{GC'_i} - \vec{GC'_j} \quad (22)$$

From equation (19):

$$\vec{GC'_i} = R_7^6 \cdot \vec{r_{56}} + \dots + R_7^{j+1} \cdot G_{j+1} \vec{G_j} + R_7^j \cdot G_j \vec{G_i} + R_7^i \cdot G_i \vec{C_i} \quad (23)$$

$$\vec{GC'_j} = R_7^6 \cdot \vec{r_{56}} + \dots + R_7^{j+1} \cdot G_{j+1} \vec{G_j} + R_7^j \cdot G_j \vec{C_j} \quad (24)$$

Figure 3-6
Vectors relative to the global coordinate system

Substitute Equation (24) in Equation (22), yields;

$$\vec{C_i C_j} = R_7^6 \cdot \vec{r_{56}} + \dots + R_7^{j+1} \cdot \vec{G_{j+1} G_j} + R_7^j \cdot \vec{G_j G_i} + R_7^i \cdot \vec{G_i C_i} - R_7^6 \cdot \vec{r_{56}} + \dots + R_7^{j+1} \cdot \vec{G_{j+1} G_j} + R_7^j \cdot \vec{G_j C_j}$$

$$\vec{C_i C_j} = R_7^j \cdot \vec{G_j G_i} + R_7^i \cdot \vec{G_i C_i} - R_7^j \cdot \vec{G_j C_j} \quad (25)$$

Next substitute Equation (25) in Equation (21);

$$\vec{\delta} = \{ | R_7^j \cdot \vec{G_j G_i} + R_7^i \cdot \vec{G_i C_i} - R_7^j \cdot \vec{G_j C_j} | - | \vec{C_i C_j} | \} \cdot \vec{u_{C_i C_j}}$$

$$\vec{F_i} = k_i \cdot \{ | R_7^j \cdot \vec{G_j G_i} + R_7^i \cdot \vec{G_i C_i} - R_7^j \cdot \vec{G_j C_j} | - | \vec{C_i C_j} | \} \cdot \vec{u_{C_i C_j}} \quad (26)$$

All forces on one vertebra in a matrix form will be:

$$\left\{ \vec{F_i} \right\} = [k_i] \begin{pmatrix} \vec{\delta_1} & \vec{\delta_2} & \dots & \vec{\delta_i} \end{pmatrix}^T \quad (27)$$

$\vec{F_i}$: The force vector on ligament i , with respect to the global coordinate system.

The outcome of Equation (26) represents an individual ligament reaction, while the resultant of the external force will be the summation of all ligaments reactions. Some rules have to be applied during the programming stage. Equation (26) is limited to measure reactions at or below C_i , when $i=j+1$, or in other terms when C_i is located on top of C_j . Also, all negative deflections resulting from Equation (25), have to be ignored and equated to zero, since the ligaments react to tension only, and to not react to compression. The above procedure is repeated to measure actions relative to the global coordinate systems of vertebrae C_i first, while the vectorial sum of one set represents the resultant force exerted on one vertebra, the sum of these results is equal to the static external force.

Similar to Equation (20) $\vec{F_{ci}}$;

$$\vec{F_{Ci}} = \sum \vec{F_i} \quad (28)$$

Where:

$\vec{F_{Ci}}$: The resultant force on C_i , with respect to the global coordinate system.

$$\vec{F} = \sum \vec{F_{Ci}} \quad (29)$$

$\vec{F}$: The resultant force the cervical spine, with respect to the global coordinate system.

CHAPTER FOUR

THE NUMERICAL SOLUTION OF THE CERVICAL SPINE

3

This chapter will integrate the mathematical model from Chapter Three and the physical properties from Chapter Two to develop a numerical solution for the cervical spine. Since there is no commercial computer packages to solve this type of modeling directly, implementing a custom code was among this project's tasks. A MAT-LAB[™] package was selected to develop a solution for a general case study.

Programming in MAT-LAB[™] is not that different from any other package code writing. They all share the same objective to solve a set of mathematical equations and present solutions in some form, and they all demand input data and a logical sequence of ordering equations. Basically, the structure of the code should follow a specific flow chart. The 1st section of this chapter will highlight the details of that, while the 2nd part will explain the data used for the input. The last part presents the outputs of a specific case, to be treated as an example demonstrating the entire project.

4-1 Flow Chart and Problem Definition:

Equations (21), (26) and (27) from the third chapter served two main functions: First to define the kinematics of vertebrae; finding positions of all points located on the cervical spine when vertebrae are moving to any arbitrary position. Secondly, to define the kinetics of the vertebra and deal with forces and stresses upon the elements of the cervical spine, starting with the force and stress applied on each individual ligaments and ending by the finding resultant force applied on each vertebra, and the resultant force vector applied on the whole cervical spine which causes the corresponding deformation (movement) of the cervical spine. The flow chart in Figure 4-1 briefly illustrates the flow chart of building the code, while the subsequent parts explain it in more detail.

Figure 4-1
Flow chart of the code used in solving the cervical spine for this project,

Step 1: Inputs

As mentioned above, the first step in writing a code is to identify constants and data input. In this study, inputs were either constants or variables changed by the case or rotational angles classified to be the variable inputs. Users of the code should be careful inputting variable data at this stage. A certain sequence must be followed when filling out the matrix of angles. Users must start with the rotation of vertebra C7, along the x-axes, followed by the rotation of the same vertebra along the y-axis, then rotation along the z-axis. Only then can users continue to the next row and input information on vertebra C6. Likewise the same sequence must be followed as with the previous vertebra. In the same manner, the subsequent rows will be filled with the right angles, ending by the seventh row for C₁. However, users should keep in mind, this sequence doesn't necessarily depict the actual vertebra rotation to reach its final position, but it does determine the final position of the cervical spine and the head regardless of which path the vertebra took, i.e., if vertebra C_i travels from position A to position B by following a certain path, it is not of the project's interest to deal with that path. In fact, the aim is to study position B itself, and to relate position B into three rotational angles around the local xyz-axes, without loosing the above condition of sequence.

Aside from the variable inputs are the constant inputs, such as initial position vectors and cross sectional areas of each ligament, the geometry and rotation centers of vertebra, spring constants, etc. All geometrical information has been obtained from the CAD model presented in chapter two, while the rest of the information has been obtained from previous studies by other researcher in this field, as demonstrated in Chapter Two.

Step 2: Calculation of Matrix Transformation

Final positions, or B positions, of each vertebra have been determined by three angles. In a certain sequence, each of these sets of angles is considered relative to a lower level of the local coordinate system. For instance, position B of vertebra "n", is the rotation of vertebra "n" around its center of rotation, relative to vertebra "n-1", while vertebra "n-1" set of rotational angles is taken relative to vertebra "n-2", and so on. Since the cervical spine consists of seven vertebrae, this will lead to seven sets of local axis and one global axis. The code in Step 2 deals with this issue and transfers all local rotations to the one global axis, following the set of the algebraic equations in chapter three.

Step 3: Solving Ligaments between Vertebra C7 and Vertebra C6

This step will be divided into sub-steps, targeting the ligament between vertebra C₇ and C₆. It starts with identifying the position vectors of the ligaments, ending by calculating forces, stresses and final resultant force vector at that increment of the cervical spine. Figure 4-1 illustrates a flow chart summarizing the following steps.

- *Sub-Step A: determine ligaments position vectors at position A: -*

In this stage of programming, the position vector of each individual ligament will be tabulated, considering the cervical spine at position A, which is assumed to be the neutral position. While the force vector due to the extension of ligaments equals zero, basically it is considered to be the absolute relaxation of the ligament in the neck. This step is obtained directly from the CAD file, it could be considered as part of the constant input.

- *Sub-Step B: determine ligaments lengths at position A:-*

The magnitude of each position vector is equal to the length of the corresponding ligament, while it is in the relaxed position, position A.

- *Sub-Step C: determine ligaments position vectors at position B:-*

It has a similar nature to sub-step A; however, it will deal with the transformation matrices from Step 2, since these vectors depict position vectors relative to the global coordinate system.

- *Sub-Step D: determine ligaments lengths at position B:-*

The magnitude of each position vector at Position B is different from the one obtained from sub-step B, due to the extension which occurs from the relative movement between the ends of the individual ligament. The difference will represent the deflection in the ligament.

- *Sub-Step E: determine ligaments unit vectors at position B ;*

Dividing each position vector along the ligament at position B by its length, which resulted in its unit vector, determines the direction of the ligament at its final position, position B.

- *Sub-Step F: determine ligaments deflection;*

The difference between Sub-Step D and Sub-Step B. At this point there is no need to use the direction of the deflection; however, this will take place in a later sub-step.

- *Sub-Step G: determine ligaments deflection sign;*

Determine whether the ligaments are under tension or relaxation. The code should ignore the ones under relaxation condition, since they have no effect on producing any type of force. If the magnitude of the deflection is positive, the code will keep the value to be used for the next steps; otherwise, it will be eliminated and given the value of zero. This is because of the fact that ligaments work in a similar way to an elastic band; it resists tension but does nothing against contraction.

- *Sub-Step H: determine ligament force magnitude & direction;*

Multiplying the result from sub-step G by corresponding results from sub-step E results in a position vector of the deflection. Force vectors due to the extension of the ligament calculated by multiplying results from sub-step G by the ligament constants “k” from step 1, force magnitude from each individual extended ligament obtained by getting the magnitude of force vectors. These values are important for finding stresses in the next sub-step.

- *Sub-Step I: stress on ligaments:*

The division of the second set of results from sub-step H by the corresponding cross sectional area, obtained from the inputs from Step 1 or each element (ligament) which resulted in stress on each ligament. Although the results are dimensionless, stresses calculated are tensile stresses along the position vector of the ligament in the final position, position B.

- *Sub-Step J: calculate resultant force at the specific increments;*

Simply summing rows in the matrix resulting from the first set of results at sub-step H yield to the resultant force vector.

Step 4: Solving The Ligaments Between Vertebra C6 And Vertebra C5

This step repeats the previous step, except input data corresponds to different increment for ligaments between vertebra C6 and vertebra C5.

Step 5: Solving The Ligaments Between Vertebra C5 And Vertebra C4

This step repeats the previous step, except input data corresponds to different increment, for ligaments between vertebra C5 and vertebra C4.

Step 6: Solving The Ligaments Between Vertebra C4 And Vertebra C3

This step repeats the previous step, except input data corresponds to different increment, for ligaments between vertebra C4 and vertebra C3.

Step 7: Solving The Ligaments Between Vertebra C3 And Vertebra C2

This step repeats the previous step, except input data corresponds to different increments for ligaments between vertebra C3 and vertebra C2.

Step 8: Solving The Ligaments Between Vertebra C2 And Vertebra C1

This step repeats the previous step, except input data corresponds to different increment, for ligaments between vertebra C2 and vertebra C1.

Step 9: Finding Result Force Vector

The final result force vector is the result of adding corresponding elements from the first set of results at sub-step H from steps 3.

By running the above, the proposed model of the cervical spine is solved. Both the model and the program are built to be flexible to any future additions and improvement. For instance, it isn't that big of an effort to add muscles, facets or joints, providing that all data is numerically available.

4-2 Inputs Data

This section will deal with two forms of input, constant input, and variable input. Constant input is either gathered from the last chapter, relying on previous research, or is obtained from the 3-D SOLID WORK™ CAD model. Ligament cross-section area and ligament stiffness data are collected from tables 2-2, 2-3 and 2-4. Position vectors between vertebrae, centers of rotation, or local coordinate systems are obtained from the CAD, as in the matrix shown in Figure 4-2. It illustrates the position vectors from G_j to G_i , (Figure 3-6). The 1st row represents the vector component in the x-axis direction (i 's), the 2nd row for the y direction (j 's), and the 3rd for the z direction (k 's). The Table in Figure 4-2 is 3-1, referred to as the $G_i G_j$ matrix, since it represent the position vectors

	G7 G6	G6 G5	G5 G4	G4 G3	G3 G2	G2 G1	G1 G0
$G_i G_j$	000 6.7509 17.694	00 5.5235 18.1631	00.00 4.2283 18.4475	00.00 2.3133 18.7792	0000 1.5233 28.5265	00 2.4701 28.2337	-- -- --

Figure 4-2
Local coordinate system initial positions

between G_i and G_j . Note that G_j is lower than G_i in level. For example, the 2nd column in this table represents the vector from the local coordinate of the 6th vertebra (C6) to the local coordinate system of the 5th vertebra (C5).

The position vector of the ligament obtained from the CAD model, in its initial position, is position "A". Standardizing work for this particular project was done by deciding the directions of local coordinate system, where ligaments start at the top faces of the vertebra C_j ending at the bottom of the next upper vertebra C_i , i.e, the ligament vectors always indexing to the top of the cervical spine or human head. Ligament abbreviations to be used in this project follows Table 4-1.

Left anterior longitudinal ligament =	ALL-L
Center anterior longitudinal ligament =	ALL-C
Right anterior longitudinal ligament =	ALL-R
Left Interspinous ligaments=	ISL- L
Center Interspinous ligaments=	ISL- C
Right Interspinous ligaments=	ISL- R
Left posterior longitudinal Ligaments=	PLL-L
Left outer posterior longitudinal Ligaments=	PLL-OL
Center posterior longitudinal Ligaments=	PLL-C
Right outer posterior longitudinal Ligaments=	PLL-OR
Right posterior longitudinal Ligaments=	PLL-R
Left intertransverse ligament =	IL-L
Right intertransverse ligament =	IL-R
Left outer flavum ligament =	FL-OL
Left anterior flavum ligament =	FL-IL
Centre outer flavum ligament =	FL-OC
Centre anterior flavum ligament =	FL-IC
Right anterior flavum ligament =	FL-IR
Right outer flavum ligament =	FL-OR
Left Alar ligament =	AL-L
Right Alar ligament =	AL-R
Left Transverse atlantal ligament=	TL-L
Right Transverse atlantal ligament=	TL-R

Table 4-1
Ligaments abbreviations

The MAT-LAB™ code is ready to be implemented then tested and verified at this stage of work. All needed data and information are available to solve the mathematical equations from Chapter Three utilizing the above procedure and data. Units used in this project follow the ISO standard; the stiffness of individual ligament is in N/mm². The cross sectional areas are in mm² and the position vectors relative to the local axis are built into the code and become part of it. However, in order to run the code each time in a different case, the variable inputs have to be fed as described in step 2; the variable inputs will be identified by filling the rotational angle matrix of each individual vertebra.

4-3 Case Study #1

The code in the previous section was written to analyze the cervical spine in any applicable case; however, this section and the following one will discuss and analyze two specific extreme cases. The first cases analyzes the cervical spine when the head attempts to rotate to the left 72 degrees, assuming the subject is rotating his/her head to the extreme side, keeping the shoulder stationary. 72 degrees is the summation of the average of vertebra maximum side axial rotation along the z-axis. Figure 4-3 shows the 2 positions of the cervical spine. Position *A* is the initial position while position *B* is the maximum axial rotation. As mentioned previously and with the aid of Table 2-6, the variable input of the rotation angle matrix will be;

$$\text{Rotation angle} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 6 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 40 \end{bmatrix}$$

Figure 4-3A
Axial rotation, Left: top view of position "B", Right: top view of position "A"

Figure 4-3 B
case study #1; Axial rotation , Left: position “B”, Right: position “A” , ,

The detailed model is the most useful for studying the injury mechanism because deformation and load of the individual soft tissue can be assessed. Figure 4-4 (A), illustrates the top projection of the right cervical anterior longitudinal ligament, ALL set in the original position of the cervical spine and in the extreme Axial rotation. The light weight lines represent the original position while the heavy lines represent the final position. The matrix below the figure implies the exact deflection for that state. For example, the maximum deflection occurs between C2-C1, which is equal to 5.4879 mm, while the lowest deflection is between C3-C2 equal to 0.0347 mm. The amount of the elongation considered to be zero is between C7-C6 since there is no ALL there. The same is true for Figure 4-4 b, which shows the top projection of the central cervical anterior longitudinal ligament, where maximum deflection occurs between C2-C1 which is equal to 3.2762 mm, while the lowest deflection is between C3-C2 equal to 0.0431 mm. Note that ALL between C7-C6 has a value since it is existing and stretched 0.3244 mm. Figure 4-4 (C), shows a similar pattern. With the top projection of the right cervical anterior flavum ligament, maximum deflection between C4-C3 2.2065 mm, and minimum between C3-C2 1.0343 mm. Note that there is no FL-IR between C2-C1. In the same manner Figure 4-4 (D), Figure 4-4 (E), Figure 4-4 (F), Figure 4-4 (G) and Figure 4-4 (H), illustrate top projected views of both deflected and original length of the right outer flavum ligament, right interspinous ligaments, central interspinous ligaments and left interspinous ligaments. Note that they all illustrate ligaments between six vertebrae except for Figure 4-4 (E) , which represents the flavum between C7-C6 only.

Figure 4-4 A

Ligaments deflection and final position for ALL-R, Light weight lines represent initial position, heavy lines represent final position , Matrix tabulate the final magnitude for ALL-R lengths

Figure 4-4 B

Ligaments deflection and final position for ALL-C, Light weight lines represent initial position, heavy lines represent final position , Matrix tabulate the final magnitude for ALL-C lengths

Figure 4-4 C

Ligaments deflection and final position for FL-IR, Light-weight lines represent initial position, heavy lines represent final position , Matrix tabulates the final magnitude for FL-IR lengths.

Figure 4-4 D

Ligaments deflection and final position for FL-OR, Light-weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for FL-OR lengths

Figure 4-4 E

Ligaments deflection and final position for FL for C6-C7, Light-weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for FL for C6-C7lengths

Figure 4-4 F

Ligaments deflection and final position for ISL-R, Light-weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for ISL-R lengths

Figure 4-4 G

Ligaments deflection and final position for ISL-C, Light-weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for ISL-C lengths

Figure 4-4 H

Ligaments deflection and final position for ISL-L, Light weight lines represent initial position, heavy lines represent final position , Matrix tabulate the final magnitude for ISL-L lengths

As mentioned previously, the main reason for this study is to develop a methodical model for the prediction and control of spinal injuries or, in less extreme, the cause of pain sustained during a verity of human body movements. Case One and Case Two represent one of these extreme actions of the human neck. However, tissue and vertebrae reach a limit before injury happens. This limit is normally translated by the human brain as a gradual pain progressing from mild and ending to extreme. Regarding the ligaments, obviously the more stress suffered, the more potential damage might occur. Pain is sustained at this point; therefore the force exerted on vertebra is the main factor, as depicted in Figure 4-5. This figure illustrates the magnitude of the results force vector exerted between vertebrae. For example, C2-C1 bears a huge amount of force, 76.6 N, while the next set of vertebra, C3-C2 bears the least amount of force, at 3.9 N. The average of the next three sets is approximately 11.8 N and the last set, C7-C6 exerts only 5.8 N. The deflection of ligaments plays a major role affecting force magnitudes; this can be seen in Figure 4-4, where the set of C7-C6 always has the highest value. Configuration of vertebra affects the ligaments displacements too.

	C7-C6	C6-C5	C5-C4	C4-C3	C3-C2	C2-C1
Force magnitude (N)	5.8404	11.4810	11.9513	12.0132	3.9085	76.5862

Figure 4-5
Magnitude of total force on vertebrae

4-3 Case Study #2

The second case study in this project was to analyze the cervical spine with the head in the position of its maximum flexion, the subject is looking at his/her chest. Refer to Figure 2-1 and Table 2-1. Following the same procedure as in Case Study #1, assuming that the information in Table 2-4 represents half of the full movement span of an average person along the X-axis. Therefore, only half of this span is considered to be the distance from the initial A position to the final B position, which accumulates a 55.5 degree counterclockwise angle on the X-axis, as in Figure 4-6. Hence, the matrix of the input will be:

$$\begin{matrix}
 \circ \\
 ROTATION - ANGLE =
 \end{matrix}
 \begin{bmatrix}
 4.5 & 0 & 0 \\
 8.5 & 0 & 0 \\
 10 & 0 & 0 \\
 10 & 0 & 0 \\
 7.5 & 0 & 0 \\
 5.0 & 0 & 0 \\
 10 & 0 & 0
 \end{bmatrix}$$

Figure 4-6 A
Left: side view of position "B", Max. Flexion, Right: side view of position "A"

Figure 4-6 B

Left: Iso view of position "B", Maximum Flexion, Right: Iso view of position "A", Case study #2

As in the previous section, the detailed model will be used to investigate the neck injury mechanism since deformation and load of the individual soft tissues can be assessed. Figure 4-7 (A), illustrates a side projection of cervical right and left Interspinous ligament sets of the cervical spine in the extreme flexion position. The figure shows only one set, either right or left, while the other set is laying on top of the first one. The light weight lines represent the original position while the heavy lines represent the final position. The matrix below the figure implies the exact deflection for that state. For example maximum deflection occurs between C6-C5, deflection between C5-C4 is equal to 8.5 mm, while the lowest deflection is between C3-C2, equal to 4.6 mm. Elongation is considered to be zero between C2-C1 since there is no ISL-R or ISL-L at that level. The same is true in Figure 4-7 (B). This shows the side projection of cervical central interspinous ligaments, with maximum deflection occurring between C6-C5. Deflection between C5-C4 is equal to 6.4 mm, while the lowest deflection is between C7-C6, equal to -2.1 mm, in which the code will ignore and equated to zero. Note that the ISL-C between C7-C6 has no value. Figure 4-7 (C) is similar to the previous ones, showing a side projection of the cervical right and left outer posterior longitudinal ligaments with a maximum deflection between C7-C6 of 1.96 mm, and minimum deflection between C3-C2 of 0.77 mm. Note there is no FL-IR between C2-C1. In the same manner Figure 4-7 (D) and Figure 4-7 (E) illustrate side-projected views of both deflected and original length of right, left and central posterior longitudinal ligaments.

Figure 4-7 A

Ligaments deflection and final position for ISL-R & ISL-L, Light weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for ISL-R & ISL-L lengths

Figure 4-7 B

Ligaments deflection and final position for ISL-C, Light weight lines represent initial position, heavy lines represent final position, Matrix tabulate the final magnitude for ISL-C lengths

Figure 4-7 C

Ligaments deflection and final position for PLL-OR & PLL-OL, Light weight lines represent initial position, heavy lines represent final position , Matrix tabulate the final magnitude for PLL-OR & PLL-OL lengths

Figure 4-7 D

Ligaments deflection and final position for PLL-C, Light weight lines represent initial position, heavy lines represent final position , Matrix tabulate the final magnitude for PLL-C lengths

Figure 4-7 E

Ligaments deflection and final position for PLL-R & PLL-L, Light weight lines represents initial position, heavy lines represents final position, Matrix tabulate the final magnitude for PLL-R & PLL-L lengths

Figure 4-8 represents the magnitude of force vectors on vertebrae with maximum flexion of 55.5 degrees which is considered to generate strain in the human neck. However, this does not exceed tissue and vertebrae limits. The more stress a ligament suffers, the more potential damage may occur. Figure 4-8 illustrates the magnitude of the results that force vector exerted between vertebrae. For example, C5-C4 bears the highest amount of force, 42.9 N, while the C6-C5 bears 41.8 N to the lowest at C3-C2, which bears only a 20.6 N. load distribution. The cervical spin in this case is more evenly distributed among the seven vertebrae compared to case #1.

Figure 4-8
Magnitude of total force on vertebrae, At 55.5 degree flexion

CHAPTER FIVE

CONCLUSIONS AND FUTURE WORK RECOMMENDATIONS

5-1 Conclusions

This study reviewed the development of mathematical influence in biomechanical modeling. It shows how modeling provides a wealth of information on our physiological performance and reduces our dependence on animal and cadaver experiments, which is a valuable complement to clinical studies. It also concludes that as computing power and software capabilities increase, it is quite conceivable that, in the future, it will be possible to generate patient-specific models that could be used for patient assessment.

A detailed biomechanical investigation of the human spine can be undertaken in two ways: by experiment or by modeling. The experimental studies are based on animal spines, cadaver spines or artificial spine models, while the modeling is mathematical or computer based. Of course, each approach has a number of advantages and disadvantages. Experimentation provides the most direct and obvious way to obtain information on spinal biomechanics and performance. However, it can be expensive, and needs more staff and equipment. Despite specimen variability and issues of repeatability, interchangeability will itself be an important parameter in the spine's characteristics.

Modeling has the advantages that large number and variety of 'specimens' can be generated and examined if required. The same specimen can be used for any number of loading conditions and combinations; the specimen cannot be overloaded or damaged, and the specimen can be modified to reflect any physiological condition. However, generating an accurate model and ensuring that it is a valid representation of the problem may be difficult and, for a problem as complex as the spine, cannot easily be undertaken. Thus, the work that has been done over the years in the establishment of representative spine models is essential if the models are to be fully understood and if truly representative spine models are to be developed.

Validation of any spinal model by experimentation is essential but extremely difficult, due to the unavoidable and often significant physiological variations among individuals and specimens. Comparison with experimental data needs careful interpretation and analysis, because validation with the simplistic loading conditions

invariably applied in a laboratory setting does not necessarily mean that the model will function correctly with all the complex loading regimes experienced in reality.

5-2 Future Directions

The future work directions of this study can be divided into three main streams; each of them directly related to this study, and complementary to each other:

- A- Expand the existing model by introducing components not included in this or other models, such as the intervertebral, facets, and muscles. The above tissues greatly influence the cervical spine. In fact, they perform three major tasks; firstly, they provide stability of the neck and head in a given posture; secondly, they enable movement of the head during physiological activities, and thirdly, they assist in the protection of the cervical spine. They should be studied to determine the passive and active effects, and the response to impacts. While the existing model dealt with static loads on vertebrae and ligaments, if dynamics are considered, then the coefficient of damping of flexible spine components should be included.
- B- A source of losing precision and potential error of any model is due to the lack of published tissue properties. Therefore, it is recommended that experiments be performed to quantify and validate the characteristics of cervical intervertebral discs, cervical spinal ligaments, facet joints and contact of spinous processes in static and dynamic conditions. This would enable verification of these tissue properties in the new model, and improve the model accordingly where necessary.
- C- Verifying angle inputs: It is quite conceivable that it will be possible to generate patient-specific models in the future that could be used for patient assessment. More advanced, powerful computer programming and software will allow more automated generation of mathematical models directly from x-ray and MRI data using feature extraction procedures to derive important geometrical details.

5-3 Proposal for Experimental Work

Criteria observed from experimental work should verify the results of the theoretical aspect of this study. However, the first step at this level of work should correspond with the start of the code, the rotational displacement of the vertebra. A simple observation of comparing two 2D pictures of the neck positions- both initial and final will not generate the required input, since this comparison represents the total rotations of vertebra. Input should be generated from the rotation of vertebra around its own local axis, i.e., the direct observation represented by the rotation of the vertebra around its own local axis plus the rotation generated by the previous (lower) vertebra angular motion. Hence, some method must be applied to distinguish the two types of

rotation and input the correct data. It could be simply an extension of the code work as an interface between the relative rotation and the absolute rotation of the vertebra.

As above mentioned, output verification can be achieved by experimentation and filed results. The experiment will subject a volunteer, who will be asked to relax on a rigid chair simulating the one on a helicopter. He/she will be asked to imitate the position of a pilot operating the navigation system on the lower right or left corner. The volunteer will then be asked to relax their neck muscles at that position. The only external forces in this scenario include the weight of the head and the weight of the helmet. Ligaments, facet and intervertebral internal force vectors react against these external forces. Since the head is subjected to the equilibrium, the vector summation of all of these internal forces is equal as a result of the external set of forces. The head will be enclosed and rested by the helmet shown in both Figures 5-1 and Figure 5-2, the load cell reading in case of totally head and helmet relaxing will be equal to the total resultant force plus the weight of the head.

In the case of muscle involvement, the test will differ slightly; however, the same testing structure is used, the head with helmets attached to a six-axis load cell, as shown in Figures 5-1 and Figure 5-2. The volunteer will resume the previous positions providing there is no muscle relaxation. The combination of muscles, along with the internal and external forces will produce the required head position. If the head with helmet is forced to a given position, the load cell will detect the necessary force to read the static equilibrium. This equilibrium force vector represents the actual external force, which, theoretically should be relative and equal to the one resulting from running the code.

Figure 5-1

A magnified side view, "no volunteer" of load cell and part of the frame; 1- Helmet, 2- Helmet holder, 3- Load cell, 4- Frame

Figure 5-2

Proposed testing instrument concept view includes the Volunteer, load cell and chair the 6-degree of freedom frame; 1- Vertical Squ. Tube, tension and compression, z-axis, 2- flexion & extension around x-axis, 3- anterior & posterior shear plate, y-axis, 4- lateral bending shaft, around y-axis, 5- lateral shear, x-axis, 6-axial rotation, around z-axis, 7- volunteer body, 8- chair

REFERENCES

- 1- H. Gray, "Anatomy of the Human Body, on line book <http://www.bartleby.com/107/19.html>, 1918.
- 2- By R.M.H. Mcminn, "Last Anatomy Regional and Applied", Ninth Edition, Churchill Livingstone, 1994.
- 3- W. A. Wolovich, "Robotics: Basic Analysis And Design", Holt, Rinehart And Winston, 1987.
- 4- M. M. Panjabi, "Three-Dimensional Mathematical Model of The Human Spine Structure", Journal Of Biomechanics, Vol. 6. Pp. 671-680, 1973.
- 5- A. B. Schultz. T. B. Belytschko, T. P. Andriacchi, "Analog Studies Of Forces In The Human Spine: Mechanical Properties And Motion Segment Behavior", Journal Of Biomechanics, Vol.6. Pp 361-371, 1973.
- 6- A. Kecskemethy, C. Lange And G. Grabner, "Simulating Cervical Vertebrae Motion Using Elementary Contact Pairs", Technical University Of Graz, 1996.
- 7- R. H. Brown, A. H. Burstein, C. L. Nash and C. C. Schock, "Spinal Analysis Using A Three-Dimensional Radiographic Technique", Journal Of Biomechanics, Vol. 9. Pp. 355-365, 1976.
- 8- A. B. Schultz & J.O. Galante, "A Mathematical Model For The Study Of The Mechanics Of The Human Vertebral Column", Journal Of Biomechanics Vol. 3. Pp. 405-416, 1970.
- 9- M. Panjabi and A. A. White, "A Mathematical Approach For Three-Dimensional Analysis Of The Mechanics Of The Spine", Journal Of Biomechanics, Vol. 4. Pp. 203-211, 1971.
- 10- J. Dulhanty, "A Simplified Conceptual Model Of The Human Cervical Spine For Evaluating Force Transmission In Upright Static Posture", A Private Practice Of Chiropractic, Mackay, Australia. 2002.
- 11- R. L. Adler, J. P. Dedieu, J. Y. Margulies, M. Martens and M. Shub, "Newton's Method On Riemannian Manifolds And Geometric Model For The Human Spine", Journal Of Numerical Analysis, Vol. 22. Pp. 359 – 390, 2002.
- 12- B. Schultz. T. B. Belytschko, T. P. Andriacchi, "Analog Studies Of Forces In The Human Spine: Mechanical Properties And Motion Segment Behavior", Journal Of Biomechanics, Vol 6. Pp. 373-383, 1973.

- 13- N. Yoganandan, F.A. Pinter, D.J Maiman, J. F. Cusick, A. Sances and P.R. Walsh, "Human Head-Neck Biomechanics Under Axial Tension", Medical Engineering, Vol. 18, Pp. 289-294, 1996.
- 14- N. Yoganandan A. S. Kumaresan And F. A. Pinter, "Biomechanics Of The Cervical Spine Part 2. Cervical Spine Soft Tissue Responses and Biomechanical Modeling", Clinical Biomechanics, Vol. 16, Pp. 1-27, 2001.
- 15- P. D. Stepeny, B. S. Acar, K. Case, M. Porter, "Computer Aided Parametric Solid Modeling Of The Spine", Proceeding Of The Eurographics UK Chapter, 14th Annual Conference, Vol. 2, Pp. 235-244, March 1996.
- 16- S. Kumaresan, N. Yoganandan and F. A. Pinter, "Finite Element Analysis For The Cervical Spine; Material Study, Sensitivity Study", Clinical Biomechanics, Vol.14, Pp. 41-53, 1999.
- 17- K. K Lee, E. C. Teo, F. K. Fuss, V. Vanneuville, T. X. Qiu, H. W. Ng, K. Yang, And R. J. Sabitzer, "Finite-Element Analysis For Lumbar Inter-Body Fusion Under Axial Loading", Transactions On Biomedical Engineering, Vol. 51, No. 3, Pp. 393-400, 2004.
- 18- E.C. Teo, H.W. Ng, "Evaluation of The Role Of Ligaments, Facets And Disc Nucleus In Lower Cervical Spine Under Compression And Sagittal Moments Using Finite Element Method", Medical Engineering & Physics, Vol. 23, Pg.155–164, 2001.
- 19- M J Fagan, S Julian and A M Mohsen, "Finite Element Analysis In Spine Research", Proc I Mechanical Engineers, Vol. 216 Part H, Pp. 281-295, 2002.
- 20- Horst, Van Der Marike J., "Human Head Neck Response In Frontal, Lateral And Rear End Impact Loading, Modeling And Validation", Universiteit Eindhoven, 2002.
- 21- M. K. De Jager, "Mathematical Head-Nick, Model For Acceleration Impact",² University Of Eindhoven, 1996.
- 22- A. B.Schultz, H. Larocca, J.O. Galante, "A Study Of Geometrical Relationships In Scoliotic Spines", Journal Of Biomechanics. Vol. 5, Pp. 409-420, 1972.
- 23- Panjabi, Manohar M., "Cervical Spine Models For Biomechanical Research", Spine, , Vol. 23(24), Pp. 2684-2699, 1998.
- 24- D. Ornes and Y. K. Llu, "A Mathematical Model Of Spinal Response To Impact, Journal of Biomechanics", Vol 4. Pp. 49-71, 1971.
- 25- A. A. White, "Kinematics Of The Normal Spine As Related To Scoliosis", Journal Of Biomechanics, Vol. 4. Pp. 405-411, 1971.

- 26- K. D. Statler, E. A. Keshner, "Effects Of Inertial Load And Cervical-Spine Orientation On A Head-Tracking Task In The Alert Cat", *Exp Brain Res*, Vol. 148 Pp. 202–210, 2003.
- 27- J. Cholewicki¹, M. M. Panjabi, K. Nibu, L. B., Babat, J. N. Grauer¹ and J. Dvorak, "Head Kinematics During In Vitro Whiplash Simulation", *Accid. Anal And Prev.*, Vol. 30, No. 4, Pp. 469–479, 1998.
- 28- E. Privitzer and T. Belytschko, "Impedance Of A Three-Dimensional Head-Spine Model", *Mathematical Modeling*, Vol. I, Pp. 189-209, 1980.
- 29- M. M. Panjabi, A. M Oearson, S. Ito, P. C. Ivancic, "Cervical Spine Curvature During Simulated Whiplash", *Clinical Biomechanics*, Vol. 19, Pp. 2004.
- 30 - T. Belytschko. R. F. Kulak and A. B. Schultz, "Finite Element Stress Analysis Of An Intervertebral Disc", *Journal Of Biomechanics*, Vol. 2, Pp. 277-285, 1974.
- 31- R. F. Kulak. T. B. Belytschko And A. B. Schultz, "Nonlinear Behavior Of The Human Intervertebral Disc Under Axial Load", *Journal Of Biomechanics*, Vol. 9. Pp. 377-386, 1976.
- 32- K. Broil and P. Leijonhufvud, "The Effect Of Muscle Activation On Neck Response", *Taylor & Francis Inc.*, 2005.
- 33- T. Zander, A. Rohlmann, J. Calisse And G. Bergmann, "Estimation Of Muscle Forces In The Lumbar Spine During Upper- Body Inclination", *Clinical Biomechanics* 16 Supplement No. 1, Pp.73-80, 2001.
- 34- V. Peter, P. Lam, K. Seng, "Neck Muscle Strength Across The Sagittal And Coronal Planes: An Isometric Study", *Clinical Biomechanics*, Vol. 17, Pp. 545–547, 2002.
- 35- A. N. Vasavada · B. W. Peterson · S. L. Belp, "Three-Dimensional Spatial Tuning of Neck Muscle Activation In Humans", *Exp Brain Res*, Vol. 147, Pp. 437–448, 2002.
- 36- D. T.Reilly. A.t H. Burstein and V. H. Frankel, "The Elastic Modulus For Bone", *Journal Of Biomechanics*, Vol. 7, Pp. 271-275, 1974.
- 37- P. M. Buechner, R. S. Lakes, "Size Effects In The Elasticity And Viscoelasticity Of Bone", *Biomechan Model Mechanobiol* 1 Pp. 295-301, 2003.

- 38- J. M. O. Starzewski, "Modeling Of Bone At A Single Lamella Level", Biomechanical Model Mechanobiology, 3: Pp.67–74, 2004.
- 39- K. Brodin, I. Leijonhufvud and P. Halldin, "The Effect of Muscle Activation on Neck Response", Traffic Injury Prevention, 6:Pp. 67–76, 2005.
- 40- C. Helleur, S. Gracovetsky, H. Farfan And E. Wingrowicz, "Modeling of The Muscular Response of The Cervical Spine", Mathematical Modeling, Vol. 6, Pp. 443-462, 1985.