COMPUTER SIMULATION AND OPTIMAL CONTROL OF CONTINUOUS INFRARED DRYERS

by

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Computer Simulation and Optimal Control of Continuous Infrared Dryers
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Abstract

This study focuses on Infrared Dryers, which are of great commercial importance in all industrial applications encompassing food, agriculture, mining and manufacturing sectors. An unsteady-state model, and a steady-state model are used to describe the operation of dryers with multiple heating stages, and fresh air injection units. The unsteady-state model is a set of three non-linear partial differential equations, which describe the three state variables: (i) humidity of sheet being dried, (ii) temperature of sheet and, (iii) temperature of air flowing above sheet for the start-up of the drying process as a function of time and dryer length. The steady-state model consists of three non-linear ordinary differential equations, which describe the three state variables as a function of dryer length. The two sizes of infrared dryers, industrial and lab scale, with a wide range of temperature of the radiants and fresh air injection have been simulated with both models. The optimal control of infrared dryers is considered for the first time in this work. As the next step, radiant temperature has been used as an optimal control function along the length of a dryer to minimize the exit humidity of sheet. A robust optimal control technique based on genetic algorithms is used to achieve the optimal control objective. The results show considerable reduction of the exit humidity of sheet, by 19.5–99.9%, with the application of optimal control.

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Chapter 1

Introduction

1.1 Background

Drying is an extremely common unit operation in chemical engineering, and is used in the production of a wide variety of products. This operation is a separation process used to remove moisture, usually water, from a solid or fluid so as to yield a dry product. Moisture removal is normally achieved by the addition of heat, which vaporizes water. Thus, heat transfer is an important phenomenon in dryers. Drying is one of the most energy intensive industrial processes. Consequently, the efficient use and management of energy is an important concern in the design and development of drying and equipment processes. A modern society requires better product quality, improved safety practices and more environmentally benign operations, higher productivity, better energy efficiency, and reduced material wastage. As drying is certainly one of the most heat-intensive operations, and operation at higher thermal efficiency is of major concerns for industry today, the development of accurate models and optimal control strategies offer opportunities for considerable and much-desired improvement in the design and operation of dryers.

Among industrial dryers, infrared dryers are very common and carry out the important unit operation of drying in chemical engineering. The mathematical model of infrared dryers is generally described by a set of partial or ordinary differential equations. Accurately modeling the drying of materials will increase the competitiveness of the drying industry by providing a means to reduce energy/utility costs and off-quality products due to overdrying. Drying by infrared radiation can be made energy-efficient if the drying process is tuned accordingly. The optimal process conditions can be determined based on process simulation. The present work targets to provide a computer-based simulation of the drying process of thin sheets for the unsteady-state and steady-state phases of its operation to

- i. Study the rate of disappearance of volatile liquids, and
- ii. Maximize the removal of moisture content in the material being dried through optimal control.

At unsteady-state, the process variables change with time. All unsteady processes may be called transient, meaning that the process variables continuously evolve over time. At steady-state, none of the process variables change with time (if we ignore small, random fluctuations). Due to the complexity of theoretical models of drying phenomenon, the design of industrial dryers is more commonly based on the examination of external drying conditions such as the humidity of the material being dried, and temperature of air or the material. Drying processes are described by various plots, which present the moisture and temperature of solid or rate of drying as a function of time and/or space. A typical drying curve for convective drying is shown in Figure 1. According to Figure 1, a drying operation can be divided into three parts:

1. Preheating or initial drying period, Curve AB or A'B
2. A constant drying rate period, Line BC
3. A falling drying rate period, Curve CDE

Curve AB illustrates the change in moisture in the material being dried along time during the initial drying period. During this period, the temperature of the material and its liquid-covered surface are lower than the equilibrium temperature (wet bulb), and as a result the initial drying rate follows Curve AB until the surface temperature reaches the temperature corresponding to Curve BC. If the temperature of the wet material is higher than the equilibrium temperature, the initial drying period follows Curve A'B. The initial period is usually very short, and is neglected in practice. The drying rate along Line BC is constant and equal to its slope because the rate of drying depends on the constant parameters such as the velocity of the sheet and heating power. Curve CDE is called a falling rate period. The moisture content at the transition Point C is called the critical moisture content X_{cr} . Below this value, the drying rate corresponds to the appearance of scattered dry areas, which give rise to internal resistance to the moisture transfer towards the solid surface. Therefore, the rate of evaporation during the falling rate slows down as a result of diffusional constraints. At this critical Point C, the straight line becomes a curve, which asymptotically approaches the equilibrium moisture of the solid X_{eq} .

Figure 1. A typical drying curve for convective drying (Strumillo& Kudra [1]).

1.2 Heat Transfer in Dryers

In almost every conceivable dryer, heat is transferred by a combination of conduction, convection and radiation, although one mode usually dominates. In infrared IR dryers, radiation is used as the primary mode of heating. IR dryers have several features that distinguish them from other types of dryers. IR radiation involves the transfer of heat by electromagnetic waves, or photons that can travel long distances without interacting with a medium while conduction and convection are short-range phenomenon. Therefore, radiation can be effective over long distances. The difference between conduction/convection and radiation is the difference in their temperature dependencies.

Conduction is normally well described by Fourier's law:

$$q_x = -k_c \frac{\partial T}{\partial x} \quad (1)$$

where q_x = flux of heat flow in direction normal to surface

k_c = thermal conductivity

T = temperature

Similarly, the convective heat flux from a surface to the process environment can usually be calculated from Newton's law of cooling:

$$q = h_c (T - T_\infty) \quad (2)$$

where q = rate of heat flow

h_c = convective heat transfer coefficient

T = surface temperature

T_∞ = temperature of process environment, far from surface

Thus, conductive and convective heat transfers are almost linearly proportional to temperature differences. Radiation heat-transfer rates, on the other hand, are generally proportional to differences in temperature raised to the fourth power e.g.,

$$q = \sigma AF(T^4 - T_{\infty}^4) \quad (3)$$

Where q = net radiation between two surfaces

A = area of either of two surfaces, chosen arbitrarily

F = dimensionless geometric factor

Radiation thus becomes increasingly important as the temperature level increases. The non-linear dependence also means that at elevated temperatures even a minor temperature increase may result in a major increase in heat flux.

1.3 Infrared Drying

The radiation emitted by a hot body consists of a spectrum of wavelengths. Some of these wavelengths are called infra-red. The term infrared is often used as a synonym for thermal radiation. Radiation heaters, so called infrared heaters, allow heat to be supplied precisely, at a specific location and often at lower costs than those with other mechanisms. All electric infrared heaters basically rely on resistive heating. A current is passed through a resistor, which heats up and either emits radiation itself or heats the emitter through conduction. This operation has been described in some details by Biau[2], Ratti and Mujumdar [3] as:

When heat is applied to a moist subject, the vapor pressure of the liquid is increased and results in a driving force for the transport of moisture from the subject to the external

atmosphere. Thus, drying occurs under the simultaneous transport of heat and mass. In infrared dryers, radiation is the dominant phenomenon, and the transfer of the energy is based on molecular collisions between molecules. There are different applications of infrared dryers in the industry e.g. the heat treatment of foodstuffs, metal and glass processing, drying of paints and drying of thin continuous sheets such as textiles and paper. New drying techniques using infrared have been developed lately to replace or improve existing continuous drying processes (Hashimoto [4], Dhib [5-8]). The successful utilization of an infrared dryer is highly dependent on the spectral characteristics of the material being dried, and how well the spectral output of the infrared source matches with that of the material. At the same time, energy efficiency of an IR dryer is a very imperative aspect and concern in the industry.

1.4 Literature Survey

There are number of publications concerning heat and mass transfer and energy use in infrared dryers. Most of the publications on infrared dryers concern possible or actual applications, operating experiences or new designs. Studies of infrared drying mechanisms and of drying kinetics are scattered, although a number of them will be referenced below as an indication of subjects that have attracted scientific interest.

The unsteady-state model used in this thesis is based on the work of Dhib et al. [6, 7]. They proposed a model of four coupled hyperbolic partial differential equations describing the variations of air and sheet temperatures, and humidity of the sheet as a function of time and space along dryer length. They showed that the model could be

reduced to a set of three equations when airflow and renewal rate were high inside a dryer. The reduced model was adaptable to digital control, and was analyzed for controllability and observability. The run-by-run calibration of the model generated parameter values, which showed significant variations for the heat and mass transfer coefficients between the humid sheet and the flowing air but somewhat less important variations for the parameter correcting the material emissivity and the shape factor. Like other chemical processes such as flow-through catalytic reactors and absorbers, the dynamics of continuous dryers are generally modeled by non-linear partial differential equations (Dhib et al.,[6-8]; Nissan et al., [9]; Kuang et. al., [10]) based on mass and energy balances and the applicable boundary conditions. Also, Coté et al. [11] developed a differential model based on simultaneous heat and mass transfer for the infrared drying of continuous thin layers.

Silventoinen and Palosaari [12] presented a method for calculation of the drying time and surface temperature of water-based paints during both convective and infrared drying. The model was based on mass and energy balances together with an experimental characteristics drying curve. Rosler et al [13] conducted drying experiments with water based paints and organic coatings using IR heaters with different emission spectra. Navarri et al [14] evaluated experimentally the influence of various parameters, including the radiative properties of a coated plate, on the infrared drying of thin paint films on impermeable supports. They presented further experimental results concerning aqueous coated films that were shrinking. Their diffusional model was capable of predicting

drying curves and temperatures well. The model parameters were determined from the drying curves.

Parouffe et al [15] in an experimental study of the combined infrared and convective drying of glass beads investigated drying characteristics, and the influence of thermal radiation on convective heat and mass transfer. Parouffe et al [16] presented results from three different experimental studies of the relationship between heat and mass transfer coefficients in combined convective and infrared drying.

Navarri et al [17] performed drying experiments on a wet sand layer under intensive infrared radiation. Heat and mass transfer processes were analyzed and a simple model was found able to predict drying rates and surface temperatures during the constant rate period. Navarri and Andrieu [18] reported similar experiments, and presented a model able to predict drying rates and temperature profiles down to zero moisture content. Hashimoto et al. [19] used three kinds of membrane filters differing in their mean pore diameter for studying the infrared drying characteristics of wet porous materials.

Blanc et al [20] described the experimental work and modeling related to the infrared drying of reactive car paints, that is of paints in which vaporization is coupled with a polymerization reaction in a paint film. Techniques for monitoring the reaction kinetics and drying kinetics were discussed and experimental and modeled drying curves were presented. Blanc et al [21] discussed methods for studying reaction kinetics during the infrared curing/drying of epoxy-based car paints, along with related changes in structure

and rheological behaviour. Blanc et al [22] presented a model including simultaneous polymerization, and heat and mass transfer for the infrared drying of epoxy-based paints on iron support. Several parameters were determined in separate experiments, and experimental drying curves were used for model validation.

Vainberg and Grabovsky [23] experimentally investigated the infrared drying of a film formation in magnetic varnish, a two-component solution with appreciably different boiling temperatures. Changes in radiation properties and in film thickness were investigated as the drying proceeded and drying curves were determined. Chen and Lin [24] developed a mathematical method for the infrared drying of a polymer solution film; including both diffusive and radiation transport inside the film in the model and examined the effect of various radiation properties on temperature profiles, and on drying rates. Chen and Lin [25] developed a similar model for intermittent infrared drying.

Le Person et al. [26] investigated the drying of a thin multicomponent film produced for pharmaceutical purposes. Internal concentration gradients were measured and the results were combined with drying rate data to gain an understanding of the drying process and of the multicomponent transport phenomenon involved. This insight was then used to explain and prevent the occurrence of mechanisms detrimental to product quality.

Sandu [27] analyzed the application of infrared drying in food engineering transport phenomenon and process applications. Yamasaki et al. [28] studied the infrared drying of food, using gelatinous materials as model substances. Factors that influenced drying rate

and shrinkage, including the radiative properties of the product and the heater, were examined experimentally and were compared with a simple drying model. Afzal and Abe [29] studied the combined convection and infrared drying of rough rice. The effects of radiation intensity, air temperature and velocity on drying kinetics were evaluated experimentally. Fasina et al. [30] proposed a set of heat and mass transfer equations to simulate the infrared drying of agricultural crops. The model was compared with experimental data on the surface temperature and average moisture content of barley kernels found on a vibrating conveyor exposed to infrared radiation. Afzal and Abe [31] performed drying experiments using a far infrared heater to examine the drying characteristics of potato slices. The effects of several parameters on drying rate and on product temperature history were investigated.

Dostie et al. [32] conducted a pilot-scale investigation of the combined convective and intermittent infrared drying of insulating wood panels and acoustic tiles. Dostie [33] investigated a numerical and experimental study of the drying of mineral board, using infrared, radio frequency, and convective drying in combination and optimized the combined use of these technologies from an economic point of view. Broadbent et al. [34] investigated the influence of various parameters on drying rates when a number of different textile fabrics were pre-dried in a pilot-scale infrared dryer.

Kuange et al. [35] developed a mathematical model for paper drying using a gas-fired IR dryer. The model included a number of different internal transport phenomenon, and involved a simple radiation absorption approach. Prediction of moisture content by the

model was compared with and fitted to pilot-machine measurements. In another paper, Kuange et al. [36] somewhat elaborated the model, and performed a sensitivity analysis. Later, Kuange et al. [37] presented experimental results from the pilot paper dryer, and the influence of various operating conditions on the drying rate. Sain et al. [38] modified and extended the model of Kuange et al. mentioned above, and made additional experimental observations. The modified model was validated on the basis of experimental drying rates, and surface temperatures of paper. The drying mechanism was discussed in light of experimental findings and model predictions. Fernandez et al. [39] proposed a model for infrared paper drying, which included a large number of different transport processes. Particular attention was directed at the complete radiative transport equation being used, although only two wavelength bands represented spectral variation in the properties. Numerical solutions were presented for a 1 mm thick sheet, using transport parameters of wood as well as radiation from a source at 398–500 K.

Gutoff [40] presented a model for the drying of coated sheets, considering the possible use of an infrared dryer. The model included the diffusion of moisture, although only a lumped heat capacity model was used for the temperature.

The efficiency and compatibility of the conventional drying processes are of major concerns for the industry today since overdrying may damage several physical and mechanical properties of material (shrinkage, tensile strength, etc.). Also for some materials such as paper, textile and granular food, insufficiently dried material may deteriorate through rotting under long-term storage conditions. In order to enhance the

capability and efficiency of infrared dryers, it is desirable to optimally control their performance. The above review includes most of the research on infrared drying concerning the mass and energy balance and modeling and simulation. The applications studied involve mainly paint, food-stuffs and continuous sheets such as of textiles and paper, although various other materials have been investigated as well, including certain model substances with idealized properties.

Under the steady-state condition, dryers are modeled by ordinary differential equations. At the start-up of the process, the process variables are changing with time and the system is modeled by non-linear partial differential equations, and is under unsteady-state operation. It would take a time period at least equal to the time for the material inside a dryer to travel its entire length to be subjected to the whole effect of the radiation field. Thus the products at the start-up will be usually off-specification. The process of continuous drying during the start-up operation period is similar to the process of a batch or rotary dryer. The main goal is to operate the continuous process so that the desired steady-state is reached from the initial state within a specific time or in another term; the scope of start-up is to change the state of the process from one condition (initial state) to one that product specifications for the new grades are satisfied. Here, the first step is to solve the steady-state equations and unsteady-state model by applying the fully centered finite difference approximation method. The second step is to apply the genetic algorithms based optimal control method Upreti [41], to minimize the final humidity of sheet, using the fifth-order Runge-Kutta Fehlberg method with Cash-Karp parameters and adaptive step-size control, Press et al., [42].

Chapter 2

Mathematical Models and Simulation

The mathematical modeling and simulation of continuous IR dryers consists of

- i. Unsteady-state
 - a) Lab Scale
 - b) Industrial Scale
- ii. Steady-state
 - a) Lab Scale
 - b) Industrial Scale

2.1 The Infrared Dryer

The schematic of an infrared dryer is presented in Figure 2. The manipulated variables for the system are the electrical power to the infrared elements, and the velocity of sheet

of the material to be dried. The main load to the process is the inlet humidity of sheet. The material-feeding system consists of a wetting bath with squeeze rollers, and a vacuum extractor, which is used to remove excess water before the material enters the infrared dryer. The vacuum extractor develops a pressure gradient up to 70 kPa. The electric infrared sources are placed in six modules situated symmetrically above and below the moving sheet. The radiant emitters are positioned 5 cm away from the sheet. The dryer may be operated at a maximum power of 20 kW. The material to be dried is typically a bleached fabric with an average width of 23 cm. Driven by the rollers, it travels inside the dryer for the drying process. Between the moving sheet and the radiating emitters, air flows co-current or counter-current with a specific velocity.

At the center of the lab scale dryer, and at equally spaced positions within commercial dryers, wet air is evacuated and replaced with fresh air to help moisture removal. The radiation warms up the sheet as well as the air flowing above and below the sheet. The humidity of sheet drops as its temperature and temperature of air increase, and the liquid inside the sheet starts to vaporize.

Figure 2. Infrared dryer, Lab Scale, Dhib [6].

2.2 Mass and Energy Balances

Figure 3 shows the differential element for the following mass and energy balances, Dhib et al. [6], for modeling the lab scale infrared dryer system:

For the air mass:

$$\left\{ \begin{array}{c} \text{Rate of change} \\ \text{Of} \\ \text{Water vapor} \end{array} \right\} + \left\{ \begin{array}{c} \text{Rate of mass} \\ \text{of water} \\ \text{vapor out} \end{array} \right\} - \left\{ \begin{array}{c} \text{Rate of mass} \\ \text{of water} \\ \text{vapor in} \end{array} \right\} = \left\{ \begin{array}{c} \text{Rate} \\ \text{of} \\ \text{evaporation} \end{array} \right\}$$

For the sheet moisture mass:

$$\left\{ \begin{array}{c} \text{Rate of change} \\ \text{Of} \\ \text{Water content} \end{array} \right\} + \left\{ \begin{array}{c} \text{Rate of mass} \\ \text{of water} \\ \text{content out} \end{array} \right\} - \left\{ \begin{array}{c} \text{Rate of mass} \\ \text{of water} \\ \text{content in} \end{array} \right\} = - \left\{ \begin{array}{c} \text{Rate} \\ \text{of} \\ \text{evaporation} \end{array} \right\}$$

For the air mixture:

$$\left\{ \begin{array}{c} \text{Rate of change} \\ \text{Of} \\ \text{energy} \end{array} \right\} + \left\{ \begin{array}{c} \text{Rate of} \\ \text{energy} \\ \text{out} \end{array} \right\} - \left\{ \begin{array}{c} \text{Rate of} \\ \text{energy} \\ \text{in} \end{array} \right\} = \left\{ \begin{array}{c} \text{Rate of heat} \\ \text{added/removed} \\ \text{by evaporation and convection.} \end{array} \right\}$$

Figure 3- Differential element of the upper symmetrical half of the dryer system for heat and mass balance. (Dhib. R 1994)

2.3 The Unsteady-state Distributed Parameter Model

Based on the work of Dhib and co-workers [5–8], the model is presented to depict the temperature and humidity dynamics of a moving, thin fiber sheet, and of the airflow, in an infrared dryer. Mainly heated by the radiation from the radiant emitters, the sheet gives up sensible heat as water vapor and exchanges convective heat with air. Considering that the sheet of material is less than 0.5 mm thick, and that it receives radiation energy on both sides, moisture and temperature gradients in the direction of its thickness are neglected. This corresponds to assuming uniform humidity and temperature profiles across the sheet thickness at any given point along the dryer's length. Also, since the velocity of sheet was relatively high, the mass of water transported in the direction of dryer length is much greater than the diffusion of water in the direction of the width of dryer, which was neglected.

The above balance can be mathematically written as follows:

$$\frac{\partial H_s}{\partial t} \pm V_s \frac{\partial H_s}{\partial z} = k_1 M_{evap} \quad (5)$$

$$\frac{\partial T_a}{\partial t} \pm V_a \frac{\partial T_a}{\partial z} = k_2 (T_r - T_a) + k_3 (T_s - T_a) + k_4 M_{evap} (T_s - T_a) \quad (6)$$

$$\frac{\partial T_s}{\partial t} \pm V_s \frac{\partial T_s}{\partial z} = [k_5 (T_r^4 - T_s^4) + k_6 (T_s - T_a) + k_7 M_{evap} (\lambda - H_{des})] / (k_8 + H_s) \quad (7)$$

In the above equations, H_s is the humidity of sheet being dried. T_a and T_s are the temperature of the inlet air and of sheet, respectively. The terms k_i , $i = 1 \dots 8$ represent model parameters, which have been defined to simplify the model. M_{evap} , λ , and H_{des}

represent the rate of evaporation of water, the latent heat of evaporation of water, and the heat of desorption of water from the material, respectively. V_a and V_s represent the velocity of air in dryer and the velocity of sheet. A positive sign for V_i denotes co-current drying and a negative sign denotes counter current drying, Yliniemi Leena et al. [43]. In this work co-current system has been implemented. A complete definition of the terms, correlations and the constants used in this model has been provided in Appendix A. The initial conditions for the above model are:

$$H_s(0,z)=H_{si} \quad (8)$$

$$T_a(0,z)=T_{ai} \quad (9)$$

$$T_s(0,z)=T_{si} \quad (10)$$

Thus, the humidity of sheet throughout the dryer at time $t=0$ is equal to its initial value at the inlet of dryer. Also the temperature of sheet, and of air at time $t=0$ throughout the dryer are equal to their inlet temperatures.

The boundary conditions for the model are:

$$H_s(0)=H_{sb} \quad (11)$$

$$T_a(0)=T_{ab} \quad (12)$$

$$T_s(0)=T_{sb} \quad (13)$$

Thus, the humidity of sheet at $z=0$ is equal to its initial value at the boundary of dryer. Also the temperature of sheet, and of air at $z=0$ is equal to their inlet temperatures. There is an additional boundary condition at the dryer length L_{inj} , where there are fresh air injections to dryer:

$$T_a(0)=T_a(L_{inj})=T_{ab} \quad (14)$$

At an injection point, the wet air is replaced by fresh air at the same inlet air temperature.

2.4 Assumptions

The assumptions made for the mathematical model of infrared dryers are as follows:

1. Air and water vapor form a homogeneous mixture with negligible absorption of radiation.
2. The physical properties, namely, the radiant source-air heat transfer coefficient, moist solid-air heat transfer coefficient, convective heat transfer coefficient, density of the material being dried, latent heat of vaporization, thermal conductivity, and density of air are constant.
3. Initially, dryer is filled with the material to be dried.
4. There are no heat interactions between the radiants in the dryer.
5. There are no radiations from the walls of the oven.
6. The heat losses to the surroundings are negligible.
7. Moisture and temperature gradients in the direction of the thickness of the material to be dried are neglected.
8. The mass of water transported in the direction of dryer length is neglected.

2.5 Model Simulation

Several finite difference approximations methods have been reported to solve partial differential equations. Oran & Boris [44] and Dhib [6] used the donor-cell and Lax-Wendroff finite difference schemes. Zalesak, [45] used the Lax-Wendroff corrected-flux method, which is a hybrid scheme of higher and lower order algorithms. In this work, the 2nd order finite difference approximation has been used. The distributed parameter model is transformed into a set of ordinary differential equations. The model was used to describe the dynamics of a lab scale, and an industrial scale infrared dryer. To simulate the model, each of the Equations 5 to 7 were finite-differenced along z direction using the following 2nd order centered difference formula:

$$\frac{\partial \xi}{\partial z} = \frac{(\xi_{i+1} - \xi_{i-1})}{2\Delta z} \quad (15)$$

where ξ is a state variable and Δz is the equispaced size of the dryer with n grid points and the length of L :

$$\Delta z = \frac{L}{n} \quad (16)$$

The following 2nd order backward finite difference formula used at the middle and exit of the dryer, and at each injection point:

$$\frac{\partial \xi}{\partial z} = \frac{(3\xi_i - 4\xi_{i-1} + \xi_{i-2})}{2\Delta z} \quad (17)$$

The resulting set of ODEs is provided in Appendix B. The equations were simultaneously solved using the 5th order Runge-Kutta Fehlberg method with Cash-Karp parameters (Appendix C), and adaptive step size control.

Figure 4 is a schematic of the grid points for the unsteady-state operation in dryers with single and multiple fresh air injections. For the lab scale dryer, fresh air replaces the wet air at the middle of the dryer. For the industrial size dryer, the injection units are equispaced along the length of the dryer.

2.6 Results

The parameters used in the simulation of the ordinary and partial differential equations are presented in Table–1. The model behavior was studied for the humidity and temperature of sheet and temperature of air and also for changes of the sheet velocity for a lab scale and an industrial IR dryer.

2.6.1 Results for the Unsteady-state Simulation

Figure 5 shows the humidity of sheet, and the temperature of air and of sheet profiles versus time for a lab size IR dryer. It shows that it takes 18 seconds for the sheet to travel the entire length of the oven when its velocity is $V_s=5$ m/min, and the temperature of the radiants is 600 °C. This figure shows that for a short heating period, the temperature of sheet reaches a constant value while the moisture of the material is still decreasing. This characterizes the constant rate period, which means that all heat reaching the wet material is used to evaporate water. Once the material moisture falls below its critical value (25–30/100g on a dry basis for cotton), the sheet temperature begins to rise. This is the falling rate period of drying.

Figure 4. Discretization along the length of dryer (Industrial Scale and Lab scale)

Table-1: Model Inputs

Properties	Lab Scale	Industrial Scale
No. of Step sizes	12	84
Range of integration (s) for Unsteady-State	1–18	1–23
Length of the dryer (cm)	144	1008
Initial temperature of the air ($^{\circ}\text{C}$)	25	25
Initial temperature of the sheet ($^{\circ}\text{C}$)	15	15
Initial humidity of the sheet (%)	50	50
Velocity of the sheet, V_s (m/min)	5	20
Velocity of the air, V_a (m/min)	40	40
(H_c) (g/100gr of dry solid)	30	30
A_a (cm^2)	362.5	362.5
W_a (cm)	47.4	47.4
α_r	0.4	0.4
α_{ra}	0.2	0.2
α_{sa}	0.3	0.3
ρ_s (g/cm^3)	158×10^{-4}	158×10^{-4}
ρ_a (g/cm^3)	1.029×10^{-3}	1.029×10^{-3}

Table-1 cont'd

Properties (cont'd)	Lab Scale	Industrial Scale
P (mmHg)	760	760
C_{ph} (J/g.K)	0.4469	0.4469
C_{ps} (J/g.K)	0.3	0.3
W_s (cm)	22.5	22.5
W_r (cm)	67.5	67.5
D_h (cm)	13.11	13.11
σ (W/cm ² .K ⁴)	1.355×10^{-12}	1.355×10^{-12}
ε	0.9774	0.9774
A_r (cm ²)	4176	4176
T_r (°C)	50 – 800	50 – 800

Figure 5. Evolution of output variables for the unsteady-state operation at the exit of a lab scale infrared dryer. $V_s=5$ m/min (Final $H_s=14.9\%$)

The final humidity of sheet with these conditions is 14.9%, which is in close range (1.7% less) with the results of the previous studies (Dhib [4-5] $H_s=16.7\%$). The final temperature of sheet and air are 59.6°C and 117°C, respectively. Comparing with the results of Dhib [6, 7] (56°C and 122°C), there is a difference of +3.6 °C and -5 °C for the temperature of sheet and of air between the two approximations. The influence of the velocity of sheet on the drying process has been investigated through simulations here, and it shows that increasing the velocity of sheet causes less reduction in the final humidity of sheet. The results for velocities of 7 and 10 m/min for a lab scale IR dryers are shown in Figures 6 and 7. As the velocity is reduced, more water is evaporated and the sheet temperature increases. Changing the velocity from 5 to 10 m/min causes increasing the final humidity of the sheet from $H_s=14.9\%$ to 31.9% from its initial value of 50%, which is a 53% increase or in a better word 53% less reduction. It takes only 9 seconds for the transition of unsteady-state to steady-state operation for the case of $V_s=10$ m/min. Table 2 summarizes these results at the exit of the lab size IR dryer in unsteady-state operation for Figures 5 to 7 at the constant temperature of the radiants of 600°C.

Figures 8 to 16 present the humidity of sheet, and temperature of air and sheet in an industrial size IR dryer with one, two and six fresh air injection units and different velocities of sheet at a constant temperature of the radiants under unsteady-state operation. Figures 8 to 10 present the profiles of the humidity of sheet and temperatures of sheet and air for the transition of unsteady-state to the steady-state in the oven when the velocity of sheet is $V_s=30, 35$ and 40 m/min, respectively. The number of air injection units is one and the temperature of the radiants is 600 °C. The final humidity of sheet

Figure 6. Evolution of output variables for the unsteady-state operation at the exit of a lab scale infrared dryer. $V_s=7$ m/min (Final $H_s=24.65\%$)

Figure 7. Evolution of output variables for the unsteady-state operation at the exit of a lab scale infrared dryer. $V_s=10$ m/min (Final $H_s=31.9\%$)

Table 2. Output variables for the unsteady-state operation at the exit of a lab scale infrared dryer for different velocity of the sheet at a constant $T_r=600^\circ\text{C}$.

V_s (m/min)	t_{travel} (Sec)	H_s (%)	T_a ($^\circ\text{C}$)	T_s ($^\circ\text{C}$)
5	18	14.9	117	59.6
7	13	24.7		51.9
10	9	34.9		48.8

Figure 8. Evolution of output variables for the unsteady-state operation at the exit of an Industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=22.6\%$)—one injection point

Figure 9. Evolution of output variables for the unsteady-state operation at the exit of an Industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=26.7\%$)—one injection point

Figure 10. Evolution of output variables for the unsteady-state operation at the exit of an Industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=27.9\%$)—one injection point

with $V_s=30$ has been determined as 22.6 % and the final temperature of sheet and of air has been found as 61.08 °C and 293.2 °C, respectively.

Finally, continued integration of the model indicated that the lab scale and industrial size dryers attained steady-state in 29-32 seconds.

Figures 11 to 13 show the profiles of sheet and air for transition of unsteady-state to the steady-state operation in an industrial IR dryer when the velocity of sheet is $V_s=30$, 35 and 40 m/min, respectively. The number of injection units is two and the temperature of the radiants is 600 °C. The final humidity of sheet with $V_s=30$ is 22.3 % and the final temperature of sheet and of air is 58.3°C and 256.3°C, respectively. Figures 14 to 16 show the mentioned variables in an industrial infrared dryer when the velocity of sheet is $V_s=30$, 35 and 40 m/min, respectively. The number of air injection units is six and the temperature of the radiants is 600 °C. The final humidity of sheet with $V_s=30$ is 24 % and the final temperature of sheet and of air is 56.9°C and 198 °C, respectively.

Table 3 summarizes these results at the exit of the industrial dryer in unsteady-state operation with one to six fresh air injection points. This table shows that increasing the number of air injections for a constant velocity of sheet does not affect the final humidity of sheet at the exit of dryer significantly. However it affects the final temperatures of air and sheet at the exit of dryer. Also this table shows that increasing the velocity of sheet from 30 to 40 m/min increases the final humidity of sheet while air temperature remains constant and sheet temperature drops to lower values.

Figure 11. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=22.01\%$)—two injection points

Figure 12. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=26.4\%$)—two injection points

Figure 13. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=27.5\%$)—two injection points

Figure 14. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=23.9\%$)—six injection points

Figure 15. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=27.1\%$)—six injection points

Figure 16. Evolution of output variables for the unsteady-state operation for the exit of an industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=28.4\%$)—six injection points

Table 3. Output variables for the unsteady-state operation at the exit of an industrial scale infrared dryer for different velocity of the sheet and injection points at a constant $T_r=600^\circ\text{C}$.

V_s (m/min)	H_s (%)	T_a ($^\circ\text{C}$)	T_s ($^\circ\text{C}$)	No. of Injection Points	t_{travel} (s)
30	22.6	293.1	61	1	21
35	26.7		58.2		18
40	27.4		57.6		15
30	22.3	256.3	58.3	2	21
35	26.4		55.1		18
40	27.5		53.9		15
30	24	198	56.9	6	21
35	27.1		55.8		18
40	28.4		54.2		15

2.6.2 Results for the Steady-state Simulation

The steady-state model is time independent and the state variables are investigated along the length of dryer. Figure 17 reflects the humidity of sheet and temperature of air and sheet along the length of dryer when a lab size infrared dryer with one fresh air injection unit is being studied with the velocity of sheet as $V_s=5$ m/min and the temperature of the radiants as 600 °C. This figure shows that at the middle of dryer and close to air fresh air injection, there is a sharp drop of air temperature to its initial value of 25°C. The wet air, which is carrying the vapor of sheet, is being replaced with fresh air. The air temperature increases again as it becomes exposed to the heat of the radiants. The temperature of sheet instead, drops very slightly and smoothly and rises again as the temperature of air increases. The humidity of sheet continues to decrease for the whole process. The final humidity of sheet at the exit of dryer with these conditions drops to 8.67 %, and the corresponding final temperature of sheet and air is 71.24 °C and 187°C, respectively.

Figures 18 and 19 show the influence of the velocity of the moving sheet on the drying process for a lab size infrared dryer with one fresh air injection unit. These two figures, and Figure 16 present state variable profiles for different velocity of sheet, (5, 7 and 10 m/min) at constant temperature of the radiants, $T_r = 600^\circ\text{C}$ for a lab scale IR dryer in steady-state operation. Changing the velocity of sheet from 5 to 10 m/min increases the final humidity of sheet from $H_s=8.67$ to 30.1% from its initial value of 50%. Table 4 summarizes these results at the exit of the lab size dryer in steady-state operation. This table shows that increasing the velocity of sheet increases its final humidity at the exit of

Figure 17. Output variables for the steady-state operation at the exit of a lab scale infrared dryer. $V_s=5$ m/min (Final $H_s=8.67\%$)

Injection Point

Figure 18. Output variables for the steady-state operation at the exit of a lab scale infrared dryer. $V_s=7$ m/min (Final $H_s=20.5\%$)

Injection Point

Figure 19. Output variables for the steady-state operation at the exit of a lab scale infrared dryer. $V_s=10$ m/min (Final $H_s=30\%$)

Injection Point

Table 4. Output variables for the steady-state operation at the exit of a lab scale infrared dryer for different velocity of the sheet at a constant $T_r=600^\circ\text{C}$.

V_s (m/min)	H_s (%)	T_a ($^\circ\text{C}$)	T_s ($^\circ\text{C}$)
5	8.67	187	71.2
7	20.5		59.8
10	30		53.7

the dryer while air temperature remains constant and sheet temperature drops to lower value.

Figures 20 to 28 present the profiles of sheet moisture and temperature and air temperature coming out of dryer versus its length for different velocities of sheet at a constant temperature of the radiants as 600°C when an industrial size IR dryer with one, two and six fresh air injection units is simulated for the steady-state operation.

Figures 20 to 22 show the profiles of the humidity of sheet and temperature of air and sheet and the influence of the velocity of sheet on the drying process for the steady-state operation when $V_s=30, 35$ and 40 m/min respectively. The number of injection units is selected as one at a fixed temperature of the radiants as 600 °C. Air temperature drops to its initial value of 25 °C at the middle of dryer sharply due to approaching air injection point. Sheet temperature also drops to lower value but not as sharp as air temperature. The final humidity of sheet for $V_s=30$ is 2.54 % and the final temperature of sheet and of air is 84 °C and 279 °C, respectively. Increasing the velocity of sheet increases the final humidity of sheet from its initial value of 50%. Table 5 summarizes these results at the exit of the industrial size dryer for steady-state operation with one to six fresh air injection points. This table shows that increasing the number of air injections for a constant velocity of sheet does not affect the final humidity of sheet at the exit of dryer significantly. However it affects the final temperatures of air and sheet at the exit of dryer. Also this table shows that increasing the velocity of sheet from 30 to 40 m/min increases the final humidity of sheet while air temperature remains constant and sheet

Figure 20. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=2.54$ %)—one injection point

Injection Point

Figure 21. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=8.35$ %)—one injection point

★ *Injection Point*

Figure 22. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=13$ %)—one injection point

★ *Injection Point*

temperature drops to lower values. Figures 23 to 25 show the same profiles and the influence of the velocity of sheet on the drying process when $V_s=30, 35$ and 40 m/min, the number of injection units is two, and the temperature of the radiants is 600 °C. For this dryer, fresh air enters at two equispaced points along dryer length.

At each injection point, the temperature of air drops to its initial value, 25 °C. However, this effect is not that severe on sheet temperature. After the fresh air has been exposed to the radiants temperature, both temperatures of sheet and of air rise again until approaching the next air injection unit. The final humidity of sheet with $V_s=30$ is 2.6% and the final temperature of sheet and of the air is 85.4 °C and 279 °C, respectively. Figures 26 to 28 show the same profiles and the influence of the sheet velocity on the drying process when $V_s=30, 35$ and 40 m/min, the number of air injection units as six and the temperature of the radiants is 600 °C. For this dryer, fresh air enters at six equispaced points and replaces wet air. These points are recognizable by every sharp drop of the temperature of air profile in these figures. The final humidity of sheet with $V_s=30$ is 2.57% , the final temperature of sheet and of air as 84.9 °C and 279 °C, respectively. Table 5 summarizes these results at the exit of an industrial size dryer in steady-state operation.

Figure 23. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=2.6$ %)—two injection points

Figure 24. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=7.85$ %)—two injection points

Figure 25. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=13$ %)—two injection points

Figure 26. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=30$ m/min (Final $H_s=2.57$ %)—six injection points

Figure 27. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=35$ m/min (Final $H_s=8.16$ %)—six injection points

Figure 28. Output variables for the steady-state operation at the exit of an Industrial scale infrared dryer. $V_s=40$ m/min (Final $H_s=12.7$ %)-six injection points

Table 5. Output variables for the steady-state operation at the exit of an industrial scale infrared dryer for different velocity of the sheet and injection points at a constant $T_r=600^\circ\text{C}$.

V_s (m/min)	H_s (%)	T_a ($^\circ\text{C}$)	T_s ($^\circ\text{C}$)	No. of Injection Points
30	2.54	279	84.4	1
35	8.35		71.3	
40	13		66.7	
30	2.6	252	81.4	2
35	7.85		70.6	
40	13		64.2	
30	2.57	207	79.9	6
35	8.16		69.6	
40	12.7		61.6	

Chapter 3

Optimal Control

Optimal control is a special case of optimization. Optimal control implies the optimization of a system described by differential equations, and at least one control function instead of variables. Optimal control finds use in many practical control problems, where it is desired to optimize a given cost functional while satisfying constraints on the states and the input. A good optimal control technique should provide consistent, good quality results and use a reasonable amount of performance index evaluations. The differential equations 5 to 7, they can be written as:

$$\frac{dx}{dz} = f(x, T_r) \quad 0 \leq t \leq t_f \quad (18)$$

Where:

x : is a vector of state variables and is known at $t=0$, and

T_r : control function, which represents the temperature of the infrared radiants.

Equation (18) is subject to satisfaction of a value of constraints and is considered as:

$$T_{r,\min} \leq T_r \leq T_{r,\max} \quad (19)$$

$$T_s \leq T_{s,\max} \quad (20)$$

Where $T_{r,\min}$ and $T_{r,\max}$ are the minimum and maximum temperatures of the radiants, and $T_{s,\max}$ is the maximum allowable temperature of the sheet being dried. The objective is to obtain the optimal control function $\hat{T}_r(z)$, which will minimize the moisture content of sheet at dryer exit. Thus, the performance index to be minimized is:

$$J_{\min} = H_s(L) \quad (21)$$

The discrete step values of T_r are considered as optimization variables. These step values form a control vector T_r . To handle this challenging optimal control problem, a robust optimal control method based on genetic algorithms, Upreti [41], was applied. The temperature of sheet and air inside dryer were represented by a series of step values of temperature (or control stages) equispaced along dryer length. The number of control stages, the mathematical model of dryer, and its parameters were input to the optimal control method. Its application generated the optimal control function by stochastically applying genetic operations on a set of optimization functions, which were evaluated through the mathematical model of the process to be optimized. Further details of this method may be found in Upreti [41].

3.1 Optimization

This section outlines optimization and genetic algorithms. It is followed by the results obtained for IR dryer. Optimization is defined as the science of determining the best solutions to certain mathematically defined problems, which are the models of physical reality. It involves the study of optimality criteria for problems, the determination of algorithmic methods of solution, the study of the structure of such methods, and computer experimentation with methods both under trial conditions and on real life

problems. There is an extremely diverse range of practical applications. The applicability of optimization methods is widespread, reaching into almost every activity in which numerical information is processed (Science, Engineering, Mathematics, Economics, Commerce, etc.). To give some idea of what is involved in optimization technique, consider any design in Chemical Engineering, which can be modeled mathematically. An *objective function* to be optimized might therefore be the quantity of the product or the profit from operating the system. The *variables* would be any condition influencing the input or the process itself, or any physical and energy conditions related to the process. These variables are subject to restrictions or interrelations of many kinds, which are referred to *constraints*. Chemical Engineering models are usually described by partial differential equations and are highly non-linear, which makes the determination of optimal control very difficult (Barton [46]). It gives rise to a multimodal, and non-continuous relation, or functional, between a performance index and a control function.

3.2 Genetic Algorithms

Genetic algorithms are optimization methods, which mimic evaluation of living animals. They were invented by J. Holland [47], and developed by him and his students. As defined in his book, to use a genetic algorithm, we should represent a solution to our problem as a genome (or chromosome). The genetic algorithm then creates a population of solutions and applies genetic operators such as mutation and crossover to evolve the solutions in order to find the best one(s). The three most important aspects of using genetic algorithms are:

1. Definition of the objective function,
2. Definition and implementation of the genetic representation, and
3. Definition and implementation of the genetic operators.

Once these three aspects have been defined, the generic genetic algorithms should work fairly well. Genetic algorithms belong to the class of stochastic search methods. Whereas most stochastic search methods operate on a single solution to the problem at hand, genetic algorithms operate on a population of solutions. If we are solving a problem, we are usually looking for some solutions, which will be the best among others. The space of all feasible solutions is called search space. Each point in search space represents one possible solution. Each possible solution can be marked by its value (or fitness) for the problem. With GA we look for the best solution among a number of possible solutions, represented by one point in the search space. All living organisms consist of cells. In each cell there is the same set of chromosomes. Complete set of genetic material (all chromosomes) is called genome. Particular set of genes in genome is called genotype. During reproduction, recombination (or crossover) first occurs. Genes from parents combine to form a whole new chromosome. The newly created offspring can then be mutated. Mutation means that the elements of DNA are a bit changed. These changes are mainly caused by errors in copying genes from parents.

The optimal control technique Upreti [41], needs the following inputs:

1. The number of state variables, their initial values, and constraints
2. The range of integration, its accuracy, the minimum step of integration, and the maximum step of integration

3. The number of step changes or stages for control function T_r
4. The minimum value of control domain, its maximum value, and a factor the size of control domain
5. A seed number to generate pseudo-random numbers
6. The number of inactive iterations needed to start the alteration of the logarithmic mapping with the linear mapping
7. The number of iterations of the optimal control technique
8. The number of bits to represent $\Delta u_{i,2}$, the number of cross-over sites, the probability of cross-over and mutation, and power index for the scaling of performance index.

3.3 Results

The genetic algorithms based optimal technique (Upreti [41]), is applied to both steady-state and unsteady-state models for two sizes of lab and industrial scale IR dryers to minimize the final humidity of sheet, H_s . Six equal-length stages of emitter heating units for the lab scale, and 42 equal-length heating units for the industrial size dryer are implemented. The maximum and the minimum ranges of the temperature of the radiants have been chosen as 800 and 50°C, respectively. Two constraints have been considered for this process. First, the maximum allowable temperature of the sheet, T_s during the drying process is 700°C. Second, the minimum humidity of sheet is 0.001 %. Violation of the first constraint, temperature of sheet, automatically deletes the chosen temperature of the radiants for that particular grid and moves on to find the next suitable temperature.

Violation of the humidity stops the program to note that there would be no more means to continue the optimization since the desire humidity has been reached.

The genetic algorithms optimal control has been considered for the following 4 cases:

1. Unsteady-state
 - c) Lab Scale
 - d) Industrial Scale
2. Steady-state
 - e) Lab Scale
 - f) Industrial Scale

3.3.1 Results for the Optimal Control of Unsteady-state Operation

For the lab scale IR dryer, six heating stages with just one fresh air injection unit have been considered. Figure 29 shows the variation of the humidity of sheet against the number of stages to demonstrate the application of the temperature of the radiants in different stages along the length of a lab scale dryer in unsteady-state operation. This figure shows that after applying the optimal control technique with the above conditions the final humidity of sheet drops to 12 %. Comparing with the results of the time when there was no optimal technique (Figure 5, $H_s=14.9\%$), there is an addition of 3% more reduction in the final humidity of sheet from the initial value of 50%. The velocity of sheet for the above section is chosen as $V_s=5$ m/min. The same time of 18 seconds is the traveling time for sheet to exit from dryer for the unsteady-state operation. These results

Figure 29. Temperature of the radiants profile versus number of heating stages for the unsteady-state operation of a lab scale infrared dryer ($V_s=5$ m/min).

show that for the lab scale dryers, the temperature of air and of sheet have higher values comparing to the time when the optimal technique has not been applied to the models. Table 6 summarizes the effects of the optimal control policy on the above state variables in lab scale dryer.

For the industrial size dryers, 42 heating stages implemented instead, with 3 different sets of fresh air injection units (1, 2 and 6). The reason for applying different injection units is to realize the effect of the fresh air application and to understand if increasing the number of air injection units helps to minimize the final humidity of sheet. Two contradictory explanations can be expressed for the advantage and disadvantage of applying multiple injection units:

First, the additional fresh air injection units help to reduce the final humidity of sheet by replacing wet air, which carries water vapor, and might undermine the effect of hot air on sheet and further evaporation of water from sheet. Second, at each injection point air, which has already been warmed up by the radiation heat, will be replaced with fresh and colder air. Therefore for a specific period of time, the sheet is under the effect of the lower temperature of the injected fresh air ($T_{a \text{ initial}}=25 \text{ }^{\circ}\text{C}$) before being heated again.

Figures 30 to 32 are the results for the multiple fresh air injection units. For an industrial scale infrared dryer in unsteady-state operation, the humidity of sheet has been plotted against the number of heating stages for different injection points and one velocity of sheet $V_s=30 \text{ m/min}$. The humidity drops to around 12 % for all three dryers with different

Table 6. The effect of the optimal control policy on the reduction of H_s for the unsteady-state operation at the exit of a lab scale infrared dryer, $V_s=5$ m/min, 1 Injection point and 6 heating stages.

Optimal Technique	H_s (%)	T_a (°C)	T_s (°C)	$t_{\text{travel}}(\text{sec})$
Benchmark	14.9	117	59.6	18
Optimal	11.994	121	63.1	

Figure 30. Temperature of the radiants profile versus number of heating stages for the unsteady-state operation of an industrial scale infrared dryer with one injection point ($V_s=30$ m/min).

Figure 31. Temperature of the radiants profile versus number of heating stages for the unsteady-state operation of an industrial scale infrared dryer with two injection points ($V_s=30$ m/min).

Figure 32. Temperature of the radiants profile versus number of heating stages for the unsteady-state operation of an industrial scale infrared dryer with six injection points ($V_s=30$ m/min).

injection units. Comparing the results with the same situation before applying the optimal technique, (Figure 8, 11 and 14), there has been a significant improvement in the reduction of the final humidity of sheet. A 10.6% additional reduction occurs in the final humidity of sheet (22.6 % to 12 % from its initial value of 50%), when there is one injection point in the dryer (Figure 8). A 10.01% additional reduction occurs (22.01 % to 12.00%) when the number of injection units is two (Figure 11), and finally 23.9 % to 12%, which is an additional reduction of 11.9% when number of injection units has been chosen as six (Figure 14).

One important achievement from these results is that for both before and after the technique application, increasing the number of the injection points does not affect the final humidity significantly, but has effects on the final temperature of sheet and of air, which could be very interesting for future investigation. For example in Table 5, which was presented earlier, temperature of the air drops to 207 °C from 279 °C if the number of the injection units increases to six from one unit in the same dryer. Temperature of sheet has the same effect and increasing the injection units from one to six causes a drop from 84.4 to 79°C in sheet temperature. This reduction is important for the final physical and mechanical properties of the material being dried, such as tensile strength and shrinkage. Cost and capital investment reduction can be studied because multiple injection units increase capital investment costs.

Variation of the state variables H_s , T_a , and T_s with respect to time and space upon the application of optimal radiant temperature function for the unsteady-state lab and

industrial size dryers are shown in Figures 33 to 36. Figure 33 shows the variation of the humidity of sheet and temperature of air and of sheet for times 4, 8, 12, 16, and 17 seconds of the drying process along the length of a lab scale dryer. The final humidity of sheet drops to 11.994% at the exit of dryer, comparing to 14.9 % before the optimal technique application. Temperature of air has a sharp drop at the middle of the dryer due to the application of the air injection unit and has a final value of 121 °C. This temperature before the application of the optimal control was calculated as 117 °C at the exit of the lab size dryer. Temperature of sheet also drops at the air injection point and finally rises to 63.1 °C at the exit of dryer. Before the technique application this temperature was calculated as 59.6 °C.

Figures 34 to 36 show the variation of the same variables for the industrial scale dryer with 1, 2, and 6 air injection units. Different drops in the temperature of air refer to the different number of air injection units. The traveling time for sheet with the velocity of 30m/min is 20 seconds and the number of grid points is chosen as 84. In these dryers, after applying the optimal technique, the final humidity of sheet has a lower value than the time before the technique application (as mentioned earlier). The final temperature of sheet and of air also have lower values comparing to the time before applying the optimal technique but the dryer with 6 air injection units has the lowest value in the final temperature of sheet and of air at the exit of dryer. It means that the number of injection units has a direct effect on the final temperature of sheet and of air. Table 7 summarizes the effects of optimal control policy on the above state variables in an industrial scale dryer for unsteady-state operation.

Figure 33- Variation of state variables with respect to time and space upon the application of optimal radiant temperature function for Figure 29.

Figure 34- Variation of state variables with respect to time and space upon the application of optimal radiant temperature function for Figure 30.

Figure 35- Variation of state variables with respect to time and space upon the application of optimal radiant temperature function for Figure 31.

Figure 36- Variation of state variables with respect to time and space upon the application of optimal radiant temperature function for Figure 32.

Table 7. The effect of the optimal control policy on the reduction of H_s for the unsteady-state operation at the exit of an industrial scale infrared dryer, $V_s=30$ m/min and 1, 2, and 6 Injection points and 42 heating stages.

Optimal Technique	t_{travel} (sec)	H_s (%)	T_a (°C)	T_s (°C)	No. of fresh air injection units
Benchmark	20	22.6	293.1	61	1
Optimal		12.002	206	60	
Benchmark		22.01	256.3	58.3	2
Optimal		12.001	160	54.2	
Benchmark		23.9	198	56.9	6
Optimal		12.076	99	44	

3.3.2 Results for the Optimal Control of Steady-state Operation

For the lab scale infrared dryer, six heating stage units with just one fresh air injection point have been implemented. The domain of control function, T_r , is within 50°C and 800°C while two constraints $T_{\text{max}} = 700^\circ\text{C}$ and $H_{\text{min}} = 0.001\%$ should also be satisfied. Figure 37 shows that for this dryer after applying the optimal control technique with all above-mentioned conditions, the final humidity of sheet has been dropped to 0.001 % (maximum reduction). Comparing with the results of the time when there was no optimal technique, (Figure 17, $H_s = 8.67\%$), there has been a significant improvement in the final humidity. The velocity of sheet for the above section is $V_s = 5\text{ m/min}$. This figure has been plotted for the humidity of sheet against the number of heating stages.

For the industrial scale dryer, 42 heating stages has been implemented, with 3 different sets of fresh air injection units (1, 2, and 6). The reason for applying different air injection units is the same as mentioned for the industrial size dryers in unsteady-state operation.

Figures 38 to 40 are the results for the steady-state industrial dryers with $V_s = 30\text{ m/min}$. The humidity of sheet has been plotted against the number of heating stages for different fresh air injection points. The final humidity of sheet has been dropped to around 0.001% (maximum reduction) for all three different number of the injection unit sets. Comparing the results under the same conditions, with the velocity of sheet as $V_s = 30\text{ m/min}$ before

Figure 37. Temperature of the radiants profile versus number of heating stages for the steady-state operation of a lab scale infrared dryer ($V_s=5$ m/min).

Figure 38. Temperature of the radiants profile versus number of heating stages for the steady-state operation of an industrial scale infrared dryer with one injection point ($V_s=30$ m/min).

Figure 39. Temperature of the radiants profile versus number of heating stages for the steady-state operation of an industrial scale infrared dryer with two injection points ($V_s=30$ m/min).

Figure 40. Temperature of the radiants profile versus number of heating stages for the steady-state operation of an industrial scale infrared dryer with six injection points ($V_s=30$ m/min).

applying the optimal technique (Figure 20, 23, and 26), shows that there has been a significant improvement in reduction of the final humidity from 2.54 % to 0.001 % for the oven with one injection point (Figure 38), and a reduction of 2.6 % to 0.001 % when the number of injection units is two (Figure 39), and finally reduction from 2.57 % to 0.001 % when number of injection units is 6 (Figure 40). As experienced over the unsteady-state condition, increasing the number of fresh air injection units does not effect the final humidity of sheet significantly even when the condition of having the second constraint $H_{smin}=0.001$ % was removed from the program to compare the final results.

Variation of the state variables H_s , T_a , and T_s with respect to space upon the application of optimal radiant temperature function for the steady-state lab and industrial size dryers are shown in Figures 41 to 44. Figure 41 shows the variation of the humidity of sheet and temperature of air and of sheet along the length of a lab scale dryer. The final humidity of sheet drops to 0.001% at the exit of dryer, comparing to 8.5 % before the optimal technique application. Temperature of air has a sharp drop at the middle of dryer due to the application of air injection unit and has a final value of 204 °C. This temperature before the application of the optimal control was calculated as 187 °C. Temperature of sheet has a smoother drop at the air injection point and finally ends on 80.28 °C at the exit of dryer. Before the technique application this temperature was calculated as 71.2 °C.

These results show that for the lab scale dryer, the temperature of air and of sheet have higher values as opposed to its non-optimal operation. Table 8 summarizes the effects of

Figure 41- Variation of state variables with respect to space upon the application of optimal radiant temperature function for Figure 37.

Figure 42- Variation of state variables with respect to space upon the application of optimal radiant temperature function for Figure 38.

Figure 43- Variation of state variables with respect to space upon the application of optimal radiant temperature function for Figure 39.

Figure 44- Variation of state variables with respect to space upon the application of optimal radiant temperature function for Figure 40.

Table 8. The effect of the optimal control policy on the reduction of H_s for the steady-state operation at the exit of a lab scale IR dryers, $V_s=5$ m/min, 1 fresh air injection point and 6 heating unit stages.

Optimal Technique	H_s (%)	T_a (°C)	T_s (°C)
Benchmark	8.5	187	71.2
Optimal	0.001	204	75.9

optimal control policy on the above state variables in the lab scale dryer for steady-state operation. Figures 42 to 44 show the variation of the same variables for the industrial scale dryer with 1, 2, and 6 air injection units. Different sharp drops of air temperature to its initial value refer to the different number of air injection units. In these dryers, the final humidity of sheet decreases after applying the optimal technique (as mentioned earlier). The final temperatures of sheet and of air also have lower values as opposed to its non-optimal operation. The dryer with 6 air injection units has the lowest value in the final temperature of sheet and of air. It means that the number of injection units has a direct affect on the final temperature of sheet and of air. Table 9 summarizes the effects of optimal control policy on the above state variables in an industrial scale dryer after the application of optimal technique.

Table 9. The effect of the optimal control policy on the reduction of H_s for the steady-state operation at the exit of an industrial scale IR dryer, $V_s=30$ m/min, 1, 2, and 6 Injection points, and 42 heating stages.

Optimal Technique	H_s (%)	T_a (°C)	T_s (°C)	No. of fresh air injection units
Benchmark	2.54	279	84.4	1
Optimal	0.001	250.7	74.5	
Benchmark	2.6	252	81.4	2
Optimal	0.001	234.45	69.4	
Benchmark	2.57	207	79.9	6
Optimal	0.001	179.8	68.9	

Chapter 4

Discussions and Conclusions

The optimal control technique has been applied to improve the efficiency of IR dryer by minimizing the exit humidity of sheet to be dried. The results for both lab and commercial size dryers with different velocity of sheet show that the final humidity of sheet at the exit of dryer drops significantly after the application of the optimal control policy. This reduction is about 19.5% for the lab scale dryer in unsteady-state operation with the velocity of sheet as $V_s=5$ m/min, where the final humidity of sheet drops to 11.994% from the value of 14.9% before the application. For the same dryer, but in a steady-state operation, the final humidity of sheet at the exit of dryer has an improvement of 99.9% after the application of the optimal control policy, when it drops to 0.001% from the non-optimal value of 8.5%. For the industrial size dryer, the final humidity of

sheet after applying the optimal technique at the exit of dryer with $V_s=30$ m/min has major improvements. In this dryer, the final humidity of sheet at the exit point in unsteady-state operation before the technique application is 22.6, 22.01, and 23.9% for 1, 2, and 6 fresh air injection units respectively. After applying the technique, the final humidity of sheet drops to 12.002, 12.001, and 12.076% for 1, 2, and 6 fresh air injection units, respectively. This is over 46% of improvement in the reduction of the final humidity for all 3 numbers of air injection units. For the steady-state operating industrial dryer, the final humidity of sheet at the exit of dryer before applying the technique is 2.54, 2.6, and 2.57% for 1, 2, and 6 fresh air injection units, respectively. After applying the optimal technique, the humidity drops to 0.001% (the ultimate reduction) for all three number of fresh air injection units, which is well over 99% improvement in the final humidity of sheet.

Increasing the velocity of sheet has a direct effect on its final humidity. The simulation of the models shows that in the lab size dryer, increasing the velocity of sheet from 5 to 10 m/min for the unsteady-state operation increases the final humidity of sheet from 14.9 to 34.9% from the initial humidity of 50%. For the steady-state operation, again in the lab scale dryer, the final humidity of sheet increases to 30% from 8.67% when the velocity of sheet increases from 5 to 10m/min.

For the unsteady-state operating commercial dryers, increasing the velocity of sheet from 30 to 40 m/min increases the final humidity of sheet from 22.6, 22.3, and 24 % to 27.4,

27.5, and 28.4% for 1, 2, and 6 fresh air injection units, respectively. For the steady-state industrial dryer, increasing the velocity of sheet from 30 to 40 m/min increases the final humidity of sheet from 2.54, 2.6, and 2.57% to 13, 13, and 12.7% for 1, 2, and 6 fresh air injection units, respectively.

The results for the transition of unsteady-state to steady-state operation were also studied. The results also show that before the transition of unsteady-state to steady-state occurs, the temperature of air has lower values, which justifies the higher humidity of sheet at the exit of dryer in unsteady-state operation. In the lab scale dryer, the final humidity of sheet at the exit of dryer in unsteady-state operation is 14.9% when the velocity of sheet is $V_s=5$ m/min. The humidity with the same conditions in the steady-state operation drops to 8.67%, which is a 42% improvement from the unsteady-state operation. For the velocity of sheet as 7 and 10 m/min, the final humidity of sheet at the exit of dryer in unsteady-state operation drops to 24.7, and 34.9%, respectively. After the transition to the steady-state operation, this reduction is 20.5, and 30% respectively, which is an improvement of 17 and 14%.

For the industrial scale dryer with one fresh injection unit, the final humidity of sheet at the exit of dryer in unsteady-state operation is 22.6% when the velocity of sheet is $V_s=30$ m/min. The same state variable with the same conditions in the steady-state operation drops to 2.54%, which is an 88% reduction from the unsteady-state operation. For the velocity of sheet as 35 and 40 m/min, and one fresh injection unit, the final humidity of

sheet at the exit of dryer reduces 68 and 52%, respectively for the transition of unsteady-state to steady-state operation, when it drops from 26.7, and 27.4% to 8.35, and 13% in steady-state operation.

For the unsteady-state industrial scale dryer with two fresh injection units, the final humidity of sheet at the exit of dryer is 22.3, 26.4, and 27.5% when the velocity of sheet is $V_s=30, 35, \text{ and } 40 \text{ m/min}$, respectively. After the transition to steady-state operation, the final humidity of sheet at the exit of dryer drops to 2.6, 7.85, and 13%, respectively. This indicates the reduction of 88, 70, and 52% from the results obtained in unsteady-state operation.

If the number of air injection units in this dryer is six, the final humidity of sheet at the exit of dryer for unsteady-state operation is 24, 27.1, and 28.4% for the velocity of sheet as $V_s=30, 35, \text{ and } 40 \text{ m/min}$, respectively. After transition to steady-state operation, the final humidity of sheet at the exit of dryer drops to 2.57, 8.16, and 12.7% for the velocity of sheet at $V_s=30, 35, \text{ and } 40 \text{ m/min}$, respectively. This is an 89, 69, and 55% reduction from the unsteady-state operation.

This work shows that increasing the number of air injection units has no significant impact on the final humidity of sheet at the exit of dryer. The reason is that replacing the wet air with fresh air helps getting rid of moisture at the expense of heat loss for dryer. This is seen in the results for the industrial size dryer, when the number of the injection

units has been increased to 2 and 6 stages. For the industrial dryer with the velocity of sheet at 30 m/min, the final humidity of sheet at the exit of dryer in unsteady-state operation is 22.6, 22.3, and 24% for 1, 2, and 6 fresh air injection units, respectively. For the same dryer with $V_s=35$ m/min, the final humidity of sheet at the exit of dryer drops to 26.7, 26.4, and 27.1% for 1, 2, and 6 fresh air injection units, respectively. If the velocity of sheet increases to $V_s=40$ m/min, the final humidity of sheet at the exit of dryer drops to 27.4, 27.5, and 28.4% for 1, 2, and 6 fresh air injection units, respectively. Like the lab size dryers, increasing the number of fresh air injection units along the length of dryer has negligible effects on the final humidity of sheet at the exit of the industrial continuous infrared dryers. For the steady-state operation, the same results are obtained.

The average temperature of the radiants, applied to the dryers during the optimal control application was also studied. Before applying the technique, the variables H_s , T_a , and T_s were studied under the constant temperature of the radiants as $T_r=600$ °C. After applying the optimal technique, for the lab scale dryer, the average temperature of the radiants applied in the unsteady-state operation is 585°C. Comparing to the temperature before the technique, there is a reduction of 15°C of the heat applied. For the lab scale infrared dryer in steady-state operation, the average temperature of the radiants applied is 602 °C, which is slightly higher than the constant $T_r=600$ °C before the application.

For the industrial scale dryer, the average temperature of the radiants applied to the dryer for 1, 2, and 6 air injection units in unsteady-state operation was 360, 389, and 459°C,

respectively. This is a significant reduction of 240, 211, and 141°C comparing to the constant temperature of the radiants $T_r=600$ °C before the optimal technique application. For the industrial dryer in steady-state operation, the average applied temperature of the radiants for 1, 2, and 6 fresh air injection units was 367, 431, and 546°C, respectively. This is an improvement of 233, 169, and 54 °C comparing to the constant temperature of the radiants $T_r=600$ °C before the optimal technique application. Comparing the state variable profiles under optimal control policy, for lab size dryers (steady-state and unsteady-state) the temperature of air and of sheet have higher values than those for non-optimal operation (Tables 6, 8 and Figures 33-36). For the industrial dryers, reverse results are observed. Due to longer drying length and more control stages, the temperature of sheet and air have lower values than those for non-optimal operation (Tables 7, 9 and Figures 41–44).

Chapter 5

Future Work

Based on this work, the following would be interesting topics for future work:

1. Multivariable optimal control problems incorporating air injections, and sheet velocity can be a major interest of the future investigation. The humidity can be affected at a constant radiant temperature if the velocity of the sheet is optimized accordingly. The constraint for this application is the production rate, since lower velocity of the sheet lowers the rate of the production.
2. Multiobjective performance index incorporating energy consumption can also be investigated as the next subject. Humidity of the sheet, energy/utility cost, number of fresh air injection units and number of heating stages can be optimized (minimized) together. The objective would be to find a minimized humidity percentage along with all or some of the other minimized mentioned functions.

Nomenclature

A	area, cm^2
C	Courant number
C_p	specific heat, J/g.K
D_a	diffusion coefficient of air, cm^2/s
D_h	hydraulic diameter, cm
E	electric power, kW
f_{sr}	shape factor
F	mass flow, g/s
h_{ra}	radiant source-air heat transfer coefficient, $\text{W/cm}^2.\text{K}$
h_{sa}	moist solid-air heat transfer coefficient, $\text{W/cm}^2.\text{K}$
h_c	convective heat transfer coefficient
H	moisture content, g/100g of dry solid
H_c	critical humidity of sheet, %
H_{si}	initial humidity of the sheet, %
H_{sb}	humidity of sheet at the boundaries, %
H_{des}	heat of desorption of water, J/g
H_{st}	saturated moisture of air (g/g on dry basis)
k_m	mass transfer coefficient, g/s.cm^2
k_c	thermal conductivity
L	length of the heating zone, cm
L_{inj}	fresh air injection point position along the dryer length, cm
M_{evap}	drying rate per unit area, g/s.cm^2

N_{Re}	Reynolds number, $\rho_a V_a D_h / \mu_a$
N_{Sc}	Schmidt number, $\mu_a / \rho_a D_a$
N_{Pr}	Prandtl number, $C_{pa} \mu_a / k$
P	atmospheric pressure, Pa
P_{st}	vapor pressure of water, Pa
q	rate of heat flow
q_x	flux of heat flow in direction normal to surface
T	temperature, K
T_a	temperature of air, K
$T_{a,b}$	temperature of air at the boundaries, K
T_{ai}	inlet temperature of air, K
T_s	solid temperature (cotton), K
T_{sb}	temperature of sheet at the boundaries, K
T_{si}	inlet temperature of sheet, K
T_r	temperature of radiant heater, K
T_{ra}	radiant emitter-air convection temperature, K
T_{rs}	radiant emitter-sheet radiation temperature, K
T_{sa}	sheet-air convection temperature, K
T_∞	temperature of process environment, far from surface
T_∞	temperature of process environment, far from surface, K
V	velocity, cm/s
W	width, cm

X_{cr}	critical moisture content
X_{eq}	equilibrium moisture of the solid
z	distance along the length of the dryer, cm
Δt	time interval, second
Δz	space interval, cm

Greek letters

α_p	parameter correcting the shape factor and emissivity
α_{ra}	parameter correcting, h_{ra}
α_{sa}	parameter correcting, h_{sa}
ε	emissivity
λ_l	latent heat of vaporization, J/g water
μ	viscosity, g/cm.s
ρ	density, g/cm ³
ρ_s	thin solid density, g/m ²
σ	Stefan-Boltzmann constant, W/cm ² .K ⁴

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Appendix A

Correlations and constants used in solving the PDE's and ODE's:

- Air is considered to be equally distributed between six ducts inside the dryer. Its average velocity is given by:

$$V_a = \frac{(F_a / 6)}{\rho_a A_a}$$

- For a flowing gas confined between two parallel planes the convective heat transfer coefficient is defined by Treybal (1980):

$$h_{ia} = \alpha_{ia} \rho_a V_a C_{p_a} R_e^{-0.29} P_r^{-0.67}, \quad i=r \text{ or } s \quad (\alpha_{ia} \text{ based on experiment}).$$

- From the Chilton-Colburn analogy, the mass transfer coefficient of vapor between the moving sheet and air is expressed by Treybal, (1980):

$$k_m = \frac{h_{sa}}{Cp_a} S_c^{-0.67} P_r^{0.67}$$

- Air in direct contact with the moving sheet is assumed to be saturated all along the dryer length and therefore its relative humidity is computed as:

$$H_{st} = \frac{18}{29} \frac{P_{st}}{P - P_{st}}$$

- Saturated air pressure P_{st} is calculated from the following relation Hougen (1974):

$$\text{Log}_{10}(P_{st}) = \frac{2034}{T_s} + 6.2151 - \exp(-4.77 \times 10^{-5} (T_s - 105.50)^2)$$

- $f_{sr} = (-1.86e - 6W_s^2 + 2.36e - 5W_s + 1.33e - 4d^2 - 7.01e - 3d + 0.113$

- $N_{Re} = \frac{\rho_a V_a D_h}{\mu_a}$

- $N_{Pr} = \frac{C_{pa} \mu_a}{K_a}$

- $N_{Sc} = \frac{\mu_a}{\rho_a D_a}$

- $H_t = \begin{cases} 1 & \text{if } H_s > 30; \\ H_s/30 & \text{otherwise} \end{cases}$

$$\bullet \quad H_{\text{des}} = \begin{cases} 0 & \text{if } H_s > 12; \quad \checkmark \\ -0.24H_s^3 + 6.18H_s^2 - 65.02H_s + 340.72 & \text{otherwise} \end{cases}$$

- $\lambda_L = -0.61T_s + 767.5$
- $k_1 = (-200k_m)/\rho_s$
- $k_2 = (2W_r h_{ra}) / (4 C_{pa} \rho_a A_a)$
- $k_3 = (2W_s h_{sa}) / (4 C_{pa} \rho_a A_a)$
- $k_4 = (2W_s k_m C_{pv}) / (4 C_{pa} \rho_a A_a)$
- $k_5 = (800W_a \sigma \epsilon f_{sr}) / (C_{ph} \rho_a W_s)$
- $k_6 = (-200W_a h_{sa}) / (C_{ph} \rho_s W_s)$
- $k_7 = (-200W_a k_m) / (C_{ph} \rho_s W_s)$
- $k_8 = 100(C_{ps} / C_{ph})$

Appendix B

Unsteady-state model:

For humidity of the sheet:

For $i=0$:

$$\frac{dH_{s,i}}{dt} = \frac{-V_s(H_{s,i+1} - H_{s,0})}{2dz} + \alpha_{Sa} k_1 H_{st} H_t$$

For $i=N_{\text{grid}}-1$:

$$\frac{dH_{s,i}}{dt} = \frac{-V_s(3H_{s,i} - 4H_{s,i-1} + H_{s,i-2})}{2dz} + \alpha_{Sa} k_1 H_{st} H_t$$

For other values of i :

$$\frac{dH_{s,i}}{dt} = \frac{-V_s(H_{s,i+1} - H_{s,i-1})}{2dz} + \alpha_{Sa} k_1 H_{st} H_t$$

For temperature of the air:

For $i=0$:

$$\frac{dT_{a,i}}{dt} = \frac{-V_a(T_{a,i+1} - T_{a,0})}{2dz} + \alpha_{Ra} k_2 (u - T_{a,i}) + \alpha_{Sa} k_3 (T_{s,i} - T_{a,i}) + \alpha_{Sa} k_4 H_{st} H_t (T_{s,i} - T_{a,i})$$

For $i=(N_{\text{grid}}/2)-2$:

$$\frac{dT_{a,i}}{dt} = \frac{-V_a(3T_{a,i} - 4T_{a,i-1} + T_{a,i-2})}{2dz} + \alpha_{Ra}k_2(u - T_{a,i}) + \alpha_{Sa}k_3(T_{s,i} - T_{a,i}) + \alpha_{Sa}k_4H_{st}H_t(T_{s,i} - T_{a,i})$$

For other values of i :

$$\frac{dT_{a,i}}{dt} = \frac{-V_a(T_{a,i+1} - T_{a,i-1})}{2dz} + \alpha_{Ra}k_2(u - T_{a,i}) + \alpha_{Sa}k_3(T_{s,i} - T_{a,i}) + \alpha_{Sa}k_4H_{st}H_t(T_{s,i} - T_{a,i})$$

For temperature of the sheet:

For $i=0$:

$$\frac{dT_{s,i}}{dt} = \frac{-V_s(T_{s,i+1} - T_{s,0})}{2dz} + (\alpha_Rk_5(u^4 - T_{s,i}^4) + \alpha_{Sa}k_6(T_{s,i} - T_{a,i}) + \alpha_{Sa}k_7H_{st}H_t$$

$$(\lambda_L + H_{des}))/ (k_8 + H_{s,i})$$

For $i=N_{\text{grid}}-1$:

$$\frac{dT_{s,i}}{dt} = \frac{-V_s(3T_{s,i} - 4T_{s,i-1} + T_{s,i-2})}{2dz} + (\alpha_Rk_5(u^4 - T_{s,i}^4) + \alpha_{Sa}k_6(T_{s,i} - T_{a,i}) + \alpha_{Sa}k_7H_{st}H_t$$

$$(\lambda_L + H_{des}))/ (k_8 + H_{s,i})$$

For other values of i :

$$\frac{dT_{s,i}}{dt} = \frac{-V_s(T_{s,i+1} - T_{s,i-1})}{2dz} + (\alpha_R k_5 (u^4 - T_{s,i}^4) + \alpha_{Sa} k_6 (T_{s,i} - T_{a,i} + \alpha_{Sa} k_7 H_{st} H_t (\lambda_L + H_{des})) / (k_8 + H_{s,i}))$$

Steady-state model

For humidity of the sheet:

$$\frac{dH_s}{dz} = \alpha_{Sa} k_1 H_{st} H_t / V_s;$$

For temperature of the air:

$$\frac{dT_a}{dz} = \{ \alpha_{Ra} k_2 (T_r - T_a) + (\alpha_{Sa} k_3 + \alpha_{Sa} k_4 H_{st} H_t) (T_s - T_a) \} / V_a;$$

For temperature of the Sheet:

$$\frac{dT_s}{dz} = (\alpha_R k_5 (T_r^4 - T_s^4) + \alpha_{Sa} k_6 (T_s - T_a) + \alpha_{Sa} k_7 H_{st} H_t (\lambda_L + H_{des})) / (k_8 + H_s) / V_s;$$

Appendix C

The fourth and fifth-order algorithm for Runge-Kutta method with Cash-Carp development.

The fourth-order Runge-Kutta method:

$$y_{i+1} = y_i + ah$$

$$h = \text{Stepsize}$$

$$a = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

and:

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1h)$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2h)$$

$$k_4 = f(x_i, y_i + k_3h)$$

The fifth-order Runge-Kutta method:

$$\boxed{y_{i+1} = y_i + ah}$$

$$h = \text{Stepsize}$$

where:

$$a = \frac{1}{90}(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6)$$

and:

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{4}h, y_i + \frac{1}{4}k_1h)$$

$$k_3 = f(x_i + \frac{1}{4}h, y_i + \frac{1}{8}k_1h + \frac{1}{8}k_2h)$$

$$k_4 = f(x_i + \frac{1}{2}h, y_i - \frac{1}{2}k_2h + k_3h)$$

$$k_5 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{16}k_1h + \frac{9}{16}k_4h)$$

$$k_6 = f(x_i + h, y_i - \frac{3}{7}k_1h + \frac{2}{7}k_2h + \frac{12}{7}k_3h - \frac{12}{7}k_4h + \frac{8}{7}k_5h)$$

The Cash-Carp Runge-Kutta method:

$$\boxed{y_{i+1} = y_i + ah}$$

$$h = \text{Stepsize}$$

$$a = \left(\frac{37}{378} k_1 + \frac{250}{621} k_3 + \frac{125}{594} k_4 + \frac{512}{1771} k_6 \right)$$

for the 4th order algorithm

where:

$$h = \text{Stepsize}$$

$$a = \left(\frac{2825}{27648} k_1 + \frac{18575}{48384} k_3 + \frac{13525}{55296} k_4 + \frac{277}{14336} k_5 + \frac{1}{4} k_6 \right)$$

for the 5th order algorithm

and:

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{5}h, y_i + \frac{1}{5}k_1h)$$

$$k_3 = f(x_i + \frac{3}{10}h, y_i + \frac{3}{40}k_1h + \frac{9}{40}k_2h)$$

$$k_4 = f(x_i + \frac{3}{5}h, y_i + \frac{3}{10}k_1h - \frac{9}{10}k_2h + \frac{6}{5}k_3h)$$

$$k_5 = f(x_i + \frac{3}{4}h, y_i - \frac{11}{54}k_1h + \frac{5}{2}k_2h - \frac{70}{27}k_3h)$$

$$k_6 = f(x_i + \frac{7}{8}h, y_i + \frac{1631}{55296}k_1h + \frac{175}{512}k_2h + \frac{575}{13824}k_3h \\ + \frac{44275}{110592}k_4h + \frac{253}{4096}k_5h)$$