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Heat Transfer To Small Cylinders Within A Packed Bed

Christopher Penny
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HEAT TRANSFER TO SMALL CYLINDERS WITHIN A PACKED BED

by

Christopher Penny

Bachelor of Engineering (BEng)

Ryerson, University

Toronto, 2007

A thesis

presented to Ryerson University

in partial fulfillment of the

requirements for the degree of

Master of Applied Science

in the Program of

Mechanical Engineering

Toronto, Ontario, Canada, 2009

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ABSTRACT

Heat Transfer to Small Cylinders Within a Packed Bed

Masters of Applied Science

2009

Christopher Penny

Mechanical Engineering

Ryerson University

Heat transfer to small cylinders within a porous media has been experimentally and analytically studied extensively over a varying degree of sample and particle sizes and fluid flow regimes. In general, the observations, trends and empirical correlations developed for these systems do not accurately extrapolate down to small cylinders operating under the packed bed condition. The objective of this research is to develop an empirical correlation that expresses the Nusselt number of small cylinders immersed horizontally within a packed bed subject to forced convection heat transfer, in terms of the pertinent test parameters and material properties.

A set of seven small cylinders ranging in size from 1.27 to 9.53mm were resistively heated within a 311mm diameter lab-scale packed bed. The porous medium in which the samples were immersed was fine alumina oxide sand, with mean particle sizes ranging from 145 to 330 μ m. Four separate Type K thermocouples were used to measure temperatures at pertinent locations within the apparatus: bed temperature, inner sample temperature, left and right sample temperatures. The apparatus was operated under flow rates up until incipient

fluidization. The trends observed in this research compared well with published data, though the correlations developed from other research consistently under-predicted the heat transfer capacity within the packed bed. The correlation that was developed for calculating the mean Nusselt number was accurate to within $\pm 15\%$ for the entire range of tested and published data.

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U_{min}	Minimum Fluidization Velocity [m/s]	dt	Throat diameter of Venturi Flow Meter [m]
V	Voltage [V]	g	Acceleration due to Gravity [m/s^2]
V_s	Volume of Sample [m^3]	G	Fluidizing Gas Mass Flow Rate [$kg/m^2 \cdot s$]
x	Position [m]	G_{min}	Fluidizing Gas Mass Flow Rate at Minimum Fluidization [$kg/m^2 \cdot s$]
		Gr	Grashof Number (Equation 2.4)
		h	Heat Transfer Coefficient [$W/m^2 \cdot K$]
		h_{avg}	Average Heat Transfer Coefficient [$W/m^2 \cdot K$]
		H_{mf}	Bed Particle Height at Minimum Fluidization [m]
B	Throat-to-Pipe Diameter Ratio		
	Expansion of Air	k	Thermal Conductivity of Sample Material [$W/m \cdot K$]
ΔP	Pressure Drop Across the Bed	K	Permeability [m^2]
ΔT	Temperature Difference Between Hot and Cold Surfaces	k_{eff}	Effective Thermal Conductivity [$W/m \cdot K$]
ϵ	Bed Voidage Fraction	k_g	Fluidizing Gas Thermal Conductivity [$W/m \cdot K$]
ϵ_b	Bed Voidage	k_p	Thermal Conductivity of Particle [$W/m \cdot K$]
E_{mf}	Bed Voidage Fraction at Minimum Fluidization	L	Sample Length [m]
μ_g	Fluidizing Gas Viscosity [$N \cdot s/m^2$]	L_c	Characteristic Length [m]
ρ_g	Density of Gas/Fluid [kg/m^3]	Nu	Nusselt Number
ρ_s	Density of Solid Particles [kg/m^3]	Nu_{exp}	Experimental Nusselt Number
ρ_{ss}	Resistivity of Stainless Steel [$\Omega \cdot m$]	Nu_{mean}	Mean Nusselt Number
ν	Kinematic Viscosity [m^2/s]	P	Perimeter of Samples [m]
ψ	Sphericity	P_m	Absolute Pressure Measured in Flow Meter Flow Tube [kPa]
θ	Excess Temperature	P_c	Power [W]
		Pe	Peclet Number ($Re \cdot Pr$)
		Pr	Prandtl Number
		q_c	Heat Conduction into the CV [W]
		q'	Heat Generation per Unit Length [W/m]
		q'''	Volumetric Heat Generation [W/m^3]
		Q	Air Volumetric Flow Rate [m^3/s or CFM or l/min]

NOMENCLATURE

A	Ergun Coefficient	82
A_c	Cross Sectional Area of Sample [m^2]	83
A_b	Effective Bed Area [m^2]	83
A_s	Outside Surface Area of Sample [m^2]	83
A_T	Throat Area of Venturi Flow Meter [m^2]	83
Ar	Archimedes Number (Equation 2.22)	83
C_v	Venturi Discharge Coefficient	83
C_{pg}	Specific Heat of Fluidizing Gas at Constant Pressure [J/kgK]	83
D	Diameter of Sample [m]	83
d_{in}	Venturi Pipe Diameter [m]	83
d_p	Particle Diameter [m]	83
d_s	Outer Diameter of Sample [m]	83
d_T	Throat diameter of Venturi Flow Meter [m]	83
g	Acceleration due to Gravity [m/s^2]	83
G	Fluidizing Gas Mass Flow Rate [$kg/m^2 \cdot s$]	83
G_{mf}	Fluidizing Gas Mass Flow Rate at Minimum Fluidization [$kg/m^2 \cdot s$]	83
Gr	Grashof Number (Equation 2.4)	83
h	Heat Transfer Coefficient [$W/m^2 \cdot K$]	83
h_{avg}	Average Heat Transfer Coefficient [$W/m^2 \cdot K$]	83
H_{mf}	Bed Particle Height at Minimum Fluidization [m]	83
I	Current [A]	83
k	Thermal Conductivity of Sample Material [$W/m \cdot K$]	83
K	Permeability [m^2]	83
k_{eff}	Effective Thermal Conductivity [$W/m \cdot K$]	83
k_g	Fluidizing Gas Thermal Conductivity [$W/m \cdot K$]	83
k_p	Thermal Conductivity of Particle [$W/m \cdot K$]	83
L	Sample Length [m]	83
L_c	Characteristic Length [m]	83
Nu	Nusselt Number	83
Nu_{exp}	Experimental Nusselt Number	83
Nu_{mean}	Mean Nusselt Number	83
P	Perimeter of Samples [m]	83
P_m	Absolute Pressure Measured in Flow Meter Flow Tube [kPa]	83
P_o	Power [W]	83
Pe	Peclet Number ($Re \cdot Pr$)	83
Pr	Prandtl Number	83
q_x	Heat Conduction into the CV [W]	83
q'	Heat Generation per Unit Length [W/m]	83
q'''	Volumetric Heat Generation [W/m^3]	83
Q	Air Volumetric Flow Rate [m^3/s or CFM or L/min]	83

r	Radius of Sample [m]
r_i	Inner Radius of Sample [m]
r_o	Outer Radius of Sample [m]
R	Resistance [Ω]
Ra	Rayleigh Number
Re_d	Particle Reynolds Number ($\rho_g U_d p / \mu_g$)
Re_D	Sample Reynolds Number ($\rho_g U_d s / \mu_g$)
Re_{mf}	Reynolds Number at Minimum Fluidization ($\rho_g U_{eff} D / \mu_g$)
t	Time [s]
T_{film}	Film Temperature [K or $^{\circ}C$]
T_c	Surface Temperature of Sample [K or $^{\circ}C$]
T_m	Air Temperature Measured in Flow Meter Flow Tube [$^{\circ}C$]
T_T	Terminal Temperature [K or $^{\circ}C$]
T_b	Bed Temperature
U	Gas Velocity [m/s]
U_{eff}	Effective Bed Velocity [m/s]
U_{mf}	Minimum Fluidization Velocity [m/s]
V	Voltage [V]
V_c	Volume of Sample [m^3]
x	Position Along Sample [m]

GREEK SYMBOLS

β	Throat-to-Pipe Diameter Ratio of Venturi Flow Meter/Coefficient of Volumetric Expansion of Air
ΔP	Pressure Drop Across the Bed/Pressure Difference Across Venturi Flow Meter
ΔT	Temperature Difference Between Sample Surface and Bed [K or $^{\circ}C$]
ϵ	Bed Voidage Fraction
ϵ_b	Bulk Bed Voidage
ϵ_{mf}	Bed Voidage Fraction at Minimum Fluidization
μ_g	Fluidizing Gas Viscosity [$N\cdot s/m^2$]
ρ_g	Density of Fluid/Gas [kg/m^3]
ρ_p	Density of Solid Particles [kg/m^3]
ρ_{res}	Resistivity of Stainless Steel [$n\Omega\cdot m$]
ν	Kinematic Viscosity [m^2/s]
ψ	Sphericity Factor
θ_T	Excess Terminal Temperature

CHAPTER 1

1 INTRODUCTION

1.1 INTRODUCTION

The heat transfer between immersed objects suspended within a porous material has been extensively studied since the nineteenth century due to the efficient and effective heat transfer capacity such a system provides. Heat transfer derived from flow through a porous medium has several engineering applications in areas such as industrial and geophysical contexts, thermal insulation techniques, geothermal processes, chemical and nuclear engineering and hydrology applications. This research looks to continue the development of augmented heat transfer through a porous medium by considering the forced convection heat transfer to an immersed cylinder in a packed bed of sand. Specifically, fluidized beds have become increasingly popular for heat-treating steel wire [1,2]. Industrial applications applying fluidized bed technology to wire heat treating results in large fluidized beds, with an approximate bed surface area of 20m^2 . There is the possibility in a fluidized bed this size that there could be areas where localized de-fluidization could occur, due to distributor clogging, control drift, or other such failures. Within these areas of de-fluidization, the wires may be exposed to a packed bed condition. Under the packed bed condition the flow rates through the porous medium do not surpass the minimum fluidization velocity (U_{mf}), or alternately do not provide enough upward force to move the particles within the bed; thus the particles are in a stagnant state. Under this condition the heat transfer within the bed is much less than that of the same system operating under the fluidized state. In some comparisons, the fluidization results in heat transfer rates are as much as 16 times greater than in the packed bed condition [3]. The effect on the wire metallurgy of these localized low-level heat transfer rates is not

known. In order to determine under which conditions unacceptable wire metallurgy will occur requires the knowledge of the heat transfer rate to the wire within the packed bed.

The heat transfer application to objects imbedded in a porous medium has been the subject of numerous scholarly research papers with respect to varying geometries, shapes, sizes and porous materials; but generally, the research has been conducted in the fluidized flow regime, with very little data pertaining to the condition in which the bed is in the packed condition. This research investigates the heat transfer rate, or Nusselt Number, to small cylinders immersed in a porous media subject to such stagnant flow conditions, ranging from essentially zero flow up to and including incipient fluidization. The goal of this study is to generate an empirical correlation in which to accurately relate the Nusselt number to the specific system performance and geometrical characteristics and ultimately understand and define the effects of low heat transfer to heat treated wires. Previous work has been conducted by a range of researchers [4 - 9] with similar geometry to the work herein, though the particle sizes were an order of magnitude greater than that used in this research. There is generally a high level of uncertainty in correlations when scaling them down to work at geometry sizes not used in its formation (See Figure 1.4). Due to this inconsistency in scaling correlations, a separate correlation must be developed for the specific geometry sizes used in this work, specifically wires sizes ranging from 1.27 – 9.53mm (0.05 - 0.375") with alumina sand particle sizes ranging from 145-330 μ m.

Conceptually, in the context of wire heat treatment, a packed bed is simply a fluidized bed operating within a specific performance range where fluidization does not occur. A basic description of the application of fluidized beds within the industry is discussed hereafter, specifically pertaining to the heat treating process within the steel wire manufacturing industry. Manufacturing of steel wires begins with the molten metal being poured into various sized molds, some such molds are Ingots, Blooms, Slabs and Billets whose use depends on the desired size, weight and shape of the final configuration. Wire rod, the raw material for wire is formed from heated Billets that are passed through a die to form a continuous rod about 12mm in diameter and wound into large coils. The wire manufacturers then cold-draw the wire by

pulling it through successively smaller dies until the final desired diameter is reached. The drawn wire is work-hardened with high strength but low ductility. Depending on the specific application of the steel wires, they may require an additional heat treatment process applied to them. Traditionally these steel wires have been immersed into a tank of molten lead (operating at approximately 730°C) and are allowed to heat to approximately tank temperature as they are continuously passed through the tank. This particular process is effective in that heat transfer from molten lead is very fast, minimizing required immersion time with little heat addition required. However, it has been noted that heating lead to above 500°C generates noticeable lead vapor and fume levels and provides for hazardous working conditions [10]. Since molten lead heat treating furnaces operate at a temperature level where such hazardous conditions are present, many jurisdictions have banned the use of such devices due to the detrimental health hazards and environmental affects generated through their use. Therefore, steel wire manufactures are required to find other means of heat-treating their steel wire. The application of fluidized beds in the heat-treating of steel wires provides comparative heat-treating capacities without the environmental and health hazards associated with the presence of the molten lead. This study extends the range of flow speeds (in the lower flow regimes) developed within [3], as the physical flow and heat transfer rate characteristics developed for fluidized bed regimes do not accurately extrapolate into the packed bed condition. As such, the work presented here studies the heat transfer to small cylinders within a lab-scale packed bed of fine grit aluminum oxide sand operating primarily at ambient conditions, while some consideration is given to packed bed operation with heated air (to approximately 400°C) as the working fluid.

1.2 LITERATURE REVIEW

A review of the literature on the applications and theories pertaining to heat transfer within a porous media with and without an immersed cylinder are presented below. Though the focus of this research is on the forced convection heat transfer to a cylinder embedded in a

packed bed, the inclusion of some research on the general heat transfer in a packed bed is included as it provides useful background knowledge of porous media flow properties and characteristics.

1.2.1 HEAT TRANSFER TO POROUS MEDIA

The original investigation of fluid flow through a porous medium dates back to the nineteenth century from the work done by Darcy [11]. He was the first to perform experiments from which mathematical formulations were derived pertaining to porous media. The Darcian model is popular in porous media investigations as its performance within its range of validity is very consistent and accurate. The Darcy model is founded on several assumptions, which lend to its simplistic nature; these assumptions include a neglect of friction due to macroscopic shear, which translates into not satisfying the condition of no-slip velocity on solid boundaries. As the Darcian model was developed based on laminar flow, where viscous forces dominate and inertial forces are essentially disregarded, the model does not handle the inertial forces present in fast flows. The final assumption is that the spatial variation of the matrix porosity is neglected. However, particle spatial variation is responsible for flow channeling at or near the solid boundaries, which results in an irregular velocity profile across the length of the bed. Several researchers have explored the use of the Darcian model and have verified its use in laminar flow systems; though as work has continued in the field it has been recorded that the effects that the Darcy Law neglects can greatly affect the predicted heat transfer rates within the system. As such the following studies reflect experimental and analytical work done on non-Darcian effects, as the approximations imposed by Darcy have since been shown to be adequate in some situations, but inadequate in others.

One particular study pertaining to the boundary and inertia effects on heat transfer in porous media was completed by Vafai and Tien [12]. The study primarily focused on flow through a porous medium near a solid boundary. The work applied the volume-averaging technique to the fundamental flow and energy equations pertinent to porous media flow. One assumption in this study that demands notice is the assumption of a constant-porosity medium,

which neglects the effect of porosity variations near the wall. A finite-difference approach was utilized to solve the linearized, volume averaged flow and energy equations. This approach, though an approximation in itself, was verified by increasing the number of grid points across the system. The results from this analysis state that Darcy's law is applicable in the domain of $\Phi < 1$ and $\Psi < 6.0 \times 10^{-3}$, where Φ and Ψ are the boundary parameter $(Pr/\gamma L)^2$ and inertia parameter $(Re/\gamma^2 L)$ respectively, where γ is the porosity shape parameter $(2d_p/D)$. Conclusions drawn from this study are that with a higher permeability and lower fluid viscosity the inertial effects increase, as the velocity gradients near the wall will also increase. Ultimately, for flows outside of the range above Vafai *et al.* determined that inertial effects are no longer negligible. Based on the ranges listed above this research falls well below the minimum values, thus the inertial effects are negligible for the parametric range covered in this study.

Another aspect, which the Darcy model does not consider, is that of transverse thermal dispersion across the length of the packed bed. Several studies have been performed, mainly theoretically on this aspect, two of which will be discussed in some detail here. Both Cheng and Vortmeyer [13] and Hsu and Cheng [14] studied the effect of thermal dispersion with a porous medium, each with a different analytical technique. Cheng *et al.* used a method of matched asymptotic expansions to provide a solution of the Brinkman equation for flow in a packed bed between parallel plates. They also included a wall function to model the transverse thermal dispersion to take into consideration the higher thermal resistance commonly noticed near the wall. This particular analysis neglects the inertial effects of the momentum equation and as such is viable only for low Re flows. Also, due to the neglect of the inertial term the velocity distribution is independent of Re and dependent primarily on the porosity shape parameter (γ), as an increase in flow speed would lead to a dependence on Re. A matching technique was applied in determining several constants, which apply to the wall function. Within this process a number of uncertainties exist as matching the exact temperature profiles from those obtained experimentally to those predicted (using an asymmetric temperature profile) is quite difficult. In addition, the surface heat flux is a difficult value to match between the theoretical and experimental work. In determining the heat flux the temperatures at the wall and at the centerline must be known, though often the temperature at the centerline is not recorded

explicitly by authors; as such an estimation is made. This estimation can be a source of error as at low Re because the temperature variation between wall and centerline is often quite far from being based on a linear relationship. Results obtained offer a prediction of the surface heat flux within the packed bed. At low γ (≈ 0.074) a smaller heat flux is observed, while at a higher value of γ (≈ 0.37) a higher predicted heat flux is obtained. Overall, the wall function introduced for the transverse thermal dispersion near the wall provides a satisfactory manner to predict wall phenomenon seen in experimental results. With respect to the present research the porosity shape factor, defined as the ratio of the particle diameter to the half-width of the packed bed, the maximum γ is equal to 0.0021; thus, the transverse thermal dispersion is negligible in the system herein and is omitted from further consideration.

An experimental analysis by Renken and Poulikakos [15] presents results from the study of forced convection in the thermally developing region of a channel. This channel is packed with fluid-saturated beads and bounded by two solid boundaries maintained at a constant temperature. The experimental method used measures the wall and the fluid/porous temperatures as they relate to proximity to the channel wall; from which the results are to be compared to the theoretical model developed by [12]. The apparatus in this study consists of a water tank, pump, fluid saturated bed (divided into heated and non-heated sections), rotameter, temperature/data acquisition hardware. The bed was uniformly filled with glass spheres with a nominal diameter of 3mm. 148 thermocouples were used throughout the bed to accurately determine the temperature variations. The experiment considered such factors as macroscopic shear, flow inertia and variable porosity; all variables neglected by Darcy Law approximations, though thermal dispersion was not considered. If the results obtained by [11] are considered accurate, then if thermal dispersion was considered in a future study the temperature profile would develop faster, due to those effects. Another important result was the effect of flow channelling, which is basically the porous medium and the variations in its matrix as related to proximity to the wall. It was noticed that the Nusselt number measurement within said region produces a higher heat transfer rate than predicted by the Darcy flow model. This result makes intuitive sense because the porous matrix is generally denser [12, 13], which

provides for more surface-to-surface contact, ultimately encouraging the increase in heat transfer rates in that specific region.

Wu and Hwang [16] experimentally and theoretically investigated the flow and heat transfer inside packed and fluidized beds, focusing on the heat transfer between the particles and fluidizing fluid. The primary components of the experimental apparatus consisted of a wind tunnel, blower, a test section and a heating plate. The test section was filled with randomly packed spherical particles with diameters in the range of 5mm – 10mm, with an average porosity in the range of 0.38-39 (for the packed bed condition). The heated inlet flow supplied thermal energy to the heat transfer surface of the porous medium. The outlet fluid temperature was recorded and the heat transfer coefficient was then estimated based on the temperature difference between the inlet and outlet. Hwang *et al.* was able to further confirm that the decrease of the porosity of the bed increased the heat transfer coefficient, which is consistent with results observed thus far [14, 15]. Results that were obtained were consistent with results obtained by previous researchers. This was attributed to the packing characteristics of the bed used, where, for the fluidized bed measurements, the bed was constructed with an aligned porous matrix; therefore stating that a randomly packed bed can produce better heat transfer rates. General conclusions compare well with other conclusions made on similar studies (Hwang *et al.* [17]). It was observed that the heat transfer is primarily affected by the particle Reynolds number (742-2530) and the porosity of the bed matrix. It was also determined that with a ratio of $L/d_p \geq 20$ that the assumption of fully developed flow can be assumed, as the flow drag continually decreases beyond this range. With respect to the present research being conducted the smallest L/d_p ratio that will arise in the testing is 775, considerably greater than the minimum value given. Therefore the assumption of fully developed flow throughout the packed bed column is considered valid.

A theoretical extension of work done by [12 , 16] was performed by Yee and Kamiuto [18]; where the effects of viscous dissipation on forced convection with cylindrical packed beds was analysed. The study included many important physical phenomenon, including: boundary layer approximations, bed porosity variation based on distance from wall, and overall fluid solid

thermal equilibrium. Ultimately, the goal was to determine whether the viscous dissipation effects within a packed bed can be neglected (within a given range), as stated by [12]. It was discerned that as the Peclet number increases past 10^4 the effects of viscous dissipation are significant. Which, when considered, is intuitive because as the Peclet number is increased the local temperature is increased, and thus the Nusselt number is further increased. For packed beds with the adiabatic wall boundary condition the effect of viscous dissipation is influential at increasing r_o/d_p and Re_D . For an isothermal wall boundary condition, and below $Pe = 10^4$, the effects of additional heat generation due to viscous dissipation is negligible, which is consistent with the conclusions of [12]. Based on the range above the analysis of this research may neglect the effects of viscous dissipation as the maximum operational Reynolds number Re_D is 82 and the Prandtl number is 0.0385, providing for a Peclet number well below the stated range.

Another study on the general effects of heat transfer in a packed bed at moderate Reynolds number values was presented by Dekhtyar *et al.* [19], in which a semi-empirical relation for the Nusselt number was determined. Glass spheres, ranging in diameter of 0.9-8.9 mm were used as the porous matrix, while a solution of filtered water and glycerine was used as the fluid medium. A cylindrical tube was used to hold the porous media. Generally the results that were obtained on the heat transfer between the flowing fluid and the porous matrix were consistent with previous results recorded by other researchers. One pertinent conclusion was that at $Re_p > 34$ the temperature profile across the channel becomes flattened, given in Figure 1.1. This flattening of the temperature profile is indicative that the wall zone contribution to the bed thermal resistance has increased. Thus at increased operational flow rates the flow channelling at the walls of the tube causes the mean temperature of the bed to 'even out' across the inner diameter. One other distinguishing result was the relation developed to determine the irregularities of the behaviour of the heat transfer on the channel wall; given by $Nu_w \approx (d_p/D)Pe^{1/2}$ for $Re_D < 100$.

Figure 1.1: Experimentally obtained temperature profile [19]

Throughout 2004 – 2006 several studies were published (Sheikh [20], Suhalka *et al.* [21], Schroder *et al.* [22], Wen and Ding [23], and Laguerre [24]) focusing on the heat transfer within porous media at low flow rates; thus within the range of our packed bed criteria. The results of the studies all conclude that correlations of low Reynolds number flow provide inconsistent results. Schroder *et al.* determined that experimental data below $Re_D < 200$ in particular produces large variances; whether being compared between theoretical and experimental results, or between experimental and experimental results. Laguerre focused on determining a heat transfer relation between the bed and the bed wall; using an experimental setup using glass spheres with a nominal diameter of 38mm. It was determined that the air velocity plays a direct role in the heat transfer in the bed while the temperature difference between the wall and the air (fluid medium) does little to influence the overall bed heat transfer rates.

1.2.2 HEAT TRANSFER FROM EMBEDDED CYLINDERS IN PACKED BEDS

1.2.2.1 FORCED CONVECTION CRITERIA

As this particular research is focusing on forced convection heat transfer to embedded wires in a porous media, the range for defining such heat transfer effects must be known. The following is a brief discussion of several researchers' work and their respective criteria for forced convection heat transfer. Considering the particle sizes and porosity of the porous medium used in this research, it is intuitive that because the pore sizes are small it would be expected that the buoyancy effects would be negligible. Further discussion and consideration of this behaviour is discussed in the following chapter; suffice to say that forced convection is indeed the means of transferring heat within this system.

One of the earliest studies performed was done by Fand and Keswani [4], where a range was determined within which the primary mode of heat transfer is forced convection as opposed to free, or natural, convection. The experiment consisted of placing a cylindrical, electrically heated test specimen horizontally in a water tunnel, such that the flow was perpendicular to the axis of the sample. The sample diameter was 12.7mm (0.5 inches). In this case there was no porous medium; instead water was used as the flowing medium. This study was an extension of a study done by Fand and Keswani [25] in 1972 where they considered the influence of property variation on forced convection to liquids. The work in that case was restricted to $Gr/Re^2 < 0.5$; whereas the newest study by Fand and Keswani [4] a range of $0.5 < Gr/Re^2 < 2$. The tests were run at bulk temperatures between 10-60°C. When the results were analyzed the variance between the theoretical and experimental predictions at a range of $Gr/Re^2 < 0.5$ was 5%, whereas at $Gr/Re^2 = 2$ the error was in excess of 10%. Generally, as stated by [4], 5% is considered an acceptable error in heat transfer; as such the range $Gr/Re^2 < 0.5$ was defined as heat transfer due to pure forced convection. This result was later verified by Fand and Phan [5] through an unbiased error reduction process. The study by [5] was an experimental analysis of heat transfer from a horizontal cylinder within a porous medium. The

cylinder was submerged in a test section filled with glass spheres, the cylinder was then resistively heated and the heat transfer between the cylinder and the bed was measured. Both forced convection ranges were determined based on a completely different experimental process and apparatus. On that basis the agreement observed between the two values can be considered accurate as the original result was verified through a separate means. Cheng [26] performed an analytical analysis on free and forced convection about inclined surfaces in porous media. Though agreement was reached in that the Gr/Re parameter governs the conventional forces present in the flow, the range at which those forces vary was different than that determined by [4, 5, 25]. Also Cheng determined that the inertial forces (represented by the Reynolds number) was less influential, as such the denominator of the parameter (Re) was not squared, as was done by [4, 5]. Cheng determined that when $Gr/Re < 0.15$ forced convection is the prominent force. This is quite conservative when compared to the value previously discussed. An explanation that justifies the difference is the fact that [26] neglected the component of buoyancy force normal to the inclined surface. As the surface is moved to a horizontal orientation that assumption will be no longer valid; thus altering the range of forced convection effects. Another theoretical study, by Badr [27], focused on mixed convection about a horizontal cylinder in a cross stream. The flow is assumed to be 2D, and as such the end effects on the velocity and temperature are neglected (though it has since been noted by Ahmed [28] that the assumption of $L \gg D$ is valid for a circular-cylinder). As a result of the analytical study it was noticed that the average Nusselt number increase did not exceed 4%, when $Gr/Re^2 < \approx 0.2$; which states that the natural convection does not play a role in the heat transfer capabilities of the system below that range. Also, a plot was developed by [29] which depicted the pressure distribution around the cylinder surface. For $Re = 20$ and $Gr = 100$ it was noticed that the forced convection distribution mirrored that of the mixed convection case; leading to the conclusion that at $Gr/Re^2 < 0.25$ the primary effects are indeed forced convection. Clearly this value is not as restrictive as the one developed by [6], but it remains that it's less than that determined by [4, 5, 26]. This can be attributed to the fact that the experiment was not based on porous media, but was a result of cylinder heat transfer to cross-flow of air. Huang *et al.* [29] developed an analytical study from which the effects of natural

convection, or buoyancy effects, were studied. A numerical scheme was employed to solve the transformed non-linear boundary layer equations. It was determined, through using the parameter $Ra/Pe^{3/2}$, that the error between pure forced convection and mixed convection heat transfer exceeded 5% above the value of 0.17 (for a horizontal cylinder, when $Ra/Pe^{3/2} > 0.17$) due to neglecting the free convection component of heat transfer. As such the range, as per [29], for forced convection to a horizontal cylinder is within the range of $Ra/Pe^{3/2} \leq 0.17$.

Table 1.1: The Range of Forced Convection Heat Transfer Influence

Range of Forced Convection Heat Transfer Influence

<i>Researcher</i>	<i>Parameter</i>	<i>Value</i>
Fand & Keswani [4]	Gr_D/Re^2	< 0.5
Fand & Phan [5]	Gr_K/Re^2	< 0.5
Cheng [26]	Gr_K/Re	< 0.15
Badr [27]	Gr_D/Re^2	< 0.25
Huang [29]	$Ra/Pe^{3/2}$	< 0.17

1.2.2.2 EXPERIMENTAL STUDY REVIEW

In the previous section the study by Fand and Phan [5] was mentioned, on the basis of their forced convection criteria; the remainder of their study will be discussed here (along with several others), as it uses similar geometry and experimental process as studied in this work. The scope of this study was to experimentally determine the effects of forced and natural convection heat transfer from a cylinder embedded in a porous medium. The apparatus used, given in Figure 1.2, consisted of a water tunnel, heated test cylinder and a pump.

Figure 1.2: Test Section of Water Tunnel Apparatus [5]

The glass spheres within the test section had a nominal diameter of 3mm (with an overall test section porosity of 0.3606) and the test specimen had a diameter of 11.45mm. To increase the accuracy of the investigation the effect of variation in porosity of the medium at the wall was included. The relation used was based on a definition constructed by [4], originally based on effects arising from natural convection heat transfer from a cylinder in a porous medium. The original relation was constrained by the particle diameter-to-test cylinder diameter (d_p/D) equal to 0.35. Table 1.2 depicts the range of experimental values used for this investigation.

Table 1.2: Range of Experimental Values for Fand and Phan [5]

Range of Experimental Values	
$19 \leq T_b \leq 27$	[°C]
$2.5 \leq \Delta T \leq 27$	[°C]
$0.034 \times 10^{-3} \leq U \leq 0.077$	[m/s]
$0.5 \leq Re_D \leq 1280$	

Flow was pumped into the test section through a series of screens to prevent clogging and to promote uniform velocity throughout the bed. A thermocouple placed upstream of the test section was used to measure the bulk temperature of the fluid, in this case: water. The cylinder was heated through a central electrical heating coil which was fed by a DC power

supply. A thermocouple mounted below the cylinder was used to determine the surface temperature. The results that were obtained indicated that within the Darcy regime that the natural convection effects are not negligible, especially when coupled with a large ΔT and a low velocity. The primary goal of this study was to provide Nusselt number correlation for the main flow scheme's seen in porous media (as commonly defined by researchers in the field), those being: Darcy, Forchheimer, turbulent, and the regimes that span between them. That goal was obtained, though little correlation with other results was present. The exact correlations determined are displayed in the section summary. The Darcy flow range is that regime where the assumptions that were originally made by Darcy [11] still apply; generally a very slow flow with $Re_d < 2.3$, as defined by [4]. The Forchheimer flow scheme is defined by the range of $5 < Re_d < 80$, where the flow is still considered laminar but inertial effects become significant. The turbulent flow range is where the flow through the system is no longer laminar, but turbulent and the inertial effects are the primary means of heat transfer within the system, bounded by $Re_d > 120$. That goal was obtained, though little correlation with other results was present. The exact correlations determined are displayed in the section summary. From Figure 1.2 the diameter of the test apparatus is 86.6mm. With a test specimen of 11.45mm there would be significant blockage within the apparatus. This blockage would limit the uniform flow through the apparatus as the flow would have to turn, or accelerate, around the test specimen thus altering the local velocity rates and ultimately the local heat transfer values around the surface of the specimen.

Badr [27], as was previously discussed, provided a theoretical investigation of laminar mixed convection from a horizontal cylinder in an air-flow cross stream, where no porous medium is present. As the analysis is based on an analytical method the results of the study will be discussed, but details pertaining to the process involved will not be mentioned. Not only does the ratio Gr/Re^2 dictate the range of heat transfer present in the system it also has great influence on the velocity and thermal boundary layer development. It was also noted that an increase in Gr related to an increase in the velocity gradient at the surface of the cylinder, which results in reducing the pressure and creating areas of suction. It was determined that for the forced convection regime the Nusselt number is at a maximum at $\theta = 180^\circ$ (the bottom of

the sample) and a minimum at $\theta = 0^\circ$ (the top of the sample). Badr and Pop [29] extended the previous study [27] by performing a similar study, but with the rod buried in a porous medium, rather than being subject only to the cross-flow of air. In this theoretical investigation Darcy's law is assumed to hold, in that the viscous drag and inertial terms within the momentum equation are ignored, based on their influence being negligible at low particle Re values. The overall result was similar to that obtained in Badr's previous study [27] in that the maximum Nusselt number occurred at $\theta = 180^\circ$ and the minimum at $\theta = 0^\circ$. In addition to those results it was determined that within the forced convection regime the temperature distribution across the radius of the cylinder approached a linear relationship, while once natural convection terms were considered the relationship became less linear and a small decrease in radial distance resulted in a large change in temperature across the surface, which is consistent with boundary layer solutions obtained by [14].

Fand *et al.* [30] continued work on non-Darcian effects regarding cylinders embedded in porous media by establishing empirical correlations that account for the duct wall effect and dispersion. This particular study was an attempt to encompass the effects that other researchers had neglected and have a large scope, in terms of flow regimes, as to fill a void in the available data base. The procedure used to conduct the experiment is as follows: two isothermal electrically heated cylinders having different diameters (see Table 1.3) are inserted transversely into the test section. The test section was packed with porous media (soda lime glass spheres) of varying diameters. Table 1.3 lists the ranges of some of the experimental parameters used in this analysis.

Table 1.3: Range of Experimental Values for Fand *et al.* [30]

Range of Experimental Values

Sample Cylinder Diameter, d_s	8.509-11.450 [mm]
Particle Diameter, d_p	2.098-5.969 [mm]
Range of ΔT	2.9-46.6 [°C]
Range of Re_d	0.23-450

The cylinders were maintained at various temperatures by controlling the electric current running through them, while being subjected to varying fluid velocities (either oil or water). Though wall effects in the channel were considered, the end effects of the sample were not considered, as it was considered an infinite cylinder. Fand and Thinakaran [31] designated that a $D_c/d_p \geq 40$ must be achieved before that assumption can be accurately applied. An empirical relation was developed based on the 565 data points that takes into account the test cylinder diameter, the particle diameters, the varying porosity and the various dimensionless flow parameters (Re_D , Pr , and Re_d); which is given in the following section. A final result worth noting, with regards to details stated earlier, is that within the Darcy flow regime the limiting Nu was found to be 2; which is consistent with results obtained by [32, 33].

Nasr *et al.* [7] studied the forced convection heat transfer from a cylinder embedded in a packed bed, on an experimental basis – which is essentially the same topic as the work reported herein. Air was the working fluid for this analysis. The effects of the particle diameter and thermal conductivity were studied to determine the overall effect upon the heat transfer rate. The particle properties that were used are given in Table 1.4.

Table 1.4: Particle Properties used in the Experimental Analysis by Nasr et al. [7]

Material	Particle Diameter, d_p [mm]	Particle-to-Sample Ratio d_p/d_s	k_p [W/m·K]	k_p/k_s	Porosity, ϵ
Aluminum	3.24	0.255	200	7605	0.37
	6.33	0.498			0.38
	12.33	0.963			0.39
Alumina	2.77	0.218	40	1520	0.36
	6.64	0.523			0.37
	9.79	0.771			0.38
Glass	2.85	0.224	1.01	38	0.37
	6.00	0.472			0.37
	13.53	1.065			0.39

Figure 1.3: Experimental Apparatus for Nasr et al. [7]

The cylinder used in the system had a diameter of 12.7mm. Figure 1.3 depicts the experimental apparatus used in this study; where the test chamber, blower and the measurement instrumentation were the primary pieces of equipment. The cylinder was heated internally by an electrical cartridge heater powered by a DC power supply. A total of five thermocouples were used around the circumference of the cylinder; three positioned at 7.66cm from the end and two positioned at 2.5cm from the end. The test section was filled with 16cm of particles (overall length of the test section was 17.8cm). This depth value was based on a conservative depth estimate to eliminate undesirable boundary effects arising from the presence of the screen. Separate runs with a depth of 12cm were completed with no variance in the results noticed. The results of the analysis showed that the heat transfer within a packed bed is up to 7 times greater than for a cylinder in cross-flow of air [32, 34]. It was also noted that a decrease in the particle size resulted in an increase in the heat transfer coefficient, which correlates with results obtained by [32, 33]. The results, when compared to work done by [35] were reasonable, though the slopes of the values were inconsistent. Cheng [35] provided a theoretical study addressing the problem of the augmented heat transfer to an object

immersed in a porous medium through employing the boundary layer approximation to the energy equation in conjunction with Darcy's law, ultimately determining an expression for the variation of the local Nusselt number over an isothermal cylinder under forced flow conditions, given in Table 1.6. This inconsistency between Nasr et al. [7] and Cheng [35] is expected though as the correlation developed by [35] does not fully account for variations in the thermal conductivity of the packing materials. It was also noted that especially with the larger particle diameters disagreement arose, which was attributed to the boundary layer assumption becoming invalid. In the case of the research reported herein that is of little consequence as the largest particle diameter is 0.330mm. The empirical correlation developed by Nasr et al., given in Table 1.6, represented 95% of the experimental data within a $\pm 15\%$ range.

Shortly after publishing their experimental study Nasr *et al.* [8] completed an analytical study to theoretically determine the effects of thermal dispersion within the test chamber. For this study, particle diameters that were used were in the range of 3.24-13.53 mm (with an average porosity of 0.38) and the diameter of the test sample was 12.7mm. It was noticed, for the larger particle diameter cases, that the effective thermal conductivity (ratio of fluid thermal conductivity to effects enhanced by thermal dispersion) was lower than for the smaller diameter cases. This was assumed to be inconsistent with the other results based on the fact that the cylinder and particle diameters are very close in size, and thus the volume averaging technique employed is no longer suitable. Through development of the empirical correlation for the effective thermal conductivity it became apparent that the dispersive conductivity was independent of the packing material, but exhibited a very strong dependence on the overall particle size. Overall the results that were obtained were consistent with predictions based on the boundary layer theory; in that: (a) increase in Reynolds number relates to an enhanced heat transfer rate, (b) an increase in the Darcy number resulted in a decrease in heat transfer and (c) the Nusselt number increases with the Prandtl number. The greatest fallback from this analysis was the neglect of the porosity variation (which has been found to influence the heat transfer rate) and the fact that effective thermal conductivity was based on a global function of variables; where a local function would produce more accurate predictions.

Studies completed by Ahmed and Yovanovich [28], Ichimiya [36], Bharti and Chhabra [37] Khan *et al.* [38], and Wakao and Kaguei [39] continued to examine the effects of forced convection heat transfer to cylinders; either in cross-flow of air or in a packed bed condition. The studies compared well with other results, though [28] and [38] noticed that at low Re the variances in data are noticeable. Ahmed and Yovanovich [28] noted that this variance was due mainly to large L/D assumptions used to simplify the problem to 2D.

Table 1.5: Overall Range of Pertinent Experimental Values

	Overall Range of Experimental Values	Average Value
Porous Particle Size [d_p]	0.095 – 38 [mm]	4.29
Porous Particle Materials	Sand, Nylon, Glass, Aluminum, Alumina, Soda-Lime Glass	N/A
Cylinder Sample Size [d_s]	0.125 – 20 [mm]	14.3
Reynolds Number	$\sim 0 < Re < 10000$	450

Table 1.6: Nusselt Number Relations as Developed by Several Researchers

Reference	Equation	Restrictions	Heat Transfer Mechanism
Fand and Keswani [4]	$Nu = (0.255 + 0.699Re_d^{0.5})Pr^{0.29}$	$d_p/d_s \leq 0.35$	Forced Convection from flowing fluid to cylinder
Fand and Phan [5]	$Nu = 2.17(0.255 + 0.299Re_d^{0.5})Pr^{0.188}Re_d^{0.230}$	$0.5 < Re_d \leq 3$ $d/D \leq 0.35$	Packed Bed, from cylinder embedded in porous medium
Fand and Phan [5]	$Nu = 2.15(0.255 + 0.299Re_d^{0.5})Pr^{0.154}Re_d^{0.126}$	$3 < Re_d \leq 100$ $d/D \leq 0.35$	Packed Bed, from cylinder embedded in porous medium
Fand and Phan [5]	$Nu = 1.48(0.255 + 0.299Re_d^{0.5})Pr^{0.290}Re_d^{0.179}$	$Re_d > 100$ $d/D \leq 0.35$	Packed Bed, from cylinder embedded in porous medium
Kunii and Levenspiel [6]	$Nu = 2 + 1.8Re^{0.5}Pr^{0.33}$	N/A	Packed Bed: from porous medium to flowing gas
Nasr <i>et al.</i> [7]	$Nu = 0.53 \left(\frac{D}{d_p} \right)^{0.114} \left[Pe_d^{0.66} \left(\frac{k_p}{k_g} \right)^{-0.0174} \right]$	$1 < (D/d_p) < 5$ $10 < (k_s/k_f) < 7600$	Packed Bed: from immersed cylinder to flowing gas
Fand <i>et al.</i> [9]	$Nu = 1.248Re_d^{0.5}Pr^{0.3534}(0.3534Re_d^{0.05355}) \left[\arctan \left(\frac{d_s}{d_p} \right)^{0.5} \right]^{0.5467}$	Darcy Flow: $Re_d > 2.3$	Packed Bed, from isothermal cylinder in liquid saturated porous medium
Cheng [35]	$Nu = 1.0157Pe_d^{0.5}$	N/A	Packed Bed: from immersed cylinder to flowing gas
Wakao and Kaguei [39]	$Nu = 2 + 1.1Pr^{\frac{1}{3}}Re^{0.6}$	N/A	Packed Bed: from porous medium to flowing gas

1.3 PARAMETRIC RANGE OF RESEARCH SUMMARY

Based on the above experimental and theoretical studies the following assumptions are imposed on the research herein:

- The inertial effects are neglected, on the basis of the range specified by [12].
- The transverse thermal dispersion is neglected, based on the range determined by [13, 14].
- With regards to the range obtained by [16, 17] the flow through the bed is assumed to be fully developed flow; as the L/d_p ratio is much greater than the minimum value.
- Viscous dissipation is neglected as the operational Peclet number is well below the maximum value, as studied by [18].
- Forced convection is assumed to be the primary means of heat transfer within the packed bed based on ranges developed by [4, 5, 7, 35]. Calculations supporting this is given in the following chapter.
- The effective thermal conductivity is used in the calculation of the Nusselt number and Prandtl number, as specified by [4, 5, 6, 7, 9].

Essentially the same assumptions that Darcy [11] originally made apply to the geometry and physical characteristics of this research, as stated above. Thus the use of the relation obtained by [11], which relates the velocity within a packed bed to the pressure gradient through the bed, given in Chapter 2, is substantiated. The following, Figure 1.4, relates several of the above correlations as they relate to the geometry used in this work, though the individual parametric ranges differ from that used in this work, most often the porous media sizes are an order of magnitude larger than that used herein. From Figure 1.4 the correlations vary greatly across the range, where the largest variance is at the point of minimum fluidization, occurring at $G/G_{mf} = 1$. Though the relations depicted have a relatively common lower (or limiting) Nusselt number the discrepancy at the higher flow rates substantiates the

research herein as there is no accurate agreement between relations. Thus an empirical correlation must be established based on the specific sample geometry and sand particle properties that the research herein entails, that is, as mentioned, wires sizes ranging from 1.27 – 9.53mm (0.05 - 0.375”) with alumina sand particle sizes ranging from 145-330 μ m.

Figure 1.4: Comparison of Published Correlations for a 7.94mm Cylindrical Sample in a Packed Bed of 145 μ m Alumina Particles [90 Grit]

CHAPTER 2

2 THEORETICAL CONSIDERATIONS

2.1 INTRODUCTION

Since a packed bed is a 'condition' of a functioning or potentially non-functioning fluidized bed there is some overlap in theoretical knowledge between packed beds and fluidized beds that is of importance to the reader. The following sections discuss the general characteristics of a packed bed, briefly mentioning the flow ranges through a fluidized bed, and continue to develop the particle properties as they pertain to the heat transfer and flow mechanics through the bed. Further development on topics solely applicable to packed beds is then discussed, ranging from: forced convection criteria, thermal conductivity, Darcy's Law, the development of the minimum fluidization velocity, the derivation of the temperature profile across the cylindrical sample and the means of correcting the interior sample temperature, as recorded by the thermocouple to the exterior, surface, temperature required.

2.2 CHARACTERISTICS OF PACKED BED SYSTEMS

A packed bed is characterized by four basic components, namely a container, a gas distributor, a powder (or particles) and a source of fluid [40]. The working fluid, usually a gas, is passed upwards into the bed of contained particles that are supported by a porous base plate, the distributor. The distributor plate provides uniform gas flow into the bed (through an even dispersion through the plate due to its inherent porosity), where the gas continues to flow until it escapes at the surface of the powder. All fine powders have a very large specific area, according to Geldart - 1 m^3 of $100\mu\text{m}$ particles has a surface area of $30,000\text{m}^2$ [40]. As the gas flow is progressively increased, the upward force acting on the contained particles reaches a

critical value at which point the drag force is equivalent to the gravitational force. Thus, the contained particles are suspended within the container based only on the force interaction. At the point where the drag and gravitational forces are equal, the state of the bed is defined as “incipient fluidization”, meaning that any further increase in gas velocity will cause the system to become fully fluidized, see Figure 2.1. The velocity at which incipient fluidization is apparent is likewise defined as the minimum fluidization velocity, U_{mf} . This value is established based on physical properties associated with both the gas and the contained particles. This parameter defines the maximum operational velocity to be implemented in this research, as velocities rates below this represent the packed bed condition.

Figure 2.1: Degree of Fluidization [40]

2.3 PARTICLE PROPERTIES

As there are only a few pertinent components within a packed bed system, the particles (or powder) play a vital role in the effectiveness of the particular heat transfer application. Several different types of particles can be used for varying applications, all having their own physical properties that influence, both explicitly and implicitly, the behavior within the packed bed system. Accurately establishing the fundamental physical parameters of the particles, such as size, shape, density and size distribution prove to be one of the most complex elements within the design and functional application of such a system.

2.3.1 PARTICLE SIZE AND SHAPE

For a particle of any shape, other than sphere, there are several ways to define its size. Allen [41] defines twelve separate ways to define a particle's size! In this case, only two definitions will be mentioned as they apply more directly to packed beds. The *volume diameter* d_v is defined as the diameter of a sphere having the same volume as the particle. The second definition, the *surface/volume diameter* d_{sv} is given as the diameter of a sphere having the same external surface area/volume ratio as the particle.

These two definitions can be related through the sphericity factor ψ , which is defined as:

$$\psi = \frac{\text{surface area of equivalent volume sphere}}{\text{surface area of the particle}} = \frac{d_{sv}}{d_v} \quad (2.1)$$

The application of such a factor is needed when the bed is composed of non-spherical particles, which is often the case, as it is for the current research. From the definition a spherical particle would be defined as having $\psi = 1$, while all other shapes would be in the range of: $0 < \psi < 1$. Table 2.1 provides a list of sphericities of some common solids, including the alumina sand used in this research.

Table 2.1: Sphericity Factor for Some Common Particle Types

Particle Type	Sphericity Factor (ψ)
Glass Beads	1.00
Round Sand	0.92-0.98
Alumina Sand	0.9
Common Salt	0.84
Crushed Coal	0.75
Crushed Glass	0.65

2.3.2 PARTICLE DENSITY AND BED VOIDAGE

The density of a particle is defined as:

$$\rho_p = \frac{\text{mass of a single particle}}{\text{volume the particle would displace if its surface were non - porous}} \quad (2.2)$$

This volume calculation includes the voids inside the particle, whether or not they are open or closed. This density definition should not be confused with the density of the entire bed, defined as the bulk density ρ_B , which includes the spaces between the particles within the bed. The particle density ρ_p is considered the hydrodynamic density, as it is defined on the basis of the size and shape that gas encounters travelling through the system. A common relationship that is defined on the basis of the two discussed density relations is the bed voidage, denoted as ϵ , defined as:

$$\epsilon = \frac{\text{volume of bed} - \text{volume of particles}}{\text{volume of bed}} = 1 - \frac{\rho_B}{\rho_p} \quad (2.3)$$

The bed voidage fraction is the ratio of total bed volume to the volume of particles; essentially defining the fraction of 'free space' within the bed. As defining the particle density is often a difficult task, especially with fine powders, it follows that determining the bed voidage fraction would also be difficult. There are several factors that affect the bed voidage: (a) particle shape – where the lower the sphericity, the higher the voidage within the bed, (b)

particle size – for a loosely packed condition the bed voidage decreases with an increase in particle size; the particle size has less influence on a densely packed bed voidage fraction, and (c) size distribution – the wider the size spread between the individual particles, the lower the bed voidage. Work completed by Partridge and Lyall (42) gives a variation of packed bed voidage in comparison to particle size, for narrow size distribution; illustrated in Table 2.2. The bed voidage fraction used in this research is $\epsilon = 0.4$.

Table 2.2: Bed Voidage Fraction as it Relates to Particle Size

$d_p (\mu m)$	550	460	390	330	230	140	82	72
ϵ	0.422	0.432	0.440	0.437	0.507	0.563	0.590	0.602

2.4 FORCED CONVECTION CRITERIA

As there is no clear dividing line between natural and forced convection heat transfer within porous media several researchers have found it helpful to design a range or criterion in which the natural convection effects can be neglected. Forced convection is applicable when the fluid flow is driven by external means, such as a fan or a pump. Natural convection is defined as the convective heat transfer when the flow is induced by the buoyancy effects that arise from density differences caused by temperature differences in the flow. An analysis of scholarly work done on this topic was summarized in Chapter 1. The forced convection ranges as developed by those authors is summarized here again for convenience.

Table 2.3: Upper Range of Forced Convection Effects

<i>Researcher</i>	<i>Parameter</i>	<i>Value</i>
Fand & Keswani [4]	Gr_D/Re^2	< 0.5
Fand & Phan [5]	Gr_K/Re^2	< 0.5
Cheng [26]	Gr_K/Re	< 0.15
Badr [27]	Gr/Re^2	< 0.25
Huang [29]	$Ra/Pe^{3/2}$	< 0.17

In order to verify that the work in this study can neglect the buoyancy effects (the effects of natural convection), the physical flow and particle properties have to be analyzed on the basis of established parametric ranges. From Table 2.3 it is apparent that the forced convection to a horizontal cylinder subjected to cross flow in a porous media is directly proportional to Gr_D/Re_D^2 , where Gr_D is the Grashof number, and Re_D is the Reynolds number. The work completed by [4] did not involve porous media, but the work completed by [5] found empirical evidence to suggest that the same range is applicable to porous media if Gr_D is replaced by Gr_K . Thus the dimensionless variables used in defining the forced convection range are:

$$Gr_K = \frac{gKd_s\beta\Delta T}{\nu^2} \quad (2.4)$$

$$Re_D = \frac{\rho_f U_{mf} d_s}{\mu_f} \quad (2.5)$$

where g = Acceleration due to gravity

$K = \frac{\varepsilon^3 a_p^2}{A(1-\varepsilon)^2}$ = permeability

A = Ergun coefficient = 150

β = Coefficient of Volumetric Expansion of Air = $1/T$

ΔT = Difference in temperature between heated sample and packed bed = $T_s - T_b$

ν = kinematic viscosity of air

When inputting the physical properties into the above two equations the lowest velocity for each sand grit was used as that would cause the Reynolds number to be as small as possible; as the Reynolds number is in the denominator of the criterion it would provide the means of obtaining the largest parameter ratio. The results for 50, 70, and 90 sand grits are given in Table 2.4. From the forced convection limits listed in Table 2.3, it is apparent that even the most conservative restriction on forced convection, imposed by [26], could be applied to the physical characteristics of this research, as the Gr/Re ratio falls significantly below the free convection heat transfer range. Based on this analysis, it can be concluded that the dominant heat transfer mechanism occurring over the entire parametric range of this research is indeed forced convection.

Table 2.4: Forced Convection Range as Applied Specifically to Research Herein

Criterion	Sample Diameter [mm/in]	50 Grit Sand 330µm	70 Grit Sand 203µm	90 Grit Sand 145µm
Gr_K/Re_D^2	1.27/0.050	2.007E-04	4.204E-04	7.518E-04
	2.11/0.083	1.209E-04	2.533E-04	4.529E-04
	3.18/0.125	8.029E-04	1.682E-04	3.007E-04
	4.76/0.1875	5.353E-05	1.121E-04	2.005E-04
	6.35/0.250	4.014E-05	8.409E-05	1.504E-04
	7.94/0.3125	3.212E-05	6.727E-05	1.203E-04
	9.53/0.375	2.676E-05	5.606E-05	1.002E-04
Gr_K/Re_D	N/A	2.178E-04	1.939E-04	1.853E-04

2.5 EFFECTIVE THERMAL CONDUCTIVITY

In the design of equipment used for interactions involving gases and solids (porous media), the proper values of the effective thermal conductivities must be estimated as simply and as accurately as possible. On the basis of several theoretical studies reported by [7,43,44] it has been found that the effective thermal conductivities can be separated into two terms; one of which is independent of the fluid flow and the second that is dependent on the lateral mixing of the fluid in the packed beds. The varying mechanisms of heat transfer within a packed bed are given in Figure 2.2. The heat transfer mechanisms that are independent of fluid flow are, given by [44]:

1. Thermal conduction through solid particle
2. Thermal conduction through the contact surfaces of the two particles
3. Radiant heat transfer between surfaces of two particles
4. Radiant heat transfer between neighbouring voids.

Figure 2.2: Heat Transfer Mechanisms in a Packed Bed [44]

The heat transfer mechanisms that are dependent on the fluid flow are:

5. Thermal conduction through the fluid film near the contact surface of the two particles
6. Heat transfer through convection, between solid-fluid-solid
7. Heat transfer by lateral mixing of fluid

Theoretical calculations were completed by [44] on heat transfer mechanism 5. It was concluded that the majority of the heat transfer is through the thin films of fluid near the contact point between the particles. Based on this, it can then be assumed that the fluid flow has little effect on heat transfer mechanism number 5, except if operating with large Reynolds numbers. Whilst operating under small Reynolds numbers the gas film, or boundary layer, between adjacent particles is proportionally larger than for faster flows and as such heat transfer mechanisms 1, 3, 4 and 5 are the predominant heat transfer components within a packed bed. On the basis of the above arguments the pertinent parameters that would affect the effective thermal conductivity within a packed bed are the gas thermal conductivity k_g , the particle thermal conductivity k_p , and the overall bed porosity ϵ . Several theoretical and empirical models for the determination of this value have been published [45 - 47]. In the review of varying effective thermal conductivity relations [9] recommends the use of the model proposed by [47], evaluating the stagnant effective thermal conductivity, given by:

$$k_{eff} = k_g \left\{ 1 - \sqrt{(1 - \varepsilon)} + \frac{2\sqrt{1 - \varepsilon}}{1 - \lambda B} \times \left[\frac{(1 - \lambda)B}{(1 - \lambda B)^2} \ln\left(\frac{1}{\lambda B}\right) - \frac{B + 1}{2} - \frac{B - 1}{1 - \lambda B} \right] \right\} \quad (2.6)$$

where

$$B = 1.25 \left[\frac{1 - \varepsilon}{\varepsilon} \right]^{\frac{10}{9}} \quad (2.7)$$

$$\lambda = \frac{k_g}{k_p} \quad (2.8)$$

Another correlation developed by [44] is based upon the same governing variables, but provides for less extensive computational efforts. It is defined based on a packed bed of fine, highly conductive solid particles with a motionless gas. Recommendations from [4] state that the use of a stagnant thermal conductivity relation in a packed bed is more suitable to the flow characteristics within the bed. The second stagnant effective thermal conductivity relation is:

$$k_{eff} = \frac{k_g(1 - \varepsilon)}{\frac{k_g}{k_p} + \varphi} \quad (2.9)$$

where φ = ratio of the effective thickness of fluid film adjacent to the contact surface of two solid particles to the average diameter of packing = l_v/D_p

The variable φ is determined from published experimental data, given in Figure 2.3. For a given bed porosity $\varepsilon = 0.4$, $\varphi = 0.032$; from Figure 2.3 and mentioned explicitly in [44]. The final effective thermal conductivity equation to be considered is also from [43], specifically pertaining to a packed bed with a stagnant gas, given as:

$$k_{eff} = \varepsilon k_g + \frac{(1 - \varepsilon)k_s}{\varphi \left(\frac{k_p}{k_g} \right) + \frac{2}{3}} \quad (2.10)$$

Again, the variable φ is determined from published experimental data (Figure 2.3), and is the same value as listed above, that is $\varphi = 0.032$. The thermal conductivities of the working fluid and porous media used in this research are given in Table 2.5 alongside the effective thermal conductivities given by Equation (2.6), (2.9) and (2.10).

Table 2.5: Packed Bed Properties and Calculated Effective Thermal Conductivities

Bed Porosity, ϵ	0.40
Air Thermal Conductivity, k_f [W/m·K] @ 20.0°C	0.0257
Alumina Sand Thermal Conductivity, k_s [W/m·K] @ 20.0°C	40
Stagnant Effective Thermal Conductivity, as per Equation (2.6), k_{eff} [W/m·K]	0.48747
Stagnant Effective Thermal Conductivity, as per Equation (2.9), k_{eff} [W/m·K]	0.47239
Stagnant Effective Thermal Conductivity, as per Equation (2.10), k_{eff} [W/m·K]	0.48579
Average Error between Eq. (2.6), Eq. (2.9), & Eq. (2.10)	1.72 %
Average Effective Thermal Conductivity, $k_{avg,eff}$ [W/m·K]	0.48188

Figure 2.3: Values of ϕ Calculated from Experimental Data Reported (for air) [44]

From Table 2.5 the three effective thermal conductivities are within less than 2% of each other, which verifies the consistency of the relations. On the basis that both [7,9] recommend the use of Equation (2.6), and the similarity in experimental apparatus and procedure used in

[7], that will be the equation used in this research. It also provides for a more accurate value as no extrapolation from a plot is needed; furthermore the porous medium used in the development of Figure 2.3 were an order of magnitude greater than those used in this research (3.54mm versus 0.226mm). Further development was completed by [44] on the effect of the particle diameter has on the effective thermal conductivity within the system. It was found that for particles greater than 0.35mm in diameter they have a substantial effect on the effective thermal conductivity which only increases with an increase in temperature. But, for particles under 0.35mm there was no change in the effective thermal conductivity across an increase in working temperature, up to 800°C, thus providing an indication that the effects of thermal radiation within a packed bed of fine particles less than 0.35mm in diameter are negligible. Therefore, the heat transfer mechanics in this packed bed analysis are (a) the thermal conduction through the individual particle and (b) the thermal conduction through the gas-film between, or near, the contact surface of two adjacent particles.

2.6 DARCY'S LAW

The earliest work completed on flow through porous media was completed by Henry Darcy in 1856, where the flow of water through sand was analyzed from which an expression of the conservation of momentum was derived. This expression is commonly known as Darcy's Law and is based on the following assumptions/limitations:

- Neglect friction through the flow, due to macroscopic shear; thus not satisfying the no-slip velocity condition on solid boundaries
- Due to development being constructed based on laminar flow conditions, Darcy's Law is valid for slow, viscous flow (Stokes's Flow). Generally with $Re < 1$, though work by [48] has stated that laminar flow may continue up into the range of $Re < 10$ within a packed bed.

- Uniform velocity profile throughout the apparatus due to the neglect of spatial variation of the matrix porosity. Commonly the particle location varies, predominantly near the solid boundaries, creating a non-uniform velocity profile through the apparatus.

The correlation developed by Darcy was basically a reduction of the Navier-Stokes (N-S) equation based on Stoke's Flow; which is slow, viscously driven flow. The N-S equation in cartesian co-ordinates with flow in the x – direction is given as:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x \quad (2.12)$$

With the assumption of stationary, creeping, and incompressible flow, where the viscous forces dominate, the velocity vector (inertial) components within the N-S equation can be neglected and Equation (2.12) becomes:

$$0 = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x \quad (2.13)$$

A further assumption that the viscous resisting force is proportional to the velocity, but opposite in direction Equation (2.13) can be re-written as:

$$0 = - \frac{\partial p}{\partial x} - \frac{\mu \phi}{k_x} u_x + \rho g_x \quad (2.14)$$

where ϕ = is the bed porosity

k_x = intrinsic permeability = $C \cdot d_p^2$

C = dimensionless constant, related to flow path configuration

d_p = average, or effective particle diameter

Rearranging Equation (2.14) to solve for the velocity, u_x , gives:

$$u_x = - \frac{k_x}{\phi \mu} \frac{(\partial p - \rho g_x)}{\partial x} \quad (2.15)$$

The velocity, u_x , (the pore velocity) is related to the Darcy flux (q) through the porosity, ϕ , where the flux is divided by the porosity to account for the fact that only a fraction of the total formation volume is available for flow:

$$q_x = u_x \cdot \phi \quad (2.16)$$

Combining this with Equation (2.15) yields Darcy's Law

$$q_x = -\frac{k}{\mu} \frac{(\partial p - \rho g_x)}{\partial x} \quad (2.17)$$

From examining Equation (2.17), it states that the Darcy flux through a porous media is directly proportional to the pressure gradient through the channel (or apparatus). Equation (2.17) can be re-written in simpler terms, from which the relationship can be more readily understood.

$$U \propto \frac{\Delta P}{H} \quad (2.18)$$

As the working fluid is passed into the bed and is continually increased, the upward force, or drag force, acting on the contained particles reaches a point at which it is in equilibrium with the gravitational force. At this point, the pressure drop across the bed is defined as:

$$\Delta P_{mf} = H_{mf} g (1 - \epsilon_{mf}) (\rho_p - \rho_g) \quad (2.19)$$

where,

ΔP_{mf} = Pressure drop across the bed

H_{mf} = Height of particles at minimum fluidization

ϵ_{mf} = Bed voidage fraction at minimum fluidization

ρ_p = Particle density

ρ_g = Gas density

This above relationship was further developed by Ergun [49]. Ergun developed a correlation (Equation (2.19)) that related the viscous and inertial terms that define the pressure gradient across the bed, from which the minimum fluidization velocity U_{mf} can be determined. The determination of U_{mf} is crucial as this parameter defines the maximum flow rate range used in a packed bed analysis, as any additional flow through the apparatus will cause the confined particles to become fluidized.

$$\frac{\Delta P_{mf}}{H_{mf}} = \left[\frac{1.75(1 - \varepsilon)\rho_g U_{mf}^2}{\varepsilon^3 \psi d_p} \right] + \left[\frac{150(1 - \varepsilon)^2 \mu_g U_{mf}}{\varepsilon^3 (\psi d_p)^2} \right] \quad (2.20)$$

where,

- ε = Bed voidage fraction
- ψ = Particle sphericity
- d_p = Particle diameter
- μ_g = Gas viscosity

If Equation (2.18) and (2.19) are combined whilst substituting $\varepsilon = 0.4$ and $\psi = 1$, and solving for U_{mf} yields [48]:

$$U_{mf} = \frac{\mu_g}{\rho_g d_p} \frac{Ar}{(1400 + 5.22\sqrt{Ar})} \quad (2.21)$$

where,

$$Ar = \frac{d_p^3 g \rho_g (\rho_p - \rho_g)}{\mu_g^2} \quad (2.22)$$

Another common definition used is that of the fluidizing gas mass flow rate G , related by:

$$G = \rho_g U = \frac{\rho_g Q}{A_b} \quad (2.23)$$

where,

- Q = Volumetric flow rate
- A_b = Effective bed area

It is often more convenient to non-dimensionalize the gas mass flow rate, by dividing it by its minimum fluidization counterpart G/G_{mf} , defined as:

$$G_{mf} = \rho_g U_{mf} \quad (2.24)$$

As the gas mass flow rate incorporates the fluid density, which is a function of the operational temperature, it is often useful to apply it as a more accurate means of comparing heat transfer data.

2.7 GENERAL CONDUCTION ANALYSIS – CALCULATION OF THE HEAT TRANSFER COEFFICIENT

To determine the heat transfer coefficient, h , of the system the temperature distribution along the sample had to be determined. As the samples used in this research have a much larger length than diameter, such that $L \gg d_s$, the process of determining the temperature distribution across a pin-type fin of uniform cross-sectional area [50] was implemented with a modification to account for the heat generation within the sample [3] specific to the geometry herein. In order to determine the temperature distribution across the sample, first an energy balance must be performed on a differential element, given in Figure 2.4.

Figure 2.4: Energy Balance for a Hollow Tube

The energy balance of the element is simplified if certain assumptions are made; primarily if only 1-D conduction in the longitudinal direction is considered. As fins are generally

thin, the temperature changes in the longitudinal are of a much greater consequence than they are in the transverse direction. The following summarizes the assumptions imposed on this differential element energy balance:

- 1-D Conduction in the longitudinal (x) direction
- Steady-state conditions
- Constant thermal conductivity
- Negligible radiation effects
- Heat transfer coefficient is uniform across the surface

Applying the conservation of energy requirement to the differential element, including the effects of heat generation, the following is obtained:

$$q_x + q'dx = q_{x+dx} + dq_{conv} \quad (2.25)$$

From Fourier's Law the heat conduction into the control volume (CV) is:

$$q_x = -kA_c \frac{dT}{dx} \quad (2.26)$$

The heat generation due to the applied electrical resistance is:

$$q'dx = \frac{VI}{L} dx \quad (2.27)$$

The heat conduction out of the CV, expressed as $x + dx$, is related by:

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx} \left(A_c \frac{dT}{dx} \right) dx \quad (2.28)$$

The heat convection from the surface of the CV is defined as:

$$dq_{conv} = h dA_s (T(x) - T_b) \quad (2.29)$$

$$\text{where: } dA_s = P dx \quad (2.30)$$

Substituting the above rate equations, (2.26)-(2.29), into the energy balance equation (2.25), yields:

$$\frac{VI}{LkA_c} = -\frac{d^2T(x)}{dx^2} + \frac{hP}{kA_c}[T(x) - T_b] \quad (2.31)$$

2.7 GENERAL CONDUCTION ANALYSIS

CALCULATION OF THE HEAT TRANSFER

COEFFICIENT

To simplify the above equation the dependent variable is altered through defining an excess temperature term $\theta(x)$:

$$\theta(x) \equiv T(x) - T_b \quad (2.32)$$

Since T_b is defined as constant:

$$\frac{d\theta}{dx} = \frac{dT}{dx} \quad (2.33)$$

Substituting Equation (2.32) into (2.31) forms a linear, homogeneous, second-order differential equation:

$$\frac{VI}{LkA_c} = -\frac{d^2\theta(x)}{dx^2} + m^2\theta(x) \quad (2.34)$$

$$\text{where: } m^2 \equiv \frac{hP}{kA_c} \quad (2.35)$$

The general solution to the above relation is of the form:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \quad (2.36)$$

To evaluate the constants, specific boundary conditions must be applied. Firstly, the excess temperature change at $x = 0$, which is constant based on symmetry, is defined:

$$\frac{d\theta(0)}{dx} = 0 \quad (2.37)$$

Applying the first boundary condition, Equation (2.37), to the general solution, Equation (2.36), results in $C_1 = C_2 = C$. Applying the identity $2\cosh(mx) = e^{mx} + e^{-mx}$ and C to the general solution yields:

$$\theta(x) = C[2\cosh(mx)] + \frac{VI}{LhP} \quad (2.38)$$

The final boundary conditions was resolved by defining an excess temperature at the point $x = \pm(L/2)$:

$$\theta_T = \theta\left(\pm\frac{L}{2}\right) \equiv T_T - T_b \quad (2.39)$$

Solving Equation (2.38) for the constant, C, and applying the second boundary condition, Equation (2.39), gives:

$$C = \frac{\theta_T - \frac{VI}{LhP}}{2\cosh\left(m\frac{L}{2}\right)} \quad (2.40)$$

Substituting Equation (2.40) into Equation (2.38), along with the excess temperature definition (2.32), yields the final form of the temperature distribution profile in which the heat transfer coefficient is given implicitly:

$$T(x) = \left[\theta_T - \frac{VI}{LhP}\right] \frac{\cosh(mx)}{\cosh\left(m\frac{L}{2}\right)} + \frac{VI}{LhP} + T_b \quad (2.41)$$

The variables in Equation (2.41) are defined as:

$T(x)$ = Temperature at position 'x' along sample length

θ_T = Excess terminal temperature (Equation 2.32)

V = Measured voltage drop across sample

I = Applied current to sample

L = Length of sample

h = Heat transfer coefficient

k = Thermal conductivity of sample

T_b = Bed temperature

Examining Equation (2.41) it is apparent that the heat transfer coefficient is given implicitly, thus iterative mathematical techniques were used to determine the value. The specific algorithm and process explanation for the use of this calculation is given in Chapter 4. From this heat transfer coefficient, the experimental Nusselt number Nu_{exp} was then calculated based on the following relation:

$$Nu_{exp} = \frac{hd_s}{k_{eff}} \quad (2.42)$$

CHAPTER 3

3 EXPERIMENTAL APPARATUS

3.1 APPARATUS OVERVIEW

As previously mentioned, the four main components of a fluidized bed system are (a) source of gas, (b) a container, (c) a distributor, and (d) a powder or granular porous medium. The following sub-sections discuss these individual components, and others, which compose the entire lab-scale fluidized bed apparatus. The discussion follows the flow of air through the system, based on the following component list.

I.	Air Delivery System
II.	Air Heater and Ducting
III.	Lab-Scale Packed Bed
IV.	Distributor Plate Analysis
V.	Steel Wire Samples & Thermocouple Setup
VI.	Resistive Heat Generation System
VII.	Data Acquisition System

3.2 AIR DELIVERY SYSTEM

Air is supplied to the system by a GAST centrifugal regenerative blower (R3105-1) with an operating power of 0.37kW (0.5 HP), running at 3450 RPM. The blower is capable of delivering a maximum airflow of 88 – 90 m³/h (52-53 SCFM) at a maximum pressure of 107-137 mbar. The air delivery system is designed to provide the fluidized bed unit with an air flow rate ranging from 1.00 – 43.8 m³/h (0.296cm³/s – 13.02cm³/s), which maintains the packed bed

condition (based on an ambient temperature of 21°C and for sand grits ranging from 50 - 90). A schematic depicting the air delivery system is shown in Figure 3.1. The air is initially drawn in through a coarse air filter and passes through a ball valve and directly into the 31.75mm (1 ¼ ") inlet. The ball valve (specifically, referring to Figure 3.1, Ball Valve 1) is used as an initial flow rate controller, providing a coarse air flow rate adjustment. The air flows from the blower outlet and is delivered through a 31.75mm (1 ¼ ") steel pipe to three separate pipe paths; two 19.05mm (¾") paths and one 31.75mm (1 ¼ ") path. The two ¾" paths have identical flow control components, consisting of two ball valves and a flow meter. The first ball valve is used to alter the air flow rate at smaller increments, to ultimately achieve the desired flow rate. For each sand grit 10 separate flow rates were considered, ranging from 10% - 100% U_{mf} ; where U_{mf} can be found using Equation (2.21). The flow meters used in the apparatus are TSI 4000 Series (4045) General Purpose Thermal Mass Flowmeters. The flow meters operate at a standard flow rate range of 0 - 18.0 m³/h (0 – 300 Std L/min) with the ability to also provide temperature, static pressure, and volume measurements. Since thermal flow sensors are sensitive to changes not only to the air density, but also to the air velocity, the flow meters are calibrated to indicate flow rates with reference to a standard set of conditions; in this case a temperature of 21.1°C (70°F) and a pressure of 101.3kPa (14.7 psia). The volumetric flow rate of the air flow was then determined using the following density correction factor which was computed manually:

$$Q = Q' \left[\frac{273.15 + T_m}{273.15 + 21.11} \right] \frac{101.3}{P_m} \quad (3.1)$$

Where,

Q = Volumetric Flow Rate

Q' = Standard Flow Rate

T_m = Air Temperature measured in flow tube [°C]

P_m = Absolute Pressure measured in flow tube [kPa]

The centre (1 ¼ ") pipe passage is used primarily for cases in which fluidization of the bed is necessary, though that was of little value in the experiments conducted with respect to this study, though for the larger sand grits (50 and 60) the additional airflow was needed to

fluidize the bed in order to properly immerse the samples. Once the flow has passed through the parallel set of ¾" paths and/or the centre 1 ¼" path the air continues through a Chromalox Precision Heater where, in some of the test cases, it is heated and continues on into the lab scale packed bed unit. A pressure gauge in this line provides a measurement of air pressure to the bed. The following discusses the air delivery system as it pertains to each individual sand grit type and Table 3.5 summarizes the flow paths used throughout this study.

Table 3.1: Pertinent Flow Conditions and Parameters for 90 Grit Sand

90 GRIT (145µm):

Archimedes Number (Ar)	421
Reynolds Number (Re_{mf})	0.279
Minimum Fluidizing Velocity (u_{mf})	2.95cm/s

Table 3.1 lists the pertinent flow conditions and parameters for the 90 grit alumina sand. The minimum fluidizing velocity of 2.95 cm/s requires a volumetric flow rate from 16.44 - 164.38 L/min (0.58-5.80CFM) to provide measurements from $0.1U_{mf}$ up to $1.0U_{mf}$. As each individual flow meter operates up to a maximum of 300L/min, only one flow meter had to be used to measure the entire range of operational speeds for this sand grit. Initially Ball Valve 1 was adjusted to obtain a coarse flow rate, and Ball Valve 2 was adjusted in finer increments to obtain the desired flow rate. Ball Valve 5 was opened all the way, as it does not alter the flow rate in the current configuration. This flow path is designated as 'Flow Path 1'

Table 3.2: Pertinent Flow Conditions and Parameters for 70 Grit Sand

70 GRIT (203µm):

Archimedes Number (Ar)	1154
Reynolds Number (Re_{mf})	0.732
Minimum Fluidizing Velocity (u_{mf})	5.52 cm/s

Table 3.2 lists the pertinent flow conditions and parameters for the 70 grit alumina sand. The range of operational flow rates for the 70 Grit sand was 30.78 - 307.83 L/min (1.08-10.86 CFM). Two different flow paths were used for this sand grit, as the 100% U_{mf} flow rate is greater than that of a single flow meters calibrated reading ability. For flow rates ranging from

10% - 90% U_{mf} the same process used for the 90 Grit sand was employed, that is the flow rates were obtained through altering both Ball Valve 1 and 2. For the final operational velocity point (100% U_{mf}) Ball Valve 1 was adjusted to provide a coarse air flow adjustment while both Ball Valve 2 and Ball Valve 3 were adjusted such that the addition of the flow rates read by Flow Meter 1 and Flow Meter 2 was equal to the desired amount (in this case, 307.83 L/min). This second flow path is designated as 'Flow Path 2'

Table 3.3: Pertinent Flow Conditions and Parameters for 50 Grit Sand

50 GRIT (330 μ m):

Archimedes Number (Ar)	4958
Reynolds Number (Re_{mf})	2.805
Minimum Fluidizing Velocity (u_{mf})	13.02 cm/s

Table 3.3 lists the pertinent flow conditions and parameters for the 50 grit alumina sand. As the 50 Grit sand represents the largest particle diameter, it also has the highest minimum fluidizing velocity requirement. The range of operational flow rates were from 72.59-725.94 L/min (2.56-25.62 CFM). Three different flow paths were used for this sand grit, the first path being identical to that used for the 90 Grit sand. This flow path, with the flow travelling only through Ball Valves 1, 2, and 5 (with adjustments at 1 and 2) was used for 10% - 40% U_{mf} . For flow rates in the range of 50% - 80% U_{mf} the same flow path that was used for the 70 Grit test cases is also employed here. That is, the flow is finely adjusted with Ball Valve 2 and Ball Valve 3 such that the readings on Flow Meter 1 and Flow Meter 2 equal the desired flow increment rates. As the flow rates for 90% and 100% U_{mf} were greater than that of the operational flow rates of the flow meters, the apparatus had to be altered to provide another means of measuring the flow speed. In this case, a 1" Venturi Flow Meter was installed in between the Coarse Air Filter and Ball Valve 1, as shown in Figure 3.1. The specifications of the Venturi are given in Table 3.4.

Table 3.4: 1" Venturi Flow Meter Specifications

C_v	Discharge Coefficient	0.9797
d_T	Throat Diameter (mm/in)	19.05/0.750
d_{in}	Pipe Diameter (mm/in)	25.4/1.00
A_T	Throat Area (mm ² /in ²)	285.2/0.442
β	Throat-to-Pipe Diameter Ratio	0.5628

The implementation of the Venturi Flow Meter measured the flow rate of the air through the apparatus in inches of water, "H₂O and thus another calculation method had to be completed to determine the flow rate. The following basic Venturi flow equation was used to relate the measured pressure difference to the Venturi Flow Meter characteristics:

$$Q = C_v A_T \sqrt{\frac{2\Delta P}{\rho_g (1 - \beta^4)}} \quad (3.2)$$

where: ΔP = measured pressure difference across Venturi

Inputting the specific Venturi specifications given in Table 3.4, Equation (3.2) reduces to the following:

$$Q = 80 \sqrt{\frac{\Delta P}{34}} \text{ [SCFM]} \quad (3.3)$$

With the Venturi Flow Meter installed the maximum measurable flow speed was recorded as approximately 5.00 in H₂O, equal to 870 L/min (30.71 CFM), which meets the required flow ranges needed for this sand grit. The flow path for the final two data points (90% and 100% U_{mf}) was controlled via Ball Valve 0, which was directly above the Venturi Flow Meter. That Ball Valve was adjusted such that the required ΔP (in H₂O) was equal to the required duty point flow rate, the flow was then passed through the center pipe (31.75mm/1 ¼") and Ball Valve 4 was opened fully to allow the controlled flow to pass on into the packed bed. This flow path is designated as 'Flow Path 3'. The following table, Table 3.5, details the

different flow paths with respect to their maximum flow capabilities and to their uses concerning the different types of sand grits.

The abbreviations used in Table 3.5 are as follows: O = opened, A = adjusted and C = closed.

Table 3.5: Flow Path Air Delivery Capabilities and Uses

Flow Path	Ball Valve							Measurement Device	Flow Rate Range (L/min)	Sand Grit Uses
	0	1	2	3	4	5	6			
1	O	A	A	C	O	C	C	Flow Meter 1	0-300	90G/70G/60G
2	O	A	A	A	C	O	O	Flow Meter 1 & 2	300-600	70G/60G/50G
3	A	O	C	C	O	C	C	Venturi Flow Meter	600-870	50G

Figure 3.1: Air Delivery Schematic

3.3 AIR HEATER AND DUCTING

Several tests were completed with heated air (up to 350°C) to observe the difference in heat transfer rates within the system between ambient air and heated air entering the packed bed. In order to heat the air to the desired temperature a high temperature air duct heater was installed in line between the regenerative blower and the packed bed. A schematic detailing the duct in which the heater was installed is given in Figure 3.5 & 3.6. The duct was designed to diffuse the flow upon entry to accommodate the heater, at which point the heated air then exits the duct and flows into the packed bed. The duct was insulated with a high temperature foiled insulation batting (Figure 3.7). This limited the heat loss from the interior of the duct and improved the efficiency and the safety of the unit. The heater that was used (Figure 3.2 & 3.3) was a Chromalox ADHT Series Duct Heater (Model No ADHT-015FV). The unit operates at 575V and has a maximum power output of 15kW and a maximum output temperature of 650°C (1200 °F). The nine heating elements are sheathed in 0.475" diameter INCOLOY® tubing.

Figure 3.2: Grayscale 3D Picture of Chromalox Duct Heater

Figure 3.3: Schematic of Chromalox Duct Heater

Power output is controlled by a PID temperature controller driving an SCR (silicon controlled rectifier) power control module. These components are mounted to a heavy ¼" thick

steel flange. The flange is attached to a housing, which provides for 3 inches of unheated sheath, which limits the conduction of heat into the terminal enclosure. There is additional 1" thick high-temperature insulation between the terminal enclosure and the wiring area. A Type K T/C is welded to the element sheath surface to determine the temperature, to prevent element overheating. The air outlet temperature is monitored by a T/C connected to the PID temperature controller. Table 3.6 lists the measurements that apply to Figure 3.3, specific to the duct heater model used in this apparatus.

Table 3.6: High Temperature Duct Heater Dimensions

Letter	Dimension (mm/in)
A	244.475/9.625
B	517.525/20.375
C	714.375/28.125
D	203.2/8.0
E	6.35/0.25
H	76.2/3.0
K	88.9/3.5
L	282.575/11.125
M	241.3/9.5

The flange of the duct heater was bolted onto the flange of the duct with 12 steel ¼" by 1 ½" bolts. High temperature silicon was spread between the two flanges to ensure a high temperature seal between the two components. The silicon used was rated to a maximum of 315 °C (600 °F), but the maximum temperature used in the actual experimental application was approximately 204 °C (400 °F). A temperature control panel (Figure 3.4) was installed and connected to the Chromalox heater. The panel was wired with 3 Fuji Electric Systems Micro-Controller X (Model PXR4) Digital Temperature Controllers, to control and display the temperature of three separate Type K thermocouples. The first controller, labeled "Temperature Controller" monitored air temperature using a T/C placed in the air line directly after the heating duct. The temperature controller varied the heater output to maintain the desired air temperature. A maximum temperature range was inputted into the controller and once the temperature reading reached that point the heater would then maintain the current heat generation rate. The second controller, labeled "Sheath Temp", displays the element sheath temperature within the heater itself. This allowed for an accurate temperature reading

within the heater. A maximum value of 385°C was imposed on this controller, which was adequate for producing the required temperature increase to the flow passing through the heating duct. The final temperature controller, "Bed Temp", was simply used to display the current bed temperature within the packed bed. The other controls on the panel were used to start and stop both the heater and the exhaust fan.

Figure 3.4: Chromalox Precision Heater Control Panel

SIDE VIEW

Figure 3.5: Schematic of Heating Duct – Side View

FRONT VIEW

Figure 3.6: Schematic of Heating Duct – Front View

Figure 3.7: Actual Picture of Insulated Heating Duct

3.4 LAB-SCALE PACKED BED

The Lab-scale packed bed that was used in this study is shown in Figure 3.8. The overall dimensions of the unit consist of an interior diameter (the working area) of 311.15mm (12 ¼") and an overall height of 914.4mm (36"). The bed is manufactured from stainless steel with 6.35mm (¼") thick walls. The flange on the bottom of the bed is bolted onto a 622mm (24 ½") square plenum with 18 ½" steel bolts. Between the square plenum and the bottom flange of the bed, high temperature silicone was spread to ensure a high temperature air-tight seal. The square plenum is situated 457mm (12 ½") above the floor. The airflow enters through the center of the plenum at the desired flow rate. The air enters the plenum and passes through the distributor plate, from which uniform flow across the bed is introduced. The sample holder is held in place within the bed by a pin and steel block assembly, which is bolted onto the top of the upper flange of the bed. The sample holder can be positioned at any height within the bed, depending on the requirement.

3.8 DISTRIBUTOR PLATE

Figure 3.8: Schematic of Packed Bed Column and Base

3.5 DISTRIBUTOR PLATE

The distributor plate, a 305mm x 305mm x 25.4mm (12" x 12" x 1") porous fused alumina hearth tile (modeled in Figure 3.9), is the initial medium through which uniform air distribution into the packed bed is imposed. The plate is mounted flush with the top of the base and the clearances (6.35mm) between the tile and the base structure is filled with a high temperature grout to hold the tile in place and to also guarantee that the entire air flow passes through the plate itself.

Due to the support structure designed to hold the plate in place (a 90 degree steel bracket), consideration into whether or not uniform air flow was actually guaranteed across the entire surface area was undertaken. The plate was divided into 144 25.4mm (1") squares (Figure 3.9) and the flow speed was measured at each of those locations. The air speed travelling through the plate was measured by holding a TSI General Purpose Thermal Mass Flowmeters perpendicular to the plate with a rubber washer between the flowmeter inlet and the one-inch square to ensure a tight seal and consequently an accurate flow rate reading. This process was completed for each individual square. The recorded flow speeds were tabulated and a 3-D area graph was made to visualize the flow across the surface. (Figure 3.10) The outer edges of the plate provided the lowest flow speed measurements, which is consistent with expectations as the 90 degree bracket impedes the flow on the outer edges of the plate. Another flow impedance that was considered was the column of the packed bed and how it sat on the plate, as detailed in Figure 3.9. The column limits the flow of several of the outer segments, the majority of which overlap the regions already impeded by the support bracket; so these segments were not further considered. Other inner segments that were compromised by 50% or more by the packed bed column were not considered to be involved with providing any air flow to the bed.

Figure 3.9: Schematic of Segmented Distributor Plate and the Compromised Flow Regions

The following plot, Figure 3.10, represents the air flow that was recorded in all of the 'un-hatched' regions in Figure 3.9. From visually inspecting the plot, the flow rates are reasonably consistent across the area of the distributor plate. Table 3.7 displays the pertinent calculated values and the average deviation across the individual flow segments.

Figure 3.10: 3D Area Plot Representing Flow Through Individual 1" Squares

Table 3.7: Error Analysis of Individual Flow Segments & Average Flow Deviation

Average Flow Speed	5.21 L/min/in ²
Maximum Error	18.47%
Minimum Error	0.24%
Average Error	6.51%
Average Deviation	±0.34 L/min/in ²

With an average deviation between flow segments of only ± 0.34 L/min/in² and an average percent error of 6.51%, it confirmed the initial assumption of uniform flow distribution through the distributor and into the packed bed. It is also expected that the layer of course-grit sand (Section 3.4) would further improve the uniformity of the flow as the large particle voidage provides ample area for the airflow to travel and disperse into the bed unhindered.

3.6 STEEL WIRE SAMPLES AND THERMOCOUPLE SETUP

The samples tested in this research were seven 302 Stainless Steel tubes of varying size, ranging from 1.27mm-9.525mm (0.050" – 0.375") outside diameter with a wall thicknesses ranging from 0.254mm-1.778mm (0.010" – 0.07"). Each sample was affixed to a sample holder, illustrated in Figure 3.11. The shaft of the sample holder was made from 19mm ($\frac{3}{4}$ ") square steel tubing with a U-shaped bracket welded to the bottom. At the end of both sides of the U-shaped bracket a 12.7mm hole was drilled to hold plastic inserts, which were used to secure the sample to the bracket while also providing electrical and thermal protection to both the samples and terminals from the steel bracket. The terminals that were used to resistively heat the sample were copper mechanical lugs, attached to both ends of the sample. The assembly is shown in Figure 3.11. Each terminal had a sandwich screw which secured the terminal T/C lead wires as well as the high current cables. The high current cables were attached to the terminals

with a screw and a bolt with a tinned copper lug wire connector. For the larger samples, larger terminals were used, and as such larger high current cables were used which meant that the tinned copper connectors also had to be larger in order to handle the supplied current, which ranged from less than 2A for the smallest (1.27mm) samples up to almost 30A for the largest (9.525mm) samples. The two leads of the center T/C were welded at their joint tip and covered with electrical tape before being fed into the center of the larger samples. Smaller T/C wires (0.08mm) were used for the smaller samples, as there was not enough clearance for the larger T/C wires to fit inside the tube. The T/C sizes used were 0.08 – 0.71mm. Silicon was used to seal the ends of the plastic inserts ensuring that no airflow would enter the tube. The high current wires and the T/C wires were all attached to the bracket and the sample holder shaft with electrical tape. A Type K T/C was affixed to the sample holder with electrical tape and the end was bent away from the sample at 90° and at an approximate distance of 76.2mm (3") away from the center of the sample. This T/C was used to monitor the temperature of the bed.

Figure 3.11: Cross-Sectional View of Cable-to-Sample Connection

Four Type K thermocouples were used on each individual sample apparatus (Figure 3.12) to measure four independent temperatures within the packed bed. One T/C was used on

each end of the sample, to measure the copper terminal temperatures (T_T) which was assumed to be the same temperature as the actual samples ends. These terminal T/C temperature readings were required inputs required as per Equation 2.41, to implicitly solve for the heat transfer coefficient of the system. The two T/C wires were twisted together and then affixed to the sample via the sandwich screw; a component of the copper terminal. The Bed T/C (T_{inf}) was encapsulated in a stainless steel sheath and was insulated by MgO. The lead T/C wires were spot welded together to provide an accurate means of directly reading the bed temperature. This T/C was affixed securely to the sample apparatus with electrical tape and was bent to a position of 76mm (3") from to the center of the sample. The fourth T/C (T_c) was used to measure the central interior temperature of the sample and was thus located along the sample centerline. The fourth T/C wires were also spot welded and then insulated with thin plastic tape before being inserted into the interior of the sample. Initially the sample T/C wires were welded to the outer surface of the sample but heat conduction out of the T/C wires (the fin effect) caused errors in the temperature readings and as such, the T/C wires were moved into the interior of the samples. The temperature correction (Equation 3.9), given in the following section, was used to alter the interior temperature reading to reflect the actual outer surface temperature. This correction relation is based solely on the sample geometry and material properties and as such this type of relation does not rely on experimentally determined values, providing for an accurate estimation of the outer surface temperature of the sample.

Figure 3.12: Data Acquisition Components and Thermocouple Setup

As the fourth T/C was located within the sample, some special consideration had to be applied to correct temperature errors due to conduction through the tube wall. In order to correct for such temperature influences, a simple correction was introduced that accounted for the temperature drop from the interior of the sample to the outer surface. The correction expression was developed based on the 1-D steady state heat conduction model [3, 50], based on Figure 3.13, beginning with the appropriate form of the heat equation, given as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{q'''}{k} = 0 \quad (3.4)$$

Figure 3.13: Conduction in a Cylinder in Cross-Flow – Adapted from [50]

Separating the variables and assuming uniform heat generation, the above expression can be integrated to obtain:

$$r \frac{dT}{dr} = -\frac{q'''}{2k} r^2 + C_1 \quad (3.5)$$

Integrating once again, the general solution of the temperature distribution becomes:

$$T(r) = -\frac{q'''}{4k} r^2 + C_1 \ln r + C_2 \quad (3.6)$$

To obtain the constants of integration (C_1 and C_2), the following boundary equations must be applied to Equation (3.6).

$$\text{At } r = r_i: \frac{\partial T}{\partial r} = 0$$

The heat transfer within the interior of the sample is equal to zero.

$$\text{At } r = r_i: T(r_i) = T_c$$

The temperature of the interior surface of the sample is known

After applying the above boundary conditions to the general solution, Equation (3.6), the constants of integration are determined to be:

$$C_1 = \frac{q'''}{2k} r_i^2 \quad (3.7)$$

$$C_2 = T_c + \frac{q'''}{4k} r_i^2 - \frac{q'''}{2k} r_i^2 \ln r_i \quad (3.8)$$

Substituting the above constants into the general solution yields the following expression for the outside tube temperature:

$$T(r_0) = \frac{-q'''}{4k} (r_0^2 - r_i^2) + \frac{q'''}{2k} r_i^2 \ln \left(\frac{r_0}{r_i} \right) + T_c \quad (3.9)$$

As previously mentioned the temperature drop between the interior of the sample and the outer surface is independent of the heat transfer coefficient and varies only with the sample geometry and material properties. Though this calculation was completed for each individual test run, the average calculated temperature difference between the interior of the sample and the outer surface was calculated to be approximately 2.0°C across all sample sizes.

3.7 RESISTIVE HEAT GENERATION SYSTEM

The samples were heated using a resistive heating technique with the current being supplied by a GW Instek® Programmable Power Supply (PSH-10100); which provides regulated current up to 100A at a maximum of 10VDC. A schematic detailing the sample heat generation system is shown in Figure 3.14.

In the experimentation process, the voltage V was measured across the sample terminals with a separate voltmeter to avoid errors associated with resistance of the power cables. The current applied to the sample was measured directly from the LCD display on the GW Instek® Programmable Power Supply. To provide a means in which to supply the current to the sample in a manual-controlled environment a Sprecher + Schuh (S+S) high current relay (CA6 – 140) was used in conjunction with a manual trigger switch. With the manual trigger switch *ON* current was supplied to the sample through high current cables, connected to large copper terminals at each end of the sample. For accurate temperature readings the manual trigger switch was turned *OFF*, which opened the S+S relay, ultimately eliminating any electrical interference to the sample. The electrical interference witnessed was due primarily to two separate components. Firstly, the applied electrical current imposed a voltage gradient across the T/C lead wires, altering their temperature readings. In addition, the electrical outlet was susceptible to ground interference effects from large motors and other large electrical equipment in adjacent labs. The samples were heated to provide a difference in temperature between the sample and the bed of approximately 10°C . The voltage drop across the sample was measured using a digital multimeter (Model GDM – 814M).

3.7 RESISTIVE HEAT GENERATION SYSTEM

Figure 3.14: Schematic of Sample Heat Generation System

3.8 DATA ACQUISITION SYSTEM

3.8.1 HARDWARE SYSTEM:

The hardware used for the acquisition of the temperature data was the IOtech Personal Daq 3000™ Series. The Personal Daq is a 16-bit multi-function data module which connected directly to the PC's USB 2.0 port, with duty-cycle measurements being recorded up to 1 MHz. The unit included 8 differential (thermocouple) or 16 single-ended analog inputs, while also providing 24 high speed digital I/O lines. The Daq provides a wide variety of triggering modes to accommodate any type of measurement application. The hardware analog triggering provides latencies guaranteed to be below 1.3μs while the digital/TTL triggering has an even lower latency range, of below 1.0μs.

The unit was factory calibrated using a digital NIST traceable calibration method, which operates on a correction factor being applied to the individual ranges on the unit. Table 8 lists the accuracy of the Daq based on using a Type K T/C.

Table 3.8: Temperature Range and Accuracy for Type-K Thermocouples

T/C Type	Temperature Range (°C)	Accuracy ($\pm^\circ\text{C}$)	Noise ($\pm^\circ\text{C}$)
K	-200 to 1200	1.8	0.2

In order for the DAQ to accurately read and record the system temperatures, the thermocouple wires had to be connected in *differential mode* to the analog input channels. Differential mode limits the noise and therefore greatly reduces any false readings the DAQ could produce compared to the standard Single-Ended connection method. A differential connection is made (Figure 3.15) by (a) connecting the red thermocouple wire to the channel's low (LO) connector and (b) connecting the other wire (with respect to this study, the other wire was yellow) to the channel's high (HI) connector.

Figure 3.15: Differential Connection (V₁) to Personal DAQ Analog Input Channels

3.8.2 SOFTWARE SYSTEM:

The software that was used in conjunction with the data acquisition system was DaqView (Version 9.1.27), which is a Windows-based 32-bit application. This software allows one to verify the signal connections, acquire and save the data directly to disk and to also graphically view real-time data. The acquired data can readily be saved (or exported) to various types of files, including MATLAB, Excel, and txt (ASCII) files. The primary means of altering the hardware, acquisition, and display parameters is through a simple spreadsheet type interface. Another pertinent feature within the software is the ability to select programmable trigger conditions (e.g., immediate, keyboard hit, external TTL, and channel value), which operate at the trigger latency values mentioned earlier. The graphical representations that are available from which to view the data are strip charts, bar graphs, dials, and digital displays.

Specifically this acquisition system was used to measure the right and left terminal, the bed and the sample's inner centered temperatures. Once the data was acquired for the duration of the run, the data was truncated to exclude the regions of resistive heating. Thus, the data set only included the data acquired immediately after the applied current was turned off and up until the sample reached a steady-state temperature, approximately equal to the bed. From each experimental test, only one data point was actually used in the calculation of the heat transfer. Once the resistive heating process was terminated, the first data point represented the maximum heat transfer coefficient of the system as the highest difference in temperature between sample and bed occurred at this point. Based on the simple heat transfer relation (Equation 7.7) the applied power is a primary variable within the heat transfer rate calculation (which is consistent with the implicit relation developed for this research (2.41)), thus obtaining the point that represents the highest sample temperature provides the most representative system calculation.

CHAPTER 4

4 EXPERIMENTAL PROCEDURE

OVERVIEW

The components discussed in the previous chapter are now further developed as they apply specifically within the experimental procedure.

4.1 LAB-SCALE PACKED BED AND SAMPLE INSTALLATION

The bed was filled with 76.2mm (3") of coarse sand, placed directly on top of the distributor plate. This additional medium was implemented to provide uniform flow in the 'working area' of the bed. As the packed bed does not displace the particles within itself, unlike a fluidized bed, a modest depth of 152.4mm (6") was chosen for the aluminum oxide sand used in the testing procedures. Three primary sand types were used: 50 Grit (330 μ m), 70 Grit (203 μ m), and 90 Grit (145 μ m). A few tests were completed with the 60 Grit (254 μ m) sand type, but only to verify and compare the recorded data trends. To switch between sand types a 4.25HP, 12 gallon wet-dry vacuum was used to remove the sand from the bed. The aluminum oxide sand was screened to remove any of the coarse sand that had become mixed with it during the removal process. To properly submerge the sample below the surface of the sand, the air system was started and the bed was allowed to reach a fluidized state, such that the sample could easily be inserted into the column and attached to the sample holder. The sample was submerged approximately 88.9mm (3.5") below the surface of the sand. It was affixed to the sample holder with a 5/16" pin, which fit into a hole in both the sample holder

and the bed apparatus holder. A comparison test was completed in order to determine whether the depth of the sample had a substantial effect on the heat transfer coefficient. The sample was placed at 50% of the standard depth, 44.45mm (1.75"), the remainder of the test conditions were left the same. The difference between the calculated Nu at the two different depths was 2.4%, verifying the limited effect the depth of the sample has on the calculated Nu. Once the sample was immersed in the bed the air flow rates were reduced below the minimum fluidization velocity to create the packed bed. The high current cables, T/C lead wires and voltmeter wires were all attached once the sample was secured in the packed bed. Once the sample was immersed, secured and connected to the appropriate devices, current was applied to the sample. The applied current was adjusted such that difference between the recorded bed temperature and sample center temperature was approximately 10 °C, given as:

$$\Delta T = T_c - T_b = 10\text{ }^{\circ}\text{C}$$

This temperature range was selected because at higher levels of ΔT natural convection effects become significant, a heat transfer mechanism, which is not considered in this study. In addition, with a larger ΔT the variation in the system properties becomes an issue, thus maintaining a conservative range value these effects are negligible. Also, a lower ΔT would not provide enough resolution to interpret the results.

For the smallest sample, an applied current of about 2A was sufficient to heat the sample, but for the larger samples, an applied current upwards of 25A was needed. The applied current was adjusted manually for each test and was increased by increments of 0.1A until the desired temperature difference was obtained. The voltmeter was used to read the voltage drop across the sample. The time it took the samples to reach a steady state temperature varied greatly both between different samples sizes and sand grit. The larger samples took much longer to reach a steady state value, up to 5 hours in some cases, which is intuitive as there is a larger volume that needs to reach a common temperature; the smaller samples took much less time. On average the smaller samples reached steady state temperature within 30-40% of the time it took the larger samples to reach a steady state temperature, based on tests operating

with the same sand grit. There was also a large difference in the time it took the samples to reach steady state between the different sand grits. The samples all reached a steady state temperature in the 50, 70, and 90 sand grits within an average of 20 minutes, 45 minutes, and 90 minutes respectively.

4.2 AIR DELIVERY SYSTEM

A detailed explanation of the air delivery system is given in earlier sections (3.2). The following further develops the flow paths and the possible flow rates with respect to their use with the different sand types, given in Table 4.1. As mentioned, the 60 Grit sand was used only for a couple of test runs, as will be discussed in subsequent sections, and as such, it is not included in this table.

Table 4.1: Fluidizing Gas Flow Rate and G/G_{mf} as Related to the Three Flow Paths for 50, 70, and 90 Grit Sand

Flow Path	Measurement Device	Flow Rate Q (L/min)	Gas Mass Flow Rate G ($\text{kg/m}^2\text{-s}$)	G/G_{mf}		
				50 G G/G_{mf}	70G G/G_{mf}	90G G/G_{mf}
1	Electronic Flow Meter 1	17	0.00367			0.1
		32	0.00690		0.1	0.2
		50	0.01078			0.3
		65	0.01402	0.1	0.2	0.4
		82	0.01768			0.5
		97	0.02092		0.3	0.6
		123	0.02653		0.4	0.7
		132	0.02847			0.8
		152	0.03278	0.2	0.5	0.9
		165	0.03559			1.0
		185	0.0399		0.6	
		215	0.04637	0.3	0.7	
		246	0.05306		0.8	
		282	0.06082	0.4	0.9	
2	Electronic Flow Meter 1 & 2	307	0.06639		1.0	
		363	0.07828	0.5		
		436	0.09394	0.6		
		508	0.10960	0.7		
		581	0.12525	0.8		
3	1" Venturi Flow Meter	653	0.14091	0.9		
		726	0.15657	1.0		

4.3 DATA ACQUISITION

Once the four Type K T/C lead wires were screwed into their respective analog terminal block locations on the Personal Daq 3005, the DaqView software was opened and the test was initiated. The software enabled oversampling of the recorded data, which essentially allows for averaging of the data as it is recorded. The range of oversampling rates is 16-16384 operating at either 50Hz or 60Hz. In implementing the oversampling mode, the noise is reduced and in the case of this research, the 60Hz pick-up noise is significantly reduced. 60Hz noise often originates from the internal resistance within circuits, or can be due to AC power line contamination. In this research, both could have been the cause of the noise. Though it is very difficult to determine to what degree or what exactly is causing the reading fluctuations, efforts to limit the variances can still be initiated; thus the oversampling technique. A negative aspect of the oversampling rate mode is that increasing the rate, ultimately decreasing the noise, results in a decrease in the maximum scan rate of the unit. Operating with no oversampling rate the Daq is able to record thousands of scans per second, on the opposite end of the spectrum with the Personal Daq operating at a oversampling rate of 16384 (the maximum possible rate) the maximum scan rate is just less than 10 scans per second. These oversampling rate numbers represent the amount of time before one 16-bit reading is recorded. For the 16384 case, 1 16-bit value is recorded across 16384 microseconds. Thus that one recorded value is an average of all the values across that time frame. Justification for the use of this oversampling rate is given in Figure 4.1, where the difference between no oversampling rates and maximum oversampling rates are given for two separate samples. The smallest sample, 1.27mm (0.050"), is subjected to a high level of noise as the T/C wires for this sample are very small and are not insulated. Also, there was no enough clearance within the smaller samples to insert the T/C wires along with the high thermal conductive chemical epoxy used for the other larger samples. Thus with the T/C wires not being insulated themselves, and lacking the insulating epoxy they were subject to noise interference, from either grounding problem inherent in the equipment or interference from the heated sample. The difference between the oversampled and non-oversampled rate for the smallest sample is overwhelming (the

maximum difference between measured temperature values was 83.8°C!) and justifies the use of such an operational mode. The other plot, Figure 4.2, gives the same comparison for the largest sample with the largest T/C wires (which are insulated). The difference between the oversampled and non-oversampled rate is much less overwhelming (where the maximum difference between measured temperature readings was only 2.2°C) and as such, the maximum oversampling rate used was 128; which relates to a maximum scan frequency of 1116Hz in comparison to only 8.72Hz for the 1.72mm (0.050") sample. Based on these rates the average number of scans per second was between 5 and 20. Though this is a modest number of scans per second, the individual tests lasted from approximately 20 minutes to 5 hours, which still allowed for an accurate representation of system temperatures. The S+S relay and manual trigger device (Figure 3.14) were used to further isolate the sample from noise once the appropriate heat generation effects had been completed. The DaqView system recorded all the temperature data from the point of applying the proper flow rate to the bed, while the sample was being heated (with the applied resistive heating) and until the sample had reached a steady-state and had dropped back down to the packed bed temperature. Having the DaqView system recording the entire time limited any latency effects from the manual and/or automatic triggers traditionally used. Also, since the tests were long and a quick change of test parameters was not needed it was just as efficient to allow the software to continually record, as limited vigilance was needed throughout the heating and cooling portions of the tests.

Figure 4.1: Temperature Reading Fluctuations and Oversampled Readings for 0.050" Sample

Figure 4.2: Temperature Reading Fluctuations and Oversampled Readings for 0.375" Sample

4.4 EXPERIMENTAL PROCEDURE AND TEST MATRIX

The experiments that were conducted to determine the heat transfer coefficients were carried out under standard atmospheric conditions. Ultimately the experiments were conducted to measure the temperature of the sample (T_s), the terminal temperature (T_T) and the bed temperature (T_b). In order to obtain these temperatures at their respective locations the following steps for each individual test were completed:

Table 4.2: Experimental Procedure Used for the Acquisition of Temperature Readings

1	Attach bed thermocouple to test sample holder
2	Position test sample into lab-scale packed bed
3	Connect high voltage cables to sample end terminals
4	Connect thermocouple wires to DAQ data acquisition unit
5	Connect voltmeter terminals to lead terminal wires
6	Adjust ball valves to acquire desired flow speed, beginning at minimum G/G_{mf}
7	The manual trigger is activated and the DaqView software begins monitoring system temperatures: T_c , T_T , and T_b .
8	The supplied current is adjusted to resistively heat the sample to a minimum of 10°C above the bed temperature (T_b)
9	Steady-state conditions are obtained by ensuring that there is no longer an increase in the sample center temperature (T_c) reading
10	At steady-state the i) Voltage [V] (as per the voltmeter) and ii) Current [A] (as per the variable power supply) are recorded
11	The manual trigger is then turned off and the sample is allowed to cool to current bed temperature
12	The ball valves are adjusted to an increased G/G_{mf} rate at 10% increments
13	Steps 6 – 11 are repeated up until 100% G/G_{mf} is achieved
14	The test sample and/or the sand type are changed. Steps 1 – 12 are repeated.

Table 4.3: Experimental Procedure Test Matrix

Sample Cylinder Diameter	Sand Grit Size	G/G _{mf}	Heated Air Tests
1.27 mm (0.050")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No
2.11 mm (0.083")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No
3.18 mm (0.125")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No
4.76 mm (0.1875")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	Yes
	90	0.1 – 1.0	No
6.35 mm (0.250")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No
7.94 mm (0.3125")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No
9.53 mm (0.375")	50	0.1 – 1.0	No
	60	0.1/0.5/1.0	No
	70	0.1 – 1.0	No
	90	0.1 – 1.0	No

4.5 AIR HEATING

The effectiveness of increasing the working gas thermal conductivity, through increasing the operating temperature was studied through the use of the heater and heater ducting discussed earlier. Within a packed bed, where particle motion is zero, it remains that the only other controllable limiting factor within the system is the working gas thermal conductivity. The heater was controlled using the control panel discussed in Chapter 3. As the data points collected from this analysis were being used as only a preliminary comparison to the data collected based on ambient working conditions it was decided that the median sized sample and median sand grit would be used. Thus, the 4.763 mm (0.1875") sample was used in conjunction with the 70 Grit (220 μm) sand. Several operational velocity points were selected over the standard working range (10% - 100% U_{mf}) in order to obtain an accurate mean heat transfer rate for this sample/sand setup. The maximum operational heater element temperature was 415°C, which resulted in an outlet (heater duct outlet) air temperature of approximately 110°C. A plot depicting the temperature trends observed throughout the heated air tests is given in Figure 4.3. In order for the bed to reach a steady state temperature in a timely manner additional flow had to be provided into the apparatus. Thus, during the stage before applying the resistive heat generation to the sample the apparatus was conducted as a fluidized bed with maximum flow rate through the bed. This enabled a faster and more effective means of carrying the heated air and mixing it into the bed, resulting faster bed heat-up time. From Figure 4.3, it is apparent that even with the additional flow into the bed the temperature in the bed takes a substantial amount of time (approximately 2.5 hours) to reach a steady state value, primarily due to its thermal mass. Once the bed had reached this steady-state temperature (of approximately 100°C) the resistive heat generation process was initiated and the remainder of the experimental procedure given in Table 4.2 was followed, beginning at Step 8.

Figure 4.3: Comparison of Heated Air T/C Temperature Rates

4.6 DATA PROCESSING

The processing of the experimental data is completed through a two step process. The first step, as illustrated by the flow chart in Figure 4.6, is an iterative calculation for the heat transfer rate using a MATLAB program. The experimental test conditions and T/C temperature readings are inputted to the program. The terminal temperatures were affected by an 'offset bias' when the manual switch was turned ON. The offset bias is a condition that once the switch is turned ON and voltage is applied across the sample through the terminals, a voltage drop is imposed upon the terminal thermocouples; altering their temperature reading. As there is

some latency throughout the system the point of terminating the current to the sample (manually turning off the power supply unit) did not necessarily relate to the point where the offset bias was no longer affecting the terminal temperature data. In order to determine an accurate terminal temperature to input into the program the temperature data (as collected by the DaqView software) was plotted in Excel and the point at which the temperature instantly increased (Figure 4.4) back to the steady state temperature was taken with the following 24 points (total of 25) and an average value was taken as the terminal temperature. From Figure 4.5, the average temperature across the 25 data points (representative of approximately 30 seconds to one minute of testing time, depending on the sample size and the oversample rate used) is very representative of the overall temperature readings, and thus was satisfactorily accurate enough to be used as the terminal temperature. The same process was applied in selecting the bed temperature from the collected data set. The centre T/C temperature reading was the first data point of the 25 averaged points, as it most accurately represented the maximum temperature the sample achieved.

Figure 4.4: Example of Offset Bias for Terminal Thermocouples

Figure 4.5: Example of Recorded Temperature and the Calculated Average Temperature Across 50 Seconds

Once the temperature values were determined, they, in conjunction with the other user-inputted variables, were entered into the MATLAB program. The equations used for the geometric calculations are given in Table 4.4.

Table 4.4: Geometric Calculations Used in Heat Transfer Calculation

Geometric Value	Equation	
Volume of Sample	V_c	$\pi(r_o^2 - r_i^2)L$
Cross-Sectional Area of Sample	A_c	$\pi(r_o^2 - r_i^2)$
Surface Area of Sample	A_s	$2\pi r_o^2 L$
Perimeter of Sample	P	$2\pi r_o$
Volumetric Heat Generation of Sample	q'''	$\frac{V \cdot I}{V_c}$

The algorithm, as depicted in Figure 4.6, is based on an iterative calculation of the heat transfer rate for the particular run being studied. The comparison between the sample T/C temperature value and the calculated surface temperature is the basis of the iterative scheme.

With the outer surface temperature T_c calculated, an initial guess for the heat transfer coefficient of 0.1 is applied to the $T(x)$ equation (Equation 2.41). That value is calculated and then compared with the experimentally recorded and corrected T_c .

Figure 4.6: MATLAB Heat Transfer Coefficient Program Algorithm Flow Chart

An absolute error value of 0.00001 is imposed and unless the deviation between the two calculated values is equal or less than that value, the calculation is run through again with an increase in the heat transfer rate by certain step rate, generally 0.1. Once the calculated deviation is below the imposed error value, the program accepts the heat transfer calculation and proceeds to calculate the heat transfer rate based on the simple equation (Equation 7.7) and declare it as h_{avg} , and the error between the iterated value and the simple equation value is then calculated, which was on average 4%. Once a heat transfer rate is determined, that value is inputted (along with other system properties and characteristics, Figure 4.7) into an Excel file; this is the second step of the data processing process. From this file Nu_{exp} vs. G/G_{mf} plots are generated.

Figure 4.7: Input Variables and Output Diagrams

CHAPTER 5

5 RESULTS

5.1 INTRODUCTION

Once Nu_{exp} is calculated, based on Equation 2.42, it is plotted against G/G_{mf} for the entire operational range, 10% - 100% (0.1 – 1.0) G/G_{mf} for 50, 70, and 90 grit sands. The properties of the aluminum sand used are given in Table 5.1, where Ar and G_{mf} are determined via Equations (2.21) and (2.23) respectively.

Table 5.1: Sand Grit Physical Properties

Sand Grit	Sand Particle Size d_p [μm]	Density ρ_p [kg/m^3]	Fluidizing Rates G/G_{mf}	Archimedes Number Ar
50	330	3970	0.1-1.0	4958
60	254	3970	0.1-1.0	2261
70	203	3970	0.1-1.0	1154
90	145	3970	0.1-1.0	421

5.2 AMBIENT AIR TESTS

Figures 5.1-5.3 display the experimentally calculated Nusselt Number, Nu_{exp} , versus G/G_{mf} for all the samples in 50, 70, and 90 sand grits operating under ambient conditions. From these figures, it is apparent that at the lowest end of the G/G_{mf} range the Nu_{exp} is at a minimum, and linearly increases as G/G_{mf} increases. Within a 10-15% range the individual samples increase linearly at the same rate (with the same linear trend line slope) for each sand grit type. This increase in heat transfer with increased flow rates is consistent with results published based on the boundary layer theory, from which an increase in the Nusselt Number is achieved with an increase in the Reynolds Number (which is representative of an increase in

velocity or system flow rate) [4-8, 26, 27, 29, 30, 50]. Consistent with other published results, the larger diameter samples resulted in larger calculated Nusselt Numbers. [7, 8, 45]

Figure 5.1: Experimental Nusselt Number vs. G/G_{mf} for all Samples Sizes in 50 Grit Sand

Figure 5.2: Experimental Nusselt Number vs. G/G_{mf} for all Samples Sizes in 70 Grit Sand

Figure 5.3: Experimental Nusselt Number vs. G/G_{mf} for all Sample Sizes in 90 Grit Sand

Figure 5.4 depicts the calculated Nusselt Numbers for the 4.76mm sample in 50, 70, and 90 grit sizes. From the linear trend lines, it is apparent that the particle diameter d_p is a predominant influence on the heat transfer rate within the system. Based on the sand particle diameters given in Table 5.1, the 50 grit sand is 2.3 times larger in diameter than the 90 grit sand and 1.6 times larger in diameter than the 70 grit sand. Operating around the maximum flow rate, $0.7 < G/G_{mf} \leq 1$, the results obtained between the various sand grits linearly follow the particle diameter ratios given above, within an average deviation of 10%. That is, the maximum calculated Nu for the 50 grit sand is (within $\pm 10\%$) 2.3 times greater than the same data point for the 90 grit sand and 1.6 times greater for the 70 grit sand. For data points operating at $G/G_{mf} = 0.6$ and less the particle diameter ratio does not predict the data points accurately, indicating that at these lower flow rates there is a variance in the heat transfer mechanisms within the system. This is consistent with published results in that whilst operating under laminar flow conditions ($Re_p < 10$) that the particle diameter does not greatly affect the heat transfer coefficient [45, 48]. With flow speed below the range of $G/G_{mf} = 0.7$ the calculated Re_p reaches a maximum of 11, verifying the defined range of laminar flow conditions and heat transfer mechanisms within that range. At low flow rates, there is generally a high uncertainty in Nusselt number values, which implies that there is an insignificant role of particle-to-fluid (particle convection) heat transfer within the overall heat transfer process. This conclusion has been considered reasonable in that at the lower flow rates a particle and its surrounding envelope of fluid are likely to be in thermal equilibrium [45]. Therefore, at the lower flow rates the heat transfer is due primarily to the forced convection through the packed bed, but as the velocity increases the particle convective component becomes more important, witnessed by the steeper increase in Nu at higher flow rates. The trend in Figure 5.4 was consistent for all samples across the varying sand grits.

Figure 5.4: Experimental Nusselt Numbers vs. G/G_{mf} for 4.76mm Sample in 50, 70, and 90 Sand Grit

An average of the calculated Nusselt Numbers was completed across the entire operational flow range for each sample and plotted against the mean particle diameters d_p , given in Figure 5.5. From Figure 5.5 it is shown that the mean Nusselt Number generally increases with increasing particle diameter. For sample sizes 1.27mm (0.050") and 2.11 mm (0.083") or $d_s/d_p \leq 8.3$, the effect of particle diameter did not play a significant role in the increase of the mean Nusselt Number. For $d_s/d_p > 8.3$ the samples increased fairly linearly in mean Nusselt Numbers and overall slopes. Upon generating this figure, additional tests were completed with the 60 grit sand to verify the collected data and to substantiate the implemented linear trend line. Upon completion of those 60 grit tests, it was found that with less than a 3% deviation the new data points coincided with the linear trend line. Additional verification was completed by determining the deviation between the mean Nusselt Number calculated based on the trend line and the experimentally determined mean Nusselt Number. It was found that the inclusion of the additional data point (60 grit sand) reduced the average deviation from 6.3% to 5.1%. Another relevant relationship depicted in Figure 5.5 is that an increase in the mean particle diameter results in an increase in the Nusselt number. Published results dictate that an increase in Nusselt number is obtained from a decrease in mean particle diameter [7, 51, 52]. In Figure 5.5 the data is not accurately linearized as the 50 grit sand operates at flow rates 4.5 times greater than the 90 grit sand. This leads to an increase in Reynolds number by the same factor. Figure 5.6 depicts all the data points that correspond to the restriction of $Re_D < 18.5$. This restriction was imposed as the largest operational Re_D for the 90 grit sand was in fact 18.5 and in order to compare between the different sand grits an equivalent maximum flow rate, or Re had to be chosen. From Figure 5.6 it is observed that the smaller particle size corresponds to an increase in experimental Nusselt number (based on the linear trendlines), consistent with [7, 28, 51, 52]. A 15% error margin bar was included in Figure 5.6 and it is apparent that the individual linear trendlines (representative of the mean data points for each sand grit) fall within the applied error range. As such, it can be concluded that within the range of $Re_D < 18.5$ or alternatively $Re_p < 1$ there is a minimal change in the experimental Nusselt number, again consistent with [43-45]. The restriction imposed by [48]

was that laminar flow conditions apply to the range of $Re_D < 10$, this research has extended that range to 18.5 as the same flow properties are witnessed herein.

Figure 5.5: Comparison of Experimental Nusselt Number vs. Particle Diameter and Particle Size for all Sample Sizes

Figure 5.6: Linearization of Experimental Nusselt Number versus Sample Reynolds Number, Re_D

Figure 5.7: Comparison of Mean Experimental Nusselt Number vs. Cylinder Diameter and Particle Diameter ratio (d_s/d_p)

5.3 HEATED AIR TESTS

Based on increase in the working gas temperature, an increase in gas thermal conductivity is achieved. But this increase in temperature also affects the working gas's density and viscosity. Specifically an increase in temperature results in a decrease in fluid density and in increase in viscosity, as shown in Figure 5.8 and 5.9 respectively.

Figure 5.8: Air Density Dependence on Temperature

Figure 5.9: Air Viscosity Dependence on Temperature

With the general equation of the Reynolds Number (Re) given as:

$$Re = \frac{\rho U D}{\mu} \quad (5.1)$$

The decrease in density and increase in viscosity leads to a significant decrease in the system Reynolds Number. Generally, the Nusselt Number of the system is proportional to Re raised to some positive power [7], which is apparent in several of the correlations given in Chapter 1. Thus, a reduction in the Nusselt Number would be expected with an increase in working gas temperature. Based on a comparison between operating conditions at ambient conditions (20°C) and at heated conditions (~100°C) a maximum reduction in Re of approximately 46.5% is realized. Furthermore, with the general equation of the Nusselt Number given as:

$$Nu = \frac{hD}{k_g} \quad (5.2)$$

The increase in gas thermal conductivity (k_g) would also tend to decrease the system Nusselt Number. As mentioned in Chapter 3 the mean experimental conditions were used, that is to say that the 70 Grit Sand and the 4.76mm (0.1875") sample were used. The operational points that were considered were 10%, 50%, and 100% U_{mf} . The experimental Nusselt Numbers calculated based on the data recorded from those test runs are plotted against the data recorded for the same test components with the system operating at ambient conditions, given in Figure 5.10.

Figure 5.10: Comparison of Experimental Nusselt Numbers Between Heated and Ambient Test Conditions

(5.2) From the above graph, there is an obvious decrease in the experimental Nusselt Numbers, though the slopes are the same. On average, there is a decrease of 36.6% between the heated and ambient temperature test cases, not as large a decrease (46.5%) as determined solely based on the Reynolds number. Therefore, it can be concluded that the increased effective thermal conductivity increases the heat transfer of the system, though at the temperatures tested the lower density and higher viscosity keep the experimental heat transfer coefficients at values lower than those calculated at ambient conditions.

The decrease in density and increase in viscosity leads to a significant decrease in the system Reynolds Number. Generally, the Nusselt Number of the system is proportional to Re raised to some positive power [7], which is apparent in several of the correlations given in Chapter 1. Thus, a reduction in the Nusselt Number would be expected with an increase in working gas temperature. Based on a comparison between operating conditions at ambient conditions and heated conditions, a decrease in Nusselt Number of approximately 46.5% is realized. Furthermore, with the general equation of the Nusselt Number given as:

CHAPTER 6

6 ANALYSIS

6.1 INTRODUCTION

As stated in the beginning of this report the ultimate goal of this research was to study the heat transfer within a packed bed and to develop an empirical correlation that represents the acquired data (as well as representing other researchers published results) with a high degree of accuracy and consistency. The method of dimensional analysis is commonly employed in the interpretation of experimental results [39]. This method enables the researcher to consider all the variables within the experimental study and group them into dimensionless parameters; from which the variables that have the largest influence on the results are revealed. The following section completes such an investigation, on the basis of Buckingham's Method.

6.2 DIMENSIONAL ANALYSIS

The Buckingham's' Method is an extension of another popular method, Rayleigh's Method. The latter encounters difficulties when more variables need be considered [39]. The number of dimensionless parameters that can be combined is given by Buckingham's pi (π) theorem, which essentially states that the number of dimensionless parameters (i) in the final relationship is at least equal to the difference between the number of experimental variables (V) and the number of fundamental dimensions (d) used to describe the variables, given by:

$$\pi = f(\pi_2, \pi_3, \dots, \pi_i) \quad (6.1)$$

where $i \geq V - d$

The fundamental dimensions used in this study are given in Table 6.1.

Table 6.1: Dimensionless Unit Designation

Dimension	Dimensionless Unit Designation
Length	L
Mass	M
Temperature	T
Heat	Q
Time	t

The following, Table 6.2, exhibits the influencing variables within this study alongside their respective dimensions.

Table 6.2: Pertinent Variables and their Respective Dimensions

Variable	Symbol	Units	Dimensions
Particle Diameter	d_p	m	L
Particle Thermal Conductivity	k_p	W/m·K	Q/t·L·T
Gas Viscosity	μ_g	kg/m·s	M/L·t
Gas Density	ρ_g	kg/m ³	M/L ³
Gas Specific Heat	C_{pg}	W·s/kg·K	Q/M·T
Gas Thermal Conductivity	k_g	W/m·K	Q/t·L·T
Characteristic Length	L_c	m	L
Effective Bed Velocity	U_{eff}	m/s	M/t
Bed Voidage	ϵ	N/A	N/A
Heat Transfer Coefficient	h	W/m ² ·K	Q/t·L ² ·T

Based on the data presented in the above tables and Equation (6.1), five fundamental dimensions and 10 influencing variables results in a total of no less than 5 different dimensionless paramters, given by.

$$\pi_1 = f(\pi_2, \pi_3, \dots, \pi_8) \quad (6.2)$$

As the heat transfer within the system is the variable of primary interest, the variables that are descriptive of the thermal and flow properties alongside the system geometry should be chosen to represent the core of the dimensionless groups. From work presented by other researchers it is a known fact that the heat transfer within a porous medium is a function of the

physical properties of the porous medium and the fluidizing gas, as well as the flow rate through the system [45]. On that basis the five core variables selected were U_{eff} , ρ_g , μ_g , k_g , and d_s . Thus the pi groups are defined as:

$$\pi_1 = U_{eff}^a \rho_g^b \mu_g^c k_g^d d_s^e k_g$$

$$\pi_2 = U_{eff}^f \rho_g^g \mu_g^h k_g^i d_s^j C_{pg}$$

$$\pi_3 = U_{eff}^k \rho_g^l \mu_g^m k_g^n d_s^o d_p$$

$$\pi_4 = U_{eff}^p \rho_g^q \mu_g^r k_g^s d_s^t \varepsilon$$

$$\pi_5 = U_{eff}^u \rho_g^v \mu_g^w k_g^x d_s^y h$$

$$\pi_5 = U_{eff}^u \rho_g^v \mu_g^w k_g^x d_s^y h$$

Following the procedure as defined in [39], writing the above pi groups in their respective dimensional forms, equating the exponents of the fundamental dimensions and solving the unknowns, yields the following dimensionless parameters:

$$\pi_1 = \frac{k_p}{k_g}$$

$$\pi_2 = \frac{\mu_g C_{pg}}{k_{eff}} \equiv Pr$$

$$\pi_3 = \frac{\rho_g U_{eff} d_p}{\mu_g} \equiv Re_d$$

$$\pi_4 = \varepsilon \quad \text{or} \quad \pi_4 = 1 - \varepsilon$$

$$\pi_5 = \frac{d_s h}{k_g} \equiv Nu$$

$$\pi_5 = \frac{d_s}{d_p}$$

As expected, the non-dimensionalized parameter pertaining to the specific heat transfer within the system is defined in terms of the Nusselt Number. It remains that the additional non-dimensionalized groups that affect the Nusselt Number need be assessed, on the basis of the physical properties of the fluid and sample alongside the flow rate through the apparatus.

6.2.1 Pi GROUP ANALYSIS:

The thermal conductivity ratio, given by π_1 , relates the two thermal conductivities involved in this study. As the working gas and the porous medium were constant across the entire testing matrix the aforementioned ratio does not represent varied parameters, thus further consideration of this dimensionless parameter is unnecessary and π_1 is removed from the analysis.

Several researchers have proposed that the flow through a packed bed is basically laminar within the range of $1 < Re \leq 10$ [45, 48]. Following this, for a bed of small particles operating under laminar conditions, there is no effective interaction between individual particles, thus ϵ is expected to remain constant with increasing Re (up to the maximum laminar flow range). The maximum Reynolds number for the operational conditions within this research was 2.81, well within the range of laminar flow and as such the non-dimensionalized group π_4 is omitted from further consideration.

With the above pi groups removed from the analysis it leaves that the Nusselt number is a function of the following:

$$Nu = f\left(Re_D, Pr, \frac{d_s}{d_p}\right) \quad (6.3)$$

The variables present in the above relation are consistent with those used in other published research papers, (see Table 1.6). [4-6] Furthermore the two remaining dimensionless parameters represent the pertinent variables within a packed bed system. As discussed in Chapter 2, there are three main components of heat transfer within a packed porous medium apparatus, those being (a) conduction through the packing (b) conductive heat transfer due to

the flowing gas and (c) conductive heat transfer due to the transport of fluidized particles. In the case of this research the primary means of heat transfer is through the movement of the gas through the bed of particle with some heat transfer also gained from the conduction through the packing of the bed. Essentially zero heat transfer is gained from particle motion within the bed. The Prandtl number, Pr , is the ratio of the momentum and thermal diffusivities of the working fluid; the inclusion of which is intuitive in that the thermal properties of the gas (providing the primary means of heat transfer) is pertinent. The Prandtl number is defined using the effective thermal conductivity within the system, not the gas thermal conductivity. As discussed earlier the inclusion of the effective thermal conductivity in the dimensionless parameters is consistent with other scholarly work [4, 5, 7, 15] and as a packed bed operates essentially as a single entity (due to no particle movement) the thermal conductive abilities of the particles must play a role in the heat transfer characteristics of the system. The Reynolds number, Re_D , considers the viscous and inertial forces present in the working fluid around the immersed sample. The inclusion of the Reynolds number in the correlation is imperative because it includes the operational flow speeds through the apparatus up until the minimum fluidizing speed U_{mf} , likewise the minimum fluidizing Reynolds number, Re_{Dmf} . In this research the ratio of sample diameters to particle diameters ranged from $3.85 \leq d_s/d_p \leq 65.69$, clearly a substantial range as other researchers had ranges of only $1 \leq d_s/d_p \leq 5$ [7], $1.4 \leq d_s/d_p \leq 5.5$ [30], and $d_s/d_p = 3.72$ [5]. From Figure 5.7 in Chapter 5, the heat transfer dependence on d_s/d_p is apparent. Thus that ratio is included in the correlation as it; a) represents a large range of diameter ratios which augment the heat transfer process differently and b) ultimately provides a correlation applicable to a large data set which is not currently available in published literature.

From Figure 5.7 the relationship for each particle size is linear. Based upon the linear dependence observed when the data is linearized, an equation of the following form is proposed to represent the entire collected data range, as employed previously by [4, 5]:

$$Nu = (A + BRe_D^x)Pr^y \left(\frac{d_s}{d_p} \right)^z \quad (6.4)$$

The constant A is often inputted into a correlation involving the calculation of Nusselt numbers as it represents the limiting Nusselt number of the system, essentially the minimum range of heat transfer due solely to pure conduction. There is much disagreement on this value as there is no exact theory which satisfactorily describes the transport phenomena in a packed bed. Many researchers adopt the theory that a continual decrease in Reynolds number (directly related to the direct decrease in operational flow rates) corresponds to a decrease in Nusselt number, with no limiting or minimum value. Most packed bed relations (Table 1.6) incorporate some limiting Nusselt number in their correlations relating to their own respective results. The published limiting Nusselt numbers range from approximately 0.255 – 10, with an average, or common, value of 2. For this correlation a log plot of the experimental Nusselt number Nu_{exp} versus the Reynolds number Re_D was completed with a linear trendline incorporated separately for each sand grit, shown in Figure 6.1. From the linear trendline the Nusselt number at zero flow, or a Reynolds number equal to zero can be found from the y intercept (If $Re_D = 0$, $Nu = b$); based on the form of a linear relation:

$$y = mx + b \quad (6)$$

or

$$Nu_{exp} = mRe_D + b \quad (6)$$

The variables present in the above relation are consistent with those used in other published research papers, (see Table 1.6). [4-6] Furthermore the two remaining dimensionless parameters represent the pertinent variables within a packed bed system. As discussed in Chapter 2, there are three main components of heat transfer within a packed porous medium apparatus, those being (a) conduction through the packing (b) convective heat transfer due to

- ◆ Experimental Data for 90 Grit Sand
- Experimental Data for 70 Grit Sand
- ▲ Experimental Data for 50 Grit Sand

Figure 6.1: Log-Plot of Nu_{exp} versus Re_D : Finding the Limiting Nusselt Number

The experimental limiting Nusselt numbers for the 90 Grit, 70 Grit and 50 Grit sands was 0.303, 0.338, 0.360 respectively; with an average of 0.333. Thus constant A is equal to 0.333.

The final constant, B, and the exponents x,y, and z, were obtained by reducing the root mean square error, given by Equation 6.5, where RMSE = 0 is indicative of a perfect fit:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N \frac{(Nu_{exp,i} - Nu_{pred,i})^2}{Nu_{exp,i}^2}}{N - 1}} \quad (6.5)$$

The unknown correlation values, B,x,y, and z were calculated to be 0.26, 0.533, 0.333, and 0.10 respectively with a RMSE equal to 0.06756. The exponents x and y (representative of the exponents for Re_D and Pr , respectively) were varied within a specific range, as defined by correlations developed within the literature. The exponent 'x' (Re_D) was varied between 0.45 and 0.55, as this range was commonly witnessed in correlations developed by other researchers [4-6]. The exponent 'y' (Pr) was varied between 0.3 and 0.4, as for air this is considered the standardized range [45,48]. Rewriting the correlation with the constants yields:

$$Nu = (0.333 + 0.26Re_D^{0.533})Pr^{0.333}\left(\frac{d_s}{d_p}\right)^{0.10} \quad (6.6)$$

Only 3% of the data fell outside of the correlation line with an error range of $\pm 25\%$, as shown in Figure 6.2, and 95% of the mean data fell within $\pm 15\%$ of the correlation line, with an average error of only 5.42%, as shown in Figure 6.3.

The above equation was established based on the mean experimental Nusselt numbers for each sand grit, on that basis Equation (6.6) has an overall uncertainty of $\pm 15\%$ with a 95% confidence level.

The above correlation was used to predict published data to verify it's reliability and parametric range limits. The following table, Table 6.3 lists the relevent values of the referenced experiments used in this calculation. Figure 43 depicts the results from such an analysis. It was found that Equation 6.6 accurately predicted 85% of the relevent data within an error range of $\pm 17.5\%$. From Figure 6.4 it can be seen that the larger particle diameters (9.79mm, 12.23mm) strayed further from the correlation line. This is mainly due to the fact that the sample diameter-to-particle diameter ratio is approaching unity. Also, the experimental Reynolds number was greater than 1000; which is, from Figure 6.4, approximately the maximum range of applicability of Equation 6.6. It is at this point that you may no longer neglect the boudary layer effects, as the continuum treatment is inappropriate [8]; thus it would be expected that there would be some variance between the predicted value and experimental value because the correlation (Equation 6.6) was developed without regard to

boundary layer effects. The data point from [5] is based on a glass-air (particle – gas) packed bed system, thus having very different physical behaviour. The correlation was satisfactorily accurate in predicting the Nu. The maximum operational Re_D used in this work was 81. All of the referenced data had Reynolds numbers at least double that used herein. Thus it can be concluded that the correlation accurately predicts the Nu for a variety of physical property combinations. On the basis of Figure 6.4 it can be concluded that Equation 6.6 predicts the Nusselt number well for, conservatively, $d_s/d_p > 3$ or $Re_D < 1000$. Figure 6.5-6.7 represent comparisons between the correlations developed by [4, 5, 7, 35] against the experimental results for 2.11mm, 6.35mm, 7.94mm samples respectively, or $d_s/d_p = 6.39, 31.28, 54.7$ respectively. The data within Figure 6.5 falls within the d_s/d_p range (< 5) stipulated by the correlations, and as such a common slope shape is observed between the correlations though they underpredict the Nusselt number by close to 30%. The developed correlation (Equation 6.6) displays an excellent fit to the data and shows a similar slope shape as the other correlations; due primarily to a similar correlation form. The same trends can be seen in Figures 6.6 and 6.7, the correlations developed by the other researchers consistently underpredict the Nusselt number for this research and as d_s/d_p decreases the deviation between the predictions and the experimental results increase. From the set of figures (6.5-6.7) it can be concluded that the developed correlation indeed represents the experimental data well in that the maximum deviation observed for the plotted data is 15% and the average is less than 6%.

Table 6.3: Relevant Experimental Values from Referenced Experiments

Reference	Material	Particle Diameter, d_p [mm]	Sample Diameter, d_s [mm]	Reynolds Number, Re_D	Prandtl Number, Pr	Effective Thermal Conductivity, k_{eff}
Nasr et al. [7]	Aluminum	3.24	12.7	568	0.0259	0.715
		6.33		892	0.0270	0.686
		12.23		1281	0.0281	0.657
Nasr et al. [7]	Alumina	2.77	12.7	653	0.0337	0.549
		9.79		1386	0.0350	0.528
Fand and Phan [5]	Soda Lime Glass	3.062	11.45	167	4.80	0.871

Figure 6.2: Calculated Nusselt Number vs. Experimentally Determined Nusselt Number for All Data, $\pm 25\%$ Error Range

Figure 6.3: Predicted Nusselt Number vs. Experimentally Determined Nusselt Number for Mean Data, ±15% Error Range

Figure 6.4: Calculated Nusselt Number versus Mean Experimental Data and Referenced Data with a $\pm 17.5\%$ Error Range

Figure 6.5: Calculated Nusselt Number and Experimental Nusselt Number for $d_s = 2.11\text{mm}$ in 50 Grit Sand ($d_s/d_p = 6.39$)

Figure 6.6: Calculated Nusselt Number and Experimental Nusselt Number for $d_s = 6.35\text{mm}$ in 70 Grit Sand
Sand
($d_s/d_p = 31.28$)

Figure 6.7: Calculated Nusselt Number and Experimental Nusselt Number for $d_s = 7.94\text{mm}$ in 90 Grit Sand ($d_s/d_p = 54.7$)

CHAPTER 7

7 ERROR EVALUATION

7.1 INTRODUCTION

Regardless of the time and care taken in the completion of experimental testing and research, some degree of uncertainty is associated with the obtained results. It is the role of the researcher to evaluate these and determine the degree to which they affect the results. Completion of such an assessment provides a means by which to verify the accuracy of the results within a given error range and furthermore allows for the components that contribute the greatest error to the results to become known. The following sub-sections discuss the errors associated with the various experimental system components.

7.2 AIR DELIVERY ERROR EVALUATION

The airflow travelling through the apparatus was measured with two separate types of flow measurement devices: one being the digital flow meters and the other being a venturi flow meter (connected to a U-Tube manometer). As specified by the manufacturer the digital flow meters were accurate within 2% of the reading or 0.05 Std L/min, whichever was greater. When the flow meters were operating at their maximum measurement range (300 L/min) the error in reading is then ± 6 L/min. The accuracy of the flow meters stated by the manufacturer is based on standard operating conditions of 21.1°C. Throughout the testing process, the temperature from the blower into the flow meters reached the upper operational temperature range of the flow meters of 50°C. As stipulated by the manufacturer an additional 0.075% of the reading is in error for every additional 1°C away from standard conditions. Based on a worst-case scenario

(operating at the maximum temperature range of the meter) the accuracy of the digital flow meters would be within approximately 4% of the reading, or 12 L/min based on maximum flow. The expression used to relate the actual volumetric flow within the system to the displayed standard flow rate was the following, based on material provided by the flow meter manufacturer (TSI):

$$Q = Q' \left[\frac{273.15 + T_m}{273.15 + 21.11} \right] \frac{101.3}{P_m} \quad (7.1)$$

The accuracy of the flow meter in terms of measuring the temperature and pressure are $\pm 1^\circ\text{C}$ and ± 1 kPa respectively. For every 10°C above standard conditions an additional 0.2 kPa of uncertainty must be added, resulting in a worst-case scenario error uncertainty of ± 1.6 kPa. In order to estimate the relative uncertainty within the volumetric flow rate calculation the uncertainties inherent within each variable must be considered. These individual uncertainties form the basis of determining the relative uncertainty, given by the following general uncertainty results, as developed by [39]:

$$w_R = \left[\left(\frac{\partial R}{\partial x_1} w_1 \right)^2 + \left(\frac{\partial R}{\partial x_2} w_2 \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} w_n \right)^2 \right]^{1/2} \quad (7.2)$$

Inputting the characteristic variables given in (7.2), the above relation becomes:

$$w_Q = \left[\left(\frac{\partial Q}{\partial Q'} w_{Q'} \right)^2 + \left(\frac{\partial Q}{\partial T_m} w_{T_m} \right)^2 + \left(\frac{\partial Q}{\partial P_m} w_{P_m} \right)^2 \right]^{1/2} \quad (7.3)$$

Solving the above relation based on the highest flow rate and temperature rates (representative of worst case error uncertainty) yields $w_Q = 4.86\%$ or 0.54 CFM. For several of the higher flow test cases a 1" venturi flow meter was used to measure the volumetric flow rate through the system. The following expression was used in determining the flow rate:

$$Q = C_v A_T \sqrt{\frac{2\Delta P}{\rho_g (1 - \beta^4)}} \quad (7.4)$$

The venturi manufacturer estimates the uncertainty in the discharge coefficient, C_v , to be $\pm 0.5\%$. Proper installation procedures were followed as to not affect the inlet flow conditions and as such all pipe burrs and adhesive residues were carefully removed before and during

assembly. The throat area of the venturi, A_T , is given as better than $\pm 0.001\%$ or 0.0254mm , and is thus considered a negligible source of error as its effect on the overall venturi uncertainty is very slight. The pressure drop, or differential pressure reading, was measured using a U-tube manometer. The error incurred from this component is limited by the user's ability to read the scale accurately. The scale on the manometer is given in increments of $0.1''$ WC. A conservative estimate of the inherent uncertainty for the manometer would be equal to the scale increments and as such the error was estimated to be $\pm 0.1''$ WC. The density of the air was based on the ideal gas law where the pressure and temperature were measured with the electronic flow meters. The errors for those measurements are given above and represent a 0.75% uncertainty for the inlet air density. The throat-to-pipe diameter ratio's contribution to the overall uncertainty is considered negligible due to proper and careful assembly of the venturi flow meter, which ensured tight tolerances between the pipe and the venturi. Adapting equation (7.2) with the variables given in (7.4) which provide considerable uncertainty levels results in the following relation:

$$w_Q = \left[\left(\frac{\partial Q}{\partial C_v} w_{C_v} \right)^2 + \left(\frac{\partial Q}{\partial \Delta P} w_{\Delta P} \right)^2 + \left(\frac{\partial Q}{\partial \rho_g} w_{\rho_g} \right)^2 \right]^{1/2} \quad (7.5)$$

The relative uncertainty within the flow rate measurements using the venturi is $w_{Qv} = 4.5\%$ or 0.6 CFM. Considering the errors in the uncertainty analysis, the component that contributes to the majority of the error is the reading (measurement) from the U-tube manometer (ΔP), as this value is based on individual discretion and reading ability.

7.3 TEMPERATURE MEASUREMENT ERROR EVALUATION

The temperatures that were measured throughout the testing process were also subject to errors arising from the thermocouples themselves and the Personal DAQ data acquisition system. As stated by the manufacturer [54] the Type K thermocouples had an inherent $\pm 2.2^\circ\text{C}$ or 0.75% (of the recorded temperature, whichever was greater) standard error limit while

operating within -200°C - $+1250^{\circ}\text{C}$. The Personal DAQ 3000 data acquisition system was accurate within $\pm 1.8^{\circ}\text{C}$ utilizing Type K thermocouples based on an operating range between -200°C - $+1200^{\circ}\text{C}$. When measuring and recording temperatures throughout the data acquisition period the monitored temperatures remained steady within a $\pm 0.5^{\circ}\text{C}$ range, thus the overall temperature measurement error was estimated at approximately $\pm 1.0^{\circ}\text{C}$. The temperature difference measured between bed and sample was consistently applied at 10°C - 15°C , thus a temperature error of $\pm 1.0^{\circ}\text{C}$ was considered quite acceptable. The final temperature component which would cause some error (in the calculation process) is the excess terminal temperature, θ_T . The highest evaluated excess terminal temperature value was $\pm 2^{\circ}\text{C}$, with the majority of the test cases having an excess terminal temperature value of $\pm 1^{\circ}\text{C}$. Based on these discussed errors the overall estimated error uncertainty in temperature measurement was $\pm 2.15^{\circ}\text{C}$ and $\pm 1.25^{\circ}\text{C}$ for θ_T . It is unlikely that the thermocouple/data acquisition unit would incur such a high temperature measurement error ($\pm 1.8^{\circ}\text{C}$), thus in order to accurately determine the inherent temperature measurement errors additional measurements of the four T/C's were completed with the sample (4.76mm) immersed in the alumina sand (70 grit) with no airflow. Under these conditions it was assumed that the bed was isothermal and the T/C temperatures were recorded. The results are plotted in Figure 7.1 alongside the average temperature of each T/C. As the bed temperature is the only T/C not affixed directly to the sample it was taken as the standard, or actual, temperature value. The average temperature readings for the center, right, left and bed T/C's are 21.65 (corrected to outer surface temperature), 22.02, 20.79, and 22.2 respectively. This results in an average deviation of 3.21%. Based on a $\Delta T \approx 10^{\circ}\text{C}$ a deviation of 3.21% relates to an error of approximately 0.3% or $\pm 0.3^{\circ}\text{C}$. Therefore, the uncertainty in the temperature measurements was estimated to be $\pm 1.0^{\circ}\text{C}$.

Figure 7.1: Thermocouple Temperature Measurement Comparison

7.4 HEAT TRANSFER CALCULATION ERROR EVALUATION

Based on the temperature distribution relation developed in Chapter 3 (Equation 3.17, reprinted below) there are only two additional variables that could affect the accuracy of the heat transfer calculation, the applied current, I , and the measured voltage, V , across the sample.

$$T(x) = \left[\theta_T - \frac{VI}{LhP} \right] \frac{\cosh(mx)}{\cosh\left(m\frac{L}{2}\right)} + \frac{VI}{LhP} + T_\infty \quad (7.6)$$

The geometric variables within Equation 17 are constant and as such, no error is associated with those values. The excess terminal temperature, θ_T , and the bed temperature, T_∞ and their respective uncertainties were discussed in the above section; leaving only the uncertainties within the current and voltage. The current applied to the sample was supplied by a GW Instek Programmable Power Supply (Model PSH-10100) operating at a constant voltage. As per the manufacturer the displayed current is accurate within $\leq 0.22\%$ or $\pm 90\text{mA}$, whichever was greater. The voltage across the sample was measured using a digital multimeter (Model GDM-814M) with an accuracy of $\pm 0.1\%$. The overall uncertainty in the power generation/dissipation term is estimated at $\pm 0.3\%$.

Applying the traditional technique, given in [39], to determine the cumulative uncertainties within a certain equation begins with solving for the heat transfer coefficient, h , presented implicitly in Equation (7.6). Solving for h yielded a root function, which makes it difficult to solve the partial derivatives for the uncertainty analysis. Based on this difficulty the above relation was examined and the variables that provided the greatest degree of uncertainty were the measured temperatures, given in the θ_T and the ΔT terms. Based on the above temperature measurement error evaluation the θ_T term was deemed to have a negligible influence on the error associated with the calculation of h , leaving only the ΔT term. A simple

correlation, Equation (7.7), was used as a comparative calculation to the more arduous relation, Equation (7.6).

$$h_{avg} = \frac{VI}{A_s \Delta T} \quad (7.7)$$

It was found that it was accurate within 5% to the results obtained from the implicit h relation. For the sake of consistency, an error uncertainty analysis on the simple h relation is performed for a series of different test operating conditions, detailed in Table 7.1-7.3, for sand grits 50, 70 and 90 respectively. Using the procedure given in [39] the overall uncertainty in the heat transfer coefficient calculation, Equation (7.7), is given by:

$$w_h = \left[\left(\frac{\partial h}{\partial V} w_V \right)^2 + \left(\frac{\partial h}{\partial I} w_I \right)^2 + \left(\frac{\partial h}{\partial \Delta T} w_{\Delta T} \right)^2 \right]^{1/2} \quad (7.8)$$

Table 7.1: 50 Grit Heat Transfer Calculation Error Evaluations

Sample	I [A]	w_I	V [V]	w_V	ΔT [°C]	$w_{\Delta T}$	h [W/m ² -K]	Uncertainty [%]
1.27 mm (0.050")	2.25	0.0005	0.6745	0.0007	11.2	±1.00°C	142.6	8.93
9.53 mm (0.375")	25.65	0.0513	0.2254	0.0002	10.0	±1.00°C	76.0	10.00

Table 7.2: 70 Grit Heat Transfer Calculation Error Evaluations

Sample	I [A]	w_I	V [V]	w_V	ΔT [°C]	$w_{\Delta T}$	h [W/m ² -K]	Uncertainty [%]
1.27 mm (0.050")	1.92	0.0004	0.5628	0.0005	9.8	±1.00°C	124.45	9.77
9.53 mm (0.375")	19.02	0.0038	0.1559	0.0001	11.6	±1.00°C	33.7	11.35

Table 7.3: 90 Grit Heat Transfer Calculation Error Evaluations

Sample	I [A]	w_I	V [V]	w_V	ΔT [°C]	$w_{\Delta T}$	h [W/m ² -K]	Uncertainty [%]
1.27 mm (0.050")	2.16	0.0004	0.6294	0.0006	9.1	±1.00°C	144.9	10.98
9.53 mm (0.375")	17.82	0.0036	0.1529	0.0001	10.3	±1.00°C	34.95	9.67

CHAPTER 8

8 CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

The heat transfer coefficients were experimentally determined for small cylindrical samples immersed within a packed bed of alumina sand particles under conditions of forced convection. The flow rates were varied from $0.1G/G_{mf}$ up to $1.0G/G_{mf}$, or up to the point of minimum fluidization, the point at which the particles are no longer stagnant but begin to move, or fluidize. From the experimental results, several trends developed, the primary trend being that an increase in Nu was directly proportional to both an increase in sample diameter and airflow rate through the bed. The secondary trend observed was that the decrease in particle size increased the heat transfer coefficient, or more commonly the Nusselt number. Both trends were consistent with published results. The increase in Nusselt number based on increased sample diameter size is intuitive based on the Nu relation (Equation 2.42), as the increase in sample diameter increases the experimental Reynolds number, which directly leads to an increase in the experimental Nusselt number. The increased heat transfer rate with the smaller diameter particles is due primarily to the change in bed permeability and gas film, or boundary layer, near the sample surface. With increased contact points between the surface and particles, additional particle conduction can occur, increasing the heat transfer. Another important observation was that, for $Re_D < 18.5$, the effect of particle diameter had very little effect on the Nusselt number, as the experimental Nusselt numbers were within 15% of each other across the three sand grits (50, 70, and 90). Some consideration was also included on the effect of working with an increased air temperature, in this case, operating at 100°C . It was found that the experimental Nusselt numbers were uniformly 36.6% below

those of the corresponding ambient air tests. This is due mainly to the increased viscosity and decrease in density, ultimately decreasing the system Reynolds number, where it has been established that Nu is directly related to Re [4-6,9]. A correlation (Equation 6.6) was developed based on the experimental results that predict the mean Nusselt number of the cylindrical samples within a $\pm 15\%$ error range with a 95% confidence limit. This correlation was also applied to data by other researchers and good agreement was found. When the correlations developed by other researchers were compared to the experimental data herein it was observed that the correlations under predicted the Nu by as much as 45%. This is consistent with packed bed phenomenon, as the tested particle sizes in those cases were an order of magnitude greater than those used here and as such the smaller particle sizes used here would produce a higher Nu . It was also observed that the correlation predicts the Nu for the heated air tests accurately. The correlation did not predict the data from experiments with $d_s/d_p \approx 1.0$, as the boundary layer assumption herein is no longer valid. As such, a conservative restriction on the developed correlation is $d_s/d_p > 3$ or $Re_D < 1000$. Therefore, the experimental results obtained compare well with established data and fills a gap in the published research for very small cylinders in very fine particles ($3.85 < d_s/d_p < 65.69$). Thus, the original objective was fulfilled, a correlation was developed that correlates the data well (for d_s/d_p ratios given above), which otherwise are not represented well by published research.

8.2 RECOMMENDATIONS

The following discusses some further work that would complement the research herein and would further the understanding of the heat transfer to small cylinders in packed beds.

Within this research, the bed porosity ϵ was assumed constant between the sand grits, whereas a variance in this value would be expected. As within a packed bed, the effective thermal conductivity is an essential value in determining the heat transfer, of

which the bed porosity is a pertinent parameter. Further consideration of the bed behavior between the different particle sizes would help highlight any differences in the bed porosity and ultimately the overall effective thermal conductivity of the system.

Further development of hot air tests should be completed for the packed bed condition. These tests should be completed over a broad range of both sand grits and increased temperatures. In doing so, further understanding of the effect of the additional heat to the bed would be achieved. Which would also provide the means to compare between the ambient air tests on a more accurate basis.

8.2 RECOMMENDATIONS

sample surface. With increased contact points between the surface and particles,

The following discuss some further work that would complement the research

performed and would further the understanding of the heat transfer to small cylinders in

packed beds.

Included in the research were Nusselt numbers within the range of 0.5 to 1.0, which is a low range for heat transfer.

Within this research, the bed porosity was assumed constant between the sand

on the effect of working with an increased air temperature. In this case, operating at

grit, whereas a variance in the value would be expected. As within a packed bed, the

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