**MULTIDISCIPLINARY CONSTRAINTS WITHIN A
TWO-DIMENSIONAL AERODYNAMIC OPTIMIZATION
METHOD**

by

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requirements for the degree of Master of Applied Science in the Program of
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ABSTRACT

MULTIDISCIPLINARY CONSTRAINTS WITHIN A TWO-DIMENSIONAL AERODYNAMIC OPTIMIZATION METHOD

Master of Applied Science, Mechanical Engineering

Ryerson University, 2006

Maureen Louise Kolla

This research demonstrates the importance of including multidisciplinary constraints within a two-dimensional aerodynamic optimization method. These constraints increase the methods flexibility and versatility by providing the aerodynamic designer with the latitude to expand the design envelope. The additional constraints include a global minimum thickness, a maximum point thickness, an area, two curvature functions and a stowability constraint. The global minimum thickness constraint is used to prevent airfoil surface crossovers. The maximum point thickness and area constraint address airfoil structural requirements. The curvature function constraints deal with the airfoils manufacturability. Finally, the stowability constraint combines flap trajectory, including the flap mechanics, together with the final airfoil shape, to ensure high-lift stowability.

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To my parents,
Because they have always believed.

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NOMENCLATURE

LATIN SYMBOLS

a	Speed of sound
a_{cf1}	Design specified curvature function <i>one</i> $\pm$ tolerance
b_{f2}	Design specified curvature function <i>two</i> tolerance
c	Airfoil chord
c_f	Flap chord
c_{f2}	Design specified curvature function <i>two</i> amplitude
c_i	Constraint equations
c_{ig}	Global minimum thickness constraint equations
$c_{i\kappa}$	Curvature constraint equations
c_{imx}	Maximum thickness constraint equations
c_{istow}	Stowability constraint equations
c_{ita}	Area constraint equations
c_w	Flap chord length corresponding to wing chord
C_L, C_D	Lift and drag coefficients
C_L^*, C_D^*	Target lift and drag coefficients

d_{f2}	Design specified curvature function <i>two</i> vertical translation
D	Design variable vector
D_{scal}	Scaled design variable vector
e	Total energy
e_{f2}	Design specified curvature function <i>two</i> horizontal translation
E	Inviscid flux vector in x -direction
E_v	Viscous flux vector in x -direction
F	Inviscid flux vector in y -direction
F_v	Viscous flux vector in y -direction
$\mathcal{G}$	Gradient vector
$\mathcal{G}_{scal}$	Scaled gradient vector
I	Objective function (scalar)
I_c	Objective function constraints
I_d	Design objective function
$h(x_i)$	Constraint value at location x_i
$h^*(x_i)$	Allowable constraint value at location x_i
$h_{tk}(x_i)$	Airfoil thickness at location x_i
$h_g(x_i)$	Global thickness at location x_i
$\mathcal{H}$	Inverse Hessian matrix approximation
J	Metric Jacobian
n_c	Total number of individual constraint equations

N_C	Total number of active constraints
p	Pressure
Pr	Laminar Prandtl number
Pr_t	Turbulent Prandtl number
Q	Vector of conservative flow variables
R	Discrete residual equation vector
R_{LD}	Lift-to-drag ratio
R_{LD}^*	Target lift-to-drag ratio
Re	Reynolds number
S	Vorticity
s	Search direction vector
t	Time
u, v	x, y -component velocity, respectively
U, V	Contravariant velocities
w_D	Designer specified drag coefficient weighting
w_L	Designer specified lift coefficient weighting
w_κ	Designer specified curvature constraint weighting
w_{mx}	Designer specified maximum thickness constraint weighting
w_{stow}	Designer specified stowability constraint weighting
w_{ta}	Designer specified area constraint weighting
x, y	Cartesian coordinate directions

GREEK SYMBOLS

$\Delta\kappa$	Difference in curvature
γ	Specific heat ratio
κ	Curvature
κ_{f1}	Curvature function <i>one</i> equation
κ_{f2}	Curvature function <i>two</i> equation
κp	Curvature peak
μ	Dynamic laminar viscosity
μ_t	Turbulent eddy viscosity
ν	Kinematic laminar viscosity
ν_t	Kinematic turbulent viscosity
$\tilde{\nu}$	Tubulence model working variable
ρ	Density
τ	Shear stress
ξ, η	General curvilinear coordinates

ABBREVIATIONS

$f1$	Curvature function <i>one</i>
$f2$	Curvature function <i>two</i>
ll	Lower surface lower curvature limit
ls	Lower surface
lu	Lower surface upper curvature limit
ul	Upper surface lower curvature limit
uu	Upper surface upper curvature limit
us	Upper surface

CHAPTER 1

1.1 Introduction

Aerodynamic design and optimization have received significant attention over the last decade, with aerodynamic optimization moving from indirect to direct design methods. The indirect method uses an inverse design approach that determines the airfoil shape based on a pressure distribution. While this method is effective in designing an airfoil to meet aerodynamic constraints, it does not contain non-aerodynamic constraints [1]. The desired direct method is an iterative procedure that requires objective functions, constraints, design variables, a flow solver and a numerical optimization method. In his paper Ref. [2], Jameson discusses the evolution of computational fluid dynamics (CFD) for aerodynamic design and optimization. The direct method is favored as it offers the airfoil designer greater flexibility to achieve an effective design strategy capable of dealing with nonlinear flow features, multiple operating conditions and multidisciplinary interactions.

The focus over the last few years has been in developing and implementing the direct optimization procedure. Each component has been well researched and is highlighted and discussed in Section 1.2. The current re-

search focusses on furthering the optimization development method by including multi-disciplinary constraints. There have been papers written by Drela [3], Jou *et al.* from the Boeing Company [4] and Venkataraman [1] on the importance of including non-aerodynamic constraints within optimization methods. Drela [3] believes that to have an effective design environment, there must be a toolbox which allows the airfoil to be modified with geometric and/or aerodynamic specifications in a natural and intuitive manner. Jou *et al.* [4] have found from their experience that optimization methods must have the flexibility to incorporate geometric constraints from other disciplines such as manufacturing and structures. Section 1.3 contains the current research objectives and expands on the multi-disciplinary constraints that have been included in the optimization method.

1.2 Aerodynamic Optimization

Aerodynamic optimization is a philosophy where the goal is to achieve an optimal airfoil shape and/or the optimal position for a set of design objectives. The various direct design method components will be discussed within this section, as well as some of the different approaches used to implement the components within the optimization procedure.

The optimization objective is typically set up as a minimization function that includes lift and drag. This function is termed the objective function. The constraints placed on the optimization can be handled in different ways but the most popular approach is to use penalty functions. Various methods used to handle constraints are presented by Michalewicz [5], Giles [6] and Drela [7]. They each provide a detailed summary of the factors involved to

select appropriate objective functions and constraints.

The design variables are used to control the airfoil shape and/or its position. The shape design variables are selected using a parameterization approach. The most common approaches are the shape-function, polynomial and spline approaches. A detailed description for the shape function developed by Hicks and Henne is provided in [8] and the function application for aerodynamic optimization is provided in [9, 10]. The polynomial approach uses airfoil parameters such as the trailing-edge angle, airfoil thickness and curvature to define the airfoil shape. PARSEC [11] and Chebyshev polynomials [12] are two examples of this approach. Finally, spline approaches can either be a Bezier spline (B-spline) [13, 14, 15] or a non-uniform rational B-spline (NURBS) [16].

Throughout the optimization, the airfoil shape or position is constantly changing requiring the grid around the airfoil to also change. A grid generator is typically used to obtain the starting mesh; however, using it at every optimization iteration would not be necessary for small changes in geometry or position. Therefore, an interior grid movement scheme is needed to adjust the grid nodes. For both two and three-dimensional cases, the mesh can be either structured [17, 18] or unstructured [19, 13]. On structured meshes, the most popular mesh movement strategy is an algebraic perturbation scheme [20, 10, 15] as it is easy to implement and requires little computational memory. Unstructured methods use a spring analogy as described in Ref. [14, 13]

An accurate flow solution around the airfoil is necessary in order to produce an accurate optimization. For two-dimensional single and multi-element airfoils, a compressible flow solver based on the Navier-Stokes equations

[21, 22, 23] is used. For more complex three-dimensional cases, the compressible Euler equations [9, 20] are solved. The turbulent effects can be modeled using several different models. For single element airfoils where the geometry is simple and for designs at low angles of attack, the Baldwin-Lomax model can be used [21] but for high-lift geometries the Spalart-Allmaras [24] turbulence model is more accurate.

A plethora of numerical optimization methods have been developed to solve various problems. Depending on the situation, a particular method will be more applicable and suitable. Nelder and Mead introduced the simplex method, which is a direct search method. Duvigneau and Visonneau [25] demonstrated this method's usefulness for aerodynamic and hydrodynamic shape optimization. The flow is governed by the two-dimensional incompressible Navier-Stokes equations. The most popular method in the last few years has been the gradient-based method. It was the introduction of the adjoint method or control theory approach by Jameson [26] and Pironneau [27] that made this method attractive. Originally, when a gradient based optimization method was used, the design sensitivity was determined using the finite difference method. This limited the number of design variables since the finite difference method required a flow solution for every design variable and was therefore computationally expensive. With the adjoint method, the sensitivity is determined from the partial differential equations or adjoint equations, which only require one flow solution per optimization iteration and is not dependent on the number of design variables. Thus, the computing time required for the sensitivity terms is greatly reduced. Newman *et al.* [28] put together a good overview illustrating various ways to calculate the

sensitivity terms within the gradient-based shape optimization. Since then, further development has been done by Jameson and Kim [29], Nemec and Zingg [30], Giles *et al.* [31] and Le Moigne and Qin [32].

Some other methods that have been used to solve the single-objective optimization problem are an exergy-based method [33], a Taguchi simulated annealing (SA) method [34] and a kriging model [35]. Finally, swarm algorithms [11] have been developed for multi-objective optimization problems. This is by no means a complete list but highlights some of the interesting methods developed in just the last couple of years.

1.3 Objectives

The current research objective is to enhance an existing two-dimensional aerodynamic optimization method by including additional multi-disciplinary constraints. This will increase the method's flexibility and provide the airfoil designer with more freedom. The optimization method used was developed by Dr. Marian Nemec, during his Ph.D. studies at the University of Toronto. The method is a gradient-based algorithm that is governed by the compressible, two-dimensional Navier-Stokes equations in conjunction with the Spalart-Allmaras turbulence model [36, 22, 30].

The multi-disciplinary constraints that have been included are:

1. A global minimum thickness constraint to help prevent the airfoil from becoming too thin, especially near the trailing edge, as well as to control surface crossovers [36] that result in unrealistic airfoil shapes being

produced.

2. A local maximum thickness constraint to complement the existing minimum thickness constraint.
3. An area objective/constraint term [37, 4, 3] to design for a specific airfoil cross-sectional area.
4. Curvature constraints to prevent undesired surface waviness [3] resulting in a simpler manufacturing process.
5. A stowability constraint to include the flap movement mechanics [38] within high lift optimizations. This constraint generates a design area that controls the high lift device shape in order to prevent fouling along the trajectory path.

Chapter 2 summarizes the governing equations for the formulation of the optimization problem and the numerical algorithm. For a detailed presentation see Ref. [36, 22]. The multi-disciplinary constraints are discussed in Chapter 3 with design examples for each constraint in Chapter 4. Finally, Chapter 5 contains the research conclusions and recommendations for future work.

CHAPTER 2

2.1 Governing Equations

Aerodynamic optimization can be defined as a minimization of a chosen objective function, I , based on selected design variables, D .

$$\min_D I(D, Q) \quad (2.1)$$

with the objective function, I , subject to the constraint equations, c_i

$$c_i(D, Q) \leq 0 \quad i = 1, \dots, n_c \quad (2.2)$$

where Q is a vector containing the conservative flow variables and n_c is the total number of constraint equations.

The following sections expand Equations 2.1 and 2.2 for the optimization method used in this research.

2.2 Design Variables

The design variable vector, D can contain parameters that control the airfoil shape, the slat or flap position in a multi-element design and/or the airfoil's angle of attack. The airfoil shape is controlled with B-spline control points.

For all the test cases in this research, cubic B-splines are used to approximate the upper and lower airfoil surface. In order to maintain the chord line orientation, the B-spline curve is forced through the leading and trailing edge points. Figure 2.1 shows an airfoil with a cubic B-spline curve on the upper and lower surfaces, both generated from 8 control points. The B-spline

Figure 2.1: Airfoil with B-spline control points.

control points are limited to one degree of freedom, namely movement in the vertical direction only. The leading and trailing edge points, LE and TE respectively in Figure 2.1, remain fixed. Any combination of B-spline control points can be selected as design variables.

The slat or flap position is controlled with translation variables that are parallel and normal to the wing chord. For example, the parallel translation variable or the flap overlap is defined from the main element trailing edge to

the flap leading edge. The normal translation variable or the gap is from the shroud trailing edge on the main element to the flap upper surface. Figure 2.2 illustrates this relationship.

Figure 2.2: Definition of gap and overlap.

2.3 Objective Function

The objective function consists of two parts, the design objectives and the constraints and can be expressed as the following equation

$$I(D, Q) = I_d(D, Q) + \sum_{j=1}^{N_C} I_{c_j}(D) \quad (2.3)$$

where N_C is the total number of constraints placed on the objective function. In this research, the constraints are only a function of the design variables.

By combining the constraints within the objective function the optimization problem can be cast as an unconstrained problem. This allows the constrained problem to be solved using unconstrained optimization techniques. Section 2.3.1 discusses the possible design objectives and Section 2.3.2 discusses the method used to include the constraints within the objective function and the constraint equations.

2.3.1 Design Objectives

This program currently allows for four design objectives. The general design objective function equation is defined as follows

$$I_d(D, Q) \equiv w_L I_L + w_D I_D \quad (2.4)$$

where

$$I_L = \left(1 - \frac{C_L}{C_L^*}\right)^2 \quad (2.5)$$

$$I_D = \begin{cases} \left(1 - \frac{C_D}{C_D^*}\right)^2, & \text{if } C_D > C_D^* \\ 0, & \text{otherwise} \end{cases} \quad (2.6)$$

where C_L and C_D are the airfoil's lift and drag coefficients, C_L^* and C_D^* are the target lift and drag coefficients and w_L and w_D are the weights. The weights are specified by the designer to keep each design objective term at a similar magnitude and to weight terms according to importance.

Selecting this design objective function formulation allows for easy implementation of other objective terms, for example an area objective for maximum fuel storage. This will be further discussed in Chapter 3.

Using Equation 2.4, the designer can either 1) reduce the drag coefficient to a chosen target value while maintaining the lift coefficient, 2) increase the lift coefficient while maintaining the drag coefficient or 3) maximize the lift coefficient. The first two objectives are achieved by selecting weighting values that allow the drag or lift coefficient terms to dominate the function. The third objective, maximizing the lift coefficient, is achieved by setting the target drag coefficient, C_D^* to a value larger than the actual drag coefficient, C_D . This will set I_D to zero, as seen in Equation 2.6, and the design objective function will only contain the lift contribution.

The last design objective, maximizing the desired lift-to-drag ratio, is achieved by modifying Equation 2.5 to become

$$I_L = \left(1 - \frac{R_{LD}}{R_{LD}^*}\right)^2 \quad (2.7)$$

where R_{LD} is the actual lift-to-drag ratio and R_{LD}^* is the target. As well, I_D in Equation 2.4 is set to zero.

2.3.2 Constraints

The objective function constraints are defined as follows

$$I_{c_j}(D) \equiv w_j \sum_{i=1}^{n_c} (c_i)_j \quad j = 1, \dots, N_C \quad (2.8)$$

where N_C is the number of active constraints imposed on the objective function, $(c_i)_j$ is the j th constraint equation for an individual constraint i , w_j is the weighting for the active constraint and n_c is the total number of constraint points for an individual constraint. For example, if the optimization problem contained four minimum and two maximum local thickness constraints with

the minimum and maximum thickness constraint weighting being 1.0 and 2.0, respectively, then Equation 2.8 would become

$$I_{c_2}(D) = 1.0 \sum_{i=1}^4 (c_i)_1 + 2.0 \sum_{i=1}^2 (c_i)_2 \quad (2.9)$$

For simplicity in the following chapter, $w_j \sum_{i=1}^{n_c} (c_i)_j$ is defined as $w_j C_j$.

With the chosen form of the design objective function, the constraints are included within the main objective function using the penalty method. The constraint equation, c_i , based on Equation 2.2, becomes

$$c_i = h^*(x_k) - h(x_k) \leq 0 \quad (2.10)$$

where $h^*(x_k)$ is the allowable value for the constraint at the x_k location and $h(x_k)$ is the actual value at that location. These equations are implemented using the quadratic penalty method given by

$$c_i = \begin{cases} \left[1 - \frac{h(x_k)}{h^*(x_k)}\right]^2, & \text{if } h(x_k) < h^*(x_k) \\ 0, & \text{otherwise} \end{cases} \quad (2.11)$$

The weighting term, w_j , in Equation 2.8 acts as the penalty parameter for the penalty method. This term's value should be chosen with care as problems within the optimization algorithm can result [39]. For example, the objective function contains both the design objective terms and the constraint terms, if these terms are not of similar magnitude, the optimization algorithm can have convergence problems. From experience, the optimization method used in this research, works best if the initial constraint term is an order of magnitude less than the design term. For example, if the initial design objective term is 10^{-2} and a constraint term is 1.0 then w_j should be 10^{-3} .

On the other hand, if the initial constraint term is 10^{-6} than a larger value for w_j is needed, such as 10^3 .

Equation 2.2 initially represented a local minimum airfoil thickness and a gap and overlap constraint. This equation now represents a global minimum airfoil thickness, local maximum airfoil thickness, cross-sectional area, curvature and stowability constraint. Chapter 3 describes these added constraints in more detail.

2.4 Flow Equations

The governing flow equations used in the program are the compressible Navier-Stokes equations, in conjunction with the one-equation Spalart-Allmaras [24] turbulence model. The thin-layer approximation [40] is used to simplify the Navier-Stokes equations.

2.4.1 Navier-Stokes Equations

The compressible Navier-Stokes equations for a two-dimensional flow with density ρ , velocities (u, v) in Cartesian coordinates (x, y) and total energy e are given by

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = \mathcal{R}e^{-1} \left(\frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} \right) \quad (2.12)$$

where $\mathcal{Re}$ is the Reynolds number and

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix} \quad (2.13)$$

is the conservative dependent flow variable vector. The convective flux vectors are

$$E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(e + p) \end{bmatrix}, \quad \text{and} \quad F = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ v(e + p) \end{bmatrix} \quad (2.14)$$

in the x and y direction, respectively and the viscous flux vectors are given by

$$E_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \varphi_1 \end{bmatrix}, \quad \text{and} \quad F_v = \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \varphi_2 \end{bmatrix} \quad (2.15)$$

with

$$\begin{aligned} \tau_{xx} &= (\mu + \mu_t)(4u_x - 2v_y)/3 \\ \tau_{xy} &= (\mu + \mu_t)(u_y - v_x) \\ \tau_{yy} &= (\mu + \mu_t)(-2u_x + 4v_y)/3 \\ \varphi_1 &= u\tau_{xx} + v\tau_{xy} + (\mu\mathcal{P}r^{-1} + \mu_t\mathcal{P}r_t^{-1})(\gamma - 1)^{-1}\partial_x(a^2) \\ \varphi_2 &= u\tau_{xy} + v\tau_{yy} + (\mu\mathcal{P}r^{-1} + \mu_t\mathcal{P}r_t^{-1})(\gamma - 1)^{-1}\partial_y(a^2) \end{aligned} \quad (2.16)$$

where μ_t is the turbulent eddy viscosity, Pr is the laminar Prandtl number, and Pr_t is the turbulent Prandtl number. The pressure, p is related to the flow variables by the equation of state for a perfect gas

$$p = (\gamma - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2) \right] \quad (2.17)$$

The speed of sound, a , is given by

$$a = \sqrt{\frac{\gamma p}{\rho}} \quad (2.18)$$

Note that Equations 2.12 - 2.18 are in a non-dimensional form. For the scaling parameters see Ref. [36].

2.4.2 Thin-Layer Approximation

The thin-layer approximation is used to simplify the Navier-Stokes equations by neglecting the viscous streamwise derivatives. The thin-layer approximation is based on curvilinear coordinates, i.e. the ξ direction follows the airfoil contours and the η direction is locally normal to the airfoil. To map the curvilinear grid into the computational domain a general transformation is used

$$\begin{aligned} \tau &= t \\ \xi &= \xi(x, y, t) \\ \eta &= \eta(x, y, t) \end{aligned} \quad (2.19)$$

Below is a brief summary of the thin-layer Navier-Stokes equations. For more details regarding the thin-layer approximation and the generalized curvilinear coordinate transformation see Ref. [40].

The thin-layer Navier-Stokes equations in generalized curvilinear coordinates are given by

$$\frac{\partial \widehat{Q}}{\partial \tau} + \frac{\partial \widehat{E}}{\partial \xi} + \frac{\partial \widehat{F}}{\partial \eta} = \mathcal{R}e^{-1} \frac{\partial \widehat{S}}{\partial \eta} \quad (2.20)$$

where the flow variable vector becomes

$$\widehat{Q} = J^{-1} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix} \quad (2.21)$$

and the convective flux vectors becomes

$$\widehat{E} = J^{-1} \begin{bmatrix} \rho U \\ \rho U u + \xi_x p \\ \rho U v + \xi_y p \\ (e + p)U - \xi_t p \end{bmatrix}, \quad \widehat{F} = J^{-1} \begin{bmatrix} \rho V \\ \rho V u + \eta_x p \\ \rho V v + \eta_y p \\ (e + p)V - \eta_t p \end{bmatrix} \quad (2.22)$$

with

$$U = \xi_t + \xi_x u + \xi_y v, \quad V = \eta_t + \eta_x u + \eta_y v \quad (2.23)$$

as the contravariant velocities. The variable J represents the metric Jacobian of the transformation

$$J^{-1} = (x_\xi y_\eta - x_\eta y_\xi) \quad (2.24)$$

The viscous flux vector becomes

$$\widehat{S} = J^{-1} \begin{bmatrix} 0 \\ \eta_x m_1 + \eta_y m_2 \\ \eta_x m_2 + \eta_y m_3 \\ \eta_x (u m_1 + v m_3 + m_4) + \eta_y (u m_2 + v m_3 + m_5) \end{bmatrix} \quad (2.25)$$

with

$$m_1 = (\mu + \mu_t)(4\eta_x u_\eta - 2\eta_y v_\eta)/3 \quad (2.26)$$

$$m_2 = (\mu + \mu_t)(\eta_y u_\eta - \eta_x v_\eta)/3 \quad (2.27)$$

$$m_3 = (\mu + \mu_t)(-2\eta_x u_\eta + 4\eta_y v_\eta)/3 \quad (2.28)$$

$$m_4 = (\mu \mathcal{P}r^{-1} + \mu_t \mathcal{P}r_t^{-1})(\gamma - 1)^{-1} \eta_x \partial_\eta (a^2) \quad (2.29)$$

$$m_5 = (\mu \mathcal{P}r^{-1} + \mu_t \mathcal{P}r_t^{-1})(\gamma - 1)^{-1} \eta_y \partial_\eta (a^2) \quad (2.30)$$

2.4.3 Turbulence Model

The turbulence effects are accounted for by the dynamic eddy viscosity, μ_t , in Equation 2.26. To determine the value of μ_t , the Spalart-Allmaras [24] turbulence model is used. This one-equation model, written in generalized curvilinear coordinates, is given by

$$\begin{aligned} \frac{\partial \tilde{\nu}}{\partial \tau} + U \frac{\partial \tilde{\nu}}{\partial \xi} + V \frac{\partial \tilde{\nu}}{\partial \eta} = \frac{1}{Re} \left[c_{b1} \tilde{S} \tilde{\nu} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d_w} \right)^2 \right. \\ \left. + \frac{1}{\sigma} [(1 + c_{b2}) T_1 - c_{b2} T_2] \right] \end{aligned} \quad (2.31)$$

where

$$\begin{aligned} T_1 = \xi_x \frac{\partial}{\partial \xi} \left[(\nu + \tilde{\nu}) \xi_x \frac{\partial \tilde{\nu}}{\partial \xi} \right] + \eta_x \frac{\partial}{\partial \eta} \left[(\nu + \tilde{\nu}) \eta_x \frac{\partial \tilde{\nu}}{\partial \eta} \right] \\ + \xi_y \frac{\partial}{\partial \xi} \left[(\nu + \tilde{\nu}) \xi_y \frac{\partial \tilde{\nu}}{\partial \xi} \right] + \eta_y \frac{\partial}{\partial \eta} \left[(\nu + \tilde{\nu}) \eta_y \frac{\partial \tilde{\nu}}{\partial \eta} \right] \end{aligned} \quad (2.32)$$

and

$$\begin{aligned} T_2 = (\nu + \tilde{\nu}) \left[\xi_x \frac{\partial}{\partial \xi} \left(\xi_x \frac{\partial \tilde{\nu}}{\partial \xi} \right) + \eta_x \frac{\partial}{\partial \eta} \left(\eta_x \frac{\partial \tilde{\nu}}{\partial \eta} \right) \right. \\ \left. + \xi_y \frac{\partial}{\partial \xi} \left(\xi_y \frac{\partial \tilde{\nu}}{\partial \xi} \right) + \eta_y \frac{\partial}{\partial \eta} \left(\eta_y \frac{\partial \tilde{\nu}}{\partial \eta} \right) \right] \end{aligned} \quad (2.33)$$

Note that the laminar-turbulent transition functions in the model have been set to zero, since the flow is assumed to be fully turbulent. For a definition of these functions see Ref. [24] and for the remaining constants see Ref. [36]

2.4.4 Flow Solver

The thin-layer Navier-Stokes spatial discretization equations consist of second-order centered-difference operators with second and fourth-order difference scalar artificial dissipation. For more details on the discretization of the flow equations and the boundary conditions see Ref. [41, 36, 24]. Overall, the spatial discretization leads to a nonlinear system of equations

$$R(\hat{Q}, D) = 0 \tag{2.34}$$

where R is the residual equations and $\hat{Q}$ is the discrete vector of conservative dependent flow variables.

Equation 2.34 is solved using Newton's method. The implicit-Euler time-marching method, in conjunction with approximate factorization, is used as the startup algorithm to obtain an initial guess for Newton's method. Then the generalized minimal residual (GMRES) Krylov subspace method is used to solve the linear system of equations. The full details of this algorithm is presented in Ref. [36, 22].

2.5 Adjoint Gradient Calculation

The gradient formulation is based on the control theory approach developed by Jameson [26]. Using this method, the governing flow field equations are introduced as a constraint within the gradient equation so that the final

gradient expression does not require flow-field re-evaluations. The gradient is calculated indirectly by solving the adjoint equation. This makes the gradient calculation independent of the number of chosen design variables. Therefore, the gradient calculation cost is one flow solution and one adjoint solution and as the adjoint equation is a linear equation the solution complexity is reduced. As previously stated, the objective function is given as

$$I = I(D, Q) \quad (2.35)$$

For flow around an airfoil, the aerodynamic properties that define the objective function are functions of flow field variables, Q , and the physical boundary location, which is represented by the design variables, D . A change in D results in a change in the objective function, I , given by

$$\delta I = \frac{\partial I^T}{\partial D} \delta D + \frac{\partial I^T}{\partial Q} \delta Q \quad (2.36)$$

To prevent re-evaluations of the flow field, δQ must be eliminated from Equation 2.36. Consider Equation 2.34 which contains the dependence of $\hat{Q}$ and D on the residual equations. Then δQ is determined from the equation

$$\delta R = \left[\frac{\partial R}{\partial \hat{Q}} \right] \delta \hat{Q} + \left[\frac{\partial R}{\partial D} \right] \delta D = 0 \quad (2.37)$$

Note that $\partial \hat{Q} / \partial Q = J^{-1} I$, where I is the identity matrix and consequently Equation 2.37 becomes

$$\delta R = \left[\frac{\partial R}{\partial Q} \right] \delta Q + \left[\frac{\partial R}{\partial D} \right] \delta D = 0 \quad (2.38)$$

Equation 2.38 is adjoined to the objective function by a Lagrange multiplier, ψ , to get

$$\begin{aligned}\delta I &= \frac{\partial I^T}{\partial D} \delta D + \frac{\partial I^T}{\partial Q} \delta Q - \psi^T \left(\left[\frac{\partial R}{\partial Q} \right] \delta Q + \left[\frac{\partial R}{\partial D} \right] \delta D \right) \\ &= \left(\frac{\partial I^T}{\partial Q} - \psi^T \left[\frac{\partial R}{\partial Q} \right] \right) \delta Q + \left(\frac{\partial I^T}{\partial D} - \psi^T \left[\frac{\partial R}{\partial D} \right] \right) \delta D\end{aligned}\tag{2.39}$$

where ψ is chosen to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial Q} \right]^T \psi = \frac{\partial I}{\partial Q}\tag{2.40}$$

which then eliminates the first term in Equation 2.39 and produces

$$\delta I = \mathcal{G} \delta D\tag{2.41}$$

where $\mathcal{G}$ is the gradient and is given as

$$\mathcal{G} = \frac{\partial I^T}{\partial D} - \psi^T \left[\frac{\partial R}{\partial D} \right]\tag{2.42}$$

The GMRES method that is used in the flow solver is used to solve the adjoint equations. The remaining two terms in Equation 2.42, $\partial I^T / \partial D$ and $\partial R / \partial D$ are evaluated using centered differences. For details on the adjoint equations solution refer to Ref. [36, 30, 22]

2.6 Optimizer

The optimization problem is cast as an unconstrained optimization since the constraints are included within the objective function as penalty terms. The unconstrained optimization algorithm is used to minimize the objective function. The sufficient conditions for a local minimum at a point x^* are

1. $\|g(x^*)\| = 0$
2. $G(x^*)$ is positive definite

where g is the first derivative (gradient) and G is the objective function second derivative [42].

To satisfy these conditions, the objective function is modeled as a quadratic equation at each optimization iteration, k .

$$m_k(s) = I_k + \mathcal{G}_k^T s + \frac{1}{2} s^T B_k s \quad (2.43)$$

where s is a vector, $\mathcal{G}_k$ is the gradient and B_k is a symmetric positive definite matrix which holds the curvature information and for Newton's method it is the Hessian matrix. Calculating the exact Hessian matrix is often difficult, so quasi-Newton methods are used to approximate the Hessian. These methods are based on the idea of building up the curvature information as the optimization proceeds, using the observed objective function behavior and its gradient [39]. The Broyden, Fletcher, Goldfarb and Shanno (BFGS) secant update is used to generate a sequence of matrices, $\mathcal{H}_k$ that approximate the inverse of the exact Hessian matrix, B_k^{-1} with increasing accuracy. The resulting quadratic equation is

$$m_k(s) = I_k + \mathcal{G}_k^T s + \frac{1}{2} s^T \mathcal{H}_k^{-1} s \quad (2.44)$$

For details on the BFGS update see Ref. [36, 39, 43].

To ensure that the objective function is being minimized at iteration k , s is selected such that the gradient of m_k goes to zero

$$\nabla m_k(s) = \mathcal{G}_k + \mathcal{H}_k^{-1} s = 0 \quad (2.45)$$

resulting in

$$s = -\mathcal{H}_k \mathcal{G}_k \quad (2.46)$$

where s is then used as the search direction vector resulting in a design variable update of

$$D_{k+1} = D_k + \beta_k s_k \quad (2.47)$$

where β represents a step length determined by the line search procedure. This guarantees that the design variables movement will be in a ‘downhill’ direction resulting in the objective function being minimized.

Ideally, for the algorithm to converge superlinearly, a full-Newton step, $\beta = 1$, is desired. However, this can result in the algorithm diverging in the first few iterations so a backtracking line-search procedure is included that systematically reduces the step length, β , until an acceptable value is found. The acceptable value must satisfy the Armijo-Goldstein-Wolfe conditions [39, 43]

$$\begin{aligned} I[D_{k+1}, Q(D_{k+1})] &\leq I[D_k, Q(D_k)] + c_1 \beta_k \mathcal{G}[D_k, Q(D_k)]^T s \\ |\mathcal{G}[D_{k+1}, Q(D_{k+1})]^T s| &\leq c_2 |\mathcal{G}[D_k, Q(D_k)]^T s| \end{aligned} \quad (2.48)$$

where $c_1 = 1 \times 10^{-4}$ and $c_2 = 0.9$. Placing these conditions on the algorithm ensures that $\mathcal{H}_{k+1}$ remains positive definite and that a sufficient decrease in the objective function value results from a sufficiently large β_k .

When the step length, β_k , does not satisfy the above conditions, β is reduced using a line search algorithm that uses the objective function value and the gradient value to construct a local cubic interpolant. See Ref. [36] for the details.

As the objective function value and the gradient can vary by orders of magnitude, these values are scaled at the optimization algorithm onset. Scaling advantages and disadvantages are discussed by Gill *et al.* in Ref. [42]. As well, Liu and Zingg [44] and Zingg *et al.* [45] have preformed comparative research for different scaling methods.

The original method uses the initial design variables as the scaling factor for the design variables and gradient. So at iteration k ,

$$\begin{aligned} D_{scal_k} &= L^{-1} D_k \\ \mathcal{G}_{scal_k} &= L \mathcal{G}_k \end{aligned} \tag{2.49}$$

where L is a diagonal matrix containing the initial design variable values.

As well, the fixed scaling method outlined in Liu and Zingg [44] has also been added into the optimization algorithm. When using the original method, the scaled gradient becomes dependent on the design variable y -coordinate magnitude. This method works well for airfoils that have design variables of similar magnitudes, such as airfoils with chords that lie near the horizontal. However, when the chosen design variables have a range of magnitudes, such as from 1×10^{-4} to 0.1, the scaled gradient becomes too dependent on the y -coordinate location. This can result in the gradient value sensitivities changing as the gradient is being scaled corresponding to the y -coordinate magnitude. For example, if the gradient value of a particular design variable is the most sensitive but is located at a very small y location, then when it is scaled this sensitivity becomes significantly less and vice versa for a small gradient value and a large y -coordinate location. As the scaled gradient value is used to determine the new design variable location, the sensitivity changes result in a change in the optimization direction. As well,

the true gradient sensitivities are not being realised. Therefore, a fixed scaled factor that corresponds to the magnitude of the averaged design variables is used as outlined in Ref [44]. At iteration k , the scaled design variables and gradient become

$$\begin{aligned} D_{scal_k} &= X_f^{-1} D_k \\ \mathcal{G}_{scal_k} &= X_f \mathcal{G}_k \end{aligned} \tag{2.50}$$

where X_f is a constant value. It is left up to the designer to choose which scaling factor works best for the problem at hand.

Since the optimization is scaled the stopping criteria is now based on the scaled gradient value. This is

$$\|\mathcal{G}\| = 0 \tag{2.51}$$

becomes

$$\|\mathcal{G}_{scal}\|_2 = 0 \tag{2.52}$$

where the L_2 norm is used since the gradient contains both positive and negative values. Since it is unrealistic to expect the optimization to decrease the gradient to machine zero, a stopping tolerance is applied instead. To ensure that the optimization has reached a minimum, the stopping tolerance should allow the scaled gradient to be reduced by at least three orders of magnitude. Note that the selected tolerance will be scaled to keep it consistent with the scaled gradient.

2.7 Algebraic Perturbation method

During every optimization iteration, the airfoil shape or position changes resulting in the need for an interior grid node adjustment. The grid nodes are adjusted through an algebraic perturbation strategy. This strategy preserves the outer boundary location and moves the interior nodes along a normal grid line, as shown below.

$$y_k^{new} = y_k^{old} + \frac{\Delta y}{2} [1 + \cos(\pi S_k)] \quad k = 1, \dots, k_{max} - 1 \quad (2.53)$$

where Δy represents the change in airfoil shape or position. S_k is the normalized arc length distance given by

$$S_1 = 0$$

$$S_k = \frac{1}{L_g} \sum_{i=2}^k L_i \quad k = 2, \dots, k_{max} - 1 \quad (2.54)$$

where L_i is the segment length between nodes k and $k - 1$. L_g is the total grid line length from the airfoil surface to the outer boundary and is defined as

$$L_g = \sum_{i=2}^{k_{max}} L_i \quad (2.55)$$

The above grid perturbation strategy holds for the streamwise direction as well but for shape optimization, only changes in the normal direction are allowed. For gap and overlap optimization, the perturbation strategy is first applied in the normal direction and then in the streamline direction.

CHAPTER 3

3.1 New Constraints

The reason for the original constraints was to prevent unrealistic geometries, such as airfoils with surface crossovers, from occurring during the optimization and to enforce structural limitations such as a minimum airfoil thickness [36]. Through testing, it was discovered that the local minimum thickness constraint was not sufficient to produce valid airfoil shapes. As the optimization method used in this research adds the constraints to the objective function as penalty terms, the addition of more constraints was possible. As well, the designer has complete control as to which constraints, if any, are active. This allows for a variety of designs based on specific criteria. The following sections outline the new constraints.

3.2 Global Minimum Thickness Constraint

The global minimum thickness is used to prevent surface crossovers and to keep the airfoil sufficiently thick near the trailing edge in order to meet structural requirements. The local minimum thickness constraint can achieve

similar results but only if the surface crossover location is known a priori. This constraint only checks for surface crossovers at the specified chordwise locations. The global thickness constraint is always active, not just when the point thickness constraints are used. All points along the airfoil are checked for surface crossovers and can therefore be avoided, thus creating realistic airfoil shapes.

In order to keep this constraint global and case dependent, the airfoil thickness is allowed to reduce to 70% of the initial airfoil shape. If the optimized shape becomes any thinner, the global thickness constraint will become active. This approach will allow large changes in thickness, typically in the region between 20% and 60% chord and minimal changes near the trailing edge where the thickness is small from the onset.

The initial airfoil thickness is determined at the start of the optimization process by first rotating the airfoil about the trailing edge to the horizontal position:

$$\begin{aligned}x_i &= x_{mid} + dx \cos \theta - dy \sin \theta \\y_i &= y_{mid} + dx \sin \theta + dy \cos \theta\end{aligned}\tag{3.1}$$

For a sharp trailing edge, x_{mid} and y_{mid} are the airfoil trailing edge coordinates and for a blunt trailing edge, they represent the middle location coordinate. dx , dy , and θ are defined in the equations below:

$$\begin{aligned}dx &= x_i - x_{mid} \\dy &= y_i - y_{mid} \\\theta &= -\arctan\left(\frac{dy}{dx}\right)\end{aligned}\tag{3.2}$$

Once the airfoil is rotated, the upper and lower surface points are splined using a cubic spline interpolation routine [46]. The thickness is then calculated at every splined surface point using Equation 3.3.

$$h_{tk}(x_i) = y_{upp}(x_i) - y_{low}(x_i) \quad i = 1, \dots, i_{max} \quad (3.3)$$

where $y_{upp}(x_i)$ and $y_{low}(x_i)$ are the upper and lower surface y coordinates at the x_i location, respectively and i_{max} is the total number of splined surface points. The allowable global minimum thickness is then calculated using the following formula.

$$h_g^*(x_i) = h_{tko}(x_i) * 0.3 \quad i = 1, \dots, i_{max} \quad (3.4)$$

where $h_g^*(x_i)$ is the minimum allowable thickness (global thickness) at x_i and $h_{tko}(x_i)$ is the initial airfoil thickness at x_i . Using the constraint equation as defined in Equation 2.2, the global thickness constraint becomes

$$h_g^*(x_i) - h_{tk}(x_i) \leq 0 \quad (3.5)$$

where $h_{tk}(x_i)$ is the current airfoil thickness at x_i

For the constraints to have the same form as the objective function the allowable thickness is divided through Equation 3.5 and the penalty method is used, giving the following equation.

$$c_{ig} = \begin{cases} \left[1 - \frac{h_{tk}(x_i)}{h_g^*(x_i)}\right]^2, & \text{if } h_{tk}(x_i) < h_g^*(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

where c_{ig} is the global thickness contribution to the overall constraint equation given in Equation 2.8. So when the actual airfoil thickness, $h_{tk}(x_i)$ becomes less than the allowable global thickness, $h_g^*(x_i)$ a penalty is added to

the objective function to account for this violation in the constraint. However, unlike the minimum thickness constraint where the total number of constraint points are set and fixed at the onset, the number of constraint points for the global thickness can vary from one optimization iteration to the next. As this constraint should never dominate the objective function and is used more as a preventive constraint, the final global thickness constraint equation becomes

$$I_{c_g}(D) = (w_g C_g) \left(\frac{1}{n_c} \right) \quad (3.7)$$

where n_c is the total number of constraint points and w_g is the weighting for the global thickness constraint. Note that w_g is fixed at 0.001 in order to keep this constraint contribution small compared to the other objective terms.

3.3 Maximum Thickness Constraint

At times it is useful to specify a maximum thickness constraint instead of a minimum. For instance, in flap design the airfoil needs to have a maximum thickness constraint to allow the flap to properly stow within the main element. As the designer will only need to specify certain maximum thickness locations rather than a global maximum thickness, a local thickness constraint was added. To keep the constraint equation, Equation 2.2 true, the maximum thickness constraint becomes

$$h_{tk}(x_i) - h_{mx}^*(x_i) \leq 0 \quad (3.8)$$

where $h_{tk}(x_i)$ is the actual thickness at x_i and $h_{mx}^*(x_i)$ is maximum allowable thickness at x_i .

Using the penalty method

$$c_{mx} = \begin{cases} \left[1 - \frac{h_{mx}^*(x_i)}{h(x_i)}\right]^2, & \text{if } h(x_i) > h_{mx}^*(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.9)$$

and the maximum thickness constraint becomes part of the objective function as follows

$$I_{c_{mx}}(D) = (w_{mx}C_{mx}) \quad (3.10)$$

where w_{mx} is the weighting for the maximum thickness constraint.

3.4 Area Objective/Constraint

When designing a wing, not only must the structural requirements be considered but also fuel volume. With the area constraint term, the minimum allowable area is specified where as with the area objective term, the target area is specified. The main difference between these terms is how they are included within the objective function. If the area is specified as a constraint, it becomes a penalty within the objective function when the constraint is violated. However, with the area as an objective for the minimization problem, it remains active within the equation until the target area is achieved. This provides the designer with more freedom.

Since the thickness is already calculated along the splined airfoil for the global minimum thickness constraint, the area term is easily implemented. The distance between x_i and x_{i+1} or Δx_i along the splined airfoil chord is small. Therefore, the airfoil can be broken up into rectangles with each rectangular area summed together to determine the total airfoil area. This

is shown in the following equations:

$$\begin{aligned}\Delta x_i &= x_{i+1} - x_i \\ A(x_i) &= \left[\frac{h_{tk}(x_i) + h_{tk}(x_{i+1})}{2} \right] \Delta x_i \\ A_{total} &= \sum_{i=1}^{i_{max}} A(x_i)\end{aligned}\tag{3.11}$$

where i_{max} is the total number of splined points along the airfoil. Applying the constraint equation. Equation 2.2, the area constraint becomes

$$A_{total}^* - A_{total} \leq 0\tag{3.12}$$

where A_{total}^* is the minimum allowable area and A_{total} is the actual area of the current airfoil. The area is included in the objective function as a penalty in the form

$$c_{ita} = \begin{cases} \left[1 - \frac{A_{total}}{A_{total}^*} \right]^2, & \text{if } A_{total} < A_{total}^* \\ 0, & \text{otherwise} \end{cases}\tag{3.13}$$

where c_{ita} is the area contribution to the overall constraint equation, Equation 2.8. When the actual airfoil area, A_{total} becomes less than the allowable area, A_{total}^* a penalty is added to the objective function to account for this violation. Note that the designer specifies the minimum allowable area where as the actual area is calculated in the optimization program. The final area constraint equation becomes

$$I_{c_{ta}}(D) = (w_{ta}C_{ta})\tag{3.14}$$

where w_{ta} is the weighting of the area constraint.

When the airfoil area is selected as an optimization objective, an area term is added to the design objective function equation, Equation 2.4, as:

$$I_d(D, Q) \equiv w_L I_L + w_D I_D + w_A I_A \quad (3.15)$$

where w_A is a weighting term specified by the designer and I_A is given as

$$I_A = \left(1 - \frac{A_{total}}{A_{total}^*}\right)^2 \quad (3.16)$$

where A_{total}^* is the target area specified by the designer and A_{total} is the calculated current airfoil area.

3.5 Curvature Constraint

To address an airfoil's manufacturability, a curvature constraint is used. The previously discussed constraints alone will not guarantee that the optimized shape can be manufactured as there is no mechanism to prevent oscillations along the airfoil surface from occurring. An ideal airfoil surface should be smooth with minimum changes in curvature. To help control unwanted oscillations along the surface, a surface curvature is employed as a constraint.

The first idea for implementing a curvature constraint was to use the radius of curvature since these changes are easy to visualize. This allows the designer to get a sense of what would be acceptable by looking at the airfoil shape. For instance, consider a sinusoidal curve with a period of 2π , now imagine drawing a circle under or over each peak as shown in Figure 3.1. The first circle at $\pi/2$ represents the starting radius of curvature and each additional circle that can be drawn in a peak or valley represents a change in the radius of curvature sign. In this example the radius of curvature sign

Figure 3.1: Sinusoidal curve.

changes once from 0 to 2π . The goal here would be to reduce the oscillation frequency by restricting the radius of curvature sign changes. This idea works well in theory but not in practice. Apply the curvature equation [47] to each point along the sinusoidal curve in Figure 3.1 as follows

$$\kappa(x_i) = \pm \frac{\frac{d^2y(x_i)}{dx^2}}{\left[1 + \left(\frac{dy(x_i)}{dx}\right)^2\right]^{\frac{3}{2}}} \quad i = 1, \dots, i_{max} \quad (3.17)$$

where i_{max} is the maximum number of splined surface points, x_i is the specified point, $d^2y(x_i)/dx^2$ is the second derivative and $dy(x_i)/dx$ is the first derivative at x_i .

Using Equation 3.17, the sinusoidal curvature is calculated and is shown in Figure 3.2

The radius of curvature is defined as the inverse of κ , κ^{-1} and by defini-

Figure 3.2: Curvature of sinusoidal curve.

tion, the second derivative is zero through an inflection point. This means that, as the sinusoidal curve changes in curvature from positive to negative, the curvature at $\theta = \pi$ is zero. The radius of curvature goes to infinity at this point and any other point that is zero along the curve. This is seen in Figure 3.3 where, numerically, dividing by zero is not possible. A limit of 100 was used to allow the range to be seen.

Since the radius of curvature is not the best choice for the constraint, the curvature value was considered. It is less intuitive than the radius of curvature but it will not cause the numerical algorithm to diverge. There are, however, a few problems associated with using just the curvature as a constraint. It would be very difficult for the designer to select a maximum allowable curvature without some prior knowledge of what is considered undesirable curvature. As well, the curvature of an airfoil can change significantly

Figure 3.3: Radius of curvature of sinusoidal curve.

from the leading edge to the trailing edge, so a single target value would not be very useful. Due to these reasons, a difference in curvature value was considered. This would be more intuitive for the designer and a single value for the target could be used. The difference in curvature value is defined as a change in curvature between the current shape and the initial shape.

$$\Delta\kappa_i = \kappa_{k_i} - \kappa_{initial_i} \quad i = 1, \dots, i_{max} \quad (3.18)$$

where κ_k is the current shape curvature, $\kappa_{initial}$ is the initial shape curvature and i_{max} is the total number of splined airfoil points. The initial shape is used for the comparison as it is assumed to have a smooth curvature without any unnecessary oscillations.

Specifying a single difference in curvature value will only produce airfoil shapes that are similar to the initial shape. To illustrate this point, consider Figure 3.4, where the solid line represents the upper surface curvature and

Figure 3.4: Symmetric airfoil upper and lower surface curvature.

the dashed line represents the lower surface curvature for a symmetric airfoil.

If this airfoil was used as the initial airfoil and a cambered airfoil was the final optimized shape, the difference in curvature constraint could be used for the upper surface without any problems occurring. The upper surface of the symmetric and cambered airfoils is similar. This is illustrated in Figure 3.5, where both the symmetric and cambered airfoils have similar curvature plots with a maximum difference of approximately 0.5 occurring at $x = 0.35$.

Now consider the lower surface curvature for the symmetric and cambered airfoils shown in Figure 3.6. These airfoils do not have a similar lower surface curvature but the cambered lower surface curvature is acceptable. Selecting a single value for the difference in curvature would be quite difficult. Since selecting a large value would allow for changes near the leading edge but would not prevent undesirable oscillations further along the airfoil where

Figure 3.5: Symmetric and cambered airfoils upper surface curvature.

Figure 3.6: Symmetric and cambered airfoils lower surface curvature.

the curvature is significantly less, and vice versa for a small difference in curvature values.

This leads to the question, what changes in curvature are acceptable and what are not? This is a difficult question to answer as the curvature of a shape is not always intuitive. Instead, consider this question, what characterizes the waviness or oscillatory behavior of an airfoil? The waviness could be defined as oscillations within the curvature, in other words, the curvature changing sign from positive to negative several times along the airfoil surface. As well, the greater the magnitude between a positive and negative peak would result in a larger oscillation. Therefore, by monitoring the magnitude of the curvature sign change, the waviness would be restricted. This idea led to the next proposed constraint which calculates where the minimum and/or maximum peaks in curvature occur along the airfoil chord. If the vertical distance between two consecutive curvature peaks of opposite sign are greater than the set tolerance the constraint becomes active and a penalty is imposed on the objective function.

Again, this idea worked better in theory than in practice. This constraint caused the norm of the gradient value within the optimization algorithm to oscillate which prevented the algorithm from converging. The previous constraints once activated slowly get minimized as the optimization proceeds but this curvature peak constraint acts more as an on/off constraint causing the gradient to oscillate. Consider Figure 3.7 where the dashed line represents the lower surface curvature at iteration j , the solid line represents the curvature at iteration $j + 1$ and the peaks are represented by a square and diamond, respectively. The area within the circle is the trouble area for

Figure 3.7: Comparison between curvature peaks for iteration j and $j + 1$.

this example and is more pronounced in Figure 3.8. For sufficiently small geometric changes, the curvature at the trouble point oscillates between a negative and positive value. Therefore at iteration j , this point doesn't contribute to the penalty term in the objective function. However, at iteration $j + 1$, there is a large penalty from this point due to the magnitude of the sign change. This results in oscillations in the gradient and prevents the algorithm from converging. Similar gradient problems were encountered by Jameson [48]. He developed the control theory using the partial differential equations to highlight ill posed formulations of the problem. For example, if the pressure sensitivity to changes in the airfoil shape was calculated at a particular location, there exists the possibility that a shape modification could cause a shock wave to pass over that location. As a shock is discon-

Figure 3.8: Target area between curvature peaks for iteration j and $j + 1$.

tinuous, the sensitivity becomes unbounded resulting in problems within the gradient calculations.

Based on these ideas, a few conclusions regarding the curvature constraint become obvious. The curvature constraint needs to be something that will not be discontinuous with small changes in the design variables. It must allow for larger curvature changes at the leading edge than along the rest of the chord line. Furthermore, it must be intuitive and simple for the designer to use. With these limitations in mind, two curvature functions were devised that vary chordwise along the airfoil and bracket the initial curvature. The first function, achieves an allowable area by bounding the initial curvature by a plus-minus tolerance of a_{cf1} such that

$$-a_{cf1} * (\kappa_{initial}) < (\kappa) < a_{cf1} * (\kappa_{initial}) \quad (3.19)$$

As this function is symmetrical, it requires the designer to input only one value to determine the curvature limits. However, this limits the designer as the upper and lower limit for the selected surface will be the same. For problems where the designer wants to restrict one limit more than the other, the second curvature function can be used. This function requires the designer to input four values for each limit thereby giving the designer more freedom. Both functions will be outlined in detail in Sections 3.5.1 and 3.5.2, respectively.

3.5.1 Curvature Function *One*

To determine the curvature from the airfoil shape, the first and second derivatives need to be calculated. When the airfoil upper and lower surfaces are splined [46] for the thickness calculation, each new value of y is approximated by a cubic polynomial given by

$$y = Ay_j + By_{j+1} + Cy_j'' + Dy_{j+1}'' \quad (3.20)$$

where

$$A \equiv \frac{x_{j+1} - x}{x_{j+1} - x_j} \quad B \equiv 1 - A = \frac{x - x_j}{x_{j+1} - x_j} \quad (3.21)$$

$$C \equiv \frac{1}{6}(A^3 - A)(x_{j+1} - x_j)^2 \quad D \equiv \frac{1}{6}(B^3 - B)(x_{j+1} - x_j)^2 \quad (3.22)$$

Therefore, for each new splined y airfoil coordinate, the first derivative result is

$$\frac{dy}{dx} = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} - \frac{3A^2 - 1}{6}(x_{j+1} - x_j)y_j'' + \frac{3B^2 - 1}{6}(x_{j+1} - x_j)y_{j+1}'' \quad (3.23)$$

and the second derivative is

$$\frac{d^2y}{dx^2} = Ay_j'' + By_{j+1}'' \quad (3.24)$$

The curvature for each splined airfoil point can be calculated using Equation 3.17 and the results from Equations 3.23 and 3.24. To produce a unique solution using the cubic spline interpolation routine, boundary conditions at x_1 and x_{jmax} need to be known. Since neither the first nor second derivative is known at these points, the second derivative is set to zero producing a natural cubic spline [46]. However, by using the natural spline, the first curvature points at the leading edge and last points at the trailing edge will be inaccurate. Since a large number of points are used to spline the airfoil, the starting and end point inaccuracies are not significant. Each curvature function addresses the inaccuracies and are discussed later in the section.

The next step was to create the curvature function. By breaking the airfoil into three sections, namely, leading edge, wing box and trailing edge, the curvature can be easily modeled. The leading edge is defined between 0% to 15% chord. the wing box between 15% to 85% chord and the trailing edge from 85% to 100% chord.

The curvature function for a symmetric airfoil is the largest in magnitude at the leading edge then exponentially descends towards zero along the airfoil's remaining shape. However, for cambered airfoils, the curvature function is again greatest in magnitude at the leading edge, followed by a sign change, then approaches zero at the trailing edge. Figures 3.5 and 3.6, as shown previously, illustrate the curvatures for a symmetric versus a cambered airfoil.

The leading edge section needs to account for airfoils with varying degrees of camber. That is, the curvature function must allow for the largest curvature occurring just after the leading edge as well for it to drop exponentially

towards zero. To account for both of these characteristics, the curvature at the leading edge is modeled as a cubic S-curve with the following equation.

$$\kappa_{f1}(x_i) = a_{m1}(x_i - x_o)^3 + b_{m1}(x_i - x_o)^2 + c_{m1} \quad i = 1, \dots, i_{0.15} \quad (3.25)$$

where κ_{f1} is the curvature function, x is the splined chordwise location, i is the total number of splined surface points and a_{m1} , b_{m1} and c_{m1} are the coefficients. To determine the coefficients, three points are chosen based on the initial curvature.

As discussed above, the calculated curvature for the first and last few points on the airfoil are not very accurate due the first and second derivative approximations at the boundary in the cubic spline routine. Therefore, the first accurate curvature point is selected to be at

$$x_{acc} = 0.01(x_1) \quad (3.26)$$

The coordinates of the first point for the model are

$$x_{m1_o} = x_1 \quad \kappa_{m1_o} = \kappa(x_{acc}) \quad (3.27)$$

The coordinates of the third point for the model will be at 15% chord, so

$$x_{m1_2} = x(1) + 0.15c \quad \kappa_{m1_2} = \pm a_{cf1}\kappa(x_{m1_2}) \quad (3.28)$$

where c is the airfoil chord and a_{cf1} is the function tolerance as shown in Equation 3.19 and the sign of a_{cf1} is determined by the curvature sign at this location. The second model point will be the average of the end points, so

$$x_{m1_1} = \frac{x_{m1_o} + x_{m1_2}}{2} \quad \kappa_{m1_1} = \frac{\kappa_{m1_o} + \kappa_{m1_2}}{2} \quad (3.29)$$

Using these three points, the coefficients of Equation 3.25 can be determined analytically as

$$\begin{aligned} a_{m1} &= \frac{\kappa_{m1_2} - \kappa_{m1_o} - Y1D2(x_{m1_2} - x_{m1_o})^2}{(x_{m1_2} - x_{m1_o})^3 - X3DX2(x_{m1_2} - x_{m1_o})^2} \\ b_{m1} &= Y1D2 - a_{m1}(X3DX2) \\ c_{m1} &= \kappa_{m1_o} \end{aligned} \quad (3.30)$$

where

$$Y1D2 = \frac{(\kappa_{m1_1} - \kappa_{m1_o})}{(x_{m1_1} - x_{m1_o})^2} \quad X3DX2 = \frac{(x_{m1_1} - x_{m1_o})^3}{(x_{m1_1} - x_{m1_o})^2} \quad (3.31)$$

The airfoil wing box section is modeled as a quadratic curve again with three points chosen based on the initial curvature. The model equation is

$$\kappa_{f1}(x_i) = a_{m2}(x_i)^2 + b_{m2}(x_i) + c_{m2} \quad i = i_{0.15}, \dots, i_{0.85} \quad (3.32)$$

The first point coordinates for this second model are equal to the first models last point. This will allow for a smooth curvature function. Therefore,

$$x_{m2_o} = x_{m1_2} \quad \kappa_{m2_o} = \kappa_{m1_2} \quad (3.33)$$

In this airfoil section, capturing the lowest curvature point is important. This is achieved by determining where the curvature peaks occur along the airfoil chord. A maximum curvature peak, $\kappa_{p_{max}}$ exists when the following inequality is true.

$$\kappa(x_{i-1}) < \kappa(x_i) > \kappa(x_{i+1}) \quad i = 1, \dots, i_{max} \quad (3.34)$$

then the maximum peak coordinates become

$$x_{p_{max_j}} = x_i \quad \kappa_{p_{max_j}} = \kappa(x_i) \quad j = 1, \dots, j_{max} \quad (3.35)$$

where j_{max} is total number of maximum peaks along the airfoil chord.

Similarly, a minimum curvature peak, κp_{min} exists when this inequality is true

$$\kappa(x_{i-1}) > \kappa(x_i) < \kappa(x_{i+1}) \quad i = 1, \dots, i_{max} \quad (3.36)$$

and the minimum peak coordinates become

$$xp_{min_m} = x_i \quad \kappa p_{min_m} = \kappa(x_i) \quad m = 1, \dots, m_{max} \quad (3.37)$$

where m_{max} is the total number of minimum peaks along the airfoil chord.

Once all the peaks have been calculated, the overall minimum peak located between 20% and 80% chord is chosen as the second point within the wing box model. The coordinates are determined as

$$x_{m2_1} = xp_{min} \quad \kappa_{m2_1} = \pm a_{cf1} \kappa(x_{m2_1}) \quad (3.38)$$

The third peak coordinates for the model will be at 85% chord, so

$$x_{m2_2} = x(1) + 0.85c \quad \kappa_{m2_2} = \pm a_{cf1} \kappa(x_{m2_2}) \quad (3.39)$$

where c is the airfoil chord and a_{cf1} is the tolerance as shown in Equation 3.19.

Using these three points, the coefficients of Equation 3.32 can be determined analytically as

$$\begin{aligned} a_{m2} &= \frac{\kappa_{m2_2} - \kappa_{m2_o} - Y2D1(x_{m2_2} - x_{m2_o})}{(x_{m2_2}^2 - x_{m2_o}^2) - X2DX1(x_{m2_2} - x_{m2_o})} \\ b_{m2} &= Y2D2 - a_{m2} * X2DX1 \\ c_{m2} &= \kappa_{m2_o} - a_{m2}(x_{m2_o})^2 - b_{m2}(x_{m2_o}) \end{aligned} \quad (3.40)$$

where

$$Y2D2 = \frac{(\kappa_{m2_1} - \kappa_{m2_o})}{(x_{m2_1}^2 - x_{m2_o}^2)} \quad X2DX1 = \frac{(x_{m2_1}^2 - x_{m2_o}^2)}{(x_{m2_1} - x_{m2_o})} \quad (3.41)$$

The third section is modeled as an exponential curve, for unlike the leading edge, the trailing edge should have no peaks in curvature. The model equation is given by

$$\kappa_{f1}(x_i) = a_{m3}e^{(m_{m3}*x_i)} \quad i = i_{0.85}, \dots, i_{max} \quad (3.42)$$

where a_{m3} and m_{m3} are determined from two points based on the initial curvature.

Like the transition from the first model to the second, the first point coordinates for the third model are equal to the second model's last point. Therefore,

$$x_{m3_o} = x_{m2_2} \quad \kappa_{m3_o} = \kappa_{m2_2} \quad (3.43)$$

The second point, the exponential curve peak will be equal to the first model starting point, so

$$x_{m3_1} = x_{m1_o} \quad \kappa_{m3_1} = \kappa_{m1_o} \quad (3.44)$$

Then the coefficients a_{m3} and m_{m3} can be determined from the following equations

$$m_{m3} = \frac{\ln\left(\frac{\kappa_{m3_1}}{\kappa_{m3_o}}\right)}{x_{m3_1} - x_{m3_o}} \quad a_{m3} = \frac{\kappa_{m3_o}}{e^{(m_{m3}*x_{m3_o})}} \quad (3.45)$$

This function, κ_{f1} , represents the lower curvature limit, κ_{f1_o} for both the upper and lower airfoil surfaces. The upper curvature limit will be the negative of the lower limit, this is

$$\kappa_{f1_{up}} = -\kappa_{f1_o} \quad (3.46)$$

As was previously mentioned, the tolerance sign a_{cf1} is dependent on the sign of κ in order to keep the lower limit negative if the curvature changes sign along the chord.

Figure 3.9 shows the upper surface curvature limits for the NACA 0012 airfoil with $a_{cf1} = 2$. The limits are presented as the dashed lines and the upper surface curvature is the solid line. Since this airfoil is symmetric, the lower and upper surface curvature limits are the same.

Figure 3.9: Upper surface curvature limits for the NACA 0012 airfoil.

Implementation within Objective Function

At the optimization program onset, the curvature at each point is calculated for the initial airfoil shape which is then used in creating the curvature function for both the upper and lower airfoil surfaces. During the optimization,

the difference in curvature is calculated as

$$\Delta\kappa(x_i) = \kappa(x_i) - \kappa(x_{i_o}) \quad i = 1, \dots, i_{max} \quad (3.47)$$

where i_{max} is the total number of splined points and $\kappa(x_{i_o})$ is the initial curvature.

The difference in curvature is used as it will eliminate the model inaccuracy near the leading and trailing edges and will assume that the initial airfoil shape has a smooth curvature. As well, it allows for the curvature limit tolerance a_{cf1} to be any fraction of the initial curvature. For example, if a very small curvature tolerance of 0.25 is selected, then the changing geometry will be restricted to very small changes in curvature. If the tolerance is set to 3, more changes in curvature will be allowed. This allows the designer to adjust the limits based on the problem at hand.

For each point on the splined airfoil, the difference in curvature is calculated for the upper and lower surfaces and compared with the respective curvature function limits. If the difference in curvature is outside the limits, a penalty is added to the objective function using the penalty method as previously discussed.

The constraint equation, Equation 2.2, outlined in Section 2.3.2 gives the following condition for the airfoil upper surface curvature limits

$$\Delta\kappa_{us}(x_i) - \kappa_{f1_{uu}}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.48)$$

$$\kappa_{f1_{ul}}(x_i) - \Delta\kappa_{us}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.49)$$

where $\Delta\kappa_{us}(x_i)$ is the difference in curvature at x_i and $\kappa_{f1_{uu}}(x_i)$ and $\kappa_{f1_{ul}}(x_i)$ are the upper and lower curvature limit value at x_i , respectively. For the lower

surface

$$\Delta\kappa_{ls}(x_i) - \kappa_{f1_{lu}}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.50)$$

$$\kappa_{f1_{ll}}(x_i) - \Delta\kappa_{ls}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.51)$$

where $\Delta\kappa_{ls}(x_i)$ is the difference in curvature at x_i and $\kappa_{f1_{lu}}(x_i)$ and $\kappa_{f1_{ll}}(x_i)$ are the upper and lower curvature limit value at x_i , respectively.

Using the penalty method, the upper and lower surface curvature function *one* constraint becomes

$$C_{i\kappa_{f1_{us}}} = \begin{cases} \left[1 - \frac{\kappa_{f1_{uu}}(x_i)}{\Delta\kappa_{us}(x_i)}\right]^2, & \text{if } \Delta\kappa_{us}(x_i) > \kappa_{f1_{uu}}(x_i) \\ \left[1 - \frac{\Delta\kappa_{us}(x_i)}{\kappa_{f1_{ul}}(x_i)}\right]^2, & \text{if } \Delta\kappa_{us}(x_i) < \kappa_{f1_{ul}}(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.52)$$

$$C_{i\kappa_{f1_{ls}}} = \begin{cases} \left[1 - \frac{\kappa_{f1_{lu}}(x_i)}{\Delta\kappa_{ls}(x_i)}\right]^2, & \text{if } \Delta\kappa_{ls}(x_i) > \kappa_{f1_{lu}}(x_i) \\ \left[1 - \frac{\Delta\kappa_{ls}(x_i)}{\kappa_{f1_{ll}}(x_i)}\right]^2, & \text{if } \Delta\kappa_{ls}(x_i) < \kappa_{f1_{ll}}(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.53)$$

respectively, and the curvature function *one* constraint becomes part of the objective function as follows

$$I_{C_{\kappa_{f1}}}(D) = w_{\kappa_{f1_{us}}}(C_{\kappa_{f1_{us}}}) + w_{\kappa_{f1_{ls}}}(C_{\kappa_{f1_{ls}}}) \quad (3.54)$$

where $w_{\kappa_{f1_{us}}}$ and $w_{\kappa_{f1_{ls}}}$ are the weightings for the upper and lower surface curvature function *one* constraint, respectively.

3.5.2 Curvature Function *Two*

In creating the second curvature function, the curvature of several different airfoils were considered to gain a better understanding of what similarities

and differences existed between airfoil curvatures. As previously noted, the largest changes in curvature occur near the leading edge with the least curvature through the wing box and slightly larger curvature at the trailing edge. This time instead of modeling each section with a different function and then combining the end points together, this curvature function combines an exponential and cosine function together to vary from leading to trailing edge. The exponential curve will dominate near the leading edge and allow for large changes in curvature and the cosine function will dominate further aft along the chordline.

The general form of the curvature function is,

$$\begin{aligned} \kappa_{f2}(x_i) = & a_{f2} * \exp [m_{f2}(x_{n_i} + e_{f2})] (1 \pm b_{f2}) \\ & + c_{f2} [\cos(Px_i + T_h)] + d_{f2} \end{aligned} \quad (3.55)$$

where x_{n_i} is the normalized x-coordinate location given by

$$x_{n_i} = \frac{x_i - x_1}{c} \quad (3.56)$$

and a_{f2} and m_{f2} are calculated based on the initial surface curvature, b_{f2} , c_{f2} , d_{f2} and e_{f2} are specified by the designer and P and T_h are preset based on the specified curvature limit. Variable, b_{f2} is used to dictate how similar the exponential curve will be to the initial conditions with values typically between 0 and 1. The $\pm$ sign is used to dictate the upper and lower limit, respectively. Variable e_{f2} controls the exponential curve horizontal placement, c_{f2} controls the cosine curve amplitude and d_{f2} controls the curve's vertical translation. Finally, P is used to indicate the cosine curve period and T_h is the horizontal translation. To illustrate how each term affects the curvature limit a series of figures are shown using the NACA 0012 lower surface cur-

vature and a corresponding baseline upper limit. The limit is given by the following equation

$$\begin{aligned} \kappa_{f2_{lu}}(x_i) = a_{f2_{ls}} * \exp [m_{f2_{ls}}(x_{ni} + e_{f2_{lu}})](1 + b_{f2_{lu}}) \\ + c_{f2_{lu}} [\cos(Px_i + T_h)] + d_{f2_{lu}} \end{aligned} \quad (3.57)$$

with the baseline limit obtained using the following values.

$$a_{f2_{ls}} = \kappa_{p_{ls}}(1) \quad m_{f2_{ls}} = \frac{\log\left(\frac{\kappa_{o_{ls}}(x_{0.05i})}{a_{f2_{ls}}}\right)}{x_{0.05i}}$$

where $\kappa_{p_{ls}}(1)$ is the first curvature peak as discussed in Section 3.5.1 and $\kappa_{o_{ls}}(x_{0.05i})$ is the initial curvature at 5% chord. The designer specified terms are set to

$$b_{f2_{lu}} = 0 \quad c_{f2_{lu}} = 5 \quad d_{f2_{lu}} = 0 \quad e_{f2_{lu}} = 0$$

and the preset values are given as

$$P = \pi \quad T_h = 0$$

Figure 3.10 shows the NACA 0012 lower surface curvature and the baseline curvature limit. Note the initial amplitude, $c_{f2_{lu}}$ is set to 5 in order to make the changes more pronounced and $P = \pi$ to have the cosine curve at its maximum at $x = 0$ and its minimum at $x = 1$. Figures 3.11 to 3.14 illustrate the effect of the designer specified terms and Figures 3.15 and 3.16 illustrate the preset terms.

In Figure 3.11, variable $b_{f2_{lu}}$ in Equation 3.57 is changed from 0 to 0.9, which adjusts the function's exponential portion. With $b_{f2_{lu}} = 0$, as in the baseline limit, the exponential function is approximated by the initial

Figure 3.10: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit.

Figure 3.11: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $b_{f2_{lu}} = 0.9$.

curvature. By adjusting b_{f2lu} , an initial curvature fraction is used to generate the exponential function.

Figure 3.12 illustrates the influence that the amplitude, c_{f2lu} has on the baseline limit. When c_{f2lu} is adjusted from 5 to 8, the curve maximum and minimum value adjusts accordingly. If c_{f2lu} is negative, then the cosine maximum and minimum values are switched. In this example, if $c_{f2lu} = -8$, the cosine minimum peak would occur at $x = 0$ and its maximum at $x = 1$.

In Figure 3.13, variable d_{f2lu} , the vertical translation, is changed from 0 to 10. This change results in the limit moving vertically in the positive direction. If d_{f2lu} were negative, the vertical translation would be in the negative direction.

Finally, by adjusting e_{f2lu} , the limit function is translated horizontally. This is shown in Figure 3.14. The value of e_{f2lu} is the percentage chord that

Figure 3.12: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $c_{f2lu} = 8$.

Figure 3.13: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $d_{f2lu} = 10$.

Figure 3.14: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $e_{f2lu} = -0.2$.

the designer wishes to shift the function, with a positive value moving it to the left and a negative to the right.

The preset terms are illustrated in order to show how and why these values were chosen. Figure 3.15 shows how the cosine curve period changes from 2 to 4 when P changes from π to a $\pi/2$, respectively. The cosine curve maximum value is still at $x = 0$ but now instead of going to its corresponding minimum value at $x = 1$, the period has been increased so that at $x = 1$ the cosine curve goes to 0.

Figure 3.16 shows the baseline limit with the cosine curve horizontal translation, $T_h = 0$ and the adjusted limit with $T_h = 2\pi/3$. This variable shifts the cosine curves starting and end points. Now instead of having the maximum and minimum values at $x = 0$ and $x = 1$, respectively the curve has been shifted by $2\pi/3$. Resulting in the adjusted limit value at $x = 0$ corre-

Figure 3.15: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $P = \pi/2$.

Figure 3.16: Lower surface curvature for NACA 0012 airfoil with baseline curvature limit and adjusted limit with $T_h = 2\pi/3$.

sponding to the baseline limit at $x = 0.67$ and the adjusted limit at $x = 1$ corresponding to the baseline limit at $x = 1.67$.

For each curvature limit, the P and T_h values were chosen based upon curvature trends observed in studying several different airfoil geometries. The curvature limit equations for the curvature function *two* are as follows:

1. Upper surface upper curvature limit (**uu**)

$$\begin{aligned} \kappa_{f2_{uu}} = & a_{f2_{us}} \exp [m_{f2_{us}}(x_{n_i} + e_{f2_{uu}})](1 + b_{f2_{uu}}) \\ & + c_{f2_{uu}} \left[\cos\left(\frac{2\pi}{3}x_i + \frac{\pi}{5}\right) \right] + d_{f2_{uu}} \end{aligned} \quad (3.58)$$

2. Upper surface lower curvature limit (**ul**)

$$\begin{aligned} \kappa_{f2_{ul}} = & a_{f2_{us}} \exp [m_{f2_{us}}(x_{n_i} + e_{f2_{ul}})](1 + b_{f2_{ul}}) \\ & + c_{f2_{ul}} \left[\cos\left(\frac{\pi}{2}x_i + \frac{\pi}{3}\right) \right] + d_{f2_{ul}} \end{aligned} \quad (3.59)$$

3. Lower surface upper curvature limit (lu)

$$\begin{aligned}\kappa_{f2lu} = a_{f2ls} \exp [m_{f2ls}(x_{n_i} + e_{f2lu})] (1 + b_{f2lu}) \\ + c_{f2lu} \left[\cos\left(\frac{\pi}{2}x_i + \frac{\pi}{3}\right) \right] + d_{f2lu}\end{aligned}\quad (3.60)$$

4. Lower surface lower curvature limit (ll)

$$\begin{aligned}\kappa_{f2ll} = a_{f2ls} \exp [m_{f2ls}(x_{n_i} + e_{f2ll})] (1 + b_{f2ll}) \\ + c_{f2ll} \left[\cos\left(\pi x_i + \frac{\pi}{2}\right) \right] + d_{f2ll}\end{aligned}\quad (3.61)$$

with a_{f2us} , m_{f2us} , a_{f2ls} and m_{f2ls} defined as

$$\begin{aligned}a_{f2us} &= \kappa p_{us}(1) & m_{f2us} &= \frac{\log\left(\frac{\kappa_{ous}(x_{0.05i})}{a_{f2us}}\right)}{x_{0.05i}} \\ a_{f2ls} &= \kappa p_{ls}(1) & m_{f2ls} &= \frac{\log\left(\frac{\kappa_{ols}(x_{0.05i})}{a_{f2ls}}\right)}{x_{0.05i}}\end{aligned}\quad (3.62)$$

where $\kappa p_{us}(1)$ and $\kappa p_{ls}(1)$ are the upper and lower surface first curvature peak and $\kappa_{ous}(x_{0.05i})$ and $\kappa_{ols}(x_{0.05i})$ are the initial upper and lower surface curvature at 5% chord.

The remaining terms in Equations 3.58 to 3.61 are specified by the designer. Figures 3.17 and 3.18 show the NACA 0012 upper and lower surface curvature and the corresponding limits, respectively. For these limits, the designer specified terms are

$$\begin{array}{llll}b_{uu} = 0.9 & c_{uu} = -0.05 & d_{uu} = -0.01 & e_{uu} = 0 \\ b_{ul} = 0.9 & c_{ul} = -6 & d_{ul} = -8 & e_{ul} = 0 \\ b_{lu} = 0.9 & c_{lu} = 2.5 & d_{lu} = 3 & e_{lu} = 0 \\ b_{ll} = 0.9 & c_{ll} = -0.25 & d_{ll} = -1.0 & e_{ll} = 0\end{array}\quad (3.63)$$

Figure 3.17: Upper surface curvature for NACA 0012 airfoil with curvature function *two* limits.

Figure 3.18: Lower surface curvature for NACA 0012 airfoil with curvature function *two* limits.

Implementation within Objective Function

Since these curvature limits are not a plus-minus bounding, using the difference in curvature as was done for the first curvature function outlined in Section 3.5.1 is not possible. For this case, the current airfoil shape curvature is calculated and compared with the respective curvature function limits. If the curvature is outside the limits, a penalty is added to the objective function using the penalty method.

The constraint equation, Equation 2.2, outlined in Section 2.3.2 gives the following condition for the airfoil upper surface curvature limits

$$\kappa_{us}(x_i) - \kappa_{f2_{uu}}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.64)$$

$$\kappa_{f2_{ul}}(x_i) - \kappa_{us}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.65)$$

where $\kappa_{us}(x_i)$ is the curvature at x_i and $\kappa_{f2_{uu}}(x_i)$ and $\kappa_{f2_{ul}}(x_i)$ are the curvature function *two* limit values at x_i for the upper and lower limits, respectively.

The conditions for the lower surface are

$$\kappa_{ls}(x_i) - \kappa_{f2_{lu}}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.66)$$

$$\kappa_{f2_{ll}}(x_i) - \kappa_{ls}(x_i) \leq 0 \quad i = 1, \dots, i_{max} \quad (3.67)$$

where $\kappa_{ls}(x_i)$ is the curvature at x_i and $\kappa_{f2_{lu}}(x_i)$ and $\kappa_{f2_{ll}}(x_i)$ are the curvature function *two* limit values at x_i for the upper and lower limits, respectively

Using the penalty method, the upper and lower curvature function *two*

constraint becomes

$$C_{i\kappa_{f2us}} = \begin{cases} \left[1 - \frac{\kappa_{f2uu}(x_i)}{\kappa_{us}(x_i)}\right]^2, & \text{if } \kappa_{us}(x_i) > \kappa_{f2uu}(x_i) \\ \left[1 - \frac{\kappa_{us}(x_i)}{\kappa_{f2ul}(x_i)}\right]^2, & \text{if } \kappa_{us}(x_i) < \kappa_{f2ul}(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.68)$$

$$C_{i\kappa_{f2ls}} = \begin{cases} \left[1 - \frac{\kappa_{ls}(x_i)}{\kappa_{f2lsul}(x_i)}\right]^2, & \text{if } \kappa_{ls}(x_i) > \kappa_{f2lsul}(x_i) \\ \left[1 - \frac{\kappa_{f2lsul}(x_i)}{\kappa_{ls}(x_i)}\right]^2, & \text{if } \kappa_{ls}(x_i) < \kappa_{f2lsul}(x_i) \\ 0, & \text{otherwise} \end{cases} \quad (3.69)$$

respectively, and the curvature function constraint *two* becomes part of the objective function as follows

$$I_{C_{\kappa_{f2}}}(D) = w_{\kappa_{f2us}}(C_{\kappa_{f2us}}) + w_{\kappa_{f2ls}}(C_{\kappa_{f2ls}}) \quad (3.70)$$

where $w_{\kappa_{f2us}}$ and $w_{\kappa_{f2ls}}$ are the weightings for the upper and lower surface curvature function constraints, respectively.

3.6 Stowability Constraint

The above constraints all deal with controlling the airfoil shape. These are necessary but not the only important component for the design of an airfoil, especially for high-lift design. As stated by Mathews [38] from Short Brothers, “The design of high lift systems cannot be based solely upon maximum aerodynamic performance but is constrained by weight, cost, ease of manufacture, maintainability, reliability and the mechanics of the flap movement.” Stowability is the constraint which addresses flap movement mechanics by controlling the flap or slat shape so that its final optimal shape can stow

within the main element. This constraint links the slat or flap path with its shape, linking the kinematics to the aerodynamics. This constraint is very important. The designer will know that, at the optimization conclusion, not only will the best aerodynamic shape be produced but this shape will also meet the structural requirements necessary to move the high lift device from extended to stowed position.

3.6.1 Linkage system

This section focuses on the flap movement kinematics. The same theory can also be applied to the slat. In traditional kinematic linkage analysis, the parameters are known and the trajectory path is determined. However, in this case, the goal is to determine the linkage parameters so that the desired path is achieved. This is known as kinematic synthesis. Both Norton [49] and Sandor [50] provide a very detailed description of basic kinematic synthesis for linkages. For practical uses of kinematic synthesis within machine design and automobiles, see [51, 52] and [53], respectively. This is by no means a complete list but demonstrates that kinematic synthesis is valid and useful in the analysis of linkages.

In motion generation problems in mechanical synthesis, a rigid body is required and is guided through a series of prescribed positions. In this research, only two positions, the initial and final, are considered but a similar analysis could be applied to incorporate up to five prescribed positions. The rigid body vertices are selected based on essential points used to stow the flap within the main element. On the main element upper surface, the trailing edge point is termed the upper shroud location. This point is where the

flap upper surface and main element meet. Equivalently, there is a lower shroud point, on the main element lower surface that is typically located at about 3% wing chord from the stowed flap leading edge. These two points were selected, as the airfoil shape aft of these points is defined by the main element high speed lines. These points are shown in Figure 3.19, where P_a and P_b are the lower and upper shroud locations, respectively, on the main element. P_1 is the flap leading edge location in its extended position. Since the flap chord is fixed throughout the optimization, the flap leading edge in the stowed or cruise position, P_2 , is known. Note that all these values do include a manufacturing tolerance to prevent the flap from fouling against the main element. As well, Figure 3.19 illustrates the stowed flap chord in line with the wing chord. The constraint is set up to deal with this case, as

Figure 3.19: Essential points for creating the rigid body.

well as cases where the flap chord is rotated below the wing chord. The rigid body points are determined using the following equations

$$\begin{aligned}
P_{1x} &= F_{te_x} - (c_w + tol) \cos \theta_d & P_{1y} &= F_{te_y} + (c_w + tol) \sin \theta_d \\
P_{2x} &= (1 - c_w) - tol & P_{2y} &= 0 \\
P_{ax} &= P_{2x} + l_{per} & P_{ay} &= M_y(P_{ax}) + tol \\
P_{bx} &= M_{te_x} & P_{by} &= M_{te_y} - tol
\end{aligned} \tag{3.71}$$

where (F_{te_x}, F_{te_y}) are the flap trailing edge coordinates, (M_{te_x}, M_{te_y}) are the main element trailing edge coordinates, tol is the manufacturing tolerance and is specified by the designer, l_{per} is percentage of wing chord that the lower shroud point will be located at and is typically 3%, c_w is the flap chord length corresponding to the wing chord and is calculated as

$$c_w = c_f \cos \theta_{wf} \tag{3.72}$$

where c_f is the flap chord and θ_{wf} is the angle between the flap chord and wing chord in the stowed position. The final term, θ_d in Equation 3.71, is the total flap deflection angle in the extended position. This is

$$\theta_d = \theta_{wf} + \theta_{df} \tag{3.73}$$

where θ_{df} is the flap deflection angle relative to the flap chord.

The final value represented in Figure 3.19, θ_{deg} , is the angle between the main element upper surface and the upper cove line. This value must be at least four degrees for manufactureability and to allow for adequate space above the flap for the spoiler to be stored. This calculated angle will be discussed later in the chapter.

The flap movement is modeled as a four-bar linkage with the rigid body included as a part of its coupler link. Figure 3.20 illustrates the linkage in the initial and final positions. Note that the pivot points and linkage bars are only for illustration and that only the coupler vertices are known at this time.

Figure 3.20: Four-bar linkage in initial and final positions.

Figure 3.21 shows the linkage system lengths and angles that will need to be solved in order to determine where the pivot points, W and U , and their corresponding angles, θ and σ , are located. The coupler lengths (s, z, v) and angles (not shown on diagram) are solved using the distance equation and the cosine law. Once the coupler has been determined, the initial and final coupler positions are calculated using the distance equation to solve for P_{21} and angle equations to solve for ψ , ϕ , α_{21} , and δ_{21} . For details on these

Figure 3.21: Four-bar linkage with necessary lengths and angles.

calculations see Appendix A. The remaining angles β_{21} and γ_{21} are adjusted to satisfy θ_{deg} . Initially, $\beta_{21} = \theta_d$ and $\gamma_{21} = \theta_d + 5^\circ$. By setting $\gamma_{21} > \beta_{21}$ the right linkage pivot point will always be ahead of the left point, thereby preventing the linkage arms from colliding.

Once all of the angles have been determined the pivot points can be found. Begin by solving for the left pivot point, W . To do this, focus on the left side of each linkage and write a vector loop equation to include the leading edge starting and ending position points. This is demonstrated in Figure 3.22 and is written as:

$$\vec{W}_2 + \vec{Z}_2 - \vec{P}_{21} - \vec{Z}_1 - \vec{W}_1 = 0 \quad (3.74)$$

The vector equation can then be rewritten in terms of its complex number

Figure 3.22: Left side of linkage system.

equivalents.

$$we^{j(\theta+\beta_{21})} + ze^{j(\phi+\alpha_{21})} - P_{21}e^{j\delta_{21}} - ze^{j\phi} - we^{j\theta} = 0 \quad (3.75)$$

Then using the Euler identity: $e^{\pm j\theta} = \cos \theta \pm j \sin \theta$, Equation 3.75 becomes:

$$\begin{aligned} w(\cos \theta + j \sin \theta)(\cos \beta_{21} + j \sin \beta_{21} - 1) \\ + z(\cos \phi + j \sin \phi)(\cos \alpha_{21} + j \sin \alpha_{21} - 1) \\ - P_{21}(\cos \delta_{21} + j \sin \delta_{21}) = 0 \end{aligned} \quad (3.76)$$

The above equation is then simplified and separated into its real and

imaginary parts to obtain the following two equations.

$$\begin{aligned}
& [w \cos \theta](\cos \beta_{21} - 1) - [w \sin \theta] \sin \beta_{21} \\
& \quad + [z \cos \phi](\cos \alpha_{21} - 1) - [z \sin \phi] \sin \alpha_{21} \\
& \quad = P_{21} \cos \delta_{21} \quad (3.77)
\end{aligned}$$

$$\begin{aligned}
& [w \sin \theta](\cos \beta_{21} - 1) - [w \cos \theta] \sin \beta_{21} \\
& \quad + [z \sin \phi](\cos \alpha_{21} - 1) - [z \cos \phi] \sin \alpha_{21} \\
& \quad = P_{21} \sin \delta_{21} \quad (3.78)
\end{aligned}$$

In the above two equations, there are eight variables; $w, \theta, \beta_{21}, z, \phi, \alpha_{21}, P_{21}$ and δ_{21} but only w and θ are unknown. To separate the unknown terms in Equations 3.77 and 3.78, let

$$\begin{aligned}
W_{1x} &= w \cos \theta & Z_{1x} &= z \cos \phi & (3.79) \\
W_{1y} &= w \sin \theta & Z_{1y} &= z \sin \phi
\end{aligned}$$

and let,

$$\begin{aligned}
A &= \cos \beta_{21} - 1 & B &= \sin \beta_{21} & C &= \cos \alpha_{21} - 1 & (3.80) \\
D &= \sin \alpha_{21} & E &= P_{21} \cos \delta_{21} & F &= P_{21} \sin \delta_{21}
\end{aligned}$$

Then Equations 3.77 and 3.78 become,

$$W_{1x}A - W_{1y}B + Z_{1x}C - Z_{1y}D = E \quad (3.81)$$

$$W_{1y}A + W_{1x}B + Z_{1y}C + Z_{1x}D = F \quad (3.82)$$

Finally, Equations 3.81 and 3.82 can be solved to find W_{1x} and W_{1y} ,

$$W_{1x} = \frac{A(E - Z_{1x}C + Z_{1y}D) + B(F - Z_{1y}C - Z_{1x}D)}{-2A} \quad (3.83)$$

$$W_{1y} = \frac{A(F - Z_{1y}C - Z_{1x}D) + B(-E + Z_{1x}C - Z_{1y}D)}{-2A} \quad (3.84)$$

The same procedure can be applied to the right hand side of each linkage in order to find U_{1x} and U_{1y} . This analysis is shown in Appendix A.

The remaining angles, θ and α , are calculated as

$$\theta = \tan^{-1} \left(\frac{W_{1y}}{W_{1x}} \right) \quad \alpha = \tan^{-1} \left(\frac{U_{1y}}{U_{1x}} \right) \quad (3.85)$$

and the length of each linkage as

$$w = \frac{W_{1x}}{\cos \theta} \quad u = \frac{U_{1x}}{\cos \alpha} \quad (3.86)$$

Now, it is possible to trace the path each coupler point will take using kinematic analysis. Figure 3.23 shows the linkage in the initial position, where the points $\vec{P}_c$ and $\vec{P}_d$ correspond to the points $\vec{P}_a$ and $\vec{P}_b$, respectively, when

Figure 3.23: Four-bar linkage in initial position.

the flap is in the stowed position. This calculation is shown in Appendix A.

$\vec{W}_c$ and $\vec{U}_c$ are the pivot point coordinates and are calculated from

$$\begin{aligned} W_{cx} &= P_{cx} - W_{1x} & W_{cy} &= P_{cy} - W_{1y} \\ U_{cx} &= P_{dx} - U_{1x} & U_{cy} &= P_{dy} - U_{1y} \end{aligned} \quad (3.87)$$

The upper shroud point dictates where the main element and the flap come together. The path of this point is used in determining the allowable stowability design space so that the flap will not collide with the main element. In order to determine this allowable area, the path of each coupler point is traced just beyond the stowed location. The main linkage angle, θ is incremented by a_{inc} through an angle of $\beta_{21} + 10^\circ$. This results in each coupler path consisting of $k = 1, \dots, a_{inc}$ points.

Using Figure 3.23 as a reference, the lower shroud path at each increment is calculated as:

$$L_{cx_k} = W_{cx} + w \cos \theta \quad L_{cy_k} = W_{cy} + w \sin \theta \quad (3.88)$$

where L_{cx} and L_{cy} are the x and y lower shroud path coordinates, respectively.

The upper shroud point path, $\vec{L}_d$ is calculated using the equation of a circle at each increment about the lower shroud path, $\vec{L}_c$ and the right pivot point, $\vec{U}_c$.

$$v^2 = (L_{dx_k} - L_{cx_k})^2 + (L_{dy_k} - L_{cy_k})^2 \quad (3.89)$$

$$u^2 = (L_{dx_k} - U_{cx})^2 + (L_{dy_k} - U_{cy})^2 \quad (3.90)$$

where v is the linkage length between $\vec{P}_c$ and $\vec{P}_d$ and u is the length between $\vec{P}_d$ and $\vec{U}_c$.

After expanding the above equations, Equation 3.90 is subtracted from 3.89 and rearranged to get

$$L_{dx_k} = \frac{v^2 - u^2 - L_{cx_k}^2 + U_{cx}^2 - L_{cy_k}^2 + U_{cy}^2}{2(U_{cx} - L_{cx_k})} - \frac{2L_{dy_k}(U_{cy} - L_{cy_k})}{2(U_{cx} - L_{cx_k})} \quad (3.91)$$

To simplify the above equation let,

$$S = \frac{v^2 - u^2 - L_{cx_k}^2 + U_{cx}^2 - L_{cy_k}^2 + U_{cy}^2}{2(U_{cx} - L_{cx_k})} \quad (3.92)$$

and

$$T = \frac{U_{cy} - L_{cy_k}}{U_{cx} - L_{cx_k}} \quad (3.93)$$

Then Equation 3.91 becomes:

$$L_{dx_k} = S - L_{dy_k}T \quad (3.94)$$

By substituting Equation 3.94 into Equation 3.90 an expression for L_{dy_k} can be determined as:

$$[T^2 + 1]L_{dy_k}^2 - [2(S - U_{cx})T + 2U_{cy}]L_{dy_k} + [(S - U_{cx})^2 + U_{cy}^2 - U_1^2] = 0 \quad (3.95)$$

and is solved using the quadratic formula

$$L_{dy_k} = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P} \quad (3.96)$$

where

$$P = T^2 + 1$$

$$Q = -[2(S - U_{cx})T + 2U_{cy}]$$

$$R = (S - U_{cx})^2 + U_{cy}^2 - U_1^2$$

To find the leading edge point path, $\vec{L}_1$, the same approach is used but now with the equation of a circle calculated about the upper and the lower

shroud points, $\vec{L}_d$ and $\vec{L}_c$, respectively. The following equations were determined to calculate L_{1x} and L_{1y} at each location, k , along the path:

$$\begin{aligned} L_{1x_k} &= M - NL_{1y_k} \\ L_{1y_k} &= \frac{-K \pm \sqrt{K^2 - 4JL}}{2J} \end{aligned} \quad (3.97)$$

where

$$\begin{aligned} J &= N^2 + 1 & K &= 2(M - L_{dx_k})N + 2L_{dy_k} \\ L &= (M - L_{dx_k})^2 + L_{dy_k}^2 - s^2 & N &= \frac{L_{dy_k} - L_{cy_1}}{L_{dx_k} - L_{cx_k}} \\ M &= \frac{z^2 - s^2 - L_{cy_k}^2 + L_{dy_k}^2 - L_{cx_k}^2 + L_{dx_k}^2}{2(L_{dx_k} - L_{cx_k})} \end{aligned}$$

Once the coupler path has been determined the last check is performed. The angle, θ_{deg} as shown in Figure 3.19, between the path of $\vec{L}_d$ and the main element upper surface at the upper shroud point is calculated. If θ_{deg} is less than four degrees, new values for β_{21} and γ_{21} are calculated as

$$\begin{aligned} \beta_{21_n} &= \beta_{21_o} & \gamma_{21_n} &= \gamma_{21_o} + 1^\circ \end{aligned} \quad (3.98)$$

where β_{21_n} and γ_{21_n} are the new angle values and β_{21_o} and γ_{21_o} are the previous values. New pivot points are then calculated and a new coupler path determined. This process is repeated until θ_{deg} becomes greater than four degrees.

3.6.2 Constraint

Now that the pivot points for the four-bar linkage have been determined and the coupler path calculated, the constraints for the optimization can be

set up. In flap design, only the airfoil portion covered by the aft shroud is allowed to be modified. The remaining flap surface shape is governed by the airfoil high speed lines. The upper shroud location is case dependent but is normally located at about 50% flap chord and the lower shroud location is approximately 10% flap chord measured from the leading edge. Beyond these points to the trailing edge, the upper surface is dictated by the main element upper surface and similarly for the lower surface.

The most important issue with this constraint is to make sure at no time does the flap collide with the main element during its expected trajectory path. Based on the linkage results, a design space domain is setup to dictate the stowable flap. On the upper surface, between $\vec{P}_1$ and $\vec{P}_d$ the design space is dictated by the path of $\vec{P}_d$ and hence is an arc of radius u . Between $\vec{P}_d$ and the flap trailing edge, and then from the trailing edge to $\vec{P}_c$, the design space is a linear constraint as these surfaces are based on the high speed lines. The design space on the flap lower surface between the lower shroud point, $\vec{P}_c$, and leading edge point, $\vec{P}_1$, is a continuation of the lower surface linear constraint. A linear constraint was used as this exposed area is small and less critical than the upper surface. In Figure 3.24 this design space is illustrated by the dashed lines.

At the moment, only the surface between the flap lower and upper shroud point is included as a constraint so it is assumed that the initial flap shape meets the main element high-speed lines. For each point on the flap between P_{1x} and P_{dx} , the length, r_1 , from the pivot point $\vec{U}$ to the current point is calculated and compared with the path radius, u of $\vec{P}_d$. If the length, r_1 is greater than the radius, u , a penalty is added to the objective function

Figure 3.24: Stowable design space.

using the penalty method previously discussed in Section 2.3.2. For the lower surface, each point between P_{1x} and P_{cx} is compared with the linear constraint, l_1 . If the y -coordinate of each point is less than l_1 , a penalty is added to the objective function.

Using the constraint equation, Equation 2.2 as outlined in Section 2.3.2 gives the following condition for the upper surface

$$r_{1j} - u \leq 0 \quad j = j_{le}, \dots, j_{upshrd} \quad (3.99)$$

where j_{le} is the point corresponding to $\vec{P}_1$ and j_{upshrd} corresponds to $\vec{P}_d$. For the lower surface the condition is

$$l_1 - y_j \leq 0 \quad j = j_{le}, \dots, j_{loshrd} \quad (3.100)$$

where j_{loshrd} corresponds to $\vec{P}_c$.

Using the penalty method, the stowability constraint becomes part of the objective function with the following equations, respectively.

$$C_{istow_{us}} = \begin{cases} \left[1 - \frac{u}{r_1}\right]^2, & \text{if } r_1 > u \\ 0, & \text{otherwise} \end{cases} \quad (3.101)$$

$$C_{istow_{ls}} = \begin{cases} \left[1 - \frac{y}{l_1}\right]^2, & \text{if } y < l_1 \\ 0, & \text{otherwise} \end{cases} \quad (3.102)$$

The final stowability constraint equation becomes

$$I_{c_{stow}}(D) = w_{stow}(C_{stow_{us}} + C_{stow_{ls}}) \quad (3.103)$$

where w_{stow} is the stowability constraint weighting.

CHAPTER 4

4.1 Design Examples

Now that the new constraints have been discussed in Chapter 3, their usefulness for design will be demonstrated. In Section 4.2, single-element airfoil optimization problems are examined for the minimum global and local maximum thickness, area and curvature constraints. The stowability constraint will be examined in the high lift optimization examples in Section 4.3. For all test examples, the exit tolerance for the optimization algorithm is $10^{-5} * \|DV_{scal}\|_2$ which results in the scaled gradient L_2 norm being decreased by three orders of magnitude. For the single-element problems, the original scaling is used whereas for the high-lift problem, a fixed scaling of 0.03 is used. Both scaling methods were outlined and discussed in Section 2.6

An H-topology grid was created around both the single element and high lift examples using AMBER2D [54]. The grids consist of approximately 25,000 and 30,000 nodes for the single element and high lift examples, respectively. The distance to the outer boundary is $12c$, the off-wall spacing is $2 \times 10^{-6}c$, the spacing at the stagnation points is $2 \times 10^{-6}c$ and the trailing-

edge clustering is $2 \times 10^{-3}c$ for all examples. The selected grids are similar to those presented in [36] and used in a detailed accuracy study presented in [55, 56].

4.2 Single Element Constraint Demonstrations

The following optimization examples use the NACA 0012 airfoil [57] as the initial geometry with subsonic conditions. The freestream conditions are $M_\infty = 0.25$, $\alpha = 9^\circ$ and $Re = 2.88 \times 10^6$. The airfoil shape is described with 20 B-spline control points and 16 of which are used as design variables. This results in all control points except the leading and trailing edge points being used. The angle of attack is fixed during the optimization process. The objection function, as described in Equation 2.4, will be case dependent. The weighting terms in the objective function equation will be used to ensure that the objective terms and the constraints are of similar magnitude. For all design examples, the flow is assumed to be fully turbulent.

4.2.1 Global Minimum Thickness

In some situations, the designer wants a solution without any constraints imposed on the problem to study the shape evolution based on the aerodynamic properties. While this approach is analytically valid, the final shape can be unrealistic. This is best shown through an example. Using the subsonic example laid out above, a lift-to-drag optimization was performed with no constraints placed on the geometry. The target lift-to-drag ratio was 60. The optimization algorithm converged to the target value as seen in Figure 4.1.

Figure 4.1: Lift-to-drag curve for the NACA 0012 optimization without constraints.

The L_2 norm of the scaled gradient dropped the required three orders of magnitude, as shown in Figure 4.2. However, for this example the error lies within the final geometry. Figure 4.3 shows the initial and final geometry along with the corresponding pressure distributions. The surface crossover that occurs between 20% and 55% chord is numerically valid. As discussed in Section 2.4.2, the thin-layer approach is used to approximate the Navier-Stokes equations. This approach transforms the curvilinear grid into the computational domain which essentially unwraps the grid and is therefore not able to detect surface crossovers.

The original method used local minimum thickness constraints to check and prevent these crossovers. However, as previously discussed this constraint only checks for crossovers at the constraint location and therefore works best when the crossover location is known a priori. To illustrate this

Figure 4.2: L_2 norm of the scaled gradient for the NACA 0012 optimization without constraints.

Figure 4.3: Initial and final geometry and pressure distribution for the NACA 0012 optimization without constraints.

point, consider the same example discussed above but now with local thickness constraint placed on the problem. Since the crossover peak occurs at 35% chord, a constraint is imposed at this point. The initial thickness at 35% chord is 11.78% and the minimum allowable thickness is set to 3.5%. This target was chosen as it corresponds to the minimum global thickness constraint value at this location, which is based on 30% of the original thickness.

This example converges by three orders of magnitude in the scaled gradient and increases the lift to drag ratio from 45.3 to 59.0 in 30 iterations, as shown in Figures 4.4 and 4.5, respectively. The oscillations between iterations 5 and 11 are due to the minimum thickness constraint activation. This is illustrated in Figure 4.6 where the objective function terms are plotted.

The final airfoil thickness at 35% chord was just above the minimum

Figure 4.4: L_2 norm of the scaled gradient for the NACA 0012 optimization with a thickness constraint.

Figure 4.5: Lift-to-drag ratio for the NACA 0012 optimization with a thickness constraint.

Figure 4.6: Objective terms for the NACA 0012 optimization with a thickness constraint.

allowable thickness at 3.51% chord. Again, even though this example converged, surface crossovers still exist along the airfoil. This constraint did prevent the surface from crossing over at 35% chord where the minimum thickness was imposed. However, the lower surface was still allowed to cross over the upper surface between 7% and 18% chord and again between 58% and 80% chord. This is shown in Figure 4.7 where the initial and final geometries and the corresponding pressure distributions are plotted.

It would be possible to place more local thickness constraints along the airfoil chord to prevent these additional crossovers but there is no guarantee that a crossover won't occur somewhere else and result in the same process having to be repeated. This can become very time consuming. Instead, the minimum global thickness constraint, as outlined in Section 3.2 is used.

For the global minimum thickness constraint, the optimization program

Figure 4.7: Initial and final geometry and pressure distribution for the NACA 0012 optimization with a thickness constraint.

reduced the scaled gradient by three orders of magnitude as shown in Figure 4.8 and produced a final lift-to-drag ratio of 58.8 as shown in Figure 4.9. A lower lift-to-drag ratio was expected since more constraints have been placed on the problem.

The final geometry has no surface crossovers but the target global thickness, 30% of the initial thickness, was slightly violated along the airfoil chord at the optimization conclusion. This is shown in Figure 4.10. The fact that the final thickness is less than the global target is insignificant, as the main purpose for this constraint is to prevent surface crossovers and the trailing edge from becoming too thin. For most examples, other constraints will be included to dictate the final geometry.

The initial and final geometry is also shown in Figure 4.11 with the corresponding pressure distributions.

Figure 4.8: L_2 norm of the scaled gradient for the NACA 0012 optimization with minimum global thickness constraint

Figure 4.9: Lift-to-drag ratio for the NACA 0012 optimization with minimum global thickness constraint.

Figure 4.10: Initial and final geometry and thickness with global constraint for NACA 0012 optimization

Figure 4.11: Initial and final geometry and pressure distribution for the NACA 0012 optimization with global thickness constraint.

As expected, both the optimization with one minimum thickness constraint and the global thickness constraint took longer to converge than the example without constraints. The global thickness constraint took 5 iterations longer to converge than the minimum thickness constraint but produced no surface crossovers. In order for an example with local minimum thickness constraints to produce a final shape without surface crossovers, it would require additional thickness constraints and this would increase the number of iterations required to converge. Therefore, there is not a penalty in terms of computational time in using the global thickness constraint compared to the minimum thickness constraint. It will, in the end, save time in the design process as the final optimized shape will always be realistic.

4.2.2 Maximum Thickness Constraint

To demonstrate the maximum thickness constraint, the same subsonic design problem is used but the objective function is changed from a lift-to-drag maximization to a lift maximization. This removes the drag influence on the shape and will allow the geometry to become thicker thereby activating this constraint. For this design example, the NACA 0012 geometry will need to decrease in thickness at 40% chord to meet the allowable maximum thickness constraint of 11% chord. This optimization criterion becomes important when an airfoil is required to fit within a specified space, for example a flap within the main airfoil. The target lift coefficient is set to 1.5.

Figure 4.12 shows the initial pressure distribution for the NACA 0012 airfoil, the final optimized pressure distribution and the corresponding airfoil geometries. The lift coefficient increased from 0.949 to 1.5 and the final

Figure 4.12: Initial and final geometry and pressure distributions for a NACA 0012 optimization with a maximum thickness constraint.

thickness at 40% chord is 11.1%. The scaled gradient was reduced three orders of magnitude in 19 iterations. Figure 4.13 shows the lift coefficient plot and Figure 4.14 shows the scaled gradient.

Figure 4.13: Lift coefficient for the NACA 0012 optimization with a maximum thickness constraint.

The oscillations in the L_2 norm of the scaled gradient are caused by the maximum thickness constraint activation. Figure 4.15 shows the maximum thickness constraint contribution to the objective function and how the constraint influences the optimization.

4.2.3 Area Constraints

Area constraints are useful when the thickness at specific locations along the airfoil chord are not essential but the airfoil needs to maintain a set area for fuel and component storage. The airfoil area can either be used as a constraint or as a design objective, as discussed in Section 3.4. Both

Figure 4.14: L_2 norm of the scaled gradient for NACA 0012 optimization with a maximum thickness constraint.

Figure 4.15: Objective function terms for the NACA 0012 optimization with a maximum thickness constraint.

scenarios will be considered using the subsonic design problem described in Section 4.2. The objective is to increase the lift-to-drag ratio while achieving a final area of 0.06. The starting NACA 0012 airfoil area is 0.0808.

The first test example demonstrates the area as a constraint. Figure 4.16 shows the initial pressure distribution for the NACA 0012 airfoil, the final optimized pressure distribution and the corresponding airfoil shapes. The lift to drag ratio increased from 45.3 to 55 and the scaled gradient decreased three orders of magnitude in 28 iterations. This can be seen in Figures 4.17 and 4.18, respectively.

The area constraint is still slightly active at the optimization conclusion with a final airfoil area of 0.05997 but the lift-to-drag term is the dominate component in the objective function. This is shown in Figure 4.19 where each terms contribution in the objective function is highlighted. As well, the

Figure 4.16: Initial and final geometry and pressure distributions for the NACA 0012 optimization with an area constraint.

Figure 4.17: Lift-to-drag ratio for the NACA 0012 optimization with an area constraint.

Figure 4.18: L_2 norm of the scaled gradient for the NACA 0012 optimization with an area constraint.

Figure 4.19: Objective function terms for the NACA 0012 optimization with an area constraint.

figure shows that the area constraint does not become active until the fourth iteration when the current area becomes less than the specified minimum.

The second example demonstrates the area as a design objective. The initial and optimized pressure distributions, as well as the corresponding shapes, are shown in Figure 4.20. Similarly to the area constraint example, this example converges to a lift-to-drag ratio of 55, as shown in Figure 4.21. The scaled gradient converged three orders of magnitude in 24 iterations shown in Figure 4.22.

The final airfoil area is 0.05995 so the area term is still active within the objective function, as seen in Figure 4.23. Both examples produced similar results with the biggest difference seen in objective function contributions. The area design objective contribution is always active and therefore always plays a role in the optimization, whereas the area constraint only plays a

Figure 4.20: Initial and final geometry and pressure distributions for the NACA 0012 optimization with an area objective.

Figure 4.21: Lift-to-drag ratio for the NACA 0012 optimization with an area design objective.

Figure 4.22: L_2 norm of the scaled gradient for the NACA 0012 optimization with an area design objective.

Figure 4.23: Objective function terms for the NACA 0012 optimization with an area objective.

part when the airfoil area becomes less than the specified allowable area. For an example where the unconstrained optimization would increase the area, the differences in these two terms would be more pronounced. The area constraint would probably remain inactive and therefore take the optimization in a different direction than the area design objective.

4.2.4 Curvature Limits Constraint

This constraint is helpful in preventing large oscillations along the airfoil surface. Three design examples will be compared to demonstrate this constraint's importance. The subsonic NACA 0012 example laid out in Section 4.2 along with two minimum thickness constraints will be used for all examples. The thickness constraints are positioned to correspond to typical spar locations in a wing structure. One is located at 20% chord with an allowable minimum thickness of 7.5% chord and the another is located at 60% chord with an allowable value of 5.5% chord. The first design example will only contain the thickness constraints. The second example will have the thickness constraints and the curvature function *one* constraint with a curvature tolerance of $a_{cf1} = 2$. The final example will use the thickness constraints and the curvature function *two* constraint with the designer specified terms given as

$$\begin{array}{llll}
b_{uu} = 0.9 & c_{uu} = -0.05 & d_{uu} = -0.01 & e_{uu} = 0 \\
b_{ul} = 0.9 & c_{usll} = -6 & d_{ul} = -8 & e_{ul} = 0 \\
b_{lu} = 0.9 & c_{lu} = 2.5 & d_{lu} = 3 & e_{lu} = 0 \\
b_{ll} = 0.9 & c_{ll} = -0.25 & d_{ll} = -1.0 & e_{ll} = 0
\end{array} \tag{4.1}$$

The optimization goal for all examples is to increase the lift-to-drag ratio.

The first example converged to a final lift-to-drag ratio of 56.7 in just over 70 iterations. Figure 4.24 shows the lift-to-drag ratio versus the iteration number and Figure 4.25 shows the L_2 norm of the scaled gradient.

The oscillations within the gradient are caused by the activation of the global minimum thickness constraint. However, the overall objective function is influenced more by both the design objective and the thickness constraint as shown in Figure 4.26.

Figure 4.27 shows the initial and final pressure distribution and the corresponding airfoil geometries. As evident in the plots, there are a number of changes in curvature along the final geometry lower surface. The oscillations occur around the imposed thickness constraints at 20% and 60% chord and produce additional wing loading. However, the last oscillation occurred

Figure 4.24: Lift-to-drag ratio for NACA 0012 optimization with two thickness constraints.

Figure 4.25: L_2 norm of the scaled gradient for NACA 0012 optimization with two thickness constraints.

Figure 4.26: Objective function terms for NACA 0012 optimization with two thickness constraints.

at 90% chord resulting in the airfoil thinning near the trailing edge. Since the trailing edge is a fixed point, the upper and lower airfoil surfaces narrow rapidly from 98% chord onward resulting in lift loss as evident in Figure 4.27.

The corresponding lower surface curvature plot is shown in Figure 4.28 along with the final geometry.

The final geometry is not a typical single element airfoil, which usually characterizes a wing section in cruise position. This shape is more indicative of a high-lift configuration, with the first 10% of the airfoil resembling a fixed slat. The advantage of a slat is the increase in the leading edge stall angle. However, with a fixed slat the drag coefficient in the cruise position increases substantially [57]. To verify that this geometry would be generated at different operating conditions, a multi-point optimization was performed.

The current operating conditions were chosen for the multi-point opti-

Figure 4.27: Initial and final pressure distributions and geometries for NACA 0012 optimization with two thickness constraints.

Figure 4.28: Final curvature and geometry for NACA 0012 optimization with two thickness constraints.

mization with the first point having an angle of attack of 9° and the second point with an angle of attack of 12° . After 42 iterations, the scaled gradient had decreased by two orders of magnitude, as seen in Figure 4.29. As this example was performed to verify that a similar wavy geometry would be produced, the optimization was stopped at this point. The initial and final geometry and pressure distributions at each multi-point operating condition are shown in Figure 4.30. Each operating point produced a similar pressure distribution with the oscillations creating additional wing loading and the lift loss evident over the last 5% chord.

Figure 4.31 compares the final geometry lower surface curvature for the single and multi-point optimizations. The lower surface curvature is very similar, with the curvature peaks occurring in the same places but the magnitudes varying slightly.

Figure 4.29: L_2 norm of the scaled gradient for NACA 0012 multi-point optimization with two thickness constraints.

Figure 4.30: Initial and final geometry and pressure distributions for NACA 0012 multi-point optimization with two thickness constraints.

Figure 4.31: Final geometry and pressure distributions for NACA 0012 multi-point optimization with two thickness constraints.

Now, even though wavy geometries are valid airfoils that produce the desired aerodynamic performance at the specified conditions, there are other factors that the designer may want to consider. For instance, the manufacturability of a geometry. With the advances in composite materials, it is now possible to manufacture more complex geometries but at a higher cost [58]. Therefore, for a company to spend additional resources, both financial and in manpower, to manufacture a more complex geometry there must be a significant advantage in performance. Note that other factors do play a role in deciding if a significant advantage exists, such as the weight, structural stability, fuel area, and control systems placement; however, these topics are beyond the scope of this work.

To determine if a less complex airfoil geometry could be produced while maintaining the same aerodynamic performance, the curvature function *one*

constraint was applied to the optimization. For this example, the curvature limit constraint tolerance, a_{cf2} is equal to 2. Figure 4.32 shows the curvature limits and the initial NACA 0012 curvature for the upper and lower surface. In this example, the upper and lower surfaces have the same limits since the initial airfoil is symmetric.

This design example takes longer to converge to the specified tolerance as more constraints are now imposed on the problem. In just over 130 iterations, the lift-to-drag ratio converged to 56.2. This is shown in Figure 4.33 and the scaled gradient is shown in Figure 4.34.

The gradient for this test example is influenced significantly by the curvature constraint within the first 40 iterations where the geometry is changing the most. As the geometry reaches its converged shape, the curvature constraint settles down and the scaled gradient can converge. This is demonstrated in Figure 4.35 where the objective function terms are plotted.

Figure 4.32: Initial curvature and geometry for NACA 0012 airfoil.

Figure 4.33: Lift-to-drag ratio for NACA 0012 optimization with two thickness and a curvature function *one* constraint.

Figure 4.34: L_2 norm of the scaled gradient for NACA 0012 optimization with two thickness and a curvature function *one* constraint.

Figure 4.35: Objective function terms for NACA 0012 optimization with two thickness and a curvature function *two* constraint.

The initial and final pressure distribution plot, with the corresponding geometries, is shown in Figure 4.36, and the final lower surface curvature, difference in curvature and the corresponding curvature limits are shown in Figure 4.37.

The final geometry for this example has a significantly smoother lower surface with smaller oscillations in the curvature than the first example, as shown in Figure 4.38. The difference in the lift-to-drag ratio between the first and second examples is approximately 1%. This difference is due to a combination of the lift and drag coefficients. This second example has a slightly lower lift coefficient and a slightly higher drag coefficient. Figure 4.39 shows the final pressure distribution for both geometries. In comparing the distributions, it is noted that the oscillations produced in the first example help to load the airfoil but, due to the constraints, the final shape loses lift

Figure 4.36: Initial and final pressure distribution and geometries for NACA 0012 optimization with two thickness and a curvature function *one* constraint.

Figure 4.37: Final lower surface curvature, difference in curvature, curvature limits and geometry for NACA 0012 optimization with two thickness and a curvature function *one* constraint.

Figure 4.38: Final curvature and geometry for NACA 0012 optimization with two thickness and either without or with a curvature constraint.

Figure 4.39: Final pressure distribution for NACA 0012 optimization with two thickness and either without or with a curvature constraint.

over the last 5% chord approaching the trailing edge. The second example has smaller oscillations resulting in less wing loading and also less lift loss over the last 5% chord. However overall, the additional loading produced from the oscillations would contribute to the slightly larger lift coefficient for the first example.

The slight difference in the drag coefficient could be explained by the airfoil thickness. Figure 4.40 shows the percent difference in thickness between the two examples with a maximum difference of 35% occurring at around 7% chord. The first example is generally thinner along the chord line than the second. This could result in a lower drag coefficient for the first example at these specific operating conditions.

Overall, the goal for this design example was achieved. The oscillations were minimized to produce a less complex geometry and the aerodynamic

Figure 4.40: Percent difference in thickness between NACA 0012 with two thickness and without or with a curvature constraint.

performance was maintained. However, the final geometry produced in the second example still has a drooped leading edge, which will produce a higher drag coefficient at smaller angles of attack, such as in cruise position. The designer may require a smooth leading edge for better performance at various angles of attack or he/she may require more area in the leading edge section to position deicing equipment. These added requirements will further restrict the airfoil geometry which will likely cause a negative effect on the performance.

The goal for the third design example is to produce a smooth leading edge with minimal surface oscillations along the chord line. This example uses the curvature function *two* constraint with the designer specified terms outlined in Equation 4.1. The curvature limits and the initial NACA 0012 curvature for the upper and lower surface are shown in Figures 4.41 and 4.42, respectively.

This example stalls just after 80 iterations but by this point the scaled gradient had decreased by two orders of magnitude, as shown in Figure 4.43. As well, the lift to drag ratio converged to a final value of 55.8 by the 50th iteration, as illustrated in Figure 4.44. Therefore, even though the scaled gradient did not quite meet the required decrease of three orders of magnitude, the optimization program has determined the best possible geometry to achieve the desired performance based on the given conditions and constraints. Figure 4.45 shows the convergence of the objective function terms. The design objective term is constant after 45 iterations, the thickness term after 55 iterations and only the lower surface curvature term is changing until the stall point.

Figure 4.41: Upper surface curvature for NACA 0012 airfoil with curvature function *two* limits.

Figure 4.42: Lower surface curvature for NACA 0012 airfoil with curvature function *two* limits.

Figure 4.43: L_2 norm of the scaled gradient for NACA 0012 optimization with two thickness and a curvature function *two* constraint.

Figure 4.44: Lift-to-drag ratio for NACA 0012 optimization with two constraints and a curvature function *two* constraint.

Figure 4.45: Objective function terms for NACA 0012 optimization with two thickness and a curvature function *two* constraint.

The initial and final geometries and pressure distributions are shown in Figure 4.46. There is still a slight waviness around 60% chord but this is due to the thickness constraint imposed at this position. As well, this final geometry does not produce a loss in the lift over the last 5% chord as was seen in Figures 4.27 and 4.36 for the first and second examples, respectively.

The final lower surface curvature and curvature bounds, as well as the final geometry, are shown in Figure 4.47. Using these curvature bounds, the leading edge dip on the lower surface has been removed. There is still a slight change in curvature between 52% and 76% chord but this can be attributed to the thickness constraint imposed at 60% chord. The change at 95% is due to the fixed sharp trailing edge. The final lower surface curvature for the initial example and this final example are compared in Figure 4.48 where the differences in the oscillations are highlighted. Unlike the comparison shown

Figure 4.46: Initial and final pressure distribution and geometries for NACA 0012 optimization with two thickness and a curvature constraint.

Figure 4.47: Final lower surface curvature, curvature limits and geometry for NACA 0012 optimization with two thickness and a curvature constraint.

Figure 4.48: Final lower surface curvature and geometry comparison for NACA 0012 optimization with two thickness constraints and either without or with a curvature function *two* constraint.

between the initial and second examples in Figure 4.38, there is more than just a difference in the oscillation magnitudes. In this example a different shape has been produced.

Comparing the performance produced by the final geometries, the first example achieved a lift-to-drag ratio of 56.7 and this final example achieved a value of 55.8. This results in a 1.6% difference in the lift-to-drag ratio. The final lift coefficient for the third example is slightly larger by 0.1% and the final drag coefficient is also larger by 2%. By comparing the pressure distributions shown in Figure 4.49, the difference in lift can be explained. The initial design example, as seen previously, has several oscillations which contribute to additional loading on the airfoil. However, due to the shape near the trailing edge, a loss of lift is generated. This third example has a

Figure 4.49: Final pressure distributions for NACA 0012 optimization with two thickness constraints and either without or with a curvature function *two* constraint.

much smoother distribution along the lower surface and, without the loss of lift at the trailing edge, a slightly larger lift could be produced.

The increase in the drag coefficient is what causes the major difference, in the lift-to-drag ratio, between the two examples. Unfortunately, determining what exactly causes an increase in drag is not as easy to answer as an increase in lift. The second example could be said to be a scaled version of the initial example and therefore explaining the trends is simplified. However, this example produces a different geometry which results in a different pressure distribution, as shown in Figure 4.49. A different pressure distribution would result in a change in the amount of form drag produced. In this example, the form drag increased with the smoother geometry. As well, the third example is generally thicker than the initial, as shown in Figure 4.50. With the thicker

Figure 4.50: Thickness distributions for NACA 0012 optimization with two thickness constraints and either without or with a curvature function *two* constraint.

airfoil, there is a slight increase in the skin friction drag. Between these two increases, the total drag coefficient is increased.

The goal for this design example was achieved. The leading edge is much smoother which will result in better performance at lower angles of attack and has more room for deicing equipment. As well, with the overall smoother geometry, the airfoil manufacturability is increased as less complex methods can be used to produce the shape. At the same time, only a slight decrease in the airfoil performance resulted.

To summarize, three design examples were examined. The goal was to maximize the lift-to-drag ratio with two thickness constraints applied at 20% and 60% chord to simulate the spar locations. The first example only had the two thickness constraints and produced a lift-to-drag ratio of 56.7. The final

geometry had several changes in curvature along the lower surface that would result in higher manufacturing costs. The second test example included the curvature function *one* constraint as outlined in Section 3.5.1. This example applied the curvature constraints in order to achieve a similar but smoother shape that maintained the same aerodynamic performance. This provided the designer with the option to achieve a smoother geometry without the loss of performance. The final lift-to-drag ratio was 56.2 and the final geometry, though generally smoother, still produced a slightly drooped leading edge. The third example includes the curvature function *two* constraint as outlined in Section 3.5.2 with the goal being to produce a different shape. This constraint further restricted the final geometry and, as such, a decrease in the aerodynamic performance resulted. Again, this constraint provided the designer with more options. The final smoother geometry produced a lift-to-drag ratio of 55.8.

4.2.5 Single Element Constraint Summary

The global minimum thickness constraint prevents surface crossovers from occurring and therefore encourages realistic shapes to be generated. Figure 4.51 shows the airfoils produced for a lift-to-drag maximization using no constraints, one local thickness constraint and the global minimum thickness constraint.

The local maximum thickness constraint is just the reverse of the local minimum constraint and is equally important. The maximum constraint will help prevent airfoils from being too thick which is very helpful in slat and flap design to ensure the airfoil retracts into the main element without colliding.

Figure 4.51: Geometries for NACA 0012 optimization with no constraints, a thickness constraint and the global thickness constraint.

These constraints can be applied together if the designer needs to bound the thickness.

The fuel for most commercial aircraft is held within the wing and, as such, maintaining a specified area is important. As well, a specific area is needed to maintain structural integrity and component storage. Therefore, an area objective/constraint term was added to the program. When the area is specified as a constraint, the objective function is only influenced when the airfoil area becomes less than the allowable. However, if the area is specified as an objective, the objective function is influenced by the area term until the target area has been reached.

Finally, two curvature function constraints were used to obtain geometries that were easier to manufacture. The curvature function *one* constraint increased the manufacturability and also maintained the desired aerodynamic

performance. The curvature function *two* constraint was more restricting which resulted in the manufacturability increasing but the aerodynamic performance decreasing. The chosen curvature function was dependent upon the designer's optimization objectives. Figure 4.52 shows the final geometries and lower surface curvatures produced with two thickness constraints and either no curvature constraint, the curvature function *one* constraint or the curvature function *two* constraint.

Figure 4.52: Lower surface curvature and geometries for NACA 0012 optimization with two thickness constraints and no curvature constraint, curvature function *one* constraint, curvature function *two* constraint.

4.3 High-lift Configuration

A high-lift configuration was tested to demonstrate the stowability constraint. This constraint focuses on modifying the flap surface between the

upper and lower shroud points to allow the airfoil to stow within the main element. Since the flap geometries available for two-element configurations already have stowable surfaces, an existing configuration was modified. The NLR 7301 [59] two-element configuration was chosen. The flap was rotated from its initial position of approximately 14° to a landing configuration of 30° with a gap of 1.7% c and an overlap of 0% c. In the stowed position, the flap is positioned to allow for a consistent transition between the main element and flap upper and lower surfaces. When the flap was added to the NLR 7301 wing, the main element was created by modifying the NLR 7301 wing with the goal being to prevent flow separation. Therefore, the shroud locations and lower surface shroud shape were chosen to achieve fully attached flow [59]. Typically, 15% flap chord is exposed in the cruise position. However, with the NLR 7301 main element and flap configuration, only 6% flap chord is exposed, so this in itself makes the concatenation between the main element and flap upper surfaces difficult. Therefore, a stow angle of 4° was chosen to achieve concatenation between the lower surface main element and flap. As the stowability constraint focusses on the flap surfaces ahead of the shroud points, this slight discrepancy in high speed lines is not significant. Note that all the final shapes generated will have this discrepancy. Figure 4.53 shows the NLR 7301 main element and flap in the chosen stowed position.

In flap design, the designer only modifies the flap surface between the upper and lower shroud points, as discussed in Section 3.6.2, so this will be termed the design space. Aft of these points, the geometry is fixed by the high speed lines. To illustrate the stowability constraint, the NLR 7301 flap was

Figure 4.53: NLR 7301 two-element configuration in the stowed position.

modified with these considerations in mind. The original NLR 7301 lower flap surface, when rotated by 4° in the stowed position, is fairly consistent with the main element high speed lines. Therefore, this portion was not modified. The upper surface from the leading edge to the blend area was modified using a polynomial spline routine in CATIA. The blend area begins at the shroud location and, for this example, ends aft at 2.5% wing chord on both the upper and lower surfaces. The original and modified NLR 7301 flap geometries are shown in Figure 4.54 and the stowability design space is shown in Figure 4.55. As the example presented here is a landing configuration, the optimization goal is to maximize the lift. The initial lift coefficient is 2.63 and the goal is to increase it by 10% to 2.9. The stowable area for this configurations is based on a flap deflection angle of $\theta_{df} = 30^\circ$ and a stowed angle between the flap and wing chord of $\theta_{wf} = 4^\circ$. A manufacturing tolerance of 0.1% wing

Figure 4.54: NLR 7301 two-element configuration with modified flap geometry.

Figure 4.55: The flap design space for the modified NLR 7301 configuration.

chord is used to prevent the surfaces from fouling. Figure 4.56 illustrates the stowable area around the modified NLR 7301 flap.

The first design example has no constraints imposed on the flap. This example converged to a lift coefficient of 2.9 in 14 iterations. The lift coefficient and scaled gradient convergence histories are shown in Figures 4.57 and 4.58, respectively.

The initial and final geometries are shown in Figure 4.59. The final geometry characteristics can be explained using the flow properties. Consider the initial and final geometry pressure distributions shown in Figure 4.60. As the shape evolves from the baseline configuration to the final design, the shape becomes thinner near the leading edge. This produces a larger pressure spike between 95% and 100% wing chord, as evident in Figure 4.60, which results in an increase in the local lift. Consequently, as the shape thins, the

Figure 4.56: Modified NLR 7301 baseline configuration with stowability constraint.

Figure 4.57: Lift Coefficient for the modified NLR 7301 example without constraints.

Figure 4.58: L_2 norm of the scaled gradient for the modified NLR 7301 example without constraints.

Figure 4.59: Initial and final geometries for the modified NLR 7301 example without constraints.

Figure 4.60: Initial and final pressure distributions for the modified NLR 7301 example without constraints.

flow separation that originally occurred on the flap at approximately 107% wing chord is moving further aft to an approximate location of 117%. At the flow separation location, the flow velocity decreases in magnitude resulting in an increase in local surface pressure. This produces an increase in the wing loading at 115% wing chord, as illustrated in Figure 4.60. These two positive influences in the lift coefficient and the imposed high speed lines on the last 5% wing chord results in the airfoil geometry narrowing at approximately 107% wing chord.

On the lower surface, the resulting droop over the first few percent flap chord is partly due to the location of the stagnation point. The flow velocity here is zero, which results in a higher pressure coefficient. As the flap is fixed from approximately 100% wing chord to the trailing edge, the lower surface geometry is restricted producing the final geometry.

The final geometry was compared to the stowable design area for the modified NLR configuration and is shown in Figure 4.61. As illustrated, it exceeds the stowability constraint on the upper surface between 115% and 119% wing chord and just slightly on the lower surface between 94% and 95% wing chord. This implies that at the current operating conditions, it is very probable that the main element and flap surfaces would foul during flap extension and retraction.

In order to improve the final geometry, the next design example imposes a stowability and thickness constraint on the optimization problem. The stowability constraint will help to ensure that the flap will not collide with the main element upper and lower surface while being retracted or extended. A thickness constraint of 2.5% chord will help to prevent the final geometry

Figure 4.61: Final geometry and stowable design area for the modified NLR 7301 example without constraints.

from becoming too narrow. For this example, the optimization converged to a final lift coefficient of 2.9 in 28 iterations as shown in Figure 4.62.

The scaled gradient results are shown in Figure 4.63. The oscillations that occur between iterations 12 and 23 are due to the local thickness and stowability constraint activation. This is illustrated in Figure 4.64 where each term in the objective function is plotted.

The initial and final geometries are shown in Figure 4.65. The final flap is significantly thicker with the two constraints but still has a slight narrowing between 102% and 112% wing chord.

The pressure distributions for the initial and final geometries are illustrated in Figure 4.66. Similarly to the first design example, the narrowing is a result of the pressure peak near the leading edge and the wing loading near the flow separation location.

Figure 4.62: Lift Coefficient for the modified NLR 7301 configuration with a stowability and thickness constraint.

Figure 4.63: L_2 norm of scaled gradient for modified NLR 7301 configuration with a stowability and thickness constraint.

Figure 4.64: Objective function terms for modified NLR 7301 configuration with a stowability and thickness constraint.

Figure 4.65: The initial and final geometries for the modified NLR 7301 configuration with a stowability and thickness constraint.

Figure 4.66: Initial and final pressure distribution for modified NLR 7301 configuration with a stowability and thickness constraint.

The final geometry is plotted with the stowability constraint in Figure 4.67. This geometry exceeds the stowability area slightly, as was indicated in the objective term plot, Figure 4.64. However, the location at which this occurs is within the blend area located between 82% and 90% flap chord. As this area is located aft of the upper shroud point, it is not extended from or retracted into the main element. Therefore, this geometry can be safely stowed based on the current conditions. To illustrate the differences in the final geometries and how they affect the stowability constraint, the final flap shapes were plotted together with the stowability constraint in Figure 4.68. Figure 4.69 focuses on the upper surface between 65% and 90% flap chord to highlight the stowability constraint violation for each geometry. The example with no constraints exceeds the constraint between 70% and 90% chord. As the upper shroud location is located at 82% flap chord, this final

Figure 4.67: Final geometry and stowability design area for modified NLR 7301 configuration with a stowability and thickness constraint.

Figure 4.68: Final geometries and stowability design area for modified NLR 7301 configuration with no constraints and with a stowability and thickness constraint.

Figure 4.69: Final geometries and stowability design area between 65% and 90% flap chord for modified NLR 7301 configuration with no constraints and with a stowability and thickness constraint.

shape has the potential to foul with the main element upper surface during flap extension and retraction. The second example containing the stowability constraint exceeds aft of the upper shroud location in the blend area. As this area remains outside of the main element in the stowed position, this slight constraint violation is not as critical as the first example violation.

Now consider the lower surface leading edge to about 20% flap chord as shown in Figure 4.70. Again, the first example exceeds the constraint between 1% and 2% flap chord which has the potential to cause fouling between the main element and flap surfaces. By including a stowability constraint, this surface section is now within the stowable area.

The changes in the geometry between the two examples results in different pressure distributions. These distributions are shown in Figure 4.71. The

Figure 4.70: Final geometries and stowability design area between 0% and 20% flap chord for modified NLR 7301 configuration without constraints and with a stowability and thickness constraint.

Figure 4.71: Final pressure distributions for modified NLR 7301 configuration without constraints and with a stowability and thickness constraint.

difference in the lift coefficient between the example with no constraints and the one with a thickness and stowability constraint is 0.03%. The first example geometry produces a larger pressure peak near the flap leading edge and, combined with the additional wing loading at approximately 115% wing chord, results in a slightly greater lift coefficient. The flow separation occurs around 117% wing chord for both examples.

Since the example with two constraints still produced a narrowing through the flap wing box, a third test example was performed. This example includes the previous two constraints, a thickness constraint at 40% chord and a stowability constraint, as well as a curvature constraint. To decide on the curvature function, the baseline curvature and final geometry curvatures for the first and second example were compared. The upper surface curvature comparison is shown in Figure 4.72 and the lower surface is shown in Figure 4.73.

As illustrated, the baseline upper surface curvature maintains the same curvature sign chord-wise along the flap, whereas both examples change curvature sign four times along the flap chord. For the lower surface curvature, all three curves change curvature sign at the leading edge but the two example cases produce a larger negative curvature followed by a curvature sign change at around 15% flap chord. The large negative curvature produces a drooped leading edge. The curvature constraint goal would be to restrict the upper surface from changing curvature sign and on the lower surface to reduce the negative curvature magnitude at the leading edge and prevent the curvature sign change at 15% chord. Due to this, the curvature function *two* constraint, as outlined in Section 3.5.2, was chosen since it is the more

Figure 4.72: Upper surface curvature for modified NLR 7301 baseline, without constraints and with a stowability and thickness constraint.

Figure 4.73: Lower surface curvature for modified NLR 7301 baseline, without constraints and with a stowability and thickness constraint.

restrictive of the two functions.

The design specified terms for the curvature function as defined in Equations 3.58 to 3.61 are given as

$$\begin{array}{llll}
 b_{uu} = 0.9 & c_{uu} = -0.05 & d_{uu} = -0.25 & e_{uu} = 0 \\
 b_{ul} = 0.9 & c_{ul} = -10 & d_{ul} = -17 & e_{ul} = 0 \\
 b_{lu} = 0.9 & c_{lu} = 1.5 & d_{lu} = 3.2 & e_{lu} = -0.015 \\
 b_{ll} = 0.9 & c_{ll} = -1.0 & d_{ll} = -5.0 & e_{ll} = 0
 \end{array} \tag{4.2}$$

The curvature function limits are plotted with the baseline curvature and the two example curvatures in Figure 4.74 and Figure 4.75 for the upper and lower surfaces, respectively.

Both the upper and lower surface curvature limits capture the undesired

Figure 4.74: Curvature function limits and upper surface curvature for modified NLR 7301 baseline, without constraints and with a stowability and thickness constraint.

Figure 4.75: Curvature function limits and lower surface curvature for modified NLR 7301 baseline, without constraints and with a stowability and thickness constraint.

effects produced in the previous examples.

The third example with a thickness, stowability and curvature function constraint reached a lift coefficient of 2.89 in 100 iterations, as shown in Figure 4.76. The optimization program concluded as the maximum number of iterations (100) specified for the problem was reached. As the scaled gradient had already decreased by two orders of magnitude and the final shape was no longer changing significantly, these results were deemed sufficiently converged. The scaled gradient results are shown in Figure 4.77. As seen, the scaled gradient oscillates quite significantly throughout the optimization; however, it is moving in a downward, converging manner. The oscillations are a result of the constraints activation but influenced mostly by the upper surface curvature constraint as seen in Figure 4.78 where the influence

Figure 4.76: Lift coefficient for the modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

of each objective function term is shown. By the optimization conclusion, only the design objective and upper surface curvature function is active. The curvature function behavior and hence the scaled gradient behavior is not surprising. The geometry is being influenced to create the narrowing through the flap wing box and hence a change in curvature. As the curvature is quite restricted, small curvature changes will strongly influence the scaled gradient value. Similar results were seen for the single element curvature example in Section 4.2.4.

The initial and final geometries for this example are shown in Figure 4.79. The final flap no longer has the narrowing through the flap wing box and remains thicker along the chord line. The final geometry is not typical of the standard flap geometries; however, the Boeing Company did perform a gap and overlap optimization study on a similar shape [60] and they achieved

Figure 4.77: L_2 norm of scaled gradient for modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

Figure 4.78: Objective function terms for modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

Figure 4.79: Initial and final geometries for the modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

their desired results. As well, with the placement of the upper shroud point so far aft on the flap, more surface on the upper flap is permitted to change. Therefore, this final shape although not typical is still valid for the current operating conditions. Further analysis would need to be performed to verify if this geometry would be valid at different operation conditions and flap deflections but that is beyond the scope of this work.

The upper and lower final curvatures with the corresponding curvature function limits are shown in Figures 4.80 and 4.81, respectively. The upper surface curvature lies along the upper curvature limit, prevented from changing curvature sign. Between 107% and 115% wing chord, the curvature becomes larger which has occurred for all three examples and is a result of the flap upper surface needing to meet the fixed flap portion aft of 119% wing chord.

Figure 4.80: Curvature function limits and final upper surface curvature for the modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

Figure 4.81: Curvature function limits and final lower surface curvature for the modified NLR 7301 configuration with a thickness, stowability and curvature constraint.

The lower surface curvature fits well within the selected curvature function limits. However, it did exceed these limits primarily through iteration 30 to 60, as was shown in Figure 4.78.

The final upper surface curvatures for the three examples are shown in Figure 4.82 and for the lower surface in Figure 4.83. Each plot illustrates the differences in the final surface curvatures.

The final lift coefficient is slightly less than the previous two examples by about 0.4%. All three flap pressure distributions are compared in Figure 4.84. All distributions have similar attributes, with a pressure peak around the leading edge and flow separation occurring further aft around 117% wing chord. The final example does not have the extra wing loading at 115% wing chord that is evident in the first and second examples.

Figure 4.82: Upper surface curvature for modified NLR 7301 configuration without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraint.

Figure 4.83: Lower surface curvature for modified NLR 7301 configuration without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraint.

Figure 4.84: Flap pressure distributions for modified NLR 7301 configuration without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraint.

The final geometries are compared with the stowability constraint in Figure 4.85. The figure illustrates the lack of narrowing that occurs around 50% flap chord for the third example compared to the first two examples. Figure 4.86 highlights the area between 60% and 90% flap chord, where the final example case geometry does not violate the stowability constraint.

The same is done for the lower surface near the leading edge, as shown in Figure 4.87. Here the drooped leading edge that was generated in the first two examples is replaced with a smooth leading edge that is well within the stowable area.

Finally, the final flap geometries are plotted in the stowed position. The first example, with no constraints, is shown in Figure 4.88. The second example, with a thickness and stowability constraint, is shown in Figure 4.89.

Figure 4.85: Geometry and design area for modified NLR 7301 without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraint.

Figure 4.86: Geometry and design area at 60% to 90% for modified NLR 7301 without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraint.

Figure 4.87: Geometry and design area at 0% to 20% for modified NLR 7301 without constraints, a thickness and stowability constraint and a thickness, stowability and curvature constraints.

Figure 4.88: Modified NLR 7301 configuration example with no constraints in stowed position.

The third example, with a thickness, stowability and curvature function *two* constraint, is shown in Figure 4.90. As expected, the lower surface main element concatenates well with the flap lower surface but due to the nature of the NLR 7301 configuration, the main element and flap upper surfaces do not concatenate as well. Another possible concern that the designer may have is the spacing between the main element and flap upper surfaces between the upper shroud point and 90% wing chord. This could be improved by adding a maximum thickness constraint into the optimization problem to influence the bump at 90% chord to decrease in magnitude. Also, remember that this configuration is not typical of an actual multi-element configuration, in that the upper shroud point is usually located around 85% wing chord.

In concluding this section, the stowability constraint was demonstrated for a modified NLR 7301 configuration. The stowability constraint influences

Figure 4.89: Modified NLR 7301 configuration example with a thickness and stowability constraint in stowed position.

Figure 4.90: Modified NLR 7301 configuration example with a thickness, stowability and curvature constraint in stowed position.

the flap geometry from the lower surface blend point to the upper surface blend point. The remaining flap geometry is restricted by the main element high-speed lines. Three design examples were considered with each example expanding on the previous results. The first example had no constraints placed on the optimization program in order to determine how the optimization would progress. This example produced a thin flap that exceed the stowability area. The second example included a thickness and stowability constraint to help resolve these issues. This example helped produce a geometry that could now be safely stowed but there still existed a narrowing at approximately mid-chord. Instead of including more thickness constraints, the curvature function *two* constraint was included with the thickness and stowability constraint to help generate a smoother airfoil. The final geometry had both a smooth upper and lower flap surface with the stowability requirements being met.

CHAPTER 5

5.1 Conclusions

This research demonstrates that multi-disciplinary constraints can be implemented within a two-dimensional aerodynamic optimization method to increase its flexibility and versatility. These added constraints provide an aerodynamic designer the latitude to expand the design envelope to consider non-aerodynamic factors that are vital to overall airfoil design. This will in turn reduce the preliminary design cycle time as it will not need to pass from one designer to another in order to address structural and manufacturing constraints. An aerodynamic optimization method now contains the tools to address design issues from several disciplines, making the design process more efficient.

The original method utilizes a local minimum thickness constraint to prevent surface crossovers. While this method penalizes the optimization for surface crossovers that occur at specified locations, it does nothing to prevent them at other locations. For this method to be effective, the surface crossover locations need to be known a priori. Therefore, a global minimum thickness constraint was incorporated to monitor and control the optimizing

airfoil from becoming too thin and to prevent surface crossovers. This constraint considers every splined point along the airfoil surface and is always active. As a result, surface crossover locations, if they exist, are always penalized. It is important to note that the calculations necessary for the global thickness constraint are based on the initial airfoil geometry. This enhances the method's flexibility by utilizing generic input data to optimize existing data.

To complement the local minimum thickness constraint, a maximum thickness constraint was included. This constraint prevents the airfoil surface from becoming too thick at a specified location. This is beneficial during flap and/or slat optimization to prevent surface fouling.

Commercial transportation aircraft utilize the unused wing volume to carry fuel. For two-dimensional airfoil design this translates to a cross-sectional area. The area was implemented as both a design objective and as a constraint. Selecting the target area as a design objective forced the airfoil area to remain active within the objective function until the target area had been achieved. This allows the designer to specify and achieve a required area needed to store the structural and system components and desired fuel. However, if the area was selected as a constraint, it only became active within the objective function when the computed airfoil area fell below the target area. This feature allows the final area to surpass the target area during the optimization process without a penalty being imposed thus focusing on the aerodynamic performance while meeting the structural requirements.

Two curvature function constraints have also been incorporated in the enhanced algorithm. The addition of these constraints directly addresses an

airfoil's manufacturability. For the curvature function *one* constraint, the airfoil is divided into three typical design sections, leading edge from 0% to 15% chord, wing box from 15% to 85% chord and trailing edge from 85% to 100% chord. The leading edge curvature was modeled as a cubic S-curve generated from the curvature extracted from the baseline geometry at 1%, 7.5% and 15% chord. The wing box curvature is modeled as a quadratic curve generated from three extracted curvature values from the baseline geometry, namely, at 15% chord, the lowest curvature peak between 20% and 80% chord and the curvature at 85% chord. Finally, the trailing edge curvature is modeled as an exponential curve generated from the extracted curvature value at 85% chord and the calculated S-curve starting point. For the upper and lower airfoil surfaces, the lower curvature limit is calculated using the curvature function and multiplied by the designer specified tolerance. The upper curvature limit is determined by multiplying the lower curvature limit by the negative specified tolerance. This produces symmetrical curvature limits that bound around zero allowing for a difference in curvature to be used to test if the constraint has been violated. This eliminated the curvature inaccuracies near the leading and trailing edges. If only a section of the airfoil surface is being designed, using a difference in curvature removes the influence the non-changing points have on the objective function. By including this curvature function as a constraint, the oscillations along an airfoil surface were reduced in magnitude while the overall airfoil performance was maintained. Reducing the oscillations produced a less complex geometry which in turn would allow for a cost-effective manufacturing process.

The curvature function *two* constraint is a single equation containing an

exponential and cosine function. The curvature function uses the exponential and cosine curve characteristics to control the curvature limits. The exponential characteristics dominate this curvature function within the first 15% chord and the cosine characteristics dominate from 15% to 100% chord. To determine the unknown variables within this curvature function equation, several different airfoil geometries were considered to determine trends in airfoil curvature. Based on this analysis, the equation variables are determined as follows: 1) the exponential curve variables are calculated from curvature values extracted from the baseline geometry, 2) the cosine curve period and phase shift are preset for each curvature limit and 3) the exponential tolerance value, cosine curve amplitude and overall horizontal and vertical transition variables are specified by the designer. For the upper and lower surfaces, each curvature limit is calculated independently from the other and unlike the curvature function *one* constraint, the limits do not necessarily bound around zero. Due to this feature, the designer is required to pay special attention when inputting the function values as the current airfoil curvature is used to determine if the constraint has been violated. If an airfoil surface section is fixed during the optimization and this section's curvature exceeds the limits imposed, a constraint penalty will remain active within the objective function throughout the optimization process, resulting in convergence problems. This function requires prior knowledge concerning the initial airfoil curvature before initiating the optimization process. This curvature function produced a different airfoil geometry compared to the curvature function *one* constraint. However, since this constraint was very restricting, the airfoil performance at the current operating conditions decreased slightly. With

that being said, the final geometry was much smoother, especially near the leading edge which would result in enhanced manufacturability and improved performance over a range of operating conditions. If the designer is willing to forego some aerodynamic performance, this constraint provides him/her with a different airfoil geometry to meet the non-aerodynamic constraints.

The final constraint included was the stowability constraint. This constraint combined flap trajectory, including the flap mechanics, together with the final airfoil shape to ensure high-lift stowability while maintaining an optimized aerodynamic performance. The flap movement was modeled as a four-bar linkage using kinematic synthesis to determine the pivot point locations, based on the linkage coupler initial and final positions. The coupler is composed of three vertices, the upper shroud point, the lower shroud point and the leading edge point in the stowed position. Kinematic analysis was used to determine the path each coupler point would follow from the flap extended to stowed position. This path is used to identify the design space for the stowability constraint. A modified NLR 7301 flap was used to demonstrate the stowability constraint. The flap surface optimization was only allowed to occur between the lower and upper shroud intersection points. Aft of these points, the flap airfoil surface was governed by the main element high-speed lines. The example illustrated that without a stowability constraint employed for flap optimization, there was a strong possibility the main element and flap could foul during retraction. When the stowability constraint was included within the optimization, this issue was addressed and prevented. With the addition of the stowability constraint, the designer can now efficiently design an optimized flap and be assured the final geometry

will not foul with the main element during retraction.

5.2 Recommendations for Future Work

Research to improve and continually enhance aerodynamic optimization methods is a never-ending process. To further enhance the above multi-disciplinary constraints, a number of improvements could be implemented. During the stowability constraint development, test cases were performed where only the upper surface, from the leading edge to the upper shroud point, was allowed to change. During the optimization, the airfoil design points are adjusted in the direction normal to the wing chord, so, in a landing configuration where the flap chord is rotated a large degree with respect to the wing chord, tangency discontinuities occurred at the leading edge. As well, a local thickness constraint was placed at a specific local chord location to maintain a specified thickness. During the program initialization, the upper and lower surface points closest to the desired location relative to the local chord are determined and fixed to calculate the airfoil thickness. At large flap deflection angles, this produces inaccurate thickness results, since the fixed thickness point on the upper and lower surface are moving normal to the wing chord not the flap chord. To address these issues, the airfoil surface points would need to be translated in a direction normal to the local chord instead of the wing chord. This improvement would require some fundamental changes to the surface, control and design point movement and to the algebraic grid movement scheme. Therefore, more testing at large flap deflection angles would also be recommended to verify these trends.

As the stowability constraint is important in flap design, further devel-

opment is necessary. Within the kinematic synthesis calculations, used to determine the flap linkage pivot points, only two positions were considered, the initial and final. This means the linkage path generated will definitely capture these two positions but not necessarily any others. To further enhance the stowability constraint, more positions within the kinematic synthesis could be used. For instance, a landing, takeoff and cruise position could be incorporated. This constraint should also be applied to the slat positions for three-element configurations.

This research concentrated on shape optimization since the additional multi-element constraints affect the airfoil geometry. However, some effort was placed on enhancing the interior grid movement scheme by including a smoothing option. This was to be used in gap and overlap studies where the grid cells can become excessively skewed. Unfortunately, due to time constraints, these studies were not performed during this research but should be investigated at some future time.

Finally, multi-point optimization was only briefly touched on for comparison purposes. However, further work should be done with multi-point optimization as airfoil's geometries need to perform at various operating conditions. And additional constraints that the designer may require could come out of this work.

APPENDIX A

A.1 Coupler lengths and angles

Figure A.1 shows the stowed position of the coupler. The coupler lengths, z ,

Figure A.1: Lengths and angles of stowed coupler.

s , and v are calculated as

$$\begin{aligned} z &= \sqrt{(P_{2x} - P_{ax})^2 + (P_{2y} - P_{ay})^2} \\ s &= \sqrt{(P_{bx} - P_{2x})^2 + (P_{by} - P_{2y})^2} \\ v &= \sqrt{(P_{ax} - P_{bx})^2 + (P_{ay} - P_{by})^2} \end{aligned} \quad (\text{A.1})$$

To calculate the angles, begin by using the cosine law

$$\begin{aligned} s^2 &= v^2 + z^2 - 2vz \cos a \\ v^2 &= z^2 + s^2 - 2zs \cos b \end{aligned} \quad (\text{A.2})$$

rearrange to get

$$\begin{aligned} a &= \cos^{-1} \left(\frac{v^2 + z^2 - s^2}{2vz} \right) \\ b &= \cos^{-1} \left(\frac{z^2 + s^2 - v^2}{2zs} \right) \end{aligned} \quad (\text{A.3})$$

and then

$$\begin{aligned} c &= 180 - (a + b) \\ \theta_c &= \sin^{-1} \left(\frac{P_{by} - P_{2y}}{s} \right) \end{aligned} \quad (\text{A.4})$$

Figure A.2 shows the coupler in the extended position. The upper and lower shroud points, $\vec{P}_d$ and $\vec{P}_c$, respectively are calculated as

$$\begin{aligned} P_{cx} &= P_{1x} + z \cos(b + \theta_d - \theta_c) & P_{cy} &= P_{1y} - z \sin(b + \theta_d - \theta_c) \\ P_{dx} &= P_{1x} + s \cos(\theta_d - \theta_c) & P_{dy} &= P_{1y} - s \sin(\theta_d - \theta_c) \end{aligned} \quad (\text{A.5})$$

The angles are calculated as

$$\begin{aligned} e &= 180 - (90 + (\theta_d - \theta_c)) \\ \psi &= 90 + e \end{aligned} \quad (\text{A.6})$$

Figure A.2: Lengths and angles of extended coupler.

and

$$\begin{aligned} f &= 180 - (\psi + c) \\ \phi &= a - f \end{aligned} \tag{A.7}$$

Once the angles at the two positions have been calculated, the two positions are related.

$$P_{21} = \sqrt{(P_{2x} - P_{1x})^2 + (P_{2y} - P_{1y})^2} \tag{A.8}$$

and the angle, δ_{21}

$$\begin{aligned} h &= \sin^{-1} \left(\frac{P_{2y} - P_{1y}}{P_{21}} \right) \\ \delta_{21} &= 180 - h \end{aligned} \tag{A.9}$$

The angle, α_{21} is calculated as

$$g = \sin^{-1} \left(\frac{P_{2y} - P_{ay}}{z} \right)$$

$$\alpha_{21} = 180 - (\phi + g) \quad (\text{A.10})$$

A.2 Right-hand side pivot points

To begin solving the right pivot point, U focus on the right side of each linkage and write a vector loop equation including the leading edge starting and ending position points. This is demonstrated in Figure A.3 and is written as:

$$\vec{U}_2 + \vec{S}_2 - \vec{P}_{21} - \vec{S}_1 - \vec{U}_1 = 0 \quad (\text{A.11})$$

Figure A.3: Right side of linkage system.

The vector equation can then be rewritten in terms of its complex number equivalents.

$$ue^{j(\sigma+\gamma_{21})} + se^{j(\psi+\alpha_{21})} - P_{21}e^{j\delta_{21}} - se^{j\psi} - ue^{j\sigma} = 0 \quad (\text{A.12})$$

Multiplying through and simplifying

$$\begin{aligned} ue^{j\sigma}e^{j\gamma_{21}} + se^{j\psi}e^{j\alpha_{21}} - P_{21}e^{j\delta_{21}} - se^{j\psi} - ue^{j\sigma} &= 0 \\ ue^{j\sigma}(e^{j\gamma_{21}} - 1) + se^{j\psi}(e^{j\alpha_{21}} - 1) - P_{21}e^{j\delta_{21}} &= 0 \end{aligned} \quad (\text{A.13})$$

Then using the Euler identity: $e^{\pm j\theta} = \cos \theta \pm j \sin \theta$, Equation A.13 becomes:

$$\begin{aligned} u(\cos \sigma + j \sin \sigma)(\cos \gamma_{21} + j \sin \gamma_{21} - 1) \\ + s(\cos \psi + j \sin \psi)(\cos \alpha_{21} + j \sin \alpha_{21} - 1) \\ - P_{21}(\cos \delta_{21} + j \sin \delta_{21}) = 0 \end{aligned} \quad (\text{A.14})$$

Expand

$$\begin{aligned} u(\cos \sigma \cos \gamma_{21} + j \cos \sigma \sin \gamma_{21} + j \sin \sigma \cos \gamma_{21} \\ - \sin \sigma \sin \gamma_{21} - \cos \sigma - j \sin \sigma) \\ + s(\cos \psi \cos \alpha_{21} + j \cos \psi \sin \alpha_{21} + j \sin \psi \cos \alpha_{21} \\ - \sin \psi \sin \alpha_{21} - \cos \psi - j \sin \psi) \\ - P_{21} \cos \delta_{21} + j P_{21} \sin \delta_{21} = 0 \end{aligned} \quad (\text{A.15})$$

The above equation is then rearranged and separated into its real and imaginary parts to obtain the following two equations.

$$\begin{aligned} [u \cos \sigma](\cos \gamma_{21} - 1) - [u \sin \sigma] \sin \gamma_{21} \\ + [s \cos \psi](\cos \alpha_{21} - 1) - [s \sin \psi] \sin \alpha_{21} \\ = P_{21} \cos \delta_{21} \end{aligned} \quad (\text{A.16})$$

$$\begin{aligned}
& [u \sin \sigma](\cos \gamma_{21} - 1) - [u \cos \sigma] \sin \gamma_{21} \\
& + [s \sin \psi](\cos \alpha_{21} - 1) - [s \cos \psi] \sin \alpha_{21} \\
& = P_{21} \sin \delta_{21} \quad (\text{A.17})
\end{aligned}$$

In the above two equations, there are eight variables; $u, \sigma, \gamma_{21}, s, \psi, \alpha_{21}, P_{21}$ and δ_{21} but only u and σ are unknown. So to separate these terms in Equations A.16 and A.17, let:

$$\begin{aligned}
U_{1x} &= u \cos \sigma & S_{1x} &= s \cos \psi \\
U_{1y} &= u \sin \sigma & S_{1y} &= s \sin \psi
\end{aligned} \quad (\text{A.18})$$

and let,

$$G = \cos \gamma_{21} - 1 \quad H = \sin \gamma_{21}$$

Using the same variables from the left side linkage

$$\begin{aligned}
C &= \cos \alpha_2 - 1 & D &= \sin \alpha_2 \\
E &= P_{21} \cos \gamma_2 & F &= P_{21} \sin \gamma_2
\end{aligned} \quad (\text{A.19})$$

Then equations A.16 and A.17 become,

$$U_{1x}G - U_{1y}H + S_{1x}C - S_{1y}D = E \quad (\text{A.20})$$

$$U_{1y}G + U_{1x}H + S_{1y}C + S_{1x}D = F \quad (\text{A.21})$$

Equations A.20 and A.21 can be solved to find U_{1x} and U_{1y} ,

$$U_{1x} = \frac{G(E - S_{1x}C + S_{1y}D) + H(F - S_{1y}C - S_{1x}D)}{-2G} \quad (\text{A.22})$$

$$U_{1y} = \frac{G(F - S_{1y}C - S_{1x}D) + H(-E + S_{1x}C - S_{1y}D)}{-2G} \quad (\text{A.23})$$

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