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PSM: A New Approach to Probabilistic Scheduling

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PSM: A NEW APPROACH TO PROBABILISTIC SCHEDULING

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by

Benyamin Tedjakusuma

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Bachelor of Engineering, Ryerson University, 2004

A thesis

presented to Ryerson University

in partial fulfillment of the
requirements for the degree of
Master of Applied Science

in the Program of
Civil Engineering

Toronto, Ontario, Canada, 2007

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By Benjamin J. Fortakue, B.A., M.Sc., Ryerson University

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A new method of scheduling, the Probabilistic Scheduling Method (PSM), can be assigned to activities and used in the project scheduling method (PSM). The PSM provides confidence intervals for the activities and the project duration.

Tests were carried out using activities 1, 2, 3, 4, 5, 6, 7, 8, and 9 activities to evaluate PSM's performance. The results of PSM to Critical Path Method (CPM), Program Evaluation and Review Technique (PERT), and Monte Carlo simulation to PSM and PSM. The results indicate that PSM results in a more realistic project duration. Further tests were implemented to evaluate PSM's performance in scheduling activities in an ongoing project. A Microsoft Excel spreadsheet was used to organize test data and calculations.

PSM can be used in industry practice. They allow for a range of activities and activities with a range of probabilities. PSM provides a more realistic project duration and a range of probabilities. PSM provides a more realistic project duration and a range of probabilities. PSM provides a more realistic project duration and a range of probabilities.

ABSTRACT

PSM: A New Approach to Probabilistic Scheduling, M.A.Sc., 2007
by **Benyamin Tedjakusuma, Civil Engineering, Ryerson University**

A new scheduling method, where probability values can be assigned to activity durations, is proposed in this thesis. Probabilistic Scheduling Method (PSM) accepts activity durations tagged with probability or confidence intervals.

Tests were carried out using examples of 3, 7, and 9 activities to evaluate PSM's practical capability. The comparisons of PSM to Critical Path Method (CPM), Performance Evaluation and Review Technique (PERT), and Monte Carlo application to PERT (MC PERT) conclude that PSM results in similar most probable duration estimation. Further tests were implemented to evaluate PSM's capability to project schedule revision on an ongoing project. A Microsoft Excel application was used to organize tests data and calculations.

PSM computations are more industry friendly. They allow for a range of duration associated with a range of probabilities. PSM provides flexibility and simplicity, and also dependency information that will benefit its user in decision making process.

ACKNOWLEDGEMENTS

The author would like to thank God for His blessings throughout his study and the development of this research. The author would also like to thank his family for their continuous prayers and encouragements for him.

The author would like to express great and sincere gratitude to his supervisor, Dr. Paul S.H. Poh, Professor, Department of Architectural Science, for his patience, inspirations, informative guidance, and encouragements given during his studies, and especially during the development of this research.

The author wishes to express his gratitude to Dr. Mui Lam, Queen's University, for his great knowledge in probability, which plays a major part in the development of this research.

Special thanks are due to Jimmy Susetyo and Cindy Kumalasari, for their help during the preparation of this document. Thanks are also due to Janet Manoy and Vivi Noviati, for their help in the theoretical study of probability and statistics.

The author also acknowledges the financial support provided by the Ryerson University in the form of a Graduate Scholarship and Assistantship.

TABLE OF CONTENTS

ABSTRACT	ii
AUTHOR'S BIOGRAPHY	14
ACKNOWLEDGMENT	xv
LIST OF TABLES	xvii
LIST OF FIGURES	xi
LIST OF APPENDICES	xiii
SYMBOLS AND ABBREVIATIONS	xviii

CHAPTER I

INTRODUCTION

1.1 HISTORY OF CONSTRUCTION PROJECT SCHEDULING	1
1.2 NEED FOR DEVELOPMENT	2
1.3 OBJECTIVES OF PRESENT RESEARCH	3
1.4 ORGANIZATION OF THE THESIS	5

CHAPTER II

LITERATURE REVIEW

2.1 CONSTRUCTION PROJECT PLANNING	5
2.1.1 SCHEDULING AND CONTROL	5
2.2 CONSTRUCTION PROJECT SCHEDULING	6
2.2.1 Gantt Chart	6
2.2.2 Critical Path Method	6
2.2.2.1 Precedence Diagram Method	6

TABLE OF CONTENTS

ABSTRACT	vii
ACKNOWLEDGEMENTS	ix
DEDICATION	xi
LIST OF TABLES	xvii
LIST OF FIGURES	xix
LIST OF APPENDICES	xxi
NOMENCLATURE	xxiii
CHAPTER	1
1 INTRODUCTION	1
1.1 HISTORY OF CONSTRUCTION PROJECT SCHEDULING	1
1.2 NEED FOR DEVELOPMENT	2
1.3 OBJECTIVES OF PRESENT RESEARCH	2
1.4 ORGANIZATION OF THE THESIS	3
2 LITERATURE REVIEW	5
2.1 CONSTRUCTION PROJECT PLANNING, SCHEDULING, AND CONTROL	5
2.2 CONSTRUCTION PROJECT SCHEDULING	6
2.2.1 Gantt Chart	6
2.2.2 Critical Path Method	9
2.2.2.1 Precedence Diagram Method	14

2.2.3	Performance Evaluation and Review Technique	16
2.2.3.1	Monte Carlo Application to PERT	22
2.2.4	Graphical Evaluation and Review Technique	26
2.3	RECENT DEVELOPMENT IN SCHEDULING RESEARCH	28
3	PROBABILISTIC SCHEDULING METHOD	31
3.1	GENERAL	31
3.2	PSM PROCEDURES	31
3.3	PROBABILISTIC SCHEDULING METHOD INPUTS	32
3.3.1	Probabilistic Inputs Calculation	33
3.4	PSM SCHEDULING	35
3.4.1	Probabilistic Principles	36
3.4.2	PSM Scheduling Patterns	37
3.4.3	PSM Scheduling Calculation	38
3.4.4	PSM Scheduling Results Analysis	42
3.4.5	Forward Pass Maximum Rule	44
4	PSM TESTS AND RESULTS	47
4.1	GENERAL	47
4.2	3 ACTIVITIES TESTING	47
4.3	7 ACTIVITIES TESTING	51
4.4	9 ACTIVITIES TESTING	56
4.5	WORKSHEET APPLICATION	61

5	PSM APPLICATION TO PROJECT SCHEDULE REVISION	69
5.1	GENERAL	69
5.2	7 ACTIVITIES SCHEDULE REVISION	69
5.3	9 ACTIVITIES SCHEDULE REVISION	72
6	CONCLUSIONS	75
7	PSM RECOMMENDATIONS FOR FUTURE STUDIES	77
7.1	OVERALL PSM EVALUATION	77
7.2	RECOMMENDATIONS FOR FUTURE RESEARCH	77
	REFERENCES	79
	APPENDICES	83
	Appendix A: Project Schedule Breakdown	83
	Appendix B: Activity Duration Comparison	83
	Appendix C: Activity Duration Inputs	83
	Appendix D: Activity Duration Results	83
	Appendix E: Activity Duration Comparison	83
	Appendix F: Activity Duration Inputs	83
	Appendix G: Activity Duration Results	83
	Appendix H: Activity Duration Comparison	83
	Appendix I: Activity Duration Inputs	83
	Appendix J: Activity Duration Results	83
	Appendix K: Activity Duration Comparison	83
	Appendix L: Activity Duration Inputs	83
	Appendix M: Activity Duration Results	83
	Appendix N: Activity Duration Comparison	83
	Appendix O: Activity Duration Inputs	83
	Appendix P: Activity Duration Results	83
	Appendix Q: Activity Duration Comparison	83
	Appendix R: Activity Duration Inputs	83
	Appendix S: Activity Duration Results	83
	Appendix T: Activity Duration Comparison	83
	Appendix U: Activity Duration Inputs	83
	Appendix V: Activity Duration Results	83
	Appendix W: Activity Duration Comparison	83
	Appendix X: Activity Duration Inputs	83
	Appendix Y: Activity Duration Results	83
	Appendix Z: Activity Duration Comparison	83

LIST OF TABLES

Table 2.1	Advantages and Disadvantages of Gantt Chart	8
Table 2.2	Critical Path Method Inputs (Foundation Construction)	9
Table 2.3	Critical Path Method Scheduling Results	12
Table 2.4	Advantages and Disadvantages of Critical Path Method	14
Table 2.5	PERT Inputs (Foundation Construction)	16
Table 2.6	PERT Expected Average and Standard Deviation	18
Table 2.7	PERT Scheduling Results	20
Table 2.8	Advantages and Disadvantages of PERT	22
Table 2.9	Advantages and Disadvantages of Monte Carlo PERT	26
Table 2.10	GERT Nodal Symbols	28
Table 3.1	PSM Probabilistic Inputs	32
Table 3.2	Probabilistic Inputs Breakdown	34
Table 3.3	PSM Calculated Inputs	35
Table 3.4	PSM Scheduling Results	42
Table 3.5	Maximum Time Comparison	45
Table 4.1	3 Activities Inputs	48
Table 4.2	Activities Distributions (3 Activities)	49
Table 4.3	Project Duration Distribution (3 Activities)	49
Table 4.4	Comparison Inputs (3 Activities)	50
Table 4.5	7 Activities Inputs	52
Table 4.6	Activities Distributions (7 Activities)	53
Table 4.7	Project Duration Distribution (7 Activities)	54
Table 4.8	Comparison Inputs (7 Activities)	54

Table 4.9	9 Activities Inputs	57
Table 4.10	Activities Distributions (9 Activities)	58
Table 4.11	Project Duration Distribution (9 Activities)	59
Table 4.12	Comparison Inputs (9 Activities)	60
Table 5.1	Duration Distribution (7 Activities to Activity D)	70
Table B1	PERT Normal Distribution (3 Activities)	96
Table B2	Monte Carlo PERT Iterations	98
Table B3	PERT Normal Distribution (7 Activities)	110
Table B4	PERT Normal Distribution (9 Activities)	124
Table C1	PERT Normal Distribution (7 Activities Revision)	130

LIST OF FIGURES

Figure 2.1	Implementation of Labour, Material, and Equipment in Construction	5
Figure 2.2	Gantt Chart	7
Figure 2.3	Adamiecki's Harmonogram	8
Figure 2.4	Critical Path Method Network Diagram (Activity-on-Arrow)	10
Figure 2.5	Critical Path Method Activity-on-Arrow Scheduling	12
Figure 2.6	Precedence Diagram Method Network	15
Figure 2.7	PERT Network Diagram (Activity-on-Node)	19
Figure 2.8	PERT Probability Distribution Chart	20
Figure 2.9	PERT Cumulative Distribution Chart	21
Figure 2.10	Basic Principle of Monte Carlo Simulation	23
Figure 2.11	Monte Carlo PERT Probability Distribution Chart	24
Figure 2.12	Monte Carlo PERT Cumulative Distribution Chart	25
Figure 2.13	GERT Looping Feature	27
Figure 2.14	GERT Routing Feature	27
Figure 2.15	Genetic Algorithm Operation	29
Figure 3.1	PSM Scheduling Pattern 1	37
Figure 3.2	PSM Scheduling Pattern 2	38
Figure 3.3	PSM Distribution Chart	43
Figure 3.4	PSM Cumulative Distribution Chart	43
Figure 3.5	Maximum Time Comparison Chart	46
Figure 4.1	3 Activities Comparison Chart	50

Figure 4.2	3 Activities Cumulative Comparison Distribution Chart	51
Figure 4.3	7 Activities Comparison Chart	55
Figure 4.4	7 Activities Cumulative Comparison Distribution Chart	56
Figure 4.5	9 Activities Comparison Chart	60
Figure 4.6	9 Activities Cumulative Comparison Distribution Chart	61
Figure 5.1	PSM Actual vs. Revised Schedule (7 Activities to Activity D)	70
Figure 5.2	PSM Actual vs. Revised Cumulative Chart (7 Activities to D)	71
Figure 5.3	PERT Actual vs. Revised Schedule (7 Activities to Activity D)	71
Figure 5.4	PERT Actual vs. Revised Cumulative Chart (7 Activities to D)	72
Figure 5.5	PSM Actual vs. Revised Schedule (9 Activities)	73
Figure 5.6	PSM Actual vs. Revised Cumulative Chart (9 Activities)	73
Figure 5.7	PERT Actual vs. Revised Schedule (9 Activities)	74
Figure 5.8	PERT Actual vs. Revised Cumulative Chart (9 Activities)	74
Figure B1	3 Activities Network Diagram	95
Figure B2	7 Activities Network Diagram	109
Figure B3	9 Activities Network Diagram	123

LIST OF APPENDICES

Appendix A	PSM INPUTS CALCULATIONS	83
Appendix B	PROBABILISTIC SCHEDULING METHOD TESTS	87
	B1 3 Activities Testing	87
	B2 7 Activities Testing	99
	B3 9 Activities Testing	111
Appendix C	PROJECT SCHEDULE REVISION APPLICATION	125
	C1 7 Activities Project Schedule Revision	125
	C2 9 Activities Project Schedule Revision	131

NOMENCLATURE

ATD	Average total duration
a	Optimistic duration
b	Pessimistic duration
CDF	Cumulative density function
D	Activity duration time
D_j	Duration time for activity $i - j$
EF	Early finish time
EF_j	Early finish time for activity $i - j$
ES	Early start time
ES_j	Early start time for activity $i - j$
F	Float duration
FS_{ij}	Finish-to-start constraint
f_x	Microsoft Excel function
ID	Activity ID number
LF	Late finish time
LF_j	Late finish time for activity $i - j$
LS	Late start time
LS_j	Late start time for activity $i - j$
m_l	Most likely duration
$P_{a,t}$	Probabilistic value of time t activity a , %
$P_{a,t \max}$	Probability of activity a maximum time, %
$P_{a,t \min}$	Probability of activity a minimum time, %
$P_{p,t \max}$	Probability of preceding activity p maximum time, %

$P_{p, t \min}$	Probability of preceding activity p minimum time, %	NOMENCLATURE
$P_{Tr, t}$	Probability of terminal time t, %	
PDF	Probability density function	
SD	Standard deviation	Average total duration
TD	Total duration	Optimistic duration
TDI	Total duration of i^{th} number of iteration	Pessimistic duration
$t_{a, \max}$	Activity a maximum time	Cumulative density function
$t_{a, \min}$	Activity a minimum time	Activity duration time
$t_{a, r \max}$	Maximum time of range r activity a	Duration time for activity i - 1
$t_{a, r \min}$	Minimum time of range r activity a	Early finish time
$t_{a, r-1 \max}$	Maximum time of range r-1 activity a	Early finish time for activity i - 1
$t_{a, r-1 \min}$	Minimum time of range r-1 activity a	Early start time
$t_{p, \max}$	Preceding activity p maximum time	Early start time for activity i - 1
$t_{p, \min}$	Preceding activity p minimum time	Float duration
t_e	Expected duration	Finish-to-start constant
t_{Tr}	Terminal time t	Microsoft Excel function
$tR_{a, r}$	Time range r activity a	Activity ID number
$tR_{a, r-1}$	Time range r-1 activity a	Late finish time
μ	Duration mean or ATD	Late finish time for activity i - 1
μ_e	Expected average duration	Late start time
μ_T	Total average duration	Late start time for activity i - 1
$\%R_{a, r}$	Percentage range r activity a, %	Most likely duration
$\%R_{a, r-1}$	Percentage range r-1 activity a, %	Probabilistic value of time t activity a
σ	Duration standard deviation or SD	Probability of activity a maximum time
σ_e	Expected standard deviation	Probability of activity a minimum time
σ_T	Total standard deviation	Probability of preceding activity p maximum time

CHAPTER 1

INTRODUCTION

1.1 HISTORY OF CONSTRUCTION PROJECT SCHEDULING

Construction Project Scheduling is a discipline in construction management, which deals with the scheduling of construction projects. The history of construction project management goes back to at least 5,000 years ago (Patrick, 2004). However, there are no records regarding what methods were used at the time.

The first ever recorded scheduling method in construction management was around the mid-1800s (O'Brien et al., 2006). It was a method developed by Henry L. Gantt and Frederick W. Taylor, which is now commonly called as the Gantt chart. The Gantt chart uses a graphical chart that represents work activities against anticipated durations.

Contemporary project scheduling began in the mid-1950s with the development of the Critical Path Method (Patrick, 2004). Most contemporary project scheduling methods, nowadays, are suited for use on a computer, including the Critical Path Method. There are currently three well-established project scheduling methods in use: Gantt chart, Critical Path Method (CPM), and Performance Evaluation and Review Technique (PERT).

Two other established methods are Monte Carlo application to PERT and Graphical Evaluation and Review Technique (GERT). These, however, are rarely used for practical construction scheduling.

Most recent methods are Self-Adapting Genetic Algorithm application to scheduling, Fuzzy Logic application to scheduling, Distributed Scheduling Model, and Module-Based construction schedule.

1.2 NEED FOR DEVELOPMENT

The Critical Path Method was developed using a deterministic approach, in which a determined duration is used. The end result is a "fixed" estimate of total project duration. It is generally simpler in comparison to others using probabilistic approaches. This deterministic approach, however, does not lend itself well towards any possible variations in activity durations. Probabilistic approaches, on the other hand, enable probability distributions for activity durations. A notable example is PERT.

Although much research has been done to develop probabilistic-based scheduling methods, none has allowed confidence intervals in their computations. To the best of the author's knowledge, there is no published research on a probabilistic scheduling method that allow a probability interval to be assigned to activity duration estimates and which resulted in a probability interval tagged to the total project duration.

1.3 OBJECTIVES OF PRESENT RESEARCH

The main objective of this research is to develop a probabilistic-based scheduling method that allows for a range of probability values to each activity duration estimate, and results in total project durations, each tagged with a level of confidence or probability.

The method, which is developed and presented, borrows certain steps from the CPM. The method is also highly amendable for implementation in a spreadsheet program, such as Microsoft Excel.

1.4 ORGANIZATION OF THE THESIS

This thesis consists of seven chapters. In Chapter 1, the need for the study and the objectives of the research are presented. Chapter 2 visits the commonly used project scheduling methods and discusses their shortfalls. Chapter 3 presents a new project scheduling method called Probabilistic Scheduling Method (PSM). Chapter 4 presents the results of PSM in comparison to other probabilistic-based scheduling methods. Chapter 4 also includes PSM's worksheet applications. PSM's application to project schedule revision is presented in Chapter 5. In Chapter 6, recommendations for future research for PSM are suggested. Finally, the conclusions are in Chapter 7.

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CHAPTER 2

LITERATURE REVIEW

2.1 CONSTRUCTION PROJECT PLANNING, SCHEDULING, AND CONTROL

For a construction project of just about any size, there is a need to plan, to schedule, and eventually to control its pace when it is in progress. Project planning includes definition of project tasks or activities, selection of techniques, and identification of resource requirements and constraints (Patrick, 2004). Construction project scheduling involves the arranging of project tasks and resources in a manner to minimize time and cost. A pictorial description of construction project scheduling is given in Figure 2.1. Controlling and monitoring are the processes of comparing the scheduled and the actual events, and correcting the differences by coordinating the actions of all which are involved in the project (Patrick, 2004).

Figure 2.1. Implementation of Labour, Material, and Equipment in Construction

Each phase of the planning, scheduling, and controlling has its own methodology. Most scheduling methods deal with time management before considering dollar values into the estimated duration. There are also methods that are cost restricted, such as a "crash" method, in which the scheduled time is adjusted to the preferred cost. By and large, resources are scheduled, levelled, and allocated based on the time and cost (Patrick, 2004).

2.2 CONSTRUCTION PROJECT SCHEDULING

As stated briefly in the introduction, there are five established construction project scheduling techniques: Gantt chart, Critical Path Method (CPM), Performance Evaluation and Review Technique (PERT), Monte Carlo PERT (MC PERT), and Graphical Evaluation and Review Technique (GERT).

Each of these has advantages and disadvantages. Texts on Gantt chart and CPM are widely available. They are simple and are generally used on small projects that do not require elaborate planning and scheduling. PERT inputs, in which an expected estimate is calculated from the weighted average of 3 activity duration estimates, are sometimes applied as alternate inputs to the CPM. Published treatments on GERT are generally non-construction related.

2.2.1 Gantt Chart

The Gantt chart (Figure 2.2) is a graphical representation of work vs. time. It is categorized as a traditional scheduling method and is a modification of the Harmonogram method, as shown in Figure 2.3, which was developed by Karol

Adamiecki (Marsh, 1975).

Gantt charts are used frequently in many construction projects due to its simple nature. Today's Gantt chart has been improved and it can allow for precedence. It is essentially a platform, where project activities and their duration are shown graphically. The advantages and disadvantages of the Gantt are presented in Table 2.1.

The chart provides a chronological overview of the project, which is valuable for determining project milestones. Its simplicity means easier understanding for individuals that are involved in the project. It, however, does not include probabilistic nor network analysis, which are important considerations in today's construction industry.

If a large number of activities are involved, the Gantt chart can become more complicated for use. Although not impossible, it can be difficult to monitor project progresses and delays using a Gantt chart.

Figure 2.2. Gantt Chart

Figure 2.3. Adamiecki's Harmonogram

Source: Marsh (1975).

Table 2.1. Advantages and Disadvantages of Gantt Chart

ADVANTAGES	DISADVANTAGES
<ol style="list-style-type: none"> 1. Simple and good graphical presentation 2. Little prerequisite skills necessary 3. Good for denoting project milestones 	<ol style="list-style-type: none"> 1. Complexity increases as the number of activity is increased 2. No probabilistic analysis 3. No network analysis 4. Unreliable for time estimation 5. When delay happens, project progress calculation becomes complicated

2.2.2 Critical Path Method

The Critical Path Method (CPM) was developed in 1956 in a joint venture between DuPont Corporation and Remington Rand Corporation for time efficiency in schedule preparation (O'Brien et al., 2006). It was the first computation-based scheduling method that uses a Forward Pass calculation. In conjunction with a Backward Pass calculation, a critical path of project activities can be determined. It is not necessary to use the Backward Pass calculation if there is no desire to define the critical path. The simplicity that CPM offers has attracted many latter scheduling methods to emulate its principles, especially its Forward Pass calculation. The method that is proposed in this thesis is no different.

The procedures for using the CPM begin with defining activities and their estimated durations, drawing a network diagram, and performing a Forward Pass and a Backward Pass calculation. Critical and non-critical activities can then be identified. Activity inter-dependencies are important and need to be defined. An example is shown in Table 2.2.

Table 2.2. Critical Path Method Inputs (Foundation Construction)

Activity	Description	Predecessor	Duration (day)
A	Excavation	—	5
B	Formwork	A	9
C	Rebar fabrication	—	3
D	Rebar installation	B, C	6
E	Foundation Casting	D	4

Figure 2.4. Critical Path Method Network Diagram (Activity-on-Arrow)

Early CPM adopts an Activity-on-Arrow (AOA) network diagram as shown in Figure 2.4. The general formulas and procedures for the Forward Pass and Backward Pass calculations are as follows (Patrick, 2004):

Forward Pass Calculation

Step 1: Compute the early start time, ES_j , of the activity (j) in question.

$$ES_j = \max_{\text{all } i} \left[\begin{matrix} \text{initial time, 0} \\ EF_i + FS_{ij} \end{matrix} \right] \quad (2.1)$$

Step 2: Compute the early finish time, EF_j .

$$EF_j = ES_j + D_j \quad (2.2)$$

Backward Pass Calculation

Step 1: Compute the late finish time, LF_i , of the activity (i) in question.

$$LF_i = \min_{\text{all } j} \left[\begin{array}{c} \text{terminal time} \\ LS_j - FS_{ij} \end{array} \right] \quad (2.3)$$

Step 2: Compute the late start time, LS_i .

$$LS_i = LF_i - D_i \quad (2.4)$$

where:

D_j = duration time for activity $i - j$

EF_j = early finish time for activity $i - j$

ES_j = early start time for activity $i - j$

FS_{ij} = finish-to-start constraint

i = starting node

j = ending node

LF_j = late finish time for activity $i - j$

LS_j = late start time for activity $i - j$

In Activity-on-Arrow diagram, Activity A can be written as Activity 10 – 20, which refers to the assigned starting and ending nodes of Activity A.

Figure 2.5. Critical Path Method Activity-on-Arrow Scheduling

Table 2.3. Critical Path Method Scheduling Results

Activity	I	j	ES	EF	LF	LS	Float
A	10	20	0	5	5	0	0
B	20	30	5	14	14	5	0
C	10	40	0	3	14	11	11
D	40	50	14	20	20	14	0
E	50	60	20	24	24	20	0

The end results of a CPM are estimated project duration, identification of critical and non-critical activities, and activities slack or float duration for the latter. In the example illustrated in Figure 2.5 and Table 2.3, the total project duration is estimated as 24 days with the completion of Activity E. The critical activities are Activity A, B, D, and E (Float time = 0). In CPM, defining the critical activities is important because any delays from the critical activity will directly delay the total project duration. Activity slack duration is calculated for the non-critical activity, which is Activity C in this example. From Figure 2.5 or Table 2.3, the earliest date Activity C can start is Day 0 and the latest finish date is Day 14, and it only requires 3 days to complete. Therefore, the slack duration is calculated as 11 days, which suggests that Activity C can afford a delay of 11 days without affecting the project duration.

From the given example, there are several points that suggest the advantages and disadvantages of the CPM. Knowing which activities are critical and which are not are useful to the project manager. The calculation procedure is the simplest among all the computation-based scheduling methods. The CPM uses only one duration estimate, which is fairly restrictive. While CPM provides an estimate of project duration, it is otherwise an inflexible method. The advantages and disadvantages of CPM are summarized in Table 2.4.

Table 2.4. Advantages and Disadvantages of Critical Path Method

ADVANTAGES	DISADVANTAGES
<ol style="list-style-type: none">1. Shows critical path of the project2. Simple in calculation and network analysis3. Provide rough estimate of the project duration	<ol style="list-style-type: none">1. No flexibility in planning2. No probabilistic analysis3. When delays happen, recalculation is required4. Using only one estimate duration

2.2.2.1 Precedence Diagram Method

Precedence Diagram Method (PDM) was developed in 1960 by Professor John W. Fondahl (O'Brien et al., 2006). The Precedence Diagram Method is a part of the Critical Path Method. It changes the appearance of the Activity-on-Arrow network diagram by using boxes instead of circular node. It directly applies the Forward Pass and Backward Pass calculations to the network instead of using separate tables. An example of the node arrangements is as follows:

where:

D = activity duration time

EF = early finish time

ES = early start time

F = float duration

ID = activity ID number

LF = late finish time

LS = late start time

The PDM scheduling the example, using inputs from Table 2.2, is performed as shown in Figure 2.6. The Precedence Diagram Method has the same advantages and disadvantages as the CPM as shown in Table 2.4.

Figure 2.6. Precedence Diagram Method Network

2.2.3 Performance Evaluation and Review Technique

Performance Evaluation and Review Technique (PERT) was developed not long after the Critical Path Method for the United States Navy Special Projects Office as a part of the Polaris missile project (O'Brien et al., 2006). The PERT borrows the Forward Pass calculation from the CPM. It, however, allows for three estimates of duration inputs: Optimistic duration (O); Pessimistic duration (P); and, Most Likely duration (ML). Table 2.5 shows a PERT evaluation for the same foundation construction example.

Table 2.5. PERT Inputs (Foundation Construction)

Activity	Description	Predecessor	Duration (day)		
			O	ML	P
A	Excavation	–	4	5	7
B	Formwork	A	6	9	12
C	Rebar fabrication	–	2	3	4
D	Rebar installation	B, C	4	6	8
E	Foundation Casting	D	2	4	5

In PERT, the optimistic, pessimistic, and most likely durations are combined as an expected duration (Equation 2.5).

$$t_e = \frac{a + 4m\ell + b}{6} \quad (2.5)$$

where:

a = optimistic duration (O)

b = pessimistic duration (P)

$m\ell$ = most likely duration (ML)

t_e = expected duration

The expected duration (t_e) is the weighted average of the three estimated durations, which is also denoted as μ_e . PERT also requires standard deviation calculations from the estimated inputs (Equation 2.6).

$$\sigma_e = \frac{b - a}{6} \quad (2.6)$$

where:

σ_e = expected standard deviation

The calculated expected durations are used as the scheduling inputs as shown in Table 2.6. The PERT scheduling is performed in the same manner as the CPM scheduling.

Table 2.6. PERT Expected Average and Standard Deviation

Activity	Predecessor	μ_e	σ_e
A	–	5.1667	0.5000
B	A	9.0000	1.0000
C	–	3.0000	0.3333
D	B, C	6.0000	0.6667
E	D	3.8333	0.5000

The PERT favours an Activity-on-Arrow (AOA) network diagram as shown in Figure 2.7. The expected durations, μ_e , are inputs on the network diagram. The resulting PERT schedule is shown in Table 2.7. The critical activities in the example are Activity A, B, D, F, and G. The procedures up to defining the critical path are the same as the CPM procedures. In PERT, the total project duration is calculated by adding the average durations of all critical activities (Patrick, 2004); therefore, the total duration (μ_T) for this example is 24 days. Standard deviation of the project is calculated using the critical activities' standard deviation as follows:

$$\sigma_T = \sqrt{0.5^2 + 1^2 + 0.6667^2 + 0.5^2} = 1.3944$$

The probability and cumulative distribution charts (Figure 2.8 and 2.9) can now be drawn using the calculated mean and standard deviation.

The charts are drawn using a normal distribution equation and a Microsoft Excel function as follows:

$$PDF = \frac{1}{\sigma_T \sqrt{2\pi}} \exp \left(-\frac{(\text{Duration} - \mu_T)^2}{2\sigma_T^2} \right) \quad (2.7)$$

$$CDF = \text{NORMDIST}(\text{Duration}, \mu_T, \sigma_T) \quad (2.8)$$

where:

CDF = cumulative density function

PDF = probability density function

μ_T = total average duration

σ_T = total standard deviation

Figure 2.7. PERT Network Diagram (Activity-on-Arrow)

Table 2.7. PERT Scheduling Results

Activity	ES	EF	LF	LS	Float
A	0	5	5	0	0
B	5	14	14	5	0
C	0	3	14	11	11
D	14	20	20	14	0
E	20	24	24	20	0

EF = early finish time

LF = late finish time

ES = early start time

LS = late start time

Figure 2.8. PERT Probability Distribution Chart

Figure 2.9. PERT Cumulative Distribution Chart

With these charts, the project duration can be estimated according to the preferred probabilistic value within the distribution. The advantages and disadvantages of PERT are given in Table 2.8. Because of its probabilistic approach, it is the author's view that PERT may provides better practical applications than the CPM. The PERT procedures are quite similar to those of the CPM's with some additions of the weighted average and standard deviation calculations. This proves that most people, who are able to use the CPM, should be able to use the PERT as easily.

The use of Equations 2.5 and 2.6 implies the assumption of a Beta distribution for the activity durations, which may or may not be correct for each individual activity. The assumptions that suggest Equations 2.5 and 2.6 simplify the Beta distribution analyses do not account any reason to assume a normal distribution for the total project duration. There are little or scant evidence to prove or justify this.

Table 2.8. Advantages and Disadvantages of PERT

ADVANTAGES	DISADVANTAGES
<ol style="list-style-type: none">1. Considers probabilistic approach2. Simple in calculation and network analysis3. Provides good estimate of project durations4. The use of three duration estimates provide flexibility	<ol style="list-style-type: none">1. Too many assumptions2. The adoption of Beta Distribution complicates probabilistic analysis of activity3. While considering probabilistic approach, PERT does not explain probabilistic dependency relationships between activities

2.2.3.1 Monte Carlo Application to PERT

Monte Carlo simulation method was developed in 1946 by Stanislaw Ulam (Eckhardt, 1987). Its application to the PERT method is a recent development in construction project scheduling. Monte Carlo PERT (MC PERT) follows the same procedures as the PERT. Its duration inputs, however, are not as using Equations 2.5 and 2.6. The MC PERT principle provides for a simulated scheduling calculation using random numbers generation as shown in Figure 2.10. Using the same input data as in Table 2.5, the random numbers are generated using the following Excel function:

$$f_x = a + \text{RAND}() \times (b - a) \quad (2.9)$$

where:

f_x = Microsoft Excel function

Figure 2.10. Basic Principle of Monte Carlo Simulation

Source: <http://vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html>

The random numbers are generated for each activity within the project and copied down to certain number of iterations (refer to Table B2 in Appendix B1). The scheduling is performed using the Forward Pass on the generated activity durations. The resulting project durations will depend on the number of iterations performed in the simulation. A larger number of iteration should provide for a better estimation of project duration.

The MC PERT distribution chart may be drawn using an Excel frequency function (rate of occurrence) or using the normal distribution equations (Equations 2.7 and 2.8) once the average and standard deviation are calculated. Figures 2.11 and 2.12 show the probability distribution and cumulative distribution of the resulting MC PERT from Table 2.5 example (in 500 iterations). The red curve shown in Figures 2.11 and 2.12 is charted from the frequency function while the blue curve is drawn using Equations 2.7 and 2.8. The results taken from the Excel frequency function produce a non-uniform distribution, which has little resemblance to the distribution produced using the normal equations.

Figure 2.11. Monte Carlo PERT Probability Distribution Chart

Figure 2.12. Monte Carlo PERT Cumulative Distribution Chart

Due to the random number generation function, the generated number changes each time an Excel command is performed, i.e. the produced distribution chart will change accordingly. This inconsistency is one of the disadvantages of the Monte Carlo PERT. The use of simulation allows MC PERT not to assume a Beta distribution for its activity durations. The basic analysis of its distribution is similar to completing one project using the CPM calculation several number of times while recording each result as past record of the project. Although the simulation should reflect on the actual construction, there is no control over the random generated inputs; thus, no probabilistic relationship is given between project activities. The advantages and disadvantages of Monte Carlo PERT are tabulated in Table 2.9.

Table 2.9. Advantages and Disadvantages of Monte Carlo PERT

ADVANTAGES	DISADVANTAGES
<ol style="list-style-type: none">1. Considers probabilistic approach2. Provides good estimates of project durations3. The use of three duration estimates provide flexibility4. Simulation method provides less assumption for input	<ol style="list-style-type: none">1. Simulation algorithm and logic become more complex as the project gets bigger2. Special skills are necessary to use the Monte Carlo method3. While considering probabilistic approach, MC PERT does not explain probabilistic relationship between activities4. Inconsistencies may occur from the use of random numbers5. The results will be more accurate as the number of iteration increases

2.2.4 Graphical Evaluation and Review Technique

Graphical (or sometimes Generalized) Evaluation and Review Technique (GERT) was introduced in 1966 by Alan B. Pritsker (Pritsker, 1966). Since its development, the GERT has not been adopted in construction scheduling. The information given in GERT principles suggests its use more in production management than in construction management.

Construction practice may, however, use its looping ability as an application to the current scheduling methods, e.g. looping application to CPM (O'Brien et al., 2006). Aside from a looping ability, GERT also has a routing feature, which provides for alternatives to its estimation. Probabilistic value can be applied in the GERT network as shown in Figures 2.13 and 2.14. The nodal symbols in used for GERT are depicted in Table 2.10.

Figure 2.13. GERT Looping Feature

Figure 2.14. GERT Routing Feature

Table 2.10. GERT Nodal Symbols

NAME	SYMBOL	CHARACTERISTIC
AND		The node will be realised only if all the branches leading into the node are realised.
DETERMINISTIC		All branches emanating from the node are taken if the node is realised.
PROBABILISTIC		Exactly one branch emanating from the node is taken if the node is realised.

Source: Pritsker (1966).

2.3 RECENT DEVELOPMENT IN SCHEDULING RESEARCH

Aside from the MC PERT and GERT, there are two notable scheduling methods that are still in the development process: Self-Adapting Genetic Algorithm (GA) application to scheduling and Fuzzy Logic application to scheduling; and other scheduling models including Distributed Scheduling and Module-Based construction schedule.

Genetic Algorithm (GA) applies some principles from biological evolution to solve optimization problems (Hartmann, 2002).

The GA interprets its inputs as a population, which are generated using random number generation. These inputs are evaluated based on defined criteria, e.g. specified time and cost. There are also other biological processes, such as crossover and mutation, to assist in the evaluation. The end result of Genetic Algorithm is an optimized time-cost solution to the scheduling. Figure 2.15 illustrates the operation process of Genetic Algorithm.

Figure 2.15. Genetic Algorithm Operation

Source: Zheng et al. (2004).

A Fuzzy Logic is defined as a soft-computing technique used to effectively solve uncertainties due to imprecise inputs to generate linguistic or quantitative outputs (Siwani et al., 2004). It offers an alternative to the probabilistic approach. Its inputs are unique in comparison to other scheduling methods. Instead of stating that an Activity A can be done in 3 weeks, the Fuzzy Logic assign the Activity A can be completed in "about 1 month". Its concept is based on the dependency logic between project activities. Due to its imprecise inputs, Fuzzy Logic offers approximate outputs, i.e. ± 5 weeks (Bonnal et al., 2004).

Distributed Scheduling Model (DSM) offers an innovative model for scheduling inputs (Hegazy et al., 2004). While most of the scheduling methods inputs are based on estimation, the Distributed Scheduling Model proposes inputs that are based on the calculated moving time and moving distance. It is developed for larger construction companies, where resources must be distributed according to the project needs. DSM implements a Genetic Algorithm for its scheduling processes to get an optimal time for the project duration. Computing each activity based on moving distance and moving time is very time consuming. Its results, however, may produce better inputs than those of estimated ones.

Module-Based construction schedule offers generalized scheduling networks. While the construction industry may be a unique industry, its construction processes can be generalized for several similar projects, e.g. Home Depot constructions at two different places. In such cases, the number of activities and their precedence are identical between the projects. Therefore, it is likely that they will use a same network diagram. The Module-Based construction schedule provides several modules of network diagrams, which illustrate some identical projects (Zheng et al., 2004).

CHAPTER 3

PROBABILISTIC SCHEDULING METHOD

3.1 GENERAL

A new scheduling method, where probability values can be assigned to activity durations, is proposed in this thesis. Probabilistic Scheduling Method (PSM) offers simplicity and flexibility in comparison to the PERT and GERT.

3.2 PSM PROCEDURES

The PSM provides for a set of procedures as follows:

1. List all activities within project,
2. List precedence relationships for all activities,
3. Estimate activities duration ranges including their corresponding probabilistic values. These estimations may be from user's experience or prior data,
4. Calculate probabilities associated with each activity using Equations 3.1, 3.2, and 3.3, or assign them from past data,
5. Schedule the project using results from procedure 1 – 4 and Equations 3.6 – 3.10 while taking note on the maximum rule,
6. Once the total project duration range is calculated, including its probabilistic values, analyze each total project time to estimate the most probable duration range.

3.3 PROBABILISTIC SCHEDULING METHOD INPUTS

PSM accepts activity durations tagged with probability or confidence intervals. The probability intervals can be from user's experience or past data collection. The probabilistic inputs can be given in sets of probability duration range as shown in Table 3.1.

Table 3.1 denotes Activity A can be completed with near 100% confidence within 2 – 8 days, 75% confidence within 3 – 6 days, and 50% confidence within 4 – 5 days. The percentages and duration ranges are estimated based on user's experience and confidence. While the maximum percentage confidence is not restricted to 100% confidence, it must be equal for all activities within a project, i.e. all activities may have a maximum of 90% confidence. As is expected, the percentage confidence decreases as the duration range is narrowed, i.e. lower confidence level on a narrower range of duration estimates.

Table 3.1. PSM Probabilistic Inputs

Activity A		
Probability	100 %	2 – 8 days
	75 %	3 – 6 days
	50 %	4 – 5 days

Activity B		
Probability	100 %	3 – 8 days
	75 %	4 – 6 days

3.3.1 Probabilistic Inputs Calculation

Inputs for each activity are distributed using Equations 3.1, 3.2, and 3.3.

$$P_{a,t} = \frac{\%R_{a,r} - \%R_{a,r-1}}{tR_{a,r} - tR_{a,r-1}} \quad (3.1)$$

$$tR_{a,r} = t_{a,r \max} - t_{a,r \min} + 1 \quad (3.2)$$

$$tR_{a,r-1} = t_{a,r-1 \max} - t_{a,r-1 \min} + 1 \quad (3.3)$$

where:

$P_{a,t}$ = probabilistic value of time t activity a , %

$t_{a,r \max}$ = maximum time of range r activity a

$t_{a,r \min}$ = minimum time of range r activity a

$t_{a,r-1 \max}$ = maximum time of range $r-1$ activity a

$t_{a,r-1 \min}$ = minimum time of range $r-1$ activity a

$tR_{a,r}$ = time range r activity a

$tR_{a,r-1}$ = time range $r-1$ activity a

$\%R_{a,r}$ = percentage confidence range r activity a , %

$\%R_{a,r-1}$ = percentage confidence range $r-1$ activity a , %

The range $r-1$ is defined as the range lower than range r . In Table 3.1, a range r of 75% confidence defines the range $r-1$ to a confidence level of 50%. The calculation begins with the definition of activity duration range, e.g. Activity $A = 100\%$ confidence within 2

– 8 days, 75% confidence within 3 – 6 days, and 50% confidence within 4 – 5 days. Starting from the lowest percentage, durations of Activity A are distributed equally within their percentage values as shown in Table 3.2, i.e. $P_{A,3}$ and $P_{A,6} = 25\% / 2 = 12.5\%$. Probabilities are then calculated for each estimated duration using the given equations. Table 3.3 shows the calculated inputs for the PSM scheduling. The detailed calculations for this example are given in Appendix A.

Table 3.2. Probabilistic Inputs Breakdown

Activity A		
Probability	100 %	2 3 4 5 6 7 8
	75 %	3 4 5 6
	50 %	4 5

Activity A		
Probability	100 % - 75 %	2 3 4 5 6 7 8
	75 % - 50 %	3 4 5 6
	50 %	4 5

Activity B		
Probability	100 %	3 4 5 6 7 8
	75 %	4 5 6

Activity B		
Probability	100 % - 75 %	3 7 8
	75 %	4 5 6

Table 3.3. PSM Calculated Inputs

Activity A			
t_A	$P_{A,t}$	t_A	$P_{A,t}$
2	8.3 %	6	12.5 %
3	12.5 %	7	8.3 %
4	25.0 %	8	8.3 %
5	25.0 %		

Activity B			
t_B	$P_{B,t}$	t_B	$P_{B,t}$
3	8.3 %	6	25.0 %
4	25.0 %	7	8.3 %
5	25.0 %	8	8.3 %

3.4 PSM SCHEDULING

The PSM scheduling consists of two computation steps: a Forward Pass calculation and a probabilistic calculation. Aside from the computed inputs from Table 3.3, PSM requires information on dependencies between activities (precedence information), e.g. Activity A precedes Activity B. The Forward Pass is used to calculate every duration inputs between one activity and its succeeding activity, i.e. Activity A + B = 2 + 3, 2 + 4, ..., 8 + 8. The resulting range of Activity A + B is from 5 to 16 days total duration with 100% probability. An easier approach to do the Forward Pass is to calculate only the minimum and maximum time of Activities A and B (2 + 3 and 8 + 8), which will also solve for 5 days total duration and 16 days total duration as minimum and maximum

time of A + B, respectively. The probabilistic calculation is done after the duration range is provided using simple probabilistic calculations and understanding on combinations. To calculate the probability of 7 days total duration of Activity A + B requires the combinations of the time of Activity A and time of Activity B that add up to 7 days, which are 2 + 5, 3 + 4, and 4 + 3 of A + B, respectively.

3.4.1 Probability Principles

PSM uses an "AND" principle and an "OR" principle to calculate the probabilistic values, i.e. probability of a 7 days total duration is (2 AND 5) OR (3 AND 4) OR (4 AND 3). The equations for "AND" and "OR" principles can be illustrated as follows:

$$P(A \text{ AND } B) = P(A) \times P(B) \quad (3.4)$$

$$P(C \text{ OR } D) = P(C) + P(D) \quad (3.5)$$

where:

A, B, C, D = events in scheduling

P = probability, %

Based on Equations 3.4 and 3.5, $P(A + B = 7) = P(2) \times P(5) + P(3) \times P(4) + P(4) \times P(3)$.

Here, Event A and Event B refer to the completion of Activity A and Activity B, which are independent events. Event C and Event D can be classified as mutually exclusive events because they refer to the same duration of Activity A + B while mutually exclusive, i.e. Event C = 2 + 5 = 7 and Event D = 3 + 4 = 7.

3.4.2 PSM Scheduling Patterns

The scheduling calculations of Activity A + B form two patterns as shown in Figures 3.1 and 3.2. The scheduling begins with the minimum time of Activities A and B (shown as red line in Figure 3.1). 6 days total duration calculations follow the blue line, which start from A = 2 and B = 4; and as A increases, B decreases. As the first pattern ends with the 10 days total duration calculation (A = 2 and B = 8, A = 3 and B = 7, ..., A = 7 and B = 3), the second pattern begins with A = 3 and B = 8 or 11 days total duration shown as the green lines in Figure 3.2 (A increases and B decreases). The second pattern ends with the maximum time of A and B calculation (purple line in Figure 3.2). All PSM scheduling will have the two patterns, and all its probabilistic calculations are done in the same manner.

Figure 3.1. PSM Scheduling Pattern 1

Figure 3.2. PSM Scheduling Pattern 2

3.4.3 PSM Scheduling Calculation

The probabilistic inputs (Table 3.3) are scheduled using Equations 3.6 – 3.10. Equation 3.6 consists of two equations. The top portion of Equation 3.6 relates to the first PSM scheduling pattern, while the bottom portion relates to the second pattern. The values for j , m , ℓ , and o in Equation 3.6 are computed using Equations 3.7 – 3.10.

$$P_{Tr,t} = \begin{cases} \sum_{i=0}^m [P_{p,t_{min}+i} \times P_{a,t_{min}+j-i}], & t_{Tr} = t_{p,min} + t_{a,min} + j, \\ \sum_{k=0}^o [P_{p,t_{min}+\ell+k} \times P_{a,t_{max}-k}], & t_{Tr} = t_{p,min} + t_{a,max} + \ell. \end{cases} \quad (3.6)$$

where:

$$j = 0 \rightarrow (t_{a,max} - t_{a,min}), \quad (3.7)$$

$$m = \min[j, \min(t_{p,max} - t_{p,min}, t_{a,max} - t_{a,min})], \quad (3.8)$$

$$\ell = 1 \rightarrow (t_{p,max} - t_{p,min}), \quad (3.9)$$

$$o = \min[(t_{p,max} - t_{p,min} - \ell), \min(t_{p,max} - t_{p,min}, t_{a,max} - t_{a,min})]. \quad (3.10)$$

where:

$P_{a, t max}$ = probability of activity a maximum time, %

$P_{a, t min}$ = probability of activity a minimum time, %

$P_{p, t max}$ = probability of preceding activity p maximum time, %

$P_{p, t min}$ = probability of preceding activity p minimum time, %

$P_{Tr, t}$ = probability of terminal time t, %

$t_{a, max}$ = activity a maximum time

$t_{a, min}$ = activity a minimum time

$t_{p, max}$ = preceding activity p maximum time

$t_{p, min}$ = preceding activity p minimum time

t_{Tr} = terminal time t

The PSM scheduling calculations are done using the Equations 3.6 through 3.10. The equations relate to two activities at a time, which are the calculated activity "a" and its preceding activity "p". Terminal time has the same definition as nodal time, which is the scheduled time after activity "a" calculation.

The calculations begin with the inputs of $t_{p, \min}$, $t_{p, \max}$, $t_{a, \min}$, and $t_{a, \max}$. Using values taken from the inputs calculation in Table 3.3, the inputs are as follows:

$$t_{p, \min} = t_{A, \min} = 2$$

$$t_{p, \max} = t_{A, \max} = 8$$

$$t_{a, \min} = t_{B, \min} = 3$$

$$t_{a, \max} = t_{B, \max} = 8$$

Equations 3.7 to 3.10 are used to calculate j , m , ℓ , and o :

$$j = 0 \rightarrow (t_{B, \max} - t_{B, \min}) = 0 \rightarrow (8 - 3) = 0 \rightarrow 5$$

$$m = \min[j, \min(t_{A, \max} - t_{A, \min}, t_{B, \max} - t_{B, \min})] = \min[j, \min(8 - 2, 8 - 3)] = \min[j, 5]$$

$$\ell = 1 \rightarrow (t_{p, \max} - t_{p, \min}) = 1 \rightarrow (t_{A, \max} - t_{A, \min}) = 1 \rightarrow (8 - 2) = 1 \rightarrow 6$$

$$o = \min[(t_{p, \max} - t_{p, \min} - \ell), \min(t_{p, \max} - t_{p, \min}, t_{a, \max} - t_{a, \min})] = \min[(6 - \ell), 5]$$

Equation 3.6 is used under the constraints of j , m , ℓ , and o . The calculations start with the top portion of Equation 3.6, which represents the first PSM scheduling pattern. The bottom portion of Equation 3.6 refers to the second scheduling pattern.

The scheduling calculations for Table 3.3 are as follows:

$$P_{1,t} = \begin{cases} \sum_{i=0}^m [P_{A,2+i} \times P_{B,3+j-i}], & t_1 = 2 + 3 + j, \\ \sum_{k=0}^o [P_{A,2+\ell+k} \times P_{B,8-k}], & t_1 = 2 + 8 + \ell. \end{cases}$$

For $j = 0 \rightarrow 5$, $t_1 = (2 + 3 + 0) \rightarrow (2 + 3 + 5) = 5 \rightarrow 10$:

$$P_{1,5} = \sum_{i=0}^0 [P_{A,2+i} \times P_{B,3+0-i}] = P_{A,2+0} \times P_{B,3+0-0} = P_{A,2} \times P_{B,3} = 8.3\% \times 8.3\% = 0.69\%$$

$$\begin{aligned} P_{1,6} &= \sum_{i=0}^1 [P_{A,2+i} \times P_{B,3+1-i}] = P_{A,2+0} \times P_{B,3+1-0} + P_{A,2+1} \times P_{B,3+1-1} \\ &= P_{A,2} \times P_{B,4} + P_{A,3} \times P_{B,3} = 8.3\% \times 25\% + 12.5\% \times 8.3\% = 3.11\% \end{aligned}$$

: : : : : :

$$\begin{aligned} P_{1,10} &= \sum_{i=0}^5 [P_{A,2+i} \times P_{B,3+5-i}] = P_{A,2+0} \times P_{B,3+5-0} + P_{A,2+1} \times P_{B,3+5-1} + \dots + P_{A,2+5} \times P_{B,3+5-5} \\ &= P_{A,2} \times P_{B,8} + P_{A,3} \times P_{B,7} + \dots + P_{A,7} \times P_{B,3} = 18.04\% \end{aligned}$$

For $\ell = 1 \rightarrow 6$, $t_1 = (2 + 8 + 1) \rightarrow (2 + 8 + 6) = 11 \rightarrow 16$:

$$\begin{aligned} P_{1,11} &= \sum_{k=0}^5 [P_{A,2+1+k} \times P_{B,8-k}] = P_{A,2+1+0} \times P_{B,8-0} + P_{A,2+1+1} \times P_{B,8-1} + \dots + P_{A,2+1+5} \times P_{B,8-5} \\ &= P_{A,3} \times P_{B,8} + P_{A,4} \times P_{B,7} + \dots + P_{A,8} \times P_{B,3} = 15.25\% \end{aligned}$$

: : : : : :

$$P_{1,16} = \sum_{k=0}^0 [P_{A,2+6+k} \times P_{B,8-k}] = P_{A,2+6+0} \times P_{B,8-0} = P_{A,8} \times P_{B,8} = 0.69\%$$

3.4.4 PSM Scheduling Results Analysis

The solution to Table 3.3 scheduling is shown in Table 3.4, and the probabilistic and cumulative distribution charts are drawn in Figures 3.3 and 3.4. From Table 3.4, Activities A and B can be completed within 5 to 16 days with 100% probability. Other estimations can be made such as 75.59% probability for 8 – 12 days total duration, 50.64% within 9 – 11 days total duration, and 51.98% within 10 – 13 days total duration.

Table 3.4. PSM Scheduling Results

Terminal 1 (Activity A + B)	
t_1	$P_{1,t}$
5	0.69 %
6	3.11 %
7	7.28 %
8	13.53 %
9	17.53 %
10	18.04 %
11	15.25 %
12	11.43 %
13	7.26 %
14	3.80 %
15	1.38 %
16	0.69 %

Figure 3.3. PSM Distribution Chart

Figure 3.4. PSM Cumulative Distribution Chart

3.4.5 Forward Pass Maximum Rule

Extra attention must be drawn regarding the Forward Pass maximum rule. As shown in Equation 2.1, a maximum time is taken between two activities which merge into one node (or terminal). Due to the duration range in PSM, deciding the activity maximum time is more difficult than it is in CPM.

In CPM, only one activity with the highest duration is chosen among two or three activities that lead into a node (e.g. Activities C and D in Figure 2.6). The highest duration is chosen under an assumption that all activity conditions are the same, i.e. CPM activities are scheduled based on their workload.

PSM must use the same assumption to choose the maximum time range between two or three activities. In PSM, aside from workload, activities are scheduled based on their probabilistic values. Thus, these probabilities may represent the conditions stated in the assumption.

An example of PSM maximum time comparison difficulties is illustrated in Table 3.5, where 2 activities are defined. The duration range of a defined Activity F is 20 – 32 days total duration, while Activity G is 19 – 34 days total duration. Solving for maximum time comparison in Table 3.5, a cumulative distribution chart is drawn in Figure 3.5 showing a higher time from the Activity G.

Table 3.5. Maximum Time Comparison

t_F	$P_{F,t}$	$C_{F,t}$	t_G	$P_{G,t}$	$C_{G,t}$
20	0.13 %	0.13 %	19	0.02 %	0.02 %
21	0.72 %	0.85 %	20	0.14 %	0.15 %
22	2.86 %	3.71 %	21	0.68 %	0.83 %
23	7.49 %	11.20 %	22	2.29 %	3.13 %
24	14.26 %	25.46 %	23	5.75 %	8.88 %
25	20.51 %	45.96 %	24	11.08 %	19.95 %
26	20.90 %	66.86 %	25	16.59 %	36.54 %
27	15.56 %	82.42 %	26	19.25 %	55.79 %
28	9.70 %	92.12 %	27	17.45 %	73.24 %
29	5.08 %	97.20 %	28	12.72 %	85.96 %
30	2.02 %	99.22 %	29	7.74 %	93.70 %
31	0.65 %	99.87 %	30	3.96 %	97.66 %
32	0.13 %	100.00 %	31	1.65 %	99.31 %
			32	0.55 %	99.85 %
			33	0.13 %	99.98 %
			34	0.02 %	100.00 %

where:

$C_{F,t}$ = cumulative density function of Activity F duration, %

$C_{G,t}$ = cumulative density function of Activity G duration, %

$P_{F,t}$ = probabilistic density function of Activity F duration, %

$P_{G,t}$ = probabilistic density function of Activity G duration, %

t_F = duration time of Activity F

t_G = duration time of Activity G

Figure 3.5. Maximum Time Comparison Chart

CHAPTER 4

PSM TESTS AND RESULTS

4.1 GENERAL

Three tests were performed using examples of 3 activities, 7 activities, and 9 activities. Results from each example were compared to the CPM, PERT, and MC PERT. Microsoft Excel worksheet was used to administer data and calculations.

4.2 3 ACTIVITIES TESTING

The test is done as outlined in PSM procedures.

1. List all activities within the project:

Activity A

Activity B

Activity C

2. List precedence relationships correspond to all activities:

Activity	Predecessor
A	-
B	-
C	A

3. Estimate activities duration ranges including their corresponding probabilistic values (Table 4.1).
4. Calculate the probabilities for each activity using Equations 3.1, 3.2, and 3.3 (Refer to Appendix B1 for calculations). The results are as in Table 4.2.
5. Schedule the project using inputs from Table 4.2 and Equations 3.6 – 3.10 (Refer to Appendix B1 for calculations and Table 4.3 for results).
6. Analyze each total project time to estimate the most probable duration range:

The distribution in Table 4.3 shows that the project can be completed within 6 to 13 days with near 100% probability, within 7 to 10 days and within 9 to 12 days with near 75% probability, and within 6 to 9 days and within 10 to 13 days with near 50% probability. The most probable duration is 9.5 days total duration.

Table 4.1. 3 Activities Inputs

Activity	Predecessor	r	$t_{a,r \min}$	$t_{a,r \max}$
A	-	100%	2	6
		75%	3	5
		50%	4	4
B	-	100%	3	9
		75%	4	8
		50%	5	7
C	A	100%	4	7
		75%	5	6
		50%	-	-

Table 4.2. Activities Distributions (3 Activities)

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
2	12.50%	3	12.50%	4	12.50%
3	12.50%	4	12.50%	5	37.50%
4	50.00%	5	16.67%	6	37.50%
5	12.50%	6	16.67%	7	12.50%
6	12.50%	7	16.67%		
		8	12.50%		
		9	12.50%		

Table 4.3. Project Duration Distribution (3 Activities)

Project Duration	Probability
6	1.56%
7	6.25%
8	15.63%
9	26.56%
10	26.56%
11	15.63%
12	6.25%
13	1.56%

The inputs of the CPM, PERT, and MC PERT for comparison are tabulated in Table 4.4. The CPM returns 9.2 days as the total project duration. Both PERT and Monte Carlo PERT distributions are drawn in comparison to the PSM distribution (Figures 4.1 and 4.2). Due to the disparity given by the MC PERT frequency distribution, only distribution drawn using the normal equations is used for comparison. While the PERT most probable duration stays constant at 9.2 days, the MC PERT duration changes between 9 and 10 days with its probability discrepancies are between 26% and 31%.

Table 4.4. Comparison Inputs (3 Activities)

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	2	4	6
B	-	3	5	9
C	A	4	5	7

Figure 4.1. 3 Activities Comparison Chart

Figure 4.2. 3 Activities Cumulative Comparison Distribution Chart

The estimation produced by the PSM has an offset of around 0.5 days in comparison to PERT and MC PERT for this particular example. It shows that, while PSM accounts all calculations based on proper probabilistic principles, its results are very close in comparison to other established methods.

4.3 7 ACTIVITIES TESTING

The second test was run using 7 activities A through G as shown in Table 4.5. The inputs are distributed in Table 4.6, and scheduling solution is tabulated in Table 4.7 (Refer to details on Appendix B2). The most probable duration for PSM is 20.5 days, while the duration ranges are 13 – 28 days (100%), 17 – 22 days and 19 – 24 days (both 77.61%), 19 – 22 days (62.26%), and 13 – 20 days and 20 – 28 days (both 50%).

Table 4.5. 7 Activities Inputs

Activity	Predecessor	r	$t_{a, r \min}$	$t_{a, r \max}$
A	-	100%	2	6
		75%	3	5
		50%	4	4
B	-	100%	3	9
		75%	4	8
		50%	5	7
C	A	100%	4	7
		75%	5	6
		50%	-	-
D	A	100%	4	10
		75%	5	9
		50%	6	8
E	B, C	100%	4	7
		75%	5	6
		50%	-	-
F	D	100%	3	8
		75%	4	7
		50%	5	6
G	E	100%	3	8
		75%	4	7
		50%	5	6

Table 4.6. Activities Distributions (7 Activities)

Activity A		Activity B		Activity C		Activity D	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$	t_D	$P_{D,t}$
2	12.50%	3	12.50%	4	12.50%	4	12.50%
3	12.50%	4	12.50%	5	37.50%	5	12.50%
4	50.00%	5	16.67%	6	37.50%	6	16.67%
5	12.50%	6	16.67%	7	12.50%	7	16.67%
6	12.50%	7	16.67%			8	16.67%
		8	12.50%			9	12.50%
		9	12.50%			10	12.50%

Activity E		Activity F		Activity G	
t_E	$P_{E,t}$	t_F	$P_{F,t}$	t_G	$P_{G,t}$
4	12.50%	3	12.50%	3	12.50%
5	37.50%	4	12.50%	4	12.50%
6	37.50%	5	25.00%	5	25.00%
7	12.50%	6	25.00%	6	25.00%
		7	12.50%	7	12.50%
		8	12.50%	8	12.50%

Table 4.7. Project Duration Distribution (7 Activities)

Project Duration	Probability
13	0.02%
14	0.20%
15	0.83%
16	2.47%
17	5.54%
18	9.81%
19	14.16%
20	16.97%
21	16.97%
22	14.16%
23	9.81%
24	5.54%
25	2.47%
26	0.83%
27	0.20%
28	0.02%

Table 4.8. Comparison Inputs (7 Activities)

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	2	4	6
B	-	3	5	9
C	A	4	5	7
D	A	4	6	10
E	B, C	4	5	7
F	D	3	4	8
G	E	3	5	8

Inputs for comparison are shown in Table 4.8. The CPM solves the 7 activities to 19.5 days total duration, which is also the most probable duration for the PERT. Figures 4.3 and 4.4 show the comparison of the distributions of PSM, PERT, and MC PERT. The percentage discrepancies in MC PERT are between 18% and 21%, while the most probable durations vary around 21 days.

Figure 4.3. 7 Activities Comparison Chart

Figure 4.4. 7 Activities Cumulative Comparison Distribution Chart

4.4 9 ACTIVITIES TESTING

A third test was run using 9 activities inputs as shown in Table 4.9. Tables 4.10 through 4.12 correspond to the inputs distributions, scheduling solution, and the comparison inputs of the 9 activities. The most probable duration of the 9 activities example is 33 days with 16.86% probability. The CPM and PERT duration is scheduled to 31 days total duration, while the MC PERT solves for 32 – 32.5 days (18% – 20.5%) as shown in Figures 4.5 and 4.6. Calculations details are provided in Appendix B3.

Table 4.9. 9 Activities Inputs

Activity	Predecessor	r	$t_a, r \text{ min}$	$t_a, r \text{ max}$
A	-	100%	3	6
		75%	4	5
		50%	-	-
B	-	100%	2	5
		75%	3	4
		50%	-	-
C	A	100%	7	12
		75%	8	10
		50%	9	9
D	A	100%	5	9
		75%	6	8
		50%	7	7
E	B, C	100%	8	12
		75%	9	11
		50%	10	10
F	B, C	100%	10	14
		75%	11	13
		50%	12	12
G	D, E	100%	1	4
		75%	2	3
		50%	-	-
H	D, E	100%	4	6
		75%	-	-
		50%	5	5
I	F, G	100%	5	9
		75%	6	8
		50%	7	7

Table 4.10. Activities Distributions (9 Activities)

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
3	12.50%	2	12.50%	7	8.33%
4	37.50%	3	37.50%	8	12.50%
5	37.50%	4	37.50%	9	50.00%
6	12.50%	5	12.50%	10	12.50%
				11	8.33%
				12	8.33%

Activity D		Activity E		Activity F	
t_D	$P_{D,t}$	t_E	$P_{E,t}$	t_F	$P_{F,t}$
5	12.50%	8	12.50%	10	12.50%
6	12.50%	9	12.50%	11	12.50%
7	50.00%	10	50.00%	12	50.00%
8	12.50%	11	12.50%	13	12.50%
9	12.50%	12	12.50%	14	12.50%

Activity G		Activity H		Activity I	
t_G	$P_{G,t}$	t_H	$P_{H,t}$	t_I	$P_{I,t}$
1	12.50%	4	25.00%	5	12.50%
2	37.50%	5	50.00%	6	12.50%
3	37.50%	6	25.00%	7	50.00%
4	12.50%			8	12.50%
				9	12.50%

Table 4.11. Project Duration Distribution (9 Activities)

Project Duration	Probability
24	0.00%
25	0.02%
26	0.11%
27	0.44%
28	1.36%
29	3.35%
30	6.71%
31	11.02%
32	14.98%
33	16.86%
34	15.76%
35	12.41%
36	8.34%
37	4.81%
38	2.37%
39	0.99%
40	0.34%
41	0.09%
42	0.02%
43	0.00%

Table 4.12. Comparison Inputs (9 Activities)

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	3	4	6
B	-	2	3	5
C	A	7	8	12
D	A	5	7	9
E	B, C	8	9	12
F	B, C	10	12	14
G	D, E	1	2	4
H	D, E	4	5	6
I	F, G	5	6	9

Figure 4.5. 9 Activities Comparison Chart

Figure 4.6. 9 Activities Cumulative Comparison Distribution Chart

4.5 WORKSHEET APPLICATION

Several worksheet functions are used to apply the PSM scheduling in Microsoft Excel. Following the same procedures as given in Chapter 3.4, Equations 3.1 – 3.3 can be written as they are using the Excel function. The problems lie with Equations 3.6 – 3.10.

Taking an example from Appendix B3, Terminal 5 is written as follows:

Table 4-12. Comparison Figure 4-1 (Terminal 5)

Terminal 5

Predecessor Activity	B, C E		
t_3	t_E	t_5	$P_{5,t}$
10	8	18	0.13%
11	9	19	0.72%
12	10	20	2.86%
13	11	21	7.49%
14	12	22	14.26%
15		23	20.51%
16		24	20.90%
17		25	15.56%
18		26	9.70%
		27	5.08%
		28	2.02%
		29	0.65%
		30	0.13%

i	j	k	l	m	o
0	0	0	1	0	4
1	1	1	2	1	4
2	2	2	3	2	4
3	3	3	4	3	4
4	4	4	5	4	3
			6		2
			7		1
			8		0

Figure 4-1-9 provides Comparison Chart.

Listing Terminal 3 and Activity E durations, values of j , ℓ , m , and o are calculated as follows:

For j (written on cell $j = 1$):

$$f_x = \text{IF}(j - 1 < (t_{a,\max} - t_{a,\min}), j, t_{a,\max} - t_{a,\min}) \quad (4.1)$$

where:

IF Function = IF(Logical Test, Values If True, Values If False)

IF Function = function to conduct conditional tests on values and formulas

For ℓ (written on cell $\ell = 2$):

$$f_x = \text{IF}(\ell - 1 < (t_{p,\max} - t_{p,\min}), \ell, t_{p,\max} - t_{p,\min}) \quad (4.2)$$

The first values of j and ℓ are written manually, i.e. $j = 0$ and $\ell = 1$. Formulas on Equations 4.1 and 4.2 are written as instructed and copied down to automatically return the desired values.

For m (written for each j value):

$$f_x = \text{MIN}(j, \text{MIN}(t_{p,\max} - t_{p,\min}, t_{a,\max} - t_{a,\min})) \quad (4.3)$$

For o (written for each ℓ value):

$$f_x = \text{MIN}((t_{p,\max} - t_{p,\min} - \ell), \text{MIN}(t_{p,\max} - t_{p,\min}, t_{a,\max} - t_{a,\min})) \quad (4.4)$$

where:

MIN Function = MIN(Number 1, Number 2, ...)

MIN Function = function that returns the smallest number in a set of values

For i and k, values are written manually from 0 to maximum m and 0 to maximum o, respectively. The t_5 calculations are as follows:

For $j = 0 \rightarrow 4$:

$$f_x = t_{p,\min} + t_{a,\min} + j \quad (4.5)$$

For $\ell = 1 \rightarrow 8$:

$$f_x = t_{p,\min} + t_{a,\max} + \ell \quad (4.6)$$

For $P_{5,t}$ calculations, combinations of VLOOKUP and SUM commands are used. The inputs must be organized before using the VLOOKUP function. The probabilistic values must be placed on the right column next to the duration values, i.e. as shown in Table 4.10 ($P_{3,t}$ column is placed right next to t_3). $P_{5,t}$ calculations are as follows:

For $i = 0$ and $j = 0$:

$$P_{5,18} = \text{SUM}(\text{VLOOKUP}(t_{3,\min} + i, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i, t_{E,\min} : P_{E,\max}, 2))$$

For $i = 1$ and $j = 1$:

$$P_{5,19} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + i - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

where:

SUM Function = SUM(Number 1, Number 2, ...)

SUM Function = function that adds all the numbers in a range of cells

VLOOKUP Function = VLOOKUP(Lookup Value, Table Array, Column Index)

VLOOKUP Function = function that searches for a value in the leftmost column of a table, and then returns a value in the same row from a column that is specified in the table.

For $i = 2$ and $j = 2$:

$$P_{5,20} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + i - 2, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 2, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

The number increases as many as i values, i.e. for $i = 4$ and $j = 4$:

$$P_{5,22} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + i - 4, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 4, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i - 3, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 3, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i - 2, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 2, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + i, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\min} + j - i, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

For $k = 4$ and $\ell = 1$:

$$P_{5,23} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + \ell + k - 4, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 4, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 3, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 3, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 2, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 2, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

For $k = 4$ and $\ell = 2 \rightarrow 4$ ($P_{5,24} - P_{5,26}$):

$$P_{5,t} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + \ell + k - 4, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 4, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 3, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 3, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 2, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 2, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

For $k = 3$ and $\ell = 5$:

$$P_{5,27} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{3,\min} + \ell + k - 3, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 3, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 2, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 2, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k - 1, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k + 1, t_{E,\min} : P_{E,\max}, 2) \\ \text{VLOOKUP}(t_{3,\min} + \ell + k, t_{3,\min} : P_{3,\max}, 2) * \text{VLOOKUP}(t_{E,\max} - k, t_{E,\min} : P_{E,\max}, 2) \end{array} \right)$$

The values of i and k correspond to the values of m and o . The key to using the worksheet application is to understand the patterns given in the functions.

4.2.7 ACTIVITY SCHEDULE REVISION

The 7 activities example is already in progress with the completion of Activities A, B, C, and D inputs from Table 4.6 were modified so that Activity A = 5 days, B = 5 days, C = 5 days, and D = 5 days, while the rest (Activities E, F, and G) are still unknown. The resulting schedule changes from Table 4.7 in Table 4.8. Figures 4.1 and 4.2 show PDM comparisons on the initial chart against the revised chart. PDM comparisons are made in Figures 4.3 and 4.4 to evaluate PDM against PERT in project schedule revision. The calculations are presented in Appendix C1.

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CHAPTER 5

PSM APPLICATION TO PROJECT SCHEDULE REVISION

5.1 GENERAL

Probabilistic Scheduling Method can also be used to revise project schedule when the project is already in progress. Five tests were conducted to investigate the resulting PSM schedule when the project is underway. Modified inputs from Tables 4.6 and 4.10 were used for this purpose. The completed activity will provide information of its duration at 100% probability.

5.2 7 ACTIVITIES SCHEDULE REVISION

The 7 activities example is already in progress with the completion of Activities A, B, C, and D. Inputs from Table 4.6 were modified so that Activity A = 5 days, B = 6 days, C = 6 days, and D = 8 days, while the rest (Activities E, F, and G) are still in progress. The resulting schedule changes from Table 4.7 to Table 5.1. Figures 5.1 and 5.2 show PSM comparisons on the actual chart against the revised chart. PERT comparisons are made in Figures 5.3 and 5.4 to evaluate PSM against PERT in project schedule revision. The calculations are presented in Appendix C1.

Table 5.1. Duration Distribution (7 Activities to Activity D)

Project Duration	Probability
18	1.56%
19	6.25%
20	12.50%
21	18.75%
22	21.88%
23	18.75%
24	12.50%
25	6.25%
26	1.56%

Figure 5.1. PSM Actual vs. Revised Schedule (7 Activities to Activity D)

Figure 5.2. PSM Actual vs. Revised Cumulative Chart (7 Activities to D)

Figure 5.3. PERT Actual vs. Revised Schedule (7 Activities to Activity D)

Figure 5.4. PERT Actual vs. Revised Cumulative Chart (7 Activities to D)

5.3 9 ACTIVITIES SCHEDULE REVISION

Inputs from Table 4.10 were modified four times to investigate any discrepancies between the completion of 3 activities, 5 activities, and 6 activities, and also to evaluate if differences in the completed activity duration will alter the revision. First modification was on Activity A = 4 days, B = 4 days, and C = 9 days. Second test was modified from the first trial by assigning Activity D = 8 days and E = 10 days. The third test assigned Activity F = 11 days in addition to the second modification, while the fourth test assigned Activity F = 13 days. Comparisons between PSM and PERT are shown in Figures 5.5 – 5.8. Calculations are shown in Appendix C2.

Figure 5.5. PSM Actual vs. Revised Schedules (9 Activities)

Figure 5.6. PSM Actual vs. Revised Cumulative Chart (9 Activities)

Figure 5.7. PERT Actual vs. Revised Schedule (9 Activities)

Figure 5.8. PERT Actual vs. Revised Cumulative Chart (9 Activities)

CHAPTER 6

CONCLUSIONS

This thesis has demonstrated a new methodology for construction project scheduling based on a range of estimated duration associated with a probability range. The Probabilistic Scheduling Method (PSM) accepts two "connected" or "related" inputs: activity duration and an associated confidence interval, the latter is processed prior to assigning them as the scheduling inputs. Two assumptions are used to simplify the inputs calculations: maximum percentages are equal for all activities and the probabilities are distributed equally within the duration range. It is also possible to ignore these assumptions to introduce more flexibility to the PSM inputs.

PSM uses "AND" principle and "OR" principle from probability and combines both with the Forward Pass calculation method for its scheduling process. Each of the PSM scheduling forms two patterns which are easily recognised. Generalized equations are provided to assist user in the calculation processes. PSM also offers the possibility of a worksheet application to administer its data and calculations. Noticeable patterns are also found in the inputs for worksheet application functions.

The comparisons of PSM to CPM, PERT, and MC PERT conclude that the quality of PSM is similar to the other three in terms of most probable duration estimation (2 days and 20% differences for the 9 activities). Moreover, PSM introduces a relationship between probability in project activities and the probability in total project duration, i.e. all values are accounted for. PSM is confident that a project can be completed nearly within the scheduled range should 100% probability is used for every activities.

CHAPTER 2

PSM shows a good result in its project schedule revision application. The PSM duration range is narrowed as the project progresses and its corresponding probability increases.

CHAPTER 7

PSM RECOMMENDATIONS FOR FUTURE STUDIES

7.1 OVERALL PSM EVALUATION

PSM computations are more industry friendly. Its inputs reflect the condition of the construction industry, where certain estimation may be made within a range of activity duration, and this estimation will become less certain as the range is confined. The PSM allows for a range of durations associated with a range of probabilities. Various tests have shown that PSM estimations are very close to the CPM and PERT estimations. As the number of activities increases, the variations get larger in comparison to PERT.

The strengths of PSM are found within its inputs and outputs flexibility and calculations simplicity. User may have more confidence by estimating certain ranges of activity duration, instead of having to decide on one or three particular duration estimates. Its calculations consist of simple probability and forward pass calculations, which are not difficult to understand. Its feature on providing dependency information will benefit its users in the decision making processes.

7.2 RECOMMENDATIONS FOR FUTURE RESEARCH

Further study is recommended to implement the PSM in real construction practice to evaluate its reliability in comparison to the CPM, PERT, or MC PERT. Comparison between a scheduled plan and the actual finish date may provide an evaluation on the PSM accuracy in probabilistic scheduling.

It is recommended to develop Windows-based computer software to accommodate the PSM calculations. The provided worksheet application is not yet developed as a user-friendly module. A user-friendly module is suggested for time efficiency in scheduling a bigger project.

The PSM provides a range of total project duration with its corresponding probability range. Many construction projects, however, are often restricted to only one completion date depending on their contract. A study is recommended to do the PSM in reverse to adjust the activity duration range based on the restricted project duration.

Another possible area of future work could be adopting PSM as a platform for calculating resources and cost estimation, potentially by using the same principles as the CPM application to resources allocation and cost estimation. Yet another could be in the development of a network diagram that is unique to the PSM schedule. In this area, a GERT routing network diagram could be used as references.

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Activity A	
Probability	100 % 2-5 days
	75 % 3-5 days
	50 % 4-5 days

Activity B	
Probability	100 % 3-5 days
	75 % 4-5 days
	50 % 5-6 days

Activity A:

$t_{A,normal} = 2$ $t_{A,crash} = 3$ $t_{A,normal} = 4$

$t_{A,normal} = 5$ $t_{A,crash} = 6$ $t_{A,normal} = 7$

$$WR_{A,2} = t_{A,normal} - t_{A,crash} + 1 = 2 - 3 + 1 = 0$$

$$WR_{A,3} = t_{A,normal} - t_{A,crash} + 1 = 3 - 4 + 1 = 0$$

$$WR_{A,4} = t_{A,normal} - t_{A,crash} + 1 = 4 - 5 + 1 = 0$$

$$P_A = \frac{WR_{A,2} \cdot WR_{A,3} \cdot WR_{A,4} \cdot (100\% - 75\% + 25\%)}{WR_{A,2} + WR_{A,3} + WR_{A,4}} = \frac{0 \cdot 0 \cdot 0}{0 + 0 + 0} = 0\%$$

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APPENDIX A – PSM INPUTS CALCULATIONS

Activity A		
Probability	100 %	2 – 8 days
	75 %	3 – 6 days
	50 %	4 – 5 days

Activity B		
Probability	100 %	3 – 8 days
	75 %	4 – 6 days

Activity A:

$$t_{A, 100 \min} = 2 \quad t_{A, 75 \min} = 3 \quad t_{A, 50 \min} = 4$$

$$t_{A, 100 \max} = 8 \quad t_{A, 75 \max} = 6 \quad t_{A, 50 \max} = 5$$

$$tR_{A, 100} = t_{A, 100 \max} - t_{A, 100 \min} + 1 = 8 - 2 + 1 = 7$$

$$tR_{A, 75} = t_{A, 75 \max} - t_{A, 75 \min} + 1 = 6 - 3 + 1 = 4$$

$$tR_{A, 50} = t_{A, 50 \max} - t_{A, 50 \min} + 1 = 5 - 4 + 1 = 2$$

$$P_{A, 2} = \frac{\%R_{A, 100} - \%R_{A, 75}}{tR_{A, 100} - tR_{A, 75}} = \frac{100\% - 75\%}{7 - 4} = \frac{25\%}{3} = 8.3\%$$

$$P_{A,3} = \frac{\%R_{A,75} - \%R_{A,50}}{tR_{A,75} - tR_{A,50}} = \frac{75\% - 50\%}{4 - 2} = \frac{25\%}{2} = 12.5\%$$

$$P_{A,4} = \frac{\%R_{A,50} - 0}{tR_{A,50} - 0} = \frac{50\% - 0\%}{2 - 0} = \frac{50\%}{2} = 25\%$$

$$P_{A,5} = \frac{\%R_{A,50} - 0}{tR_{A,50} - 0} = \frac{50\% - 0\%}{2 - 0} = \frac{50\%}{2} = 25\%$$

$$P_{A,6} = \frac{\%R_{A,75} - \%R_{A,50}}{tR_{A,75} - tR_{A,50}} = \frac{75\% - 50\%}{4 - 2} = \frac{25\%}{2} = 12.5\%$$

$$P_{A,7} = \frac{\%R_{A,100} - \%R_{A,75}}{tR_{A,100} - tR_{A,75}} = \frac{100\% - 75\%}{7 - 4} = \frac{25\%}{3} = 8.3\%$$

$$P_{A,8} = \frac{\%R_{A,100} - \%R_{A,75}}{tR_{A,100} - tR_{A,75}} = \frac{100\% - 75\%}{7 - 4} = \frac{25\%}{3} = 8.3\%$$

Activity A			
t_A	$P_{A,t}$	t_A	$P_{A,t}$
2	8.3 %	6	12.5 %
3	12.5 %	7	8.3 %
4	25.0 %	8	8.3 %
5	25.0 %		

Activity B:

$$t_{B, 100 \min} = 3 \quad t_{B, 75 \min} = 4$$

$$t_{B, 100 \max} = 8 \quad t_{B, 75 \max} = 6$$

$$tR_{B, 100} = t_{B, 100 \max} - t_{B, 100 \min} + 1 = 8 - 3 + 1 = 6$$

$$tR_{B, 75} = t_{B, 75 \max} - t_{B, 75 \min} + 1 = 6 - 4 + 1 = 3$$

$$P_{B, 3} = P_{B, 7} = P_{B, 8} = \frac{\%R_{B, 100} - \%R_{B, 75}}{tR_{B, 100} - tR_{B, 75}} = \frac{25\%}{3} = 8.3\%$$

$$P_{B, 4} = P_{B, 5} = P_{B, 6} = \frac{\%R_{B, 75} - 0}{tR_{B, 75} - 0} = \frac{75\%}{3} = 25\%$$

Activity B			
t_B	$P_{B, t}$	t_B	$P_{B, t}$
3	8.3 %	6	25.0 %
4	25.0 %	7	8.3 %
5	25.0 %	8	8.3 %

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APPENDIX B – PROBABILISTIC SCHEDULING METHOD TESTS

B1. 3 Activities Testing

Activity	Predecessor	r	$t_{a,r \min}$	$t_{a,r \max}$
A	-	100%	2	6
		75%	3	5
		50%	4	4
B	-	100%	3	9
		75%	4	8
		50%	5	7
C	A	100%	4	7
		75%	5	6
		50%	-	-

Probabilistic Inputs Calculation:

$$P_{a,t} = \frac{\%R_{a,r} - \%R_{a,r-1}}{tR_{a,r} - tR_{a,r-1}} \quad (3.1)$$

$$tR_{a,r} = t_{a,r \max} - t_{a,r \min} + 1 \quad (3.2)$$

$$tR_{a,r-1} = t_{a,r-1 \max} - t_{a,r-1 \min} + 1 \quad (3.3)$$

Where:

$P_{a,t}$ = probabilistic value of time t activity a, %

$t_{a,r \max}$ = maximum time of range r activity a

$t_{a, r \min}$ = minimum time of range r activity a

$t_{a, r-1 \max}$ = maximum time of range r-1 activity a

$t_{a, r-1 \min}$ = minimum time of range r-1 activity a

$tR_{a, r}$ = time range r activity a

$tR_{a, r-1}$ = time range r-1 activity a

$\%R_{a, r}$ = percentage range r activity a, %

$\%R_{a, r-1}$ = percentage range r-1 activity a, %

Activity A Sample Calculation:

Given:

$$\%R_{A, 100} = 100 \%$$

$$t_{A, 100 \min} = 2$$

$$\%R_{A, 75} = 75 \%$$

$$t_{A, 100 \max} = 6$$

$$\%R_{A, 50} = 50 \%$$

$$t_{A, 75 \min} = 3$$

$$t_{A, 75 \max} = 5$$

$$t_{A, 50 \min} = 4$$

$$t_{A, 50 \max} = 4$$

Solution:

$$tR_{A, 100} = t_{A, 100 \max} - t_{A, 100 \min} + 1 = 6 - 2 + 1 = 5$$

$$tR_{A, 75} = t_{A, 75 \max} - t_{A, 75 \min} + 1 = 5 - 3 + 1 = 3$$

$$tR_{A, 50} = t_{A, 50 \max} - t_{A, 50 \min} + 1 = 4 - 4 + 1 = 1$$

$$P_{A,2} = \frac{\%R_{A,100} - \%R_{A,75}}{tR_{A,100} - tR_{A,75}} = \frac{100\% - 75\%}{5 - 3} = \frac{25\%}{2} = 12.5\%$$

$$P_{A,3} = \frac{\%R_{A,75} - \%R_{A,50}}{tR_{A,75} - tR_{A,50}} = \frac{75\% - 50\%}{3 - 1} = \frac{25\%}{2} = 12.5\%$$

$$P_{A,4} = \frac{\%R_{A,50} - 0}{tR_{A,50} - 0} = \frac{50\% - 0\%}{1 - 0} = \frac{50\%}{1} = 50\%$$

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
2	12.50%	3	12.50%	4	12.50%
3	12.50%	4	12.50%	5	37.50%
4	50.00%	5	16.67%	6	37.50%
5	12.50%	6	16.67%	7	12.50%
6	12.50%	7	16.67%		
		8	12.50%		
		9	12.50%		

PSM Scheduling Calculation:

$$t_1 = t_0 + t_A$$

Where:

t_0 = terminal 0 time t

t_1 = terminal 1 time t

t_A = activity A time t

Terminal 1 Sample Calculation:

$$t_1 = t_0 + t_A = 0 + 2 = 2 \quad P_{1,2} = P_{A,2} = 12.5\%$$

$$t_1 = t_0 + t_A = 0 + 3 = 3 \quad P_{1,3} = P_{A,3} = 12.5\%$$

Terminal 1

Predecessor -
Activity A

t_0	t_A	t_1	$P_{1,t}$
0	2	2	12.50%
0	3	3	12.50%
0	4	4	50.00%
0	5	5	12.50%
0	6	6	12.50%

Note: Activity A does not have predecessor, i.e. there are no $P_{p,t}$.

Terminal 0 is the starting terminal of the project, i.e. $t_0 = 0$.

Terminal 2

Predecessor -
Activity B

t_0	t_B	t_2	$P_{2,t}$
0	3	3	12.50%
0	4	4	12.50%
0	5	5	16.67%
0	6	6	16.67%
0	7	7	16.67%
0	8	8	12.50%
0	9	9	12.50%

Terminal 3

Predecessor
Activity

A
C

t_1	t_c	t_3	$P_{3,t}$
2	4	6	1.56%
3	5	7	6.25%
4	6	8	15.63%
5	7	9	26.56%
6		10	26.56%
		11	15.63%
		12	6.25%
		13	1.56%

i	j	k	ℓ	m	o
0	0	0	1	0	3
1	1	1	2	1	2
2	2	2	3	2	1
3	3	3	4	3	0

Terminal 3 Sample Calculation:

$$j = 0 \rightarrow (t_{c,\max} - t_{c,\min}) = 0 \rightarrow (7 - 4) = 0 \rightarrow 3$$

$$\begin{aligned} m &= \min[j, \min(t_{1,\max} - t_{1,\min}, t_{c,\max} - t_{c,\min})] \\ &= \min[j, \min(6 - 2, 7 - 4)] = \min[j, \min(4, 3)] = \min[j, 3] \end{aligned}$$

$$\ell = 1 \rightarrow (t_{1,\max} - t_{1,\min}) = 1 \rightarrow (6 - 2) = 1 \rightarrow 4$$

$$\begin{aligned} o &= \min[(t_{1,\max} - t_{1,\min} - \ell), \min(t_{1,\max} - t_{1,\min}, t_{c,\max} - t_{c,\min})] \\ &= \min[(6 - 2 - \ell), \min(6 - 2, 7 - 4)] = \min[(4 - \ell), 3] \end{aligned}$$

$$t_3 = \begin{cases} t_{1,\min} + t_{C,\min} + j = 2 + 4 + j = 6 + j, & j = 0 \rightarrow 3, \\ t_{1,\min} + t_{C,\max} + l = 2 + 7 + l = 9 + l, & l = 1 \rightarrow 4. \end{cases}$$

$$P_{3,t} = \begin{cases} \sum_{i=0}^m [P_{1,2+i} \times P_{C,4+j-i}], & t_3 = 6 + j, \\ \sum_{k=0}^o [P_{1,2+l+k} \times P_{C,7-k}], & t_3 = 9 + l. \end{cases}$$

For $j = 0 \rightarrow 3$:

$$P_{3,6} = \sum_{i=0}^0 [P_{1,2+i} \times P_{C,4+0-i}] = P_{1,2+0} \times P_{C,4+0-0} = P_{1,2} \times P_{C,4} = 12.5\% \times 12.5\% = 1.56\%$$

$$P_{3,7} = \sum_{i=0}^1 [P_{1,2+i} \times P_{C,4+1-i}] = P_{1,2+0} \times P_{C,4+1-0} + P_{1,2+1} \times P_{C,4+1-1}$$

$$= P_{1,2} \times P_{C,5} + P_{1,3} \times P_{C,4} = 12.5\% \times 37.5\% + 12.5\% \times 12.5\% = 6.25\%$$

$$P_{3,8} = \sum_{i=0}^2 [P_{1,2+i} \times P_{C,4+2-i}] = P_{1,2+0} \times P_{C,4+2-0} + P_{1,2+1} \times P_{C,4+2-1} + P_{1,2+2} \times P_{C,4+2-2}$$

$$= P_{1,2} \times P_{C,6} + P_{1,3} \times P_{C,5} + P_{1,4} \times P_{C,4} = 12.5\% \times 37.5\% + 12.5\% \times 37.5\% + 50\% \times 12.5\%$$

$$= 15.63\%$$

$$\begin{aligned}
P_{3,9} &= \sum_{i=0}^3 [P_{1,2+i} \times P_{C,4+3-i}] = P_{1,2+0} \times P_{C,4+3-0} + P_{1,2+1} \times P_{C,4+3-1} + P_{1,2+2} \times P_{C,4+3-2} + P_{1,2+3} \times P_{C,4+3-3} \\
&= P_{1,2} \times P_{C,7} + P_{1,3} \times P_{C,6} + P_{1,4} \times P_{C,5} + P_{1,5} \times P_{C,4} \\
&= 12.5\% \times 12.5\% + 12.5\% \times 37.5\% + 50\% \times 37.5\% + 12.5\% \times 12.5\% = 26.56\%
\end{aligned}$$

For $\ell = 1 \rightarrow 4$:

$$\begin{aligned}
P_{3,10} &= \sum_{k=0}^3 [P_{1,2+1+k} \times P_{C,7-k}] \\
&= P_{1,2+1+0} \times P_{C,7-0} + P_{1,2+1+1} \times P_{C,7-1} + P_{1,2+1+2} \times P_{C,7-2} + P_{1,2+1+3} \times P_{C,7-3} \\
&= P_{1,3} \times P_{C,7} + P_{1,4} \times P_{C,6} + P_{1,5} \times P_{C,5} + P_{1,6} \times P_{C,4} \\
&= 12.5\% \times 12.5\% + 50\% \times 37.5\% + 12.5\% \times 37.5\% + 12.5\% \times 12.5\% \\
&= 26.56\%
\end{aligned}$$

$$\begin{aligned}
P_{3,11} &= \sum_{k=0}^2 [P_{1,2+2+k} \times P_{C,7-k}] = P_{1,2+2+0} \times P_{C,7-0} + P_{1,2+2+1} \times P_{C,7-1} + P_{1,2+2+2} \times P_{C,7-2} \\
&= P_{1,4} \times P_{C,7} + P_{1,5} \times P_{C,6} + P_{1,6} \times P_{C,5} \\
&= 50\% \times 12.5\% + 12.5\% \times 37.5\% + 12.5\% \times 37.5\% = 15.63\%
\end{aligned}$$

$$\begin{aligned}
P_{3,12} &= \sum_{k=0}^1 [P_{1,2+3+k} \times P_{C,7-k}] = P_{1,2+3+0} \times P_{C,7-0} + P_{1,2+3+1} \times P_{C,7-1} \\
&= P_{1,5} \times P_{C,7} + P_{1,6} \times P_{C,6} = 12.5\% \times 12.5\% + 12.5\% \times 37.5\% = 6.25\%
\end{aligned}$$

$$P_{3,13} = \sum_{k=0}^0 [P_{1,2+4+k} \times P_{C,7-k}] = P_{1,2+4+0} \times P_{C,7-0} = P_{1,6} \times P_{C,7} = 12.5\% \times 12.5\% = 1.56\%$$

Note: Terminal 3 is the last terminal, i.e. total project durations = Terminal 3 durations.

Comparison to CPM, PERT, and MC PERT:

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	2	4	6
B	-	3	5	9
C	A	4	5	7

Activity	t_e	μ_e	σ_e
A	4.0	4	0.666666667
B	5.3	5.333333333	1
C	5.2	5.166666667	0.5

CPM (Calculated using t_e):

Node	T_N
0	0
1	4.0
2	9.2

TD = 9.2

Figure B1. 3 Activities Network Diagram

PERT (Calculated using μ_e):

Node	T_N
0	0
1	4
2	9.166666667

ATD = 9.166666667 (Average Total Duration)

SD = 1.201850425 (Standard Deviation)

Normal Distribution Formula:

$$PDF = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(TD - \mu)^2}{2\sigma^2}\right) \quad (2.7)$$

$$\text{CDF} = \text{NORMDIST}(\text{TD}, \mu, \sigma, \text{TRUE}) \quad (2.8)$$

Where:

PDF = probability density function

CDF = cumulative density function

TD = total duration

μ = duration mean or ATD

π = 3.1415929

σ = duration standard deviation or SD

Table B1. PERT Normal Distribution (3 Activities)

TD	PDF	CDF
4	3.22115E-05	8.5875E-06
5	0.000814924	0.000263319
6	0.010317077	0.004209062
7	0.06536287	0.035711675
8	0.207224036	0.165842528
9	0.328763606	0.444853439
10	0.261012876	0.755963027
11	0.103699036	0.936423484
12	0.02061684	0.99080019
13	0.002051181	0.999287404
14	0.000102122	0.999971081
15	2.54432E-06	0.999999393

Monte Carlo PERT (MC PERT):

Random Number Function:

$$t_a = O + \text{RAND}() * (P - O) \quad (2.9)$$

Sample Calculation:

$$t_A = 2 + \text{RAND}() * (6 - 2)$$

Note: Duration of A, B, C are written in the 1st row of iteration and copied 499 times.

The scheduling is done using Forward Pass calculation.

The normal distribution is calculated using the same formula as in the PERT's. The mean and standard deviation are calculated as follows:

$$\mu = \text{AVERAGE}(\text{TDI1} : \text{TDI500})$$

$$\sigma = \text{STDEV}(\text{TDI1} : \text{TDI500})$$

Where:

TDI = total duration of ith number of iteration

Table B2. Monte Carlo PERT Iterations

Ite.	ACTIVITY			NODE			TD
	A	B	C	0	1	2	
1	5.014208959	3.501909515	6.543622799	0	5.014208959	11.55783176	11.55783176
2	2.579948283	4.106975049	4.388043423	0	2.579948283	6.967991706	6.967991706
3	4.83516769	6.216880642	4.770396394	0	4.83516769	9.605564084	9.605564084
4	5.34986229	4.393926976	6.882140555	0	5.34986229	12.23200284	12.23200284
5	3.258332823	5.272744073	6.198970723	0	3.258332823	9.457303546	9.457303546
6	3.764566587	8.450909163	5.990322694	0	3.764566587	9.754889282	9.754889282
7	3.418372427	8.186321107	6.010685286	0	3.418372427	9.429057713	9.429057713
8	4.173889389	6.148354839	5.959746607	0	4.173889389	10.133636	10.133636
9	3.403061768	7.610817293	4.236196376	0	3.403061768	7.639258144	7.639258144
10	5.321060145	5.637176828	4.732747169	0	5.321060145	10.05380731	10.05380731
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
500	5.308782485	5.250452623	4.298567537	0	5.308782485	9.607350022	9.607350022

B2. 7 Activities Testing

Activity	Predecessor	r	$t_a, r \text{ min}$	$t_a, r \text{ max}$
A	-	100%	2	6
		75%	3	5
		50%	4	4
B	-	100%	3	9
		75%	4	8
		50%	5	7
C	A	100%	4	7
		75%	5	6
		50%	-	-
D	A	100%	4	10
		75%	5	9
		50%	6	8
E	B, C	100%	4	7
		75%	5	6
		50%	-	-
F	D	100%	3	8
		75%	4	7
		50%	5	6
G	E	100%	3	8
		75%	4	7
		50%	5	6

Activity A		Activity B		Activity C		Activity D	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$	t_D	$P_{D,t}$
2	12.50%	3	12.50%	4	12.50%	4	12.50%
3	12.50%	4	12.50%	5	37.50%	5	12.50%
4	50.00%	5	16.67%	6	37.50%	6	16.67%
5	12.50%	6	16.67%	7	12.50%	7	16.67%
6	12.50%	7	16.67%			8	16.67%
		8	12.50%			9	12.50%
		9	12.50%			10	12.50%

Activity E		Activity F		Activity G	
t_E	$P_{E,t}$	t_F	$P_{F,t}$	t_G	$P_{G,t}$
4	12.50%	3	12.50%	3	12.50%
5	37.50%	4	12.50%	4	12.50%
6	37.50%	5	25.00%	5	25.00%
7	12.50%	6	25.00%	6	25.00%
		7	12.50%	7	12.50%
		8	12.50%	8	12.50%

Terminal 1

Predecessor
Activity

-
A

t_0	t_A	t_1	$P_{1,t}$
0	2	2	12.50%
0	3	3	12.50%
0	4	4	50.00%
0	5	5	12.50%
0	6	6	12.50%

Terminal 2

Predecessor
Activity

-
B

t_0	t_B	t_2	$P_{2,t}$
0	3	3	12.50%
0	4	4	12.50%
0	5	5	16.67%
0	6	6	16.67%
0	7	7	16.67%
0	8	8	12.50%
0	9	9	12.50%

Terminal 3

Predecessor
Activity

A
C

t_1	t_c	t_3	$P_{3,t}$
2	4	6	1.56%
3	5	7	6.25%
4	6	8	15.63%
5	7	9	26.56%
6		10	26.56%
		11	15.63%
		12	6.25%
		13	1.56%

i	j	k	l	m	o
0	0	0	1	0	3
1	1	1	2	1	2
2	2	2	3	2	1
3	3	3	4	3	0

Terminal 4

Predecessor
Activity

A
D

t_1	t_D	t_4	$P_{4,t}$
2	4	6	1.56%
3	5	7	3.13%
4	6	8	9.90%
5	7	9	11.98%
6	8	10	15.63%
	9	11	15.63%
	10	12	15.63%
		13	11.98%
		14	9.90%
		15	3.13%
		16	1.56%

i	j	k	l	m	o
0	0	0	1	0	3
1	1	1	2	1	2
2	2	2	3	2	1
3	3	3	4	3	0
4	4			4	
	5			4	
	6			4	

Terminal 5

Predecessor
Activity

B, C
E

t_3	t_E	t_5	$P_{5,t}$
6	4	10	0.20%
7	5	11	1.37%
8	6	12	4.88%
9	7	13	11.72%
10		14	19.92%
11		15	23.83%
12		16	19.92%
13		17	11.72%
		18	4.88%
		19	1.37%
		20	0.20%

i	j	k	l	m	o
0	0	0	1	0	3
1	1	1	2	1	3
2	2	2	3	2	3
3	3	3	4	3	3
			5		2
			6		1
			7		0

Note: Activity B and C lead to Activity E, i.e. Terminal 3 durations are chosen.

Terminal 6 Sample Calculations:

$$j = 0 \rightarrow (t_{F,\max} - t_{F,\min}) = 0 \rightarrow (8 - 3) = 0 \rightarrow 5$$

$$m = \min[j, \min(t_{4,\max} - t_{4,\min}, t_{F,\max} - t_{F,\min})]$$

$$= \min[j, \min(16 - 6, 8 - 3)] = \min[j, \min(10, 5)] = \min[j, 5]$$

$$\ell = 1 \rightarrow (t_{4,\max} - t_{4,\min}) = 1 \rightarrow (16 - 6) = 1 \rightarrow 10$$

$$o = \min[(t_{4,\max} - t_{4,\min} - \ell), \min(t_{4,\max} - t_{4,\min}, t_{F,\max} - t_{F,\min})]$$

$$= \min[(16 - 6 - \ell), \min(10, 5)] = \min[(10 - \ell), 5]$$

$$t_6 = \begin{cases} t_{4,\min} + t_{F,\min} + j = 6 + 3 + j = 9 + j, & j = 0 \rightarrow 5, \\ t_{4,\min} + t_{F,\max} + \ell = 6 + 8 + \ell = 14 + \ell, & \ell = 1 \rightarrow 10. \end{cases}$$

$$P_{6,t} = \begin{cases} \sum_{i=0}^m [P_{4,6+i} \times P_{F,3+j-i}], & t_6 = 9 + j, \\ \sum_{k=0}^0 [P_{4,6+\ell+k} \times P_{F,8-k}], & t_6 = 14 + \ell. \end{cases}$$

For $j = 0 \rightarrow 5$:

$$P_{6,9} = \sum_{i=0}^0 [P_{4,6+i} \times P_{F,3+0-i}] = P_{4,6+0} \times P_{F,3+0-0} = P_{4,6} \times P_{F,3} = 1.56\% \times 12.5\% = 0.20\%$$

$$P_{6,11} = \sum_{i=0}^2 [P_{4,6+i} \times P_{F,3+2-i}] = P_{4,6+0} \times P_{F,3+2-0} + P_{4,6+1} \times P_{F,3+2-1} + P_{4,6+2} \times P_{F,3+2-2}$$

$$= P_{4,6} \times P_{F,5} + P_{4,7} \times P_{F,4} + P_{4,8} \times P_{F,3}$$

$$= 1.56\% \times 25\% + 3.13\% \times 12.5\% + 9.9\% \times 12.5\% = 2.02\%$$

$$P_{6,14} = \sum_{i=0}^5 [P_{4,6+i} \times P_{F,3+5-i}]$$

$$= P_{4,6+0} \times P_{F,3+5-0} + P_{4,6+1} \times P_{F,3+5-1} + P_{4,6+2} \times P_{F,3+5-2} \\ + P_{4,6+3} \times P_{F,3+5-3} + P_{4,6+4} \times P_{F,3+5-4} + P_{4,6+5} \times P_{F,3+5-5}$$

$$= P_{4,6} \times P_{F,8} + P_{4,7} \times P_{F,7} + P_{4,8} \times P_{F,6} + P_{4,9} \times P_{F,5} \\ + P_{4,10} \times P_{F,4} + P_{4,11} \times P_{F,3}$$

$$= 1.56\% \times 12.5\% + 3.13\% \times 12.5\% + 9.9\% \times 25\% \\ + 11.98\% \times 25\% + 15.63\% \times 12.5\% + 15.63\% \times 12.5\%$$

$$= 9.96\%$$

For $\ell = 1 \rightarrow 10$:

$$P_{6,15} = \sum_{k=0}^5 [P_{4,6+1+k} \times P_{F,8-k}]$$

$$= P_{4,6+1+0} \times P_{F,8-0} + P_{4,6+1+1} \times P_{F,8-1} + P_{4,6+1+2} \times P_{F,8-2} \\ + P_{4,6+1+3} \times P_{F,8-3} + P_{4,6+1+4} \times P_{F,8-4} + P_{4,6+1+5} \times P_{F,8-5}$$

$$= P_{4,7} \times P_{F,8} + P_{4,8} \times P_{F,7} + P_{4,9} \times P_{F,6} + P_{4,10} \times P_{F,5} \\ + P_{4,11} \times P_{F,4} + P_{4,12} \times P_{F,3}$$

$$= 3.13\% \times 12.5\% + 9.9\% \times 12.5\% + 11.98\% \times 25\% \\ + 15.63\% \times 25\% + 15.63\% \times 12.5\% + 15.63\% \times 12.5\%$$

$$= 12.43\%$$

$$P_{6,18} = \sum_{k=0}^5 [P_{4,6+4+k} \times P_{F,8-k}]$$

$$= P_{4,6+4+0} \times P_{F,8-0} + P_{4,6+4+1} \times P_{F,8-1} + P_{4,6+4+2} \times P_{F,8-2} \\ + P_{4,6+4+3} \times P_{F,8-3} + P_{4,6+4+4} \times P_{F,8-4} + P_{4,6+4+5} \times P_{F,8-5}$$

$$= P_{4,10} \times P_{F,8} + P_{4,11} \times P_{F,7} + P_{4,12} \times P_{F,6} + P_{4,13} \times P_{F,5} \\ + P_{4,14} \times P_{F,4} + P_{4,15} \times P_{F,3}$$

$$= 15.63\% \times 12.5\% + 15.63\% \times 12.5\% + 15.63\% \times 25\% \\ + 11.98\% \times 25\% + 9.9\% \times 12.5\% + 3.13\% \times 12.5\%$$

$$= 12.43\%$$

$$P_{6,19} = \sum_{k=0}^4 [P_{4,6+5+k} \times P_{F,8-k}]$$

$$= P_{4,6+5+0} \times P_{F,8-0} + P_{4,6+5+1} \times P_{F,8-1} + P_{4,6+5+2} \times P_{F,8-2} \\ + P_{4,6+5+3} \times P_{F,8-3} + P_{4,6+5+4} \times P_{F,8-4}$$

$$= P_{4,11} \times P_{F,8} + P_{4,12} \times P_{F,7} + P_{4,13} \times P_{F,6} + P_{4,14} \times P_{F,5} \\ + P_{4,15} \times P_{F,4}$$

$$= 15.63\% \times 12.5\% + 15.63\% \times 12.5\% + 11.98\% \times 25\% \\ + 9.9\% \times 25\% + 3.13\% \times 12.5\%$$

$$= 9.96\%$$

$$P_{6,22} = \sum_{k=0}^2 [P_{4,6+8+k} \times P_{F,8-k}] = P_{4,6+8+0} \times P_{F,8-0} + P_{4,6+8+1} \times P_{F,8-1} + P_{4,6+8+2} \times P_{F,8-2}$$

$$= P_{4,14} \times P_{F,8} + P_{4,15} \times P_{F,7} + P_{4,16} \times P_{F,6}$$

$$= 9.9\% \times 12.5\% + 3.13\% \times 12.5\% + 1.56\% \times 25\% = 2.02\%$$

$$P_{6,24} = \sum_{k=0}^0 [P_{4,6+10+k} \times P_{F,8-k}] = P_{4,6+10+0} \times P_{F,8-0} = P_{4,16} \times P_{F,8} = 1.56\% \times 12.5\% = 0.20\%$$

Terminal 6

Predecessor
Activity

D
F

t_4	t_F	t_6	$P_{6,t}$
6	3	9	0.20%
7	4	10	0.59%
8	5	11	2.02%
9	6	12	3.91%
10	7	13	6.90%
11	8	14	9.96%
12		15	12.43%
13		16	14.00%
14		17	14.00%
15		18	12.43%
16		19	9.96%
		20	6.90%
		21	3.91%
		22	2.02%
		23	0.59%
		24	0.20%

i	j	k	l	m	o
0	0	0	1	0	5
1	1	1	2	1	5
2	2	2	3	2	5
3	3	3	4	3	5
4	4	4	5	4	5
5	5	5	6	5	4
			7		3
			8		2
			9		1
			10		0

Terminal 7

Predecessor
Activity

E
G

t_5

10
11
12
13
14
15
16
17
18
19
20

t_6

3
4
5
6
7
8

t_7

13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28

$P_{7,t}$

0.02%
0.20%
0.83%
2.47%
5.54%
9.81%
14.16%
16.97%
16.97%
14.16%
9.81%
5.54%
2.47%
0.83%
0.20%
0.02%

i

0
1
2
3
4
5

j

0
1
2
3
4
5

k

0
1
2
3
4
5

l

1
2
3
4
5
6
7
8
9
10

m

0
1
2
3
4
5

o

5
5
5
5
5
4
3
2
1
0

Comparison to CPM, PERT, and MC PERT:

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	2	4	6
B	-	3	5	9
C	A	4	5	7
D	A	4	6	10
E	B, C	4	5	7
F	D	3	4	8
G	E	3	5	8

Activity	t_o	μ_o	σ_o
A	4.0	4	0.666666667
B	5.3	5.333333333	1
C	5.2	5.166666667	0.5
D	6.3	6.333333333	1
E	5.2	5.166666667	0.5
F	4.5	4.5	0.833333333
G	5.2	5.166666667	0.833333333

CPM (Calculated using t_o):

Node	T_N
0	0
1	4.0
2	9.2
3	10.3
4	14.3
5	19.5

TD = 19.5

Figure B2. 7 Activities Network Diagram

PERT (Calculated using μ_e):

Node	T_N
0	0
1	4
2	9.166666667
3	10.33333333
4	14.33333333
5	19.5

ATD = 19.5

SD = 1.280190958

Table B3. PERT Normal Distribution (7 Activities)

TD	PDF	CDF
14	3.05923E-05	8.6924E-06
15	0.000646515	0.000219837
16	0.007422525	0.003128868
17	0.046294716	0.025419584
18	0.156862059	0.120658779
19	0.288742794	0.348058809
20	0.288742794	0.651941191
21	0.156862059	0.879341221
22	0.046294716	0.974580416
23	0.007422525	0.996871132
24	0.000646515	0.999780163
25	3.05923E-05	0.999991308

B3. 9 Activities Testing

Activity	Predecessor	r	$t_{a, r \min}$	$t_{a, r \max}$
A	-	100%	3	6
		75%	4	5
		50%	-	-
B	-	100%	2	5
		75%	3	4
		50%	-	-
C	A	100%	7	12
		75%	8	10
		50%	9	9
D	A	100%	5	9
		75%	6	8
		50%	7	7
E	B, C	100%	8	12
		75%	9	11
		50%	10	10
F	B, C	100%	10	14
		75%	11	13
		50%	12	12
G	D, E	100%	1	4
		75%	2	3
		50%	-	-
H	D, E	100%	4	6
		75%	-	-
		50%	5	5
I	F, G	100%	5	9
		75%	6	8
		50%	7	7

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
3	12.50%	2	12.50%	7	8.33%
4	37.50%	3	37.50%	8	12.50%
5	37.50%	4	37.50%	9	50.00%
6	12.50%	5	12.50%	10	12.50%
				11	8.33%
				12	8.33%

Activity D		Activity E		Activity F	
t_D	$P_{D,t}$	t_E	$P_{E,t}$	t_F	$P_{F,t}$
5	12.50%	8	12.50%	10	12.50%
6	12.50%	9	12.50%	11	12.50%
7	50.00%	10	50.00%	12	50.00%
8	12.50%	11	12.50%	13	12.50%
9	12.50%	12	12.50%	14	12.50%

Activity G		Activity H		Activity I	
t_G	$P_{G,t}$	t_H	$P_{H,t}$	t_I	$P_{I,t}$
1	12.50%	4	25.00%	5	12.50%
2	37.50%	5	50.00%	6	12.50%
3	37.50%	6	25.00%	7	50.00%
4	12.50%			8	12.50%
				9	12.50%

Terminal 1

Predecessor
Activity

-
A

t_0	t_A	t_1	$P_{1,t}$
0	3	3	12.50%
0	4	4	37.50%
0	5	5	37.50%
0	6	6	12.50%

Terminal 2

Predecessor
Activity

-
B

t_0	t_B	t_2	$P_{2,t}$
0	2	2	12.50%
0	3	3	37.50%
0	4	4	37.50%
0	5	5	12.50%

Terminal 3

Predecessor
Activity

A
C

t_1

t_c

t_3

$P_{3,t}$

3

7

10

1.04%

4

8

11

4.69%

5

9

12

14.06%

6

10

13

26.04%

11

14

26.04%

12

15

15.10%

16

7.81%

17

4.17%

18

1.04%

i

j

k

l

m

o

0

0

0

1

0

2

1

1

1

2

1

1

2

2

2

3

2

0

3

3

3

4

3

5

3

Terminal 4

Predecessor
Activity

A
D

t_1

t_b

t_4

$P_{4,t}$

3

5

8

1.56%

4

6

9

6.25%

5

7

10

15.63%

6

8

11

26.56%

9

12

26.56%

13

15.63%

14

6.25%

15

1.56%

i

j

k

l

m

o

0

0

0

1

0

2

1

1

1

2

1

1

2

2

2

3

2

0

3

3

3

4

3

Terminal 5

Predecessor
Activity

B, C
E

t_3	t_E	t_5	$P_{5,t}$
10	8	18	0.13%
11	9	19	0.72%
12	10	20	2.86%
13	11	21	7.49%
14	12	22	14.26%
15		23	20.51%
16		24	20.90%
17		25	15.56%
18		26	9.70%
		27	5.08%
		28	2.02%
		29	0.65%
		30	0.13%

i	j	k	l	m	o
0	0	0	1	0	4
1	1	1	2	1	4
2	2	2	3	2	4
3	3	3	4	3	4
4	4	4	5	4	3
			6		2
			7		1
			8		0

Terminal 6

Predecessor
Activity

B, C
F

t_3	t_F	t_6	$P_{6,t}$
10	10	20	0.13%
11	11	21	0.72%
12	12	22	2.86%
13	13	23	7.49%
14	14	24	14.26%
15		25	20.51%
16		26	20.90%
17		27	15.56%
18		28	9.70%
		29	5.08%
		30	2.02%
		31	0.65%
		32	0.13%

i	j	k	l	m	o
0	0	0	1	0	4
1	1	1	2	1	4
2	2	2	3	2	4
3	3	3	4	3	4
4	4	4	5	4	3
			6		2
			7		1
			8		0

Terminal 7

Predecessor
Activity

D, E
G

t_s	t_G	t_T	$P_{7,t}$
18	1	19	0.02%
19	2	20	0.14%
20	3	21	0.68%
21	4	22	2.29%
22		23	5.75%
23		24	11.08%
24		25	16.59%
25		26	19.25%
26		27	17.45%
27		28	12.72%
28		29	7.74%
29		30	3.96%
30		31	1.65%
		32	0.55%
		33	0.13%
		34	0.02%

i	j	k	l	m	o
0	0	0	1	0	3
1	1	1	2	1	3
2	2	2	3	2	3
3	3	3	4	3	3
			5		3
			6		3
			7		3
			8		3
			9		3
			10		2
			11		1
			12		0

Terminal 7 Excel Inputs:

For j column (at j = 1):

$$f_x = \text{IF}(j - 1 < (t_{G,\max} - t_{G,\min}), j, t_{G,\max} - t_{G,\min}) = \text{IF}(j - 1 < (4 - 1), j, 4 - 1) = \text{IF}(j - 1 < 3, j, 3)$$

For ℓ column (at $\ell = 2$):

$$f_x = \text{IF}(\ell - 1 < (t_{5,\max} - t_{5,\min}), \ell, t_{5,\max} - t_{5,\min}) = \text{IF}(\ell - 1 < 12, \ell, 12)$$

For m column (for j):

$$f_x = \text{MIN}(j, \text{MIN}(12, 3)) = \text{MIN}(j, 3)$$

For o column (for ℓ):

$$f_x = \text{MIN}((12 - \ell), \text{MIN}(12, 3)) = \text{MIN}((12 - \ell), 3)$$

For t_7 column:

For j = 0 $\rightarrow$ 3:

$$f_x = t_{5,\min} + t_{G,\min} + j = 18 + 1 + j = 19 + j$$

For $\ell = 1 \rightarrow 12$:

$$f_x = 18 + 4 + \ell = 22 + \ell$$

For $i = 0$ and $j = 0$:

$$P_{7,19} = \text{SUM}(\text{VLOOKUP}(t_{5,\min} + i, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i, t_{G,\min} : P_{G,\max}, 2))$$

For $i = 1$ and $j = 1$:

$$P_{7,20} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + i - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $i = 2$ and $j = 2$:

$$P_{7,21} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + i - 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $i = 3$ and $j = 3$:

$$P_{7,22} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + i - 3, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 3, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i - 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + i, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\min} + j - i, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 3$ and $\ell = 1$ ($o = 3$):

$$P_{7,23} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + \ell + k - 3, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 3, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k - 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 3$ and $\ell = 2$ ($o = 3$):

$$P_{7,24} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + \ell + k - 3, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 3, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k - 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 3$ and $\ell = 3 \rightarrow 9$ ($o = 3$):

$$P_{7,25 \rightarrow 31} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + \ell, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max}, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + 3, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - 3, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 2$ and $\ell = 10$ ($o = 2$):

$$P_{7,32} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + \ell + k - 2, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 2, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 1$ and $\ell = 11$:

$$P_{7,33} = \text{SUM} \left(\begin{array}{l} \text{VLOOKUP}(t_{5,\min} + \ell + k - 1, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k + 1, t_{G,\min} : P_{G,\max}, 2) \\ \text{VLOOKUP}(t_{5,\min} + \ell + k, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k, t_{G,\min} : P_{G,\max}, 2) \end{array} \right)$$

For $k = 0$ and $\ell = 12$:

$$P_{7,34} = \text{SUM}(\text{VLOOKUP}(t_{5,\min} + \ell + k, t_{5,\min} : P_{5,\max}, 2) * \text{VLOOKUP}(t_{G,\max} - k, t_{G,\min} : P_{G,\max}, 2))$$

Terminal 8

Predecessor
Activity

D, E
H

t_s	t_H	t_8	$P_{8,t}$
18	4	22	0.03%
19	5	23	0.24%
20	6	24	1.11%
21		25	3.48%
22		26	8.02%
23		27	14.13%
24		28	19.04%
25		29	19.47%
26		30	15.43%
27		31	10.01%
28		32	5.47%
29		33	2.44%
30		34	0.86%
		35	0.23%
		36	0.03%

i	j	k	l	m	o
0	0	0	1	0	2
1	1	1	2	1	2
2	2	2	3	2	2
			4		2
			5		2
			6		2
			7		2
			8		2
			9		2
			10		2
			11		1
			12		0

Terminal 9

Predecessor
Activity

F, G
I

t_7	t_1	t_9	$P_{9,t}$
19	5	24	0.00%
20	6	25	0.02%
21	7	26	0.11%
22	8	27	0.44%
23	9	28	1.36%
24		29	3.35%
25		30	6.71%
26		31	11.02%
27		32	14.98%
28		33	16.86%
29		34	15.76%
30		35	12.41%
31		36	8.34%
32		37	4.81%
33		38	2.37%
34		39	0.99%
		40	0.34%
		41	0.09%
		42	0.02%
		43	0.00%

i	j	k	l	m	o
0	0	0	1	0	4
1	1	1	2	1	4
2	2	2	3	2	4
3	3	3	4	3	4
4	4	4	5	4	4
			6		4
			7		4
			8		4
			9		4
			10		4
			11		4
			12		3
			13		2
			14		1
			15		0

Comparison to CPM, PERT, and MC PERT:

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	3	4	6
B	-	2	3	5
C	A	7	8	12
D	A	5	7	9
E	B, C	8	9	12
F	B, C	10	12	14
G	D, E	1	2	4
H	D, E	4	5	6
I	F, G	5	6	9

Activity	t_e	μ_e	σ_e
A	4.2	4.166666667	0.5
B	3.2	3.166666667	0.5
C	8.5	8.5	0.833333333
D	7.0	7	0.666666667
E	9.3	9.333333333	0.666666667
F	12.0	12	0.666666667
G	2.2	2.166666667	0.5
H	5.0	5	0.333333333
I	6.3	6.333333333	0.666666667

CPM (Calculated using t_e):

Node	T_N
0	0
1	4.2
2	12.7
3	22.0
4	24.7
5	31.0

TD = 31

Figure B3. 9 Activities Network Diagram

PERT (Calculated using μ_0):

Node	T_N
0	0
1	4.166666667
2	12.66666667
3	22
4	24.66666667
5	31

ATD = 31

SD = 1.354006401

Table B4. PERT Normal Distribution (9 Activities)

TD	PDF	CDF
25	1.60438E-05	4.68347E-06
26	0.000322248	0.000110923
27	0.003751338	0.001567425
28	0.025310097	0.013357849
29	0.098972274	0.069824699
30	0.224308328	0.230090468
31	0.294638401	0.5
32	0.224308328	0.769909532
33	0.098972274	0.930175301
34	0.025310097	0.986642151
35	0.003751338	0.998432575
36	0.000322248	0.999889077
37	1.60438E-05	0.999995317

APPENDIX C – PROJECT SCHEDULE REVISION APPLICATION

C1. 7 Activities Project Schedule Revision

Activity A		Activity B		Activity C		Activity D	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$	t_D	$P_{D,t}$
5	100%	6	100%	6	100%	8	100%

Activity E		Activity F		Activity G	
t_E	$P_{E,t}$	t_F	$P_{F,t}$	t_G	$P_{G,t}$
4	12.50%	3	12.50%	3	12.50%
5	37.50%	4	12.50%	4	12.50%
6	37.50%	5	25.00%	5	25.00%
7	12.50%	6	25.00%	6	25.00%
		7	12.50%	7	12.50%
		8	12.50%	8	12.50%

Terminal 1

Predecessor -
Activity A

t_0	t_A	t_1	$P_{1,t}$
0	5	5	100%

Terminal 2

Predecessor -
Activity B

t_0	t_B	t_2	$P_{2,t}$
0	6	6	100%

Terminal 3

Predecessor
Activity

A
C

t_1	t_c	t_3	$P_{3,t}$
5	6	11	100%

Terminal 4

Predecessor
Activity

A
D

t_1	t_D	t_4	$P_{4,t}$
5	8	13	100%

Terminal 5

Predecessor
Activity

B, C
E

t_3	t_E	t_5	$P_{5,t}$
11	4	15	12.50%
	5	16	37.50%
	6	17	37.50%
	7	18	12.50%

i	j	m
0	0	0
	1	0
	2	0
	3	0

Terminal 5 Sample Calculation:

$$j = 0 \rightarrow (t_{E,\max} - t_{E,\min}) = 0 \rightarrow (7 - 4) = 0 \rightarrow 3$$

$$m = \min[j, \min(t_{3,\max} - t_{3,\min}, t_{E,\max} - t_{E,\min})]$$

$$= \min[j, \min(11 - 11, 7 - 4)] = \min[j, \min(0, 3)] = \min[j, 0] = 0$$

Note: ℓ and o is omitted because $t_{3,\max} - t_{3,\min} = 0$

$$t_5 = t_{3,\min} + t_{E,\min} + j = 11 + 4 + j = 15 + j, \quad j = 0 \rightarrow 3$$

$$P_{5,t} = \sum_{i=0}^m [P_{3,11+i} \times P_{E,4+j-i}], \quad t_5 = 15 + j$$

For $j = 0 \rightarrow 3$, $t_5 = 15 \rightarrow 18$:

$$P_{5,15} = \sum_{i=0}^0 [P_{3,11+i} \times P_{E,4+0-i}] = P_{3,11+0} \times P_{E,4+0-0} = P_{3,11} \times P_{E,4} = 100\% \times 12.5\% = 12.5\%$$

$$P_{5,16} = \sum_{i=0}^0 [P_{3,11+i} \times P_{E,4+1-i}] = P_{3,11+0} \times P_{E,4+1-0} = P_{3,11} \times P_{E,5} = 100\% \times 37.5\% = 37.5\%$$

$$P_{5,17} = \sum_{i=0}^0 [P_{3,11+i} \times P_{E,4+2-i}] = P_{3,11+0} \times P_{E,4+2-0} = P_{3,11} \times P_{E,6} = 100\% \times 37.5\% = 37.5\%$$

$$P_{5,18} = \sum_{i=0}^0 [P_{3,11+i} \times P_{E,4+3-i}] = P_{3,11+0} \times P_{E,4+3-0} = P_{3,11} \times P_{E,7} = 100\% \times 12.5\% = 12.5\%$$

Terminal 6

Predecessor
Activity

D
F

t_4
13

t_F
3
4
5
6
7
8

t_6
16
17
18
19
20
21

$P_{6,t}$
12.50%
12.50%
25.00%
25.00%
12.50%
12.50%

i
0

j
0
1
2
3
4
5

m
0
0
0
0
0
0

Terminal 7

Predecessor
Activity

E
G

t_5
15
16
17
18

t_G
3
4
5
6
7
8

t_7
18
19
20
21
22
23
24
25
26

$P_{7,t}$
1.56%
6.25%
12.50%
18.75%
21.88%
18.75%
12.50%
6.25%
1.56%

i
0
1
2
3

j
0
1
2
3
4
5

k
0
1
2

l
1
2
3

m
0
1
2
3
3
3

o
2
1
0

Comparison to CPM and PERT:

Activity	Predecessor	Optimistic	Most Likely	Pessimistic
A	-	5	5	5
B	-	6	6	6
C	A	6	6	6
D	A	8	8	8
E	B, C	4	5	7
F	D	3	4	8
G	E	3	5	8

Activity	t_e	μ_e	σ_e
A	5.0	5	0
B	6.0	6	0
C	6.0	6	0
D	8.0	8	0
E	5.2	5.166666667	0.5
F	4.5	4.5	0.833333333
G	5.2	5.166666667	0.833333333

CPM (Calculated using t_e):

Node	T_N
0	0
1	5.0
2	11.0
3	13.0
4	16.2
5	21.3

TD = 21.3

PERT (Calculated using μ_e):

Node	T_N
0	0
1	5
2	11
3	13
4	16.16666667
5	21.33333333

ATD = 21.33333333

SD = 0.971825316

Table C1. PERT Normal Distribution (7 Activities Revision)

TD	PDF	CDF
17	1.97662E-05	4.12146E-06
18	0.001144586	0.000301871
19	0.022989625	0.008175604
20	0.160167394	0.085033538
21	0.387057176	0.365800357
22	0.324440394	0.753641734
23	0.094330797	0.956826124
24	0.009513293	0.996964959
25	0.000332788	0.999919316
26	4.03796E-06	0.999999213
27	1.69948E-08	0.999999997
28	2.48101E-11	1

C2. 9 Activities Project Schedule Revision

Project to Activity C:

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
4	100%	4	100%	9	100%

Activity D		Activity E		Activity F	
t_D	$P_{D,t}$	t_E	$P_{E,t}$	t_F	$P_{F,t}$
5	12.50%	8	12.50%	10	12.50%
6	12.50%	9	12.50%	11	12.50%
7	50.00%	10	50.00%	12	50.00%
8	12.50%	11	12.50%	13	12.50%
9	12.50%	12	12.50%	14	12.50%

Activity G		Activity H		Activity I	
t_G	$P_{G,t}$	t_H	$P_{H,t}$	t_I	$P_{I,t}$
1	12.50%	4	25.00%	5	12.50%
2	37.50%	5	50.00%	6	12.50%
3	37.50%	6	25.00%	7	50.00%
4	12.50%			8	12.50%
				9	12.50%

Terminal 1

Predecessor
Activity

-
A

t_0
0

t_A
4

t_1
4

$P_{1,t}$
100%

Terminal 2

Predecessor Activity - B

t_0	t_B	t_2	$P_{2,t}$
0	4	4	100%

Terminal 3

Predecessor Activity A C

t_1	t_C	t_3	$P_{3,t}$
4	9	13	100%

Terminal 4

Predecessor Activity A D

t_1	t_D	t_4	$P_{4,t}$
4	5	9	12.50%
	6	10	12.50%
	7	11	50.00%
	8	12	12.50%
	9	13	12.50%

i	j	m
0	0	0
	1	0
	2	0
	3	0
	4	0

Terminal 4 Excel Functions:

For j column (at j = 1):

$$f_x = IF(j-1 <> (t_{D,max} - t_{D,min}), j, t_{D,max} - t_{D,min}) = IF(j-1 <> (9-5), j, 9-5) = IF(j-1 <> 4, j, 4)$$

For m column (for j):

$$f_x = \text{MIN}(j, \text{MIN}(0, 4)) = \text{MIN}(j, 0) = 0$$

For t_4 column:

For $j = 0 \rightarrow 4$:

$$f_x = t_{1,\min} + t_{D,\min} + j = 4 + 5 + j = 9 + j$$

For $i = 0$ and $j = 0$:

$$P_{4,9} = \text{SUM}(\text{VLOOKUP}(t_{1,\min} + i, t_{1,\min} : P_{1,\max}, 2) * \text{VLOOKUP}(t_{D,\min} + j - i, t_{D,\min} : P_{D,\max}, 2))$$

For $i = 0$ and $j = 1 \rightarrow 4$:

$$P_{4,10 \rightarrow 13} = \text{SUM}(\text{VLOOKUP}(t_{1,\min}, t_{1,\min} : P_{1,\max}, 2) * \text{VLOOKUP}(t_{D,\min} + j, t_{D,\min} : P_{D,\max}, 2))$$

Terminal 5

Predecessor
Activity

B, C
E

t_3
13

t_E
8
9
10
11
12

t_5
21
22
23
24
25

$P_{5,t}$
12.50%
12.50%
50.00%
12.50%
12.50%

i	j	m
0	0	0
	1	0
	2	0
	3	0
	4	0

Terminal 6

Predecessor
Activity

B, C
F

t_3
13

t_F
10
11
12
13
14

t_6
23
24
25
26
27

$P_{6,t}$
12.50%
12.50%
50.00%
12.50%
12.50%

i	j	m
0	0	0
	1	0
	2	0
	3	0
	4	0

Terminal 7

Predecessor
Activity

D, E
G

t_5
21
22
23
24
25

t_G
1
2
3
4

t_7
22
23
24
25
26
27
28
29

$P_{7,t}$
1.56%
6.25%
15.63%
26.56%
26.56%
15.63%
6.25%
1.56%

i	j	k	l	m	o
0	0	0	1	0	3
1	1	1	2	1	2
2	2	2	3	2	1
3	3	3	4	3	0

Terminal 8

Predecessor
Activity

D, E
H

t_s	t_H	t_s	$P_{s,t}$
21	4	25	3.13%
22	5	26	9.38%
23	6	27	21.88%
24		28	31.25%
25		29	21.88%
		30	9.38%
		31	3.13%

i	j	k	l	m	o
0	0	0	1	0	2
1	1	1	2	1	2
2	2	2	3	2	1
			4		0

Terminal 9

Predecessor
Activity

F, G
I

t_7	t_i	t_s	$P_{s,t}$
22	5	27	0.20%
23	6	28	0.98%
24	7	29	3.52%
25	8	30	8.59%
26	9	31	15.43%
27		32	21.29%
28		33	21.29%
29		34	15.43%
		35	8.59%
		36	3.52%
		37	0.98%
		38	0.20%

i	j	k	l	m	o
0	0	0	1	0	4
1	1	1	2	1	4
2	2	2	3	2	4
3	3	3	4	3	3
4	4	4	5	4	2
			6		1
			7		0

Project to Activity E:

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
4	100%	4	100%	9	100%

Activity D		Activity E		Activity F	
t_D	$P_{D,t}$	t_E	$P_{E,t}$	t_F	$P_{F,t}$
8	100%	10	100%	10	12.50%
				11	12.50%
				12	50.00%
				13	12.50%
				14	12.50%

Activity G		Activity H		Activity I	
t_G	$P_{G,t}$	t_H	$P_{H,t}$	t_I	$P_{I,t}$
1	12.50%	4	25.00%	5	12.50%
2	37.50%	5	50.00%	6	12.50%
3	37.50%	6	25.00%	7	50.00%
4	12.50%			8	12.50%
				9	12.50%

Terminal 1

Predecessor
Activity

-
A

t_0
0

t_A
4

t_1
4

$P_{1,t}$
100%

Terminal 2

Predecessor
Activity

-
B

t_0
0

t_B
4

t_2
4

$P_{2,t}$
100%

Terminal 3

Predecessor
Activity

A
C

t_1
4

t_C
9

t_3
13

$P_{3,t}$
100%

Terminal 4

Predecessor
Activity

A
D

t_1
4

t_D
8

t_4
12

$P_{4,t}$
100%

Terminal 5

Predecessor
Activity

B, C
E

t_3
13

t_E
10

t_5
23

$P_{5,t}$
100%

Terminal 6

Predecessor
Activity

B, C
F

t_3
13

t_F
10
11
12
13
14

t_6
23
24
25
26
27

$P_{6,t}$
12.50%
12.50%
50.00%
12.50%
12.50%

i
0

j
0
1
2
3
4

m
0
0
0
0
0

Terminal 7

Predecessor
Activity

D, E
G

t_5
23

t_6
1
2
3
4

t_7
24
25
26
27

$P_{7,t}$
12.50%
37.50%
37.50%
12.50%

i
0

j
0
1
2
3

m
0
0
0
0

Terminal 8

Predecessor
Activity

D, E
H

t_5
23

t_H
4
5
6

t_8
27
28
29

$P_{8,t}$
25.00%
50.00%
25.00%

Activity I			Activity H			Activity G		
i	j	m	i	j	m	i	j	m
0	0	0	0	0	0	0	0	0
	1	0		1	0		1	0
	2	0		2	0		2	0

Terminal 9

Predecessor
Activity

F, G
I

t_i	t_j	t_g	$P_{g,t}$
24	5	29	1.56%
25	6	30	6.25%
26	7	31	15.63%
27	8	32	26.56%
	9	33	26.56%
		34	15.63%
		35	6.25%
		36	1.56%

i	j	k	l	m	o
0	0	0	1	0	2
1	1	1	2	1	1
2	2	2	3	2	0
3	3			3	
	4			3	

Project to Activity F ($t_F = 11$):

Activity A		Activity B		Activity C	
t_A	$P_{A,t}$	t_B	$P_{B,t}$	t_C	$P_{C,t}$
4	100%	4	100%	9	100%

Activity D		Activity E		Activity F	
t_D	$P_{D,t}$	t_E	$P_{E,t}$	t_F	$P_{F,t}$
8	100%	10	100%	11	100%

Activity G		Activity H		Activity I	
t_G	$P_{G,t}$	t_H	$P_{H,t}$	t_I	$P_{I,t}$
1	12.50%	4	25.00%	5	12.50%
2	37.50%	5	50.00%	6	12.50%
3	37.50%	6	25.00%	7	50.00%
4	12.50%			8	12.50%
				9	12.50%

Terminal 1

Predecessor
Activity

-
A

t_0
0

t_A
4

t_1
4

$P_{1,t}$
100%

Terminal 2

Predecessor
Activity

-
B

t_0
0

t_B
4

t_2
4

$P_{2,t}$
100%

Terminal 3

Predecessor
Activity

A
C

t_1
4

t_C
9

t_3
13

$P_{3,t}$
100%

Terminal 4

Predecessor
Activity

A
D

t_1
4

t_D
8

t_4
12

$P_{4,t}$
100%

Terminal 5

Predecessor
Activity

B, C
E

t_3
13

t_E
10

t_5
23

$P_{5,t}$
100%

Terminal 6

Predecessor
Activity

B, C
F

t_3
13

t_F
11

t_6
24

$P_{6,t}$
100%

Terminal 7

Predecessor
Activity

D, E
G

t_5
23

t_G
1

t_7
24

$P_{7,t}$
12.50%
37.50%
37.50%
12.50%

i
0

j
0
1
2
3

m
0
0
0
0

Terminal 8

Predecessor
Activity

D, E
H

t_5
23

t_H
4
5
6

t_8
27
28
29

$P_{8,t}$
25.00%
50.00%
25.00%