PERFORMANCE OF PIEZOCERAMIC SENSORS

by

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Abstract

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This study is in continuation of the previous research conducted at Ryerson University to develop a model for active monitoring of reinforced concrete members using piezo-ceramic sensor. In the previous research large concrete beam were investigated. The following study examines the performance of piezo-ceramic sensor to monitor maturity of concrete and to evaluate crack width of large base width concrete plates. Small scale beams and large scale slabs specimens were tested under three and four points bending respectively to evaluate the effectiveness of piezo-sensors at different failure modes. Test results from small scale beams were used to model correlation between concrete crack width/strains and sensor signals. The test result indicated that piezo-sensor can be used for active concrete strength monitoring at later stages. It can also be used as an effective indicator of crack width for reinforced concrete members with small base width.

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Dedication

*This project is dedicated to my beloved parents and family for their understanding,
encouragement and undying support throughout the project.*

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List of Notations

A	Harmonic amplitude
A_r	received signal amplitude
A_s	Area of steel reinforcement
A_a	maximum amplitude of the actuation signal
$A_{f\ n,0}$	Initial Amplitude
$A_{f\ n,t}$	Final Amplitude
a	Shear span
a_1	distance to the top of the outer layer of reinforcement
a_2	distance to the inner layer of reinforcement
a/d	Shear span to depth ratio
b	base width
BPS	Band Pass Filter
C	wave velocity
C_P	Compressive wave (P-wave) velocity
C_S	Shear wave (S-wave) velocity
C_R	Rayleigh wave (R-wave) velocity
CG	Crack-Gauge
C.W.	Crack Width
d_{be}	bar diameter
h_{ef}	effective embedded thickness
E	Young's Modulus of elasticity
EM	Electro-magnetic waves
f	frequency
f_p	frequency of P-waves
f_s	frequency of S-waves
f'_c	Compressive strength of concrete
f_y	Yield strength
FOS	Fibre Optics Sensor
FBG	Fibre Bragg Grating
FFT	Fourier Transformation

GPR	Ground Penetrating Radar
HMI	Health Monitoring Index
I	amplitude of incident energy
IE	Impact Echo
IR	Impulse Response
I_p	Stress associated with incident P-wave
k_I	bond properties of coefficient
k_2	strain gradient
kN	Kilo Newton
kHz	Kilo Hertz
LVDT	linear Variable Differential Transformer
MSR	Miniature seismic reflection
mV	millivolt
n	refractive index of fibre core
nm	nano-meter
P_w	maximum propagated value
PZT	lead zirconate titanate
PVDF	polyvinylidene fluoride
r	tension stiffening factor
R	Amplitude of reflected energy
R_v	Amplitude attenuation coefficient
RC	Reinforced Concrete
S	maximum strength
SPR	Short Pulse radar
T	Time period
t	travel time
SASW	Spectral Analysis of Surface waves
SHM	Structural Health Monitoring
SSP	Split spectrum processing
S_{rm}	Average crack spacing
UTT	Ultrasonic through transmission
w_k	maximum crack width
w_m	average crack width

X	Distance
Z	acoustic impedance
Z_1	acoustic impedance of the first medium
Z_2	acoustic impedance of the second medium
σ_{sr}	stress in reinforcement at cracking load
σ_s	Stress in reinforcement at applied load
β	factor that takes into account loading procedure
ρ	density
ε_1	lowest tensile strain on the tension reinforcement
ε_2	largest tensile strain on the tension reinforcement
ϵ_1	di-electric constant of material-1
ϵ_2	di-electric constant of material-2
ν	Poisson's ratio
λ	wavelength
ω	angular frequency
φ	phase difference
Λ	fringe spacing
μ	dynamic Poisson's ratio

Chapter 1

INTRODUCTION

Reinforced concrete (RC) structures are widely used in the world due to their durability and economy. Offshore platforms, nuclear power containment, bridges and other critical structures require the most reliable and durable structural health monitoring system to maintain overall serviceability and safety throughout their life-span. This research investigates the development and implementation of piezo-ceramic sensor as a part of an active health monitoring system for reinforced concrete member. The proposed system uses embedded piezo-ceramic sensor to assess the structural integrity of RC structures throughout their lifetime. The proposed system not only provides a mean of establishing a damage index relative to its initial condition but also specifies the location of the damage. Due to increasing world economic demands, a large number of sensitive structure have come into existence. These structures are very expensive and are required to have extended lifespans (exceeding 50 years). It is therefore required to design cheap, yet durable and reliable method that can be used to efficiently monitor the overall health and performance of the structure. The current study investigates the performance of piezo-ceramic sensor to detect the propagation of cracks in a structure. Although reinforced concrete is designed to crack, the amount of cracking a structural member can experience is limited. The limits are set indicating the serviceability of the structure. Concrete's low tensile stress makes it easily prone to cracking. Structures exposed to harsh environment, such as off-shore oil platforms and bridges, have firm limitation due to salt degradation, and can quickly become unserviceable. The presence of cracks will allow ingress of chloride as well as sulfate ions into concrete matrix. These salts will accelerate rebar's corrosion and will in turn results in concrete deterioration. The allowed serviceability limit of the structure is dependent upon its use and is thereby governed by design limitations. Structures such as oil tanks, containment structures, and other nuclear reactor facilities, have zero tolerance for cracking. The appearance of cracking in such a facility would make it unserviceable, and cause devastating environmental problems. The development of such a system to detect the presence of cracking in these critical structures would ensure its safe and reliable operation. The proposed piezo-ceramic system in this

investigation is developed to eliminate the drawbacks of conventional non-destructive and monitoring techniques, and to facilitate fair evaluation of reinforced concrete mega structures

1.1 Scope

The scope of the study is:

- To verify the effectiveness of embedded piezo-sensor for structural health monitoring.

1.2 Objective

The objective of the study is:

- Verify the previous developed model for embedded piezo-sensor for evaluation of concrete early strength and concrete strain.
- To correlate the piezo-sensor signal with equivalent concrete strain/crack width.
- Verify the workability of piezo-sensors on reinforced concrete member with large base width.

.

1.3 Methodology

In order to achieve the above mentioned objectives within the identified scope experiments were designed after critically reviewing the related literature. The specimens were tested and obtained results were analysed and discussed, before finally presenting the conclusion and recommendations:

The sequence of the chapter along with its summarized information is given as under:

Chapter 2 deals with the well-defined review of literature and various books that enabled us to collect knowledge relevant to principal of propagation of ultrasonic waves, application of piezo-sensors in structural health monitoring and other comparable non-destructive techniques.

Chapter 3 identifies the experimental program that was undertaken to achieve the desired objective and experimental basis on which this program is formulated. The detailing, construction and testing procedure of small-scale and large scale samples are discussed along the measurement devices installation.

Chapter 4 tabulates the observed results of small scale and large scale samples in terms of graph and tables. The member behaviour is first presented herein followed by the analysis of results that are derived on the basis of observed data from different measurement devices. The results are discussed for effectiveness of piezo-sensors in crack-width calculation and also in early strength monitoring.

Chapter 5 contains the conclusion that we have drawn from our derived results in the light of literature review and recommendations that we will be proposed for any other study in the pertinent field.

Chapter 2

LITREATURE REVIEW

2.1 Introduction

Reinforced concrete (RC) structures are widely used in civil infrastructure as they are low cost and possess long service life under various conditions. With good loading capacity and relatively low requirements for skilled constructions, its applications are in bridges, buildings, highway systems, underground structures, water tank, and dams.

An RC Structure is strong in compression but weak in tension, frequently leading to cracks and failure. In particular, debond damage of steel reinforcing bars cause serious problems. Due to deterioration or overloading, the debond damage weakens or destroys the structure's integrity. Debond damage could also greatly reduce the tensile resistance of the structure, causing it to become more vulnerable to damage when there are earthquake and other dynamic forces. Therefore, it is necessary to apply a damage detection strategy for engineering structure, especially in offshore and other important massive structure.

To ensure structural integrity and safety, civil structures have to be equipped with Structural Health Monitoring (SHM), which aims to develop automated systems for the continuous monitoring, inspection, and damage detection of structures with minimum labour involvement. An effective SHM system can in real time detect various defects and monitor strain, stress, and temperature so that the optimum maintenance of the structures can be carried out to ensure safety and durable service life (Sun, *et al.*, 2010)

2.2 Structural Health Monitoring

Structural health monitoring (SHM), structural sensing, structural health and performance diagnosis are all related to modern structural engineering. Due to advancement in signal

processing, communications techniques and sensors, it is now possible to monitor precisely its structural properties (physical and mechanical) and behavior to assess the damage level and predict its structural health. SHM enhance the safety of structure by giving warning of impending failure so that resources can be utilized more efficiently for maintenance as well as safety of structure (Huston, 2011a).

Intelligent sensor helps determining the early detection of damage of structure by taking into account a potentially diverse array of data and imaging its condition. Thus it's imperative to know wide varieties of sensors that can be deployed for the inspection of structure to increase their safety and reliability. The sensors must be robust and should have good signal to noise ratio as compared to the level of potential damage that it would assess. They also need to be highly reliable so that they can operate in the long run without constant monitoring (Mukhopadhyay & Ihara, 2011).

2.2.1 Components of Structural Health Monitoring

In general SHM System consists of the following aspects, and are also illustrated in Figure 2.1 (Sun & Yan, 2008)

1. The transducer system, including the signal projector and receiver.
2. The signal transmitted and received system.
3. The damage identification system.
4. The warning system

Figure 2.1: The SHM system composition (W. Sun & Yan, 2008)

W. Sun and Yan (2008) in their paper discussed three basic concept of health monitoring strategy. Firstly, transducer should be placed in the form of an array to cover the region effectively. The transducer array divided the structure into many sub-regions where the damage locations were approximately determined. The accuracy of the damage location will be better and finer with the increasing of the transducer array resolution. Secondly, it was proposed that high frequency signal should be used as it is sensitive to micro cracks. Lastly, a feasible damage identification method was proposed.

2.3 Stress Wave

Ultrasonic testing is based on time varying vibration or deformation generally referred as acoustics. When a solid body is disturbed by means of an impact or electrical mechanical impedance (as in piezoelectric transducers), it causes the entire body to respond by linear and angular accelerations. If the applied force is rapid and changes with time, such as piezoelectric transducers or the collision of two solid bodies and occurs within elastic strange, stress waves are generated. Primarily, three types of stress waves propagate in concrete:

compression waves or P waves and shear waves or S wave and third type of stress waves which propagates along the surface is known as Rayleigh waves or R-wave.

2.3.1 Longitudinal Waves

The compression wave or P wave propagates through a solid medium in a similar fashion like sound waves travel through air. The polarization of P wave is longitudinal and vibrates parallel to the direction of propagation, which produces compressive or tensile stress. They are also referred as density waves because their particle density fluctuates as they travel through medium. P waves travel at higher velocities, followed by S wave and R waves, in the order. These types of waves are used to access one dimensional characteristic of internal structure, hence used in non-destructive techniques like impact-echo. For concrete, the compression waves velocity ranges from 3000 to 5000 m/s. Particle movement responsible for propagation of compression and shear waves are illustrated in Figure 2.2:

Figure 2.2: Compression and Shear waves propagation (NDT Resource Centre)

2.3.2 Shear Waves

For Shear waves or S waves, particle motion is vertical or perpendicular to the direction of propagation, which produces shear stress. S waves provide much vibrant image of the internal structure due to their perpendicular propagation as they possess higher chance to come in contact with the internal imperfection of the medium through which they propagates. Shear waves requires an acoustically solid material for effective propagation and hence are not effective in material like liquid or gases. In concrete, the velocities of shear waves are typically 60% of the compressional waves velocity. The MIRA 3D topographer is a non-destructive instrument that utilizes stress waves to inspect structural flaws. Particle movement responsible for propagation of compression and shear waves are illustrated in Figure 2.2.

2.3.3 Rayleigh Waves

Rayleigh waves, surface waves or R waves travel the surface of relatively thick solid material penetrating to a depth of one wavelength. Surface waves combine with compressional and shear waves to create an elliptic orbit particle motion. Rayleigh waves are useful as they are sensitive to surface defects and they follow surface around curves. Cracks that penetrate deeper into the surface than the wavelength will completely attenuate this wave mode. In concrete, the velocity of surface waves is typically 55% of the compression waves velocity.

2.3.4 Guided Waves

Guide wave are structure borne ultrasonic waves that propagates along a structure confined and guided by its geometric boundaries. GW has been found efficient to detect damages due to its capacity of relatively long propagation range as well as its flexibility in selecting sensitive mode frequency combination.(Wang *et al.*, 2009) The operating frequency of GWT is usually low 5 to 250 kHz compared to ultrasonic testing. (Guided Wave Testing)

2.3.5 Stress Wave Propagation Principles

The waves generated by piezoelectric transducer must conform to the fundamental equation of stress and electromagnetic waves propagation. These waves principal are hence useful in ultrasonic testing and data analyzing. Among the properties of waves propagating in isotropic solid material are wavelength, frequency and velocity. Stress wave follow the fundamental equation of waves given by:

$$C = f \cdot \lambda \quad - (2.1)$$

Where, C is the wave velocity, f is the wave frequency and λ is the wavelength. The velocity of stress waves in a material depends upon the properties of material it travels through, its density as well as elastic properties of material from which it is composed. In an infinite elastic solid, the velocity P wave is computed by the following equation:

$$C_P = \sqrt{\frac{E(1 - \nu)}{(1 + \nu)(1 + 2\nu)\rho}} \quad - (2.2)$$

where E is the Young's modulus of elasticity, C_P is the P wave velocity, ρ is the density and ν is the Poisson's ration. In rod-shaped structures, where the diameter of the cylinder is much smaller than its length, $d \ll l$, the transverse strains are similar to axial strain. Therefore, velocity P wave is now given by:

$$C_P = \sqrt{\frac{E}{\rho}} \quad - (2.3)$$

The shear wave velocity C_S is computed by the following equation:

$$C_S = \sqrt{\frac{E}{2(1 + \nu)\rho}} \quad - (2.4)$$

Rayleigh wave velocity C_R , can be determined by the following equation following equation:

$$C_R = \frac{0.87 + 1.12\nu}{1 + \nu} \quad - (2.5)$$

2.3.6 Waves Reflection/Refraction Mechanism

The interaction of waves between various materials boundaries is an important characteristic of ultrasonic wave's analysis. When stress waves encounter a medium of different acoustic interface, it causes reflection, refraction and mode conversion of the waveform. An acoustic interface is the boundary between two materials with different acoustic impedance. Acoustic impedance, Z is defined by the following equation:

$$Z = \rho \cdot C_p \quad - (2.6)$$

where, ρ is the density of the material and C_p is the compression wave velocity.

For a P wave with normal angle of incidence, the incidence and reflected stresses are computed by the following equation:

$$R_p = I_p \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad - (2.7)$$

Where I_p is the stress associated with incident P waves, R_p is the stress associated with reflected P waves. Z_1 is the acoustic impedance of the first medium and Z_2 is the acoustic impedance of the second medium.

As if compression P waves are incident on a material with lower acoustic impedance Z_2 i.e. concrete/air interface, the waves sign polarity changes this means that a compression waves changes to a tension waves and most of the waves is reflected back. Similarly, if the second medium has higher acoustic impedance ($Z_2 \gg Z_1$) the reflected waves remain with same polarity. The acoustic impedance of each material and the wave's angle of incidence, control the stresses associated with the wave reflection and refraction (Sadri & Mirkhani, 2009).

2.3.7 Factors Affecting Waves Propagation

The particle velocity of the wave depends on the elastic properties and density of the medium through which it propagates. For elastic, homogenous solid media the compressional wave velocity is given by (Naik *et al.*, 2003)

$$C_p = \sqrt{\frac{KE}{\rho}} \quad - (2.8)$$

where, C_p is compressional wave velocity; E is the dynamic modulus of elasticity; ρ is density; μ = dynamic Poisson's ratio and $K = (1-\mu)/((1+\mu)(1-2\mu))$.

Thus, the variation in E and ρ has more significant effect on ' C_p ' than variation in ' μ '. The magnitude of the scattering wave is more if the wavelength of the propagating waves is of the same size or smaller than the size of the scatterer, resulting in rapid wave attenuation. A material causes higher rate of attenuation of any transmitted waves when it passes through non-homogeneous material. The total attenuation is the summation of the absorption and scattering and is commonly measured as loss in sound per distance of travel. Scattering in concrete is a function of the maximum particle diameter; the smaller the particle the higher the scattering experienced by the signal. Coarse aggregate will facilitate the transfer of signal within the concrete as they are homogeneous in nature. The effects of scattering and absorption will largely reduce the signal strength reaching the receiver; as well the spectral response period will increase, due to signal lag, and leading to skewed results.

2.4 Piezo-Sensors

Piezoelectric sensor uses electrical/mechanical transformation. Piezoelectric effect is the strain-induced voltage or the voltage-induced strain in solids. Three main types of materials appears in piezoelectric transducer (1) *single-crystal* like quartz (2) *polycrystalline*

piezoceramic, for example, lead zirconate titanate (PZT); (3) *piezopolymers*, for example , polyvinylidene fluoride (PVDF) (Huston, 2011b). Piezoelectric sensors are actively used in SHM as smart sensing technology based on the measurement of electrical impedance and by using elastic waves.

The impedance method utilizes higher frequency waves greater than 20 kHz to monitor mechanical impedance of the structure by the sensors. Physical change in the structure induces change in the mechanical properties of concrete and hence electrical impedance. Thus it can be used to identify damage in the structure. This method relies on the electrical impedance generated by a single sensor and hence can be proved ineffective when large concrete member are used. Costly impedance also restricted its application (Zhu *et al.*, 2012).

The second method of elastic waves relies on the propagation of waves. It is found active in monitoring damage in structure and is also relatively cheap. The principle of elastic wave method relies on the decomposition of frequency using FFT (Fourier Transformation) or by Wavelet Transformation into frequency amplitude domain which are analyzed.

Piezo-sensors are used as an integral part of modern structural engineering. Many researches have been conducted to determine the effectiveness of sensors to help determining the material property, in strength monitoring, measuring deboning and impact evaluation.

2.4.1 Application of Piezo-electric Sensots

2.4.1.1 Cracks and Inclusion Detection

Hu *et al.* (2013) in their research studied the effectiveness of embedded piezoelectric sensor or smart aggregates for detecting flaws and inclusions in concrete structure. A total of five specimens with identical dimensions of 5x5x16 in (127x127 x406.4mm) were casted, having smart aggregates embedded on central axis near two ends. One specimen were used as control specimen, three contained different types of defects: notch, hole, inclusion (while the last one tested under two point loading for detecting real cracks in concrete. Band pass filter (BPS) is used so that only certain range of frequencies can be generated. Hinkley criterion

was adopted for arrival time determination, which measure arrival time from the angle of its energy content. However, energy contents fluctuate due to presence of noise so a certain value is subtracted to account for it. Continuous wavelet transform (CWT) is used for wavelet packet decomposition into time and frequency domain which represented Damage index for the structure as:

$$DI = \sqrt{\frac{\sum_{j=1}^{2n} (E_{i,j} - E_{0,j})^2}{\sum_{j=1}^{2n} E_{0,j}^2}} \quad - (2.9)$$

where, $E_{0,j}$ is the energy of a specimen at its healthy state, i is an index that corresponds to specimen's health status and j is the number of signal set.

From their experimental investigation they concluded that notch and hole type defects affects the time of flight and signal amplitude. Signal amplitude decreased while the time of flight increased irrespective of defect type. However, impurity inclusion in concrete doesn't affect the time of flight but reduces signal amplitude. Bending test results exhibited similar behavior, time of flight increased and amplitude decreased. It was also proposed that wavelet packet decomposition based damage index is very sensitive to damage in concrete.

Zhu et al. (2012) in their research also evaluated wave propagation based active crack detection technique with surface bonded and embedded piezoelectric ceramic (PZT) patches on concrete structure. A series of eight beams with one control beam was tested with different crack lengths. In their study they discussed the influence of excited frequencies; actuator sensor distance, crack depth and the type of wave propagation on the received sensor signals. Amplitude attenuation coefficient (R_v) is used as an indicator of the damage existence in concrete structure in contrast to the baseline of the intact structure. Amplitude attenuation coefficient R_v is expressed as:

$$R_v = \frac{A_r}{A_a} \quad - (2.10)$$

where, A_r is the received signal amplitude and A_a is the maximum amplitude of the actuation signal.

Based on their experimental results they concluded that the signal in range of 10-100 kHz follows a trend of exponential attenuation with 10 kHz is stronger than those at excitation central frequencies more than 20 kHz. However, contradictory results were obtained while examining effect of embedded and surfaced bonded sensors on stress wave's propagation. It was shown that the nearest sensor for the surface bonded PZT received maximum voltage of 350 mV, while for the embedded PZT maximum voltage appeared maximum in the farthest sensor i.e. 1500 mV. Higher signal intensities actuated by the embedded PZT sensor make them more suitable for structural health monitoring. It was also concluded that relative attenuation factor decreases with increasing sensor distance as well as crack depth as it will decreased signal amplitude received at sensor.

2.4.1.2 Material Properties/Early Strength Evaluation

The stress waves generated by PZT actuators carry information about the host structure and thus can be used to identify the properties of it. The harmonic amplitude 'A' and angular frequency 'ω' is correlated with material properties by equation

$$A = \left(\frac{1}{\omega}\right) \left(\frac{4p^2}{E\rho}\right)^{1/4} \quad - (2.11)$$

where, E is young modulus and ρ is the density of the material. From this equation, it can be seen that as Young modulus 'E' of the material increases, harmonic amplitude 'A' decreases. Concrete gains strength during the hydration process which leads to the increase in young's modulus of concrete. Thus, the amplitude of stress waves generated is a representation of compressive strength gain by concrete. This equation is also used in literature to estimate material properties like density and young modulus.

Jinsong and Likun (2011) also conducted research to investigate the mechanical properties of concrete structure using piezoelectric sensor based wave propagation method. The experimental results of cube and cylinder specimen were used to investigate the effect of excitation frequencies; excitation amplitude, wave propagation path and curing age on the

output signals of sensors are evaluated. For their research they concluded that RVAC sharply decreased to a relatively steady level after the 28days curing age. The influence of the input signal was observed by varying it over the range of 1-50 kHz and found that RVAC is independent of the amplitude. The results indicated that RVAC decreased as the excitation frequency increased as the waves serves greater attenuation at higher frequencies.

Kim *et al.* (2011) also monitored real time concrete strength by using guided waves. The guided waves signals were measured by embedded piezoelectric sensors in an unreinforced concrete member. The test was processed at 2, 3, 5, 7, 10, 14 and 28 days. The maximum propagation power in each signal is extracted and related with measure compressive strength using linear regression method represented by

$$S(MPA) = 333.2P_w + 8.692 \quad - (2.12)$$

Where S represents the estimated strength and P_w represents the maximum propagation value. They examined that the maximum power of the guided waves signal is increasing with the signal amplitude.

2.4.1.3 Debonding Detection

Wu and Chang (2006) in their study utilized built-in piezoelectric ceramic discs as sensors and actuator to diagnose debond damage in an RC structure. In their test 20x4x4 in. beams were used with a steel reinforcing bar embedded in middle having diameter of 0.75 in. Three types of test were performed on it. Firstly, debond test on RC beam to observe signal changes from debond damage in beams. Secondly, tensile test on reinforcement bars to observe signal change to load and bar deformation. Lastly, Bending test on RC beam to provide combined effect of debond damage and tensile test which provide better simulation of real application. From their test result they concluded that piezoelectric sensors are effective in determining the debonding damage. Further they showed that the signal's amplitude increases as the extent of debond increases and signal's arrival time is sensitive to rebar elongation.

2.4.2 Optical Fibre Bragg Grating (FBG Sensor)

FOS has gained prime importance to effectively monitor structural health performance. FOS is generally embedded into the structure or surface mounted to yield information about strain (static or dynamic), temperature and defects (cracks and delamination) (Kreuzer). The sensor works in way that stretching a fiber optic sensor causes change in the grating period and hence change in wavelength of ultraviolet reflected light. The operating wavelength also referred as Bragg wavelength can vary from 153-1550nm.

The grating consists of a number of small fringes. Bragg grating reflects specific wavelength of light, called Bragg wavelength from there fringes as shown in Figure 2.3. This caused FBG to reflect specific frequency while transmitting other. The Bragg wavelength is thus a function of ‘n’ the refractive index of the fiber core and Λ is the spacing between fringes knows as grating period as shown in Figure 2.4 given by:

$$\lambda_b = 2n\Lambda \quad - (2.13)$$

Figure 2.3: Operation of an FBG Optical Sensor (National Instrument, 2011)

A change in strain or temperature affect both the refractive index ‘n’ and grating period ‘ Λ ’ which causes shift in reflected wavelength. This change is then correlated to strain and temperature (National Instrument, 2011).

Figure 2.4: FBG Sensors and its component (National Instrument, 2011)

Many literatures are found evident of the accuracy and stability of FOS in measuring crack and strain in reinforced concrete structure. Quirion and Ballivy (2000) evaluated the performance of FOS embedded in high performance concrete. They concluded that FOS give exact measure of strain at high level as compared to LVDT (linear Variable Differential Transformer), electrical strain gauge and vibrating wire.

2.5 Other Concrete Non-Destructive Techniques

2.5.1 Impact Echo (IE)

The impact echo system is composed of three systems: an impact source, a receiving transducer and wave form analyzer, which is used to store the output of the transducer in the waveform and perform signal analysis. This technique measures the surface displacement close to the impact point which is generated by mechanical impact i.e. hammer. The stress wave generated by it propagates into the objects, which undergoes multiple reflections between the test surface and the internal defects of the opposite boundary of the test objects. The reflected wave 'P' travels twice distance from the surface to the internal defects i.e. $2T$ when the wave reflects from a free or low stress boundary (i.e. concrete/air or concrete/soil boundary) (Sadri & Mirkhani, 2009). Hence, the travel time 't' between the successive arrivals of reflected 'P' waves that depend on on the P wave velocity C_p , and is computed by:

$$t = \frac{2T}{C_p} \quad - (2.14)$$

However, where the secondary boundary has higher acoustics (i.e. concrete/steel or concrete/rock), the reflection characteristics of the signal changes and arrivals of the signal on the surface further doubles i.e. $4T$.

Figure 2.5: Impact echo principle (Sadri & Mirkhani, 2009)

Thus, the depth of the object can be determined by measuring the travel/contact time 't' of the multiple P wave reflection. Generally, the frequency value of the highest frequency component with significant amplitude are used to calculate the contact time 't' which is equal to inverse of frequency and the shortest wavelength should corresponds to the size of the smallest flaw that has to be detected. Thus, the receiving transducer must be broadband so that it can detect wide range of frequencies (Sadri & Mirkhani, 2009).

2.5.2 Spectral Analysis of Surface Waves (SASW)

The SASW system consists of an impact device, two surface displacements transducers, and a waveform analyzer. It is used to determine stiffness profile of layered structures. It's based on the principle that various wavelength components penetrates to different depths in the layered pavement in the impact generated surface. Unlike P waves, the velocity of given R waves, C_R is also a function of the property of the material it propagates through. Therefore, the frequency component of the R waves will propagate at different speeds in the layered structure. The velocity component of R wave is also termed as 'phase velocity'. The velocity is calculated by measuring the travel time between two transducers of known separation distance 'X', when they reach at the surface (Sadri & Mirkhani, 2009).

The wavelength is related to the frequency and phase velocity as,

$$\lambda = \frac{C_R}{f} = \frac{360}{\phi} \cdot X \quad - (2.15)$$

where, ϕ is the phase difference

Figure 2.6: Principle of Spectral analysis of surface waves (SASW) system (Sadri & Mirkhani, 2009)

2.5.3 Ultrasonic Through Transmission (UTT)

The equipment of a UTT consists of a transmitting & receiving transducer and a digital waveform analyzer. This technique utilizes the relationship between the quality of concrete and the velocity of an electronic pulse through the material. The pulse velocity is computed by measuring the travel time 't' between the two transducer separated by distance 'X'. It is given by the following equation

$$C_p = \frac{X}{t} \quad - (2.16)$$

Fairly extensive research has been conducted to relate the pulse velocity with the compressive strength. Compressive strength is based on number of variables i.e. w/c ratio, aggregate size etc., however not all of these variables affects the pulse velocity. The UTT is used to evaluate the uniformity of concrete, thus it yields more qualitative results than quantitative.

Figure 2.7: Ultrasonic Through Transmission (Sadri & Mirkhani, 2009)

Ultrasonic tomography is often used to create an acoustic image of the structure. It uses spatial distribution of propagated stress wave's velocity, which depends on characteristic of medium through which it propagates, to reconstruct the acoustic image of the object.

The Ultrasonic pulse-echo system is another technique which measures the change in the travel of the ultrasonic signal to detect the position of the cracks and discontinuities in the structure. In this technique, both transmitting and receiving transducers are placed on the same face of the structure. In the recent developments, particular filters (Split Spectrum Processing or SSP) are used in order to remove the effect of inherent noise caused by aggregates and air bubbles (Sadri & Mirkhani, 2009).

2.5.4 Miniature Seismic Reflection (MSR)

The MSR System consists of a vertical and tangential transducer, impact source and waveform analyzer. Its function is based on the principle of impact-echo principle. A supplementary tangential transducer helps evaluate the dynamic elastic properties of concrete. The maximum peak of the frequency spectrum generated by tangential transducer is

the indicator of shear wave reflection. Figure 2.8 represents typical MSR system schematic diagram.

Figure 2.8 : The MSR System (Sadri & Mirkhani, 2009)

If the thickness ‘T’ and frequency of P & S waves are known, the wave’s velocity can be computed by following relation

$$C_{P=} = 2T \cdot f_p \quad - (2.17)$$

$$C_{S=} = 2T \cdot f_s \quad - (2.18)$$

Thus, the wave’s (P & S) velocity and concrete density helps evaluate Poisson ration, shear modulus, young’s modulus and bulk modulus of elasticity of concrete. MSR advantage is that it can operate from one free surface and also helps determine internal flaws.(Sadri & Mirkhani, 2009)

2.5.5 Impulse Response (IR)

The Impulse response (IR) method is a surface reflection technique that depends on the identification of compression wave reflection. The test involves impacting the drilled shaft with low strain hammer so that structure responds to impact in bending mode i.e. it vibrates at low frequency. The FFT algorithm is used to process both signals to give, force vs frequency for the hammer impact and velocity vs frequency for the geophone responses. The frequency for the geophone is the actual structural vibration frequency at the test point in response to the impact. As the response is dependent on the impact force a “transfer function” termed as Mobility plot is derived by dividing the velocity spectrum (structure response) by the force spectrum (hammer input) with its unit expressed in ‘m/sN’ (Clausen *et al.*, 2012). Figure 2.9 shows typical mobility plot for pile.

Figure 2.9 : Mobility plot for pile (Sadri & Mirkhani, 2009)

The impulse response function depends upon the type of structure, its geometry, support condition and existence of flaws in structure. At low frequencies, pile and soil vibrate together and the mobility plot thus provides information of the dynamic stiffness of the soil/structure.

2.5.6 Short Pulse Radar (SPR) OR Ground Penetrating Radar (GPR)

Short pulse radar (SPR) is widely used for testing of concrete. SPR system is based on SPR wave's i.e. electromagnetic waves (EM) that propagate through air at the speed of light. EM wave's behavior is governed by material dielectric and conductivity properties. If the EM waves encounter two material of different dielectric constant, they will have part of it reflected from the interface and part of it transmitted through it. Any defect in concrete such as air, water, salt water or air gap, will cause a new dielectric constant in concrete. Any reflection that is caused by this new interface of different dielectric constant will be recorded by the equipment and type of severity will be assessed. Reflection coefficient of the materials is given by

$$\rho = \frac{R}{I} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \quad - (2.19)$$

where, R is the amplitude of reflected energy, I is the amplitude of incident energy, ϵ_1 and ϵ_2 are the dielectric constant of the material 1 and material 2.

The larger the difference in dielectric constant larger will be the reflection. However, only defects that are perpendicular to the direction of travel of radar pulse are detected. Figure 2.10 shows schematic diagram of basic of short pulse radar technique (Sadri & Mirkhani, 2009).

Figure 2.10 : Basics of Short Pulse Radar technique (Sadri & Mirkhani, 2009)

2.6 Cracks Width

Concrete is strong in tension and weak in compression and its tensile capacity is about 5% of its compressive strength. Due to its low tensile capacity, it can be easily prone to cracks if applied load or stress exceeds the threshold value to its tensile capacity. Further, Concrete suffers many types of deterioration and distress i.e. sulphate attack, freezing/Thawing, shrinkage, corrosion etc. particularly in offshore structures. These types of distress and deterioration lead concrete to expand and contraction, ultimately leads to cracks. Cracks in concrete are not only aesthetically appealing but can also create structural problems. The Canadian Standard Association (CSA) offshore code CSA-S474-04 and Norwegian Council for Building Standardization code 3473E (E-1992) are found to be most precise in estimating crack spacing relevant to reinforced concrete offshore structure (Marzouk & Hossin, 2008; Marzouk *et al.*, 2010). The Canadian Standard Association (CSA) offshore code CSA-S474-04 provides the following expression for calculating the crack spacing:

$$S_{rm} = 2.0(c + 0.1S) + k_1 k_2 d_{be} h_{ef} b / A_s \quad - (2.20)$$

where, S_{rm} =Average crack spacing (mm); k_1 is the bond properties coefficient (0.4 for deformed bar, 0.8 for plain bar); k_2 is coefficient to account for Strain gradient= ; S is the bar spacing in outer layer (mm); ϵ_1 and ϵ_2 =Smallest and largest tensile strain in embedded zone; d_{be} =bar diameter (mm²); h_{ef} =effective embedded thickness (mm); b is the base width (mm); A_s =Area of reinforcement in effective thickness ; a_1 is the distance to the top of the outer layer of reinforcement (mm); a_2 =distance to the inner layer of reinforcement (mm). This equation takes into account the effect of geometry as well as the effects of bond properties of bars and strain gradient. The Canadian Code suggests that crack width is a function of the average tensile strain (f_t) multiplied by the average crack spacing (S_m).

$$f_t = \frac{0.33\sqrt{f'c}}{1 + \sqrt{500\epsilon_1}} \quad - (2.21)$$

where, f'_c is equal to concrete compressive strength and ε_1 is the tensile strain on the tension reinforcement.

Figure 2.11: Effective embedded thickness (CSA-S474-04, 2004)

Norwegian Council for Building Standardization code 3473E (E-1992) on the other hands takes into account the tensioning stiffening effect for calculating represented by factor r . The average and characteristic spacing of the cracks are given by

$$w_k = 1.7w_m \quad - (2.22)$$

$$w_m = r\varepsilon_1 S_{rm} \quad - (2.23)$$

$$r = 1 - \frac{\beta}{2.5k_1} \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2 \quad - (2.24)$$

where, w_k is the maximum characteristic width (mm); w_m is the average crack width (mm); S_{rm} is the average crack spacing (mm); r is the factor that account for tension stiffening; ε_1 is the principal strain of the tensile reinforcement; β is the factor that takes into account loading procedure; σ_{sr} , σ_s is the stress in the reinforcement at cracking load and applied load respectively.

Similar equation is given by both, Norwegian Code 3473E and Canadian code CSA-S474-04 for calculating crack spacing. Further, Norwegian Code 3473E estimates the crack width at the level of reinforcement and Canadian code CSA-S474-04 estimates it at the surface of member. (Marzouk & Hossin, 2008)

2.7 Effect of Shear Span to Depth Ratio

The cracking pattern and failure of beam is largely governed by the effect of shear and flexure stress magnitude. Many literatures have shown that shear-span to depth ratio (a/d) is the governing factors that influence mode of failure. This factor is directly proportional to the shear and flexure stresses at particular location in the beam (Brzev & Pao, 2009).

In general, three types of failure mode or their combination occurs depending on the a/d ratio: flexural failure, diagonal tension failure, and shear compression failure as show in figure 2.12.

Figure 2.12: Failure mode of concrete beam without shear reinforcement (a)Flexure Failure (b) Diagonal tension failure (c) crack pattern of diagonal tension failure (d) Shear compression failure (ASCE ACI Committee 426, 1973)

Short Beam: with a/d in the range of 1 to 2.5, failure is initiated by an inclined crack usually a flexure shear crack. The actual failure may take place either by: (1). Crushing of the reduced concrete section above the tip of the crack under combined shear and compression, termed shear tension failure.(2). Secondary cracking along the tension reinforcement termed as shear tension failure. Both of these types of failure occur before the flexural strength of the beam is achieved

Normal Beam: have a/d ratio in excess of 2.5. Such beams may fail either in shear or in flexure. The limiting a/d ratio above which flexure failure is certain depends on the tension steel area well as strength of concrete and steel.

Long Beam: with a/d ratio greater than 5.5. Vertical flexure shear cracks develop at the midspan after approximately 50% of the load. The mode of failure depends on the amount ore reinforcement as either (concrete controlled or steel controlled). (Menon, 1998)

Lee and Kim (2008) in their considered the effect of shear span to depth (a/d) longitudinal tensile ratio and compressive strength on deflection and reserve strength after diagonal cracking. From their test result of 26RC beam, they concluded that by increasing a/d ratio from 3 to 5 and longitudinal ratio from .0093 to 0.0279, reserve strength increased by 8% and 16% respectively. They also concluded that shear reinforcement ratio should be increased with increasing a/d ratio to have adequate reserve strength after cracking. Similarly, high strength concrete decreased the diagonal concrete strength. Constantinescu and Magureanu (2010) in their research on high strength concrete observed that by decreasing a/d ratio from 1.7 to 1.5, an increase in 12% bearing resistance is achieved. Similarly, the bearing strength increased when stirrups spacing is decreased irrespective of a/d ratio.

Gurutzeaga *et al.* studied the effect of longitudinal bar spacing on one-way slab. They observed that with short longitudinal bar spacing, the shear failure was cylindrical and uniform. On contrary, in the slab with large spacing between longitudinal bars, the shear failure was irregular and curled due to three dimensional resisting mechanisms.

Chapter 3

EXPERIMENTAL INVESTIGATION

The following chapter details the experimental investigation undertaken to validate the effective of embedded piezo-ceramic sensors for structural health monitoring of reinforced concrete members. The experimental program including specimen preparation, experimental setup and test procedure are presented.

Previous research dealt with small and large beams. This experimental program consisted of two small-scale beams and two full-scale slabs that were tested under flexural loading. The small-scale beams were used to develop a model to predict and quantify the appearance of cracking in a reinforced concrete member. The full scale slabs are then used to verify the model's effectiveness for predictive health monitoring of reinforced concrete plates. This experimental program was carried out to establish a correlation between sensor signal and concrete strain/crack width as well as to monitor the development of concrete early age strength.

3.1 Material Strength Gain Identification

Previous research conducted by Hugi (2012) proved that the piezo-sensor provide an effective means of non-destructive material strength gain evaluation. This research will further investigate the effectiveness of piezo-sensor in concrete strength maturity. A total of 30 standard cylinders; 100 mm into 200 mm were casted. Three cylinders will be tested each day at the curing age of 1, 2, 3, 4, 5, 6, 7, and 14 day. The piezo-sensor signals are also processed in small scale specimen at the same time during testing of cylinder specimen to have fair evaluation of concrete strength.

3.2 Structural Specimen Identification

A total of four specimens were cast and tested as part of this experimental investigation, two full-scale slabs and two small-scale beams. Their specifications are summarized in Table 3.1. A series of steps were undertaken to investigate the piezo-ceramic sensors' effectiveness. These steps include theoretical capacity failure mode calculation, formwork design and mouldings, reinforcement layout, sensors mounting, casting, curing and testing procedures.

Table 3.1: Specimen Specification

<i>Specimen</i>	<i>Member Size</i>	<i>Identification</i>	<i>Cross section</i>	<i>Length</i>	<i>Stirrups Spacing</i>
Beam	Small Scale (SS)	SS-B1	200 x 300 mm.	1000 mm.	125 mm
		SS-B2			
Slab	Large Scale (LS)	LS-S1	1100 x 300 mm.	3000 mm.	-
		LS-S2			

3.3 Small-Scale Beam Specimen

The scope of the small-scale experimental program was to determine the piezo-ceramic sensors ability to predict, detect and quantify the appearance of flexural cracks in a reinforced concrete member. Two small scale beams were designed so that one flexural crack would appear directly at the centre. The results of these specimens are used to compare the effectiveness of piezo-ceramic sensors with conventional means of determining strains and crack widths. These specimens were also used for prediction of concrete early strength.

3.3.1 Member Specification

The beams were designed to resist an ultimate bending moment of 52 kN.m, a cracking moment of 16 kN.m, and exceed a shear capacity of 150 kN; thus ensuring flexural failure. The beams had a height of 300 mm, base width of 200 mm, and unsupported span of 800

mm. The design requirements dictated that each beam would have three 10 M bars to act in the tension zone, two 10 M bars in the compression zone, and stirrup spaced at 125 mm, as illustrated by Figure 3.1.

Two beams were loaded in 5 kN load step upto failure. Reading were taken using steel strain gauges, FBG Fibre optic sensor, concrete crack gauges and piezo-sensors. The beams cracking pattern was also visually observed throughout the loading program.

Figure 3.1: Reinforcement Details - SS Beams

3.3.2 Member Fabrication

The following section explains the member fabrication process which includes formwork fabrication, reinforcement placement, casting and curing process. Ready mix concrete having a minimum of 60 MPa compressive strength was required to achieve the desired flexural capacity.

3.3.3 Formwork

The formworks used in the casting of the beams were constructed out from timber. They were first modelled on Google SketchUp to have precise dimension and adequate stability as illustrated in Figure 3.2.

Figure 3.2: Formwork Model SS Beams (a) Assembled (b) Google Sketchup Model

The beam wooden moulds consisted of five panels assembled together using nuts and bolts. The beams sides and bottom were made from 5-ply plywood of 19 mm (3/4 in.) thickness, reinforced with timber stiffeners longitudinally and vertically bearing a cross-section of 38 x 89 mm (1.5 x 3.5 in). The vertical and longitudinal stiffeners are used to avoid distortion in the member's dimensions during the casting process. The joints between each beams side were water sealed using silicone, as to keep the moisture of the concrete confined inside the mould. The actual and modelled beam formworks are shown in Figure 3.2. A notch of 3 mm is introduced on the tension face of the beam at midspan by placing cardboard during casting, which provided a weak zone and thus facilitates the initiation of cracking

3.3.4 Reinforcement

Beams reinforcement cage was made with 10 M bars of yield strength 450 MPa. The cage is made with three flexure bars in tension; two bars in compression attached with seven prefabricated stirrups attached using zip ties. The three tensile flexural bars in the specimen were attached to plastic seats to provide the 30 mm clear concrete cover required. Two hangers were also fabricated and implemented as part of the reinforcement cage. The hangers were placed one at either end of the beam, as to provide a means of transporting the member

once it is cast and ready to test. The placement of the reinforcement cage in the wooden mold is illustrated in Figure 3.3

Figure 3.3: Reinforcement of Small Scale Beams

3.3.5 Casting/Curing

Both small scale beams were casted with Ready Mix concrete of strength of 70 MPa, minimum aggregate size of 14 mm. and with slump of 150 mm. A total of forty cylinders were also cast to obtain the compressive strength of the samples at various time intervals. Moulds were sprayed with oil lubricant before placing of concrete so that the concrete doesn't adhere to the plywood during de-moulding. The concrete was poured into the moulds, a third of the height at a time, and vibrated for two minutes to ensure an even distribution of aggregate and paste throughout the reinforced concrete beams, and to avoid the occurrence of honeycombing during the casting process.

The samples were covered with damp burlap for the duration of the first seven days after casting, and sealed underneath a plastic tarp. This would ensure the proper curing of the samples in the critical first seven days. On the seventh day after the concrete reached approximately 80% of its strength, the beams were de-moulded and tested. Figure 3.4 illustrated the beams/slabs on the day of demolding.

Figure 3.4: Beam demolded and stacked aside each other

3.3.6 Loading Procedure

Small Scale beams were tested under three point flexure tests as illustrated in Figure 3.5. Crack-Gauges are attached to the surface of the beam on the either side at 80 mm from tension surface to measure the crack width. The beam is loaded at load steps of 5kN at the midspan of the beam up to failure. Load application at the midspan would ensure that crack would first appear from the artificial notch provided Figure 3.5 illustrates the test setup.

Deflection is measured using LVDT (Linear Variable Displacement Transducer), while rebar and concrete strains were recorded with different measurement devices at every load step. Piezosensors signals are recorded initially before load application and are then recorded at every increment of load, which is converted to concrete strains after.

Figure 3.5: Small Scale Beams (a) Test Setup (b) Three Point bending loading condition

3.3.7 Measurement Devices

The following section explains the placement of the various measuring devices used in determining member cracking response during its loading program. These devices include steel strain gages, FBG fiber optic sensors, piezo-ceramic sensors, and concrete crack gages.

3.3.7.1 Steel Strain Guages

A steel strain-gauge is attached at the centre of the middle tension reinforcement to measure strains experienced by the reinforcement at time of load application as shown in Figure 3.6. The steel strain will also be used to measure the rebar strains at the midspan, which is the probable cracking region, and determine once the beam has exceeded its serviceability limit, and ultimate limit state.

Figure 3.6: SS-Beams with Strain Guages

3.3.7.2 Crack Gage

Crack gauges were attached over the midspan of the beam where the expected cracking region is. The crack width was measured 80 mm above the tension face of the member. These were surface mounted and were used to correlate to the results obtained from the piezoceramic sensors. The installed crack gauge is illustrated in Figure 3.7.

Figure 3.7: SS-Beams with Crack-Gauges on either face

3.3.7.3 FBG Fiber Optic Sensors

The beams were also equipped with Fibre Optics Sensor (FBG) at midspan to measure concrete strains precisely. The FBG sensor was placed at 70 mm above the tension face of the beam, which is also the centre of the piezo sensors. The strains from embedded piezo-sensor will be a useful tool to correlate concrete strains with the sensors signals. The placement of Fibre Optics Sensor (FBG) is demonstrated in Figure 3.8.

Figure 3.8: SS-Beams with FBG Fibre Optics Sensor

3.3.7.4 Piezo-Ceramic Sensors

All the beams were equipped with embedded piezosensors to determine its effectiveness as an active structural health monitoring system. The piezosensors were attached at the centre over the tension reinforcement 235 mm apart, measuring the transmitted signal at 75mm above the tension face of the beam as shown in Figure 3.9. The sensors were isolated from the tension reinforcement from the rubber pad so that a sensors signal doesn't transit through the reinforcement bars.

Figure 3.9: SS-Beams with Piezo-Ceramics Sensor

3.4 Full-Scale Slab Specimens

3.4.1 Member Specification

Two slabs were cast as a part of full scale program. They were used to determine effectiveness of piezosensor and also validate the model determined from small scale experimental program. The piezosensors were arranged to depict different cracking mechanism (Shear and Flexure) experienced by the slab by varying (a/d) ratio.

3.4.2 Member Fabrication

The slabs were designed to resist an ultimate bending moment of 238.38 kN.m , and a cracking moment of 76.68 kN.m . One slab was forced to failure in shear and the second slab was forced to fail in flexure. This was achieved by varying the shear span (a/d) ratios as discussed in Section 2.7. Both slabs had a height of 300 mm, base width of 1100 mm and unsupported span of 2500 mm. Based on CSA A-23 requirements each slab was dictated to have five 30 M bars in the tension zone. Minimum reinforcement was provided with 10M bars spaced at 300 mm perpendicular to the flexural reinforcement in the tension zone and in both direction in the compression zone as shown in Figure 3.10

Figure 3.10: Reinforcement Detail – LS Slab

3.4.2.1 Shear Span to Depth Ratio

Many literatures have shown that shear-*span to depth ratio* (a/d) is the governing factors that influence mode of failure. This factor is directly proportional to the shear and flexure stresses at particular location in the beam (Brzev & Pao, 2009). LS-S1 has (a/d) ratio of 4.04 which falls into the category of normal beam, having (a/d) ratio exceeding 2.5. Normal beam are beams either fails in shear or flexure depending on concrete strength and steel ratio. LS-S2 also has (a/d) ratio of 2.67 and governs the limitation for normal beam. (Menon, 1998).

Gurutzeaga et al. studied the effect of longitudinal bar spacing on one-way slab. They observed that with short longitudinal bar spacing, the shear failure was cylindrical and uniform. On contrary, in the slab with large spacing between longitudinal bars, the shear failure was irregular and curled due to three dimensional resisting mechanisms.

3.4.3 Formwork

The slabs were modelled precisely on the Google SketchUp as shown in Figure 3.11. Two slabs formwork were moulded side by side resting on three 5-ply plywood having dimension of 1219 x 2438 mm (4x8 ft.) to ensure an even surface. The slab sides were made of the same 5-ply plywood cut to dimension reinforced laterally with 18 x 38 mm (1.5 x 3.5 in.) at bottom and with 18 x 18 mm (1.5 x 1.5 in.) at top. These parts were joined to each other and at the bottom plywood surface using screws of varying length. The sides were sealed using silicone and duct tape was applied at the joining surfaces of the bottom plywood. The actual and modelled beam formwork as illustrated in Figure 3.11.

Figure 3.11: Formwork Model LS Slabs (a) Assembled (b) Google Sketchup Model

3.4.4 Reinforcement

Slab reinforcement cage was made by placing 30 M bar in bottom longitudinally with minimum shrinkage reinforcement of 10 M bar placed in transverse direction with reinforcement ratio of 1.3%. Top layer comprises of minimum flexural and shrinkage reinforcement i.e. 10 M bar placed in both direction. Longitudinal reinforcement is attached to the transverse reinforcement by means of zip ties. Bottom and top layer were attached to the plastic seats as to provide clearance of 50 mm. and 230 mm. with the bottom respectively. Eight hangers in pairs were placed at each corner of the slab so it can be used to transport the slab. The reinforcement cage of the slab is illustrated in Figure 3.12.

Figure 3.12: Reinforcement Large Scale Slabs

3.4.5 Casting/Curing

Both Large scale slabs were casted with the same Ready Mix concrete as that of beam having compressive strength of 60 MPa, minimum aggregate size of 14 mm. and with slump of 150 mm. Moulds were sprayed with oil lubricant before placing of concrete so that the concrete doesn't adhere to the plywood during de-moulding. The concrete was poured into the moulds, a third of the height at a time, and vibrated for two minutes to ensure an even distribution of aggregate and paste throughout the reinforced concrete beams, and to avoid the occurrence of honeycombing during the casting process.

The samples were covered with damp burlap for the duration of the first seven days after casting, and sealed underneath a plastic tarp as shown in Figure 3.13. This would ensure the proper curing of the samples in the critical first seven days. On the seventh day after the concrete reached approximately 80% of its strength, the beams were de-moulded and tested. Figure 3.14 illustrated the slabs on the day of demolding.

Figure 3.13: Slabs and beams cured and covered with buralps

Figure 3.14: Slabs and Beam demolded and stacked on each other

3.4.6 Loading Procedure

Large scale slabs were tested under four point bending as illustrated in Figure 3.15. Shear and flexure mode of failure were obtained by varying span to depth ratio of the slabs. LS-S1 and LS-S2 were tested with shear span to depth ratio (a/d) of 4.04 and 2.76 respectively. Slabs were loaded in 10kN steps up to failure. Crack gauges are attached over the surface after the first crack become visible at the surface. Three LVDT's are attached; two at the quarter spans and one at the midspans to measure displacement through the loading program. The signals are recorded from different set of embedded piezosensors to monitor the whole region of slab forming an array. An initial signals reading from piezosensors was noted before load is applied. These signals are then correlated to the rebar strains, chiefly at the midspan to validate the effectiveness of the model.

Figure 3.15: Large Scale Slabs (a) Test Setup (b) Four Point bending/ Two Point loading Condition

3.4.7 Measurement Devices

3.4.7.1 Steel Strain Gage

Rebar's in LS-S1 and LS-S2 were equipped with six and nine strain gauges respectively. The steel strain gauges were used to measure strains in rebar's between sensors up to serviceability limit of $2/3 f_y$ as indicated by CSA code. The LS-S1 is loaded in a way to fail in flexure and is thereby equipped with six strain gauges. Three strains gauges were attached to second and to fourth rebar's at quarter and middle spans as shown in Figure. LS-S2 is loaded in a way to fail in shear and is equipped with nine strain gauges i.e. three in second rebar and six in fourth rebar to measure as shown in Figure 3.16-3.17

Figure 3.16: Rebar's Strain-Gauge placement (LS-S1)

Figure 3.17: Rebar's Strain-Gauge placement (LS-S2)

3.4.7.2 Crack Gauge

Crack Gauges were attached to the surface of the concrete once concrete cracking had initiated. These crack-gages are intended to measure the crack width from the initiation of cracking to the serviceability limit of the slab. The length of the crack gauge used was 10cm and measure the strains in μm .

3.4.7.3 Piezo-Ceramic Sensors

Large scales slabs (LS-S1/LS-S2) were equipped with equal number of number of piezo-sensors (i.e. fifteen sensors) to monitor the shear and flexure cracks in the reinforced concrete members. Twelve piezo-sensors are distributed along two longitudinal bars (i.e. second and fourth bar) in tension, while two of them are mounted on the top longitudinal compression reinforcement bars in each slab as shown in Figure 3.18-3.21. Figure 3.22 illustrates the final placement of all measuring devices in large scale samples. These piezo-sensors work as a pair of transmitting and receiving sensors to make an array over the whole slab. The sensor signals are transmitted and received in the following manner as shown in Table 3.2

Table 3.2: Piezo-sensors signal path

<i>Transmit</i>	2	2	4	4	6	8	8	9	15	15	12	13	13
<i>Receive</i>	1	3	3	5	5	7	10	10	10	11	13	11	14

Figure 3.18: Piezosensor placement (LS-S1) – Planview

a)

b)

Figure 3.19: Piezosensor placement (LS-S1) – Cross Section (a) A-A' (b) B-B'

Figure 3.20: Piezosensor placement (LS-S2) – Planview

a)

b)

Figure 3.21: Piezosensor placement (LS-S2) – Cross Section (a) A-A' (b) B-B'

Figure 3.22: Placement of Piezosensors and Steel Strain-Gauges in Large scale slabs

Chapter 4

RESULT ANALYSIS AND DISCUSSION

4.1 Introduction

This chapter will discuss the results obtained from the experimental program for the small scale and large-scale specimens. This research work is in continuations of the past research conducted by Hughi (2012) on the piezoceramic sensor. The sensor signal output determined are used establish relation between concrete strength maturity and frequency shift as well as to correlate the sensor signals with the concrete strain/crack width.

4.2 Early Strength Development

An investigation was carried out for quantifying the development of concrete's early strength using the piezo-sensors on the small scale experimental beams. This section will illustrate the results obtained from multiple standard concrete cylinders testing during the first seven days after concrete casting (for results see Appendix A). The result obtained for the concrete compressive strength is plotted in Figure 4.1 and and tabulated in Table 4.1.

Figure 4.1: Cylinder Compressive Strength

Table 4.1 : Standrad Cylinder Compressive Strength

Day	Date	Specimen Type	Compressive Strength MPa (kN)						Average Compressive Strength Mpa (kN)
			1st Specimen		2nd Specimen		3rd Specimen		
1	May, 29	Cylinder (100x200 mm.)	26.076	204.8	24.552	192.85	23.408	183.83	24.68
2	May, 30		36.025	282.96	34.35	269.77	33.24	261.08	34.54
3	May, 31		35.92	282.11	38.67	303.71	44.106	346.4	39.57
4	June, 1		31.461	247.08	42.651	335	47.671	374.43	40.59
5	June, 2		46.45	364.81	45.423	356.76	52.214	410.07	48.03
6	June, 3		51.4	403.7	46	361.2	45.368	356.32	47.59
7	June, 4		49.884	391.79	50.227	394.89	49.229	386.63	49.78
14	June, 11		54.041	424.45	60.943	478.65	63.66	499.99	59.55
34	July, 4		58.999	463.33	69.313	544.38	63.85	501.48	64.05
41	July, 11		57.16	448.98	52.904	415.4	59.888	470.38	56.65
49	July, 19	75.53	593.24	67.479	529.96	71.327	560.22	71.45	

Figure 4.2 represents the vibration characteristic or frequency shift for SS-B1 and SS-B2 between two piezo-sensors in the first seven days of casting. The initial reading was taken after 24 hours of the mix casting as is used as a reference for shift in frequency for the days after. Four control points were taken on vibration characteristic curve as a reference of frequency shift. These points were tracked for the first seven days on the vibration characteristic curve and the resulting shift are illustrated in Figure 4.2, and tabulated in Table 4.2. The proposed method of determining compressive strength by Hughli (2012) is based on initial compressive strength after 24hours of casting and on the average spectral shift of frequency domain with respect to initial condition. The equation used to predict the compressive strength is given by:

$$f'cd = 163.65 \left(\frac{F_d}{F_i} \right) - (121.47 + f'_{c1}) \quad \text{--(4.1)}$$

Figure 4.2: Beam Frequency Spectral Shift; (a) SS-B1 - 1-4 days, (b) SS-B1 - 5-14 days, (c) SS-B2 - 1-4 days, (d) SS-B2 - 5-14 days,

The vibration characteristic of the beam were taken daily for the first seven days to accurately measure the natural frequency shift of the concrete mix. Three standard cylinders are also tested daily for the first seven to have reliable compressive strength for comparison. Specified points representing the highs and lows of spectrum were monitored throughout the development of compressive strength. Table 4.3 tabulates the average spectral shift of the small scale beams. It can be seen from the table that the frequency domain has shifted from 1.04 to 1.07 for SS-B1 and 1.05 to 1.07 for SS-B2 in the first seven days. The reading was also taken on the test day i.e. fourteenth day and sixteenth day for SS-B1 and SS-B2 respectively. The average spectral shift found was 1.27 and 1.22 for SS-B1 and SS-B2 respectively.

Table 4.2: Beam Frequency Spectral Shift Results; (a) SS-B1, (b) SS-B2

a)

SS - B1								
Control Points	Day1	Day2	Day3	Day4	Day5	Day6	Day7	Day14 (Test Day)
	<i>Frequency (kHz)</i>							
1	73333.3	76666.7	78000	78666.7	79333.3	79333.3	80000	94666.66
2	80666.6	82000	82000	82666.7	83333.3	84000	84000	97333.3
3	69333	73333.3	74666.7	75333.3	76000	76666.7	76666.7	93333.3
4	80000	84000	84000	84666.7	85333.3	85333.3	85333.3	101333.3

b)

SS - B2								
Control Points	Day1	Day2	Day3	Day4	Day5	Day6	Day7	Day16 (Test Day)
	<i>Frequency (kHz)</i>							
1	71333.3	75333.3	76666.7	77333.3	78000	78000	78666.6	89333.33
2	74666.7	78666.7	80000	80666.7	82666.7	82000	82000	91333.33
3	70000	73333.3	74666.7	75333.3	76000	76000	76666.7	86666.66
4	77333.3	82000	83333.3	84000	84000	84666.7	84666.7	92000

Table 4.3: Average Beam Frequency Spectral Shift; (a) SS-B1, (b) SS-B2

a)

SS - B1								
Control Points	Day1	Day2	Day3	Day4	Day5	Day6	Day7	Day14 (Test Day)
	Average Shift							
1		1.04545	1.06364	1.07273	1.08182	1.08182	1.09091	1.290909
2		1.01653	1.01653	1.02479	1.03306	1.04132	1.04132	1.2066122
3		1.0577	1.07693	1.08654	1.09616	1.10577	1.10577	1.3461598
4		1.05	1.05	1.05833	1.06667	1.06667	1.06667	1.2666666
Average Shift		1.0424	1.0518	1.0606	1.0694	1.0739	1.0762	1.27759

b)

SS - B2								
Control Points	Day1	Day2	Day3	Day4	Day5	Day6	Day7	Day16 (Test Day)
	Average Shift							
1		1.05607	1.07477	1.08411	1.09346	1.09346	1.1028	1.2523365
2		1.05357	1.07143	1.08036	1.10714	1.09821	1.09821	1.2232144
3		1.04762	1.06667	1.07619	1.08571	1.08571	1.09524	1.2380951
4		1.06034	1.07759	1.08621	1.08621	1.09483	1.09483	1.1896552
Average Shift		1.0544	1.0726	1.0817	1.0931	1.0931	1.0978	1.22583

To verify equation 4.1, the spectral shift obtained in Table 4.3 are processed using equation the equation into the respective compressive strength as to compare the standard cylinder compressive strength to that obtained from piezo-system. The results of the compressive strength for the first seven days along with that obtained from piezo-sensor system are tabulated in Table 4.4 and illustrated in Figure 4.3.

Table 4.4: Average Piezo-Sensor Compressive strength vs. Actual Compressive Strength; (a) SS-B1, (b) SS-B2

a)

SS - B1							
Day	2	3	4	5	6	7	14
Average Compressive Strength (MPa)	24.4421	25.9727	27.4171	28.8615	29.593	29.9649	62.927097
Actual Compressive Strength (MPa)	34.54	39.57	40.59	48.03	47.59	49.78	59.55

b)

SS - B2							
Day	2	3	4	5	6	7	16
Average Compressive Strength (MPa)	26.403	29.3829	30.8729	32.7408	32.7282	33.5002	54.456309
Actual Compressive Strength (MPa)	34.54	39.57	40.59	48.03	47.59	49.78	59.55

Note: Both beams have same actual compressive strength as they are cast with one mix and cured together. Their strength is based on Standrad cylinder test from particular mix.

The results obtained from the analysis suggest that piezo-system give good evaluation of concrete compressive strength at later stages. It is also observed that the results obtained during first seven days are far from those obtained using standard cylinder testing. These results are in contrary to that obtained by Hughie and Marzouk (2013) as their results experienced deviation at latter stages. Thus it can be concluded that the proposed equation is inconsistent for variety of concrete mix. Other parameter influencing the evaluation of concrete compressive strength using piezo-system must be investigated and equation must be modified.

Figure 4.3: Actual Compressive strength vs. Average piezo-sensor based compressive strength

4.3 Concrete Strain/Crack Width Monitoring

The test result obtained here is presented in terms of HMI value, as represented by:

$$HMI = \sum_{n=0}^{\infty} \sqrt{\frac{(A_{f\ n,0} - A_{f\ n,t})^2}{(A_{f\ n,0})^2}} \quad --(4.2)$$

where. $A_{f\ n,0}$, $A_{f\ n,t}$ is the initial and final amplitude, respectively. The value of HMI obtained by piezo-ceramic sensors will be interpreted to predict and quantify the cracks as well as to monitor the early strength development in reinforced concrete member.

4.3.1 Small Scale Beams

The small scale program consisted of two beams tested under three points loading. These members were equipped with pair of piezo-sensors, FBG Fibre Optic sensor, steel strain gauge and a crack gauges on either face of the beam. The beams were loaded to failure in 5 kN load step and the results obtained from the measurement devices are used to correlates concrete strain with piezo-sensor signals. The HMI value obtained from the piezosensors signals are interpreted to concrete strains using relationship proposed by Hugi (2012) in their past research.

4.3.1.1 Member Behaviour

The two beams tested in this experimental program were identical with respect to geometry, reinforcement detailing, material properties and loading procedure. The beams mid-span load deflection curves are presented in Figure 4.4, which demonstrate that both beams exhibited similar behaviour on application of loads. Table 4.5 tabulates the failure load, corresponding mid-span deflection, observed and actual cracking load. SS-B1 has the failure load of 170 kN with midspan deflection of 2.45 mm; similarly SS-B2 has the failure load of 170 kN with midspan deflection of 1.98 mm. Overall the member has comparable durability as well as failure mode. Both beams failed in flexure as dictated by their design. The crack was first initiated at the midspan and their corresponding observed loads are tabulated in Table 4.5. Actual cracking load corresponds to average concrete strain of 110 μm that was experienced by crack gauges attached to either face of the beam.

Figure 4.4: Load Deflection Curve (SS)

Table 4.5: Ultimate Load and Deflection (SS)

<i>Member</i>	<i>Failure Load (kN)</i>	<i>Failure Deflection (mm)</i>	<i>Observed Cracking Load (kN)</i>	<i>Actual Cracking Load (kN)</i>
SS- B1	170 kN	2.45	65	50
SS- B2	170 kN	1.98	85	55

Figure 4.5 represents the strain experienced by the rebars at the midspan on the application of load. It can be seen from the graph that both beam depicted similar behaviour for load and strain curve. For both beam, the steel started to yield after about 2500 μm which corresponds to strain for 450 MPa steel. The crack width formulations as indicted by CSA-S474 corresponds to 2/3 yield stress of rebar which is primary purpose in calculating crack width. The corresponding strain at the serviceability limit is 1650 μm which is experienced by SS-B1 at around 110 kN and SS-B2 by around 120 kN.

Figure 4.5: Load and Rebar Strain Curve (SS)

4.3.1.2 Piezo-Sensor Results

The piezo-sensors were to monitor the development of cracks across the midsection of the member. The sensors are used to compare the signal output from the piezo-sensors and strain reading from crack gauges and FBG (Fibre Optics Sensor). The sensors signals are recorded as HMI (Health Monitoring Index) which depends upon the initial and final amplitude. The results obtained from the measurement devices are used to locate the point at which the beam has reached its cracking load and to correlates the piezo-sensor strains with concrete strains.

The crack width equation as given by Canadian Offshore structures code (CSA-S474-04) gives the average crack width as a function of average crack spacing (S_{rm}) and total average tensile concrete strain considering the contribution of tension stiffening. CSA-S474-04 and NS 3473 E provides similar expression for calculating crack spacing with Canadian Code estimating the crack spacing at the surface of the member while Norwegian Code estimates it at the level of reinforcement. Equation 2.20-2.23 are used to calculated the crack spacing, average crack width and character crack width and are given by following expressions:

$$S_{rm} = 2.0(c + 0.1S) + k_1 k_2 d_{be} h_{ef} b / A_s$$

$$w_k = 1.7w_m$$

$$w_m = r \varepsilon_1 S_{rm}$$

The values of HMI are used to approximate average concrete strains between two piezo-sensors for pre and post cracking stage is given by equation: (Hughi, 2012)

$$HMI = \begin{cases} 0.1217 \ln(\varepsilon) - 0.2664; & 0 \leq \varepsilon \leq 110 \\ 0.1335 \ln(\varepsilon) - 0.2875; & 0 \leq \varepsilon \leq 3500 \end{cases} \quad --(4.3)$$

and the HMI crack width equation ($C.W$) is given by the equation: (Hughi, 2012)

$$C.W. = \frac{(e^{\frac{HMI+0.2875}{0.1335}} - 110)}{10000} \quad --(4.4)$$

where, HMI is the Health Monitoring Index and 110 represents the average cracking concrete strain for a reinforced concrete member. Figure 4.6 represents the HMI obtained using equation 4.2 at various load steps.

Figure 4.6: Load – HMI curve; (a) SS-B1 (b) SS-B2

Equation (4.3) is used to calculate piezo-sensor strains. These strains are related to the values of strains measured by FBG Fibre Optic Sensor (FBG) and crack gauges on either face. Table 4.6-4.7 tabulates the actual and observed cracking loads and their respective strains. It can be seen from the table that actual cracking load corresponding to 110 μm strain is experienced by SS-B1 at around 50 kN and SS-B2 at around 55 kN. At the actual cracking load on SS-B1,

the value of strain from crack gauges is 109.5 μm while piezo-sensors strains came out to be 132.50 μm with the percentage difference of 17.36%. Similarly, crack gauges on SS-B2 experienced strains of 127.50 μm with 223 μm piezo-sensor strains with 74.90% percentage difference. Table 4.6-4.7 tabulates the strains measured by crack gauges and interpreted piezo-sensor strains. SS-B1 experienced the crack at the load of 65 kN with 665 μm crack gauges strain and 543 μm piezo-sensor strains with overall difference of 18.35%. Similarly, SS-B2 experienced the crack at 85 kN load with 1152 μm and 1597 μm strain for crack gauges and piezo-sensor respectively. The percentage difference in strain measured by both was 38.62%.

Table 4.6: Actual Cracking Load and Corresponding Strains

<i>Member</i>	<i>Actual Cracking Load (kN)</i>	<i>Piezo Sensor Strains</i>	<i>Crack Gauge Strains</i>	<i>Percentage Difference (%)</i>
SS- B1	50	132.50	109.50	17.36
SS- B2	55	223.00	127.50	74.90

Table 4.7: Observed Cracking Load and Corresponding Strains

<i>Member</i>	<i>Observed Cracking Load (kN)</i>	<i>Piezo Sensor Strains</i>	<i>Crack Gauge Strains</i>	<i>Percentage Difference (%)</i>
SS- B1	65	543.00	665.00	18.35
SS- B2	85	1597.62	1152.50	38.62

Figure 4.7 – 4.8 represents the piezo-strains, average crack gauge strain and FBG strains. It can be seen from the Figure 4.7 that piezo-sensors strains of SS-B1 were in good comparison to that of crack gauges strain up to 500 μm but starts to deviate from it afterwards. Similarly,

the piezo-sensor strain of SS-B2 doesn't correspond to that of strain from crack-gauges and FBG from start as show in Figure 4.8. The probable reason for this deviation is that the equation derived by Hughli (2012) based on different concrete strength and the piezo-sensors are made from different epoxy coated material. The proposed equation is dependent only on the HMI values which result in erroneous strains.

Figure 4.7: Load Strains Graph (SS-B1)

Figure 4.8: Load Strains Graph (SS-B2)

Figure 4.9-4.10 represents the comparison of crack-width measured from crack-gauges, average crack width equation and piezo-sensors HMI based crack width equation. It can be seen that piezo-sensors in SS-B1 gave accurate crack width until the actual cracking load

strain i.e. 50 kN and similar is the results from SS-B2 which gives precise crack width upto a load of 55 N. After the concrete has reached the 110 μm strain, the piezo-sensor crack width deviated from that experienced by the crack-gauges in both beams. SS-B1 piezo-sensor crack width didn't experience as much deviation as that experienced by SS-B2 but results obtained were not in good agreement as those measured by crack-gauges. Similarly, the average crack width equation given by CSA-S474-04 showed that concrete start to crack at about 80 kN load and increased linearly at a steady rate. It can be seen that average crack width equation hasn't give good justification of crack width as in comparison to reading measured by crack-gauge. On the other hand in SS-B2, it can be seen the piezo-sensor crack width appears to give close relation to the crack width after the load of 120 kN. The piezo crack width appears to whirl around the graphed average crack width line as given by equation 2.21.

Figure 4.9: Load - Crack width (SS-B1)

Figure 4.10: Load - Crack width (SS-B2)

4.3.2 Large Scale Slabs

Two slabs were tested under large scale experimental program which have similar dimension, longitudinal and transverse reinforcement. Slabs were tested under four point bending at 10 kN load steps under varying shear span to have different mode of failure i.e. shear and flexure. Slabs were equipped with steel strain gauges to illustrate member behaviour throughout the loading procedure. The results from piezo-sensors were used to correlate it to concrete strain/crack width and are also verified by small scale experimental results.

4.3.2.1 Member Behaviour

The load deflection curve for both slabs LS-S1 and LS-S2 is shown in Figure 4.11. Three LVDT were installed in each slab at mid and the two quarter span of the beams. From the load deflection graph, it can be seen that LS-S1 has achieved higher deflection at midspans at lower load than LS-S2. The reason for this is that LS-S1 was loaded with higher span to depth ratio to have flexure failure and has thereby experienced more deflection. LS-S2 has lower shear span to acquire flexure failure and hence it can be seen that quarter span deflections are higher in LS-S2 than LS-S1 at lower load.

Figure 4.11: Load Deflection Curve; (a) LS-S1 midspan (b) LS-S1 Quarter Span (c) LS-S2 midspan (d) LS-S2 Quarter Span

Figure 4.12-4.13 represents the rebar strain of LS-S1 and LS-S2 at every 10kN load step. From the graphs of Figure 4.12, it can be seen that midspan strains recorded on second and fourth rebar were consistent up to load of 80 kN and then it started to deviate. Overall the trend of strain increment was found consistent for strain gauge ‘B’ and ‘E’ following a linear behaviour at the end. Similar behaviour of strain was experienced by strain gauges on left quarter span ‘F’ and ‘C’ and on the right quarter span ‘D’ and ‘A’. The strain gauges at quarter span achieved a plateau at around 160 kN following by yielding of rebar at constant load and then responded in a linear fashion to the load.

Figure 4.13 indicate the rebar strain experienced by LS-S2 placed at different location as indicated in Figure 3.18. Strain gauge 'B', 'D' and 'A', 'E' are placed at equidistance from the middle of rebar and hence are plotted together for comparison. Whereas strain gauge 'C' is located close to midspan and strain gauge 'F' is placed near to support to have adequate strains measurement throughout the span. All strain gauges other than 'F' exhibited similar trend indicating strain plateau achieved around load of 120 kN followed by linear increase of strain thereafter. Similar behaviour was exhibited by LS-S1 which showed that both slabs have comparable durability for results comparison.

Figure 4.12: Load Strain Graph (LS-S1); (a) Midspan, (b) Left Quarter Span, (c) Right Quarter Span

Figure 4.13: Load Strain Graph (LS-S2); Strain Gauge (a) B and D, (b) A and E, (c)C, (d)F

4.3.2.2 Piezo-Sensor Results

Large scale slabs LS-S1 and LS-S2 were equipped with pairs of piezo-sensors throughout the second and fourth tension rebar to record initiation of cracks. Absurd reading of piezo-sensor signals were obtained on application of load on both slabs as shown in Figure 4.14-4.15. HMI reading of LS-S1 showed that the sensor worked initially giving change in amplitude upto load of 100 kN and then achieved a plateau. Similar, results were obtained by LS-S2 showing no signs of change in amplitude on application of load. These results showed that the signal are either completely attenuated by using reinforced concrete members of higher width as

compare to those of beams. Other probable reason could be that the crack might not have run through the whole width of slab cross-section thereby no change of amplitude was observed after certain point.

Figure 4.14: Piezo-Sensor pair HMI Reading measured at every loadstep (LS-S1)

Figure 4.15: Piezo-Sensor pair HMI Reading measured at every loadstep (LS-S2)

Chapter 5

CONCLUSION

This project was intended to determine the effectiveness of embedded piezo-sensor to monitor structural health. Determination of crack width is important ensure the serviceability of concrete. The test comprises of small scale beam and large scale slabs used to determine the piezo-sensor workability in reinforced concrete members. Small scale beams were tested to verify the effectiveness of sensor as well to correlate the piezo-sensor signals with concrete strain/crack width. The developed model was verified in the light of past research as well as verified by applying it to large scale members.

5.1 Research Findings

Throughout the casting process, the sensor proved to be very durable and the rugged epoxy coating made it ideal for mass concrete structure. The piezo-sensors proved to be effective in determining concrete strength maturity for varying dimension structural member. The equation presented for determining concrete early strength appeared to give good estimate of concrete strength at the later stage in this research. In the previous research, Hughli (2012) showed that the piezo-sensor showed good strength evaluation in early six days. Thus, it can be concluded that piezo-sensor provide good evaluation of concrete strength. However the proposed equation is very sensitive by means picking control points and analysing their shift which can result in erroneous results.

Small scale beam and large scale slabs were also tested under three and four point bending respectively. The signals from sensor were recorded at every load in terms of difference in amplitude with respect to initial condition referred as HMI. For concrete strain/crack width, the data from different measuring devices was carefully processed to correlation it to signal change. From the experimental program, it can be concluded that small scale beam piezo-sensor strain showed good comparison with crack-gauges strain. However, piezo-sensor

system doesn't appear to work for reinforced concrete members with large base width more than a meter width due to rapid attenuation of signal. Further, piezo-system proved to be a useful indicator of concrete maturity at latter stages which is in contrary to results obtained by in their research. The equation for piezo-system concrete strain and early strength as derived by Hughi (2012) is very much dependent on their particular testing condition. In this research, different concrete mix and epoxy coating used for encasing piezo-sensor might have led to erroneous results for piezo-strains as well as piezo dictated early strength.

5.2 Recommendation

In the light of this experimental program, following recommendation can be made:

- The value of HMI interpreted by piezo-sensors at different condition need to be evaluated carefully.
- The complete attenuation of piezo-signal in large base width reinforced members need to be evaluated and governing factor should be considered before designing any health monitoring system.

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APPENDIX

DATA

Table A-1 Small Scale Beam-1 – Measurement Device Result

SS- B1											
ID	Load (kN)	CrackGauge-1 (µm)	CrackGauge-2 (µm)	Average CG (µm)	Avg - CW (mm)	Piezo-HMI	Piezo - CW (mm)	Piezo-Strain (µm)	Steel - B1 (µm)	FBG - B1 (µm)	Deflection - B1 (mm)
505	5.03	5	5	5	0.0005	0.012048158	-0.009057085	11.20937106	10	0	0.12
4122	10.06	10	10	10	0.001	0.032111284	-0.008904176	13.37997339	23	0.172	0.14
4756	15.02	15	25	20	0.002	0.06691371	-0.008577808	18.1888169	34	0.71	0.26
5626	20.05	20	40	30	0.003	0.090186086	-0.008306963	22.33443848	47	2.45	0.25
6257	25	25	50	37.5	0.00375	0.126388546	-0.007779564	30.73892102	61	3.12	0.25
6671	30.11	40	60	50	0.005	0.156395158	-0.007219941	40.0554992	80	4.17	0.38
7119	35.14	50	75	62.5	0.00625	0.188162767	-0.006473056	53.01309081	100	6.03	0.35
7533	40.02	70	85	77.5	0.00775	0.252509501	-0.004288796	93.5264585	121	7.83	0.4
8005	45.12	105	100	102.5	0.01025	0.291841612	-0.002332026	132.3243172	152	10.342	0.42
9410	50.08	140	125	132.5	0.01325	0.330023301	0.000206835	109.5099564	182	12.03	0.45
15451	55.11	200	170	185	0.0185	0.415274435	0.009329702	189.0844823	240	16.02	0.49
20750	60.07	535	545	540	0.054	0.544134274	0.040748655	431.7126659	511	61.3	0.71
21233	65.02	675	655	665	0.0665	0.58010767	0.056443288	543.6094129	616	80.13	0.68
23677	70.2	800	810	805	0.0805	0.653193351	0.104870689	868.2407435	748	116.3	0.68
24262	75.01	830	935	882.5	0.08825	0.6706293	0.120897347	970.8538334	894	165.81	0.8
24662	80.04	905	1010	957.5	0.09575	0.670473379	0.120744555	969.8844938	987	198.98	0.8
26321	85.07	995	1085	1040	0.104	0.688747425	0.13992418	1090.349228	1102	254.05	0.82
26706	90.03	1025	1140	1082.5	0.10825	0.657919274	0.109009964	894.930983	1188	308.17	0.86
27539	95.06	1025	1190	1107.5	0.11075	0.669432585	0.119729207	963.4387648	1293	364.46	0.96
27869	100.01	1065	1235	1150	0.115	0.705192675	0.159578355	1211.498255	1370	406.17	1.03
28304	105.04	1120	1270	1195	0.1195	0.679369494	0.129753901	1026.768454	1459	434.08	1.06
28995	110	1165	1335	1250	0.125	0.665852865	0.116296822	941.5945226	1552	472.34	1.09
29606	115.03	1205	1380	1292.5	0.12925	0.681704508	0.132219803	1042.244104	1632	498.2	1.34
30041	120.28	1255	1415	1335	0.1335	0.710522164	0.166485084	1253.578495	1713	525.67	1.45
30438	125.16	1305	1450	1377.5	0.13775	0.727956463	0.191105644	1401.717935	1809	545.37	1.54
30983	130.34	1340	1475	1407.5	0.14075	0.729659918	0.193688186	1417.0994	1914	498.01	1.67
32242	135.08	1385	1510	1447.5	0.14475	0.728977432	0.192649537	1410.916683	1990	507.42	1.71
33295	140.11	1450	1535	1492.5	0.14925	0.708232717	0.163484272	1235.32545	2076	533.91	1.69
33729	145.06	1520	1555	1537.5	0.15375	0.716582877	0.174681914	1303.211272	2159	561.04	1.78
34050	150.32	1570	1585	1577.5	0.15775	0.723441911	0.184418585	1361.756214	2246	587.76	1.89
34698	155.42	1640	1630	1635	0.1635	0.744441726	0.217537595	1557.862541	2345	602.25	1.97
35097	160.15	1680	1675	1677.5	0.16775	0.758670108	0.243127956	1706.546419	2428	626.8	2
37259	164.67	1800	1715	1757.5	0.17575	0.75454121	0.235419022	1661.995651	2672	637.57	2.44
37772	170.66	1940	1765	1852.5	0.18525	0.789296771	0.308399958	2076.507783	3650	693.4	2.45
37951	175.39	5155	5050	5102.5	0.51025	0.815791335	0.378296598	2460.661597	12058	759.18	2.95
38850	179.9	7925	7995	7960	0.796	0.829506039	0.420308047	2686.653185	11270	4969.8	3.31
39898	185.45	10125	10275	10200	1.02	0.832728156	0.430820185	2742.690538	11202	6626.98	4.89
40436	190.85	12330	12645	12487.5	1.24875	0.817909754	0.384507355	2494.285538	11247	551.7	5.19
40459	195.74	13775	14265	14020	1.402	0.827508465	0.413917237	2652.488862	11411	558.69	5.45
40984	200.25	14855	15585	15220	1.522	0.832517672	0.43012571	2738.994491	10951	2116.2	5.65
42111	205.06	16000	16525	16262.5	1.62625	0.845033611	0.473384664	2967.668849	4738	3928.9	7.79
42210	210.09	16765	16790	16777.5	1.67775	0.840385428	0.456843877	2880.59604	4806	3927.15	8.52
42651	215.34	20265	19625	19945	1.9945	0.858309203	0.523925	3231.122114	4751	5660.94	9.88
44171	220	23865	22730	23297.5	2.32975		-0.009138454		4556	4941.32	12.76
45624	225.55	1700	5715	3707.5			-0.009138454		4495	3906.49	15.22
48068	230.21	1565	6010	3787.5			-0.009138454		4426	4390.92	17.42
48543	235.09	1710	6375	4042.5			-0.009138454		4328	2377.4	20.24
49228	239.08	2040	6905	4472.5			-0.009138454		4369	3421.28	24.59

Table A-2: Small Scale Beam-2 – Measurement Device Result

SS - B2											
ID	Load (kN)	CrackGauge-1 (µm)	CrackGauge-2 (µm)	Average CG (µm)	Avg - CW (mm)	Piezo-HMI	Piezo-Strain (µm)	Piezo - CW (mm)	Steel - B2 (µm)	FBG - B2 (µm)	Deflection - B2 (mm)
398	5.03	5	10	7.5	0.00075	0	10.079	-0.00913845	15	0.81	0.13
1633	10.36	10	10	10	0.001	0.042149	14.61893811	-0.00881861	28	4.41	0.16
2372	15.16	20	20	20	0.002	0.056982089	16.66289224	-0.00867977	44	10.61	0.29
2802	20.27	25	25	25	0.0025	0.08441534	21.22578742	-0.00837859	58	13.36	0.41
3319	25.3	30	35	32.5	0.00325	0.111498568	26.95479795	-0.00801391	81	16.75	0.38
3709	30.48	35	40	37.5	0.00375	0.140361891	34.77194441	-0.00753455	89	21.06	0.43
4362	35.43	45	55	50	0.005	0.169868364	45.11138597	-0.00692472	112	83.37	0.46
4791	40.02	55	65	60	0.006	0.196964715	57.29394498	-0.00623268	130	93.81	0.55
5174	45.05	70	75	72.5	0.00725	0.253912394	94.6912443	-0.00422846	153	106.52	0.53
5576	50.38	95	105	100	0.01	0.301934541	91.47440555	-0.00172983	186	134.149	0.53
5817	55.04	120	135	127.5	0.01275	0.441440925	223.5946385	0.013515189	217	210.37	0.64
6830	60.14	175	270	222.5	0.02225	0.518502579	366.3340644	0.03188331	302	222.16	0.59
7249	65.1	255	385	320	0.032			-0.00913845	387	236.74	0.63
7751	69.91	750	680	715	0.0715	0.66717869	949.6266447	0.11755736	670	466.85	0.73
8413	75.01	990	815	902.5	0.09025	0.71591246	1297.625774	0.173756794	782	664.55	0.81
8925	80.11	1145	935	1040	0.104	0.74413828	1554.836862	0.217020988	879	680.66	0.81
9341	85	1280	1025	1152.5	0.11525	0.748375341	1597.621893	0.224341806	953	721.84	0.85
10519	90.17	1375	1140	1257.5	0.12575	0.701520436	1183.328094	0.154977267	1038	807.03	0.98
10794	94.91	1410	1220	1315	0.1315	0.697125185	1150.47147	0.149634122	1109	830.11	0.99
11274	100.16	1390	1330	1360	0.136	0.735686119	1472.880737	0.203093365	1200	900.88	1.09
11625	105.04	1395	1410	1402.5	0.14025	0.745691086	1570.382139	0.219677001	1273	907.9	1.06
12041	110.15	1440	1465	1452.5	0.14525	0.763988861	1765.700235	0.253416377	1344	920.6	1.15
13317	115.18	1565	1435	1500	0.15	0.745372379	1567.17891	0.219129343	1440	934.46	1.22
13617	120.13	1655	1440	1547.5	0.15475	0.728417423	1405.863653	0.191801238	1505	954.31	1.25
14054	125.02	1780	1405	1592.5	0.15925	0.762951654	1754.005924	0.251377735	1581	1030.18	1.37
14616	129.9	1870	1395	1632.5	0.16325	0.780681926	1965.00503	0.288502303	1657	1045.71	1.43
16065	135.22	1855	1415	1635	0.1635	0.793573355	2134.188164	0.318764819	1761	1046.97	1.54
16501	140.48	1900	1455	1677.5	0.16775	0.789169619	2074.816898	0.308096843	1830	1063.61	1.53
16994	145.06	1955	1485	1720	0.172	0.779539442	1950.674583	0.28595864	1895	1135.03	1.63
17447	150.02	1995	1515	1755	0.1755	0.795541443	2161.268408	0.323647449	1963	1147.94	1.68
17970	155.05	2055	1545	1800	0.18	0.803464697	2273.810785	0.344048925	2050	1169.53	1.72
18534	160.15	2100	1585	1842.5	0.18425	0.805382338	2301.918566	0.34917131	2118	1179.84	1.84
19177	165.18	2225	1615	1920	0.192	0.816832947	2477.137231	0.381338069	2223	1274.09	1.9
19624	170.51	2455	1625	2040	0.204	0.82695125	2643.036599	0.412151538	2362	1376.57	1.98
20261	175.39	5500	4270	4885	0.4885	0.851483859	3092.876454	0.497313526		2440.78	2.32
20859	180.13	8090	6625	7357.5	0.73575	0.860094538	3268.292202	0.531113295		4595.13	2.7
21301	185.23	10305	8570	9437.5	0.94375	0.867205895	3420.640852	0.56071917		4816.41	2.99
21809	190.48	12260	10260	11260	1.126			-0.00913845		5065.09	3.23
22293	195.14	14430	12210	13320	1.332			-0.00913845		5493.41	3.61
23270	200.03	16400	13835	15117.5	1.51175			-0.00913845		5722.14	3.9
24013	205.06	17620	14595	16107.5	1.61075			-0.00913845		5594.3	5.31
24370	210.31	19920	16390	18155	1.8155			-0.00913845		6499.1	6.19
24884	215.26	24225	19120	21672.5	2.16725			-0.00913845		7610.68	7.56
25353	220.15	27625	21980	24802.5	2.48025			-0.00913845		10818.7	9.04
26383	225.18	410	6710	3560				-0.00913845		13272.5	10.98
26952	230.06	380	6585	3482.5				-0.00913845		4357.76	12.9
27095	235.31	360	6575	3467.5				-0.00913845		4618.29	14.37
27914	240.27	275	6430	3352.5				-0.00913845		5597.18	19.65

Table A-3: Small Scale Beam-Average and characteristic width

<i>Load (kN)</i>	<i>w_m (mm)</i>	<i>w_k (mm)</i>	<i>Load (kN)</i>	<i>w_m (mm)</i>	<i>w_k (mm)</i>
5.00	0.00	0.00	125.00	0.22	0.38
10.00	0.00	0.00	130.00	0.25	0.42
15.00	0.00	0.00	135.00	0.27	0.45
20.00	0.00	0.00	140.00	0.29	0.49
25.00	0.00	0.00	145.00	0.31	0.52
30.00	0.00	0.00	150.00	0.33	0.56
35.00	0.00	0.00	155.00	0.35	0.59
40.00	0.00	0.00	160.00	0.37	0.62
45.00	0.00	0.00	165.00	0.38	0.65
50.00	0.00	0.00	170.00	0.40	0.69
55.00	0.00	0.00	175.00	0.42	0.72
60.00	0.00	0.00	180.00	0.44	0.75
65.00	0.00	0.00	185.00	0.46	0.78
70.00	0.00	0.00	190.00	0.48	0.81
75.00	0.00	0.00	195.00	0.49	0.84
80.00	0.00	0.00	200.00	0.51	0.87
85.00	0.03	0.05	205.00	0.53	0.90
90.00	0.06	0.10	210.00	0.55	0.93
95.00	0.08	0.14	215.00	0.56	0.96
100.00	0.11	0.19	220.00	0.58	0.99
105.00	0.13	0.23	225.00	0.60	1.02
110.00	0.16	0.27	230.00	0.62	1.05
115.00	0.18	0.31	235.00	0.63	1.08
120.00	0.20	0.35	240.00	0.65	1.11

Table A-4: Large Scale Slab-1 – Measurement Device Result

LS - S1											
ID	SecondsElapsed	POT-1(mm)	POT-2(mm)	POT-3(mm)	Load Cell (kN)	Strain Gauge 'A' (µm)	Strain Gauge 'B' (µm)	Strain Gauge 'C' (µm)	Strain Gauge 'D' (µm)	Strain Gauge 'E' (µm)	Strain Gauge 'F' (µm)
27	13.5	0.03	-0.06	0.02	5.44	1	2	1	1	2	0
352	176	0.06	-0.04	0.05	10.11	2	3	2	3	4	2
619	309.5	0.11	0.03	0.12	20.77	5	8	6	5	11	4
896	448	0.18	0.19	0.18	30.22	9	13	9	7	16	7
1180	590	0.23	0.22	0.25	40.55	11	17	11	10	22	11
1424	712	0.28	0.23	0.31	50.1	15	22	15	12	26	13
1652	826	0.34	0.39	0.38	60.54	19	28	17	16	34	17
1928	964	0.41	0.44	0.44	70.21	24	36	21	20	44	21
2252	1126	0.48	0.6	0.54	80.09	30	48	26	22	56	25
2531	1265.5	0.55	0.71	0.66	90.43	35	56	32	27	73	27
2773	1386.5	0.66	0.81	0.8	100.76	42	73	36	29	147	31
3031	1515.5	0.75	1.09	0.91	110.42	47	96	40	36	204	35
3394	1697	0.85	1.16	1.04	120.86	60	122	46	42	249	40
3638	1819	0.93	1.38	1.18	130.08	80	142	55	63	290	54
5967	2983.5	1.18	1.7	1.42	140.97	151	182	80	128	349	174
6194	3097	1.3	1.86	1.53	150.41	162	205	93	141	412	203
6451	3225.5	1.47	2.21	1.81	160.86	187	256	283	173	586	357
6749	3374.5	1.67	2.27	1.99	170.08	377	277	349	392	621	415
6993	3496.5	1.8	2.5	2.12	182.52	417	305	391	436	669	453
7250	3625	1.91	2.75	2.24	191.52	445	336	424	466	712	486
7493	3746.5	2.01	2.73	2.36	200.51	472	368	459	496	753	519
7747	3873.5	2.12	3.03	2.49	210.07	500	400	489	524	795	550
7977	3988.5	2.23	3.23	2.62	220.51	525	429	522	552	835	574
8305	4152.5	2.35	3.29	2.75	230.28	555	462	556	582	881	597
8561	4280.5	2.45	3.53	2.88	240.54	581	496	585	606	922	621
8810	4405	2.56	3.61	3.01	250.73	606	540	613	629	969	644
9054	4527	2.68	3.84	3.16	261.83	636	584	644	657	1016	673
9286	4643	2.77	4.09	3.29	271.94	666	624	673	686	1056	700
9512	4756	2.87	4.2	3.42	282.05	694	666	702	712	1099	726
9743	4871.5	3.02	4.47	3.57	290.16	720	977	726	731	1234	752
10038	5019	3.17	4.59	3.72	301.27	754	1048	759	761	1301	785
10325	5162.5	3.3	4.79	3.86	312.38	780	1093	788	782	1355	816
10567	5283.5	3.46	4.99	4.01	321.93	804	1131	815	810	1397	841
10839	5419.5	3.59	5.19	4.13	330.04	821	1167	840	842	1435	865
11224	5612	3.74	5.41	4.29	341.71	853	1213	871	876	1484	895
11464	5732	3.84	5.62	4.41	351.37	877	1250	898	902	1526	922
11708	5854	3.95	5.64	4.53	360.7	901	1285	925	927	1565	947
11962	5981	4.09	5.93	4.66	372.7	932	1327	954	956	1613	977
12231	6115.5	4.2	6.04	4.77	380.81	955	1364	979	983	1650	1001
12474	6237	4.32	6.24	4.9	391.14	981	1403	1006	1012	1693	1028
12730	6365	4.43	6.32	5.04	402.47	1008	1441	1038	1040	1739	1057
12949	6474.5	4.53	6.5	5.15	410.36	1032	1471	1061	1063	1771	1078
13225	6612.5	4.66	6.68	5.28	421.02	1061	1510	1091	1091	1812	1108
13464	6732	4.78	6.83	5.41	430.58	1089	1550	1111	1119	1843	1139
13686	6843	4.89	7	5.53	440.35	1116	1584	1135	1146	1877	1163
13943	6971.5	5.01	7.18	5.66	450.79	1147	1624	1164	1175	1916	1192

14207	7103.5	5.13	7.31	5.79	460.24	1174	1660	1191	1202	1950	1221
14494	7247	5.29	7.61	5.94	472.01	1209	1705	1225	1237	1991	1255
14729	7364.5	5.4	7.8	6.07	480.23	1238	1738	1248	1261	2022	1282
14962	7481	5.52	7.92	6.2	491.12	1276	1774	1278	1300	2063	1314
15247	7623.5	5.63	8.14	6.32	500.12	1311	1807	1305	1328	2097	1342
15515	7757.5	5.77	8.35	6.46	510.78	1346	1844	1334	1362	2136	1370
15761	7880.5	5.9	8.5	6.6	521.11	1378	1882	1364	1395	2173	1400
16061	8030.5	6.04	8.72	6.73	530.22	1410	1917	1393	1427	2207	1428
16364	8182	6.16	8.91	6.89	541.11	1443	1955	1426	1461	2248	1457
16616	8308	6.31	9.07	7.06	550.22	1472	1987	1452	1491	2282	1489
16899	8449.5	6.46	9.32	7.26	560.66	1501	2022	1479	1520	2317	1519
17149	8574.5	6.62	9.57	7.44	570.32	1529	2057	1504	1551	2353	1549
17422	8711	6.77	9.85	7.66	580.54	1560	2094	1529	1584	2389	1578
17687	8843.5	6.96	10.07	7.92	591.1	1593	2135	1556	1615	2420	1611

Table A-5: Large Scale Slab-2 – Measurement Device Result

LS - S2											
ID	SecondsElapsed	POT-1 (mm)	POT-2 (mm)	POT-3 (mm)	Load Cell (kN)	Strain Gauge 'A' (μm)	Strain Gauge 'B' (μm)	Strain Gauge 'C' (μm)	Strain Gauge 'D' (μm)	Strain Gauge 'E' (μm)	Strain Gauge 'F' (μm)
878	439	0.04	0.22	0.02	5.33	1	1	32	3	1	1
884	442	0.07	0.14	0.06	10.55	3	3	33	5	3	2
1141	570.5	0.1	0.28	0.11	15.22	5	5	36	7	4	3
1154	577	0.14	0.3	0.15	20.88	7	8	37	9	6	3
1388	694	0.17	0.31	0.19	25.44	8	8	40	10	8	4
1399	699.5	0.2	0.41	0.23	30.55	10	10	42	12	10	5
1637	818.5	0.27	0.41	0.29	40.55	13	15	9	16	13	5
1893	946.5	0.34	0.49	0.35	50.21	17	18	17	20	16	6
2120	1060	0.39	0.49	0.42	60.21	20	21	37	24	21	7
2357	1178.5	0.46	0.68	0.49	70.43	24	25	43	29	25	9
2597	1298.5	0.52	0.67	0.54	80.76	30	30	51	34	29	11
2829	1414.5	0.57	0.83	0.6	90.76	32	34	65	38	33	13
2844	1422	0.63	0.85	0.65	100.09	35	38	67	43	36	13
3342	1671	0.7	0.9	0.72	110.31	41	44	90	49	44	14
3639	1819.5	0.76	1.04	0.79	120.86	45	47	117	55	49	16
3882	1941	0.92	1.13	0.93	130.08	51	65	340	68	59	18
4214	2107	1.14	1.47	1.08	140.86	69	307	410	76	66	18
5039	2519.5	1.32	1.64	1.25	150.3	102	380	433	203	94	21
5603	2801.5	1.44	1.78	1.37	160.74	135	423	462	225	144	22
6211	3105.5	1.63	2.16	1.65	170.3	205	461	499	260	486	29
6478	3239	1.72	2.31	1.73	180.3	248	489	524	283	517	32
6733	3366.5	1.86	2.5	1.87	191.41	291	527	559	359	605	34
7140	3570	1.99	2.69	2.01	200.18	329	571	602	418	651	37
7407	3703.5	2.12	2.77	2.14	210.52	361	608	630	474	709	39
7657	3828.5	2.23	2.9	2.24	220.51	396	644	644	513	753	42
7948	3974	2.34	3.09	2.34	231.29	435	688	673	545	793	44
9485	4742.5	2.55	3.27	2.49	240.51	492	818	725	587	833	56
9962	4981	2.65	3.5	2.61	251.62	519	858	749	612	854	80
10209	5104.5	2.75	3.66	2.7	261.17	543	889	768	639	879	93
10450	5225	2.85	3.68	2.81	271.06	575	925	795	675	909	108
10701	5350.5	2.96	3.8	2.91	280.28	603	958	815	708	938	124
10956	5478	3.07	3.96	3.02	292.28	636	997	777	743	972	137
11203	5601.5	3.19	4.18	3.13	301.61	665	1031	807	781	1004	149
11450	5725	3.3	4.28	3.23	311.94	695	1068	890	820	1038	162
11717	5858.5	3.43	4.47	3.35	322.49	722	1105	918	887	1046	174
11981	5990.5	3.53	4.61	3.46	330.38	748	1140	943	961	1075	185
12231	6115.5	3.65	4.71	3.58	341.16	779	1182	967	1009	1114	209
12511	6255.5	3.77	4.83	3.7	351.26	806	1222	992	1041	1141	230
12761	6380.5	3.89	4.97	3.8	361.37	834	1258	1017	1072	1168	255
12992	6496	3.99	5.16	3.93	371.82	863	1294	1042	1104	1203	422
13223	6611.5	4.11	5.26	4.06	380.92	892	1329	1063	1135	1238	631
13456	6728	4.24	5.38	4.17	391.48	926	1368	1089	1168	1269	737
13725	6862.5	4.34	5.61	4.27	400.14	957	1404	1107	1196	1295	838
13962	6981	4.46	5.67	4.38	410.7	992	1441	1133	1228	1326	922
14197	7098.5	4.6	5.9	4.49	422.36	1025	1484	1162	1260	1361	1001
14443	7221.5	4.71	5.99	4.57	430.47	1056	1517	1177	1288	1387	1088
14706	7353	4.85	6.13	4.7	440.91	1076	1552	1200	1319	1418	1180
14973	7486.5	4.96	6.38	4.81	450.69	1102	1588	1221	1349	1449	1269
15217	7608.5	5.08	6.37	4.92	460.91	1129	1626	1248	1379	1480	1345
15477	7738.5	5.19	6.66	5.03	470.68	1147	1663	1269	1409	1511	1420
15732	7866	5.32	6.68	5.14	480.68	1178	1699	1289	1440	1542	1503
16001	8000.5	5.44	6.86	5.25	491.01	1211	1734	1310	1468	1575	1583

16254	8127	5.56	7.02	5.36	500.68	1240	1768	1331	1501	1607	1659
16497	8248.5	5.7	7.15	5.48	512.12	1274	1804	1356	1534	1639	1746
16741	8370.5	5.81	7.23	5.57	520.23	1315	1834	1363	1560	1664	1840
17006	8503	5.93	7.52	5.69	530.67	1354	1871	1384	1590	1696	1939
17295	8647.5	6.05	7.65	5.8	541.34	1386	1908	1407	1619	1726	2051
17549	8774.5	6.18	7.75	5.92	550.44	1418	1940	1427	1646	1751	2238
17807	8903.5	6.32	7.9	6.08	561.33	1439	1975	1451	1675	1778	2546
18069	9034.5	6.45	8.08	6.22	570.55	1487	2002	1424	1702	1803	2770
18343	9171.5	6.58	8.27	6.35	580.77	1523	2036	1444	1731	1829	2831
18357	9178.5	6.67	8.36	6.44	591.55	1547	2065	1473	1754	1856	2858
18375	9187.5	6.76	8.6	6.55	601.1	1574	2096	1498	1781	1883	2909
18432	9216	6.93	8.74	6.75	610.65	1616	2131	1520	1810	1901	3188
18484	9242	7.08	8.93	6.93	620.65	1657	2166	1541	1838	1923	3421
18549	9274.5	7.2	9.08	7.08	630.65	1692	2199	1560	1865	1949	3579
18843	9421.5	7.34	9.27	7.24	640.2	1715	2233	1577	1898	1983	3682
18984	9492	7.54	9.6	7.68	650.2	1751	2273	1594	1949	2046	4991
19170	9585	7.67	9.84	7.87	660.31	1787	2305	1613	1980	2079	5047

Calculation

$$d = 300 - 50 (\text{cover}) - 30/2 \left(\frac{d_b}{2} \right) = 235 \text{ mm}$$

Effective Shear Span (d_v)

$$d_v = \max \left\{ \frac{0.9 d}{0.72 h} \right\} = \max \left\{ \frac{0.9 \times 235}{0.72 \times 300} \right\} = \max \left\{ \frac{211.5}{216} \right\} = 216 \text{ mm}$$

$$V_c = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v = (0.65)(1)(0.21) \cdot \sqrt{60} \cdot 1100 \cdot 216 = 250.8 \text{ kN}$$

where, $\beta=0.21$ If slab thickness is less than 350 mm. (A23.3 C1.11.3.6.2)

Ultimate Moment:-

$$\begin{aligned} M_u &= A_s \cdot f_y \left(d - \frac{A_s \cdot f_y}{2 \cdot \alpha \cdot f'_c \cdot b} \right) = 3500 \times 400 \left(235 - \frac{3500 \times 400}{2 \times 0.76 \times 60 \times 1100} \right) \\ &= 309.46 \text{ kN.m} \end{aligned}$$

Resisting Moment:-

$$\begin{aligned} M_r &= \phi_s A_s \cdot f_y \left(d - \frac{\phi_s \cdot A_s \cdot f_y}{2 \cdot \alpha \cdot \phi_c \cdot f'_c \cdot b} \right) \\ &= 0.85 \times 3500 \times 400 \left(235 - \frac{0.85 \times 3500 \times 400}{2 \times 0.76 \times 0.65 \times 60 \times 1100} \right) \\ &= 257.93 \text{ kN.m} \end{aligned}$$

Serviceability Moment: - evaluated at $\frac{2}{3} \cdot f_y = 266.6$

$$\begin{aligned}
M_r &= \phi_s A_s \cdot (2/3 \cdot f_y) \left(d - \frac{\phi_s \cdot A_s \cdot (2/3 \cdot f_y)}{2 \cdot \alpha \cdot \phi_c \cdot f'_c \cdot b} \right) \\
&= 0.85 \times 3500 \times 266.6 \left(235 - \frac{0.85 \times 3500 \times 266.6}{2 \times 0.76 \times 0.65 \times 60 \times 1100} \right) \\
&= 176.73 \text{ kN.m}
\end{aligned}$$

Cracking Moment

$$M_{cr} = \frac{0.6\lambda \cdot \sqrt{f'_c} \cdot I}{C} = \frac{0.6 \times 1 \times \sqrt{60} \times \frac{1100 \times 300^3}{12}}{\frac{300}{2}} = 76.68 \text{ kN.m}$$

$$A_{smin} = \frac{0.2 \times \sqrt{f'_c} \cdot b \cdot h}{f_y} = \frac{0.2 \times \sqrt{60} \times 1100 \times 300}{400} = 1278.08 \text{ mm}^2$$

$$\rho_{min} = \frac{A_{smin}}{b \cdot d} = \frac{1278}{1100 \times 235} = 4.9 \times 10^{-3} = 0.49\%$$

$A_{smax} =$

$$\frac{c}{d} \leq \frac{700}{700 + f_y} = \frac{700}{700 + 400} = 0.63$$

$$c = 0.63 \cdot d = 0.63 \times 235 = 148.05 \text{ mm}$$

$$a = \beta \cdot c = 0.82 \times 148.05 = 121.40 \text{ mm}$$

$$A_{sb} = \frac{\alpha_1 \cdot \phi_c \cdot f'_c \cdot a \cdot b}{\phi_s \cdot f_y} = \frac{0.76 \times 0.65 \times 60 \times 121.40 \times 1100}{0.85 \times 400} = 11641.54 \text{ mm}^2$$

$$\rho_b = \frac{11641.45}{1100 \times 235} = 0.045 = 4.5\%$$

$$A_{sprovided} = 5 \times 700 = 3500 \text{ mm}^2$$

$$\rho = \frac{3500}{1100 \times 235} = 0.013 = 1.3\%$$

Average Crack Spacing – 100 kN Load

$$h_{eff} = c + d_{be} + 7.5d_{be} = 30 + 11.3 + 7.5(11.3) = 126.05 \text{ mm}$$

$$Mf = P \cdot \frac{L}{4} = 100 \cdot \frac{0.8}{4} = 20 \text{ kN.m}$$

$$\begin{aligned} f_s &= \frac{M_f}{\Phi_s(A_{sb} - A_{st})(d - a/2) + \Phi_s(A_{st})(d - c - d_b/2)} \\ &= \frac{20}{0.85(300 - 200)(264.35 - 19.47/2) + 0.85(200)(264.35 - 30 - 11.3/2)} \\ &= 330.46 \text{ MPa} \end{aligned}$$

$$\begin{aligned} f_s &= \frac{M_{cr}}{\Phi_s(A_{sb} - A_{st})(d - a/2) + \Phi_s(A_{st})(d - c - d_b/2)} \\ &= \frac{16}{0.85(300 - 200)(264.35 - 19.47/2) + 0.85(200)(264.35 - 30 - 11.3/2)} \\ &= 264.37 \text{ MPa} \end{aligned}$$

$$\xi_b = \frac{f_s}{E_s} = \frac{330.64}{200000} = 0.00165;$$

$$\xi_1 = \frac{\xi_b(h - a)}{d - a} = \frac{0.00165(300 - 19.47)}{264.35 - 19.47} = 0.001896;$$

$$\xi_2 = \frac{\xi_1(h - h_{eff} - a)}{d - a} = \frac{0.001896(300 - 126.05 - 19.47)}{264.35 - 19.47} = 0.00119;$$

$$k_1 = 0.4; k_2 = \frac{0.25(\xi_1 + \xi_2)}{2\xi_1} = \frac{0.25(0.00119 + 0.00189)}{2(0.00119)} = 0.2038$$

$$\begin{aligned} S_{rm} &= 2.0(c + 0.1S) + k_1 k_2 d_{be} h_{ef} \frac{b}{A_s} \\ &= 2.0 \left(30 + 0.1 \left(\frac{200 - 2(20) - 5(11.3)}{2} \right) \right) + 0.4(0.2038)(11.3)(126.05) \left(\frac{200}{300} \right) = 184.49 \text{ mm} \end{aligned}$$

$$w_m = r\varepsilon_1 S_{rm} = (0.36)(184.49) \left(\frac{330.64}{200000} \right) = 0.1097 \text{ mm}$$

$$w_k = 1.7w_m = 1.7(0.1097) = 0.1865 \text{ mm}$$