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SEQUENTIAL OPTIMIZATION METHOD FOR TRANSPORTATION PLANNING

by

Panteha Safavi- Momini

B.Eng., Ryerson University, Toronto, 2003

A report

presented to Ryerson University

in partial fulfillment of the
requirement for the degree of

Master of Engineering

in the Program of

Mechanical Engineering

Toronto, Ontario, Canada, 2005

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ABSTRACT

A fundamental requirement for pollution control is to quantify the emissions being released through emission values. This report expanded the research of Nagurney (2000) by calculating the coefficient of PM_{10} (Particulate Matter < 10 micrometers) and incorporating that into both single and multi-period case studies. Sensitivity analysis was done on reducing the emission constraint and noting the changes to the cost for each path. Several observations were made: (1) single and multi-period scenarios of the same case study followed similar patterns with the addition of emission and further reduction of the emission constraint; (2) for fixed demand, separate cost and variable demand, un-separate cost the cheapest path remained the same despite the addition of emission; (3) for variable demand with separate cost scenario there existed two economical paths at 15% emission reduction; (4) emission reductions of $> 5\%$ did not affect the optimal path for fixed demand with separate cost systems.

ACKNOWLEDGMENTS

This report is dedicated to my parents for their unconditional support throughout the years. Their constant patience and guidance has allowed me to pursue my dreams and accomplish my objectives. I would also like to express my heartfelt gratitude to my sister **Pania** for putting up with me all these years.

I am truly in debt to **Dr. Saeed Zolfaghari**, who generously took over the role of my supervisor at the last minute. This report would not have been possible without his encouragement, guidance and critical comments. My heartfelt appreciation also goes to all my teachers and colleagues at Ryerson University who encouraged me to continue with my research specifically those from the Department of Industrial Engineering. A special thanks should also go to **Dr. Wu** for introducing me to the wonderful world of *Optimization Modeling*.

Finally, I would like to thank my best friend **Karolina** for her unconditional loyalty over the years. I am deeply indebted to her for her much appreciated criticism over the years.

Nomenclature

G	General Network
N	The set of nodes
L	The set of directed links
W	The set of directed paths
a	A link of the network connecting a pair of nodes
p	A path consisting of a sequence of links connecting O/D pairs
P_w	Set of paths connecting the O/D pair of nodes w
P	Set of all paths in network
J	Number of O/D pairs of nodes
x_p	Flow on path p
f_a	The load on link a
np	Number of paths in a network
n	Number of links in a network
d_w	Demand associated with O/D pair w
δ_{ap}	Link to path indicator (equals 1 if path taken; 0 otherwise)
c_a	User link travel cost associated with link a
C_p	User cost associated with path p
C_a	Increasing function of the link load f_a
$\hat{c}_a$	Total cost on link a
$\hat{C}_p$	Total cost on path p
$\hat{C}'_p$	The marginal of the total cost on path p
λ_w	Indicator for the user optimality condition
μ_w	Indicator for the system optimality condition

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CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

The movement of humans and commodities is vital to the economic well being of our society. The key to the success of this movement is the existence of transportation networks. With the explosion of urbanization, traveling has become a serious concern for commuters. Every day, radio stations and newspapers report traffic stories of delays due to construction or congestion. As Rodrigue (2005) reports, the most important transport problems are often related to urban areas, especially when urban transport systems cannot satisfy the numerous requirements of urban circulation. Urban productivity is highly dependent on the efficiency of its transport system, notably to move labour, consumers and freight between several origins and destinations. Thus, the issue of urban traffic planning and the future of public transportation has become a key topic for discussion among city officials and transportation engineers. The key to successful planning is to ensure less traffic and less environmental emissions. This can be done through proper equilibrium modeling and network topology.

1.2 GROWING POPULATION

The population of cities around the world is growing at the rate of about 180,000 a day (Metropolis, 2005). Many regard this massive drift from rural to urban areas as a threat with disastrous consequences for developing countries. The statistics are staggering. According to

UN-HABITAT report (2004/05), the world's urban population will grow from 2.86 billion in 2000 to 4.98 billion by 2030, of which high-income countries will account for only 28 million out of the expected increase of 2.12 billion. The world's annual urban growth rate is projected at 1.8 per cent in contrast to the rural growth rate of 0.1 per cent (Hove, 2000). The report also states that urban growth in middle-and low-income countries accounted for about two billion of this increase. Much of this growth will be in cities over one million persons. Consequently, the concept of urbanization has been one of outmost importance during the past few years. The problem of urbanization have let to an even bigger problem of urban transportation. One of the major causes of urban transport problem is linked to the ever-increasing usage of the private car. Several factors influence the growth of vehicle fleets worldwide. These factors could include sustained economic growth, flexible workforce and simply more leisure time. As the following figure indicates, there is a continuous growth of the fleet with an annual car production of about 34-40 million vehicles in the 1990s. For example, there were over 550 million automobiles in circulation in 2001, an increase of more than 100 million since 1991 (Rodrigue, 2005).

Figure 1-1: World Automobile Production and Fleet, 1965-2002
SOURCE: Worldwatch Institute

1.3 CONGESTION

The growing complexity of cities has been accompanied by a wide array of urban transportation problems. These problems consist of congestion and environmental impacts such as CO₂ emissions linked with the diffusion of the internal combustion engines (Rodrigue, 2005). Congestion is one of the most prevalent transport problems in large urban centers. It occurs when the transport demand exceeds the transport supply. Under this circumstance, each vehicle impairs the mobility of others. During the last decade there has been a tremendous extension of roads in rural and urban areas. Road congestion has resulted in \$100 billion in lost productivity a year in the United States and an estimated \$150 billion in Europe (Nagurney, 2000). According to US Department of Transportation, between 1980 and 1999, route miles of highways have increased 1.5% while vehicle miles of travel have increased by 76%. Traffic delays are commonly measured by comparing the actual travel time with the theoretical travel time, which is the amount of time it would take if there were no delays. The difference equals to the amount of time lost because of traffic delays (Rodrigue, 2005). Further evidence of congestion can be seen through a study done by the Texas Transportation Institute. According to their findings, in the early 1980s, the majority of urban circulation took place in un-congested settings. However, during the late 1990s, 65% of the urban circulation occurred under congestion and half of it under heavily congested situations. Again, the major factors linked with this situation are related to urban sprawl and a growing fleet of trucks and automobiles (Texas Transportation Institute, 1999). Congestion costs are also increasing. According to a report by the Texas Transportation Institute (2005), congestion caused 3.7 billion hours of travel delay and 8.7 billion litres of wasted fuel, an increase of 79 million hours and 262 million litres from 2002 to a total cost of more than \$63 billion for the year.

1.4 ENVIRONMENTAL CONCERNS

During the last decade, global warming caused by an increase in greenhouse gas emissions has become a major issue across the globe. The transportation sector is currently responsible for approximately one-quarter of greenhouse gas emissions in the United States and is expected to be one of the fastest growing sources of greenhouse gas emissions in the near future (Turchetta, 2004). The following figure shows the percentage of U.S. greenhouse gas emissions for 2002:

Figure 1-2: Percentage of U.S. Greenhouse Gas Emissions, 2002
SOURCE: US Department of Transportation

1.5 EMISSION VALUES

A fundamental requirement in the effort to control pollution in any form is to quantify the emissions being released. Emission values are important for developing emission control strategies and determining applicability control programs. An emission value is a representative value that attempts to relate the quantity of a pollutant released to the atmosphere with an activity associated with the release of that pollutant. These values are usually expressed as the weight of pollutant divided by a unit weight, volume, or distance. In most cases, these values are simply averages of all available data and are generally assumed to be representative of long-term averages (Environmental Protection Agency, 2005). Emission values vary depending on several characteristic travel activities such as vehicle type, age mix, vehicle speed, trip length

distribution, operating mode, and ambient temperature (Decorla-Souza et al., 1995). The following table summarizes the listing of variables that affect vehicular emissions:

Table 1-1: Variable Affecting Vehicular Emissions

Variables
Age distribution of vehicular fleet
Number of miles each age cohort is driven
Average emission by cohort
Rate of decay of emission controls
Degree of tampering
Inspection and maintenance effectiveness
Ambient temperature
Driving speed
Frequency of open-loop
Number of cold starts
Idling time
Type of fuel utilized

SOURCE: Compiled by Hall (1995)

1.6 PARTICULATE MATTER (PM₁₀)

In the past there have been numerous studies regarding emission controls of carbon monoxide, sulphur oxides and nitrogen oxides. However, very little research has been done regarding PM₁₀ (particulate matter less than 10 micrometers in diameter).

Only recently the harmful effect of these particulates has gathered attention. Recent studies have found that fine particulates at current levels can pose a greater danger to our health than the better-level ozone, sulphur dioxide and carbon monoxide (Path Finder Science, 2004). Because of their small size, these particles can be inhaled and deposited in the thoracic region of the lungs. Exposure to PM₁₀ can chronically and acutely affect human health, particularly aggravate existing pulmonary and cardiovascular disease, affect mucocilliary clearance and increase mortality. High PM₁₀ levels can also increase corrosion and soiling of materials, and may damage vegetation (Vancouver District, 2004). There has also been a well-established link between PM₁₀ and premature death (see Pope et al., 1995 and Dockery et al., 1993). PM₁₀ is now

considered one of the worst air pollution problems in the U.S., as hazardous as ground-level ozone and all other outdoor air pollutants. The findings are quite alarming. Senior citizens and people who already have lung or heart problems are most at risk, but normal healthy adults and children can also be affected (Path Finder Science, 2004). PM_{10} is generated by diesel and petrol motor vehicles and other combustion processes that burn fossil fuels, such as power generation, industrial processes and domestic solid fuel heaters (Environmental Management, 2004).

Figure 1-3: Source of PM_{10}
SOURCE: www.sparetheair.org/.../bigpicture/IIIAii.html

For obvious reasons, PM_{10} is a bigger concern in major urban areas. The following table lists PM_{10} levels in various areas in Ontario:

Table 1-2: PM_{10} Levels in Ontario

City	PM_{10} Level
Toronto	25.0
Etobicoke	24.0
Oakville	18.5
Hamilton	19.1
London	15.4
Windsor	22.2

SOURCE: U of T Map Collection

The accumulation of PM₁₀ has been a global problem. Figure 1-4 illustrates the levels of PM₁₀ for various areas around the globe:

Figure 1-4: Annual PM₁₀ Levels For Selected Cities (1995)
SOURCE: EPA Victoria (2000) Draft Air Quality Improvement Plan Port Phillip 2000

There have been studies in Australia, Egypt and Canada regarding the harmful effects of these particulates. According to a study done in Melbourne, Australia, on a summer weekday, motor vehicles contribute to about 25% of PM₁₀ (EPA Victoria, 2005). Similar study conducted in Egypt by the Egyptian Environmental Affairs Agency indicated that PM₁₀ presents the most critical air quality problem in Egyp (Egyptian Environmental Agency, 2001). A review of medical studies in Canada has shown that there is no apparent lower threshold for adverse health effects related to PM₁₀.

This has recently prompted the Canadian government to review and strengthen air quality criteria for PM₁₀ in order to reduce the risks to Canadians (Ministry of Environment BC, 2005). Steps

have only been taken recently by the Canadian Environmental Protection Act (CEPA) to declare PM_{10} as a toxic. The U.S. Environmental Protection Agency (EPA) has also recently labeled particulate matter a probable human carcinogen. In addition, in 1998, following an exhaustive 10-year scientific assessment process, the California Air Resources Board (CARB) identified particulate matter from diesel-fuelled engines as a toxic air contaminant (Lung Association, 2005). Australia has also taken measures to control the amount of PM_{10} by issuing the National Environment Protection Measures (NEPM) in 1998. Thus, a trend can be seen around the globe in monitoring the levels of PM_{10} .

1.7 LITERATURE REVIEW

In the past there have been numerous studies regarding transportation networks and methodology for solving these networks. However, very little research exists regarding the harmful effects of PM_{10} . The following sections summarize the various contributions to these fields and provide a justification for the research presented in this report.

1.7.1 Previous Transportation Research

One of the earlier people to discuss the topic of transportation was Hitchcock (1941). The Hitchcock transportation problem is given as follows:

$$\begin{aligned}
 & \text{Minimize} && \sum_{i \in U} \sum_{j \in V} c_{ij} x_{ij} \\
 & \text{Subject to:} && \sum_{j \in V} x_{ij} = a_i && \forall i \in U \\
 & && \sum_{i \in U} x_{ij} = b_j && \forall j \in V \\
 & && x_{ij} \geq 0 && \forall (i, j) \in U \times V
 \end{aligned} \tag{1.1}$$

where $U = \{1, 2, \dots, n\}$ and $V = \{1, 2, \dots, m\}$. Each element in V is called demand point and each element in U is called supply point. The classical example for this problem is the distribution of products from depots to customers; where the nodes in U represent the depots, the nodes in V represent the customers and an edge $(i, j) \in U \times V$ represents a distribution link from the depot i to the customer j . The model finds the minimal cost of product shipment through the complete bipartite graph in order to satisfy the demand at the nodes V and the supplies at the node U (Matsui, 1993).

A few years later, Tjalling Koopmans (1949) discovered that transporting a homogeneous commodity from a set of origins to a set of destinations so as to minimize the total cost of transportation could be formulated as a problem of minimizing a linear function of a number of variables, subject to a set of linear inequalities constraining the values assumed by these variables (Nadal, 2005). Koopmans published these findings in 1949 (Koopmans, 1949). During his research, Koopmans also met Leonid V. Kantorovich who had developed a test for optimality and an outline of an algorithm for linear programming that was similar to but more cumbersome than the simplex method. Kantorovich's work helped Koopmans to outline the final form of his model. In 1975, both Leonid Kantorovich and Tjalling Koopmans received the Nobel Prize in Economics for their contributions on the theory of the optimum allocation of resources (Nadal, 2005).

In the models of Hitchcock (1941), Kantorovich (1942) and Koopmans (1947) no consideration was given to the problem of congestion. Congestion was first examined by Wardrop (1952) where he set forth two principles of transportation network utilization, which have come to be termed, respectively user-optimization and system-optimization (Nagurney, 2005). The first

principle expressed that travelers select their routes of travel from origins to destinations independently and ultimately the journey times of all routes actually used between an origin/destination pair are equal and less than those, which would be experienced by a single vehicle on any unused route. The second principle, in contrast, reflected the situation in which there is a central controller who routes the traffic flows in an optimal manner from origins to the destinations so as to minimize the total cost in the network. That optimum is reached when the marginals of the total costs on used paths connecting an origin/destination pair are equal and minimal (Nagurney, 2005).

Beckmann et al. (1956), however, were the first to provide a rigorous mathematical formulation of the conditions set forth by Wardrop's first principle that allowed for the ultimate solution of the traffic network equilibrium problem in the context of certain link cost functions, which were increasing functions of the flows on the links. In particular, they demonstrated that the optimality conditions in the form of Kuhn- Tucker (1951) conditions of an appropriately constructed mathematical programming/optimization problem coincided with the statement that the travel costs on utilized routes/paths connecting each origin/destination pair of nodes in a transportation network have equal and minimal travel costs. Hence, no traveler, acting unilaterally will have any incentive to alter his path (assuming rational behaviour) since his travel cost is minimal (Nagurney, 2005).

As in market equilibrium problems, the concept of equilibrium can also be present in the transportation equilibrium models. One of the first people to look at this concept was Sheffi (1985). The traditional quantitative method for transportation planning is a *four-step process*. The four steps are trip generation, trip distribution, mode choice, and route assignment. In the

first two steps, the origins (e.g. residential areas) and destinations (e.g. workplaces) are determined and then connected. The typical result of these steps is an origin/destination matrix, which gives, for each zone in the simulated region, the number of trips to each other zone in a given time slice. The last two steps then determine which mode and which path these trips take (Zhang and Levinson, 2005).

As can be seen from above, most of the research has revolved around general congestion and traffic analysis. Only recently, there has been interest in the environmental aspect of traffic control and analysis. Some of the more recent literature has looked at ways of incorporating environmental concerns in traffic control analysis. For example, Tzeng and Chen (1993) proposed a multi-criteria traffic network model using system optimization criteria that incorporated explicit pollution minimization criterion. In case of user optimization, Rilett and Benedek (1994) considered environmental criterion for a single class of user. Bueler (1997) investigated CO₂ emission permits in the framework of a competitive economic equilibrium model.

Nagurney (1999), identified several distinct paradoxical phenomena that could occur in congested transportation networks and their effects on emissions generated. In particular she illustrated through specific examples that the addition of a road may actually result in an increase in total emissions with no change in travel demand and that the total emissions may increase with a decrease in travel demand. This paper demonstrated that network topology as well as cost structure and travel demand must be considered for policies of emission reduction.

Nagurney et al. (1998) were the first however, to explicitly construct a system of pollution permits for congested transportation networks. Nagurney (2000) introduced two new pollution

permit systems for congested transportation networks based on origin/destination pairs and on paths. More recently, Nagurney et al. (2002) published a paper on traffic network equilibrium and the environment where a multi-criteria traffic network equilibrium with an explicit environmental criterion was developed. In this model, the travelers considered two criteria: travel cost and pollution generated on each link in the network. Unfortunately, to date there has been very little research done on PM_{10} and its harmful effect. No previous research was found on PM_{10} and its effect on environmentally friendly transportation networks. The following section provides what limited research exists on PM_{10} .

1.7.2 Previous PM_{10} Research

Limited epidemiological studies have been carried out throughout the world regarding the health impacts of particles in the air. The most common methodology for assessing the relationships has been the standard time series epidemiological studies. Historically, these have shown increases in mortality of around 0-1% per $10 \mu g m^{-3}$ increase in the 24-hour average PM_{10} concentration (New Zealand Ministry of Environment, 2005). An alternative study methodology was used in two US studies, the Harvard Six Cities study (Dockery et al., 1993) and the American Cancer Society study (Pope et al., 1995). These studies followed groups of 8111 and 552,138 adult subjects for 14-16 and seven years respectively. Higher concentrations of particles were found to be associated with increased mortality in both studies (New Zealand Ministry of Environment, 2005). Kunzli et al. (2000) used the results from these studies to estimate the impact of PM_{10} concentrations on mortality in Switzerland, France and Austria. This assessment was based on an increase of 4.3% in mortality per $10 \mu g m^{-3}$ increases in annual average PM_{10} concentrations assuming a no-effect threshold of $7.5 \mu g m^{-3}$. The basis was primarily the Dockery et al. (1993) study which indicated an increase in mortality of around 26% from the

least polluted to the most polluted city, giving a 4.3% increase per $10 \mu\text{gm}^{-3}$ increase in annual average PM_{10} (New Zealand Ministry of Environment, 2005).

In New Zealand, two studies were carried out. The first study considered the impact of both PM_{10} concentrations and temperature on mortality (Hales et al., 1999). Results indicated an increase in both all cause mortality and respiratory mortality associated with a one-day lag in PM_{10} concentrations. An association between respiratory mortality and PM_{10} concentrations of 4% (1.5-5.9%) per $10 \mu\text{gm}^{-3}$ increases in daily PM_{10} concentrations was also recorded (New Zealand Ministry of Environment, 2005). In addition to the Hales et al. (1999) mortality study, a study of the relationship between hospital admissions and particle pollution was also carried out by McGowan et al. (2002). Results indicated a 3.37% increase in respiratory hospital admissions and a 1.26% rise in cardiac admissions for each inter-quartile rise in PM_{10} concentrations (inter-quartile value $14.8 \mu\text{gm}^{-3}$).

As shown, only recently has environmental constraints been added to transportation networks and most research revolve around CO_2 emissions. The author could not find any previous research that specifically looked at PM_{10} and its incorporation into transportation networks. Thus, a need arises to expand the existing research by analyzing system optimized transportation models with the addition of PM_{10} constraint.

1.8 OBJECTIVES

Transportation networks and methodology for solving these networks are very well established. The research can be traced back to the early 1920's where Pigou (1920) considered a two-node, two-link simple network. The research has come a long way recently because of the work of Sheffi (1985) and Nagurney (2000). However, as shown very limited research exists regarding

the harmful effects of PM_{10} concentration. No previous research was found to focus on the effects of PM_{10} levels on urban transportation. Additionally, no research exists to provide guideline on how to monitor and control these harmful particulates in urban areas. Thus, the objectives of this study are as follows:

- Obtain case studies that illustrate different types of transportation networks such as fixed demand, variable demand, separate and un-separate cost
- Expand the research done by Nagurney (2000) by calculating the coefficient of PM_{10} and incorporating this new constraint into each of the case studies
- Perform sensitivity analysis on possible PM_{10} level reductions that a transportation governing body could put on the network and repeat the same procedure for single period and multi-period models with discounted rates

1.9 ORGANIZATION OF REPORT

This report is organized as follows. In Chapter 2, networks and equilibrium models are introduced. Chapter 3 presents an overview of the case studies looked at. In Chapter 4, single period models are discussed while Chapter 5 presents multi-period models. Chapter 6 provides sensitivity analysis for both single period and multi-period models. Finally, the last chapter of this report (Chapter 7) provides conclusions and recommendations for further study required to advance the knowledge.

CHAPTER 2

TRANSPORTATION NETWORKS

2.1 OVERVIEW

The first few section of this chapter provides a background on equilibrium theory as well as transportation theory. The emphasis on this chapter, however, is on transportation networks, specifically urban networks. This chapter ends with a discussion on system-optimized systems versus user-optimized systems.

2.2 GENERAL EQUILIBRIUM THEORY

General equilibrium theory provides a distributed method for efficiently allocating goods and resources among agents. The commodities in an equilibrium market can be physical, or they can be more abstract. Figure 2-1 illustrates a basic form of an equilibrium market mechanism.

Figure 2-1: High-level view of an equilibrium market mechanism.
SOURCE: Sandholm and Ygge, 1999

First the auctioneer announces a new auction, possibly accompanied by some information for example about expected prices. Then agents respond with demand functions, telling what change in allocation they desire at different price alternatives (the demand may be negative, and hence represent supply). Based on the functions submitted by the agents, the auctioneer tries to establish a market-clearing price (or rather a set of prices) such that demand equals supply for each commodity. Optionally, the auctioneer may need to iteratively request further demand and supply information from the agents before equilibrium can be established. When an equilibrium price has been found, the commodities are reallocated among the agents as described by their demands and the market clearing prices (Sandholm and Ygge, 1999).

General equilibrium theory is a branch of microeconomics. It describes the relationship among production, consumption and prices in the economy. Under capitalism, the prices and production of all goods are interrelated. A change in the price of one good, for example bread, may affect another price, for example, the wages of bakers. If bakers differ in tastes from others, the demand for bread might be affected by a change in bakers' wages, with a consequent effect on the price of bread. Calculating the equilibrium price of just one good, in theory, requires an analysis that accounts for all of the millions of different goods that are available (Wikipedia, 2005).

In equilibrium markets, the behaviour of the producers is characterized by a supply function and the behaviour of the consumers is characterized by a demand function. The supply function expresses the amount of goods that the suppliers produce as a function of the price of the product. As the price increases, it becomes profitable to produce more and the quantity supplied increases. The demand function describes the aggregate behaviour of consumers by relating the

amount of the product consumed to its price. As the price increases, the amount of consumed also decreases (Sheffi, 1985). Figure 2-2 illustrates a simple demand and supply function for a certain product. The point where the two curves intersect is called *market clearing* price P^* and quantity produced Q^* . This is the point where the price of the product is just right so that the entire quantity produced is consumed. If the price is higher than P^* , production will be higher than consumption (Figure 2-2b). Such an imbalance cannot be sustained since not all of the product sells and inventories will grow indefinitely. Prices will eventually fall and consumption will increase accordingly. If the price is lower than P^* , the quantity demanded is higher than the production. Such a situation is again unstable since producers will try to increase prices in order to capture the consumers' willingness to pay more. Such price increases will lead to higher production and lower demand. Thus, if the prices are either lower or higher than P^* , market forces will tend to push the price toward its *market clearing* level. At this point the price will be stable and thus the point (P^*, Q^*) is known as the *equilibrium* point (Sheffi, 1985).

Figure 2-2: Demand/Supply Equilibrium:
(a) Market-clearing quantity and price

(b) Price too high

(c) Price too low

SOURCE: Urban Transportation Networks

2.3 TRANSPORTATION

According to Sheffi (1985), transportation is a service that the transportation industry offers travelers. Furthermore, transportation level of service can be measured in terms of travel time, schedule convenience, reliability, safety, comfort, spatial coverage, and accessibility to the service. Many of the components of transportation level of service are not fixed but rather are flow dependent. In other words, their value depends on the usage of the transportation system. An example of a transit line connecting two points in an urban area illustrates this. If many

individuals choose to use this line, the buses will get congested, the waiting time will increase, the probability of obtaining a seat will be lower and the driver is less likely to be friendly. The level of service then will be lower with increased patronage even if the fare system remains fixed. In response to this, some patrons may drive instead of taking transit, other may change the timing of the trips, and yet others may forgo the trip altogether (Sheffi, 1985). According to Sheffi, this situation can be analyzed by using two functions: (1) a performance function that describes how the level of service deteriorates with increasing volume; and (2) a demand function that describes how the volume of passengers increases with improved level of service. These functions are defined for a given situation in terms of fare structure, schedule etc.

The dependence of the level of service on the flow is a fundamental characteristic of the transportation market. In the context of urban transportation, performance functions are rooted in the congestion phenomenon, which increases delays and costs with increased flow. The equilibrium in the urban transportation market is necessarily reached over the urban network of streets and transit lines (Sheffi, 1985). In order to define the notion of equilibrium over a transportation network, the network concept has to be presented and discussed. The following section will cover this topic thoroughly.

2.4 NETWORK REPRESENTATION

The term network is commonly used to describe a structure that can be either physical (e.g., streets and intersections) or conceptual (e.g., information lines and people). Each of these networks includes two types of elements: a set of points and a set of line segments connecting these points. This observation leads to the mathematical definition of a network as a set of *nodes* and a set of *links* connecting these nodes. Figure 2-3 illustrates a network including five nodes

connected by 10 links. Each line in this network is associated with a direction of flow. For example, link 5 represents flow from node 3 to node 2 while link 6 represents the reverse flow.

Figure 2-3: Directed network of links and nodes

Each network link is typically associated with some impedance that affects the flow using it. Impedance can represent electrical resistance, time, costs, utility or any other relevant measure. When the flow involves people, the term *level of service* is usually used instead of impedance. It should be noted that only links could be associated with impedance. Nodes represent merely the intersection of links and are not associated with any impedance to flow (Sheffi, 1985).

Further, the networks discussed in this report are *connected networks*. In other words, to get from one to the next, the entity will follow a *path*. Thus, a path is a sequence of directed links leading from one node to another. For example, looking back at Figure 2-3, to get from node 2 to node 4, the following paths (designated by link numbers) could be followed:

- Path 1 → 9
- Path 2 → 6,7
- Path 3 → 6,3,4
- Path 4 → 1,2,7
- Path 5 → 1,4

For obvious economic reasons, each node will only be used once. Thus, path 1,2,3,4 will not be an economically viable path to take since the driver will have to double back to node 1.

2.5 URBAN NETWORKS

The focus of this study is on urban transportation. Thus, it is important to briefly discuss urban networks. The roadway network in an urban area includes intersections and streets through which traffic moves. A relevant measure for links in urban networks is the time required to traverse the street. The movement of vehicular traffic through streets and intersections is not the only flow in an urban network. It is also important to look at the flow of transit through the links (Sheffi, 1985). It should be noted that each traffic zone is represented by a node known as a *centroid*. The centroids are the source and sink nodes where traffic originates and to which traffic is destined. According to Sheffi (1985), once the set of centroids is defined, the desired *destination matrix* can be expressed. The destination matrix specifies the flow between every origin centroid and every destination centroid in the network (Sheffi, 1985).

2.6 LINK PERFORMANCE FUNCTIONS

A *performance function* rather than a constant travel time measure should be associated with each of the links representing the urban network. The performance function relates the travel time on each link to the flow traversing this link as illustrated in the following figure:

Figure 2-4: Typical link performance function
SOURCE: Sheffi (1985)

The above function captures both the time spent in traveling along the approach under consideration and the delay at the down-stream intersection. The travel time at zero flow is known as the *free-flow* travel time. At this point, a traveling car would not be delayed because of interaction with any other car moving along the link. The only source of delay at this point is the time associated with traversing the link and the expected delay associated with the probability of being stopped by a red signal. As the flow increases, the travel time monotonically increases since both the travel time along the approach increases and the intersection delay increases with the flow. The capacity of the transportation network is also important. The *capacity* is the maximum flow that can go through any transportation facility. When the flow approaches capacity, the queues at the intersection will start growing and clogging the system until traffic finally comes to a halt (Sheffi, 1985).

2.7 SYSTEM OPTIMIZATION VS. USER OPTIMIZATION

The first person to look at alternative possible behaviours of users of transportation networks was Wardrop (1952). These two principles of transportation network utilization are referred to as user optimization and system optimization.

2.7.1 SYSTEM OPTIMIZATION

In *system-optimized* situations, there is a central controller who routes the traffic flows in an optimal manner from origins to the destinations so as to minimize the total cost in the network. That optimum is reached when the marginals of the total costs on used paths connecting an origin/destination pair are equal and minimal (Nagurney, 2003).

Consider a general network $G = [N, L]$, where N denotes the set of nodes and L the set of directed

links. Let a denote a link of the network connecting a pair of nodes and let p denote a path consisting of a sequence of links connecting an origin-destination (O/D) pair. It should be noted that in transportation networks, nodes correspond to origins and destinations as well as to intersections. Links, on the other hand, correspond to roads/streets. A path, thus, is a sequence of roads, which comprise a route from an origin to a destination. P_w denotes the set of paths connecting the O/D pair of nodes w . Let P denote the set of all paths in the network and assume that there are J O/D pairs of nodes. Let x_p represent the flow on path p and let f_a be the load on link a . The path flows on the network are grouped in the column vector $x \in R_+^{n_p}$ where n_p denotes the number of paths in the network. The link nodes are grouped together in column vector $f \in R_+^n$ where n denotes the number of links in the network (Nagurney, 2000). The following conservation of flow equation must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap} \quad \forall a \in L \quad (2.1)$$

Where $\delta_{ap} = 1$ if link a is contained in path p , and 0 otherwise. The above equation indicates that the load on a link a is equal to the sum of all the path flows on path p that contain link a . Additionally, if d_w denotes the demand associated with O/D pair w , then

$$d_w = \sum_{p \in P_w} x_p \quad \forall w \in W \quad (2.2)$$

Let c_a denote the user link travel cost associated with traversing link a and let C_p denote the user cost associated with traversing the path p . Assume that the user link travel cost function is given by

$$C_a = c_a(f_a) \quad \forall a \in L \quad (2.3)$$

Where C_a is assumed to be an increasing function of the link load f_a in order to model the effect of the link load on the travel cost. The total cost on link a denoted by $\hat{c}_a$ is given by:

$$\hat{c}_a = c_a(f_a) \times f_a \quad \forall a \in L \quad (2.4)$$

In other words, the total cost on a link is equal to the user link travel cost on the link times the flow on the link. In a system-optimized problem, there exists a central controller who seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a) \quad (2.5)$$

Thus, the system optimization problem is given by:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) \quad (2.6)$$

$$\text{Subject To:} \quad d_w = \sum_{p \in P_w} x_p \quad \forall w \in W \quad (2.7)$$

$$f_a = \sum_{p \in P} x_p \delta_{ap} \quad \forall a \in L \quad (2.8)$$

$$x_p \geq 0 \quad \forall p \in P \quad (2.9)$$

The total cost on a path is denoted by $\hat{C}_p$ and is the user link travel cost on a path times the flow on a path:

$$\hat{C}_p = C_p x_p \quad \forall p \in P \quad (2.10)$$

Where the user travel cost on a path C_p is given by the sum of the user link travel cost on the links that comprise the path:

$$C_p = \sum_{a \in L} c_a(f_a) \delta_{ap} \quad \forall a \in L \quad (2.11)$$

In addition, one may express the cost on a path p as a function of the path flow variables and thus an alternative version of the above system-optimization problem can be stated in path flow variable only (Nagurney, 2000):

$$\text{Minimize} \quad \sum_{p \in P} C_p(x) x_p \quad (2.12)$$

For each O/D pair $w \in W$ and each path $p \in P$ the flow pattern x must satisfy the following:

$$\hat{C}'_p \begin{cases} = \mu_w, & \text{if } x_p > 0 \\ \geq \mu_w, & \text{if } x_p = 0 \end{cases} \quad (2.13)$$

Where

$$\hat{C}'_p = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap} \quad (2.14)$$

is the marginal of the total cost on path p .

2.7.2 USER OPTIMIZATION

In *user optimized* situations, travelers select their routes of travel from origins to destinations independently and ultimately the journey times of all routes actually used between an origin/destination pair are equal and less than those which would be experienced by a single

vehicle on any unused route. The user-optimized solution is also referred to as a traffic network equilibrium or a traffic assignment (Nagurney, 2003).

Consider again the general network $G = [N, L]$, given in section 2.7.1. As with the *system optimized* problem, the travel demands associated with the origin/destination pairs as well as the user link travel cost functions are assumed as given. In this type of problem, however, one tries to determine the path flow pattern x^* and link load pattern f^* which would satisfy the conservation of flow equations of 2.1 and 2.2 and the non-negativity assumption on the path flows. For each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p \begin{cases} = \lambda_w, & \text{if } x_p^* > 0 \\ \geq \lambda_w, & \text{if } x_p^* = 0 \end{cases} \quad (2.15)$$

In other words, in the user-optimization problem, there is no explicit optimization concept since now travelers act independently in a no-cooperative manner until they cannot improve on their situations unilaterally and thus, equilibrium is achieved. In order to obtain a solution, Beckmann et al. (1956) established that the solution to the equilibrium problem in the case of separable user link travel cost functions could be obtained by solving the following optimization problem (Nagurney, 2000):

$$\text{Minimize} \quad \sum_{a \in L} \int_0^{f_a} c_a(x) dx \quad (2.16)$$

$$\text{Subject to:} \quad \sum_{p \in P_w} x_p = d_w, \quad \forall w \in W \quad (2.17)$$

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L \quad (2.18)$$

$$x_p \geq 0, \quad \forall p \in P \quad (2.19)$$

It should be noted that the conservation of flow equations are identical in both the user-optimized problem (2.17-2.19) and the system-optimized problem (2.7-2.9), but the behaviour of the travelers is different.

CHAPTER 3

SUMMARY OF EXAMPLES

3.1 OVERVIEW

For the purpose of this research project, five examples taken from various sources were used.

The first source is *Sustainable Transportation Networks* (Nagurney, 2000) and *Network Economics: A Variational Inequality Approach* (Nagurney, 1998). In addition, a previous thesis work, *Multi-Period Transportation Equilibrium Approach for Transportation Planning* (Wu, 2003) was also used for another source of examples.

The following table summarizes the examples used for the purpose of this report:

Table 3-1: Summary of Examples

EX	Reference	Fixed Demand	Variable Demand	Separate Cost	Un-Separate Cost
1	STN-24	√		√	
2	STN-156	√		√	
3	NE-160/176	√			√
4	MW		√	√	
5	STN-32		√		√

*NOTE: STN: Sustainable Transportation Networks

NE: Network Economics

MW: Meng Wu's Thesis

The following figures represent the networks used for the above examples:

Figure 3-1: Network for Example 1 (Fixed Demand and Separate Cost)

Figure 3-2: Network for Example 2 (Fixed Demand and Separate Cost)

Figure 3-3: Network for Example 3 (Fixed Demand and Un-Separate Cost)

Figure 3-4: Network for Example 4 (Variable Demand and Separate Cost)

Figure 3-5: Network for Example 5 (Variable Demand and Un-Separate Cost)

Please refer to Appendix A for a detailed solution of these examples.

3.2 LINGO

All models in this thesis were solved using LINGO software, a product of LINDO Systems™. LINGO is a comprehensive tool designed to build and solve linear, nonlinear, and integer optimization models faster, easier, and more efficient. LINGO lets users formulate linear, non-linear, and integer problems quickly in a highly readable form. LINGO's modeling language allows models to be expressed in a straightforward manner using summations and subscripted variables (LINGO, 2000).

3.3 EMISSION VALUES

As previously mentioned, emission values depend on numerous variables, including vehicle type, age, and speed. However, calculating the exact emission value for PM_{10} levels is beyond the scope of this project. Thus, research was done to find an estimate of this value to be used for this project.

For example, one source indicates that PM_{10} levels should be set at 0.02 (EPA, 2005). Another source indicates that PM_{10} emission factors should range from 0.030 (+/-0.009) to 0.131 (+/-0.024) g/vehicle.km with an average of 0.069 (+/-0.030) g/vehicle.km (Gillies et al., 2001). The U.S. Environmental Protection Agency uses 1998 Ford Taurus as an example and gives the emission factor as 0.08 (Montgomery EPA, 2005). Finally, another study breaks down the emission factor to incorporate break and tire wear, running exhaust and speed. The following table summarizes their findings (California Air Resource Board, 2004).

Table 3-2: Average Emission Factor for Light Vehicles

Source	Emission Factor g/vehicle.km
Running Exhaust	0.014
Tire and Break Wear	0.021
Speed of 40 miles/hr	0.02
TOTAL	0.055

From the above, it can be seen that different sources give different values for PM_{10} emission factors. Thus, it was decided to use a range of 0.03-0.07 g/vehicle.km for the scope of this project. The following tables summarize the emission factors used for the different links of each example used in this report:

Table 3-3: Average Emission Factor for Different Links

Links	Emission Factor (g/vehicle.km)
a	0.03
b	0.05
c	0.04
d	0.04
e	0.07

CHAPTER 4

SINGLE PERIOD MODELS

4.1 OVERVIEW

As mentioned in the pervious chapters, equilibrium concepts can be readily applied in transportation networks. In this chapter, single-period transportation problems are discussed for system-optimized situations. In addition, emission constraints are added to the models to note the changes to the cost for the system.

4.2 SINGLE-PERIOD OPTIMIZATION MODELS

In single-period system optimization models, there exists a central controller who seeks to minimize the total time spent in the network. The system achieves system equilibrium status when the marginal time spent in used paths is less than or equal to marginal time on unused paths. In other words, the system achieves equilibrium status from the whole system to minimize the total drivers' travel time rather than individual user travel time.

As mentioned in chapter 2, the single-period system optimized model can be given by:

$$\begin{aligned}
 & \text{Minimize} && \sum_{a \in L} \hat{c}_a(f_a) \\
 & d_w = \sum_{p \in P_w} x_p && \forall w \in W \\
 & f_a = \sum_{p \in P} x_p && \forall a \in L
 \end{aligned}$$

$$x_p \geq 0 \quad \forall p \in P \quad (4.1)$$

It should be noted that in scenarios where the demand is not constant, the objective function changes to include the demand disutility function:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) - \sum_{w \in W} \lambda_w(d_w) d_w \quad (4.2)$$

Where λ_w is the travel disutility function and is dependent on the entire vector of travel demands, which are no longer, fixed but are now variables.

4.3 SINGLE PERIOD – NO EMISSION

For illustration purposes, this section provides a full solution for the formulation of Example 1 and Example 5. These models along with the rest of examples are provided in Appendix A.

4.3.1 Formulation of Example 1

Nodes = 2

Links = 2

Paths = 2 (a or b)

Travel Cost Function $\rightarrow C_a = c_a(f_a) = 2f_a + 5$

$C_b = c_b(f_b) = f_b + 10$

Travel Demand $\rightarrow d_w = 10$

Path to Link Relationship:

		Path 1	Path 2
Path 1 → Link <i>a</i>	Link <i>a</i>	Yes (1)	No (0)
Path 2 → Link <i>b</i>	Link <i>b</i>	No (0)	Yes (1)

$$\delta_{ap} \rightarrow \begin{array}{ll} \delta_{a,1} = 1 & \delta_{b,1} = 0 \\ \delta_{a,2} = 0 & \delta_{b,2} = 1 \end{array}$$

Total Travel Cost on each Path:

$$C_p = \sum_{a \in L} c_a(f_a) \delta_{ap}$$

$$C_1 = c_a(f_a) \delta_{a,1} + c_b(f_b) \delta_{b,1} = c_a(f_a) \quad \therefore \text{cost of Path 1} = \text{cost of Link } a$$

$$C_2 = c_a(f_a) \delta_{a,2} + c_b(f_b) \delta_{b,2} = c_b(f_b) \quad \therefore \text{cost of Path 2} = \text{cost of Link } b$$

Flows on each Link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}$$

$$f_a = x_1 \delta_{a,1} + x_2 \delta_{a,2} = x_1$$

$$f_b = x_1 \delta_{b,1} + x_2 \delta_{b,2} = x_2$$

Total Cost on Link *a*:

$$\hat{c}_a = c_a(f_a) \times f_a$$

$$= (2f_a + 5) \times f_a$$

$$= 2f_a^2 + 5f_a$$

Total Cost on Link b:

$$\begin{aligned}\hat{c}_b &= c_b(f_b) \times f_b \\ &= (f_b + 10) \times f_b \\ &= f_b^2 + 10f_b\end{aligned}$$

Model:

$$\text{Minimize } z = 2f_a^2 + 5f_a + f_b^2 + 10f_b$$

$$\text{Subject To: } f_a + f_b = 10$$

$$f_a, f_b \geq 0$$

The above section illustrated how models where cost of each path is equal to the cost of their corresponding link is formulated. Same approach was used for Examples 2 and 3 of this report. However, in situations where multiple links add to a single path, a different approach is used to calculate the cost for each path. The following section illustrates the formulation of Example 5.

4.3.2 Formulation of Example 5

Nodes = 3

Links = 3

Paths = 2 (ac or bc)

$$\text{Travel Cost Function} \rightarrow C_a = c_a(f_a) = 5f_a + 2f_b + 5$$

$$C_b = c_b(f_b) = 7f_b + f_a + 5$$

$$C_c = c_c(f_c) = 3f_c + f_a + f_b + 7$$

$$\text{Demand Disutility Function} \rightarrow \lambda_{12} = 99 - d_1$$

Path to Link Relationship:

		Path 1	Path 2
Path 1 → Link <i>a</i> + Link <i>c</i>	Link <i>a</i>	Yes (1)	No (0)
Path 2 → Link <i>b</i> + Link <i>c</i>	Link <i>b</i>	No (0)	Yes (1)
	Link <i>c</i>	Yes (1)	Yes (1)

$$\delta_{ap} \rightarrow \begin{array}{lll} \delta_{a,1} = 1 & \delta_{b,1} = 0 & \delta_{c,1} = 1 \\ \delta_{a,2} = 0 & \delta_{b,2} = 1 & \delta_{c,2} = 1 \end{array}$$

Total Travel Cost on each Path:

$$C_p = \sum_{a \in L} c_a(f_a) \delta_{ap}$$

$$C_1 = c_a(f_a) \delta_{a,1} + \cancel{c_b(f_b) \delta_{b,1}} + c_c(f_c) \delta_{c,1} = c_a(f_a) + c_c(f_c)$$

$$C_2 = \cancel{c_a(f_a) \delta_{a,2}} + c_b(f_b) \delta_{b,2} + c_c(f_c) \delta_{c,2} = c_b(f_b) + c_c(f_c)$$

Flows on each Link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}$$

$$f_a = x_1 \delta_{a,1} + \cancel{x_2 \delta_{a,2}} = x_1$$

$$\cancel{f_b = x_1 \delta_{b,1}} + x_2 \delta_{b,2} = x_2$$

$$f_c = x_1 \delta_{c,1} + x_2 \delta_{c,2} = x_1 + x_2$$

Model:

$$\text{Minimize } z = [5f_a^2 + 2f_a f_b + 5f_a + 7f_b^2 + f_a f_b + 5f_b + 3f_c^2 + f_a f_c + f_b f_c + 7f_c] - [99d_1 - 2d_1^2]$$

$$\text{Subject To: } f_a + f_b = f_c$$

$$f_a + f_b = d_1$$

$$f_a, f_b, f_c \geq 0$$

The following table summarizes the results calculated for the models formulated:

Table 4-1: Summary of Single Period – No Emission

Example	1	2	3	4	5
z	147.92	108.78	283.33	-86.74	-196.31
f_a	4.16	2.65	3.33	1.13	2.76
f_b	5.83	3.80	1.67	0.63	1.75
f_c	-	3.54	3.33	1.31	4.51
f_d	-	-	0.00	1.81	-
f_e	-	-	5.00	-	-

The cost on each link for each of the examples was also calculated. Table 4-2 summarizes these costs:

Table 4-2: Summary of Costs for each Link

Example	C_a	C_b	C_c	C_d	C_e
1	13.32	15.83	-	-	-
2	10.30	11.80	10.31	-	-
3	21.65	21.70	168.3	156.33	33.33
4	2.13	2.63	3.31	2.81	-
5	22.30	20.01	25.04	-	-

4.4 SINGLE PERIOD – WITH EMISSION

4.4.1 Example 1

Using *Example 1* of Appendix A, the following models were modified to incorporate the emission constraint:

$$\text{For Link a} \rightarrow \text{Total Emission} = 0.03 * f_a$$

$$\begin{aligned}
 \text{For Link b} \rightarrow \text{Total Emission} &= 0.03 * 4.16 &= 0.12 \\
 &= 0.05 * f_b \\
 &= 0.05 * 5.83 &= 0.29
 \end{aligned}$$

$$\text{Upper limit for model} = 0.41 \text{ g/vehicle.km}$$

New Model Formulated for Example 1:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

New Results:

Optimal solution found at step: 4
Objective value: 148.25

Variable	Value	Reduced Cost
X1	4.50	0
X2	5.50	0

Row	Slack or Surplus	Dual Price
1	148.250	1.00
2	0.00	-26.00
3	0.00	100.00
4	4.50	0.00
5	5.50	0.00

Table 4-3: No Emission vs. Emission Scenario for Example 1

Comparison	Z	X ₁	X ₂	C _a	C _b
No Emission	147.92	4.16	5.83	13.32	15.83
With Emission	148.25	4.50	5.50	14.00	15.50

In this example, the cost for each path is equal to the cost for each link since each path has only one link. For instance, the cost for *path a* equals the cost for *Link a*. Table 4-3 illustrates that the cost for *path a* increased from 13.32 to 14.00. However, the cost of *path b* decreased from 15.83

to 15.50. In other words, despite the decreases observed in the two scenarios, *path a* remains the cheapest path to take.

4.4.2 Example 2

The following model was created for example two:

For Link a → Total Emission	$= 0.03 * f_a$	
	$= 0.03 * 2.65$	$= 0.079$
For Link b → Total Emission	$= 0.05 * f_b$	
	$= 0.05 * 3.80$	$= 0.190$
For Link c → Total Emission	$= 0.04 * f_c$	
	$= 0.04 * 3.54$	$= 0.141$
Upper limit for model		$= 0.41 \text{ g/vehicle.km}$

New Model Formulated:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

New Results:

Optimal solution found at step: 4
Objective value: 108.80

Variable	Value	Reduced Cost
X1	2.72	0.00
X2	3.72	0.00
X3	3.55	0.00

Row	Slack or Surplus	Dual Price
1	108.80	1.00
2	0.00	-16.55
3	0.00	22.22
4	2.72	0.31
5	3.72	0.00
6	3.55	0.00

Table 4-4: No Emission vs. Emission Scenario for Example 2

Comparison	Z	X ₁	X ₂	X ₃	C _a	C _b	C _c
No Emission	108.78	2.65	3.80	3.54	10.30	11.80	10.31
With Emission	108.80	2.72	3.72	3.55	10.44	11.72	10.32

This example is similar to example 1 in which the cost for each path equals the cost for each link. From above table it can be seen that the cost for *path a* and *path c* increased, while the cost for *path b* decreased. In other words, originally *path a* was the cheapest path to take but with the addition of emission to the system, *path c* became the cheapest path.

4.4.3 Example 3

The following model was created for example three:

For Link a → Total Emission	= 0.03 * f _a	
	= 0.03 * 3.33	= 0.099
For Link b → Total Emission	= 0.05 * f _b	
	= 0.05 * 1.67	= 0.083
For Link c → Total Emission	= 0.04 * f _c	
	= 0.04 * 0.00	= 0.00
For Link d → Total Emission	= 0.04 * f _d	
	= 0.04 * 0.00	= 0.00
For Link e → Total Emission	= 0.07 * f _e	
	= 0.07 * 5.00	= 0.35
Upper limit for model		= 0.53 g/vehicle.km

New Model Formulated:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } 0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.53$$

$$X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 7
 Objective value: 258.40

Variable	Value	Reduced Cost
X1	3.40	0.00
X4	0.00	115.20
X2	1.60	0.00
X3	0.00	152.00
X5	5.00	0.00

Row	Slack or Surplus	Dual Price
1	258.40	1.00
2	0.00	-41.99
3	0.00	-56.99
4	0.00	99.99
5	3.40	0.00
6	1.60	0.00
7	0.00	0.00
8	0.00	0.00
9	5.00	0.00

Table 4-5: No Emission vs. Emission Scenario for Example 3

Comparison	Z	X ₁	X ₂	X ₃	X ₄	X ₅	C ₁	C ₂	C ₃	C ₄	C ₅
No Emission	258.33	3.33	1.67	0.00	0.00	5.00	21.65	21.70	168.30	156.33	33.33
With Emission	258.40	3.40	1.60	0.00	0.00	5.00	22.00	21.00	135.00	156.80	23.60

Similar to Examples 1 and 2, the cost for each path is equal to the cost for each link in this example. However, in this example there are three forward links (*Link a*, *Link b* and *Link c*) and two reverse links (*Link d* and *Link e*). For the forward link with no emission it was found that the cheapest path would be *path a*. When emission was added to the system, the cheapest path changes to *Path b*. In the case of the reverse links, under no circumstances would *path d* be the cheapest path to take. In other words, the cheapest path to take with and without emission would be *Path e*.

4.4.4 Example 4

The following model was created for example four:

For Link a → Total Emission	= 0.03 * f_a = 0.03 * 1.13	= 0.03
For Link b → Total Emission	= 0.05 * f_b = 0.05 * 0.63	= 0.03
For Link c → Total Emission	= 0.04 * f_c = 0.04 * 1.31	= 0.05
For Link d → Total Emission	= 0.04 * f_d = 0.01 * 1.81	= 0.02
Upper limit for model		= 0.13 g/vehicle.km

New Model Formulated:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject to: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.13$$

Results:

Optimal solution found at step: 10
 Objective value: -80.29

Variable	Value	Reduced Cost
X1	1.21	0.1011E-05
X2	0.00	3.616
X3	0.919	0.1678E-06
X4	1.419	0.00
D1	1.214	0.00
D2	2.339	0.00
F1	1.214	0.00
F2	2.339	0.00
F3	2.134	0.00
F4	1.419	0.00

Row	Slack or Surplus	Dual Price
1	-80.286	1.00
2	0.00	-10.99
3	0.00	-13.93
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	252.27

Table 4-6: No Emission vs. Emission Scenario for Example 4

Comparison	Z	X ₁	X ₂	X ₃	X ₄	C _{p1}	C _{p2}	C _{p3}	C _{p4}
No Emission	-86.74	1.13	0.63	1.31	1.81	5.44	4.94	5.94	5.44
With Emission	-80.29	1.21	0.00	0.92	1.42	5.13	4.63	4.92	4.42

Cost of Path Calculations (No Emission):

C_{p1} = Cost of Link ($a + c$)

$$= 2.13 + 3.31$$

$$= 5.44$$

C_{p2} = Cost of Link ($a + d$)

$$= 2.13 + 2.81$$

$$= 4.94$$

$Cp_3 = \text{Cost of Link } (b + c)$

$$= 2.63 + 3.31$$

$$= 5.94$$

$Cp_4 = \text{Cost of Link } (b + d)$

$$= 2.63 + 2.81$$

$$= 5.44$$

In this example, the cost for each path is not identical to the cost for each link. This is because each path contains more than one link. According to the above calculations, the cheapest path to take is Path 2 (*Link a + Link d*) for the no emission scenario. Similar calculations were done to find the cheapest path when emission is added to the system. In this scenario the cheapest path changes to Path 4 (*Link b + Link d*).

4.4.5 Example 5

The following model was created for example five:

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 2.76 &= 0.08 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 1.75 &= 0.09 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 4.51 &= 0.18 \end{aligned}$$

$$\text{Upper limit for model} = 0.35 \text{ g/vehicle.km}$$

New Model Formulated:

$$\text{MIN } Z = (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) +$$

$$\text{Subject to: } 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1)$$

$$X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 6
 Objective value: -196.31

Variable	Value	Reduced Cost
X1	2.75	0.00
X2	1.75	0.00
X3	4.50	0.00
D1	4.50	0.00

Row	Slack or Surplus	Dual Price
1	-196.31	1.00
2	0.00	38.62
3	0.00	-80.99
4	0.00	3.00
5	2.75	0.00
6	1.75	0.205E-06
7	4.50	0.00

Table 4-7: No Emission vs. Emission Scenario for Example 5

Comparison	Z	X ₁	X ₂	X ₃	C _{p1}	C _{p2}
No Emission	-196.31	2.76	1.75	4.51	47.34	45.05
With Emission	-196.31	2.75	1.75	4.50	47.25	45.00

Cost of Path Calculations (No Emission):

$$\begin{aligned}
 \text{Cp}_1 &= \text{Cost of Link } (a + c) & \text{Cp}_2 &= \text{Cost of Link } (b + c) \\
 &= 22.30 + 25.04 & &= 20.01 + 25.04 \\
 &= 47.34 & &= 45.05
 \end{aligned}$$

As with the previous example, the cost for each path is not identical to the cost for each link.

According to Table 4-7, the cheapest path to take is Path 2 (*Link b + Link c*) for the no emission scenario. Similar calculations were done to find the cheapest path when emission is added to the system. It was found that the cheapest path remained the same with the addition of emission to the system (Path 2).

CHAPTER 5

MULTI PERIOD MODELS

5.1 OVERVIEW

This chapter covers multi-period transportation problems for system-optimized situations. Three period problems with and without the emission constraint will be looked at. Similar to single-period optimization models, in multi-period models there exists a central controller who seeks to minimize the total time spent in the network. The system achieves system equilibrium status when the marginal time spent in used paths is less than or equal to marginal time on unused paths for the total amount of periods (Wu, 2003).

5.2 MULTI-PERIOD

As previously mentioned three period situations were looked at for the purpose of this report. The following sections summarize the models created for $t = 3$ period models. For a complete solution of this section, refer to Appendix C found at the end of this report.

5.3 MULTI PERIOD – NO EMISSION

The following table summarizes the values obtained for two period models ($t = 3$) with no emission and no discount factor considered.

Table 5-1: Summary of Multi Period ($t = 3$) – No Emission

Example	1	2	3	4	5
Z	443.75	326.36	775.00	-260.23	-588.94
X ₁₁	4.16	2.65	3.33	1.13	2.76
X ₁₂	4.16	2.65	3.33	1.13	2.76
X ₁₃	4.16	2.65	3.33	1.13	2.76
X ₂₁	5.83	3.80	1.67	0.63	1.75
X ₂₂	5.83	3.80	1.67	0.63	1.75
X ₂₃	5.83	3.80	1.67	0.63	1.75
X ₃₁		3.54	0.00	1.31	4.51
X ₃₂		3.54	0.00	1.31	4.51
X ₃₃		3.54	0.00	1.31	4.51
X ₄₁			0.00	1.81	-
X ₄₂			0.00	1.81	-
X ₄₃			0.00	1.81	-
X ₅₁			5.00	-	-
X ₅₂			5.00	-	-
X ₅₃			5.00	-	-
d ₁₁				1.77	4.51
d ₁₂				1.77	4.51
d ₁₃				1.77	4.51
d ₂₁				3.11	
d ₂₂				3.11	
d ₂₃				3.11	
f ₁₁				1.77	
f ₁₂				1.77	
f ₁₃				1.77	
f ₂₁				3.11	
f ₂₂				3.11	
f ₂₃				3.11	
f ₄₁				2.44	
f ₄₂				2.44	
f ₄₃				2.44	

The cost on each path for each of the examples was also calculated and given in Table 5-2:

Table 5-2: Total Cost for t=3

Example	C_a	C_b	C_c	C_d	C_e
1	39.96	47.49			
2	30.90	35.40	30.93		
3	64.95	65.10	405.00	469.98	69.96
4	6.39	7.89	9.93	8.43	
5	66.90	60.03	75.21		

5.4 MULTI PERIOD – EMISSION

Using the multi-period models ($t = 3$) of Appendix C, the following models were modified to incorporate the emission constraint:

5.4.1 Example 1

The following model was created for example one:

For $t = 3$

$$E(1) = 0.03X_{11} + 0.05X_{21} \leq 0.41$$

$$E(2) = 0.03X_{12} + 0.05X_{22} \leq 0.41$$

$$E(3) = 0.03X_{13} + 0.05X_{23} \leq 0.41$$

New Model for $t = 3$

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.41$$

$$0.03X_{12} + 0.05X_{22} \leq 0.41$$

$$0.03X_{13} + 0.05X_{23} \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: 444.750

Variable	Value	Reduced Cost
X11	4.50	0.00
X21	5.50	0.00
X12	4.50	0.00
X22	5.50	0.00
X13	4.50	0.00
X23	5.50	0.00
X1	0.249E-05	0.00
X2	0.249E-05	0.00

Row	Slack or Surplus	Dual Price
1	444.75	1.00
2	0.00	-26.00
3	0.00	-26.00
4	0.00	-26.00
5	0.00	100.00
6	0.00	100.00
7	0.00	100.00
8	0.249E-05	0.00
9	0.249E-05	0.00

Table 5-3: Multi- Period No Emission vs. Emission for Example 1

Comparison	Z	X ₁	X ₂	C _a	C _b
No Emission	443.75	4.16	5.83	39.96	47.49
With Emission	444.75	4.50	5.50	42.00	46.50

It can be seen that by the addition of the emission constraint to the model, the cost for *path a* increased from 39.96 to 42.00. However, the cost of *path b* decreased from 47.49 to 46.50. The optimal path for this example remains *path a* despite the changes seen as a result of the additional emission constraint to the system. It should be noted that for the single period scenario of this same example (Table 4-3), the addition of the emission constraint also did not change the cheapest path.

5.4.2 Example 2

The following model was created for example two:

For $t = 3$

$$E(1) = 0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.41$$

$$E(2) = 0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.41$$

$$E(3) = 0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.41$$

New Model for $t = 3$

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.41$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.41$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 7
Objective value: 326.416

Variable	Value	Reduced Cost
X11	2.72	0.00
X21	3.72	0.00
X31	3.55	-0.284E-06
X12	2.72	0.00
X22	3.72	0.00
X32	3.55	-0.694E-06
X13	2.72	0.00
X23	3.72	0.00

X33	3.55	-0.437E-06
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	326.416	1.00
2	0.00	-16.55
3	0.00	-16.55
4	0.00	-16.55
5	0.00	22.22
6	0.00	22.22
7	0.00	22.22
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

Table 5-4: Multi- Period No Emission vs. Emission for Example 2

Comparison	Z	X ₁	X ₂	X ₃	C _a	C _b	C _c
No Emission	326.36	2.65	3.80	3.54	30.90	35.40	30.93
With Emission	326.42	2.72	3.55	3.72	31.32	34.65	31.74

It can be seen that by the addition of the emission constraint to the model, the cost for *path a* and *path c* increased. However, the cost of *path b* decreased from 35.40 to 34.65. The optimal path for this example remains *path a* despite the changes seen as a result of the additional emission constraint to the system. It should be noted that for the single period scenario of this same example (Table 4-4), the addition of emission constrained changed the optimal path from *path a* to *path c*.

5.4.3 Example 3

For $t = 3$

$$E(1) = 0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.53$$

$$E(2) = 0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.53$$

$$E(3) = 0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.53$$

New Model for $t = 3$:

$$\begin{aligned} \text{MIN } Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\ & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\ & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\ & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\ & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\ & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 5$$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.53$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.53$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.53$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 776.25

Variable	Value	Reduced Cost
X11	3.50	0.00
X41	0.00	110.49
X21	1.49	0.00
X31	0.00	152.50
X51	5.00	0.00
X12	3.50	0.00
X42	0.00	110.49
X22	1.49	0.00
X32	0.00	152.50
X52	5.00	0.00

X13	3.50	0.00
X43	0.00	110.49
X23	1.49	0.00
X33	0.00	152.50
X53	5.00	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	776.250	1.00
2	0.00	-47.50
3	0.00	-47.50
4	0.00	-47.50
5	0.00	-67.50
6	0.00	-67.50
7	0.00	-67.50
8	0.286E-07	250.00
9	0.286E-07	250.00
10	0.286E-07	250.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

Table 5-5: Multi- Period No Emission vs. Emission for Example 3

Comparison	Z	X ₁	X ₂	X ₃	X ₄	X ₅	C _a	C _b	C _c	C _d	C _e
No Emission	775.00	3.33	1.67	0.00	0.00	5.00	64.95	65.10	405.00	469.98	90.00
With Emission	776.25	3.50	1.50	0.00	0.00	5.00	67.50	60.00	405.00	471.00	90.00

For the forward links with no emission, it was found that the cheapest path would be *path a*.

When emission was added to the system, the cheapest path changes to *Path b*. In the case of the reverse links, the cheapest path to take with and without emission would be *Path e*. These results are in agreement with the single period models of the same example. Thus, both the multi-period and single period scenarios of this example provide the same optimal path.

5.4.4 Example 4

The following model was created for example four:

For $t = 3$

$$E(1) = 0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} \leq 0.13$$

$$E(2) = 0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} \leq 0.13$$

$$E(3) = 0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} \leq 0.13$$

New Model for $t = 3$:

$$\begin{aligned} \text{MIN } Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\ & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\ & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\ & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\ & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$X_{31} + X_{41} = d_{21}$$

$$X_{32} + X_{42} = d_{22}$$

$$X_{33} + X_{43} = d_{23}$$

$$X_{11} + X_{21} = f_{11}$$

$$X_{12} + X_{22} = f_{12}$$

$$X_{13} + X_{23} = f_{13}$$

$$X_{31} + X_{41} = f_{21}$$

$$X_{32} + X_{42} = f_{22}$$

$$X_{33} + X_{43} = f_{23}$$

$$X_{11} + X_{31} = f_{31}$$

$$X_{12} + X_{32} = f_{32}$$

$$X_{13} + X_{33} = f_{33}$$

$$X_{21} + X_{41} = f_{41}$$

$$X_{22} + X_{42} = f_{42}$$

$$X_{23} + X_{43} = f_{43}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} \leq 0.13$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} \leq 0.13$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} \leq 0.13$$

Results:

Optimal solution found at step: 32
 Objective value: -240.858

Variable	Value	Reduced Cost
X11	1.21	-0.540E-03
X21	0.222E-15	3.61
X31	0.92	-0.213E-02
X41	1.42	0.00
D11	1.21	0.00
D21	2.34	0.00
X12	1.21	0.247E-02
X22	0.00	3.619
X32	0.92	0.444E-03
X42	1.42	0.00
D12	1.21	0.00
D22	2.34	0.00
X13	1.21	-0.530E-02
X23	0.00	3.610
X33	0.92	0.00
X43	1.42	-0.126E-02
D13	1.21	0.00
D23	2.34	0.00
F11	1.21	0.00
F12	1.21	0.00
F13	1.21	0.00
F21	2.34	0.00
F22	2.34	0.00
F23	2.34	0.00
F41	1.42	0.00
F42	1.42	0.00
F43	1.42	0.00

Row	Slack or Surplus	Dual Price
1	-240.86	1.00
2	-10.99	0.00
3	-10.99	0.00
4	0.00	-11.00

5	0.00	-13.93
6	0.00	-13.93
7	0.00	-13.93
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.00
20	0.00	252.25
21	0.00	252.30
22	0.00	252.21

Table 5-6: Multi- Period No Emission vs. Emission for Example 4

Comparison	Z	X ₁	X ₂	X ₃	X ₄	C _{p1}	C _{p2}	C _{p3}	C _{p4}
No Emission	-260.23	1.13	0.63	1.31	1.81	16.32	14.82	17.82	16.32
With Emission	-240.86	1.21	0.22e-15	0.92	1.42	15.39	13.89	14.76	13.26

According to the cost of path calculation, it was observed that the cheapest path to take is Path 2 (*Link a + Link d*) for the no emission scenario. Using similar calculations, it was found that with the addition of the emission constraint the cheapest path changes to Path 4 (*Link b + Link d*). This is in agreement with the single period scenario of the same example.

5.4.5 Example 5

The following model was created for example five:

$$E(1) = 0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.35$$

$$E(2) = 0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.35$$

$$E(3) = 0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.35$$

New Model for $t = 3$:

$$\begin{aligned} \text{MIN } Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\ & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\ & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\ & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\ & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\ & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13}) \end{aligned}$$

Subject to: $X_{11} + X_{21} = X_{31}$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.35$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.35$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 11
Objective value: -588.937

Variable	Value	Reduced Cost
X11	2.75	0.00
X21	1.75	0.234E-05
X31	4.50	0.00
D11	4.50	0.00
X12	2.75	0.00
X22	1.75	0.220E-05
X32	4.50	0.00
D12	4.50	0.00
X13	2.75	0.00
X23	1.75	0.220E-05
X33	4.50	0.00
D13	4.50	0.00

X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-588.937	1.00
2	0.00	38.62
3	0.00	38.62
4	0.00	38.62
5	0.00	-80.99
6	0.00	-80.99
7	0.00	-80.99
8	0.00	3.00
9	0.00	3.00
10	0.00	3.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

Table 5-7: Multi- Period No Emission vs. Emission for Example 5

Comparison	Z	X ₁	X ₂	X ₃	C ₀₁	C ₀₂
No Emission	-588.94	2.76	1.75	4.52	142.02	135.15
With Emission	-588.94	2.75	1.75	4.50	141.75	135.00

For the no emission scenario of this example, the cheapest path was found to be Path 2 (*Link b* + *Link c*). Using similar calculations it was found that the cheapest path remained the same with the addition of emission to the system.

CHAPTER 6

SENSITIVITY ANALYSIS

6.1 OVERVIEW

Sensitivity analysis is concerned with how changes in linear programming parameters affect the overall optimal solution. In this chapter, changes are made to the right hand side of the emission constraint to see the resulting effects on the cost of each path. Thus, the right hand side of the emission constraint for each example was changed from 5% to 25%.

6.2 SINGLE PERIOD MODELS

6.2.1 Example 1

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 1*:

Table 6-1: Sensitivity Analysis for Example 1

Emission Reduction	Z	X ₁	X ₂
Original - No Emission	147.92	4.16	5.83
0 % reduction	148.25	4.50	5.50
5 %	153.25	5.50	4.50
10 %	164.25	6.50	3.50
15 %	181.25	7.50	2.50
20 %	204.25	8.50	1.50
25%	233.25	9.50	0.50

For a complete listing of these results, please refer to Appendix B-1. The changes in costs for

Example 1 as a result of the addition of the emission constraint are given below:

Table 6-2: Summary of Costs for each Link - Example 1

Emission Reduction	C_a	C_b
Original- No Emission	13.32	15.83
0 % reduction	14.00	15.50
5 %	16.00	14.50
10 %	18.00	13.50
15 %	20.00	12.50
20 %	22.00	11.50
25 %	24.00	10.50

Figure 6-1: Summary of Costs for each Link - Example 1

The results of Figure 6-1 illustrate that with no emission up to 0% emission reduction, the favourable path is *path a* because of its lower cost. However, once emission is changed by a factor greater than 5%, the favourable path becomes *path b*.

6.2.2 Example 2

The following table illustrates the changes observed to the parameters by reducing the emission constrain for *Example 2*:

Table 6-3: Sensitivity Analysis for Example 2

Emission Reduction	Z_i	X_1	X_2	X_3
Original – No Emission	108.78	2.65	3.80	3.54
0 % reduction	108.80	2.72	3.72	3.55
5 %	115.97	4.05	2.05	3.88
10 %	121.25	4.50	1.50	4.00
15 %	136.14	5.38	0.38	4.22
20 %	161.5	7.00	0.00	3.00
25%	213.5	9.00	0.00	1.00

For a complete listing of these results, please refer to Appendix B-2. The changes in costs for

Example 2 as a result of the addition of the emission constraint are given below:

Table 6-4: Summary of Costs for each Link - Example 2

Emission Reduction	C_a	C_b	C_c
Original- No Emission	10.30	11.80	10.31
0 % reduction	10.44	11.72	10.32
5 %	13.10	10.05	10.82
10 %	14.00	9.50	9.00
15 %	15.76	8.38	11.33
20 %	19.00	8.00	9.50
25 %	23.00	8.00	6.50

Figure 6-2: Summary of Costs for each Link - Example 2

The results of Figure 6-2 illustrate that with no emission, the favourable path is *path a*, and with 0% emission reduction the most favoured path is *path c*. In other words, *path a* will no longer remain the cheapest path once emission is introduced to the model. Further, if the emission were to be changed by 5%, 15% and 20%, *path b* will remain the cheapest path to take.

6.2.3 Example 3

The following table illustrates the changes observed to the parameters by reducing the emission constrain for *Example 3*:

Table 6-5: Sensitivity Analysis for Example 3

Emission Reduction	Z	X ₁	X ₂	X ₃	X ₄	X ₅
Original – No Emission	283.33	3.33	1.67	3.33	0.00	5.00
0 %	258.40	3.40	1.60	0.00	0.00	5.00
5 %	300.00	5.00	0.00	0.00	0.00	5.00
10 %	380.67	5.00	0.00	0.00	0.67	4.33
15 %	516.67	5.00	0.00	0.00	1.67	3.33
20 %	670.67	5.00	0.00	0.00	2.67	2.33
25%	783.33	5.00	0.00	0.00	3.33	1.67

For a complete listing of these results, please refer to Appendix B-3. The changes in costs for *Example 3* as a result of the addition of the emission constraint are given below:

Table 6-6: Summary of Costs for each Link – Example 3

Emission Reduction	C _a	C _b	C _c	C _d	C _e
Original- No Emission	21.65	21.70	168.3	156.33	33.33
0 % reduction	22.00	21.00	135.00	156.80	23.6
5 %	30.00	5.00	135.00	160.00	30.00
10 %	30.67	8.35	131.65	163.35	30.00
15 %	31.67	13.35	126.65	168.35	30.00
20 %	32.67	18.35	121.65	173.35	30.00
25 %	33.33	21.65	118.35	176.65	30.00

Figure 6-3: Summary of Costs for each Link - Example 3

The results of Figure 6-3 illustrate that under no circumstances will *path d* and *path c* will be taken because of their high cost. If the emission were to be changed by anything greater than 5% the most economical path would be *path b* for the forward links and *path e* for the reverse link.

6.2.4 Example 4

The following table illustrates the changes observed to the parameters by reducing the emission constrain for *Example 4*:

Table 6-7: Sensitivity Analysis for Example 4

Emission Reduction	Z	X ₁	X ₂	X ₃	X ₄
Original – No Emission	-86.74	1.13	0.63	1.31	1.81
0 % reduction	-80.29	1.21	0.00	0.92	1.42
5 %	-77.48	1.11	0.00	0.83	1.33
10 %	-76.53	1.08	0.00	0.81	1.31
15 %	-74.12	1.01	0.00	0.75	1.25
20 %	-70.21	0.90	0.00	0.66	1.16
25%	-68.92	0.87	0.00	0.63	1.14

For a complete listing of these results, please refer to Appendix B-4. The changes in costs for *Example 4* as a result of the addition of the emission constraint are given below:

Table 6-8: Summary of Costs for each Link - Example 4

Emission Reduction	C_{p1}	C_{p2}	C_{p3}	C_{p4}
Original- No Emission	5.44	4.94	5.94	5.44
0 % reduction	5.13	4.63	4.92	4.42
5 %	4.94	4.44	4.83	4.33
10 %	4.89	4.39	4.81	4.31
15 %	4.76	4.26	4.75	4.25
20 %	4.56	4.06	4.66	4.14
25 %	4.50	4.01	4.63	4.14

Figure 6-4: Summary of Costs for each Link - Example 4

The results of Figure 6-4 illustrate that with no emission, the optimum path to take would be *path 2*. With the addition of emission to the system and up to 15% emission reductions, the cheapest path becomes *path 4*. However, once emission is reduced by a factor greater than 20%, the cheapest path changes back to *path 2*.

6.2.5 Example 5

The following table illustrates the changes observed to the parameters by reducing the emission constrain for *Example 5*:

Table 6-9: Sensitivity for Example 5

Emission Reduction	Z	X ₁	X ₂	X ₃
Original – No Emission	-196.31	2.76	1.75	4.51
0 % reduction	-196.31	2.75	1.75	4.50
5 %	-195.66	2.67	1.59	4.26
10 %	-194.38	2.61	1.47	4.08
15 %	-191.97	2.54	1.32	3.86
20 %	-188.81	2.47	1.19	3.66
25%	-183.99	2.39	1.03	3.42

For a complete listing of these results, please refer to Appendix B-5. The changes in costs for *Example 5* as a result of the addition of the emission constraint are given below:

Table 6-10: Summary of Costs for each Link - Example 5

Emission Reduction	C _{p1}	C _{p2}
Original- No Emission	47.34	45.05
0 % reduction	47.25	45.00
5 %	45.48	42.75
10 %	44.31	41.22
15 %	42.78	39.22
20 %	41.37	37.44
25 %	39.69	35.28

Figure 6-5: Summary of Costs for each Link - Example 5

The results of Figure 6-5 illustrate that under all different reduction scenarios, *path 2* would be the cheapest path to take.

6.3 MULTI-PERIOD MODELS ($t = 3$)

6.3.1 Example 1

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 1*:

Table 6-11: Multi-Period Sensitivity Analysis for Example 1

Emission Reduction	Z	X ₁	X ₂
Original – No Emission	443.75	4.16	5.83
0 % reduction	444.75	4.50	5.50
5 %	460.97	5.55	4.45
10 %	494.87	6.55	3.45
15 %	549.84	7.60	2.40
20 %	620.64	8.60	1.40
25%	714.35	9.65	0.35

For a complete listing of these results, please refer to Appendix D-1. The changes in costs for *Example 1* as a result of the addition of the emission constraint are given below:

Table 6-12: Summary of Costs for each Link - Example 1

Emission Reduction	C_a	C_b
Original- No Emission	39.96	47.49
0 % reduction	42.00	46.50
5 %	48.30	43.35
10 %	54.30	40.35
15 %	60.60	37.20
20 %	66.60	34.20
25 %	72.90	31.05

Figure 6-6: Multi – Period Summary of Costs for each Link - Example 1

The results of Figure 6-6 illustrate that with no emission up to 0% emission, the favorable path is *path a* because of its lower cost. However, once emission is changed by a factor greater than 5%, the favorable path becomes *path b*.

6.3.2 Example 2

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 2*:

Table 6-13: Multi-Period Sensitivity Analysis for Example 2

Emission Reduction	Z	X ₁	X ₂	X ₃
Original – No Emission	326.36	2.65	3.80	3.54
0 % reduction	326.42	2.72	3.72	3.55
5 %	337.37	3.65	2.55	3.79
10 %	365.57	4.54	1.44	4.01
15 %	413.84	5.48	0.28	4.24
20 %	496.32	7.20	0.00	2.80
25%	671.14	9.30	0.00	0.70

For a complete listing of these results, please refer to Appendix D-2. The changes in costs for *Example 2* as a result of the addition of the emission constraint are given below:

Table 6-14: Summary of Costs for each Link - Example 2

Emission Reduction	C _a	C _b	C _c
Original- No Emission	30.90	35.40	30.93
0 % reduction	31.32	34.65	31.77
5 %	36.90	31.65	32.05
10 %	42.24	28.32	33.04
15 %	47.88	24.75	34.08
20 %	58.20	24.00	27.60
25 %	70.80	24.00	18.15

Figure 6-7: Multi – Period Summary of Costs for each Link - Example 2

The results of Figure 6-7 illustrate that with no emission and 0% emission reduction, the favorable path is *path a*. However, with the addition of emission reduction to the system, the cheapest path becomes *path b* (5-20% emission reduction).

6.3.3 Example 3

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 3*:

Table 6-15: Multi-Period Sensitivity Analysis for Example 3

Emission Reduction	Z	X ₁	X ₂	X ₃	X ₄	X ₅
Original – No Emission	775.00	3.33	1.67	0.00	0.00	5.00
0 % reduction	776.25	3.50	1.49	0.00	0.00	5.00
5 %	878.51	4.85	0.15	0.00	0.00	5.00
10 %	1180.37	5.00	0.00	0.00	0.76	4.23
15 %	1550.0	5.00	0.00	0.00	1.67	3.33
20 %	1947.28	5.00	0.00	0.00	2.53	2.47
25%	2402.77	5.00	0.00	0.00	3.43	1.57

For a complete listing of these results, please refer to Appendix D-3. The changes in costs for *Example 3* as a result of the addition of the emission constraint are given below:

Table 6-16: Summary of Costs for each Link - Example 3

Emission Reduction	C _a	C _b	C _c	C _d	C _e
Original- No Emission	64.95	65.10	405.00	469.98	69.96
0 % reduction	67.50	59.70	405.00	471.00	72.00
5 %	87.75	19.50	405.00	479.10	88.20
10 %	92.28	26.40	393.45	491.40	90.00
15 %	95.01	40.05	379.95	505.05	90.00
20 %	97.59	52.95	367.05	517.95	90.00
25 %	100.29	66.45	353.55	531.45	90.00

Figure 6-8: Multi – Period Summary of Costs for each Link - Example 3

The results of Figure 6-8 illustrate that under no circumstances will *path d* and *path c* should be taken because of their high cost. If the emission were to be changed by anything greater than 5% the most economical path would be *path b* for the forward pass and *path e* for the reverse pass.

6.3.4 Example 4

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 4*:

Table 6-17: Multi-Period Sensitivity Analysis for Example 4

Emission Reduction	Z	X ₁	X ₂	X ₃	X ₄
Original – No Emission	-260.23	1.13	0.63	1.31	1.81
0 %	-240.86	1.21	0.22e-15	0.92	1.42
5 %	-232.45	1.11	0.00	0.83	1.33
10 %	-229.60	1.08	0.00	0.81	1.31
15 %	-222.37	1.00	0.00	0.75	1.25
20 %	-210.62	0.90	0.00	0.66	1.16
25%	-206.77	0.87	0.00	0.64	1.14

For a complete listing of these results, please refer to Appendix D-4. The changes in costs for *Example 4* as a result of the addition of the emission constraint are given below:

Table 6-18: Summary of Costs for each Link - Example 4

Emission Reduction	C_{p1}	C_{p2}	C_{p3}	C_{p4}
Original- No Emission	16.32	14.82	17.82	16.32
0 % reduction	15.39	13.89	14.76	13.26
5 %	14.82	13.32	14.49	12.99
10 %	14.67	13.17	14.43	12.93
15 %	14.25	12.00	14.25	12.00
20 %	13.68	12.18	13.98	12.48
25 %	13.53	12.03	13.92	12.42

Figure 6-9: Multi – Period Summary of Costs for each Link - Example 4

The results of Figure 6-9 illustrate that without the emission constraint, the cheapest path would be *path 2*. However, once emission is added to the model and changed to up to 10%, the cheapest path is *path 4*. For 15% emission reduction, there are two economical paths (*path 2* or *path 4*). For 20% and 25% reduction in emission, the cheapest path is *path 2* once again.

6.3.5 Example 5

The following table illustrates the changes observed to the parameters by reducing the emission constraint for *Example 5*:

Table 6-19: Multi-Period Sensitivity Analysis for Example 5

Emission Reduction	Z	X ₁	X ₂	X ₃
0 %	-588.94	2.75	1.75	4.50
5 %	-586.97	2.67	1.59	4.26
10 %	-583.15	2.61	1.47	4.08
15 %	-575.92	2.54	1.32	3.86
20 %	-566.43	2.47	1.19	3.66
25%	-551.97	2.39	1.03	3.42

For a complete listing of these results, please refer to Appendix D-5. The changes in costs for *Example 5* as a result of the addition of the emission constraint are given below:

Table 6-20: Summary of Costs for each Link - Example 5

Emission Reduction	C _{p1}	C _{p2}
Original- No Emission	142.02	135.15
0 % reduction	141.75	135.00
5 %	136.71	128.52
10 %	132.93	123.66
15 %	128.34	117.66
20 %	124.11	112.32
25 %	119.07	105.84

Figure 6-10: Multi – Period Summary of Costs for each Link - Example 5

The results of Figure 6-10 illustrate that with or without the emission factor, *path b* would be the cheapest path to take.

6.4 DISCUSSION

Sensitivity analysis of the models provided several valuable observations. In the first three examples, the cheapest path equaled the cheapest link. For example, in the case of fixed demand and separate cost, the cheapest path changed with the addition of an extra path or link to the system. This was illustrated with *Example 1*. In *Example 2* (also fixed demand with separate cost) the addition of a third path caused the cheapest path to change with the introduction of emission to the system. *Example 3* was unique because there were forward and reverse paths available, making it a more realistic scenario. In this fixed demand with un-separate cost example, the addition of emission to the system only caused the forward path to change. Thus, for the reverse flow, the optimal path remained the same despite addition of emission to the system.

The last two examples were illustrations of scenarios where the cheapest path did not equal the cheapest link. This was because each path contained more than one link. In *Example 4* (variable demand with separate cost) there existed four paths with each containing 2 links. This example, illustrated that with the addition of emission to the system the optimal path would change. Lastly, for the variable demand with un-separate cost scenario represented with *Example 5* (also multi link for each path), addition of emission to the system did not cause the cheapest path to change. It should be noted that in the last two examples, the z value was a negative number due to the nature of the objective function where the demand had to be subtracted from the cost (variable demand scenarios).

For multi-period models, all the cheapest paths changed in parallel to their single period counterparts. In other words, the same pattern of change could be seen with and without the emission constraint as it was seen for single period models. The only exception was for *Example 2* where a slight change in values resulted in a different economical path. For the single period scenario of this example, the cheaper path was found to remain *path a* despite the addition of emission constraint to the model. Yet, for the multi period scenario of the same model, the cheapest path changed from *path a* to *path c*. Despite the change in optimal path, the difference between the costs of two paths for the multi-period scenario was found to be very small (0.42), thus indicating an error caused by the level of significance used for this study. Table 7-1 summarizes these findings:

Table 7-1: Comparison for Single Period vs. Multi-Period

Examples	SINGLE PERIOD		MULTI PERIOD	
	With Emission	No Emission	With Emission	No Emission
1	Link a	Link a	Link a	Link a
2	Link a	Link a	Link c	Link a
3*	Link a	Link b	Link a	Link b
4	Path 4	Path 2	Path 4	Path 2
5	Path 2	Path 2	Path 2	Path 2

NOTE*: Results for forward pass only

Sensitivity analysis was also carried out to see the effects of emission reduction on the system and view the changes to cost as a result of reducing the right hand side of the emission constraint. In the case of *Example 1* (fixed demand with separate cost), emission reductions greater than 5% caused the cheapest path to shift from *path a* to *path b*. The same pattern of change could be seen in *Example 2* with the exception of 25% emission reduction where the cheapest path switched to the third path (*path c*). In the case of *Example 3* (fixed demand with un-separate cost), the cheapest path remained *path b* (for the forward pass) and *path e* for the reverse path despite the emission reductions forced on the system. For the case of variable demand with separate cost (*Example 4*) emission reduction up to 15% caused the optimal path to switch. Any reductions greater than 15% caused the cheapest path to switch back to the original no emission scenario. It should be noted that for the multi-period scenario of this example, at 15% emission reductions, there was a tie for the cheapest path (*path 2* and *path 4*). Finally, for the case of variable demand with un-separate cost (*Example 5*), *path 2* remained the cheapest path despite the emission reductions enforced on the system.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1 OVERVIEW

This final chapter will provide conclusions, summary of contributions and direction for future research.

7.2 CONCLUSIONS

This report looked at five different case studies that illustrate different types of transportation networks such as fixed demand, variable demand, separate cost and un-separate cost. Each case study was solved using LINGO and then single period versus multi-period scenarios were set up and also solved. Next, the coefficient of emission for PM_{10} was determined and added to each model. The models were then resolved and compared to the non-emission scenarios. Finally, sensitivity analysis was done to note the changes to cost as a result of emission reduction.

Several observations were made: (1) single and multi-period scenarios of the same case study followed similar patterns with the addition of emission and further reduction of the emission constraint; (2) for fixed demand with separate cost and variable demand with un-separate cost the cheapest path remained the same despite the addition of emission; (3) for variable demand with separate cost scenario there existed two economical paths at 15% emission reduction; (4)

emission reductions of $> 5\%$ did not affect the optimal path for fixed demand with separate cost systems.

7.3 SUMMARY OF CONTRIBUTIONS

The main contributions of this report can be summarized as follows:

1. Introduction of emission factors to single and multi period transportation models
2. Estimation of PM_{10} emission value and its addition to the models
3. Sensitivity analysis on possible PM_{10} level reductions and their effects on overall cost for each path

7.4 FURTHER RESEARCH

This report set out to note the effects of emission constraints to system optimized transportation networks. It was decided to use PM_{10} as a constraint for the models due to increasing concern regarding high exposure to this particulate matter. As previously mentioned, however, emission values depend on numerous variables, including vehicle type, age, and speed and their exact calculations were beyond the scope of this report. Further study is required to actually calculate the values of these emission factors and incorporate them into the systems provided.

APPENDIX A

The following is a summary of the five different examples used in this report. A network diagram plus full solution in LINGO is provided for each example. The following notation should also be noted:

$$\begin{aligned} X_1 &= f_a \\ X_2 &= f_b \\ X_3 &= f_c \\ X_4 &= f_d \\ X_5 &= f_e \end{aligned}$$

Example 1 – Fixed Demand + Separate Cost

INFO:

$$C_a = c_a(f_a) = 2f_a + 5$$

$$C_b = c_b(f_b) = f_b + 10$$

$$d_{12} = 10$$

MODEL:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

RESULTS:

Local optimal solution found at step: 8
Objective value: 147.92

Variable	Value	Reduced Cost
X1	4.16	0.697E-08
X2	5.83	0.00

Row	Slack or Surplus	Dual Price
1	147.92	1.00
2	0.00	-21.66
3	4.16	0.00
4	5.83	0.00

SUMMARY:

$$Z = 147.92$$

$$X_1 = 4.16$$

$$X_2 = 5.83$$

Example 2 – Fixed Demand + Separate Cost

INFO:

$$C_a = c_a(f_a) = 2f_a + 5$$

$$C_b = c_b(f_b) = f_b + 8$$

$$C_c = c_c(f_c) = 1.5f_c + 5$$

$$d_{12} = 10$$

MODEL:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

RESULTS:

Local optimal solution found at step: 10
 Objective value: 108.78

Variable	Value	Reduced Cost
X1	2.65	0.975E-08
X2	3.80	0.129E-07
X3	3.53	0.00

Row	Slack or Surplus	Dual Price
1	108.78	1.00
2	0.00	-15.615
3	2.65	0.00
4	3.80	0.00
5	3.53	0.00

SUMMARY:

$$Z = 108.78$$

$$X_1 = 2.65$$

$$X_2 = 3.80$$

$$X_3 = 3.54$$

Example 3 – Fixed Demand + Un-Separate Cost **INFO:**

$$C_a = c_a(f_a) = 5f_a + f_d + 5$$

$$C_b = c_b(f_b) = 10f_b + 5f_d + 5$$

$$C_c = c_c(f_c) = 10f_c + 5f_e + 110$$

$$C_d = c_d(f_d) = 5f_d + 2f_a + 150$$

$$C_e = c_e(f_e) = 4f_e + 3f_c + 10$$

$$d_{w1} = 5$$

$$d_{w2} = 5$$

MODEL:

$$\text{MIN } Z = 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 +$$

$$\text{Subject to: } 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5$$

$$X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4
 Objective value: 258.33

Variable	Value	Reduced Cost
X1	3.33	0.00
X4	0.00	118.33
X2	1.67	0.00
X3	0.00	151.67
X5	5.00	0.00

Row	Slack or Surplus	Dual Price
1	258.33	1.00
2	0.00	-38.33
3	0.00	-50.00
4	3.33	-0.12E-12
5	1.67	0.00
6	0.00	0.00
7	0.00	0.00
8	5.00	0.00

SUMMARY:

$$Z = 258.33$$

$$X_1 = 3.33$$

$$X_2 = 1.67$$

$$X_3 = 0.00$$

$$X_4 = 0.00$$

$$X_5 = 5.00$$

Example 4 – Variable Demand + Separate Cost **INFO:**

$$C_a = c_a(f_a) = f_a + 1$$

$$C_b = c_b(f_b) = f_b + 2$$

$$C_c = c_c(f_c) = f_c + 2$$

$$C_d = c_d(f_d) = f_d + 1$$

$$\lambda_{ad} = 28 - 7d_{14}$$

$$\lambda_{bd} = 42 - 6d_{24}$$

MODEL:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject to: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

RESULTS:

Optimal solution found at step: 10

Objective value: -86.74

Variable	Value	Reduced Cost
X1	1.13	0.00
X2	0.63	-0.815E-07
X3	1.31	0.151E-06
X4	1.81	0.522E-07
D1	1.77	0.181E-06
D2	3.11	0.00
F1	1.77	0.00

F2	3.11	0.00
F3	2.44	0.00
F4	2.44	0.00

Row	Slack or Surplus	Dual Price
1	-86.74	1.00
2	0.00	-3.27
3	0.00	-4.61
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00

SUMMARY:

$$Z = -86.74$$

$$X_1 = 1.13$$

$$X_2 = 0.63$$

$$X_3 = 1.31$$

$$X_4 = 1.81$$

$$d_1 = 1.77$$

$$d_2 = 3.11$$

$$f_1 = 1.77$$

$$f_2 = 3.11$$

$$f_3 = 2.44$$

$$f_4 = 2.44$$

Example 5 – Variable Demand + Un-Separate Cost

INFO:

$$C_a = c_a(f_a) = 5f_a + 2f_b + 5$$

$$C_b = c_b(f_b) = 7f_b + f_a + 5$$

$$C_c = c_c(f_c) = 3f_c + f_a + f_b + 7$$

$$\lambda_{12} = 99 - 2d_1$$

MODEL:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

RESULTS:

Local optimal solution found at step: 9
 Objective value: -196.31

Variable	Value	Reduced Cost
X1	2.76	-0.499E-07
X2	1.75	-0.510E-07
X3	4.51	0.00
D1	4.51	0.00

Row	Slack or Surplus	Dual Price
1	-196.31	1.00
2	0.00	38.59
3	0.00	-80.94
4	2.76	0.00
5	1.75	0.00
6	4.51	0.00

SUMMARY:

$$Z = -196.31$$

$$X_1 = 2.76$$

$$X_2 = 1.75$$

$$X_3 = 4.51$$

$$d_1 = 4.51$$

APPENDIX B-1

Example 1

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 4.16 &= 0.12 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 5.83 &= 0.29 \end{aligned}$$

$$\text{Upper limit for model} = 0.41$$

(1) New Model Formulated:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

New Results:

Optimal solution found at step: 4

Objective value: 148.25

Variable	Value	Reduced Cost
X1	4.50	0.00
X2	5.50	0.00

Row	Slack or Surplus	Dual Price
1	148.250	1.00
2	0.00	-26.00
3	0.00	100.00
4	4.50	0.00
5	5.50	0.00

(2) New Model with 5% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.39$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 153.25

Variable	Value	Reduced Cost
X1	5.50	0.00
X2	4.50	0.00

Row	Slack or Surplus	Dual Price
1	153.250	1.00
2	0.00	-39.00
3	0.00	400.00
4	5.50	0.00
5	4.50	0.00

(3) New Model with 10% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.37$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 164.25

Variable	Value	Reduced Cost
X1	6.50	0.00
X2	3.50	0.00

Row	Slack or Surplus	Dual Price
1	164.25	1.00
2	0.00	-52.00
3	0.00	700.00
4	6.50	0.00
5	3.50	0.00

(4) New Model with 15% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 181.25

Variable	Value	Reduced Cost
X1	7.50	0.00
X2	2.50	0.00

Row	Slack or Surplus	Dual Price
1	181.25	1.00
2	0.00	-65.00
3	0.00	1000.00
4	7.50	0.00
5	2.50	0.00

(5) New Model with 20% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.33$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

New Results:

Optimal solution found at step: 4

Objective value: 204.25

Variable	Value	Reduced Cost
X1	8.50	0.00
X2	1.50	0.00

Row	Slack or Surplus	Dual Price
1	204.25	1.00
2	0.00	-78.00
3	0.00	1300.00

4	8.50	0.00
5	1.50	0.00

(6) New Model with 25% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$0.03X_1 + 0.05X_2 \leq 0.31$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

New Results:

Optimal solution found at step: 4

Objective value: 233.25

Variable	Value	Reduced Cost
X1	9.50	0.00
X2	0.50	0.00

Row	Slack or Surplus	Dual Price
1	233.25	1.00
2	0.00	-91.00
3	0.00	1600.00
4	9.50	0.00
5	0.50	0.00

APPENDIX B-2

Example 2

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 2.65 &= 0.0795 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 3.80 &= 0.19 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 3.54 &= 0.4116 \end{aligned}$$

$$\text{Upper limit for model} = 0.41$$

(1) New Model Formulated:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

New Results:

$$\begin{aligned} \text{Optimal solution found at step:} & 4 \\ \text{Objective value:} & 108.80 \end{aligned}$$

Variable	Value	Reduced Cost
X1	2.72	0.00
X2	3.72	0.00
X3	3.55	0.00

Row	Slack or Surplus	Dual Price
1	108.80	1.00
2	0.00	-16.55
3	0.00	22.22

4	2.72	0.310E-06
5	3.72	0.00
6	3.55	0.00

(2) New Model with 5% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.39$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 115.97

Variable	Value	Reduced Cost
X1	4.05	0.00
X2	2.05	0.00
X3	3.88	0.00

Row	Slack or Surplus	Dual Price
1	115.97	1.00
2	0.00	-34.88
3	0.00	455.55
4	4.05	0.00
5	2.05	0.11E-06
6	3.88	0.00

(3) New Model with 10% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.37$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 121.25

Variable	Value	Reduced Cost
X1	4.50	0.00
X2	1.50	0.00
X3	4.00	0.00

Row	Slack or Surplus	Dual Price
1	121.25	1.00
2	0.00	-41.00
3	0.00	600.00
4	4.50	0.00
5	1.50	0.21E-06
6	4.00	0.00

(4) New Model with 15% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to : } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 136.14

Variable	Value	Reduced Cost
X1	5.38	0.00
X2	0.38	0.00
X3	4.22	0.00

Row	Slack or Surplus	Dual Price
1	136.14	1.00
2	0.00	-53.22
3	0.00	888.88
4	5.38	0.00
5	0.38	-0.108E-05
6	4.22	0.00

(5) New Model with 20% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.33$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: 161.50

Variable	Value	Reduced Cost
X1	7.00	0.00
X2	0.00	13.00
X3	3.00	0.00

Row	Slack or Surplus	Dual Price
1	161.50	1.00
2	0.00	-90.00
3	0.00	1900.00
4	7.00	0.00
5	0.00	0.00
6	3.00	0.00

(6) New Model with 25% Less Emission:

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.31$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: 213.50

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	9.00	0.00
X2	0.00	33.00
X3	1.00	0.00

<u>Row</u>	<u>Slack or Surplus</u>	<u>Dual Price</u>
1	213.50	1.00
2	0.00	-140.00
3	0.00	3300.00
4	9.00	0.00
5	0.00	0.00
6	1.00	0.00

APPENDIX B-3

Example 3

For Link a → Total Emission	$= 0.03 * f_a$ $= 0.03 * 3.33$	$= 0.099$
For Link b → Total Emission	$= 0.05 * f_b$ $= 0.05 * 1.67$	$= 0.083$
For Link c → Total Emission	$= 0.04 * f_c$ $= 0.04 * 0.00$	$= 0.00$
For Link d → Total Emission	$= 0.04 * f_d$ $= 0.04 * 0.00$	$= 0.00$
For Link e → Total Emission	$= 0.07 * f_e$ $= 0.07 * 5.00$	$= 0.35$
Upper limit for model		$= 0.53$

(1) New model Formulated:

$$\begin{aligned}
 \text{MIN } Z &= 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\
 &\quad 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \\
 \text{Subject To: } &X_1 + X_2 + X_3 = 5 \\
 &X_4 + X_5 = 5 \\
 &0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.53
 \end{aligned}$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 7
Objective value: 258.40

Variable	Value	Reduced Cost
X1	3.40	0.00
X4	0.00	115.20
X2	1.60	0.00
X3	0.00	152.00
X5	5.00	0.00

Row	Slack or Surplus	Dual Price
1	258.4000	1.00
2	0.00	-41.99
3	0.00	-56.99
4	0.00	99.99
5	3.40	0.00
6	1.60	0.00
7	0.00	0.00
8	0.00	0.00
9	5.00	0.00

(2) New Model with 5% Less Emission:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.50$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4
Objective value: 300.00

Variable	Value	Reduced Cost
X1	5.00	0.00
X4	0.00	0.00
X2	0.00	26.67
X3	0.00	173.33
X5	5.00	0.00

Row	Slack or Surplus	Dual Price
1	300.00	1.00
2	0.00	-170.00
3	0.00	-318.33
4	0.00	3833.33
5	5.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	0.00
9	5.00	0.00

(3) New Model with 10% Less Emission:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.48$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4
Objective value: 380.67

Variable	Value	Reduced Cost
X1	5.00	0.00
X4	0.67	0.00
X2	0.00	36.00

X3	0.00	170.00
X5	4.33	0.00

Row	Slack or Surplus	Dual Price
1	380.67	1.00
2	0.00	-184.00
3	0.00	-341.00
4	0.107E-07	4233.33
5	5.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.67	0.00
9	4.33	0.00

(4) New Model with 15% Less Emission:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.45$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4
Objective value: 516.67

Variable	Value	Reduced Cost
X1	5.00	0.00
X4	1.67	0.00
X2	0.00	49.99
X3	0.00	165.00
X5	3.33	0.00

Row	Slack or Surplus	Dual Price
1	516.67	1.00
2	0.00	-205.00
3	0.00	-375.00
4	0.119E-07	4833.33
5	5.00	0.00
6	0.00	0.00

7	0.00	0.00
8	1.67	0.00
9	3.33	0.00

(5) New Model with 20% Less Emission:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.42$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4
Objective value: 670.67

Variable	Value	Reduced Cost
X1	5.00	0.00
X4	2.67	0.00
X2	0.00	64.01
X3	0.00	160.00
X5	2.33	0.00

Row	Slack or Surplus	Dual Price
1	670.67	1.00
2	0.00	-226.00
3	0.00	-409.00
4	0.00	5433.33
5	5.00	0.00
6	0.00	0.00
7	0.00	0.00
8	2.67	0.00
9	2.33	0.00

(6) New Model with 25% Less Emission:

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 + 0.075X_5 \leq 0.40$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

RESULTS:

Optimal solution found at step: 4

Objective value: 783.33

Variable	Value	Reduced Cost
X1	5.00	0.00
X4	3.33	0.00
X2	0.00	73.34
X3	0.00	156.67
X5	1.67	0.00

Row	Slack or Surplus	Dual Price
1	783.33	1.00
2	0.00	-240.00
3	0.00	-431.67
4	0.00	5833.33
5	5.00	0.00
6	0.00	0.00
7	0.00	0.00
8	3.33	0.00
9	1.67	0.00

APPENDIX B-4

Example 4

For Link a → Total Emission	$= 0.03 * f_a$ $= 0.03 * 1.13$	$= 0.03$
For Link b → Total Emission	$= 0.05 * f_b$ $= 0.05 * 0.63$	$= 0.03$
For Link c → Total Emission	$= 0.04 * f_c$ $= 0.04 * 1.31$	$= 0.05$
For Link d → Total Emission	$= 0.04 * f_d$ $= 0.01 * 1.81$	$= 0.02$
Upper limit for model		$= 0.13$

(1) New Model Formulated:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.13$$

Results:

Optimal solution found at step: 10
Objective value: -80.29

Variable	Value	Reduced Cost
X1	1.21	0.10E-05
X2	0.00	3.616
X3	0.919	0.16E-06
X4	1.419	0.00
D1	1.214	0.00
D2	2.339	0.00
F1	1.214	0.00
F2	2.339	0.00
F3	2.134	0.00
F4	1.419	0.00

Row	Slack or Surplus	Dual Price
1	-80.286	1.00
2	0.00	-10.99
3	0.00	-13.93
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	252.27

(2) New Model with 5% Less Emission:

$$\text{MIN } Z = (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2)$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.12$$

Results:

Optimal solution found at step: 10
Objective value: -77.48

Variable	Value	Reduced Cost
X1	1.109	0.220E-06
X2	0.00	4.94
X3	0.83	-0.339E-06
X4	1.33	0.00
D1	1.11	0.00
D2	2.17	0.00

F1	1.109	0.00
F2	2.167	0.00
F3	1.943	0.00
F4	1.333	0.00

Row	Slack or Surplus	Dual Price
1	-77.48	1.00
2	0.00	-12.461
3	0.00	-15.989
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	308.04

(3) New Model with 10% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.117$$

Results:

Optimal solution found at step: 11

Objective value: -76.53

Variable	Value	Reduced Cost
X1	1.08	-0.102E-06
X2	0.00	5.338
X3	0.81	-0.263E-06
X4	1.31	0.00
D1	1.08	0.00
D2	2.12	0.00
F1	1.08	0.00
F2	2.12	0.00
F3	1.89	0.00
F4	1.31	0.00

Row	Slack or Surplus	Dual Price
1	-76.53	1.00
2	0.00	-12.90
3	0.00	-16.61

4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	324.77

(4) New Model with 15% Less Emission:

$$\text{MIN } Z = (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2)$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.11$$

Results:

Optimal solution found at step: 10
 Objective value: -74.12

Variable	Value	Reduced Cost
X1	1.01	-0.729E-05
X2	0.00	6.26
X3	0.75	-0.141E-05
X4	1.25	0.00
D1	1.01	0.00
D2	1.99	0.00
F1	1.01	0.00
F2	1.99	0.00
F3	1.75	0.00
F4	1.25	0.00

Row	Slack or Surplus	Dual Price
1	-74.12	1.00
2	0.00	-13.93
3	0.00	-18.05
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	363.81

(5) New Model with 20% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.10$$

Results:

Optimal solution found at step: 10
Objective value: -70.21

Variable	Value	Reduced Cost
X1	0.90	0.57E-06
X2	0.00	7.589
X3	0.66	0.77E-06
X4	1.16	0.00
D1	0.90	0.00
D2	1.82	0.00
F1	0.90	0.00
F2	1.82	0.00
F3	1.56	0.00
F4	1.16	0.00

Row	Slack or Surplus	Dual Price
1	-70.21	1.00
2	0.00	-15.39
3	0.00	-20.11
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	419.57

(6) New Model with 25% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = d_1$$

$$X_3 + X_4 = d_2$$

$$X_1 + X_2 = f_1$$

$$X_3 + X_4 = f_2$$

$$X_1 + X_3 = f_3$$

$$X_2 + X_4 = f_4$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 + 0.04X_4 \leq 0.097$$

Results:

Optimal solution found at step: 7

Objective value: -68.92

Variable	Value	Reduced Cost
X1	0.87	0.00
X2	0.00	7.99
X3	0.63	-0.453E-05
X4	1.14	0.00
D1	0.87	0.00
D2	1.77	-0.665E-05
F1	0.87	0.00
F2	1.77	0.00
F3	1.50	0.00
F4	1.14	0.00

Row	Slack or Surplus	Dual Price
1	-68.92	1.00
2	0.00	-15.83
3	0.00	-20.72
4	0.00	0.00
5	0.00	0.00
6	0.00	0.00
7	0.00	0.00
8	0.00	436.30

APPENDIX B-5

Example 5

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 2.76 &= 0.08 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 1.75 &= 0.09 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 4.51 &= 0.18 \end{aligned}$$

$$\text{Upper limit for model} = 0.35$$

(1) New Model Formulated:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 6

Objective value: -196.31

Variable	Value	Reduced Cost
X1	2.75	0.00
X2	1.75	0.00
X3	4.50	0.00
D1	4.50	0.00

Row	Slack or Surplus	Dual Price
1	-196.31	1.00
2	0.00	38.63
3	0.00	-80.99
4	0.00	3.00
5	2.75	0.00
6	1.75	0.205E-06
7	4.50	0.00

(2) New Model with 5% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.33$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: -195.66

Variable	Value	Reduced Cost
X1	2.67	0.00
X2	1.59	0.00
X3	4.26	0.00
D1	4.26	0.00

Row	Slack or Surplus	Dual Price
1	-195.66	1.00
2	0.00	39.32
3	0.00	-81.95
4	-0.131E-07	62.52
5	2.67	0.00
6	1.59	0.1070E-05
7	4.26	0.00

(3) New Model with 10% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.315$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: -194.38

Variable	Value	Reduced Cost
X1	2.61	0.00
X2	1.46	0.00
X3	4.08	0.00
D1	4.08	0.00

Row	Slack or Surplus	Dual Price
1	-194.38	1.00
2	0.00	39.85
3	0.00	-82.67
4	0.00	107.16
5	2.61	0.00
6	1.46	0.139E-05
7	4.08	0.00

(4) New Model with 15% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.297$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: -191.97

Variable	Value	Reduced Cost
X1	2.54	0.00
X2	1.32	0.00
X3	3.86	0.00
D1	3.86	0.00
Row	Slack or Surplus	Dual Price
1	-191.97	1.00
2	0.00	40.48
3	0.00	-83.54
4	0.00	160.72
5	2.54	0.00
6	1.32	0.171E-05
7	3.86	0.00

(5) New Model with 20% Less Emission:

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.28$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: -188.81

Variable	Value	Reduced Cost
X1	2.47	0.00
X2	1.19	0.00
X3	3.66	0.00
D1	3.66	0.00
Row	Slack or Surplus	Dual Price
1	-188.81	1.00
2	0.00	41.07
3	0.00	-84.35

4	0.00	211.32
5	2.47	0.00
6	1.18	0.105E-05
7	3.66	0.00

(6) New Model with 25% Less Emission:

$$\text{MIN } Z = (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + 3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1)$$

$$\text{Subject To: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$0.03X_1 + 0.05X_2 + 0.04X_3 \leq 0.26$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: -183.99

Variable	Value	Reduced Cost
X1	2.39	0.00
X2	1.03	0.00
X3	3.42	0.00
D1	3.42	0.00

Row	Slack or Surplus	Dual Price
1	-183.99	1.00
2	0.00	41.77
3	0.00	-85.31
4	0.00	270.84
5	2.39	-0.114E-05
6	1.03	0.00
7	3.42	0.00

APPENDIX C

Example 1

Original Model:

$$\text{MIN } Z = 2X_1^2 + 1X_2^2 + 5X_1 + 10X_2$$

$$\text{Subject to: } X_1 + X_2 = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

For $t = 3$

$$\text{MIN } Z = (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23})$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 443.75

Variable	Value	Reduced Cost
X11	4.17	0.00
X21	5.83	-0.656E-06
X12	4.17	0.00
X22	5.83	-0.65664E-06
X13	4.17	0.00
X23	5.83	-0.656E-06
X1	0.249E-06	0.00
X2	0.249E-06	0.00

Row	Slack or Surplus	Dual Price
1	443.750	1.00

2	0.00	-21.67
3	0.00	-21.67
4	0.00	-21.67
5	0.249E-06	0.00
6	0.249E-06	0.00

Example 2**Original Model:**

$$\text{MIN } Z = 2X_1^2 + 5X_1 + 1X_2^2 + 8X_2 + 1.5X_3^2 + 5X_3$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

New Model for t = 3:

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 7

Objective value: 326.36

Variable	Value	Reduced Cost
X11	2.65	0.00
X21	3.81	0.932E-07
X31	3.54	0.895E-07
X12	2.65	0.00
X22	3.81	0.932E-07
X32	3.55	0.895E-07
X13	2.65	0.00

X23	3.81	0.932E-07
X33	3.54	0.895E-07
X1	0.109E-05	0.00
X2	0.109E-05	0.00
X3	0.109E-05	0.00

Row	Slack or Surplus	Dual Price
1	326.3654	1.00
2	0.00	-15.61
3	0.00	-15.61
4	0.00	-15.61
5	0.109E-05	0.00
6	0.109E-05	0.00
7	0.109E-05	0.00

Example 3**Original Model:**

$$\begin{aligned} \text{MIN } Z = & 5X_1^2 + X_1X_4 + 5X_1 + 10X_2^2 + 5X_2X_4 + 5X_2 + 10X_3^2 + 5X_3X_5 + \\ & 150X_3 + 5X_4 + 2X_1X_4 + 150X_4 + 4X_5^2 + 3X_3X_5 + 10X_5 \end{aligned}$$

$$\text{Subject to: } X_1 + X_2 + X_3 = 5$$

$$X_4 + X_5 = 5$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

New Model for t = 3:

$$\begin{aligned} \text{MIN } Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\ & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\ & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\ & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\ & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\ & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53}) \end{aligned}$$

Subject to: $X_{11} + X_{21} + X_{31} = 5$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 5
Objective value: 775.00

Variable	Value	Reduced Cost
X11	3.33	0.00
X41	0.00	118.33
X21	1.67	-0.177E-06
X31	0.00	151.67
X51	5.00	0.00
X12	3.33	0.00
X42	0.00	118.33
X22	1.67	-0.177E-06
X32	0.00	151.67
X52	5.00	0.00
X13	3.33	0.00
X43	0.00	118.33
X23	1.67	-0.601E-06
X33	0.00	151.67
X53	5.00	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	775.00	1.00
2	0.00	-38.33
3	0.00	-38.33
4	0.00	-38.33
5	0.00	-50.00
6	0.00	-50.00
7	0.00	-50.00
8	0.00	0.00

9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00

Example 4**Original Model:**

$$\begin{aligned} \text{MIN } Z = & (X_1^2 + X_2^2 + X_3^2 + X_4^2) + (X_1 + 2X_2 + 2X_3 + X_4) + \\ & (7d_1^2 + 6d_2^2 - 28d_1 - 42d_2) \end{aligned}$$

$$\begin{aligned} \text{Subject to: } & X_1 + X_2 = d_1 \\ & X_3 + X_4 = d_2 \\ & X_1 + X_2 = f_1 \\ & X_3 + X_4 = f_2 \\ & X_1 + X_3 = f_3 \\ & X_2 + X_4 = f_4 \end{aligned}$$

New Model for t = 3:

$$\begin{aligned} \text{MIN } Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\ & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\ & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\ & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\ & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23}) \end{aligned}$$

$$\begin{aligned} \text{Subject to: } & X_{11} + X_{21} = d_{11} \\ & X_{12} + X_{22} = d_{12} \\ & X_{13} + X_{23} = d_{13} \\ & X_{31} + X_{41} = d_{21} \\ & X_{32} + X_{42} = d_{22} \\ & X_{33} + X_{43} = d_{23} \\ & X_{11} + X_{21} = f_{11} \\ & X_{12} + X_{22} = f_{12} \\ & X_{13} + X_{23} = f_{13} \end{aligned}$$

$$X_{31} + X_{41} = f_{21}$$

$$X_{32} + X_{42} = f_{22}$$

$$X_{33} + X_{43} = f_{23}$$

$$X_{11} + X_{31} = f_{31}$$

$$X_{12} + X_{32} = f_{32}$$

$$X_{13} + X_{33} = f_{33}$$

$$X_{21} + X_{41} = f_{41}$$

$$X_{22} + X_{42} = f_{42}$$

$$X_{23} + X_{43} = f_{43}$$

Results:

Optimal solution found at step: 10
Objective value: -260.23

Variable	Value	Reduced Cost
X11	1.13	0.00
X21	0.63	-0.217E-06
X31	1.31	-0.793E-07
X41	1.80	0.697E-07
D11	1.77	0.173E-05
D21	3.11	0.00
X12	1.13	0.00
X22	0.63	-0.545E-06
X32	1.31	-0.793E-07
X42	1.81	0.697E-07
D12	1.77	-0.814E-07
D22	3.11	0.00
X13	1.13	0.00
X23	0.63	-0.144E-06
X33	1.31	0.268E-06
X43	1.81	0.763E-06
D13	1.77	0.441E-06
D23	3.11	0.00
F11	1.77	0.00
F12	1.77	0.00
F13	1.77	0.00
F21	3.11	0.00
F22	3.11	0.00
F23	3.11	0.00
F41	2.44	0.00
F42	2.44	0.00
F43	2.44	0.00

Row	Slack or Surplus	Dual Price
1	-260.23	1.00
2	0.00	-3.27
3	0.00	-3.27

4	0.00	-3.27
5	0.00	-4.61
6	0.00	-4.61
7	0.00	-4.61
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.00

Example 5**Original Model:**

$$\begin{aligned} \text{MIN } Z = & (5X_1^2 + 2X_1X_2 + 5X_1) + (7X_2^2 + X_1X_2 + 5X_2) + \\ & (3X_3^2 + X_1X_3 + X_2X_3 + 7X_3) + (2d_1^2 - 99d_1) \end{aligned}$$

$$\text{Subject to: } X_1 + X_2 = X_3$$

$$X_1 + X_2 = d_1$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

New Model for t = 3:

$$\begin{aligned} \text{MIN } Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\ & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\ & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\ & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\ & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\ & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13}) \end{aligned}$$

Subject to: $X_{11} + X_{21} = X_{31}$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 8
Objective value: -588.9424

Variable	Value	Reduced Cost
X11	2.76	0.00
X21	1.75	-0.134E-05
X31	4.51	0.00
D11	4.51	0.884E-06
X12	2.76	0.00
X22	1.75	-0.130E-05
X32	4.51	0.00
D12	4.51	0.859E-06
X13	2.76	0.00
X23	1.75	-0.115E-05
X33	4.51	0.00
D13	4.51	0.102E-05
X1	0.109E-05	0.00
X2	0.109E-05	0.00
X3	0.1092E-05	0.00

Row	Slack or Surplus	Dual Price
1	-588.94	1.00
2	0.00	38.59
3	0.00	38.59
4	0.00	38.59
5	0.00	-80.95
6	0.00	-80.95
7	0.00	-80.95
8	0.109E-05	0.00
9	0.109E-05	0.00
10	0.109E-05	0.00

APPENDIX D-1

Example 1

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 4.16 &= 0.12 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 5.83 &= 0.29 \end{aligned}$$

$$\text{Upper limit for model} = 0.41$$

(1) New Model Formulated:

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.41$$

$$0.03X_{12} + 0.05X_{22} \leq 0.41$$

$$0.03X_{13} + 0.05X_{23} \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

New Results:

$$\begin{aligned} \text{Optimal solution found at step:} & 4 \\ \text{Objective value:} & 444.75 \end{aligned}$$

Variable	Value	Reduced Cost
X11	4.50	0.00
X21	5.50	0.00
X12	4.50	0.00
X22	5.50	0.00
X13	4.50	0.00

X23	5.50	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	444.75	1.00
2	0.00	-26.00
3	0.00	-26.00
4	0.00	-26.00
5	0.00	100.00
6	0.00	100.00
7	0.00	100.00
8	0.25E-05	0.00
9	0.25E-05	0.00

(2) New Model with 5% Less Emission:

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.389$$

$$0.03X_{12} + 0.05X_{22} \leq 0.389$$

$$0.03X_{13} + 0.05X_{23} \leq 0.389$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 460.97

Variable	Value	Reduced Cost
X11	5.55	0.00
X21	4.45	0.00
X12	5.55	0.00
X22	4.45	0.00
X13	5.55	0.00
X23	4.45	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	460.97	1.00
2	0.00	-39.65
3	0.00	-39.65

4	0.00	-39.65
5	0.00	415.00
6	0.00	415.00
7	0.00	415.00
8	0.25E-05	0.00
9	0.25E-05	0.00

(3) New model for $t = 3$ with 10% less emission

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.369$$

$$0.03X_{12} + 0.05X_{22} \leq 0.369$$

$$0.03X_{13} + 0.05X_{23} \leq 0.369$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 494.87

Variable	Value	Reduced Cost
X11	6.55	0.00
X21	3.45	0.00
X12	6.55	0.00
X22	3.45	0.00
X13	6.55	0.00
X23	3.45	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	494.87	1.00
2	0.00	-52.65
3	0.00	-52.65
4	0.00	-52.65
5	0.00	715.00
6	0.00	715.00
7	0.00	715.00
8	0.25E-05	0.00
9	0.25E-05	0.00

(4) New model for $t = 3$ with 15% less emission

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.348$$

$$0.03X_{12} + 0.05X_{22} \leq 0.348$$

$$0.03X_{13} + 0.05X_{23} \leq 0.348$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 549.84

Variable	Value	Reduced Cost
X11	7.60	0.00
X21	2.40	0.00
X12	7.60	0.00
X22	2.40	0.00
X13	7.60	0.00
X23	2.40	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	549.84	1.00
2	0.00	-66.30
3	0.00	-66.30
4	0.00	-66.30
5	0.00	1030.00
6	0.00	1030.00
7	0.00	1030.00
8	0.25E-05	0.00
9	0.25E-05	0.00

(5) New model for $t = 3$ with 20% less emission

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\begin{aligned} \text{Subject to: } &X_{11} + X_{21} = 10 \\ &X_{12} + X_{22} = 10 \\ &X_{13} + X_{23} = 10 \\ &0.03X_{11} + 0.05X_{21} \leq 0.328 \\ &0.03X_{12} + 0.05X_{22} \leq 0.328 \\ &0.03X_{13} + 0.05X_{23} \leq 0.328 \\ &X_1 \geq 0 \\ &X_2 \geq 0 \end{aligned}$$

Results:

Optimal solution found at step: 4
Objective value: 620.64

Variable	Value	Reduced Cost
X11	8.60	0.00
X21	1.40	0.00
X12	8.60	0.00
X22	1.40	0.00
X13	8.60	0.00
X23	1.40	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	620.64	1.00
2	0.00	-79.30
3	0.00	-79.30
4	0.00	-79.30
5	0.00	1330.00
6	0.00	1330.00
7	0.00	1330.00
8	0.25E-05	0.00
9	0.25E-05	0.00

(6) New model for $t = 3$ with 25% less emission

$$\begin{aligned} \text{MIN } Z &= (2X_{11}^2 + X_{21}^2 + 5X_{11} + 10X_{21}) + (2X_{12}^2 + X_{22}^2 + 5X_{12} + 10X_{22}) + \\ &\quad (2X_{13}^2 + X_{23}^2 + 5X_{13} + 10X_{23}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = 10$$

$$X_{12} + X_{22} = 10$$

$$X_{13} + X_{23} = 10$$

$$0.03X_{11} + 0.05X_{21} \leq 0.307$$

$$0.03X_{12} + 0.05X_{22} \leq 0.307$$

$$0.03X_{13} + 0.05X_{23} \leq 0.307$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 714.35

Variable	Value	Reduced Cost
X11	9.65	0.00
X21	0.35	0.00
X12	9.65	0.00
X22	0.35	0.00
X13	9.65	0.00
X23	0.35	0.00
X1	0.25E-05	0.00
X2	0.25E-05	0.00

Row	Slack or Surplus	Dual Price
1	714.35	1.00
2	0.00	-92.95
3	0.00	-92.95
4	0.00	-92.95
5	0.00	1645.00
6	0.00	1645.00
7	0.00	1645.00
8	0.25E-05	0.00
9	0.25E-05	0.00

APPENDIX D-2

Example 2

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 2.65 &= 0.0795 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 3.80 &= 0.19 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 3.54 &= 0.4116 \end{aligned}$$

$$\text{Upper limit for model} = 0.41$$

(1) New Model Formulated:

$$\begin{aligned} MIN \ Z &= (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ &\quad (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ &\quad (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.41$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.41$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.41$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

$$\begin{aligned} \text{Optimal solution found at step:} &7 \\ \text{Objective value:} &326.42 \end{aligned}$$

Variable	Value	Reduced Cost
X11	2.72	0.00
X21	3.72	0.00
X31	3.55	-0.28E-06
X12	2.72	0.00
X22	3.72	0.00
X32	3.55	-0.69E-06
X13	2.72	0.00
X23	3.72	0.00
X33	3.55	-0.43E-06
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	326.42	1.00
2	0.00	-16.55
3	0.00	-16.55
4	0.00	-16.55
5	0.00	22.22
6	0.00	22.22
7	0.00	22.22
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

(2) New model for $t = 3$ with 5% less emission

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.389$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.389$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.389$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 17
Objective value: 337.37

Variable	Value	Reduced Cost
X11	3.65	0.00
X21	2.55	-0.12E-05
X31	3.79	0.00
X12	3.65	0.00
X22	2.55	-0.12E-05
X32	3.79	0.00
X13	3.65	0.00
X23	2.55	-0.13E-05
X33	3.79	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	337.37	1.00
2	0.00	-29.39
3	0.00	-29.39
4	0.00	-29.39
5	0.00	325.55
6	0.00	325.55
7	0.00	325.55
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

(3) New model for t = 3 with 10% less emission

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.369$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.369$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.369$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 5
Objective value: 365.57

Variable	Value	Reduced Cost
X11	4.54	0.00
X21	1.44	0.12E-05
X31	4.01	0.00
X12	4.54	0.00
X22	1.44	0.14E-05
X32	4.01	0.00
X13	4.54	0.00
X23	1.44	-0.23E-05
X33	4.01	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	365.57	1.00
2	0.00	-41.61
3	0.00	-41.61
4	0.00	-41.61
5	0.00	614.44
6	0.00	614.44
7	0.00	614.44
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

(4) New model for t = 3 with 15% less emission

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.348$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.348$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.348$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 413.84

Variable	Value	Reduced Cost
X11	5.48	0.00
X21	0.28	0.21E-05
X31	4.24	0.00
X12	5.48	0.00
X22	0.28	-0.49E-05
X32	4.24	0.00
X13	5.48	0.00
X23	0.28	-0.28E-05
X33	4.24	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	413.84	1.000
2	0.00	-54.44
3	0.00	-54.44
4	0.00	-54.44
5	0.00	917.77
6	0.00	917.77
7	0.00	917.77
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

(5) New model for t = 3 with 20% less emission

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.328$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.328$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.328$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 496.32

Variable	Value	Reduced Cost
X11	7.20	0.00
X21	0.00	14.99
X31	2.80	0.00
X12	7.20	0.00
X22	0.00	14.99
X32	2.80	0.00
X13	7.20	0.00
X23	0.00	14.99
X33	2.80	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	496.32	1.00
2	0.00	-95.00
3	0.00	-95.00
4	0.00	-95.00
5	0.00	2040.00
6	0.00	2040.00
7	0.00	2040.00
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

(6) New model for t = 3 with 25% less emission

$$\begin{aligned} \text{MIN } Z = & (2X_{11}^2 + 5X_{11} + 1X_{21}^2 + 8X_{21} + 1.5X_{31}^2 + 5X_{31}) + \\ & (2X_{12}^2 + 5X_{12} + 1X_{22}^2 + 8X_{22} + 1.5X_{32}^2 + 5X_{32}) + \\ & (2X_{13}^2 + 5X_{13} + 1X_{23}^2 + 8X_{23} + 1.5X_{33}^2 + 5X_{33}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 10$$

$$X_{12} + X_{22} + X_{32} = 10$$

$$X_{13} + X_{23} + X_{33} = 10$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.307$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.307$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.307$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 671.14

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X11	9.30	0.00
X21	0.00	35.99
X31	0.70	0.00
X12	9.30	0.00
X22	0.00	35.99
X32	0.70	0.00
X13	9.30	0.00
X23	0.00	35.99
X33	0.70	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

<u>Row</u>	<u>Slack or Surplus</u>	<u>Dual Price</u>
1	671.14	1.00
2	0.00	-147.50
3	0.00	-147.50
4	0.00	-147.50
5	0.00	3510.00
6	0.00	3510.00
7	0.00	3510.00
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00

APPENDIX D-3

Example 3

For Link a → Total Emission	$= 0.03 * f_a$ $= 0.03 * 3.33$	$= 0.099$
For Link b → Total Emission	$= 0.05 * f_b$ $= 0.05 * 1.67$	$= 0.083$
For Link c → Total Emission	$= 0.04 * f_c$ $= 0.04 * 0.00$	$= 0.00$
For Link d → Total Emission	$= 0.04 * f_d$ $= 0.04 * 0.00$	$= 0.00$
For Link e → Total Emission	$= 0.07 * f_e$ $= 0.07 * 5.00$	$= 0.35$
Upper limit for model		$= 0.53$

(1) New model Formulated:

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
 & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
 & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
 & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\
 & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
 & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53}) \\
 Subject \ to: & X_{11} + X_{21} + X_{31} = 5 \\
 & X_{12} + X_{22} + X_{32} = 5 \\
 & X_{13} + X_{23} + X_{33} = 5
 \end{aligned}$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.53$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.53$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.53$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: 776.25

Variable	Value	Reduced Cost
X11	3.50	0.00
X41	0.00	110.49
X21	1.49	0.00
X31	0.00	152.50
X51	5.00	0.00
X12	3.50	0.00
X42	0.00	110.49
X22	1.49	0.00
X32	0.00	152.50
X52	5.00	0.00
X13	3.50	0.00
X43	0.00	110.49
X23	1.49	0.00
X33	0.00	152.50
X53	5.00	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	776.25	1.00
2	0.00	-47.50
3	0.00	-47.50
4	0.00	-47.50
5	0.00	-67.50
6	0.00	-67.50
7	0.00	-67.50
8	0.28E-07	250.00

9	0.28E-07	250.00
10	0.28E-07	250.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

(2) New model for $t = 3$ with 5% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
 & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
 & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
 & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\
 & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
 & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53})
 \end{aligned}$$

$$Subject \ to: X_{11} + X_{21} + X_{31} = 5$$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.503$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.503$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.503$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4
 Objective value: 878.51

Variable	Value	Reduced Cost
X11	4.85	0.00
X41	0.00	47.05
X21	0.15	0.00

X31	0.00	159.25
X51	5.00	0.00
X12	4.85	0.00
X42	0.00	47.05
X22	0.15	0.00
X32	0.00	159.25
X52	5.00	0.00
X13	4.85	0.00
X43	0.00	47.05
X23	0.15	0.00
X33	0.00	159.25
X53	5.00	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	878.51	1.00
2	0.00	-121.75
3	0.00	-121.75
4	0.00	-121.75
5	0.00	-209.25
6	0.00	-209.25
7	0.00	-209.25
8	-0.20E-07	2274.99
9	-0.20E-07	2274.99
10	-0.20E-07	2274.99
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

(3) New model for t = 3 with 10% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
 & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
 & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
 & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\
 & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
 & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53})
 \end{aligned}$$

Subject to: $X_{11} + X_{21} + X_{31} = 5$

$X_{12} + X_{22} + X_{32} = 5$

$X_{13} + X_{23} + X_{33} = 5$

$X_{41} + X_{51} = 5$

$X_{42} + X_{52} = 5$

$X_{43} + X_{53} = 5$

$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.477$

$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.477$

$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.477$

$X_1 \geq 0$

$X_2 \geq 0$

$X_3 \geq 0$

$X_4 \geq 0$

$X_5 \geq 0$

Results:

Optimal solution found at step: 4

Objective value: 1180.37

Variable	Value	Reduced Cost
X11	5.00	0.00
X41	0.76	0.00
X21	0.00	37.40
X31	0.00	169.50
X51	4.23	0.00
X12	5.00	0.00
X42	0.76	0.00
X22	0.00	37.40
X32	0.00	169.50
X52	4.23	0.00
X13	5.00	0.00
X43	0.76	0.00
X23	0.00	37.40
X33	0.00	169.50
X53	4.23	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	1180.37	1.00
2	0.00	-186.10
3	0.00	-186.10

4	0.00	-186.10
5	0.00	-344.40
6	0.00	-344.40
7	0.00	-344.40
8	0.00	4293.33
9	0.00	4293.33
10	0.00	4293.33
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

(4) New model for $t = 3$ with 15% less emission

$$\begin{aligned}
MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
& 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
& (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
& 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) + \\
& (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
& 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53})
\end{aligned}$$

$$Subject \ to: X_{11} + X_{21} + X_{31} = 5$$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.45$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.45$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.45$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4
Objective value: 1550.00

Variable	Value	Reduced Cost
X11	5.00	0.00
X41	1.67	0.00
X21	0.00	49.99
X31	0.00	165.00
X51	3.33	0.00
X12	5.00	0.00
X42	1.67	0.00
X22	0.00	49.99
X32	0.00	165.00
X52	3.33	0.00
X13	5.00	0.00
X43	1.67	0.00
X23	0.00	49.99
X33	0.00	165.00
X53	3.33	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	1550.00	1.00
2	0.00	-205.00
3	0.00	-205.00
4	0.00	-205.00
5	0.00	-375.00
6	0.00	-375.00
7	0.00	-375.00
8	0.12E-07	4833.33
9	0.12E-07	4833.33
10	0.12E-07	4833.33
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

(5) New model for t = 3 with 20% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
 & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
 & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
 & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\
 & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
 & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53})
 \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} + X_{31} = 5$$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.424$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.424$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.424$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 1947.28

Variable	Value	Reduced Cost
X11	5.00	0.00
X41	2.53	0.00
X21	0.00	62.13
X31	0.00	160.67
X51	2.47	0.00
X12	5.00	0.00
X42	2.53	0.00
X22	0.00	62.13
X32	0.00	160.67
X52	2.47	0.00
X13	5.00	0.00
X43	2.53	0.00
X23	0.00	62.13
X33	0.00	160.67
X53	2.47	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

Row	Slack or Surplus	Dual Price
1	1947.28	1.00
2	0.00	-223.20
3	0.00	-223.20

4	0.00	-223.20
5	0.00	-404.47
6	0.00	-404.47
7	0.00	-404.47
8	0.00	5353.33
9	0.00	5353.33
10	0.00	5353.33
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

(6) New model for $t = 3$ with 25% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + X_{11}X_{41} + 5X_{11} + 10X_{21}^2 + 5X_{21}X_{41} + 5X_{21} + 10X_{31}^2 + 5X_{31}X_{51} + \\
 & 150X_{31} + 5X_{41} + 2X_{11}X_{41} + 150X_{41} + 4X_{51}^2 + 3X_{31}X_{51} + 10X_{51}) + \\
 & (5X_{12}^2 + X_{12}X_{42} + 5X_{12} + 10X_{22}^2 + 5X_{22}X_{42} + 5X_{22} + 10X_{32}^2 + 5X_{32}X_{52} + \\
 & 150X_{32} + 5X_{42} + 2X_{12}X_{42} + 150X_{42} + 4X_{52}^2 + 3X_{32}X_{52} + 10X_{52}) \\
 & (5X_{13}^2 + X_{13}X_{43} + 5X_{13} + 10X_{23}^2 + 5X_{23}X_{43} + 5X_{23} + 10X_{33}^2 + 5X_{33}X_{53} + \\
 & 150X_{33} + 5X_{43} + 2X_{13}X_{43} + 150X_{43} + 4X_{53}^2 + 3X_{33}X_{53} + 10X_{53})
 \end{aligned}$$

$$Subject \ to: X_{11} + X_{21} + X_{31} = 5$$

$$X_{12} + X_{22} + X_{32} = 5$$

$$X_{13} + X_{23} + X_{33} = 5$$

$$X_{41} + X_{51} = 5$$

$$X_{42} + X_{52} = 5$$

$$X_{43} + X_{53} = 5$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} + 0.07X_{51} \leq 0.397$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} + 0.07X_{52} \leq 0.397$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} + 0.07X_{53} \leq 0.397$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$

Results:

Optimal solution found at step: 4

Objective value: 2402.77

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X11	5.00	0.00
X41	3.43	0.00
X21	0.00	74.73
X31	0.00	156.17
X51	1.57	0.00
X12	5.00	0.00
X42	3.43	0.00
X22	0.00	74.73
X32	0.00	156.17
X52	1.57	0.00
X13	5.00	0.00
X43	3.43	0.00
X23	0.00	74.73
X33	0.00	156.17
X53	1.57	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00
X4	0.00	0.00
X5	0.00	0.00

<u>Row</u>	<u>Slack or Surplus</u>	<u>Dual Price</u>
1	2402.77	1.00
2	0.00	-242.10
3	0.00	-242.10
4	0.00	-242.10
5	0.00	-435.07
6	0.00	-435.07
7	0.00	-435.07
8	0.00	5893.33
9	0.00	5893.33
10	0.00	5893.33
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

APPENDIX D-4

Example 4

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 1.13 &= 0.03 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 0.63 &= 0.03 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 1.31 &= 0.05 \end{aligned}$$

$$\begin{aligned} \text{For Link d} \rightarrow \text{Total Emission} &= 0.04 * f_d \\ &= 0.01 * 1.81 &= 0.02 \end{aligned}$$

$$\text{Upper limit for model} = 0.13$$

(1) New Model Formulated:

$$\begin{aligned} MIN \quad Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\ & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\ & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\ & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\ & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23}) \end{aligned}$$

$$\begin{aligned} \text{Subject to: } & X_{11} + X_{21} = d_{11} \\ & X_{12} + X_{22} = d_{12} \\ & X_{13} + X_{23} = d_{13} \\ & \dots \end{aligned}$$

$$X_{31} + X_{41} = d_{21}$$

$$X_{32} + X_{42} = d_{22}$$

$$X_{33} + X_{43} = d_{23}$$

$$X_{11} + X_{21} = f_{11}$$

$$X_{12} + X_{22} = f_{12}$$

$$X_{13} + X_{23} = f_{13}$$

$$X_{31} + X_{41} = f_{21}$$

$$X_{32} + X_{42} = f_{22}$$

$$X_{33} + X_{43} = f_{23}$$

$$X_{11} + X_{31} = f_{31}$$

$$X_{12} + X_{32} = f_{32}$$

$$X_{13} + X_{33} = f_{33}$$

$$X_{21} + X_{41} = f_{41}$$

$$X_{22} + X_{42} = f_{42}$$

$$X_{23} + X_{43} = f_{43}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} \leq 0.13$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} \leq 0.13$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} \leq 0.13$$

Results:

Optimal solution found at step: 32
Objective value: -240.86

Variable	Value	Reduced Cost
X11	1.21	-0.54E-03
X21	0.22E-15	3.61
X31	0.92	-0.21E-02
X41	1.42	0.00
D11	1.21	0.00
D21	2.34	0.00
X12	1.21	0.25E-02
X22	0.00	3.62
X32	0.92	0.44E-03
X42	1.42	0.00
D12	1.21	0.00
D22	2.34	0.00
X13	1.21	-0.53E-02
X23	0.00	3.61
X33	0.92	0.00
X43	1.42	-0.12E-02
D13	1.21	0.00
D23	2.34	0.00
F11	1.21	0.00

F12	1.21	0.00
F13	1.21	0.00
F21	2.34	0.00
F22	2.34	0.00
F23	2.34	0.00
F41	1.42	0.00
F42	1.42	0.00
F43	1.42	0.00

Row	Slack or Surplus	Dual Price
1	-240.86	1.00
2	0.00	-10.99
3	0.00	-10.99
4	0.00	-11.00
5	0.00	-13.93
6	0.00	-13.93
7	0.00	-13.93
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.00
20	0.00	252.25
21	0.00	252.30
22	0.00	252.21

(2) New model for t = 3 with 5% less emission

$$\begin{aligned}
 MIN \quad Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\
 & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\
 & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\
 & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\
 & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23})
 \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$X_{31} + X_{41} = d_{21}$$

$$X_{32} + X_{42} = d_{22}$$

$$X_{33} + X_{43} = d_{23}$$

$$X_{11} + X_{21} = f_{11}$$

$$X_{12} + X_{22} = f_{12}$$

$$X_{13} + X_{23} = f_{13}$$

$$X_{31} + X_{41} = f_{21}$$

$$X_{32} + X_{42} = f_{22}$$

$$X_{33} + X_{43} = f_{23}$$

$$X_{11} + X_{31} = f_{31}$$

$$X_{12} + X_{32} = f_{32}$$

$$X_{13} + X_{33} = f_{33}$$

$$X_{21} + X_{41} = f_{41}$$

$$X_{22} + X_{42} = f_{42}$$

$$X_{23} + X_{43} = f_{43}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} \leq 0.12$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} \leq 0.12$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} \leq 0.12$$

Results:

Optimal solution found at step: 30
Objective value: -232.45

Variable	Value	Reduced Cost
X11	1.11	0.29E-05
X21	0.00	4.94
X31	0.83	-0.13E-04
X41	1.33	0.00
D11	1.11	0.00
D21	2.16	0.00
X12	1.11	0.23E-05
X22	0.00	4.94
X32	0.83	-0.62E-05
X42	1.33	0.00
D12	1.11	0.00
D22	2.16	0.00
X13	1.11	-0.27E-05
X23	0.00	4.94

X33	0.83	0.00
X43	1.33	0.00
D13	1.11	0.00
D23	2.16	0.00
F11	1.11	0.00
F12	1.11	0.00
F13	1.11	0.00
F21	2.16	0.00
F22	2.16	0.00
F23	2.16	0.00
F41	1.33	0.00
F42	1.33	0.00
F43	1.33	0.13E-04

Row	Slack or Surplus	Dual Price
1	-232.45	1.00
2	0.00	-12.46
3	0.00	-12.46
4	0.00	-12.46
5	0.00	-15.99
6	0.00	-15.99
7	0.00	-15.99
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.13E-04
20	0.00	308.04
21	0.00	308.04
22	0.00	308.04

(3) New model for t = 3 with 10% less emission

$$\begin{aligned}
 MIN \quad Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\
 & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\
 & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\
 & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\
 & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23})
 \end{aligned}$$

Subject to:

$$\begin{aligned}
 X_{11} + X_{21} &= d_{11} \\
 X_{12} + X_{22} &= d_{12} \\
 X_{13} + X_{23} &= d_{13} \\
 X_{31} + X_{41} &= d_{21} \\
 X_{32} + X_{42} &= d_{22} \\
 X_{33} + X_{43} &= d_{23} \\
 X_{11} + X_{21} &= f_{11} \\
 X_{12} + X_{22} &= f_{12} \\
 X_{13} + X_{23} &= f_{13} \\
 X_{31} + X_{41} &= f_{21} \\
 X_{32} + X_{42} &= f_{22} \\
 X_{33} + X_{43} &= f_{23} \\
 X_{11} + X_{31} &= f_{31} \\
 X_{12} + X_{32} &= f_{32} \\
 X_{13} + X_{33} &= f_{33} \\
 X_{21} + X_{41} &= f_{41} \\
 X_{22} + X_{42} &= f_{42} \\
 X_{23} + X_{43} &= f_{43} \\
 0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} &\leq 0.117 \\
 0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} &\leq 0.117 \\
 0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} &\leq 0.117
 \end{aligned}$$

Results:

Optimal solution found at step: 27
 Objective value: -229.60

Variable	Value	Reduced Cost
X11	1.08	0.18E-06
X21	0.00	5.34
X31	0.81	0.21E-07
X41	1.31	0.00
D11	1.08	0.00
D21	2.11	0.00
X12	1.08	-0.35E-06
X22	0.00	5.34
X32	0.81	0.21E-06
X42	1.31	0.00
D12	1.08	0.00
D22	2.11	0.00
X13	1.08	0.14E-06

X23	0.00	5.34
X33	0.81	0.00
X43	1.31	0.00
D13	1.08	0.00
D23	2.11	0.00
F11	1.08	0.00
F12	1.08	0.00
F13	1.08	0.00
F21	2.11	0.00
F22	2.11	0.00
F23	2.11	0.00
F41	1.31	0.00
F42	1.31	0.00
F43	1.31	-0.42E-06

Row	Slack or Surplus	Dual Price
1	-229.60	1.00
2	0.00	-12.90
3	0.00	-12.90
4	0.00	-12.90
5	0.00	-16.60
6	0.00	-16.60
7	0.00	-16.60
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	-0.41E-06
20	0.00	324.77
21	0.00	324.77
22	0.00	324.77

(4) New model for t = 3 with 15% less emission

$$\begin{aligned}
 MIN \ Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\
 & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\
 & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\
 & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\
 & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23})
 \end{aligned}$$

Subject to: $X_{11} + X_{21} = d_{11}$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$X_{31} + X_{41} = d_{21}$$

$$X_{32} + X_{42} = d_{22}$$

$$X_{33} + X_{43} = d_{23}$$

$$X_{11} + X_{21} = f_{11}$$

$$X_{12} + X_{22} = f_{12}$$

$$X_{13} + X_{23} = f_{13}$$

$$X_{31} + X_{41} = f_{21}$$

$$X_{32} + X_{42} = f_{22}$$

$$X_{33} + X_{43} = f_{23}$$

$$X_{11} + X_{31} = f_{31}$$

$$X_{12} + X_{32} = f_{32}$$

$$X_{13} + X_{33} = f_{33}$$

$$X_{21} + X_{41} = f_{41}$$

$$X_{22} + X_{42} = f_{42}$$

$$X_{23} + X_{43} = f_{43}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} \leq 0.11$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} \leq 0.11$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} \leq 0.11$$

Results:

Optimal solution found at step: 21
Objective value: -222.37

Variable	Value	Reduced Cost
X11	1.00	-0.27E-06
X21	0.00	6.26
X31	0.75	0.49E-06
X41	1.25	0.00
D11	1.00	0.00
D21	1.99	0.00
X12	1.00	0.32E-07
X22	0.00	6.26
X32	0.75	0.38E-06
X42	1.25	0.00
D12	1.00	0.00
D22	1.99	0.00
X13	1.00	-0.17E-06
X23	0.00	6.26

X33	0.75	0.00
X43	1.25	0.00
D13	1.00	0.00
D23	1.99	0.00
F11	1.00	0.00
F12	1.00	0.00
F13	1.00	0.00
F21	1.99	0.00
F22	1.99	0.00
F23	1.99	0.00
F41	1.25	0.00
F42	1.25	0.00
F43	1.25	0.12E-05

Row	Slack or Surplus	Dual Price
1	-222.37	1.00
2	0.00	-13.92
3	0.00	-13.92
4	0.00	-13.92
5	0.00	-18.04
6	0.00	-18.04
7	0.00	-18.04
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.12E-05
20	0.00	363.80
21	0.00	363.80
22	0.00	363.80

(5) New model for t = 3 with 20% less emission

$$\begin{aligned}
 \text{MIN } Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\
 & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\
 & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\
 & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\
 & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23})
 \end{aligned}$$

Subject to:

$$\begin{aligned}
 X_{11} + X_{21} &= d_{11} \\
 X_{12} + X_{22} &= d_{12} \\
 X_{13} + X_{23} &= d_{13} \\
 X_{31} + X_{41} &= d_{21} \\
 X_{32} + X_{42} &= d_{22} \\
 X_{33} + X_{43} &= d_{23} \\
 X_{11} + X_{21} &= f_{11} \\
 X_{12} + X_{22} &= f_{12} \\
 X_{13} + X_{23} &= f_{13} \\
 X_{31} + X_{41} &= f_{21} \\
 X_{32} + X_{42} &= f_{22} \\
 X_{33} + X_{43} &= f_{23} \\
 X_{11} + X_{31} &= f_{31} \\
 X_{12} + X_{32} &= f_{32} \\
 X_{13} + X_{33} &= f_{33} \\
 X_{21} + X_{41} &= f_{41} \\
 X_{22} + X_{42} &= f_{42} \\
 X_{23} + X_{43} &= f_{43} \\
 0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} &\leq 0.10 \\
 0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} &\leq 0.10 \\
 0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} &\leq 0.10
 \end{aligned}$$

Results:

Optimal solution found at step: 20
 Objective value: -210.62

Variable	Value	Reduced Cost
X11	0.90	0.70E-06
X21	0.00	7.58
X31	0.66	-0.17E-05,
X41	1.16	0.00
D11	0.90	0.00
D21	1.82	0.00
X12	0.90	0.61E-06
X22	0.00	7.58
X32	0.66	-0.19E-06
X42	1.16	0.00
D12	0.90	0.00
D22	1.82	0.00
X13	0.90	0.20E-06
X23	0.00	7.58

X33	0.66	0.00
X43	1.16	0.00
D13	0.90	0.00
D23	1.82	0.00
F11	0.90	0.00
F12	0.90	0.00
F13	0.90	0.00
F21	1.82	0.00
F22	1.82	0.00
F23	1.82	0.00
F41	1.16	0.00
F42	1.16	0.00
F43	1.16	0.17E-05

Row	Slack or Surplus	Dual Price
1	-210.62	1.00
2	0.00	-15.38
3	0.00	-15.38
4	0.00	-15.38
5	0.00	-20.10
6	0.00	-20.10
7	0.00	-20.10
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.17E-05
20	0.00	419.57
21	0.00	419.57
22	0.00	419.57

(6) New model for t = 3 with 25% less emission

$$\begin{aligned}
 MIN \ Z = & (X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2) + (X_{11} + 2X_{21} + 2X_{31} + X_{41}) + \\
 & (7d_{11}^2 + 6d_{21}^2 - 28d_{11} - 42d_{21}) + (X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2) + \\
 & (X_{12} + 2X_{22} + 2X_{32} + X_{42}) + (7d_{12}^2 + 6d_{22}^2 - 28d_{12} - 42d_{22}) + \\
 & (X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2) + (X_{13} + 2X_{23} + 2X_{33} + X_{43}) + \\
 & (7d_{13}^2 + 6d_{23}^2 - 28d_{13} - 42d_{23})
 \end{aligned}$$

Subject to:

$$\begin{aligned}
 X_{11} + X_{21} &= d_{11} \\
 X_{12} + X_{22} &= d_{12} \\
 X_{13} + X_{23} &= d_{13} \\
 X_{31} + X_{41} &= d_{21} \\
 X_{32} + X_{42} &= d_{22} \\
 X_{33} + X_{43} &= d_{23} \\
 X_{11} + X_{21} &= f_{11} \\
 X_{12} + X_{22} &= f_{12} \\
 X_{13} + X_{23} &= f_{13} \\
 X_{31} + X_{41} &= f_{21} \\
 X_{32} + X_{42} &= f_{22} \\
 X_{33} + X_{43} &= f_{23} \\
 X_{11} + X_{31} &= f_{31} \\
 X_{12} + X_{32} &= f_{32} \\
 X_{13} + X_{33} &= f_{33} \\
 X_{21} + X_{41} &= f_{41} \\
 X_{22} + X_{42} &= f_{42} \\
 X_{23} + X_{43} &= f_{43} \\
 0.03X_{11} + 0.05X_{21} + 0.04X_{31} + 0.04X_{41} &\leq 0.097 \\
 0.03X_{12} + 0.05X_{22} + 0.04X_{32} + 0.04X_{42} &\leq 0.097 \\
 0.03X_{13} + 0.05X_{23} + 0.04X_{33} + 0.04X_{43} &\leq 0.097
 \end{aligned}$$

Results:

Optimal solution found at step: 20
 Objective value: -206.77

Variable	Value	Reduced Cost
X11	0.87	-0.16E-06
X21	0.00	7.98
X31	0.64	-0.79E-06
X41	1.14	0.00
D11	0.87	0.00
D21	1.77	0.00
X12	0.87	-0.54E-06
X22	0.00	7.98
X32	0.64	-0.23E-05
X42	1.14	0.00
D12	0.87	0.00
D22	1.77	0.00
X13	0.87	-0.928E-06
X23	0.00	7.98

X33	0.64	0.00
X43	1.14	0.00
D13	0.87	0.00
D23	1.77	0.00
F11	0.87	0.00
F12	0.87	0.00
F13	0.87	0.00
F21	1.77	0.00
F22	1.77	0.00
F23	1.77	0.00
F41	1.14	0.00
F42	1.14	0.00
F43	1.14	0.51E-06

Row	Slack or Surplus	Dual Price
1	-206.77	1.00
2	0.00	-15.83
3	0.00	-15.83
4	0.00	-15.83
5	0.00	-20.73
6	0.00	-20.73
7	0.00	-20.73
8	0.00	0.00
9	0.00	0.00
10	0.00	0.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00
16	0.00	0.00
17	0.00	0.00
18	0.00	0.00
19	0.00	0.517E-06
20	0.00	436.30
21	0.00	436.30
22	0.00	436.30

APPENDIX D-5

Example 5

$$\begin{aligned} \text{For Link a} \rightarrow \text{Total Emission} &= 0.03 * f_a \\ &= 0.03 * 2.76 &= 0.08 \end{aligned}$$

$$\begin{aligned} \text{For Link b} \rightarrow \text{Total Emission} &= 0.05 * f_b \\ &= 0.05 * 1.75 &= 0.09 \end{aligned}$$

$$\begin{aligned} \text{For Link c} \rightarrow \text{Total Emission} &= 0.04 * f_c \\ &= 0.04 * 4.51 &= 0.18 \end{aligned}$$

$$\text{Upper limit for model} = 0.35$$

(1) New Model Formulated:

$$\begin{aligned} \text{MIN } Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\ & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\ & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\ & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\ & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\ & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13}) \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = X_{31}$$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.35$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.35$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.35$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 11

Objective value: -588.94

Variable	Value	Reduced Cost
X11	2.75	0.00
X21	1.75	0.23E-05
X31	4.50	0.00
D11	4.50	0.00
X12	2.75	0.00
X22	1.75	0.22E-05
X32	4.50	0.00
D12	4.50	0.00
X13	2.75	0.00
X23	1.75	0.22E-05
X33	4.50	0.00
D13	4.50	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-588.94	1.00
2	0.00	38.62
3	0.00	38.62
4	0.00	38.62
5	0.00	-80.99
6	0.00	-80.99
7	0.00	-80.99
8	0.00	3.00
10	0.00	3.00
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

(2) New model for $t = 3$ with 5% less emission

$$\begin{aligned}
 \text{MIN } Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\
 & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\
 & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\
 & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\
 & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\
 & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13})
 \end{aligned}$$

$$\begin{aligned}
 \text{Subject to : } & X_{11} + X_{21} = X_{31} \\
 & X_{12} + X_{22} = X_{32} \\
 & X_{13} + X_{23} = X_{33} \\
 & X_{11} + X_{21} = d_{11} \\
 & X_{12} + X_{22} = d_{12} \\
 & X_{13} + X_{23} = d_{13} \\
 & 0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.33 \\
 & 0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.33 \\
 & 0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.33 \\
 & X_1 \geq 0 \\
 & X_2 \geq 0 \\
 & X_3 \geq 0
 \end{aligned}$$

Results:

Optimal solution found at step: 5
 Objective value: -586.97

Variable	Value	Reduced Cost
X11	2.67	0.00
X21	1.59	0.36E-05
X31	4.26	0.00
D11	4.26	0.00
X12	2.67	0.00
X22	1.59	0.282E-05
X32	4.26	0.00
D12	4.26	0.00
X13	2.67	0.00
X23	1.59	0.36E-05
X33	4.26	0.00
D13	4.26	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-586.97	1.00
2	0.00	39.32
3	0.00	39.32
4	0.00	39.32
5	0.00	-81.95
6	0.00	-81.95
7	0.00	-81.95
8	-0.13E-07	62.52
9	-0.13E-07	62.52
10	-0.13E-07	62.52
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

(3) New model for $t = 3$ with 10% less emission

$$\begin{aligned}
 \text{MIN } Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\
 & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\
 & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\
 & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\
 & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\
 & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13})
 \end{aligned}$$

$$\text{Subject to: } X_{11} + X_{21} = X_{31}$$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.315$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.315$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.315$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 7

Objective value: -583.15

Variable	Value	Reduced Cost
X11	2.61	0.00
X21	1.47	0.23E-05
X31	4.08	0.00
D11	4.08	0.00
X12	2.61	0.00
X22	1.47	0.19E-05
X32	4.08	0.00
D12	4.08	0.00
X13	2.61	0.00
X23	1.47	0.22E-05
X33	4.08	0.00
D13	4.08	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-583.15	1.00
2	0.00	39.85
3	0.00	39.85
4	0.00	39.85
5	0.00	-82.67
6	0.00	-82.67
7	0.00	-82.67
8	0.00	107.16
9	0.00	107.16
10	0.00	107.16
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

(4) New model for t = 3 with 15% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\
 & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\
 & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\
 & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\
 & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\
 & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13})
 \end{aligned}$$

$$Subject \ to: \ X_{11} + X_{21} = X_{31}$$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.297$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.297$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.297$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 5
 Objective value: -575.92

Variable	Value	Reduced Cost
X11	2.54	0.00
X21	1.32	0.19E-05
X31	3.86	0.00
D11	3.86	0.00
X12	2.54	0.00
X22	1.32	0.57E-06
X32	3.86	0.00
D12	3.86	0.00
X13	2.54	0.00
X23	1.32	0.16E-05
X33	3.86	0.00
D13	3.86	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-575.92	1.00
2	0.00	40.48
3	0.00	40.48
4	0.00	40.48
5	0.00	-83.54
6	0.00	-83.54
7	0.00	-83.54
8	0.00	160.72
9	0.00	160.72
10	0.00	160.72
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

(4) New model for $t = 3$ with 20% less emission

$$\begin{aligned} MIN \ Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\ & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\ & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\ & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\ & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\ & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13}) \end{aligned}$$

$$Subject \ to: \ X_{11} + X_{21} = X_{31}$$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.28$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.28$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.28$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 5
Objective value: -566.43

Variable	Value	Reduced Cost
X11	2.47	0.00
X21	1.19	-0.16E-05
X31	3.66	0.00
D11	3.66	0.00
X12	2.47	0.00
X22	1.19	-0.16E-05
X32	3.66	0.00
D12	3.66	0.00
X13	2.47	0.00
X23	1.19	-0.13E-05
X33	3.66	0.00
D13	3.66	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

Row	Slack or Surplus	Dual Price
1	-566.43	1.00
2	0.00	41.07
3	0.00	41.07
4	0.00	41.07
5	0.00	-84.36
6	0.00	-84.36
7	0.00	-84.36
8	0.00	211.32
9	0.00	211.32
10	0.00	211.32
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

(6) New model for t = 3 with 25% less emission

$$\begin{aligned}
 MIN \ Z = & (5X_{11}^2 + 2X_{11}X_{21} + 5X_{11}) + (7X_{21}^2 + X_{11}X_{21} + 5X_{21}) + \\
 & (3X_{31}^2 + X_{11}X_{31} + X_{21}X_{31} + 7X_{31}) + (2d_{11}^2 - 99d_{11}) + \\
 & (5X_{12}^2 + 2X_{12}X_{22} + 5X_{12}) + (7X_{22}^2 + X_{12}X_{22} + 5X_{22}) + \\
 & 3X_{32}^2 + X_{12}X_{32} + X_{22}X_{32} + 7X_{32}) + (2d_{12}^2 - 99d_{12}) + \\
 & (5X_{13}^2 + 2X_{13}X_{23} + 5X_{13}) + (7X_{23}^2 + X_{13}X_{23} + 5X_{23}) + \\
 & 3X_{33}^2 + X_{13}X_{33} + X_{23}X_{33} + 7X_{33}) + (2d_{13}^2 - 99d_{13})
 \end{aligned}$$

$$Subject \ to: \ X_{11} + X_{21} = X_{31}$$

$$X_{12} + X_{22} = X_{32}$$

$$X_{13} + X_{23} = X_{33}$$

$$X_{11} + X_{21} = d_{11}$$

$$X_{12} + X_{22} = d_{12}$$

$$X_{13} + X_{23} = d_{13}$$

$$0.03X_{11} + 0.05X_{21} + 0.04X_{31} \leq 0.26$$

$$0.03X_{12} + 0.05X_{22} + 0.04X_{32} \leq 0.26$$

$$0.03X_{13} + 0.05X_{23} + 0.04X_{33} \leq 0.26$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

Results:

Optimal solution found at step: 5

Objective value: -551.97

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X11	2.39	0.00
X21	1.03	-0.10E-05
X31	3.40	0.00
D11	3.42	0.00
X12	2.39	0.00
X22	1.03	-0.99E-06
X32	3.42	0.00
D12	3.42	0.00
X13	2.39	0.00
X23	1.03	-0.17E-05
X33	3.42	0.00
D13	3.42	0.00
X1	0.00	0.00
X2	0.00	0.00
X3	0.00	0.00

<u>Row</u>	<u>Slack or Surplus</u>	<u>Dual Price</u>
1	-551.96	1.00
2	0.00	41.77
3	0.00	41.77
4	0.00	41.77
5	0.00	-85.31
6	0.00	-85.31
7	0.00	-85.31
8	0.00	270.84
9	0.00	270.84
10	0.00	270.84
11	0.00	0.00
12	0.00	0.00
13	0.00	0.00

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