**VERIFICATION OF A CONTROL SYSTEMS DESIGN
FOR A SERIES RESONANT CONVERTER
BASED ON A SMALL-SIGNAL MODEL**

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by

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Bachelor of Engineering, Ryerson University, Canada, 1997

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Verification of a Control Systems Design for a Series Resonant Converter

Based on a Small-Signal Model

Master of Engineering

2006

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Electrical and Computer Engineering

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ABSTRACT

The project verifies the performance of a control system designed for a Series Resonant Converter (SRC). A discrete small-signal model of an SRC was derived and used to design the control system. An accurate control system was designed in the frequency domain to eliminate the need for trial-and-error methods or other time domain methods. A PSPICE simulation of a regulated SRC circuit was used to test the control system. A prototype of the regulated SRC was developed to verify the simulated results. The tests performed demonstrated that the control system design provides a quick response. The tests determined that when a step input is applied to the reference input the control system adjusts the SRC's output voltage to the new reference input value. Also, the control system is able to regulate output load voltage in the presence of sudden load changes.

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NOMENCLATURE

Generic Variable Usage Conventions

Variable Format	Meaning
X	Upper Case: steady-state value
x	Lower Case: instantaneous value
$\hat{x}$	Small-signal variable
$\mathbf{x}(k)$	State vector in discrete time, where $k = 0, 1, 2, \dots$
X_{1n}	Normalized value of X_1

Specific Variable Definitions

Variable	Meaning
ω_o	Angular resonant frequency
f_o	resonant frequency (Hz)
F_s, f_s, f_{sn}	switching frequency
V_c, v_c, v_{cn}	resonant capacitor voltage
I_l, i_l, i_{ln}	resonant inductor current
V_o, v_o, v_{on}	converter output voltage
V_{ref}	converter output control voltage
v_f	input control signal
V_g, v_g, v_{gn}	input DC voltage
$[i_{ln}(k) \ v_{cn}(k) \ v_{on}(k)]^T$	state vector variables
$c(k)$	discrete control signal
$\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0$	coefficient matrices for discrete small-signal model (even values of k)
$\mathbf{A}_1, \mathbf{B}_1, \mathbf{C}_1$	coefficient matrices for discrete small-signal model (odd values of k)
$\mathbf{A}_2, \mathbf{B}_2, \mathbf{C}_2$	coefficient matrices for discrete small-signal model (closed form solution, all values of k)
C_o	output filter capacitor
R_o	output load resistance
C_r	resonant capacitor
L_r	resonant inductor
Z_o	resonant impedance
α	switching transistor conduction angle
β	switching transistor's anti-parallel diode conduction angle
Q	load quality factor
Q_1, Q_2	inverter MOSFET switches

CHAPTER 1: INTRODUCTION

1.1 Background

Many electronic products today are designed to be more compact and mobile yet provide better performance than their bulky predecessors. A perfect example of this is the evolution of the cell phone. Today it is smaller than the original and has better performance and additional features. Similarly, there is a demand on power converters to have greater power density with improved efficiency. In fact, many power converters today can be designed into small surface mount packages. High performance DC-DC converters for industrial, residential and aerospace applications are required to be lighter and smaller. Traditional switch-mode DC-DC converters are bulky due to their large magnetic components. Older converter designs employ the PWM (pulse-width modulation) technique. This means that the output of the converter is regulated by varying the pulse-width (duty cycle) of the gate control signal and keeping the switching frequency constant. Many high power PWM-based switching converters experience high switching losses and component stresses because the switching devices turn ‘off’ and ‘on’ under high current demand. These converters produce quasi-square wave voltages/currents resulting in high EMI (electromagnetic interference). The above traditional converter limitations have led to recent interest and research in resonant converters.

Resonant converters consist of capacitors and inductors in the power stage which are arranged to produce nearly sinusoidal current and voltage waveforms, to allow for zero-voltage switching (ZVS) and/or zero-current switching (ZCS) [1-5]. Since the voltages and currents are sinusoidal the converter can operate at higher frequencies. The

increased switching frequency results in smaller magnetic component designs and a lighter (higher power density) power supply. Since the resonant converter is operating at a higher switching frequency the system response to sudden transients is much quicker. Resonant converters can be controlled by variable switching frequency instead of PWM. This is another advantage of resonant converters. By employing variable switching frequency control with gating signals kept at a constant 50% duty cycle, there is less THD (total harmonic distortion) and EMI (electromagnetic interference). THD is reduced because of the sinusoidal currents and voltages present in the converter. EMI is reduced because of ZCS which means there is no current flowing through the switching device at the time it turns off with. This eliminates high di/dt when the switching device turns off.

Conventional resonant converters can be classified in two categories: parallel and series resonant converters. The parallel resonant converter has an inductor and capacitor arrangement such that the capacitor voltage is used as the energy buffer between the source and the output. In the simpler series resonant converter (SRC) arrangement the inductor current is used as the energy buffer between the source and the output. The SRC also requires the fewest components of the two topologies. As mentioned before, both converters can be controlled by variable switching frequency. However, as will be seen later, the control characteristic curve (V_o/V_g vs. switching frequency) for the resonant converter is non-linear. Figure 1.1 shows an example of a SRC (series resonant converter) circuit.

Figure 1.1: DC-DC Series Resonant Converter Circuit

These control characteristic curves have been thoroughly analyzed using a steady-state analysis of resonant converters based on a sinusoidal approximation technique [1], where the following equation is developed for the control characteristic curve:

$$M = \frac{V_o}{V_g} = \frac{1}{1 + j \frac{\pi^2}{8} Q \left(\frac{w_s}{w_o} - \frac{w_o}{w_s} \right)} \quad (1.1)$$

Equation 1.1 shows that V_o is linked to the switch frequency (where $f_s = \frac{w_s}{2\pi}$).

The control characteristic curve is the magnitude of M plotted against the normalized switch frequency ($f_{sn} = f_s / f_o$).

Figure 1.2: DC-DC SRC Control Characteristic Curve

It can be seen in Figure 1.2 that the SRC is nonlinear and therefore a small-signal model needs to be determined in order to linearize the converter around a particular operating point [7]-[12].

The dynamic behavior of resonant converters has also been analyzed to show how these converters respond to sudden changes (or perturbations) in input, load and component variations. Designing a good control system for the resonant converter is dependent upon the analysis of the converter's dynamic behavior. A technique used to describe the dynamic behavior of the resonant converter is small-signal analysis.

Several small-signal analysis techniques have been investigated based on time averaging, sample-data modeling, and approximation methods. Time averaging assumes that the switching period is much smaller than any time constants in a circuit [6]. For instance, the time constant formed by the output filter capacitor and load is much larger

than the switching period of the converter. With this assumption, the output voltage can be seen as a straight line approximation with a very small ripple. This method provides an averaged differential equation that describes the average response of the converter. In resonant converters the straight line approximation method does not work because the time constants for resonant converters are of the same order as the switching period resulting in an incorrect phase response at high frequencies.

In the sampled-data modeling method a linear model is developed that provides a better overall (high/low) frequency response for current-mode controlled converters that have a constant switching frequency. It uses a sampling process that samples the analog control input whereby the small-signal response of the converter due to the control input can be accurately predicted. The sampled-data modeling method uses the same assumptions as the time-averaging method: small ripple voltage and the straight line approximation [7]. This method provides better high frequency response, but cannot be applied directly to resonant converters.

Approximation techniques for small-signal modeling combine both the time-averaging and sample-data methods and have been applied to series and parallel resonant converters [8]. The operating point is determined from the steady state analysis. Once an operating point is determined, the control-to-output and line-to-output transfer functions can be approximated by inspection of the previous experimental and analytical (sample-data and time averaging) bode-plot results. The output low-pass filter determines the dominant behavior of the transfer function. It is also seen as internal converter impedance. The high frequency behavior of the transfer function can be modeled as a second-order response whose frequency is the difference between the resonant frequency

and switching frequency. Also, the $Q (w_o L_r / R_o)$ value used in the high frequency response approximation is the original resonant Q modified by the internal impedance of the converter. This method provides a good approximation to the exact and experimental results for the conventional series and parallel resonant converter; however, there is no analytical solution that can be generalized to other converter topologies.

Analytical results for small-signal modeling of the series resonant converter have been investigated [9]-[13]. Their derivations are very complex and not applicable to other resonant converters. A generalized approach to small-signal modeling of DC-DC resonant converters that operate in a continuous conduction mode (CCM) has been derived using the Taylor series expansion and superposition theory [14]. It is shown that the steady-state V-I trajectory of the series resonant converter reveals a simple closed contour on a state-plane diagram that repeats itself every switching period (T_s). Also, the trajectory has symmetry through origin. This means that the discrete small-signal model can be recursively formulated. The discrete small-signal model for the series resonant converter is derived analytically based on a perturbation method applied to a steady-state trajectory.

The state-plane approach mentioned above can be linked to the Taylor's series expansion. This can be used to develop a generalized solution to other half-bridge or full-bridge topologies. The analytical solution of the small-signal transfer functions (line-to-input and control-to-output) has been verified by simulation and experimentation with good results [14]. However, the parasitic effects of the converter devices (MOSFET's, diodes, output transformer, etc.) used in the experiment caused the frequency response at high frequencies to deviate from the analytical results.

In summary, the generalized small-signal analysis approach based on the Taylor's series expansion is a straightforward method of developing small-signal models of resonant converters. Once the state-plane diagram for a particular converter is known, systematic steps can be taken to derive its small-signal transfer function. A simple Matlab program for the converter can be written to produce the required transfer functions. The results of the small-signal analyses can then be used to design a regulated converter. This method allows one to design a resonant converter system without the need for building costly prototypes.

Small-signal models can be used to produce bode-plots that reveal the frequency response of the SRC's power stage designed at a particular operating point. Having knowledge of the exact frequency response characteristics of a converter allows for control system design in the frequency domain instead of the time domain (which involves making estimations, approximations and predictions). This project will verify through computer simulation and prototype tests a control system design for an SRC based on a small signal model.

1.2 Motivation and Objective

In the past, control system design for resonant converters was done by trail-and-error due to the lack of a good system model. This project will demonstrate and verify a methodical control system design for a series resonant DC-DC converter based on its small-signal model derived from a perturbation method [14]. Verification of the design will be done by simulation and practical implementation experiments. The small-signal model will allow for the control system to be designed in the frequency domain, instead of using time domain techniques. Time domain techniques are less reliable since they

involve approximation and iterative processes. The small-signal model will produce an input-to-output transfer function and a control-to-output transfer function where the input is the DC voltage source, the output is the load voltage and the control signal refers to the variable frequency gating signal.

1.3 Project Organization

The work of the project is organized into 5 chapters.

Following the introduction of chapter 1, chapter two will present the theoretical background necessary to obtain the SRC model. First, the steady-state analysis of the SRC will be presented. A state-plane diagram of the SRC based on a particular operating point will be shown along with the equations associated with the state-plane diagram. The equations developed through the steady-state analysis will be used in the small-signal analysis. A mathematical representation of the discrete small-signal model will be developed based on a Taylor's series expansion method.

In chapter three a control system for the SRC will be designed based on its small-signal model. A control system design objective will be specified. Based on some design specifications, the SRC components will be designed using the steady-state analysis derived in chapter two. The small-signal model of the SRC at the operating point specified in the design specifications will also be developed based upon the results of the small-signal analysis in chapter two. The line-to-output and control-to-output frequency responses of the SRC small-signal model will be used to design the control system. Simulation results of the complete SRC system will be presented to show how the design specifications have been met. These results will also illustrate how the control system responds to sudden disturbances.

In chapter four experimental results of the SRC prototype will be shown and used to verify the simulation results. Waveforms will be shown illustrating the impact of sudden changes in 'reference input' and load on the SRC's output.

Finally, a summary of the project results and potential future work topics will be presented in the conclusions section.

CHAPTER 2: MODELING AND ANALYSIS OF SERIES RESONANT CONVERTER

This chapter will provide both steady-state and small-signal analyses of Series Resonant Converter (SRC). The steady-state analysis will be used to develop a state-plane diagram whereby the steady-state operation of the SRC can be visualized. Using the state-plane diagram of the SRC under steady-state conditions we derive the control characteristics curve (relating the converter output voltage to the switching frequency). Because the resonant inductor current and capacitor voltage are sinusoidal and repeat every switching period we can plot one against the other to reveal a set of closed-form trajectories. Each trajectory represents a set of equations that are used to obtain an analytical solution for the converter DC gain ($M=V_o/V_g$). The equations developed in steady-state analysis will be used as part of the small-signal analysis.

Small-signal analysis will be used to develop a discrete small-signal model by means of a perturbation method. Perturbation of the state-plane diagram is accomplished by adding small disturbances (i.e. ' $\hat{x}$ ' is the small-signal representation of the instantaneous variable ' x ') to the state variables ($\hat{i}_{ln}$, $\hat{v}_{cn}$, $\hat{v}_{on}$). By using the Taylor series expansion and the symmetrical properties associated with state-plane diagram, the small-signal model (input-to-output and control-to-output transfer functions) can be obtained analytically.

2.1 Steady-State Analysis

2.1.1 Steady-State Circuit Operation Below Resonance ($1/2f_o < f_s < f_o$)

Figure 2.1 shows the power stage circuit of the SRC. The transformer in this circuit is used to isolate the output from the input. Assuming the output capacitor is large

enough such that V_o is a constant with no ripple then the voltage across the transformer (V_b) can be seen as a square wave with magnitude $\pm V_o$.

Figure 2.1: Series Resonant Converter Power Stage Circuit

The waveforms of the SRC under steady-state conditions are shown in Figure 2.2. The first waveform is the gate signal for Q_1 (gating signal for Q_2 is complementary to Q_1). The second waveform is the resonant capacitor voltage and the third waveform is the resonant inductor current. The last waveform is the voltage across the isolation transformer. In a balanced system, the coordinates at t_0 will be equal to the coordinates at $t_0 + T_s$ where t_0 is the time of the initial condition and T_s is the switching period. From t_0 to t_2 , the converter completes half of the switching period. From t_2 to $t_0 + T_s$ the converter completes the remaining half of the switching period. Both periods are symmetrical therefore only half the cycle is needed for the analysis. Q_1 conducts during $t_0 < t < t_1$, D_{Q1} conducts during $t_1 < t < t_2$, Q_2 conducts during $t_2 < t < t_3$ and D_{Q2} conducts during $t_3 < t < t_0 + T_s$.

Figure 2.2: Waveforms of the SRC During Steady-State Condition

The switching period (T_s) can be divided into four time intervals: M_1 , M_2 , M_3 , and M_4 . Each time interval has an equivalent circuit representation as shown in Figure 2.3.

Figure 2.3: Equivalent Circuit at Each Time Interval: (a) M_1 , (b) M_2 , (c) M_3 and (d) M_4

Each equivalent circuit mode can be analyzed individually to derive a set of differential equations as follows:

$$\frac{di_l}{dv_c} = \frac{C_r}{L_r} \frac{V_g - v_c - V_o}{i_l}, \quad t_0 \leq t < t_1 \quad (M_1) \quad (2.1)$$

$$\frac{di_l}{dv_c} = \frac{C_r}{L_r} \frac{V_g - v_c + V_o}{i_l}, \quad t_1 \leq t < t_2 \quad (M_2) \quad (2.2)$$

$$\frac{di_l}{dv_c} = \frac{C_r}{L_r} \frac{-V_g - v_c + V_o}{i_l}, \quad t_1 \leq t < t_2 \quad (M_3) \quad (2.3)$$

$$\frac{di_l}{dv_c} = \frac{C_r}{L_r} \frac{-V_g - v_c - V_o}{i_l}, \quad t_3 \leq t < t_0 + T_s \quad (M_4) \quad (2.4)$$

where V_g, v_c, V_o and i_l are the steady-state supply voltage, instantaneous resonant capacitor voltage, steady-state output voltage, and instantaneous resonant inductor current respectively. By solving equations (2.1)-(2.4), the following set of equations is obtained:

$$Z_o^2 i_l^2 + (v_c - V_g + V_o)^2 = V_{M1}^2 \quad t_0 \leq t < t_1 \quad (2.5)$$

$$Z_o^2 i_l^2 + (v_c - V_g - V_o)^2 = V_{M2}^2 \quad t_1 \leq t < t_2 \quad (2.6)$$

$$Z_o^2 i_l^2 + (v_c + V_g - V_o)^2 = V_{M1}^2 \quad t_2 \leq t < t_3 \quad (2.7)$$

$$Z_o^2 i_l^2 + (v_c + V_g + V_o)^2 = V_{M2}^2 \quad t_3 \leq t < t_0 + T_s \quad (2.8)$$

where

$$\begin{aligned} V_{M1} &= V_{c,max} - V_g + V_o, \\ V_{M2} &= V_{c,max} - V_g - V_o, \\ Z_o &= \sqrt{L_r / C_r} \end{aligned}$$

and $V_{c,max}$ is the value of the resonant capacitor voltage when the inductor current is zero.

These equations will be used in the next section to derive the state-plane diagram for the SRC.

2.1.2 State-Plane Diagram Derivation

The state-plane diagram is the graphical representation of equations of (2.5)-(2.8), but in normalized form. Once the state-plane diagram of converter type is known the operating parameters can be obtained graphically instead of repeating the equation derivations. In order to convert equations (2.5)-(2.8) to their normalized form, the equations must be divided by the square of the base voltage V_g to obtain the equivalent normalized equations as follows:

$$i_{ln}^2 + (v_{cn} - 1 + V_{on})^2 = V_{M1n}^2 t_0 \leq t < t_1 \quad (2.9)$$

$$i_{ln}^2 + (v_{cn} - 1 - V_{on})^2 = V_{M2n}^2 t_1 \leq t < t_2 \quad (2.10)$$

$$i_{ln}^2 + (v_{cn} + 1 - V_{on})^2 = V_{M1n}^2 t_2 \leq t < t_3 \quad (2.11)$$

$$i_{ln}^2 + (v_{cn} + 1 + V_{on})^2 = V_{M2n}^2 t_3 \leq t < t_0 + T_s \quad (2.12)$$

where the subscript 'n' represents normalized variables. The conduction angles of the transistors (Q_1 and Q_2) and anti-parallel diodes (D_{Q1} and D_{Q2}) are α and β respectively.

These angles are related to the switching frequency as follows:

$$\alpha + \beta = \frac{\pi}{f_{sn}} = \gamma \quad (2.13)$$

where $f_{sn} = f_s/f_o$ is the normalized switching frequency and γ is the sum of the conduction angles. Using the above relationships and equations (2.9)-(2.13), V_{M1n} and V_{M2n} can be determined. Once V_{M1n} and V_{M2n} are determined, the initial coordinates $[v_{cn}(t_0), i_{ln}(t_0)]$ can be determined as follows:

$$|v_{cn}(t_0)| = V_{on} (V_{M1n} + 1 - V_{on}) \quad (2.14)$$

$$|i_{ln}(t_0)| = \sqrt{(1 - V_{on}^2)[(V_{M1n} - V_{on})^2 - 1]} \quad (2.15)$$

The normalized state equations, along with the initial conditions derived above can now be used to plot the state-plane diagram shown in Figure 2.4.

Figure 2.4: Steady-State State Plane Diagram for the SRC

When the converter reaches steady-state, the resonant circuit forms a closed trajectory which can be seen in Figure 2.4. “ I_{ln} ” and “ V_{cn} ” are the normalized resonant inductor current and capacitor voltage respectively. These two variables are used in the state-plane diagram because in a properly designed converter the resonant current and voltage are sinusoidal and have half wave symmetry. If the steady-state parameters of the converter are known then the state-plane diagram can be drawn by hand; otherwise, a simulation package such as Matlab or PSPICE can be used to generate the V-I trajectory. These normalized variables are normalized with respect to the following base values:

$$\begin{aligned} V_{base} &= V_g \\ Z_{base} &= Z_o \\ I_{base} &= V_{base}/Z_{base} = V_g/Z_o \\ f_{base} &= f_o \end{aligned}$$

The parameters of the state-plane diagram can be used to find a solution for the DC gain analytically for the SRC. The next section will provide the solution for the converter DC gain which will be used to plot the control characteristic curve.

2.1.3 Series Resonant Converter DC Gain

The SRC converter DC gain ($M=V_o/V_g$) as a function of the switching frequency (from $1/2f_o < f_s < f_o$) can be obtained analytically using the known parameters from the previous sections. The results are as follows:

$$aM^2 + bM + c = 0 \quad (2.16)$$

and

$$a = \gamma^2 Q^2 \cos^2(\gamma/2) + 4 \sin^2(\gamma/2) \quad (2.17)$$

$$b = -4\gamma Q \cos^2(\gamma/2) \quad (2.18)$$

$$c = 4[\cos^2(\gamma/2) - 1] \quad (2.19)$$

where ‘ Q ’ is the load quality factor. By obtaining an analytical solution for M the control characteristics curve can be seen by plotting M as a function of f_{sn} at different loads as shown in Figure 2.5.

Figure 2.5: Control Characteristics Curve for the SRC Operating Below Resonance

It can be seen from Figure 2.5 that the SRC is nonlinear. The output voltage is uncontrollable at no-load ($Q = 0$). The output voltage is proportional to the switching frequency when operating the SRC below resonance frequency. The figure also illustrates that a steady-state operating point can be selected from the control characteristic curve.

In summary the SRC under steady-state conditions can be graphically represented on a state-plane diagram. Curves on the state-plane trajectory represent specific time segments during the switching period. The state-plane trajectory has half-wave symmetry. This means that the resonant inductor current and capacitor voltage during the first half of the switching is equal in magnitude but opposite in ‘sign’ during the second half of the switching period. The state-plane diagram is derived from a set of steady-state equations which lead to an analytical solution for the converter DC gain.

2.2 Small-Signal Analysis

Once the steady-state operating conditions are known from the state-plane diagram and control characteristics curve then the small-signal model of the SRC can be determined analytically. A small-signal model is needed in order to design a proper control system for the SRC. Small-signal analysis provides insight into the converters response to disturbances and perturbations in the steady-state trajectory. The response of the converter to steady-state conditions is a trajectory of curves that form a closed contour and repeat every switching period as shown in Figure 2.4. The response of the converter to any disturbances such as a sudden change in control signal or input voltage will lead to a trajectory that deviates from the steady-state trajectory.

Using a perturbation method on the steady-state trajectory a discrete small signal model can be obtained geometrically. The perturbation variables that will be considered for the SRC are the following: normalized input voltage (v_{gn}), normalized control switching frequency (f_{sn}), and the state-variables. The state variables are variables that characterize the dynamics of the converter. For the SRC these state variables are as follows: normalized resonant inductor current (i_{ln}), normalized resonant capacitor voltage (v_{cn}) and the normalized output load voltage (v_{on}). The perturbed signals can be represented as follows:

$$\hat{i}_{ln}(t) = i_{ln}(t) - I_{ln}(t) \quad (2.20)$$

$$\hat{v}_{cn}(t) = v_{cn}(t) - V_{cn}(t) \quad (2.21)$$

$$\hat{v}_{on}(t) = v_{on}(t) - V_{on}(t) \quad (2.22)$$

$$\hat{f}_{sn}(t) = f_{sn}(t) - F_{sn}(t) \quad (2.23)$$

$$\hat{v}_{gn}(t) = v_{gn}(t) - 1 \quad (2.24)$$

The uppercase variables (i.e. $V_{on}(t)$) represent the steady-steady conditions. The analysis is based on the premise that at the beginning of the switching period any of the perturbations can occur; however, the response of the system to the perturbation is in the next half switching period. The perturbation state vector is $\hat{\mathbf{x}} = [\hat{i}_{in} \ \hat{v}_{cn} \ \hat{v}_{on}]^T$ is assumed to be small enough such that the principle of superposition can be used to determine the effect of each perturbed state variable independently. $\hat{\mathbf{x}}(0)$ and $\hat{\mathbf{x}}(T_s/2)$ represent the perturbation vectors at the turn-on instants of Q_1 and Q_2 respectively. Using the odd symmetry properties of the steady-state state-plane diagram, the discrete small-signal model can be written as follows:

$$\hat{\mathbf{x}}\left(\frac{T_s}{2}\right) = \mathbf{A}_0 \hat{\mathbf{x}}(0) + \mathbf{B}_0 \hat{f}_{sn}(0) + \mathbf{C}_0 \hat{v}_{gn}(0), \text{ for } t_0 \leq t < T_s/2 \quad (2.25)$$

$$\begin{aligned} \hat{\mathbf{x}}(T_s) &= \mathbf{A}_1 \hat{\mathbf{x}}\left(\frac{T_s}{2}\right) + \mathbf{B}_1 \hat{f}_{sn}\left(\frac{T_s}{2}\right) + \mathbf{C}_0 \hat{v}_{gn}\left(\frac{T_s}{2}\right), \\ &\text{for } T_s/2 \leq t < T_s \end{aligned} \quad (2.26)$$

Since the steady-state trajectory repeats every switching period and has odd symmetry it would be necessary to express Equations 2.25 and 2.26 as a function of a discrete variable as follows:

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}_0 \hat{\mathbf{x}}(k) + \mathbf{B}_0 \hat{f}_{sn}(k) + \mathbf{C}_0 \hat{v}_{gn}(k), \text{ for even } k \quad (2.27)$$

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}_1 \hat{\mathbf{x}}(k) + \mathbf{B}_1 \hat{f}_{sn}(k) + \mathbf{C}_0 \hat{v}_{gn}(k), \text{ for odd } k \quad (2.28)$$

where k is a integer (i.e. 0, 1, 2, 3,...) variable representing time $kT_s/2$.

A_0 , B_0 and C_0 are the coefficient matrices. They appear as follows:

$$A_0 = \begin{bmatrix} \frac{\partial i_{ln}(1)}{\partial i_{ln}(0)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(0)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} \\ \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} \\ \frac{\partial i_{ln}(1)}{\partial i_{ln}(0)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(0)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} \\ \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} & \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} \frac{\partial i_{ln}(1)}{\partial f_{sn}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} \\ \frac{\partial i_{ln}(1)}{\partial f_{sn}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} \\ \frac{\partial i_{ln}(1)}{\partial f_{sn}(0)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \text{ and } C_0 = \begin{bmatrix} \frac{\partial i_{ln}(1)}{\partial v_{gn}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{cn}(1)} \\ \frac{\partial i_{ln}(1)}{\partial v_{gn}(0)} \\ \frac{\partial i_{ln}(1)}{\partial v_{on}(1)} \\ \frac{\partial i_{ln}(1)}{\partial v_{gn}(0)} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

A_1 , B_1 and C_1 have the same magnitude as A_0 , B_0 and C_0 respectively, but have different polarities as follows:

$$A_1 = \begin{bmatrix} a_{11} & a_{12} & -a_{13} \\ a_{21} & a_{22} & -a_{23} \\ -a_{31} & -a_{32} & a_{33} \end{bmatrix}, B_1 = \begin{bmatrix} -b_1 \\ -b_2 \\ b_3 \end{bmatrix}, \text{ and } C_1 = \begin{bmatrix} -c_1 \\ -c_2 \\ c_3 \end{bmatrix}$$

The full derivations of the matrices coefficients are shown in Appendix A. Appendix A shows how to derive the coefficients based on the assumption that since the signal perturbations are small, the Taylor Series expansion can be implemented on the perturbation state equations whereby the nonlinear terms of the expansion can be eliminated. This leads to a linear solution for the small-signal model.

The elements of each matrix are derived based on parameters from the previous steady-state analysis. Details of the equations for each coefficient (see Appendix A) are determined based on perturbations of the state-plane diagram of Figure 2.4. By analyzing the state-plane diagram during the steady-state condition it can be seen that SRC behaves

as a discrete system where the trajectory during the first half of the switching period (even symmetry) equals the exact opposite in the second half of the switch period (odd symmetry) that repeats every half switch period ($T_s/2$). Matrices $\mathbf{A}_0$, $\mathbf{B}_0$ and $\mathbf{C}_0$ represent the solution for the even portion of the trajectory (i.e. first half of the switch period). Matrices $\mathbf{A}_1$, $\mathbf{B}_1$ and $\mathbf{C}_1$ represent the solution for the odd portion of the trajectory. The magnitude of the coefficients for $\mathbf{A}_1$, $\mathbf{B}_1$ and $\mathbf{C}_1$ are the same as $\mathbf{A}_0$, $\mathbf{B}_0$ and $\mathbf{C}_0$. A closed form solution is required to provide a complete discrete small-signal model of the SRC. Using the even/odd symmetry principals mentioned above and some matrix transformations (see [14]), a closed-form solution is found in $\mathbf{A}_2$, $\mathbf{B}_2$ and $\mathbf{C}_2$ whose coefficients have the same magnitude as $\mathbf{A}_0$, $\mathbf{B}_0$ and $\mathbf{C}_0$ respectively with different polarities as follows:

$$\mathbf{A}_2 = \begin{bmatrix} -a_{11} & a_{12} & -a_{13} \\ a_{21} & -a_{22} & a_{23} \\ a_{31} & -a_{32} & a_{33} \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -b_1 \\ b_2 \\ b_3 \end{bmatrix} \text{ and } \mathbf{C}_2 = \begin{bmatrix} -c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

such that:

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}_2 \hat{\mathbf{x}}(k) + \mathbf{B}_2 \hat{f}_{sn}(k) + \mathbf{C}_2 \hat{v}_{gn}(k), \text{ for all } k \quad (2.29)$$

The state-space representation of the discrete small-signal model (z-transform) using the matrices $\mathbf{A}_2$, $\mathbf{B}_2$ and $\mathbf{C}_2$ can be used to find the line-to-output $\hat{v}_{on}(s)/\hat{v}_{gn}(s)$ and control-to-output $\hat{v}_{on}(s)/\hat{f}_{sn}(s)$ transfer functions as follows.

$$\frac{\hat{v}_{on}(s)}{\hat{v}_{gn}(s)} = [0 \quad 0 \quad 1][z\mathbf{I} - \mathbf{A}_2]^{-1} \mathbf{C}_2 \quad (2.30)$$

$$\frac{\hat{v}_{on}(s)}{\hat{f}_{sn}(s)} = [0 \quad 0 \quad 1][z\mathbf{I} - \mathbf{A}_2]^{-1}\mathbf{B}_2 \quad (2.31)$$

where 'I' is the identity matrix and $z = \frac{1 + sT_s/4}{1 - sT_s/4}$ (bi-linear transformation).

2.3 Example: Small-Signal Model Verification

In order to verify the accuracy of the small signal model derived in the previous section, a series resonant converter circuit with small-signal perturbation was simulated using PSPICE. For this example, the SRC has the following operating conditions:

$$V_g = 40\text{V, input voltage}$$

$$V_o = 15\text{V, output voltage}$$

$$F_s = 100\text{kHz, switching frequency}$$

$$F_o = 122\text{kHz, resonant frequency}$$

$$L_r = 133.5\mu\text{H, resonant inductor}$$

$$C_r = 12.8\text{nF, resonant capacitor}$$

$$C_o = 32\mu\text{F, output filter capacitor}$$

A small-signal disturbance of 10kHz will be added to the switching frequency (control signal). A bode plot of equation 2.31 will reveal the theoretical magnitude and phase response of the SRC to a 10kHz disturbance. Using Equation 2.31 and the Matlab program in Appendix B, the control-to-output transfer function for the operating conditions listed above is as follows:

$$\frac{\hat{v}_{on}(s)}{\hat{f}_{sn}(s)} = \frac{0.00153s^3 - 282.2s^2 - 5.167 \times 10^7 s + 9.383 \times 10^{12}}{s^3 + 2.947 \times 10^4 s^2 + 5.232 \times 10^9 s + 4.716 \times 10^{12}} \quad (2.32)$$

Figure 2.6 shows the magnitude and phase response of the control-to-output transfer function in equation 2.32.

Figure 2.6: SRC Control-to-Output Small-Signal Response (theoretical)

The small-signal response of the SRC to a 10kHz disturbance in the switching frequency can be predicted by reading the bode plot of the control-to-output transfer function. At 10kHz on the frequency axis, the magnitude and phase responses are as follows:

$$\left| \frac{\hat{v}_{on}}{\hat{f}_{sn}} \right| = 21.9dB,$$

$$\angle \left(\frac{\hat{v}_{on}}{\hat{f}_{sn}} \right) = 110^\circ$$

Figure 2.7 shows the SRC circuit used to verify the theoretical results obtained above.

The circuit is driven by a frequency modulated control signal.

voltage ($\hat{v}_o$) which represents the output response of the SRC to the small-signal disturbance. The magnitude and phase of the control-to-output response at $f_m = 10\text{kHz}$ can be obtained from Figure 2.8 as follows:

$$\left| \frac{\hat{v}_o}{\hat{v}_f} \right| = 2.494, \text{ and } \angle(\hat{v}_o/\hat{v}_f) = 102.13^\circ$$

Figure 2.8: SRC Control-to-Output Small Signal Simulation Response at 10kHz

The control-to-output response $\hat{v}_{on}(s)/\hat{f}_{sn}(s)$ from the analytical solution is related to $\hat{v}_o/\hat{v}_f$ from the simulation results as follows:

$$\left| \frac{\hat{v}_{on}(s)}{\hat{f}_{sn}(s)} \right| = \left| \frac{\hat{v}_o}{\hat{v}_f} \right| \frac{f_o}{K_m V_g}, \text{ and}$$

$$\angle \left(\frac{\hat{v}_{on}(s)}{\hat{f}_{sn}(s)} \right) \cong \angle \left(\frac{\hat{v}_o}{\hat{v}_f} \right)$$

where K_m is the VCO (voltage controlled oscillator) gain. Using the above scaling, the magnitude of the control-to-output response in decibels is 22.4dB, compared to 21.9dB from the theoretical results.

Since the theoretical results are in good agreement with the simulation results the discrete small-signal model of the SRC can be used to design a control system in the frequency domain. The next chapter will illustrate a complete SRC power stage and control system design based on the steady-state and small-signal analysis discussed in this chapter.

2.4 Summary

The steady-state analysis of the SRC begins with its circuit operation under steady-state conditions. The operation of the SRC during steady-state can be divided into four equivalent circuits for four different time intervals in one switching cycle. These four equivalent circuits lead to the solution of four differential equations. The normalized version of these equations along with solution to the initial conditions of the resonant capacitor voltage and inductor current are used to plot the state-plane diagram of the SRC. One benefit of having a graphical representation of the SRC during steady-state is that the operating conditions can be solved directly from the state-plane diagram. A second benefit is that the perturbation method can be directly applied to the state-plane diagram in order to develop the discrete small-signal model of the SRC.

The small-signal model is derived based on the perturbation method applied to the state variables. Assuming the disturbances are small enough then the principle of superposition can be used to analyze the effects of input disturbances on each state variable. The result of the analysis is a discrete small-signal model of the SRC. The input-to-output and control-to-output transfer functions can then be derived using the discrete small-signal model. The analytical results for the control-to-output transfer function were verified by using a simulation circuit of the SRC and injecting a small-

signal disturbance into the control signal (switch frequency). The output response of the simulation circuit was compared to the frequency response of the analytical control-to-output transfer function at the frequency of the disturbance signal. Both the simulation and analytical results were comparable and in good agreement.

CHAPTER 3: CONTROL SYSTEM DESIGN

Chapter 2 showed the theoretical analysis necessary to obtain the discrete small-signal model of the SRC analytically without using approximation techniques. The outcomes of the analysis results in the two following transfer functions: control-to-output ($\hat{v}_{on}(s)/\hat{f}_{sn}(s)$) and input-to-output ($\hat{v}_{on}(s)/\hat{v}_{gn}(s)$). This chapter will utilize these two transfer functions to design a control system in the frequency domain by using bode diagrams to analyze the magnitude and phase response of the small-signal transfer functions.

3.1 Compensation Network Design

Before a compensation network can be designed it is necessary to define what is being controlled or regulated. In a typical DC-DC converter application the output voltage is usually controlled around a particular set point. The compensation network should prevent any abnormal system behaviors from affecting the output voltage set point.

Figure 3.1 shows a block diagram of a regulated SRC system.

Figure 3.1: Block Diagram of a Regulated DC-DC SRC

The output voltage of the converter is the only significant parameter needed in the feedback loop; hence, a single loop feedback control scheme can be used to regulate the output voltage. The output voltage is negatively fed back to the controller. This means that the compensator uses the error signal between V_{ref} and V_{out} to modulate the switching frequency (control signal) and keep V_{out} at the DC level set point determined by V_{ref} . The performance of a closed-loop converter system can be determined using time domain criteria such as percent overshoot, rise time and settling time. When the time domain techniques are used to design a control system for the SRC they provide no insight into the converters behavior. This results in a control system design that is based on trial-and-error. By using the small-signal model discussed in the previous chapter the frequency response of the SRC can be realized. A design specification for the control system can then be specified in the frequency domain. Figure 3.2 shows a bode plot of a typical SRC small-signal control-to-output transfer function ($\hat{v}_{on}(s)/\hat{f}_{sn}(s)$).

Figure 3.2: Control-to-Output Transfer Function Bode Diagram

The frequency response of the SRC's control-to-output transfer function shows a system that is frequency-dependent requiring the control system to be frequency-dependent. If a constant-gain compensation network were used to control the system in Figure 3.2 the controller would perform adequately during low frequency disturbances. If a sudden step in the line voltage were applied then the high frequency components of the step voltage would cause a ringing transient in the output voltage due to the phase shift at higher frequencies. A 'PI' (proportional-integral) regulator may be considered as a possible compensation network. This regulator will provide zero steady-state error and run into the same stability problem as the constant-gain compensator for heavy loads. Figure 3.3 shows a compensation network that can provide good DC-gain, integration as well as lead-lag compensation.

Figure 3.3: Integral Lag-Lead Compensation Network

The integral lag-lead network provides zero steady-state error by including an integrator. The phase margin of the compensated loop is also good due to the lead-lag network. Additional lead compensators at high frequencies can be added to improve the step response speed due to a sudden line voltage change. The mathematical expression for the network in Figure 3.3 is as follows:

$$\frac{\hat{v}_o(s)}{\hat{v}_e(s)} = A(s) = -\frac{1 + sC_2(R_1 + R_2)}{sC_1R_1} \cdot \frac{1 + sC_1R_3}{1 + sC_2R_2} \quad (3.1)$$

where $\hat{v}_e$ represents the error signal and is the input to the voltage-controlled oscillator (VCO). The integral portion is most effective at low frequencies and provides the phase-lag. The integral and lead compensation take effect beyond $1/2\pi C_2(R_1 + R_2)$ Hz up to the gain crossover frequency $f_m = 1/(2\pi C_2R_2)$. Depending on what the phase response (ϕ_{fm}) is at for the gain crossover frequency of the control-to-output transfer function, additional lead or lag compensation can be added. Figure 3.4 shows an example of the magnitude response of the integral lag-lead compensation network. A design example, discussed later, will demonstrate how to design the parameters for a compensation network based on closed loop gain and phase margin specifications.

Figure 3.4: Magnitude Response of Integral Lag-Lead Network

3.2 Control Methodology

Many power converter's (i.e. buck, boost) regulators control the output voltage by varying the pulse width of the gating signal from 0 to 100%. This is better known as pulse-width modulation (PWM). PWM can produce highly non-sinusoidal currents and voltages in the power stage that result in high THD (total harmonic distortion). The SRC incorporates a half-bridge inverter (see Figure 3.1) which converts a DC input voltage into a sinusoidal current flowing through the series resonant tank circuit. Since the resonant tank impedance (Z_o) is a function of the switching frequency so is the output voltage/current of the inverter. The output voltage of the SRC is also a function of the switching frequency. This means the control system design for a SRC must incorporate a variable-switching frequency controller. In Figure 3.1, the VCO represents the variable-switching frequency circuit. The VCO technique used in the simulation and prototype experiments is based on an error voltage-to-oscillator current conversion technique used in the MC34066 Resonant Controller (see [15]). Figure 3.5 shows the VCO circuit used to produce the variable switching frequency. I_{osc} is equal to the voltage across R_{vfo} ($V_{err} - 1.4V$) divide by R_{vfo} . V_{err} is the output voltage of the error amplifier. The maximum voltage across R_{vfo} is 2.5V; therefore, the maximum value of I_{osc} is $2.5V / R_{vfo}$. In the 'Current Mirror' circuit the discharge current out of the V_{osc} terminal is equal to I_{osc} . The oscillator amplifier represents a hysteresis comparator that allows C_{osc} to discharge and charge between 3.6 to 5.1V. ' I_{chg} ' is a current source and is used to quickly charge the oscillator capacitor C_{osc} . The charge time of C_{osc} is set to be approximately 50ns.

The charge time of C_{osc} is very small compared to the discharge time. The hysteresis comparator is used to produce the ramp voltage of V_{osc} . For the SRC a constant 50% duty cycle is need for the gating signal of the half-bridge switches as the switching frequency varies. In Figure 3.6 each ramp represents the on-time of one switch and the off-time of the other switch. The charge time of C_{osc} represents the dead-time between switches in order to prevent shoot-through (where both switches are on at the same time causing a virtual short circuit). The width of the ramp is controlled by the value of I_{osc} which is a function of V_{err} . Therefore, as V_{err} increases, I_{osc} increases and the ramp width becomes smaller resulting in an increase in switching frequency. The reverse occurs when V_{err} decreases. The benefit of the VCO circuit of Figure 3.6 is that the minimum and maximum oscillator frequencies can be programmed. The following equations show how the components in Figure 3.6 can be used to set the minimum and maximum oscillator periods:

$$t_{dchg(min)} = R_{osc} \cdot C_{osc} \cdot \ln \left[\frac{\frac{2.5R_{osc}}{R_{vfo}} + 5.1}{\frac{2.5R_{osc}}{R_{vfo}} + 3.6} \right] \quad (3.2)$$

$$t_{dchg(max)} = R_{osc} \cdot C_{osc} \cdot \ln \left[\frac{5.1}{3.6} \right] \quad (3.3)$$

The VCO technique used in this project is current-dependent as opposed to the typical VCO used in communication circuits and can be easily programmed and simulated.

3.3 Example: Design and Implementation of a Regulated SRC

The first step is to design the power stage using the control characteristic curve and the state-plane diagram under steady-state conditions as shown in chapter two. The following are the design specifications for the SRC:

Input Voltage (V_g): 37VDC

Switch Frequency (F_s): 100kHz

Output Voltage (V_o): 12V

Rated Load (R_o): 23 ohms.

Switch Frequency range: $0.5F_o < F_s < 1 F_o$,

where F_o is the resonant frequency.

Two more parameters need to be determined in order to complete the design of the power stage: the resonant capacitor (C_r) and inductor (L_r). The DC converter gain (M) is a function of the load quality factor (Q) and the switching frequency according to equations 2.16 - 2.19. Figure 2.2 in the previous chapter shows the control characteristic curves of the SRC for different values of ' Q '. The curve that offers the most control range is the curve where Q is approximately equal to 5.4. In [12], it shown that when $Q \geq 2.5$ the resonant capacitor voltage and resonant inductor current are approximately sinusoidal. From the above design specifications we can calculate that $M = V_o/V_g = 0.32$. From the control characteristic curve when $M = 0.32$ the normalized switching frequency is approximately 0.8 on the curve for $Q = 5.4$. The resonant frequency can be solved from the normalized switching frequency as follows:

$$F_o = F_s/0.8 = 125\text{kHz}$$

Now that ' F_o ' is known the resonant tank components can be determined from 'Q' as follows:

$$\begin{aligned} L_r &= Q \cdot R_o / \omega_o = 156 \mu\text{H}, \\ C_r &= 10.2 \text{ nF} \end{aligned}$$

where

$$\omega_o = 2\pi F_o \text{ and } F_o = 1/(2\pi\sqrt{L_r C_r})$$

The low-pass filter capacitor (C_o) must not be too large or else the response of the controller will be slow. ' C_o ' must be able to carry the equivalent of at least 48% in magnitude of the output DC current in high frequency ripple current. For this converter design $C_o = 40 \mu\text{H}$ is sufficient to filter out the high frequency ripple current and allow for a quick control system response to a sudden disturbance.

In Appendix A the discrete small signal model can be determined based on the steady-state parameters selected above. Appendix B shows a sample Matlab program used to generate the small-signal control-to-output and input-to-output transfer functions. The program uses the power stage parameters previously solved and the equations derived from the state-plane analysis. After running the program the following transfer functions are written to an array parameter 'sys' in the code:

$$\frac{\hat{v}_{on}(s)}{\hat{f}_{sn}(s)} = \frac{0.00158s^3 - 367.1s^2 - 2.822 \times 10^7 s + 7.625 \times 10^{12}}{s^3 + 4.322 \times 10^4 s^2 + 6.731 \times 10^9 s + 4.741 \times 10^{12}} \quad (3.4)$$

$$\frac{\hat{v}_{on}(s)}{\hat{v}_{gn}(s)} = \frac{2.975 \times 10^{-5} s^3 - 52.2s^2 + 1.512 \times 10^7 s + 1.548 \times 10^{12}}{s^3 + 4.322 \times 10^4 s^2 + 6.731 \times 10^9 s + 4.741 \times 10^{12}} \quad (3.5)$$

The bode plot of the control-to-output transfer function is the same as the one shown in Figure 3.2. The control system can now be designed in the frequency domain based on the small-signal model (equations 3.4 and 3.5) of the SRC. A typical control system specification for a DC-DC converter is as follows:

$$\begin{aligned}\text{Loop-gain bandwidth} &= f_m = 10\text{kHz} \\ \text{Loop-gain phase margin} &= \phi_{mc} = 45 \text{ degrees}\end{aligned}$$

The ‘integral-lag-lead’ network shown in Figure 3.3 will be used for the compensation network. The next step will be to derive the component values (i.e. resistors and capacitors) of the compensation network. From the control-to-output transfer function bode plot in Figure 3.2 the magnitude response at f_m is $1/K_{fm} = 0.0309$ while the phase response is $\phi_{fm} = -147$ degrees. From the design specifications the compensation network phase response at f_m can be found as follows:

$$\phi_{com} = \phi_{mc} - \phi_{fm} - 180 = 45 - (-147) - 180 = 12 \text{ deg.} \quad (3.6)$$

The closed-loop gain at the cross-over frequency f_m should be 0dB therefore the compensation network magnitude response at f_m should be K_{fm} . From the previous section the first zero ($f_{z1} = 1/2\pi C_2(R_1 + R_2)$) should be set to:

$$f_{z1} \leq \frac{f_m}{10} = \frac{10\text{kHz}}{10} = 1\text{kHz} \quad (3.7)$$

and the second zero ($f_{z2} = 1/2\pi C_1 R_3$) should be set to:

$$f_{z2} = f_m 10^{\left[1 - \frac{\phi_{com}}{45}\right]} = 54.116\text{kHz} \quad (3.8)$$

The high frequency gain (K_p) of the compensation network is set to:

$$K_p = 10K_{fm} = \frac{R_3}{R_1 // R_2} \quad (3.9)$$

The high frequency gain pole (f_p) should be set to:

$$f_p = 10f_{z2} = 1/(2\pi R_2 C_2) \quad (3.10)$$

A choice for C_2 was made to be 20nF. The rest of the components were solved using equations 3.7 – 3.10 as follows:

$$R_1 = 7.94\text{k}\Omega, R_2 = 14.7\Omega, R_3 = 4.75\text{k}\Omega, \text{ and } C_1 = 619.2\text{pF}$$

The compensation network bode plot is similar to the one shown in Figure 3.4. The steep slope at the beginning of the graph represents the integral portion of the network. The integral is needed to provide zero steady-state error and phase-lag. The lead portion (flat section of the graph) begins at f_{z1} and ends at f_{z2} (where the gain is K_{fm}). Additional lead compensation is represented by the sudden rise in slope up to the second flat portion (where the gain is K_p).

3.4 Control System Verification

In order to evaluate the performance of the control system in the time domain a control objective needs to be defined. According to the converter design specifications in the previous section the controller must regulate the output voltage to 12V (or $V_{on} = 0.32$) by varying the switching frequency (F_s). If a sudden load change occurs the controller must be able to regulate the converters output voltage at 12V by updating the switching frequency accordingly.

The performance of the SRC's control system design will be evaluated based on the step response of the controller due to a sudden load (R_o) change or a sudden reference input (V_{ref}) change. The time domain performance results should be as follows:

- 1) Zero steady state error
- 2) 10% or less percent overshoot
- 3) Settling time of less than 1msec

Figure 3.7: PSPICE Simulation Circuit of the Regulated SRC: (a) Power Stage and (b) Control System

Figure 3.7 shows the complete (PSPICE) simulation circuit used to test the SRC's control system design.

Figure 3.8: Simulation Response of V_o and I_o to Sudden Change in R_o (47 to 22 ohms)

Figure 3.8 shows the simulation results for a sudden load change. The output voltage remains constant at 12V while the output load current step response has no percent overshoot and the rise time is approximately 5us. Figure 3.9 shows the simulation result for a sudden increase in reference input (V_{ref}) at $R_o = 47$ ohms respectively. A step increase of 0.5V was applied to the reference input (12V to 12.5V) and the output voltage response was measured.

The simulation results meet the performance specifications for the sudden change in reference input (V_{ref}) with a percent overshoot (P.O.) of less than 10%. The control system is able to provide a zero steady-state error in the output voltage with a settling time of less than 200us.

Figure 3.9: Response of V_o due to Step Input of V_{ref} at $R_o = 47$ ohms (simulation)

3.5 Summary

This chapter presented the process of designing and verifying a complete SRC system as shown in Figure 3.1. The converter power stage was designed based on the steady-state conditions and control characteristics curve. Then the SRC's small-signal model was generated based on the analytical results shown chapter 2. The control system was designed in the frequency domain based on the frequency response of the small-signal model. Lastly the complete SRC system was verified by simulation waveforms using PSPICE. The simulation yielded the following conclusions:

- (1) The SRC's discrete small-signal model provides an accurate way to design a control system (in the frequency domain)
- (2) The control system design process does not require trail-and-error or any iterative processes
- (3) The control system met the requirement to maintain V_o constant in the presence of sudden parameter changes

- (4) The control system provides a quick response to abnormal dynamic behaviors in the SRC.

CHAPTER 4: EXPERIMENTS

An experimental prototype was constructed to verify the performance of the control system design for the SRC. The tests performed on the experimental prototype were the same as the tests performed on the simulation circuit in the previous chapter. The prototype's response to a sudden change in load (R_o) and a sudden change in reference input (V_{ref}) will be shown. The prototype's experimental results will be compared to the simulation results to determine how close the simulation circuit model is to the real world situation.

4.1 Experiment Setup

The PCB designed for the experimental prototype is shown in Appendix C. The schematic circuit used to develop the PCB is shown in Figure 4.1.

The input DC voltage was created by first isolating the offline AC voltage with an isolation transformer then rectifying the AC voltage. The output of the AC rectifier is connected across two input DC capacitors to produce the input DC voltage (V_g).

The inverter switches (MOSFET's) are controlled by an analog resonant mode controller. The resonant controller contains the feedback error amplifier, VCO, and the gate drive circuitry for inverter switches. The resonant controller's switch frequency range is programmed with external resistors and capacitors. The resonant controller drives the MOSFET's with a 50% duty cycle square wave that gate signal. The dead time in between switching signals is also programmable but it was set to its minimum value (approx. 50ns) for this experiment.

The output voltage of the SRC was fed back to the error amplifier through an optocoupler that provided two functions. The first function was to isolate the connected

load from the control system. The second function was to step down the output voltage to the reference input voltage (V_{ref}) level. Any connected load was also isolated from the input DC voltage by using a 1:1 output isolation transformer. Table 4.1 shows a list of major components used in the experimental prototype. Inductive components were assembled and wound by hand.

The test equipment used to obtain the experimental results included the following: a Tektronix digital TDS3000B oscilloscope, a Tektronix differential probe and a Tektronix high frequency current probe. A decade resistor box was used to simulate the load and a DC power was used to provide 15V for the control circuitry. An auto-transformer connected across the offline AC voltage source provided an adjustable input AC voltage.

Table 4.1: Experimental Prototype Major Components

Component	Description	Manufacturer/Part No.
Resonant Inductor (L_r)	- 156uH - 55 turns of Litz wire, 40AWG (170 strands)	- Core: Toriod, Magnetics, 77440-A7 - Litz wire: New England Wire NELC170/40SPUL
Output Isolation Transformer	- 1:1, 16 turns on primary and secondary windings	- Litz wire: New England Wire NELC170/40SPUL - E-core, Ferroxcube, E30/15/7-3F3 - Core Bobbin, ACC04- 0659
Resonant Capacitor (C_r)	- 10nF	- Electrocube, 950B1G103K
Control Isolation Optocoupler	- Control/output Isolation	NEC, P2501-1
Controller	- resonant mode controller	- ON Semiconductor, MC34066 (see reference [15])
MOSFET's (Q_1, Q_2)	- inverter switches	- International Rectifier, IRFPC60
Output Rectifier Diodes	- schottky rectifier diodes	- International Rectifier, 20CTQ150
Input Rectifier Diodes	- input line frequency diodes	- International Rectifier, 10ETS08
Input DC Capacitors	- 330uF	- CDE, 250V, 300331U250JT2E

(a)

(b)

(c)

(d)

Figure 4.2: Response of V_o and I_o to Sudden Change in R_o : (a) Prototype: 48 to 23 ohms, (b) Simulation: 48 to 23 ohms, (c) Prototype: 35 to 45 ohms, (d) Simulation: 35 to 45 ohms

As illustrated above the experiment results are in agreement with the simulated results. In Figures 4.2a and 4.2b, the rise time of the prototype and simulation waveforms was approximately 5μs in the output current waveform while the output voltage remained a constant 12V and met the requirements of the control system, which fulfills the objective of the control system. Figures 4.2c and 4.2d illustrate the fall time of both the prototype

and simulation waveforms was approximately 2 μ s while the output voltage remained constant at 12V.

The second test of the experiment prototype was to verify the response of the output voltage (V_o) when the reference input (V_{ref}) was suddenly increased. Since the control system for the SRC is based on a small-signal model V_{ref} was increased a small amount from 12V to 12.5V. This verified the SRC's small-signal model presented in chapter two that attempted to linearize a non-linear system based on the assumption that the disturbances or perturbations would be small.

Figure 4.3 shows the response of V_o to a 0.5V step increase in V_{ref} under a 47 ohms load. Under these conditions no steady-state error resulted and the settling time was less than 1ms. The result agreed with the simulation result.

Figure 4.3: Response of V_o to the Step Input of V_{ref} at $R_o = 47$ ohms (experiment)

4.3 Summary

Chapter four has shown the experimental results of the prototype SRC system. The SRC system was tested to verify the control system's performance against sudden

load changes and against a sudden change in reference input voltage. The outcome of these experiments brings forth the following conclusions:

- (1) The control system designed in the frequency domain has a quick response
- (2) The control system provides zero-steady-state error and settling times less than 1ms
- (3) The experimental prototype results verified the control system designed for the SRC based on a small-signal model was accurate and agreed with the simulations results.

CHAPTER 5: CONCLUSIONS

This project has verified that a control system can be designed for a Series Resonant Converter (SRC) based on a small-signal model. The investigation was divided into three parts: 1) the development of a SRC model, 2) the control system design in the frequency domain based on the SRC model and 3) the verification of the control system design using both a computer simulation and a prototype test.

Designing a control system for the SRC based on a small-signal model provides several benefits. This project illustrates that a control system can be designed without using a prototype or simulation circuit to test the design. This method will result in both cost and time savings. The small-signal model shown in this project is an adequate model of the SRC's dynamic behavior. The control-to-output and input-to-output transfer functions can be obtained directly from the small-signal model and provides a graphical representation of the frequency response of the SRC. The method followed in this project can be extended to other converter topologies that incorporate a half-bridge or full-bridge inverter. The small-signal analysis developed in this project can be applied to other converter topologies given the converter's state-plane diagram.

The simulation and experimental results verified the control systems performance with standard performance specifications (i.e. zero steady state error and less than 10% percent overshoot). The following statements summarize the conclusions obtained as a result of the experiments performed:

- (1) The derivation of a small-signal model analytically saves time and cost in designing the SRC system since it provides a control system design in the

frequency domain and eliminates the need for trial-and-error or iterative techniques.

- (2) The control system regulated the output voltage under variable load conditions including a step change in the load.
- (3) The control system provides a quick response that is able to follow a sudden change in 'reference input'. The control system provides a zero steady-state error at the new reference input voltage and provides an output voltage settling time within one millisecond.

The prototype test results agreed with the simulations results when tested under sudden load and reference input changes. The designed control system was able to control the output of a highly non-linear system (SRC) based on a small-signal model.

Further Work

Future research in SRC control system designs may investigate non-linear control systems and adaptive controllers. The mathematics involved in obtaining a non-linear robust control system is complicated. Simpler analytical methods must be developed to obtain an effective non-linear control system design. Adaptive controllers are another area that should be examined further. This project illustrates that certain parameters can suddenly change while other components (i.e. opto-coupler) may degrade over time. Future work could examine not only the small perturbations but significant parameter changes (load, LC tank, output filter capacitor) when designing a control system. This leads into the application of adaptive controllers. Future research could include how to estimate parameters based on measurements of current and voltage at different points in

the circuit. Adaptive controllers may provide a better system to adapt to changes in circuit parameters and ultimately produce a more reliable control system.

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APPENDIX A: DERIVATION OF THE DISCRETE SMALL-SIGNAL MODEL'S MATRIX COEFFICIENTS FOR A SERIES RESONANT CONVERTER (SRC)

Let p and q be the number of inductor and capacitor components in the resonant tank circuit. In the case of the series resonant converter, $p = 1$ and $q = 1$. In order to determine the small-signal properties of the resonant converter the number of independent state variables must be determined. The output of the series resonant converter (SRC) can be seen as a first order independent state. The total independent states will be $p + q + 1$. For any resonant converter in general the normalized independent instantaneous state vector can be written as:

$$\mathbf{x}(t) = [i_{ln1}(t) \dots i_{lnp}(t) \ v_{cn1}(t) \dots v_{cnq}(t) \ v_{on}(t)]^T \quad (\text{A.1})$$

where i_{ln} , v_{cn} , and v_{on} are the normalized state variables with respect to $V_{base} = V_g$, $I_{base} = V_g/Z_o$ and $Z_o = \sqrt{L_r/C_r}$. For the SRC the state vector is written as follows:

$$\mathbf{x}(t) = [i_{ln}(t) \ v_{cn}(t) \ v_{on}(t)]^T \quad (\text{A.2})$$

Figure A.1 reveals that the steady-state trajectory of the SRC is symmetrical about the origin and it repeats itself every switching period. ' $\mathbf{x}(t)$ ' under steady-state can be written as $\mathbf{x}_{ss}(t)$ where subscript 'ss' denotes steady-state and can be assigned to all the state variables. The resonant tank circuit is driven by an equivalent symmetrical source which goes from V_g to $-V_g$. As a result the resonant inductor current and capacitor voltage shown in Figure A.1 have odd symmetry about the origin.

Figure A.1: Steady-State Trajectory of the SRC

This means that value of the resonant inductor current and capacitor voltage is the same in magnitude every half switching period ($kT_s/2$, where k is any integer 0, 1, 2,...) but opposite in sign (i.e. $v_{cn}(0) = -v_{cn}(T_s/2)$). On the other hand since the output voltage (v_{on}) is driven by a DC current the output state variable has even symmetry (i.e. $v_{on}(t) = v_{on}(t+T_s/2)$). Assuming the switching frequency is high and using the symmetrical characteristics of the SRC the instantaneous and steady-state vectors can be made discrete by replacing time (t) with $kT_s/2$. This will allow for the analysis to take place in one half of the switch period since the results will be similar in the other half of the switching period. The discrete instantaneous and steady-state vectors are as follows:

$$\mathbf{x}(k) = [i_{ln}(k) \quad v_{cn}(k) \quad v_{on}(k)]^T \quad (\text{A.3})$$

$$\mathbf{x}_{ss}(k) = [I_{ln}(k) \quad V_{cn}(k) \quad V_{on}(k)]^T \quad (\text{A.4})$$

The response of a state variable at the end of a half switching period can be expressed as a function of the state variables, control signal $c(k)$ (which represents the switching frequency), and input voltage $v_{gn}(k)$ at the beginning of the switching period. The instantaneous discrete state $x_i(k+1)$ at the end of each half switching period can be expressed as follows:

$$x_i(k+1) = y_i(x_i(k), x_2(k), \dots, x_{r+1}(k), c(k), v_{gn}(k)) = y_i(k) \quad (\text{A.5})$$

for $i \leq 1 \leq r+1$

where $r = p + q$. For the SRC $r = 2$. The instantaneous discrete response vector can be written as follows:

$$\mathbf{x}(k+1) = [y_1(k) \ y_2(k) \ \dots \ y_{r+1}(k)]^T = \mathbf{y}(k) \quad (\text{A.6})$$

Similar results can be shown for the steady-state response vector by adding the subscript 'ss' to equations A.5 and A.6.

The small-signal analysis is based on perturbations or disturbances to the input voltage and control signal that cause the state variables (i_{ln} , v_{cn} and v_{on}) to deviate from their steady-state conditions. The small-signal perturbations are defined as follows:

$$\hat{x}_i(k+1) = x_i(k+1) - x_{ss,i}(k+1) \quad (\text{A.7a})$$

$$\hat{x}_i(k) = x_i(k) - x_{ss,i}(k) \quad (\text{A.7b})$$

$$\hat{y}_i(k) = y_i(k) - y_{ss,i}(k) \quad (\text{A.7c})$$

$$\hat{c}(k) = c(k) - C(k) \quad (\text{A.7d})$$

$$\hat{v}_{gn}(k) = v_{gn}(k) - V_{gn}(k) \quad (\text{A.7e})$$

$C(k)$ and $V_{gn}(k)$ are the steady-state values of the instantaneous variables $c(k)$ and $v_{gn}(k)$ respectively. The discrete small-signal model must be developed from a linearization of a nonlinear system. In order to generate a linear model the small-signal perturbations on the state-plane diagram of a SRC are assumed to be so small that a linear response can be developed for a nonlinear system. A method used to generate linear equations for a nonlinear system is the Taylor Series expansion. Since the small-signal perturbations are assumed to be small the higher order nonlinear terms in the expansion are negligible. Applying the Taylor Series expansion to equation A.7 gives the following result:

$$y_i(k) = y_{ss,i}(k) + \sum_{j=1}^{r+1} \frac{\partial y_i(k)}{\partial x_j(k)} \hat{x}_j(k) + \frac{\partial y_i(k)}{\partial c(k)} \hat{c}_j(k) + \frac{\partial y_i(k)}{\partial v_{gn}(k)} \hat{v}_{gn}(k) \quad (A.8)$$

where the partial derivative terms are evaluated under steady-state conditions. Using equations A.5 and A.6 the perturbed instantaneous state can be written as follows:

$$\hat{x}_i(k+1) = \hat{y}_i(k) = y_i(k) - y_{ss,i}(k) \quad (A.9)$$

Substituting equation A.9 into A.8 results in the following

$$\hat{x}_i(k+1) = \sum_{j=1}^{r+1} \frac{\partial y_i(k)}{\partial x_j(k)} \hat{x}_j(k) + \frac{\partial y_i(k)}{\partial c(k)} \hat{c}_j(k) + \frac{\partial y_i(k)}{\partial v_{gn}(k)} \hat{v}_{gn}(k) \quad (A.10)$$

By examining equation A.10 it can be seen that the discrete small-signal response can be expressed as a linear function of the state variables, control signal and input voltage.

Equation A.10 can be further simplified as follows:

$$\hat{x}_i(k+1) = \mathbf{A}_0 \hat{x}_j(k) + \mathbf{B}_0 \hat{c}_j(k) + \mathbf{C}_0 \hat{v}_{gn}(k) \text{ for even } k \quad (A.11)$$

$$\hat{x}_i(k+1) = \mathbf{A}_1 \hat{x}_j(k) + \mathbf{B}_1 \hat{c}_j(k) + \mathbf{C}_1 \hat{v}_{gn}(k) \text{ for odd } k \quad (A.12)$$

where

$$\mathbf{A}_0 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \mathbf{B}_0 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \mathbf{C}_0 = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \text{ and}$$

$$\mathbf{A}_1 = \begin{bmatrix} a_{11} & a_{12} & -a_{13} \\ a_{21} & a_{22} & -a_{23} \\ -a_{31} & -a_{32} & a_{33} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} -b_1 \\ -b_2 \\ b_3 \end{bmatrix}, \mathbf{C}_1 = \begin{bmatrix} -c_1 \\ -c_2 \\ c_3 \end{bmatrix}$$

for the SRC.

The derivation of each coefficient is based on the partial derivative terms of equation A.10. The functions for the state variables

$(y_1(k) = i_{ln}(k), y_2(k) = v_{cn}(k), y_3 = v_{on}(k))$ of the SRC can be derived directly from the steady-state trajectory shown in Figure A.1 as follows:

$$y_1(k) = i_{ln}(k+1) = -(-1)^k \left[\sqrt{i_{ln}^2(k) + ((-1)^k (v_{gn}(k) - v_{on}(k)) - v_{cn}(k))^2} - 2v_{on}(k) \right] \quad (\text{A.13})$$

$$\times \sin\left(\frac{\pi}{f_{sn}(k)} - \beta(k)\right)$$

$$y_2(k) = v_{cn}(k+1) = (-1)^k \left[\sqrt{i_{ln}^2(k) + ((-1)^k (v_{gn}(k) - v_{on}(k)) - v_{cn}(k))^2} - 2v_{on}(k) \right] \quad (\text{A.14})$$

$$\times \cos\left(\frac{\pi}{f_{sn}(k)} - \beta(k)\right) + v_{gn}(k) + v_{on}(k) \right]$$

$$y_3(k) = v_{on}(k+1) = \frac{4\tau - T_s}{4\tau + T_s} v_{on}(k) + \frac{4R_0 C_r}{4\tau + T_s} [2v_{cn}(t_1) - v_{cn}(k) - v_{cn}(k+1)] (-1)^k \quad (\text{A.15})$$

where

$$v_{cn}(t_1) = (-1)^k \left[v_{gn}(k) - v_{on}(k) + \sqrt{i_{ln}^2(k) + ((-1)^k (v_{gn}(k) - v_{on}(k)) - v_{cn}(k))^2} \right],$$

$\tau = R_0 C_0$ (output filter time constant) and C_r is the series resonant capacitor. Once the equations for the state variables are known the matrix coefficients for $\mathbf{A}_0$, $\mathbf{B}_0$ and $\mathbf{C}_0$ can be derived as follows:

$$a_{11} = \frac{\partial y_1(0)}{\partial i_{ln}(0)} = \frac{V_{M2n}}{V_{M1n}} \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad (\text{A.16})$$

$$a_{12} = \frac{\partial y_1(0)}{\partial v_{cn}(0)} = -\sin \alpha \sin \beta - \frac{V_{M2n}}{V_{M1n}} \cos \alpha \sin \beta \quad (\text{A.17})$$

$$a_{13} = \frac{\partial y_1(0)}{\partial v_{on}(0)} = \sin \alpha (2 - \cos \beta) - \sin \beta \cos \alpha \frac{V_{M2n}}{V_{M1n}} \quad (\text{A.18})$$

$$a_{21} = \frac{\partial y_2(0)}{\partial i_{ln}(0)} = \frac{V_{M2n}}{V_{M1n}} \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad (\text{A.19})$$

$$a_{22} = \frac{\partial y_2(0)}{\partial v_{cn}(0)} = -\frac{V_{M2n}}{V_{M1n}} \sin \alpha \sin \beta + \cos \alpha \cos \beta \quad (\text{A.20})$$

$$a_{23} = \frac{\partial y_2(0)}{\partial v_{on}(0)} = 1 + \cos \alpha (\cos \beta - 2) - \frac{V_{M2n}}{V_{M1n}} \sin \alpha \sin \beta \quad (\text{A.21})$$

$$a_{31} = \frac{\partial y_3(0)}{\partial i_{ln}(0)} = \frac{4R_0 C_r}{4\tau + T_s} \left[\sin \beta (2 - \cos \alpha) - \frac{V_{M2n}}{V_{M1n}} \cos \beta \sin \alpha \right] \quad (\text{A.22})$$

$$a_{32} = \frac{\partial y_3(0)}{\partial v_{cn}(0)} = \frac{4R_0 C_r}{4\tau + T_s} \left[2 \cos \beta - \cos \alpha \cos \beta - \frac{V_{M2n}}{V_{M1n}} \sin \beta \sin \alpha - 1 \right] \quad (\text{A.23})$$

$$a_{33} = \frac{\partial y_3(0)}{\partial v_{on}(0)} = \frac{4\tau - T_s}{4\tau + T_s} + \frac{4R_0 C_r}{4\tau + T_s} \left[2 \cos \beta - \cos \alpha (\cos \beta - 2) + \frac{V_{M2n}}{V_{M1n}} \sin \beta \sin \alpha - 3 \right] \quad (\text{A.24})$$

$$b_1 = \frac{\partial y_1(0)}{\partial f_{sn}(0)} = V_{M2n} \frac{\pi}{F_{sn}^2} \cos \alpha \quad (\text{A.25})$$

$$b_2 = \frac{\partial y_2(0)}{\partial f_{sn}(0)} = V_{M2n} \frac{\pi}{F_{sn}^2} \sin \alpha \quad (\text{A.26})$$

$$b_3 = \frac{\partial y_3(0)}{\partial f_{sn}(0)} = \frac{4R_0 C_r}{4\tau + T_s} V_{M2n} \frac{\pi}{F_{sn}^2} \sin \alpha + \frac{2V_{on}}{F_0 F_{sn}^2} \frac{1}{4\tau + T_s} \quad (\text{A.27})$$

$$c_1 = \frac{\partial y_1(0)}{\partial v_{gn}(0)} = \sin \alpha \cos \beta + \frac{V_{M2n}}{V_{M1n}} \cos \alpha \sin \beta \quad (\text{A.28})$$

$$c_2 = \frac{\partial y_2(0)}{\partial v_{gn}(0)} = 1 - \cos \alpha \cos \beta + \frac{V_{M2n}}{V_{M1n}} \sin \alpha \sin \beta \quad (\text{A.29})$$

$$c_3 = \frac{\partial y_3(0)}{\partial v_{gn}(0)} = \frac{4R_0 C_r}{4\tau + T_s} \left[-2 \cos \beta - \frac{V_{M2n}}{V_{M1n}} \sin \alpha \sin \beta + \cos \alpha \cos \beta + 1 \right] \quad (\text{A.30})$$

APPENDIX B: MATLAB CODE FOR GENERATING A DISCRETE SMALL-SIGNAL MODEL OF AN SRC

The proceeding code was developed in Matlab to automatically generate the small-signal control-to-output and input-to-output transfer functions. The inputs to the file are the steady-state values of the input voltage V_g , resonant tank parameters (L_r & C_r), switching frequency F_s , output voltage V_o , the output filter capacitor (C_o) and load (R_o).

$L_r=0.1561e-3;$	resonant inductor
$C_r=10.217e-9;$	resonant capacitor
$C_o=32e-6;$	output filter capacitor
$R_o=23;$	load resistance
$V_g=37.5;$	input voltage
$V_o=12;$	output voltage
$F_s=100000;$	switching frequency
$F_o=1/(2*\pi*(L_r*C_r)^{.5});$	resonant frequency
$F_{sn}=F_s/F_o;$	normalized switching frequency
$t_l=R_o*C_o;$	output filter time constant
$t_s=1/F_s;$	switch period
$V_{gn}=1;$	normalized input voltage
$V_{on}=V_o/V_g;$	normalized output voltage
$Z_o=(L_r/C_r)^{.5};$	resonant tank impedance
$Q=Z_o/R_o;$	load quality factor
$\epsilon_l=\pi/F_{sn};$	sum of conduction angles

$i_{base}=V_g/Z_o;$	base current
$J=V_o/R_o/i_{base};$	normalized output current
$VM1n=((1-V_{on}^2*\sin(\epsilon l f/2)^2)/\cos(\epsilon l f/2)^2)^{.5}+V_{on};$	state-plane curve radius during transistor conduction
$VM2n=VM1n-2*V_{on};$	state-plane curve radius during anti-parallel diode conduction
$i_{ln0}=\text{abs}(((1-V_{on}^2)*((VM1n-V_{on})^2-1))^{.5});$	initial normalized resonant inductor current
$v_{cn0}=\text{abs}(V_{on}*(VM1n+1-V_{on}));$	initial normalized resonant capacitor voltage
$\beta=\pi-\text{asin}(i_{ln0}/VM1n);$	transistor conduction angle
$\alpha=\epsilon l f-\beta;$	anti-parallel diode conduction angle
$a_{11}=\cos(\beta)*\cos(\alpha)*VM2n/VM1n-\sin(\alpha)*\sin(\beta);$	matrices coefficients
$a_{21}=\sin(\beta)*\cos(\alpha)+\cos(\beta)*\sin(\alpha)*VM2n/VM1n;$	
$a_{31}=(4*R_o*Cr)/(4*t_l+ts)*(\sin(\beta)*(2-\cos(\alpha))-\cos(\beta)*\sin(\alpha)*VM2n/VM1n);$	
$a_{12}=-(\cos(\beta)*\sin(\alpha)-\sin(\beta)*\cos(\alpha)*VM2n/VM1n);$	
$a_{22}=\cos(\beta)*\cos(\alpha)-\sin(\alpha)*\sin(\beta)*VM2n/VM1n;$	
$a_{32}=(4*R_o*Cr)/(4*t_l+ts)*(2*\cos(\beta)-1-$	
$\cos(\beta)*\cos(\alpha)+\sin(\beta)*\sin(\alpha)*VM2n/VM1n);$	
$a_{13}=\sin(\alpha)*(2-\cos(\beta))-\sin(\beta)*\cos(\alpha)*VM2n/VM1n;$	
$a_{23}=1-2*\cos(\alpha)+\cos(\beta)*\cos(\alpha)-VM2n/VM1n*\sin(\beta)*\sin(\alpha);$	

```

a33=(4*ro*cr)/(4*tl+ts)*(2*cos(beta)-3-2*cos(alpha)-
cos(beta)*cos(alpha)+VM2n/VM1n*sin(alpha)*sin(beta))+(4*tl-ts)/(4*tl+ts);
b11=pi/Fsn^2*VM2n*cos(alpha);
b21=pi/Fsn^2*VM2n*sin(alpha);
b31=(Von/Fo-2*pi*Ro*Cr*VM2n*sin(alpha))*1/Fs^2*2/(4*tl+ts);
b12=cos(beta)*sin(alpha)+cos(alpha)*sin(beta)*VM2n/VM1n;
b22=1-cos(beta)*cos(alpha)+VM2n/VM1n*sin(beta)*sin(alpha);
b32=(1-2*cos(beta)+cos(beta)*cos(alpha)-
VM2n/VM1n*sin(beta)*sin(alpha))*(4*Ro*Cr)/(4*tl+ts);
A=[-a11 a12 -a13; a21 -a22 a23; a31 -a32 a33];          closed form matrices
B=[-b11 -b12; b21 b22; b31 b32];
C=[0 0 1];
D=zeros(1,2);
[num,den]=ss2tf(A,B,C,D,2);
x=elf^2*Q^2*cos(elf/2)^2+4*sin(elf/2)^2;
y=-4*elf*Q*cos(elf/2)^2;
z=4*(cos(elf/2)^2-1);
m1=(-y+sqrt(y^2-4*x*z))/(2*x);          converter DC gain
m2=(-y-sqrt(y^2-4*x*z))/(2*x);
h1=b31;
h2=-b11*a31-b21*a32+b31*(a11+a22);
h3=b11*(a21*a32-a31*a22)+b21*(a12*a31-a11*a32)+b31*(a11*a22-a12*a21);
p1=a11+a22-a33;

```



```

p2=a11*a22-a22*a33-a11*a33+a31*a13+a32*a23-a21*a12;
p3=a11*a23*a32+a21*a12*a33+a31*a13*a22-a21*a32*a13-a12*a23*a31-a11*a22*a33;
nd=[h1 h2 h3];
dd=[1 p1 p2 p3];
q1=b32;
q2=-b12*a31-b22*a32+b32*(a11+a22);
q3=b12*(a21*a32-a31*a22)+b22*(a12*a31-a11*a32)+b32*(a11*a22-a12*a21);
nd2=[q1 q2 q3];
sysd=ss(A,B,C,D,ts);                                discrete-time LTI model
sysc=d2c(sysd,'tustin');                              equivalent continuous time model
sys=tf(sysc)                                           's' domain transfer functions

```

The last three commands in the code are used to convert the discrete small-signal model to the s-domain control-to-output and input-to-output transfer functions.

APPENDIX C: PROTOTYPE PCB LAYOUT

③

36-85-108