1-1-2010

Inventory control in a two-level supply chain with learning, quality and inspection errors

Mehmood Khan
Ryerson University

INVENTORY CONTROL IN A TWO-LEVEL SUPPLY CHAIN WITH LEARNING, QUALITY AND INSPECTION ERRORS

by

MEHMOOD KHAN

Bachelor of Engineering in Mechanical Engineering

NED University of Engineering & Technology, Karachi, Pakistan, 1997

Master of Science in Systems Engineering,

King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia, 2000

A dissertation presented to

RYERSON UNIVERSITY

in partial fulfillment of the

requirements for the degree of

DOCTOR OF PHILOSOPHY

in the program of

MECHANICAL ENGINEERING

Toronto, Ontario, Canada, 2010

Author's Declaration

I hereby declare that I am the sole author of this thesis or dissertation.

I authorize Ryerson University to lend this thesis or dissertation to other institutions or individuals for the purpose of scholarly research.

Mehmood Khan

I further authorize Ryerson University to reproduce this thesis or dissertation by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Mehmood Khan

Inventory Control in a Two-Level Supply Chain with Learning, Quality and Inspection Errors

PhD 2010

Mehmood Khan

Mechanical Engineering

Ryerson University

Abstract:

A common measure of quality for a buyer or a vendor is the defect rate. Defects may represent an attribute, a dimension or a quantity. They may be classified as product quality defects or process quality defects. Product quality defects may be caused by human error which can be due to fatigue, lack of proper training, or other reasons. For example, an inspector may misclassify a defective fuel tank of a car as good. On the other hand, process quality defects maybe caused by a machine going out-of-control.

While many researchers assume that the screening processes which separate the defective items are error-free, it would be realistic to consider misclassification errors in this process. Beside inspection errors, learning is another human factor that brings in enhancement in the overall performance of a supply chain. Learning is inherent when there are workers involved in a repetitive type of production process. Learning and forgetting are even more important in manufacturing environments that emphasize on flexibility where workers are cross-trained to do different tasks and where products have a short life cycle.

Inventory management with learning in quality, inspection and processing time will be the focus of this thesis. A number of models will be developed for a buyer and/or a two level supply chain to incorporate these human factors. The key findings of this work may be summarized as

1. Inspection errors significantly affect the annual profit.
2. An increase in the unit screening cost reduces the annual profit to a great extent at slower rates of learning.
3. For the two-level supply chain we investigated, learning in production drops the annual cost significantly while the learning in supplier’s quality results in a situation where there are no defectives from the suppliers.
4. Type II error may seem to be beneficial for a two level supply chain as the order/lot size goes down and thus affects the costs of ordering, production and screening.
5. Consignment stocking policy performs better than conventional stocking when holding costs go higher than a threshold value.

Acknowledgements

Achieving a remarkable goal like a PhD degree can never be a straightforward task. My gratitude is due to almighty God, that the journey through this demanding avenue has been quite enthusiastic. During the course of about three and a half years, I have been supported and inspired by a number of people. I am grateful to all those who not only mentored and encouraged me but also made my life here in Toronto a pleasant experience.

First of all, I am thankful to my supervisor Professor Mohamad Y. Jaber, without whose persuasion, I may not have joined Ryerson University. His guidance, encouragement, motivation and knowledgeable advice have always kept me on track to finish this tough job. This document in your hand won't have been possible without his generous dedication towards my research. He has always been available to advise and to provide valuable feedback on my work. I am also grateful to the other members of the supervising committee and the external reviewers for their timely response and advice on my research.

Secondly, I am grateful to the Dean of the Faculty of Engineering, Architecture and Science at Ryerson University and the Natural Science and Engineering Research Council of Canada (NSERC) for the financial support required for my research. Needless to say that, this support was vital for me to work with peace of mind. I am also thankful to Dr. Ahmed El-Saadany, who has always been a sincere friend.

Finally, I am bound to thank my wife and my parents who have been a great help and constant support for me. Their patience and understanding all through my work was quite priceless.

Mehmood Khan

Toronto, December 2010

to my parents, my wife and two lovely sons

Table of Contents

List of Tables

List of Figures

Nomenclature

D	=	Number of units demanded per year
n	=	Number of shipments/batches in a production cycle
Q	=	Optimal order size (referred to as Q_i in some models)
c_1	=	Unit variable cost
A_b	=	Buyer's fixed ordering cost
V	=	A parameter used for simplifying the holding cost in Eq. (3.5)
s_1	=	Unit selling price of a nondefective item
v	=	Unit selling price of a defective item
x	=	Screening rate
x_1	=	Initial inspection rate in case of learning in inspection ($x_1 \neq D$)
d	=	Unit screening cost
d_1	=	Screening cost per year
T	=	Cycle length
C_s	=	Total cost of a supplier in a cycle
C_v	=	Total cost of a vendor in a cycle
C_b	=	Total cost of a buyer in a cycle
TCU	=	Total annual cost (TCU (Q) or $TCU(Q_i)$, TCU (Q, n))
TPU	=	Total annual profit (TPU (Q) or $TPU(Q_i)$)
m_1	=	Random variable representing the possibility of committing Type I error (classifying a nondefective item as defective)
m_2	=	Random variable representing the possibility of committing Type II error (classifying a defective item as nondefective)
γ	=	Probability that an item is defective
τ_1	=	Inspection time in a cycle
t_2	=	The remaining time in a cycle, after the defective items are screened out
$f_1(\gamma)$	=	Probability density function of γ
f_2 (m_1)	=	Probability density function of m_1
$f_3(m_2)$	=	Probability density function of m_2.
B_1	=	Number of items that are classified as defective in one cycle

B_2	=	Number of defective items that are returned from the market in one cycle
c_a	=	Cost of accepting a defective item
c_r	=	Cost of rejecting a nondefective item
i	=	An index to represent a cycle of learning
b	=	Learning exponent
β_i	=	Forgetting exponent in the i^{th} learning cycle
L	=	Time for total forgetting to occur
u_i	=	Experience of screening remembered from the previous $i-1$ cycles
Z_i	=	Lost sales quantity in the i^{th} cycle
B_i	=	Backorder quantity in the i^{th} cycle
t_{si}	=	Time at which the screening rate is equal to the demand in the i^{th} cycle
t_{Bi}	=	Time to inspect B_i and Dt_{Bi} units in the i^{th} cycle
x_{si}	=	Inspection rate by time t_{si} in the i^{th} cycle
Q_{si}	=	Units inspected and consumed by t_{si}, where $x_{si} = x_1 (x_{si} + u_i)^b$
Q_{Bi}	=	Units inspected and consumed by t_{Bi} in the i^{th} cycle
$TP_{iB}(Q_i)$	=	Total profit in a cycle of learning in screening in case of backorders
$TP_{iL}(Q_i)$	=	Total profit in a cycle of learning in screening in case of lost sales
T_{iB}	=	Length of a cycle of learning in screening in case of backorders
T_{iL}	=	Length a cycle of learning in screening in case of lost sales
$I(t)$	=	Inventory level at time t
c_L	=	Cost of lost sales per unit
c_B	=	Cost of backorders per unit per unit time
s	=	A notation representing suppliers, $s = 1, \ldots, m$
K_s	=	Integer multiplier for a coordination mechanism in chapter 5
μ_s	=	Number of parts from supplier s required for a product
γ_s	=	Percentage of defective parts supplied by supplier s
γ_{max}	=	The highest percentage of defectives in suppliers' items
π_s	=	The fraction used to accommodate the leftovers before ordering parts of type s, in a cycle $(1-(\gamma_{\max}-\gamma_s))$
D_s	=	Demand for the supplier s ($D_s = D\mu_s$)

Z_{1s}	=	Inventory level of raw material from supplier s, after finishing its screening
Z_{2s}	=	Inventory level of raw material from supplier s, after screening out the defectives
P	=	Vendor's initial production rate
c	=	Vendor's production cost per unit time
T_1	=	Vendor's time to produce the first unit, in case of learning (=1/P)
Q_s	=	Number of nondefective items provided by suppliers s; $Q_s = Q\mu_s(1 - \gamma_s)$
z	=	Minimum number of items that can be produced in a production cycle; $z = \mathrm{Min}(\mathrm{Int}(Q\,(1 - \gamma_s)))$
l_s	=	Number of unused non-defective parts of type s in a cycle; $l_s = Q_s - z\mu_s$
A_v	=	Vendor's fixed ordering/setup cost
$a_{v,s}$	=	Vendor's variable cost of ordering an item from supplier s
c_f	=	Vendor's cost per unit of making a defective product
A_s	=	Suppliers' setup cost
M_s	=	Percentage of defective parts supplied by supplier s, accommodating the inspection errors at the vendor's end $\{= (1-\gamma_s)e_1 + \gamma_s(1-e_2)\}$
$M_{\max}$	=	The highest percentage of defectives in suppliers' items, accommodating the inspection errors at the vendor's end
π_{se}	=	The fraction used to accommodate the leftovers before ordering parts of type s, in a cycle, accommodating inspection errors $\{= 1 - (M_{\max} - M_s)\}$
h_b	=	Buyer's unit holding cost (non financial component in *CS* policy)
h_v	=	Vendor's unit holding cost (financial and non financial components)
h'_v	=	Vendor's unit holding cost (only financial component)
h_{v1}	=	Vendor's unit holding cost for raw material
h_{v2}	=	Vendor's unit holding cost for finished products
h_s	=	Suppliers' unit holding cost
γ_e	=	Percentage of defective products observed by the buyer through screening $= (1-\gamma)m_1 + \gamma(1-m_2)$
M_1	=	A term representing the expected value of percentage of defectives, = E[γ],

($M_{1\mathrm{e}}$ in case of inspection error)

M_2 = A term involving the expected value of percentage of defectives, ($M_{2\mathrm{e}}$ in case of inspection error) $= 1/(1 - \mathrm{E}[\gamma])$

CHAPTER 1 BACKGROUND OF THE RESEARCH

1.1 Introduction

The use of the economic/production quantity (EOQ/EPQ) model has been quite popular among researchers and industries for the last hundred years (Simpson, 2001). This model is essentially a summary of ordering/setup and holding/carrying costs in a vendor/buyer setup/order that presents an economic order/lot size by balancing or trading off these costs. Although this model has been widely cited, accepted and utilized, it has several weaknesses (e.g., Jaber *et al.*, 2004). The inherent idealistic assumptions claim there is a perfectly steady demand known with certainty and all the items received from the suppliers are of a perfect quality (e.g., Jaber, 2006*b*). These assumptions initiated a huge arena of research for many in the industry and academics. The result was a vast literature that studies the basic EOQ/EPQ model under real life situations, e.g., Porteus (1986*b*), Rosenblatt and Lee (1986) and Silver (1976).

Another challenging aspect of the above model is ignoring the role of human factors like inspection errors, fatigue and learning (both in production and quality). These factors have never been modeled in the context of supply chain management though they play a vital role in measuring the performance of a supply chain. That is, although the impact of learning has been studied in production (Salameh *et al.*, 1993 and Jaber and Bonney, 1999), its combined effect on screening and production in a two-level supply chain has never been studied before. Besides, an effective depiction of these human factors helps to ensure quality in a global supply chain where the cost of a defective item is relatively higher.

Human errors in inspection are found in the literature for single sampling and repeat inspection plans. These errors can be fatal in the case of some critical components, for example, parts of an aircraft or a complex gas ignition system. Repeating the inspection process is believed to reduce the effect of human error at a nominal increase in the inspection cost (Swain, 1970). The other prominent literature in the prevailing line of research is Bennett *et al*. (1974) for single sampling plan (Raouf *et al*., 1983) for repeated inspections for multicharacteristic components, and Duffuaa and Khan (2002) for repeated inspections with general classifications.

Another interesting human aspect that is important in the area of inventory management is learning. This aspect is not new in determining the optimal lot sizes (Jaber and Bonney, 1999), but it has yet to gain its due place in the supply chain models (Jaber *et al*., 2010). Learning is inherent when there are workers involved in a repetitive type of production process. The learning process affects production time, product quality and the inspection errors too, with the passage of time. A learning process is described by a power curve suggested by Wright (1936). The first who investigated the lot sizing problem with learning and forgetting effects is Keachie and Fontana (1966), who considered a simplistic forgetting relationship. Adler and Nanda (1974) developed two models of optimal lot sizes with learning. The first one was restricted to the equal lot sizes while the second one was restricted to equal production intervals. The other studies, but not limited to, in this line of research are surveyed in Jaber and Bonney (1999). While learning is significant in production, it is also important in improving the quality of a product with the passage of time. It is again inherent that human beings tend to become more and more accustomed to the processes, thus resulting in better quality of the product. Recently, Jaber et al. (2008) presented a model for the case where the quality of supplier's items improves following a learning curve. They based their study on the data collected from an automotive industry. Learning curve has been referred to in the literature by different names. For example, the 'Experience curve' is a similar term which explains an improvement over *n* cycles (Dar-El, 2000) but is more commonly used to show a decrease in the cost of performing a task with experience. In this thesis, the improvement in learning is time based and the term "learning curve" will be used throughout.

Salameh and Jaber (2000) exposed a new course of research to the field of inventory and logistics management that ensures quality of the suppliers' items. This model has been getting more and more attention as it touched upon a vital limitation to the earlier literature. It has been widely extended to address the issues of shortages/backorders, quality, fuzziness input parameters (e.g., demand), and joint lot sizing in two-level supply chains. Few of these extensions are Goyal and Cárdenas-Barrón (2002), Goyal *et al*. (2003), Wang (2005), Papachristos and Konstantaras (2006), Wee *et al*. (2007), Eroglu and Ozdemir (2007), Konstantaras *et al*. (2007), and Maddah and Jaber (2008*b*).

Goyal and Gupta (1989) suggested that coordination between a vendor and a buyer (i.e., integrated inventory models) can be attained through joint replenishment policies. Other types of coordination mechanisms have been used in supply chain literature such as quantity discount (Munson and Rosenblatt, 2001), buy-back and revenue sharing contracts (Cachon, 2003), common replenishment epochs (Viswanathan and Piplani, 2001), permissible delay in payments, (Jaber and Osman, 2006). Readers may refer to the works of Sarmah *et al*. (2006), Ben-Daya *et al*. (2008) and Jaber and Zolfaghari (2008) for reviews. Coordination may involve several decision makers in supply chains. These decision makers may belong to different firms and thus may have conflicting objectives. This is the case of decentralized decision making process. This thesis assumes that the decision making process is centralized where the players in a two-level supply chain (vendor-buyer) belong to one firm.

With this formal introduction to the various issues in supply chains, this thesis will aim at studying these issues in isolated and integrated models in a two-level supply chain. The objective of the work would be to extend the model of Salameh and Jaber (2000) to present a realistic approach of studying supply chain inventory models. A brief review of the various topics covered in the thesis is presented in the following sections. The model of Salameh and Jaber (2000) will be explained at the end of this chapter.

1.2 Supply Chains

In today's competitive markets, there is an increase in the willingness on the part of a vendor to pay close attention to the design and assembly processes of its suppliers, to ensure certain level of quality. They have also become keen in accomplishing the needs of their end-consumer. The more they become involved with other stakeholders downstream or upstream in the supply chain, the more are the overall benefits to the supply chain.

Some supply chains also involve a number of other companies that play a very important role in providing information (upstream) or products (downstream). These companies can be the providers of service, warehouses, trucking, shipping or just information systems. With this background, a supply chain can be defined as the alignment of firms that bring products or services to market (Lambart *et al*., 1998). A supply chain can be classified on the basis of the number of stakeholders, their relationships, or coordination mechanisms (Stadtler and Kilger, 2008).

The focus in this thesis would be on a two-level (2 stakeholders) supply chain. A number of supplier-vendor and a vendor-buyer supply chain in models will be considered in the thesis. The number of stakeholders in most of the supply chain literature has been two, three or four. A typical four level (or tier) supply chain would be composed of a supplier, a vendor, a distributer and a buyer. A number of coordination mechanisms will be illustrated for these supply chains to compare their performance. An internal supply chain at the vendor's site and its incorporation into the two level supply chain will be left for future research.

1.2.1 *Coordination in a Supply Chain*

Responsibilities in a firm are usually divided among different departments such as engineering, purchasing, marketing and logistics (Mentzer, 1993). An inter-functional or inter-firm coordination in itself is not enough to manage a supply chain. Effective coordination is associated by an increased contact with other departments and firms, through information flows (Urban and Hauser, 1993). Several strategies are used to align the business processes and activities of the members of a supply chain to ensure better coordination (Sarmah *et al.*, 2006). These strategies tend to improve the performance in terms of cost or response time. There is not a single coordination strategy effective for all supply chains. As noted earlier, various coordination mechanisms have been used in supply chain coordination literature such as buy back or return policy, option to credit, quantity discount, or delay in payments.

A number of researchers have illustrated the coordination between a single vendor and a single buyer. For example, Goyal (1977), Banerjee (1986*a*), Goyal (1988), Goyal and Gupta (1989), Lu (1995) and Goyal (1995). Hill (1997) described a general inventory policy for the coordination between a single vendor and a single buyer. The behavior of a vendor's inventory with this approach is shown in Figure 1.1 below.

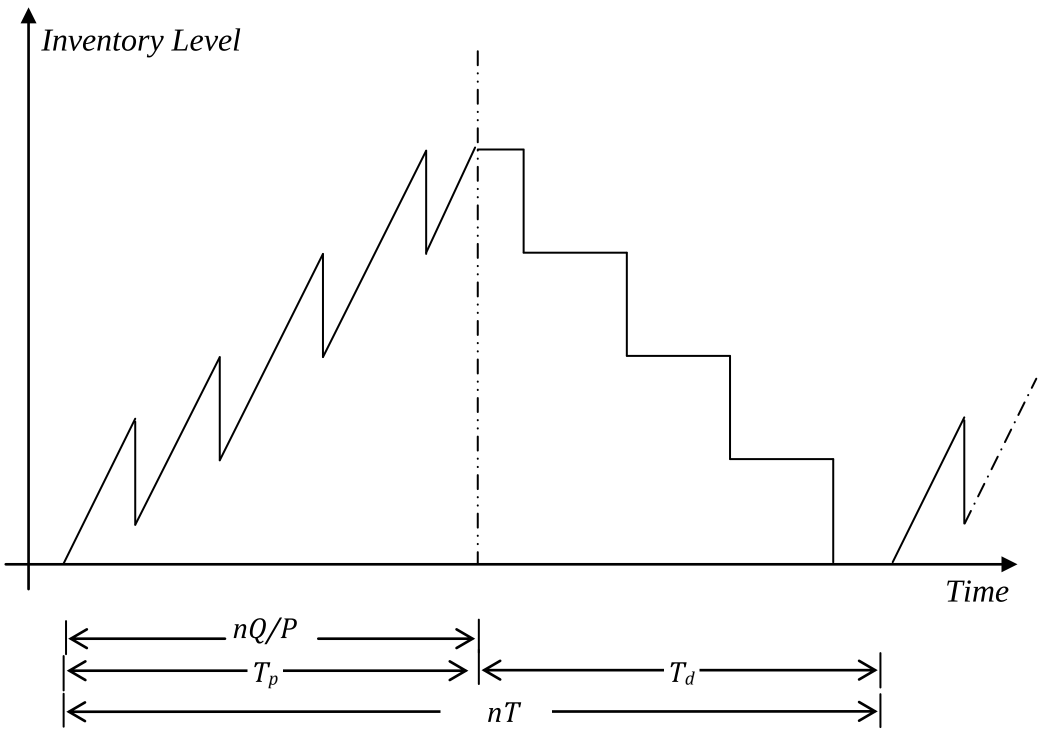

Figure 1.1 Behavior of a vendor’s inventory (similar to Hill's (1997) model)

Another single vendor and single buyer setup that is applied in practice, is Consignment Stock (CS) (Braglia and Zavanella, 2003). Although an old practice, (Kisner *et al*., 1975) CS started gaining attention in recent years. It requires a continuous exchange of information between the two parties. The utility of this approach comes from the fact that leads to a reduction in the vendor’s inventory costs, as this party will use the buyer’s facility or warehouse to stock its material, usually because it is cheaper. This warehouse is assumed to be close to the buyer’s production line so that the material may be picked up when needed. Besides, it would be the vendor’s responsibility to ensure that no stock-out situation will occur. The buyer will take from the store the quantity of material necessary to cover the production planned. The continuous exchange of this information keeps the vendor aware of the consumption rate. The behavior of a vendor’s inventory with this approach is shown in Figure 1.2.

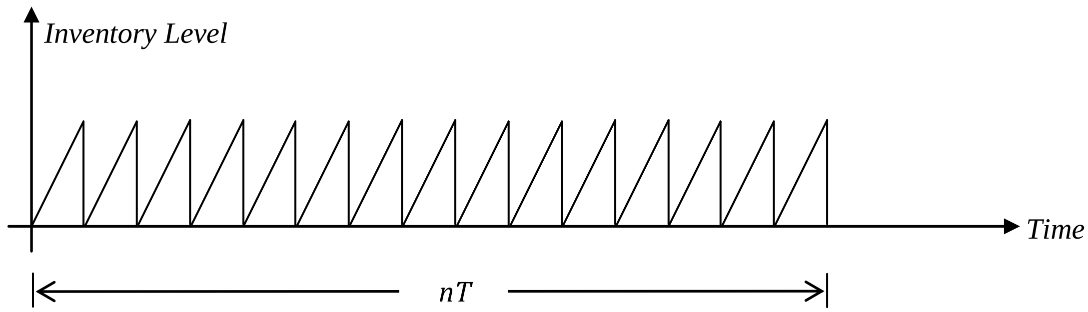

Figure 1.2 Behavior of a vendor's inventory in Braglia and Zavanella (2003) model

Khouja (2003) discussed three different coordination schemes for a three level supply chain comprising of a supplier, a vendor and a buyer. He developed closed form solutions for the optimal cycle time for the three coordination mechanisms. In the first mechanism, they assumed that the cycle time used throughout the chain is the same for all its stakeholders; in the second mechanism, each stakeholder has a cycle time which is an integer multiplier of that of the adjacent downstream stage; while in the third mechanism, each stakeholder has a cycle time which is an integer-power-of-two multiplier of that of the adjacent downstream stage.

Two types of supply chain coordination schemes will be adopted in this thesis. That is, variables of interest will be evaluated as a single decision maker for the whole supply chain. In other words, their annual cost and profit would be evaluated through a joint decision. Coordination may result in one or more player benefiting more than the others in the chain. These players will compensate the loosing ones. To understand this point, consider a simple vendor-buyer supply chain for a single product. The total annual costs of the two stakeholders are given by

$$TCU_b(Q) = \frac{A_b D}{Q} + \frac{h_b Q}{2} \tag{1.1}$$

$$TCU_v(Q, n) = \frac{A_v D}{nQ} + \frac{h_v Q(n-1)}{2} \tag{1.2}$$

Assume that $D = 1{,}000\ per\ year$, $A_b = \$25$, $A_v = \$400$, $h_b = \$5\ per\ unit\ per\ year$, $h_v = \$4\ per\ unit\ per\ year$. In case of no coordination, the optimal order size Q is determined by solving Eq. (1.1). Eq. (1.2) is solved for the optimal number of shipments n with this Q. This way the values of Q and n would be 100 and 4 respectively while the total cost of the supply chain ends up being \$2100. The annual costs of the buyer and the vendor in this case are \$500 and \$1600 respectively. On the other hand, with coordination, the sum of Eqn. (1.1) and (1.2) will be solved for Q in terms of n and an optimal n will be determined through iteration. Following this, the optimal Q and n would be 224 and 2 respectively with an annual cost of supply chain being \$2013. The share of the buyer and the vendor in this cost is \$671 and \$1342 respectively. It should be noticed that although the annual cost of the supply chain does not change noticeably but the vendor benefits from coordination while the buyer does not. That is why the vendor will have to compensate the buyer. This compensation may be in the form of lump payments or some sort of discount (e.g., quantity discounts).

Firstly, we will study different human factors with the first two coordination mechanisms given by Khouja (2003) in a supplier-vendor supply chain. Then we will compare the two coordination schemes given by Hill (1997) and Braglia and Zavanella (2003) respectively, for a vendor-buyer supply chain.

1.3 Quality

There are many ways to define quality. A common definition in the industry is “meeting or exceeding customer expectations” (Sontrop and MacKenzie, 1995). Many companies refine their processes/products to meet the customer expectations based on their surveys. The customers can be internal or external. For example, the internal customers for a fuel tank would be an assembly line or the paint shop while its external customers would be a car dealer or the purchaser.

The definition of quality emphasized in this thesis is “conformity to specifications”. Specifications are target values and tolerances such as the length of trunk lid can be 150 ± 1 cm. That is, a conforming length falls in the interval 149 to 151 cm. Targets and tolerances are set by the design and manufacturing engineers in a plant. The other characteristics of interest can be design configuration like weight, thickness, reliability and ease of fitness.

The most successful organizations today have learned that statistical process control (SPC) only works when the operating philosophy is that everyone in the organization is responsible for and committed to quality. SPC focuses on the methods by which results are generated – on improvements to the processes that create products and services of the least variability. The traditional tools that SPC uses to improve on variability are (*i*) flow charts, (*ii*) cause-and-effect diagrams, (*iii*) pareto charts, (*iv*) histograms, (*v*) scatter plots and (*vi*) control charts (Sontrop and MacKenzie, 1995).

Quality starts at the design stage (Hollins, 1995). A designer takes information from the customer (market) to define what the customer wants, needs, and expects from a particular product. These requirements are then translated into specifications and tolerances. The production department uses this information, along with the prescribed machinery, to fabricate the product. The product is then delivered via marketing channels to the customer, after passing through quality checks. To satisfy the customer, the product must perform the specified or associated functions. Thus, quality can also be demonstrated through a happy customer. Customer feedback to the designers and makers comes in terms of the number of products sold and the warranty, repair, and complaint rate. Increasing sales volume and market share with low warranty, repair, or complaint rates indicates high quality (i.e. happy customers).

1.3.1 *Fraction of Defectives as Quality*

With this notion of quality or quality control in mind, the industry today refers to the fraction of defective items in their production or in the supplier-lots as the "quality" of the lot produced or received. A defective unit denotes that a part or a product (assembly of parts) is unfit for use (Sontrop and MacKenzie 1995). The in-built defectiveness is an imperfection in the whole product or one or more parts. A common practice in the industry to describe this statistic is to use a *p*-chart. This chart tracks the proportion of defectives in a collection of units taken from shipments or batches. The utility of the chart is that it is based on a pass/fail test that can be applied to a single characteristic or multiple characteristics in a part and/or a product. Typically, a single point on this chart represents the outcome for all the parts/products received in a single batch/shipment.

This thesis will explore the relationship of this quality with errors in screening and learning in production and screening process. A formal understanding of inspection errors and learning process, will be developed later in this chapter.

1.3.2 *Inspections Errors*

Inspection means comparing or determining the conformance of products to established specifications. Inspection tasks may be classified into three basic categories; (*a*) tasks involving visual scanning, (*b*) tasks involving measurements, and (*c*) tasks involving monitoring of a process. Although there have been advancements in technology there are still wastes generated by errors in screening. In addition, ignoring the presence of inspection error can severely distort the performance measures of any inspection activity. It is not unusual to find inspection error rates of 25% or 30% in complex inspection activities (Jaraiedi *et al*., (1987).

The inspection accuracy is influenced by a number of factors. These factors can be categorized into three groups:

1. Inspector related factors, such as the age, experience, sex, visual activity, intelligence, level of training, and psychological factors.

2. Task related factors, such as task pacing, task perception, task complexity, design of work place and rate of defects.

3. Environmental and organizational factors, such as illumination, noise, temperature, humidity, motivation and incentives.

Inspectors make two types of errors. That is Type I error, in which a conforming quality characteristic is classified as nonconforming; and Type II error, in which a nonconforming characteristic is passed as conforming. This is illustrated in Figure 1.3 below. An inspector screens a lot with γ percent defectives and the probabilities that he will commit as Type I and Type II errors are m_1 and m_2, respectively.

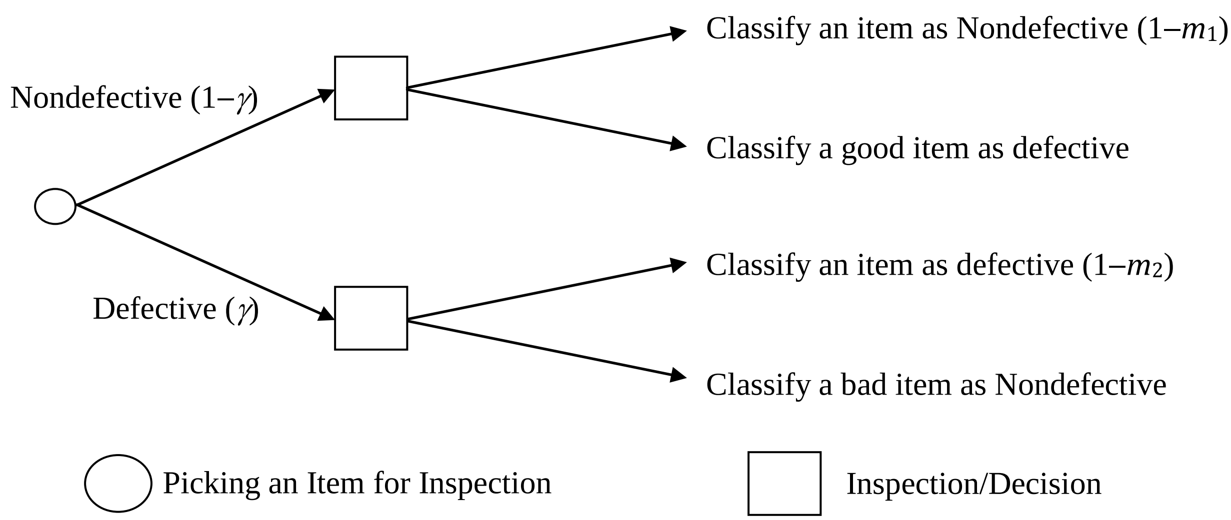

Figure 1.3 Four possibilities in an inspection process

1.3.3 *Quality Investment*

There are a number of articles in the literature that highlight the importance of investment in reducing the fraction of defectives or in other words, improving the quality, for example, Lee (2005). In the context of a supply chain, the decision maker has to find a trade-off between this investment and the savings by the reduction of the defective items. The vendors learn to select optimal investment levels by developing cost/benefit models for different investment strategies (Lee, 2005). This optimal investment affects the management performance of a vendor or a supply chain.

1.4 Learning

Learning in an organization has been receiving more and more attention. Steven (1999) presented examples from electronics, construction and aerospace to conclude that learning curves will gain more interest in high technology systems. Wright (1936) was the first to model the learning relationship in an industrial setting. This complex behavior has had different names; such as start-up curves, (Baloff, 1970), progress functions, (Glover,1965), and improvement curves, (Steedman, 1970). But researchers have agreed that the power-form learning curve are the most widely used to depict the learning phenomenon, (Yelle,1979; Jaber, 2006*b*).

It is very hard to define this complex behavior. But practitioners and researchers mostly believe that it is the trend of improvement in performance achieved by virtue of practice. The Wright (1936) learning curve states that time to produce every successive unit in repetition keeps on decreasing till plateauing occurs. Plateauing is a state where a system or a worker ceases to improve in his performance. The reason for this could be the worker ceasing to learn or the unwillingness of the organization to invest any more capital. The mathematical form of Wright's model is given by

$$T_x = T_1 x^{-b} \tag{1.3}$$

where x is the tally of the unit being produced, T_x and T_1 are the times to produce the x^{th} and the first unit respectively, and b is the learning exponent. This model is illustrated in Figure 1.4. The learning exponent in this expression is often referred to as an index called learning rate (LR).

Learning occurs each time the production quantity is doubled; such as

$$LR = \frac{T_{2x}}{T_{1x}} = \frac{T_1(2x)^{-b}}{T_1(x)^{-b}} = 2^{-b} \tag{1.4}$$

To illustrate, if T_1=10 minutes, b = 0.3218 (LR=80%), then assembling the second unit would take T_2 = 10×$2^{-0.3219}$ = (0.8)(10) = 8 minutes, the fourth unit would take T_4 = 10×$4^{-0.3219}$ (0.8)(8) = 6.4 minutes, and so on. Thus the Wright's learning curve can be drawn as Figure 1.4.

Thus, following the above learning curve, time to produce x units, $t\,(x)$, is given by

$$t(x) = \sum_{i=1}^{x} T_1 i^{-b} = \int_{i=0}^{x} T_1\, i^{-b} di = \frac{T_1}{1-b} x^{1-b} \tag{1.5}$$

Note that (1.4) is valid when $0 \le b < 1$, which what has been observed in practice (Dutton and Thomson, 1985; Dar-El, 2000). Extreme cases where $b \geq 1$ (Jaber and Guiffrida, 2007) are not discussed in this thesis.

Although Figure 1.4 and the above expression represent the improvement in the time to process a unit, learning can be shown in the cost, productivity and other similar measures of a production system. The outcome of applying learning theory is that a firm can expect continuous improvements of its productivity ratio as a consequence of increasing its experience or stock of knowledge (Dar-El,2000). Besides, a good learning may be required for determining labor costing, manpower policies, time standards and the optimal cycle times for assembly of a product.

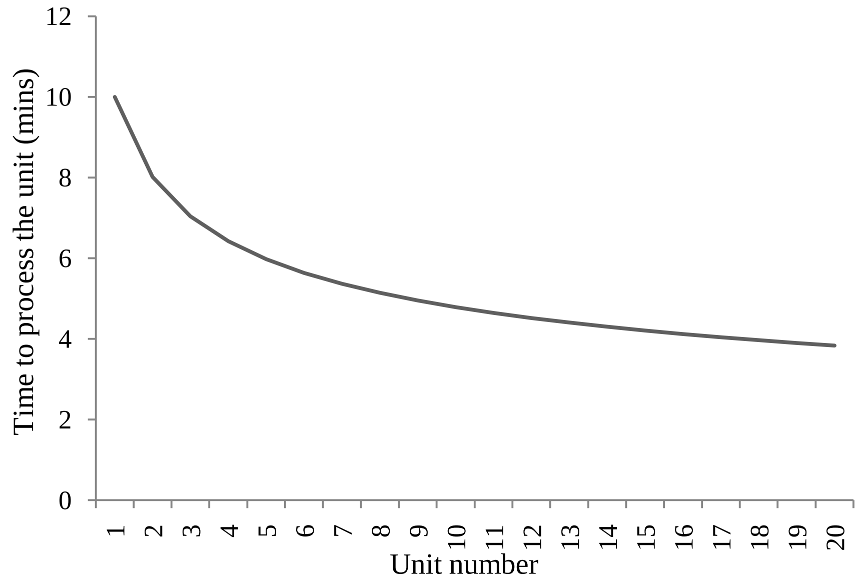

Figure 1.4 Wright's learning model

Learning can be categorized into four groups; (*a*) individual learning that includes factors as forgetting and relearning, (*b*) product learning where individual learning is integrated with all aspects of a product improvement, (*c*) learning in design and development where learning introduces rapid changes and (*d*) organizational learning that incorporates almost all the functions of improvement in an organization (Dar-El, 2000). The level of learning attained by a worker in a repetitive task is governed by a number of factors. These factors are previous experience, training, motivation, job complexity, number of repetitions, length of the task, errors and forgetting (e.g. Jaber, 2006*b*). Quantifying these factors would help in a better prediction of the learning rate for a worker.

It should be noticed that there are some limitations to the use of learning curves (Rea and Kerzner, 1997): (*i*) the learning curve plateaus after some cycles, (*ii*) an inspector may have different learning rate for different products or processes, (*iii*) the results of a learning curve may be overestimated if the loss of learning (forgetting) is ignored, (*iv*) the benefits of learning are more visible in long-term horizons.

1.5 Forgetting

Most of the literature on learning assumes continuous learning though interruptions occur regularly and frequently. This brings in another related notion into light which is forgetting. That is the loss of skill or knowledge because of interruption (a production break). The effects of forgetting are important where estimates of man-hours, production schedules and manpower are required. A distinction is made between procedural and psychomotor tasks in this context (Dar-El, 2000). Procedural tasks consist of a series of discrete motor tasks as found in most industrial activities. Psychomotor tasks involve repetitive movements without a clear beginning or end - such as riding a bicycle. It is obvious that the later exhibits lesser forgetting. Besides, the time required to teach a psychomotor task is longer and as a result they are retained better. Another fact that affects forgetting is the level of learning before the break. That is, if a worker is highly skilled at an activity for a long time, his performance would not be affected by even a long interruption. Another distinction can be made between automatic and controlled tasks. Automatic skills are related to quick and unconscious activities while controlled skills are the systematic response to failure of a system. Fisk et al. (1987) found that there is a large decline with the performance of controlled tasks after a non-practice of one year, whereas automatic tasks have good retention even after a year. The tendency in the industry is to automate mechanical tasks as much as possible and to utilize people for control and decision-making purposes. A lot more research is yet to come for a better understanding of the relationship between the break length and the forgetting process.

The loss of learning (forgetting) in screening is studied in chapter 4. The forgetting curve (Jaber and Bonney,1996), is usually taken to be a mirror image of the learning curve. To determine the forgetting exponent β_i in cycle i, it is customary to equate the learning and forgetting time at the instant a worker has inspected Q_i units. This determines the value of the intercept of the forgetting curve $\widehat{\pi_1}$, where the '^' represents mirror image. That is, the forgetting curve takes the form

$$\widehat{\pi_m} = \widehat{\pi_1} m^{\beta} \tag{1.6}$$

where $\widehat{\pi_1} = \pi_1 m^{-(\beta+b)}$and $\widehat{\pi_m} = \pi_m$, where *m* is the equivalent number of items that could have been screened. The forgetting exponent in cycle *i* is determined by Jaber and Bonney (1996) as

$$\beta_i = \frac{b(1-b)\log(u_i + y_i)}{\log(1 + {}^{L}\!/_{\lambda_i})} \tag{1.7}$$

where L is the time for total forgetting to occur and is assumed to be an input parameter, u_i is the experience remembered in cycle i, log is logarithm base 10, and λ_i is the time to inspect ($u_i + y_i$) items without interruption.

So, from Jaber and Bonney (1996): $\lambda_i = \frac{(u_i+Q_i)^{1-b}}{x_1(1-b)}$, $u_i = (u_i + Q_i)^{\frac{(\beta_i+b)}{b}} R_i^{\frac{-\beta_i}{b}}$

with

$$R_i = \left[x_1(1-b)(T-\tau_1)(u_i+Q_i)^{(1-b)}\right]^{\frac{1}{(1-b)}} \tag{1.8}$$

where R_i is the equivalent number of items that could have been screened if no interruption occurs of length τ_i, and τ_i is the screening time in cycle i. In case of breaks in screening, one should note that $0 < u_i < \sum_{j=1}^{i-1} Q_j$ when $T - \tau_i < L$, for partial forgetting; $u_i = 0$ when $T - \tau_i \geq L$, for total forgetting; and $u_i = \sum_{j=1}^{i-1} Q_j$ when L becomes infinite, for total transfer of learning. Detailed derivations of Eqs. (1.6) – (1.7) can be found in Jaber and Bonney (1996). Eq. (1.5) will be used to determine the equivalent experience remembered at the start of each cycle, in case of partial and total transfer of learning. For a relationship between the forgetting slope (β_i) and learning slope (b), Jaber and Kher (2002, p.241–242) have shown that the forgetting slope is a concave function over the interval of learning slope from zero to one ($\beta_i > 0$, $\forall\ 0 \leq b < 1$) with a unique maximum at learning slope = 0.5.

1.6 Why Learning and Quality Together?

Most of the researchers and practitioners using learning curves have assumed that all the units produced are of perfect quality requirements (Jaber, 2006*b*). However, most of the manufacturing processes end up in a fraction of defective units which are either reworked or scrapped (e.g., Jaber and Khan, 2010). Thus, it is significant to address these two issues together for a production facility though a little attention has been paid to this area. For example, Dance and Jarvis (1992*a*) studied the importance of learning curves in the semiconductor industry. Besides, Jaber and Guiffrida (2004) extended Wright (1936) learning curve by incorporating quality. They discussed an imperfect production in which defective items can be reworked and proposed a composite learning curve, which is the sum of two learning curves, i.e. learning in production and reworks. However, their quality learning curve had some limitations. That is, their model (a) did not apply to cases where defective items are discarded or scrapped, (b) assumed the rate of generating defective items is constant, and (c) was for a single stage and a single cycle production facility only. They tackled this third assumption in their later work (Jaber and Guiffrida, 2008) by interrupting a production process to restore its quality (Khouja, 2005).

1.7 Salameh and Jaber's (2000) Model

Porteus (1986) and Lee and Rosenblatt (1987) had assumed that all defective items are reworkable. The work of Salameh and Jaber (2000) is the first model that treats imperfect quality items (not defectives) to be salvaged at a discounted price. They implied that imperfect quality items are functional items that fo not meet the quality requirements for a given product/task; however, they do for a lower grade one. Salameh and Jaber (2000) extended the traditional EOQ model by assuming that each lot received from a vendor contains imperfect quality items. They assumed that (i) the demand is deterministic and that it occurs parallel to the inspection process and is fulfilled from goods found to be perfect by the inspection process, (ii) the orders are replenished instantaneously, (iii) there are no shortages, (iv) the lot contains a fixed fraction γ of imperfect items with known probability density function, (v) a 100% screening is performed to separate these defective items, and (vi) items of poor quality are kept in stock and sold prior to receiving the next shipment as a single batch at a discounted price. The behavior of inventory is as described in Figure 1.5.

Note that the behavior in Figure 1.5 is an average one. If the model is simulated for different values of γ, the average ending inventory by time T is either positive or negative. The average access inventory can be assumed to be zero. A simulation study was conducted to verify this with ten thousand runs with parameters: Q=100, D=10, $\gamma \sim U\ (0,\ 0.2)$ and $\gamma \sim U(0,0.5)$. In other words, access ending inventory in one cycle would be used to cover for a stock out in a subsequent cycle. This behavior incurs additional costs (extra holding and shortage costs). It is assumed that the value of h in the work of Salameh and Jaber (2000) and this thesis is significant enough to account for these additional costs. A detailed study of this issue may be addressed in a technical note or short communication. This will be left for a future work.

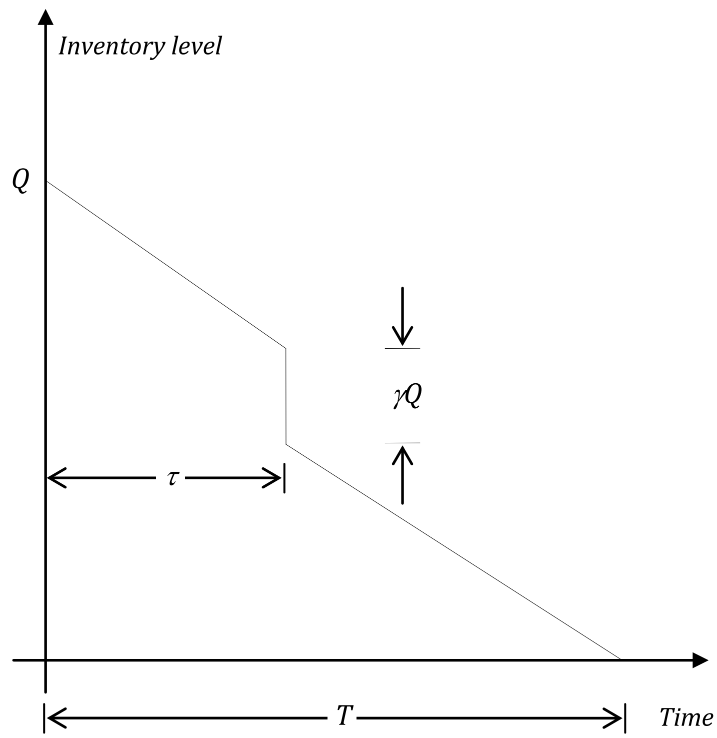

Figure 1.5 Behavior of inventory in Salameh and Jaber (2000) model

where Q is the lot size γ is the percentage of imperfection in Q, τ is the inspection time and T is the cycle time. The screening and cycle time are shown by τ and T respectively. To guarantee there are no shortages, Salameh and Jaber (2000) set the condition $\gamma = 1 - D/x$ where x is the screening rate ($x > D$) The number of good items in each order of size Q, is

$$N(Q,\gamma) = Q - \gamma Q = (1-\gamma)Q \tag{1.9}$$

The total revenue per cycle is given by the sum of the revenue from selling defective and nondefective items, as

$$TR(Q) = s_1 Q(1-\gamma) + vQ\gamma \tag{1.10}$$

where s_1 and v are the unit selling price of a good item and the unit salvage price of an imperfect quality item.

The total cost per cycle is the sum of ordering, purchasing, screening and holding costs. It is given by

$$TC(Q) = A_b + c_1 Q + dQ + h_b\left(\frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right) \tag{1.11}$$

where A_b is the buyer's order cost, c_1 is the unit purchase cost, d is the unit screening cost, and h is the unit holding cost. Detailed derivations of Eqs. (1.9) and (1.10) are provided in Salameh and Jaber (2000). So, the total profit per cycle would be

$$TP(Q) = TR(Q) - TC(Q) \tag{1.12}$$

Total profit per unit time is

$TPU(Q) = \frac{TP(Q)}{T}$ where

$T = \frac{(1-\gamma)Q}{D}$. So,

$$TPU(Q) = D\left(s_1 - v + \frac{h_b Q}{x}\right) + D\left(v - \frac{h_b Q}{x} - c_1 - d - \frac{A_b}{Q}\right)\left(\frac{1}{1-\gamma}\right) - \frac{h_b Q(1-\gamma)}{2} \tag{1.13}$$

Taking the fraction of defectives as a random variable, the expected annual profit would then be

$$\mathrm{E}[TPU(Q)] = D\left(s_1 - v + \frac{h_b Q}{x}\right) + D\left(v - \frac{h_b Q}{x} - c_1 - d - \frac{A_b}{Q}\right)\mathrm{E}\left[\frac{1}{1-\gamma}\right] - \frac{h_b Q(1-\mathrm{E}[\gamma])}{2} \tag{1.14}$$

Refer to Salameh and Jaber (2000) for detailed derivations of $TPU(Q)$ and $\mathrm{E}[TPU(Q)]$.

To find the optimal value of the above profit, they differentiated the above equation w.r.t Q and equated that to zero. The second derivative of the above equation remains negative for all values Q which implies that there exists a unique value Q^* that maximizes the above profit. That value would be

$$Q^* = \sqrt{\frac{2A_b D\mathrm{E}[1/(1-\gamma)]}{h_b[1-\mathrm{E}[\gamma]-2D(1-\mathrm{E}[1/(1-\gamma)])/x]}} \tag{1.15}$$

Note that when γ=0, the denominator reduces to h and the numerator reduces to $2A_b D$.

It should be noted that the above equation reduces to the traditional EOQ model when percentage defective γ is zero. They concluded that economic lot size quantity tends to increase as the average percentage of imperfect quality items increase. The number "2" in the denominator was missing in the above equation in Salameh and Jaber (2000) which was pointed out by Cárdenas-Barrón (2000). For a detailed derivation of the above model, refer to Salameh and Jaber (2000), page 61. This model can be referred to as a base EOQ model for imperfect items. This model was extended in a number of directions. Few of these extended models are outlined here: Goyal *et al*. (2003) used the base model to develop a two level supply chain model for imperfect items, Chan *et al*. (2003) introduced a number of quality classifications in the base model, Papachristos and Konstantaras (2006) developed the sufficient conditions for the base model and that of Chan *et al*. (2003), Wee *et al*. (2007) incorporated backordering, and Maddah and Jaber (2008) suggested using renewal reward theorem to estimate the expected annual profit.

Goyal and Cárdenas-Barrón (2002) simplified the above base model by ignoring the screening and purchasing costs. They showed that this simplification results in almost zero penalty. Maddah and Jaber (2008) used this simplified and corrected a flaw in the base model. They noticed that the annual profit in the base model is not exact as the cycle profit is a renewal process. They suggested using renewal reward theory to compute it. That is, the annual profit should be calculated as a ratio of profit per cycle and the cycle time, in the presence of imperfect items in the lot being screened. So, the expected time unit profit function is written as

$$\begin{aligned}\mathrm{E}[TPU(Q)] &= \frac{\mathrm{E}[TP(Q)]}{\mathrm{E}[T]} \\ &= \frac{1}{1-\mathrm{E}[\gamma]}\left\{[s_1(1-\mathrm{E}[\gamma])+vE[\gamma]-c_1-d]-\frac{A_b D}{Q}-\frac{h_b Q}{2}\mathrm{E}[(1-\gamma)^2]-\frac{h_b D}{x}\mathrm{E}[\gamma]\right\}\end{aligned} \tag{1.16}$$

Thus, they obtained simpler expressions for the annual profit and the optimal order size while the penalty for using their annual profit function instead of that in Salameh and Jaber (2000) was negligible. They also showed that the optimal order size has a direct relation with the screening rate and the fraction of defectives.

A possible issue with the base model is if it addresses the uncertainty in a supplier's quality well with a uniform distributed fraction of defectives. Numerous examples were tested to study the difference in the annual profit in Maddah and Jaber (2008) for (*i*) uniformly and (*ii*) normally distributed fraction of defectives. The mean and variance of the uniform distribution were used as input parameters for normal distribution. A snapshot of this experiment is shown in Appendix 1. It was observed that the difference in expected cost and the lot size quantity were insignificant, which reasonably justifies the use of the uniform distribution in the base model. Besides, a uniform distribution was adopted in all the studies that extended or modified the model of Salameh and Jaber (2000). The same distribution will be used for the numerical analysis throughout this thesis.

1.8 Summary

A number of human factors have been introduced in this chapter that will be used throughout the thesis. These factors are screening errors, learning and forgetting in processing time. A detailed description of the model of Salameh and Jaber (2000) is given as all the models developed in this thesis are an extension of this base model. A number of possible questions related to uncertainty of results in the base model have also been addressed.

The direct extensions studied in the thesis include (*i*) errors in the screening process, (*ii*) learning and forgetting in the screening process, and (*iii*) two different designs of two-level supply chains under a number of coordination mechanisms. The thesis is structured as follows. Chapter three and four present extensions for the errors and learning/forgetting in the screening process, respectively. Chapter five presents a two-level supplier-vendor supply chain with (*i*) errors in buyer's screening process, (*ii*) learning in vendor's production process and (*iii*) learning in suppliers' quality. Chapter six and seven present a two-level vendor-buyer supply chain with conventional and consignment stocking schemes, respectively. Errors in buyer's screening and learning in vendor's production process are studied in these two chapters. Lastly, chapter eight presents results and avenues of possible future research.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

In this Chapter, a thorough review of the related research is presented. This survey will cover the different aspects of research covered in the thesis. These aspects are, as discussed in Chapter One, supply chains, quality management and learning and forgetting. The survey classifies the literature in several categories that encompass these aspects of research. That is, (*i*) EOQ/EPQ models for imperfect items (*ii*) rework and scrap items (*iii*) supply chains (*iv*) quality and investment to improve quality (*v*) imperfect inspection (*vi*) shortages and backordering (*vii*) learning and forgetting (*viii*) consignment stock and (*ix*) contemporary trends in EOQ/EPQ modeling.

The work of Salameh and Jaber (2000) has been a major source of inspiration for the research in this thesis. They extended the EOQ model for the case where each lot contains a random fraction of imperfect items. They assumed that (*i*) this fraction has a known probability distribution, (*ii*) each shipment goes through complete screening, and (*iii*) the defective items are all separated and sold at a discounted price in each cycle. Recently, this model has got noticeable attention by the researchers in this field. This model has been re-examined and extended in a number of ways. The broad scope and application of this work indicates its importance in the concerned literature. The literature review in this chapter will include, but is not limited to, all the important extensions of the work of Salameh and Jaber (2000) that are relevant to the scope of the thesis.

2.2 EOQ/EPQ Models for Imperfect Items

Szendrovits and Drizner (1975) illustrated the functional relationship between the manufacturing cycle time and process inventory. The production system consisted of a fixed sequence of manufacturing operations, with constant times per unit at various operations. Each operation was performed by a single facility with no capacity constraint and there was only one setup for the whole production lot size. It was assumed that a constant lot size is manufactured through several operations with only one set-up at each stage, but transportation of sub-batches was allowed which resulted in an overlap between operations to reduce the manufacturing cycle time. The inventory system assumed an infinite planning horizon with fixed per-lot costs and linear carrying costs. A process inventory must be carried until products are finished at the last operation during the manufacturing cycle time. The economic lot size was determined by minimizing the sum of the fixed costs per lot and the costs of the average finished product and process inventories.

Silver (1976) extended the economic order quantity model for the case where there is a difference between quantity received and quantity ordered by a buyer. That is, the supplied quantity can be more or less than the ordered quantity. The reason for this difference could be: (*i*) inadequate raw material, (*ii*) human error in counting, (*iii*) scrapped items, (*iv*) exceptionally good production, (*v*) rounding of the items, and (*vi*) damage in transit. Two situations were discussed where the standard deviation of the received quantity is independent or proportional to the ordered quantity.

Rosenblatt and Lee (1986) assumed that the time between the in-control and the out-of-control state of a process follows an exponential distribution and that the defective items are reworked instantaneously. They suggested producing in smaller lots when the process is not perfect. In a later model, Lee and Rosenblatt (1987) studied a joint lot sizing and inspection policy for an EOQ model with a fixed percentage of defective products. A simple relationship was developed to determine whether maintenance by inspection is worthwhile or not. They showed that the optimal inspection schedule is equally spaced throughout the production run. Gerchak *et al.* (1988) analyzed a single period production problem and extended it to an *n*-period problem where the production process has a variable yield and the demand is uncertain.

Cheng (1991) proposed an extension to the EOQ model in which demand exceeds supply and the production process is imperfect. Thus, the amount of demand the firm chooses to satisfy was a decision variable. Unit production cost was assumed to decrease as more demand is satisfied (economies of scale) and to increase as process reliability is improved (due to investment in technology). He derived a closed form expressions for the optimal demand to satisfy ordering quantity and process reliability.

Khouja and Mehrez (1994) formulated and solved an economic production lot size problem of an imperfect production process. They assumed that (*i*) production rate is a decision variable, (*ii*) unit production cost is a function of the production rate, (*iii*) quality of the production process deteriorates with increased production rate. They showed that for both weak and strong relationships between the production rate and process quality, the optimal production rate may be considerably different from the production rate that minimizes unit production cost. For cases where the mean time until the process shifts 'out-of-control' is strongly dependent on the production rate, the optimal production rate is smaller than the production rate that minimizes unit production cost. The opposite is true when the mean time until the process shifts 'out-of-control' is mostly independent of the production rate.

Yano and Lee (1995) reviewed and identified shortcomings in the literature dealing with determining lot sizes where production or procurement yields are stochastic. They claimed that good models for lot sizing decisions in the presence of random yields require adequate treatment of these elements. That is, (*i*) characterization of the yield loss process and the distribution of yield losses; (*ii*) characterization of the inspection process and its effect on timing and costs; (*iii*) possible recourse actions in response to defects; (*iv*) objective function and constraints that capture the consequences of yield losses; (*v*) proper accounting of the time and timing elements of the problem; and (*vi*) linkages across products and among different parts of a production system.

Ben-Daya and Hariga (2000) discussed the effects of imperfect quality and process restoration on the economic lot scheduling problem. They assumed that the production facility starts in the in-control state and shifts at a random time to an out of control state and begins to produce nonconforming items. Moon *et al*. (2002) extended the work of Ben-Daya and Hariga (2000) for the capacity constraints of setup and production lengths for non-zero setup times of a machine which is used for multiple products.

Inderfurth (2004) determined an optimal production policy for a uniformly distributed demand and yield rate and discussed some managerial aspects of this policy. Wang (2005) considered a production system with random deterioration from an in-control state to the out-of-control state while the in-control period follows a general probability distribution. They developed an optimal policy for production run length and inspection, and showed that the run length is an increasing function of the restoration cost. Rekik *et al.* (2007) extended the work of Inderfurth (2004) for two cases: (*a*) an additive errors case where the variability of errors is independent of the order quantity, and (*b*) a multiplicative errors case where the variability of errors is proportional to the order quantity. They showed that, depending on values that system parameters take, the optimal quantity to order may not be in the form of a newsvendor type solution adjusted by the average error rate. They developed a complete analysis that enables to determine the optimal order quantity in the presence of errors for all values of system parameters. Besides, they evaluated the benefit that would stem from eliminating the uncertainty on the quantity received by comparing the optimal costs associated with a model without errors and a model where errors disturb the quantity effectively received from the supplier.

Sana *et al*. (2007*a*) considered a flexible manufacturing system with two types of demand rates, i.e. one for perfect quality items and the other for imperfect items. The imperfect items, as in Salameh and Jaber (2000), were sold in a secondary market at a discounted price. They assumed the unit production cost to be a function of production rate which was treated as a decision variable. In a similar work, Sana *et al*. (2007*b*) assumed that the demand for imperfect items is a function of the reduction in selling price. Tsou (2007) introduced the cost of quality into the model of Salameh and Jaber (2000) and model the cost of quality for the items within the specification limits using a Taguchi loss function. They also compared their results to those of the traditional EOQ model and showed that the economic order quantity is larger due to the amount of defectives and the Taguchi loss.

Maddah and Jaber (2008) revisited the work of Salameh and Jaber (2000) and present a renewal reward theorem for determining the expected annual profit. They further extended the model by allowing for several batches of imperfect quality items to be consolidated and shipped in a single lot. Nahavandi and Haghighirad (2008) investigated an inventory system of two products where one of the products is an ingredient for the other.

El-Kassar (2009) discussed a situation where the imperfect items received by the buyer in Salameh and Jaber (2000) model are not defective. Rather, they assumed that the demands for both perfect and imperfect quality items are continuous during the inventory cycle. Yoo *et al*. (2009) incorporated inspection error and sales-return option to address the scenario where buyers receive defective products even after the inspection at suppliers' end. They showed that the annual profit may even increase by increasing the fraction of defectives and the Type I error and conclude that managers need to make more careful decisions to control the production/inspection process.

Continuing their line of research Sana (2010) generalized the model of Khouja and Mehrez (1994) by assuming that the percentage of defective items varies nonlinearly with production rate and production-run-time. Under this generalization, production starts with a variable production rate that shifts to an 'out-of-control' state. The duration of time leading to the production rate shift follows an exponential distribution function with defective items being reworked immediately and incurring a rework cost. This practical model could be used by managers to make 'what-if' analysis in order to gain specific insights into their cost reduction manufacturing problems.

2.3 Rework and Scrap

Hayek and Salameh (2001) studied the effect of imperfect quality on a finite production process. They suggested reworking imperfect items after the production process stops. Chiu *et al*. (2004) extended the work of Hayek and Salameh (2001) to study an EPQ model with imperfect rework in every cycle. They assumed that the scrap items are produced randomly during production and rework processes and showed that various models in the literature dealing with perfect rework and no-scrap are special cases of their work.

Chan *et al*. (2003) integrated lower pricing, rework and reject situations into a single EPQ model. They developed three scenarios on when to dispose the defective items. That is, selling them at a discounted price (*i*) when identified, (*ii*) at the end of production and (*iii*) just before the start of the next cycle. Jamal *et al*. (2004) determined an optimal batch quantity in a single-stage system with reworking under two different operational policies to minimize the total system cost. The first policy dealt with reworking within the same cycle, after production. The second policy dealt with the reworking *N* cycles of production.

Chiu and Chiu (2005) studied the effect of rework process in Hayek and Salameh (2001) model without backlogging. Papachristos and Konstantaras (2006) revisited the work of Salameh and Jaber (2000) and Chan *et al*. (2003) and showed that the conditions given in these models are not enough to ensure non-shortages. They presented a model where defective items are withdrawn at the end of a cycle. They showed that the results given in Salameh and Jaber (2000) can be obtained by considering the costs only rather than the revenues as in there. Sheu *et al.* (2006) discussed production correction in an imperfect process. Their model also assumed two outcomes of inspection of the system, (*i*) in-control state or (*ii*) out-of-control state. They assumed that correction may worsen the system or bring it back to the initial state. They determined an optimal number of successive out-of-control states after which the system must be maintained.

Konstantaras *et al*. (2007) extended the work of Salameh and Jaber (2000) in two directions. One is by assuming that the acceptable items are sent to working inventory in batches and not on unit-by-unit basis. The other is by reworking the defective items to bring them to acceptable quality standards and then using them to meet the demand. Ojha *et al*. (2007) assumed a 100% inspection and rework of the defectives, after production, before delivering a product to the buyer. They assumed three scenarios for this; (*a*) a single lot of raw material for multiple lots of finished product and delivery of the product in multiple batches, (*b*) a single lot of raw material for multiple lots of finished product and delivery of the product in a single batch, and (*c*) lot-for-lot of raw material and delivery of finished product in single batch. Liao (2007*b*) proposed investigating a system after each production cycle and that the system may either be in an in-control (Type I) or out-of-control (Type II) state depending on whether the correction process might be either imperfect or perfect. He suggested that after N Type I states, the operating system must be reorganized and returned to the initial condition. The model of the optimum N states was examined. Liao (2007*a*) also studied a production process that is maintained and renewed after N corrections.

Hejazi *et al*. (2008) extended the work of Chan et al. (2003) and Jamal *et al*. (2004) for the fact that reworking an imperfect item cannot be instantaneous and it takes time and money as does the processing/production of a product. He followed Chan *et al*. (2003) assumption that processing leads to four groups of products, i.e. (*i*) perfect products, (*ii*) imperfect products, (*iii*) defective but reworkable products, and, finally (*iv*) non-reworkable defective products. Biswas and Sarker (2008) considered a single stage production of reworkable and scrap items along with the finished product. Their model assumed reworking the defective items within each production cycle and that the scrap was detected (*i*) 'before' rework, (*ii*) 'during' rework, and (*iii*) 'after' rework. To avoid shortage of finished products due to scrap production, there was a buffer of finished products. They showed that the lowest total cost is obtained when the scrap is detected before rework. Sarker *et al.* (2008) developed models similar to that in Jamal *et al.* (2004) for the optimum batch quantity in a multi-stage system with rework process. They observed that the second policy incurs shortages and the optimal batch quantity increases with defects, in both policies, to compensate for the loss of planned products. Lo and Yang (2008) developed an integrated inventory model for a single vendor single buyer setup, with process unreliability consideration and permissible delay in payments. The imperfect quality items were reworked immediately after production. They showed that permissible delay in payments can promote the profit improvement and cost reduction.

Tsou *et al*. (2009) followed the studies of Salameh and Jaber (2000), Chan *et al*. (2003) and Papachristos and Konstantaras (2006) to develop an EPQ model with continuous quality characteristic, rework and reject situations. Chiu and Chang (2009) studied a JIT lot sizing problem with rework and variable setup cost. They showed that the lot size and the rework cost have a direct relation with the fraction of defectives. Their results also showed the setup cost influences the overall cost in a lean manufacturing environment. Haji *et al.* (2009*a*) discussed an economic batch quantity model for a single machine system in which defective items in N equal cycles of production are reworked in the N+1th cycle called the rework cycle. They assumed that there is a limit on the number of defective items and that the rework process is defect free. Haji *et al*. (2009*b*) extended their study in Haji *et al.* (2009*a*) to the case where reworks are performed in every cycle.

Haji *et al*. (2010) discussed an imperfect production system in which all the defective items are reworked in the same cycle after production. A 100% inspection of the produced and reworked items, with Type I and Type II errors was included in their model. It should be noted that the vendor in this thesis is assumed to have a perfect production and an imperfect inspection process. The items found defectives after rework were scrapped. They showed that fraction of defectives and Type I error have a significant impact on the batch size and the total cost.

2.4 Supply Chains

Goyal (1977) stressed that the requirement of raw material in a product dictates the ordering policy of a buyer/vendor. He developed a joint inventory policy for the raw material and the finished product assuming no lead time. Goyal (1977) pointed out that the analysis of a single product with variable lead time would have similar results.

Banerjee (1986*a*) demonstrated the advantage of the joint economic lot size (JELS) approach through an analysis of the cost trade-offs and appropriate price adjustment from the perspective of each party’s optimal position. It was shown that by adopting a jointly optimal ordering policy, one party's loss is more than offset by the gain of the other, and the net benefit can be shared by both parties in some equitable fashion. Deterministic demand and lead times were the limitations to their work. Banerjee (1986*b*) developed a pricing model from the perspective of a supplier who produces a product for a single customer. The objective was to determine the product's price in order to achieve a stated level of gross profit. He assumed that the supplier follows a lot-for-lot production strategy. This assumption would suit the instances where there is an unusually high inventory carrying cost or a lack of storage space.

Goyal and Satir (1989) reviewed joint replenishment models for deterministic and stochastic problems. The deterministic joint replenishment models approach the objective of minimization of inventory related costs by calculating the (near) optimum values of cycle time of ordering and positive integer number for ordering cycle frequency for each item. Thus, they can be considered as periodic review inventory models. On the other hand, stochastic joint replenishment models are continuous review models with decision variables of can-order, must-order and order-up-to points. They suggested that the future research should focus on (*a*) flexible manufacturing and just-in-time environments; (*b*) operational systems with resource restrictions such as transportation or production capacities, shelf life and demand interactions due to product substitutions; (*c*) multi-echelon production/inventory systems. Furthermore, the applicability of the above models could be enhanced by considering (*i*) multiple coordinated parallel cycles, thereby facilitating non-integer values of multipliers; (*ii*) interactions in the replenishment cost (*iii*) stochastic replenishment lead times.

The inventory problems of vendor and buyer have long been treated in isolation. Goyal and Gupta (1989) reviewed the literature that deals with the interaction between a buyer and a vendor. The integrated models were classified as (*i*) models that deal with joint economic lot sizing policies; (*ii*) models that deal with coordination of inventory by simultaneously determining the order quantity of the buyer and the vendor; (*iii*) models that deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor; (*iv*) models that deal with buyer-vendor coordination due to marketing considerations. They also discussed the incentives for the vendor to offer price quantity discount as: (*i*) larger orders from buyers may result in lower sales costs since there would be fewer sales cells necessary and fewer orders to be processed, (*ii*) the vendor may in turn save by obtaining quantity discounts on raw materials from his suppliers, (*iii*) larger orders, if produced to order, will result in fewer manufacturing setups per year and longer production runs leading to higher production efficiency, (*iv*) larger orders may allow the vendor to take advantage of transportation discounts that may be available, and (*v*) larger orders will exhibit a change in the pattern of orders placed throughout the years which may mean a shift in the timing of order payments that in turn may lead to larger revenues available to the vendor for reinvestment for longer periods.

Hill (1997) discussed the benefits of multiple deliveries for a single order in an integrated inventory model, showing that a cooperative batching policy can significantly reduce total costs in a JIT environment. It was assumed that the supplier could start shipping even before completing the entire lot as soon as the production quantity becomes greater than the shipping size.

Munson and Rosenblatt (2001) proposed a mechanism by which a company can coordinate its purchasing and production functions and create an integrated plan that dictates order and production quantities throughout a three layer channel. That is, they modeled a company that attempts to dictate channel lot sizes by obtaining a quantity discount from its supplier while offering a different one to its customer. The model is quite useful as it takes care of a three level chain (supplier – vendor – buyer) and explores the benefits of using quantity discounts on both ends of the supply chain to decrease costs. They showed that incorporating quantity discounts into both ends of the supply chain can significantly decrease costs compared to concentrating only on the lower end.

Kim and Ha (2003) developed a JIT lot-splitting model that deals with buyer–supplier coordination. They discussed a simple JIT environment, i.e., single buyer and single supplier, under deterministic conditions for a single product. They examined the benefits of the proposed JIT lot-splitting policy of multiple deliveries over the lot-for-lot delivery policy. It was shown that regardless of the order size, the delivery size converges to an optimal size that can be used as a basis for determining a standard transportation vehicle size. Singer *et al*. (2003) considered the strategic behavior regarding quality within a supplier–buyer partnership in a disposable product industry. They studied a single-product distribution channel where a supplier manufactures items of a given type, some of which are defective. The buyer detects only a subset of the defective items, passing the rest along to customers. They were concerned with questions such as, (*i*) what is the effect of the different cost and demand parameters? (*ii*) in what circumstances can non-observable parameters be inferred? (*iii*) how quality performance is affected by observable parameters in a vertical integration? (*iv*) if agents should remain autonomous, is it possible to devise transfer contracts that can lead to quality improvements? and (*v*) can they reach optimality in a non-cooperative setup?

Goyal *et al.* (2003) performed a sensitivity analysis of different parameters on the integrated model. They concluded that the number of shipments has a direct relationship with the holding cost for the buyer while it has an inverse one with the holding cost for the vendor. Khouja (2003) formulated a three-stage supply chain model where a firm can supply many customers. They dealt with three inventory coordination mechanisms between chain members and solved a cost minimization model for each. In the first mechanism the same cycle time is used throughout the supply chain. In the second mechanism the cycle time at each stage of the chain is an integer multiple of the cycle time of the adjacent downstream stage. In the third mechanism the cycle time of each firm is integer powers of two multiples of a basic cycle time. Their analysis showed that the integer multipliers mechanism has lower total cost than the equal cycle time mechanism, and the integer powers of two multipliers mechanism have lower cost than the integer multipliers mechanism. These models had several limitations, such as: (*i*) each buyer received its raw material from only one supplier, (*ii*) a deterministic demand was assumed, and (*iii*) they did not describe how to distribute the savings from the implementation of their models, among the stakeholders in the supply chain.

Huang (2004) extended the work of Salameh and Jaber (2000) and Ha and Kim (1997) to determine a joint inventory policy for a single-vendor single-buyer case. Under the assumption that the buyer's order quantity was replenished in n shipments of identical size, a mathematical model for the optimal order size and the number of shipments was developed and illustrated using the example found in Salameh and Jaber (2000).

Sarmah *et al*. (2006) investigated supply chain coordination models that have used quantity discount as coordination tool under deterministic environment, considering quantity discount to be one of the most popular mechanisms of coordination between the business entities. They categorized the coordination models as (*i*) vendor perspective models, (*ii*) joint buyer and supplier perspective models, (*iii*) models using joint buyer and supplier coordination through cooperative and non-cooperative game, and (*iv*) models with single vendor and multiple buyers using any of the first three categories.

Jaber *et al*. (2006) extended the model of Munson and Rosenblatt (2001) to investigate the coordination in a supplier-vendor-buyer supply chain. They assumed a unit price discounts, profit maximisation approach. A price elastic demand and a profit sharing mechanism were adopted. They showed that as demand becomes more sensitive to price discount, the replenishment policies for the players were to order in larger quantities.

Wu *et al*. (2007) studied a buyer-vendor inventory system where the buyer inspects sublots. They derived two integrated models with backorders, where the first model assumed a normal distribution while the second model used a distribution-free minimax approach, for the lead time demand. Lo *et al*. (2007) developed an integrated production-inventory model from the perspectives of both a vendor and a buyer. They assumed a varying rate of deterioration with partial backordering, inflation, imperfect production processes and multiple deliveries. They showed that the integrated decision results in a lower optimal joint cost when compared with an independent decision by the vendor or the buyer.

Jaber and Zolfaghari (2008) reviewed the literature on quantitative models for the coordination in a centralized supply chain, for the period between 1990 and 2007. They classified the models on the basis of coordination schemes, supply chain, assumptions and levels of supply chain. They also provided directions for future research in the respective areas based on the assumptions and limitations in the available literature. Chung and Wee (2008) developed an integrated two-stage production-inventory deteriorating model for the buyer and the supplier with stock-dependent selling rate. They considered a single setup and multiple JIT deliveries strategy for a single product. They derived optimal results for the number of inspections and deliveries as well as the optimal delivery-time interval.

Huang (2002) used the approach of Salameh and Jaber (2000) to develop an optimal integrated vendor-buyer inventory policy for imperfect items in a just-in-time (JIT) manufacturing environment. Lin (2009) extended Huang (2004) model to present a production/inventory model with imperfect quality and screening errors. He showed that the lot size per shipment and the number of deliveries per cycle have a direct relationship with Type I screening error while they have an inverse one with Type II screening error.

Chung *et al*. (2009) generalized the work of Salameh and Jaber (2000) to develop an inventory model for two warehouses that are constrained with limited storage capacity. An interesting feature of their model is the use of a piecewise concave function for determining the expected total profit per unit time.

2.5 Learning, Quality and Investment

Towill (1985) studied the effect of industry learning on batch production policy and found that production management should take steps to reduce slippage in performance in a manufacturing firm. Slippage is caused by the elapsed time between the completion of one batch and the start of the next. This effect causes a significant increase in manufacturing times, and hence in direct labour costs.

Chiu (1997) incorporated both learning and forgetting into discrete time-varying demand lot-sizing model. The production cost required to produce the first unit of each successive lot over a finite planning horizon was derived. Besides, an optimal lot sizing model and three heuristic models were developed by extending the existing models without learning and forgetting considerations. He showed that larger lot sizes are needed when learning and forgetting are considered. An alternate to this would be to use automated equipment to avoid heavy reliance on worker proficiency.

Jaber and Bonney (1998) examined the impact of learning and forgetting on the optimal production lot size in an infinite and a finite planning horizons, where production runs are intermittent. They assumed that the optimal production quantity depends on (*i*) the maximum inventory accumulated prior to interruption; (*ii*) the length of the interruption required for total forgetting; and (*iii*) the level of experience in equivalent units remembered at the start-up of the next production run. They showed that under the partial transfer of learning (PTL), the optimal policy was to carry less inventory in later lots. Jaber and Bonney (1999) surveyed the literature that deals with the effect of learning on the lot-size problem. They suggested that the traditional lot size problem can be extended in a number ways. That is, by (*i*) investigating the impact of learning/forgetting on the lot size and the setup frequency, (*ii*) acquiring the optimal rate of learning to minimize the cost of learning, the cost of reworking defective items and the total cost of inventory system, and by (*iii*) studying the effects of learning and forgetting on the lot-size problem with non-stationary demand rates.

Huan *et al.* (2003) extended Chiu (1997) in a number of ways. That is, (*i*) they considered both learning and forgetting in setups and production simultaneously, (*ii*) they assumed that the forgetting rate in production is a function of the break length and the level of experience gained before the break, and (*iii*) they assumed that each production batch is completed as close as possible to the time of delivery in order to reduce the inventory carrying cost. Chiu and Chen (2005) also incorporated both learning and forgetting in setups and production into the dynamic lot-sizing model to obtain an optimal production policy. They showed that the effects of production learning on the number of production runs and total cost are more influential than that of setup learning.

Tsou and Chen (2005*a*) developed a quality improvement model based on the classical EOQ theory. They assumed that the mean and standard deviation of a quality distribution for the product can be improved through an investment. They verified their model through analyzing a car seat assembly line and proved that the quality investment ends up reducing the overall cost of the production system. Tsou and Chen (2005*b*) considered a defective production system and compared the total cost with and without quality improvement (Poka-Yoke). They verified their model through a case study and showed that the total cost increases with the number of production runs. Comeaux and Sarker (2005) addressed shortcomings and assumptions regarding inspection policy, scarp generation and inspection errors (Type I and Type II) in models for vendor-buyer coordination. They developed models for the optimal joint economic lot size and optimal fraction conforming to quality and showed that increasing the fraction of product inspected increases the total cost while decreasing the optimal batch size.

Dobos and Richter (2006) investigated a production-recycling model with quality consideration. They showed that it is better to outsource the quality control and repurchase only reusable products. They claim that a mixed strategy would be economical compared to the pure strategies, that is either production or recycling. Their results motivate an examination of the trade-off between an investment in improving quality and cost of handling defective items. Lee (2008) developed cost/benefit models for investments made in quality improvements in a multi-level assembly system. He suggested investment in preventive maintenance to reduce variance and the deviation of mean from the target value of the quality characteristic, hence to reduce the proportion of defectives and also to increase reliability.

Tsou and Chen (2008) studied the impact of preventive activities on the economics of defective production systems. They showed through a practical case that return of the improvement depends on the cost of poor quality and that of the preventive activities. Tomlin (2009) tried to understand the operational implications of supply learning. He argued that assuming that a firm knows the supply distribution of its suppliers with certainty is a strong standpoint. His objective was to investigate how an uncertainty about a supplier's true reliability influences a firm's optimal sourcing and inventory decisions. That is, (*i*) if uncertain reliability makes a supplier more or less attractive? and (*ii*) if it increase or decrease the firm's incentive to invest in inventory? Using a Bayesian approach for supply learning (i.e., reliability-forecast updating), he characterized the firm's optimal sourcing and inventory decisions. He proved that for a given expected supplier reliability (i.e., the mean of the firm's forecast for a supplier's reliability), increasing forecast uncertainty (coefficient of variation) increases the attractiveness of a supplier, but it reduces the firm's wish to invest in inventory to protect against future supply failures. Kulkarni (2008) considered an inspection, inventory planning and process investment problem when process yield is random and backorders are permitted. He concluded through numerical analysis that modified reflected normal loss in the lot-sizing problem serves better than Taguchi loss function.

Wang (2009) presented an investment model with nonlinear stochastic functional differential equations. He also established criteria for the exponential stability of nonlinear stochastic neutral technical progress and investment system. Hsu *et al.* (2009) derived a replenishment policy for an EOQ model of deteriorating items where investment is made to improve the production process. They studied sensitivity of the investment to the fraction of defectives. Mehdi *et al*. (2009) presented a joint strategy of quality control and preventive maintenance for an imperfect production process. They used three decision variables, i.e. (*i*) the rate of producing defective units, (*ii*) the size of the buffer stock, and (*iii*) the threshold level of the rate of rejection.

Although learning is commonly believed to cause improvement in the time to process a unit only, learning can be shown in the cost, productivity and other similar measures of a production system. In this section the literature that links quality and learning will be categorized through exploring the literature that finds a (*i*) mathematical relationship, and (*ii*) empirical relationship.

2.5.1 *Mathematical Relationship*

Wright (1936) was probably the first one to come up with a relationship that signifies the importance of experience or learning in a production facility. He studied the variation in production cost with production quantity on cost. A curve depicting such variation was worked up empirically from the two or three points which previous production experience of the same model in differing quantities made possible. This curve was found to take an exponential form.

Levy (1965) is among the first few who captured the linkage between quality and learning. Fine (1986) demonstrated that learning is the bridge between quality improvement and productivity increase. This supports the observation of Deming (1982) that quality and productivity are not to be traded off against each other; instead, productivity increases follow from quality improvement efforts. He introduced and formally modeled a theory of quality-based learning. He was of the view that when costs are affected by quality-based learning curve, product quality favourably influences the rate of cost reduction due to learning. Thus, costs decline more rapidly with the experience of producing higher quality products. He presented two formulations for quality based learning. The first assumes that quality-based experience affects direct manufacturing costs. The second formulation assumes that quality-based experience affects quality control costs. In this case, the optimal quality level is always increasing over time.

A key feature of the second quality-based model is that it resolves the controversy between the economic conformance level model of Juran (1978), which asserts that one should use cost-tradeoff analysis to find the optimal quality level, and the claims of Deming (1982) and Crosby (1979), that zero defects is always the optimal quality level. Fine claimed that firms choosing to produce high quality products will learn faster or go down a steeper experience curve than firms producing lower quality products (where quality is defined as degree of conformance to design specification). Fine's quality-based learning theory added a dynamic learning curve effect to the static economic conformance level model so that the modified model is consistent with the "higher quality costs less" school of thought and is consistent with the evidence from Japan. In the quality-based learning formulation, both induced learning and autonomous learning are modeled. Induced learning, in contrast to autonomous learning, depends on conscious actions and efforts by management and technical people to improve the efficiency of the production system.

Porteus (1986*a*) modeled the relationship between quality and learning in a different way. He assumed that for a very small probability that a process goes out of control, the expected number of defective items in a lot is proportional to this probability. Tapiero (1987) discussed the practice in manufacturing in which quality control is integrated into the process of production in altering both the product design and manufacturing techniques to prevent defective units. He developed a stochastic dynamic programming problem for determining the optimal parameters of a given quality control policy with learning. He defined the knowledge in a production facility with quality control as a learning function. It was concluded that the optimal quality control policy a vendor may use is not only a function of the costs of inspection and the costs of products' failures but also of the vendor's ability to use the inspected samples in improving (through "experience") the production technology. This observation is of course in line with the Japanese practice of "full inspection" and of learning "as much as possible" to obtain finally a zero-defects production technology.

To help explain how one firm can have both higher quality and lower costs than its competitors, Fine (1988) explored the role of inspection policies in quality-based learning, a concept introduced in Fine (1986) and extended by Tapiero (1987). He discussed an imperfect production process, i.e., one that at times produces defective items. The process may produce defective items for any of a large number of reasons, such as poorly designed or poorly constructed materials and/or components, substandard workmanship, faulty or poorly maintained equipment, or ineffective process controls. This model permits the production process to be improved through "quality-based learning" by the operators. It has tight quality control standards for the time in the "out of control" state, enforced by intensive inspection procedures, thus resulting in faster learning. This leads to lower long-run failure costs than the short-run inspection costs. This work is different from that of Fine (1986) and Tapiero (1987) in that it attempts to model more explicitly how learning events arise rather than using an aggregate measure of cumulative experience or learning. The model points out to managers responsible for quality policies that they may be choosing suboptimal inspection policies if they are ignoring potential learning benefits from inspection.

Chand (1989) used the same approach to model the expected number of defective units in a production lot. It was assumed that the process is in control at the start of a production lot and no corrective steps are taken if the process goes out of control while producing a lot. Before starting a new production lot, the process was reset so that it is in control. Chand (1989) elaborated the benefits of small lot sizes in terms of reduced setup costs and improved process quality due to worker learning. He discussed a lot sizing problem with learning in process quality and in setups but no learning in processing times. He showed that the lot sizes do not have to be equal in the optimal solution even if the demand rate is constant.

Urban (1998) modeled the defect rate of a process as a function of the run length, and derived closed-form solutions for the economic production quantity. This model accounted for either positive or negative learning effects in production processes. To elaborate the learning effect of run length on product quality and thus on production costs, a generalization of the basic economic production quantity (EPQ) model was examined by taking the defect rate as a function of the production lot size. A constant and deterministic demand of a single item was discussed without any backorders. This functional form was found to be very useful for a number of reasons, (*i*) using appropriate parameters, this functional form can represent the JIT philosophy, the disruptive philosophy, or a constant defect rate independent of the lot size, (*ii*) it provides a bound for the defect rate i.e. as the lot size increases, the defect rate approaches a given value, (*iii*) it is straightforward to estimate the model parameters in practice, using simple linear regression and generally readily available data on lot sizes and defect rates, and (*iv*) a closed-form solution can easily be obtained, which can then be examined to gain important insights into the problem.

It should be noticed that the traditional way to obtain data for a learning cycle is usually erroneous as the individual data is composed of some variance. To counter this, Zangwill and Kantor (1998) proposed to measure the individual improvements directly and to use the learning cycle repeatedly. This would make the management observe which techniques are producing greater improvement and thus learn how to improve processes faster and faster. They came up with a differential equation which was composed of three forms of learning, i.e. power form, exponential form, and the finite form. Zangwill and Kantor (2000) extended upon their earlier work and emphasized that traditional learning curve cannot identify which techniques are making improvements and which are not, on a period by period basis. Their approach helps to boost rate of improvement in every cycle and makes the learning a dynamic process.

Jaber and Guiffrida (2004) presented a quality learning curve (QLC) which is a modification of Wright's learning curve (WLC) for imperfect processes where defectives can be reworked. They incorporated process quality into the learning curve by assuming no improvement in reworks and then modeled the same situation by relaxing this assumption. Their approach was similar to that of Porteus (1986*a*) in that the process remains in control at the beginning of the lot and generates no defects. In the course of learning, there is a point where there is no more improvement in performance. This phenomenon is known as plateauing. The possible reasons for plateauing may be (*i*) labor ceases to learn, (*ii*) management becomes unwilling to invest in learning efforts, and (*iii*) management becomes sceptical that learning improvement can continue.

Their composite learning model resulted in the following findings: (*i*) for a learning in reworks such that the learning exponent is less than half, composite learning curve was found to be convex with a local minimum that represents cumulative production in a given cycle, (*ii*) for the learning exponent equal to half, the composite learning curve would plateau at a fixed value, as cumulative production approaches infinity, (*iii*) when the learning exponent remains between half and one, learning was found to behave in a similar manner to that of Wright (1936). That is, as cumulative production approaches infinity, the time to produce a unit would approach zero. Their work had some limitations too. That is, (*i*) it cannot be applied to cases where defects are discarded, (*ii*) the rate of generating defects is constant, (*iii*) the process can go out-of-control with a given probability each time an item is produced, Porteus (1986*a*), (*iv*) there is only one stage of production and rework considered.

Jaber and Guiffrida (2008) assumed that an imperfect process can be interrupted to restore quality. This way they addressed the shortcomings in Wright's learning curve (WLC), i.e., (*i*) the learning phenomenon continues indefinitely and (*ii*) all units produced are of acceptable quality. This model also addressed the third limitation in the work of Jaber and Guiffrida (2004), as stated above. They assumed that (*i*) a lot is divided into n equal sub-lots corresponding to $(n - 1)$ interruptions, and (*ii*) the restoration time is a constant percentage of the production time. This way they modified the equations for total time to produce x units, in Jaber and Guiffrida (2004), for the cases of with and without learning in reworks. Their results indicated that introducing interruptions into the learning process to restore the production process quality, may improve performance when the percentage of the production time that represents the process restoration time is smaller than the production learning rate. Otherwise, they recommended the use of QLC (Jaber and Guiffrida, 2004). One important outcome of this research was that restoring the production process breaks the plateau barrier and thus provides opportunities to improve performance.

2.5.2 *Empirical Relationships*

Foster and Adam (1996) included speed of quality improvement in Fine (1986) quality-based learning curve model. The model demonstrated that under different circumstances, rapid quality improvement effects are either beneficial or unfavourable to improvement in quality-related costs. They demonstrated that rapid quality improvement, if sustained and permanent, can lead to higher levels of learning. However, under certain conditions, rapid speed of quality improvement can also impede organizational learning. They developed two hypotheses from this analysis and tested them in an automotive parts manufacturing company with five similar plants.

It was found that rapid speed of quality improvement resulted in reductions in the rate of improvement in quality-related costs. In addition, slower quality improvement was more closely associated with reductions in quality-related costs. This behavior was named as organizational learning. This type of learning is found in many organizations. For example, with the passage of time, (*i*) inspection related costs are reduced, (*ii*) need for acceptance sampling of raw materials is reduced, (*iii*) prevention related costs decline, and (*iv*) prevention activities become more focused and specific. The overall result of all this is the reduction in quality or improvement related costs due to learning.

This model demonstrated that some quality related efforts may go ineffective. They proposed that a slower, steadier speed of quality improvement with learning was more strongly associated with decreases in quality-related costs than with rapid improvement. That is, rapid speed of quality improvement can be detrimental on the cost side of the business. The model was supported by the empirical findings.

Forker (1997) examined the results of a survey of 348 aerospace component vendors to investigate the factors that affect supplier quality performance. He discussed the process view of quality management to describe the inconsistent association between practice and performance in a supplier firm. This helped him link the quality management with process optimization to address both effectiveness and efficiency concerns. The quality performance of a supplier was linked with a variety of dimensions such as features, reliability, conformance, durability, serviceability and aesthetics.

The role of human learning in his theory and hypotheses was important as (*i*) learning curve affected the transaction cost of different supplies, and (*ii*) suppliers' attitude towards learning and thus their efficiency impacted quality magnitude. The study showed that as processes become more streamlined and capable, firms should invest their resources in product design and in training all employees in quality improvement concepts and techniques.

Li and Rajagopalan (1997) collected about three years of data on quality levels, production, and labor hours from two manufacturing firms, to answer the three questions related to the impact of quality on learning. That is, (*i*) how well does cumulative output of defective or good units explain learning curve effects? (*ii*) do defective units explain learning curve effects better than good units? (*iii*) how should cumulative experience be represented in the learning curve model when the quality level may have an impact on learning effects? The data was taken from two plants making tire tread and medical instruments (kits and fixtures) respectively.

They used defect levels as a substitute for the effort devoted to process improvement. In another model, Li and Rajagopalan (1998) had shown that the optimal investment in process improvement effort is proportional to the defect levels. In the traditional learning curve model, defective units and good units were assumed to be equivalent in explaining learning curve effects. Therefore, cumulative production volume was used as a proxy for knowledge or experience. They proposed a model complementary to the one in Fine (1986) and demonstrated that if the defective level in a period is very high, it immediately gets the attention and considerable effort is directed at identifying the source of defectives. If defect levels continue to be high for a few consecutive periods, increased attention is paid and additional resources are devoted to investigate the cause of the defects. They concluded that defective and good units do not explain learning curve equally as is implicitly assumed in traditional learning curve models. In other words, defective units are statistically more significant than good units in explaining learning curve effects.

Badiru (1995) claimed that quality is a hidden factor in learning curve analysis. Quality was considered to be a function of performance which in turn is a function of production rate. Forgetting was believed to affect the product quality in the sense that it can impair the proficiency of a worker in performing certain tasks. That is, the loss in worker performance due to forgetting is reflected in product quality through poor workmanship. It was demonstrated that forgetting can take different forms as (*i*) intermittent forgetting (i.e. in scheduled production breaks), (*ii*) random forgetting (e.g. machine breakdown), and (*iii*) natural forgetting (i.e. effect of ageing)

Badiru (1995) formulated a multivariate learning curve and tested it on the 4-year record of a troublesome production line. This production line was a new addition to an electronics plant and thus was subject to significant learning. The company used to stop the production line temporarily if the quality problems would arise. It was hypothesized that the quality problems could be overcome if the downtime (forgetting) could be reduced so that workers could have a more consistent operation. The variables of interest were: production level, number of workers, and number of hours of production downtime. The dependent variable was the average production cost per unit. An analysis of variance of the regression model showed the fit is highly significant with 95% variability in the average cost per unit. It was noticed that the average cost per unit would be underestimated if the effect of downtime hours is not considered. Thus, the multivariate model would provide a more accurate picture of the process when there are many factors associated with the process.

Mukherjee *et al*. (1998) studied why some quality improvement projects are more effective than others. They explored this by studying 62 quality improvement projects undertaken in one factory over a decade, and identified three learning constructs that characterize the learning process. That is, scope, conceptual learning, and operational learning. The purpose of their study was to establish a link between pursuit of knowledge and pursuit of quality.

This study followed Kim (1993) approach of distinguishing between two types of effort, conceptual learning and operational learning. Conceptual learning is in essence trying to understand why events occur, i.e., the acquisition of know-why. In contrast, operational learning consists of implementing changes and observing the results of these changes. Operational learning is basically developing a skill of how to deal with experienced events, i.e., the acquisition of know-how.

In order to establish links between learning and quality, field researchers tried to control potentially confusing factors such as variations in product and resource markets, general management policies, corporate culture, production technology, and geographical location. They must have access to detailed data about the systems used to improve quality. They concluded that (*i*) management plays a role in addressing 80-85% of quality problems, (*ii*) in dynamic production environments a cross-functional project team is in a better position to create technological knowledge, and that (*iii*) operational and conceptual learning have different potentials in a plant.

Lapre *et al*. (2000) explored the learning curve of total quality management (TQM) in a factory. In this model, they extended the link between learning and quality from a cross-sectional, project-level analysis to a longitudinal, factory-level analysis. They focused on waste which is a key driver of both quality and productivity. The learning rate was not taken as constant. It was rather modeled as a variable that depends on autonomous and induced learning. The parameters of this model were determined by analyzing a number of projects in a factory. These projects were coded on questions that dealt with their learning process and their performance by giving a response on a five-point likert scale.

Jaber and Bonney (2003) used the record of an electronics production line in Badiru (1995) to show that it follows two hypotheses; (*i*) the time required to rework a defective item reduces as production increases and the rework times conform to a learning relationship, (*ii*) quality deteriorates as forgetting increases due to interruptions in the production process.

To validate the first hypothesis, they analyzed the effect of cumulative production level on the average time to rework a unit. The analysis of variance showed that 84% of the variability in the average rework time per unit is explained by cumulative production as an independent variable. Similarly, to validate the second hypothesis, they analyzed the impact of forgetting due to production down time on the average rework time for a unit and cumulative production level. In this case, the analysis of variance showed that 88% of the variability in the average rework time per unit is explained by cumulative production and production down time as independent variables.

Hyland (2003) reported their research into continuous improvement and learning in logistics of a supply chain. This research was based on a model of continuous innovation in the product development process and a methodology for mapping learning behaviors. Learning was taken to be crucial to innovation and improvement. To build innovative capabilities, organizations need to develop and encourage learning behaviors. People could question, through learning, the existing standard behavioral patterns. He believed that capabilities could only be developed over time by the progressive consolidation of behaviors, or by strategic actions aimed at the stock of existing resources. He identified four key capabilities that are central to learning and continuous improvement (CI) in a supply chain. That is (*i*) the management of knowledge; (*ii*) the management of information; (*iii*) the ability to accommodate and manage technologies and the associated issues; and (*iv*) the ability to manage collaborative operations.

Salameh and Jaber (2000) had assumed the fraction of defectives to be following a known probability density function. Jaber *et al*. (2008) noticed that the data for this fraction from an automotive industry reduces according to a learning curve, over the number of shipments. They tried to fit several learning curve models to the collected data and found that the S-shaped logistic learning curve fitted the data well. Two models similar to that of Salameh and Jaber (2000) were developed. That is, one for an infinite planning horizon and the other for a finite planning horizon. They found that in the infinite planning model, the number of defective units, the shipment size, and total cost reduces with an increase in learning increases. On the other hand, for the finite learning model, an increase in learning recommends larger lot sizes less frequently.

In this thesis, learning from cycle to cycle will be considered in a vendor's production process (time) and in supplier's quality (fraction of defectives). That is, vendor's production time and supplier's fraction of defectives are assumed to decrease while moving from one cycle to another in learning.

2.6 Imperfect Inspection

Collins *et al*. (1973) considered the effects of inspection error on the probability of acceptance, average outgoing quality and average total inspection. They examined these measures under both replacement and non-replacement assumptions. Bennett *et al*. (1974) investigated the effect of error on a single sampling plan with known incoming quality.

Raouf *et al*. (1983) were the first to develop a model for determining the optimal number of repeat inspections for multi-characteristic components to minimize the total expected cost per accepted component due to Type I error, Type II error and the cost of inspection. Lee (1988) provided a simplified version of the model given by Raouf *et al*. (1983) to evaluate the costs involved in the multiple-cycle inspection schemes for multi-characteristic components. He extended the results for the case where the probabilities of defectives are random.

Chandra and Schall (1988) studied the effect of replicate measurements on average outgoing quality and the average total inspection. They obtained the optimum number of replications based on the total cost of inspection. Duffuaa and Raouf (1989) developed three mathematical optimization models for multi-characteristic repeat inspection. Duffuaa and Raouf (1990) established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf *et al*. (1983). Duffuaa and Nadeem (1994) extended the model proposed by Raouf *et al*. (1983) to cases where defective rates are statistically dependent. They proposed an algorithm to determine the optimal number of repeat inspections and sequenced the characteristics for inspection in order to minimize the total expected cost.

Sylla and Drury (1995) proposed a model that uses a form of SDT to predict inspector performance in order to improve system performance. They presented the concept of lability to characterize the inspector's ability to respond to the costs, penalties and probabilities involved in the inspection decision.

Duffuaa and Al-Najjar (1995) proposed a new inspection plan for critical multi-characteristic components with a variable number of inspections for different characteristics. They proposed an algorithm to determine the optimal number of repeat inspections and sequenced the characteristics for inspection in order to minimize the total expected cost. Duffuaa (1996) investigated the statistical and economic impact of inspector errors on the performance measures, i.e. ATI, AOQ and ETC, of a complete inspection plan. He concluded that Type I and Type II errors have a significant effect on the performance measures of repeat inspection plans. Hong *et al*. (1998) developed economic screening procedures when the rejected items are reworked. Screening procedures based on the performance variable of interest and a correlated variable are considered. They considered the cost incurred by imperfect quality, reprocessing cost and inspection cost.

Duffuaa and Khan (2002) provided a more realistic formulation to the models for inspecting multi-characteristic components. Their inspection plan assumed three classifications for the components under inspection, which were: characteristics meet specifications (good), need rework or are scrap. In this situation an inspector could make six types of errors. These are (*i*) a good characteristic is classified as rework or scrap, (*ii*) a rework characteristic is classified as good or scrap, and (*iii*) a scrap characteristic (defective) is classified as good or rework. So, there could be six types of misclassifications in this plan. A number of practical applications of the model were identified such as gas ignition systems, aircraft avionics systems, space shuttle and nuclear reactors.

Duffuaa and Khan (2005) quantified the effect of inspection errors, mainly on the performance measures of a complete repeat inspection plan (Duffuaa and Khan, 2002). This was accomplished by conducting a sensitivity analysis on the errors and then observing the changes in expected total cost (ETC), average total inspection (ATI) and average outgoing quality (AOQ). They showed that the errors of classifying good items as scrap and scrap as good have significant effect on the performance measures of the repeat inspection plan.

Duffuaa and Khan (2008) presented an inspection plan for the case where the characteristics' defective rates are statistically dependent. This model was an extension of the work of Duffuaa and Khan (2002) and Duffuaa and Nadeem (1994). They assumed that characteristic defective rates can be obtained from historical data or the production process capability.

In this thesis, Type I and Type II errors have been incorporated in the screening process both at a vendor's and a buyer's end in a two level supply chain context.

2.7 Fuzzy Set Theory

The fraction of defectives in Salameh and Jaber (2000) is usually taken from historical data. In the absence of such historical evidence, fuzzy set theory can be used to parameterize the level of quality of a supplier. Chang (2004) reformulated the EOQ model of Salameh and Jaber (2000) to capture the uncertainty in the defective rate using fuzzy set theory. In this extension, the complement of the defect rate, $\varphi = 1 - \gamma$, was used as a triangular fuzzy number $\tilde{\varphi}$ of the form $\tilde{\varphi} = (\varphi - \Delta_1, \varphi, \varphi + \Delta_2)$. The parameters $\Delta_i (i = 1,2)$ are determined by the decision maker's interpretations of the magnitude of the complement of the defective rate and are constrained as $0 < \Delta_1 < \varphi$ and $0 < \Delta_2 < 1 - \varphi$ respectively. Ouyang *et al.* (2006) extended the integrated vendor-buyer lot sizing model of Huang (2002) with imperfect quality by treating the defective rate as the triangular fuzzy number $\tilde{\gamma} = (\gamma - \Delta_1, \gamma, \gamma + \Delta_2)$ where $0 < \Delta_1, < \gamma$ and$0 < \Delta_2, < 1 - \gamma$. The fuzzy set theory was originally introduced by Zadeh (1965) and has been implemented in inventory and production management environments. See Guiffrida and Nagi (1998) and Guiffrida (2009) for reviews. Fuzzy set theory will not be considered in this thesis.

2.8 Shortage and Backordering

Montgomery *et al*. (1973) studied a periodic review inventory model where lost sales and backorders are caused by the stock-out of inventory in a production system. Kim and Park (1985) studied a stochastic inventory model for situations in which, during a stock out period, a fraction of the demand is backordered and the remaining fraction of the demand is lost. The model is suggested by the different reactions of a buyer to the stock out condition. That is, during the stock out period, some buyers wait until their demand is satisfied, while others cannot wait and have to fill their demand from another source. The cost of a backorder was assumed to be proportional to the length of time for which the backorder exists, and a fixed penalty cost incurred per unit of lost demand. They developed a mathematical model for the average annual cost and treated the reorder point with a heuristic approach. It was shown that erroneous assumption of the fraction backordered is very sensitive to the average annual cost.

Nahmias and Smith (1994) discussed a similar model with partial lost sales. They assumed instantaneous deliveries from the warehouse to the buyers. Andersson and Melchiors (2001) derived an approximate solution for the case where customer demand is a Poisson process, demand during a stockout is lost, buyers operate (one for one) base stock control policies and the warehouse lead time is fixed. They adjusted the arrival rate of buyer orders to allow for the effect of stockouts at the buyers and they estimate and then use the mean extra delay at the warehouse. Yeh and Chen (2003) extended the work of Salameh and Jaber (2000) to account for shortages backordered. They assumed that the shortages are fulfilled without screening, once the order arrives. This would result in a warranty cost. Chiu (2003) extended the work of Hayek and Salameh (2001) to present a modified EPQ model, for the fact that only a fraction of (instead of all) the defective items are reworkable.

Rezaei (2005) also extended Salameh and Jaber (2000) to include shortages. He optimized simultaneously the lot size and the shortage level and showed that the lower the percentage of defectives, the higher the savings in the expected total profit per unit time for proposed model with respect to Salameh and Jaber (2000) model. Yu *et al*. (2005) extended the work of Salameh and Jaber (2000) to include deterioration in the process and partial backordering. They considered a ratio of the backorder and lost sales in their model and determined the lower bound of this ratio through iteration, for a feasible profit function. They showed that the management of an enterprise can select suppliers based on the percentage of defectives and the deterioration rate of the products supplied by each supplier. Wee *et al*. (2006) extended the work of Salameh and Jaber (2000) for a single-vendor single-buyer supply chain with product deterioration. They assumed complete backordering of the shortages. They showed that imperfect quality, deterioration and backordering have a significant effect on the supply chain performance. Chiu and Chiu (2006) extended the work of Chiu (2003) for an imperfect repair process. They assumed that a portion of the reworked items fails the repairing process and thus becomes scrap.

Hill *et al*. (2007) considered a single-item, two-echelon, continuous-review inventory model. The unfulfilled demand at a buyer was considered as lost. They approximated the arrival of orders at the warehouse from the buyers as a Poisson process. The results of simulation proved that this approximation has excellent results in describing the behavior of the system. The steady state of the system was determined by an iterative scheme. An important feature of this research was the finite state space at the warehouse. Eroglu and Ozdemir (2007) extended the work of Salameh and Jaber (2000) by allowing shortages that are backordered. They also examined the effects of different levels of fraction of defectives on lot size and expected total profit and showed that the optimal total profit per unit time decreases when defective and scrap rates increase individually. Wee *et al*. (2007) also extended the work of Salameh and Jaber (2000) by considering permissible shortage backordering. They studied the effect of varying backordering cost and showed that the traditional EOQ and Salameh and Jaber (2000) models become special cases of their model as the backordering cost tends to infinity. Chiou and Chen (2007) also dealt with the failure in repair.

Haji *et al*. (2009) discussed the collaboration in a single-vendor single-buyer inventory system with lost sale and backordering, without defective items. They suggested that while JIT and collaboration keep the inventory level of the players low, they make the whole supply chain more vulnerable to lost sales and/or backorders. They showed that the joint inventory cost decreases by implementing their model as compared to a usual joint economic lot size (JELS) model. Chang and Ho (2009) revisited the work of Wee *et al*. (2007) and adopted renewal reward theorem to derive the expected profit per unit time, as suggested by Maddah and Jaber

(2008). They used algebraic methods to derive the exact closed-form solutions for optimal lot size, backordering quantity and maximum expected profit. Cárdenas-Barrón (2009) extended the work of Jamal *et al*. (2004) by bringing in backorders to the EPQ model for a single stage, single product model. Both the models suggest reworking the defective products in the same cycle.

Maddah *et al*. (2010) brought in the concept of order overlapping to avoid shortages in Salameh and Jaber (2000). They assumed that a new order is placed when the inventory is just enough to cover the demand during the screening period. They showed that the loss profit due to this order overlapping is negligible.

Two cases of shortages, i.e. lost sales and backordering will be introduced in chapter four where the screening for defective items starts at a slower rate as compared to the demand. Three cases for the transfer of learning will be discussed to enhance the study.

2.9 Consignment Stocks

Although not new, this technique has recently been revived in the inventory control literature. There has been a trend to adopt this strategy for storing inventory in a supply chain. This technique has been named as vendor-managed-inventory or consignment stock (e.g. Braglia and Zavanella, 2003; Persona *et al*., 2005). Consignment stocks can be defined as 'stocks owned by the vendor' but on the customer's premises and managed by the buyer. The importance of this stocking policy comes from the fact that it (*i*) saves the buyer the investment in inventory value, (*ii*) assures the supplier/vendor of an almost captive customer/buyer, and (*iii*) reassures the buyer that the supply is conveniently available (Wild, 2004). In the following, a brief review of the recent literature on consignment stock is presented.

Corbett (2001) analyzed two models of contracting on inventory policies for a supplier-buyer setup. He suggested that if supplier's long production cycles are the key driver of inventory, the supplier should be made to bear the costs of the resulting cycle stocks, e.g., through a consignment scheme. On the other hand, if uncertainty about down-stream demand is the main driver, the buyer should bear the costs of the resulting safety stock. He concluded that consignment stock helps reduce cycle stock by providing the supplier with an additional incentive to decrease batch size, but simultaneously gives the buyer an incentive to increase safety stock by exaggerating backorder costs. Valentini and Zavanella (2003) showed how consignment stock can be used in joint-profit maximizing models that use (s, S) policies. They used simulation experiments to demonstrate the rationale behind the implementation of a CS policy. They provided through an industrial case the tactical issues that a company has to address once it decides to adopt the CS policy.

Braglia and Zavanella (2003) demonstrated that the most evident difference between consignment stock and Hill (1997) conventional model lies in the location of the stocks. They modeled the CS problem analytically for a single vendor and a single buyer and compared their results with those of Hill (1997). They showed how CS policy might be profitable in situations where demand and delivery lead times are uncertain. Zanoni and Grubbstrom (2004) used Grubbstrom and Erdem (1999) approach to extend the theoretical solution proposed by Braglia and Zavanella (2003). They provided explicit analytical expressions for the optimal lot size, the optimal total number of deliveries, the number of deliveries to be delayed and the corresponding cost incurred. Srinivas and Rao (2004) provided a framework for the application of CS strategy for the stochastic systems. They reduced the effect of stochastic lead times by adding a crashing cost. They observed that the supply chain ends up having lower costs than the CS model while Hill's model always offers lower cost due to its deterministic demand profile.

Persona *et al*. (2005) proposed an analytical model to take into account the effects of obsolescence in a supply chain managed with a CS policy. They used Braglia and Zavanella (2003) deterministic model to show that (*i*) obsolescence reduces the optimal inventory level, particularly in the case of a short period of life, (*ii*) the effects of obsolescence on the correct estimation of the optimal shipment dimension are higher when the production rate is close to the demand rate. Li and Hong (2006) extended the model of Braglia and Zavanella (2003) for the case of defective deliveries from the vendor. They also studied the effect of the number of delayed deliveries on the annual cost of vendor-buyer supply chain. Liu *et al*. (2007) extended the model of Persona *et al*. (2005) under two different transportation policies. That is, transporting to two buyers simultaneously each time and transporting to one buyer alternately each time. They also analyzed the situation of indeterminate cycle time with obsolescence of the products, to show that high production flexibility and keeping outdated products in the vendor's site can lead to lower system costs.

Srinivas and Rao (2007) developed an inventory model for a single vendor single buyer where stochastic lead time in a CS strategy is controlled to minimize the joint expected total cost.

They suggested that a consignment stock-lead time (CS-LT) policy is best suitable for low or reasonable price items and when the demand is stochastic in nature. Lee and Wang (2008) studied the impact that the buyer's warehouse space capacity constraint has on the vendor's total set-up, inventory holding, and replenishment costs when there is a consignment purchasing policy between the vendor and the buyer. They derived a joint economic lot size model for the vendor's production lot and replenishment lot sizes.

Zavanella and Zanoni (2009) extended the model of Braglia and Zavanella (2003) for the case of a single-vendor, multi-buyer system, under the shared management of the buyers' inventory. They showed how the CS policy works better than the uncoordinated optimization. A sensitivity analysis was also carried out to study the influence of the parameters relevant to the economic performance of the supply chain. Huang and Chen (2009) extended the model of Braglia and Zavanella (2003) to highlight the fact that that all financial costs are borne by the vendor until the goods are used or sold. They divided the unit holding cost into a financial and a storage one to show that whether the CS model offers lower costs depends on the comparative values of buyer's and vendor's storage costs. Savasaneril and Erkip (2010) analyzed two settings of inventory management for a supply chain of a single vendor and a single buyer. These settings were: a traditional system where the buyer manages and owns the inventory, and a vendor-managed system. They modeled two cases for the vendor-managed system based on the ownership of stock. Under the no-consignment stock model (VM-NC), the stock was managed by the vendor while owned by the buyer. On the other hand, under the consignment stock model (VM-C), the inventory was both managed and owned by the vendor. They showed that under the vendor-managed system, a vendor can take a proactive approach in responding to a buyer's demand and thus can increase the capacity utilization.

Srinivas and Rao (2010) developed four analytical models with consignment stock strategy for a single-vendor–multi-buyer supply chain. That is (*i*) CS policy without delay deliveries, (*ii*) CS with delay deliveries, (*iii*) CS with information sharing and with delay model, and (iv) CS with controllable lead time. These scenarios are suitable for the contemporary supply chains having stochastic demand. Battini *et al.* (2010) extended the work of Braglia and Zavanella (2003), Valentini and Zavanella (2003), and Persona *et al.* (2005) to bring in the issues such as safety stocks, stock-out risk and restricted space availability. They demonstrated that consignment stock policy is always convenient when compared with the EOQ policy materials.

Consignment stock policy will be adopted in chapter seven for a two level (vendor-buyer) supply chain. Learning in production and errors in screening will be introduced to enhance the study.

2.10 Contemporary Trends in EOQ/EPQ Modelling

Balkhi (2004) investigated the effects of inflation and time value of money in a general EPQ model for an imperfect quality product. He assumed that production, demand, and deterioration rates are all known and differentiable functions of time. The model allowed for shortages but only a fraction of the stock out was backordered, and the rest was lost. Neither imperfect nor the deteriorated items were replaced or repaired. He used differential calculus to guarantee that the solution is minimal (optimal).

Tsou and Chen (2005*c*) proposed a power function to model the relationship between cost and yield rate of a JIT production system. They used geometric programming approach to obtain the optimal production cost and setup cost. Chung and Huang (2006) extended the work of Salameh and Jaber (2000) and Goyal (1985) by considering permissible delay in payments and interest charges on inventory costs of a buyer. They developed two theorems to determine buyer's ordering quantity. Chiu *et al*. (2006) examined a model for stationary and random defective rates.

Chiu *et al*. (2007) proposed a revised cost-benefit algorithm for solving the expediting completion time of end product (ECTEP) problem with defective components/materials in the product structure diagram. They used adjusted the unit procurement cost and the required quantity to address the defective materials procured. The critical path method and time-costing method were utilized in the proposed solution procedures for finding the optimal material procurement alternatives that minimize the end product unit manufacturing cost.

Darwish (2008) claimed that the setup cost and the cycle length in the classical EPQ model can be related to the process deterioration, learning and forgetting effects. He used the simplified form of the models given by Jaber and Bonney (2003) and developed two models for the optimal cost, that is, with and without shortages. He showed that the setup shape parameter defines the category of the production system under consideration, that is, smaller or larger lots.

Liao *et al*. (2009) presented an integrated EPQ and maintenance model by considering the impact of restoration actions such as imperfect repair, rework and preventive maintenance (PM) on the damage of a deteriorating production system. They claimed that if the PM learning rate is estimated based on the actual data, learning curves can be used to project the PM costs in the integrated EPQ model. Kelle *et al*. (2009) discussed yield uncertainty in a buyer – supplier cooperation. They extended Huang (2004) and Salameh and Jaber (2000) by assuming a known mean and variance of the random proportion of defective items. They examined two scenarios, that is (*i*) the buyer does the 100% inspection, and (*ii*) the supplier does the 100% inspection, to discuss the circumstances where the yield characteristics are (*i*) important or (*ii*) negligible. Maddah *et al.* (2009) extended the classical single period (newsvendor) and the economic order quantity (EOQ) models by accounting for random supply and imperfect quality items. The imperfect items were assumed to have their own demand and cost structure. They showed that incorporation of imperfect quality items can significantly increase the expected profit in many cases.

Hsu and Yu (2009) studied the situation where a supplier faces a surplus in inventory, or a change in the production run of a product. In either case, he/she may offer a special price discount to motivate buyers to buy in larger than normal order quantities. This work is an extension of Salameh and Jaber (2000) to investigate an EOQ model with imperfect items under a one-time-only sale. Hsu and Yu (2011) also extended the model of Salameh and Jaber (2000) to describe three scenarios for the time at which a price increase is introduced to motivate the buyer to order in larger than normal order quantities. Chen and Kang (2009) considered trade credit and imperfect quality in the integrated vendor–buyer model for a single product. They developed theorems to determine the optimal solutions of buyer's optimal replenishment period and frequency. They also suggested a profit sharing approach in which the vendor raises the warranty cost per defective item for the buyer to balance the cost saving between the vendor and buyer based on a coefficient of negotiation. The cost saving for the buyer would be negative in the integrated model if the warranty cost is the same as that in the non-integrated model. Under a long-term relationship, the vendor would raise the warranty cost per unit to compensate the buyer to achieve a win–win situation.

Nadjafi and Abbasi (2009) discussed another realistic situation where (*i*) the value or utility of goods, while in stock, may decrease in case of deteriorating items, and (*ii*) production run length influences the quality of goods. They developed an economic production quantity (EPQ) model considering both the depreciation cost of stored items and process quality cost. They assumed depreciation cost to be a continuous non-decreasing function of holding time, and process quality cost to be a continuous convex function of production run length. They solved the problem through simulated annealing (SA) and Iterated Local Search (ILS) and found ILS to be better.

2.11 Research Questions

The rationale for academic research extends far beyond the so-called economic benefits usually considered in our society. If on one hand it extends the depth and horizon of the area studied, the education of students on the other hand, which occurs in many academic projects turns out to be socially important as well (Mansfield, 1991). Inventory management is one of the most important areas of interest among modern researchers and practitioners. The industry has developed a lot in its course from using mere MS Access programs to advanced packages like ERP or SAP MM (Material Management Module), which is quite common today. The analytical research in this thesis can be of great help for engineers developing such packages. In other words, industry can benefit from this research instead of relying only on past data.

The survey of the related literature is summarized in Table 2.1. A cell with a cross means that the corresponding research issue has been addressed in the literature, whereas an empty cell reflects a research scarcity in the corresponding area. Table 2.1 shows that research addressing inventory control with learning in quality and processing time with inspection errors is quite limited and thus is a fertile field to be explored. As shown in Table 2.1, there are no available models in the literature that link inventory control in a supply chain with inspection errors along with learning in production and demand. Thus, there are following research questions to be answered:

1. What happens to the lot size policy if there are errors in inspection?
2. How does learning in inspection affect the inventory policy for a lot sizing problem?
3. How inspection errors affect order policies in a supplier-vendor supply chain?
4. What if there is learning in production and in suppliers' defective items?

5. What is the impact of inspection errors and learning in production in the traditional vendor buyer supply chain?

6. Is consignment stock (CS) a better policy than a centralized coordination one to manage inventory when defective items and errors in inspection exist in a vendor-buyer supply chain?

Thus, the shaded cells at the bottom of the table reflect the areas covered in the thesis.

Table 2.1 Research overview

Paper No	Area of Research / Author & Year of Publication	Imperfect Items	Inspection	Two Echelon Supply Chain	Rework	Piling up the Imperfect Items	Learning (Defectives)	Learning (Production/Rework)	Inspection Error	Three Echelon Supply Chain	Consignment Stock	Backlogging
1	Goyal 1977			X								
2	Raouf *et al.* 1983		X						X			
3	Banerjee 1986			X								
4	Porteus 1986	X			X							
5	Salameh & Jaber 2000	X	X									
6	Hayek & Salameh 2001	X	X		X							X
7	Goyal et al. 2003	X	X	X								
8	Kim & Ha 2003			X								
9	Khouja 2003									X		
10	Braglia & Zavanella 2003			X							X	
11	Jamal *et al.* 2004	X	X		X	X						
12	Jaber & Guiffrida 2004	X			X			X				
13	Maddah & Jaber 2008	X	X			X						
14	Jaber *et al.* 2008	X	X				X					
	The Thesis											

2.12 Research Methodology

The research objective in the thesis is to develop inventory models for a buyer or in a supply chain context with defective items. The issues to be explored in the thesis are: learning in production and rework, inspection errors, learning in inspection and learning in suppliers' defective items. A consignment stock alongside a centralized coordination strategy will also be studied to better manage inventory in the presence of defective items. Besides, the thesis will also present the behavior of the developed models with respect to parameters in learning and inspection. Analytical models would be developed to answer most of the research questions mentioned above. These models will be solved through closed forms or optimization in Mathematica or Excel Solver enhanced with VBA codes when necessary. Theorems explaining the behavior of the presented models may also be developed. Simulation will be used to carry out the statistical and sensitivity analysis of the developed models.

The layout for the thesis is given in Figure 2.1. Two models to extend Salameh and Jaber (2000) for the case of 'inspection errors in screening' and 'learning in screening' will be presented in chapters three and four respectively. The case of 'inspection errors in screening' will then be incorporated in a supplier-vendor supply chain with 'learning in production' and 'learning in supplier's quality' for two different coordination mechanisms in chapter five. A different two level supply chain will then be considered in chapters six and seven. That is the same human factors (learning in production and inspection errors in screening) will be studied in a vendor-buyer supply chain. A classical stocking policy and a consignment stock will be adopted in the two chapters respectively. The conclusion of the research and directions for future research will be outlined in chapter eight. More specifically, the research carried out is distributed in the following steps:

Chapter 3 This chapter will

1. Extend Salameh and Jaber (2000) for the case of inspection errors in the screening process instituted by a buyer.
2. Discuss managerial implications of the model.

Chapter 4 This chapter will

1. Extend Salameh and Jaber (2000) for the case of learning in inspection when there is total forgetting, partial and total transfer of learning from cycle to cycle.
2. Discuss managerial implications of the model.

Chapter 5 This chapter will

1. Extend Salameh and Jaber (2000) for a two level supplier-vendor supply chain to explore the three different coordination schemes given by Khouja (2003).
2. Study the effect of inspection errors on the schemes.
3. Study the effect of learning in production at the vendor's end.
4. Study the effect of learning in the supplier's defective items.

Chapter 6 This chapter will

1. Present a different two level supply chain (vendor-buyer) model, extending the model of Salameh and Jaber (2000) in the context of Hill (1997) approach.
2. Incorporate human factors like learning in production and inspection errors in the supply chain.
3. Discuss managerial implications of the model.

Chapter 7 This chapter will

1. Extend Salameh and Jaber (2000) for a vendor-buyer supply chain with consignment stock strategy given by Braglia and Zavanella (2003).
2. Incorporate human factors like learning in production and inspection errors in the supply chain.
3. Discuss managerial implications of the model and its differences with the model in chapter 6.

Chapter 8 This chapter will

1. Present the results and outcomes of the research carried out in the thesis.
2. Outline the future directions of potential research.

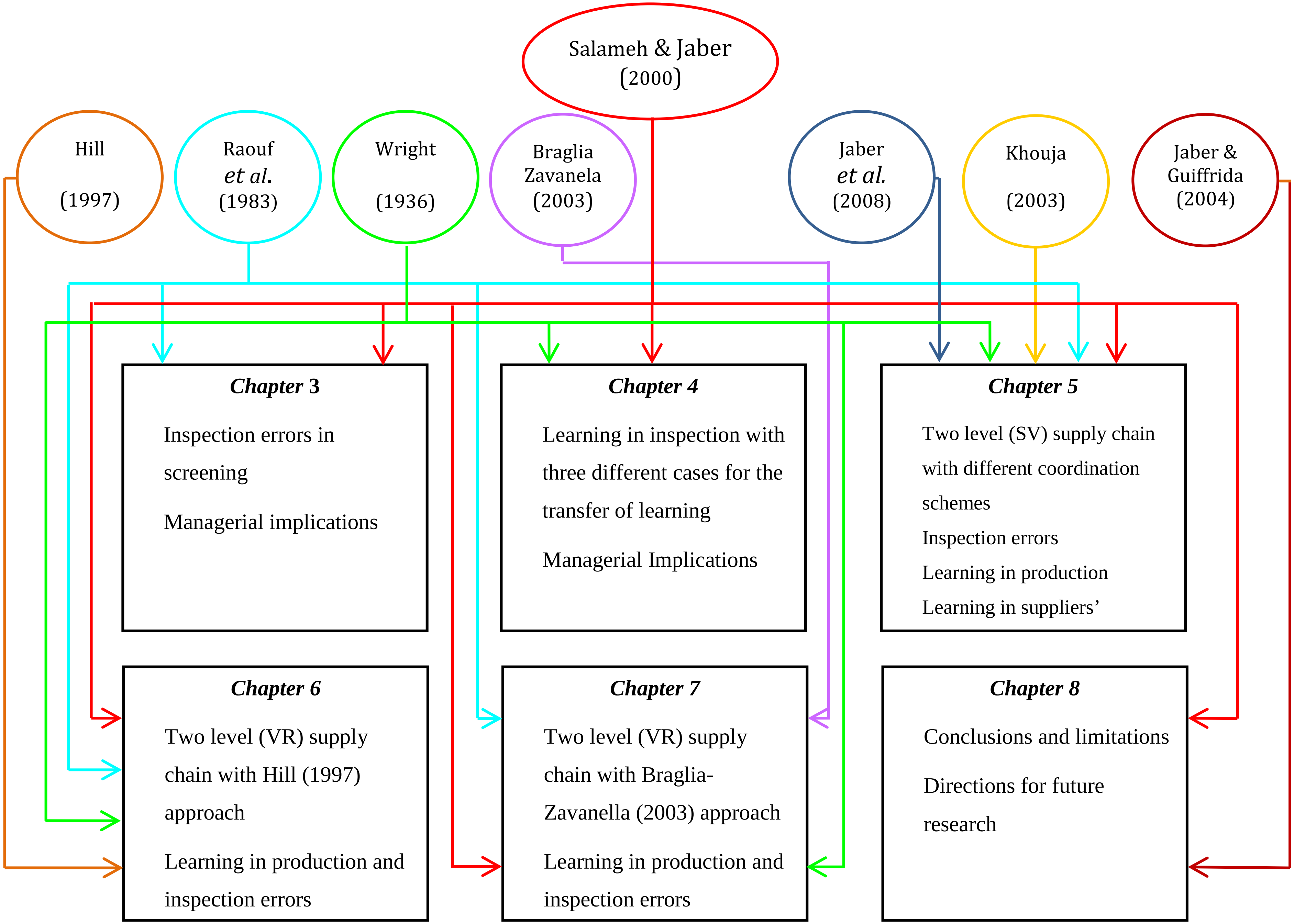

Figure 2.1 Layout of the thesis

CHAPTER 3 ECONOMIC ORDER QUANTITY (EOQ) FOR ITEMS WITH IMPERFECT QUALITY AND INSPECTION ERRORS

3.1 Introduction

Traditional inventory models tend to obtain an economic optimal order quantity (EOQ) or economic production quantity (EPQ) based on the ordering/setup cost and the inventory carrying cost. They make many assumptions while coming up with a closed form solution for the most economical batch size in a stock or a production facility. One assumption is that the items produced by the facility are all of a perfect quality. Another is that the screening process that identifies the defective items in a lot is error-free. That is, the defective items from a lot can be screened out through 100% inspection. This is an idealistic approach. In practice, the production lot may contain a substantial number of defective items, possibly because of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit (Rahim 1985). Also, the screening process, e.g. at the end of an assembly line, is never perfect. Hence, a need exists to determine an optimal order quantity when the inspection process is prone to making errors while screening a defective lot.

A number of researchers have worked to counter the perfect quality assumption above. Porteus (1986*a*) studied the effect of defective items on the basic EOQ model. He assumed that there is a fixed probability that the process would go out of control. Rosenblatt and Lee (1986) assumed that the time between the in-control and the out-of-control state of a process follows an exponential distribution and that the defective items are reworked instantaneously. They suggested producing in smaller lots when the process is not perfect. In a later paper, Lee and Rosenblatt (1987) studied a joint lot sizing and inspection policy for an EOQ model with a fixed percentage of defective products.

While most of the literature in this area deals with deterministic problems, many researchers have discussed stochastic production yield and demand rates. Gerchak *et al*. (1988) analyzed a single period production problem and extended it to an *n*-period problem where the production process has a variable yield and the demand is uncertain. Yano and Lee (1995) reviewed and identified shortcomings in the literature dealing with determining lot sizes where production or procurement yields are stochastic. Grosfeld-Nir and Gerchak (2004) also reviewed the literature on single stage and multistage imperfect production systems that involve random yield and inspection. Inderfurth (2004) determined an optimal production policy for a uniformly distributed demand and yield rate and discussed some managerial aspects of this policy. Rekik *et al*. (2007) extended the work of Inderfurth (2004) for two cases: (*a*) an additive errors case where the variability of errors is independent of the order quantity, and (*b*) a multiplicative errors case where the variability of errors is proportional to the order quantity.

The model of Salameh and Jaber (2000), suggested that the imperfect items are not reworked but just withdrawn from the received lot. It is also assumed that there is no human error in the screening process. Raouf *et al*. (1983) studied human error in inspection planning. They came up with one of the first inspection plans with misclassifications for multi-characteristic critical components. They suggested repeating the cycle of inspection to ensure the product quality and determined an optimal number of inspection cycles based on the cost of inspection and misclassifications. Duffuaa and Khan (2002) suggested an inspection plan for these critical components where an inspector can commit a number of misclassifications. They extended the Raouf *et al*. (1983) inspection plan for the case of a number of misclassifications. This was a realistic approach where an inspector can classify an item to be nondefective, reworkable or scrap. In a later paper, Duffuaa and Khan (2005) carried out a sensitivity analysis to study the effect of different types of misclassifications on the optimal inspection plan.

There may be many sources of errors in inspection, one of which is inaccuracy in records. Kök and Shang (2007) discussed inaccuracies in inventory records. They proved that an inspection adjusted base-stock policy is optimal for a single period problem, where inspection is performed if the inventory recorded is less than a threshold level. Atali *et al*. (2009) also modeled the discrepancies between actual and the recorded inventories in retail and distribution environments. They quantified the value of RFID (radio-frequency identification) that reduces the amount of such discrepancies. We leave this issue here for future research.

This chapter extends the work of Salameh and Jaber (2000) model by assuming that the screening process is not error-free. The Raouf *et al*. (1983) approach is used to suggest that an inspector could make two classifications while screening, i.e. an item may be classified as defective or nondefective. However, because the inspection process is not error free, a good item may be classified as defective, i.e. a Type I error, while a defective item may be classified as good, i.e. a Type II error (Jacobson, 1952). The rest of the chapter is organized as follows. Section 3.2 describes the model in the chapter. Section 3.3 produces a mathematical model. Section 3.4 presents a numerical example and discusses the results. Section 3.5 presents a summary and conclusions.

3.2 Model Description

Consider a lot of size Q being delivered to a buyer. It is assumed that each lot contains a fixed proportion γ of defective items. An inspector screens out the defective items from the lot with fixed rate of misclassifications. That is, a proportion m_1 of nondefective items are classified to be defective and a proportion m_2 of defective items are classified to be nondefective. It is assumed that γ, m_1 and m_2 are independent and identically distributed (iid) continuous random variables with known probability density functions, f_1, f_2 and f_3 (Appendix 3). Besides, it is quite intuitive that the suppliers' fraction of defectives would be independent of buyer's screening errors (Comeaux and Sarker, 2005). It is also assumed that the items that are returned from the market are stored with those that are classified as defective by the inspector. This is a reasonable assumption as the returned items are not repaired and are sold as defectives. They are all sold as a single batch at a discounted price in each cycle. The behavior of the inventory level is illustrated in Figure 3.1, where T is the cycle length, B_1 is the batch classified as defective by the inspector while B_2 is the batch of returned units from the market accumulated over T.

An optimal inventory policy is determined using the total revenues and the total costs. The costs considered in the model are the procurement cost, screening cost, misclassification cost and the inventory carrying cost.

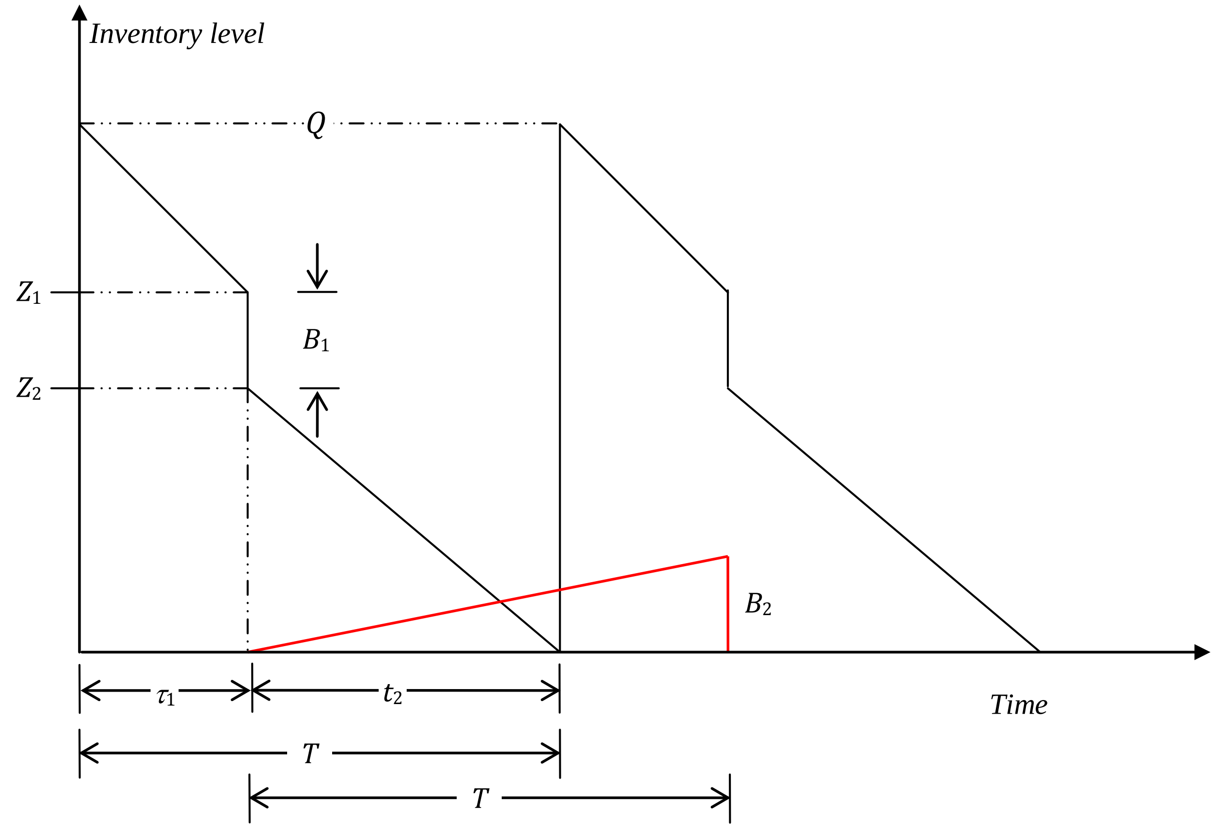

Figure 3.1 Behavior of the inventory level over time.

3.3 Mathematical Model

Figure 3.1 shows how the inventory behaves with a buyer. It should be noted here that Salameh and Jaber (2000) suggested this behavior. The screening and consumption of the inventory continues until time τ_1, after which all the defectives (B_1) are withdrawn from inventory as a single batch and are sold to the secondary market. The consumption process continues at the demand rate until the end of cycle time T. Due to inspection error, some of the items used to fulfill the demand would be defective. These defective items are later returned to the inventory and are shown in Figure 3.1 as B_2. The position of B_2 is chosen to ensure the disposal of two defective batches (B_1 and B_2) at the same time. The cycle inventory is kept at a minimum level this way. It may be beneficial to sell the defective lot in two batches (B_2' and B_2'') in a cycle, as shown in Figure 3.2, where the holding cost of the returned inventory is less; however, implicitly, this may incur additional cost such as transportation (fixed and variable cost), which makes the savings in inventory less than the additional costs incurred. So, in this chapter, it is assumed here that the two types of defective items (screened and misclassified) are sold at the same time, as shown in Figure 3.1. The model in Figure 3.2 may be addressed in a technical note sometime in the near future.

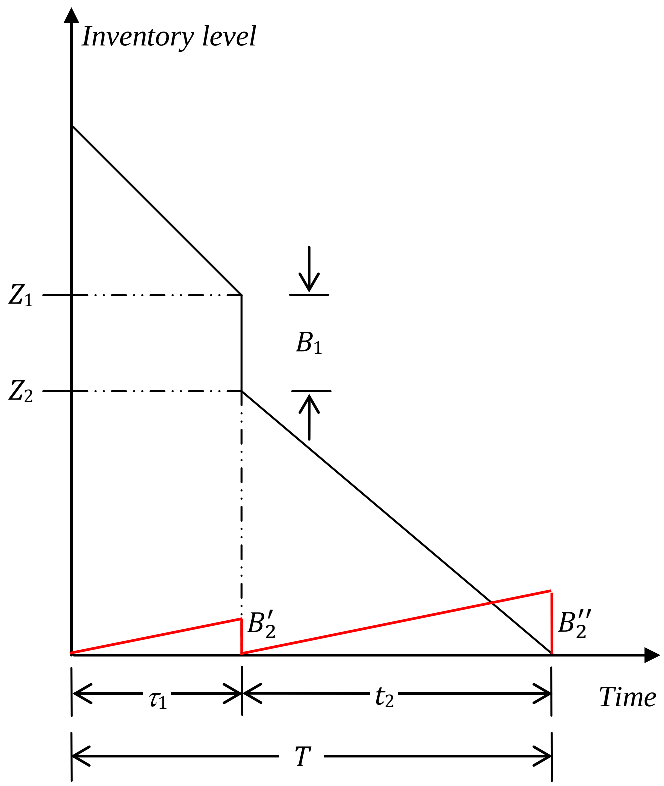

Figure 3.2 Selling the defective items twice in a cycle

To avoid shortages, it is assumed that the number of nondefective items is at least equal to the adjusted demand, that is the sum of the actual demand (D) and items that are replaced for the ones returned ($\gamma m_2 D$) from the market over T. Thus

$$Q - Q(1-\gamma)m_1 - Q\gamma(1-m_2) \geq DT + \gamma m_2 Q$$

$$Q(1-\gamma) - Qm_1(1-\gamma) \geq DT$$

$$Q(1-\gamma)(1-m_1) \geq DT$$

So, for the limiting case, the cycle length (which includes the screening time) can be written as

$$T = \frac{Q(1-\gamma)(1-m_1)}{D} \tag{3.1}$$

It should be noted that the above expression is unaffected by the Type II error and reduces to the cycle length in Salameh and Jaber (2000) if the Type I error becomes zero.

Consider now the different cases of misclassifications that an inspection process can have. There are four possibilities in such an inspection process. Those are: Case (1) A nondefective item is classified as nondefective; Case (2) A nondefective item is classified as defective; Case (3) A defective item is classified as nondefective and Case (4) A defective item is classified as defective. This scenario is depicted in Figure 3.3 below. The number of items going into different categories following these cases is given by:

Case (1): $Q(1-\gamma)(1-m_1)$

Case (2): $Q(1-\gamma)m_1$

Case (3): $Q\gamma m_2$

Case (4): $Q\gamma(1-m_2)$

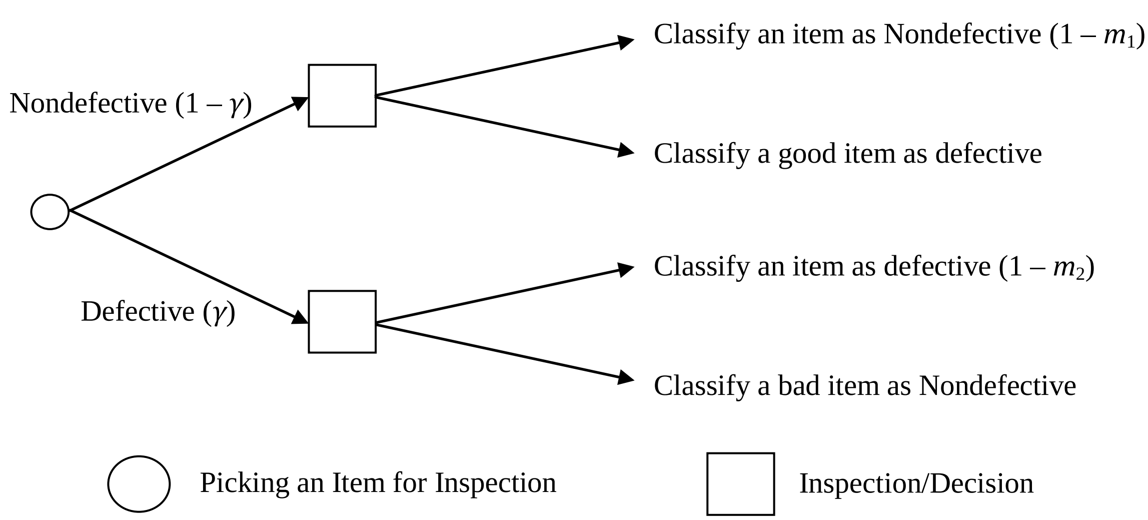

Figure 3.3 Four possibilities in the inspection process

Now B_1 and B_2 are given by

$$B_1 = Q(1-\gamma)m_1 + Q\gamma(1-m_2)$$

$$B_2 = Q\gamma m_2$$

The items in batch B_2 are returned from the market at the rate $Q\gamma m_2/T$ and are taken from the inventory with batch B_1. Therefore the revenue from salvaging $B = B_1 + B_2$ items within a cycle are given by:

$$R_1 = v(B_1 + B_2) = vQ(1-\gamma)m_1 + vQ\gamma(1-m_2) + vQ\gamma m_2$$

or

$$R_1 = vQ(1-\gamma)m_1 + vQ\gamma$$

The revenue from selling the good items is computed as

$$R_2 = s_1Q(1-\gamma)(1-m_1) + s_1Q\gamma m_2$$

So, the total revenue is given as

$$R = R_1 + R_2$$

$$R = s_1Q(1-\gamma)(1-m_1) + s_1Q\gamma m_2 + vQ(1-\gamma)m_1 + vQ\gamma \tag{3.2}$$

Consider now the different costs of the inventory system. The procurement cost per cycle is

$$PC = A_b + c_1Q \tag{3.3}$$

where c_1 is the variable cost. The screening cost per cycle is the sum of the costs of inspection and misclassifications which is given by

$$IC(Q) = dQ + c_r(1-\gamma)Qm_1 + c_aQ\gamma m_2 \tag{3.4}$$

The holding cost per cycle is the cost of carrying the (*i*) nondefective lot, (*ii*) defective lot and (*iii*) returned lot. So, from Figure 3.1 the holding cost for a cycle can be written as

$$HC(Q) = h\left\{\frac{(Q-Z_1)\tau_1}{2} + Z_1\tau_1 + \frac{Z_2t_2}{2}\right\} + h\left(\frac{B_2T}{2}\right)$$

where Z represents the stock level before and after the screening/inspection process. Replacing B_2 with $Q\gamma m_2$, Z_2 with $Z_1 - B_1$, Z_1 with $Q - D\tau_1$ and τ_1 with D/x, it can be written as

$$HC(Q) = \frac{h_b Q^2}{2}\left\{\frac{2}{x} - \frac{D}{x^2} + \frac{V^2}{D}\right\} + h\left(\frac{Q\gamma m_2 T}{2}\right) \tag{3.5}$$

where $V = 1 - \frac{D}{x} - (m_1 - \gamma) + \gamma(m_1 + m_2)$

Therefore the total cost per cycle is given by summing up Eq. (3.3), Eq. (3.4) and Eq. (3.5), i.e.

$C_b = PC + IC + HC$ or

$$C_b(Q) = A_b + c_1 Q + dQ + c_r(1-\gamma)Qm_1 + c_a Q\gamma m_2 + \frac{h_b}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{V^2}{D}\right)Q^2 + Q\gamma m_2 T\right\}$$

The term V^2 is simplified in Appendix 2. Figure 3.1 depicts the behavior of different types of inventory in the order cycle. The triangle at the bottom represents the defective lot that is returned by the market and is accumulated into the salvaged lot.

The total profit per cycle can now be written as the difference between the total revenue and total cost per cycle, that is

$$\begin{aligned} TP(Q) = {} & s_1 Q(1-\gamma)(1-m_1) + sQ\gamma m_2 + vQ(1-\gamma)m_1 + vQ\gamma \\ & - \left[A_b + c_1 Q + dQ + c_r(1-\gamma)Qm_1 + c_a Q\gamma m_2 + \frac{h_b}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{V^2}{D}\right)Q^2 + Q\gamma m_2 T\right\}\right] \end{aligned} \tag{3.6}$$

Since γ, m_1 and m_2 are assumed in this thesis to be independent and identically distributed random variables with probability density functions $f_1(\gamma)$, $f_2(m_1)$ and $f_3(m_2)$, respectively. The expected total profit can be written as

$$\begin{aligned} \mathrm{E}[TP(Q)] = {} & s_1 Q(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1]) + sQ\mathrm{E}[m_2]\mathrm{E}[\gamma] + vQ(1-\mathrm{E}[\gamma])\mathrm{E}[m_1] \\ & + vQ\mathrm{E}[\gamma] - A_b - c_1 Q - dQ - c_r(1-\mathrm{E}[\gamma])Q\mathrm{E}[m_1] - c_a Q\mathrm{E}[m_2]\mathrm{E}[\gamma] \\ & - \frac{h_b}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{\mathrm{E}[V^2]}{D}\right)Q^2 + Q\mathrm{E}[m_2]\mathrm{E}[T]\mathrm{E}[\gamma]\right\} \end{aligned} \tag{3.7}$$

Now from Eq. (3.1), the expected cycle length would be:

$$\mathrm{E}[T] = \frac{Q(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])}{D} \tag{3.8}$$

Maddah and Jaber (2008*b*) corrected the approach in the Salameh and Jaber (2000) model to determine the annual profit. They suggested using renewal reward theorem. Using this new approach, the expected annual profit for our model, is written as

$$\mathrm{E}[TPU(Q)] = \frac{\mathrm{E}[TP(Q)]}{\mathrm{E}[T]}$$

or

$$\mathrm{E}[TPU(Q)] = s_1 D + \frac{sD\mathrm{E}[\gamma]\mathrm{E}[m_2]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])} + \frac{vD\mathrm{E}[m_1]}{(1-\mathrm{E}[m_1])} + \frac{vD\mathrm{E}[\gamma]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])} -$$

$$\frac{D\left[\frac{A_b}{Q} + c_1 + d + c_r(1-\mathrm{E}[\gamma])\mathrm{E}[m_1] + c_a\mathrm{E}[\gamma]\mathrm{E}[m_2] + \frac{h_b}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{\mathrm{E}[V^2]}{D}\right)Q\right\}\right]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])} \quad (3.9)$$

$$-\frac{h_b Q\mathrm{E}[\gamma]\mathrm{E}[m_2]}{2}$$

It should be noted that Eq. (3.9) converges to the expected annual profit equation in Maddah and Jaber (2008*b*) once the value of errors goes to zero. It can be demonstrated that this expected annual profit follows a concave function. The first derivative of Eq. (3.9) is given by

$$\frac{\mathrm{d}}{\mathrm{d}Q}\mathrm{E}[TPU(Q)] = -\frac{D\left[-\frac{A_b}{Q^2} + \frac{h_b}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{\mathrm{E}[V^2]}{D}\right)\right\}\right]}{(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])} - \frac{h\mathrm{E}[\gamma]\mathrm{E}[m_2]}{2}$$

The second derivative of Eq. (3.9) is

$$\frac{\mathrm{d}^2}{\mathrm{d}Q^2}\mathrm{E}[TPU(Q)] = -\frac{2A_b D}{Q^3(1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1])}$$

Since $A_b > 0$, $D > 0$, $0 < E[\gamma] < 1$, and $0 < E[m_1] < 1$, then $\frac{\mathrm{d}^2}{\mathrm{d}Q^2}\mathrm{E}[TPU(Q)] < 0$ for every $Q > 0$, suggesting that the annual profit in Eq. (3.9) is concave. This fact is also demonstrated graphically in the next section. Thus, the optimal order size that represents the maximum annual profit, is determined by setting the first derivative equal to zero and solving for Q to get

$$Q^* = \sqrt{\frac{2A_bD}{h_b\mathrm{E}[\gamma]\mathrm{E}[m_2](1-\mathrm{E}[\gamma])(1-\mathrm{E}[m_1]) + h_bD\left(\frac{2}{x} - \frac{D}{x^2} + \frac{\mathrm{E}[V^2]}{D}\right)}} \tag{3.10}$$

It should be noticed that the denominator in Eq. (3.10) remains always positive, since $x > D$, 0 , $\mathrm{E}[\gamma] < 1$, $0 < \mathrm{E}[m_1] < 1$, $h_b > 0$, and $\mathrm{E}[V^2] > 0$ from Appendix 2. Note also that when $\gamma = m_1 = m_2 = 0$, Eq. (3.10) reduces to the traditional EOQ formula.

3.4 Numerical Analysis

Consider a production system that replenishes the buyer's orders instantly. This system is not perfect, i.e. it produces some defective items. The inspection process that screens out the defective items is also imperfect. The probability density functions for the fraction of defective items and the inspection errors are mostly taken from the history of a supplier/machine and workers. In the case when these values are not known, the fraction of defectives in a lot can be determined by using the lot size, as in Porteus (1986*a*) and Urban (1998), or the time at which a process goes out-of-control in a cycle, as in Rosenblatt and Lee (1986). Similarly, the parameters for inspection errors can be determined by the methods suggested by Cary *et al*. (1994) or Jaraiedi (1983). In the following analysis, most of the data is taken from the Salameh and Jaber (2000) model.

D	=	50000 units/year,	x	=	1 unit/min,
c_1	=	\$ 25/unit,	d	=	\$ 0.5/unit,
A_b	=	\$ 100/cycle,	h_b	=	\$ 5/ unit,
s_1	=	\$ 50/unit,	c_a	=	\$ 500/unit,
v	=	\$ 20/unit,	c_r	=	\$ 100/unit,

$$f_1(\gamma) = \begin{cases} 1/(0.04-0), & 0 \le \gamma \le 0.04 \\ 0, & otherwise \end{cases} \Rightarrow \mathrm{E}(\gamma) = 0.02$$

$$f_2(m_1) = \begin{cases} 1/(0.04-0), & 0 \le m_1 \le 0.04 \\ 0, & otherwise \end{cases} \Rightarrow \mathrm{E}(m_1) = 0.02$$

$$f_3(m_2) = \begin{cases} 1/(0.04-0), & 0 \le m_2 \le 0.04 \\ 0, & otherwise \end{cases} \Rightarrow \mathrm{E}(m_2) = 0.02$$

Assuming that the buyer operates for 8 hours per day for 365 days per year, the annual screening rate would be, x = 1(60)(8)(365) = 175200 units. Substituting above values in Eq. (3.10) and (3.9) respectively, we obtain the optimal values of the order size and the annual profit as:

$Q*$ = 1454 units

$\mathrm{E}[TPU(Q)]$ = \$ 1095090/year,

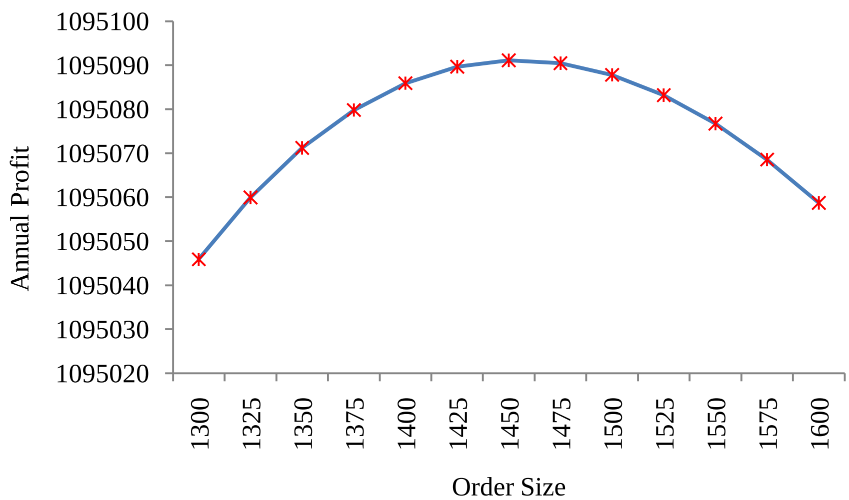

Figure 3.4 Expected annual profit is a concave function of the order size

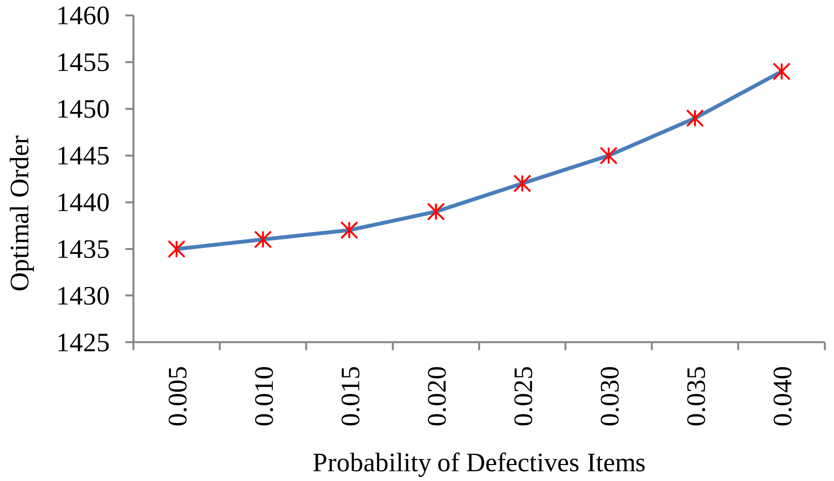

Figure 3.5 Optimal order size with respect to the probability of defectives

Figure 3.4 demonstrates the concavity of the annual profit with respect to the order size. That is, there is an optimal order size for the input parameters taken here, with respect to the annual profit. One should notice that although Figure 3.4 does not show much variation in annual profit with the order size, there is a noticeable deviation in the costs and profits per cycle as the order size moves away from the optimal one. For example, the cycle profit changes by 11% ($27344 from $30584) and 10% ($33654 from $30584) respectively if the order size per cycle is moved from the optimal value to 1300 and 1600.

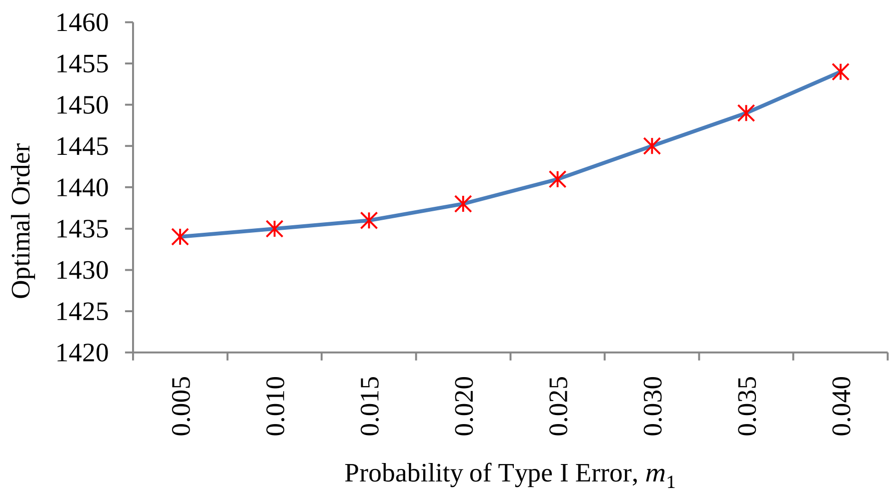

Figure 3.6 Optimal order size with respect to Type I Error

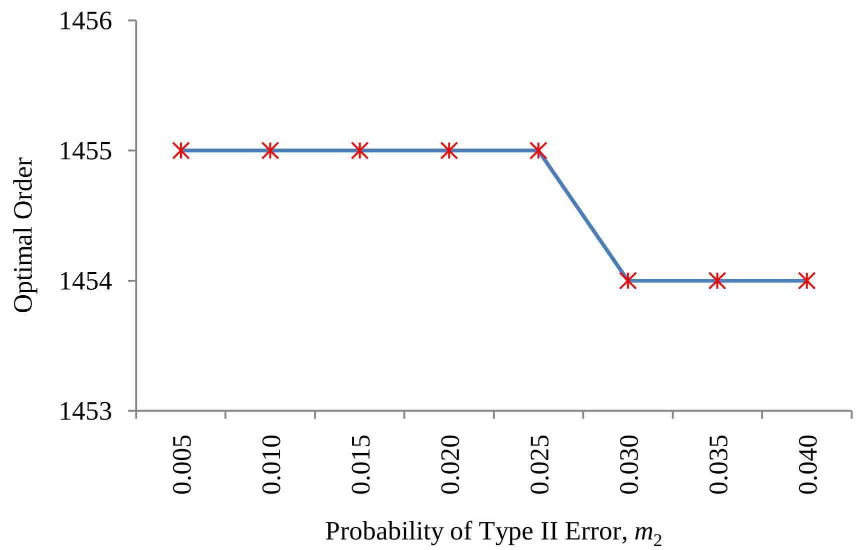

Figure 3.7 Optimal order size with respect to Type II Error

The results in our numerical example indicate that the optimal order quantity is almost the same as it was in the Salameh and Jaber (2000) model but the cost of misclassifications, i.e. false rejection and false acceptance cause a huge drop in the annual profit as compared to that in the Salameh and Jaber (2000) model. This specifies the significance of the human errors in inspection. A different example was tried with a lower profit margin (s_1 = \$30) but it made no difference on the order size though the annual profit is reduced by 91%. The rationale for this is that the profit margin ($s_1 - c_1$) in Eq. (3.9) is independent of the order size. This fact prevails throughout the thesis. One should also observe that the loss in profit has a direct relation to the fraction of defectives in the supplier's lot. To further elaborate on the impact of the different random fractions in the model, Figures 3.5, 3.6 and 3.7 represent the effect of the fraction of defectives, Type I error and Type II error respectively, on the optimal order size. We noticed that the first two of these three factors tend to increase the order size while the third one slightly decreases it. The rationale for this is that the more the defective items the more is the number of non-defective items needed by the buyer. Similarly, Type I error takes away more and more non-defective items and makes the buyer order more. But on the other hand, Type II error makes the buyer order less. It should be noticed that the order quantity is insensitive to changes in the value of Type II error

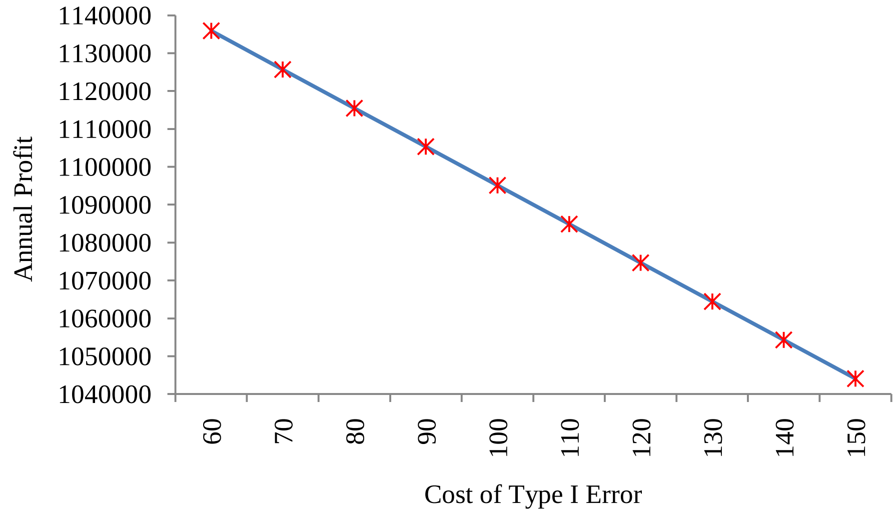

Figure 3.8 Annual profit with respect to the cost of false rejection

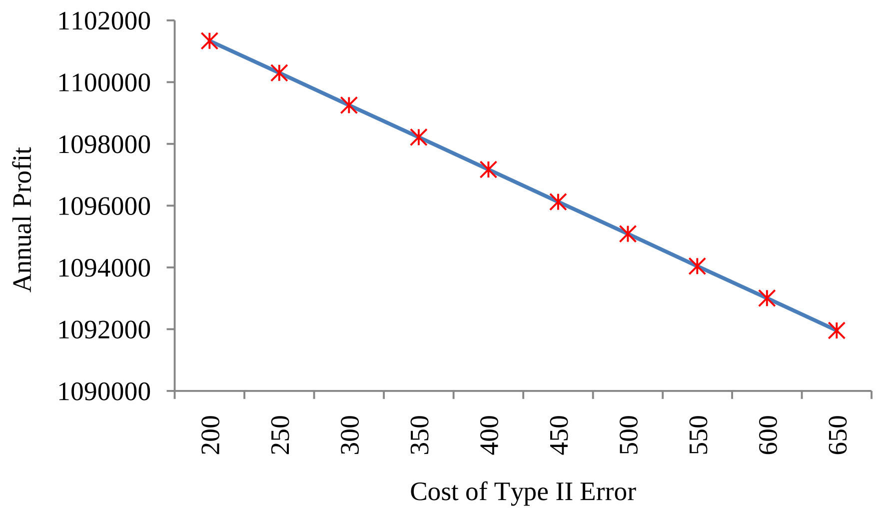

Figure 3.9 Annual profit with respect to the cost of false acceptance

Figure 3.8 and 3.9 show the impact of the cost of false rejection and false acceptance respectively, on the annual profit. As one should expect, they both tend to decrease the annual profit. It should be noticed that the two costs of error have different impact on the profit and thus have different range of values. That is why, they were shown in separate figures.

Figure 3.10 shows the impact of fraction of defectives on the behavior of annual profit, for the suggested model. This curve is obtained by varying the upper limit of the pdf of the fraction of defectives at a fixed (0.04) level of both inspection errors (Type I and Type II). The order size per cycle is kept fixed at 1439, i.e. of Salameh and Jaber (2000). It is clear that the annual profit tends to decrease with the fraction of defectives as more and more items would have to be sold at a discounted price.

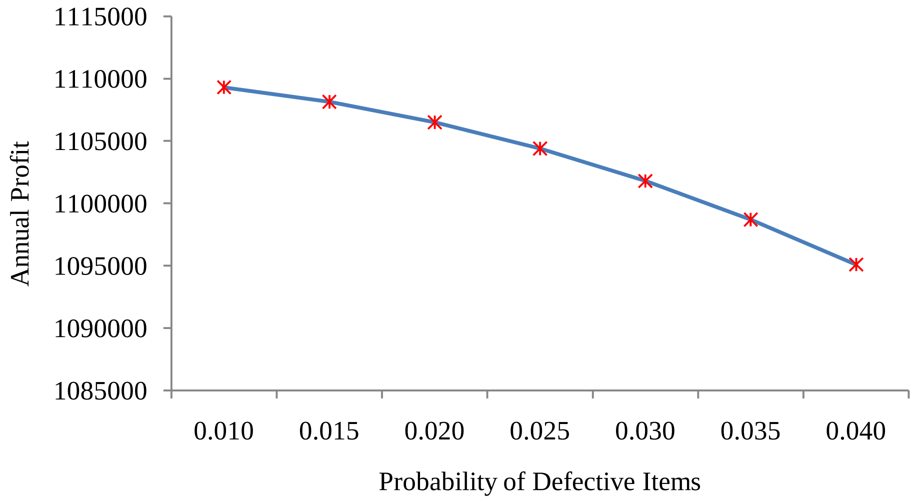

Figure 3.10 Relationship between the annual profit and the fraction of defectives

3.5 Managerial Implications

The model of Salameh and Jaber (2000) has recently been extended for a number of practical circumstances such as reworks, shortages and supply chain. The model in this chapter is an addition to this line of research. The approach adopted here provides the practitioners with a more functional alternative to the Salameh and Jaber (2000) model because it incorporates the screening costs more accurately in the economic order sizing decision.

It should be emphasized that the inspection part in the Salameh and Jaber (2000) model is suitable for buyers that have an automated screening system where one can expect no errors. On the other hand, if the characteristic of interest cannot be screened through a machine and is inspected by human beings, the screening process is bound to have misclassifications. For example, a superstore would screen the incoming products and sell the nondefective ones to its customers. It would usually sell the items of low quality at a discount price. The returned defective items, that were misclassified, would also be sold at the discount price. On the other hand, the misclassified nondefective items would incur the store a loss. The same course of action applies to a manufacturing firm that utilizes supplier items in its assembly lines. after inspection and It sells the defective items to a secondary market, after inspection. These misclassifications are critical if the parts under inspection are of an aircraft, a space shuttle or a complex gas ignition system. The reason is that the results of misclassification in such systems could be fatal. That is why the quality requirements for such components are very tight, Zunzanyika and Drury (1975). A common practice in the industry is to institute multiple inspections to reduce the effect of errors, for these components, Chandra and Schall (1988). So, it is vital for a buyer to be aware of not only the accurate parameters of error about his inspectors but also the ways to mitigate these errors.

3.6 Summary and Conclusions

This chapter makes use of Salameh and Jaber (2000) and Duffuaa and Khan (2002) models to determine an inventory policy for imperfect items received by a buyer. A realistic approach of screening is adopted. That is, an inspector may classify a nondefective item to be defective (Type I error) and he may also classify a defective item to be nondefective (Type II error). The defective items classified by the inspector and those returned from the market are accumulated and sold at a discounted price at the end of each procurement cycle.

The model in this chapter suggests that the annual profit with inspection errors remains concave with respect to the order size. Comparing the results with those in Salameh and Jaber (2000), the optimal order size is almost the same but the annual profit is much smaller. This signifies the effect of inspection errors. The increasing fraction of defectives keeps on reducing the annual profit. The fraction of defectives and Type I error both increase the order size while Type II error decreases the order size. Our results also suggested that the costs of Type I and Type II errors both cut off some of the annual profit.

The model in this chapter could be extended for the case where demand is uncertain. Furthermore, learning in the inspection rate would also enhance the usefulness of the model presented here.

CHAPTER 4 ECONOMIC ORDER QUANTITY MODEL FOR ITEMS WITH IMPERFECT QUALITY WITH LEARNING IN INSPECTION

4.1 Introduction

The EOQ is the earliest, simplest and most appreciated inventory model in the literature, (Osteryoung *et al*., 1986). Its popularity amongst academicians and businesspeople has been attributed to the ease of manipulation and calculation (Woolsey, 1990). However, some of its assumptions are never met in practice (Jaber and Sikström, 2004). One of these assumptions is that items ordered (or produced) are of perfect quality (Cheng, 1991).

One fertile area in this line of research is that the production facilities do overcome their shortages through a human phenomenon known as learning. That is, the workers tend to produce faster as they spend more time on the same machine in a line. This natural phenomenon was shown by Wright (1936) learning curve. With an interruption in the production, the workers tend to forget part of their skills. The earliest work in the literature that investigated the lot sizing problem with learning and forgetting is Keachie and Fontana (1966). Since there has been some interest in this subject, Jaber and Bonney (1998) provided almost a comprehensive survey (for the period 1966-1998). Some of the works in this line of research are, but not limited to, Balkhi (2003); Chiu *et al*. (2003), Chiu and Chen (2005), Jaber and Bonney (2007), Alamri and Balkhi (2007), Jaber and Bonney (2003), Jaber and Bonney (2007) and Jaber *et al*. (2009). Although there is consensus among these works on the form of the learning curve, it was not so for forgetting, Jaber (2006*b*). However, Jaber and Bonney (1996) developed a learn-forget curve model (LFCM) that, properly represents the learning-forgetting process, Jaber and Bonney (1997); Jaber *et al*. (2003); Jaber and Sikström (2004).

There are situations where the time to inspect defective items follows a learning curve, e.g. (Sikström and Jaber, 2002). None of the above surveyed articles and those available in the literature investigated the model of Salameh and Jaber (2000) for learning in inspection. This chapter addresses this research scarcity and extends Salameh and Jaber (2000) in two directions. First, this chapter considers that the screening rate of defectives follows a learning curve where stock-out may occur when the screening rate is slower than the demand rate. These stock-outs can be treated as either lost-sales or backorders. Second, this chapter also includes the transfer of knowledge in learning when the production moves from one cycle to another in three possible scenarios: (*i*) no transfer of learning, (*ii*) complete transfer of learning, and (*iii*) partial transfer of learning.

The remainder of this chapter is organized as follows. Section 4.2 provides a brief introduction to the model of Salameh and Jaber (2000) and to the learning and forgetting theory. Section 4.3 and 4.4 are for mathematical modeling. Section 4.5 provides numerical examples and discusses results. Section 4.6 is for the conclusions of the research in this chapter.

4.2 Mathematical Model

Consider a buyer, as in Salameh and Jaber (2000) where the inspection time in a cycle is τ_i. Assume that workers screen these items at the rate x, which goes up with the passage of time, following learning. Unlike their work, it is assumed that the screening rate is less than the demand D, in the beginning, and catches up with time by virtue of learning. The demand is not fulfilled during time t_{si} and there is a shortage of y_{si} items in cycle i. These shortages will be dealt with as both lost sales and backorders in this chapter. It is important to note that the time t_{si} may become zero in the subsequent cycles due to learning. Three scenarios will be considered for the transfer of learning from cycle to cycle: (*i*) partial forgetting (partial transfer of learning), (*ii*) total learning (total transfer of learning) and (*iii*) total forgetting (no transfer of learning). The optimum cycle time T_i and the order size Q_i will be obtained through maximizing the expected annual profit.

Inspection is usually a manual task where an inspector tests incoming units for specific quality characteristics to determine if the units conform to the quality requirements. Time to inspect (or screen) each unit reduces as the number of inspected units increase and is represented as $x_n = x_1 n^{-b}$, where $x_1 = {}^{1}\!/_{\pi_1}$and $x_n = {}^{1}\!/_{\pi_n}$ (assuming π_n to be the time to screen the *nth* unit); whereas Salameh and Jaber (2000) assumed $x_1 = x_n > D$. Here, it is assumed that $x_1 < D$ for $y_s < y$. It is also assumed that items are subject to 100% screening and defective units will never make it to customers, meaning that if $x_n < D$, then the demand met is x_n and the rate at which units lost or backordered is $D - x_n$, and D otherwise. The length of the stock-out period or the period over which backorders are accumulated is given as

$$t_{si} = \int_{u_i}^{Q_i+u_i} \frac{1}{x_1} y^{-b} dy - \int_{Q_{si}+u_i}^{Q_i+u_i} \frac{1}{x_1} y^{-b} dy = \frac{\left[(Q_{si}+u_i)^{1-b} - {u_i}^{1-b}\right]}{(1-b)x_1} \tag{4.1}$$

where $Q_{si} = [(1-b)x_1 t_{si} + {u_i}^{1-b}]^{\frac{1}{1-b}} - u_i$. Based on the fact that Q_{si} is utilized to fulfill the demand during time t_{si} , one can write

$$Q_{si} = \left({}^{D}/{}_{x_1}\right)^{\frac{1}{b}} - u_i.$$

Now the time to screen Q_i items in a cycle.

$$\tau_i = \frac{\left[(Q_i+u_i)^{1-b} - {u_i}^{1-b}\right]}{(1-b)x_1} \tag{4.2}$$

In case of no transfer of learning, that is, a worker does not retain any knowledge from earlier cycles ($u_i = 0$), it will be taken as

$$Q_s = Q_{si} = \left({}^{D}/{}_{x_1}\right)^{\frac{1}{b}} \tag{4.3}$$

Substituting it in Eq. (4.1)

$$t_s = t_{si} = \frac{D^{\frac{1-b}{b}}}{(1-b)x_1^{1/b}} \tag{4.4}$$

$$\tau = \tau_i = \frac{Q^{1-b}}{(1-b)x_1} \tag{4.5}$$

Similarly, in the case of total transfer of learning, Eqs. (4.1) and (4.2) are used with $u_i = \sum_{n=1}^{i} Q_n$.

Two cases (lost sales and backorders) will be considered now to deal with the shortages, in each of the three scenarios for the transfer of learning from one cycle to another. These models are a direct extension to the work of Salameh and Jaber (2000).

4.3 Lost Sales

In this case, the demand that cannot be fulfilled due to slow screening will be taken as lost sale. By virtue of learning, the screening will become equal to or more than the demand and there won't be any lost sales after some cycles. Three scenarios of learning will be considered now to develop a mathematical model for the expected annual profit of a buyer.

4.3.1 *Lost sales for partial transfer of learning*

The behavior of inventory in the lost sales case is shown in Figure 4.1, which is duplicated in subsequent cycles. In the beginning, the screening rate is slower than the demand rate. Thus, the demand is lost till the time t_{si}. At this point, the screening rate and the demand rate become equal. In the rest of the cycle, inventory behaves as it does in Salameh and Jaber (2000). In case of partial transfer of learning (i.e. partial forgetting), a worker loses part of his/her knowledge while he/she is not screening during the break which is T_i – τ_i. It should be noted that the inspection time keeps on decreasing due to transfer of learning. This affects the overall cycle length and the annual profit.

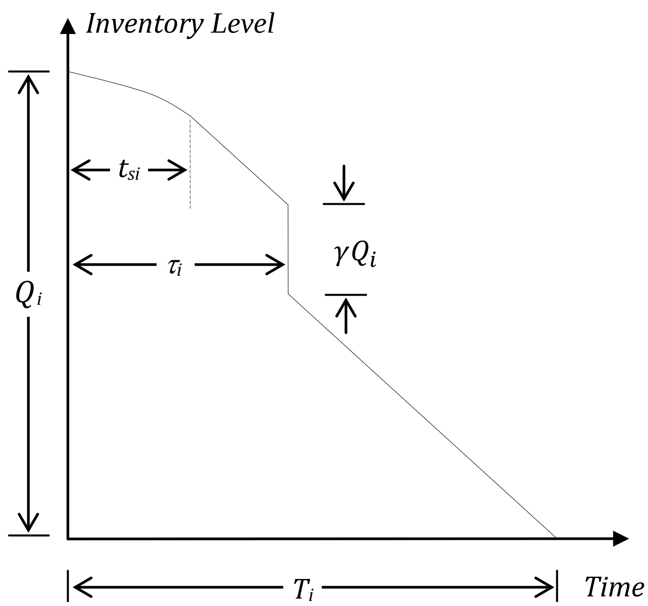

Figure 4.1 Learning in inspection with lost sales

This model differs from that of Salameh and Jaber (2000) in two ways; slower screening in the beginning of the cycle changes the inventory profile of the cycle, and it introduces a stock-out cost. The inventory level in a cycle, at time *t* in Figure 4.1 is represented as:

$$I_i(t) = \begin{cases} Q_i - [(1-b)x_1 t]^{1/(1-b)}, & 0 \le t < t_{si} \\ Q_i - Q_{si} - D(t - t_{si}), & t_{si} \le t < \tau_i \\ (1-\gamma)Q_i - Q_{si} - D(t - t_{si}), & \tau_i \le t < T_i \end{cases} \tag{4.6}$$

At time t = T_i, the inventory level can be assumed to be zero as discussed in chapter 1 directly above Figure 1.5, i.e. $(1-\gamma)Q_i - Q_{si} - D(t - t_{si}) = 0$, and the cycle time is given as

$$T_i = \frac{(1-\gamma)Q_i}{D} - \frac{Q_{si}}{D} + t_{si} \tag{4.7}$$

The holding costs for the three different behaviors of inventory shown in Figure 4.1 are determined respectively from (4.6) as:

$$\begin{aligned} HC_{1l}(Q_i) &= h_b \int_0^{t_{si}} \left[Q_i - \{(1-b)x_1 t\}^{1/(1-b)}\right] dt \\ &= hQ_i t_{si} - h\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} \end{aligned} \tag{4.8}$$

$$\begin{aligned} HC_{2l}(Q_i) &= h_b \int_{t_{si}}^{\tau_i} [Q_i - Q_{si} - D(t - t_{si})]dt \\ &= h_b(Q_i - Q_{si} + Dt_{si})(\tau_i - t_{si}) - \frac{h_b D}{2}(\tau_i^2 - t_{si}^2) \end{aligned} \tag{4.9}$$

$$\begin{aligned} HC_{3l}(Q_i) &= h_b \int_{\tau_i}^{T_i} [(1-\gamma)Q_i - Q_{si} - D(t - t_{si})]dt \\ &= h_b(Q_i - Q_{si} + Dt_{si})(T_i - \tau_i) - h_b\gamma Q_i(T_i - \tau_i) - \frac{h_b D}{2}(T_i^2 - \tau_i^2) \end{aligned} \tag{4.10}$$

Adding the costs for three different behaviors, we can get the total holding cost for the lost sales case as

$$\begin{aligned} HC_L(Q_i) &= h_b(1-\gamma)Q_i T_i + h_b\gamma Q_i \tau_i - hT_i(Q_{si} - Dt_{si}) + h_b Q_{si} t_{si} - \\ &\frac{h_b D}{2}(T_i^2 + t_{si}^2) \\ &-h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} \end{aligned} \tag{4.11}$$

Substituting τ_i and T_i in terms of Q_i from Eqs. (4.2) and (4.7) respectively, and replacing (Dt_{si} – Q_{si}) with Z_i (the lost inventory), the above expression can be simplified as

$$HC_L(Q_i) = \frac{h_b}{2}[Q_i^2(1-\gamma)^2 + 2Q_i Z_i(1-\gamma) + Z_i^2] - \frac{h_b D}{2} t_{si}^2 + h_b Q_{si} t_{si} \tag{4.12}$$

$$+\frac{h_b\gamma Q_i\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}-h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}{}^{\frac{2-b}{1-b}}$$

It should be noted that the above holding cost reduces to the one in Salameh and Jaber (2000) once b, t_{si}, Q_{si} and u_i become zero. Now,

Cost of the lost sales = $c_L(Dt_{si}-Q_{si})$

Cost of inspection = $d_1\tau_i=\frac{d_1\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}$

It should be noticed that the experience u_i will be taken as an input for every cycle of learning. The total profit per cycle for the lost sales case with partial transfer of learning, is

$$TP_L(Q_i)=[s(1-\gamma)+\nu\gamma-c_1]Q_i-K-c_LZ_i-\frac{h_b}{2D}[Q_i^2(1-\gamma)^2+2Q_iZ_i(1-\gamma)+Z_i^2$$
$$+\frac{h_bD}{2}t_{si}^2-\frac{(h_b\gamma Q_i+d_1)\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}-h_bQ_{si}t_{si}+h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}{}^{\frac{2-b}{1-b}} \tag{4.13}$$

The expected total profit per cycle is

$$\mathrm{E}[TP_{iL}(Q_i)]=\{s(1-\mathrm{E}[\gamma])+\nu\mathrm{E}[\gamma]-c_1\}Q_i-K-c_LZ_i-\frac{h_b}{2D}\{Q_i^2\mathrm{E}[(1-\gamma)^2]$$
$$+2Q_iZ_i(1-\mathrm{E}[\gamma])+Z_i^2\}+\frac{h_bD}{2}t_{si}^2-\frac{(h_bQ_i\mathrm{E}[\gamma]+d_1)\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)}-h_bQ_{si}t_{si} \tag{4.14}$$
$$+h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)}t_{si}{}^{\frac{2-b}{1-b}}$$

The expected cycle time $\mathrm{E}[T_{iL}]$ can be written as

$$\mathrm{E}[T_{iL}]=\frac{(1-\mathrm{E}[\gamma])Q_i}{D}-\frac{Q_{si}}{D}+t_{si}$$

Salameh and Jaber (2000) determined the expected annual profit from the expected value of the annual profit of the buyer. Maddah and Jaber (2008*b*) corrected this flaw and suggested the used of renewal reward theorem. That is, the expected annual profit should be a ratio of the expected profit per cycle and the expected cycle time. So,

$$\mathrm{E}[TPU_{iL}(Q_i)]=\frac{\mathrm{E}[TP_{iL}(Q_i)]}{\mathrm{E}[T_{iL}]} \tag{4.15}$$

It should be noted that the length of a cycle with learning in screening is independent of the fraction of defectives. Appendix 4 provides a proof of concavity of Eq. (4.15).

4.3.2 *Lost sales for total transfer of learning*

In case of total transfer of learning, the screening will start in each cycle with a cumulative experience gained from earlier cycles. The total screening time will tend to decrease from cycle to another. This will bring a change in the subsequent cycle lengths and the costs associated with each cycle. The experience gained in each cycle *i* will be taken as

$$u_i = \sum_{j=1}^{i-1} Q_j \tag{4.16}$$

This implies that the worker does not lose any knowledge in his break while he is not screening. The holding cost and the expected profit in Eqs. (4.12) and (4.14), respectively, will be determined using Eq. (4.16). This will change the annual profit in Eq. (4.15) accordingly.

4.3.3 *Lost sales for total forgetting*

In case of total forgetting, screening in each cycle starts with no prior knowledge, which means that the worker loses all the experience gained in the earlier cycles. This implies that u_i in cycle *i* will be taken as zero. Therefore, the order quantity and the screening time in every cycle remains the same. This will change the inspection time in Eq. (4.2) and the holding cost in Eq. (4.12), respectively, to

$$\tau = \frac{Q^{1-b}}{(1-b)x_1} \tag{4.17}$$

$$\begin{aligned} HC_L(Q) = \frac{h_b}{2D}[Q^2(1-\gamma)^2 + 2QZ(1-\gamma) + Z^2] - \frac{h_b D}{2} t_s^2 + \frac{h_b \gamma Q^{2-b}}{x_1(1-b)} + h_b Q_{si} t_{si} \\ - h_b \left(\frac{1-b}{2-b}\right) [(1-b)x_1]^{1/(1-b)} t_s^{\frac{2-b}{1-b}} \end{aligned} \tag{4.18}$$

The expected profit per cycle and the expected annual profit in Eqs. (4.13) and (4.14), respectively, will be determined using Eqs. (4.17) and (4.18).

4.4 Backorders

The behavior of inventory for the backorders case is shown in Figure 4.2 where B_i represents the size of backorder in cycle i. The screening rate becomes equal to the demand rate at t_{si}. The dotted line shows that the backorder that piles up till t_{si} is fulfilled at the time ($t_{si}+t_{Bi}$). Inventory in the rest of the cycle, behaves as it does in Salameh and Jaber (2000). This behavior repeats itself in the subsequent cycles. The maximum backorder level in Figure 4.2 is

$$B_i = Dt_{si} - Q_{si}$$

Now, following its definition in the notations, Q_{Bi} can be written as

$$Q_{Bi} = Dt_{Bi} + B_i + Q_{si} = D(Dt_{Bi} + t_{si})$$

The time t_{Bi} can be written as

$$t_{Bi} = \int_0^{Q_{Bi}} \frac{1}{x_1} y^{-b} dy - \int_0^{Q_{si}} \frac{1}{x_1} n^{-b} dn = \frac{{Q_{Bi}}^{1-b}}{x_1(1-b)} - \frac{{Q_{si}}^{1-b}}{x_1(1-b)}$$

Substituting Q_{Bi}:

$$t_{Bi} = \frac{[D(t_{Bi}+t_{si})]^{1-b}}{x_1(1-b)} - t_{si} \text{ or } t_{Bi} + t_{si} = \frac{[D(t_{Bi}+t_{si})]^{1-b}}{x_1(1-b)}$$

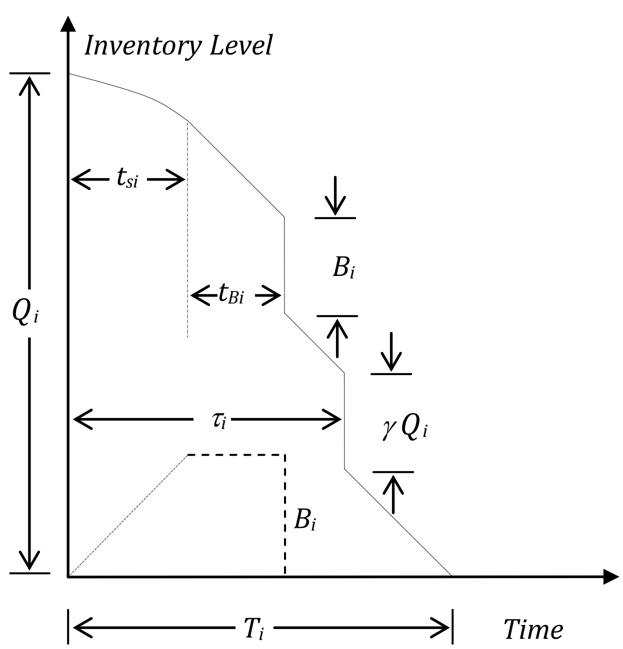

Figure 4.2 Learning in inspection with backorders

This time can be taken as

$$t_X = t_{Bi} + t_{si} = \frac{D^{\frac{1-b}{b}}}{x_1{}^{1/b}(1-b)^{1/b}} \tag{4.19}$$

Again, with the help of learning, the screening will become equal to or more than the demand and there won't be any backorders after some cycles. The three scenarios of learning discussed in the lost sales case, will be considered here to develop the expected annual profit of a buyer.

4.4.1 *Backorders for partial transfer of learning*

The inventory level in Figure 4.2 can be represented as

$$I_B(t) = \begin{cases} Q_i - [(1-b)x_1 t]^{1/(1-b)}, & 0 \le t < t_{si} \\ Q_i - Q_{si} - D(t - t_{si}), & t_{si} \le t < t_{si} + t_{Bi} \\ Q_i - Dt, & t_{si} + t_{Bi} \le t < \tau_i \\ (1-\gamma)Q_i - Dt, & \tau_i \le t < T_i \end{cases} \tag{4.20}$$

At time $t = T_i$, the inventory level is zero, i.e. $(1-\gamma)Q_i - Dt_i = 0$. The cycle time T_i is

$$T_i = \frac{(1-\gamma)Q_i}{D} \tag{4.21}$$

The holding costs for the four different time intervals shown in Figure 4.2 are determined respectively from Eq. (4.20) as:

$$HC_{1Bi}(Q_i) = h_b Q t_{si} = h_b \left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}{}^{\frac{2-b}{1-b}} \tag{4.22}$$

$$HC_{2Bi}(Q_i) = h_b(Q_i - Q_{si} + Dt_{si})t_{Bi} - \frac{h_b D}{2}[(t_{si} + t_{Bi})^2 - t_{si}^2] \tag{4.23}$$

$$HC_{3Bi}(Q_i) = h_b[\tau_i - (t_{si} + t_{Bi})]Q_i - \frac{h_b D}{2}[\tau_i{}^2 - (t_{si} + t_{Bi})^2] \tag{4.24}$$

$$HC_{4Bi} = h_b(1-\gamma)(T_i - \tau_i)Q_i - \frac{h_b D}{2}\left(T_i{}^2 - \tau_i{}^2\right) \tag{4.25}$$

Adding Eqs. (4.22) – (4.25) to get the holding cost for the backorder case:

$$HC_{Bi}(Q_i) = h_b Q_i t_{si} - h_b \left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} + h_b(Q_i - Q_{si} + Dt_{si})t_{Bi} - \frac{h_b D}{2}[(t_{si}+t_{Bi})^2 - t_{si}^2] + h_b[\tau_i - (t_{si}+t_{Bi})]Q_i - \frac{h_b D}{2}[{\tau_i}^2 - (t_{si}+t_{Bi})^2] + h_b(1-\gamma)(T_i - \tau_i)Q_i - \frac{hD}{2}\left({T_i}^2 - {\tau_i}^2\right) \tag{4.26}$$

Using Eq. (4.19) and simplifying the above expression results in:

$$HC_{Bi}(Q_i) = h_b(1-\gamma)Q_i T_i + h_b\gamma Q_i \tau_i + h_b t_X B + h_b Q_{si} t_{si} - \frac{h_b D}{2}(T_i^2 + t_{si}^2) - h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} \tag{4.27}$$

Substituting τ_i and T_i in terms of Q_i from Eqs. (4.2) and (4.21) respectively, the above expression can be simplified as

$$HC_{Bi}(Q_i) = \frac{h_b}{2D}\left[{Q_i}^2(1-\gamma)^2\right] - \frac{h_b D}{2} t_{si}^2 + \frac{h_b \gamma Q_i\{(Q_i+u_i)^{1-b} - {u_i}^{1-b}\}}{x_1(1-b)} + h_b t_X B + h_b Q_{si} t_{si} - h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{1/(1-b)} t_{si}^{\frac{2-b}{1-b}} \tag{4.28}$$

It should be noted that the above holding cost reduces to the one in Salameh and Jaber (2000) once b, t_x , t_{si}, Q_{si} and u_i become zero.

Now the backorder cost in a cycle is $= \frac{c_B t_s B_i}{2} + c_B(t_X - t_{si})B_i = c_B\left(t_X - \frac{t_{si}}{2}\right)B_i$

So, the total profit per cycle is given by

$$TP_{iB}(Q_i) = [s(1-\gamma) + \nu\gamma - c_1]Q_i - K - c_B\left(t_X - \frac{t_{si}}{2}\right)B_i - \frac{h_b}{2D}[Q_i^2(1-\gamma)^2] + \frac{h_b D}{2} t_{si}^2 - \frac{(h_b\gamma Q_i + d_1)\{(Q_i+u_i)^{1-b} - {u_i}^{1-b}\}}{x_1(1-b)} - h_b Q_{si} t_{si} - h_b t_X B + h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{\frac{1}{1-b}} t_{si}^{\frac{2-b}{1-b}} \tag{4.29}$$

and the expected total profit per cycle is

$$\mathrm{E}[TP_{iB}(Q_i)] = \{s(1-\mathrm{E}[\gamma]) + \nu\mathrm{E}[\gamma] - c_1\}Q_i - K - c_B\left(t_X - \frac{t_{si}}{2}\right)B_i - \frac{h_b}{2D}\{Q_i^2\mathrm{E}[(1-\gamma)^2]\}$$
$$+\frac{h_bD}{2}t_{si}^2 - \frac{\{h_bQ_i\mathrm{E}[\gamma]+d_1\}\{(Q_i+u_i)^{1-b} - u_i{}^{1-b}\}}{x_1(1-b)} - h_bQ_{si}t_{si} - h_bt_XB + h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{\frac{1}{1-b}}t_{si}{}^{\frac{2-b}{1-b}} \quad (4.30)$$

The expected cycle time $\mathrm{E}[T_{iB}]$ can be written using Eq. (4.21) as

$$\mathrm{E}[T_{iB}(Q_i)] = \frac{(1-\mathrm{E}[\gamma])Q_i}{D}$$

So, the expected annual profit can be written as

$$\mathrm{E}\left[TPU_{iB}\left(Q_i\right)\right] = \frac{\mathrm{E}[TP_{iB}(Q_i)]}{\mathrm{E}[T_{iB}]} \quad (4.31)$$

Eq. (4.31) is concave (refer to Appendix 4 for proof).

4.4.2 *Backorders for total transfer of learning*

In this case, the worker will retain all the experience gained in the earlier cycles. This experience will be calculated using Eq. (4.16). The holding cost, the expected profit per cycle, and the expected annual profit in Eqs. (4.28), (4.30) and (4.31), respectively, will be determined using this experience in each cycle.

4.4.3 *Backorders for total forgetting*

In this case, the experience u_i in cycle i becomes zero and the inspection time will be determined by Eq. (4.17). The holding cost in Eq. (4.28) will be written as

$$HC_B(Q) = \frac{h_b}{2D}[Q^2(1-\gamma)^2] - \frac{h_bD}{2}t_s^2 + \frac{h_b\gamma Q^{2-b}}{x_1(1-b)} + h_bQ_st_s + h_bt_XB$$
$$-h_b\left(\frac{1-b}{2-b}\right)[(1-b)x_1]^{\frac{1}{1-b}}t_s{}^{\frac{2-b}{1-b}} \quad (4.32)$$

The expected profit per cycle and the expected annual profit in Eqs. (4.30) and (4.31) respectively, will be determined using Eqs. (4.16) and (4.32).

4.5 Numerical Analysis

Consider a buyer who receives a lot of y units with defective items and the lot is screened out by workers. Unlike Salameh and Jaber (2000), at the beginning of the cycle, the screening rate is less than the demand rate and results in lost sales or backorders. As learning takes place in the screening process, the worker catches up with demand. Thus, there won't be any lost sales or backorders after some cycles of procurement.

Learning rates vary from industry to industry. For example, Cunningham (1980) collected learning rates reported in different industries that ranged from 95% (b = 0.074) in electric power generation to 60% (b = 0.737) in semiconductor manufacturing. Dutton and Thomas (1984) plotted learning rates found in 108 manufacturing firms, with 55% (b = 0.862) being the fastest learning rate reported. Dar-El (2000) on page 58, tabulated learning rates reported from previous studies that ranged from 95% (b = 0.074) to 68% (b = 0.556). Some studies observed total forgetting in a period ranging from few months to more than a year, e.g., Anderlohr (1969), McKenna and Glendon (1985). The learning rate in our example is taken to be 80% (b = 0.32) as observed in many manufacturing industries, Argote and Epple (1990).

Besides, Salameh and Jaber (2000) used an inspection cost of \$0.5/unit and obtained an optimal order size of 1439 units per cycle. They assumed a very high value of the inspection rate (175200 units per year) that represents a plateau in a learning curve. That is, when the screening rate on the learning curve exceeds 175200, the models developed herein converge to that of Salameh and Jaber (2000) whose closed form solution is given by Eq. (4.2). The time and the cost of inspection per cycle, using the parameters in Salameh and Jaber (2000), imply that an annual cost of \$87600 be used in our example. An analysis of how unit inspection cost affects the annual profit will be carried out at the end of this section.

The unit cost of backordering depends on when a buyer fulfills the backordered demand. Its value is taken to be twice as the unit holding cost, as in Wee *et al*. (2007). On the other hand, cost of lost sales is the sum of the lost revenue and the cost of goodwill: Goyal and Giri (2001); Ouyang *et al.* (2006*a*). Thus, it should be greater than or equal to the unit selling price. The unit cost of lost sales is taken to be equal to the unit selling price, in this numerical example. The effect of these two costs on the annual profit will be analyzed later in this section. Four percent defective items (γ) are assumed while the time for total forgetting (L) is taken to be 300 days.

The sensitivity of the total forgetting time will also be analyzed later. The rest of the data for this numerical analysis is taken from Salameh and Jaber (2000). The probability density function for the percentage of defectives is taken to be

$$f_1(\gamma) = \begin{cases} 1/(0.04-0), & 0 \le \gamma \le 0.04 \\ 0, & otherwise \end{cases}$$

Microsoft Excel Solver is used to obtain the optimal annual profit for a certain learning exponent and the percentage of defectives. It is observed that the annual profit plateaus to some extent after ten cycles, as a result of learning and forgetting. Therefore, in case of partial or total transfer of learning, an average of the results from the first ten cycles is used to compare them with those from the other scenarios of learning. To illustrate this, Table 4.1 shows the changes in the cycle length, the order size, the forgetting exponent and the annual profit from cycle to cycle, for the case of partial transfer of learning.

Table 4.1 Results of partial transfer of learning from cycle to cycle

Lost Sales				*Backorders*			
T_i	Q_i	β_i	$E[TPU(Q_i)]$	T_i	Q_i	β_i	$E[TPU(Q_i)]$
0.0711	3625	0.4277	1216192	0.0573	2923	0.4025	1218890
0.0493	2517	0.3914	1220513	0.0488	2491	0.3909	1220626
0.0450	2297	0.3886	1221502	0.0447	2279	0.3885	1221589
0.0419	2137	0.3889	1222305	0.0416	2123	0.3890	1222379
0.0394	2012	0.3913	1223009	0.0392	2001	0.3917	1223076
0.0375	1912	0.3956	1223650	0.0373	1903	0.3961	1223711
0.0359	1831	0.4013	1224244	0.0357	1824	0.4020	1224301
0.0346	1765	0.4084	1224802	0.0345	1759	0.4092	1224856
0.0335	1710	0.4168	1225332	0.0334	1705	0.4177	1225383
0.0431	**2201**	**0.4011**	**1222394**	**0.0414**	**2112**	**0.3986**	**1222757**

(the bold values are the averages of the columns)

The input data and the results of the numerical example are shown in Table 4.2. It should be noted that the total transfer of learning is a better approach for both lost sales and backorders. The rationale is that while retaining the previous knowledge in screening, the buyer orders less. Total forgetting in both lost sales and the backorders does not result as a profitable option which is an understandable finding as the buyer has to screen with no previous experience.

The difference between lost sales and the backorders is not huge in case of total or partial transfer of learning. The reason is that the reported results are an average of the values in ten cycles. This minimizes the difference in the two approaches to a great extent. On the other hand, this difference in case of no transfer of learning is a noticeable one.

Table 4.2 Input data and results of the numerical examples

D	x_1	K	d_1	c_1	s	v	c_B	c_L	h_b	γ	b	L
50000	30000	100	87600	25	50	20	10	50	5	0.04	0.32	0.82
units/ yr	units/ yr	$/cycle	$/yr	$/unit	$/unit	$/unit	$/unit/ yr	$/unit	$/unit/ yr	-	-	yr

	Lost Sales		*Backorders*	
	Q_i^*	*Annual Profit*	Q_i^*	*Annual Profit*
Partial Forgetting	2201	1222394	2112	1222757
Total Learning	1701	1228448	8085	1228516
Total Forgetting	3625	1216192	2923	1218890

To enhance the above example, the effect of the change in learning exponent was studied on the three scenarios discussed, for lost sales and backorders, at a fixed percentage of defectives. The average of the expected annual profit from the first ten cycles of learning was obtained at different values of the learning exponent. The results are shown in Figures 4.3 and 4.4. It can be seen that the annual profit in both the cases, in all the scenarios of learning, tends to be increasing with the learning exponent except only for total forgetting in case of lost sales. That is, learning in screening makes the buyer order less and less. This saves him some of the screening cost and the holding cost, and causes increase in the annual profit. The unusual behavior in case of lost sales with total forgetting tells us that at a very high learning exponent, the buyer tends to pile inventory with him, i.e. an increase in the holding cost which is not there in other scenarios of learning. This counters the increase in the annual profit. In general one can say that the more the knowledge in screening is retained the more is the annual profit. Furthermore, total transfer of learning remains to be the best of the three scenarios discussed, both for lost sales and backorders, in terms of annual profit.

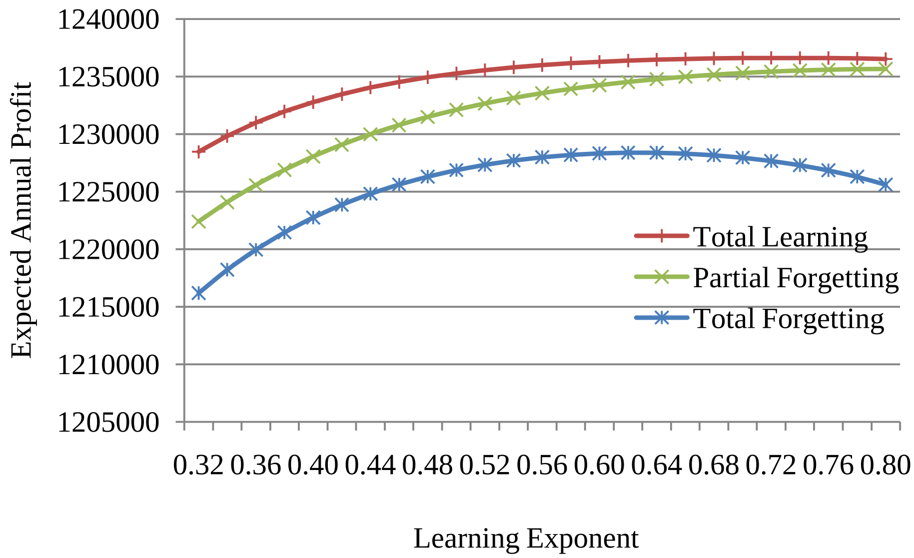

Figure 4.3 Annual profit w.r.t. b for lost sales at $\gamma = 0.04$

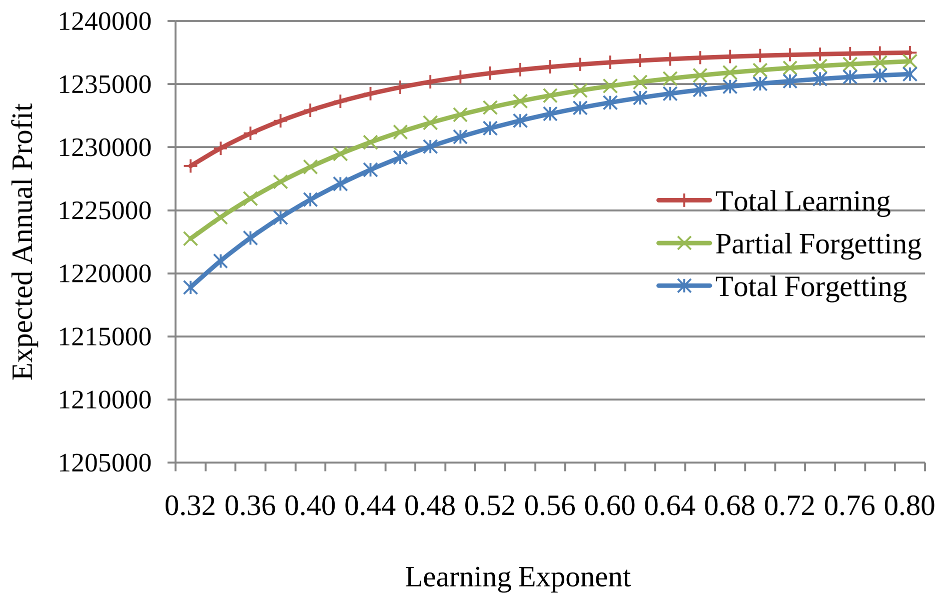

Figure 4.4 Annual profit w.r.t. b for backorders at $\gamma = 0.04$

Similarly, to understand the effect of the shortage costs, annual profit was obtained for varying lost sales and backorder costs, for the case of partial transfer of learning. Again, an average of the annual profit from ten consecutive cycles of learning was obtained at different unit-costs of lost sales, at a fixed value of the learning exponent. Figure 4.5 shows that, as intuitively expected, the annual profit tends to decline with the increasing unit lost sales cost. That is, the lost sales cost offsets the annual profit margin of the buyer. On the other hand, the annual profit for the backorder case showed almost no variation when the unit backorder cost was varied from twice to four-times the unit holding cost. This indicates that this much change in the unit backorder cost is just not enough to hint a clear difference in the annual profit, especially when the learning rate is fixed. It should be noted that in the backorder case, the demand is eventually met and hence profit is not lost. However, the only additional cost is that of back orders, which is more than the holding cost, for the quantity backordered. In order to investigate the effect of defective items on the annual profit, the sensitivity of the model to γ, was tested for the lost sales case with partial transfer of learning. Figure 4.6 shows that at a fixed learning exponent, the annual profit tends to be decreasing with the percentage of defectives. This is an expected result as the imperfect items are likely to slash the profit obtained from the non-defective items.

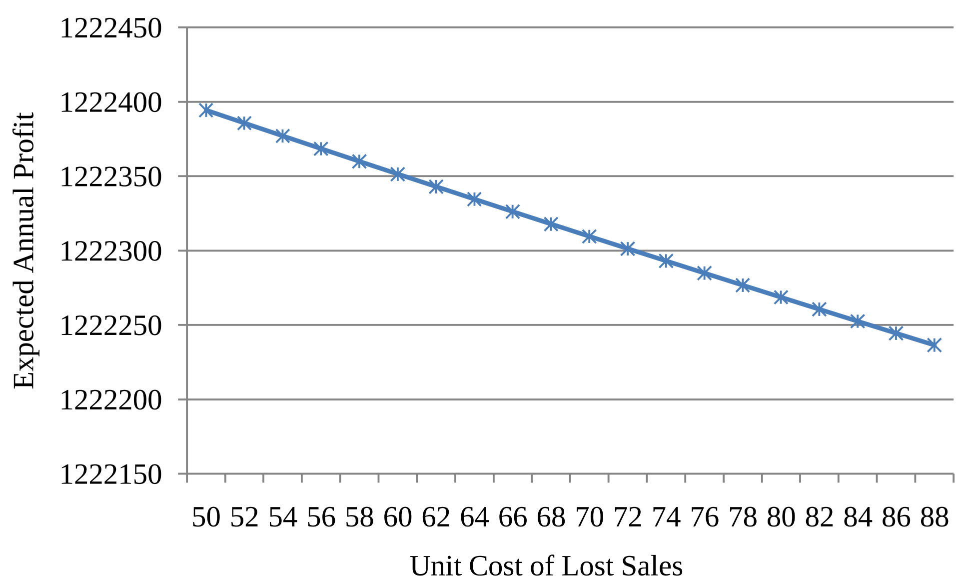

Figure 4.5 Annual profit w.r.t. c_L for lost sales with partial forgetting

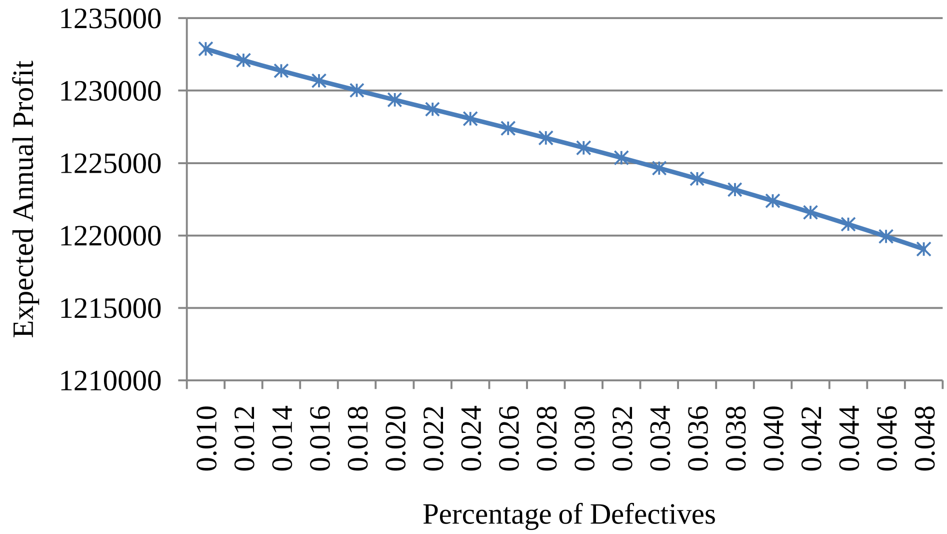

Figure 4.6 Annual profit w.r.t. γ for lost sales with partial forgetting

To investigate the sensitivity of the time for total forgetting, the response of the model to varying L was tested for the lost sales case with partial transfer of learning. The three curves in Figure 4.7 indicate that as the worker retains his/her skills for a longer period, the increase in the profit by virtue of learning ends up at a higher level than for the case where he loses his experience in shorter spells of time. As shown in Table 4.1, it can also be seen here (Figure 4.7) that the most of the benefit from learning shows in the earlier cycles (e.g., from 1 to 2).

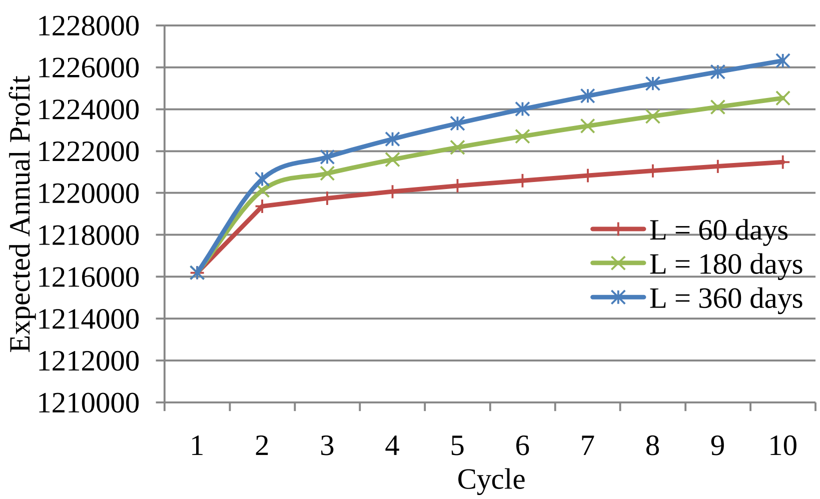

Figure 4.7 Annual profit w.r.t. L for lost sales with partial forgetting

Finally, the sensitivity of the model to the unit inspection cost is explored. The response of the annual profit for the lost sales case, with total transfer of learning, is plotted in Figure 4.8, at different levels of learning. That is, the unit inspection cost is varied from \$0.5 to \$2.5 for *b* between 0.05 and 0.4. It should be noted here that the unit inspection cost in all the above analyses was \$0.5. Figure 9 indicates that the annual profit drops by a great extent by varying the unit inspection cost at lower levels of *b* (i.e. when the learning is slow). This difference in the annual profit starts diminishing as learning becomes faster (higher values of *b*). This indicates that the unit inspection cost affects the annual profit more at the higher values of screening time. One should notice that the screening time per cycle gets shorter and shorter with learning which tends to increase the annual profit. Besides, this screening time has an inverse relation to the learning exponent *b*.

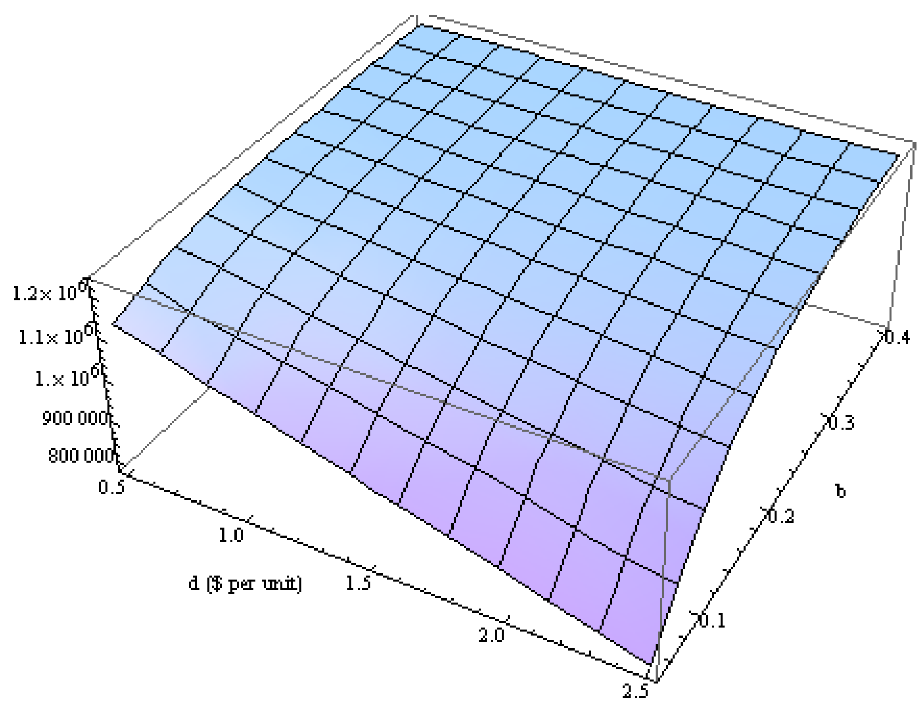

Figure 4.8 Annual profit w.r.t. *d* and *b* for lost sales with total transfer of learning

4.6 Summary and Conclusions

This chapter is an extension of Salameh and Jaber (2000) for the case where the buyer's inspection process undergoes learning while screening for defective items in a lot. It is assumed that a 100% inspection is carried out with an error free screening and the rate of screening tends to increase by virtue of learning. This counters an assumption in Salameh and Jaber (2000) that the inspection rate is fixed and is always greater than demand. This chapter makes a realistic move and contributes to the area by giving some insights to a practical scenario. Having a screening rate lesser than demand rate in the beginning of the screening process, incurs shortages which are tackled in the chapter as both lost sales and backorders. Three scenarios of learning, available in the literature, are compared for the above set-up. These scenarios are (*i*) total forgetting, where an inspector starts in every cycle with no prior experience, (*ii*) total transfer of learning, where the inspector does not lose any knowledge or skills in the breaks and the learning curve continues as if there were no interruptions, and (*iii*) partial transfer of learning, where an inspector carries part of his experience to the subsequent cycles. The last situation is the most generalized and the realistic one. The results indicate that total transfer of learning remains better for both the lost sales and the backorders set-up. The reason is that the buyer orders less and pays less in inventory and screening cost, by virtue of learning. The sensitivity of the model was tested for a number of parameters. The results indicated that the annual profit tends to increase with the learning exponent in screening. That is, the faster the learning in screening the lesser is the screening time. A similar finding of the research is that the annual profit can be increased by retaining more and more knowledge in screening process. This was pointed out by experimenting different levels of time for total forgetting, at fixed values of the learning exponent.

It was noticed that an increase in the percentage of defectives decreases the annual profit at a fixed exponent of learning. The unit cost of lost sales is also shown to have a similar effect on the annual profit. It was shown that an increase in the unit screening cost reduces the annual profit to great extent at the slower rates of learning.

The study could be enhanced in a number of ways. For example, one could study the effect of learning in suppliers' proportion of defectives. The buyer and the supplier can agree on a fixed proportion of defective items, and look for a coordinated order size per cycle.

CHAPTER 5 THE EFFECT OF HUMAN FACTORS ON THE PERFORMANCE OF A SUPPLIER–VENDOR SUPPLY CHAIN

5.1 Introduction

The EOQ/EPQ model has been quite popular among researchers and industries ever since its inception in the beginning of the last century (Simpson, 2001). This model can be summarized as determining an order quantity that makes a balance or trade-off between the ordering costs and the holding costs. Regardless of such a wide acceptance, this basic model has several weaknesses. The main idealistic assumptions claim there is a perfectly steady demand known with certainty and all the items received from the suppliers are of a perfect quality. These assumptions initiated a huge arena of research for many in the industry and academics. The result was a vast literature that studies the basic EOQ/EPQ model under real life situations, e.g., Porteus (1986*a*), Rosenblatt and Lee (1986) and Silver (1976).

In this chapter, a two-level (supplier-vendor) supply chain is considered. The vendor follows an EPQ policy to manufacture a single product. The coordination mechanism is such that (*i*) the vendor receives the market demand and orders for the different components that are needed to make the single product; (*ii*) every supplier provides a single type of a part/item required for the product; (*iii*) all the suppliers replenish the orders at the same time, i.e. at the beginning of the vendor's production cycle. The raw material from the suppliers is assumed to follow the assumptions of Salameh and Jaber (2000) where each shipment contains imperfect quality items. These defective items received from the suppliers may be a result of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit: (Ouyang *et al*., 2006*b*). Two coordination mechanisms are considered here, as in Khouja (2003). An optimal production quantity and the annual cost of the whole supply chain are determined for each of the mechanism. The model is then extended to introduce human factors such as inspection error, and learning in production and quality. In the first extension, the screening process is assumed to have Type I and Type II errors. In the second extension, the vendor's production process is assumed to follow Wright (1936) learning curve, thus affecting the production time. In the third extension, the quality of suppliers' items is assumed to follow learning. An optimal lot size and the annual cost of the supply chain are determined for each of these extensions. Thus, the major contributions of this chapter can be summarized as

1. It brings in the concept of defective items in a supplier-vendor supply chain whereas Huang (2002) and Goyal *et al*. (2003) have done the same for a vendor-buyer supply chain.

2. It extends the coordination mechanisms in Khouja (2003) for defective items from suppliers, i.e. Salameh and Jaber (2000), learning in vendor's production process, i.e. Jaber and Bonney (2003), inspection error in vendor's screening process, as in Raouf *et al*. (1983) and learning in suppliers' defective items, as in Jaber *et al*. (2008).

3. Results of this research have endorsed Khouja (2003) finding for all the extensions, that integer multiplier mechanism remains better than the equal cycle time mechanism.

The rest of the chapter is arranged as follows: In section 5.2, the description and formulation of the models are given. An approximate solution procedure to determine the multipliers of the cycle time is also presented in this section. A number of extensions of the model in section 5.2, for different human factors are introduced in section 5.3. Section 5.4 presents numerical examples and the sensitivity analysis of the different parameters used in the models. Section 5.5 presents conclusions, limitations and some suggestions for future research.

5.2 Model Description and Development

Consider a two-level supply chain scenario where a vendor has to make Q (a deterministic quantity) assembled items of a product in each production segment of length T_p at a rate P, which is consumed at a rate D ($P > D$) over the cycle time $T = Q/D$. Each of the finished products needs μ_s parts from supplier s, where $s = 1, 2, \ldots, m$. A fixed percentage γ_s of these parts is believed to be defective. For this, the vendor institutes a 100% inspection and screens out all the defective items from the lots provided by the suppliers, at a rate of x per unit time. Though we take the same inspection rate x here, it is quite reasonable to assume different screening rates for a number of parts provided by the suppliers. The rationale for this is that the parts may have a different level of complexity (Duffuaa and Khan, 2005). Since every part is assumed to have a different rate of being defective, the vendor would end up with some parts left in each cycle that would be utilized in the subsequent cycles.

An optimal production quantity for each mechanism, as described above, will be determined by minimizing the total costs experienced by all the stakeholders of the supply chain. The costs considered in the model are ordering/setup cost, screening cost, and the inventory carrying cost. The objective of the study is to minimize the total annual cost through (*i*) an optimal production quantity and (*ii*) an optimal multiplier for each supplier.

Figure 5.1 illustrates the behavior of inventory for raw material (dotted lines) and finished goods (solid lines). The behavior of raw material inventory is similar to that described in Salameh and Jaber (2000). Production and screening start at time zero. The figure represents the inventory of the raw material from three suppliers but the notations are given for the first supplier only. The inspection process, that takes time t_{11}, results in a defective sub-lot, B_1. As each supplier may have a different percentage of defectives, the vendor may have a number of unused parts l_1, l_2 left at the end of each cycle. It should be noticed that the lot with the highest number of defective items (shown by a red line) would not have any leftovers. These parts are used in the subsequent cycle.

5.2.1 *Equal Cycle Time Mechanism*

In this mechanism, the supplier is assumed to follow a lot-for-lot policy. Following an equal cycle time for all the stakeholders in such a system, the total annual cost will be computed in this section. It should be noted that the vendor has to accommodate the minimal number of parts left, l_s in each cycle. For simplicity, we relax the integer number restriction and compute l_s as

$$\mathrm{E}[l_s] = Q_s(\mathrm{E}[\gamma_{\max}] - \mathrm{E}[\gamma_s])$$

Therefore, the order quantity of the raw material/parts of type s, in a cycle, would be

$\mathrm{E}[Q_s] = Q\mu_s - \mathrm{E}[l_s]$ or

$$\mathrm{E}[Q_s] = Q\mu_s\{1 - (\mathrm{E}[\gamma_{\max}] - \mathrm{E}[\gamma_s])\} = Q\mu_s\mathrm{E}[\pi_s] \tag{5.1}$$

where $\gamma_{\max} = \max\{\gamma_s,\ s = 1,2,\dots,m\}$

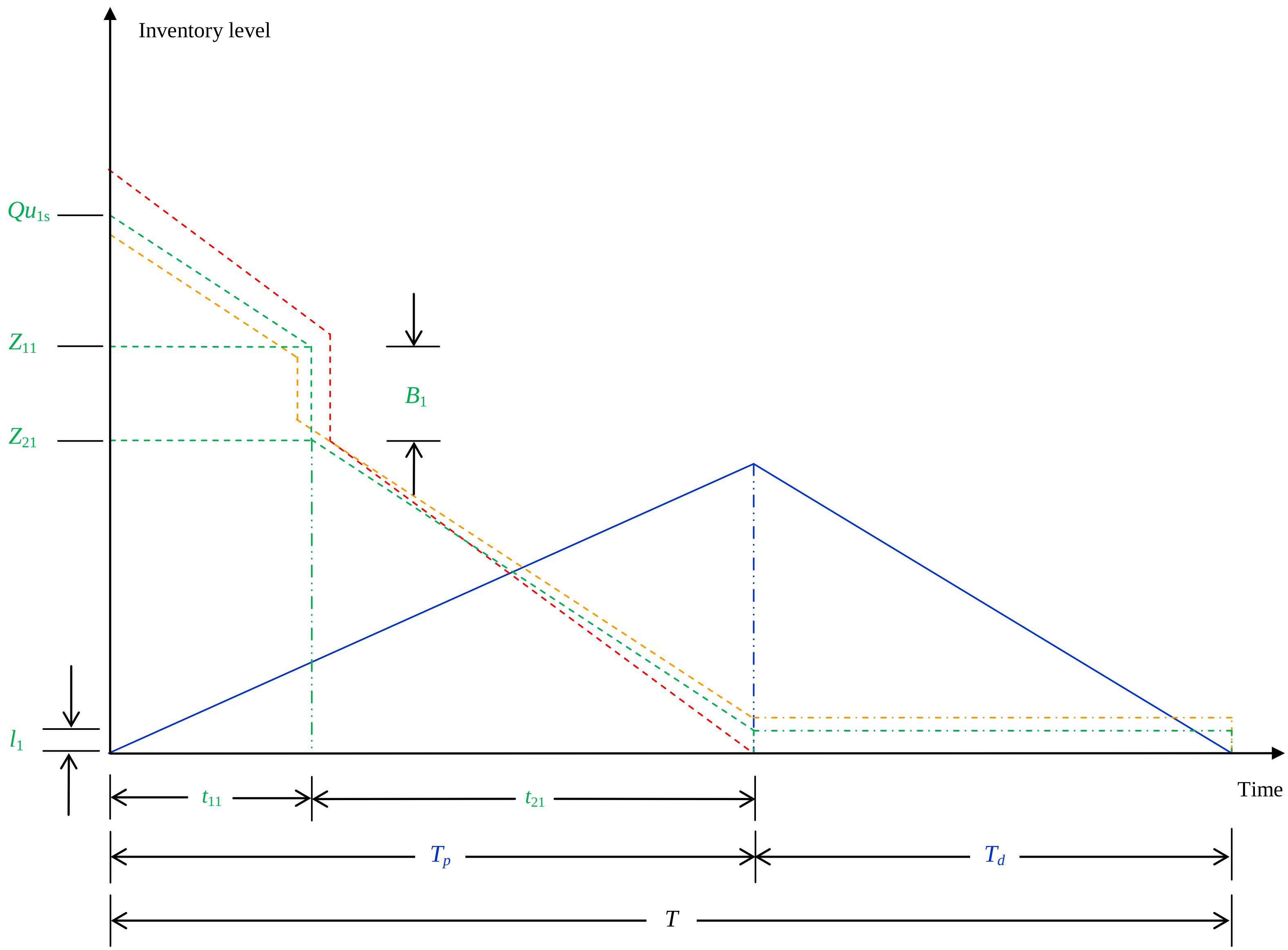

Figure 5.1 Vendor's inventory level for raw material and finished product (s = 1, 2, 3)

The fraction π_s balances for the leftovers of type *s* in a cycle. Therefore the raw material in each cycle would consist of (*i*) nondefective parts, (*ii*) defective parts and (*iii*) the leftovers. It should be noted that production and screening processes, both, start at time zero in each cycle. The defective raw material from supplier 1 is screened out at time t_{11}, whereas the inventory level for this raw material drops from Z_{11} to Z_{21}, as described in Figure 5.1.The different costs of the raw material/parts of type *s*, for the vendor, in a cycle, are given in the following:

Ordering cost = $a_{vs}Q\mu_s\mathrm{E}[\pi_s]$

Holding cost = $h_{v1s}\left\{\frac{(Q+\mathrm{E}[Z_{1s}])\mathrm{E}[t_{1s}]}{2}+\frac{\mathrm{E}[Z_{2s}](\mathrm{E}[T_p]-\mathrm{E}[t_{1s}])}{2}+\mathrm{E}[l_s]\mathrm{E}[T_d]\right\}$

where a_{vs} and h_{v1s} are respectively the vendor's unit variable cost for ordering and storing one unit and from supplier *s*. As defined in the notations, the terms in the above expression are:

$$\mathrm{E}[t_{1s}]=\frac{\mathrm{E}[Q_s]}{x}=\frac{Q\mu_s}{x}-\frac{\mathrm{E}[l_s]}{x}$$

$\mathrm{E}[Z_{1s}]=\mathrm{E}[Q_s]-D\mathrm{E}[t_{1s}]=\mathrm{E}[Q_s]\left(1-\frac{D}{x}\right)$ and

$$\mathrm{E}[Z_{2s}]=\mathrm{E}[Z_{1s}]-\mathrm{E}[\gamma_s]\mathrm{E}[Q_s]=\mathrm{E}[Q_s]\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)$$

where $D<x$ and $\mathrm{E}[\gamma_s]+\frac{D}{x}<1$.

Using Eq. (5.1), it can be simplified as

Holding cost = $\frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(\mathrm{E}[T]-\frac{Q}{P}\right)$

Screening cost = $d_sQ\mu_s\mathrm{E}[\pi_s]$

where d_s is the cost to screen one item received from supplier *s*. Thus, the vendor's total cost of the raw material, in a cycle, would be:

$$C_{vr}(Q)=\sum_{s=1}^{m}\left[(a_{vs}+d_s)Q\mu_s\mathrm{E}[\pi_s]+\frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(\mathrm{E}[T]-\frac{Q}{P}\right)\right] \quad (5.2)$$

Similarly, the vendor's cost of the finished products, in a cycle, is the sum of its setup cost, production cost and holding cost. That is

$$C_{vf}(Q) = A_v + \frac{cQ}{P} + \frac{h_{v2}Q\mathrm{E}[T]}{2}\left(1 - \frac{D}{P}\right)$$

where A_v, c and h_{v2} are, respectively, the vendor's setup cost, unit production cost, and the unit holding cost for a finished item. So, the total cost of the vendor, per cycle, can be written as

$$C_v(Q) = A_v + \frac{cQ}{P} + \frac{h_{v2}Q\mathrm{E}[T]}{2}\left(1 - \frac{D}{P}\right) + \sum_{s=1}^{m}\left[(a_{vs} + d_s)Q\mu_s\mathrm{E}[\pi_s] + \frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)\right\} + h_{v1s}Q\mu_s(1 - \mathrm{E}[\pi_s])\left(\mathrm{E}[T] - \frac{Q}{P}\right)\right] \tag{5.3}$$

Using $\mathrm{E}[T]$ or Q/D as vendor's cycle time, its annual cost can be written as

$$TCU_v(Q) = \frac{A_vD}{Q} + \frac{cD}{P} + \frac{h_{v2}Q}{2}\left(1 - \frac{D}{P}\right) + D\sum_{s=1}^{m}\left[(a_{vs} + d_s)\mu_s\mathrm{E}[\pi_s] + \frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)\right\}\right] + h_{v1s}Q\mu_s(1 - \mathrm{E}[\pi_s])\left(1 - \frac{D}{P}\right) \tag{5.4}$$

The term $\mathrm{E}[\pi_s^2]$ is simplified in Appendix 5. Now, for a lot-for-lot (LFL) case, the suppliers have order costs, A_s, but no carrying costs. That is, they supply all their raw material at the beginning of each cycle to the vendor. Thus the annual cost for a supplier s is

$$C_s = A_s$$

So, the annual cost of all the suppliers would be

$$TCU_s(Q) = \frac{D\sum_{s=1}^{m} A_s}{Q} \tag{5.5}$$

Thus, the total annual cost of the whole supply chain for an equal cycle time mechanism, would be

$$\mathrm{E}[TCU(Q)] = \frac{D(A_v+\sum_{s=1}^{m} A_s)}{Q} + \frac{cD}{P} + \frac{h_{v2}Q}{2}\left(1-\frac{D}{P}\right) + D\sum_{s=1}^{m}\Big[(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s] + \frac{h_{v1s}Q}{2}\left\{\frac{u_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}\Big] + \sum_{s=1}^{m} h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right) \tag{5.6}$$

The second derivative of the above expression with respect to the production size in a cycle, is $\frac{2D(A_v+\sum_{s=1}^{m} A_s)}{Q^3}$ which turns out to be positive ($\forall\ D > 0$, $A_v > 0$, $A_s > 0$ and $Q > 0$) and proves its convexity of Eq. (5.6). Taking the first derivative of this total annual cost and solving for $\mathrm{dE}[TCU(Q)]/\mathrm{d}Q = 0$ gives the following optimal production quantity

$$Q = \sqrt{\frac{2D\left(A_v+\sum_{s=1}^{m} A_s\right)}{h_{v2}\left(1-\frac{D}{P}\right)+D\sum_{s=1}^{m} h_{v1s}\left\{\mu_s^2\mathrm{E}[\pi_s^2]\left(\frac{1+\mathrm{E}[\gamma_s]}{x}\right)+\frac{\mu_s\mathrm{E}[\pi_s]}{P}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+2\sum_{s=1}^{m} h_{v1s}\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)}} \tag{5.7}$$

It should be noticed that an expected value of the percentage of defective parts is used to indicate uncertainty of the model. The denominator of Eq. (5.7) is positive since $D < P$, $0 < \mathrm{E}[\pi_s] < 1$, and $\mathrm{E}[\gamma_s] + \frac{D}{x} < 1$.

5.2.2 *Integer-Multiplier Cycle Time Mechanism*

Many researchers have found that the equal cycle time mechanism is not an optimal solution to the problem considered. Thus, it is now assumed that the cycle time for the suppliers and the vendor is not the same but all the suppliers still follow the same cycle time. The suppliers' cycle time is an integer multiplier of the basic cycle time T used by the vendor. As the vendor in this case follows the basic cycle time T, its annual cost is given by Eq. (5.4). On the other hand, for a supplier, during the non-production time, the inventory drops every T years by $TD_s = TQu_s$. So, its inventory level in the non production portion of the cycle is $(K_s–1)TD_s$, $(K_s–2)TD_s$ ......, TD_s, and 0 as shown in Figure 5.2.

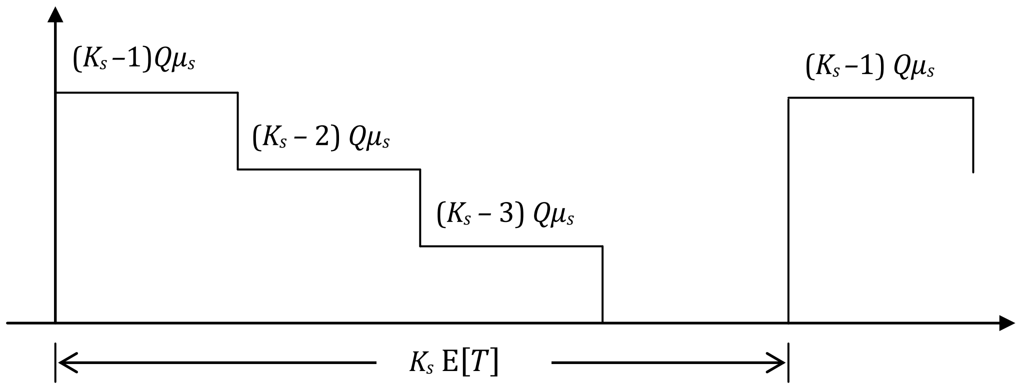

Figure 5.2 Supplier's inventory level in integer-multiplier mechanism, K_s = 4

Therefore, the inventory level for a supplier for its non-production period is $K_s\mathrm{E}[T][(K_s-1)Q\mu_s h_s]/2$. The total cost for a supplier *s*, in a cycle, can be written as

$$C_s(Q) = A_s + K_s(K_s-1)\frac{Q\mu_s h_s \mathrm{E}[T]}{2}$$

where A_s and h_s are, respectively, supplier's *s* order cost and unit holding costs. Using a supplier's cycle time as K_s E[T], its annual cost would be

$$CU_s(Q) = \frac{A_s}{K_s\mathrm{E}[T]} + (K_s-1)\frac{Q\mu_s h_s}{2}$$

Thus, the total annual cost of all the suppliers for integer multiplier mechanism, would be

$$\mathrm{E}[TCU_s(Q)] = \frac{\sum_{s=1}^{m}\frac{A_s}{K_s}}{\mathrm{E}[T]} + \frac{Q\sum_{s=1}^{m}(K_s-1)\mu_s h_s}{2} = \frac{D\sum_{s=1}^{m}\frac{A_s}{K_s}}{Q} + \frac{Q\sum_{s=1}^{m}(K_s-1)\mu_s h_s}{2} \tag{5.8}$$

It should be noted that the above expression reduces to Eq. (5.5) when K_s = 1. That is, the integer multiplier mechanism reduces to the LFL policy for an equal cycle time mechanism when K_s = 1. Using Eq. (5.4), the annual cost of the whole supply chain in case of an integer-multiplier cycle time mechanism would be

$$\mathrm{E}[TCU(Q)] = \frac{D\left(A_v+\sum_{s=1}^{m}\frac{A_s}{K_s}\right)}{Q} + \frac{cD}{P} + \frac{Q}{2}\left[h_{v2}\left(1-\frac{D}{P}\right) + \sum_{s=1}^{m}(K_s-1)h_s\mu_s\right]$$
$$+D\sum_{s=1}^{m}\left[(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s] + \frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right) \tag{5.9}$$

As in Eq. (5.6), the above expression is also convex in Q. Taking the first derivative of this total annual cost and solving for $\mathrm{d}[TCU(Q)]/\mathrm{d}Q$ = 0 gives the following optimal production quantity:

$$Q = \sqrt{\frac{2D\left(A_v+\sum_{s=1}^{m}\frac{A_s}{K_s}\right)}{h_{v2}\left(1-\frac{D}{P}\right)+\sum_{s=1}^{m}(K_s-1)h_s\mu_s+D\sum_{s=1}^{m}h_{v1s}\left\{\mu_s^2\mathrm{E}[\pi_s^2]\left(\frac{1+\mathrm{E}[\gamma_s]}{x}\right)+\frac{\mu_s\mathrm{E}[\pi_s]}{P}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+2\sum_{s=1}^{m}h_{v1s}\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)}} \tag{5.10}$$

The denominator of Eq. (5.10) is positive since $D < P$, $0 < \mathrm{E}[\pi_s] < 1$, and $\mathrm{E}[\gamma_s] + \frac{D}{x} < 1$.

5.2.3 *Approximation for K_s*

In this section, an analytical approach would be used to determine the optimal multipliers for mechanism two, discussed in section 5.2.2. Consider the total cost in Eq. (5.9) for the mechanism two that is the integer multiplier K_s mechanism. Assume for simplicity that K_s is a real number, then an approximate multiplier for supplier s can be determined by setting the first derivative of TC in Eq. (5.9) w.r.t K_s equal to zero and solving for K_s to get

$$K_s = \frac{1}{Q}\sqrt{\frac{2A_sD}{h_s\mu_s}} = \frac{H_s}{Q} \tag{5.11}$$

where $H_s \cong \sqrt{\frac{2A_sD}{h_s\mu_s}}$

Substituting Eq. (5.11) in Eq. (5.9) and simplifying results in

$$\mathrm{E}[TCU(Q)] = \frac{A_vD}{Q} + \sum_{s=1}^{m}\sqrt{2DA_sh_s\mu_s} + \frac{cD}{P} + \frac{Q}{2}\left[h_{v2}\left(1-\frac{D}{P}\right) - \sum_{s=1}^{m}h_s\mu_s\right]$$
$$+D\sum_{s=1}^{m}\left[(a_{vs}+d_s)\mu_s\pi_s + \frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)$$

As in Eq. (5.6), the second derivative test proves the convexity of the above expression. An approximate production quantity, which is independent of the multipliers, would then be

$$Q = \sqrt{\frac{2A_vD}{h_{v2}\left(1-\frac{D}{P}\right)-\sum_{s=1}^{m} h_s\mu_s+D\sum_{s=1}^{m} h_{v1s}\left\{\mu_s^2\mathrm{E}[\pi_s^2]\left(\frac{1+\mathrm{E}[\gamma_s]}{x}\right)+\frac{\mu_s\mathrm{E}[\pi_s]}{P}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+2\sum_{s=1}^{m} h_{v1s}\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)}} \quad (5.12)$$

The denominator of Eq. (5.10) is positive since $D < P$, $0 < \mathrm{E}[\pi_s] < 1$, and $\mathrm{E}[\gamma_s] + \frac{D}{x} < 1$.

To validate this approximation, one thousand examples were tried. That is, the exact and approximate production quantities were computed through Eq. (5.10) and Eq. (5.12) respectively. The difference between the costs calculated with these production quantities, was almost zero in one thousand cases. Thus, a solution procedure for the last two mechanisms would be as follows:

1. Estimate an approximate production quantity, using Eq. (5.12).
2. Estimate an approximate multiplier for supplier *s* using Eq. (5.11).
3. Determine integer values of the multipliers as $\lfloor K_s \rfloor$ and $\lceil K_s \rceil$
4. Determine an exact production quantity for each combination of the multipliers from step 3, using Eq. (5.10).
5. Determine an annual cost using Eq. (5.9), for each combination from step 3, using Q from step 4.
6. Determine an optimal annual cost of the supply chain as the minimum of the costs from step 4. This will indicate the optimal production quantity and the optimal set of multipliers.

5.3 Model Extensions

In this section, a number of extensions as outlined in section 5.1 will be discussed for the model in section 5.2.

5.3.1 *Type I and Type II Errors in Screening*

The screening process in most of the supply chain literature is assumed to be error-free, for example Huang (2002) and Goyal *et al*. (2003). But it is quite realistic to account for Type I and Type II errors committed by inspectors in this process with probabilities m_1 and m_2 respectively. In this section, it is assumed that the inspectors at the vendor's end commit errors while screening the suppliers' items. That is, they will classify some non-defective items as defectives while some defective items as non-defectives. In other words, they will attribute a percentage of defective to each supplier, different from the actual one. Thus, the defective items of type *s* classified by the inspection process would be

$$Q'_s = Q\mu_s(1-\mathrm{E}[\gamma_s])\mathrm{E}[m_1] + Q\mu_s\mathrm{E}[\gamma_s](1-\mathrm{E}[m_2])$$
$$= Q\mu_s\{(1-\mathrm{E}[\gamma_s])\mathrm{E}[m_1] + \mathrm{E}[\gamma_s](1-\mathrm{E}[m_2])\} = Q\mu_s\mathrm{E}[M_s]$$

Thus, the fraction accommodating the leftovers of type *s* in a cycle, will be given as

$$\mathrm{E}[\pi_{se}] = 1 - (\mathrm{E}[M_{\max}] - \mathrm{E}[M_s])$$

where $\mathrm{E}[M_{\max}] = \max\{\mathrm{E}[M_s],\ s = 1,2,\dots,m\}$ and Eq. (5.1) can be written as

$$Q_s = Q\mu_s\{1 - (\mathrm{E}[M_{\max}] - \mathrm{E}[M_s])\} = Q\mu_s\mathrm{E}[\pi_{se}] \tag{5.13}$$

So, the vendor's total cost of the raw materials can now be written as

$$C_{vr}(Q) = \sum_{s=1}^{m}\left[(a_{vs}+d_s)Q\mu_s\mathrm{E}[\pi_{se}] + \frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_{se}^2](1+\mathrm{E}[M_s])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_{se}]}{P}\right)\left(1 - \mathrm{E}[M_s] - \frac{D}{x}\right)\right\} + h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_{se}])\left(\mathrm{E}[T] - \frac{Q}{P}\right)\right] \tag{5.14}$$

The defective raw material misclassified by an inspector ends up making a defective product. This is assumed to cost the vendor an extra $c_f\mathrm{E}[m_2]Q\sum_{s=1}^{m}\mathrm{E}[\gamma_s]$. This may be taken as a goodwill cost or warranty cost. The loss due to misclassifying nondefective raw material is neglected for simplicity here. The rest of the model remains the same as in section 5.2.

5.3.2 *Learning in Vendor's Production Process*

In this section, it is assumed that the vendor's production process follows Wright (1936) learning curve. That is, vendor produces the final product at an increasing production rate which is consumed at a constant rate. Let us assume that T_{pi}, T_{di} and T_i are the production time, depletion time and the cycle time, respectively, in any cycle, as shown in Figure 5.3. The process produces a fixed quantity Q and builds up a maximum inventory Z_i, in each cycle *i*. The level of inventory in each cycle can be expressed as a function of time as

$$\Phi_i(t) = \begin{cases} Q(t) - Dt & 0 < t < T_{pi} \\ DT_i - Dt & T_{pi} < t < T_i \end{cases} \tag{5.15}$$

Let us now assume that *b* is the learning exponent, while $0 \leq b_i < 1$ is the learning exponent in cycle *i* of production. Faster learning is associated with higher values of b. The production time in a cycle *i* is written as

$T_{pi} = \int_{(i-1)Q}^{iQ} T_1 x^{-b_i} dx$ or

$$T_{pi} = \frac{T_1 Q^{1-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)} \tag{5.16}$$

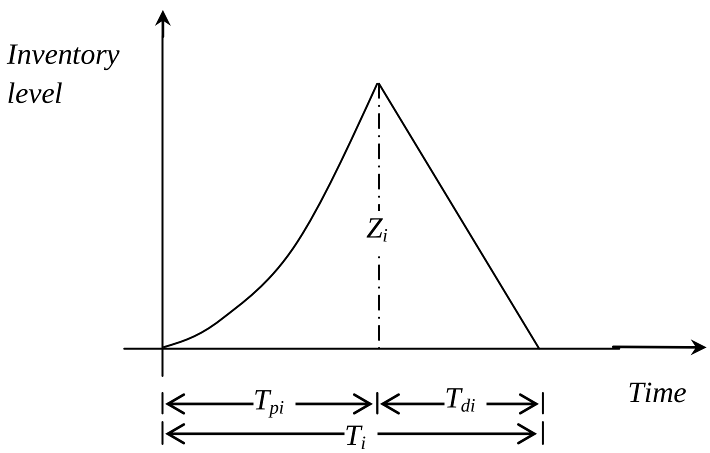

Figure 5.3 Vendor's inventory of the final product with learning in *i*th cycle

where $T_1 = 1/P$ is the time to assemble (produce) the first unit on the learning curve. To understand the changing learning rate, assume that the vendor produces *x* units in one cycle and *y* units in the next. If T_1 and T_2 are times to produce the first unit in the two cycles, the time to produce the y^{th} unit can be written as

$$T_y = T_1(x + y)^{-b} \tag{5.17}$$

and

$T_y = T_2 y^{-b_2}$ or

$$T_y = T_1(x+1)^{-b} y^{-b_2} \tag{5.18}$$

Equating expressions (5.17) and (5.18), it can be written as

$$b_2 = \frac{b[\log(x+y) - \log(x+1)]}{\log(y)}$$

In case of producing a fixed quantity Q in each cycle, the new learning rate can be written as

$$b_i = \frac{b[\log(iQ) - \log\{(i-1)Q+1\}]}{\log(Q)} \tag{5.19}$$

Now, the average inventory of finished products in a cycle *i* can be written as

$$\int_0^{T_i} \Phi_i(t)dt = \int_0^{T_{pi}} (Q - Dt)dt + \frac{Z_i T_{di}}{2}$$

After simplification, it can be written as

$$\int_0^{T_i} \Phi_i(t)dt = \frac{Q^2}{2D} - \frac{T_1 Q^{2-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)} \tag{5.20}$$

So, the vendor's total cost of the finished products in cycle *i* would be

$$C_{vf}(Q) = A_v + \frac{cT_1 Q^{1-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)} + h_{v2}\left[\frac{Q^2}{2D} - \frac{T_1 Q^{2-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]$$

Inventory buildup behaves linearly when learning in production is fast. Therefore, using Eq. (5.2), the vendor's total cost of the raw material, in cycle *i* would be

$$C_{vr}(Q) = \sum_{s=1}^{m}\left[(a_{vs} + d_s)Q\mu_s \mathrm{E}[\pi_s] + h_{v1s}Q\mu_s(1 - \mathrm{E}[\pi_s])\left(\mathrm{E}[T] - \frac{Q}{P}\right) + \right.$$
$$\left. \frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2 \mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \frac{Q^{-b_i}\mu_s \mathrm{E}[\pi_s] T_1\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)}\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)\right\}\right]$$

So, the vendor's total cost in a cycle would be

$C_v(Q)$=$A_v +$
$\sum_{s=1}^{m}\left[(a_{vs}+d_s)Q\mu_s\mathrm{E}[\pi_s]+\frac{h_{v1s}Q^2}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_s]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(\mathrm{E}[T]-\frac{Q}{P}\right)\right]+\frac{cT_1Q^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}+$
$h_{v2}\left[\frac{Q^2}{2D}-\frac{T_1Q^{2-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]$

and the vendor's annual cost would be

$$\mathrm{E}[TCU_v(Q)]=\frac{A_vD}{Q}+h_{v2}\left[\frac{Q}{2}-\frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]+D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_s]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s]\right]+\sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)+\frac{cDT_1Q^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \quad (5.21)$$

Using Eq. (5.5), the total annual cost of the supply chain for the equal cycle time can be written as

$$\mathrm{E}[TCU(Q)]=\frac{(A_v+\sum_{s=1}^{m}A_s)D}{Q}+h_{v2}\left[\frac{Q}{2}-\frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]+D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_s]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s]\right]+\sum_{s=1}^{m}h_{v1s}Qu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)+\frac{cT_1DQ^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \quad (5.22)$$

Eq. (5.22) is convex in Q (see Appendix 6 for proof). An iterative procedure will be carried out to determine the level of learning, production quantity and the annual cost for ten cycles of learning. An average of these measures will be used to compare the results with those for the other scenarios studied in the chapter.

Using Eqs. (5.8) and (5.22), the annual cost of the supply chain in a cycle, for the integer multiplier mechanism, would be

$$\mathrm{E}[TCU(Q)]=\frac{D\left(A_v+\sum_{s=1}^{m}\frac{A_s}{K_s}\right)}{Q}+h_{v2}\left[\frac{Q}{2}-\frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]+ \quad (5.23)$$

$$D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_s]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+\right.$$

$$\left.(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s]\right]+\sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)+$$

$$\frac{cDT_1Q^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}+\frac{Q\sum_{s=1}^{m}(K_s-1)h_s\mu_s}{2}$$

For an approximate value of the multipliers in Eq. (5.23), substituting K_s from Eq. (5.11) in Eq. (5.23), we get

$$\mathrm{E}[TCU(Q)]=\frac{A_vD}{Q}+\sum_{s=1}^{m}\left(\sqrt{2A_sDh_s\mu_s}-\frac{Qh_s\mu_s}{2}\right)+h_{v2}\left[\frac{Q}{2}-\frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right]+$$
$$D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_s]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s]-\frac{D}{x}\right)\right\}+\right.$$
$$\left.(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s]\right]+\sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right)+$$
$$\frac{cDT_1Q^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \tag{5.24}$$

The convexity of Eq. (5.24) can be shown in a similar manner to that of Eq. (5.22) (see Appendix 6 for proof). Again, an approximate value of the production quantity and the learning rate would be computed through iterating Eq. (5.25). This will be used to calculate the real-numbered values of multipliers by employing Eq. (5.11). The integer multipliers would be the values $\lfloor K_s \rfloor$, $\lceil K_s \rceil$ respectively. The minimum for the second policy will be found by plugging these integer values in Eq. (5.24). Again, an average of the ten cycles of learning will be calculated here.

5.3.3 *Learning in Suppliers' Quality*

In this section, it is assumed that the percentage of defectives per lot, from each supplier decreases following a learning curve. This improved quality may be attributed to the human learning in production and/or inspection at suppliers' end. Jaber *et al*. (2008) discovered this behavior in the items of an automotive industry. They showed that the data follows a logistic learning curve of the form

$$\gamma(i) = \frac{a}{g+e^{b_i}} \tag{5.25}$$

where *a* and *g* are the fit parameters while *b* is the learning rate and *i* is the number of shipments. Substituting this in Eq. (5.9), it becomes

$$\begin{aligned} \mathrm{E}[TCU(Q)] = {} & \frac{D\left(A_v+\sum_{s=1}^{m}\frac{A_s}{K_s}\right)}{Q} + \frac{cD}{P} + \frac{Q}{2}\left[h_{v2}\left(1-\frac{D}{P}\right) + \sum_{s=1}^{m}(K_s-1)h_s\mu_s\right] \\ & +D\sum_{s=1}^{m}\left[(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_s] + \frac{h_{v1s}Q}{2}\left\{\frac{u_s^2\mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s(i)])}{x} + \left(\frac{\mu_s\mathrm{E}[\pi_s]}{P}\right)\left(1-\right.\right.\right. \\ & \left.\left.\left.\mathrm{E}[\gamma_s(i)]-\frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m} h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_s])\left(1-\frac{D}{P}\right) \end{aligned} \tag{5.26}$$

The second derivative test proves the convexity of above cost function, as in Eq. (5.6). This annual cost function will be iterated through ten shipments. The average level of quality (γ_s) will be used to approximate Eq. (5.12) and the procedure to find the multipliers will remain the same as in section 5.2.

5.3.4 *Integrated Model*

It would be more realistic to consider all the human factors at the same time in our model. To do this, the inspection errors and learning in supplier's quality will have to be incorporated in Eq. (5.22), i.e. the case of learning in production. The resulting equation would be

$$\begin{aligned} \mathrm{E}[TCU(Q)] = {} & \frac{(A_v+\sum_{s=1}^{m}A_s)D}{Q} + h_{v2}\left[\frac{Q}{2} - \frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right] + \\ & D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_{se}^2](1+\mathrm{E}[\gamma_s(i)])}{x} + \frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_{se}]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s(i)]-\right.\right.\right. \\ & \left.\left.\left.\frac{D}{x}\right)\right\} + (a_{vs}+d_s)\mu_s\mathrm{E}[\pi_{se}]\right] + \sum_{s=1}^{m} h_{v1s}Qu_s(1-\mathrm{E}[\pi_{se}])\left(1-\frac{D}{P}\right) + \\ & \frac{cDT_1Q^{-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)} \end{aligned} \tag{5.27}$$

Similarly, for the integer multiplier mechanism, the annual cost would be

$$\mathrm{E}[TCU(Q)] = \frac{D\left(A_v+\sum_{s=1}^{m}\frac{A_s}{K_s}\right)}{Q} + h_{v2}\left[\frac{Q}{2} - \frac{T_1DQ^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right] + \tag{5.28}$$

$$D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2\mathrm{E}[\pi_{se}^2](1+\mathrm{E}[\gamma_s(i)])}{x}+\frac{Q^{-b_i}\mu_s\mathrm{E}[\pi_{se}]T_1\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}\left(1-\mathrm{E}[\gamma_s(i)]-\frac{D}{x}\right)\right\}+(a_{vs}+d_s)\mu_s\mathrm{E}[\pi_{se}]\right]+\sum_{s=1}^{m}h_{v1s}Q\mu_s(1-\mathrm{E}[\pi_{se}])\left(1-\frac{D}{P}\right)+\frac{cT_1Q^{1-b_i}\{i^{1-b_i}-(i-1)^{1-b_i}\}}{(1-b_i)}+\frac{Q\sum_{s=1}^{m}(K_s-1)h_s\mu_s}{2}$$

Eq. (5.28) is convex in Q. A similar proof to the one in Appendix 6 can be applied here. An average of ten cycles of learning will be computed here for the production size and the annual cost.

5.4 Numerical Analysis

Consider a two-level supply chain with two suppliers and a vendor. The vendor produces a single product and procures its parts from suppliers. Most of the data is obtained from Khouja (2003) and Salameh and Jaber (2000). The vendor's production rate is taken to be three times its demand while its unit holding cost of the final product is taken to be much more than that of the raw material. This accounts for the value added during the production process. For simplicity of the analysis, it is assumed that all the suppliers have the same setup cost. Besides, the percentage of defectives and the inspection errors are assumed to be uniformly distributed and are given respectively by

$$f_1(\gamma)=\begin{cases}1/(0.1-0), & 0\le\gamma\le 0.1\\ 0, & otherwise\end{cases}$$

$$f_2(m_1)=\begin{cases}1/(0.1-0), & 0\le m_1\le 0.1\\ 0, & otherwise\end{cases}$$

$$f_3(m_2)=\begin{cases}1/(0.1-0), & 0\le m_2\le 0.1\\ 0, & otherwise\end{cases}$$

The learning curve for the two suppliers is taken to be (Jaber *et al.*, 2008)

$\gamma_1(n)=\frac{40}{999+e^{0.8n}}$ and

$\gamma_2(n)=\frac{80}{999+e^{0.8n}}$ respectively.

5.4.1 *Effect of Human Factors*

The data and results for the different mechanisms under a number of human factors discussed in this chapter are given in Table 5.1. A base case for this data is solved to have a bench mark where there are no defective items from the suppliers, no learning and there is no inspection error.

Table 5.1 Input data and results of the numerical examples

A_v	A_1	A_2	a_{v1}	a_{v2}	h_1	h_2	u_1	u_2	d	h_{v1}
200	400	400	1	1	0.5	1	10	3	0.1	2
h_{v2}	γ_1	γ_2	P	c	D	x	m_1	m_2	b	c_f
15	0.04	0.08	30000	1000000	10000	175200	0.04	0.05	0.1	50

Base Case (No Defective Items)

			Q	TC
Mechanism 1			808	501078
Mechanism 2	K_s	3,4	418	497070

Defective Items

			Q	TC
Mechanism 1			809	498418
Mechanism 2	K_s	3,4	418	494420

Inspection Error

			Q	TC
Mechanism 1			809	499034
Mechanism 2	K_s	3,4	418	495030

Learning in Production (Vendor)

			Q	TC
Mechanism 1			924	308951
Mechanism 2	K_s	3,3	462	160547

Learning in Quality (Supplier)

			Q	TC
Mechanism 1			809	499248
Mechanism 2	K_s	3,4	418	496902

Integrated Model

			Q	TC
Mechanism 1			923	310215
Mechanism 2	K_s	3,3	463	163034

It should be noted that for this base case, as one moves from mechanism 1 (equal cycle time) to mechanism 2 (integer multiplier cycle time), both the production quantity per cycle and the annual cost decrease. The rationale for this is that increased number of shipments forces both the vendor and the suppliers to carry smaller inventories, thus decreasing the overall inventory cost. This indicates that there exists an optimal set of multipliers for the given data and the overall performance of the supply chain starts deteriorating as we move away from that set of multipliers. The outcome of this example is that once the multiplier (K_s) goes beyond a certain number, the annual cost starts increasing again. One should observe that mechanism 1 is a special case of mechanism 2, for $K_s = 1$.

Khouja (2003) proposed another coordination mechanism, integer-power-of-two or 2^{M_s}, where $K_s = 2^{M_s} = 1, 2, 4, 8$, when $M_s = 0, 1, 2, 3$. One thousand data sets for the model input parameters (i.e., A_s, a_{vs}, μ_s, A_v, h_{v1s}, h_{v2}, h_s) were randomly generated and used to optimize a corresponding number of numerical examples for the model described in section 5.2 first by optimizing for K_s and then by replacing K_s in Eqs. (5.9) and (5.10) by 2^{M_s}. The results showed that the 2^{M_s} policy never performs better, which is why this policy was not considered in this chapter.

The above trend remained the same for all the cases (human factors) discussed in the chapter, as shown in Table 5.1. That is, the case with (*i*) defective items from the suppliers, (*ii*) vendor's inspection error, (*iii*) vendor's learning in production, (*iv*) suppliers' learning in quality, (*v*) the integrated model. As we include the defectives in our study, it is seen that there is a noticeable drop in the annual cost while the production quantity has a minor change. This indicates that the screened items at the vendor's end decrease its carrying cost.

5.4.2 *Sensitivity Analysis*

An interesting outcome of this example suggests that including the defective items results in the reduction of the annual cost of the supply chain. The rationale for this unusual behavior is that the leftovers considered in our example tend to decrease the annual cost, in case there are some defective items from the suppliers.

To further illustrate the effect of the defective items on the annual cost, the variation of the cost for the first mechanism, with respect to the percentage of defectives from the suppliers is shown in Figure 5.4. While varying the fraction of defectives from one supplier, the other supplier is assumed to supply 10% defective parts. It can be seen from Figure 5.4 that the defective items from supplier 2 have more variation in the impact on the annual cost of the supply chain while those from supplier 1 have little variation on this cost. This is attributed to the number of leftover parts from the other suppliers in each cycle. That is, the more the number of parts, the more are the leftovers which reduces the holding cost of the parts and thus the annual cost of the supply chain.

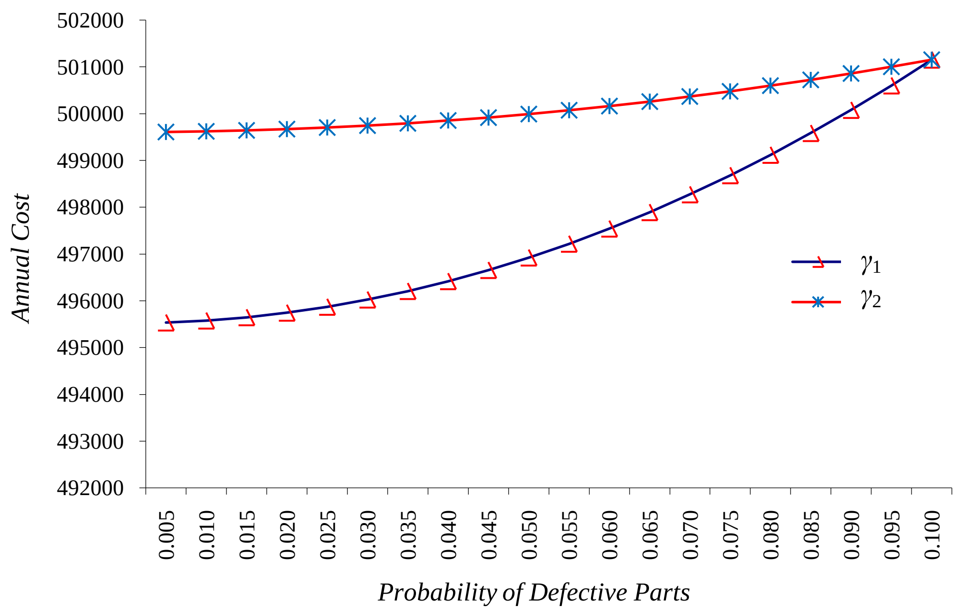

Figure 5.4 Effect of percentage of defective parts from suppliers

Introducing inspection error to the model brings in a slight increase in the annual cost. This accounts for the non-defective parts misclassified. To illustrate the effect of the inspection errors, the variation of the annual cost with these errors, for the first mechanism is drawn in Figure 5.5 and 5.6. Both the errors tend to raise the annual cost but Type II error has a pronounced effect as compared to that of Type I error. That is, the misclassified defective items cause a major impact on the overall cost of the supply chain. However, if the loss due to misclassification of perfect items is included, it may change the magnitude of the effect of Type 1 error.

Learning at the vendor's end brings in the most savings when compared to the base case (about 38%). The rationale for this is that learning decreases the production time and results in a major cut down in the overall cost for the whole supply chain. On the other hand, learning in the suppliers' quality brings the supply chain close to the state where there are no defectives from the suppliers.

For the integrated model, it can be seen that the savings from learning decrease which is quite intuitive as this is the effect of inspection errors.

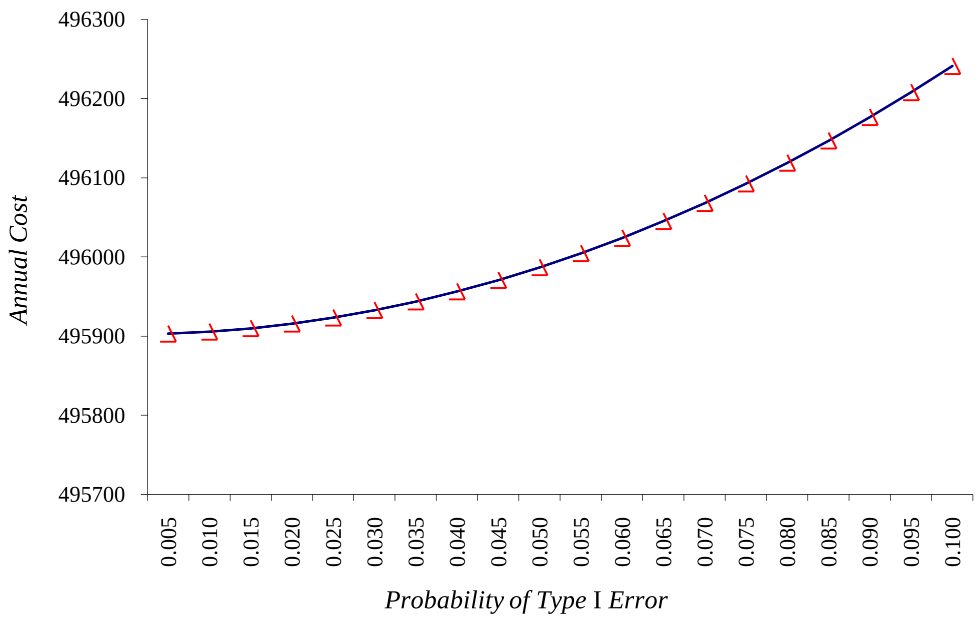

Figure 5.5 Effect of Type I error on the cost of supply chain with Type II error = 5%

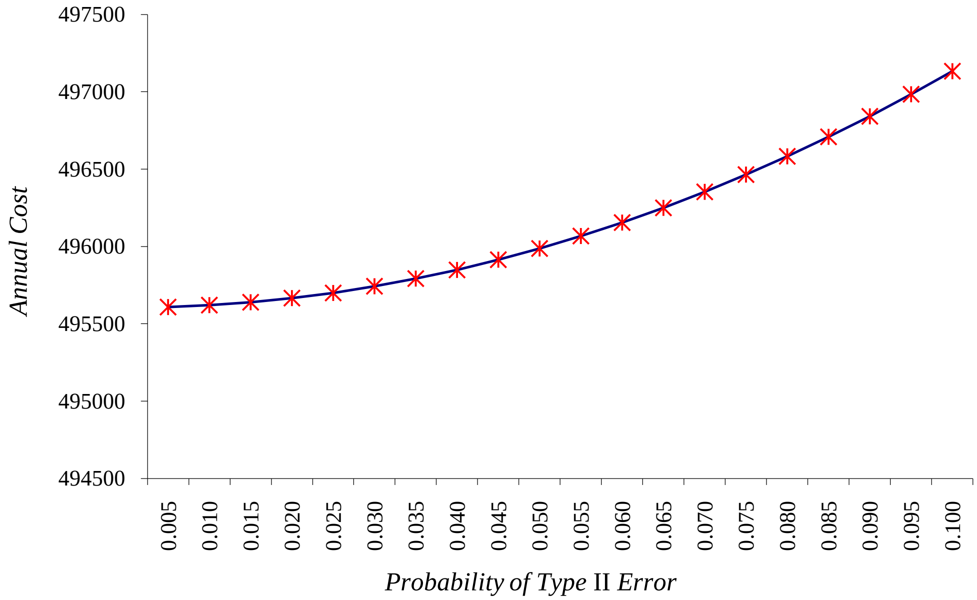

Figure 5.6 Effect of Type II error on the cost of supply chain with Type I error = 5%

5.5 Summary and Conclusions

In this chapter, a two-stage, multi-supplier, single-vendor supply chain is formulated. A vendor is supposed to ask for a number of components from different suppliers, which are needed to make a single product. Suppliers are believed to be providing a certain fixed percentage of defectives in their supplies. As it is costlier to send the defective items back to the suppliers, the vendor institutes a 100% inspection process and sells the defectives in the local market at a discounted price.

Two mechanisms, as in Khouja (2003) were studied for the coordination between suppliers and the vendor. The first mechanism is governed by an equal cycle time for all the stakeholders of the supply chain. In the second mechanism, each supplier's cycle time is taken to be an integer multiplier of the vendor's cycle time. It was observed that mechanism 1 is a special case of mechanism 2, for certain values of the multiplier. An approximate solution method is provided to attain the multipliers in the second mechanism. This approximation is validated through numerical examples and is found to work well. The third mechanism in Khouja (2003) was ignored in this research as it does not result in better costs than those of the second policy. The numerical examples presented showed that the integer multipliers mechanism behaves better than the equal cycle time mechanism. That is, the suppliers are supposed to follow a relaxed and practical approach of the integer multipliers cycle time rather than forcing themselves to follow an equal cycle time. The rationale is that after certain level of multiples of the cycle time, the total cost of the supply chain starts rising up. Besides, integer multiplier presents a practical scenario for coordination in a supply chain.

A number of human factors are brought into the picture in this chapter. First of all, a scenario is considered in which the inspectors at the vendor's end make misclassifications. This factor is shown to increase the carrying cost of the vendor and thus the overall annual cost of the supply chain. Next, the production process of the vendor is assumed to follow learning as workers tend to perform the same job at a faster pace. This brings in a substantial drop in the annual cost of the supply chain. Lastly, the quality of the suppliers' items is assumed to follow a logistic learning curve. This effect brings the supply closer to the state in which there are no defectives from the suppliers. Finally, the integrated model results in savings in the annual cost which are lesser than those experienced in the case of learning in production only.

An avenue for future research is incorporating a stochastic percentage of defectives. This would further enhance the coordination mechanisms. Another possible study could be to investigate learning in inspection errors. One could also study the effects of a probabilistic demand from the vendor in response to the market's behavior.

CHAPTER 6 A VENDOR-BUYER SUPPLY CHAIN WITH INSPECTION ERRORS AND LEARNING IN PRODUCTION

6.1 Introduction

The use of the economic order/production quantity model is quite common for about hundred years in the literature concerning inventory: Simpson (2001). This model is a trade-off between the holding cost and ordering cost of a buyer. Although this model has been so widely used it has several weaknesses. For example a steady demand from the buyer or the supply of perfect quality products from the vendor, is usually out of question. These assumptions paved the path for many researchers. The example of such literature could be Porteus (1986*b*), Rosenblatt and Lee (1986) and Silver (1976).

In this chapter an equal size policy is adopted as in Huang (2002) for a two-level vendor-buyer supply chain. The vendor follows an EPQ policy to manufacture a single product. The coordination mechanism is such that (*i*) the vendor receives the buyer's demand produces the single product; (*ii*) the vendor replenishes the order in a number of equal-sized shipments. Besides, we assume that (*i*) the vendor experiences learning in the production process, (*ii*) some of the products are defective (*iii*) the buyer institutes an inspection process as suggested by Salameh and Jaber (2000), and that (*iv*) this inspection process is prone to Type I and Type II errors. The defective products from the vendor may be a result of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit: Ouyang *et al*. (2006*b*). An equal-shipment-size coordination between vendor and buyer is described with fixed and variable transportation cost. An optimal lot size and the annual cost are determined for the two level supply chain. The model is then extended to include Type I and Type II errors in the buyer's screening process. In the second extension, the vendor's production process is assumed to follow Wright (1936) learning curve, thus affecting the production time. An optimal lot size, the number of shipments and the annual cost of the supply chain are determined for each of these extensions.

The rest of the chapter is arranged as follows: In section 6.2, a description and formulation of the model and its extensions are given. Section 6.3 presents numerical examples for the base model and the extensions. Section 6.4 presents conclusions, limitations and some suggestions for future research.

6.2 Model Description

Consider a two level supply chain scenario with one vendor and one buyer. Let us suppose that the vendor has to make nQ products in each cycle of production. A fixed percentage γ of these products is believed to be defective. For this, the buyer institutes a 100% inspection and screens out all the defective products from the lots provided by the vendor, at a rate of x per unit time.

Following Salameh and Jaber (2000) approach for defective products, we assume that each lot received by a buyer contains a fixed percentage of defectives, γ, with a known probability density function, f_1. Figure 6.1 and Figure 6.3 depict the behavior of the vendor's and buyer's inventory level, respectively. Figure 6.2 shows the accumulation of inventory at the buyer's and vendor's end.

The costs considered in the model would be ordering/setup cost, screening cost, the inventory carrying cost, shipment cost and the production cost. The buyer bears the fixed and variable cost (per unit) of shipment to the buyer. An optimal production quantity and number of shipments per cycle will be determined through the total cost of the supply chain.

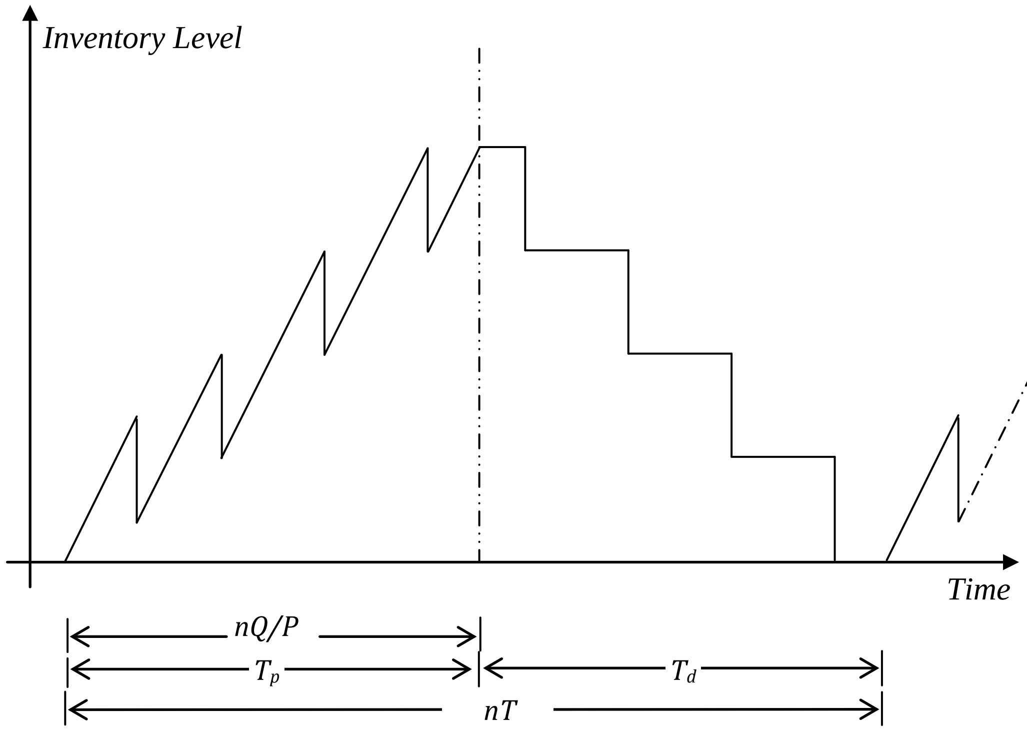

Figure 6.1 Vendor's ınventory level in a cycle with time

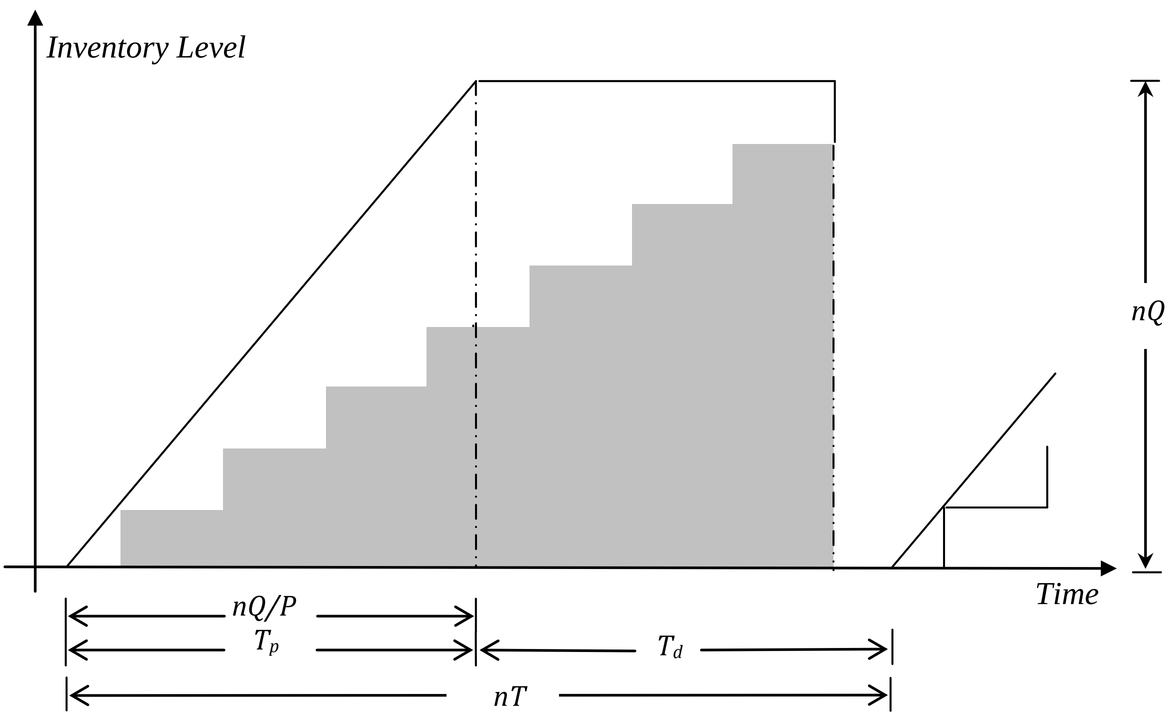

Figure 6.2 Accumulation and depletion of vendor's inventory in a cycle

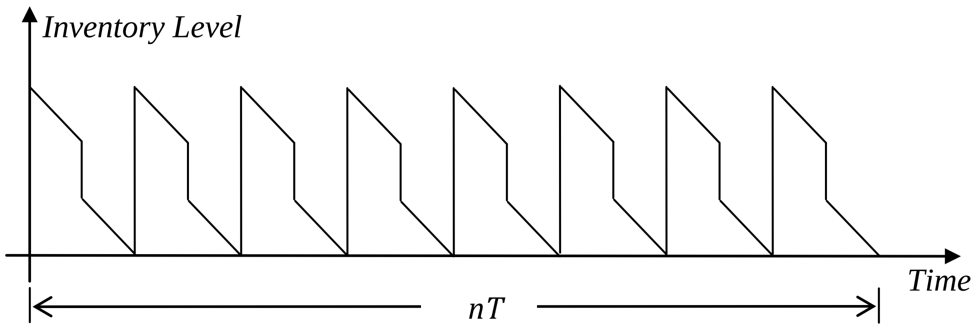

Figure 6.3 Buyer's inventory in vendor's one cycle

The objective of the study would be to minimize the total annual cost through an optimal production quantity.

6.2.1 *The Base Model*

Total inventory with the vendor in a cycle, is the sum of areas of the triangle and the rectangle with dotted line as one side, in Figure 6.1, i.e.

$$Area\ 1 = \frac{1}{2}(nQ/P)(nQ) = \frac{n^2Q^2}{2P} \tag{6.1}$$

$$Area\ 2 = nQ\left[(n-1)\left(\frac{Q}{D}-\frac{Q}{P}\right)\right] = \frac{nQ^2(n-1)(P-D)}{PD} \tag{6.2}$$

The total inventory moved to the buyer in a cycle by vendor is $n(n-1)Q^2/2D$. So, vendor's total inventory in a cycle is

$$I_v(Q,n) = \frac{n^2Q^2}{2P} + \frac{nQ^2(n-1)(P-D)}{PD} - \frac{n(n-1)Q^2}{2D} = \frac{nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\} \tag{6.3}$$

Vendor's total cost in a cycle is the sum of setup, carrying and production costs:

$$C_v(Q,n) = A_v + \frac{h_v nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + \frac{ncQ}{P}$$

The buyer's total cost in vendor's one cycle is the sum of ordering, carrying, screening and the shipment costs:

$$C_b(Q,n) = nA_b + nh_b\left\{\frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right\} + ndQ$$

Total cost of the two level (vendor-buyer) supply chain in a cycle is

$$TC = A_v + \frac{h_v nQ^2}{2D}\left\{(n-1) - (n-2)\frac{D}{P}\right\} + \frac{ncQ}{P} + nA_b + nh_b\left\{\frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right\} + ndQ \tag{6.4}$$

Since γ is a random variable with probability density function f_1, the expected total cost of the supply chain per cycle, after rearranging the terms, is

$$TC(Q,n) = A_v + nA_b + \frac{nQ^2}{2D}\left[h_v\left\{(n-1) - (n-2)\frac{D}{P}\right\} + \frac{2h_b D\mathrm{E}[\gamma]}{x}\right] + \frac{ncQ}{P} + \frac{nh_b Q(1-\mathrm{E}[\gamma])\mathrm{E}[T]}{2} + ndQ$$

As we have

$\mathrm{E}[T] = \frac{(1-\mathrm{E}[\gamma])Q}{D}$, using $n\mathrm{E}[T]$ as the total cycle time, the expected annual cost, using Maddah and Jaber (2008) approach, would be

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{(1-\mathrm{E}[\gamma])Q}\left\{\frac{A_v}{n} + A_b\right\} + \frac{dD}{(1-\mathrm{E}[\gamma])} + \frac{Q}{2(1-\mathrm{E}[\gamma])}\left[h_v\left\{(n-1) - (n-2)\frac{D}{P}\right\} + \frac{2h_b D\mathrm{E}[\gamma]}{x}\right] + \frac{cD}{P(1-\mathrm{E}[\gamma])} + \frac{h_b Q(1-\mathrm{E}[\gamma])}{2}$$

or

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{(1-\mathrm{E}[\gamma])Q}\left\{\frac{A_v}{n} + A_b\right\} + \frac{D}{(1-\mathrm{E}[\gamma])}\left(d + \frac{c}{P}\right) + \frac{Q}{2}\left[\frac{h_v}{(1-\mathrm{E}[\gamma])}\left\{(n-1) - (n-2)\frac{D}{P}\right\} + h_b\{(1-\mathrm{E}[\gamma]) +\}\frac{2D\mathrm{E}[\gamma]}{x(1-\mathrm{E}[\gamma])}\right] \tag{6.5}$$

Let $M_1 = \mathrm{E}[\gamma]$ and $M_2 = {}^{1}/_{(1-\mathrm{E}[\gamma])}$

$$\mathrm{E}[TCU(Q,n)] = \frac{DM_2}{Q}\left\{\frac{A_v}{n} + A_b\right\} + DM_2\left(d + \frac{c}{P}\right)$$
$$+\frac{Q}{2}\left[h_v M_2\left\{(n-1) - (n-2)\frac{D}{P}\right\} + h_b\{(1-M_1)+\}\frac{2DM_1M_2}{x}\right] \tag{6.6}$$

The second derivative of the above expression is positive $\frac{d^2}{dQ^2}E[TCU] = \frac{2DM_2}{Q^3}\left\{\frac{A_v}{n} + A_b\right\} > 0$, $\forall\, Q > 0,$ which shows that Eq. (6.6) convex in Q. To obtain an optimal batch size and the number of shipments per cycle from the above expression, we would equate its first derivative with respect to Q, to zero and then find the value of *n* through iteration.

6.2.2 *Inspection Error*

The screening process in most of the supply chain literature is assumed to be error-free, for example Huang (2002) and Goyal *et al*. (2003). But it is quite realistic to account for Type I and Type II errors committed by inspectors in this process. In this section, it is assumed that the inspectors at the buyer's end commit errors while screening the vendor's product. That is, they will classify some non-defective products as defectives while some defective products as non-defectives. In other words, they will attribute a percentage of defective to the vendor which is different from the actual one. Thus, the fraction of defective products as perceived by the inspectors would be

$$\gamma_e = (1-\gamma)m_1 + \gamma(1-m_2)$$

and

$$\mathrm{E}[\gamma_e] = (1-\mathrm{E}[\gamma])\mathrm{E}[m_1] + \mathrm{E}[\gamma](1-\mathrm{E}[m_2])$$

So the time interval between successive shipments would now be

$$\mathrm{E}[T] = \frac{(1-\mathrm{E}[\gamma_e])Q}{D} = \frac{\{(1-\mathrm{E}[\gamma])\mathrm{E}[m_1] + \mathrm{E}[\gamma](1-\mathrm{E}[m_2])\}Q}{D}$$

Similarly $M_{1e} = \mathrm{E}[\gamma_e]$ and $M_{2e} = {}^{1}/_{(1-\mathrm{E}[\gamma_e])}$

Thus Eqs. (6.5) and (6.6), for the expected annual cost of the supply chain would now be written as

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{(1-\mathrm{E}[\gamma_e])Q}\left\{\frac{A_v}{n} + A_b\right\} + \frac{D}{(1-\mathrm{E}[\gamma_e])}\left(d + \frac{c}{P}\right)$$
$$+ \frac{Q}{2}\left[\frac{h_v}{(1-\mathrm{E}[\gamma_e])}\left\{(n-1) - (n-2)\frac{D}{P}\right\} + h_b\{(1-\mathrm{E}[\gamma_e]) +\}\frac{2D\mathrm{E}[\gamma_e]}{x(1-\mathrm{E}[\gamma_e])}\right] \quad (6.7)$$

$$E[TCU(Q,n)] = \frac{DM_{2e}}{Q}\left\{\frac{A_v}{n} + A_b\right\} + DM_{2e}\left(d + \frac{c}{P}\right)$$
$$+ \frac{Q}{2}\left[h_v M_{2e}\left\{(n-1) - (n-2)\frac{D}{P}\right\} + h_b\{(1-M_{1e}) +\}\frac{2DM_{1e}M_{2e}}{x}\right] \quad (6.8)$$

The second derivative of the above expression is positive $\frac{d^2}{dQ^2}E[TCU] = \frac{D}{(1-\mathrm{E}[\gamma_e])Q^3}\left\{\frac{A_v}{n} + A_b\right\} > 0$, $\forall\, Q > 0$ where $0 < \mathrm{E}[\gamma_e] < 1$, which shows that Eq. (6.8) is convex in Q. We will use the procedure described above to determine the optimal batch size and the number of shipments with screening errors.

6.2.3 *Learning in Production*

In this section, it is assumed that the vendor's production process follows Wright (1936) learning curve. That is, vendor produces the product at an increasing production rate which is consumed at a constant rate. Assume that every cycle of production makes Q_p (nQ) products, with a learning rate b. This situation for cycle *i* is described in Figure 6.4 and Figure 6.5. The production time in cycle *i* is

$$T_{pi} = \int_{(i-1)Q}^{Q} T_1 x^{-b} dx$$

$$T_{pi} = \frac{T_1 x^{1-b}\{i^{1-b} - (i-1)^{1-b}\}}{1-b} \quad (6.9)$$

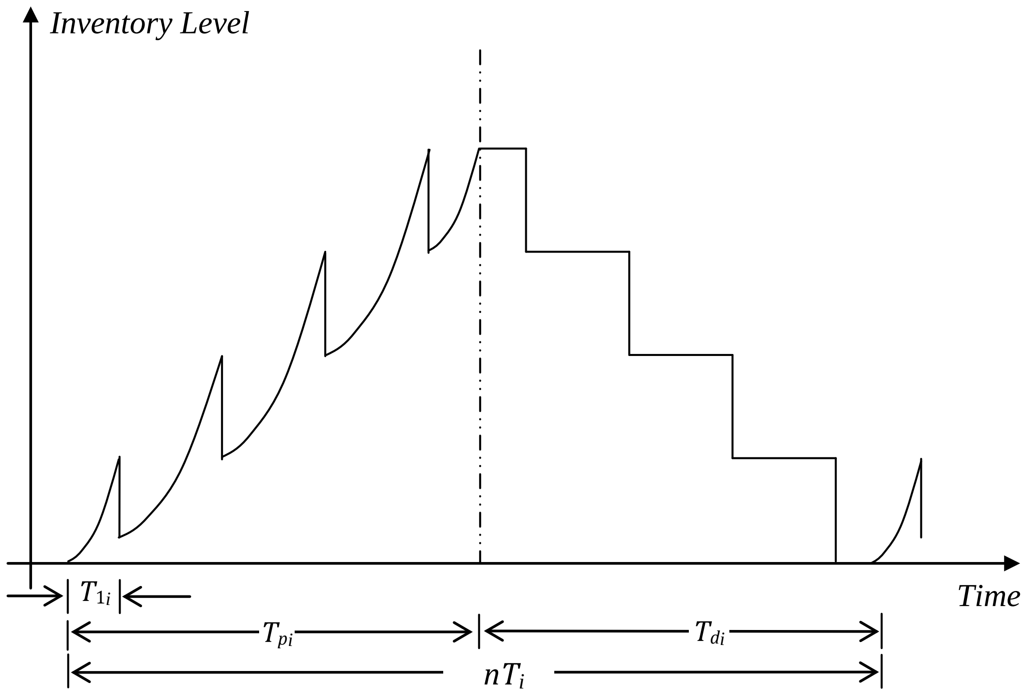

Figure 6.4 Vendor's inventory level in i^{th} cycle with learning in production

So, the production quantity in cycle *i* can be written as

$$Q(t) = \left[\frac{(1-b)t}{T_1}\left\{\frac{1}{i^{1-b}-(i-1)^{1-b}}\right\}\right]^{\frac{1}{1-b}} \tag{6.10}$$

Now, the average inventory of products in a cycle *i* during production is

$$I_{T_{pi}} = \int_0^{T_{pi}} Q(t)dt$$

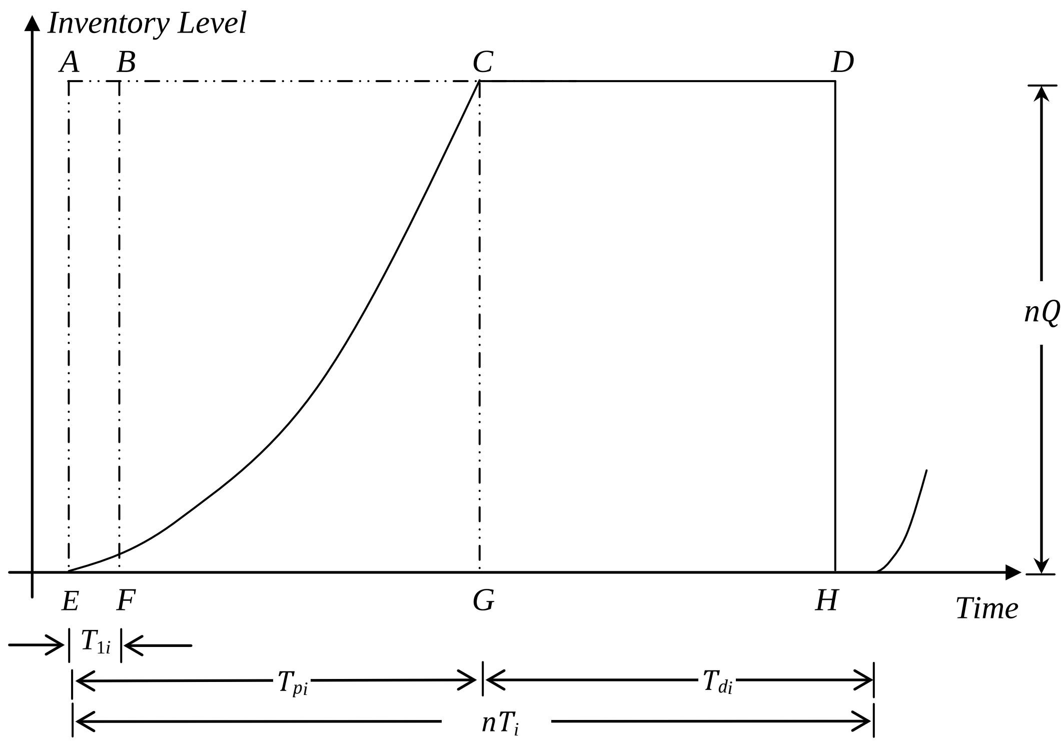

Figure 6.5 Vendor's total inventory in i^{th} cycle with learning in production

or

$$I_{T_{pi}} = \frac{T_1\{i^{1-b} - (i-1)^{1-b}\}(nQ)^{2-b}}{2-b} \tag{6.11}$$

Time for the first dispatch after the start of production in i^{th} cycle, in Figures 6.4, 6.5 would be:

$$T_{1i} = \int_{(i-1)nQ}^{Q+(i-1)nQ} T_1 x^{-b} dx$$

$$T_{1i} = \frac{T_1 Q^{1-b}[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b}$$

Now we determine vendor's average inventory in a cycle. The area of the rectangles *ABEF*, *BFDH* and *AECG* are given by:

$$Area_{ABEF} = nQT_{1i} = \frac{nT_1 Q^{2-b}[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b}$$

$$Area_{BFDH} = nQ\left\{(n-1)\frac{Q}{D}\right\} = \frac{n(n-1)Q^2}{D}$$

$$Area_{AECG} = nQT_{pi} = \frac{T_1(nQ)^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{1-b}$$

So, the vendor's average inventory in the depletion period in i^{th} cycle is determined from the three areas above as

$$I_{T_{di}} = Area_{ABEF} + Area_{BFDH} - Area_{AECG}$$

$$I_{T_{di}}(Q,n) = \frac{nT_1Q^{2-b}[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b} + \frac{n(n-1)Q^2}{D} - \frac{T_1(nQ)^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{1-b} \tag{6.12}$$

As in Eq. (6.3), vendor's average inventory in a cycle would be

$$I_{vi} = I_{T_{pi}} + I_{T_{di}} - \frac{n(n-1)Q^2}{2D}$$

Using Eqs. (6.11) and (6.12):

$$I_{vi}(Q,n) = \frac{T_1\{i^{1-b} - (i-1)^{1-b}\}(nQ)^{2-b}}{2-b} + \frac{nT_1Q^{2-b}[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b} + \frac{n(n-1)Q^2}{D} - \frac{T_1(nQ)^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{1-b} - \frac{n(n-1)Q^2}{2D}$$

or

$$I_{T_{di}}(Q,n) = \frac{nT_1Q^{2-b}[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b} + \frac{n(n-1)Q^2}{2D} - \frac{T_1(nQ)^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{(2-b)(1-b)} \tag{6.13}$$

It should be noted that by substituting T_1 by the initial production rate P and b by zero, the above expression reduces to Eq. (6.3), where there is no learning. In other words, Eq. (6.13) represents an extension of Hill (1997) model of equal shipments, for learning in production.

Now, vendor's total cost in a cycle is

$$C_{vi}(Q) = A_v + h_v \left[\frac{nT_1 Q^{2-b}[\{1+(i-1)\lambda\}^{1-b} - \{(i-1)n\}^{1-b}]}{1-b} + \frac{n(n-1)Q^2}{2D} \right.$$

$$\left. - \frac{T_1(nQ)^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{(2-b)(1-b)} \right] + cT_{pi}$$

or

$$C_{vi}(Q) = A_v + \frac{h_v Q^{2-b}}{P(1-b)} \left[n[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}] - \frac{n^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{(2-b)} \right]$$

$$+ \frac{h_v n(n-1)Q^2}{2D} + \frac{c(nQ)^{1-b}\{i^{1-b} - (i-1)^{1-b}\}}{P(1-b)}$$

So, the total cost of the two level supply chain, with learning in production would be

$$TC_i(Q) = A_v + \frac{h_v Q^{2-b}}{P(1-b)} \left[n[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}] - \frac{n^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{(2-b)} \right]$$

$$+ \frac{h_v n(n-1)Q^2}{2D} + \frac{c(nQ)^{1-b}\{i^{1-b} - (i-1)^{1-b}\}}{P(1-b)} + A_b + nh_b \left\{ \frac{Q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x} \right\}$$

$$+ndQ$$

The expected total cost in a cycle is

$$\mathrm{E}[TC_i(Q,n)] = A_v + nA_b$$

$$+ \frac{h_v Q^{2-b}}{P(1-b)} \left[n[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}] - \frac{n^{2-b}\{i^{1-b} - (i-1)^{1-b}\}}{(2-b)} \right]$$

$$+ \frac{h_v n(n-1)Q^2}{2D} + \frac{c(nQ)^{1-b}\{i^{1-b} - (i-1)^{1-b}\}}{P(1-b)} + nh_b \left\{ \frac{Q(1-\mathrm{E}[\gamma])\mathrm{E}[T]}{2} + \frac{\mathrm{E}[\gamma]Q^2}{x} \right\}$$

$$+ndQ$$

and the expected annual cost is

$$\mathrm{E}[TCU_i(Q,n)] = \frac{D}{(1-\mathrm{E}[\gamma])Q}\left\{\frac{A_v}{n}+A_b\right\} + \frac{dD}{(1-\mathrm{E}[\gamma])}$$

$$+\frac{h_v D Q^{1-b}}{P(1-\mathrm{E}[\gamma])(1-b)}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right]$$

$$+\frac{h_v(n-1)Q}{2(1-\mathrm{E}[\gamma])D} + \frac{c(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{P(1-\mathrm{E}[\gamma])(1-b)} + \frac{h_b Q(1-\mathrm{E}[\gamma])}{2} + \frac{h_b \mathrm{E}[\gamma]QD}{x(1-\mathrm{E}[\gamma])}$$

Replacing $M_1 = \mathrm{E}[\gamma]$ and $M_2 = {}^{1}/_{(1-\mathrm{E}[\gamma])}$:

$$\mathrm{E}[TCU_i(Q,n)] = \frac{DM_2}{Q}\left\{\frac{A_v}{n}+A_b\right\} + dDM_2 + \frac{h_b QDM_1M_2}{x}$$
$$+\frac{h_v DM_2 Q^{1-b}}{P(1-b)}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right] \qquad (6.14)$$
$$+\frac{h_v(n-1)QM_2}{2D} + \frac{cM_2(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{P(1-b)} + \frac{h_b Q(1-M_1)}{2}$$

Eq. (6.14) is convex in Q (see Appendix 7 for proof). We assume that the learning in the vendor's production process plateaus to some extent after ten cycles. Mathematica 5 will be used to find an optimal batch size and the number of shipments from Eq. (6.14), with the steps given below:

1. Set i = 1 and n = 1.
2. Find an optimal value of Q and annual cost from Eq. (6.14) through iteration. Set n = 2.
3. Repeat steps 2 till the annual cost for n shipments is more than that of (n – 1) shipments.
4. Record the optimal values of Q and annual cost for n^* = n – 1.
5. Set i = i + 1 and n = 1.
6. Repeat steps 2 through 4 till i = 10. (A maximum of 10 cycles is considered)

7. Find an average of the number of shipments, batch size and the annual cost from the values recorded in step 4.

It should be noted that for an integrated model that incorporates learning in production and the inspection errors at the same time, the fraction of defective (M_1 and M_2) in Eq. (6.14) will have to be updated.

6.3 Numerical Example

Consider a two-level vendor-buyer supply chain. The vendor produces a single product to fulfill the demand of the buyer. Most of the data is obtained from Goyal *et al*. (2003) and Salameh and Jaber (2000). The unit holding cost of the product at buyer's end is taken to be more than that the vendor's end which accounts for the value added during the production process. The percentage of defectives and the inspection errors (Type I and Type II) are assumed to be uniformly distributed as given by

$$f_1(\gamma) = \begin{cases} 1/(0.04-0), & 0 \le \gamma \le 0.04 \\ 0, & otherwise \end{cases}$$

$$f_2(m_1)= \begin{cases} 1/(0.04-0), & 0 \le m_1 \le 0.04 \\ 0, & otherwise \end{cases}$$

$$f_3(m_2)= \begin{cases} 1/(0.04-0), & 0 \le m_2 \le 0.04 \\ 0, & otherwise \end{cases}$$

respectively.

To illustrate the iterative procedure for a cycle of learning in section 6.2.3, an example is provided in Table 6.1*a* and *b*. The Table 6.1*a* shows the results of the first cycle of learning while Table 6.1*b* indicates how we take an average of ten cycles. The optimal values used in the analysis are shown in bold. It can be seen that for the first cycle of learning, the optimal number of shipments is 5 while the size of each shipment and the annual cost are 206 and \$ 8255 respectively. It should be noticed in Table 6.1*b* that by virtue of learning, the total inventory in a cycle reduces from 1030 (5×206) to 639 (3×213) i.e. 38% while the annual cost is reduced from 8255 to 4611, i.e. about 44%. This indicates a reduction in the average cost per unit from 8.01 (8255/1030) to 7.22 (4611/639).

Table 6.1*a* Iterative procedure to find n and Q with learning

i	n	Q	E[TCU (Q)]
1	1	880	8592
	2	478	8356
	3	330	8289
	4	253	8263
	5	**206**	**8255**
	6	174	8258

Table 6.1*b* The optimal policies for 10 consecutive cycles

i	n	Q	E[TCU(Q)]
1	5	206	8255
2	4	201	6153
3	3	245	5635
4	3	235	5337
5	3	229	5133
6	3	225	4981
7	3	221	4862
8	3	218	4764
9	3	215	4681
10	3	213	4611
Average	**3**	**221**	**5441**

The results in Table 6.2 indicate that bringing in screening errors at the buyer's end makes the annual cost of the supply chain a little higher which accounts for the increase in the order size. That is, the inspectors classify some non-defective items as defectives and some defective items as non-defectives. To illustrate the effect of the inspection errors, the variation of the annual cost with these errors, is drawn in Figure 6.6 and 6.7. It can be seen that for this vendor-buyer supply chain, Type I error has a pronounced effect as compared to that of Type II error. That is, the more non-defective items are misclassified the higher is the order size and so is the annual cost of the supply chain. Type II errors reduces the annual cost of the supply chain because it tends to bring in more and more defective items into the system and thus reduces the order/lot size per cycle.

Table 6.2 Input data and results of the numerical example

D	P	A_v	A_b	h_v	h_b	c	d	x	b
1000	3200	400	25	4	5	100000	0.5	175200	0.32
units/yr	units/yr	$/cycle	$/cycle	$/unit/yr	$/unit/yr	$/yr	$/unit	unit/yr	-

	# of Shipments per Cycle	Batch Size per Shipment	Expected Annual Cost
Base Model	7	779	34328
Inspection Errors	7	785	35001
Learning in Production	5	943	5441
Integrated Model	5	951	5529

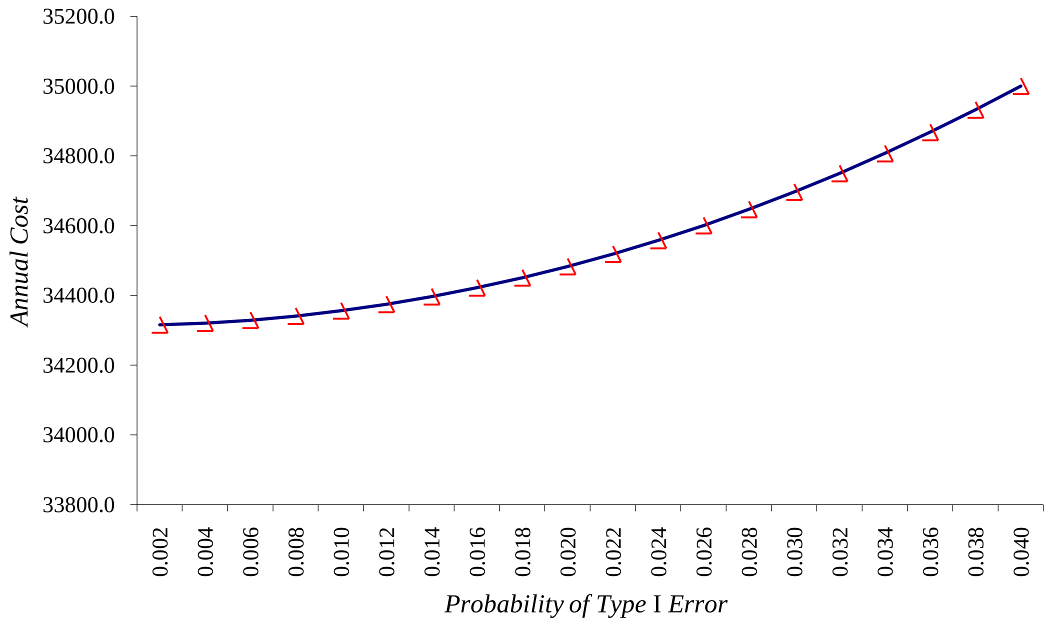

Figure 6.6 Effect of Type I errors on the annual cost of supply chain

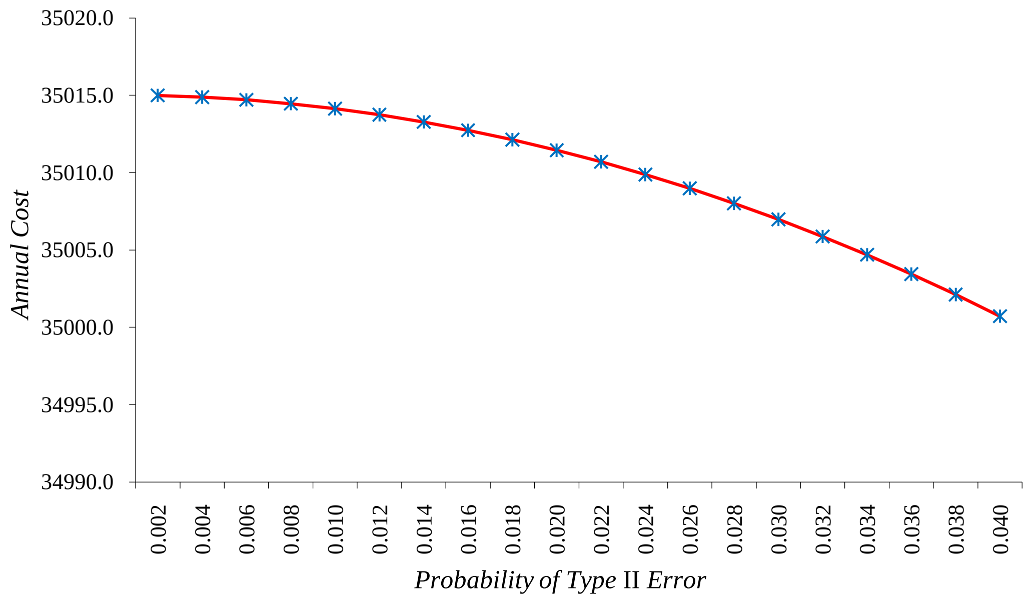

Figure 6.7 Effect of Type II errors on the annual cost of supply chain

To study the impact of learning, an average of ten cycles of production is reported in Table 6.2. Learning in the vendor's production process brings in a substantial amount of saving (about 86%) to the supply chain as the production time becomes shorter and shorter in the subsequent cycles. Figure 6.8 indicates that this saving increases as the learning becomes faster. The batch size in this case goes up as the vendor replenishes the order in lesser number of shipments. A number of experiments with the fraction of defectives and inspection errors showed that the trend in Figure 6.8 remains the same as the curve always tends to plateau at a certain annual cost.

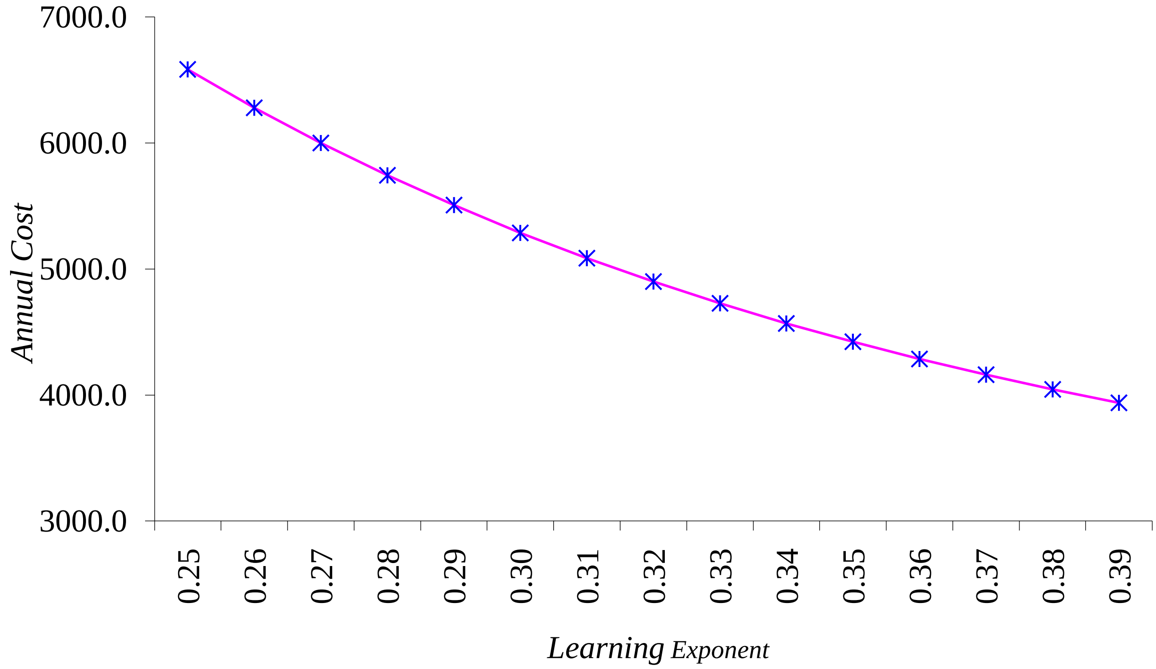

Figure 6.8 Effect of learning in production on the annual cost of supply chain

Studying the effect of both the above factors, that is, screening errors and learning indicates a slight loss in the savings of the supply chain. The rationale for this is the screening errors at buyer's end.

6.4 Conclusions

In this chapter, a two-stage, single-vendor, single-buyer supply chain is formulated. A vendor is supposed to make a single product for its buyer and it is believed that a known fraction of its lots is defective. The buyer institutes a 100% inspection process to separate these defective products. A model depicting this scenario is formulated to find an optimal batch size and the number of shipments for each order.

Two human factors are brought into the picture in this chapter. First of all, a scenario is considered in which the inspectors at the buyer's end make misclassifications. This factor is shown to increase the inspection cost and thus the overall annual cost of the supply chain. Next, the production process of the vendor is assumed to follow learning as workers tend to perform the same job at a faster pace. This brings in a substantial drop in the annual cost of the supply chain.

Analysis of the parameters indicated that Type I error has a pronounced effect on the supply chain as compared to the Type II errors. The rationale for this is an increased order size and thus the inspection cost. There is a limitation to the model that it does not consider costs of Type I and Type II errors. On the other hand, increasing the level of learning at vendor's production process ends up in more and more savings to the supply chain.

This study can be enhanced in a number of ways. For example, one could investigate the effect of learning in buyer's inspection errors. Another practical situation would be to study the effects of a probabilistic demand from the buyer in response to the market's behavior.

CHAPTER 7 CONSIGNMENT STOCK POLICY FOR A VENDOR-BUYER SUPPLY CHAIN WITH INSPECTION ERRORS AND LEARNING IN PRODUCTION

7.1 Introduction

The industry today is striving hard to achieve more and more coordination in their joint businesses. Information technology has had a substantial impact in achieving this goal in contemporary supply chains. Scanners collect sales data at the point-of-sale and electronic data interchange (EDI) allows these data to be shared immediately with all stages of the supply chain. The application of these technologies, especially in the grocery industry, has substantially lowered the time and cost to process an order, leading to impressive improvements in supply chain performance (Cachon and Fisher, 2000).

Inventory includes a company's raw material, work in process, supplies used in operations, and finished goods. Inventory can be something as simple as a bottle of glass cleaner, or something as complex as a mix of raw materials and subassemblies used as part of a manufacturing process. *Raw material* inventory is used to produce partial products or completed goods. On the other hand, a finished product is a product ready for current customer sale. It can also be used to buffer manufacturing from predictable and unpredictable market demand. A third category in inventory is *work-in-process* (*WIP*) is the raw material that is being converted into partial product, subassemblies, and finished product (Muller, 2003).

This chapter extends the work of Braglia and Zavanella (2003) to incorporate the approach of Salameh and Jaber (2000) for the products received at the buyer's warehouse. That is, the buyer would institute an inspection process for each lot to screen out the imperfect products. These imperfect items would be separated from the inventory after the screening is finished. There are some limitations to this work. That is, (*i*) the vendor's production process is not considered in this model, (*ii*) the buyer's screening process is assumed to be free of errors, (*iii*) there is no strategy to dispose the imperfect quality items, and (*iv*) the transportation cost of the deliveries to the buyer is ignored. The rest of the chapter is organized as follows. Section 7.2 describes the mathematical model. Section 7.3 presents a numerical example and discusses the results of the model in section 7.2. Section 7.4 introduces different human factors into the model in section 7.2. Section 7.5 presents a numerical analysis of the extensions in section 7.4. Section 7.6 presents a summary and conclusions.

7.2 Model Description

Consider a vendor-buyer supply chain in which vendor produces in batches. Each batch is transported to the buyer in a number of deliveries. Following the consignment stock policy the vendor uses buyer's warehouse to store/stock its products. The buyer withdraws from this inventory based on the market demand. The vendor makes sure that the quantity stored in the buyer's warehouse is always between a maximum level (S) and a minimum one (s). To extend the model in Braglia and Zavanella (2003) it is assumed that each lot withdrawn by the buyer, contains a fixed proportion γ of defective items. An inspector screens out the defective items from the lot with fixed rate of inspection. These defective products are disposed off from each lot after finishing the screening process. The behavior of the inventory level is illustrated in Figure 7.1, where T is the cycle length. An optimal inventory policy and the number of deliveries per batch, is determined using the total annual cost of the supply chain with this setup. The following nomenclature is used throughout the chapter.

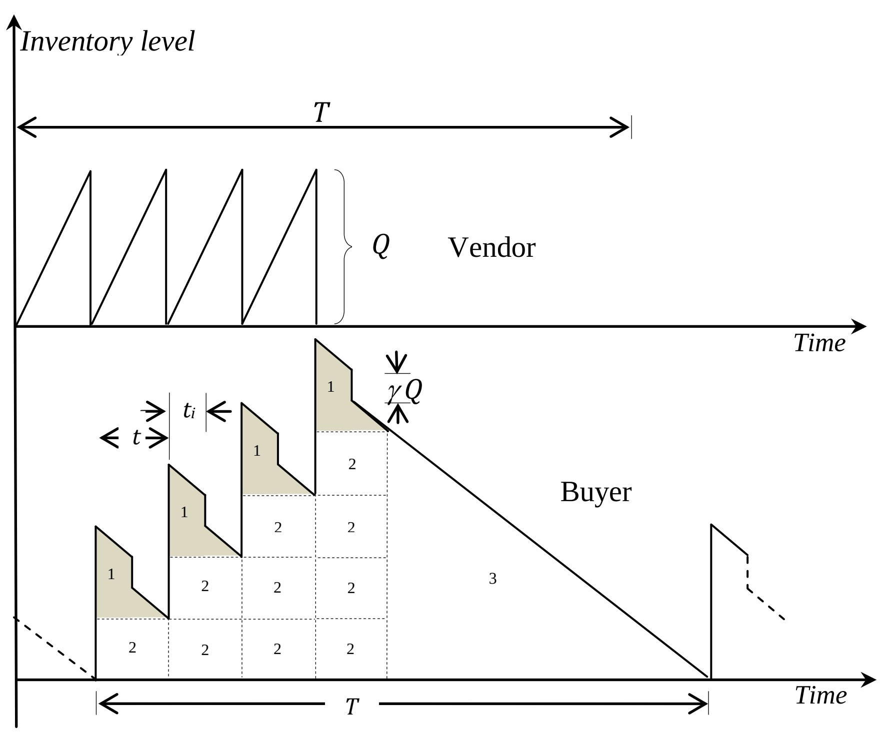

Figure 7.1 The behavior of inventory for vendor and buyer (n = 4)

As shown in Figure 7.1, the inventory with the buyer contains several profiles similar to those given by Salameh and Jaber (2000). Let us first of all determine the area of buyer's inventory profile in Figure 7.1. The area of the shaded profiles '1' would be

$$Area\ 1 = \frac{nDQ^2}{2P^2} + \frac{n\gamma Q^2}{x} \tag{7.1}$$

Now for the rectangles '2', we notice that they increase by one every time the buyer starts his replenishment process. So, their area is given by

$$Area\ 2 = \frac{n(n+1)Q^2}{2P}\left(1 - \gamma - \frac{D}{P}\right) \tag{7.2}$$

The rest of the inventory profile is the big triangle '3'. Its area is

$$Area\ 3 = \frac{1}{2}\left\{nQ\left(\frac{1}{D} - \frac{1}{P}\right)\right\}\left\{nQ\left(1 - \gamma - \frac{D}{P}\right)\right\}$$

or

$$Area\ 3 = \frac{n^2Q^2}{2}\left(\frac{1}{D} - \frac{1}{P}\right)\left(1 - \gamma - \frac{D}{P}\right) \tag{7.3}$$

Therefore, the buyer's holding cost in vendor's one cycle of production is computed using the areas in Eqs. (7.1), (7.2) and (7.3), as

$$HC_b(Q) = h_b nQ^2\left(\frac{D}{2P^2} + \frac{\gamma}{x}\right) + \frac{h_b Q^2}{2}\left(1 - \gamma - \frac{D}{P}\right)\left\{\frac{n(n+1)}{P} + n^2\left(\frac{1}{D} - \frac{1}{P}\right)\right\} \tag{7.4}$$

The expected holding cost per cycle would be:

$$\mathrm{E}[HC_b(Q)] = h_b nQ^2\left(\frac{D}{2P^2} + \frac{\mathrm{E}[\gamma]}{x}\right) + \frac{h_b Q^2}{2}\left(1 - \mathrm{E}[\gamma] - \frac{D}{P}\right)\left\{\frac{n(n+1)}{P} + n^2\left(\frac{1}{D} - \frac{1}{P}\right)\right\}$$

Using $\frac{nQ(1-\mathrm{E}[\gamma])}{D}$ as the cycle length, buyer's annual holding cost can be written as

$$\mathrm{E}[HCU_b(Q)] = \frac{\mathrm{E}[HC_b]}{\mathrm{E}[T]} = \frac{h_b Q}{2}\left\{n + \frac{D}{P} + \left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x} - \frac{n}{P}\right)\right\} \tag{7.5}$$

For the case of no defectives, the above equation reduces to buyer's annual cost in Braglia and Zavanella (2003), i.e. Eq. (7), on page 3798. For the purpose of comparison, the inventory behavior with respect to the two stakeholders in Hill's (1997) model is given in Appendix 8.

Having computed the above holding cost, the buyer's total annual cost which is composed of ordering, holding, inspecting will be given by

$$\mathrm{E}[CU_b(Q,n)] = \frac{A_b D}{Q(1-\mathrm{E}[\gamma])} + \frac{dD}{(1-\mathrm{E}[\gamma])} + \frac{h_b Q}{2}\left\{n + \frac{D}{P} + \left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x} - \frac{n}{P}\right)\right\} \tag{7.6}$$

Note that the above expression follows renewal reward theory. Similarly, the vendor's annual cost would be

$$\mathrm{E}[CU_v(Q,n)] = \frac{A_v D}{nQ(1-\mathrm{E}[\gamma])} + \frac{h_v Q D}{2P(1-\mathrm{E}[\gamma])} + \frac{h_v' Q}{2}\left\{n + \frac{D}{P} + \left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x} - \frac{n}{P}\right)\right\} \tag{7.7}$$

It should be noted from Eqs. (7.6) and (7.7) that the holding costs for stocking items at the buyer's warehouse, according to the Consignment Stock agreement, are carried by the buyer for the non-financial component while for the financial component are carried by the vendor. One may need to understand that the financial component includes the investment while the non-financial component has the storage cost only. This point was not explicitly considered in the analytical model of Braglia and Zavanella (2003) and the readers might end up underestimating the total costs, while it is clearly addressed in Valentini and Zavanella (2003) Thus, the total annual cost of the supply chain, after simplification is

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{Q(1-\mathrm{E}[\gamma])}\left(\frac{A_v}{n}+A_b\right) + \frac{dD}{(1-\mathrm{E}[\gamma])}$$
$$+\frac{Q}{2}\left[\frac{h_v D}{P(1-\mathrm{E}[\gamma])} + (h_b + h'_v)\left\{n + \frac{D}{P} + \left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x} - \frac{n}{P}\right)\right\}\right] \tag{7.8}$$

The expected annual cost of the supply chain, after simplification is

$$\mathrm{E}[TCU(Q,n)] = \frac{DM_2}{Q}\left(\frac{A_v}{n}+A_b\right) + dDM_2$$
$$+\frac{QM_2}{2}\left[\frac{h_v D}{P} + (h_b + h'_v)\left\{(1-M_1)\left(n+\frac{D}{P}\right) + D\left(\frac{2M_1}{x} - \frac{n}{P}\right)\right\}\right] \tag{7.9}$$

where

$M_1 = \mathrm{E}[\gamma]$ and $M_2 = {}^{1}\!/_{(1-\mathrm{E}[\gamma])}$

Eq. (7.9) is convex in Q, since $\frac{\partial^2}{\partial Q^2}\mathrm{E}[TCU(Q,n)] = \frac{DM_2}{Q^3}\left(\frac{A_v}{n}+A_b\right) > 0$, $\forall\ Q > 0$. Eq. (7.9) is also convex in *n*, since $\frac{\partial^2}{\partial n^2}E[TCU(Q,n)] = \frac{DM_2}{Q}\left(\frac{A_v}{n^3}\right) > 0$, $\forall\ n > 0$ indicating that $\mathrm{E}[TCU(Q,n^*-1)] > \mathrm{E}[TCU(Q,n^*)] < \mathrm{E}[TCU(Q,n^*+1)]$. To obtain an optimal batch size and the number of shipments per cycle from the above expression, we would equate its first derivative with respect to Q, to zero and then find the value of *n* through iteration. Thus, the optimal lot size Q in terms of *n* is

$$Q(n) = \sqrt{\frac{2D\left(\frac{A_v}{n}+A_b\right)}{\frac{h_v D}{P} + (h_b + h'_v)\left\{(1-M_1)\left(n+\frac{D}{P}\right) + D\left(\frac{2M_1}{x}-\frac{n}{P}\right)\right\}}} \tag{7.10}$$

7.3 Numerical Example 1

Consider a two-level vendor-buyer supply chain. The vendor produces a single product to fulfill the demand of the buyer. The supply chain is assumed to have a consignment stock policy where the vendor keeps on supplying the inventory the buyer's warehouse with a regular interval while the buyer withdraws from this warehouse following the market demand. The problem here would be to determine the optimal size of each shipment, the number of shipments per batch and the total annual cost of the supply chain. Most of the data is obtained from Goyal (1988) and Salameh and Jaber (2000). The unit holding cost of the product at buyer's end is taken to be more than that the vendor's end which accounts for the value added during the production process. More specifically, the non-financial component at the buyer's end is taken to be higher than that at the vendor's end. The percentage of defectives is assumed to be uniformly distributed as given by

$$f_1(\gamma) = \begin{cases} 1/(0.04-0), & 0 \le \gamma \le 0.04 \\ 0, & otherwise \end{cases}$$

The results indicate that following Salameh and Jaber (2000) approach with the consignment stock (CS) policy addressed by Braglia and Zavanella (2003), turns up increasing the annual cost of the supply chain. The major contributors of this difference are the costs of inspection and that of carrying the defective items. Figure 7.2 shows the convex behavior of the model in this chapter and that of the above two models in the literature. It should be noticed that moving from Hill's approach to that of Braglia and Zavanella, the shift in the cost is mostly governed by the difference in carrying cost of the two stakeholders. On the other hand, when we move from Braglia and Zavanella's policy to ours, the escalation in the cost is dominated by the inspection cost to screen out the defective products. In short, the practical methodology introduced in this chapter does increase the overall cost. Establishing a warranty from the vendor for its products can enforce the reduction not only in the fraction of defective products but the total cost of the supply chain too.

Table 7.1 Input data and results of the numerical example 1

D	P	A_v	A_b	h_v	h'_v	h_b	d	x
1000 units/yr	3200 units/yr	400 $/cycle	25 $/cycle	4 $/unit/yr	2 $/unit/yr	3 $/unit/yr	0.5 $/unit	175200 unit/yr

	# of Shipments per Cycle	Lot Size per Shipment	Expected Annual Cost
Model in the Chapter	4	124	2559.80
Zavanella's Model	4	123	2034.85
Hill's Model	5	131	1903.29

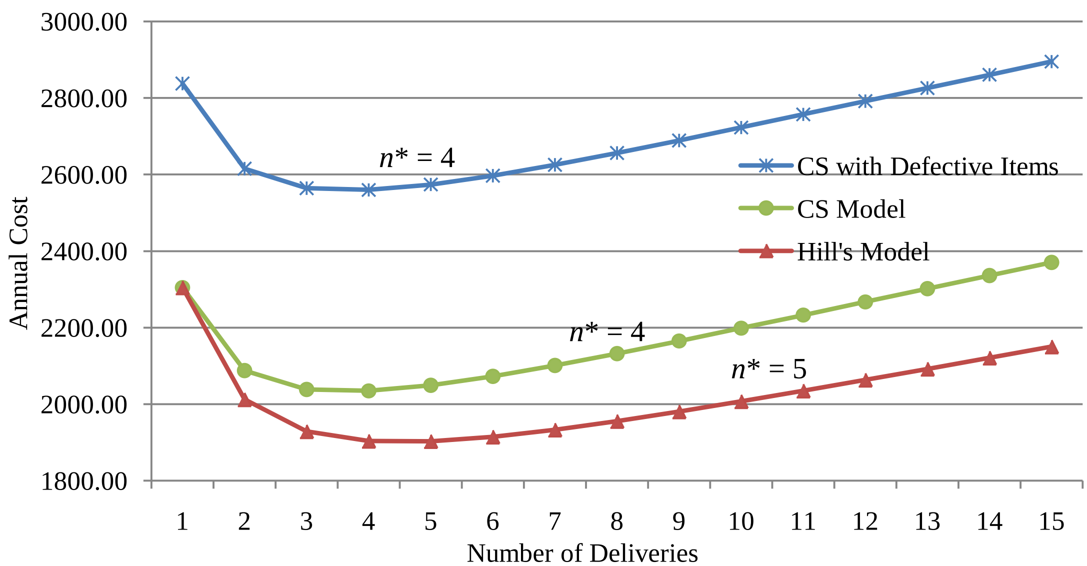

Figure 7.2 Variation in the annual cost of the three models

To further enhance the results, Figure 7.3 compares the annual cost of the model in this chapter to that of Hill, assuming that it has defective items too (model in chapter 6). It can be seen that the difference in the two costs reduces as the defective products tend to increase the other costs of the two stakeholders.

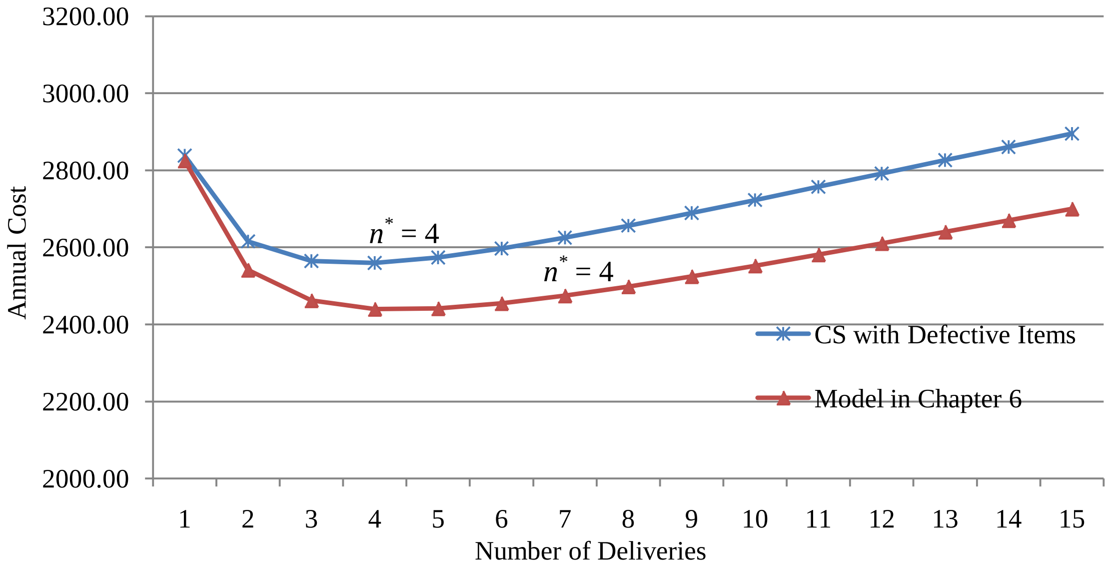

Figure 7.3 Comparison of the results with those in chapter 6

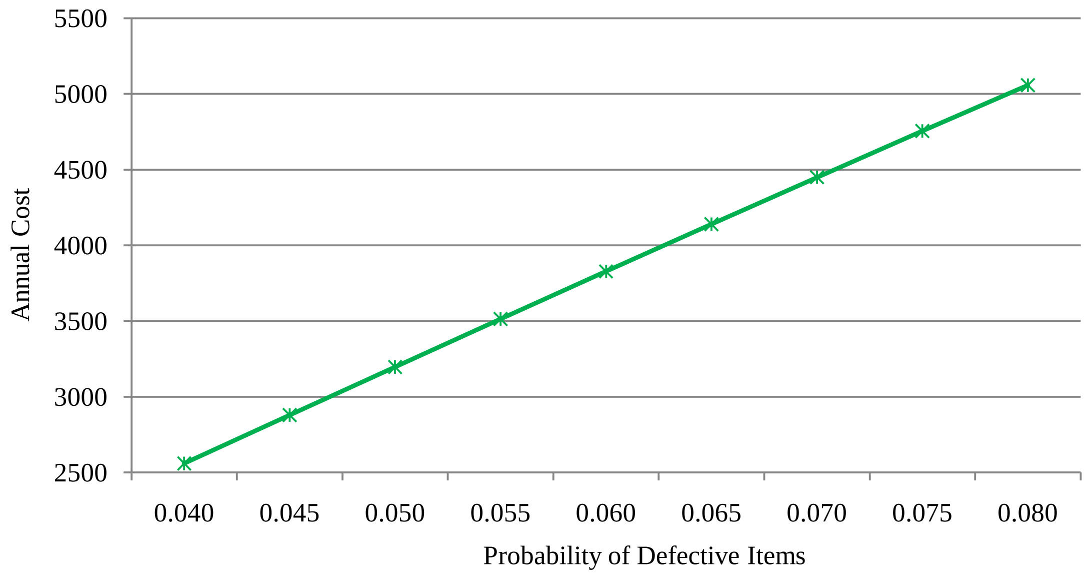

Figure 7.4 Variation of the annual cost with fraction of defectives

Figure 7.4 shows the sensitivity of our model to the fraction of defective products. We noticed that an increasing fraction of defective products goes on enhancing all the cost elements for the buyer and the vendor in this setup though the inspection cost and buyer's carrying costs are affected the most.

One may wonder if there would be a set of holding costs at the ends of the two stakeholders that make the costs in our model and in that of Hill (1997) with defective items and inspection, equal. This later model is described in chapter 6. To investigate this, we varied the financial (investment) component of the holding costs at the vendor's and the buyer's end, and computed the difference of the annual cost in our models in this chapter and that in chapter 6. The result is shown in Figure 7.5 that shows that the two annual costs become approximately equal when the financial component of their holding costs is 2.61. It should be noticed that the model in this chapter performs well even with inspection costs when this component of holding costs goes beyond 2.61.

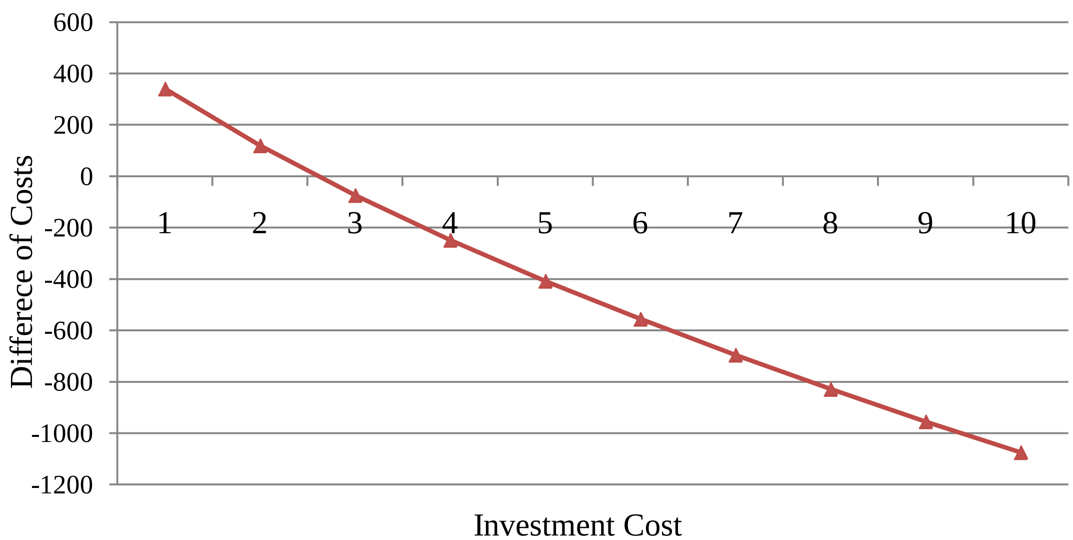

Figure 7.5 Difference of annual costs of the models in chapter 6 and chapter 7

7.4 Inspection Error and Learning

Let us now consider the two human factors for the model depicted in section 7.2. It should be noted that in section 7.2, vendor's production cost was ignored. Thus, incorporating it in Eq. (7.8) becomes

$$\mathrm{E}[TCU(Q,n)] = \frac{D}{Q(1-\mathrm{E}[\gamma])}\left(\frac{A_v}{n} + A_b\right) + \frac{D}{(1-\mathrm{E}[\gamma])}\left(d + \frac{c}{P}\right) \tag{7.11}$$

$$+\frac{Q}{2}\left[\frac{h_v D}{P(1-\mathrm{E}[\gamma])}+\left(h_b+h_v^{'}\right)\left\{n+\frac{D}{P}+\left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x}-\frac{n}{P}\right)\right\}\right]$$

Similarly, the expected annual cost of the supply chain, in Eq. (7.9), after simplification is

$$\mathrm{E}[TCU(Q,n)]=\frac{DM_2}{Q}\left(\frac{A_v}{n}+A_b\right)+DM_2\left(d+\frac{c}{P}\right)$$
$$+\frac{QM_2}{2}\left[\frac{h_v D}{P}+\left(h_b+h_v^{'}\right)\left\{(1-M_1)\left(n+\frac{D}{P}\right)+D\left(\frac{2M_1}{x}-\frac{n}{P}\right)\right\}\right] \tag{7.12}$$

Eq. (7.12) is convex in Q, since $\frac{\partial^2}{\partial Q^2}\mathrm{E}[TCU(Q,n]=\frac{DM_2}{Q^3}\left(\frac{A_v}{n}+A_b\right)>0$, $\forall\ Q>0$. Eq. (7.12) is also convex in n, since $\frac{\partial^2}{\partial n^2}\mathrm{E}[TCU(Q,n)]=\frac{DM_2}{Q}\left(\frac{A_v}{n^3}\right)>0$, $\forall\ n>0$ indicating that $\mathrm{E}[TCU(Q,n^*-1)]>\mathrm{E}[TCU(Q,n^*)]<\mathrm{E}[TCU(Q,n^*+1)]$.

In the following subsections, the two human factors namely, inspection errors in the buyer's screening process and the learning in vendor's production process will be considered for the consignment stock policy described above.

7.4.1 *Inspection Errors*

The screening process in most of the supply chain literature is assumed to be error-free, for example Huang (2002) and Goyal *et al*. (2003). But it is quite realistic to account for Type I and Type II errors committed by inspectors in this process. In this section, it is assumed that the inspectors at the buyer's end commit errors while screening the vendor's product. That is, they will classify some non-defective products as defectives while some defective products as non-defectives. In other words, they will attribute a percentage of defective to the vendor which is different from the actual one. Thus, the fraction of defective products as perceived by the inspectors at the buyer's facility would be

$$\gamma_e=(1-\gamma)m_1+\gamma(1-m_2)$$

and

$$\mathrm{E}[\gamma_e]=(1-\mathrm{E}[\gamma])\mathrm{E}[m_1]+\mathrm{E}[\gamma](1-\mathrm{E}[m_2])$$

So the time interval between successive shipments would now be $\frac{(1-\mathrm{E}[\gamma_e])Q}{D}$, and we can write

$M_{1e} = \mathrm{E}[\gamma_e]$ and $M_{2e} = {}^{1}\!/_{(1-\mathrm{E}[\gamma_e])}$

Thus Eq. (7.12), for the expected annual cost of the supply chain would now be written as

$$\begin{aligned} \mathrm{E}[TCU(Q,n)] &= \frac{DM_{2e}}{Q}\left(\frac{A_v}{n} + A_b\right) + DM_{2e}\left(d + \frac{c}{P}\right) \\ &+ \frac{QM_{2e}}{2}\left[\frac{h_v D}{P} + \left(h_b + h_v^{'}\right)\left\{(1 - M_{1e})\left(n + \frac{D}{P}\right) + D\left(\frac{2M_{1e}}{x} - \frac{n}{P}\right)\right\}\right] \end{aligned} \tag{7.13}$$

Eq. (7.13) is convex in Q, since $\frac{\partial^2}{\partial Q^2}\mathrm{E}[TCU(Q,n] = \frac{DM_{2e}}{Q^3}\left(\frac{A_v}{n} + A_b\right) > 0,\ \forall\ Q > 0$. Beside, Eq. (7.13) is also convex in n, since $\frac{\partial^2}{\partial n^2}\mathrm{E}[TCU(Q,n)] = \frac{DM_{2e}}{Q}\left(\frac{A_v}{n^3}\right) > 0,\ \forall\ n > 0$ indicating that $TCU(Q, n^* - 1) > TCU(Q, n^*) < TCU(Q, n^* + 1)$.

The procedure described above will be used to determine the optimal batch size and the number of shipments with screening errors.

7.4.2 *Learning in Production*

In this section, it is assumed that the vendor's production process follows Wright (1936) learning curve. That is, vendor produces the product at an increasing production rate which is consumed at a constant rate. Assume that every cycle of production makes Q_p (nQ) products, with a learning rate *b*. This situation for cycle *i* is described in Figure 7.6. Note that the replenishments of equal shipments are still made at equal intervals, as in Figure 7.1. The production time in cycle *i* is

$$T_{pi} = \int_{(i-1)Q_p}^{iQ_p} T_1 x^{-b}\, dx$$

$$T_{pi} = \frac{T_1 {Q_p}^{1-b}\{i^{1-b} - (i-1)^{1-b}\}}{(1-b)} \tag{7.14}$$

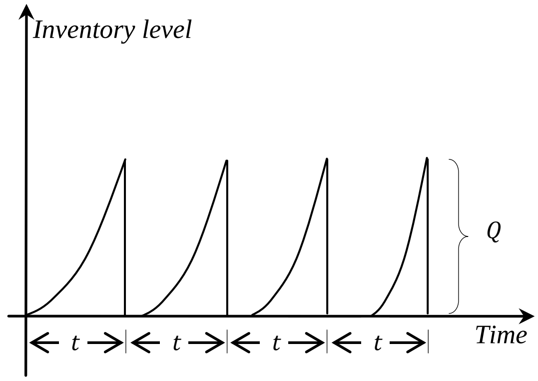

Figure 7.6 Vendor's inventory level in i^{th} cycle with learning in production

So, the production quantity in cycle i can be written as

$$Q_p(t) = \left[\frac{(1-b)t}{T_1}\left\{\frac{1}{i^{1-b}-(i-1)^{1-b}}\right\}\right]^{\frac{1}{1-b}} \tag{7.15}$$

Now, the average inventory of products in a cycle i during production is

$$I_{Tpi} = \int_0^{T_{pi}} Q_p(t)dt$$

Using Eqs (7.14) and (7.15), the vendor's average inventory in i^{th} cycle can be simplified as

$$I_{Tpi} = \frac{T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{2-b}}{2-b}$$

So, vendor's total cost in a cycle would be

$$C_{vi} = A_v + \frac{h_v T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{2-b}}{2-b} + cT_{pi}$$

or

$$C_{vi} = A_v + \frac{h_v T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{2-b}}{2-b} + \frac{cT_1(nQ)^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{1-b}$$

and vendor's expected annual cost is

$$\mathrm{E}[CU_{vi}] = \frac{D}{nQ(1-\mathrm{E}[\gamma])}\left[A_v + \frac{h_v T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{2-b}}{2-b} + \frac{cT_1(nQ)^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{1-b}\right]$$

or

$$\mathrm{E}[CU_{vi}] = \frac{D}{(1-\mathrm{E}[\gamma])}\left[\frac{A_v}{nQ} + \frac{h_v T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{1-b}}{2-b} + \frac{cT_1(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{1-b}\right]$$

Using Eq. (7.6), the total expected annual cost of the supply chain in a cycle, would be

$$\mathrm{E}[TCU_i(Q,n)] = \frac{A_b D}{q(1-\mathrm{E}[\gamma])} + \frac{dD}{(1-\gamma)} + \frac{(h_b+h_v^{'})Q}{2}\left\{n + \frac{D}{P} + \left(\frac{D}{1-\mathrm{E}[\gamma]}\right)\left(\frac{2\mathrm{E}[\gamma]}{x} - \frac{n}{P}\right)\right\} + \frac{D}{(1-\mathrm{E}[\gamma])}\left[\frac{A_v}{nQ} + \frac{h_v T_1\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{1-b}}{2-b} + \frac{cT_1(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{1-b}\right]$$

or

$$\mathrm{E}[TCU_i(Q,n)] = \frac{DM_2}{Q}\left(\frac{A_v}{n} + A_b\right) + dDM_2 + \frac{(h_b+h_v^{'})Q}{2}\left\{n + \frac{D}{P} + DM_2\left(\frac{2M_1}{x} - \frac{n}{P}\right)\right\} + \frac{DM_2}{P}\left[\frac{h_v\{i^{1-b}-(i-1)^{1-b}\}(nQ)^{1-b}}{2-b} + \frac{c(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{1-b}\right] \tag{7.16}$$

Eq. (7.16) is convex in Q (see Appendix 9 for proof). We assume that the learning in the vendor's production process plateaus to some extent after ten cycles. The following steps will be used to find an optimal batch size and the number of shipments from Eq. (7.16),

1. Set i = 1 and n = 1.
2. Find an optimal value of Q and annual cost from Eq. (7.16) through iteration. Set n = 2.
3. Repeat Step 2 till the annual cost for n shipments is more than that of (n – 1) shipments.
4. Record the optimal values of Q and annual cost for n^* = n – 1.
5. Set i = i + 1 and n = 1.

6. Repeat steps 2 through 4 till i = 10. (A maximum of 10 cycles is considered)
7. Find an average of the number of shipments, batch size and the annual cost from the values recorded in step 4.

7.5 Numerical Example 2

Consider the same numerical values as in Example 1 with annual production cost of 100, 000 and

$$f_2(m_1) = \begin{cases} 1/(0.04 - 0), & 0 \leq m_1 \leq 0.04 \\ 0, & otherwise \end{cases}$$

$$f_3(m_2) = \begin{cases} 1/(0.04 - 0), & 0 \leq m_2 \leq 0.04 \\ 0, & otherwise \end{cases}$$

To illustrate the iterative procedure for a cycle of learning in section 7.4.2, an example is provided in Table 7.2a and b. The Table 7.2a shows the results of the first cycle of learning while Table 7.2b indicates how we take an average of ten cycles. The optimal values used in the analysis are shown in bold. It can be seen that for the first cycle of learning, the optimal number of shipments is 6 while the size of each shipment and the annual cost are 185 and \$ 8113 respectively. It should be noticed in Table 7.2b that by virtue of learning, the total inventory in a cycle reduces from 1110 (6×185) to 700 (4×175) i.e. 37% while the annual cost is reduced from 8113 to 4497, i.e. about 45%. This indicates a reduction in the average cost per unit from 7.31 (8113/1110) to 6.42 (4497/700).

Table 7.2*a* Iterative procedure to find n and Q with learning

i	n	Q	E[$TCU(Q)$]
1	1	872	8622
	2	493	8288
	3	346	8181
	4	267	8137
	5	218	8118
	6	**185**	**8113**
	7	161	8115

Table 7.2*b* The optimal policies for 10 consecutive cycles

i	n	Q	E[$TCU(n,Q)$]
1	6	185	8113
2	5	174	6026
3	5	164	5512
4	4	192	5216
5	4	187	5014
6	4	183	4864
7	4	181	4745
8	4	178	4648
9	4	176	4567
10	4	175	4497
Average	**4**	**180**	**5320**

The results in Table 7.3 indicate that bringing in screening errors increases the lot size per delivery that ultimately affects all the cost factors and thus the annual cost of the supply chain. The level of this annual cost in the base case is higher as compared to that in example 1, which accounts for the increased holding cost in the consignment stock policy and the reduced number of deliveries per batch. Besides, the impact of inspection errors there was not so prominent because the number of deliveries did not change while moving from the base case. It should also be noticed that introducing vendor's production cost causes huge increase in the annual cost of consignment stock policy in section 7.2. The benefit of this practical change in the analytical model is more evident when learning is brought into the picture. The average of ten cycles of learning in our model showed that this could save the annual cost of the supply chain up to 86%. That is, accounting for learning in vendor's production process keeps on decreasing the supply chain's annual cost in every successive cycle. This saving reduces to some extent when inspection errors and learning in production are considered at the same time. This accounts for a little change in the lot size per delivery.

Table 7.3 Results of the numerical example 2

	# of Shipments per Cycle	Lot Size per Shipment	Expected Annual Cost
Base Model	4	124	34452
Inspection Errors	4	126	35109
Learning in Production	4	180	5320
Integrated Model	4	183	5384

Figures 7.7 and 7.8 show the sensitivity of annual cost of the supply chain to the Type I and Type II errors respectively, in buyer's screening process. While one error is varied, the other is kept at the level of 0.05. Obviously, when the two errors are at this level, the annual cost in Table 7.3 is obtained. The effect of Type II error gives an impression that it is favourable for the supply chain to make this error. The rationale is that the inspectors keep on adding the defective products to the lot and thus force the vendor to produce lesser. This affects every cost factor in the two level supply chain, the most critical of which is vendor's production cost. In other words, the buyer has to carry lesser inventory to fulfil the demand in this case. An interesting avenue of research would be to study the impact of warranty costs on the overall costs and the production quantity.

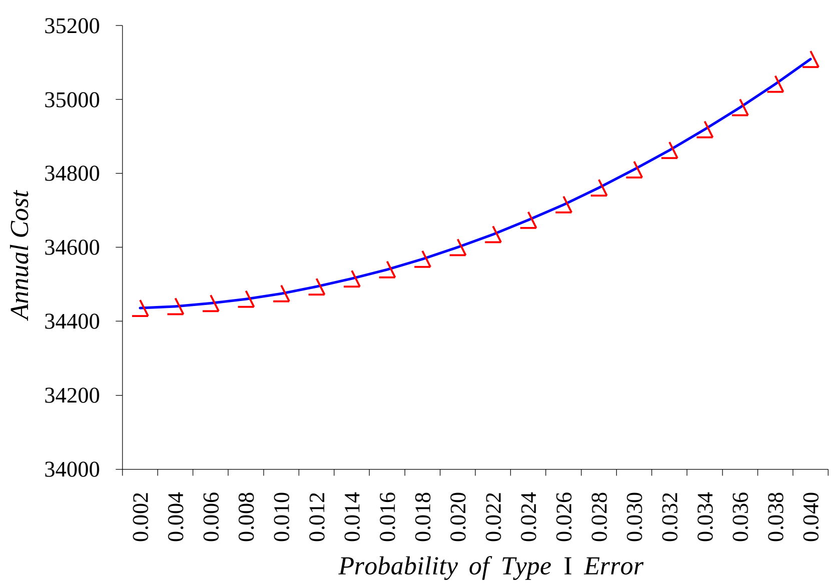

Figure 7.7 Variation in the annual cost with Type I error

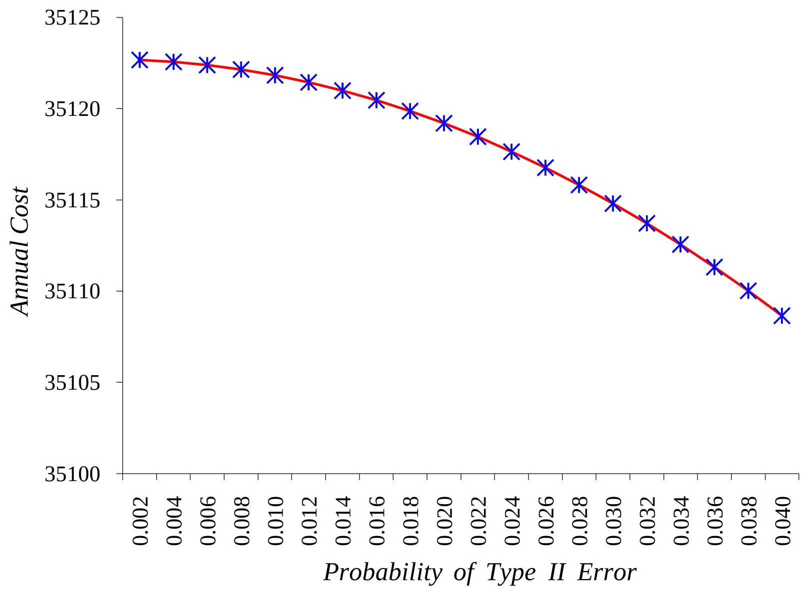

Figure 7.8 Variation in the annual cost with Type II error

Type I error has an opposite impact on the annual cost as shown by Figure 7.7. It could be inferred that the buyer's screening process rejects a good number of non-defective products and thus forces the vendor to produce more. This adds up not only the production cost but the carrying cost of both stakeholders too. A proper disposal of such components is another avenue of research.

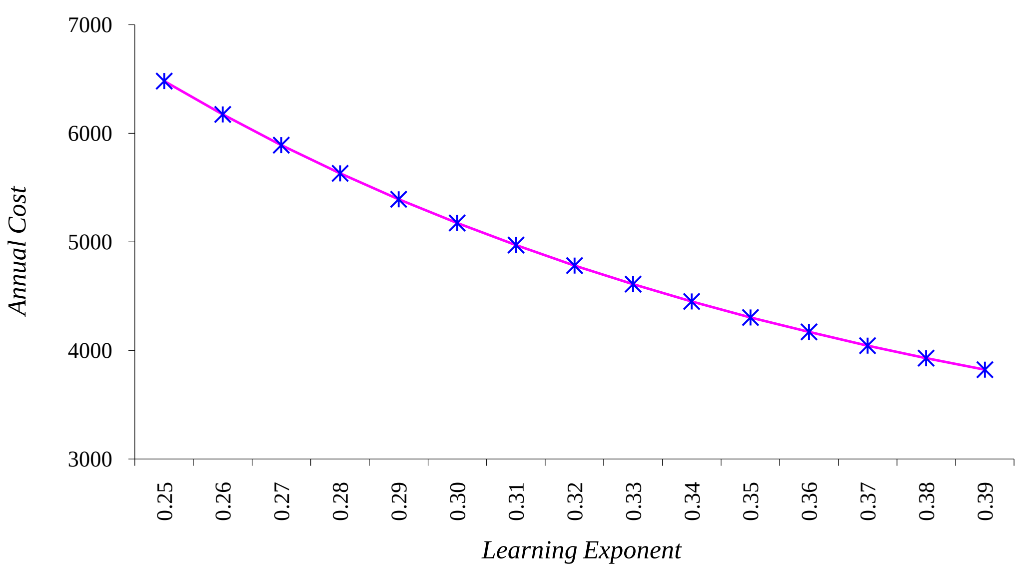

Figure 7.9 Variation in the annual cost with learning exponent

Figure 7.9 shows the drop in the annual cost of the supply chain by virtue of learning in vendor's production process. That is, the production time in a cycle of the supply chain becomes lesser and lesser by increasing the learning exponent. This affects the major cost component, the annual production cost. A more realistic approach would be to integrate loss and continuation of the learning process from cycle to cycle.

7.6 Conclusions

A single vendor, single buyer supply chain is studied in this chapter. The vendor is supposed to make a single product and it is believed that a known fraction of its lots is defective. The buyer institutes a 100% inspection process to separate these defective products. They follow a consignment stock policy according to which the vendor keeps on supplying its inventory to the buyer's warehouse with regular intervals. The buyer withdraws from this warehouse according to the market demand. A model depicting this scenario is formulated to find an optimal lot size and the number of shipments per batch for the vendor. This work is a extension of the work of Braglia and Zavanella (2003) and Salameh and Jaber (2000).

The results indicate that incorporating screening for the defective items results in an increase in the overall supply chain cost. An increase in the annual cost was observed when we moved from Hill (1997) model to Braglia and Zavanella (2003) and then to our model in this chapter. The difference in the cost of our model with that of Hill (1997) shrinks when it is assumed to have defective products too. This accounts for the increase in the cost governed by the defective lot. The model shows sharp increase in the cost with respect to the fraction of defectives. The results also showed that the annual costs in this model are better than that in chapter 6 when holding costs go higher than a threshold value.

The chapter deals with some interesting issues which are related to operative practice in supply chains. In particular, some issues addressed here are:

1. Finding an appropriate lot size when the cost for intercepting defects is considered.
2. Balancing the time for inspection cycle and defective items percentage. This time may be reduced by sacrificing the efficiency in intercepting defective items, as in Braglia and Zavanella (1994).
3. Correcting the production capacity at the vendor's side with respect to process defectiveness (when defects are intercepted at the buyer's side).

In the next part of the chapter, buyer’s screening process is taken to be error prone. That is, the inspectors make Type I and Type II errors in screening. Besides, the vendor’s production process follows learning from cycle to cycle. A model depicting this scenario is formulated to find an optimal lot size and the number of shipments per batch for the vendor. The results indicate that incorporating screening errors increases the annual cost of the supply chain due to an increase in the number of deliveries per cycle. On the other hand, learning in production results in huge savings as the major cost component in the model studied is the cost of production. The more is the level of learning, the more is this saving. Type I error resulted in increasing the lot size and thus the annual cost while Type II error had an opposite impact on the supply chain.

This study could be enhanced in a number of ways. For example the effect of partial and transfer of learning would increase the usefulness of the present work. Warranty costs and different scenarios of disposing the defective products are other possible avenues of research.

CHAPTER 8 CONCLUSIONS AND FUTURE DIRECTIONS

8.1 Introduction

In spite of its wide usage, EOQ/EPQ model has several flaws and limitations. The result is that this model fails to capture and remains away from the real-world production lines and the coordination mechanisms practiced today. These limitations have opened doors for many researchers in the recent literature on inventory and supply chains.

One of the major drawbacks of the model is that it takes all the products in a production line to be defect free. This strong assumption not only ends up in higher costs of warranty but also results in losing customer satisfaction level. Another challenging aspect of the above model is ignoring the role of human factors like inspection errors, fatigue and learning (both in production and quality). These factors have never been modeled in the context of supply chain management though they play a vital role in measuring the performance of a supply chain.

While there is a need to screen out defective items from a lot, human errors in screening can be fatal in case of some critical components, for example, parts of an aircraft or a complex gas ignition system. Repeating the inspection process is believed to reduce the effect of human error (Swain, 1970) at a nominal increase in the inspection cost. Another interesting human aspect that is important in the area of inventory management is learning. Learning is inherent when there are workers involved in a repetitive type of production process. The learning process affects production time, product quality and the inspection errors too, with the passage of time. That is, human beings tend to become more and more accustomed to the processes, thus resulting in better quality of the product.

To address these limitations, the work of Salameh and Jaber (2000) is used in this thesis as a base model. This model has been getting more and more attention recently as it touched upon screening of the defective items in the EOQ model. Therefore, through a series of contributions in this thesis, several models were developed to address the above mentioned limitations in the context of EOQ/EPQ and a two level supply chain. This way, the research in the area of inventory management is brought closer to reality which will help the students and practitioners understand the human behaviors and coordination mechanisms of a supply chain in an extensive manner. The developed models assume unlimited storage capacity, infinite planning horizon and a single product case.

The area of research covered in this thesis is still fertile and there are several limitations which entice future research. These directions will be outlined at the end of this chapter.

8.2 Contributions

A thorough review of the literature is presented in chapter two. This review covers the work concerning different aspects studied in the thesis in the context of EOQ model for imperfect items. The review in chapter two provides a useful resource for researchers currently engaged in the work on inventory systems with imperfect items, and hopefully may provide new ideas for further stimulating this field of research.

The first contribution in the thesis is presented in chapter three. This chapter makes use of Salameh and Jaber (2000) and Duffuaa and Khan (2002) models to suggest that an inspector may make Type I and Type II errors while screening for defective items. The annual profit with inspection errors remained concave with respect to the order size. The significance of inspection errors was indicated by the fact that annual profit becomes much smaller than that in Salameh and Jaber (2000) and that it keeps on reducing with an increase in fraction of defectives. It was emphasized that the misclassifications are critical if the parts under inspection are of an aircraft, a space shuttle or a complex gas ignition system. So, it is vital for a buyer to be aware of not only the accurate parameters of error about his inspectors but also the ways to mitigate these errors. This work has been published in a journal (Khan *et al*., 2011*a*).

The second contribution in the thesis is presented in chapter four. This chapter is an extension of Salameh and Jaber (2000) for the case where the buyer's inspection process undergoes learning while screening for defective items in a lot. A 100% inspection is carried out with an error free screening and the rate of screening tends to increase by virtue of learning. This counters an assumption in Salameh and Jaber (2000) that the inspection rate is fixed and is always greater than demand. Having a screening rate lesser than demand rate in the beginning of the screening process, incurs shortages which are tackled in the chapter as both lost sales and backorders. Three scenarios of learning, available in the literature, are compared for the above set-up. These scenarios are (*i*) total forgetting, where an inspector starts in every cycle with no prior experience, (*ii*) total transfer of learning, where the inspector does not lose any knowledge or skills in the breaks and the learning curve continues as if there were no interruptions, and (*iii*) partial transfer of learning, where an inspector carries part of his experience to the subsequent cycles. The last situation is the most generalized and the realistic one. The results indicate that total transfer of learning remains better for both the lost sales and the backorders set-up. Besides, the annual profit tends to increase with the learning exponent in screening. That is, the faster the learning in screening the lesser is the screening time. A similar finding of the research is that the annual profit can be increased by retaining more and more knowledge in screening process. This was pointed out by experimenting different levels of time for total forgetting, at fixed values of the learning exponent. It was noticed that an increase in the percentage of defectives decreases the annual profit at a fixed exponent of learning. The unit cost of lost sales was also shown to have a similar effect on the annual profit. It was shown that an increase in the unit screening cost reduces the annual profit to great extent at the slower rates of learning. This work has been published in a journal (Khan *et al*., 2010*a*).

The third contribution in the thesis is presented in chapter five. In this chapter, a two-stage, multi-supplier, single-vendor supply chain is formulated. A vendor is supposed to ask for a number of components from different suppliers, which are needed to make a single product. Suppliers are believed to be providing a certain fixed percentage of defectives in their supplies. The vendor institutes a 100% inspection process and sells the defectives in the local market at a discounted price. Two mechanisms, as in Khouja (2003) were studied for the coordination between suppliers and the vendor. The first mechanism is governed by an equal cycle time for all the stakeholders of the supply chain. In the second mechanism, each supplier's cycle time is taken to be an integer multiplier of the vendor's cycle time. The results indicated that the suppliers are supposed to follow a relaxed and practical approach of the integer multipliers cycle time rather than forcing themselves to follow an equal cycle time. A number of human factors are brought into the picture in this chapter. First of all, a scenario is considered in which the inspectors at the vendor's end make misclassifications. Next, the production process of the vendor is assumed to follow learning as workers tend to perform the same job at a faster pace. Lastly, the quality of the suppliers' items is assumed to follow a logistic learning curve. It was observed that the inspection errors tend to increase the annual cost of the supply chain, learning in production drops this cost to great extent while the learning in supplier's quality results in a situation as if there are no defectives from the suppliers. The savings in the annual cost of a model that incorporates all of the above human factors were lesser than those experienced in the case of learning in production only. Parts of this work have been accepted for publication in two journal articles: Khan and Jaber (2011) and Khan *et al*. (2011*b*).

The fourth contribution in the thesis is presented in chapter six. In this chapter, a two-stage, single-vendor, single-buyer supply chain is formulated. A vendor is supposed to make a single product for its buyer and it is believed that a known fraction of its lots is defective. The buyer institutes a 100% inspection process to separate these defective products. A model depicting this scenario is formulated to find an optimal batch size and the number of shipments for each order. Two human factors are brought into the picture in this chapter. First of all, a scenario is considered in which the inspectors at the buyer's end make misclassifications. Next, the production process of the vendor is assumed to follow learning as workers tend to perform the same job at a faster pace. The results showed that inspection errors increase the inspection cost and thus the overall annual cost of the supply chain. Type I error has a pronounced effect on the supply chain as compared to the Type II errors because of an increased order size and the inspection cost. On the other hand, increasing the level of learning at vendor's production process brought more and more savings to the supply chain. Part of this work has been presented in a conference (Khan *et al*., 2010*b*).

The fifth contribution in the thesis is presented in chapter seven. This chapter again takes a two-stage, single-vendor, single-buyer supply chain. The vendor is supposed to make a single product and it is believed that a known fraction of its lots is defective. The buyer institutes a 100% inspection process to separate these defective products. They follow a consignment stock policy according to which the vendor keeps on supplying its inventory to the buyer's warehouse with regular intervals. The buyer withdraws from this warehouse according to the market demand. A model depicting this scenario is formulated to find an optimal lot size and the number of shipments per batch for the vendor. This work is an extension of the work of Braglia and Zavanella (2003) and Salameh and Jaber (2000). In the next part of the chapter, buyer's screening process is taken to be error prone. That is, the inspectors make Type I and Type II errors in screening. Besides, the vendor's production process follows learning from cycle to cycle. A model depicting this scenario is formulated to find an optimal lot size and the number of shipments per batch for the vendor. The results showed that the annual cost increases when one moves from Hill (1997) model to Braglia and Zavanella (2003) and then to our model in this chapter. Besides, it was observed that the annual costs in this model are better than that in chapter 6 when holding costs go higher than a threshold value. The chapter dealt with some interesting issues such as:

1. Finding an appropriate lot size when the cost for intercepting defects is considered.
2. Balancing the time for inspection cycle and defective items percentage. This time may be reduced by sacrificing the efficiency in intercepting defective items, as inBraglia and Zavanella (1994).
3. Correcting the production capacity at the vendor's side with respect to process defectiveness (when defects are intercepted at the buyer's side).

Part of this work has been presented in a conference (Khan *et al.*, 2010*c*).

8.3 Future Directions

This thesis has covered several limitations in the EOQ/EPQ and supply chain modeling but there is still room for much more work to be done. Some important limitations to this work are known and deterministic demand, fraction of defectives, and zero delivery-lead-time. Besides, issues of transportation, warehouse capacity and a proper disposal of defective items still remain the untouched aspects.

In the scenario studied in the thesis, the buyer and the supplier can agree on a fixed proportion of defective items, and look for a coordinated order size per cycle. Another interesting feature would be to consider learning and transfer of learning in the rate of inspection errors. The effect of partial transfer of learning could also enhance the consignment stock policy. Lastly, an internal supply chain for the vendor's serial production line could be studied with learning in production and reworks (Jaber and Khan, 2010). This could be extended to consider the case of dual resource constrained (DRC) system where the number of machines on a production line exceeds the number of workers in which case the workers are trained for a number of work stations.

More specifically, the following directions of research can be recommended:

1. The model in chapter three could be extended for the case where demand is uncertain. Furthermore, learning in the inspection rate would also enhance the usefulness of the model presented here.
2. The model in chapter four could be enhanced by studying the effect of learning in suppliers' proportion of defectives. The buyer and the supplier can also agree on a fixed proportion of defective items, and look for a coordinated order size per cycle.
3. The model in chapter five could be extended for stochastic fractions of defectives. One could also investigate learning in inspection errors there. Besides, the effects of a probabilistic demand from the vendor in response to the market's behavior would help in presenting a realistic scenario.
4. The model in chapter six can be extended to investigate the effect of learning in buyer's inspection errors. Another practical situation would be to study the effects of a probabilistic demand in this model.
5. The model in chapter seven could be extended for partial and transfer of learning in production from cycle to cycle. Besides, one could also study the impact of warranty costs and different scenarios of disposing the defective products, in this model.

To summarize, this thesis contributes to the area of inventory control in today's production environment that is constantly trying to address increased competitiveness, shorter lead times, shorter product life cycles and the need for greater responsiveness (Bonney *et al*., 2003). The contemporary production environment requires more product variety, better quality products, items delivered on time at a competitive price and a better planning and control. More and more literature is bound to appear in this particular area of research as there is a need to address issues such as stochastic demand, cross training, warranty, remanufacturing and transportation. These models for multi-tier systems are aimed at helping engineers and practitioners developing MRP and SAP systems for better inventory control.

APPENDIX 1 Uncertainty of Results in Salameh and Jaber (2000)

	Uniform Distribution			Normal Distribution			% Error	
γ	$Avg.\ \gamma$	Q	EC	$Avg.\ \gamma$	Q	EC	$\%\ Q$	$\%\ EC$
0.01	0.0050	1432	7074.2	0.0050	1419	7081.4	0.86%	-0.10%
0.02	0.0100	1434	7084.7	0.0100	1424	7091.7	0.65%	-0.10%
0.03	0.0150	1440	7098.0	0.0150	1429	7101.8	0.75%	-0.05%
0.04	0.0200	1442	7108.9	0.0200	1435	7114.8	0.48%	-0.08%
0.05	0.0250	1445	7115.2	0.0250	1440	7124.8	0.37%	-0.13%
0.06	0.0300	1452	7132.0	0.0300	1445	7136.7	0.46%	-0.07%
0.07	0.0350	1456	7144.6	0.0350	1449	7145.9	0.47%	-0.02%
0.08	0.0399	1459	7153.9	0.0399	1455	7159.8	0.28%	-0.08%
0.09	0.0450	1464	7166.5	0.0449	1460	7170.9	0.32%	-0.06%
0.10	0.0500	1471	7181.5	0.0501	1466	7186.6	0.30%	-0.07%
0.11	0.0549	1477	7195.9	0.0551	1472	7201.0	0.35%	-0.07%
0.12	0.0600	1480	7205.9	0.0600	1474	7207.1	0.37%	-0.02%
0.13	0.0650	1484	7221.3	0.0649	1482	7225.6	0.17%	-0.06%
0.14	0.0699	1500	7235.4	0.0701	1487	7240.3	0.84%	-0.07%
0.15	0.0750	1496	7248.5	0.0749	1490	7249.5	0.38%	-0.01%
0.16	0.0800	1509	7262.2	0.0800	1496	7267.1	0.84%	-0.07%
0.17	0.0850	1508	7284.8	0.0849	1501	7280.3	0.43%	0.06%
0.18	0.0900	1507	7287.9	0.0902	1510	7306.0	-0.20%	-0.25%
0.19	0.0951	1516	7315.3	0.0950	1516	7321.7	0.04%	-0.09%
0.20	0.1000	1523	7333.0	0.1003	1520	7334.8	0.20%	-0.03%

APPENDIX 2 Expected Value of *A* in Equation (3.5)

From Eq. (3.5):

$$A = 1 - \frac{D}{x} - (m_1 + \gamma) + \gamma(m_1 + m_2)$$

$$A^2 = \left[\left(1 - \frac{D}{x}\right) - (m_1 + \gamma) + \gamma(m_1 + m_2)\right]^2$$

$$A^2 = \left(1 - \frac{D}{x}\right)^2 + (m_1 + \gamma)^2 + \gamma^2(m_1 + m_2)^2 - 2\left(1 - \frac{D}{x}\right)(m_1 + \gamma)$$
$$-2\gamma(m_1 + \gamma)(m_1 + m_2) + 2\gamma\left(1 - \frac{D}{x}\right)(m_1 + m_2)$$

$$A^2 = \left(1 - \frac{D}{x}\right)^2 + ({m_1}^2 + 2m_1\gamma + \gamma^2) + \gamma^2({m_1}^2 + 2m_1m_2 + {m_2}^2) - 2\left(1 - \frac{D}{x}\right)(m_1 + \gamma)$$
$$-2\gamma({m_1}^2 + m_1m_2 + \gamma m_1 + \gamma m_2) + 2\gamma\left(1 - \frac{D}{x}\right)(m_1 + m_2)$$

Now, the expected value of above expression including three random variables, is

$$\begin{aligned}\mathrm{E}[A^2] = {} & \left(1 - \frac{D}{x}\right)^2 + (\mathrm{E}[{m_1}^2] + 2\mathrm{E}[m_1]\mathrm{E}[\gamma] + \mathrm{E}[\gamma^2]) \\ & +\mathrm{E}[\gamma^2](\mathrm{E}[{m_1}^2] + 2\mathrm{E}[m_1]\mathrm{E}[m_2] + \mathrm{E}[{m_2}^2]) - 2\left(1 - \frac{D}{x}\right)(\mathrm{E}[m_1] + \mathrm{E}[\gamma]) \\ & -2\mathrm{E}[\gamma](\mathrm{E}[{m_1}^2] + \mathrm{E}[m_1]\mathrm{E}[m_2] + \mathrm{E}[\gamma]\mathrm{E}[m_1] + \mathrm{E}[\gamma]\mathrm{E}[m_2]) \\ & +2\left(1 - \frac{D}{x}\right)(\mathrm{E}[\gamma]\mathrm{E}[m_1] + \mathrm{E}[\gamma]\mathrm{E}[m_2])\end{aligned} \qquad \text{(A2.1)}$$

APPENDIX 3 Independence of γ, m_1 and m_2

Assume x and y are independent random variables that are uniformly distributed; x~[a,b] and y~[d,c]. Then,

$$\int_a^b \int_c^d \frac{(1-x)}{b-a}\frac{(1-y)}{d-c}\,\mathrm{d}y\,\mathrm{d}x = \frac{(c+d)(a+b)}{4} - \frac{(a+b+c+d)}{2} + 1 \tag{A3.1}$$

Now, $(1-\mathrm{E}[x])(1-\mathrm{E}[y]) = \left(1-\frac{a+b}{2}\right)\left(1-\frac{c+d}{2}\right) = 1 - \frac{(a+b+c+d)}{2} + \frac{(c+d)(a+b)}{4}$

Therefore

$$(1-\mathrm{E}[x])(1-\mathrm{E}[y]) = \int_a^b \int_c^d \frac{(1-x)}{b-a}\frac{(1-y)}{d-c}\,\mathrm{d}y\,\mathrm{d}x \tag{A3.2}$$

Similarly,

$$\int_a^b \int_c^d \frac{x}{b-a}\frac{y}{d-c}\,\mathrm{d}y\,\mathrm{d}x = \frac{1}{4}\frac{\left(d^2-c^2\right)\left(b^2-a^2\right)}{(b-a)(d-c)} = \frac{(d+c)(b+a)}{4} \tag{A3.3}$$

and

$$\mathrm{E}[x]\mathrm{E}[y] = \left(\frac{a+b}{2}\right)\left(\frac{c+d}{2}\right) = \frac{(d+c)(b+a)}{4}$$

So,

$$\mathrm{E}[x]\mathrm{E}[y] = \int_a^b \int_c^d \frac{x}{b-a}\frac{y}{d-c}\,\mathrm{d}y\,\mathrm{d}x \tag{A3.4}$$

Now, let us assume that x and y are independent random variables that are exponentially distributed; x~[0,∞) and y~[0,∞), where λ and μ are the parameters of their exponential distributions. Then,

$$\int_0^\infty \int_0^\infty (1-x)\,(1-y)\lambda\mu e^{-\mu y} e^{-\lambda x}\,\mathrm{d}y\,\mathrm{d}x = \left(\lim_{x\to\infty}\lim_{y\to\infty}\frac{(1-\lambda+\lambda x)(1-\mu+\mu y)}{\lambda\mu e^{\mu y+\lambda x}}\right) - \frac{(1-\lambda)(1-\mu)}{\mu\lambda}$$

$$\int_0^\infty \int_0^\infty (1-x)\,(1-y)\lambda\mu e^{-\mu y} e^{-\lambda x}\,\mathrm{d}y\,\mathrm{d}x = \frac{(\lambda-1)(\mu-1)}{\mu\lambda} \tag{A3.5}$$

and

$$(1-\mathrm{E}[x])(1-\mathrm{E}[y]) = \left(1-\frac{1}{\lambda}\right)\left(1-\frac{1}{\mu}\right) = \frac{(\lambda-1)(\mu-1)}{\mu\lambda}$$

Therefore,

$$(1-\mathrm{E}[x])(1-\mathrm{E}[y]) = \int_0^\infty \int_0^\infty (1-x)\,(1-y)\lambda\mu e^{-\mu y}e^{-\lambda x}\mathrm{d}y\,\mathrm{d}x \tag{A3.6}$$

Similarly,

$$\mathrm{E}[x]\mathrm{E}[y] = \int_0^\infty \int_0^\infty x\,y\lambda\mu e^{-\mu y}e^{-\lambda x}\mathrm{d}y\,\mathrm{d}x = \left(\lim_{x\to\infty}\lim_{y\to\infty}\frac{(1+\lambda x)(1+\mu y)}{\lambda\mu e^{\mu y+\lambda x}}\right) + \frac{1}{\mu\lambda}$$

$$\mathrm{E}[x]\mathrm{E}[y] = \int_0^\infty \int_0^\infty x\,y\lambda\mu e^{-\mu y}e^{-\lambda x}\mathrm{d}y\,\mathrm{d}x = \frac{1}{\mu\lambda} \tag{A3.7}$$

and

$$\mathrm{E}[x]\mathrm{E}[y] = \left(\frac{1}{\lambda}\right)\left(\frac{1}{\mu}\right) = \frac{1}{\mu\lambda}$$

Therefore,

$$\mathrm{E}[x]\mathrm{E}[y] = \int_0^\infty \int_0^\infty x\,y\lambda\mu e^{-\mu y}e^{-\lambda x}\mathrm{d}y\,\mathrm{d}x \tag{A3.8}$$

Based on the above, it can be cautiously assumed that when x and y are independent random variables that are normally distributed with probability density functions $f(x)$ and $f(y)$, then, $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(1-x)(1-y)f(x)g(y)\mathrm{d}y\mathrm{d}x = (1-E[x])(1-E[y])$.

Note that, for simplicity, the uniform distribution will be used throughout the thesis.

APPENDIX 4 Concavity of the Annual Profit in Eq. (4.15)

The expected total net revenue per cycle is:

$E[R_{iL}] = \{s(1-\mathrm{E}[\gamma]) + \nu\mathrm{E}[\gamma] - c_1\}Q_i = AQ_i$, where $A > 0$ since $s > c_1$.

The expected total cost per cycle is:

$$E[TC_{iL}(Q_i)] = K + c_L Z_i + \frac{h}{2D}\{Q_i^2\mathrm{E}[(1-\gamma)^2] + 2Q_iZ_i(1-\mathrm{E}[\gamma]) + Z_i^2\}$$

$$-\frac{h}{2D}\left(\frac{Q_{si}}{1-b}\right)^2 + \frac{(hQ_i\mathrm{E}[\gamma]+d_1)\{(Q_i+u_i)^{1-b}-u_i{}^{1-b}\}}{x_1(1-b)} + h\frac{Q_{si}^2}{D(1-b)} - h\left(\frac{x_1Q_{si}}{D}\right)^{\frac{1}{1-b}}\frac{Q_{si}}{D(1-b)}$$

The expected cycle time $\mathrm{E}[T_{iL}]$ is:

$$\mathrm{E}[TP_{iL}(Q_i)] = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} - \frac{Q_{si}}{D} + t_{si} = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} - \frac{D^{\frac{1-b}{b}}}{x_1^{1/b}} + \frac{D^{\frac{1-b}{b}}}{(1-b)x_1^{1/b}}$$

$$= \frac{(1-\mathrm{E}[\gamma])Q_i}{D} + \frac{D^{\frac{1-b}{b}}}{x_1^{1/b}}\left(\frac{b}{1-b}\right) = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} + \frac{Q_{si}}{D}\left(\frac{b}{1-b}\right) = \frac{(1-\mathrm{E}[\gamma])Q_i + Z_i}{D}$$

$$\mathrm{E}[T_{iL}] = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} - \frac{Q_{si}}{D} + t_{si} = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} - \frac{D^{\frac{1-b}{b}}}{x_1^{1/b}} + \frac{D^{\frac{1-b}{b}}}{(1-b)x_1^{1/b}}$$

$$= \frac{(1-\mathrm{E}[\gamma])Q_i}{D} + \frac{D^{\frac{1-b}{b}}}{x_1^{1/b}}\left(\frac{b}{1-b}\right) = \frac{(1-\mathrm{E}[\gamma])Q_i}{D} + \frac{Q_{si}}{D}\left(\frac{b}{1-b}\right) = FQ_i + G,$$

where $F = \frac{(1-\mathrm{E}[\gamma])}{D} > 0$ and $G = \frac{Q_{si}}{D}\left(\frac{b}{1-b}\right) > 0$.

and $Z_i =$ $(Dt_{si} - Q_{si})$, $Q_s = Q_{si} = \left({D}/{x_1}\right)^{\frac{1}{b}}$, $t_s = t_{si} = \frac{D^{\frac{1-b}{b}}}{(1-b)x_1^{1/b}} = \frac{Q_{si}}{D(1-b)}$, $\tau = \tau_i$

or $Z_i =$ $\left[D\frac{D^{\frac{1-b}{b}}}{(1-b)x_1^{1/b}} - \left({D}/{x_1}\right)^{\frac{1}{b}}\right] = \left(\frac{D^{\frac{1}{b}}}{(1-b)x_1^{1/b}} - \frac{D^{\frac{1}{b}}}{x_1^{1/b}}\right) = \frac{b}{1-b}\left({D}/{x_1}\right)^{\frac{1}{b}} = \frac{b}{1-b}Q_{si},$

Therefore, Z_i is a constant.

The expected total profit per cycle is $\mathrm{E}[TPU_{iL}] = \frac{\mathrm{E}[TP_{iL}]}{\mathrm{E}[T_{iL}]} = \frac{\mathrm{E}[R_{iL}] - E[TC_{iL}]}{\mathrm{E}[T_{iL}]} = \frac{\mathrm{E}[R_{iL}]}{\mathrm{E}[T_{iL}]} - \frac{E[TC_{iL}]}{\mathrm{E}[T_{iL}]}$.

The per unit time revenue $Y=\frac{\mathrm{E}[R_{iL}]}{\mathrm{E}[T_{iL}]}=\frac{AQ_i}{FQ_i+G}$, $\frac{dY}{dQ_i}=\frac{A(FQ_i+G)-F(AQ_i)}{(FQ_i+G)^2}=\frac{G}{(FQ_i+G)^2}>0, \forall Q_i>0$, $\frac{d^2Y}{dQ_i^2}=-\frac{G}{(FQ_i+G)^3}<0, \forall Q_i>0$ is strictly increasing function in Q_i. Furthermore, $\lim_{Q_i\to\infty} Y \to \frac{A}{F}>0$.

We need now to show that $W=\frac{\mathrm{E}[TC_{iL}]}{\mathrm{E}[T_{iL}]}$ is a convex function. To do so, it is easier to tackle each term or few terms together but not the whole function as this is a bit complex.

$$U_1=\frac{K+c_L Z_i}{FQ_i+G}, \frac{d}{dQ_i}U_1=-\frac{F(K+c_L Z_i)}{(FQ_i+G)^2}, \frac{d^2}{dQ_i^2}U_1=\frac{2F^2(K+c_L Z_i)^2}{(FQ_i+G)^3}>0, \forall Q_i>0 \tag{A4.1}$$

$$U_2=\frac{h}{2D}\frac{\{Q_i^2\mathrm{E}[(1-\gamma)^2]+2Q_iZ_i(1-\mathrm{E}[\gamma])+Z_i^2\}}{FQ_i+G} \tag{A4.2}$$

$$U_2'=\frac{Q_i^2\mathrm{E}[(1-\gamma)^2]}{FQ_i+G}, U_2''=\frac{2Q_iZ_i(1-\mathrm{E}[\gamma])}{FQ_i+G}, U_2'''=\frac{Z_i^2}{FQ_i+G}$$

$$\frac{d}{dQ_i}U_2'=\frac{\mathrm{E}[(1-\gamma)^2]\{2Q_i(FQ_i+G)-FQ_i^2\}}{(FQ_i+G)^2}=\frac{\mathrm{E}[(1-\gamma)^2](FQ_i^2+2Q_iG)}{(FQ_i+G)^2}$$

$$\frac{d^2}{dQ_i^2}U_2'=\frac{2\mathrm{E}[(1-\gamma)^2]\{2(FQ_i+G)^2-2FQ_i(FQ_i+2G)\}}{(FQ_i+G)^4}=\frac{2\mathrm{E}[(1-\gamma)^2]G^2}{(FQ_i+G)^3}>0, \forall Q_i \tag{A4.2a}$$

$$\frac{d}{dQ_i}U_2''=\frac{2Z_i(1-\mathrm{E}[\gamma])\{(FQ_i+G)-FQ_i\}}{(FQ_i+G)^2}=\frac{2Z_i(1-\mathrm{E}[\gamma])G}{(FQ_i+G)^2}$$

$$\frac{d^2}{dQ_i^2}U_2''=-\frac{4FGZ_i(1-\mathrm{E}[\gamma])}{(FQ_i+G)^3}<0, \forall Q_i$$

$$\frac{d}{dQ_i}U_2'''=-\frac{FZ_i^2}{(FQ_i+G)^2}, \quad \frac{d^2}{dQ_i^2}U_2'''=\frac{2F^2Z_i^2}{(FQ_i+G)^3}>0, \forall Q_i$$

Now since, $Z_i=\frac{b}{1-b}Q_{si}$, $F=\frac{(1-\mathrm{E}[\gamma])}{D}$ and $G=\frac{Q_{si}}{D}\left(\frac{b}{1-b}\right)=\frac{Z_i}{D}$

$$\frac{d^2}{dQ_i^2}U_2''+\frac{d}{dQ_i}U_2'''=-\frac{4FZ_iG(1-\mathrm{E}[\gamma])}{(FQ_i+G)^3}+\frac{2F^2Z_i^2}{(FQ_i+G)^3}=\frac{2F^2Z_i^2}{(FQ_i+G)^3}-\frac{4FGZ_i(1-\mathrm{E}[\gamma])}{(FQ_i+G)^3}$$

$$\frac{d^2}{dQ_i^2}U_2''+\frac{d^2}{dQ_i^2}U_2'''=\frac{2FZ_i}{(FQ_i+G)^3}\{FZ_i-2G(1-\mathrm{E}[\gamma])\}=\frac{2FZ_i}{(FQ_i+G)^3}[FZ_i-2FZ_i]=-\frac{2F^2Z_i^2}{(FQ_i+G)^3},$$

$U_3 = \frac{1}{FQ_i+G}\left\{h\frac{Q_{si}^2}{D(1-b)} - h\left(\frac{x_1 Q_{si}}{D}\right)^{\frac{1}{1-b}}\frac{Q_{si}}{D(1-b)} - \frac{h}{2D}\left(\frac{Q_{si}}{1-b}\right)^2\right\} = \frac{1}{FQ_i+G}\left\{h\frac{Q_{si}}{D(1-b)}\left[Q_{si} - \left(\frac{x_1 Q_{si}}{D}\right)^{\frac{1}{1-b}} - \frac{Q_{si}}{2(1-b)}\right]\right\} = \frac{1}{FQ_i+G}\left\{\frac{h}{2D}\frac{Q_{si}^2}{(1-b)^2}\right\} = \frac{1}{FQ_i+G}\left\{\frac{h}{2D}\frac{Z_i^2}{(1-b)b^2}\right\} = \frac{h}{2D}\frac{HZ_i^2}{FQ_i+G}$

$\frac{d}{dQ_i}U_3 = -\frac{h}{2D}\frac{FHZ_i^2}{(FQ_i+G)^2}, \frac{d^2}{dQ_i^2}U_4 = +\frac{F^2HZ_i^2}{(FQ_i+G)^3}$ (A4.2b)

$\frac{d^2}{dQ_i^2}U_2'' + \frac{d}{dQ_i}U_2''' + \frac{d^2}{dQ_i^2}U_3 = -\frac{2F^2Z_i^2}{(FQ_i+G)^3} + \frac{F^2HZ_i^2}{(FQ_i+G)^3} > 0 \Rightarrow -2 + H > 0 \Rightarrow -2 + \frac{1}{(1-b)b^2} > 0$

Note that, $(1-b)b^2$ holds its maximum value when b = 2/3 (set the first derivative to zero), where $\frac{1}{(1-b)b^2}$ holds a minimum value of $\frac{1}{(1-2/3)(2/3)^2} = \frac{27}{4}$, which means that $-2 + \frac{1}{(1-b)b^2}$ is always positive for $0 < b < 1$. Therefore, $\frac{d^2}{dQ_i^2}U_2 + \frac{d^2}{dQ_i^2}U_4 = \frac{d^2}{dQ_i^2}U_2' + \frac{d^2}{dQ_i^2}U_2'' + \frac{d^2}{dQ_i^2}U_2''' + \frac{d^2}{dQ_i^2}U_3$ $>0, \forall Q_i$.

Let $U_4 = \frac{(hQ_i\mathrm{E}[\gamma]+d_1)\{(Q_i+u_i)^{1-b} - u_i{}^{1-b}\}}{x_1(1-b)(FQ_i+G)} = \frac{h\mathrm{E}[\gamma]}{x_1(1-b)}\frac{Q_i\{(Q_i+u_i)^{1-b} - u_i{}^{1-b}\}}{(FQ_i+G)} + \frac{hd_1}{x_1(1-b)}\frac{\{(Q_i+u_i)^{1-b} - u_i{}^{1-b}\}}{(FQ_i+G)}$

For simplicity, and without loss of generality, assume $Q_i = Q$, and $u_i = (i-1)Q$, where $i \geq 0$, $FQ + G \approx \theta Q$ and

Let $U'_4 = \alpha\frac{Q^{2-b}}{\theta Q}$, $U''_4 = \beta\frac{Q^{1-b}}{\theta Q}$, where $\alpha = \frac{hE[\gamma]\left(i^{1-b} - (i-1)^{1-b}\right)}{x_1(1-b)}$

and $\beta = \frac{hd_1\left(i^{1-b} - (i-1)^{1-b}\right)}{x_1(1-b)}$

$\frac{d^2}{dQ^2}U'_4 = -\alpha\frac{Q^{-b}b(1-b)}{\theta Q}, \frac{d^2}{dQ^2}U''_4 = \beta\frac{(1+b)bQ^{-1-b}}{\theta Q}$

$\frac{d^2}{dQ^2}U'_4 + \frac{d^2}{dQ^2}U''_4 = -\alpha\frac{Q^{-b}b(1-b)}{\theta Q} + \beta\frac{(1+b)bQ^{-1-b}}{\theta Q}$,

$$\frac{d^2}{dQ^2}U'_4 + \frac{d^2}{dQ^2}U''_4 = \frac{1}{\theta Q^{1+b}}\left\{-\alpha b(1-b) + \beta\frac{(1+b)b}{Q}\right\}$$

$-\alpha b(1-b) + \beta\frac{(1+b)b}{Q} > 0 \Rightarrow \frac{1}{Q} > \frac{\alpha}{\beta}\frac{(1-b)}{1+b} \Rightarrow Q < \frac{d_1(1+b)}{E[\gamma](1-b)}$, which implies that $\frac{d^2}{dQ^2}U'_4 + \frac{d^2}{dQ^2}U''_4$

is positive for $0 < Q < \frac{d_1(1+b)}{E[\gamma](1-b)}$. For larger values of Q, we notice that $\frac{d^2}{dQ^2}U'_5 + \frac{d^2}{dQ^2}U''_5$ will go asymptotic to zero. Showing that $\frac{d^2}{dQ^2}U'_5 + \frac{d^2}{dQ^2}U''_5$>0 $\forall Q > 0$ my not be as strong as the other terms, but we can reasonably conclude that the sum of

$\frac{d^2}{dQ_i^2}U_1 + \frac{d^2}{dQ_i^2}U_2 + \frac{d^2}{dQ_i^2}U_3 + \frac{d^2}{dQ_i^2}U_4 > 0, \forall Q_i > 0$, suggesting that the expected unit time cost function, $\frac{\mathrm{E}[TC_{iL}]}{\mathrm{E}[T_{iL}]}$, is convex in Q, of the form $\frac{A}{x} + Bx.$ It was shown that the expected unit time net revenue, $Y = \frac{\mathrm{E}[R_{iL}]}{\mathrm{E}[T_{iL}]}$, is a monotonically increasing function in Q, Cx The difference of these two functions, $f = Cx - \frac{A}{x} - Bx$, would be a concave function, $\frac{d^2 f}{dx^2} = -\frac{2A}{x^2}$<0, $\forall x > 0$, with a unique maximizer. Therefore Eq. (4.14) is a concave function in Q > 0.

APPENDIX 5 $\mathbf{E}[\boldsymbol{\pi}^2]$ in Chapter 5

It is assumed that the fraction of imperfect quality items per shipment from supplier s, γ_s, is independent of those from other suppliers. So, it can be reasonably assumed that γ_s are independent and identically distributed (i.i.d.) random variables with known probability density functions $f(\gamma_s)$. The expected leftovers in a cycle are

$$\mathrm{E}[l_s] = Q_s(\mathrm{E}[\gamma_{\max}] - \mathrm{E}[\gamma_s]) = Q_s \int_{-\infty}^{+\infty} \gamma_{\max} f(\gamma_{\max}) d\,\gamma_{\max} - Q_s \int_{-\infty}^{+\infty} \gamma_{\mathrm{s}} f(\gamma_s) d\,\gamma_s \tag{A5.1}$$

The fraction π_s is defined as

$\pi_s = 1 - (\gamma_{\max} - \gamma_s)$

$(\pi_s)^2 = [1 - (\gamma_{\max} - \gamma_s)]^2$

$(\pi_s)^2 = 1 - 2(\gamma_{\max} - \gamma_s) + (\gamma_{\max} - \gamma_s)^2$

$(\pi_s)^2 = 1 - 2\gamma_{\max} - 2\gamma_s + ({\gamma_{\max}}^2 - 2\gamma_{\max}\gamma_s + {\gamma_s}^2)$

For i.i.d. fractions of defectives, the expected value of the above expression is

$$\mathrm{E}[{\pi_s}^2] = 1 - 2\mathrm{E}[\gamma_{\max}] - 2\mathrm{E}[\gamma_s] + \mathrm{E}[{\gamma_{\max}}^2] - 2\mathrm{E}[\gamma_{\max}]\mathrm{E}[\gamma_s] + \mathrm{E}[{\gamma_s}^2] \tag{A5.2}$$

APPENDIX 6 Convexity of the Annual Cost in Eq. (5.22)

Let us assume

$AA =$

$$\mathrm{E}[TCU(Q)] = \frac{(A_v + \sum_{s=1}^{m} A_s)D}{Q} + h_{v2}\left[\frac{Q}{2} - \frac{T_1 D Q^{1-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)(2-b_i)}\right] +$$

$$D\sum_{s=1}^{m}\left[\frac{h_{v1s}Q}{2}\left\{\frac{\mu_s^2 \mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} + \frac{Q^{-b_i}\mu_s \mathrm{E}[\pi_s] T_1\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)}\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)\right\} +\right.$$

$$\left.(a_{vs} + d_s)\mu_s \mathrm{E}[\pi_s]\right] + \sum_{s=1}^{m} h_{v1s} Q u_s (1 - \mathrm{E}[\pi_s]) + \frac{cT_1 D Q^{-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)}$$

It should be noticed that the production quantity in every cycle of learning was taken to be Q.

$$\frac{dAA}{dQ} = -\frac{(A_v + \sum_{s=1}^{m} A_s)D}{Q^2} + h_{v2}\left[\frac{1}{2} - \frac{T_1 D Q^{-b_i}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(2-b_i)}\right] + D\sum_{s=1}^{m}\left[\frac{h_{v1s}}{2}\left\{\frac{\mu_s^2 \mathrm{E}[\pi_s^2](1+\mathrm{E}[\gamma_s])}{x} +\right.\right.$$
$$\left.\left.Q^{-b_i}\mu_s \mathrm{E}[\pi_s] T_1\{i^{1-b_i} - (i-1)^{1-b_i}\}\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)\right\}\right] + \sum_{s=1}^{m} h_{v1s} u_s (1 - \mathrm{E}[\pi_s]) -$$
$$\frac{b_i c T_1 D Q^{-b_i - 1}\{i^{1-b_i} - (i-1)^{1-b_i}\}}{(1-b_i)}$$

$$\frac{d^2 AA}{dQ^2} = \frac{2(A_v + \sum_{s=1}^{m} A_s)D}{Q^3} + h_{v2}\left[\frac{b_i T_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}}{Q^{1+b_i}(2-b_i)}\right] -$$
$$\sum_{s=1}^{m}\left[\frac{b_i h_{v1s}\mu_s \mathrm{E}[\pi_s] T_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)}{2Q^{1+b_i}}\right] + \frac{b_i(1+b_i)cT_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}}{Q^{2+b_i}(1-b_i)} \tag{A6.1}$$

where

$\frac{2(A_v + \sum_{s=1}^{m} A_s)D}{Q^3} > 0$, $\forall Q > 0$ since $D > 0$, $A_v > 0$, and $A_s > 0$.

$c\,\frac{b_i(1+b_i)T_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}}{Q^{2+b_i}(1-b_i)} > 0$, since $0 \le b_i < 1$, $T_1 > 0$, $c > 0$, and $i^{1-b_i} - (i-1)^{1-b_i} \ge 0$

Now we need to need to prove that:

$$\frac{b_i T_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}}{Q^{1+b_i}}\left[\frac{h_{v2}}{(2-b_i)}\right] - \sum_{s=1}^{m}\left[\frac{h_{v1s}\mu_s \mathrm{E}[\pi_s]\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)}{2}\right] > 0$$

As $\frac{b_i T_1 D\{i^{1-b_i} - (i-1)^{1-b_i}\}}{Q^{1+b_i}} > 0$, it reduces to

$$\left[\frac{h_{v2}}{(2-b_i)}\right] - \sum_{s=1}^{m}\left[\frac{h_{v1s}\mu_s \mathrm{E}[\pi_s]\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)}{2}\right] > 0 \tag{A6.2}$$

For the case of the supplier with maximum fraction of defectives, where $\gamma_{max} - \gamma_s$=0, $\mathrm{E}[\pi_s] = 1$. Besides, we can assume without loss of generality, that $\mu_s = \mu$, $h_{v1s} = h_{v1}$, $b_i = b$ and $\gamma_s = \gamma$. Expression (A6.2) can then be written as

$$\left[\frac{h_{v2}}{(2-b)}\right] - h_{v1}\frac{\mu m}{2}\left(1 - E[\gamma] - \frac{D}{x}\right) \tag{A6.3}$$

By definition, each unit of a finished product consist of $m\mu$ units with unit cost of a finished product being $mc_u\mu + c_p$ where c_u and c_p are the unit purchase cost of raw material and the unit production cost of the finished product respectively. Therefore, $h_{v2} = k\left(c_u m\mu + c_p\right)$ and $h_{v1} = c_u k$, where k is the interest rate. So, following the Eq. (A6.3), we are left to prove

$\left[\frac{k\left(c_u m\mu + c_p\right)}{(2-b)}\right] - kc_u\frac{m\mu}{2}\left(1 - E[\gamma] - \frac{D}{x}\right) > 0$ or

$\frac{kc_p}{(2-b)} + \frac{kc_u m\mu}{(2-b)} - \frac{kc_u m\mu}{2}\left(1 - E[\gamma] - \frac{D}{x}\right) > 0$ or

$\frac{kc_p}{(2-b)} + kc_u m\mu\left[\frac{1}{(2-b)} - \frac{1}{2}\left(1 - E[\gamma] - \frac{D}{x}\right)\right] > 0$

which is true if

$\frac{1}{2-b} - \frac{1}{2}\left(1 - E[\gamma] - \frac{D}{x}\right) > 0$ or

$\frac{1}{2-b} - \frac{W}{2} > 0$ where $W = 1 - E[\gamma] - \frac{D}{x}$

$\Rightarrow \frac{1}{2-b} > \frac{W}{2}$ or

$\frac{1}{W} > \frac{2-b}{2}$.

Since $0 \le b < 1$ and $W < 1$, then $\frac{2-b}{2} < 1$, which makes $\frac{1}{W} > \frac{2-b}{2}$ True.

Therefore,

$\left[\frac{h_{v2}}{(2-b_i)}\right] - \sum_{S=1}^{m}\left[\frac{h_{v1s}\mu_s \mathrm{E}[\pi_s]\left(1 - \mathrm{E}[\gamma_s] - \frac{D}{x}\right)}{2}\right] > 0$, and $\frac{d^2A}{dQ^2} > 0 \forall Q > 0$, suggesting that AA is convex in Q.

APPENDIX 7 Convexity of the Annual Cost in Eq. (6.14)

$$\mathrm{E}[TCU_i] = \frac{DM_2}{Q}\left\{\frac{A_v}{n} + A_r\right\} + dDM_2 + \frac{h_r QDM_1M_2}{x}$$

$$+\frac{h_v DM_2 Q^{1-b}}{P(1-b)}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right]$$

$$+\frac{h_v(n-1)QM_2}{2D} + \frac{cM_2(nQ)^{-b}\{i^{1-b}-(i-1)^{1-b}\}}{P(1-b)} + \frac{h_r Q(1-M_1)}{2}$$

$$\frac{\mathrm{d}}{\mathrm{dQ}}\mathrm{E}[TCU_i] = -\frac{DM_2}{Q^2}\left\{\frac{A_v}{n} + A_r\right\} + \frac{h_r DM_1M_2}{x}$$

$$+\frac{h_v DM_2 Q^{-b}}{P}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right]$$

$$+\frac{h_v(n-1)M_2}{2D} - \frac{cbM_2 n^{-b}Q^{-1-b}\{i^{1-b}-(i-1)^{1-b}\}}{P(1-b)} + \frac{h_r(1-M_1)}{2}$$

$$\frac{\mathrm{d}^2}{\mathrm{dQ}^2}\mathrm{E}[TCU_i] = \frac{2DM_2}{Q^3}\left\{\frac{A_v}{n} + A_r\right\}$$

$$-b\,\frac{h_v DM_2 Q^{-1-b}}{P}\left[\{1+(i-1)n\}^{1-b} - \{(i-1)n\}^{1-b} - \frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right]$$

$$+\frac{cbM_2 n^{-b}Q^{-2-b}\{i^{1-b}-(i-1)^{1-b}\}}{P}$$

$n^{1-b}\{i^{1-b}-(i-1)^{1-b}\} > 0$, since $n \geq 1$, $i \geq 1$, and $0 \leq b < 1$.

$\frac{2DM_2}{Q^3}\left\{\frac{A_v}{n} + A_r\right\} > 0, \forall Q > 0$, since all the other parameters are > 0

$\frac{cbM_2 n^{-b}Q^{-2-b}\{i^{1-b}-(i-1)^{1-b}\}}{P} > 0, \forall Q > 0$, since all the parameter are positive and $i^{1-b} - (i-1)^{1-b} > 0, \forall i \geq 1\ and\ 0 \leq b < 1$

$+b\,\frac{h_v DM_2 Q^{-1-b}}{P}\left[\frac{n^{1-b}\{i^{1-b}-(i-1)^{1-b}\}}{(2-b)}\right] > 0, \forall Q > 0,$, since all the parameter are positive and $i^{1-b} - (i-1)^{1-b} > 0, \forall i \geq 1\ and\ 0 \leq b < 1$.

Observation 1:

$f = \{1 + (i-1)n\}^{1-b} - \{(i-1)n\}^{1-b}$ holds a maximum value of 1 when $i = 1$ when $n \geq 1, 0 \leq b < 1$. As i increase, f approaches zero. It approaches zero faster when n is large and b is close to 1. (the proof is below)

$f' = (1-b)(1-i)\{1 + (i-1)n\}^{-b} - (1-b)(1-i)^{1-b}n^{-b} \leq 0, \Rightarrow$

$\{1 + (i-1)n\}^{-b} - (1-i)^{-b}n^{-b} \leq 0 \Rightarrow \{1 + (i-1)n\}^{-b} \leq (1-i)^{-b}n^{-b} \Rightarrow$

$\left\{\frac{1+(i-1)n}{(1-i)n}\right\}^{-b} \leq 1 \Rightarrow \left\{\frac{1}{(1-i)n} + 1\right\}^{-b} \leq 1$, True

$f'' = -b(1-b)(1-i)^2\{1 + (i-1)n\}^{-1-b} + (1-b)b(1-i)^{1-b}n^{-1-b} < 0 \Rightarrow$

$-(1-i)\{1 + (i-1)n\}^{-1-b} + (1-i)^{-b}n^{-1-b} < 0 \Rightarrow -(1-i)\{1 + (i-1)n\}^{-1-b} < -(1-i)^{-b}n^{-1-b} \Rightarrow \Rightarrow (i-1)^{1+b}\left\{\frac{1+(i-1)n}{(i-1)n}\right\}^{-1-b} > 1$ for every $i > 1$

Observation 2:

Since the term $b\frac{h_v D M_2 Q^{-1-b}}{P} \to 0$ as Q increases (where $0<b<1$, $(D/P<1)$, $M_2>0$, $h_v>0$), then from observations 1 and 2, we can safely assume that the impact of $b\frac{h_v D M_2 Q^{-1-b}}{P} f$ in $\frac{\mathrm{d}^2}{\mathrm{dQ}^2}\mathrm{E}[TCU_i]$ is insignificant as it is very close to zero.

Therefore, $\frac{d^2}{dQ^2}\mathrm{E}[TCU_i] > 0, \forall Q > 0$, and Eq. (6.14) is convex with a unique minimizer.

APPENDIX 8 Hill (1997) Generalized Model

The behavior of vendor's inventory in the Hill's (1997) generalized model is described in Figure 6.1.

The vendor's total inventory in a cycle is

$$I_v = \frac{n^2Q^2}{2P} + \frac{nQ^2(n-1)(P-D)}{PD} - \frac{n(n-1)Q^2}{2D} = \frac{nQ^2}{2D}\left\{(n-1)-(n-2)\frac{D}{P}\right\}$$

So, the total annual cost of the supply chain would be

$$TC(Q,n) = \frac{(A_v + nA_b)D}{nQ} + \frac{h_vQ}{2}\left\{(n-1)-(n-2)\frac{D}{P}\right\} + \frac{h_bQ}{2}$$

or

$$TC(Q,n) = \frac{(A_v + nA_b)D}{nQ} + h_v\left\{\frac{DQ}{P} + \frac{(P-D)nQ}{2P}\right\} + \frac{(h_b - h_v)Q}{2}$$

APPENDIX 9 Optimality of the Annual Cost in Eq. 7.16

Let

$$C = \mathrm{E}[TCU_i] = \frac{DM_2}{Q}\left(\frac{A_v}{n} + A_b\right) + dDM_2 + \frac{\left(h_b + h_v^{'}\right)Q}{2}\left\{n + \frac{D}{P} + DM_2\left(\frac{2M_1}{x} - \frac{n}{P}\right)\right\}$$
$$+ \frac{DM_2}{P}\left[\frac{h_v\{i^{1-b} - (i-1)^{1-b}\}(nQ)^{1-b}}{2-b} + \frac{c(nQ)^{-b}\{i^{1-b} - (i-1)^{1-b}\}}{1-b}\right] \quad \text{(A9.1)}$$

$$X = DM_2\left(\frac{A_v}{n} + A_b\right), Y = \frac{DM_2\{i^{1-b} - (i-1)^{1-b}\}}{P}$$

Differentiating Eq.(A9.1) with respect to Q:

$$\frac{dC}{dQ} = -\frac{X}{Q^2} + \frac{\left(h_b + h_v^{'}\right)}{2}\left\{n + \frac{D}{P} + DM_2\left(\frac{2M_1}{x} - \frac{n}{P}\right)\right\} + \frac{(1-b)h_vYn^{1-b}Q^{-b}}{2-b} - \frac{bcn^{-b}YQ^{-b-1}}{1-b}$$

$$\frac{d^2C}{dQ^2} = \frac{2X}{Q^3} - \frac{b(1-b)h_vYn^{1-b}Q^{-b-1}}{2-b} + \frac{b(1+b)cYn^{-b}Q^{-b-2}}{1-b} \quad \text{(A9.2)}$$

From the other Appendices, we know that the bracket of i in Y is positive. The first term in Eq. (A9.2) is positive. So, we are left to prove that

$$-\frac{b(1-b)h_vYn^{1-b}Q^{-b-1}}{2-b} + \frac{b(1+b)cYn^{-b}Q^{-b-2}}{1-b} > 0$$

$$\frac{b(1+b)cY}{n^b(1-b)Q^{b+2}} - \frac{b(1-b)h_vY}{n^{b-1}(2-b)Q^{b+1}} > 0$$

$$\frac{(1+b)c}{(1-b)Q} - \frac{(1-b)nh_v}{(2-b)} > 0$$

Substituting ic/D for h_v, where i is the interest rate:

$$\frac{(1+b)c}{(1-b)Q} - \frac{n(1-b)}{(2-b)}\left(\frac{ic}{D}\right) > 0$$

$$\frac{(1+b)}{(1-b)Q} - \frac{n(1-b)}{(2-b)}\left(\frac{i}{D}\right) > 0 \quad \text{(A9.3)}$$

Rewriting expression (A9.3)

$$\frac{(1+b)}{(1-b)} > \frac{ni(1-b)}{(2-b)}\left(\frac{Q}{D}\right)$$

$$\frac{(1+b)(2-b)}{i(1-b)^2} > \frac{nQ}{D}$$

The right hand side in the above expression is the cycle time (T) in years, in our model in chapter seven. So, the condition for a convex annual cost would be:

$$T < \frac{(1+b)(2-b)}{i(1-b)^2} \tag{A9.4}$$

The above condition was tested for ten thousand examples through a simulation by choosing random interest rate (between 1% and 50%) and learning exponent (between 0 and 0.95), which resulted in a reasonable number for T (in years). Thus, the annual cost can be assumed to be positive.

REFERENCES

Adler, G.L. and Nanda, R. 1974. The effects of learning on optimal lot size determination-single product case. *AIIE Transactions*, 6(1): 14–20.

Alamri, A.A. and Balkhi, Z.T. 2007. The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. *International Journal of Production Economics*, 107(1): 125–138.

Allwood, J.M. and Lee, W.L. 2004. The impact of job rotation on problem solving skills. *International Journal of Production Research*, 42(5): 865–881.

Anderlohr, G. 1969. What production breaks cost? *Industrial Engineering*, 1(9): 34–36.

Andersson, J. and Melchiors, P. 2001. Two-echelon inventory model with lost sales. *International Journal of Production Economics*, 69(3): 307–315.

Argote, L. and Epple, D. 1990. Learning curves in manufacturing. *Science*, 247(4945): 920–924.

Atali, A., Lee, H. and Ozer, O. 2009. If the inventory manager knew: Value of RFID under imperfect inventory information. Graduate School of Business, Stanford University, Working Paper.

Badiru, A.B. 1995. Multivariate analysis of the effect of learning and forgetting on product quality. *International Journal of Production Research*, 33(3): 777–794.

Balkhi, Z.T. 2003. The effects of learning on the optimal production lot size for deteriorating and partially backordered items with time varying demand and deterioration rates. *Applied Mathematical Modeling*, 27(10): 763–779.

Balkhi, Z.T. 2004. An optimal solution of a general lot size inventory model with deteriorated and imperfect products, taking into account inflation and time value of money. *International Journal of Systems Science*, 35(2): 87–96.

Baloff, N. 1970. Startup management. *IEEE Transactions on Engineering Management*, 17(4): 132–141.

Banerjee, A. 1986a. A joint economic-lot-size model for purchaser and vendor. *Decision Sciences*, 17(3): 292–311.

Banerjee, A. 1986b. A supplier's pricing model under a customer's economic purchasing policy. *Omega*, 14(5): 409–414.

Battini, D., Grassi, A., Persona, A. and Sgarbossa, F. 2010. Consignment stock inventory policy: Methodological framework and model. *International Journal of Production Research*, 48(7): 2055–2079.

Ben-Daya, M. and Hariga, M. 2000. Economic lot scheduling problem with imperfect production processes. *Journal of the Operational Research Society*, 51(7): 875–881.

Ben-Daya, M., Darwish, M.A. and Ertogral, K. 2008. Joint economic lot sizing problem: A review and extensions. *European Journal of Operational Research*, 185(2): 726–742.

Bennett, G.K., Case, K.E. and Schmidt, J.W. 1974. Economic effects of inspection error on attribute sampling plans. *Naval Research Logistics*, 21(3): 431–443.

Bijari, M., Barzoki, M.R. and Jahanbazi, M. 2007. Effects of imperfect products and rework on lot sizing with work in process inventory. *Proceedings of the Industrial Engineering Research Conference*, May 19–23, Nashville, TN, USA, pp. 919–925.

Biswas, P. and Sarker, B.R. 2008. Optimal batch quantity models for a lean production system with in-cycle rework and scrap. *International Journal of Production Research*, 46(23): 6585–6610.

Bonney, M., Ratchev, S. and Moualek, I. 2003. The changing relationship between production and inventory examined in a concurrent engineering context. *International Journal of Production Economics*, 81–82: 243–254.

Braglia, M., Zavanella, L., 1994. Optimal sampling techniques in off-line quality control. *Production Planning and Control*, 5(1): 59–65.

Braglia, M. and Zavanella, L. 2003. Modeling an industrial strategy for inventory management in supply chains: The 'consignment stock' case. *International Journal of Production Research*, 41(16): 3793–3808.

Cachon, G.P. and Fisher, M. 2000. Supply chain inventory management and the value of shared information. *Management Science*, 46(8): 1032–1048.

Cachon, G.P. 2003. Supply chain coordination with contracts. In: Anonymous (Ed.), Handbooks in Operations Research and Management Science: *Supply Chain Management*. North Holland.

Cárdenas-Barrón, L.E. 2000. Observation on: Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 67(2): 201.

Cárdenas-Barrón, L.E. 2009. Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Computers and Industrial Engineering*, 57(3): 1105–1113.

Cary, D.T., Torrence, W.D. and Schniederjans, M. 1994. Quality assurance in human resource management for computer-integrated manufacturing. *International Journal of Operations & Production Management*, 14(2): 18–30.

Chan, W.M., Ibrahim, R.N. and Lochert, P.B. 2003. A new EPQ model: Integrating lower pricing, rework and reject situations. *Production Planning and Control*, 14(7): 588–595.

Chand, S. 1989. Lot sizes and setup frequency with learning in setups and process quality. *European Journal of Operational Research*, 42(2): 190–202.

Chandra, J. and Schall, S., 1988. The use of repeated measurements to reduce the effect of measurement errors. *IIE Transactions*, 20(1): 83–87.

Chang, H.C. 2004. An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Computers and Operations Research*, 31(12): 2079–2092.

Chang, H.C. and Ho, C.H., 2010. Exact closed-form solutions for ‘‘optimal inventory model for items with imperfect quality and shortage backordering’’, *Omega*, 38(3–4): 233–237.

Chang, J.H. and Chiu, H.N., 2005. A comprehensive review of lot streaming. *International Journal of Production Economics*, 43(8): 1515–1536.

Chen, L.H. and Kang, F.S. 2010. Coordination between vendor and buyer considering trade credit and items of imperfect quality. *International Journal of Production Economics*, 123(1): 52–61.

Cheng, T. 1991. An economic order quantity model with demand-dependent unit production cost and imperfect production processes. *IIE Transactions*, 23(1): 23–28.

Chiou, C.C. and Chen, L.H.C. 2007. Determining the optimal production policy for a system with imperfect production and shortage. *Proceedings in Applied Mathematics and Mechanics*, 7(1): 2060077–2060078.

Chiu, H.N. 1997. Discrete time-varying demand lot-sizing models with learning and forgetting effects. *Production Planning and Control*, 8(5): 484–493.

Chiu, H.N., Chen, H.M. and Weng, L. 2003. Determining time-varying demand lot-sizing models with learning and forgetting in setups and production. *Production and Operations Management*, 12(1): 120–127.

Chiu, H.N. and Chen, H.M. 2005. An optimal algorithm for solving the dynamic lot-sizing model with learning and forgetting in setups and production. *International Journal of Production Economics*, 95(2): 179–193.

Chiu, S.W., Gong, D. and Wee, H.M. 2004. Effects of random defective rate and imperfect rework process on economic production quantity model. *Japan Journal of Industrial and Applied Mathematics*, 21(3): 375–389.

Chiu, S.W. and Chiu, Y.S.P. 2006. Mathematical modeling for production system with backlogging and failure in repair. *Journal of Scientific and Industrial Research*, 65(6): 499–506.

Chiu, S.W., Chiu, Y.S.P. and Shih, C.C. 2006. Determining expedited time and cost of the end product with defective component parts using critical path method (CPM) and time-costing method. *Journal of Scientific and Industrial Research*, 65(9): 695–701.

Chiu, S.W. 2007. Optimal replenishment policy for imperfect quality EMQ model with rework and backlogging. *Applied Stochastic Models in Business and Industry*, 23(2): 165–178.

Chiu, S.W., Shih, C.C. and Chiu, Y.S.P. 2007. A revised cost-benefit algorithm for solving the expediting completion time of end product problem with defective materials in the product structure diagram. *Proceedings of the Institution of Mechanical Engineers*, Part B: Journal of Engineering Manufacture, 221(3): 489–497.

Chiu, Y. and Chang, C.Y. 2009. Optimal JIT production and delivery lot size model that considers vendor reworking process and variable setup cost. *Journal of the Chinese Institute of Industrial Engineers*, 26(2): 99–108.

Chiu, Y.P. 2003. Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering Optimization*, 35(4): 427–437.

Chiu, Y.S.P. and Chiu, S.W. 2005. The finite production model with the reworking of defective items. *International Journal of Industrial Engineering*, 12(1): 16–22.

Chopra, S. and Meindl, P., 2003. *Supply chain management: Strategy, planning and operation*. USA: Prentice Hall.

Chung, C.J. and Wee, H.M. 2008. Scheduling and replenishment plan for an integrated deteriorating inventory model with stock-dependent selling rate. *International Journal of Advanced Manufacturing Technology*, 35(7–8): 665–679.

Chung, K.J. and Huang, Y.F. 2006. Retailer's optimal cycle times in the EOQ model with imperfect quality and a permissible credit period. *Quality and Quantity*, 40(1): 59–77.

Chung, K.J., Her, C.C. and Lin, S.D. 2009. A two-warehouse inventory model with imperfect quality production processes. *Computers and Industrial Engineering*, 56(1): 193–197.

Collins, R.D., Case, K.E. and Bennett, G.K. 1973. Effects of inspection error on single sampling inspection plans. *International Journal of Production Research*, 11(3): 289–298.

Comeaux, E.J. and Sarker, B.R. 2005. Joint optimization of process improvement investments for supplier-buyer cooperative commerce. *Journal of the Operational Research Society*, 56(11): 1310–1324.

Copacino, W.C.1997. *Supply chain management: The basics and beyond*. Florida: CRC Press LLC.

Corbett, C.J. 2001. Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Operations Research*, 49(4): 487–500.

Crosby, P.B. 1979. *Quality is free*. New York: McGraw-Hill.

Cunningham, J. 1980. Using the learning curve as a management tool. *IEEE Spectrum*, 17(6): 43–48.

Dance, D. and Jarvis, R. 1992a. Using yield models to accelerate learning curve progress. *IEEE Transactions on Semiconductor Manufacturing*, 5(1): 41–46.

Dar-El, E.2000. *Human learning: From learning curves to learning organizations*. USA: Kluwer.

Darwish, M.A. 2008. EPQ models with varying setup cost. *International Journal of Production Economics*, 113(1): 297–306.

Deming, W.E. 1982. *Quality, productivity, and competitive position*. In: Anonymous (Ed.), M.I.T. Center for advanced Engineering Study.

Dobos, I. and Richter, K. 2006. A production/recycling model with quality consideration. *International Journal of Production Economics*, 104(2): 571–579.

Duffuaa, S.O. and Raouf, A. 1989. Mathematical optimization models for multi-characteristic repeat inspections. *Applied Mathematical Modeling*, 13(7): 408–412.

Duffuaa, S.O. and Raouf, A. 1990. An optimal sequence in multi-characteristics inspection. *Journal of Optimization Theory and Applications*, 67(1): 79–86.

Duffuaa, S.O. and Nadeem, I.A. 1994. A complete inspection plan for dependent multi-characteristic critical components. *International Journal of Production Research*, 32(8): 1897–1907.

Duffuaa, S.O. and Al-Najjar, H.J. 1995. Optimal complete inspection plan for critical multicharacteristic components. *Journal of the Operational Research Society*, 46(8): 930–942.

Duffuaa, S.O. 1996. Impact of inspection errors on performance measures of a complete repeat inspection plan. *International Journal of Production Research*, 34(7): 2035–2049.

Duffuaa, S.O. and Khan, M. 2002. An optimal repeat inspection plan with several classifications. *Journal of the Operational Research Society*, 53(9): 1016–1026.

Duffuaa, S.O. and Khan, M. 2005. Impact of inspection errors on the performance measures of a general repeat inspection plan. *International Journal of Production Research*, 43(23): 4945–4967.

Duffuaa, S.O. and Khan, M. 2008. A general repeat inspection plan for dependent multicharacteristic critical components. *European Journal of Operational Research*, 191(2): 374–385.

Dutton, J.M. and Thomas, A. 1984. Treating progress functions as a managerial opportunity. *Academy of Management Review*, 9(2): 235–247.

El-Kassar, A.N.M. 2009. Optimal order quantity for imperfect quality items. *Proceedings of the Academy of Information and Management Sciences*, 13(1): 24–30.

Eroglu, A. and Ozdemir, G. 2007. An economic order quantity model with defective items and shortages. *International Journal of Production Economics*, 106(2): 544–549.

Fine, C.H. 1986. Quality improvement and learning in productive systems. *Management Science*, 32(10): 1301–1315.

Fine, C.H. 1988. A quality control model with learning effects. *Operations Research*, 36(3): 437–444.

Fisk, A.D., Ackerman, P.L. and Schneider, W. 1987. Automatic and controlled processing theory and its applications to human factors problems. *Human Factors Psychology*, 47: 159–197.

Forker, F.B. 1997. Factors affecting supplier quality performance. *Journal of Operations Management*, 15(4): 243–269.

Foster, S.J. and Adam, E.E. 1996. Examining the impact of speed of quality improvement on quality-related costs. *Decision Sciences*, 27(4): 623–646.

Gerchak, Y., Vickson, R.G. and Parlar, M. 1988. Periodic review production models with variable yield and uncertain demand. *IIE Transactions*, 20(2): 144–150.

Glover, J.H. 1965. Manufacturing progress functions I. an alternative model and its comparison with existing functions. *International Journal of Production Research*, 4(4): 279–300.

Goyal, S.K. 1977. An integrated inventory for a single product system. *Operational Research Quarterly*, 28(3): 539–545.

Goyal, S.K. 1985. Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, 36(4): 335–338.

Goyal, S.K. 1988. A joint economic-lot-size model for purchaser and vendor: A comment. *Decision Sciences*, 19(1): 236–241.

Goyal, S.K. and Gupta, Y.P. 1989. Integrated inventory models: The buyer-vendor coordination. *European Journal of Operational Research*, 41(3): 261–269.

Goyal, S.K. and Satir, A.T. 1989. Joint replenishment inventory control: Deterministic and stochastic models. *European Journal of Operational Research*, 38(1): 2–13.

Goyal, S.K. 1995. A one-vendor multi-buyer integrated inventory model: A comment. *European Journal of Operational Research*, 82(1): 209–210.

Goyal, S.K. and Giri, B.C. 2001. A comment on Chang and Dye (1999): EOQ model for deteriorating items with time-varying demand and partial backlogging. *Journal of the Operational Research Society*, 52(2): 238–239.

Goyal, S.K. and Cárdenas-Barrón, L.E. 2002. Note on: Economic production quantity model for items with imperfect quality - A practical approach. *International Journal of Production Economics*, 77(1): 85–87.

Goyal, S.K., Huang, C.K. and Chen, K.C. 2003. A simple integrated production policy of an imperfect item for vendor and buyer. *Production Planning and Control*, 14(7): 596–602.

Grosfeld-Nir, A. and Gerchak, Y. 2004. Multiple lot sizing in production to order with random yields: Review of recent advances. *Annals of Operations Research*, 126(1–4): 43–69.

Grubbström, R.W. and Erdem, A. 1999. The EOQ with backlogging derived without derivatives. *International Journal of Production Economics*, 59(1–3): 529–530.

Guiffrida, A.L., 2009. Fuzzy inventory models. In M.Y. Jaber (Ed.), *Inventory Management: Non-Classical Views*, CRC Press, FL, Boca Raton, pp. 173–190 (Chapter 8).

Guiffrida, A.L. and Nagi, R. 1998. Fuzzy set theory applications in production management research: A literature survey. *Journal of Intelligent Manufacturing*, 9(1): 39–56.

Ha, D. and Kim, S.L. 1997. Implementation of JIT purchasing: An integrated approach. *Production Planning and Control*, 8(2): 152–156.

Haji, A., Elyasi, K.A. and Shemshaki, A.S. 2009. Optimal integrated inventory model with lost sale and backordering. *International Conference on Computers & Industrial Engineering*. July 6–9. Troyes, France, pp. 825–830.

Haji, A., Sikari, S.S. and Shamsi, R. 2010. The effect of inspection errors on the optimal batch size in reworkable production systems with scraps. *International Journal of Product Development*, 10(1–3): 201–215.

Haji, B., Haji, A. and Haji, R. 2009a. Optimal batch production with minimum rework cycles and constraint on accumulated defective units. *IEEE/INFORMS International Conference on Service Operations, Logistics and Informatics (SOLI)*, July 22–24, Chicago, USA, pp. 633–638.

Haji, B., Haji, R. and Haji, A. 2009b. Optimal batch production with rework and non-zero setup cost for rework. *International Conference on Computers & Industrial Engineering*. July 6–9, Troyes, France, pp. 857–862.

Hayek, P.A. and Salameh, M.K. 2001. Production lot sizing with the reworking of imperfect quality items produced. *Production Planning and Control*, 12(6): 584–90.

Hejazi, S.R., Tsou, J.C. and Barzoki, M.R. 2008. Optimal lot size of EPQ model considering imperfect and defective products. *International Journal of Industrial Engineering*, 4(7): 59–68.

Hill, R.M. 1997. The single-vendor single-buyer integrated production-inventory model with a generalized policy. *European Journal of Operational Research*, 97(3): 493–499.

Hill, R.M., Seifbarghy, M. and Smith, D.K. 2007. A two-echelon inventory model with lost sales. *European Journal of Operational Research*, 181(2): 753–766.

Hollins, B. 1995. *Quality starts with designers*. The TQM Magazine, 7(2): 33–35.

Hong, S.H., Kim, S.B., Kwon, H.M. and Lee, M.K. 1998. Economic design of screening procedures when the rejected items are reprocessed. *European Journal of Operational Research*, 108(1): 65–73.

Hsu, P.H., Teng, M.H. and Wee, H.M. 2009. Optimal production decisions for deteriorating items with investment on production processes. *First Asian Conference on Intelligent Information and Database Systems*. April 1–3, Dong Hoi, Vietnam, pp. 410–413.

Hsu, W.K.K. and Yu, H.F. 2009. EOQ model for imperfective items under a one-time-only discount. *Omega*, 37(5): 1018–1026.

Hsu, W.K. and Yu, H.F., 2011. An EOQ model with imperfective quality items under an announced price increase. *Journal of the Chinese Institute of Industrial Engineers*, 28(1): 34–44.

Huan, N.C., Hsin, M.C. and Li, C.W. 2003. Deterministic time-varying demand lot-sizing models with learning and forgetting in setups and production. *Production and Operations Management*, 12(1): 120–127.

Huang, C.K. 2002. An integrated vendor-buyer cooperative inventory model for items with imperfect quality. *Production Planning and Control*, 13(4): 355–361.

Huang, C.K. 2004. An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. *International Journal of Production Economics*, 91(1): 91–98.

Huang, Q. and Chen, J. 2009. A note on modeling an industrial strategy for inventory management in supply chains: The 'consignment stock' case. *International Journal of Production Research*, 47(22): 6469–6475.

Hugos, M.2006. *Essentials of supply chain management*. Canada: Wiley.

Hyland, P.W. 2003. Continuous improvement and learning in the supply chain. *International Journal of Physical Distribution & Logistics Management*, 33(4): 316–335.

Inderfurth, K. 2004. Analytical solution for a single-period production-inventory problem with uniformly distributed yield and demand. *Central European Journal of Operations Research*, 12(2): 117–127.

Irvin, D.A. and Klenow, P.J. 1994. Learning-by-doing spillovers in the semiconductor industry. *The Journal of Political Economy*, 102(6): 1200–1227.

Jaber, M. and Bonney, M. 1996. Production breaks and the learning curve: The forgetting phenomenon. *Applied Mathematical Modeling*, 20(2): 162–169.

Jaber, M.Y. and Bonney, M. 1997. A comparative study of learning curves with forgetting. *Applied Mathematical Modeling*, 21(8): 523–531.

Jaber, M.Y. and Bonney, M. 1998. The effects of learning and forgetting on the optimal lot size quantity of intermittent production runs. *Production Planning & Control*, 9(1): 20–27.

Jaber, M.Y. and Bonney, M. 1999. The economic manufacture/order quantity (EMQ/EOQ) and the learning curve: Past, present, and future. *International Journal of Production Economics*, 59(1–3): 93–102.

Jaber, M.Y, Kher, H.V, 2002. A dual-phase learning-forgetting model. *International Journal of Production Economics*, 76(3): 229–242.

Jaber, M., Kher, H. and Davis, D. 2003. Countering forgetting through training and deployment. *International Journal of Production Economics*, 85(1): 33–46.

Jaber, M.Y. and Bonney, M. 2003. Lot sizing with learning and forgetting in set-ups and in product quality. *International Journal of Production Economics*, 83(1): 95–111.

Jaber, M.Y. and Guiffrida, A.L. 2004. Learning curves for processes generating defects requiring reworks. *European Journal of Operational Research*, 159(3): 663–672.

Jaber, M.Y., Nuwayhid, R.Y., Rosen, M.A., 2004. Price-driven economic order systems from a thermodynamic point of view. *International Journal of Production Research*, 42(24): 5167–5184.

Jaber, M.Y. and Sikström, S. 2004. A note on "an empirical comparison of forgetting models". *IEEE Transactions on Engineering Management*, 51(2): 233–235.

Jaber, M.Y. 2006*a*. Learning and forgetting models and their applications. In: A. B. Badiru. (Ed.), *Handbook of Industrial & Systems Engineering*. CRC Press/Taylor & Francis, FL: Baco Raton: pp. 30:1–30:27

Jaber, M.Y. 2006*b*. Lot sizing for an imperfect production process with quality corrective interruptions and improvements, and reduction in setups. *Computers and Industrial Engineering*, 51(4): 781–790.

Jaber, M.Y. and Osman, I.H. 2006. Coordinating a two-level supply chain with delay in payments. *Computers and Industrial Engineering*, 50(4): 385–400.

Jaber, M.Y., Osman, I.H. and Guiffrida, A. L. 2006. Coordinating a three-level supply chain with price discounts, price dependent demand, and profit sharing. *International Journal of Integrated Supply Chain Management*, 2(1/2): 28–48.

Jaber, M.Y. and Bonney, M. 2007. Economic manufacture quantity (EMQ) model with lot-size dependent learning and forgetting rates. *International Journal of Production Economics*, 108(1–2): 359–367.

Jaber, M.Y., Guiffrida, A.L., 2007. Observations on the economic order (manufacture) quantity model with learning and forgetting. *International Transactions in Operational Research*, 14(2): 91–104.

Jaber, M.Y., Goyal, S.K. and Imran, M. 2008. Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics*, 115(1): 143–150.

Jaber, M.Y. and Guiffrida, A.L. 2008. Learning curves for imperfect production processes with reworks and process restoration interruptions. *European Journal of Operational Research*, 189(1): 93–104.

Jaber, M.Y. and Zolfaghari, S. 2008. Quantitative models for centralized supply chain coordination. In: S. Kordic. (Ed.), *Supply Chains: Theory and Applications*. I-Tech Education and Publishing, Vienna, Austria, EU: pp. 307–338

Jaber, M.Y., Bonney, M. and Moualek, I. 2009. Lot sizing with learning, forgetting and entropy cost. *International Journal of Production Economics*, 118(1): 19–25.

Jaber, M.Y. and Khan, M. (2010). Managing yield by lot splitting in a serial production line with learning, rework and scrap, *International Journal of Production Economics*, 124(1): 32–39.

Jaber, M.Y., Bonney, M., Guiffrida, A.L., 2010. Coordinating a three level supply chain with learning-based continuous improvement. *International Journal of Production Economics*, 127(1): 27–38.

Jacobson, H.L. 1952. A study of inspector accuracy. *Industrial Quality Control*, 9: 16–25.

Jamal, A.M.M., Sarker, B.R. and Mondal, S. 2004. Optimal manufacturing batch size with rework process at a single-stage production system. *Computers & Industrial Engineering*, 47(1): 77–89.

Jaraiedi, M., Kochhar, D.S. and Jaisingh, S.C. 1987. Multiple inspections to meet desired outgoing quality. *Journal of Quality Technology*, 19(1): 46–51.

Jaraiedi, M. 1983. Inspection error modelling and economic design of sampling plans subject to inspection error. PhD Dissertation, University of Michigan, Ann Arbor.

Juran, J.M. 1978. Japanese and western quality - A contrast. *Quality Progress*, 11(12): 10–18.

Keachie, E. and Fontana, R. 1966. Production lot sizing under a learning effect. *Management Science*, 13(2): 102–108.

Kelle, P., Transchel, S. and Minner, S. 2009. Buyer-supplier cooperation and negotiation support with random yield consideration. *International Journal of Production Economics*, 118(1): 152–159.

Khan, M. Jaber, M.Y. and Wahab, M.I.M. 2010*a*. Economic order quantity model for Items with imperfect quality with learning in inspection. *International Journal of Production Economics*, 124 (1): 87–96.

Khan, M., Jaber, M.Y., and Ahmad, A.R. 2010*b*. A two-stage supply chain with inspection errors and learning in production. *Proceedings of 40th Atlantic Schools of Business Conference*, October 1–3, Halifax, Canada, pp. 525–540

Khan, M., Jaber, M.Y., Zanoni, S. and Zavanella, L. 2010*c*. Consignment stock policy for a two-level supply chain with defective items. *Proceedings of 16th International Symposium on Inventories*, August 23–27, Budapest, Hungary.

Khan, M., and Jaber, M.Y. 2011. Optimal inventory cycle in a two-stage supply chain incorporating imperfect items from suppliers. *International Journal of Operational Research*, (In Press).

Khan, M. Jaber, M.Y. and Bonney, M. 2011*a*. An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, doi:10.1016/j.ijpe.2010.01.023.

Khan, M., Jaber, M.Y. and Guiffrida, A.L., 2011*b*. Effect of human factors on the performance of a two Level supply chain. *International Journal of Production Research*, (In Press).

Khouja, M. and Mehrez, A. 1994. Economic production lot size model with variable production rate and imperfect quality. *Journal of the Operational Research Society*, 45(12): 1405–17.

Khouja, M. 2003. Optimizing inventory decisions in a multi-stage multi-customer supply chain. *Transportation Research Part E, Logistics & Transportation Review*, 39(3): 193–208.

Khouja, M. 2005. The use of minor setups within production cycles to improve product quality and yield. *International Transactions in Operational Research*, 12(4): 403–416.

Kim, D.H. and Park, K.S. 1985. (Q, r) inventory model with a mixture of lost sales and time-weighted backorders. *Journal of the Operational Research Society*, 36(3): 231–238.

Kim, D.H. 1993. The link between individual and organizational learning. *Sloan Management Review*, 35(1): 37–50.

Kim, S.L. and Ha, D. 2003. A JIT lot-splitting model for supply chain management: Enhancing buyer-supplier linkage. *International Journal of Production Economics*, 86(1): 1–10.

Kiser, G.E., Rao, C.P. and Rao, S.R.G., 1975. Vendor attribute evaluations of "buying center" members other than purchasing executives. *Industrial Marketing Management*, 4(1): 45–54.

Kök, A.G. and Shang, K.H. 2007. Inspection and replenishment policies for systems with inventory record inaccuracy. *Manufacturing and Service Operations Management*, 9(2): 185–205.

Konstantaras, I., Goyal, S.K. and Papachristos, S. 2007. Economic ordering policy for an item with imperfect quality subject to the in-house inspection. *International Journal of Systems Science*, 38(6): 473–482.

Kulkarni, S.S. 2008. Loss-based quality costs and inventory planning: General models and insights. *European Journal of Operational Research*, 188(2): 428–449.

Lambart, D.M., Stock, J.R. and Elram, L.M., 1998. *Fundamentals of logistics management*. Higher Education: McGraw-Hill.

Lapre, A.M., Mukherjee, A.S. and Wassenhove, L.N.V. 2000. Behind the learning curve: Linking learning activities to waste reduction. *Management Science*, 46(5): 597–611.

Lee, H.H. 2005. A Cost/Benefit model for investments in inventory and preventive maintenance in an imperfect production system. *Computers & Industrial Engineering*, 48(1): 55–68.

Lee, H.H. 2008. The investment model in preventive maintenance in multi-level production systems. *International Journal of Production Economics*, 112(2): 816–828.

Lee, H.L. and Rosenblatt, M.J. 1987. Simultaneous determination of production cycles and inspection schedules in a production system. *Management Science*, 33(9): 1125–1136.

Lee, H.L. 1988. On the optimality of a simplified multi-characteristic component inspection model. *IIE Transactions*, 20(4): 392–398.

Lee, W. and Wang, S. 2008. Managing level of consigned inventory with buyer's warehouse capacity constraint. *Production Planning and Control*, 19(7): 677–685.

Levy, F.K. 1965. Adaptation in the production process. *Management Science*, 11(6): B136–B154.

Li, G. and Rajagopalan, S. 1997. The impact of quality on learning. *Journal of Operations Management*, 15(3): 181–191.

Li, G. and Rajagopalan, S. 1998. Process improvement, quality and learning effects. *Management Science*, 44(11): 1517–1532.

Li, X.M. and Hong, C.X. 2006. Consignment stock policy with defective items. *International Conference on Management Science and Engineering (ICMSE)*. October 5–7, Lille, France, pp. 540–544.

Liao, G.L. 2007a. Optimal production correction and maintenance policy for imperfect process. *European Journal of Operational Research*, 182(3): 1140–1149.

Liao, G.L. 2007b. An imperfect process policy with production correction and reorganization. International Journal of Reliability, *Quality and Safety Engineering*, 14(2): 157–168.

Liao, G.L., Chen, Y.H. and Sheu, S.H. 2009. Optimal economic production quantity policy for imperfect process with imperfect repair and maintenance. *European Journal of Operational Research*, 195(2): 348–357.

Lin, T.Y. 2009. Optimal policy for a simple supply chain system with defective items and returned cost under screening errors. *Journal of the Operations Research Society of Japan*, 52(3): 307–320.

Liu, L., Sun, Y. and Yao, J. 2007. An extended analysis of consignment stock policy in the presence of product obsolescence. *IEEE International Conference on Automation and Logistics*. August 18–21, Jinan, China, pp. 2193–2198.

Lo, M.C. and Yang, M.F. 2008. Imperfect reworking process consideration in integrated inventory model under permissible delay in payments. *Mathematical Problems in Engineering*, doi:10.1155/2008/386252.

Lo, S.T., Wee, H.M. and Huang, W.C. 2007. An integrated production-inventory model with imperfect production processes and weibull distribution deterioration under inflation. *International Journal of Production Economics*, 106(2): 493–505.

Lu, L. 1995. A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research*, 81(2): 312–323.

Maddah, B. and Jaber, M.Y. 2008. Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics*, 112(2): 808–815.

Maddah, B., Moussawi, L. and Jaber, M.Y. 2010. Lot sizing with a markov production process and imperfect items scrapped. *International Journal of Production Economics*, 124(2): 340–347.

Maddah, B., Salameh, M.K. and Karame, G.M. 2009. Lot sizing with random yield and different qualities. *Applied Mathematical Modeling*, 33(4): 1997–2009.

Maddah, B., Salameh, M.K. and Moussawi, L. 2010. Order overlapping: A practical approach for preventing shortages during screening. *Computers & Industrial Engineering*, 58(4): 691–695.

Mansfield, E. 1991. Academic research and industrial innovation. *Research Policy*, 20(1): 1–12.

McKenna, S.P. and Glendon, A.I. 1985. Occupational first aid training: Decay in cardiopulmonary resuscitation (CPR) skills. *Journal of Occupational Psychology*, 58(2): 109–117.

Mehdi, R., Nidhal, R. and Anis, C. 2009. Integrated maintenance and control policy based on quality control. *Computers & Industrial Engineering*, 58(3): 443–451

Mentzer, J.T. 1993. Managing channel relations in the 21st century. *Journal of Business Logistics*, 14(1): 27–42.

Montgomery, D.C., Bazaraa, M.S. and Keswani, A.K. 1973. Inventory models with a mixture of backorders and and lost sales. *Naval Research Logistics*, 20(2): 255–263.

Moon, I., Giri, B.C. and Choi, K. 2002. Economic lot scheduling problem with imperfect production processes and setup times. *Journal of the Operational Research Society*, 53(6): 620–629.

Mukherjee, A.S., Lapre, M.A. and Wassenhove, L.N.V. 1998. Knowledge driven quality improvement. *Management Science*, 44(11): 36–49.

Muller, M.2003. *Essentials of inventory management*. USA: Amacom Press.

Munson, C.L. and Rosenblatt, M.J. 2001. Coordinating a three-level supply chain with quantity discounts. *IIE Transactions*, 33(5): 371–384.

Nadjafi, B.A. and Abbasi, B. 2009. EPQ model with depreciation cost and process quality cost as continuous functions of time. *International Journal of Industrial Engineering*, 5(8): 77–89.

Nahavandi, N. and Haghighirad, F. 2008. An integrated inventory model for two products with internal and external demands. *World Applied Sciences Journal*, 5(2): 258–266.

Nahmias, S. and Smith, S.A. 1994. Optimizing inventory levels in a two-echelon retailer system with partial lost sales. *Management Science*, 40(5): 582–596.

Ojha, D., Sarker, B.R. and Biswas, P. 2007. An optimal batch size for an imperfect production system with quality assurance and rework. *International Journal of Production Research*, 45(14): 3191–3214.

Osteryoung, J., Nosari, E., McCarty, D. and Reinh, W. 1986. Use of the EOQ model for inventory analysis. *Production and Inventory Management*, 27(3): 39–46.

Ouyang, L.Y., Teng, J.T. and Chen, L.H. 2006*a*. Optimal ordering policy for deteriorating items with partial backlogging under permissible delay in payments. *Journal of Global Optimization*, 34(2): 245–271.

Ouyang, L.Y., Wu, K.S. and Ho, C.H. 2006*b*. Analysis of optimal vendor-buyer integrated inventory policy involving defective items. *International Journal of Advanced Manufacturing Technology*, 29(11–12): 1232–1245.

Papachristos, S. and Konstantaras, I. 2006. Economic ordering quantity models for items with imperfect quality. *International Journal of Production Economics*, 100(1): 148–154.

Persona, A., Grassi, A. and Catena, M. 2005. Consignment stock of inventories in the presence of obsolescence. *International Journal of Production Research*, 43(23): 4969–4988.

Porteus, E.L. 1986a. Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, 34(1): 137–144.

Porteus, E.L. 1986b. Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, 34(1): 137–144.

Rahim, M.A. 1985. Economic model of x-chart under non-normality and measurement errors. *Computers & Operations Research*, 12(3): 291.

Raouf, A., Jain, J.K. and Sathe, P.T. 1983. A cost-minimization model for multi-characteristic component inspection. *IIE Transactions*, 15(3): 187–194.

Rea, P. and Kerzner, H., 1997. *Strategic planning: A practical guide*. Canada: John Wiley & Sons.

Rekik, Y., Sahin, E. and Dallery, Y. 2007. A comprehensive analysis of the newsvendor model with unreliable supply. *OR Spectrum*, 29(2): 207–233.

Rezaei, J. 2005. Economic order quantity model with backorder for imperfect quality items. *Proceedings of IEEE International Engineering Management Conference*. September 11–13, St. John's, Newfoundland, Canada, pp. 466–470.

Rezaei, J. and Davoodi, M. 2008. A deterministic, multi-item inventory model with supplier selection and imperfect quality. *Applied Mathematical Modeling*, 32(10): 2106–2116.

Rosenblatt, M.J. and Lee, H.L. 1986. Economic production cycles with imperfect production processes. *IIE Transactions*, 18(1): 48–55.

Ross, P.J.1996. *Taguchi techniques for quality engineering*. New York: McGraw Hill.

Salameh, M.K., Malak, M.A.U.A. and Jaber, M.Y. 1993. Mathematical modelling of the effect of human learning in the finite production inventory model. *Applied Mathematical Modeling*, 17(11): 613–615.

Salameh, M.K. and Jaber, M.Y. 2000. Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64(1): 59–64.

Sana, S.S., Goyal, S.K. and Chaudhuri, K. 2007a. An imperfect production process in a volume flexible inventory model. *International Journal of Production Economics*, 105(2): 548–559.

Sana, S.S., Goyal, S.K. and Chaudhuri, K. 2007b. On a volume flexible inventory model for items with an imperfect production system. *International Journal of Operational Research*, 2(1): 64–80.

Sana, S.S. 2010. An economic production lot size model in an imperfect production system. *European Journal of Operational Research*, 201(1): 158–170.

Sarker, B.R., Jamal, A.M.M. and Mondal, S. 2008. Optimal batch sizing in a multi-stage production system with rework consideration. *European Journal of Operational Research*, 184(3): 915–929.

Sarmah, S.P., Acharya, D. and Goyal, S.K. 2006. Buyer vendor coordination models in supply chain management. *European Journal of Operational Research*, 175(1): 1–15.

Savasaneril, S. and Erkip, N. 2010. An analysis of manufacturer benefits under vendor-managed systems. *IIE Transactions*, 42(7): 455–477.

Sheu, S.H., Lin, Y.B. and Liao, G.L. 2006. Optimal number of production corrections before maintenance of imperfect production processes. *International Journal of Advanced Manufacturing Technology*, 30(3–4): 319–327.

Sikström, S. and Jaber, M.Y. 2002. The power integration diffusion model for production breaks. *Journal of Experimental Psychology: Applied*, 8(2): 118–126.

Silver, E., Pyke, D. and Peterson, R., 1998. *Inventory management and production planning and scheduling*. New York: Wiley.

Silver, E.A. 1976. Establishing the order quantity when the amount received is uncertain. *Information Systems and Operational Research*, 14(1): 32–39.

Simpson, N.C. 2001. Questioning the rules of dynamic lot sizing rules. *Computers & Operations Research*, 28(9): 899–914.

Singer, M., Patricio, D. and Pedro, T. 2003. Quality strategies in supply chain alliances of disposable items. *Omega*, 31(6): 499–509.

Sontrop, J.W. and MacKenzie, K., 1995. *Introduction to technical statistics and quality control*. Canada: Addison Wesley.

Srinivas, C. and Rao, C.S.P. 2004. An improved consignment stock policy under stochastic lead times for effective inventory management in supply chains. *IEEE International Engineering Management Conference*, October 18–21. Singapore, pp. 1222–1225.

Srinivas, C. and Rao, C.S.P. 2010. Optimization of supply chains for single-vendor-multibuyer consignment stock policy with genetic algorithm. *International Journal of Advanced Manufacturing Technology*, 48(1–4): 407–420.

Srinivas, C. and Rao, C.S.P. 2007. Consignment stock policy with controllable lead time for effective inventory management in supply chains. *International Journal of Manufacturing Technology and Management*, 10(2–3): 161–176.

Stadtler, H. and Kilger, C., 2008. *Supply chain management and advanced planning* (Ed.). Berlin: Springer.

Steedman, I. 1970. Some improvement curve theory. *International Journal of Production Research*, 8(3): 189–206.

Steven, G.J. 1999. The learning curve: From aircraft to spacecraft? *Management Accounting*, 77(5): 64–65.

Swain, A.D. 1970. Development of a human error rate data bank. *US Navy Human Reliability Workshop*. Washington, D.C., pp. 112–148.

Sylla, C. and Drury, C.G. 1995. Signal detection for human error correction in quality control. *Computers in Industry*, 26(2): 147–159.

Szendrovits, A.Z. and Drizner, Z. 1975. Manufacturing cycle time determination for a multi-stage economic production quantity model. *Management Science*, 22(3): 298–308.

Tapiero, C.S. 1987. Production learning and quality control. *IIE Transactions*, 19(4): 362–370.

Tomlin, B. 2009. Impact of supply learning when suppliers are unreliable. *Manufacturing and Service Operations Management*, 11(2): 192–209.

Towill, D.R. 1985. The use of learning curve models for prediction of batch production performance. *International Journal of Operations & Production Management*, 5(2): 13–24.

Tsou, J.C. and Chen, J.M. 2005a. Case study: Quality improvement model in a car seat assembly line. *Production Planning and Control*, 16(7): 681–690.

Tsou, J.C. and Chen, J.M. 2005b. Dynamic model for a defective production system with poka-yoke. *Journal of the Operational Research Society*, 56(7): 799–803.

Tsou, J.C. and Chen, J.M. 2005c. The relation between cost and yield rate in a JIT production system. *International Journal of Information and Management Sciences*, 16(1): 83–93.

Tsou, J.C. 2007. Economic order quantity model and Taguchi's cost of poor quality. *Applied Mathematical Modeling*, 31(2): 283–291.

Tsou, J.C. and Chen, W.J. 2008. The impact of preventive activities on the economics of production systems: Modeling and application. *Applied Mathematical Modeling*, 32(6): 1056–1065.

Tsou, J.C., Hejazi, S.R. and Barzoki, M.R. 2009. Economic production quantity model for items with continuous quality characteristic, rework and reject. *International Journal of Information, Business and Management*, 1(1): 1–14.

Urban, G.L. and Hauser, J.R., 1993. *Design and marketing of new products*, 2nd ed. NJ: Prentice Hall.

Urban, T.L. 1998. Analysis of production systems when run length influences product quality. *International Journal of Production Research*, 36(11): 3085–3094.

Valentini, G. and Zavanella, L. 2003. The consignment stock of inventories: Industrial case and performance analysis. *International Journal of Production Economics*, 81–82: 215–224.

Viswanathan, S. and Piplani, R. 2001. Coordinating supply chain inventories through common replenishment epochs. *European Journal of Operational Research*, 129(2): 277–286.

Wang, C.H. 2005. Integrated production and product inspection policy for a deteriorating production system. *International Journal of Production Economics*, 95(1): 123–134.

Wang, Z. 2009. Stability of solution to a class of investment system. *Applied Mathematics and Computation*, 207(2): 340–345.

Wee, H.M., Yu, J.C.P. and Wang, K.J. 2006. An integrated production-inventory model for deteriorating items with imperfect quality and shortage backordering considerations. *Lecture Notes in Computer Science*, 3982: 885–897.

Wee, H.M., Yu, J. and Chen, M.C. 2007. Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, 35(1): 7–11.

Wild, T., 2004. *Improving inventory records accuracy*. USA: Amacom Press.

Wisner, J.D., Tan, K.C. and Leong, G.K., 2009. *Principles of supply chain management, A balanced approach*. USA: Cenage Learning.

Woolsey, G. 1990. A requiem for the EOQ: An editorial. *Hospital Material Management Quarterly*, 12(1): 82–90.

Wortham, A.W. and Mayyasi, A.M. 1972. Learning considerations with economic order quantity. *AIIE Transactions*, 4(1): 69–71.

Wright, T. 1936. Factors affecting the cost of airplanes. *Journal of Aeronautical Science*, 3(2): 122–128.

Wu, K.S., Ouyang, L.Y. and Ho, C.H. 2007. Integrated vendor-buyer inventory system with sublot sampling inspection policy and controllable lead time. *International Journal of Systems Science*, 38(4): 339–350.

Yano, C.A. and Lee, H.L. 1995. Lot sizing with random yields: A review. *Operations Research*, 43(2): 311–334.

Yeh, R.H. and Chen, T.H. 2003. Optimal ordering policy with backordering for products of imperfect quality. *International Journal of Operations & Quantitative Management*, 9(4): 227–239.

Yelle, L.E. 1979. The learning curve: Historical review and comprehensive survey. *Decision Sciences*, 10(2): 302–328.

Yoo, S.H., Kim, D. and Park, M.S. 2009. Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. *International Journal of Production Economics*, 121(1): 255–265.

Yu, J.C.R., Wee, H.M. and Chen, J.M. 2005. Optimal ordering policy for a deteriorating item with imperfect quality and partial backordering. *Journal of the Chinese Institute of Industrial Engineers*, 22(6): 509–520.

Zadeh, L.A. 1965. Fuzzy sets. *Information and Control*, 8(3): 338–353.

Zangwill, W.I. and Kantor, P.B. 1998. Toward a theory of continuous improvement and the learning curve. *Management Science*, 44(7): 910–920.

Zangwill, W.I. and Kantor, P.B. 2000. The learning curve: A new perspective. *International Transactions in Operational Research*, 7(6): 595–607.

Zanoni, S. and Grubbstrom, R.W. 2004. A note on an industrial strategy for stock management in supply chains: Modeling and performance evaluation. *International Journal of Production Research*, 42(20): 4421–4426.

Zavanella, L. and Zanoni, S. 2009. A one-vendor multi-buyer integrated production-inventory model: The 'consignment stock' case. *International Journal of Production Economics*, 118(1): 225–232.

Zunzanyika, X.K. and Drury, C.G. 1975. Effects of information on industrial inspection performance. In: C. G. Drury and J. G. Fox. (Ed.), *Human Reliability in Quality Control*. Taylor & Francis Ltd., London.