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Inventory management in reverse logistics with imperfect production, learning, lost sales, subassemblies, and price/quality considerations

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INVENTORY MANAGEMENT IN REVERSE LOGISTICS WITH IMPERFECT PRODUCTION, LEARNING, LOST SALES, SUBASSEMBLIES, AND PRICE/QUALITY CONSIDERATIONS

by

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Inventory Management in Reverse Logistics with Imperfect Production, Learning, Lost Sales, Subassemblies, and Price/Quality Considerations

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Abstract:

Reverse Logistics is the flow and management of products, packaging, components, and information from the point of consumption (i.e., the market) to the point of origin (i.e., manufacturers and suppliers). It is a collection of practices similar to those of supply chain management, but in the opposite direction, from downstream to upstream. Reverse logistics is a valuable solution to the hazards jeopardizing the environment, and it involves activities such as reuse, repair, remanufacture, refurbish, reclaim and recycle.

Reverse logistics became an established line of research, covering several areas, including inventory control; though, several research gaps still exist, such as: ignoring switching costs between production and remanufacturing processes and learning effects, the assumption that production and remanufacturing processes are of perfect quality, remanufactured products are assumed to be as-good-as new, the assumption that returned products are treated as whole products while ignoring disassembly, collection rate of used items is independent of price and quality, and the assumption that pure remanufacturing and production policies are optimal. These research gaps are addressed in mathematical models to bring reverse logistics optimization closer to reality. Deterministic and stochastic components are considered here with numerical examples and results discussed. The key conclusions are as follows:

The inclusion of the first time interval where no remanufacturing/repair exists, results in preventing the overestimation of inventory holding costs in the repairable stock. Assuming

production and remanufacturing processes to be perfect, or ignoring learning effects in these processes, might not capture the benefits that product recovery programs are supposed to bring. Although works in the literature assumed pure remanufacturing is mathematically attainable but not feasible, this study shows that the pure remanufacturing case is not valid mathematically, which proves it to be infeasible. It is favourable to compensate customers to settle for remanufactured products instead of new ones. Considering disassembly of returns in the modelling of reverse logistics is proven beneficial. Finally, mixed production and remanufacturing policies are optimal rather than pure ones; and the inclusion of price and quality to determine return and collection rates is crucial.

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Working as a PhD student at Ryerson University was a challenging and enthusiastic journey. In the past three and a half years, which were my first in Canada, I met many brilliant and inspiring people who supported, encouraged and influenced me to reach my goals. I would like to thank those who helped me and made this part of my life a pleasant and a rich experience for me.

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Ahmed M. A. El Saadany

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to the most wonderful people in my life…

…my Mom and Dad

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Nomenclature

Parameters

d	Demand rate, (units per unit of time)
D_p	Primary market demand for newly produced items, (units per unit of time)
D_r	Secondary market demand for remanufactured/repaired items, (units per unit of time)
μ	Mean lead time demand, (units)
σ	Standard deviation of lead time demand, (units)
P_r	Production process rate, where $P_r > d$, (units per unit of time)
π	Demand rate to production rate ratio, i.e., $P_r = d/\pi$ $(0 < \pi < 1)$
R_r	Remanufacturing process rate, where $R_r > d$, (units per unit of time)
δ	Demand rate to remanufacturing rate ratio, i.e., $R_r = d/\delta$ $(0 < \delta < 1)$
R_s	Screening rate, where $R_s > d$, (units per unit of time)
y_1	Time to produce the first unit with no previous experience, (time per unit)
γ	Initial production rate with no learning effects, where $\gamma = 1/y_1 > d$, (units per unit of time)
r_1	Time to remanufacture (recover/repair) the first unit with no previous experience, (time per unit)
v	Initial remanufacturing rate with no learning effects, where $v = 1/\ r_1 > d$, (units per unit of time)
Δ_p	Percentage increase in the production learning exponent, *b*, per one dollar invested in the production segment
Δ_r	Percentage increase in the remanufacturing learning exponent, *a*, per one dollar invested in the remanufacturing segment
u	Number of subassemblies of a returned item
k_i	Number of components for subassembly *i* of the end product, where $i = 1, 2, 3 \ldots u$
ρ	Percentage of defective items in a batch of produced items
λ	Percentage of defective items in a batch of remanufactured items
h_p	Holding cost per unit per unit of time for newly produced units in the serviceable stock, (dollars per unit per unit time)

h_r	Holding cost per unit per unit of time for repaired units in the serviceable stock, where $h_p \neq h_r$, (dollars per unit per unit time)
h_1	Holding cost per unit per unit of time for all serviceable stock (i.e., $h_1 = h_p + h_r$),
h_u	Holding cost per unit per unit of time for repairable stock, where $h_p \geq h_r$ and $h_r \geq h_u$, (dollars per unit per unit time)
h_i	Holding cost per unit per unit of time for a returned subassembly i in the repairable stock, (dollars per unit per unit time)
S_p	Production (manufacturing) setup cost per cycle, (dollars)
S_r	Repair setup cost per cycle, (dollars)
s_p	Cost of a minor setup during production, where $s_p < S_p$, (dollars)
s_r	Cost of a minor setup during remanufacturing, where $s_r < S_r$, (dollars)
S_i	Ordering cost for a lot of subassembly i, (dollars)
α	Waste disposal percentage, where $0 \leq \alpha \leq 1$
q_r	Probability of the remanufacturing process going out-of-control
q_p	Probability of the production process going out-of-control
C_p	Unit production (manufacturing) cost, (\$/unit)
C_r	Unit remanufacturing/repair cost, (\$/unit)
C_w	Unit disposal cost, (\$/unit)
C_s	Unit screening cost, (\$/unit)
Cr_p	Cost of reworking one defective unit as a result of production process, (\$/unit)
Cr_r	Cost of reworking one defective unit as a result of remanufacturing process, (\$/unit)
C_{lp}	Cost per unit of a lost demand for a produced item, (\$/unit)
C_{lr}	Cost per unit of a lost demand for a remanufactured item, (\$/unit)
c_p	Production labour cost per unit of time, (\$/unit of time)
c_r	Remanufacturing labour cost per unit of time, where $c_p \neq c_r$, (\$/unit of time)
C_n	Cost of raw materials required to produce a newly produced unit, and $P \times C_n$ is the purchasing price for a single returned item, (\$/unit)
C_B	Backorder cost, (\$/unit)
T	Length of a manufacturing and repairing time interval (units of time), except

	the first time interval, where $T > 0$, (units of time)
T_1	Length of the first manufacturing time interval (units of time), where $T_1 \leq T$ and $T_1 > 0$, (units of time)
T_p	Length of a production interval, (units of time)
T_r	Length of a remanufacturing interval, (units of time)
τ_p	Time required for a production minor setup, during which production is stopped, (units of time)
τ_r	Time required for a remanufacturing minor setup, during which remanufacturing is stopped, (units of time)
$\beta_{\max}$	Maximum percentage of collected used units that could be recovered as a percentage of demand rate d, $(0 \leq \beta_{\max} \leq 1)$
R_x	Return percentage from the market to the repairable inventory prior to sorting
R_i	Return percentage of subassembly i after sorting, where $0 \leq R_i \leq 1$
β_x	Remanufacturing to production ratio

Decision variables

β	Return percentage, which represents the collection of repairable used items as a percentage of demand rate d, $(0 \leq \beta \leq 1)$
n	Number of newly produced (manufactured) cycles in an interval of length T
m	Number of repaired/remanufactured cycles in an interval of length T
Y_i	Binary decision variable indicating the inventory control policy for subassembly i
λ_n	Number of minor setups in the production cycle following an interruption to restore process quality
λ_m	Number of minor setups in the remanufacturing cycle following an interruption to restore process quality
P	Purchasing price for a single returned item as a percentage of the cost of raw materials required to produce a new item of the product $(0 < P < 1)$
q	Acceptance quality level of returned (collected used) items, representing the percentage of useful parts in a remanufacturable item, $(0 < q < 1)$
r_p	Reorder point for production cycle to trigger a remanufacturing order, (units)

r_r Reorder point for remanufacturing cycle to trigger a production order, (units)

CHAPTER 1: INTRODUCTION

1.1. Supply Chain Management

Firms have been developing business relationships to overcome challenges in distribution or marketing channels since the early days of the industrial revolution. Earliest distribution channels lacked efficient information on demand and customers' consumption behaviours, which resulted in these channels having conflicting goals and lengthy and unpredictable time to deliver products to customers. These channels were also characterised by being costly due to overstocking of inventory, which was the common practice to counter any kind of disorganisation (e.g., variation in a shipment delivery time, quantity and quality, machine breakdown…etc).

The introduction of the Internet as a business technology in the 1990s, with its inexpensive information transformation capabilities, revolutionized business transactions through Business-to-Business (B2B) connectivity. Coordination between members of a distribution channel had risen to new competitive levels as a result of this technology, where adjacent members in a supply chain became able to match supply and demand that resulted in reducing inventory levels and costs. The B2B connectivity gave birth to Supply Chain in the business terminology.

A Supply Chain is a network over which raw material complements semi-finished products and finished products flow forward to the customer while information and cash flow backward. The complexities in structure and the management of these networks are documented in the literature. Supply Chain Management is synonymous with network sourcing, supply pipeline management, value chain management, and value stream management (Croom et al., 2000).

Perhaps one of the reasons for a lack of a universal definition of Supply Chain Management is the multidisciplinary nature and the continuous evolution of the concept. The Council of Supply Chain Management Professionals provided the following definition:

> *Supply Chain Management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics*

management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, Supply Chain Management integrates supply and demand management within and across companies.

Several other definitions are found in Lee & Billington (1992), Pohlen & Theodore Farris II (1992), Berry et al. (1994), Kroon & Vrijens (1995), Hicks (1997), Tan et al. (1998), Dowlatshahi (2000), Ritchie et al. (2000), Fleischmann (2001a) and Lambert (2008). Despite some variations in these definitions, the main objective has always been to maximize the supply chain profitability where there is only one source of revenue, the customer purchase. Profits, or savings, generated from the supply chain operations are usually shared amongst the players in a chain.

Supply chain management is managing all the processes, functions, activities, and relationships along which products, services, information and monetary transactions move in and among enterprises. Managing the supply chain is a very challenging task (Lambert 2008); in particular, the flow of inventories from sources of supply to end-users (Ellram, 1991). As part of supply chain management, logistics includes all activities to move products and information to, from, and among members of a supply chain. The popular implementation of Enterprise Resource Planning (ERP) presented homogenous and transactional databases that facilitated the integration of supply chain activities (Shapiro, 2001).

Supply chain management expanded the borders of competition and elevated customers' expectations to higher levels. Today, products are manufactured to exact specifications and are rapidly delivered to customers at locations throughout the globe in hours, with their orders delivered to customers in the desired assortment, quantity, quality, location, time, condition, and price. What once was the exception became the expectation. Modern markets characterized by product customization and fierce competition exerted pressures on companies to deliver their best products and services while reducing their costs. This elevated competition to new heights, i.e., from a company versus company to a supply chain versus supply chain (Cottrill, 1997). Bowersox et al. (2002) discussed the main forces driving supply chain management. These forces are information technology, integrative management, responsiveness, financial sophistication, and globalization.

The supply chain provides the framework for businesses and their suppliers that join to bring goods, services, and information efficiently and effectively to customers (Bowersox et al., 2002). The value chain of any company starts by new product development, then marketing & sales, then operations, then distribution and services. Thus, a company must have a new product development strategy, a marketing strategy, together with a supply chain strategy, which covers procurement, transportation, manufacturing, inventory, distribution, and follow-up services. However, for the customer, a competitive strategy defines the set of customer needs to be satisfied. To optimize the supply chain performance, a strategic fit between the competitive strategy and the supply chain strategy has to be attained. This fit is achieved through supply chain coordination (Chopra & Meindl, 2001).

The supply chain is coordinated when all the supply chain stages optimize their decisions jointly to maximize the total supply chain profits. When there is no coordination, each stage in a supply chain assumes to maximize its local profits independently from other stages in the chain. This approach impedes supply chain profitability. Optimizing a supply chain requires making decisions at the strategic, tactical and operational levels (Chopra & Meindl, 2001). The strategic level includes decisions related to location, capacities of production and warehousing facilities, network design, products to be manufactured or stored at various locations, modes of transportation, and the type of information sharing system to be utilized. The tactical level includes decisions related to forecasting of demand in different markets, subcontracting of manufacturing, replenishment and inventory policies, backlogging policies in case of stock-out, aggregate planning, and timing as well as size of marketing policies. The operational, the third and the last level includes decisions related to allocating individual orders to inventory or production, assigning dates for orders filling, setting schedules of trucks, deciding orders sizes, and timing replenishment orders.

Although the objective of supply chain management is to integrate the firms in this chain into a single entity, research on coordinating supply chain management at the inter-organizational level is limited (Sachan & Datta, 2005). The complexity of relationships within a supply chain and the number of factors that need to be understood and managed in order to improve overall effectiveness is a significant challenge (Sohal et al., 2002). Integrating supply chains results in enhancing customer service levels where higher levels of coordination are associated with mutually fulfilled expectations of different supply chain members (Ellinger et al., 1997).

Coupled with the surplus inventory and excessive lead-time, it has been long known that the dynamics of supply chain systems create particular problems for trading partners as stand-alone entities (Sohal et al., 2002). Forrester (1961) identified the need for organizations to understand the potential for distortion as demand patterns are transferred backward along the chain. This came to be known as the Bullwhip Effect, a term first coined by the logistics executives of Procter & Gamble (Lee et al., 1997, 2004), which is one of the main challenges facing supply chain management. The Bull-Whip Effect represents the behaviour where the steady demand at the consumer's side at the end of the supply chain translates into amplified variability in orders as this demand moves upstream along the chain. The Bull-Whip Effect captured the attention of many researchers, e.g., Sterman (1989), de Kok & Graves (2003), Towill (1997), Paik & Bagchi (2007), and Huang et al. (2009).

The growing popularity of supply chain management in the 1990's coupled with environmental concerns led to the evolution of Closed Loop Supply Chains and Reverse Logistics. This is explained in the following section.

1.2. From Recycling to Reverse Logistics

Recycling is a natural phenomenon that has been a human practice for centuries especially during periods when resources were scarce. Recycling is collecting used products, disassembling them (when necessary), sorting them into categories of like components and/or materials for processing (Beamon, 1999). The main driver for recycling is the conservation of natural resources and the economic advantage of obtaining recycled stock instead of acquiring new virgin material. Resource shortages caused by wars encouraged recycling, where metallic parts were collected to be melted and reused for their high value (Steven, 2004). For example, in World War II, European governments carried out campaigns to entice their citizens to donate metals as a matter of significant patriotic importance. Like recycling, remanufacturing was performed in the past for economic reasons.

Calls to reduce waste to preserve natural resources and to protect the environment were not profound before the 1950's. The outstanding book of Vance Packard in 1960 "The Waste Makers" (Packard, 1960) is one of the pioneering calls to warn of the problem of waste and its effects on the environment. Packard (1960) showed how marketing men across the USA realised that the country's capacity to produce outstripped the country's capacity to consume. Accordingly, they innovated various ways and excuses to market and change the behaviour of the American citizen to create a "super customer". As a result, the average citizen in 1960 was consuming twice what an average citizen was consuming before World War II. Packard (1960) emphasised how companies produced unwanted, prettier, fashionable products to boom sales, and the author highlighted Eisenhower's National Advice to combat the 1958's recession by saying, "Buy… buy anything". This over-consumption behaviour resulted in tremendous amounts of waste, and yet, companies tried and succeeded in innovating new ways to encourage consumers of dumping their products, so that customers will buy more (Packard, 1960).

Companies reduced quality and designed a "planned obsolescence", where a durable product is the one that can survive the last instalment payment. As a result, Americans consumed resources in 40 years (1920~1960) more than the peoples of the world have used in 4000 years of recorded history. The fall of communism in 1990 and the emergence and escalation of capitalism as the dominant economic system resulted in an enormous increase in consumption behaviour and the subsequent rapid depletion of natural resources (e.g., Dijkgraaf & Vollebergh, 2004; Inderfurth & Langella, 2008). Shorter product life cycles and changes in customers' consumption behaviours resulted in faster product flows and subsequently faster generation of waste and depletion of natural resources (e.g., Cairncross, 1992; Prendergast, 1995; van Hoek, 1999; Blackburn et al., 2004). Beamon (1999) reported that the amount of environmental waste generated to be huge and equivalent to 10 pounds per person per day. Other calls to protect our planet, to reduce consumption and reduce the hazardous emissions are documented in recent history (e.g., the incident of the Three Mile Island in 1979 and the disaster of Chernobyl in 1986) (Gupta & Lambert 2008). Today's customers are more educated and demanding for less tolerance to products that damage the environment (Krikke et al., 2004).

One of the solutions to protect the environment alongside with repair, refurbishing and remanufacturing is the adoption of recycling (e.g., King et al., 2006). Recycling became a popular resolution in the 1950's where environmental concerns encouraged manufacturers to do more recycling, reuse and reconditioning. However, studies that considered the management of recycling activities started in the 1970s, in particular, with the beginning of the world energy crisis.

Zikmund & Stanton (1971) are believed to be the first to discuss the management of recycling introducing the term "reverse-distribution". The authors described recycling as finding new ways of using previously discarded materials to provide a sound solution to cleaning up the cluttered environment. Although recycling requires innovative scientific and technological solutions, it is as well a marketing and distribution challenge because major recycling costs are attributed to collection, sorting, and transportation processes. The authors concluded that customer awareness and government role are major factors in making recycling programs successful.

Fisk (1973) warned business leaders of the uniqueness of the environmental crisis and its implications on their marketing decisions. He further cautioned business leaders of the risks that they might face if they ignore responsible consumption for their companies. El-Ansary (1974) discussed the Marketing Mix concept, differentiated between controllable (price/promotion) and uncontrollable (ecological/environment) variables, and highlighted the importance of a marketing shift from reducing costs to enhancing social and production factors. To achieve a successful shift, the author suggested that it is necessary to manage the reverse distribution channels as a marketing function.

Reverse distribution channels for recycling within the United States were discussed by Guiltinan & Nwokoye (1975) with the key reverse distribution functions, primary factors and functional performance measures of these distribution channels identified. The authors advocated for better physical distribution and marketing programmes, stricter legislations and more joint venture resource-recovery centres to improve reverse distribution channels. For example, the Belgian Ecotax law improved industry's awareness and concern for the environment (de Clercq, 1996). The lagging on the part of industries to adopt recycling in their operations is due to legislative and economical reasons. For example, Syring (1976) recorded that product recovery rates were decreasing in the previous two decades because of the decrease in prices of virgin materials, the inappropriate tax legislations that were applied on secondary materials, and unintentionally, the favouring of virgin materials. In order to make recycling favourable to industries, Syring (1976) recommended a redesign of product return networks of secondary materials and revising governmental procurement policies.

"Reverse Marketing Channels" was the term used by Fuller (1978) to represent the mechanism to maintain adequate flow of secondary materials from consumers to manufacturers. The author identified recycling as a special case of reverse marketing channels with several potentials. Along the same line of research, Ginter & Starling (1978) showed that as the amount of solid household waste in the USA had quadrupled in 50 years. New governmental policies were introduced to entice manufacturers to implement recycling, while meeting marketing strategies with new challenges. The authors advised implementing reverse channels of distribution to consider consumers not only as potential buyers, but also as suppliers.

Barnes (1982) was the first to coin the term "Reverse Logistics". The author considered recycling as a problem in "Retrologistics" (Backwards Logistics or Reverse Logistics). The author emphasised the fact that although technology offered several solutions for implementing recycling, marketing failed to provide sound answers to implement recycling successfully and that marketing plays a critical role in the future of recycling. Barnes (1982) was one of the first researchers to show that, besides transportation, inventory of secondary materials is another challenge facing successful implementation of reverse logistics and that one of the benefits of implementing reverse logistics is enhancing a firm's image.

As products move faster along supply chains, waste is generated faster. Environmental and sustainability concerns hindered the implementation of product and material recovery programs that later took more complex forms of networks in the reverse flow. In the 1990s, the network, which captivated the interest of business and researchers, became to be known by the term "Reverse Logistics". In the following sections, a broader view of reverse logistics is presented.

1.3. Understanding Reverse Logistics

For a variety of economic, environmental, or legislative reasons, product disposal may no longer be the consumer's responsibility as products come to be recycled or remanufactured by their manufacturers. Figure 1.1 illustrates the forward (Supply Chain) and backward (Reverse Logistics) flows of products.

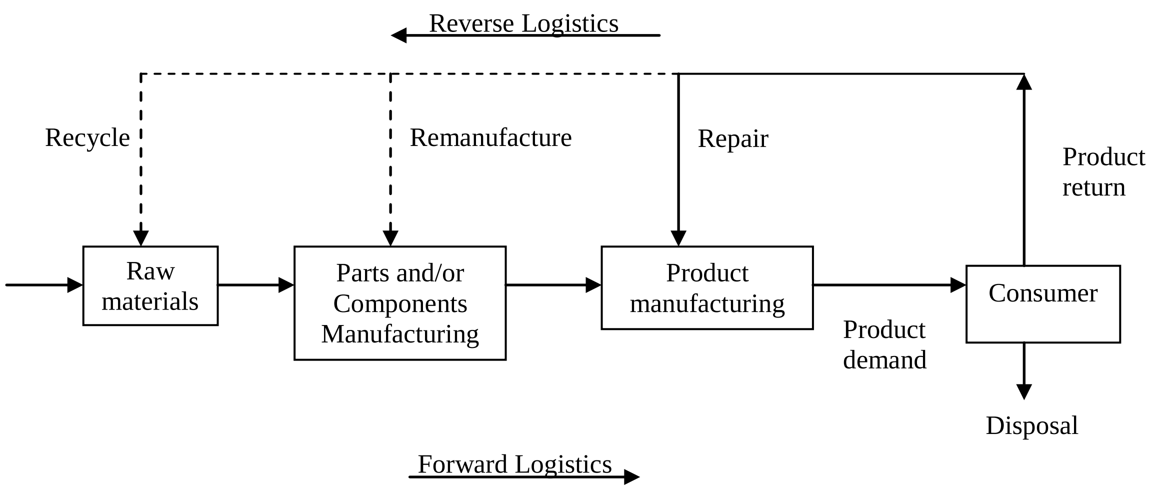


Figure 1.1. Forward and reverse logistics system

For the conventional forward logistics systems, the flow starts upstream (suppliers) as raw materials, later as manufactured parts and components to be assembled and continues downstream to reach customers as final products to be disposed once they reach their economic or useful lives.

In reverse logistics, the disposed products are pushed upstream to be repaired, remanufactured, refurbished, disassembled into components to be reused, or as raw material to be recycled for later use. Besides, by-products (hazardous and non-hazardous waste, packaging, etc) and information are also generated (Kroon & Vrijens, 1995).

Reverse Logistics provides companies with cost and strategic advantages. Companies that remanufacture may save up to 60 % of the estimated cost of a completely new product (Dowlatshahi, 2000). Reverse Logistics promotes alternative uses of resources that can be cost effective and ecologically correct by extending a product's life cycle. Having an ecological image benefits the organization as customers may be willing to pay more for products that do not harm the environment. The importance of reverse logistics practices led many manufacturers to design–for–disassembly and remanufacturing as part of a sustainable development initiative (Grenchus et al., 2001; Min et al., 2006).

Similar to supply chains, there are three decision-making phases in reverse logistics systems, which are the Design, Planning and Operational phases. The Design phase (also called strategic planning, or network design, or location allocation), is a long-term, decision-making process (more than a year) where decisions are very expensive to alter. This phase involves decision related to determining the locations of manufacturing facilities, distribution and collection centers, the products to be manufactured/remanufactured, the transportation routes and modes, collection (returns) policies, marketing strategies, selection of second hand markets, and supply chain performance measures (Pochampally et al., 2009). In the planning phase (also called tactical planning), medium-term decisions (six months to a year) are made regarding policies affecting manufacturing, remanufacturing, inventory, uncertainty in demand, exchange rates, etc. In the Operational, the third and final phase, decisions relating to short-term periods (less than six months) are made, which involve daily operations such as filling customer orders, tracking inventory levels, short-term scheduling, job rotation, etc.

Like supply chains, coordination in reverse logistics results in reducing costs and enhancing customer service (e.g., Ellinger et al., 1997; Ben-Daya et al., 2008). Some of the definitions of the term "reverse logistics" are presented in the following section.

1.4. Reverse Logistics Definitions

Reverse Logistics, or Reverse Supply Chains, is more than a backward flow (Shapiro, 2001). It involves activities such as the management of transportation, warehousing and inventory activities. Some of the available definitions in the literature of the term "Reverse Logistics" are:

> "*Reverse (distribution) Logistics is the movement of goods from a consumer (back) towards a producer in a channel of distribution.*" (Murphy, 1986; Pohlen & Theodore Farris II, 1992).
>
> "*Reverse Logistics refers to the logistics management skills and activities involved in reducing, managing and disposing of hazardous and non-*

hazardous waste from packaging and products. It includes reverse distribution, which causes goods and information to flow in the opposite direction from normal logistic activities." (Kroon & Vrijens, 1995).

"*The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal.*" (Rogers & Tibben-Lembke, 1998).

"*A process in which a manufacturer accepts previously shipped products or parts from the point of consumption for possible recycling, remanufacturing or disposal.*" (Dowlatshahi, 2000).

"*Reverse Logistics is the process of planning, implementing and controlling the efficient, effective inbound flow and storage of secondary goods and related information opposite to the traditional supply chain direction for the purpose of recovering value or proper disposal*." (Fleischmann, 2001a).

"*Reverse logistics comprises all activities involved in managing, processing, reducing and disposing of hazardous or non-hazardous waste from production, packaging and use of products, including the processes of reverse distribution.*" (Steven, 2004)

Murphy (1986) and Pohlen & Theodore Farris (1992) emphasized the direction of materials from the consumer back to the producers. Kroon & Vrijens (1995) extended the definition of reverse logistics to include the skills and activities required, types of materials (hazardous and/or non-hazardous), and the flow of information. The definition provided by Rogers & Tibben-Lembke (1998) is the most common one and it described reverse logistics as a combination of three actions: plan, implement, and control, where the definition emphasised the importance of recapturing product value and properly disposing waste. The definition introduced by Dowlatshahi (2000) expressed the case of Original Equipment Manufacturers and excluded third party logistics companies and introduced the function of remanufacturing as a possible reverse logistics activity. Fleischmann (2001) provided a similar definition with one difference; he emphasized the role of secondary goods rather than finished goods. Steven (2004) added a green perspective to the definition of reverse logistics by focusing on reducing the disposal of hazardous or non-hazardous waste (e.g., reducing use and consumption).

One should not confuse the term “Reverse Logistics” with terms like “Waste Management” or “Green Supply Chain”. Waste management deals with waste collecting and processing. Waste implies that it has legal consequences (de Brito & Dekker, 2003, 2004). However, reverse logistics deals with all kinds of returns including obsolete unused products, product recalls, and end-of lease items. Reverse logistics focuses on recapturing value besides reducing waste. Green logistics refers to practices within the supply chain that aim to reduce energy, materials and waste, through measuring environmental impact of logistics and ISO 14000 certification. However, these green activities are not necessarily related to reverse logistics. For example, choosing an environmentally friendly transportation means for the forward supply chain (e.g., an electric-gasoline hybrid vehicle) is a green logistics alternative, but it has no reverse logistics perspective. Reverse logistics is sometimes seen as a part of a bigger picture: “Green supply chain management” (Srivastava, 2007). This term, however, is rarely used.

Reverse logistics is sometimes defined as an extension from Reverse Distribution. Carter & Ellram (1998) defined Reverse Distribution as the return movement of a good or material resulting from reuse, recycling, or disposal. This upstream movement can be associated with environmental as well as quality and wear-dating issues, and, it is often performed auxiliary channel members. The authors later refined their definition of reverse logistics by including resource reduction that should result in more efficient forward and reverse distribution processes.

Reverse logistics is sometimes considered as a subset of Closed Loop Supply Chains (CLSC). CLSC is the integration of both the Forward Supply Chain and Backward Logistics, where it is usually organised and managed by an Original Equipment Manufacturer (OEM) that supports its own product line (Blumberg, 2005). Some researchers refer to the term reverse logistics to mean CLSC.

1.5. Benefits and Objectives of Reverse Logistics

Implementing Reverse Logistics has numerous benefits. These benefits (or objectives) are economical, environmental, marketing image, market share and exposure, asset protection, and reducing bullwhip effect.

Economical motives:

The main motive behind reverse logistics is economical (minimizing cost and increasing profits). Reverse logistics provides alternative economical sources of materials that are substitutes for virgin ones, thus saving natural resources, effort, and energy. Reverse logistics is a chance to recapture value from returned products through recovery/reuse/repair options. The dollar size of reverse logistics is enormous. Logistics accounts for 9.9 percent of the whole economy in the USA, equivalent to $1.3 trillion in 2006 (Wilson, 2007). Although, reverse logistics is not developed enough to be estimated, Rogers and Tibben-Lembke (1998) provided a rough figure of $35 billion in 1997, reaching $56 billion in 2007, which is equivalent to 4 percent of total logistics costs of the USA alone (Beltran, 2002; Bowersox et al., 2002; Schatteman, 2003; Lambert, 2008). Remanufacturing may have the advantage of shortening the lead-time, where the demand of spare parts is supplied by remanufacturing returned parts instead of initiating a resupply with a substantial lead-time (Minner, 2003). For example, at IBM, lease & take back programs substantially reduced the uncertainty in the reverse flow (Fleischmann, 2001a).

Environmental motives:

The presence of hazardous toxic materials such as hexavalent chromium, polybrominated diphenyl ether, mercury and lead in electronic discarded components, represents a serious threat to the environment (Pochampally et al., 2009). In the USA, 29 States have ten years or more of landfill capacity left, fifteen States have between five and ten years, and six States have less than five years (Michael Knemeyer et al., 2002). Reverse logistics reduces the amount of waste disposed into the environment (materials are reused instead of consuming new resources), thus, reducing the reliance on landfills and energy. Saving energy is another valuable environmental motive. For example, an aluminum recycling process requires only 5% as much energy as the processing of the same amount of aluminum stock from virgin ore (Fuller, 1978). Costs of directing unwanted materials to landfills continue to increase, which encouraged organizations and firms to explore other economically viable alternatives for disposal (Johnson, 1998). Besides, many products can no longer be disposed in landfill sites because of environmental regulations (Rogers & Tibben-Lembke, 1998). By implementing reverse logistics, companies save money, reduce energy usage, emit less pollutants into air and water, save natural resources, reduce disposal, and avoid landfill options. Some of reverse logistics models consider financial and environmental multi-objective functions to enhance the environmental performance of these models (Bras & McIntosh, 1999).

Marketing image motives:

Many companies today want to appeal to their customers as being "environmentally responsible", a part of their marketing image. This has become more pressing as customers became selective of green and/or environmentally friendly products. Companies in the last two decades have been spending efforts in building green profiles, especially in markets dominated by environmentally conscious customers. In addition, companies are liable to their suppliers' environmental performance, since customers and stakeholders do not distinguish between companies and their suppliers (Sarkis, 2006). For example, in 2005, Dell increased its recovery rate of used computers collected from their customers by 72 percent over 2004 through recycling events and seminars that involve suppliers and customers (Kulwiec, 2006). Public environmental concern, coupled with sustainable development, has created opportunities for organizations to differentiate their products from their competitors' by being "greener" (Johnson, 1998).

Market share & exposure motives:

Besides reducing cost, many organizations take back their competitors' products in addition to theirs as a strategy to increase their market share. Some companies offer to collect all brands of a certain product in exchange for a price discount for the company's own brand. For example, Dell accepts and picks up all computer brands and accessories (e.g., key board, mouse, monitor, or printer) from the consumer's home for a fee, and offers this service free of charge with the purchase of a new Dell computer (Kulwiec, 2006).

Asset protection motives:

High-tech companies entice their customers to return their product fearing that sensitive technological knowledge would leak to competitors, and to avoid competing with secondary markets selling used products of these companies (Fleischmann, 2001a). IBM, for example, applies reverse logistics by recovering valuable parts from retired products so as to avoid brokers and third party recyclers doing that themselves (de Brito & Dekker, 2003).

Reducing bullwhip effect motives:

In the traditional forward supply chain, the bullwhip effect is the amplification of order variance in a supply chain from a downstream stage to an upstream stage. Zhou et al. (2004) and Zhou & Disney (2006) are perhaps the only studies that discussed the bullwhip effect in a reverse logistics context. The authors showed that the returned products can reduce the bullwhip effect, experienced by absorbing demand fluctuations in the forward chain, to the extent that reverse logistics is more cost efficient than a traditional one, even if variable costs of recovery are higher than that of producing a new product.

1.6. Reverse Logistics Actors

Guiltinan & Nwokoye (1975) divided the actors in reverse logistics into four types: "traditional middle man" (e.g., retailers collecting reusable bottles), "secondary materials dealers" (e.g., scrap metals collectors), "resource recovery centers" (e.g., municipal waste centres) and "manufacturer controlled recycling centers" (e.g., a company remanufacturing its own products). A similar classification was also presented by Fuller (1978). Fleischmann et al. (1997) divided actors into two main categories: forward supply chain members (i.e., manufacturers, suppliers, wholesalers and retailers), and reverse (or backward) logistics actors (i.e., recycling specialists, municipal waste collectors). de Brito & Dekker (2003) presented a reverse logistics framework where the authors divided the actors into three main groups: forward supply chain actors, (e.g., supplier, manufacturer, distributer and retailer), specialized reverse chain players (e.g., jobbers, recycling specialists, etc.) and, opportunistic players (e.g., charity organization).

The structure of the reverse logistics network defines the roles of actors in the distribution network, which is either an open or a closed loop. An open loop network is a network where the forward and the reverse flows are managed separately involving more than one actor. It is characterized by the existence of a secondary market for products after the end of their lifetimes. Apple, Inc., is an example of an open loop network where the company is not involved in the recycling process of its products. However, Apple considered designing and manufacturing their products to support further disassembly and recycling (Sweatman et al., 2000). In addition, Apple encourages its customers to make Apple's products available for any recovery/remanufacturing option after the end of the lifetime of these products. A closed loop network is a coordinated and integrated network of forward and reverse logistics. Examples of closed loop supply chains are Xerox and BMW, where these companies are involved in the recycling of their products while considering recovery processes as integral parts of their networks.

Supply chains are either centralized (i.e., one actor will be the decision maker) or decentralized. The centralized decision making results in reducing costs for the whole network through coordination between the different actors of the supply chain (e.g., El Saadany & Jaber, 2008a). The same holds for reverse logistics. If cost efficiency is the objective, then reverse logistics should be centralised and better coordination will be achieved. On the other hand, if responsiveness is the objective, then decentralized evaluation activity is more suited to minimize time delays in processing returns (Blackburn et al., 2004).

Buyers' expectations regarding customer service have increased dramatically in recent years. Ellinger et al. (1997) studied how well firms are doing in responding to customers' requests and if there is a link between integrated logistics and customer service efficiency. The authors found that centralized companies are accommodating service requests better than decentralized ones, especially in handling returned goods requests.

1.7. Reverse Logistics Functions and Recovery Options

This section presents functions to manage and execute reverse logistics, and explains and discusses various product recovery options. Guiltinan & Nwokoye (1975) investigated collection and sorting, storage and logistics, and vendor/buyers relationships as reverse logistics functions. Pohlen & Theodore Farris II (1992) expanded the functions to include compacting or shredding, communications with buyers, processing, and retromanufacturing.

Giuntini & Andel (1995) discussed in depth the management and implementation of reverse logistics. The authors listed six steps (six R's) for a successful execution of reverse logistics. The six R’s are: Recognition (recognizing returns and information sharing between different actors); Recovery (setting policies regarding management of returns); Review (vary from physical inspection to financial decisions of returned items); Renewal (directing the returned item to the appropriate recovery option); Removal (directing returns to resale, repair or disposal); and, Re-engineering (improving the whole supply chain). Reverse logistics activities are widely applicable for products with non-consumable cores where a “core” is the main component of a product that retains the value of that product (Parkinson & Thompson, 2003). For products of consumable cores, reverse logistics activities can be applied, e.g., packaging (Kodak, 2009).

There are many options and routes for collected products along a reverse chain. Thierry et al. (1995) differentiated among product recovery options (which are: repair, refurbishing, remanufacturing, cannibalization, and recycling) by showing the level of disassembly involved in each option. Repair involves disassembly at the product level, refurbishing at the module level, remanufacturing at the component level, retrieval or cannibalisation at parts’ level, recycling at material level, and incineration at energy level. These recovery processes involve product disassembly, which is a systematic method of separating a product into its constituent parts, components, subassemblies, or other groupings (Gupta & Taleb, 1994). Rework is not a recovery process, because it is an activity performed before the selling of a product or an item. Gungor & Gupta (1999) categorized the recovery process into material recovery (recycling) and product recovery (remanufacturing/refurbishing/reclaim).

Carter & Ellram (1998) stressed that resource reduction should be the ultimate goal of reverse logistics and should be prioritized over recycling. The authors introduced a hierarchy where the resource reduction option should be exhausted before reuse, then recycling, then disposal with energy recovery, and finally, disposal in landfill as the last resort. van der Lann et al. (2004) differentiated between push (autonomous) recovery, where customers push returns as in the case of mail-order companies, and pull (managed) recovery, where the OEM manages and entices customers to return their products after the end of its lifetime to recover it. Other recovery options are available in the literature (Pohlen & Theodore Farris II, 1992; Fleischmann et al., 1997; Bras & McIntosh, 1999; de Brito & Dekker, 2003; Steven, 2004).

Carter & Ellram (1998) and Steven (2004) introduced a hierarchy of recovery options. Recovery options proposed in the literature are listed, starting from the most environmentally friendly option, and are shown in an advanced hierarchy in Figure 1.2.

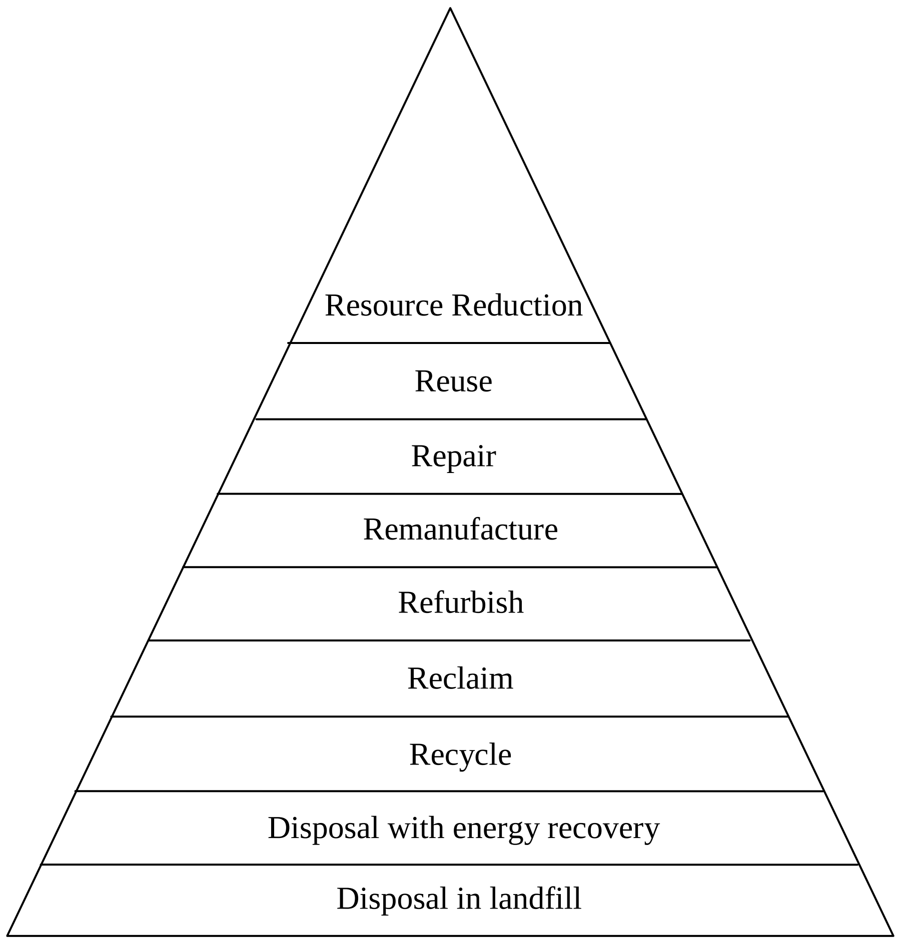


Figure 1.2. Hierarchy of recovery options

Resources reduction is a global call to reduce the inevitable increase in consumption. Consumer awareness and governmental legislations are crucial for its success. For example, the use of reusable bags in grocery shops instead of plastic bags is one useful example, where consumer awareness initiated the call to reduce the use of plastic bags. Municipal laws in some cities (e.g., Toronto, ON) and counties (e.g., Ireland) compel consumers to pay for plastic shopping bags based on this initiative (Convery et al., 2007).

Reuse or *resale* is using the unit in its "as-is" condition for the same or different purpose though it is not always feasible (de Brito & Dekker, 2003). It is the best possible option as it involves the least energy use and no wastage of material. This option is most common in catalogue retailing where a high rate of product returns is experienced. Returned products usually require minimal activities (e.g., cleaning, repackaging) to bring them back to their initial quality. Examples are refrigerator shelves, returnable packaging and, returnable bottling systems. There is another recovery option that could be considered as reuse, which is reconfiguring where a returned product is considered for another lower grade function (e.g., using a computer's component as a toy's component) (Giuntini & Andel, 1995).

Repair is defined as the process of returning a used product to its original working condition through fixing or replacing broken parts. It involves limited disassembly and reassembly. Repaired items are usually perceived by customers to be less than the quality of new ones unless the quality of repaired items is claimed and guaranteed by the manufacturer to be "as-good-as-new". Corporate-owned transportation systems, such as Wal-Mart, or third-party logistics companies, such as Consolidated Freight Inc., return a product to the original supplier or send it to a liquidation centers or a secondary market outlet (Krumwiede & Sheu, 2002).

Remanufacturing and refurbishing are better options than recycling, because remanufacturing (or refurbishing) reclaims a larger share of value than recycling. Remanufacturing reclaims material and value of parts while recycling reclaims materials only. Unless an Original Equipment Manufacturer (OEM) is involved, there will be little incentive to design products for remanufacturability or refurbishability.

Remanufacturing brings the quality of returned used items up to the quality standards of the new similar items and provides products at lower cost, and is an additional source of replacement parts (Bras & McIntosh, 1999). Remanufacturing involves the removal and replacement of all high wear-and-tear parts to extend the life span of these products. It requires high level of disassembly, sorted, inspected and classified as either good or defective. Defective subassemblies are repaired, machined, or substituted with approved quality ones. Remanufacturing can be associated with features upgrading. Volvo offers its dealers to buy remanufactured spare parts instead of buying new spare parts and guarantees the remanufactured units to have the same quality and performance and offer the dealers the same warranty. Olovsson & Khalil (2008) estimated that Volvo offers the remanufacturing components to dealers around the world for 70 % of the cost of a new part, and that 17 % of Volvo's aftermarket sales of spare parts are remanufactured products. BMW has also been remanufacturing engines and applying strict quality standards to bring them to "as-good-as-new" state (Thierry et al., 1995). Bosch sells remanufactured power hand tools (Glenn Richey et al., 2004).

Refurbishing is similar to remanufacturing, though it brings used products quality up to a specific level, less than the level of new "virgin" products. Refurbishing is very common in aircraft and military industry. NASA saved 40-60 percent in building spacecrafts by applying remanufacturing and refurbishing (Beltran, 2002). Dell's customers can refurbish existing computers or buy new parts (Ravi et al., 2005). Thomson Consumer Electronics ships recoverable items from the US to Mexico for refurbishing through a third-party distributor, Genco, to facilitate returns. The company chooses the best option between refurbishing to the Mexican market or disposal in the USA (Dhanda & Hill, 2005). Refurbished products are sometimes used to fill warranty pools of sold products, with the remaining refurbished units sold in secondary markets (Krikke et al., 2004).

Reclaim or *cannibalization*, is unlike the previous options, where it uses a small proportion of parts of the returned unit in the recovery process. Reclaim is the process of extracting parts from returned products, testing them, then the good ones are reused in new products. The remaining subassemblies of the returned product are either recycled or disposed. This practice is common in the automotive and the electronics industry. For example, memory boards used in obsolete computers are reclaimed for use in other electronic products. Aurora, a US company, reclaims integrated circuit boards of computers and sells them as separate chips (Thierry et al., 1995). In Germany, the plastic parts in park benches are reclaimed (Steven, 2004).

Not all rework activities are reverse logistics activities. Flapper et al. (2002) defined rework as all activities required to transform products that have not been produced or packaged according to preset qualifications into products that are. When rework is performed before products being distributed to customers, it is not considered as a reverse logistics activity, because the reworked product did not leave the production floor. However, when a defective unit of a product makes it to the customer, and later returned by the customer because of faulty conditions, then the defective unit is pushed backward to the manufacturer to be reworked, in this case, rework is considered a reverse logistics activity.

Recycling is the process of turning material back to its original configuration from used and non-functioning products. Unlike the previous recovery options, the identity and functionality of products and parts are lost. The retrieved material can be used either in the production of similar products or in different lower grade products. The success and effectiveness of recycling programs depend on the willingness of customers, their communities, and governmental legislations (Alter, 1993).

Disposal with energy recovery makes use of the caloric value in the disposed products and reduces its volume to reduce space consumed of the landfill. Direct disposal in the landfill, is the last and most undesirable harmful option.

The sources of returned items are numerous, for example:

1) End-of-life returns: consumers return products after excessive use and the products are no longer functioning;
2) End-of-use returns: the product is technology obsolete (e.g., computers) or outdated (e.g., carpets and garments). The returned product is sold in a secondary market;
3) Failure: products that functionally failed but can be recovered or repaired;
4) Commercial returns: end-of-shelf-life returns, obsolete products, unsold items, or wrong/damaged deliveries;
5) Warranty returns and lease returns;
6) Production scrap;
7) Stock adjustments;
8) Packaging: for example returnable containers;
9) Commercial returns: reimbursement guarantees;
10) Service returns: for example repairs and spare-parts;
11) Recalled products;
12) Used repair parts: for example parts used for service or maintenance;
13) Asset returns: for example oil drilling equipment, returnable containers;
14) Environmental returns: returns due to environmental legislations (ex: cathode ray tube monitors).

The options and sources of returns are opportunities for reverse logistics. The factors required for successful implementation of reverse logistics are accompanied with challenges that face managers and executives. These factors and challenges are discussed in the following section.

1.8. Reverse Logistics Factors and Challenges

This section highlights the factors that are essential for a successful implementation of reverse logistics and presents some of the challenges facing this implementation. There are few research works in the literature that address these factors and challenges, which makes investigating them attractive to researchers in the field. Some researchers listed several factors. Almost a unanimous agreement exists amongst them that customer awareness and legislations are the main factors for a successful implementation of reverse logistics. Consumer awareness forms legislations, which in turn lead to changing consumers' behaviour. When both of these factors are targeting a certain goal, better and faster results are achieved. According to Ginter & Starling (1978), legislations had taken a broader view of the environment in the late 1960's.

In the USA, the National Environmental Policy Act of 1969 had its effect on major waste disposal and materials recovery projects, by requiring them to assess the environmental impacts of their projects through which many solid waste disposal and resources recycling planning projects were initiated. One of the earliest legislations is a bill introduced in the US Congress in 1972 calling for tax deductions for production and packaging companies that use recycled materials (Guiltinan & Nwokoye, 1975). A good example can be seen in the beverage industry, where a large sector of consumers of the 1970's were unwilling to give up the convenience of the throw-away metal containers versus using refillable bottles which is cost attractive for producers and distributers. In 1985, it was estimated that 80% of packaged soft drinks were sold as one-way bottles or cans in the USA. However, environmental concerns pushed for stringent legislations as one of the few available ways, if not the only one, to change this behaviour. Guiltinan & Nwokoye (1975) recommended that encouraging these legislations would not only provide a change in consumer behaviour, but also the drive for faster development of biodegradable packaging containers by manufacturers. By the 1990's, consumer behaviour in this industry had changed due to the rising public concerns against the littering of these containers. This change, coupled with the beverage container deposit legislation which allowed customers to return empty beverage containers in return for a deposit, diverted a large amount of solid waste from landfills (Alter, 1993). In some other cases, retailers are responsible for collecting empty beverage containers for the sake of mandatory recycling. Michigan beverage distributors and retailers have been mandated by law to collect empty beverage containers for recycling purposes (Goldsby & Closs, 2000). Hence, since the 1990's, consumer awareness became one of the main valuable drivers to preserving the environment by implementing packaging return programs.

In Europe, legislations have been suited to have better environmentally friendly designs and processes. Besides, several countries (e.g., Germany, Denmark, and the Netherlands) introduced pollution taxes and waste management regulations (Gungor & Gupta, 1999) where the public pushed for several environmentally friendly legislations. In 1986, the German Waste Management Act emphasised the importance of waste reduction and required manufacturers to seek novel techniques and products that minimize waste and promote the reuse of non-avoidable wastes. Several countries followed Germany's steps in issuing legislations for the same purpose (Guide Jr. et al., 2000). Cairncross (1992) discussed how European companies were required to redesign their products and their networks to comply with governmental environmental legislations and regulations.

The European Union Waste Electrical and Electronic Equipment Directive (WEEE) is a legislation that has been in force since February 2003. The WEEE restricts the use of hazardous substances in electrical and electric equipment and promotes the collection and recycling of such equipment. Original Equipment Manufacturers are required to take back their products after the end of its life cycle. Prendergast (1995) discussed the European Community (EC) involvement in environmental issues and focused on the proposed EC Directive on Packaging and Packaging Waste. Although the directive focused on recycling, yet it encouraged package reduction and reuse as valuable approaches to deal with the packaging problem. In 1991, the German Packaging Ordinance compelled industries to take back all packaging materials of their sales and imposed a minimum percentage recycling (Fleischmann et al., 1997).

The environmental performance of a company is a function linked to the effectiveness and efficiency of product recovery processes (Vachon et al., 2000). Companies do not have to have necessary expertise in Reverse Logistics to implement complex green marketing tactics and strategies. Therefore, environmental groups can be a valuable source in helping other firms in understanding environmental issues, develop appropriate solutions, and implement associated strategies and tactics, which is known as “Green Alliances” (Polonsky & Rosenberger III, 2001).

Customers’ behaviour and its relation to the type of product have huge effects on returns rate. For example, in the catalogue retail industry, there is a huge deviation between clothing catalogues and electronics catalogues. Many of the customers of clothing catalogues consider trying out the products and, accordingly, the return rate is 18 to 35 % of the delivered goods, while in the electronics catalogues, the return rate is 4 to 5 % of the delivered goods (Emmett, 2005).

Removing a returned item from a Reverse Logistics system can be through resale of the unit to a secondary market or through disposal. Disposal may incur additional extra costs when disposing hazardous materials. These costs are associated with transporting, storing, ownership, and liability of hazardous material. All contributors to waste liable for any future clean-ups of dumpsites (Giuntini & Andel, 1995). A summary of the essential factors for a successful reverse logistics system are:

Design and production level:

1) Product design at early development stages must consider recovery options after the end of lifetime of the product;
2) Redesigning manufacturing processes if necessary;
3) Remanufacturing operations must be compatible with current manufacturing processes;
4) An improved or restructured Bill of Materials is essential;
5) Packaging used in reverse logistics should reduce materials, costs and transportation requirements.

Operations management level:

1) Transportation modes and networks must be well integrated with current transportation system;
2) Reduce uncertainties in the delivery time and size of a shipment of returned items; this factor is the most endorsed in literature, and the concern about it is massive;
3) Collect and analyse information for effective monitoring and control of the reverse logistics process;
4) Prediction and control of supply of used products;
5) Ensure the flexibility of management policies and procedures;
6) Develop appropriate performance measures to monitor reverse logistics efficiency through information collection and analysis.

Quality and Organisational level:

1) Consumers expect consistent quality from the manufacturer regardless of the nature of the product, either new or remanufactured, or the company has to state its level of quality performance;
2) Managers should believe in the importance of reverse logistics;
3) Create a sense of ownership among the staff;
4) Effective managerial and organizational procedures for dealing with reverse logistics must be in place;
5) Successfully develop and implement ethical standards.

Financial & Marketing level:

1) Capital investments and long-term commitment are important for recovery programs. Recovery options require investments, and investments require vision and commitment;
2) Marketing plans to present company's efforts in being environmentally responsible and its commitment of going green;
3) Educating programs to all the actors involved in the reverse logistics process including customers (Helms & Hervani, 2006);
4) Although many companies guarantee the quality of remanufactured items to be the same to that of newly produced items, not all customers generally perceive

remanufactured and new items to be the same. A price differential is offered to customers to entice customers to purchase remanufactured products.

Table 1.1 presents a comparison between recoverable manufacturing systems and traditional manufacturing systems (Guide Jr. et al., 2000; Jayaraman & Luo, 2007).

Table 1.1. Differences between recoverable and traditional manufacturing systems

Factor		Traditional manufacturing environment	Recoverable manufacturing environment
1	Environmental effect	Focus is on pre-production Pollution prevention-remediation Reduce use	Seeks to prevent postproduction waste
2	Logistics	Open forward flow Demand-driven flows No returns	Forward and reverse flows Supply-driven flows Uncertain in quantity and timing of returns
3	Production planning and control	No need to balance demand with returns Manufacturing systems are fabrication and assembly	Need to balance demand with returns Manufacturing systems are disassembly, testing remanufacturing and reassembly
4	Forecasting	Forecast only end products Standard Purchasing	Forecast core and end products Need to forecast part requirements
5	Purchasing	Materials requirements are deterministic	Uncertain materials requirements due to uncertain recovery rates
6	Inventory control	Raw materials, new parts and components	Cores, remanufactured, new parts, remanufactured substitute parts, and original manufactured.
7	Design	Focus in on environmentally conscious design, fabrication and assembly.	Design for disassembly is crucial and costly Pay-off will occur after the first life cycle
8	“Low Fashion”	Novelty is a key marketing issue. Fashion and trendiness are very important is several industries	Remanufacturing is used for functional and technical industrial applications where performance is more important than looks.

Reverse Supply Chains are complex, and managing it is a challenge (Vachon et al., 2000). Challenges that face managers and executives of reverse logistics are numerous and are summarized as follows:

- Unknown conditions of returns until these returns are disassembled and inspected;
- Part matching problem (i.e., different parts and different serial numbers);

- The complex structure of the remanufacturing shop adds to the difficulty of integrating remanufactured products with newly manufactured ones;
- Imperfect correlation among flows of new and returned products;
- Quality of a remanufactured item is usually perceived as an inferior one;
- Forecasting forward demand and reverse flows where seasonality appears in both forward and reverse flows (Aylen & Albertson, 2006);
- Delay between forward demand and returns: demand is satisfied in the early stage of its life cycle without any returns, while at the end of the life cycle, there are more returns than demand;
- Managing transportation and storage is complex. Transportation is the most visible cost in any reverse logistics network (Barnes, 1982);
- Reverse logistics is fairly a new subject, and many research gaps have to be addressed (ex: Learning curve effects, effect of reworking defectives on other production and remanufacturing processes, imperfect production and remanufacturing processes, etc).

Areas and fields that are popular environments for successful implementation of reverse logistics are introduced in the following section.

1.9. Areas of Reverse Logistics

This section presents examples of reverse logistics gathered from different industries. Some companies publish their green profiles as part of enhancing their ecological images. The application of reverse logistics is in various fields, including, and not exclusive to metals, sand, carpet industry, online and catalogue retailers, electronics, photocopiers, photography, automotive industry, pharmaceutical industry and returnable containers.

Steel: In the steel industry, reverse logistics systems are operated by ferrous scrap processing companies to collect scrap generated by metal working companies (Vachon et al., 2000). The reverse logistics of scrap metals is a complicated network. In North America, the ferrous scrap industry is composed of steel mills and foundries, metalworking companies and around one thousand brokers and processors, who collect, reprocess, transport, and resell scrap (Johnson, 1998). The global ferrous scrap market is the world's biggest and oldest recycling market, and it was about 406 million tons in 2003, which represents 40% of ferrous materials world consumption for steelmaking (Aylen & Albertson, 2006).

Aluminum: Being a major producer of primary aluminum and beverage cans, Reynolds Metals Company began the development of manufacturer controlled recycling centers in 1967, to create another steady source of Aluminum supply than the virgin core, for the company's smelting and can manufacturing operations. In 1975, Reynolds managed to redeem 24 million pounds of aluminum included in 495 million all-aluminum containers, an amount equivalent to 60% of the Reynolds manufactured cans market (Fuller, 1978).

Sand: In the Netherlands, 14 millions tons of waste products are produced per year from the construction industry, of which 1 million tons is sand. The construction waste is sieved and separated into components on construction sites or in a separate facility. The system successfully reused 80% of the construction waste, 40% of which is reused as clean sand, with the remaining 40% is reused in certain construction operations as half-clean sand (Barros et al., 1998).

Carpet industry: Louwers et al. (1999) modelled the reverse logistics problem of re-using carpet waste generated in Western Europe; estimated to be 1.6 million tons in 1996. Helms & Hervani (2006) explored the challenges of reverse logistics through the success story of carpet recycling that resulted in reducing water consumption by 46%, energy usage by 70%, and waste by 2.88 billion kilograms per year.

Online & catalogue retailers: Processing returns cost web merchants $3.2 billion in year 2001 (Jayaraman & Luo, 2007). According to a Reverse Logistics Executive Council study, 12% of all goods ordered online during the holiday season were going to be returned. Sales returns take various forms (Anonymous, 2000); refund (59%), exchanges (27%) and credit (11%). The leading products returned were clothing (27%), computer software (20%), and books (15%). In the online retail, returns are about 11% of all revenues (Grewal et al., 2004). Hallmark Cards representatives collected and destroyed out-of-season merchandise in the early to mid-1990s. By the 2000's, Hallmark used a third-party logistics company to collect excess seasonal inventory and repackage it for sale, in secondary markets (Autry et al., 2001). Wehkam, a large mail order company in the Netherlands, has some 10,000 items per day, 28% of its total sales, as product returns (van Nunen & Zuidwijk, 2004). On average, 20% of the products sold by e-retailers are returned and this figure could be as high as 35% for certain products such as clothing (Trebilcock, 2002).

Electronics industry: One of the most popular areas of reverse logistics is the electronics industry. The electronics industry characterises with high market volume, short product life cycles, and technical feasibility due to absence of wear and tear in most of the components compared to mechanical products, making these parts easier to be recovered. The total value of products returned by consumers in the U.S. was estimated at $100 billion annually with the computer industry being a main contributor (Blackburn et al., 2004); over 12 million computers are disposed every year in the USA alone (Ravi et al., 2005). IBM incorporated reverse logistics by making it easier for the customers to refurbish existing computers or buy new parts (Kumar & Craig, 2007). Dell considered each phase of the life of a product, from design to disposal, as an environmentally sensitive phase. Dell started a Design for the Environment (DfE) program to minimize environmental consequences of actions taken in each phase of the product life cycle. Dell follows a reduce, reuse, and recycle (R3) policy that achieved recycling and reuse of 77,000 tons of material, and diverted over 80 percent of non-hazardous solid wastes away from landfills (Kulwiec, 2006). Sun Microsystems remanufactures and refurbishes spare parts, through collecting, refurbishing, and then restocking for reuse at a central distribution point, to be later reused for repairs (Rogers & Tibben-Lembke, 1998).

Photocopiers: Xerox estimated its total cost savings due to recoverable manufacturing operations to over $20 million per year (Guide Jr. et al., 2000). Xerox considered partnership with its suppliers of its products to foster the design of environmentally friendly products. It continued investing in new technologies for its colour printers that produced up to 90% less waste than conventional office color printers did. Xerox achieved 98% reuse/recycling of equipment and supplies (Xerox, 2008). Canon implemented a “Return-Service Programs” that manages returns and enhances customer loyalty, and ultimately, to increases sales. The recovery program deals with returns through receiving and inspection, followed by credit authorization and issuance. The company considers repair, refurbishing, or remanufacturing before proper determination of product disposition (Canon, 2009).

Photography: Kodak considered reclaiming parts from its one-time-use cameras, and achieved a level of 77% to 90% (by weight) of the product that could be reused or remanufactured, and the rest is recycled (i.e., nothing is sent to landfills). The result is a recycling rate greater than 75%, corresponding to more than 1 billion one-time-use cameras recycled, including those of Kodak's competitors (Kodak, 2009).

Automotive: The Automotive manufacturers paid many efforts to consider design for disassembly and to include recyclable materials in its products, and recovering as many components as possible (Anonymous, 1991). Bigness (1995) stated that the automobile is one of the most recycled products in USA, where 20% of glass, 30% of paper products and 61% of aluminum cans are recycled, 95% of the 10 million cars and trucks that retire every year go are directed to recyclers of which 75% by weight is recovered for reuse. Chrysler, Ford, Volkswagen, Toyota and General Motors researchers are improving the ability to disassemble their automobiles to integrate "ease of destruction" with "ease of construction" into consideration (Gungor & Gupta, 1999; van der Laan et al., 2004; Marsillac, 2008).

BMW's strategic goal is to design a fully reclaimable automobile in the 21st century, where reusable packaging is preferred over disposable packaging, and recyclable materials are used in its newly produced vehicles, with about 15% of the total plastic components are previously disassembled components. The company promotes the culture of reducing waste by preventing it. (Dowlatshahi, 2000; BMW, 2007a, b).

Pharmaceutical: Schering AG is a pharmaceutical company that considers recycling of by-products obtained from the stages of production processes because they contain valuable materials. The company also reuses and recycles impure solvents. The implementation and planning of a reverse logistics system to support these activities resulted in annual savings of approximately DM 25 million, about 8.5 % of the total production cost (Teunter et al., 2003). Similar findings were reported in Ritchie et al. (2000).

Returnable containers & Packaging: Kroon & Vrijens (1995) studied the application of reverse logistics in the area of secondary packaging material, which are materials used for packaging products during transportation. In these systems, a service fee is paid for the use of collapsible containers in a reverse logistics framework, instead of using one-way cardboard boxes. Herman Miller Inc. saved over $600,000 in two years using returnable packaging material for steel shelves. Other companies that successfully adopted the use of returnable containers are IBM and Ford, General Motors and Toyota. Another example is a shared returnable containers system adopted by Canada Post, which resulted in significant time-savings in handling mailing orders (Duhaime et al., 2001). John Deere has invested $20 million in a returnable container program with its suppliers of assembly parts (van Nunen & Zuidwijk, 2004). In addition, there exists a huge market for third party providers where third party providers in several industries (Blumberg, 2005) run about 30% of reverse logistics businesses.

Several other examples including, but not limited to, paper, batteries, toner cartridges, lubrication oils, shoes, tires, monitors, dairy products, books, glass, power tools, aircrafts, and office equipment (see Rogers & Tibben-Lembke, 1998; Dowlatshahi, 2000; Bloemhof-Ruwaard et al., 2001; de Brito et al., 2002; González-Torre et al., 2004; de la Fuente et al., 2008; Krikke et al., 1999a, b).

Reverse logistics opportunities are not limited to the western world. Although, most of the case studies and examples in the literature on reverse logistics are in North America and Europe, there are some cases in China and India. Srivastava (2008) argued that most organizations mistakenly assume that poor markets do not have any reverse logistics opportunities.

Given these premises, a review of the research work related to reverse logistics is presented in the next chapter. The review is later narrowed to focus on inventory management research issues in reverse logistics, which will be the theme of this dissertation. An exhaustive review of the literature will identify the research gaps that relate to inventory management in reverse logistics. Some of these gaps are addressed in this dissertation, while other gaps are left for future research. Addressing the research gaps of interest led to the development of several mathematical models, which are analysed. Numerical examples and discussion of results are presented. Each chapter, which may contain more than one model, has a concluding section that highlights the main findings and brings forth some managerial insights. The last chapter provides a summary of the dissertation, the conclusions attained in this work, the managerial implications in these models, the limitations of the work presented herein, and some recommendations for future research.

CHAPTER 2: LITERATURE REVIEW OF REVERSE LOGISTICS

In this Chapter, a literature overview of the research related to reverse logistics is presented. This overview is then followed by an extensive literature review of inventory management issues in reverse logistics. The survey classifies surveyed research works to reverse logistics exposition, qualitative models and qualitative models. Reverse logistics exposition is a collection of research works that introduced and established general frameworks of reverse logistics. Qualitative models are explanatory models and are not based on mathematical laws and formulas. These models portray systems in terms of causal, fundamental, compositional, or arranged relationships among objects and events (Clancey, 1989). Quantitative models are models that depend largely on mathematical laws and logic equations to represent ideas, find and optimize solutions. Quantitative models can be in the form of mathematical equations or algorithms. Simulation models are models used to mimic and imitate a system or other mathematical models, and because these models usually contain mathematical relationships, the author considered simulation models as a subset of quantitative models. Quantitative models are divided into production planning models, network design models and inventory models.

2.1. Reverse Logistics Exposition

Several researchers examined, surveyed, and analysed reverse logistics and presented general concepts, definitions, factors, actors, benefits and challenges of reverse logistics from real life situations. In this section, research works in the literature are discussed, including early research work that recognized reverse logistics and later review papers that established reverse logistics as a separate line of research.

The introduction of reverse distribution networks was the transitional phase of research work describing recycling activities to research work of environmentally friendly supply chain management. Marketing strategies merged with public concern and developed a new opportunity and solution: reverse logistics. Research started by experienced researchers in marketing and logistics, who were trying to find better solutions and efficient methods to manage recycling.

The term “Reverse-distribution” was first introduced by Zikmund & Stanton (1971). The authors discussed management of recycling activities, and examined alternative methods in handling material waste generated by consumers. These alternatives are backward channels with a typical intermediary (i.e., without a middleman), backward channels with traditional middleman, and indirect backward channels using trash-collection specialists. The author highlighted consumer motivation and channel conflict and cooperation are crucial factors for successful recycling. The authors recommended new thinking, new packaging and design, new governmental roles, and considering recycling as part of marketing strategy for a recycling friendly future.

The work of Guiltinan & Nwokoye (1975) discussed the reverse channel concept and identified key dimensions for reverse distribution improvement, such as identifying potential markets, customer awareness, expanding capacities to maintain economies of scale, transportation flexibility, and collection incentives. Fuller (1978) discussed market opportunities for companies considering reverse distribution channels and changes in marketing strategies caused by higher energy costs, depletion of resources, increase of disposal costs, and legislations. These changes developed recycling as a marketing alternative opposed to the direct disposal of waste. The author identified three kinds of reverse marketing channels; traditional middleman channels, manufacturer controlled resource recovery channels, public resource recovery channels. Other researcher continued along this line of research, such as Fisk (1973), El-Ansary (1974), Syring (1976), and Ginter & Starling (1978).

Barnes (1982) was the first to use the term “Reverse Logistics”. The author listed the benefits of recycling by providing a second source of supply of raw materials, saving energy, and enhancing firm’s image. The author further examined factors affecting logistics of waste prepared to be recycled, e.g., the size of the shipment should be economical to recycle, the dispersion of this size should be less (i.e., collecting a mass from one place is easier than collecting the same mass from different places), and the less contamination and the higher homogeneity the better. The author recommended a better understanding of logistics and marketing for a better future of recycling.

Despite the interest for recycling in the late 1960's and 1970's, research in the 1980s focused on recycling technologies rather than managing the logistics of recycling.

Murphy (1986), who used the term "reverse distribution", noted that the topic had received limited attention. Murphy recognized that all the research in the area of reverse distribution was recycling driven, and introduced what believed to be the first work on product recalls; traffic and distribution aspects of a product recalls. For example, the recall of milk by Gatorade and the Tylenol recall in 1982 which cost $100 million.

Following a slowdown, the research on reverse logistics picked up in the 1990's. Rubio et al. (2008) surveyed reverse logistics and believed that the years 1995-2005 represented the first decade in which reverse logistics research started evolving as a field of its own. This started with the work of Muller (1991), who discussed greening the logistics, and advocated for environmentally friendly transportation, warehousing, and packaging. Pohlen & Theodore Farris II (1992) examined recycling of plastics, and showed that the need for efficient reverse logistics network became evident and the reverse logistics membership, functions, issues and future directions have remained relatively unnoticed. The authors illustrated differences between forward and reverse logistics, and showed that recyclables do not necessarily flow in the reverse direction along the same channel, and recyclable materials do not follow a clear defined path.

Cairncross (1992) discussed how European countries redesigned their products and their networks to obey the new environmental regulations that resulted in recovering 80% of collected packaging materials. Wu & Dunn (1995) examined logistics issues relative to the environment, and discussed measures that could assist in designing an environmentally accountable logistics system. The authors introduced the environmental movement that affects the activities of a firm. The authors also defined the environmental logistics system and discussed how logistics executives can make environmentally responsible decisions. Thierry et al. (1995) showed that adopting Product Recovery Management (PRM) policies are economically and ecologically sound.

Jahre (1995) was concerned about household waste collection and its disposition. The author surveyed several collection schemes and classified these reverse channels into distribution levels (vertical dimension), and each level to a number of points (horizontal dimension). The author also studied the relation between the design of the reverse distribution channels and the principle of postponement, where postponing some operations in the supply and logistics chain enhances efficiency.

Giuntini & Andel (1995) discussed the management of reverse logistics and considered "renewal" as a vital managerial task to extend returns life span. They categorized renewal into two main life span extensions, which are product and material. Product life span extension includes remanufacturing, repair and reuse, while material life span extensions include recycling, reclaiming, and reconfiguring. Prendergast (1995) discussed the relation between government legislation and reverse logistics actions to reduce packaging and subsequently the waste it generates. Managers and executives of reverse logistics should act so that their objectives are to reduce costs and to reduce the impact of waste on the environment (e.g., reducing landfill sites).

Fleischmann et al. (1997), in one of the most popular review papers of reverse logistics. The authors surveyed quantitative studies that had operational research perspective and presented a general framework for reverse logistics and divided quantitative models of reverse logistics into three groups: distribution planning, inventory control and production planning, and the authors noted that one of the main challenges facing reverse logistics is the uncertainty in systems involved in reuse/remanufacturing.

Rogers & Tibben-Lembke (1998) provided insights on how to manage reverse logistics. They authors noted the main difference between reverse logistics and green supply chain and wrote (p. 3): "*If no goods or materials are being sent backward, the activity probably is not a reverse logistics activity*". They advocated that reverse logistics is a strategic weapon to face several business risks and a competitive tool in a growing market and observed a challenge to managers of reverse logistics operations, which is the pricing of returns. In the forward logistics, brand managers and marketing specialists often set prices; however, in reverse logistics it often includes a bargaining stage, where the value of returns is negotiated without pricing guidelines and a specialized third party member usually handles these negotiations.

Design is another essential factor in the implementation of waste management programs, which enables manufacturers to produce environmentally friendly products.

Gungor & Gupta (1999) discussed that environmentally friendly designs and life cycle analysis, driven by the escalating deterioration of the environment, and are conducted over the development, manufacturing, use and disposal stages of the product. They examined reverse logistics recovery options and reviewed some inventory models pertaining to these options.

For a successful implementation of reverse logistics, the integration of reverse with forward flow is crucial. Van Hoek (1999) discussed the challenges of lowering the "ecologic footprint" of supply chains by improving the impact of business practices on the environment. Reverse logistics studies are not enough, and the focus should be on understanding the entire supply chain. He explained the differences between reverse logistics and green supply chain and recommended marketing "green" as a selling technique for environmentally conscious customers.

Ritchie et al. (2000) studied a reverse logistics system within Manchester Royal Infirmary, to evaluate and improve the recycling and disposing of pharmaceutical products. The research involved the analysis of returned stock from 28 hospital units. The authors concluded that factors of development and implementing reverse logistics are the collection and analysis of data, redesign of reverse logistics process to be integrated in the original system, and developing a sense of ownership among staff. Advantages from implementing reverse logistics are increasing profit, reducing impact of wastes on environment, and strengthening relations with customers.

Goldsby & Closs (2000) applied Activity Based Costing (ABC) as a different way of calculating costs to reverse logistics activities performed across any supply chain organization, and they applied their analysis on a beverage containers reverse logistics system in Michigan. Their analysis identified the costs associated with reverse flow, and showed that co-operation efforts among supply chain firms resulted insignificant cost savings.

Autry et al. (2001) discussed how reverse logistics performance, satisfaction and services are influenced by industry, firm size/sales volume, and assignment of responsibility for disposition (arrangement). They examined an example of an electronics catalogue retailer and found that the performance of reverse logistics is significantly impacted by sales volume while the environmental profile significantly affects clients' satisfaction. Neither performance nor satisfaction was influenced by assignment of responsibility of disposition.

Dowlatshahi (2000) discussed strategic and operational factors that are essential for the successful implementation of reverse logistics systems. The author introduced a holistic view of reverse logistics and considered cost, quality, customer service, environmental concerns, and legislations as strategic factors, as well as cost-and-benefits structure, transportation, inventory, design for remanufacturing, manufacturing-remanufacturing integration, and packaging as operational factors.

Guide Jr. et al. (2000) discussed supply chain management for recoverable manufacturing systems, and emphasized the uncertainties that affect reverse logistic activities. The authors discussed complicating characteristics that increase the uncertainty in the timing and quantity of returns, and advocated actions to reduce uncertainty, balance return rates with demand rates, and make material recovery more predictable. The authors recommended the use of information systems with innovative production planning and control techniques to increase the predictability of these tasks.

De Koster et al. (2001) discussed the factors contributing to the decision of combining versus separating inbound and outbound logistics in the return handling process. A comparative analysis of operations in nine retail warehouses identified the challenges obstructing return handling. The authors showed that quantity, quality, lead-time, and diversity of products are relevant sources of uncertainty in the reverse flow, and recommended integrating forward and reversing flows for transportation and storage activities.

Tibben-Lembke (2002) found that it is important to re-think reverse logistics in terms of the product life cycle, and to consider how reverse logistics is impacted by it. He further studied the needs of reverse logistics over three different forms of product life cycle: the product model, the product form, and the product class; concluding that it is important to know which stage the product occupies in order to prepare for the logistics challenges it will face before moving to the next phase.

De Brito & Dekker (2003) proposed a reverse logistics framework and identified the driving forces and return reasons, types of products streaming in the reverse flow, modes and options of recovery, and who is executing and managing reverse logistics operations. The driving forces identified were economical, legislations, and corporate citizenship reasons. The author grouped types of returns into manufacturing returns, customers' returns, and distribution returns, and listed recovery options and emphasised the idea of considering reuse as favoured to remanufacturing and recycling is the least preferred recovery option.

Rubio et al. (2008) evaluated the first decade of research on reverse logistics (1995-2005) and observed what has been done and how, where and by whom it has been carried out. The authors described and analysed the main characteristics of published articles on reverse logistics, developed reverse logistics concepts, outlined some directions of future research.

Several publications and monographs have appeared in the literature reflecting the growing interest of both the academic and practitioners in reverse logistics (e.g., Willits & Giuntini, 1994; Bloemhof-Ruwaard et al., 1995; Wu & Dunn, 1995; Shrivastava, 1995; Spengler et al., 1997; Tibben-Lembke, 1998; Fleischmann, 2001; Dekker et al., 2004; Dyckhoff et al., 2004a; Blumberg, 2005; Kleber, 2006; Pochampally et al., 2009). As researchers introduced definitions and established frameworks, reverse logistics became a viable business opportunity and a useful environmental solution. However, as practitioners realised the benefits, they encountered several challenges in executing their projects and realised that there are numerous unanswered questions. This raised the interest of more researchers and caught the attention of academics and practitioners to join the field to try bringing reverse logistics closer to reality, to attempt covering several research gaps, and to explore distant frontiers of the new research area. These attempts resulted in developing qualitative and quantitative models. Surveys of qualitative and quantitative models are presented in the next two sections.

2.2. Qualitative Reverse Logistics Models

Qualitative, or non-mathematical, models render systems through schemes that describe connections and relations among internal and external factors. Examples of these models are models that address issues relating to information technology and management, life cycle, decision-making processes, network complexity, etc.

Information technology usage in reverse logistics is limited due to the little available software developed for reverse logistics (Dhanda & Hill, 2005). Such software requires extensive customization and IT companies rarely consider reverse logistics as a priority (Beltran, 2002).

Although information technology has been one of the driving forces that led to the rapid expansion of supply chain networks and the development of partnerships (Bowersox et al., 2002), reverse logistics has not evolved in the same manner. Kokkinaki et al. (2000) presented a reverse logistics network focusing on the dual relation between reverse logistics activities and e-commerce. In light of the reverse logistics context, e-commerce is examined in terms of technologies and emerging services, which are used to improve trading of used products and parts, including marketing, purchasing, sales and post sales. In a following paper, Kokkinaki et al. (2001) explained how world-wide-web technologies improved business models particularly in a reverse logistics context. In a follow-up paper, Kokkinaki et al. (2002) identified key factors for reverse logistics competitive advantage and discussed opportunities for e-business reverse logistics models and how these models may thrive and advance.

Chouinard et al. (2005) considered a better control and management of the integration of reverse logistics activities within an organization, and presented a framework that describes how an information system can integrate the organizational structure of reverse logistics activities with the original organization structure. While organizations usually do not possess a complete set of data for successful decision-making at each stage of the products life cycle, an information system should ensure the effectiveness and efficiency of the recovery process. The authors applied the system to the case of a rehabilitation institute in Quebec, Canada, that distributes and collects mobility aids. They proposed two ‘‘push–pull’’ approaches to coordinate the demand, with the supply of returns. Similar works are those of Glenn Richey et al. (2004, 2005), Ketzenberg et al. (2004), Daugherty et al. (2005), and Wu & Liu (2008).

The introduction of life cycle design concept in a reverse logistics environment has grown in the past decade. Ferguson & Browne (2001) examined the information requirements for a better end-of-life decision making within the extended enterprise. The extended enterprise is a term used to reflect the high level of interdependence that exists among organizations conducting business while considering the customer. The authors noted that Enterprise Resources Planning (ERP) systems provided enterprise level visibility. It did capture, however, the information flow among partners in the value chain. The authors also developed a decision support system (DSS) for dismantling End-of-Life Vehicles (ELV) where the system uses information on quality, sales history and removal costs to support recovery processes.

Some other qualitative reverse logistics models are for decision-making purposes. Meade & Sarkis (2002) considered the problem of choosing third party logistics providers and analysed a reverse logistics network using a multiattribute decision-making model. They reported that K-mart saved $6 million per $1 billion in sales by outsourcing reverse logistics.

Krumwiede & Sheu (2002) presented a reverse logistics strategic decision-making model to guide the process of examining the feasibility of implementing reverse logistics in third-party providers, such as logistics companies, to help them pursue reverse logistics as a new market. Another decision making model is that of Karagiannidis & Moussiopoulos (1997) which discussed how to manage the household waste of Greater Athens Area. The authors used a multi-criteria analysis method considering social, political, environmental, financial, technological, and resource conversation criteria.

Dyckhoff et al. (2004b) discussed expanding supply chains to closed loop supply chains. The authors introduced strategic problems that might face managers and executives, and recommended solutions to integrate the forward and the reverse logistics, by introducing a two-layer closed loop model as an orientation framework with applications in the automotive industry. The authors also highlighted the fact that different components and materials (of size, shape and quality) that are recovered from products' waste complicate the material flow, responsibility assignments, and processes to close the loop. Besides the uncertainty of the quantity of returns.

Vachon et al. (2000) discussed complexities in reverse logistics operations. A complex system is the one when a large number of its elements interact with one another, with complexity having one of two dimensions: tangibility and uncertainty. Tangibility relates to materials and information, and uncertainty relates to how deterministic or stochastic the system parameters are. The authors introduced two examples from the electronics and the steel industries and suggested integrating the forward and the reverse flows to minimize the negative effects of complexity.

The majority of qualitative reverse logistics models consider the bigger picture, the long-term plans, which make these models very useful for the design of reverse logistics on the strategic level. When it comes for the tactical and operational levels, the importance and popularity of quantitative models are evident, with these models discussed in the next section.

2.3. Quantitative Reverse Logistics Models

Reverse logistics became more of a business opportunity, with economic benefits being considered the most important and attractive motive for reverse logistics. Reverse logistics is not only an effective environmental solution, but also a profit option. Accordingly, the majority of the models' parameters are cost parameters, and therefore, the majority of reverse logistics models are quantitative models.

Like supply chains, reverse logistics performance measures are responsiveness, efficiency, and delivery reliability. Efficiency relates to cost while responsiveness is the ability of a chain in responding to changes in customers' specifications, handling uncertainties, and, in the effectiveness of services they provide. Delivery reliability is the ratio of the number of orders delivered by their due-dates to the total number of orders made (Pochampally et al., 2009). Accordingly, reverse logistics performance is measured in economic terms where the objective function is either minimizing costs or maximizing profits. However, the challenges that face a successful implementation of reverse logistics add much complexity to these models. A survey of the works that developed quantitative models is presented below.

The supply chain management literature addressed problems relating to production planning, inventory management models, network design (i.e., facility location allocation), and demand forecasting, with the common research methodologies being simulation and/or mathematical modelling (Sachan & Datta, 2005). The same problems were addressed in the reverse logistics literature (Thierry et al., 1995; Fleischmann et al., 1997; Dowlatshahi, 2000;

de Brito & Dekker, 2003; Rubio et al., 2008). Besides, some quantitative studies discussed different aspects of reverse logistics, such as decision-making, bullwhip effect, competition in reverse logistics, and forecasting.

Sarkis (2003) developed a decision-making framework related to the relationships among organizations and considering components and elements of green supply chain management. The author modelled a dynamic non-linear multiattribute decision model to optimize partnership options, technology adopted, and type of organizational practice to execute. Repoussis et al. (2009) presented a web-based decision support system for managing the collection and regeneration of waste from used lubricants. The model enabled schedulers to achieve a considerable increase in the amounts of waste collected amounts and a decrease in transportation costs by up to 30 percent.

The bullwhip effect is rarely discussed in a reverse logistics context. It is a popular term in the forward supply chain, and it represents the increasing amplification of orders' variances occurring within a supply chain the more one moves upstream (Forrester, 1961; Sterman, 1989; Lee et al., 2004). Zhou et al. (2004) investigated the bullwhip effect in a dynamic model of a hybrid manufacturing/remanufacturing. The authors showed that the larger the return rate of used items, the less the bullwhip effect encountered in the forward logistics, and that the returned products in the reverse logistics reduce the bullwhip effect experienced by the manufacturer, compared to a manufacturer in a forward supply chain without reverse logistics. The model of Zhou et al. (2004) was extended by Zhou & Disney (2006) who concluded that, although, the length of the lead-time in the reverse logistics has little effect on total costs, shorter remanufacturing lead-time slightly reduced the bullwhip effect. However, they concluded that reverse logistics is efficient despite remanufacturing being more expensive than production.

Competition in reverse logistics is another interesting topic that received little attention. Majumder & Groenevelt (2001) presented a two-period competition model. In their model, a manufacturer, who may also remanufacture, competes with a local remanufacturer on who can provide lower recovery costs. The authors also found that reducing the cost for the manufacturer is beneficial for both competitors as it induces the manufacturer to produce more, and eventually, increase the total number of products, including the products sold to the local remanufacturer.

Forecasting the volume of used items that are returned for recovery received little attention too. Kelle & Silver (1989a) discussed forecasting procedures for returnable containers issued, returned or reissued. They suggested using a data aggregation method to predict the number of returned containers. Beril Toktay et al. (2004) discussed ways of influencing returns' flow and reviewed forecasting methods for flow in the reverse direction.

Reverse logistics quantitative models are classified into three groups: network optimization, production planning, and inventory management models. These groups are discussed in the following sections respectively.

2.4. Network Design Models

Network design deals with location allocation, strategic planning or distribution planning problems. Optimising such networks helps determine which products should be processed, in what amounts, where to be stored, and how they are transported (Pochampally et al., 2009). Networks have arcs connecting nodes together to display the structure of a reverse logistics system, with decision variables are for allocation (i.e., amounts to be processed), transportation, and facility locations. Solving such problems requires building complex mathematical models, such as Mixed Integer Linear Programming (MILP), which has been proven to be a powerful tool (Akinc & Khumawala, 1977; Baker, 1982).

Although it was recognized in the 1970s that network design is an important issue for reverse distribution networks (Fuller 1978), research in this area did not materialise until the late 1990's. Madu (1988) presented a closed queuing maintenance network of parts flowing in a reverse direction, which is an allocation decision problem with two fixed locations. Bloemhof-Ruwaard et al. (1996) used environmental impact data to optimize a reverse logistics network in the pulp and paper industry. The authors used the network model to analyse different scenarios of different recycling strategies in order to choose the best economic and ecological network. The authors found their model to be a useful tool for evaluating environmental policies. The authors were the first to investigate the effect of price on the market share and the return rate.

Spengler et al. (1997) investigated available operations research models, e.g., MILP, for production planning and network design (i.e., location allocation) problems for recycling steel by-products in Germany. The authors recommended a centralized national recycling network connecting all steel companies to reach a more cost-effective and fully utilized recycling system. Return Plant Location Problem (RPLP) is a similar problem to that of Spengler et al. (1997) and was investigated by Marin & Pelegrin (1998).

Barros et al. (1998) considered a case study of recycling sand in the Netherlands where sieved sand is collected from construction works and is moved to regional depots for sorting into three types. The problem is to determine the number of treatment facilities or regional depots, where they should be located, and what capacity of sand these facilities can process. A heuristics solution procedure was considered to decrease the computational time, and the quality of the obtained feasible solutions was assessed by means of linear relaxation.

Krikke et al. (1999a) considered a business case at Oće copier manufacturer, where network re-design for copiers was developed to collect process and recover used products. The study considered the choice between three locations and the method of processing returned products. The problem was modelled as a MILP model, which minimizes the sum of all costs (setup, processing, recycling, distribution, and inventory costs). The solution recommended in this case is centralizing the preparation and re-assembly processes.

Along the same line of research, Jayaraman et al. (1999) analyzed the logistics network of an electronic equipment remanufacturing company and recommended designing a multi-period model and studying the managerial actions to reduce uncertainty in product returns. Louwers et al. (1999) described the reverse logistics problem of re-using carpet waste. The developed model minimized the cost of transportation, storage, processing and disposal processes. Krikke et al. (2001, 2003) presented a case study of a refrigerator with the objective to integrate product design concepts and logistics management together (i.e., a model to support optimal design structure of a product, modularity, reparability, recyclability, and location allocation). Their solution recommended a centralized supply chain network rather than a decentralized one. Fleischmann (2001a, b), Fleishmann et al. (2001) and Fleischmann & van Nunen (2003) studied the decision making pertaining to plant location-allocation in reverse logistics, and recommended accounting for inventories and batch transportation and the integration of the forward and reverse flow to better optimize production and recycling activities.

The particularity of a reverse logistic network must first be considered before implementing a general solution. Schultmann et al. (2004) discussed a large size real life problem that combines facility location planning and vehicle routing and solved the problem using a heuristics method (Tabu search). Spengler et al. (2004) used an activity-based analysis to model a real life problem and showed that information sharing is important to enhance the collaborative partnership among producers and recovery companies. Steven (2004) addressed networks in reverse logistics and differentiated between voluntarily and compulsory recycling networks. Voluntarily recycling networks are networks operated by the Original Equipment Manufacturers (OEM) because of the economic profit generated, and, compulsory recycling networks are networks imposed by legislations.

El Saadany & El-Kharbotly (2004) reviewed location allocation models and proposed a location allocation model to study the effect of the batch size on the network design. The authors recommended that multi-period stochastic programming might be the appropriate tool to solve network design problems. El-Sayed et al. (2009) extended the work of El Saadany & El-Kharbotly (2004) to a multi-period multi-echelon stochastic model. Lieckens & Vandaele (2007) also presented a stochastic model for reverse logistics where they considered dynamic aspects, like stochastic lead-time and inventory, and developed a mixed integer nonlinear program model (MINLP) with queuing characteristics for a single product-single-level case. The model was solved using differential evolution technique.

Srivastava (2008) developed a conceptual multi-period location-allocation model and tried to combine descriptive modeling with optimization techniques. The author assumed two types of facilities in the reverse direction: repair facilities and refurbishing/remanufacturing facilities; where candidate facilities' locations are at several collection centres.

Similar works along the same line of research were presented by Ammons et al. (1999), Canan Savaskan & Van Wassenhove (2006) and Lu & Bostel (2007).

2.5. Production Planning Models

Production planning is a short to medium term planning task that coordinates different activities (e.g., manufacturing, purchasing, transportation, etc.) to satisfy demand and to control the resultant flow of goods and materials (Van Dierdonck & Miller, 1980). In reverse logistics, production planning is the planning of all the processes associated with the production/remanufacturing environment, including manufacturing, packaging, collection, disassembly, sorting, disposal, reassembly, reuse, remanufacturing, repackaging, etc. Production planning decisions usually fall under tactical and operational levels of reverse logistics. The uncertainty in the quality, quantity, shapes, and features of returns complicates the planning of reverse logistics. In comparison to the traditional forward logistics, there hardly exists a well-determined sequence of remanufacturing steps in reverse logistics; which complicates the recovery process and develops additional problems like capacity problems (Fleischmann et al., 1997). Managers and operators of reverse logistics face a huge variety of recovery options once a returned item is disassembled. In this section, a review of efforts in reverse logistics production planning is presented. The surveyed models tend to integrate design of the product with the feasibility of disassembling the product after the end of its lifetime.

Driven by environmental calls, Chen et al. (1993) discussed the product design for disassembly concept and presented a cost-benefit analysis model to assess the feasibility of designing for recyclability, where the potential of recyclability of a product is determined at the design stage. It was found that it was not feasible to disassemble and recycle the whole product, as with material recycling technology and market prices, costs outweighed revenues. However, the model suggested that partial recovery might be a valuable solution.

Hentschel (1994) introduced a predictive and reactive planning approach for recycling processes of discarded complex products where recycling planning involves recovery and treatment processes for cathode ray tubes form discarded monitors. It was shown that the deviations in returned product conditions lead to the need for more flexibility in designing recovery plans.

Johnson & Wang (1995) discussed disassembly for material recovery opportunities and its relation to product design and life-cycle design. An economic model was presented which focused on improving the efficiency of disassembly planning through optimizing the disassembly sequence for a 3.5” computer diskette. Criteria to evaluate marginal benefits of subassemblies were developed. The authors included total processing/upgrading costs, disposal state, availability of recovery, and disposal fees.

Kroon & Vrijens (1995) considered the reuse of secondary packaging material, used for packaging products while shipping it from a sender to a recipient. They suggested that after delivering a shipment, the container (i.e., the returnable secondary packaging material) has to be transported from the recipient to the next sender, who may or may not be the first sender.

Penev & de Ron (1996) believed that starting with a disassembly and recycling strategy is important, where the structure of the product to be disassembled is to be considered as a tree. The feasibility of any disassembly activity has to be linked to its economic feasibility, i.e., the disassembly is not the reverse actions of production. The authors applied their model to the case of a roller bearing to maximize the difference between revenues generated from selling disassembled parts and the costs of disassembling and recycling.

Krikke et al. (1998) presented a model to optimize product recovery and disposal strategy for a product by constructing its disassembly tree. This tree is composed of levels of assemblies and subassemblies, which are sorted and screened. If the assembly is to be disposed (recovered), then all its subassemblies are to be disposed (recovered). In a later paper, Krikke et al. (1999b) discussed a copier manufacturer who redesigned its copiers to streamline its reverse logistics operations, such as take back, processing and recovery of discarded products. The authors argued that savings from a recovered product could reach 40%. They argued, however, that better forecasting of the return flows could enhance savings.

Few researchers considered forecasting of returns. Kelle & Silver (1989b) examined four methods to forecast returns in a returnable containers system, where random return and random demand occurs in successive periods of time. Other researchers considered applying Materials Requirement Planning (MRP) in reverse logistics. The MRP approach uses the product demand to determine the optimum disassembly strategy and delivers a schedule of the disassembly process. Panisset (1988) applied MRP II for production planning in the repair/refurbish industries where a workshop they investigated had 80% of its activities as recovery activities. The author cautioned that MRPs were not designed for reverse activities, such as disassembly and that returned products might have several disassembly routs according to the quality of that product. Gupta & Taleb (1994) corroborated the findings of Panisset (1988).

On a larger scale, Spengler et al. (1997) discussed recovering material from demolition building and recommended increasing the disposal fees to increase the percentage of recovery and recycling of demolished buildings making the recovery program successful. Teunter (2001a) proposed a method for valuing new, recoverable, and recovered assemblies in production systems with reverse logistics. The author emphasized that values of assemblies influence their opportunity holding cost rates and are essential for comparing inventory strategies, making the accurate estimation of the holding cost essential for the success of recovery programs.

For the case of industrial waste, Hu et al. (2002) developed a cost minimization model to determine the amount of raw materials associated with hazardous waste to be collected, stored, treated and distributed. The authors reported that 1.47 million tons of hazardous waste materials are produced in Taiwan every year. The authors recommended that the public/government and the companies must identify the relationship between costs and benefits associated with any hazardous-waste reverse logistics system before applying it.

In a business case, Teunter et al. (2003) discussed two main reverse logistics processes at the pharmaceutical company, Schering. First, by-products are reused because they contain valuable materials. Second, impure solvents are recycled or thermally utilized. This reverse logistics production planning problem introduced closed cycles in the Bill-of-Materials (BOM) and the Materials Requirements Plan (MRP). This complicated the BOM and MRP, as the BOM and MRP do not allow cyclic structures. The sum of holding and setup costs were optimized in a mixed integer linear programming (MILP) model.

Fleischmann & van Nunen (2003) discussed the integration of closed loop supply chain at IBM through a project considering product returns as a source of spare parts. The model was designed to determine the dismantling channel, to choose parts to be recovered from a returned machine, and to coordinate disassembly output with spare parts' demand. The authors compared two channel alternatives: a pull channel, where the company builds up inventory of disassembled parts but not recovered (i.e., not ready as a spare part); and a push channel, where the company adds recovered parts to the serviceable stock. The authors recommended gathering more information regarding return rates to reduce total costs; and, companies should allocate their returns carefully to benefit from any reverse logistics' recovery option.

For the case of reworking defects, Flapper & Teunter (2004) discussed a type of production planning problems: reworks of production rejects. The authors considered a single product that uses the same facilities for production and rework, where reworkable defectives deteriorate over time. The authors analysed two polices: full disposal policy, which is used when processing costs are more than material costs; and, full rework policy, which is used when processing costs are less than material costs.

Production planning is a complex process, especially if the situation involves more than one part/product to recover. Kim et al. (2006) considered the case of a manufacturer with two sources of supply: ordering parts from an external suppliers or recovering parts from returned products to "as-good-as-new" condition. There are several different parts to be assembled together to form the final product, and, there is a remanufacturing facility for each part. The model maximized savings from remanufacturing and the utilization of the different remanufacturing facilities (i.e., remanufacturing costs includes set-up, operation cost, and idle cost). Their results showed that, for each facility, there is an optimal remanufacturing capacity to maximize cost savings.

Inderfurth & Langella (2008) examined the planning for disassembly problem and introduced the disassembled-to-order system. The authors formulated the problem twice: as a deterministic linear program, and, as a stochastic model where disassembling a core of uncertain quality results in releasing of several parts with an uncertain input-output relationship. They concluded that the solution of real life problems requires a very long solution time; therefore, a heuristic solution was proposed to solve the problem. The authors relaxed the assumption of deterministic demand to examine its uncertainty, and suggested including disassembly labour capacity limits as possible future work.

Scheduling is a subset of production planning. However, boundaries between lot sizing and scheduling are fading (Jans & Degraeve, 2008) and an interesting future line of research is the integration of lot sizing, sequencing and loading in a reverse logistics context.

Similar works along the same line of research were presented by Hentschel et al. (1995); Zussman (1995), Ferrer (1997), Guide Jr. (2000), Guide Jr. & Van Wassenhove (2001), and Guide Jr. et al. (2003).

2.6. Inventory Management Models

Companies consider inventory to create buffers, to overcome uncertainties in supply and demand, to take advantage of economies of scale associated with production of products in batches, to build reserves, or to facilitate the flow of products from one point to another in a supply chain (Shapiro, 2001). The efficiency of inventory policies is one of the main methods to judge the performance of any manufacturing or retail company. About 56% of the research done in the area of reverse logistics addressed production planning and inventory management problems (Rubio et al., 2008). Silver (1981) stressed that it is better for analysts to formulate accurate, but simple, models and to obtain good solutions, rather than formulating complex and unrealistically accurate models.

The evolution of supply chain management in the 1990s resulted in the establishment of reverse logistics as a separate line of research. Reverse logistics is all about the reverse flow of materials, and falls under closed loop supply chain and includes all aspects of managing a business, including inventory. Schrady (1967) and Sherbrooke (1968) reported that repaired/recovered items accounted for more than 50% of the dollars invested in inventory. Inventories represent about one-third of all assets of a typical company (Diaz & Fu, 1997).

Reverse logistics inventory models share several features including the double supply feature: the newly produced products (i.e., virgin production supply) and returns that are repaired/remanufactured (i.e., repairable stock supply), where the later usually offers a cost advantage (Minner, 2003). As demand can be met from the two sources, reverse logistics complicates the supply chain as it gives another challenge, i.e., from where to order (van der Laan et al., 2004). Managing inventory in reverse logistics has been stressed in several studies (e.g., Dowlatshahi, 2000; Krumwiede & Sheu, 2002), as it represents a clear challenge to successful implementation of reverse logistics (Dawe, 1995). In this section, a review of inventory models developed for reverse logistics operations is presented.

Inventory models fall into two main categories: deterministic and stochastic (Fleischmann et al., 1997; Guide Jr. & Srivastava, 1997; Dong et al., 2005). Deterministic models are models where the input parameters such as demand are deterministic. Stochastic models capture the variation in these input parameters. Deterministic models are developed for benchmarking purposes and to deliver managerial insights. Stochastic models provide better understanding of an inventory system as they have better description of real-life situations.

2.6.1. Deterministic models:

Research on reverse logistics grew in the late twentieth century, including inventory of reverse logistics. Inventory research started in the beginning of the same century. Harris (1913) was the first to study the economic quantity of parts, which can be manufactured at one time. The author developed the foundation of inventory: the Economic Order Quantity (EOQ). His model optimised ordering costs and holding costs and developed a closed form solution. The EOQ model has been so popular amongst academicians and practitioners (e.g., Osteryoung et al., 1986). Roach (2005) argued that although Harris (1913) was the first to consider inventory equation and introduced the EOQ, he did not develop the mathematics himself but used the mathematics of Kelvin's Law of electrical engineering, which was developed in 1881. Wilson (1934) developed a similar formula and named it the "ordering amount". Although many refer Wilson to be the first to introduce the famous formula, Erlenkotter (1989) illustrated that Harris, who was referenced with a mistake in a published work in 1931, was the first to develop the basic inventory equation, the EOQ.

The EOQ is considered to be the simplest model of cycle stocks where repeated cycles in which inventory is built up then depleted in a deterministic fashion (Porteus, 2002). It answers the basic two questions to control inventory: *how* much should be ordered, and, *when* to order (Zipkin, 2000). The EOQ is popular because of its practicality and easiness. Yet there are several other non-classical methods to evaluate inventory, such as information theory and entropy approaches (Jaber et al., 2004). All deterministic inventory models in supply chain and reverse logistics management are modified or extended versions on the EOQ formula.

Research in inventory grew as world business flourished (Hadley & Whitin, 1963; Silver et al., 1998). Recycling popularity advocated for a new line of research, which is the inventory management of recycling/repair activities. Initial attempts to address the inventory of repaired items or products dates back to the 1960s, with Schrady (1967) being the first to investigate a repair-inventory system. He developed a deterministic EOQ model for repaired items, and assumed that the manufacturing and recovery rates are instantaneous, with no disposal cost, a single manufacturing cycle, multiple repair cycles, no backorders and fixed return rate as a function of demand. Inventory stock is in two separable stock points: the ready for issue point, that contains the repaired items ready to satisfy demand, and, the non-ready for issue point: where items to be repaired are collected. Schrady made assumed that repairable items are presumably more economical than replacing it with new items. The model of Schrady (1967) was extended by Nahmias & Rivera (1979) to allow for a finite repair rate with the assumption of limited storage in the repair and production shops. The authors explained the interactions between procurement and repair functions and named the two inventory stock points as “supply depot” and “repair depot”.

Along the same line of research, Mabini et al. (1992) presented two models. The first is a model similar to that of Schrady’s (1967) which considers shortages. The second is for multiple items that share a common and limited repair capacity without shortages. The objective was to determine the optimum production (or purchase) and repair quantities. Similar to Schrady (1967), the authors assumed one cycle of purchased “new” items and several repair cycles per interval, with fixed return rate as a function of demand.

The popularity of supply chain in the 1990s enriched this line of research. Richter (1996a) investigated the case of stationary demand being satisfied by repaired used products and by newly produced ones. The author assumed multiple production and multiple repair cycles within some collection time interval. Similar to Schrady (1967), there are two shops: the “first shop”, serviceable stock, is for stocking produced and repaired units, while the “second shop”, repairable stock, is for collecting used/returned items and not yet been repaired. Richter (1996b) extended Richter (1996a) to draw additional economic conclusions. The author found that as the model's interval length tends to be very large as the return rate approached the extreme values (i.e., 0 or 1). In a follow up paper, Richter (1997) extended the cost analysis of his earlier works (Richter 1996a and 1996b) and showed that a pure bang-bang policy of either no waste disposal (total repair) or no repair (total waste disposal) is the dominant strategy. The results from this study showed that although there are bounds on the sizes of repair and manufacturing lots, the numbers of these lots per interval are unlimited. Richter & Dobos (1999) extended Richter (1997) to determine the characteristics of these optimal solutions and confirmed that the pure strategies are the optimum strategies. However, excluding the pure strategies for feasibility reasons, a mixed strategy is the optimal and its characteristics are far from the pure strategies.

Like Richter, Teunter (2001b) extended the work of Schrady (1967) and considered the case of one manufacturing cycle and multiple repair cycles. The author compared it with the case of one repair cycle and multiple manufacturing cycles and distinguished between the holding costs for manufactured and recovered items. The author addressed the same question of Richter's (1997) of whether the pure (either production or recycling) or mixed strategies are optimal. The author proved that having both numbers of repair and manufacturing lots even is never optimal, and reached the same conclusion: pure strategies are the optimal policies (return rate is equal to "0" or "1").

Dobos & Richter (2003) extended Richter's (1997) work by considering finite production and repair rates instead of instantaneous (infinite) rates as in the works of Schrady (1967) and Richter (1996a, 1996b, 1997). The author assumed a single production and a single repair cycles per time interval. Dobos & Richter (2004) generalized their earlier model (Dobos & Richter, 2003) to consider the case of multiple cycles per interval, and compared their results to those of Schrady's (1967) model, Richter's (1996a) model and Teunter's (2001b). They concluded that the pure bang-bang policies continue to prevail although not practically feasible. To generalise the model, the authors recommended either applying an upper bound on the reverse flow, or considering the quality of returned items.

Dobos & Richter (2006) extended Dobos & Richter (2004) and assumed that the quality of collected used/returned items is not suitable for further recycling and some are disposed. The authors limited the return rate to an upper bound. The authors assumed that the return rate is a product of marginal and variable return and use rates; however, their product is fixed. The authors believed that the mixed strategy could be the optimal strategy, because the extreme cases have changed: instead of "0" and "1", to "0" and the "upper bound". Minner & Lindner (2004) confirmed the results of Schrady (1967), Richter (1997) and Teunter (2001b) where it is optimum to have a single remanufacturing cycle followed by several production cycles or to have several remanufacturing cycles followed by a single production cycle. The authors extended models in literature to the case of non-equal lot sizes and proved that the final remanufacturing batch is smaller than the previous ones. The reason for that returned items have to be stored over the production interval until the next remanufacturing resumes, thus it is better to store less remanufactured items in the non-serviceable stock.

The above mentioned deterministic inventory models assumed: (1) there is no difference between newly produced and recycled items, and the authors applied the ''as-good as-new'' principle; (2) the return rate is fixed; (3) the recovery cost is less than the cost of producing/ordering a new item.

Dynamic lot sizing formulation is used to represent problems of varying parameters over time. Lot sizing problems, at large, are challenging because these problems are hard to solve (Jans & Degraeve, 2008). Dynamic lot sizing programming was introduced simultaneously by Manne (1958) and Wagner & Whitin (1958). Wagner & Whitin (1958) extended the single item EOQ model to allow demands, holding costs and setup costs to vary over a finite number of intervals of time. The authors proposed an algorithm that resulted in more efficient results than the EOQ model. Silver & Meal (1973) extended the Wagner & Whitin (1958) model by developing a heuristics to deal with the case of deterministic time-varying demand that is replenished at the beginning of discrete time periods. The presented heuristic is simpler and more generic than the Wagner-Whiten approach.

Kelle & Silver (1989a) presented a returnable containers inventory system with a stochastic issue-to-return time. Some containers might be lost and a new batch had to be issued to substitute them. A stochastic model was formulated to optimize the ordering policy for periods of finite time intervals by minimizing inventory and purchasing costs. Under a service level constraint, this model was reduced to a deterministic dynamic lot-sizing problem with the possibility of returns being greater than the demand.

Jans & Degraeve (2008) reviewed deterministic single-level dynamic lot sizing problems and found that there have been few dynamic lot sizing models proposed for the reverse logistics inventory problem. Richter & Sombrutzki (2000) presented a reverse version of Wagner & Whitin’s (1958) model and used an algorithm to determine when to repair and when to produce new products. The authors discussed the stability of their model using the Silver & Meal (1973) heuristics and proved that their model is efficient for certain cases. Richter & Weber (2001) extended Richter & Sombrutzki (2000) to the case of variable unit manufacturing and remanufacturing costs and the possibility of disposal of some of the returned products.

Minner & Kleber (2001) optimized a production, remanufacturing, and disposal strategy to represent seasonal behaviour and life cycle patterns in product demands and returns. The authors presented a dynamic lot size inventory reverse logistics model with linear costs, defined optimality conditions, outlined a solution algorithm, and provided managerial insights for different scenarios. The authors also recommended the inclusion of interest rate based opportunity cost of capital and out-of-pocket holding cost in a discounted cost/cash flow approach for future research work. Kleber (2006) extended the work of Minner & Kleber (2001) by including an anticipation stock for out-of-stock periods to reduce overall costs and examined the effect of rigid production and remanufacturing capacity constraints on the production-remanufacturing policies.

Beltran & Krass (2002) proposed a dynamic lot sizing algorithm for positive and negative demands (i.e., demand and returns) where the holding cost is a concave function, with procurement and disposal costs. The authors explained the advantage of dynamic programming over stochastic modeling, especially in the case of modeling catalogue retailing. Golany et al. (2001) studied dynamic lot size problem and showed conditions of NP-hardness. Yang et al. (2005) extended the work of Golany et al. (2001) by considering concave cost functions that was solved as a heuristics NP-hard problem. Teunter et al. (2006) surveyed the dynamic lot sizing problems in reverse logistics, compared different heuristics, and noted the difficulties that might arise when, for a certain period, returns are more than demand.

2.6.2. Stochastic Models:

The attention to recycling in the 1960's initiated the integration of research on inventory with repair/recycling systems. One of the efforts to bring reverse logistics models to reality is through representing randomness, which is a real life attribute. In stochastic models, demand and return rates follow a distribution that represents randomness. The first stochastic investigation started for repairable inventory models, which are systems with demand satisfied with repaired items. Inventory research was then expanded to cover all kinds of product recovery management. In this section, stochastic models are divided into "periodic review" and "continuous review" models.

Periodic review models:

Simpson (1978) was the first to use the two terms: "serviceable" and "repairable". The "serviceable stock" is the ready to issue stock, containing newly ordered/manufactured units and recovered items. The "repairable stock" is the ready for repair stock, containing collected and returned units. The author presented a dynamic repairable inventory model where the serviceable stock is replenished by repairing units available in the repairable stock, and only repairable units are returned. Simpson's model is a multi-period, periodic review, where unsatisfied demand is backlogged at a cost; and, cost function is differentiable and convex.

Cohen et al. (1980) introduced a stochastic periodic review inventory model and considered recycling as another recovery option in addition to repair. Demand is normally distributed random variable with the amount returned being a fixed proportion of demand. Returned units followed a lead-time with shortage costs considered.

Inderfurth (1997) considered a periodic review inventory model with stochastic demand and return rates, fixed deterministic lead-times, with no fixed costs and unsatisfied demand backordered. The authors recommended heuristics approach to determine the optimal policies, especially when there are variations in the lead times of demand and supply.

Kiesmüller & van der Laan (2001) considered the case of dependent returns and demands, which is common in the case of rented or leased products or in the case when the original manufacturer manages the reverse flow to collect its own products. The authors considered a periodic review single-echelon inventory system with lead-times, finite planning horizon, and a Markov-chain approach to determine the optimal full-up order policy (i.e. (*s*, *S*) policy). The authors showed that neglecting the dependency of the returns on demands might lead to poor order policy performance.

Kiesmüller & Minner (2003) considered a periodic review, single echelon inventory model to determine the produce-up-to level and the remanufacture-up-to level; and considered identical and non-identical lead times for production and remanufacturing. The authors believed that assuming no setup costs simplified the calculations and the presented solution heuristics produced near optimal solutions.

Continuous review models

Sherbrooke (1968) is believed to be the first to present a stochastic inventory in a repair/recycling context. The author reported the importance of recoverable items system and developed a mathematical model of a two-echelon supply system in which demand follows a Poisson distribution. Items returned for repair with a fixed probability to undergo a fixed repair time. The objective is to minimize the expected number of backorders. Graves (1985) extended the work of Sherbrooke (1968) to consider the case of multiple echelons. Later, Moinzadeh & Lee (1986) presented an approximation technique for an effective solution of a similar multi-echelon model.

In a similar work, Allen & D'Esopo (1968) introduced a repairable inventory system, and considered a system of identical items in use, and items are subject to failure according to a Poisson distribution. Like Sherbrooke (1968), the authors assumed a fixed probability of repair and fixed repair time. The objective is to minimize inventory, shortage and ordering costs. Simpson (1970) was the first to present a simulation model to study repairable inventory systems. The author assumed both demand and repair rates to follow the normal distribution, but independent, with unfilled demand backlogged. Two mathematical models were presented; the first optimizes the system subject to a service level constraint while the second minimizes the inventory and shortage costs.

Heyman (1977) showed the difficulty of having returns more than demand, which might result in having too many units in stock. Recovery in this case is not economical because the cost of repair and inventory is more than the savings from recovery. The author modeled a two-echelon inventory system with no fixed costs, and showed that the problem is equivalent to a single-server queue model. If the demand and return rates are Poisson distributed, an optimal solution can be determined. An approximate solution was found for general form distributions. In a follow-up work, Heyman (1978) developed a stochastic model of a single storage facility where used items are returned to a central warehouse, with demand and returns rates and lead times being independent but identically distributed.

Muckstadt & Isaac (1981) considered a single echelon continuous review model and expanded it to the two-echelon case. The single type of item model assumed zero lead time, returns and demand follow the Poisson distribution, and the inventory policy is a continuous review (Q, s) policy. When inventory drops to s, order or remanufacture Q, with a constant lead time. Optimal values of s and Q were determined by an approximation procedure based on the distribution of net inventory. The two-echelon case consisted of a warehouse, which had repair and storage facilities. For the two-echelon case, the optimal policy was ($S-1$, S).

van der Laan et al. (1996a) developed a single review, single echelon, continuous review inventory model and considered three inventory control policies where demand and returns follow the Poisson process. In a follow-up work, van der Laan et al. (1996b) presented a single item, single location (s, Q) inventory model and compared it with models in literature. The authors extended their model with the option of disposal because returns can be larger than required and assumed demands and returns follow the Poisson distribution with a fixed lead-time. A heuristics procedure was presented to reduce the required computational effort. Along the same line of research, van der Laan & Salomon (1997) extended their previous work to two stocking points and studied Push and Pull disposal strategies, and showed the conditions for profitable planned disposals with some product life cycle insights. The authors concluded that the Push policy is favoured over the Pull policy, unless when repairable inventory is sufficiently lower than serviceable one, then the Pull policy is favoured. However, both policies require revaluation if fluctuations occur during the life cycle of the product.

van der Laan et al. (1999a, b) compared Push and Pull inventory strategies to derive managerial insights for manufacturing firms. They considered dependent repair and demand rates with returns less than demand, and all returns are repairable. The authors recommended not to remanufacture all returned items, even if the return rate is less than demand rate, and stressed the importance of lead-time in solving these problems.

Teunter et al. (2000) compared different methods for setting the holding cost rates of returned items and showed the differences between forward and reverse logistics when calculating these rates. The authors proposed five methods and developed a finite horizon inventory model to compare these methods. They also considered the return rate to be less than the demand rate. It was shown that for returned items, opportunity costs should be charged to represent inventory in an average cost inventory approach. Along the same line of research, Teunter & Vlachos (2002) examined the trade-off of including a disposal option for returned items that results in cost reduction and additional modeling complexity. The authors concluded that, in general, it is not recommended to include a disposal option, except for items with remanufacturing cost is as expensive as manufacturing.

Teunter (2002) extended his earlier work (Teunter, 2001b) by considering a discounted cost inventory system with stochastic demand and return rates where the lead-times for manufacturing and remanufacturing are negligible. He proposed a pair of simple economic order quantity formulae for calculating the manufacturing order quantity and the remanufacturing order quantity. Using simulation, Teunter (2002) showed that these formulae provided near-optimal solutions. Fleischmann et al. (2002) presented an inventory model with returns and compared their model with classical (s, Q) inventory models, and found that these classical policies remain optimal when introducing Poisson return flows.

Reverse logistics inventory models assume that returns are non-perishable products. In some cases, selling price of remanufactured products drops over time, turning the remanufactured items as perishable items. However, none of the models in literature considered the case when returns are perishable. For a review of perishable inventory problems, see Nahmias (1982).

The prices for forward and retuned products were ignored in most of the above surveyed models. Some researchers suggested their inclusion when analysing reverse logistics systems. For example, in their study about carpet recycling, Helms & Hervani (2006) noted that the estimation of price elasticity is essential for successful reclamation of resources.

2.7. Research Gaps and Questions

The purpose of scientific research is to explain and explore the world around us, and to try to understand how any field, mechanism and industry work and operate. Research is a systematic and precise process, employed to find new theories, to test theories, to gain solutions to problems, or to interpret new facts and relationships in order to contribute to the human knowledge (Wilson, 1990; Waltz & Bausell, 1981). Reverse logistics is one of the venues to protect our environment. It is a field that has started not long ago and has many questions that need to be answered. Inventory management research in reverse logistics context has several gaps.

The above survey of the literature shows that there are several research gaps in inventory management of reverse logistics. The survey is summarized in Table 2.1, where a grey cell means that the corresponding research issue has been addressed in the literature, whereas a white cell means that a corresponding research issue has not been addressed yet and, therefore, it represents a research gap.

Table 2.1 shows that addressing inventory management in a reverse logistics context is a fertile research area. Table 2.1 shows, that switching costs, imperfect production, learning, quality of remanufactured products compared to new products, disassembly effects, and the effect of price and quality on the flow of returns in the reverse direction have not been adequately addressed in the reverse logistics research. Addressing these research issues give rise to the following questions:

- How does switching from production processes to remanufacturing processes affect the inventory policies?
- How do imperfect repair and production processes affect inventory policies? And how should they be addressed?
- How does the learning in production and remanufacturing phenomena affect inventory policies?

Table 2.1. Comparison between several works in the literature

No	Research criterion / Author	Production/repair context	Stochastic component	Disposal component	Non-instantaneous replenishment	Multiple cycles	Switching costs	Imperfect Quality	Reworks consideration	Inspection of defects	Learning effect	Returns of different quality	Subassemblies	Price and Quality relation
1	Schrady, 1967	■				■								
2	Nahimas, 1979	■			■	■								
3	Richter, 1996a, b	■		■		■								
4	Dobos & Richter, 2003	■		■	■									
5	Dobos & Richter, 2004	■		■	■	■								
6	Dobos & Richter, 2006	■		■	■	■								
7	Teunter, 2001b	■				■								
8	Teunter, 2002	■	■			■								
9	Vörös, 2002													■
10	Porteus, 1986							■						
11	Rosenblatt & Lee, 1986							■						
12	Khouja, 2005							■	■					
13	Salameh & Jaber, 2000		■					■		■				
14	Hadley & Whitin, 1963		■											
15	Jaber & Bonney, 1998										■			
16	Hajji et al., 2004						■							

- In the literature, recovered units are assumed to be as-good-as new. How is the inventory policy affected when they are not?
- In the literature, used items, that are either returned or collected, are recovered as whole units, i.e., no considerations are given when these units are disassembled to recover components/parts. How is the inventory policy affected when inventories of disassembled components are considered part of the inventory?

- The returns (used items) are treated in the literature as a percentage of the demand rate without any consideration to their quality and corresponding price. How is the inventory policy affected when the rate of returned items is price and quality dependent?
- While addressing the above research questions, the demand rate is assumed constant over time. How does including a stochastic component (e.g., demand) affect the results of any of the previous research topics?

Therefore, there is a need to fill these gaps, and the author exercised a significant effort, following solid principles of integrity, in addressing these research gaps. To answer the abovementioned questions, several models are developed and investigated, and the effects of different input parameters on the performance of the developed models are examined. Models are optimized using suitable optimization methods such as, linear and non-linear programming, and differential calculus. Simulation is used to verify some of the developed models. Theorems explaining the behaviour of the presented models are also developed. The research presented herein is as follows:

In Chapter 3: A modified production remanufacturing EOQ model is presented. Costs due to switching between production and remanufacturing processes are introduced in a reverse logistics context. It also corrects a flaw in the work of Richter (1996a, b), where Richter's method to calculate the holding cost for the repairable stock results in an overestimation error. In this chapter, imperfect production, learning, lost sales, subassemblies and price/quality considerations are not addressed.

In Chapter 4: In the literature, production and remanufacturing processes are assumed defect free, which is hardly true. This chapter addresses imperfect quality of production and remanufacturing processes in two directions: (1) defects are screened, inspected and removed from the system by the end of the inspection period; and, (2) defects are reworked while production and remanufacturing process are interrupted to restore quality. In this chapter, switching, learning, lost sales, subassemblies and price/quality considerations are not addressed.

In Chapter 5: Learning is a human attribute that is represented by the learning curve. Although this phenomenon was observed in the reverse logistics literature, it has never been addressed in the context presented herein. The effects of learning in production and remanufacturing on the inventory policy are investigated. In this chapter, switching, imperfect production, lost sales, subassemblies and price/quality considerations are not addressed.

In Chapter 6: In the literature, researchers have assumed that remanufactured items are of the same quality as newly produced ones. Many studies cautioned that, in some cases, customers do not perceive remanufactured products to be "as-good-as-new". In this chapter, the assumption of "as-good-as-new" is relaxed, which results in the case of lost sales. The production, remanufacture and waste disposal model is investigated in this context. In this chapter, switching, imperfect production, learning, subassemblies and price/quality considerations are not addressed.

In Chapter 7: The studies that investigated the production, remanufacture and waste disposal model assumed that a returned unit is recovered as a whole unit without consideration to some of its components that may not be salvageable. In this chapter, returns are assumed to be disassembled and the subassemblies are considered part of the inventory. In this chapter, switching, imperfect production, learning, lost sales and price/quality considerations are not addressed.

In Chapter 8: A common observation in material flow is that demand increases as price decreases or as quality increases. This reality is not considered in the reverse logistics literature. In this chapter, the return rate of used items is assumed price and quality dependent. In this chapter, switching, imperfect production, learning, lost sales, and subassemblies considerations are not addressed.

At the end, Chapter 9 outlines final reflections and presents directions and recommendations for future research work.

The models developed in this dissertation assume: (1) unlimited storage capacity is available; (2) an infinite planning horizon; and, (3) a single product case. For the sake of simplicity, the term "remanufacturing" will be used to refer to remanufacturing, refurbishing, repair, reclamation, cannibalization, reuse or recycling, and this referral was adopted in some works (e.g., King et al., 2006).

CHAPTER 3: REVERSE LOGISTICS WITH SWITCHING COSTS AND HOLDING COSTS CONSIDERATIONS

3.1. Introduction

This chapter addresses the problem of determining the optimal lot sizes for production and recovery in an EOQ (economic order/production quantity) repair and waste disposal model context. There are two stock points, a serviceable stock (i.e., first shop), which is used to store new produced products as well as repaired ones, and the repairable stock, (i.e., second shop), which is used to store returned used products. If the used products are not repaired, these products are disposed outside the system.

Richter (1996a, b, 1997) studied the EOQ model in several works and his works has limitations, and two of these limitations are addressed in this chapter. First, this chapter modifies the method that Richter (1996a, b, 1997) considered for calculating the holding costs in the repairable stock, where they assumed that collected items are transferred from serviceable stock to repairable stock in m cycles to be repaired at the start of each time interval. He considered a general time interval and ignored the effect of the first time interval. The first time interval has production without repair because there is nothing manufactured before that to be repaired. This assumption resulted in a residual inventory and thus overestimates the holding cost. Second, this chapter accounts for switching costs (e.g., production loss, deterioration in quality, additional labour) when alternating between production and recovery cycles. When shifting a task (production) to another (repair) in the same facility, the facility may incur additional costs referred to as switching costs (e.g., Glazebrook, 1980; Paul et al., 1980; Kella, 1991; Yan & Zhang, 1997; Teunter & Flapper, 2003; Hajji et al., 2004; Song et al., 2004; Khouja & Mehrez, 2005). An additional classification by reference of different types of switching (changeover) costs is provided in section 3.4 of this chapter.

The remainder of this chapter is organized as follows. In the next section, Section 3.2, a brief background to the models of Richter (1996a, b, 1997) is presented and the correction of the holding cost expression is shown, followed by a numerical example in section 3.3. In section 3.4, the extension and the corrected model to account for switching costs is presented, followed by numerical examples in Section 3.5. Finally, a summary and conclusions are presented in Section 3.6. This chapter assumes: (1) infinite manufacturing and recovery rates, (2) repaired items are as-good-as new, (3) demand is known, constant and independent, (4) lead time is zero and (5) no shortages are allowed.

3.2. Considering The First Time Interval

The presented model in this chapter considers the production and remanufacturing system, as shown in Figure 3.1, where a manufacturing environment (production, remanufacturing and collection of used items) consists of two stocks. The serviceable stock stores new products and remanufactured/repaired ones, where demand d is satisfied from this stock. From the market, $(\alpha + \beta)$ percentage of the demand is collected in the repairable stock, where sorting and disassembly operations are performed. α percentage of the collected products is disposed, and the rest of these products (i.e., βd) are remanufactured/repaired, are as shown in Figure 3.1.

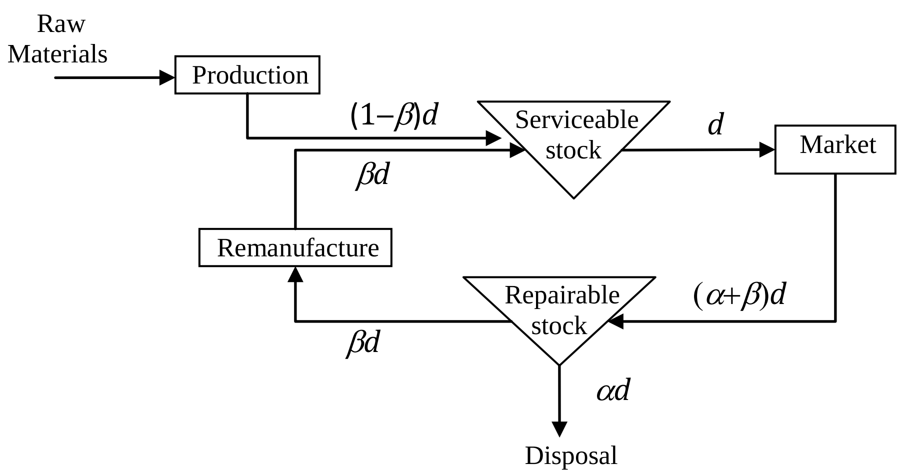


Figure 3.1. Material flow for a production and remanufacturing system

Richter (1996a, b, 1997) introduced in his model demand to be satisfied by manufacturing "new" and repairing/recovering "used" items of a certain product. There are n cycles of newly produced items and m cycles of repair/recovery items in some collection time interval T, with x the demand size per interval. Richter (1996a, b, 1997) assumed instantaneous production and repair rates, repaired/recovered items are as-good-as-new and that each time interval T starts with repair cycles, which are followed by production cycles, as shown in Figure 3.1.

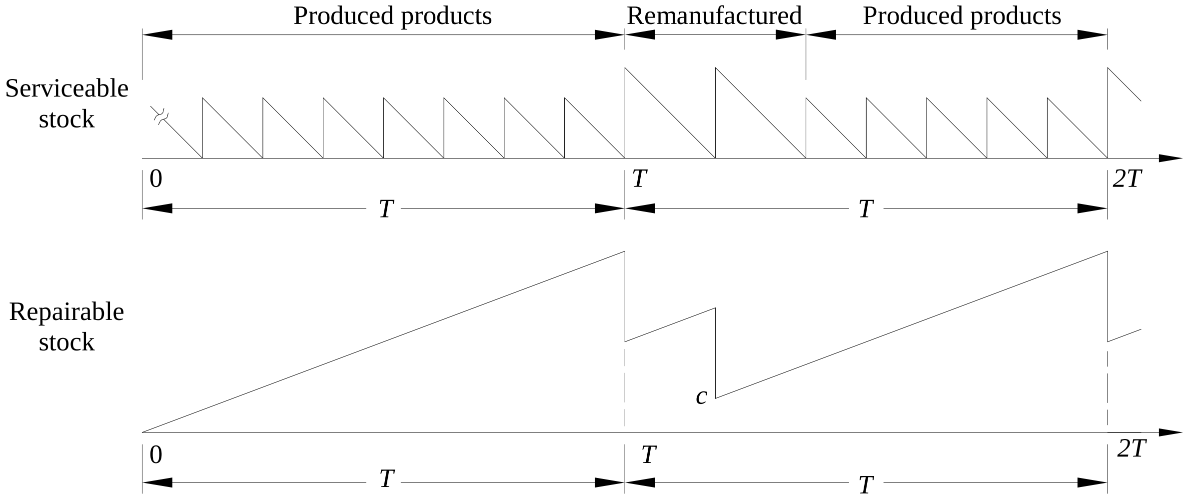


Figure 3.2. The behaviour of inventory for produced and remanufactured products as suggested by Richter (m = 2 and n = 5)

Figure 3.2 is different than the one presented in Richter (1996a, b, 1997) as the inventory in repairable stock is shown inverted and the figure is demonstrated for the case of more than one repair cycle per interval. Note that Richter presented his model starting at time 0 and assumed that repair occurs starting from the first time interval, although, for the first interval, pure production should not be accompanied with repair, as there are no returns to be repaired. The inventory in the serviceable and the repairable stocks for Richter's (1996a, b, 1997) general case (i.e., starting at any time T), is shown in Figure 3.3.

Richter's model tried to minimize the holding costs; however, his model overestimated the holding costs of the repairable stock by accounting for unnecessary amount of residual inventory, i.e., level c. This residual inventory is depicted by the shaded area in Figure 3.4.

The cost function per interval in Richter (1996a, b, 1997) is expressed as

$$K_1 = \left(mS_r + nS_p\right) + \frac{h_1}{2d}\left(\frac{(1-\beta)^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{h_u \beta T x}{2} + \frac{h_u \beta^2 x^2 (m-1)}{2dm} \qquad (3.1)$$

where $T = x/d$, S_r and S_p are respectively the repair and manufacturing setup costs, h_1 and h_u are respectively the unit holding costs of a recovered/manufactured and a collected used item. Repair (production) cycles are of length equal to $\beta T/m$ ($T(1-\beta)/n$) and of inventory level equal to $x\beta/m$ ($x(1-\beta)/n$).

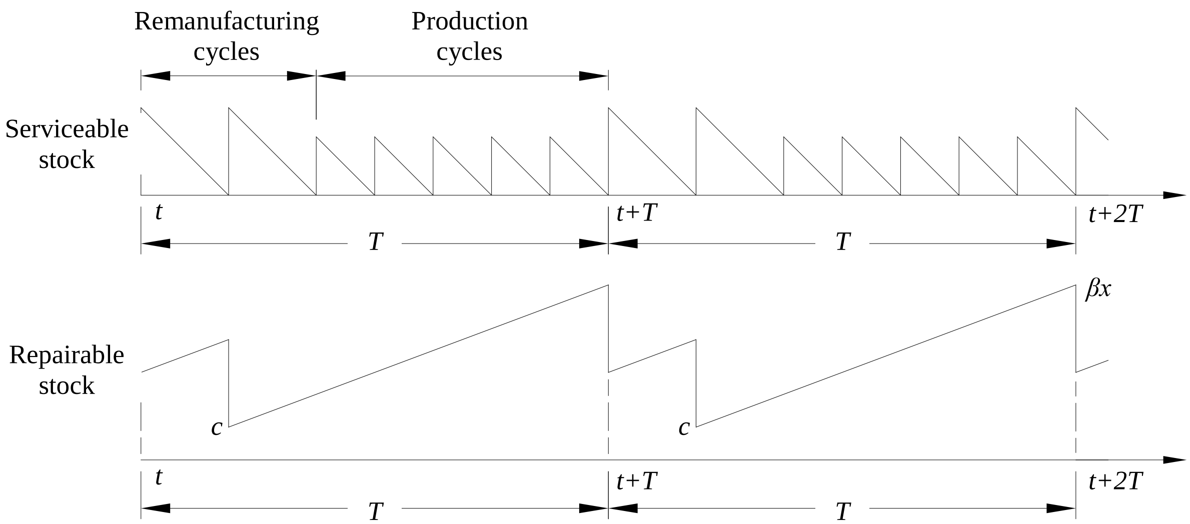


Figure 3.3. The behaviour of inventory for produced, repaired, and collected items as suggested by Richter (m = 2 and n = 5) starting at any time "T"

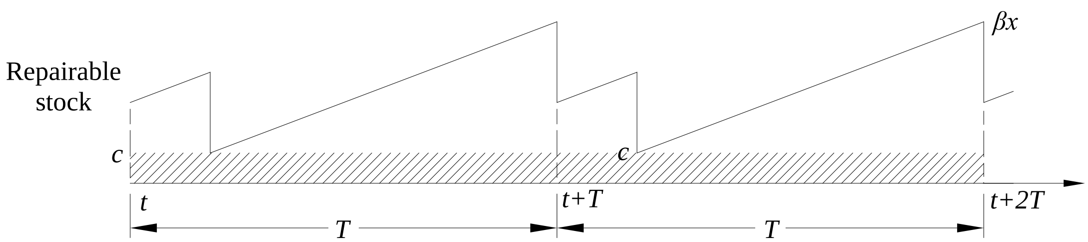


Figure 3.4. The residual inventory accumulated in repairable stock

As shown in Figures 3.2, 3.3 and 3.4, the level of inventory in the repairable stock does not fall below level c, which represents the amount of the residual inventory causing an overestimation in the holding costs of the repairable stock. Accordingly, the level of inventory at point c =

$$\beta x\left(\frac{\beta T(m-1)/m}{T}\right) = \beta x\left(\frac{\beta}{m}(m-1)\right) = \beta^2 x\left(1-\frac{1}{m}\right) > 0, \qquad \forall m > 1 \tag{3.2}$$

Therefore, as long as (m > 1) in Richter's model, there will be an overestimation of the holding costs in the repairable stock. The cost per unit time is given by dividing the above expression by T as (subscript R = Richter's model)

$$K_R(x,m,n,\beta)=\frac{K_1}{T}=\frac{d}{x}\left(mS_r+nS_p\right)+\frac{x}{2}\left[h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+h_u\beta+h_u\beta^2\,\frac{(m-1)}{m}\right] \quad (3.3)$$

The cost function given in Equation (3.3) is convex and differentiable in x, i.e., $\partial^2 K_R / \partial x^2 > 0$ for every $x > 0$. Therefore, and for given values of m, n and β, Equation (3.3) has a unique minimum value, and is derived by setting its first derivative equals to zero, $\partial K_R / \partial x = 0$, to get

$$x(m,n,\beta)=\sqrt{\frac{2d\left(mS_r+nS_p\right)}{h_1\left((1-\beta)^2/n+\beta^2/m\right)+h_u\beta+h_u\beta^2\left(\dfrac{m-1}{m}\right)}} \quad (3.4)$$

Substituting Equation (3.4) in Equation (3.3) reduces Equation (3.3) to

$$K_R(m,n,\beta)=\sqrt{2d\left(mS_r+nS_p\right)\left[h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+h_u\beta+h_u\beta^2\left(1-1/m\right)\right]} \quad (3.5)$$

Richter's model did not recognize the difference between the first time interval, T_1, and of subsequent time intervals T, which resulted in accumulating an unavoidable residual inventory. Subsequently, the holding cost expressions in Richter's model must be changed.

In the presented model, it is assumed that demand is supplied by m repair/recovery cycles and n newly produced cycles per time interval T. For the first time interval [0, T_1], production with no repair will take place in the serviceable stock, while in the repairable stock, accumulation of used items takes place to be repaired in the second time interval. After the first time interval, m repair cycles will precede n newly produced cycles. The used collected items are repaired and delivered to the serviceable stock for repair/recovery once the last produced cycle is depleted.

The first time interval T_1 was not accounted for in Richter's work, as well as other studies in the literature, perhaps because it is usually assumed that there is an infinite planning horizon. However, the exclusion of the first time interval results in a residual inventory in the repairable stock that overestimates the holding costs. Logically, the first time interval should be shorter than the subsequent time intervals allowing the inventory in the repairable stock to reach zero as shown in Figure 3.5.

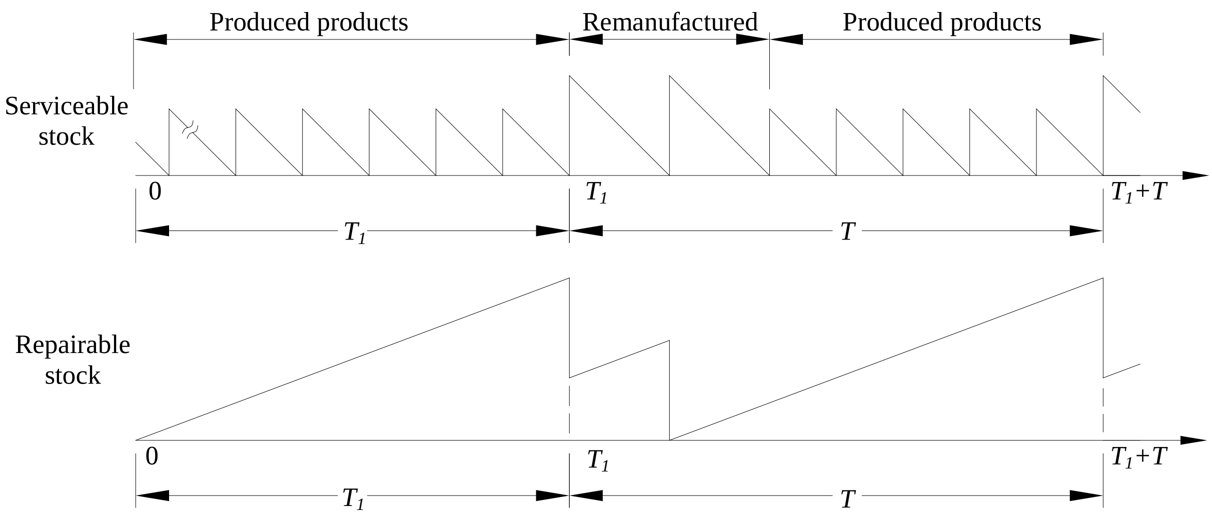


Figure 3.5. The modified behaviour of inventory in the serviceable and the repairable stocks (m = 2 and n = 5) showing the difference between the first time interval "T_1" and the general time interval "T"

Modelling the holding cost expressions for the repairable and serviceable stocks and considering the effect of the first time interval, T_1, on a general time interval of length T is given as

$$K_2 = \left(mS_r + nS_p\right) + \frac{h_1}{2d}\left(\frac{(1-\beta)^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{h_u \beta T x}{2} - \frac{h_u \beta^2 T x (m-1)}{2m}$$

$$= \left(mS_r + nS_p\right) + \frac{h_1}{2d}\left(\frac{(1-\beta)^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{h_u \beta x^2}{2d}\left(1 - \beta\frac{(m-1)}{m}\right) \qquad (3.6)$$

where the terms $mS_r + nS_p$, $h_1((1-\beta)^2x^2/n + \beta^2x^2/m)/ 2d$, and $h_u\beta x^2\left(1-\beta\left(m-1\right)/m\right)/2d$ in Equation (3.6) represent the total setup costs, the total holding costs in the serviceable stock and the repairable stock respectively. A detailed derivation of Equation (3.6) is shown in Appendix 1.

Demand is satisfied by repairing $\beta x = \beta dT$ units in m cycles of size $\beta x/m$ each, and by producing $(1-\beta)x$ units in n cycles of size $(1-\beta)x/n$ units each. Since T_1 is shorter than T, then

$$T_1 = (1-\beta)T + \frac{\beta}{m}T = T\left(1-\beta+\frac{\beta}{m}\right) \tag{3.7}$$

where for m =1, $T_1 = (1-\beta+\beta)T = T$. For this case, the suggested model in Equation (3.6) and that of Richter given in Equation (3.3) are indifferent. The behaviour of inventory in the case of general time interval T is shown in Figure 3.6.

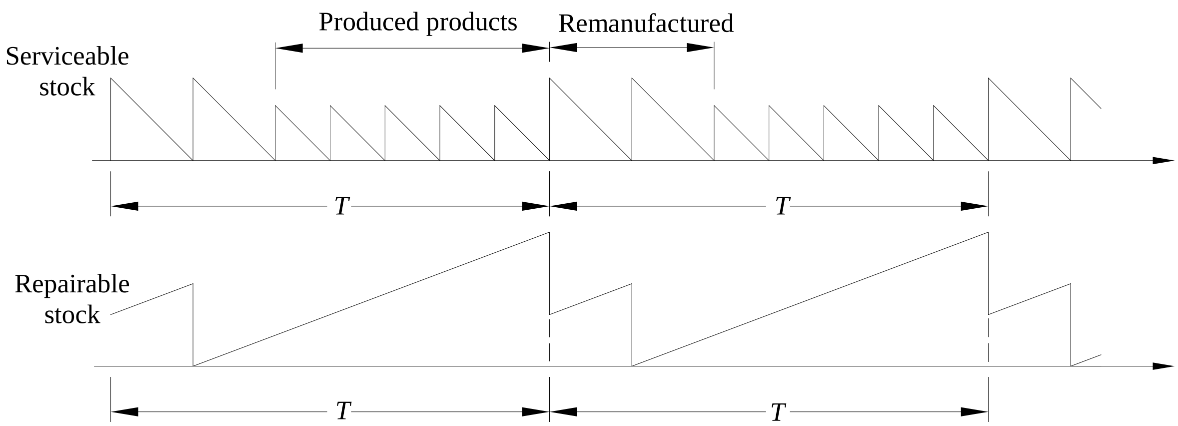


Figure 3.6. The modified behaviour of inventory in the serviceable and the repairable stocks (m = 2 and n = 5) in the case of general time interval "T"

The unit-time cost function is given by dividing Equation (3.6) by the cycle length, $T = x/d$, as

$$K(x,m,n,\beta) = \frac{K_2}{T} = \frac{d}{x}\left(mS_r + nS_p\right) + \frac{x}{2}\left[h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right) + h_u\beta - h_u\beta^2\frac{(m-1)}{m}\right] \tag{3.8}$$

Equation (3.8) is convex and differentiable in x, i.e., $\partial^2 K/\partial x^2 > 0$ for every $x > 0$. Therefore, and for given values of m, n and β, Equation (3.8) has a unique minimum and is derived by setting its first derivative equal to zero, i.e., $\partial K/\partial x = 0$, to get

$$x(m,n,\beta) = \sqrt{\frac{2d\left(mS_r + nS_p\right)}{h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right) + h_u\beta\left(1-\beta+\frac{\beta}{m}\right)}} \tag{3.9}$$

Substituting Equation (3.9) in Equation (3.8), yields

$$K(m,n,\beta) = \sqrt{2d\left(mS_r + nS_p\right)\left(h_1\left((1-\beta)^2/n + \beta^2/m\right) + h_u\beta\left(1-\beta(1-1/m)\right)\right)} \qquad (3.10)$$

The expression $K(\beta)$ is determined from Equation (3.10) by minimizing it subject to $m, n \in \{1,2,...\}$.

The difference between Richter's model, Equation (3.3), and the one suggested herein, Equation (3.8), is the term $h_u\beta^2x^2(m-1)/2m \geq 0$ in Equation (3.8) which is subtracted rather than added, and this term represents the residual overestimated inventory. Subtracting this term in Equation (3.8) resulted in reducing the holding costs, and this is achieved by considering the length of the first time interval. To illustrate, let us take the ratio of Equation (3.5) to Equation (3.10) to get

$$\frac{K_R}{K} = \sqrt{\frac{h_1\left((1-\beta)^2/n + \beta^2/m\right) + h_u\beta\left(1+\beta-\beta/m\right)}{h_1\left((1-\beta)^2/n + \beta^2/m\right) + h_u\beta\left(1-\beta+\beta/m\right)}} \qquad (3.11)$$

Equation (3.11) suggests that $K_R > K$, $\forall m > 1$. For the case of m = 1, K_R / K = 1 making the presented model and that of Richter (1996a, b, 1997) indifferent from one another. Richter's model accidentally included the residual inventory by showing the proof when m = 1.

Note that a case may occur where it is not feasible to repair. For such a case, the model described above reduces to the classical EOQ whose unit time cost function is $K_{EOQ} = dS_p/x + xh_1/2$, where $x = \sqrt{2dS_p/h_1}$ and $K_{EOQ}(m=0) = \sqrt{2dS_ph_1}$.

The two-dimensional nonlinear integer optimization problem $K(m,n,\beta)$ in Equation (3.10) is minimized as

$$Min \in \left\{\left(K(m,n,\beta) \to \min \forall m,n > 0\right), \sqrt{2dS_ph_1}\right\} \qquad \forall x, m, n, \beta \qquad (3.12)$$

This model is also compared to $K_{EOQ}(m=0) = \sqrt{2dS_ph_1}$.

THEOREM 1: A policy $K(m,n,\beta)$ with both m and n being even integers can never be optimal, since the average total cost rate associated with policy $K(m/2, n/2, \beta)$ is smaller.

PROOF: The inventories of manufactured and repaired items associated with policies $K(m,n,\beta)$, *Case a*, and $K(m/2,n/2,\beta)$, *Case b*, are shown in Figure 3.7.

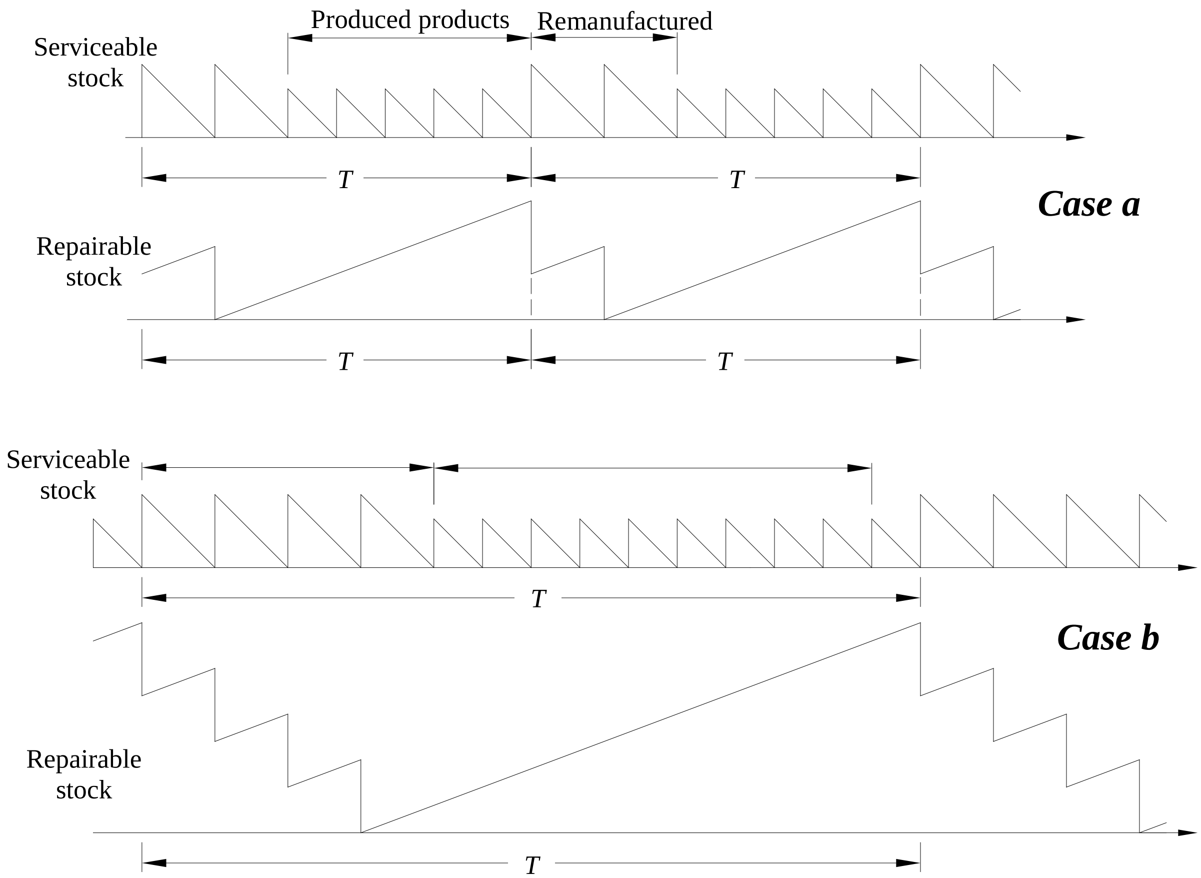


Figure 3.7. Inventory stocks for Case *a*: *K* (*m*, *n*, *β*) and Case *b*: *K* (*m*/2, *n*/2, *β*)

The parameters *d, m, n, r, s, h, β* and *u* in "*Case a*" correspond respectively to the parameters *d*, *m*/2, *n*/2*, r, s, h, β,* and *u* in "*Case b*". For "*Case a*", the unit time cost function is

$$K(m,n,\beta)=\sqrt{2d\left(mS_r+nS_p\right)\left(h_1\left((1-\beta)^2/n+\beta^2/m\right)+h_u\beta(1-\beta+\beta/m)\right)}$$

$$=\sqrt{d\left(mS_r+nS_p\right)\left(2h_1\left((1-\beta)^2/n+\beta^2/m\right)+2h_u\beta(1-\beta)+2h_u\beta^2/m\right)}$$

For "*Case b*", the unit time cost function is

$$K\left(\frac{m}{2},\frac{n}{2},\beta\right)=\sqrt{2d\left(\frac{m}{2}S_r+\frac{n}{2}S_p\right)\left(h_1\left(2(1-\beta)^2/n+2\beta^2/m\right)+h_u\beta(1-\beta)+2h_u\beta^2/m\right)}$$

$$= \sqrt{d\left(mS_r + nS_p\right)\left(2h_1\left((1-\beta)^2/n + \beta^2/m\right) + h_u\beta(1-\beta) + 2h_u\beta^2/m\right)}.$$

The difference in the square of unit time cost functions is given as

$$K_a(m,n,\beta)^2 - K_b\left(\frac{m}{2},\frac{n}{2},\beta\right)^2 = d\left(mS_r + nS_p\right)h_u\beta(1-\beta) > 0, \qquad \forall m,n,\beta > 0. \ \blacksquare$$

Note that for given n and β, the cost function given in Equation (3.8) is convex and differentiable in m, i.e., $\partial^2 K/\partial m^2 > 0$ for every $m > 0$. Setting $\partial K/\partial m = 0$ and solving for m gives

$$m = x\beta\sqrt{\frac{(h_1+h_u)}{2dS_r}} \tag{3.13}$$

Similarly, the cost function Equation (3.8) is convex and differentiable in n, i.e., $\partial^2 K/\partial n^2 >$ 0 for every $n > 0$. Setting $\partial K/\partial n = 0$ and solving for n gives

$$n = x(1-\beta)\sqrt{\frac{h_1}{2dS_p}} \tag{3.14}$$

Dividing Equation (3.13) by Equation (3.14) gives

$$\frac{m}{n} = \frac{\beta}{(1-\beta)}\sqrt{\frac{(h_1+h_u)}{h_1}\frac{S_p}{S_r}} \tag{3.15}$$

Equation (3.15) indicates that the factors affecting the ratio of m to n, are β, h_1, h_u, S_p, and S_r. To increase m with respect to n, β and hr should be increased, and S_p should be increased with respect to S_r.

3.3. Numerical Examples I

Example 3.1

Consider a case with the parameters d = 10000, h_1 = 5, h_u = 2, S_r = 3, S_p = 9, β = 0.7. The total cost K in Equation (3.10) is calculated for various m and n values. The results are plotted in Figure 3.8.

Figure 3.8 shows that for a given value of m, the total cost increases as n increases. Whereas, for a given value of n, the total cost has a minimum value at some m value. From Figure 3.8, the optimal policy occurs at n = 1 and m = 3, corresponding to an optimal lot size of 422.86 units, i.e., x(3, 1, 0.7) = 422.86 computed from Equation (3.9), and a cost of

851.35, *K* (3, 1, 0.7)= 851.35, computed from Equation (3.10). Comparing the cost value of the optimal policy to other cost values in Figure 3.8, e.g., *K* (6, 2, 0.7) = 935.95 and *K* (3, 1, 0.7) = 851.35, which confirms Theorem 1.

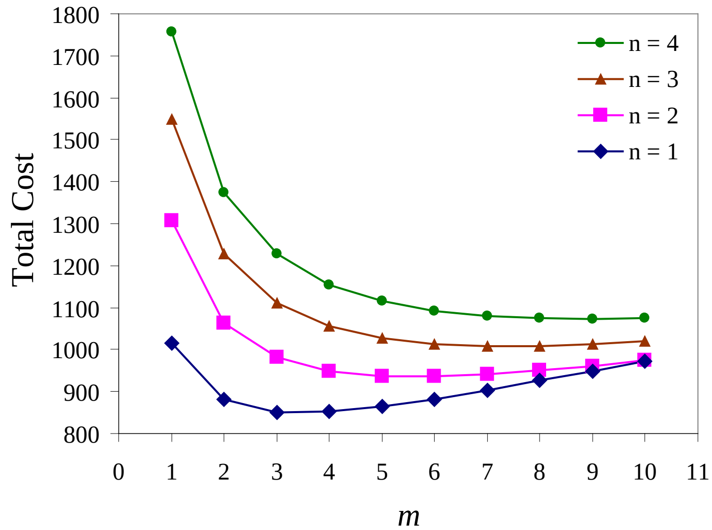


Figure 3.8. Total Cost function plotted for different values of recovery (*m*) and manufacturing (*n*) cycles, in an interval of length "*T*"

The numerical examples in Figure 3.8 were repeated using the model of Richter, provided in Equation (3.5), with results plotted in Figure 3.9 to compare Richter's results against those from the suggested model.

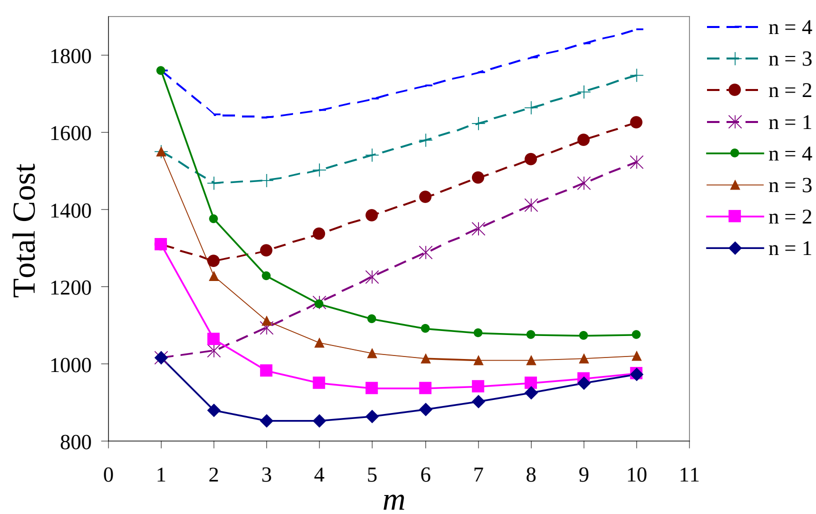


Figure 3.9. Total Cost values for the suggested (solid lines) and Richter's (dashed lines) models plotted for different recovery (*m*) and manufacturing (*n*) cycles

As shown in Figure 3.9, the suggested model and that of Richter produced identical results when m = 1; however, the suggested model produced better results than that of Richter for m > 1. For example, the global optimum solution for the suggested model produced a lower cost, K (3, 1, 0.7) = 851.35, than that of Richter's, K_R(1, 1, 0.7) = 1015.87, by 16.19% ((1015.87– 851.35)/1015.87 = 0.1619). It is worth noting that shall a situation occur where it is not feasible to repair used items, then the suggested model and Richter's model reduce to the classical EOQ model whose cost is 948.68 ($K_{EOQ}(m=0)=\sqrt{2dS_p h_1}$ = $\sqrt{2\times10000\times9\times5}$ = 948.68). By comparing Richter's optimal policy to that of the EOQ model, one may notice that Richter's model suggests that it is not feasible to repair used items since K_R(1, 1, 0.7) = 1015.87 > $K_{EOQ}(m=0)$ = 948.68. Whereas, for the suggested model it was found that it is always feasible (m > 1) to repair used items where K (m, n, β) < $K_{EOQ}(m=0)$.

3.4. Considering Switching (Changeover) Costs

As indicated before, it might be necessary to account for an additional cost when switching among products or jobs in a manufacturing facility. Switching costs to shift from producing product A to producing product B, is different from the switching costs to shift from product B to product A (e.g., Paul et al., 1980; Teunter & Flapper, 2003). Sethi & Sethi (1990) considered switching costs from one action in a manufacturing period to another and discussed its effect on manufacturing flexibility. Gascon & Leachman (1988) differentiated between setup and changeover costs and identified setup cost of an item as the cost incurred every time production of the item is started, and the changeover cost is the cost incurred when production is switched to that item. There are no switching costs for stopping and resuming any manufacturing process.

Changeover cost is defined as the cost incurred whenever two consecutive jobs do not share the same feature (Lahmar et al., 2003). Switching (changeover) costs may include cleaning cost, machine adjustment/fine tuning cost, changing tools, changing product family, changing production supplies, equipment start-up/shutdown, etc. A classification by reference of the possible types of switching (changeover) cost is provided in Table 3.1.

Table 3.1. A list of possible types of switching costs

Types of switching costs	References
Additional setup cost to switch between workstations, jobs, or products.	Galvin (1987), Gascon & Leachman (1988), Sethi & Sethi (1990), Yan & Zhang (1997), Wolsey (1997), Kim & Van Oyen (1998), Inman (1999), Tsubone & Horikawa (1999), Robinson & Sahin (2001), Lahmar & Ergan (2001), Lahmar et al. (2003), Han et al. (2003), Hur et al. (2003), de Matta & Miller (2004), Hajji et al. (2004), Wu et al. (2004), Brahimi et al. (2006)
Switching the production rate	Hall (1988); Kim & Han (2001), Kamrad & Ernst (2001); Larsen (2005)
Machine Start-up/shutdown	Pattloch & Schmidt (1996); Liaee & Emmons (1997); Kim & Han (2001); Khouja (2005); Sim et al. (2006)
Machine cleaning	Kumar (1995); Wolsey (1997); Olson & Schniederjans (2001); Robinson & Sahin (2001); de Matta & Miller (2004); Brahimi et al. (2006)
Production loss	Watkins (1957); Paul et al. (1980); Weber (2006)
Tooling	Kumar (1995); Sim et al. (2006); Brahimi et al. (2006)
Deterioration in quality	Paul et al. (1980); Khouja (2005)
Machine adjustment	Koulamas (1993); Robinson & Sahin (2001); de Matta & Miller (2004); Khouja (2005)
Additional Labour	Watkins (1957); Paul et al. (1980); Kumar (1995); Robinson & Sahin (2001); Weber (2006)
Wastage of material	Watkins (1957); Kumar (1995)

In this chapter, switching costs are considered when a process shifts from repair to production, or vice versa. Now, denote S_{r_1} as S_{p_1} as the setup and switching costs of the first repair and the first production cycles. That is, $S_{r_1} = S_r +$ "switching costs" from production to repair, and $S_{p_1} = S_p +$ "switching" costs from repair to production. When there are more than one repair cycle per interval, the setup cost per cycle is denoted as S_r except for the first cycle. When there are more than one production cycle per interval, the setup costs per cycle is denoted as S_p except for the first cycle. The process incurs $(S_{r_1} - S_r)$ and $(S_{p_1} - S_p)$ costs when switching from repair to production and from production to repair, respectively. The total setup-switching cost for *m* repair and *n* production cycles per interval of length *T*, is computed as $S_{m.n} = (m-1)S_r + S_{r_1} + (n-1)S_p + S_{p_1}$. For the case of a single repair cycle (*m* =

1) and a single production cycle (n = 1), the total setup and switching costs in an interval of length T are equal to $S_{r_1} + S_{p_1}$.

As indicated in Equation (3.7), $T_1 = (1-\beta)T + (\beta/m)T = T(1-\beta+\beta/m)$, where T_1 < T. Therefore, Equations (3.6) and (3.8) are altered to account for switching costs and are rewritten as (subscript S = including switching costs)

$$K_{2S} = (m-1)S_r + S_{r_1} + (n-1)S_p + S_{p_1} + \frac{h_1}{2d}\left(\frac{(1-\beta)^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{h_u \beta x^2}{2d}\left(1-\beta\frac{(m-1)}{m}\right) \tag{3.16}$$

$$K_S(x,m,n,\beta) = \frac{d\left[(m-1)S_r + S_{r_1} + (n-1)S_p + S_{p_1}\right]}{x} + \frac{h_1 x}{2}\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + \frac{h_u \beta x}{2}\left(1-\beta\frac{(m-1)}{m}\right) \tag{3.17}$$

Note that for given x, n and β, the cost function given in (3.17) is convex and differentiable in x, i.e., $\partial^2 K_S / \partial x^2 > 0$ for every $x > 0$. Equation (3.17) has a unique minimum and is derived by setting its first derivative equals to zero, $\partial K_S / \partial x = 0$, to get

$$x(m,n,\beta) = \sqrt{\frac{2d\left((m-1)S_r + S_{r_1} + (n-1)S_p + S_{p_1}\right)}{h_1\left((1-\beta)^2/n + \beta^2/m\right) + h_u\beta\left(1-\beta(1-1/m)\right)}} \tag{3.18}$$

Substitute Equation (3.18) in Equation (3.17) reduces it to

$$K_S(m,n,\beta) = \sqrt{2d\left((m-1)S_r + S_{r_1} + (n-1)S_{p_1} + S_{p_1}\right)\left[h_1\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + h_u\beta\left(1-\beta\left(1-\frac{1}{m}\right)\right)\right]} \tag{3.19}$$

The two-dimensional nonlinear integer optimization problem $K_S(m,n,\beta)$ in Equation (3.19) is minimized as

$$Min \in \left\{\left(K_S(m,n,\beta) \rightarrow \min \forall m,n > 0\right), \sqrt{2dS_p h_1}\right\} \qquad \forall x,m,n,\beta \tag{3.20}$$

Where $K_{EOQ}(m=0) = \sqrt{2dS_p h_1}$ and $m,n \in \{1,2,\ldots\}$.

When there is no switching, i.e., $S_{r_1} = S_r, S_{p_1} = S_r$, Equations (3.16-3.20) reduce to Equations (3.6, 3.8-3.11).

THEOREM 2: When switching costs are accounted for, an optimal policy $K_S(m,n,\beta)$ with both m and n being even integers is valid when

$$(mS_r + nS_p)(h_u\beta(1-\beta)) - 2\left(S_{r_1} - S_r + S_{p_1} - S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + \frac{h_u\beta^2}{m}\right) < 0;$$

while a policy $K_S(m,n,\beta)$ with both m and n being even integers cannot be optimal when

$$(mS_r + nS_p)(h_u\beta(1-\beta)) - 2\left(S_{r_1} - S_r + S_{p_1} - S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + \frac{h_u\beta^2}{m}\right) \geq 0,$$

where in this case, the average total cost rate associated with policy $K_S(m/2, n/2, \beta)$ is smaller.

PROOF: The inventories of manufactured and repaired items associated with policies $K_S(m,n,\beta)$, *Case a*, and $K_S(m/2, n/2, \beta)$, *Case b*, are shown in Figure 3.10.

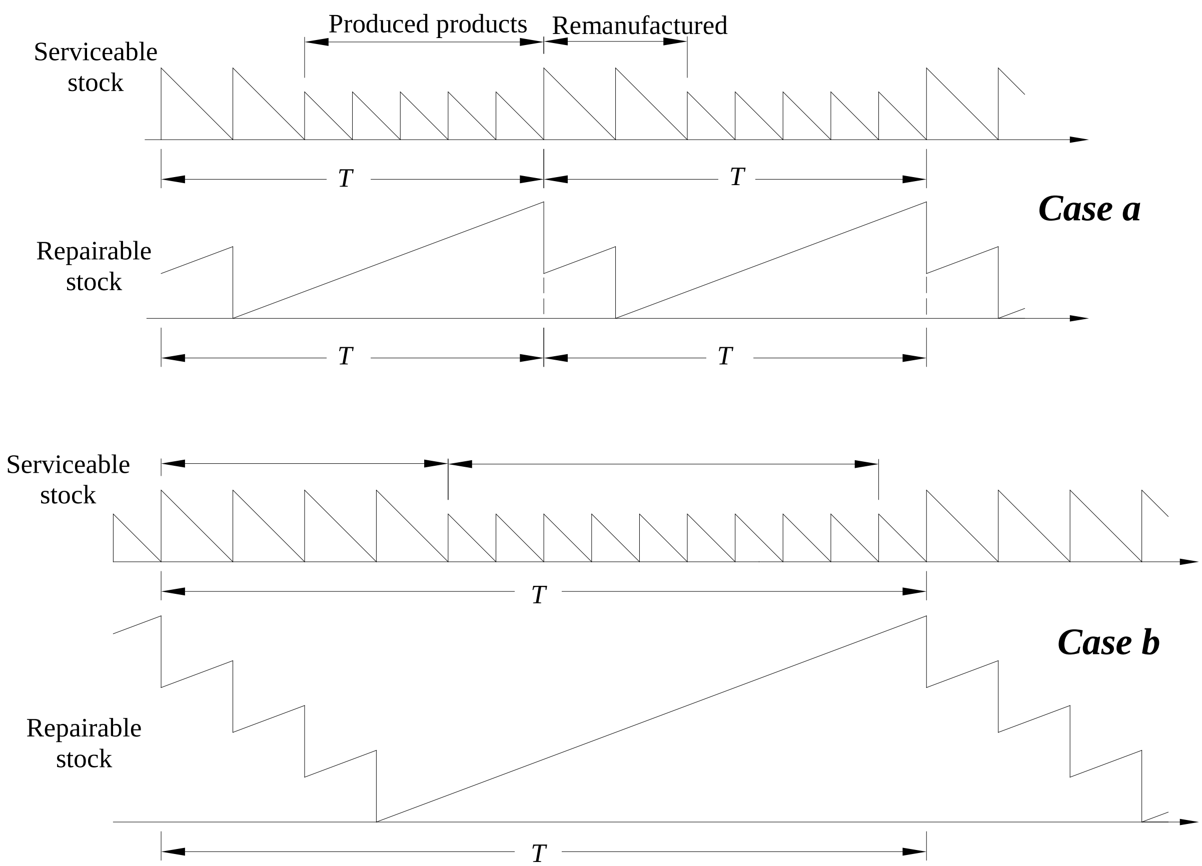


Figure 3.10. Inventory stocks for Case *a*: $K_{S,a}$ (m, n, β) and Case *b*: $K_{S,b}$ $(m/2, n/2, \beta)$

The parameters d, m, n, S_r, S_p, h_1, β and h_u in "*Case a*" correspond respectively to the parameters d, $m/2$, $n/2$, Sr, Sp, h_1, β, and h_u in "*Case b*". For "*Case a*", the unit time cost function is

$$K_{S,a}(m,n,\beta)=\sqrt{2d\left((m-1)S_r+S_{r_1}+(n-1)S_p+S_{p_1}\right)\left(h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+h_u\beta(1-\beta(1-1/m))\right)}$$

$$K_{S,a}(m,n,\beta)^2=2d\left(\left(mS_r+nS_p\right)+\left(S_{r_1}-S_r+S_{p_1}-S_p\right)\right)\left(h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+h_u\beta(1-\beta)+\frac{h_u\beta^2}{m}\right)$$

For "*Case b*", the unit time cost function is

$$K_{S,b}\left(\frac{m}{2},\frac{n}{2},\beta\right)=\sqrt{2d\left(\left(\frac{m}{2}-1\right)S_r+S_{r_1}+\left(\frac{n}{2}-1\right)S_p+S_{p_1}\right)\left(h_1\left(\frac{2(1-\beta)^2}{n}+\frac{2\beta^2}{m}\right)+h_u\beta(1-\beta(1-2/m))\right)}$$

$$K_{S,b}\left(\frac{m}{2},\frac{n}{2},\beta\right)^2=2d\left(\frac{\left(mS_r+nS_p\right)}{2}+\left(S_{r_1}-S_r+S_{p_1}-S_p\right)\right)\left(2h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+h_u\beta(1-\beta)+\frac{2h_u\beta^2}{m}\right)$$

The difference in the square of the unit time cost functions is given as

$$K_{S,a}(m,n,\beta)^2-K_{S,b}\left(\frac{m}{2},\frac{n}{2},\beta\right)^2$$

$$=d\left(mS_r+nS_p\right)\left(h_u\beta(1-\beta)\right)-2d\left(S_{r_1}-S_r+S_{p_1}-S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+\frac{h_u\beta^2}{m}\right)$$

The average total cost rate associated with policy $K_S(m/2,n/2,\beta)$ is less than $K_S(x,m,n,\beta)$ when

$$\left(mS_r+nS_p\right)\left(h_u\beta(1-\beta)\right)-2\left(S_{r_1}-S_r+S_{p_1}-S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+\frac{h_u\beta^2}{m}\right)\geq 0$$

$\forall m,n,\beta>0$.

In addition, an optimal policy $K_S(x,m,n,\beta)$ with both *m* and *n* being even integers has less associated costs than a policy $K_S(m/2,n/2,\beta)$ when

$$\left(mS_r+nS_p\right)\left(h_u\beta(1-\beta)\right)-2\left(S_{r_1}-S_r+S_{p_1}-S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n}+\frac{\beta^2}{m}\right)+\frac{h_u\beta^2}{m}\right)<0$$

$\forall m,n,\beta>0$.

3.5. Numerical Examples II

Example 3.2

Consider the input parameters from numerical example 3.1, where d = 10000, h_1 = 5, h_u = 2, S_r = 3, S_p = 9, β = 0.7. In addition to these inputs, consider two additional input parameters which are S_{r_1} = 4 and S_{p_1} = 11. The total cost K_S is computed for various *m* and *n* values and plotted in Figure 3.11.

The same behaviour as Figure 3.8 (no switching costs) is established. Switching costs push the solution for longer cycles as shown in Figure 3.11. This results from an increase in holding and setup costs. The optimal solution of 910.6 cost units was attained for n = 1 and m = 4, for a demand size per interval x = 527.1 units. In the case of n = 2 and m = 8, total costs are 983 cost units, and the demand size per interval x = 915.5 units. The optimum solution was not attained at m and n even numbers. The term

$\left(mS_r + nS_p\right)\left(h_u\beta(1-\beta)\right) - 2\left(S_{r_1} - S_r + S_{p_1} - S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + \frac{h_u\beta^2}{m}\right) > 0$, which

confirms Theorem 2.

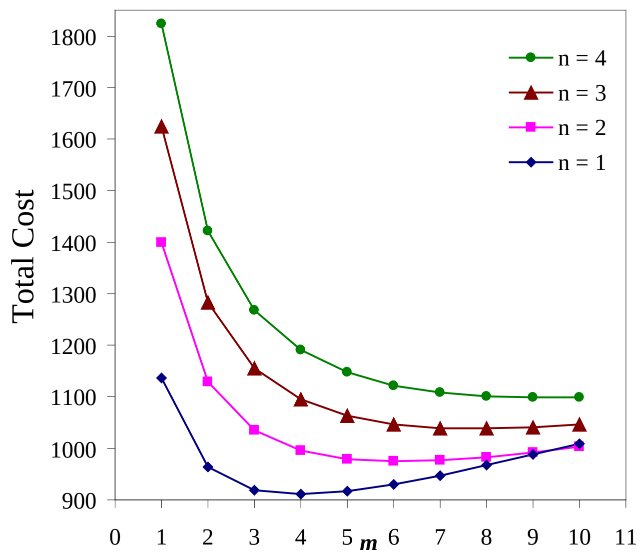


Figure 3.11. Total Cost function plotted for different values of recovery (m) and manufacturing (n) cycles in an interval of length "T"

Example 3.3

Consider a case with the parameters d = 10000, h_1 = 5, h_u = 2, S_r = 30, S_p = 90, S_{r_1} = 50 and S_{p_1} = 150, β = 0.3.

An optimal solution of 3541.7 cost units was attained for n = 2 and m = 2 (n, m even numbers), for a demand size per interval x = 1807 units. In the case of n = 1 and m = 1, total costs are 3741 cost units, and the demand size per interval x = 1069 units. The optimum solution was attained at m and n even numbers. The term

$$\left(mS_r + nS_p\right)\left(h_u\beta(1-\beta)\right) - 2\left(S_{r_1} - S_r + S_{p_1} - S_p\right)\left(h_1\left(\frac{(1-\beta)^2}{n} + \frac{\beta^2}{m}\right) + \frac{h_u\beta^2}{m}\right) < 0,$$ which

confirms Theorem 2.

3.6. Summary and Conclusions

In this chapter, a modification and an extension of the EOQ production, repair and waste disposal model (Richter, 1996a, b, 1997) is presented. First, and unlike the work available in the literature, this chapter includes the very first time interval where no repair occurs, to avoid unnecessary residual inventory and consequently avoid an overestimation of the holding costs in the repairable stock. Second, in this chapter, the work of Richter (1996a, b, 1997) was extended to consider switching costs when alternating between production and repair cycles.

The presented model showed that a policy with even numbers of production and repair cycles is never optimal, which was also found by (Teunter, 2001b). However, when accounting for switching costs, there are conditions to be met for the validity of this policy.

In this chapter, imperfect production and remanufacturing processes were not considered. The next chapter presents these considerations in a reverse logistics context.

CHAPTER 4: REVERSE LOGISTICS WITH IMPERFECT QUALITY AND PRODUCTION INTERRUPTIONS CONSIDERATIONS

4.1 Introduction

The EOQ/EPQ model has been widely used by practitioners and researchers and has been subjected to several extensions (e.g., supply chain and reverse logistics). However, the EOQ/EPQ model has been criticised because its assumptions are rarely met (e.g., Woolsey, 1990; Jones, 1991; Jaber et al., 2004). One of these assumptions is that production processes are perfect and defect free and that all items ordered/produced conform to quality. This assumption is not realistic (e.g., Gopalan & Kannan, 1995; Buzacott, 1999). Production and remanufacturing processes are not perfect and result in defective products that require reworking, that result in additional efforts and costs which should be eliminated (e.g., Wacker, 1987; Lee, 1992; Tang & Lo, 1993; Agnihothri & Kenett, 1995).

Rework is defined as all activities required transforming products that have not been produced or packaged according to preset qualifications into products that are (Flapper et al., 2002). Rework is the transformation of production rejects into re-usable products of the same or lower quality and could be defined as doing something at least one extra time (Love et al., 1999; Bohn & Terwiesch, 1999; Flapper & Teunter, 2004). Therefore, the reworked product does not have to be exactly as it was initially meant to be produced. This agrees with the Just in Time (JIT) approach, which cuts manufacturing costs by reducing production cycles to improve production quality by faster identification of defects, besides; inventory is a blanket that covers problems in production and quality (Reid, 1995; Waters, 2003). This encouraged manufacturers to reduce lot sizes to reduce defective units, and accordingly, reduce reworks.

Porteus (1986) and Rosenblatt & Lee (1986) are believed to be the first to capture the negative relationship between lot size and quality by extending the EOQ/EPQ models by considering that production processes are not defect free. Although both works approached the deterioration of the production process differently, they resulted in similar conclusions (Urban, 1998). Porteus (1986) assumed that while producing a lot, there is a possibility that the process goes out of control and once it does so, the process produces defective units until the end of the production lot. With similar assumptions, Rosenblatt & Lee (1986) compared between continuous and periodic inspection strategies and the system incurs additional costs because of these defectives. Both works realized that there is an incentive to produce smaller lots, to reduce the number of defective units. Other research works considered defective items are scrapped (e.g., Yano & Lee, 1995; Mohan & Ritzman, 1998).

Lee (1992) extended the work of Rosenblatt & Lee (1986) and assumed the time to detect the out-of-control process is a random process. Flapper et al. (2002) reviewed production planning works in the literature that considered rework processes, and identified the characteristics of the process industries that usually include rework in its production processes. Teunter & Flapper (2003) and Flapper & Teunter (2004) presented a single product production planning model that uses the same facilities for production and rework, assumed production rejects deteriorate while waiting to be reworked (e.g., food industry), and further assumed that the entire demand can never be satisfied. Inderfurth et al. (2005) extended it to the case of satisfying the entire demand by reworking all the defects, which their state deteriorates by time, and their rework cost increase by time. However, the authors restricted their model to the deterministic case. Teunter and Flapper (2006) presented a case study of lot sizing problem with planning and control of rework, and with the existence of storage space restrictions. Buscher & Lindner (2007) considered a production-rework lot sizing problem with each changeover from production to rework causes a fixed set-up cost.

The works of Porteus (1986) and Rosenblatt & Lee (1986) caught the attention of many researchers. Salameh & Jaber (2000) showed that although EOQ/EPQ inventory control model is widely used, it has some weaknesses, of which is the assumption of perfect production. The authors proposed a lot sizing and inspection policy assuming instantaneous replenishment and a random percentage of units replenished are defective. Salameh & Jaber (2000) was extended by Jaber (2006a) to the case where learning effects reduces the rate of generating defects and was later extended by Maddah & Jaber (2008) to address additional managerial insights.

In another line of research, Khouja (2005) addressed the non-perfect production assumption, however, the author assumed that production minor interruptions are allowed to adjust quality and to bring the production process to “in control” state. The author assumed non-instantaneous replenishment and derived closed-form formulas for the optimal number of minor adjustments and optimal lot sizes per cycle.

In reverse logistics literature, it has been assumed that production and remanufacturing/repair processes are perfect. This chapter investigates the imperfect production and remanufacturing processes in a reverse logistics context. Two models are developed; Model I integrates the work of Richter (1996a, b, 1997) with the works of Salameh & Jaber (2000) and Maddah & Jaber (2008). Model II, integrates the work of Dobos & Richter (2003, 2004) with the work of Khouja (2005).

4.2 Model I: A Production/Remanufacturing Model for Items with Imperfect Quality

Salameh & Jaber (2000) argued that although the EOQ/EPQ is widely used, the assumption of defect free replenishment is not true. In an instantaneous replenishment model, the authors presented a lot sizing and inspection policy, given a random percentage of units replenished is defective (i.e., electronics industry). Inspection is assumed an expensive process and there is a financial penalty for uninspected defectives. There is a cost to rework defective items, and the rework takes place instantaneously.

The model of Salameh & Jaber (2000) has been receiving the attention of researchers and was extended in several works (e.g., Goyal & Cárdenas-Barrón (2002); Papachristos & Konstantaras (2006); Wee et al. (2007); Eroglua & Ozdemir (2007); Chang, 2004; Wang et al., 2007. The model was investigated in an integrated vendor–buyer inventory context (Huang, 2002, 2004; Goyal et al., 2003; Chung & Huang, 2006; Ouyang et al., 2006). Maddah & Jaber (2008) corrected a flaw in the work of Salameh & Jaber (2000) and analyzed screening speed and variability of the supply process on the optimal order quantity. The authors presented simpler expressions for the order quantity and expected annual profit and found that the optimal order quantity is larger than the classical EOQ even when considering the variability of the fraction of imperfect items to be reasonably low. The models of Salameh & Jaber (2000), and Maddah & Jaber (2008) have never been investigated in a reverse logistics context.

Model I is presented with demand being satisfied from newly produced and remanufactures units. From the market, returns are collected in the repairable stock, and screened into two groups: acceptable items that are remanufactured and non-acceptable items that are disposed-off at a cost. Each production (remanufacturing) cycle $x(y)$ contains a percentage of imperfect quality items, ρ (λ), that are screened at a rate R_s. It is also assumed that the screening rate is faster than the demand rate, and it takes time t (t') to screen $y(x)$, where $R_s > d/(1\text{-}\rho)$ and $R_s > d/(1\text{-}\lambda)$, and therefore, no shortages occur. The nonconforming items are sold as a single cycle ρx (λy), where t and t' are respectively the times it take to screen x and y units respectively. v_p (v_r) is the unit selling price of a defective produced (remanufactured) item, k_p (k_r) unit selling price of a good produced (remanufactured item ($k > v$), and replenishment rates are assumed to be instantaneous. Q is the lot size quantity in an interval of length T, where $Q = my + nx$, where y (x) is the remanufacturing (production) cycle size

Model I considers the production and remanufacturing with imperfect quality considerations, where the flow of materials and products is similar to the production and remanufacturing system shown in Figure 3.1.

Collected returns composed of repairable returns and non-repairable items that are disposed at a cost. Collected returns are expressed as a percentage of the forward demand, and is equal to $(\alpha+\beta)d$. For further illustration, assume that the production system collects 80% of d in used items to be remanufactured or repaired. Suppose 70% of these used items are repairable, then β = 0.8x0.7 =0.56 or 56% of d are remanufactured/repaired. While, α = 0.8×0.3 = 0.24 or 24% of d are disposed outside the system at a cost.

In Model I, the remanufactured items are assumed as-good-as-new, demand is known, constant and independent, and lead time is zero with no shortages are allowed. The behaviour of inventory is illustrated in Figure 4.1. Profit maximization is adopted rather than cost minimization. The total profit per interval T is the sum of revenues from the remanufacturing and production process minus the total associated cost in all processes including inventory costs in the serviceable stock. The total revenue (TR) and total cost (TC) per a production cycle are computed respectively as

$$TR_p(x) = k_p(1-\rho)x + v_p x\rho \tag{4.1}$$

$$TC_p(x) = S_p + C_p x + C_s x + h_p\left(\frac{x^2}{2d}(1-\rho)^2 + \rho\frac{x^2}{R_s}\right) \tag{4.2}$$

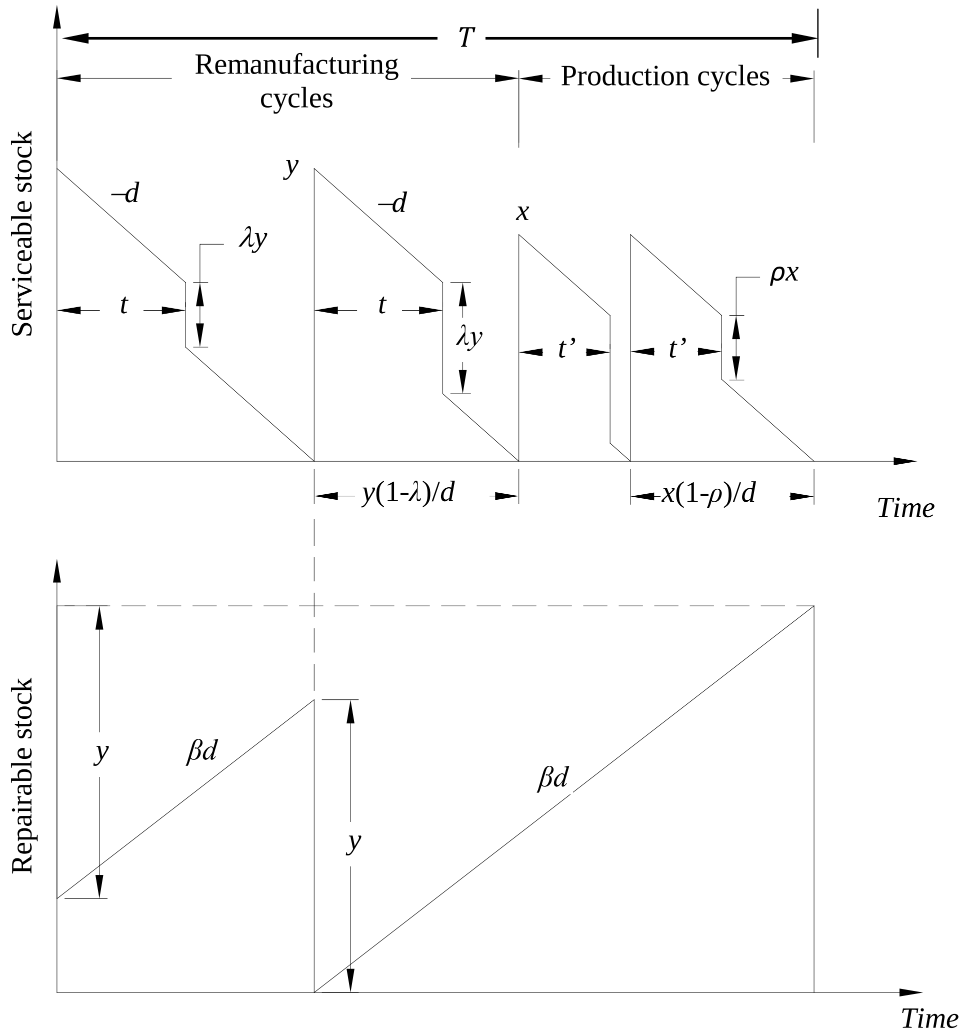


Figure 4.1. Inventory in serviceable and repairable stocks for m = 2, n = 2

The total profit (TP) per a production cycle is computed from (4.1) and (4.2) as

$$TP_p(x) = TR_p(x) - TC_p(x)$$

$$= k_p(1-\rho)x + v_p x\rho - \left[S_p + C_p x + C_s x + h_p\left(\frac{x^2}{2d}(1-\rho)^2 + \rho\frac{x^2}{R_s}\right)\right] \tag{4.3}$$

The revenue and cost per a remanufacturing cycle are determined similar to (4.1) and (4.2), and are given respectively as

$$TR_r(y) = k_r(1-\lambda)y + v_r y\lambda \tag{4.4}$$

$$TC_r(y) = S_r + C_r y + C_s y + h_r\left(\frac{y^2}{2d}(1-\lambda)^2 + \lambda\frac{y^2}{R_s}\right) \tag{4.5}$$

The total profit per a remanufacturing cycle is computed from (4.4) and (4.5) as

$$TP_r(y) = TR_r(y) - TC_r(y)$$

$$= k_r(1-\lambda)y + v_r y\lambda - \left[S_r + C_r y + C_s y + h_r\left(\frac{y^2}{2d}(1-\lambda)^2 + \lambda\frac{y^2}{R_s}\right)\right] \tag{4.6}$$

The decision variables are n, m, Q, and β, and the percentages of imperfect quality items, ρ and λ, are stochastic, as these percentages vary from cycle to cycle over T, which requires calculation the expected values of ρ and λ. Accordingly, and from Figure 3.1, $Q = dT = my(1-E[\lambda]) + nx(1-E[\rho])$, and the number of used units collected from the market over T that are remanufacturable is $\beta dE[T] = my$. Since $E[T] = T = my/\beta d$, therefore,

$$E[y] = y = \beta Q/m \tag{4.7}$$

$$E[x] = x = \frac{my(1-\beta(1-E[\lambda]))}{n\beta(1-E[\rho])} = \frac{1-\beta(1-E[\lambda])}{(1-E[\rho])n}Q \tag{4.8}$$

Note that for random variables $E[1-x] = (1-E[x])$ and $E[1-x]^2 \neq (1-E[x])^2$ (Ross, 1980).

The holding cost of collected used items (returns) in repairable stock over T is computed from Figure 4.1 as

$$H_u(m, y, \beta) = h_u\left[\frac{\beta dT^2}{2} - \frac{m(m-1)}{2} \times \frac{y(1-E[\lambda])}{d} \times y\right] \tag{4.9}$$

In their MILP model to optimize a municipal solid waste management system, Baetz & Neebe (1994) suggested expressions to estimate waste disposal either by incineration or by landfill. In the case of incineration, the (unit incineration cost) = (collection cost) + (waste-to-energy processing cost) – (energy revenue) + (residue factor) × (transport cost). In the case of landfill, the landfill disposal cost = compaction density (tons) × cost ($/ton) + unit collection cost ($/ton) + transport cost ($/ton). One may deduce that a waste disposal cost function per unit of time could be presented as

$$W = w\frac{\alpha d}{N} = C_w \alpha d \tag{4.10}$$

where w is the disposal cost per ton ($/ton) of returns that are not suitable for remanufacturing, N is the number of units per ton (units/ton) of waste, and $C_w = w/N$ is the unit disposal cost ($/unit). Several studies suggest a direct relationship similar to the one presented in (4.10).

Substituting (4.8) in (4.3), (4.6), (4.9) and (4.10) to calculate total profit for m remanufacturing cycles and n production cycles in an interval T,

$$\begin{aligned}
TP(n,m,Q,\beta) &= k_p(1-E[\rho])\left(\frac{1-\beta(1-E[\lambda])}{1-E[\rho]}\right)\frac{Q}{n}n + v_p E[\rho]\left(\frac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])}\right)\frac{Q}{n}n \\
&+ k_r(1-E[\lambda])\frac{\beta Q}{m}m + v_p E[\lambda]\frac{\beta Q}{m}m - \Bigg[nS_p + mS_r + (C_p + C_s)\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)\frac{Q}{n}n \\
&+ (C_r + C_s)\frac{\beta Q}{m}m + nh_p\left(\frac{E\left[(1-\rho)^2\right]}{2d}\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2\frac{Q^2}{n^2} + \frac{E[\rho]}{R_s}\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2\frac{Q^2}{n^2}\right) \\
&+ mh_r\left(\frac{E\left[(1-\lambda)^2\right]}{2d}\frac{\beta^2 Q^2}{m^2} + \frac{E[\lambda]}{R_s}\frac{\beta^2 Q^2}{m^2}\right) + h_u\left(\frac{\beta Q^2}{2d} - \frac{m(m-1)}{2}\frac{(1-E[\lambda])}{d}\frac{\beta^2 Q^2}{m^2}\right)\Bigg] - C_w \alpha d T
\end{aligned} \tag{4.11}$$

This total unit time profit function (ψ) of the production and remanufacturing system, where $T = Q/d$, is given as

$$\psi = \frac{TP(n,m,Q,\beta)}{T}, \text{ or}$$

$$\begin{aligned}
\psi(n,m,Q,\beta) &= \left(k_p(1-E[\rho]) + v_p E[\rho]\right)\frac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])}d + \left(k_r(1-E[\lambda]) + v_r E[\lambda]\right)\beta d \\
&- \left[(nS_p + mS_r)\frac{d}{Q} + \left((C_p + C_s)\frac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])} + (C_r + C_s)\beta\right)d\right] \\
&- \left[h_p\left(\frac{E\left[(1-\rho)^2\right]}{2d} + \frac{E[\rho]}{R_s}\right)\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2\frac{d}{n} + h_r\left(\frac{E\left[(1-\lambda)^2\right]}{2d} + \frac{E[\lambda]}{R_s}\right)\frac{\beta^2 d}{m}\right]Q \\
&- \frac{h_u \beta}{2}\left(1 - \frac{(m-1)}{m}(1-E[\lambda])\beta\right)Q - C_w \alpha d
\end{aligned} \tag{4.12}$$

To determine the optimum Q, (4.12) is differentiated with respect to Q.

Let $C = \left(k_p(1-E[\rho]) + v_p E[\rho]\right)\dfrac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])} d + \left(k_r(1-E[\lambda]) + v_r E[\lambda]\right)\beta d$

$$-\left((C_p + C_s)\frac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])} + (C_r + C_s)\beta\right)d - C_w \alpha d,$$

$$H = \left[h_p\left(\frac{E\left[(1-\rho)^2\right]}{2d} + \frac{E[\rho]}{R_s}\right)\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2 \frac{d}{n} + h_r\left(\frac{E\left[(1-\lambda)^2\right]}{2d} + \frac{E[\lambda]}{R_s}\right)\frac{\beta^2 d}{m}\right]Q$$

$$+\frac{h_u\beta}{2}\left(1 - \frac{(m-1)}{m}(1-E[\lambda])\beta\right)Q, \text{ and}$$

$$S = (nS_p + mS_r)$$

$\dfrac{\partial\psi}{\partial Q} = S\dfrac{d}{Q^2} - H$. Since $\dfrac{\partial^2\psi}{\partial Q^2} = -\dfrac{2Sd}{Q^3} < 0$ for every $Q > 0$, then $\dfrac{\partial\psi}{\partial Q} = S(n,m)\dfrac{d}{Q^2} - H = 0$,

and, $Q^*(n,m,\beta) = \sqrt{\dfrac{Sd}{H}}$, whose expanded form is

$$Q^*(n,m,\beta) = \sqrt{\frac{(nS_p + mS_r)d}{\left[h_p\left(\frac{E\left[(1-\rho)^2\right]}{2d} + \frac{E[\rho]}{R_s}\right)\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2 \frac{d}{n} + h_r\left(\frac{E\left[(1-\lambda)^2\right]}{2d} + \frac{E[\lambda]}{R_s}\right)\frac{\beta^2 d}{m} + \frac{h_u\beta}{2}\left(1 - \frac{(m-1)}{m}(1-E[\lambda])\beta\right)\right]}} \tag{4.13}$$

Substitute (4.13) in (4.12), then (4.12) could be expressed as

$$\psi(n,m,\beta) = \left(k_p(1-E[\rho]) + v_p E[\rho] - (C_p + C_s)\right)\frac{(1-\beta(1-E[\lambda]))}{(1-E[\rho])} d$$

$$+\left(k_r(1-E[\lambda]) + v_r E[\lambda] - (C_r + C_s)\right)\beta d - C_w\alpha d - \sqrt{d(nS_p + mS_r)} \times$$

$$\sqrt{\left[h_p\left(\frac{E\left[(1-\rho)^2\right]}{2d} + \frac{E[\rho]}{R_s}\right)\left(\frac{1-\beta(1-E[\lambda])}{(1-E[\rho])}\right)^2 \frac{d}{n} + h_r\left(\frac{E\left[(1-\lambda)^2\right]}{2d} + \frac{E[\lambda]}{R_s}\right)\frac{\beta^2 d}{m} + \frac{h_u\beta}{2}\left(1 - \frac{(m-1)}{m}(1-E[\lambda])\beta\right)\right]} \tag{4.14}$$

The cases when $\beta = 0$ and $\beta = 1$ are the extreme ones representing the pure production and the pure remanufacturing policies respectively. The pure production policy (i.e., $\beta = 0$) is represented as

$$\psi_p = \left(k_p(1-E[\rho]) + v_p E[\rho] - (C_p + C_s)\right)\frac{d}{(1-E[\rho])} - \frac{2d}{(1-E[\rho])}\sqrt{S_p h_p\left(\frac{E\left[(1-\rho)^2\right]}{2d} + \frac{E[\rho]}{R_s}\right)} \tag{4.15}$$

where $\beta = 0$, $m = 0, n \to \infty$; representing the EOQ model for items with imperfect quality. The pure remanufacturing policy is similar to (4.14) while β = 1 and the optimum solution is given from (4.14) and (4.15) as

Min $\{\psi(n, m, \beta), \psi_P\}$ (4.16)

Solution Procedure

STEP 1: For the set of input parameters d, S_p, S_r, h_s, h_r, h_u, R_s, λ, ρ, k_p, k_r, v_p, v_r, C_p, C_r, C_w, C_s, and α. Set m = 1 and i = 1 (i.e., n = 1), and optimize $E[\psi(1,1,\beta)]$ using (4.16), where $E[\psi(1,1,\beta^*_{1,1})]$ is the optimal profit for $i = n = 1$, m =1 and $\beta = \beta^*_{1,1}$.

STEP 2: Repeat STEP 1 for m = 2 and record $E[\psi(1,2,\beta^*_{1,2})]$. Compare $E[\psi(1,1,\beta^*_{1,1})]$ and $E[\psi(1,2,\beta^*_{1,2})]$. If $E[\psi(1,1,\beta^*_{1,1})] \geq E[\psi(1,2,\beta^*_{1,2})]$, terminate the search for (n = 1) and record the value of $E[\psi(1,1,\beta^*_{1,1})]$. If $E[\psi(1,1,\beta^*_{1,1})] < E[\psi(1,2,\beta^*_{1,2})]$, repeat for ($m$ = 3), (m = 4), etc. Terminate once $E[\psi(1, m_1^* - 1, \beta^*_{1,m_1^*-1})] < E[\psi(1, m_1^*, \beta^*_{1,m_1^*})] > E[\psi(1, m_1^* + 1, \beta^*_{1,m_1^*+1})]$, where m_1^* is the optimal value for the number of remanufacturing cycles when there is 1 production cycle. Record the values of $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})]$, m_1^*, and β^*_{1,m_1^*}.

STEP 3: Repeat STEPS 1 and 2 for $i = n = 2$. Compare $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})]$ and $E[\psi(2, m_2^*, \beta^*_{2,m_2^*})]$. If $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})] > E[\psi(2, m_2^*, \beta^*_{2,m_2^*})]$, terminate the search and $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})]$ is the optimum solution. If $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})] < E[\psi(2, m_2^*, \beta^*_{2,m_2^*})]$, then drop the value of $E[\psi(1, m_1^*, \beta^*_{1,m_1^*})]$ and repeat steps 1 and 2 for $i = n$ = 3, 4, 5,…, etc.

STEP 4: Terminate the search once $E[\psi(i-1, m_{i-1}^*, \beta^*_{i-1,m_{i-1}^*})] < E[\psi(i, m_i^*, \beta^*_{i,m_i^*})] > E[\psi(i+1, m_{i+1}^*, \beta^*_{i+1,m_{i+1}^*})]$, where i is the optimal value for the number of production cycles when there are m_i^* remanufacturing cycles at a profit $E[\psi(i, m_i^*, \beta^*_{i,m_i^*})]$ with an optimal β^*_{i,m_i^*}.

4.3 Numerical Examples I

Several numerical examples are solved to facilitate understanding the behaviour of the mathematical models developed and to draw some conclusions. Examples 4.1, 4.2 and 4.3 has similar input data to an example in Salameh & Jaber (2000), and these three examples highlight the main features of Model I. Example 4.4 has similar input data to an example in Dobos & Richter (2004) to show the similarities and differences features between Model I and that of Dobos & Richter (2004).

Example 4.1

This example is similar to those in Salameh & Jaber (2000) and Maddah & Jaber (2008). The fraction of imperfect quality items for production and remanufacturing are assumed to be uniformly distributed random variables over (a, b), where $0 < a < b < 1$, i.e., $\lambda \sim U(a, b)$. Therefore

$$E[\lambda] = a + b/2\,,\; E\left[(1-\lambda)^2\right] = \left(a^2 + ab + b^2\right)/3 + 1 - a - b\,,$$
$$E\left[(1-\beta(1-\lambda))^2\right] = 1 - 2\beta + \beta(b+a) + \beta^2\left(\frac{a^2+ab+b^2}{3} + 1 - a - b\right),$$
$$E\left[\lambda^2\right] = \frac{a^2+ab+b^2}{3}\,,$$
$$E\left[\frac{1}{(1-\lambda}\right] = \frac{\ln(1-a) - \ln(1-b)}{b-a}\,,$$
$$E\left[\frac{1}{(1-\lambda)^2}\right] = \frac{1}{(1-a)(1-b)}\,,$$

The same applies to ρ. In this example, the demand rate is d = 50,000 units/year, the setup costs are $S_p = S_r$ = 100/cycle,the holding costs are $h_p = h_r = h_u$ = 5/unit/year, the screening rate is R_s = 175,200 units/year, the screening cost is C_s = 0.5/unit, the production and remanufacturing costs are $C_p = C_r$ = 25/unit, the selling price of good quality items are $k_p = k_r$ =50/unit, the selling price of imperfect quality items, $v_p = v_r$ = 20/unit, the disposal cost C_w = 0, and $\lambda \sim U(a, b)$ and $\rho \sim U(a, b)$, where a = 0, b = 0.04.

The model is optimized using the presented solution procedure for maximum profit (ψ), and the optimum order quantity is found to be equal to Q^*= 1405, and optimum profit is ψ^*=1,121,274 that corresponds to total costs TC = 7,113, where β^* = 0. In this example, the input cost parameters for remanufacturing were higher than those of production, with the optimal solution favouring a policy of pure production. The numerical results from this example conforms to those of Salameh & Jaber (2000) and Maddah & Jaber (2008). As β = 0, the model represents the forward case, without any remanufacturing (no returns). The model was investigated for varying values of $\beta \in (0, 1)$ while optimizing for the total profit and the total costs independently. The results are shown in Figure 4.2.

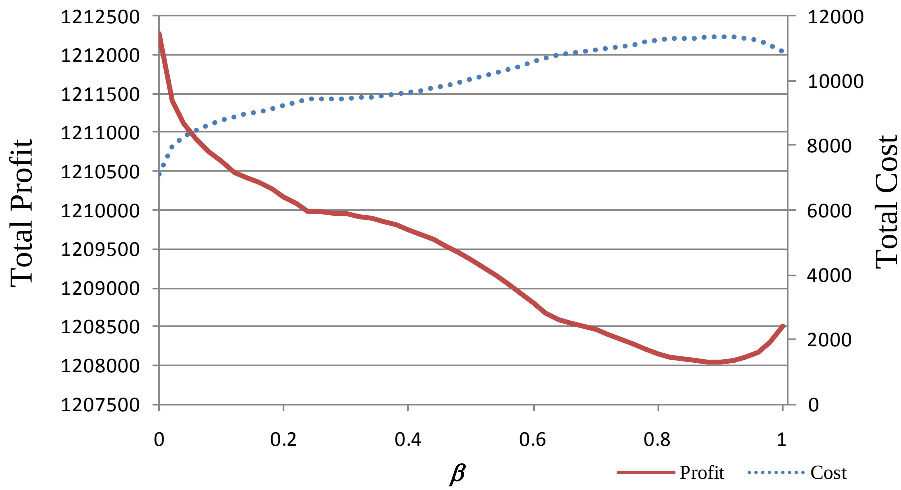


Figure 4.2. The behaviour of Total Profit and Total Cost of Example 4.1 for varying values of "β"

Figure 4.2 shows that as β increases the percentage of returns increases, and subsequently increasing the remanufacturing costs. The results show that for the total profit curve and total costs curve are mirror images of one another, where the maximum profit and minimum costs corresponds to the same $\beta^* = 0$.

Example 4.2

This example considers input parameters similar to those in Example 4.1, except for the production holding cost $h_p = 12 > h_r = 5$. The model is optimized for maximum profit (ψ), and the optimum order quantity is found to be equal to Q^*= 10088, β^* = 1 (i.e., pure production), and optimum profit is ψ^*= 1,208,484 that corresponds to total costs TC^* = 10903.7, n^* =1, m^* =10. Similar to Example 4.1, the model was also investigated for varying values of $\beta \in (0, 1)$ while optimizing the total profit and total costs independently. The results are shown in Figure 4.3.

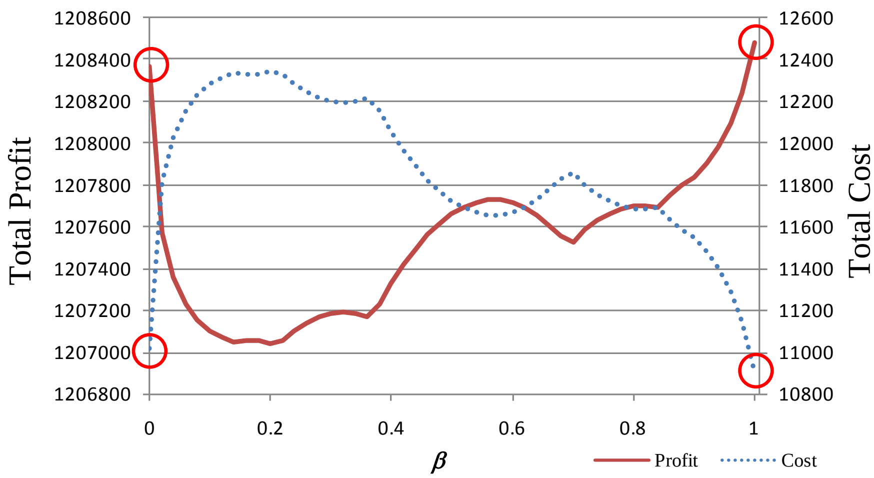


Figure 4.3. The behaviour of Total Profit and Total Cost of Example 4.2 for varying values of "β" with the solutions for the extreme cases (β = 0, 1) are circled

The results in Figure 4.3 show that the pure remanufacturing policy is more profitable than the pure production policy, and the optimum solution is attained at β = 1. In this example, at β = 0, the solution represents the forward case, where only the production setup cost is included in the total cost function. As β increases, a mixture of production and remanufacturing occurs, where multiple production and remanufacturing setup costs are added to the total cost. This explains the rapid increase in the total cost as β increases beyond 0. As β continues to increase approaching the value 1, the percentage of returns increases, decreasing the remanufacturing costs to a level less than those of production, resulting in a reduction of total costs. This result suggests that the bang-bang policy (Richter, 1997; Dobos & Richter, 2003; 2004) of pure production or pure remanufacturing is optimal when imperfect production and remanufacturing is accounted for. Note that the irregularity of the trend in the range of (β = 0.2) to (β = 0.8) is due to the integer nature of *m* and *n*.

The total profit is composed of the sum of revenues from production and remanufacturing minus the some of the total costs for both processes. When the input parameter of revenues and total costs for remanufacturing and production are equal, then the optimum solution obtained from either optimizing the profit function or the total cost function is the same. This is a special case, and the same may not hold when the input parameters for revenues and costs are not equal. This will be investigated in the Example 4.3.

Example 4.3

Input parameters used are similar to those of Example 4.2, except for k_p = 50.01 (in example 4.2, k_p = 50). The model is investigated for varying values of $\beta \in (0, 1)$ while optimizing for total profit and total costs independently. The results are shown in Figure 4.4.

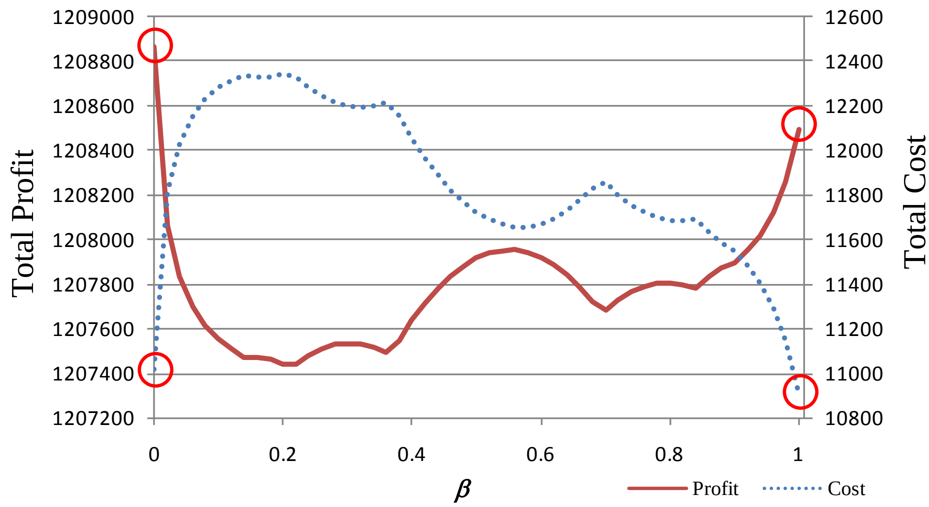


Figure 4.4. The behaviour of Total Profit and Total Cost of Example 4.3 for varying values of "β" with the solutions for the extreme cases (β = 0, 1) are circled

The optimum profit (i.e., optimum solution) is attained at $\beta^* = 0$, where the policy is pure production, however, the least cost is at pure remanufacturing ($\beta^* = 1$). One may notice that in this example, the optimal solution obtained from optimizing the total profit does not coincide with that obtained from optimizing the total costs. This suggests that a cost minimization approach (e.g., Richter, 1996a, b) may not always guarantee the best solution, and therefore, a profit approach is more appropriate.

Example 4.4

The input parameters for this numerical example are adopted from Dobos & Richter (2004, example 5, pp.321), and the fraction of imperfect quality is considered uniformly distributed over (a, b), $0 < a < b < 1$ for both remanufacturing and production. Demand rate is d = 1000 units/year, the setup cost are S_p = 1960/cycle and S_r = 440/cycle, the serviceable holding costs are $h_p = h_r$ = 850/unit/year, the repairable holding cost is h_u = 80/unit/year, screening rate is R_s = 5000 units/year, a = 0 and b = 0.45. In this example, a cost minimization approach is adopted.

The optimum solution occurs at $Q^* = 63.4$ where $TC^* = 40591$, $\beta^* = 1$, $m^* = 7$, and $n^* = 1$. The solution conforms to the solution of Dobos & Richter (2004), where the savings from remanufacturing favours a pure remanufacturing policy. The model was also investigated for varying $\beta \in (0, 1)$, while optimizing the total costs. The results are shown in Figure 4.5.

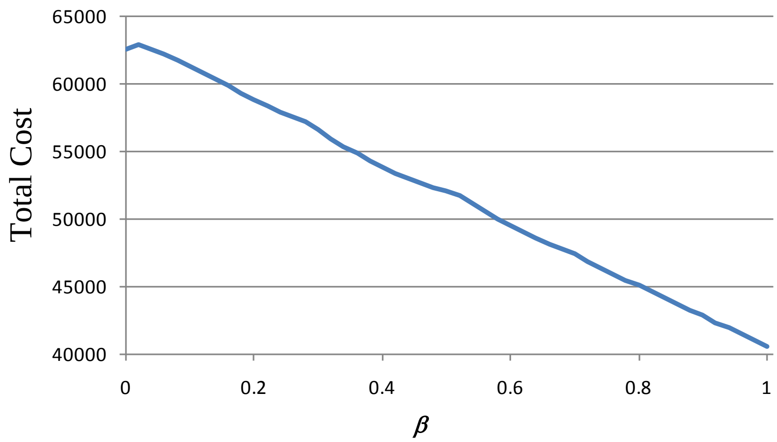


Figure 4.5. The behaviour of Total Cost of Example 4.4 for varying values of "β"

Figure 4.5 shows that the total cost decreases linearly from pure production to pure manufacturing with the optimum solution obtained at the extreme case $\beta = 1$. This is a similar behaviour to that produced by Dobos & Richter (2004), which brings us to their conclusion, that the pure bang-bang policy still dominates the production repair lot sizing problem. However, solution of Model I is not equal to that of Dobos & Richter (2004). The pure remanufacturing case (i.e., $\beta = 1$) in Richter (1996a, b, 1997) and Dobos & Richter (2003, 2004) means that only remanufacturing occurs without any production.

In Model I, the ($\beta = 1$) case represents near remanufacturing solution, where a portion of the remanufactured lots is defective, which is substituted by newly produced units. Hence, the pure remanufacturing case is still a mixture of remanufacturing and production. This is explained by the existence of surge of costs at the ($\beta = 0$) point and not at the ($\beta = 1$) case. At the ($\beta = 0$), production setup cost only exists, and remanufacturing setup cost is added once β exceeds 0, which makes it a pure production case. However, both setup costs exist at the ($\beta = 1$) case, which proves that the existence of pure remanufacturing case is technically impossible.

4.4 Model II: Production/Remanufacturing Corrective Interruptions to Improve Quality in a Reverse Logistics Environment

Classical inventory models, mostly EOQ models, assume production results in no defective items; however, production (and remanufacturing) processes are not perfect. Defective items generated from imperfect production and remanufacturing processes are reworked. Model II presents reworking as a method to correct defectives, in a reverse logistics inventory context.

Dobos & Richter (2003, 2004) addressed reverse logistics in a production remanufacturing lot sizing model that consists of two inventory stocks, serviceable and repairable stocks. In reverse logistics literature, it has always been assumed that production and remanufacturing/repair processes are defect free. Production, as well as remanufacturing, processes are not defect-free. These processes result in items that require reworking (Wein, 1992). Rework is defined as doing something at least one extra time due to a non-conformance (e.g., Love et al., 1999).

Khouja (2005) extended the work of Porteus (1986) and showed that reworks are reduced by interrupting the production process to bring it to "in-control" state. Interrupting the production process is a common practice in JIT manufacturing environments where line workers have the authority to stop the line to bring production quality to "in control" state (Inman & Brandon, 1992). Khouja (2005) showed that interrupting the production process reduces the number of defectives, and subsequently the total system costs. The model developed by Khouja (2005) has never been investigated in a reverse logistics context.

Model II introduces the imperfect production and remanufacturing concept in a reverse logistics context by integrating the works of Richter (2003, 2004) and Khouja (2005). Model II assumes that the quality of the production and remanufacturing processes are interrupted to be restored to in-control state, in a production/remanufacturing environment that consists of two stocks; the serviceable stock (newly produced and remanufactured items), and the repairable stock (collecting returned items for repair). In Model II, the decision variables are β, m, n, λ_m, λ_n, and Y. Y is the remanufacturing lot size quantity for used units that are collected in an interval of length T, and λ_m (λ_n) is the number of minor setups in the remanufacturing (production) cycle following an interruption to restore the process. X

represents the production lot size quantity for newly produced units, where X is dependent on Y and β, and therefore it is not a decision variable. There are Y remanufactured and X newly produced units in time interval T.

Model II considers the production and remanufacture model described in Figure 3.1, where a manufacturing environment consists of two stocks. The serviceable stock stores remanufactured used and newly produced units of a product. The used units (returns) are collected in the repairable stock. In each interval of length T, there are m remanufacturing cycles of repairable items and n production cycles for newly produced items.

Market demand of rate d is satisfied by remanufacturing and production cycles. Returns are collected in the repairable stock , sorting and filtering is performed to dispose a quantity of αd units, and the remaining amount (βd) is remanufactured. Accordingly, the remanufactured amount satisfy part of market demand equivalent to (βd), and the rest ($(1-\beta)d$) is satisfied by newly produced items. The ratio between the remanufactured amount Y per cycle for m cycles and the newly produced amount X per cycle for n cycles is equal to the ratio of $\beta dT/(1-\beta)dT$. Therefore,

$$X(Y,\beta)=\frac{(1-\beta)m}{\beta n}Y \tag{4.17}$$

In Model II, shortages are not allowed, and it is assumed that lead-time is zero, repairable items are as good as new and defective units are reworked at a fixed cost per unit, no defective units are scrapped, and that the production and recoverable repair processes are always in control. Figure 4.6 illustrates the behaviour of inventory over a cycle of length T.

For each interval T, inventory starts to build up by remanufacturing a lot at a rate R_r and at the same time inventory is depleted at a rate d. Interruption as a minor setup takes place for a period of τ_r to restore the remanufacturing process quality and to bring it to "in control" state. This interruption occurs at time = $Y/R_r(\lambda_m+1)$ and during which inventory depletes. Afterwards, the remanufacturing process resumes for another period of length $Y/R_r(\lambda_m+1)$ and is interrupted for another (λ_m-1) minor setups, until inventory reaches a maximum level at $Y(1-d/R_r)-\lambda_m\tau_r d$ at time $(Y/R_r)+\lambda_m\tau_r$.

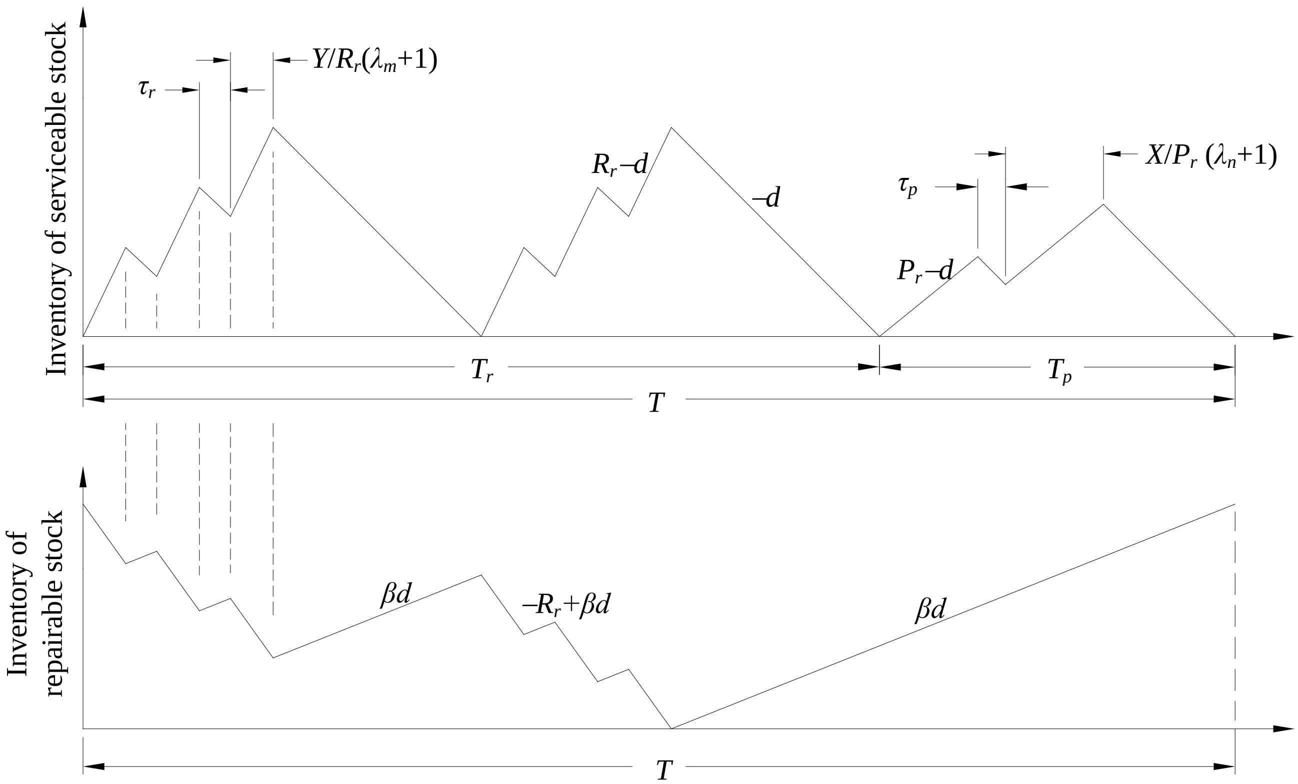


Figure 4.6. Inventory in serviceable stock and repairable stock for m = 2, n = 1

Afterwards, remanufacturing ceases and the maximum accumulated inventory of remanufactured units start to deplete at a rate d, until it reaches the zero level. The same behaviour is repeated for (m–1) remanufacturing cycles that are performed in T_r. The production cycle commences at time T_r, and the inventory of newly produced items start to build at a rate P_r–D. The production ceases for a minor setup at $X/P_r(\lambda_n+1)$ for a τ_p period of time. Similar to the remanufacturing cycle, the accumulated inventory of newly produced items reaches its maximum level at $X(1-d/P_r)-\lambda_n\tau_p d$, and then depletes at a rate d until it reaches the zero level. The same behaviour is repeated for (n–1) production cycles that are performed in T_p.

At the start of each interval T, the inventory level in the repairable stock is at its maximum level, which is the total amount of used items collected from the previous time interval. Inventory starts to deplete by repairing R_r units and at the same time starts to increase by collecting used items at a rate of βd. The remanufacturing process is interrupted after $Y/R_r(\lambda_m+1)$ units of time to restore quality through a minor setup. During the minor setup, inventory builds up at a rate βd for a period of length τ_r, then, remanufacturing resumes for another period of length $Y/R_r(\lambda_m+1)$ while being interrupted for ($m\lambda_m-1$) minor setups, until the inventory reach its minimum zero level at time $(m-1)(Y/d)+(Y/R_r)+\lambda_m\tau_r$. Accordingly, remanufacturing process halts, and inventory increases by receiving returned collected items to reach the maximum level $=Y(m(1-\beta)+\beta(1-d/R_r))-\lambda_m\tau_r\beta d$, at the end of the production/repair interval of length *T*.

Porteus (1986) suggested that smaller lot sizes improves the output quality, and estimated the number of defective units in a lot of size *Y'* that needs reworking to be *j*, where $j=Y'-(1-q)(1-(1-q)^{Y'})/q$ and *q* is the probability of the process going out-of-control. For small values of *q*, the expected percentage of defectives in a lot of size *Y'* could be approximated to $qY'/2$. If the production process is interrupted to be restored to an in-control state, the percentage of defects generated is reduced to $qY'/(2(\lambda+1))$, where λ is the number of interruptions (Khouja, 2005). This concept is introduced in a reverse logistics context where the total cost is the summation of the setup costs, holding costs, and rework costs for the serviceable and repairable stocks.

Inventory is calculated for the repairable and the serviceable stocks. For the serviceable stock, the average inventory of remanufacturing and production cycles is $(Y(1-d/R_r)-\lambda_m\tau_r d)/2$ and $(X(1-d/P_r)-\lambda_n\tau_p d)/2$ respectively. For the repairable stock, holding costs are represented by two components: the inventory of remanufactured units provided from the repairable to the serviceable stock and the inventory of used items collected to form the repairable stock, shown in Figures 4.7-*a* and 4.7-*b* respectively. The result of adding the areas in Figures 4.7-*a* and 4.7-*b* is shown in Figure 4.7-*c*, where the residual inventory from this addition (rectangle *E*) should be eliminated to produce Figure 4.7-*d*, otherwise, the maximum inventory will be overestimated to be *Z′* instead of *Z*.

The very first time interval where no repairs are performed is usually ignored in literature, because a general time interval is assumed, which results in an overestimation of the average inventory level and, subsequently, of an overestimation of the holding costs, i.e., area E in Figure 4.7-c. A similar approach was taken by Minner & Lindner (2004), who showed that an overestimation of inventory in the serviceable stock resulted in having the last remanufacturing lot in the time interval smaller than the other lots.

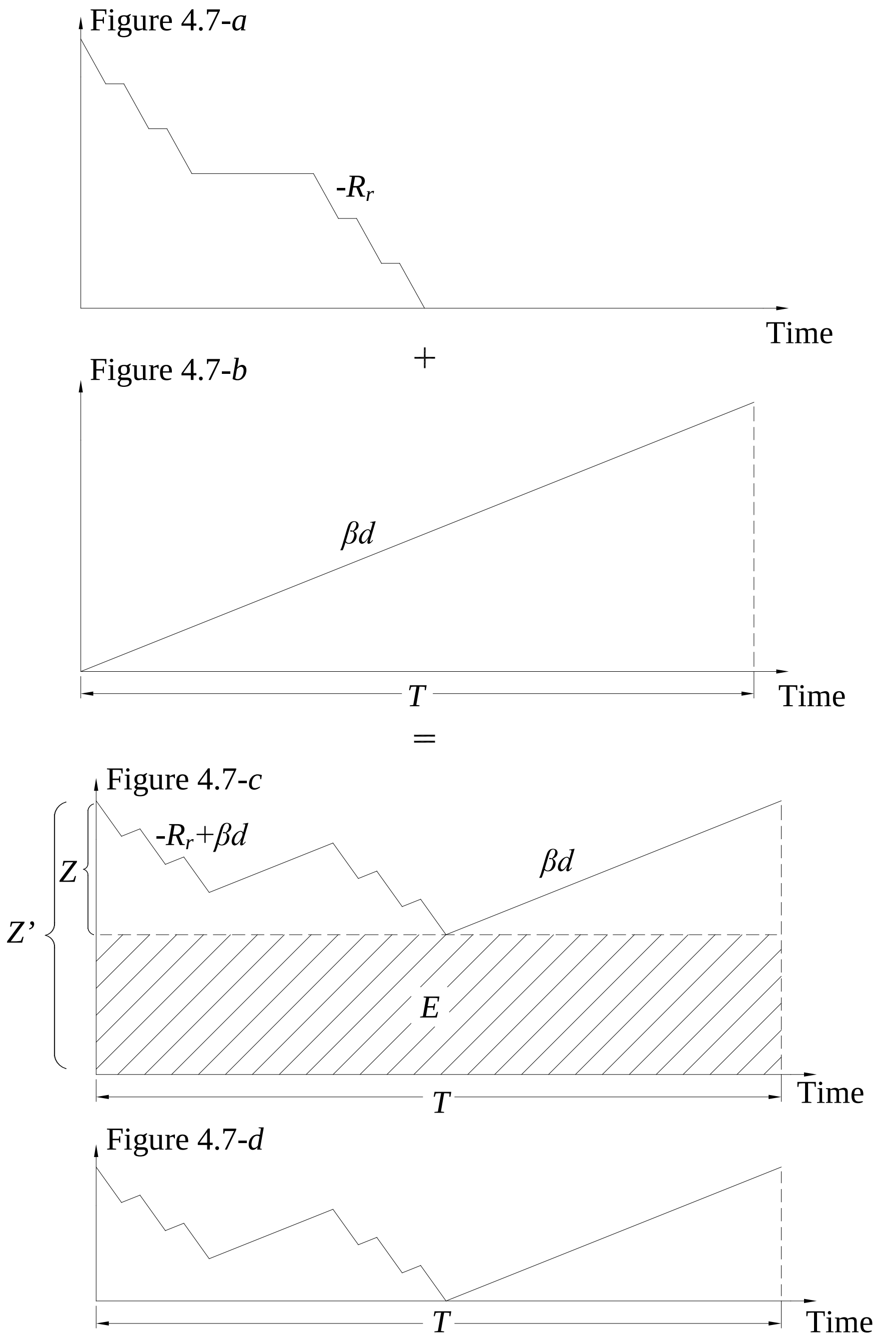


Figure 4.7. Breakdown of inventory in the repairable stock

The length of the first time interval consists of production and no repair lots is determined as

$$T_1(Y,\beta)=T-(m-1)(Y/d)-(Y/R_r)-\lambda_m\tau_r=Y\left(\frac{m-\beta(m-1)}{\beta d}-\frac{1}{R_r}\right)-\lambda_m\tau_r \qquad (4.18)$$

The average inventory of the repairable stock is equal to $\left(Y(m(1-\beta)+\beta(1-d/R_r))-\lambda_m\tau_r\beta d\right)/2$. Note that time interval T could be represented as a function of m, n, and Y; i.e., as $T_r+T_p=mY/d+nX/d=mY/\beta d$. Therefore, the unit time cost function can then be written as

$$\psi(n,m,\lambda_n,\lambda_m,Y,\beta)=\frac{\beta d}{Y}\left(m(S_r+\lambda_m s_r)+n(S_p+\lambda_n s_p)\right)+h_r\left(Y(1-d/R_r)-\lambda_m\tau_r d\right)/2$$
$$+h_p\left((1-d/P_r)\frac{(1-\beta)}{\beta}\frac{m}{n}Y-\lambda_n\tau_p d\right)\Big/2+h_u\left(Y(m(1-\beta)+\beta(1-d/R_r))-\lambda_m\tau_r\beta d\right)/2$$
$$+\frac{\beta dY}{2}\left(\frac{q_rCr_r}{(\lambda_m+1)}+\frac{q_pCr_p}{(\lambda_n+1)}\frac{(1-\beta)^2}{\beta^2}\frac{m}{n}\right) \qquad (4.19)$$

where Cr_r (Cr_p) is the cost of reworking one defective unit as a result of remanufacturing (production) process, and q_r (q_p) is the probability of the remanufacturing (production) process going out-of-control.

Equation (4.19) is convex over Y, $\because \partial^2\psi(.,Y,.)/\partial Y^2 = 2\beta d\left(S_r+\lambda_m s_r+\frac{n}{m}(S_p+\lambda_n s_p)\right)\Big/Y^3 >$ 0 $\forall\, Y>0$. Setting the first derivative of (4.19) equal to zero and solving for Y to get

$$Y(n,m,\lambda_n,\lambda_m,\beta)=\sqrt{\frac{2\beta d\left(S_r+\lambda_m s_r+\frac{n}{m}(S_p+\lambda_n s_p)\right)}{\left[\beta d\left(\frac{q_rCr_r}{(\lambda_m+1)}+\frac{q_pCr_p}{(\lambda_n+1)}\frac{(1-\beta)^2}{\beta^2}\frac{m}{n}\right)+h_rY\left(1-\frac{d}{R_r}\right)+h_p\left(1-\frac{d}{P_r}\right)\frac{(1-\beta)}{\beta}\frac{m}{n}+h_u\left(m(1-\beta)+\beta\left(1-\frac{d}{R_r}\right)\right)\right]}} \qquad (4.20)$$

Substituting (4.20) in (4.19), reduces (4.19) to

$$\psi(n,m,\lambda_m,\lambda_n,Y,\beta)=\sqrt{2\beta d\left(S_r+\lambda_m s_r+\frac{n}{m}(S_p+\lambda_n s_p)\right)}\,.$$
$$\sqrt{\left[\beta d\left(\frac{q_rCr_r}{(\lambda_m+1)}+\frac{q_pCr_p}{(\lambda_n+1)}\frac{(1-\beta)^2}{\beta^2}\frac{m}{n}\right)+h_rY\left(1-\frac{d}{R_r}\right)+h_p\left(1-\frac{d}{P_r}\right)\frac{(1-\beta)}{\beta}\frac{m}{n}+h_u\left(m(1-\beta)+\beta\left(1-\frac{d}{R_r}\right)\right)\right]}$$

$$-d\left(h_r\lambda_m\tau_r + h_p\lambda_n\tau_p + h_u\lambda_m\beta\tau_r\right)/2 \tag{4.21}$$

The objective is to minimize the total cost per unit time (4.30), as

$$\text{Minimize}\,\psi\left(n, m, \lambda_n, \lambda_m, \beta\right) \tag{4.22a}$$

Subject to:

$$0 < \beta < 1 \tag{4.22b}$$

$$\tau_r \le \frac{Y\left(1/d - 1/R_r\right)}{\lambda_m} \tag{4.22c}$$

$$\tau_p \le \frac{Y\left(1/d - 1/P_r\right)}{\lambda_n}\left(\frac{m}{n}\frac{\left(1-\beta\right)}{\beta}\right) \tag{4.22d}$$

$$n, m, \lambda_n, \lambda_m \in \{0,1\} \tag{4.22e}$$

Constraints (4.22c) and (4.22d) prevent stock-outs from occurring in the serviceable stock.

The extreme cases of $\beta = 0$ and $\beta = 1$ represent, respectively, the policies of pure production and the pure remanufacturing. Therefore, the unit time cost for a pure production policy is given as

$$\psi_p\left(\lambda_n\right) = \sqrt{2d\left(S_p + \lambda_n s_p\right)\left(\frac{q_p Cr_p d}{\left(\lambda_n + 1\right)} + h_p\left(1 - \frac{d}{P_r}\right)\right)} - h_p\lambda_n\tau_p d/2 \tag{4.23}$$

where $\beta = 0$, $m = 0, n \to \infty$; representing the EOQ model with minor setups to improve product quality. The unit time cost for a pure remanufacturing policy is given as

$$\psi_R\left(\lambda_m\right) = \sqrt{2d\left(S_r + \lambda_m s_r\right)\left(\frac{q_r Cr_r d}{\left(\lambda_m + 1\right)} + \left(h_r + h_u\right)\left(1 - \frac{d}{R_r}\right)\right)} - \left(h_r + h_u\right)\lambda_m\tau_r d/2 \tag{4.24}$$

where $\beta = 1$, and the optimum solution is given from (4.22a-4.22e), (4.23) and (4.24) as

$$\text{Min}\ \left\{\psi\left(n, m, \lambda_n, \lambda_m, \beta\right), \psi_p\left(\lambda_n\right), \psi_R\left(\lambda_m\right)\right\} \tag{4.25}$$

4.5 Numerical Examples II

This section provides numerical examples to illustrate the different behaviours of the presented model and to draw some conclusions.

Example 4.5

Let d = 60 units/day, R_r = 80 units/day, P_r = 100 units/day , S_p = 100 \$/lot, S_r = 80 \$/lot, s_p = 20 \$/setup, s_r = 20 \$/setup, h_p = 0.6 \$/unit/day, h_r = 0.5 \$/unit/day, h_u = 0.1 \$/unit/day, Cr_p = 0.3 \$/unit, Cr_r = 0.2 \$/unit, q_p = 0.01, q_r = 0.01, τ_p = 0.001 day, τ_r = 0.001 day. Substituting the values determined above in Equation (4.25), the optimum cost is ψ^* = 50.18, where Y^* = 239, λ_m= 1, and β^* = 1 (pure remanufacturing). In this example, pure remanufacturing is the optimum solution because costs associated with remanufacturing are less than those of production. The model was investigated for varying values of $\beta \in(0, 1)$ while optimizing for the total cost, the results are shown in Figure 4.8.

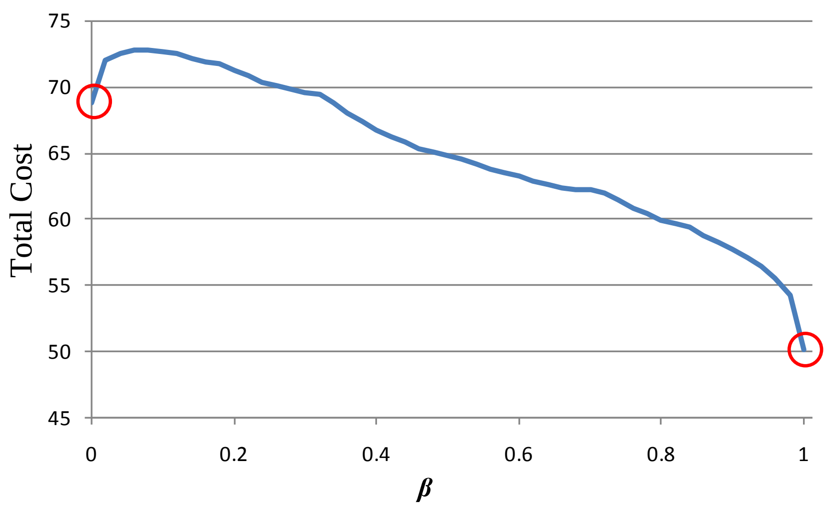


Figure 4.8. The behaviour of Total Cost of Example 4.5 for varying values of "β"

The total cost decreases as β increases, because total costs associated with remanufacturing are less than total costs associated with production. There is a surge in total cost as the return rate deviates from the extreme cases (i.e., β = 0 or 1).

For the same input parameters, except for S_r = 150 and h_r = 0.8, and by optimizing Equation (4.25), the optimum solution shifts to β^* = 0, where ψ^* = 70.99, λ_n = 0 and X^* = 208.8 (pure production). In this case, pure production is the optimum solution because costs associated with remanufacturing is more than that of production. The model was investigated for varying values of $\beta \in (0, 1)$ while optimizing for the total costs, the results are shown in Figure 4.9.

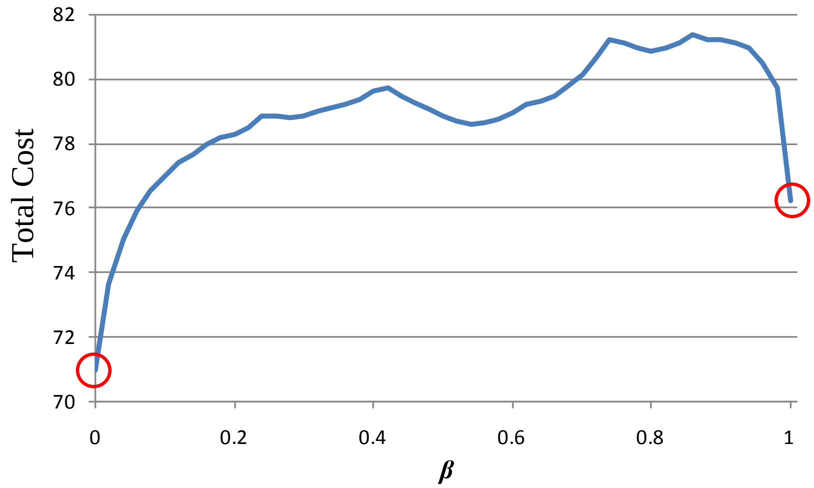


Figure 4.9. The behaviour of Total Cost of Example 4.5 (S_r = 150, h_r = 0.8) for varying values of "β"

Figure 4.8 and Figure 4.9 show that the total cost drops sharply when $\beta = 0$ and $\beta = 1$, since only one type of setup cost is included, i.e., either S_p or S_r. Whereas, for the case when $0 < \beta < 1$, which represents a mixed policy of production and remanufacturing, both types of setup costs are added to the total cost. This explains the surge in the total cost when β shifts from 0 or 1, and brings us to the finding of Richter & Dobos (1997), where either a pure production or a pure remanufacturing policy (bang-bang) is always optimum (designated by circles in Figure 4.9 and Figure 4.9). The unevenness found in Figure 4.9 especially in the range of $0.2 \leq \beta \leq 0.9$, is due to the integer nature of λ_m and λ_n, where the number of minor setups changes at β = 0.4, 0.5, 0.6, 0.75 and 0.85. However, the optimum solution behaviour remains consistent with the bang-bang policy.

Example 4.6

For the same input parameters as in Example 4.5, the model was investigated for varying values of $h_r \in (0.4, 1.8)$ while optimizing for the total costs, the results are shown in Figure 4.10.

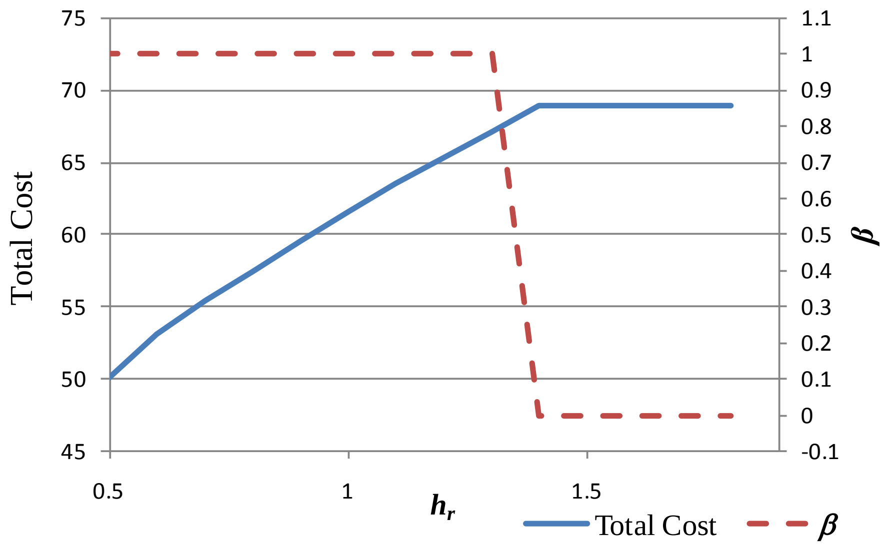


Figure 4.10. The behaviour of Total Cost and "β" of Example 4.6 for varying values of "h_r"

For $h_r \leq 1.3$, the total associated costs with remanufacturing is less than that of production, and the solution in this case favours remanufacturing, where β is equal to 1. As h_r increases, remanufacturing associated costs increases to a limit that remanufacturing is not favoured, and pure production is the optimum policy for $h_r > 1.3$.

For the same input parameters as Example 4.5 except for S_r = 200, and investigating the model for varying values of $h_p \in (0,\ 1.3)$ while optimizing for the total costs, a similar behaviour is observed as shown in Figure 4.11.

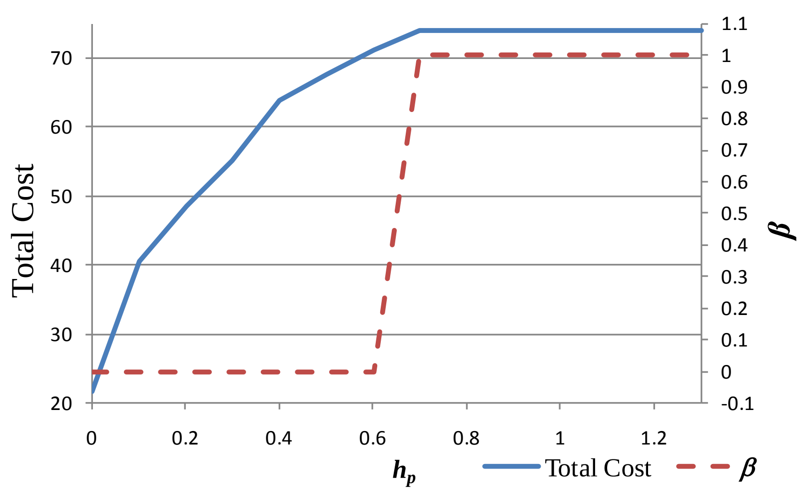


Figure 4.11. The behaviour of Total Cost and "β" of Example 4.6 (S_r = 200) for varying values of "h_p"

For $h_p \leq 0.6$, the total associated costs with remanufacturing are more than that of production, and the solution in this case favours pure production, where β is equal to zero. As h_p increases, production associated costs increases to a limit that remanufacturing is favoured, and pure remanufacturing is the optimum policy for $h_p > 0.6$.

Example 4.7

For the same values of the input parameters in Example 4.5, Equation (4.25) was optimized by varying τ_r over the range (0−4). The optima values of ψ^* and the corresponding values of λ_m are plotted in Figure 4.12.

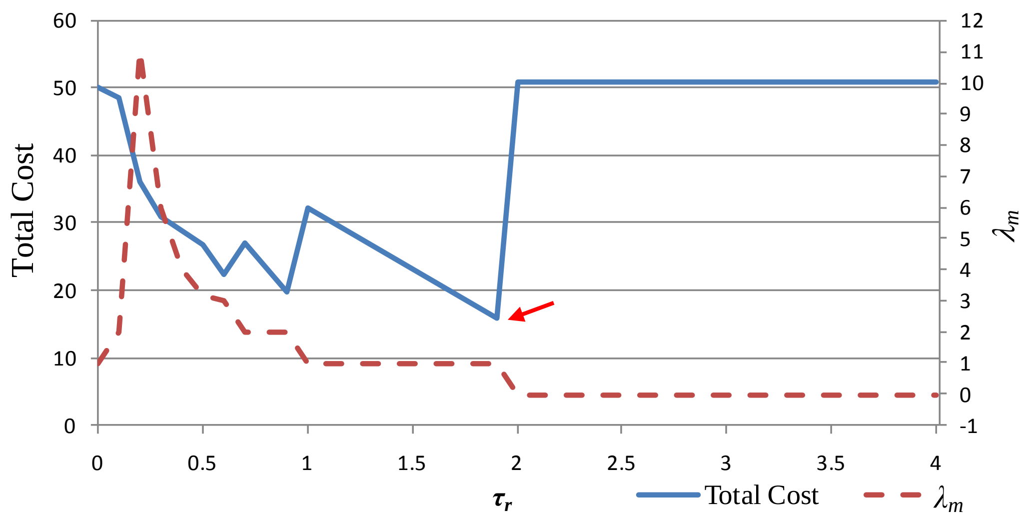


Figure 4.12. The behaviour of Total Cost and "λ_m" of Example 4.7 for varying values of "τ_r"

The optimum solution occurs at τ_r = 1.9, where ψ^* = 15.9 and λ_m = 1. There is an optimum τ_r value where average holding costs and ordering and minor setup costs are minimized (marked by an arrow). This finding suggests that quality managers should consider the effect of the length of minor setup on the robustness of the solution, especially when the time required to perform a minor setup (i.e., due to technological reasons) is larger than τ_r optimum. Note that the fluctuations in the minimum cost occur due to the integer nature of λ_m. A similar behaviour is observed for varying τ_p.

4.6 Summary and Conclusions

Model I presented a production and remanufacturing lot sizing model with imperfect production and remanufacturing processes by incorporating the works of Salameh & Jaber (2000), and Maddah & Jaber (2008) into that of Richter (1996a, b, 1997). Model I assumed that there are multiple production cycles and multiple remanufacturing cycles per time interval. Items of imperfect quality were utilized in another production/inventory situation, and for each cycle, these items were withdrawn from inventory and sold at a discounted price as a single batch by the end of the 100% screening process. In Model II, the imperfect production and reworking concept was introduced in a reverse logistics context by extending the work of Dobos & Richter (2003, 2004). Minor setups are performed to restore quality of imperfect production and remanufacturing processes. The length of the first time interval in a mathematical model of multiple remanufacturing and multiple production cycles was considered to avoid overestimating the holding costs in the repairable stock.

The results suggested that assuming production and remanufacturing processes to be perfect, may not capture the benefits that product recovery programs are supposed to bring. It is shown that the optimum system policy switches between two extreme cases; either pure production or pure remanufacturing, which brings us to the bang-bang policy of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal) discussed in Dobos & Richter (2003, 2004). However, there are two limitations. First, different optimal policies are reached if the profit or the cost functions are independently optimized, which suggests that a profit maximization approach produces better solutions than what is commonly practiced in literature; i.e., minimizing costs. Second, the case of pure remanufacturing is not mathematically attainable, as there have to be a minimal amount of newly produced units replenished to the system. This conforms to the findings of Dobos & Richter (2004), where although the authors showed that the pure policy dominates the production remanufacturing lot-sizing problems, the authors believed that it is technologically infeasible. For fixed price and quality, the competition between production and remanufacturing associated costs determine the optimal solution. Therefore, unless quality and price of returns (collected used items) are considered, the “bang-bang” rule prevails. Investigating the effects of minor setup showed that there is an optimal length for the minor setup, where average holding costs, ordering and rework costs are minimized.

In this chapter, learning phenomena was not considered. The next chapter presents learning and its effect on inventory in a reverse logistics context.

CHAPTER 5: LEARNING EFFECTS WITH INVESTMENT IN A REVERSE LOGISTICS CONTEXT

5.1 Introduction

Performance of an individual, a group of workers, or an organization can be described by the learning curve. The learning curve predicts improvements in the performance of an individual or a system as this individual or system executes its tasks. Technological progress is a kind of learning. The learning curve represents the increasing skill of an individual by repetition of a simple operation, and describes the output of more complex organism − the collective efforts of many people (Jaber, 2006b). The learning curve received more attention during World War II, as contractors searched for ways to predict costs and time requirements to build military equipment (Yelle, 1979).

Reverse logistics is all the logistics management activities involved in managing the flow of materials and information from consumers back to producers, including inventory management, which has been receiving much attention in recent years. Richter (1996a, b, 1997) developed an inventory repair/disposal model assuming demand is satisfied by newly produced and repaired items as well, and analyzed the extreme waste disposal rates showing that pure strategies dominate the repair and waste disposal strategies. Along the same line of research, Dobos & Richter (2003, 2004, 2006) reached similar conclusions and developed inventory models with finite production and repair/remanufacturing rates. The investigation of learning curve effects in a production/remanufacturing environment is very important for industries with expensive labour and learning costs so that management can use learning models to improve the utilization of resources and to coordinate and manage the supply chain (Macher & Mowery, 2003).

Learning phenomenon is clear and obvious in production and manufacturing environments (Levitt & March, 1988). Wright (1936) is believed to be the first to study the learning effect, where the author developed the learning curve theory after observing that as the quantity of units manufactured doubles, the time it takes to produce an individual unit decreases at a uniform rate. Hirsch (1952) examined the case of a large manufacturing company, and found that learning caused dynamic cost aspects and irreversible changes in technology that resulted in remarkable progress in production processes. Keachie & Fontana (1966) questioned the assumption of fixed manufacturing costs and its effect on the optimum production quantity and the authors were the first to address the effect of learning on lot sizing problems. Majd & Pindyck (1989) modeled production leaning with uncertain future prices and found that uncertainty reduces the effect of learning, and accordingly reduces the incentive to invest in learning. Dorroh et al. (1994) illustrated the importance of investment in learning and assumed a producer that chooses to invest in learning of a production process with a decreasing rate of investment over time. Jaber & Bonney (1999) surveyed works in literature that deals with the effect of learning on the lot sizing problems and showed the possibility of including the Just in Time (JIT) concept in these works, which requires smaller set up costs and times and an initial capital investment to result in a more flexible production. One of the ways to achieve lower set up costs is through learning (Jans & Degraeve, 2008). Reverse logistics activities usually include activities that involve manual skills such as disassembly and sorting, where the effects of learning on lot size are more obvious. Johnson & Wang (1998) emphasised the importance of considering the learning curve effects in calculating the time of disassembly activities, which are usually manual tasks. Similar works that incorporated the learning curve into the lot sizing problem are those of Ben-Daya & Hariga (2003), Chiu & Chen (2005), Alamri & Balkhi (2007), Jaber & Bonney (2007), and Jaber & Guiffrida (2007, 2008).

Kiesmüller et al. (2004) affirmed the importance of investigating learning in reverse logistics context, however, the author did not model the relation between learning and inventory. Kleber (2006) extended Kiesmüller et al. (2004) and showed the effect of knowledge acquisition in reverse logistics. However, the author accounted for learning in the remanufacturing process only and not in the production process. The author accounted for learning as a decreasing remanufacturing unit cost function, without representing the effect of learning on lot sizes or accounting for learning in production. The authors also studied the effect of learning for a special case when the level of inventory in the serviceable stock is zero. The effect of learning on lot size in a reverse logistic context has not been addressed in literature.

In this chapter, a model is presented to investigate the works of Dobos & Richter (2003, 2004) for learning effects, where finite production and remanufacturing rates are improved due to learning, and required capital investments are considered. It is assumed that learning does not transfer among time intervals; however, learning is transferable (i.e. accounted for) among cycles in the same time interval. Two extreme cases are presented which are the case of no learning and the case of maximum learning, where, in the later case, the model approaches the EOQ production and remanufacturing model. A description of the learning curve is presented in Appendix 2.

5.2 Learning Effects with Investment: Mathematical Modeling

In the presented model, decision variables are n, m, Q and β, where Q is the production lot size, and is measured in units, and Y is the remanufacturing lot size, which is dependent on n, m, Q and β, and therefore, is not a decision variable. The model considers production and remanufacturing as described in Figure 3.1, where a manufacturing environment (production, remanufacturing and collection of used items) consists of two stocks: the serviceable stock for new and remanufactured products, and the repairable stock where returns from the market are collected. Each interval T consists of n production lots and m remanufacturing that satisfy market demand at a constant rate d. A quantity αd is disposed, and a quantity βd is remanufactured, while $\beta_{\max}$ is the maximum percentage of collected used units that could be recovered. The ratio between the total remanufactured quantity mY and the newly produced quantity nQ is equal to the ratio $\beta/(1-\beta)$. Therefore,

$$Y(n,m,Q,\beta)=\frac{\beta nQ}{(1-\beta)m} \tag{5.1}$$

and the length of the time interval T is given as

$$T=\frac{nQ+mY}{d}=\frac{nQ+m\left(\beta nQ/(1-\beta)m\right)}{d}=\frac{nQ}{(1-\beta)d} \tag{5.2}$$

The model assumes no shortages, zero lead-time, remanufactured (recovered/repaired) items are as-good-as new, and production and remanufacturing processes are always in control. Figure 5.1 illustrates the behaviour of inventory in the serviceable and the repairable stocks over an interval T.

At the beginning of each interval (i.e., t = 0), the serviceable inventory builds up by remanufacturing the first lot of size Y while depletes at a constant rate d. This trend ends at the maximum level of the inventory of remanufactured items, t_{p_1}. This maximum level is depleted at a constant rate d until it reaches the zero level at $t_{c_1} > t_{p_1}$. This inventory behaviour is repeated in the remaining remanufacturing lots (m – 1 = 2 in Figure 5.1). The last remanufacturing lot ends at $t_{c_{m=3}}$, where the system switches from remanufacturing to production, and the inventory of newly produced items starts to build up and deplete in a similar manner to that of remanufacturing for n consecutive production lots. In the first production lot, production ceases at $t_{p_{n=1}}$ or $t_{p_{m+1=4}}$, which is followed by an inventory depletion period that ends at $t_{c_{n=1}} = t_{c_{m+1=4}}$. The nth production lot ends at $t_{c_{n+m=5}}$, and a new interval T commences.

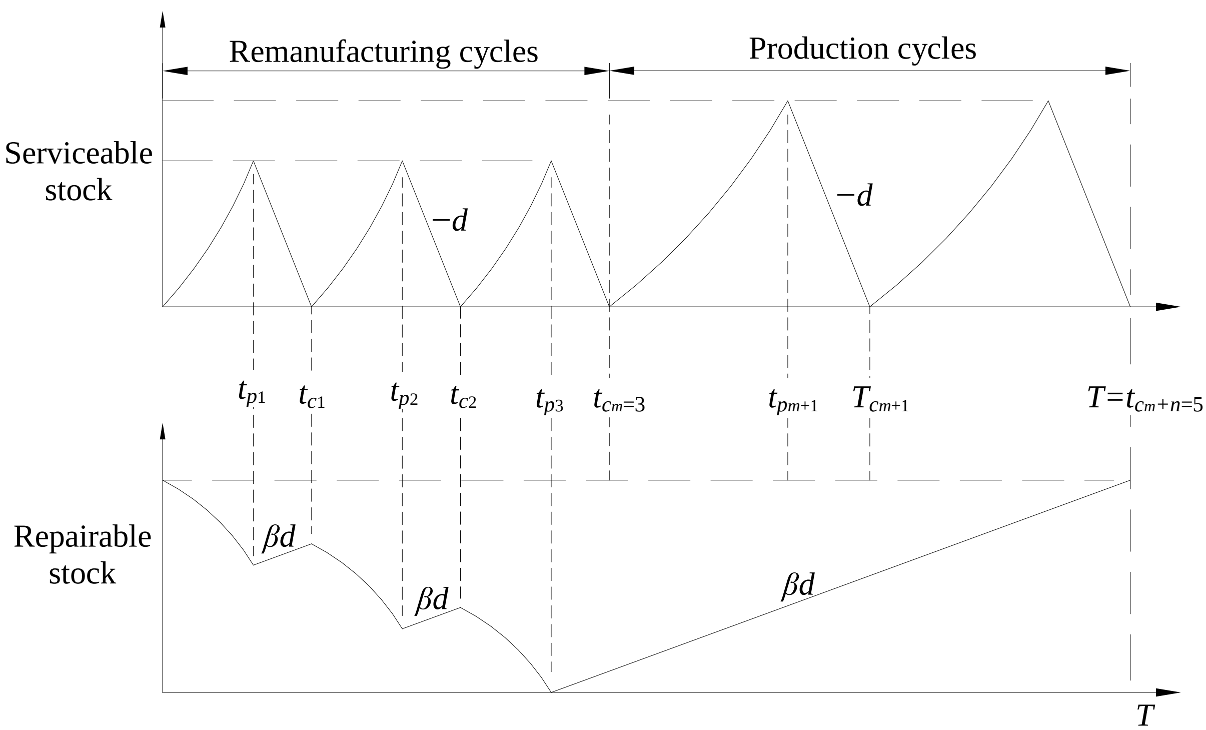


Figure 5.1. Inventory in the serviceable and repairable stocks for m = 3, n = 2

The repairable stock is at its maximum level at the start of each interval T, which is the total amount of used items collected from the previous time interval, and is equal to $I(t = 0) = Z$. Inventory in the repairable stock depletes each time remanufacturing resumes, while at the same time inventory increases by collecting used items (returns) from the market at a constant rate βd . At time t_{p_1} , the inventory reaches a value of $I(t_{p1}) = Z - Y + \beta d t_{p_1}$, and remanufacturing ceases. The inventory in the repairable stock builds as used items are collected reaching a peak of $I(t_{c_1})$ = $Z - Y + \beta d t_{c_1}$ at time t_{c_1} . The same behaviour is repeated until the inventory level reaches the zero level at time $t_{p_{m=3}}$. At that time, inventory in the repairable stock builds at a constant rate βd reaching the maximum level of $I(T) = I(t_{c_{n+m}}) = Z$ at time *T*. This accumulated inventory is used in the following interval *T*, and the behaviour of inventory in the serviceable and the repairable stocks duplicates itself in each subsequent interval *T* indefinitely.

The holding cost for a single production (remanufacturing) cycle *i* (*j*) of length t_{pi} (t_{rj}) is given from (Jaber & Bonney, 1998) as

$$H(Q_i) = h_p\left(\frac{Q_i^2}{2d} - \frac{y_1\left((Q_i + u_i)^{2-b} - u_i^{2-b}\right)}{(1-b)(2-b)} + \frac{y_1 Q_i u_i^{1-b}}{1-b}\right) \tag{5.3}$$

where u_i (w_j) is the equivalent number of units experienced at the beginning of production (remanufacturing) lot *i* (*j*), and *b* (*a*) is the learning exponent for the production (remanufacturing) process, where (0 ≤ *b*, *a* < 1) and *a* ≠ *b*. Costs per unit time for a cycle is

$$TCU(Q_i) = \frac{c_p y_1 d}{(1-b)}\left(\frac{(Q_i + u_i)^{1-b} - u_i^{1-b}}{Q_i}\right) + \frac{S_p d}{Q_i}$$
$$+ h_p\left(\frac{Q_i}{2} - \frac{y_1 d\left((Q_i + u_i)^{2-b} - u_i^{2-b}\right)}{(1-b)(2-b)Q_i} + \frac{y_1 d u_i^{1-b}}{(1-b)}\right) \tag{5.4}$$

where $u_i = \sum_{n=1}^{i-1} Q_n$ with $u_1 = 0$. Each interval *T* has *n* production cycles of length t_{pi} (*i* = 1, 2,…, *n*) each and *m* remanufacturing cycles of length t_{rj} (*j* = 1, 2,…, *m*) each, i.e., $T = \sum_{i=1}^{n} t_{pi} + \sum_{j=1}^{m} t_{rj}$.

For *m* remanufacturing cycles and *n* production cycles, the holding cost in the serviceable stock is determined as

$$H_1(m,n,Y_i,Q_i)=\sum_{i=1}^{n}h_p\left(\frac{Q_i^2}{2d}-\frac{y_1\left(\left(Q_i+u_i\right)^{2-b}-u_i^{\,2-b}\right)}{(1-b)(2-b)}+\frac{y_1Q_iu_i^{\,1-b}}{1-b}\right)$$

$$+\sum\nolimits_{j=i}^{m}h_r\left(\frac{Y_j^2}{2d}-\frac{r_1\left(\left(Y_j+w_j\right)^{2-a}-w_j^{\,2-a}\right)}{(1-a)(2-a)}+\frac{r_1Y_jw_j^{\,1-a}}{1-a}\right) \tag{5.5}$$

Without loss of generality, and for simplicity, assume that $Q_i=Q$ for all lots in *T*, then $u_i=(i-1)Q$ or $u_n=(n-1)Q$. Similarly, assume $Y_j=Y$ and $w_j=(j-1)Y$ or $w_m=(m-1)Y$, then Equation (5.5) is rewritten as

$$H_1(n,m,Q,\beta)=\sum_{i=1}^{n}h_p\left(\frac{Q^2}{2d}-\frac{y_1\left((iQ)^{2-b}-((i-1)Q)^{2-b}\right)}{(1-b)(2-b)}+\frac{y_1Q((i-1)Q)^{1-b}}{1-b}\right)+$$

$$\sum_{j=1}^{m}h_r\left(\frac{Y^2}{2d}-\frac{r_1\left((jY)^{2-a}-((j-1)Y)^{2-a}\right)}{(1-a)(2-a)}+\frac{r_1Y((j-1)Y)^{1-a}}{1-a}\right)$$

$$=h_pn\frac{Q^2}{2d}-h_p\frac{y_1n^{2-b}Q^{2-b}}{(1-b)(2-b)}+h_p\frac{y_1Q^{2-b}}{(1-b)(2-b)}\left[(n-0.5)^{2-b}-0.5^{2-b}\right]$$

$$+h_rm\frac{Y^2}{2d}-h_r\frac{r_1m^{2-a}Y^{2-a}}{(1-a)(2-a)}+h_r\frac{r_1Y^{2-a}}{(1-a)(2-a)}\left[(m-0.5)^{2-a}-0.5^{2-a}\right] \tag{5.6}$$

where $Y(n,m,Q,\beta)=\dfrac{\beta nQ}{(1-\beta)m}$.

Note that $\sum_{k=1}^{N}\left[k^{\alpha}-(k-1)^{\alpha}\right]=N^{\alpha}$, and $\sum_{i=1}^{k}i^{s}\cong\int_{0.5}^{k+0.5}i^{s}\mathrm{d}i=\frac{1}{1+s}\left[(k+0.5)^{1+s}-0.5^{1+s}\right]$, when $0\le s\le 1$. Assuming equal lot sizes is reasonable since it simplifies the mathematical modelling and the computations, and most importantly since the cost is insensitive to changes in the lot size quantity, with results confirming this insensitivity (e.g., Li & Cheng, 1994; Jaber et al., 2008).

Similar to the previous chapter, this model does not ignore the immediate time interval (or cycle) before the system starts collecting used items to be recovered, to avoid overestimating the holding cost (El Saadany & Jaber, 2008b). Taking into consideration the last time interval prior to remanufacturing will preclude any overestimation of the holding cost as shown in Figure A.3d in Appendix 3. Accordingly, the holding cost in the repairable stock is given as

$$H_2(n,m,Q,\beta)=h_u\left\{mr_1Y^{2-a}\left(\frac{m^{2-a}-(m-1)^{2-a}}{(1-a)(2-a)}-\frac{(m-1)^{1-a}}{(1-a)}\right)+\frac{m(m-1)Y^2}{2d}\right.$$
$$\left.+\frac{\beta}{2d}(mY+nQ)^2-n\frac{\beta Q}{1-\beta}\left((m-1)\frac{Y}{d}+\frac{r_1Y^{1-a}}{1-a}\left(m^{1-a}-(m-1)^{1-a}\right)\right)\right\} \tag{5.7}$$

where $Y(n,m,Q,\beta)$ is as indicated in (5.6). Refer to Appendix 3 for the derivation of (5.7). The total setup cost in T is given as

$$S(n,m)=mS_r+nS_p \tag{5.8}$$

From (Jaber & Bonney, 1998), production (remanufacturing) time t_{pi} (t_{ri}) for a cycle of size $Q_i(Y_i)$ is written as

$$t_{p_i}=\frac{y_1}{1-b}\left((Q_i+u_i)^{1-b}-u_i^{1-b}\right) \tag{5.9a}$$

$$t_{r_i}=\frac{r_1}{1-a}\left((Y_i+w_i)^{1-a}-w_i^{1-a}\right) \tag{5.9b}$$

The total labour cost is $\sum_{i=1}^{n}c_pt_{p_i}+\sum_{j=1}^{m}c_rt_{r_i}$, where t_p and t_r are the production time per lot and the remanufacturing time per lot respectively. Labour costs are determined from (5.9a, b), and in a similar to developing (5.6), as

$$C_L(n,m,Q,\beta)=\sum_{i=1}^{n}c_p\frac{y_1Q^{1-b}}{1-b}\left(i^{1-b}-(i-1)^{1-b}\right)+\sum_{j=1}^{m}c_r\frac{r_1Y^{1-a}}{1-a}\left(j^{1-a}-(j-1)^{1-a}\right)$$
$$=c_p\frac{y_1}{1-b}(nQ)^{1-b}+c_r\frac{r_1}{1-a}(mY)^{1-a} \tag{5.10}$$

The transfer of learning among subsequent production (remanufacturing) lots in an interval is allowed, as shown in Equations (5.9a) and (5.9b). The behaviour of inventory over T is duplicated in every subsequent interval. An argument may suggest that some of the learning in production (remanufacturing) is transferred from one interval to another; however, this model is not considering the transfer of learning between intervals and the mathematical modelling was restricted to a single interval case.

Proper investment is crucial for improvements by learning (e.g., Cohen & Levinthal, 1990; Levinthal & March, 1993). In the literature, there are several learning investment functions (e.g., Killingsworth, 1982; Walter & Ritter, 1996; Moskowitz et al., 2001; Affisco et al., 2002; Demeester & Qi, 2005). In this model, the learning investment in production (remanufacturing) is accounted for as C_{Ip} (C_{Ir}), where C_{Ip} (C_{Ir}) is convex and strictly increasing over *b* (*a*). Affisco et al. (2002) adopted a similar function, and considered the option of investing to improve the quality of the vendor's production process. It is assumed that learning becomes faster (*b* increases) as the amount of dollars invested increases, which has been considered in the literature in a manner that induced learning is the result of deliberate (management induced) investments in production process improvements (Jørgensen & Kort, 2002). The learning investment function is expressed as

$$b = b_o e^{\Delta_p C_{Ip}} \tag{5.11a}$$

where Δ_p (Δ_r) is the percent increase in *b* (*a*) per dollar and C_{Ip} (C_{Ir}) is the amount of dollars invested per process (i.e., production or remanufacturing, and it is independent of *n* and *m*), and b_o (a_o) is the initial learning rate for the production processes. Taking "log" of both sides for (13a) gives

$$C_{Ip} = \frac{1}{\Delta_p} \ln b - \frac{1}{\Delta_p} \ln b_o \tag{5.11b}$$

Similarly, the investment function for remanufacturing is

$$C_{Ir} = \frac{1}{\Delta_r} \ln a - \frac{1}{\Delta_r} \ln a_o \tag{5.11c}$$

Therefore, the total investment cost in an interval *T* for learning in production and remanufacturing is given from (5.11b) and (5.11c) as

$$C_I = \frac{1}{\Delta_p} \ln b - \frac{1}{\Delta_p} \ln b_o + \frac{1}{\Delta_r} \ln a - \frac{1}{\Delta_r} \ln a_o \tag{5.12}$$

The objective is to minimize the total cost per unit time , which is the summation of (5.6) + (5.7) + (5.8) + (5.10) + (5.12), and is expressed as

$$\textbf{\textit{Minimize}}\ \omega(n,m,Q,\beta) = \frac{(1-\beta)d}{nQ}\left[H_1(n,m,Q,\beta) + H_2(n,m,Q,\beta)\right.$$

$$\left. + S(n,m) + C_L(n,m,Q,\beta) + C_I\right] \tag{5.13a}$$

Subject to:

$$0 \le \beta \le \beta_{\max} \tag{5.13b}$$

$$n, m \in \{1,2,...\} \tag{5.13c}$$

$$Q > 0 \tag{5.13d}$$

The solution for the model in (5.13a – 5.13d) is bound by two extreme cases, which are the case of no learning in production and remanufacturing (CASE I: $b = a = 0$), and the case when no further improvement in learning is possible (CASE II: $y_1 \approx 0$, $r_1 \approx 0$). These two cases are discussed below.

CASE I: $b = a = 0$

For this case, Equations (5.6), (5.7), and (5.10) reduce respectively to

$$H_1(m,n,Q,\beta) = h_p n \frac{Q^2}{2d}(1 - d/\gamma) + h_r m \frac{Y^2}{2d}(1 - d/v)$$

$$= \frac{nQ^2}{2d}\left[h_p(1 - d/\gamma) + h_r \frac{\beta^2 n}{(1-\beta)^2 m}(1 - d/v)\right] \tag{5.14}$$

$$H_2(n,m,Q,\beta) = h_u\left[\frac{\beta}{2d}(mY + nQ)^2 - \frac{mY^2}{2d}\left(m - \left(1 - \frac{d}{v}\right)\right)\right]$$

$$= \frac{h_u \beta n^2 Q^2}{2d(1-\beta)^2}\left[1 - \beta\left(1 - \frac{(1 - d/v)}{m}\right)\right] \tag{5.15}$$

$$C_L(n,m,Q,\beta) = c_p \frac{n}{\gamma} Q + c_r \frac{m}{v} Y = c_p \frac{n}{\gamma} Q + c_r \frac{m}{v}\left(\frac{\beta n Q}{(1-\beta)m}\right) = nQ\left(\frac{c_p}{\gamma} + \frac{c_r \beta}{v(1-\beta)}\right) \tag{5.16}$$

and $T = nQ/d(1-\beta)$, and total cost becomes

$$\omega(n,m,Q,\beta) = \frac{d(1-\beta)}{nQ}\left[\frac{nQ^2}{2d}\left(h_p(1 - d/\gamma) + h_r \frac{\beta^2 n}{(1-\beta)^2 m}(1 - d/v)\right)\right.$$

$$\left. + \frac{h_u \beta n^2 Q^2}{2d(1-\beta)^2}\left[1 - \beta\left(1 - \frac{(1 - d/v)}{m}\right)\right] + mS_r + nS_p + nQ\left(\frac{c_p}{\gamma} + \frac{c_r \beta}{v(1-\beta)}\right)\right]$$

Differentiating with respect to Q, total cost reduces to:

$$\omega(n,m,\beta)=$$

$$\sqrt{2\left(mS_r+nS_p\right)\frac{d}{n}\left[h_p(1-\beta)^2(1-d/\gamma)+h_r\frac{\beta^2 n}{m}(1-d/\nu)+h_u\beta n\left(1-\beta\left(1-\frac{(1-d/\nu)}{m}\right)\right)\right]}$$

$$+d\left(\frac{c_p(1-\beta)}{\gamma}+\frac{c_r\beta}{\nu}\right) \tag{5.17}$$

where

$$Q=\sqrt{\frac{2\left(mS_r+nS_p\right)\frac{d(1-\beta)}{n}}{h_p(1-\beta)(1-d/\gamma)+h_r\frac{\beta^2 n}{(1-\beta)m}(1-d/\nu)+\frac{h_u\beta n}{(1-\beta)}\left[1-\beta\left(1-\frac{(1-d/\nu)}{m}\right)\right]}} \tag{5.18}$$

CASE II: $y_1\approx 0,\ r_1\approx 0.$

For this case, Equations (5.6), (5.7), and (5.10) reduce respectively to

$$H_1(n,m,Q,\beta)=h_p n\frac{Q^2}{2d}+h_r m\frac{Y^2}{2d}=\frac{nQ^2}{2d}\left[h_p+h_r\frac{n\beta^2}{m(1-\beta)^2}\right] \tag{5.19}$$

$$H_2(n,m,Q,\beta)=h_u\frac{n^2Q^2\beta}{2d(1-\beta)}\left(1-\frac{m-1}{m}\beta\right) \tag{5.20}$$

where $C_L(n,m,Q,\beta)=0$, and $T=nQ/d(1-\beta)$. The total cost becomes

$$\omega(n,m,Q,\beta)=$$

$$\frac{d(1-\beta)}{nQ}\left[mS_r+nS_p+\frac{nQ^2}{2d}\left(h_p+h_r\frac{n\beta^2}{m(1-\beta)^2}\right)+h_u\frac{n^2Q^2\beta}{2d(1-\beta)}\left(1-\frac{m-1}{m}\beta\right)\right]$$

Differentiating with respect to Q, total cost reduces to:

$$\omega(n,m,\beta)=\sqrt{2\left(mS_r+nS_p\right)\frac{d}{n}\left[h_p(1-\beta)^2+h_r\frac{\beta^2 n}{m}+h_u\beta n\left(1-\beta\frac{(m-1)}{m}\right)\right]} \tag{5.21}$$

where
$$Q=\sqrt{\frac{2\left(mS_r+nS_p\right)\frac{d(1-\beta)}{n}}{h_p(1-\beta)+h_r\frac{\beta^2 n}{(1-\beta)m}+\frac{h_u\beta n}{(1-\beta)}\left(1-\beta\frac{(m-1)}{m}\right)}} \tag{5.22}$$

5.3 Numerical Examples

To investigate the presented model, three numerical examples are solved, two of them have parameters similar to examples of Dobos & Richter (2003) and Teunter (2004). Note that these studies, like other studies in the literature, assumed that the production and remanufacturing processes are not subject to learning effects, and assumed that $h_p = h_r$. Accordingly, additional values for b, b_o, a, a_o, Δ_p, Δ_r and h_r are given, and $b_o = a_o = 0.0001$, are set as default values in Equations (5.11b) and (5.11c). Subsequently, when there is no learning in production and remanufacturing, then $b = a = 0$ and $C_{Ip} = C_{Ir} = 0$. The numerical examples were solved using Excel Solver enhanced by VBA codes.

Example 5.1

This example uses data similar to those of Dobos & Richter (2003, p. 44). Let $d = 200$, $\gamma = 300$ ($y_1 = 1/300$), $v = 300$ ($r_1 = 1/300$), $S_p = 144$, $S_r = 72$, $h_p = 12$, $h_r = 7$, $h_u = 3$, $c_p = 500$, $c_r = 500$, $b = 0$, $a = 0$, $\Delta_p = 1$ %/\$, $\Delta_r = 1$ %/\$, and $\beta_{\max} = 0.667$. The model presented herein, and previous models, consider β is a decision variable, whereas in Dobos & Richter (2003) it is an input parameter, and in the presented model, a maximum value of β is adopted; i.e., $\beta_{\max}$. In this numerical example, β in constraint (5.13b) was assumed not to be less than 1%, i.e., some collection must occur and $0.01 \leq \beta \leq \beta_{\max} = 0.667$.

Solving (5.13a) − (5.13d), the optimal policy occurred when $m^* = 2$, $n^* = 1$, $T^* = 1.247$ $Q^* = 83.14$, $Y^* = 83.14$, and $\beta^* = 0.667$ whose cost is $\omega^* = 795.21$. The optimal solution falls between the solutions of the two extreme cases. The values of ω for CASE I and CASE II are 795.21 (Equation 5.17) and 697.42 (Equation 5.21) respectively (i.e., $\omega_I^* = 795.21 > \omega^* > \omega_{II}^* = 697.42$), and the values of Q for CASE I (Equation 5.18) and CASE II (Equation 5.22) are 83.13 and 55.05 respectively. Example 5.1 was solved for different values of b = 0.1, 0.2, 0.3,…, 0.8, with results summarised in Table 5.1. The values of the learning rate were selected in conformance with industrial evidence that learning rates for different types of tasks fall in the range of 95% ($b = 0.074$) to 60% ($b = 0.734$), with the majority of tasks have a learning rate in the range of 90-70%. (e.g., Cunningham, 1980; Dutton & Thomas, 1984; Camm, 1985; Dar-El, 2000).

Table 5.1 shows that investment in learning is feasible at learning rate faster than 93.3% ($b > 0.1$), where the savings from learning are more than the amount invested to accelerate it. That is, a firm must determine its target of learning rate before investing to accelerate it. Table 5.1 shows that faster learning (b increases from 0 to 0.8) results in lowering the collection rate of used items (β decreases from 66.67% to 65.1%). This is because faster learning recommends shorter production cycle and subsequently smaller lot size and shorter time interval T. If there are governmental incentives for firms to increase the collection rate β, accelerating the learning process in production may not be a profitable choice. For each value of b, ω^* values fell between the values ω_I^* and ω_{II}^*, except for b = 0.1, where the savings from learning are less than the amount invested to accelerate the process.

Table 5.1. Optimal policy for varying values of the production learning rate "*b*"

b	T^*	m^*	n^*	Q^*	Y^*	β^*	ω^*	ω_I^*	ω_{II}^*
0.0	1.247	2	1	83.14	83.14	0.667	795.21	795.21	697.42
0.1	1.194	2	1	79.57	79.57	0.667	796.02	795.21	697.42
0.2	1.153	2	1	76.84	76.84	0.667	793.15	795.21	697.42
0.3	0.854	1	1	61.00	109.74	0.643	790.65	805.14	740.77
0.4	0.844	1	1	59.38	109.46	0.648	787.28	805.44	741.92
0.5	0.838	1	1	58.29	109.22	0.652	784.85	805.66	742.72
0.6	0.833	1	1	57.63	108.98	0.654	783.03	805.78	743.19
0.7	0.830	1	1	57.41	108.61	0.654	781.46	805.79	743.21
0.8	0.829	1	1	57.92	107.82	0.651	779.62	805.57	742.40

The numerical example in Table 5.1 was solved for learning in remanufacturing (a = 0, 0.1,…, 0.8) and no learning in production (b = 0), with results summarised in Table 5.2.

Table 5.2 shows that a maximum collection rate ($\beta_{\max}$ = 0.667) was attained for all learning rates in remanufacturing. Further, the ω^* values in Table 5.2 are less than those in Table 5.1 suggesting that investing to accelerate learning in remanufacturing may bring more benefits than investing to accelerate that in production.

Table 5.2. Optimal policy for varying values of the production learning rate "*a*"

a	T^*	m^*	n^*	Q^*	Y^*	β^*	ω^*
0.0	1.247	2	1	83.14	83.14	0.667	795.21
0.1	1.132	2	1	75.45	75.45	0.667	785.46
0.2	1.069	2	1	71.26	71.26	0.667	773.08
0.3	1.286	3	1	85.74	57.16	0.667	759.98
0.4	1.262	3	1	84.11	56.07	0.667	750.99
0.5	1.245	3	1	83.03	55.35	0.667	744.91
0.6	1.235	3	1	82.32	54.88	0.667	740.86
0.7	1.229	3	1	81.91	54.61	0.667	738.26
0.8	1.227	3	1	81.81	54.54	0.667	736.84

How would labour cost affect the results? Doubling the production and remanufacturing labour costs; i.e., $c_p = c_r$ = 1000, while keeping the other input parameters unchanged and solving Example 5.1 for b = 0, 0.1, 0.2,…, 0.8 (while keeping a = 0), the results showed that learning in production is not recommended, where the optimal policy shifted from m^* = 2, n^* =1, T^* = 1.247 Q^* = 83.14, Y^* = 83.14, and β^* = 0.667 whose cost is ω^*= 1128.55 when b = 0.1 , to m^* = 1, n^* =19, T^* = 8.301, Q^* = 86.5, Y^* = 16.61, and β^* = 0.01 whose cost is ω^*= 1051.28. These examples were solved for a = 0, 0.1, 0.2,…, 0.8 (while keeping b = 0). The results showed that investment to accelerate learning in remanufacturing was recommended as the cost decreased (from ω^*= 1128.55 to 866.60) when leaning became faster (increased from a = 0 to 0.8). For this case, the optimal collection rate remained equal to the maximum of 66.67% ($\beta^* = \beta_{\max}$ = 0.667). These results show that increasing the labour cost for production (from c_p =500 to 1000) has an adverse affect on the production process more than a similar increase would have on the remanufacturing process. This further consolidates the findings in Table 5.2 that speeding up the remanufacturing process would bring more savings than speeding up the production process. Example 5.1 was solved for a = 0, 0.1, 0.2,…, 0.8 (while keeping b = 0), $c_p = c_r$ = 1000, and Δ_r = 0.02 %/$, with results summarised in Table 5.3.

Table 5.3. Optimal policy for varying values of the production learning rate "a" (when c_p = c_r = 1000 and Δ_r = 0.02 %/$)

a	T^*	m^*	n^*	Q^*	Y^*	β^*	ω^*
0.0	1.247	2	1	83.14	83.14	0.667	1128.55
0.1	12.085	1	20	119.64	24.17	0.010	1211.56
0.2	2.840	6	2	94.67	63.11	0.667	1189.83
0.3	2.972	7	2	99.07	56.61	0.667	1153.11
0.4	2.942	7	2	98.07	56.04	0.667	1130.40
0.5	2.922	7	2	97.41	55.66	0.667	1117.20
0.6	2.911	7	2	97.03	55.45	0.667	1109.86
0.7	2.906	7	2	96.87	55.35	0.667	1106.26
0.8	2.908	7	2	96.93	55.39	0.667	1105.34

Table 5.3 shows that there is a shift from m^* = 2, n^* =1, T^* = 1.247 Q^* = 83.14, Y^* = 83.14, and β^* = 0.667 whose cost is ω^*= 1128.55, to m^* = 1, n^* =20, T^* = 12.085, Q^* =

119.64, Y^* = 24.17, and β^* = 0.01 whose cost is ω^*= 1211.56, to m^* = 6, n^* =2, T^* = 2.840, Q^* = 94.67, Y^* = 63.11, and β^* = 0.667 whose cost is ω^*= 1189.83, when learning became faster (increased from a = 0 to 0.1 to 0.2), where the optimal solution is at the extreme values of β. This behaviour brings us to the bang-bang policy of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal) of Dobos & Richter (2003, 2004).

Is there an optimal learning rate? The model in (5.13a)−(5.13d) was modified to allow for the learning rates to be decision variables. This required adding the following constraints $0 \le b \le 0.9$ and $0 \le a \le 0.9$. Example 5.1 was optimized while considering a and b as decision variables, together with n, m, Q and β, and the optimal policy occurred at m^* = 2, n^* =1, T^* = 0.916, Q^* = 61.06, Y^* = 61.06, b^* = 0.9, a^* = 0.852, and β^* = 0.667 whose cost is ω^*= 718.01, which is lower than all the policies in Tables 5.1 and 5.2. This example showed that there exist optimum values for the learning rates for both production and remanufacturing.

When should one invest in learning? The numerical examples in Table 5.1 were solved for Δ_p = 0.1, while keeping the other input parameters unchanged, and it was found that it is expensive to invest in learning, e.g., when b = 0 the cost was ω^*= 795.21 (m^* = 2, n^* =1, T^* = 0.916, Q^* = 61.06, Y^* = 61.06, β^*= 0.667), whereas b increases to 0.1, the cost increases to ω^*= 845.69 (m^* = 2, n^* =1, T^* = 1.31, Q^* = 87.31, Y^* = 87.31, β^* = 0.667) and continue to increase for higher values of b. Accordingly, savings from speeding up the learning process are less than the required amount invested, which makes it unattractive to invest in learning. For further illustration, each cost term in the serviceable and repairable stocks is investigated. Increasing b increased the investment cost from 0 to 52.75, the serviceable stock holding cost from 120.09 to 155.44, the repairable sock from 110.85 to 116.41, decreased the total setup cost from 230.94 to 219.91, and the labour cost from 111.11 to 78.96, with a total change in cost of 52.75 + (155.44 – 120.09) + (116.41 – 110.85) + (219.91 – 230.94) + (78.96 – 111.11) = 50.48 (i.e., 845.69 – 795.21 = 50.48).

There may be a case where the initial production and remanufacturing rates are much higher than the demand rate, e.g., γ = 3000 (y_1= $1/\gamma$=1/3000), v = 3000 (r_1=$1/v$ = 1/3000). Solving the numerical examples in Table 5.1 for these values of γ and v and a similar result to the one above was found. That is, it is best not to invest in learning when d/γ and d/v are significantly low as the margin of benefit from investing in learning would be minimal. However, it may be possible to invest to improve the learning process, if and only if, these ratios are associated with high labour costs; e.g., $c_p = 10{,}000$.

Example 5.2

This example uses data similar to those of Teunter (2004; p.438). Let $d = 1000$, $\gamma = 5000$ ($y_1 = 1/5000$), $v = 3000$($r_1 = 1/3000$), $S_p = 20$, $S_r = 5$, $h_p = 10$, $h_r = 6$, $h_u = 2$, $c_p = 500$, $c_r = 500$, $b = 0$, $a = 0$, $\Delta_p = 0.1$ %/$, $\Delta_r = 0.1$ %/$, and $\beta_{max} = 0.8$. Solving the mathematical programming problem given in Equations (5.13a)–(5.13d), the optimal policy occurred when $m^* = 5$, $n^* = 1$, $T^* = 0.261$ $Q^* = 52.17$, $Y^* = 41.74$, and $\beta^* = 0.8$ whose cost is $\omega^* = 498.36$. Solving for $b = a = 0.1$ the optimal policy occurred when $m^* = 20$, $n^* = 3$, $T^* = 0.972$, $Q^* = 62.77$, $Y^* = 38.86$, and $\beta^* = 0.8$ whose cost is $\omega^* = 708.74$. Solving for $b = a = 0.3$ the optimal policy occurred when $m^* = 20$, $n^* = 3$, $T^* = 0.976$, $Q^* = 65.04$, $Y^* = 39.02$, and $\beta^* = 0.8$ whose cost is $\omega^* = 693.93$. These results show that learning allows for longer intervals with multiple remanufacturing and production runs.

How transfer of learning among time intervals affect the inventory policy in subsequent intervals? In the presented model, learning does not transfer between time intervals, learning is transferable within cycles of an interval. However, an example is presented to illustrate the effects of transfer of learning on inventory policies in subsequent intervals. Salameh et al. (1993) suggested that the time to perform the first unit in a given cycle captures the transfer of learning from cycle to cycle (or from interval to interval). Although this assumption simplifies the mathematics, it will be used here for illustrative purposes. For example, if $y_1 = 1/10$, $n = 2$, $Q = 100$, and $b = 0.2$, then $y_{1,\text{next}} = y_1(nQ + 1)^{-b} = 0.1\times(201)^{-0.2} = 0.0346$. Some of the input parameters were manipulated for the purpose of this example. Example 5.2 was solved for the same input parameters except for $b = a = 0.05$, $\Delta_p = \Delta_r = 0.3$ %/$. The optimal solution occurs when $m^* = 11$, $n^* = 2$, $T^* = 0.577$, $Q^* = 57.71$, $Y^* = 41.97$, and $\beta^* = 0.8$ whose cost is $\omega^* = 596.48$.

For example, the time to remanufacture the first unit in the first remanufacturing cycle of the next interval is computed as $y_{1,\text{next}} = y_1(nQ + 1)^{-b} = (1/5000) \times(2\times57.71 + 1)^{-0.05} =$

0.000158. This problem was solved for 4 consecutive intervals with results summarised in Table 5.4.

Table 5.4 shows that the learning process slows after the first interval and the transfer of learning has little effect. This explains Case II discussed above where $y_{1,i}$ and $r_{1,i}$ become insignificantly low; i.e., close to zero. This situation may be more appropriate to be discussed in the presence of forgetting where not all of the knowledge accumulated in one interval is transferred to the next interval; see for instance Jaber & Bonney (1998, 1999).

Table 5.4. Optimal policies for four consecutive intervals with transfer of learning

i	$1/y_{1,i}$	$1/r_{1,i}$	T_i	m_i	n_i	Q_i	Y_i	β_i	ω_i
1	5000.00	3000.00	0.577	11	2	57.71	41.97	0.8	596.48
2	6342.66	4077.42	0.584	12	2	58.39	38.92	0.8	576.39
3	6566.83	4222.21	0.582	12	2	58.25	38.83	0.8	574.41
4	6701.22	4308.85	0.582	12	2	58.17	38.78	0.8	573.28

Example 5.3

How sensitive the production/remanufacture inventory policy is to changes in the learning rate? Let d = 12 units/day, y_1 = 0.0625 day (γ = 16 > d), r_1 = 0.05 day (v = 20 > d), β_{max} = 0.6, S_p = 400 \$/lot, S_r = 300 \$/lot, h_p = 1.5 \$/unit/day, h_r = 0.2 \$/unit/day, h_u = 0.1 \$/unit/day, c_p = 700 \$/day, c_r = 500 \$/day, a = 0, Δ_p = 0.0004 %/\$, Δ_r = 0.0004 %/\$. Varying b over the range of (0.08-0.72) for three values of c_p (c_p = 700, 800, 900), and optimizing the model, the optimum n, m, Q and β are determined. Results are shown in Figure 5.2.

Figure 5.2 shows that as b increases, the effects of learning increase reducing total costs. This reduction in total costs ceases once learning investment costs reaches its limit. Figure 5.2 shows that for each c_p value there is an optimal b value at which total costs are minimum. The optimal value of b increases as the value of c_p increases. For example, when c_p = 700, b^* = 0.4, when c_p = 800, b^* = 0.48, and when c_p = 900, b^* = 0.56. These results verifies the presented model, and shows that as labour costs increase, the importance of learning increase.

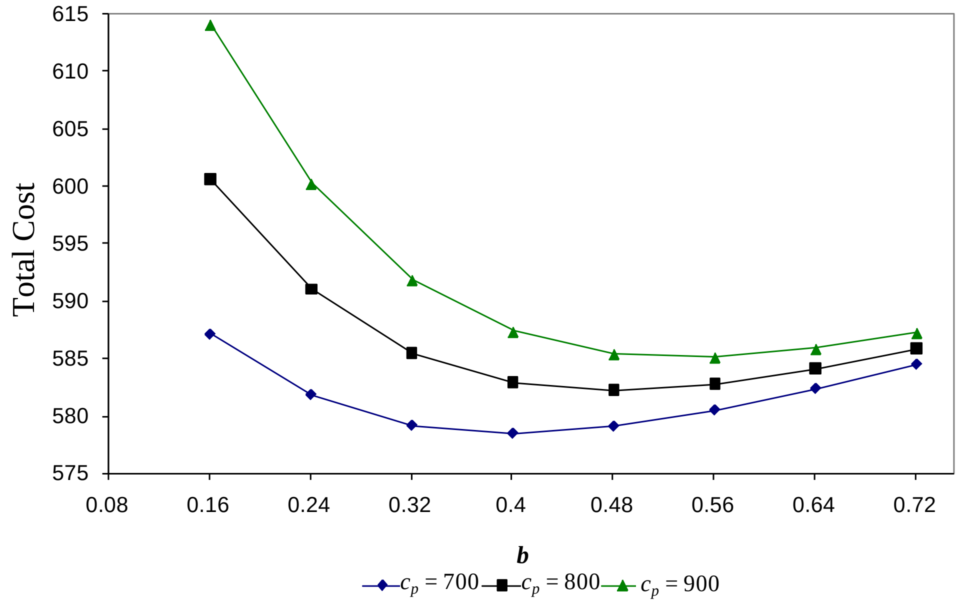


Figure 5.2. Varying "*b*" and its effect on Total Cost

5.4 Summary and Conclusions

This chapter investigated learning in a reverse logistics context. A model was presented as an extension to the work of Dobos & Richter (2003, 2004), and the effects of learning in production and remanufacturing processes were studied. Management can use established learning models to better utilize capacity, manage inventories and coordinate production and distribution throughout the chain. The presented model considered multiple production cycles and multiple remanufacturing cycles. The parameters of the numerical examples were selected from Dobos & Richter (2003) and from Teunter (2004). An extensive numerical analysis was carried to draw useful insights.

The numerical results showed that there exists a threshold learning rate beyond which investing in learning may bring savings. That is, unless the learning process is accelerated beyond the threshold value then, investment in leaning may not worth it. It was also shown that faster learning above the optimum level lowers the collection rate of used items, therefore, if there is a chance to increase the collection rate (e.g., due to new legislations or governmental incentives), then accelerating the learning process may not be desirable. It was generally found that learning reduces the lot size quantity and subsequently the time interval over which new items are produced and used ones are remanufactured. In addition, our results showed that there is an optimum learning rate to which the sum of production, remanufacturing, and learning investment costs is minimum. This suggests that there is a trade-off point between how fast an organization can go and how much that organization is willing to invest in its learning process. An important observation is that the system policy flips between two extreme policies of either pure production ($\beta = 0$) or maximum collection of used items ($\beta = \beta_{\max}$). This behaviour is referred to as the bang-bang policy, which states that the mixed strategy is not optimal and the optimal strategy is either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal), as discussed in Richter (1997) and Dobos & Richter (2003, 2004).

This chapter assumed that learning transfers among production (remanufacturing) lots in a time interval and that the time interval duplicates itself indefinitely. Although it does so, the model presented in this chapter provided the reader with an example to illustrate the effects of transfer of learning on inventory policies in subsequent intervals.

In this chapter, remanufactured products are considered to be as-good-as new. The next chapter presents the case when customers perceive remanufactured products and newly produced products differently.

CHAPTER 6: PRODUCTION AND REMANUFACTURING LOT SIZING MODEL FOR TWO MARKETS WITH LOST SALES

6.1 Introduction

Reverse logistics is becoming a popular solution to environmental waste problems. Instead of dumping in landfills, collecting and remanufacturing used products to extend their useable lives and thus reducing waste and conserving natural resources is crucial to save the environment. Research on waste management and coordinating repair with manufacturing started in the 1970's, and was established as a separate line of research by the 1990's. Inventory models addressing this line of research usually assume that customers' demand is satisfied from newly manufactured (produced) items and from remanufactured (repaired) items.

Several researchers considered inventory models to solve repair/remanufacturing and waste disposal issues. Richter (1996a, b) presented a deterministic EOQ model for repaired items where production and repair rates are instantaneous. Richter (1997) and Richter & Dobos (1999) showed that a pure policy of either no waste disposal (total repair) or no repair (total waste disposal) is always optimal, and similar results were presented by Teunter (2001b). Korugan & Gupta (1998) considered a queuing inventory system with probalistic demands and returns, and accounted for lost sales if demand is not satisfied, however, they considered remanufactured items are as-good-as new.

In all the inventory research works in the literature of reverse logistics, it is assumed that returns that are repaired or remanufactured have the same quality as the newly produced ones (i.e., returns are "as-good-as new"). This may be true in few industries, but not in other industries where customers do not consider "new" ("manufactured") and "remanufactured" ("repaired") items to be equal. Remanufactured goods sell at lower price points in secondary markets, or in different channels than new products that sell in primary markets (e.g., Tibben-Lembke & Rogers, 2002; Blackburn et al., 2004).

In this chapter, the works of Richter (1996a, 1996b) are extended by assuming that demand for manufactured items is different from that for remanufactured (repaired) ones, because customers perceive the new and remanufactured products differently. Production and remanufacturing processes are executed in the same facility, therefore, stock-out periods takes place for manufactured and remanufactured items, and accordingly, lost sales occur when demand can not be supplied (e.g., Hill et al., 2007), i.e., demand for newly manufactured items is lost during remanufacturing cycles and vice versa.

6.2 Reverse Logistics with Lost Sales: Mathematical Modelling

In this chapter, two models are presented for two cases: the total lost sales case and the partial lost sales case. The models presented herein assume: (1) infinite production and recovery rates, (2) remanufactured items are perceived by some customers to be of lower quality than newly manufactured items, (3) demand for produced and remanufactured items are known, constant but different, (4) constant but different collection rates for previously used manufactured and remanufactured items, (5) lead time is zero, (6) inventory stock-out occurs and unsatisfied demand (manufactured or remanufactured) is lost, (7) the collection of used items from previously remanufactured ones occur in the remanufacturing period, and used items from previously produced items occur during the production period.

The presented models consider the production and remanufacturing (repair) situation similar to that described in Richter (1996a, 1996b), where there are two inventories: the serviceable stock for stocking manufactured new products and the repairable stock for collecting remanufactured used items. It is assumed that remanufactured (repaired) items are not perceived by customers to be of the same quality as newly produced items. Accordingly, two demands are to be satisfied: primary market demand for newly produced items D_p, and secondary market demand for remanufactured/repaired items D_r, where D_r is not necessarily equal to D_p (e.g., Tibben-Lembke & Rogers, 2002; Blackburn et al., 2004), as shown in Figure 6.1.

In each interval of length T, there are two sub-intervals representing m remanufacturing cycles and n production cycles of lengths T_r and T_p respectively. Decision variables considered are n, m, γ_p and γ_r, where γ_p (γ_r) is the collection percentage of available returns of newly produced (previously remanufactured) items ($0 < \gamma_p, \gamma_r < 1$), and β_p (β_r) is the percentage of available returns from the primary (secondary) market for produced (remanufactured) items. Note that ($1- \gamma_r \beta_r$) and ($1- \gamma_p \beta_p$) are the waste disposal rates, where ($0 < \beta_r \leq \beta_p < 1$). It is reasonable to assume that it is less likely to recover components from a used item that was previously remanufactured than from a previously produced one, and therefore, this model assumes $\beta_r \leq \beta_p$.

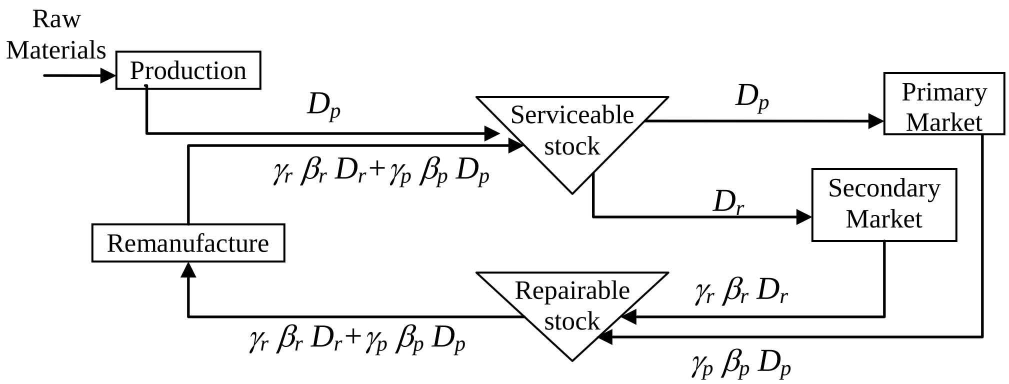


Figure 6.1. Material flow for a production and remanufacture system satisfying two markets

Each repair interval T_r consists of m remanufactured lots of size x_2/m each, and each production interval T_p consists of n production lots of size x_1/n, where x_1 (x_2) is the lot size quantity (in units) to be produced (remanufactured/repaired) in an interval of length T. Therefore, $x_2 = D_r T_r$ and $x_1 = D_p T_p$ represent the total remanufactured and production quantities per interval T. Used items are collected at rates $\gamma_r \beta_r$ over T_r and $\gamma_p \beta_p$ over T_p accumulating $x_2 = D_r T_r = \gamma_r \beta_r D_r T_r + \gamma_p \beta_p D_p T_p$ units. Accordingly,

$$x_1/x_2 = (1 - \gamma_r \beta_r)/(\gamma_p \beta_p) \tag{6.1}$$

where x_1/x_2 is the ratio of produced to remanufactured units

In addition, the presented models consider a general time interval and do not ignore the effect of the first time interval that has no remanufacturing cycles, similar to previous models. The rationale for this assumption is that nothing has been produced before to be collected and remanufactured, and ignoring this assumption results in a residual inventory and thus overestimates the holding cost. For the first time interval $[0, T_1]$, production with no remanufacturing will take place in the serviceable stock, while in the repairable stock, the accumulation of used items that are collected to be remanufactured in the second time interval. Starting from the second time interval, m remanufacturing cycles will precede n

production cycles. The behaviour of inventory for remanufactured, produced, and collected used items over interval T is illustrated in Figure 6.2.

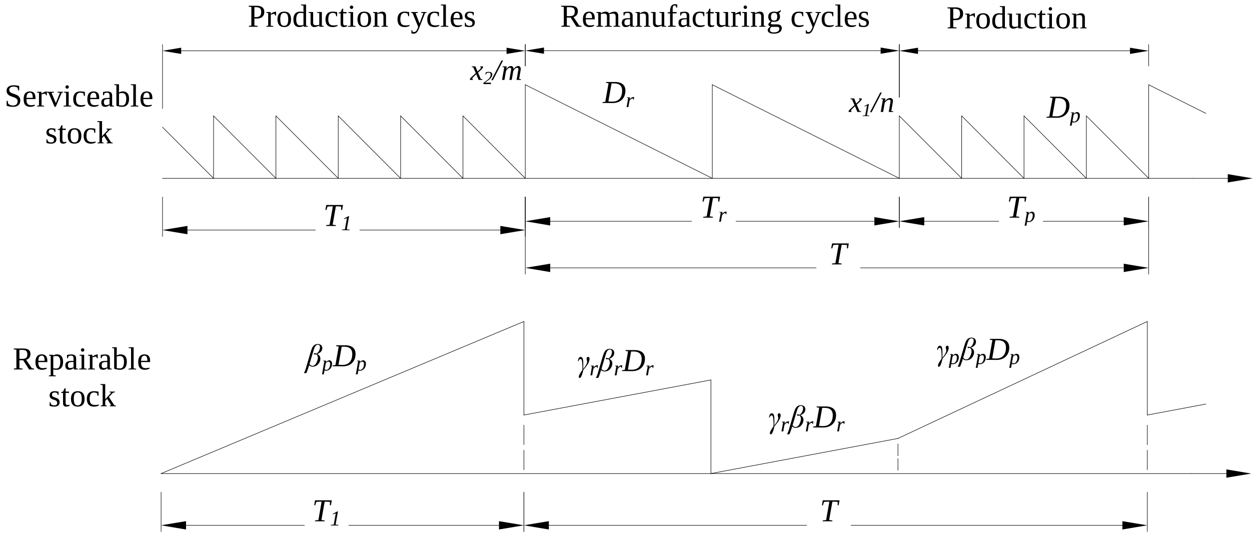


Figure 6.2. The behaviour of inventory for remanufactured, produced and collected used items over interval "T"

Two models are presented for two lost sales cases. The first model assumes that demand for newly manufactured (produced) items are lost over T_r, and that demand for remanufactured items are lost over T_p. This case is referred to as the total lost sales case. The second model assumes that it may be possible to entice some customers to settle for a remanufactured (manufactured) item at a cost, and this case is referred to as the partial lost sales case.

6.3 Model I: Total Lost Sales Case

The total cost per interval is the sum of the setup costs for remanufacturing and production, the holding costs for used, remanufactured, and newly produced items, and the stock-out costs for remanufactured and produced items, and is given as

$$\Psi(n, m, x_2, \gamma_p, \gamma_r) = mS_r + nS_p + \frac{h_r}{2mD_r} x_2^2 + \frac{h_p}{2nD_p} x_1^2 + \frac{C_{lr} D_r}{D_p} x_1 + \frac{C_{lp} D_p}{D_r} x_2$$
$$+ h_u \left[\left(\frac{m(\gamma_r \beta_r - 1) + 1}{2mD_r} \right) x_2^2 + \frac{\gamma_p \beta_p}{2D_p} x_1^2 + \left(\frac{\gamma_r \beta_r}{mD_p} + \frac{\gamma_p \beta_p (m-1)}{mD_r} \right) x_1 x_2 \right] \quad (6.2)$$

where $x_1 = \dfrac{x_2(1-\gamma_r\beta_r)}{\gamma_p\beta_p}$ and C_{lp} (C_{lr}) is the cost per unit of a lost demand for a produced (remanufactured) item. Since T_1 is shorter than T, then

$$T_1 = \frac{x_2}{\beta_p D_p}\left(1-\frac{\gamma_r\beta_r(m-1)}{m}\right) \tag{6.3}$$

The derivation of T_1 is provided in Appendix 4. The sum of the terms mS_r and nS_p in (6.2) represents the total setup cost in an interval of length T. In each interval T, there are m remanufacturing cycles each of length T_r/m and of size x_2/m, and n production cycles each of length T_p/n and of size x_1/n. The remanufactured (manufactured/produced) quantity of x_2 (x_1) units is consumed over sub-interval T_r (T_p) at a demand rate of D_r (D_p) units per unit of time, where $x_2 = D_rT_r$ ($x_1 = D_pT_p$). The average inventory per unit of time for remanufactured (manufactured/produced) items over a remanufacturing (manufacturing/production) sub-interval of length T_r (T_p), is $x_2/2m$ ($x_1/2n$). Therefore, the total holding cost for remanufactured items in (6.2) is computed as $h_r \times \dfrac{x_2}{2m} \times \dfrac{T_r}{D_r} \times m = \dfrac{h_r}{2mD_r}x_2^2$. Similarly, the total holding cost for produced items in (6.2) is computed as $h_p \times \dfrac{x_1}{2n} \times \dfrac{T_p}{D_p} \times n = \dfrac{h_p}{2nD_p}x_1^2$. It is assumed in the presented models that demand for newly produced items over T_r is lost, and demand for remanufactured items over T_p is lost too. Therefore, the total lost sales cost in an interval of length T is the sum of $C_{lr}D_rT_p$ and $C_{lp}D_pT_r$. The holding cost for collected used items is computed as shown in Appendix 5. Then, the total cost per unit time function is $\Psi(n,m,x_2,\gamma_p,\gamma_r)/T$, where $T = T_r + T_p = x_2/D_r + x_1/D_p$, and is derived from (6.2) as

$$\psi(n,m,x_2,\gamma_p,\gamma_r) = \frac{1}{x_1/D_p + x_2/D_r}\Bigg\{mS_r + nS_p + \frac{h_r}{2mD_r}x_2^2 + \frac{h_p}{2nD_p}x_1^2 + \frac{C_{lr}D_r}{D_p}x_1 + \frac{C_{lp}D_p}{D_r}x_2$$
$$+h_u\left[\left(\frac{m(\gamma_r\beta_r-1)+1}{2mD_r}\right)x_2^2 + \frac{\gamma_p\beta_p}{2D_p}x_1^2 + \left(\frac{\gamma_r\beta_r}{mD_p} + \frac{\gamma_p\beta_p(m-1)}{mD_r}\right)x_1x_2\right]\Bigg\} \tag{6.4}$$

where $x_1/x_2 = (1-\gamma_r\beta_r)/(\gamma_p\beta_p)$. Now, and for the simplicity of the presentation, let,

$$A = \frac{1}{x_1/D_p + x_2/D_r} = \frac{1}{\left(\frac{(1-\gamma_r\beta_r)}{\gamma_p\beta_p D_p} + \frac{1}{D_r}\right)x_2},\ C_{pr}(\gamma_p,\gamma_r) = \frac{C_{lr}D_r}{D_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right) + \frac{C_{lp}D_p}{D_r}, \text{ and}$$

$$H(n,m,\gamma_p,\gamma_r) = \frac{h_r}{2mD_r} + \frac{h_p}{2nD_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)^2 + h_u\left[\left(\frac{m(\gamma_r\beta_r - 1)+1}{2mD_r}\right) + \frac{\gamma_p\beta_p}{2D_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)^2 + \left(\frac{\gamma_r\beta_r}{mD_p} + \frac{\gamma_p\beta_p(m-1)}{mD_r}\right)\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)\right]$$

Therefore, (6.4) could be presented as

$$\psi(n,m,x_2,\gamma_p,\gamma_r) = \frac{A}{x_2}\left[mS_r + nS_p + x_2C_{pr}(\gamma_p,\gamma_r) + x_2^2H(n,m,\gamma_p,\gamma_r)\right] \tag{6.4a}$$

Equation (6.4) is convex over x_2, since $\partial^2\psi(.,x_2,.)/\partial x_2^2 = 2A(mS_r + nS_p)/x_2^3 > 0 \quad \forall\, x_2 > 0$. Setting the first derivative of (6.4a) equal to zero and solving for x_2 to get

$$x_2(n,m,\gamma_p,\gamma_r) = \sqrt{\frac{mS_r + nS_p}{H(n,m,\gamma_r,\gamma_p)}} \tag{6.5}$$

Substitute (6.5) in (6.4), reduces (6.4) to

$$\psi(n,m,\gamma_p,\gamma_r) = A\left(2\sqrt{(mS_r + nS_p)H(n,m,\gamma_p,\gamma_r)} + C_{pr}(\gamma_p,\gamma_r)\right) \tag{6.6}$$

The mathematical programming problem is therefore written from (6.6) as

Minimize $\psi(n,m,\gamma_p,\gamma_r)$ (6.7a)

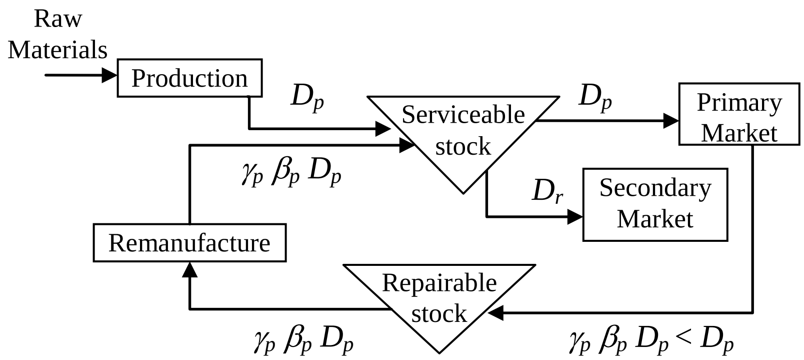
Subject to:

n, $m \geq 1$, are integers (6.7b)

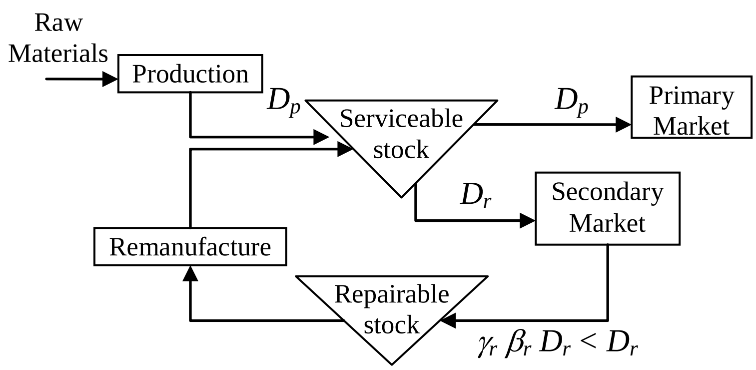
$\gamma_{\min} \leq \gamma_p \leq 1$ (6.7c)

$0 \leq \gamma_r \leq 1$ (6.7d)

The lower bounds introduced in constraints (6.7c) and (6.7d) are justified as shown in Figure 6.3. Consider case (a), $\gamma_r = 0$ and $\gamma_p > 0$, where used items are collected only from the primary market at a rate $\gamma_p\beta_p > 0$ ($\gamma_r\beta_r = 0$). For case (b), $\gamma_r > 0$ and $\gamma_p = 0$, used items are collected only from the secondary market at a rate $\gamma_r\beta_r > 0$ ($\gamma_p\beta_p = 0$).



Case (a): $\gamma_r = 0$ and $\gamma_p > 0$



Case (b): $\gamma_r > 0$ and $\gamma_p = 0$

Figure 6.3. Material flow for a production and remanufacturing system with two extreme collection cases

Case (b) is infeasible since the return flow will represent a closed loop of demand D_r that has a finite and reducing supply compared to demand of secondary market, and at some point in the future will reach a zero level; i.e., $\lim_{t \to +\infty} \beta_r^t D_r \to 0$ since $0 < \beta_r < 1$. Therefore, the secondary market must be fed by remanufacturing items collected from the primary market, subsequently, $\gamma_{\min}$ can never equal to zero and $\gamma_p \geq \gamma_{\min}$. In addition, in many countries (e.g., Germany), governmental legislations compel companies to initiate product recovery programs (e.g., Fleischmann et al., 1997; Chung & Poon, 2001). Therefore, the presented models assume that some collection must occur; i.e., $\gamma_{\min} > 0$.

6.4 Model II: Partial Lost Sales Case

In Model II, it is assumed that customers are enticed to settle for substitution between products. That is, over period T_r a percentage of demand, b_r, for newly produced items is substituted by remanufactured items at a cost v_p. Similarly, over period T_p a percentage of demand b_p, for remanufactured items is substituted by newly produced items at a cost v_r. Accordingly, the demand rates for remanufactured and newly produced items are adjusted respectively as $\tilde{D}_r = D_r + b_r D_p - b_p D_r$ and $\tilde{D}_p = D_p - b_r D_p + b_p D_r$. Therefore, Equations (6.5) and (6.6) are modified respectively as

$$\tilde{x}_2(n,m,\gamma_p,\gamma_r) = \sqrt{\frac{mS_r + nS_p}{\tilde{H}(n,m,\gamma_r,\gamma_p)}} \tag{6.8}$$

$$\tilde{\psi}(n,m,\gamma_p,\gamma_r) = \tilde{A}\left(2\sqrt{(mS_r + nS_p)\tilde{H}(n,m,\gamma_p,\gamma_r)} + \tilde{C}_{pr}(\gamma_p,\gamma_r)\right) \tag{6.9}$$

where, $\tilde{A} = \dfrac{\gamma_p\beta_p\tilde{D}_p\tilde{D}_r}{(1-\gamma_r\beta_r)\tilde{D}_r + \gamma_p\beta_p\tilde{D}_p}$, $\tilde{H}(n,m,\gamma_p,\gamma_r) = \dfrac{h_r}{2m\tilde{D}_r} + \dfrac{h_p}{2n\tilde{D}_p}\left(\dfrac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)^2$

$$+ h_u\left[\left(\frac{m(\gamma_r\beta_r - 1)+1}{2m\tilde{D}_r}\right) + \frac{\gamma_p\beta_p}{2\tilde{D}_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)^2 + \left(\frac{\gamma_r\beta_r}{m\tilde{D}_p} + \frac{\gamma_p\beta_p(m-1)}{m\tilde{D}_r}\right)\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)\right],$$

and

$$\tilde{C}_{pr}(\gamma_p,\gamma_r) = \frac{C_{lr}(1-b_p)D_r}{\tilde{D}_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right) + \frac{C_{lp}(1-b_r)D_p}{\tilde{D}_r} + \frac{v_p b_r D_p}{\tilde{D}_r} + \frac{v_r b_p D_r}{\tilde{D}_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)$$

The following solution procedure is applied to solve Equations (6.7a-6.7d) and (6.9).

6.5 Solution Procedure

Step 1: For the set of input parameters D_p, D_r, S_p, S_r, h_p, h_r, h_u, C_{lp}, C_{lr}, β_p, β_r, b_p, b_r, v_p and v_r. Set n = 1 and m = 1, and optimize $\psi(1,1,\gamma_p,\gamma_r)$. Record the values of $\psi(1,1,\gamma_p,\gamma_r)$, $\gamma^*_{p(1,1)}$ and $\gamma^*_{r(1,1)}$, where $\gamma^*_{p(1,1)}$ and $\gamma^*_{r(1,1)}$ are the optimum values of γ_p and γ_r for the case of $i = n = 1$ and $m = 1$.

Step 2: Repeat Step 1 for m = 2 and record $\psi(1,2,\gamma_p,\gamma_r)$, $\gamma^*_{p(1,2)}$ and $\gamma^*_{r(1,2)}$. Compare $\psi(1,1,\gamma_p,\gamma_r)$ and $\psi(1,2,\gamma_p,\gamma_r)$. If $\psi(1,1,\gamma_p,\gamma_r) < \psi(1,2,\gamma_p,\gamma_r)$, terminate the search for (i = n = 1) and record the value of $\psi(1,1,\gamma_p,\gamma_r)$. If $\psi(1,1,\gamma_p,\gamma_r) > \psi(1,2,\gamma_p,\gamma_r)$, repeat for ($m$ = 3), (m = 4), etc. Terminate once $\psi(1,m_1^*-1,\gamma_p,\gamma_r) > \psi(1,m_1^*,\gamma_p,\gamma_r) < \psi(1,m_1^*+1,\gamma_p,\gamma_r)$, where m_1^* is the optimal value for the number of remanufacturing cycles when there is 1 production cycle. Record the values of $\psi(1,m_1^*,\gamma_p,\gamma_r)$, m_1^*, $\gamma^*_{p(1,m_1^*)}$ and $\gamma^*_{r(1,m_1^*)}$.

Step 3: Repeat Steps 1 and 2 for n = 2. Compare $\psi(1,m_1^*,\gamma_p,\gamma_r)$ and $\psi(2,m_2^*,\gamma_p,\gamma_r)$. If $\psi(1,m_1^*,\gamma_p,\gamma_r) < \psi(2,m_2^*,\gamma_p,\gamma_r)$, terminate the search and $\psi(1,m_1^*,\gamma_p,\gamma_r)$ is the optimum solution. If $\psi(1,m_1^*,\gamma_p,\gamma_r) > \psi(2,m_2^*,\gamma_p,\gamma_r)$, then drop the value of $\psi(1,m_1^*,\gamma_p,\gamma_r)$ and repeat steps 1 and 2 for i = n = 3, 4, 5, …, etc.

Step 4: Terminate the search once $\psi(i-1,m_{i-1}^*,\gamma_p,\gamma_r) \geq \psi(i,m_i^*,\gamma_p,\gamma_r) <$ $\psi(i+1,m_{i+1}^*,\gamma_p,\gamma_r)$, where i is the optimal value for the number of production cycles when there are m_i^* remanufacturing cycles at a profit $\psi(i,m_i^*,\gamma_p,\gamma_r)$.

6.6 Numerical Examples

In this section, several numerical examples are solved whose parameters were collected from the literature. Three numerical examples were selected from Dobos & Richter (2003), Dobos & Richter (2004), and Teunter (2004). Note that these studies, like other studies in the literature, assumed that the remanufactured items are as-good-as-new, i.e., $d = D_p = D_r$ and $\beta = \beta_p = \beta_r$, $h_p = h_r$, and $C_{lp} = C_{lr} = 0$ and there is no need for substitution between products. Therefore, when solving the above indicated numerical examples, it is necessary to assume values for h_r, C_{lp}, C_{lr} and β_r. In these numerical examples, it was assumed that $\gamma_{\min} = 0.01$ (or 1%). The reason for assuming it this low is that the model may favour an optimal solution closer to a pure production and no remanufacturing, where the model moves closer to an EPQ/EOQ model. After solving these numerical examples, a simulation study to investigate the behaviour of Model I was conducted where its input parameters were randomized each over its range (of minimum and maximum values). These minimum/maximum values were determined from the above mentioned studies.

Example 6.1

This example illustrates Model I, by using data similar to those of Dobos & Richter (2003, p.44). Let $D_p = D_r = 200$, $S_p = 144$, $S_r = 72$, $h_p = 12$, $h_r = 3$, $h_u = 3$, $C_{lp} = 5$, $C_{lr} = 5$, and $\beta_p = \beta_r = 0.667$. Using the presented solution procedure, the optimal policy occurs when $\gamma_r = 1$, $\gamma_p = 1$, $m = 2$, and $n = 1$, the cost is 1,619.68, $x_2 = 123.94$, and $x_1 = 61.97$. The search results from the solution procedure described in Section 6.5 are shown in Table 6.1.

Table 6.1. A numerical example illustrating the solution procedure

Trial	n	m	γ_p	γ_r	Cost ($\psi_{n,m}$)	Notes
1	1	1	1	0.995	$\psi_{1,1} = 1634.99$	
2	**1**	**2**	**1**	**1**	$\psi_{1,2}$ **= 1619.98**	$\psi_{1,1} > \psi_{1,2}$, continue
3	1	3	1	1	$\psi_{1,3} = 1644.98$	$\psi_{1,2} < \psi_{1,3}$, Therefore $m^*_1 = 2$, $\psi_{1,2}$ **= 1619.98**
4	2	1	1	0	$\psi_{2,1} = 1695.59$	
5	2	2	1	1	$\psi_{2,2} = 1678.82$	$\psi_{2,1} > \psi_{2,2}$ continue
6	2	3	1	1	$\psi_{2,3} = 1669.33$	$\psi_{2,2} > \psi_{2,3}$ continue
7	2	4	1	1	$\psi_{2,4} = 1678.82$	$\psi_{2,3} < \psi_{2,4}$ Therefore $m^*_2 = 3$, $\psi_{2,3} = 1669.33$
8						Since $\psi_{1,2} < \psi_{2,3}$, Terminate

Values of S_r are increased (from 1 to 500) while keeping the other parameters fixed at their values to investigate the sensitivity of the Model I. The results showed that the optimal solution flips from $\gamma_r = 1$ and $\gamma_p = 1$ ($1 \le S_r \le 225$), to $\gamma_r = 0$ and $\gamma_p = 1$ ($226 \le S_r \le 292$), to $\gamma_r = 0$ and $\gamma_p = 0.01$ ($293 \le S_r \le 500$). This suggests that the optimal policy is to collect all available used items from the primary and secondary markets when $S_r \le 225$, from the primary market when $226 \le S_r \le 292$, and collect the minimum available used items from the primary market when $293 \le S_r \le 500$. Similarly, values of S_p are increased (from 1 to 500) while keeping the other parameters fixed at their values. The results showed that the optimal solution flips from $\gamma_r = 0$ and $\gamma_p = 0.01$ ($1 \le S_p \le 35$), to $\gamma_r = 0$ and $\gamma_p = 1$ ($36 \le S_r \le 46$), to $\gamma_r = 1$ and $\gamma_p = 1$ ($47 \le S_r \le 500$). The above results and the corresponding values of n, m, x_1, x_2 and $\psi_{n,m}$ are summarised in Table 6.2.

The sensitivity of the model was investigated by changing h_r (from $h_r = h_u = 3$ to $h_r = h_p = 12$), while keeping the other parameters fixed at their values. The optimal solution flipped from $\gamma_r = 1$ and $\gamma_p = 1$ to $\gamma_r = 0$ and $\gamma_p = 1$ when h_r increased from 11.1 to 11.2. A similar behaviour to that of h_r was observed when h_p increased from $h_p = h_r = 3$ to 12, with a sudden shift from $\gamma_r = 0$ and $\gamma_p = 0.01$ to $\gamma_r = 0$ and $\gamma_p = 1$ when h_p increased from 3.2 to 3.3.

Table 6.2. Optimal policies for changing values of "S_r" and "S_p"

		M	n	γ_r	γ_p	x_2	x_1	$\psi_{n,m}$
S_r	1	14	1	100%	100%	113.62	56.53	1371.44
	225	1	1	100%	100%	96.38	48.28	1674.65
	226	1	2	0%	100%	99.32	148.23	1830.53
	292	1	3	0%	100%	132.93	198.41	1874.03
	293	1	35	0%	1%	16.24	2423.86	1874.23
	500	1	45	0%	1%	20.90	3119.55	1889.05
S_p	1	1	205	0%	1%	7.95	1186	1092.8
	35	1	35	0%	1%	8.01	1195.34	1431.13
	36	1	3	0%	100%	66.28	98.93	1435.8
	46	1	2	0%	100%	55.76	83.22	1466.26
	47	1	1	100%	100%	67.31	33.72	1471.15
	500	3	1	100%	100%	210.49	104.72	1908.62

The results also showed a slow increase in γ_r when $6.6 \le h_p < 7$ followed by a sudden increase from $\gamma_r = 0$ and $\gamma_p = 1$ to $\gamma_r = 1$ and $\gamma_p = 1$ for values of $h_p \ge 7$. The sensitivity of the model was investigated by changing C_{lr} (from 0.25 to 10), while keeping the other parameters fixed at their values. The results showed a gradually increase in γ_r from $\gamma_r = 0$ to $\gamma_r =1$, when $3 \le C_{lr} < 3.9$, with a sudden increase from $\gamma_r = 0.56$ to $\gamma_r = 1$ when C_{lr} increased from 3.8 to 3.9. On the other hand, γ_p suddenly increased from $\gamma_p = \gamma_{\min} = 0.01$ to $\gamma_p =1$ when C_{lr} increased from 2.9 to 3. An opposite behaviour was recorded when the sensitivity of the model was investigated by changing C_{lp} (from 0.25 to 10). The results showed a gradual decrease in γ_r from $\gamma_r = 1$ to $\gamma_r = 0$, when C_{lp} increased from 6.1 to 7, while γ_p suddenly drops to $\gamma_p = \gamma_{\min} = 0.01$ from $\gamma_p = 1$ ($0.25 \le C_{lp} \le 7$), when $C_{lp} > 7$. The above results and the corresponding values of m, n, x_2, x_1 and $\psi_{n,m}$ are summarised in Table 6.3.

Table 6.3. Optimal policies for changing values of "h_r", "h_p", "C_{lr}" and "C_{lp}"

		m	n	γ_r	γ_p	x_2	x_1	$\psi_{n,m}$
h_r	3	2	1	100%	100%	123.94	61.96	1619.66
	11.1	3	1	100%	100%	125.12	62.55	1767.32
	11.2	1	1	0%	100%	44.98	67.47	1768.30
	12	1	1	0%	100%	44.57	66.85	1775.46
h_p	3	1	9	0%	1%	8.29	1244.22	1436.88
	3.2	1	9	0%	1%	8.05	1206.87	1450.40
	3.3	1	1	0%	100%	76.09	114.13	1454.23
	6.6	1	1	0%	100%	62.18	93.26	1555.85
	6.7	1	1	2%	100%	62.32	92.34	1558.63
	6.8	1	1	7%	100%	63.37	90.54	1561.34
	6.9	1	1	12%	100%	64.38	88.82	1563.96
	7	2	1	100%	100%	135.76	67.88	1565.69
	12	2	1	100%	100%	123.94	61.97	1619.68
C_{lr}	0.1	1	17	0%	1%	7.86	1178.30	876.29
	2.9	1	17	0%	1%	7.86	1178.30	1432.58
	3	1	1	0%	100%	50.08	75.05	1450.60
	3.1	1	1	9%	100%	52.49	73.75	1462.44
	3.2	1	1	18%	100%	54.85	72.43	1473.97
	3.3	1	1	26%	100%	57.17	71.09	1485.20
	3.4	1	1	33%	100%	59.44	69.73	1496.14
	3.5	1	1	39%	100%	61.68	68.34	1506.80
	3.6	1	1	45%	100%	63.88	66.94	1517.17
	3.7	1	1	51%	100%	66.03	65.52	1527.27
	3.8	1	1	56%	100%	68.15	64.08	1537.09
	3.9	2	1	100%	100%	123.94	61.97	1546.35
	10	2	1	100%	100%	123.94	61.97	1953.01
C_{lp}	3	2	1	100%	100%	123.94	61.97	966.35
	6.1	2	1	100%	100%	123.94	61.97	1766.35
	6.2	1	1	56%	100%	68.15	64.08	1777.09
	6.3	1	1	51%	100%	66.03	65.52	1787.27
	6.4	1	1	45%	100%	63.88	66.94	1797.17
	6.5	1	1	39%	100%	61.68	68.34	1806.80
	6.6	1	1	33%	100%	59.44	69.73	1816.14
	6.7	1	1	26%	100%	57.17	71.09	1825.20
	6.8	1	1	18%	100%	54.85	72.43	1833.97
	6.9	1	1	9%	100%	52.49	73.75	1842.44
	7	1	1	0%	100%	50.08	75.05	1850.60
	7.1	1	17	0%	1%	7.86	1178.30	1852.58
	10	1	17	0%	1%	7.86	1178.30	1856.42

The behaviour of γ_r and γ_p was also investigated for varying values of D_r (D_p) over the range 50 to 1000, while keeping the other parameters fixed at their values. The results were $\gamma_r = 0$ and $\gamma_p = \gamma_{\min} = 0.01$ ($D_r = 50$), $\gamma_r = 0$ and $\gamma_p = 1$ ($D_r = 100$), and $\gamma_r = 1$ and $\gamma_p = 1$ ($100 < D_r \leq 1000$). Whereas for changes in D_p, the results were $\gamma_r = 1$ and $\gamma_p = 1$ ($50 \leq D_p \leq 250$), $\gamma_r = 0.13$ and $\gamma_p = 1$ ($D_p = 300$), and $\gamma_r = 0$ and $\gamma_p = 0.01$ ($300 < D_p \leq 1000$).

Finally, the sensitivity of the model for changes in β_p and β_r was investigated. The results showed that the collection policy was $\gamma_r = 0$ and $\gamma_p = 0.01$ when $1\% \leq \beta_p \leq 7\%$, shifting to $\gamma_r = 1$ and $\gamma_p = 1$ when $7\% < \beta_p \leq 66.67\%$, with a sudden shift occurring when β_p was increased from 6 to 7 percent. On the other hand, the model showed to be insensitive to changes in β_r, where the policy was $\gamma_r = 1$ and $\gamma_p = 1$ for $1\% \leq \beta_p \leq 66.67$ percent. The above results suggest that there exits S_p, S_r, h_p, h_r, D_p, D_r, C_{lp} and C_{lr} values beyond which collection of used units from the secondary market is not optimal and remanufacturing is to be fed from the primary market only. The above results and the corresponding values of n, m, x_1, x_2 and $\psi_{n,m}$ are summarised in Table 6.4.

Table 6.4. Optimal policies for changing values of “D_p”, “D_r”, “β_p” and “β_r”

		m	n	γ_r	γ_p	x_2	x_1	$\psi_{n,m}$
D_r	50	1	17	0%	1%	7.85	1177.98	1102.96
	100	1	1	0%	100%	48.00	72.00	1300.00
	150	2	1	100%	100%	116.42	58.21	1471.57
	1000	1	1	100%	100%	115.29	57.65	4927.72
D_p	50	1	1	100%	100%	56.57	28.28	1259.12
	250	2	1	100%	100%	132.12	66.06	1801.40
	300	1	1	13%	100%	63.69	87.40	1969.55
	350	1	17	0%	1%	10.39	1558.64	2127.35
	1000	1	17	0%	1%	17.56	2633.80	2980.89
β_p	1%	1	140	0%	1%	0.97	9699.83	1834.24
	7%	1	53	0%	1%	2.57	3672.23	1838.58
	8%	1	2	100%	100%	33.28	138.68	1837.42
	67%	2	1	100%	100%	123.94	61.97	1619.68
β_r	1%	1	1	100%	100%	50.42	74.87	1689.64
	67%	2	1	100%	100%	124.56	61.66	1618.65

Example 6.2

This example illustrates Model I by using data similar to those of Dobos & Richter (2004, p.321). Let $D_p = D_r = 1000$, $S_p = 360$, $S_r = 440$, $h_p = 85$, $h_r = 80$, $h_u = 80$, $C_{lp} = 50$, $C_{lr} =$

50, and β_p = β_r = 0.2. The optimal policy occurs when γ_r = 0, γ_p = $\gamma_{\min}$ =0.01, m = 1, and n = 25, where the cost is 58,182.35, x_2 = 4.61, and x_1 = 2,302.80. Comparing these values with those in Example 6.1 (Dobos & Richter, 2003), where the policy for Example 6.1 recommends to collect all, the optimal policy in for Example 6.2 recommends not to collect used items to be remanufactured from the secondary market and only collect from the primary market at the lowest rate possible, which is γ_p = $\gamma_{\min}$ = 0.01. This is why it is economical to remanufacture once (m =1) every 25 production cycles (n =25), whereas in Example 6.1, it is recommended to collect all available used items in both markets. This extreme switch in decision falls in line with the results from sensitivity analysis performed on Example 6.1.

In Example 6.2, $\gamma_{\min}$ was set at 1% ($\gamma_{\min}$ =0.01), now what if the government legislation compel the manufacturer to accept say 20% ($\gamma_{\min}$ =0.2) instead of 1% ($\gamma_{\min}$ =0.01). In this case, the optimal policy occurs when γ_r = 0, γ_p = $\gamma_{\min}$ =0.2, m = 1, and n = 6, where the cost is 59,171.09, x_2 = 21.81, and x_1 = 545.19. Whereas, in Example 6.1, the optimal policy was not affected by the value of $\gamma_{\min}$ since $\gamma_p \geq \gamma_{\min}$ is an abundant constraint, while it is a binding constraint in Example 6.2. Example 6.2 was replicated for reducing values of h_r and h_p, the results showed that the optimal policy moves from γ_r = 0 and γ_p = $\gamma_{\min}$ = 0.01 when h_r = 80 and h_u = 80 to γ_r = 1 and γ_p = 1 > $\gamma_{\min}$ =0.01 when h_r = 10 and h_u = 10. This suggests that a firm may have to consider other storage options to lower its holding cost to be able to force $\gamma_p \geq \gamma_{\min}$ to be an abundant rather than a binding constraint.

Example 6.3

This example uses data similar to those of Teunter (2004, p.438). Let D_p = D_r = 1000, S_p = 20, S_r = 5, h_p =10, h_r = 2, h_u = 2, C_{lp} = 5, C_{lr} = 5, and β_p = β_r = 0.8. The optimal policy occurs when γ_r = 1, γ_p = 1, m = 4, and n = 1, where the cost is 5,329.85, x_2 = 194.03, and x_1 = 48.51. This result is similar to that obtained from Example 6.1.

Example 6.4

From Example 6.3, a reasonable range is set for the changes in the input parameters. These input parameters will be randomized each over its specified range tabulated below in Table 6.5. One thousand randomized data sets were generated form the input parameters range, corresponding to 1000 numerical examples that were optimized using the solution procedure described in Section 6.5. Note that when generating these values, the following conditions were implemented, which are $h_u \le h_r \le h_p$, $C_{lr} \le C_{lp}$, and $\beta_r \le \beta_p$.

Table 6.5. Randomized input parameters

	S_r / S_p	h_u / h_r / h_p	D_r / D_p	β_r / β_p	C_{lr} / C_{lp}
Maximum	500	100	1000	0.8	100
Minimum	5	1	100	0.1	10

The results showed that in 20.8% (208 of 1000) of the examples the optimal policy occurred when $\gamma_r = 1$ and $\gamma_p = 1$, in 78% (780 of 1000) of the examples the optimal policy occurred when $\gamma_r = 0$ and $\gamma_p = \gamma_{\min} = 0.01$, in 0.8% (8 of 1000) of the examples the optimal policy occurred when $\gamma_r = 0$ and $\gamma_p = 1$, in 0.2% (2 of 1000) of the examples the optimal policy occurred when $0 < \gamma_r < 1$ and $\gamma_p = 1$, and in 0.2% (2 of 1000) of the examples it occurred when $\gamma_r = 0$ and $0 < \gamma_p < 1$. These results suggest that the model presented in (7) could further be simplified by solving three special cases, which are $\gamma_r = 1$, $\gamma_p = 1$ (Case I), $\gamma_r = 0$ and $\gamma_p = \gamma_{\min} = 0.01$ (Case II), and $\gamma_r = 0$, $\gamma_p = 1$ (Case III). There is therefore 99.6% chance that the optimal solution will reside with one of the three cases.

Finally, Cases I and Case II respectively suggest either no disposal ($\gamma_r = 1$ and $\gamma_p = 1$; total remanufacturing) or almost total disposal ($\gamma_r = 0$ and $\gamma_p = \gamma_{\min} = 0.01$; maximum production) of collected used items. These extreme cases are in line with the bang-bang policy of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal) discussed in Dobos & Richter (2003, 2004). In the authors' opinion, to have an optimal policy were $0 < \gamma_r < 1$ and $0 < \gamma_p < 1$ the quality and price of returned items must be considered. That is, the collection rate will be price and quality driven.

Example 6.5

A simulation is done using data similar to those of Example 6.2 (Dobos & Richter, 2004; p.321). Assume that some (say 25%, $b_r = 0.25$ and $b_p = 0.25$) of the customers demanding newly produced (remanufactured) items are not totally lost during a remanufacturing (production) period. Also assume that the system's manager will pay 10%

of C_{lp} = 50 and C_{lr} = 50 as compensations (v_p = 5, v_r = 5) for customers who will settle for a remanufacturing (or new) item instead. The optimal policy occurs when γ_r = 0, γ_p = 0.025, m = 1, and n = 16, where the cost is 46,127.18, x_2 = 7.30, and x_1 = 1473.53. Now assuming that 25% is associated with a 20% pay of C_{lp} = 50 and C_{lr} = 50, the optimal policy occurs when γ_r = 0, γ_p = 0.04, m = 1, and n = 13, where the cost is 46,323.62, x_2 = 9.48, and x_1 = 1194.38. The model was tested for 30%, 40% and 50% pay and for different values of b_p and b_r, with the cost behaving in a similar manner as before. That is, the model slowly favours remanufacturing, where x_2 (increases) and x_1 (decreases). Furthermore, it would be interesting to investigate Model II where b_r and b_p are dependent of v_r and v_p respectively; i.e., $b_r(v_r)$ and $b_p(v_p)$. Although we leave this investigation for a future work, we would like to entice the readers by a numerical example.

Assume $b_i(v_i) = 1 - e^{-\alpha_i v_i}$, where i = r, p and α_i = 0.2, and solve Example 6.2 for different values of $v_r = v_p$, with the result plotted in Figure 6.4. Figure 6.4 shows that there are optimal compensation values (v_p^*, v_r^*), corresponding to $b_p\left(v_p^*\right)$ and $b_r\left(v_r^*\right)$, for substitution between products that minimizes the total cost.

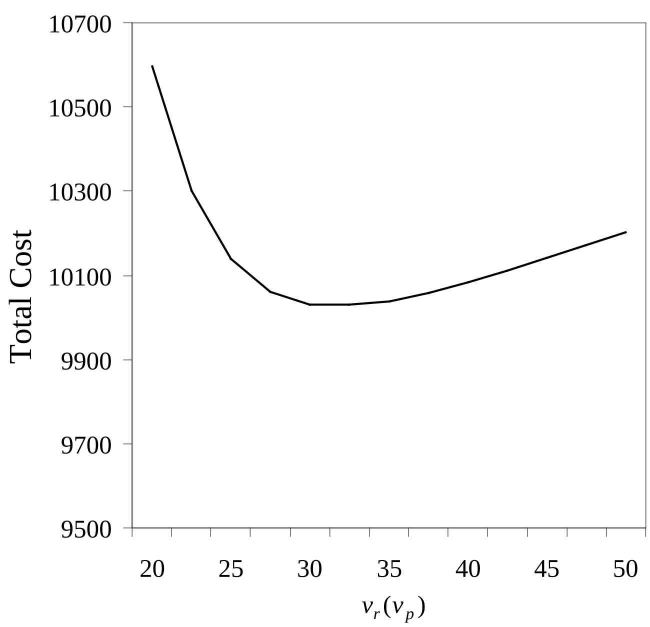


Figure 6.4. The behaviour of the total cost for increasing values of “v_r (v_p)”

6.7 Summary and Conclusions

In this chapter, two models were presented to bring reverse logistics closer to reality. Works in the literature assume remanufactured products to be "as good as new", however, customers perceive remanufactured and new products differently. In this chapter, production/ remanufacturing inventory models were extended to consider remanufactured items have different quality and value than new ones, and accordingly, demand for newly produced items is different from that for remanufactured (repaired) ones. Two mathematical models were developed; the first model assumes that demand for remanufactured (newly produced) items is lost over the production (remanufacturing) cycle. The second model assumed that a percentage of the demand is met and therefore partial lost sales occur, and in this case, it was assumed that customers are enticed (e.g., providing financial incentives) to agree to substitute between products.

Sensitivity analysis of the presented models showed that there exits values of the input parameters (e.g., setup costs, holding cost, etc) beyond which the collection of used items from the secondary market is not optimal and remanufacturing is to be fed from the primary market only. Models developed were simulated and the results showed that it is either to collect all available returns from primary and secondary markets (i.e., no disposal) or to collect from the primary market at the minimum collection rate with no collection from the secondary market (i.e., total disposal). These extreme cases are in line with the bang-bang policy (Dobos & Richter, 2003; 2004) of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal). Having a mixed collection strategy that is optimal may require modelling the collection rate of used products from secondary and primary markets as price and quality dependent variables, which will be dealt with in Chapter 8. The case of partial lost sales suggested that compensating customers to minimize the impact of lost sales is beneficial, because a reduction in total cost occurs when compensation is paid to reduce the effect of lost sales. This compensation amount was shown to have an optimal value.

In this chapter, disassembly processes were not considered. In the next chapter, products are assumed to be collected and disassembled, and the inventory of each subassembly is managed differently, to bring the modelling efforts closer to reality.

CHAPTER 7: MANAGING RETURNS' SUBASSEMBLIES INVENTORY PROBLEM

7.1. Introduction

Environmental consciousness started to grow in the 1950's, and waste management represented one of the main resolutions to save this planet. This allowed technology advancement in processes such as recycling, refurbishing, remanufacturing, repair and related activities (e.g., disassembly), giving rise to a new business term: Reverse Logistics. Reverse logistics and closed loop supply chain management became an established line of research in the late 1990's, yet, there are many research gaps in this field.

A usual modelling approach in these studies is to assume that products are returned after use at some rate and these returned units are remanufactured (or repaired). Although the available models in the literature imply that collected used units (or returns) are disassembled for recovery purposes, these models do not treat them as such. Contrary, the usual assumption is that returns are recovered as whole units, perhaps, for simplicity, although, it is a typical practice in product recovery programs, accordingly, this assumption results in missing the benefits reaped from product recovery programs.

At Volkswagen, broken parts are collected and remanufactured and resold as spare parts, and due to fluctuations in supply and demand, sometimes demand is more than supply of remanufactured spare parts. In this case, recovered spare parts satisfies demand for these parts, and in the case of shortage, newly produced spare parts are produced (van der Laan et al., 2004). Disassembly can be non-destructive or destructive. Non-destructive is the process of removing a part from an assembly with no impairment during the process. Destructive is disassembly with the objective of sorting the different materials for recycling purposes (Pochampally et al., 2009). Disassembly can be full, or partial, where certain items are recovered. In the later case, disassembly is also called reclamation or cannibalization. Research is needed into how companies should process, sort, store and dispose returned goods (Tibben-Lembke, 2002). This research need was echoed in other studies too (e.g., Klausner & Hendrickson, 2000; Walther & Spengler, 2005; Galbreth & Blackburn, 2006).

This chapter, similar to previous ones, considers a production-remanufacturing inventory model for a single product, where constant demand is satisfied from the inventory of newly produced and remanufactured items. However, this chapter assumes each unit of a collected used item is disassembled into components, where these components are sorted into subassemblies, and later their inventories are managed to be fed back into the production-remanufacturing processes. The multi-component inventory model to be presented continues the endeavour to bring reverse logistics closer to real life situations. Inventory policies will be investigated and managerial insights will be discussed.

7.2. Managing Returns' Subassemblies: Mathematical Modelling

In the presented model, it is assumed the following: (1) infinite production and remanufacturing rates, (3) remanufactured items are considered as-good-as-new, (4) demand is known constant and independent, (5) lead time is zero, and (6) no shortages are allowed.

The presented production/remanufacturing inventory model is shown in Figure 7.1. It illustrates the flow of remanufactured and newly produced items from the system to the market, and the flow of returned used items from the market to the repairable stock, where returns are disassembled and screened to verify their quality and those not conforming to remanufacturing requirements are disposed outside the system.

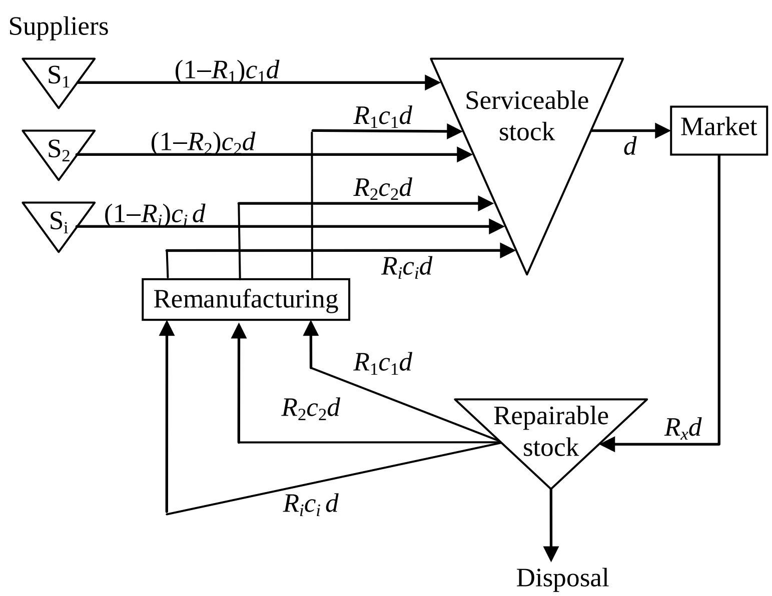


Figure 7.1. Material flow in a production and remanufacturing system

In Figure 7.1, the product's market demand d is satisfied from the serviceable stock, which holds the inventory of newly produced and remanufactured items. Returns are collected from the market according to a return percentage R_x, to be stored in the repairable stock, where $0 \leq R_x \leq 1$, where $R_x d$ is the flow rate of returns in the reverse direction. A single remanufacturing cycle and a single production cycle are considered for each time interval T.

For the single-product case considered herein, there are u different types of subassemblies that compose the product, according to a single level manufacturing Bill of Materials (BOM). Each subassembly i consists of k_i components, where $i = 1, 2, 3 \ldots u$. The term c_i represents the share of each subassembly i in the end product, where $c_i = \frac{k_i}{\sum_{i=1}^{u} k_i} \leq 1$ and $\sum_{i=1}^{u} c_i = 1$. Figure 7.2 provides a simple illustrative diagram for a BOM of an end product, which consists of 3 subassemblies, i.e., $u = 3$, where $c_1 = \frac{k_i}{\sum_{i=1}^{u} k_i} = \frac{3}{10} = 0.3$, $c_2 = 0.5$, $c_3 = 0.2$.

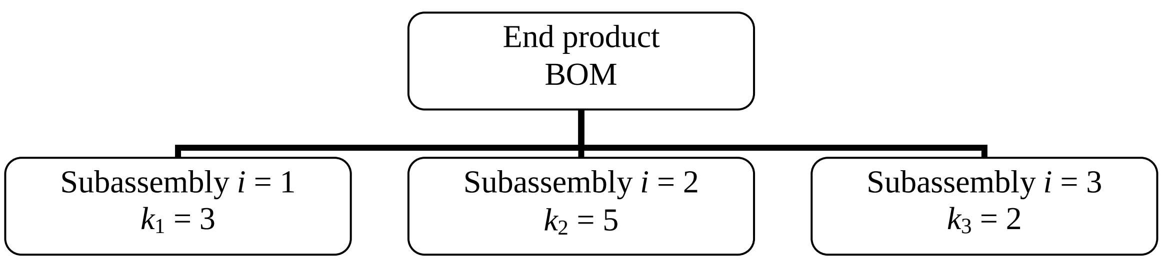


Figure 7.2. A single level BOM for an end product

The model's decision variables are β_x and Y_i, where $i = 1, 2, 3 \ldots u$. β_x refers to the ratio of the remanufactured items to the newly produced items, accordingly, the quantity required to be remanufactured for each subassembly i in interval T is $\beta_x c_i dT$. After disassembly, subassemblies are sorted and managed independently according to different usage rates, obsolescence conditions, and environmental concerns. The required quantity to be remanufactured ($\beta_x c_i dT$) might be different from the available repairable quantities for each subassembly, i.e., β_x is different from R_i. This is a common practice in the electronics recovery industry (Reimer et al., 2000; Smith et al., 1996). For the remanufacturing cycle, a quantity of ($\beta_x dT$) units is remanufactured during time T_r, where for each subassembly i a quantity of ($\beta_x c_i dT$) units is either collected, or ordered, or both. For the production cycle, a quantity of $(1-\beta_x)dT$ units is produced during time interval T_p, and for each subassembly i, a quantity of $(1-\beta_x)c_i dT$ units is produced, where $T = T_r + T_p$.

According to different usage and obsolescence rates, each subassembly i inventory management and disposal might be unique, and accordingly, a different quantity of $R_i c_i d$ units is ready to be remanufactured for each subassembly i. The remanufactured collected quantity per time interval, $R_i c_i dT$, may, or may not be enough to provide the required quantities of subassemblies for the remanufacturing cycle, $\beta_x c_i dT$. Therefore, when $R_i < \beta_x$, there is a possibility of ordering an additional quantity of $(\beta_x - R_i)c_i dT$ units for a specific subassembly i at cost S_i. The collected and ordered subassemblies are to satisfy demand rates of $\beta_x d$ and $(1-\beta_x)d$ for remanufactured and produced items respectively. The inventory behaviour of the serviceable stock, of a single remanufacturing cycle followed by a single production cycle, is presented in Figure 7.3.

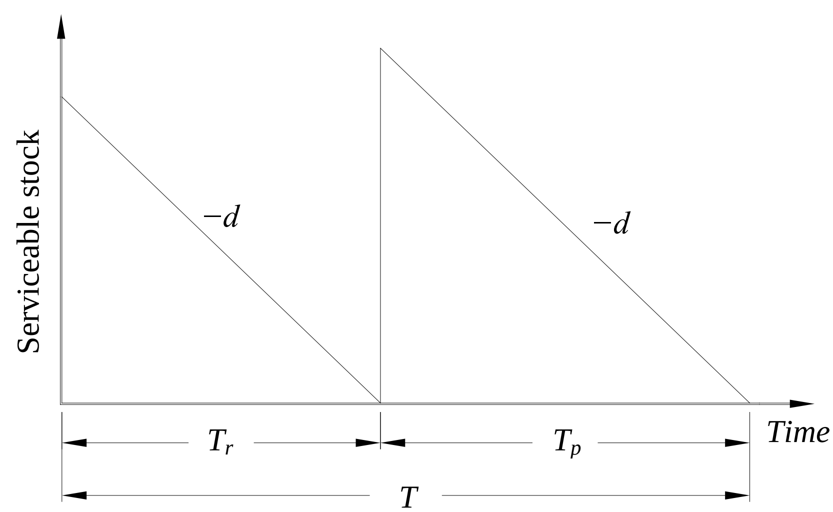


Figure 7.3. Inventory behaviour of the serviceable stock

For the repairable stock, it will be divided into u different stocks for u different subassemblies. There are three different inventory behaviours or cases according to the relationship between R_i and β_x for each subassembly i, namely Case 1, which is shown in Figure 7.4, Case 2 and Case 3.

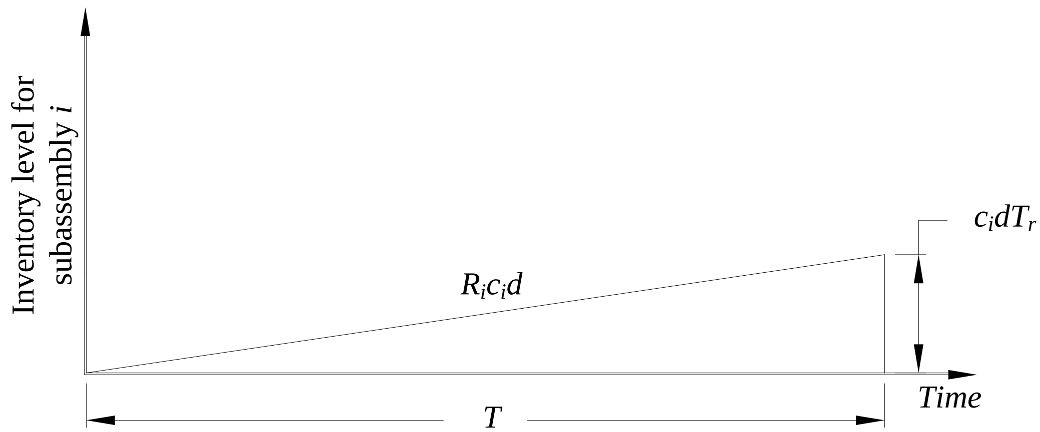


Figure 7.4. Inventory behaviour of the repairable stock for Case 1

For Case 1, $R_i = \beta_x$, and the collected quantity is exactly equal to the required quantity for remanufacturing. Case 2 is shown in Figure 7.5.

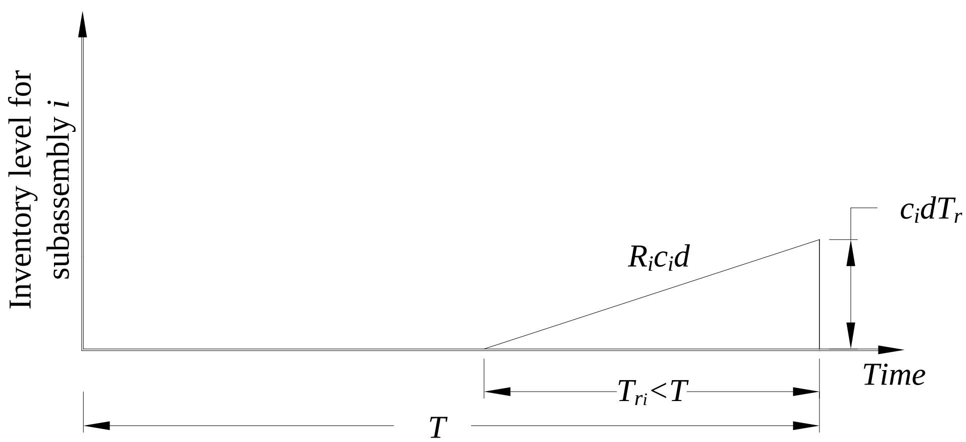


Figure 7.5. Inventory behaviour of the repairable stock for Case 2

For Case 2, $R_i > \beta_x$, and the required quantity of subassembly i is collected over interval $T_{ri} < T$. To avoid holding unnecessary returns in inventory, it is assumed that collection occurs at the end of interval T. Case 3 is shown in Figure 7.6.

For Case 3, $R_i < \beta_x$, and the collected quantity of subassembly i is less than the required quantity for remanufacturing, with the remaining quantity needed for subassembly i is ordered from a supplier at an additional ordering cost S_i.

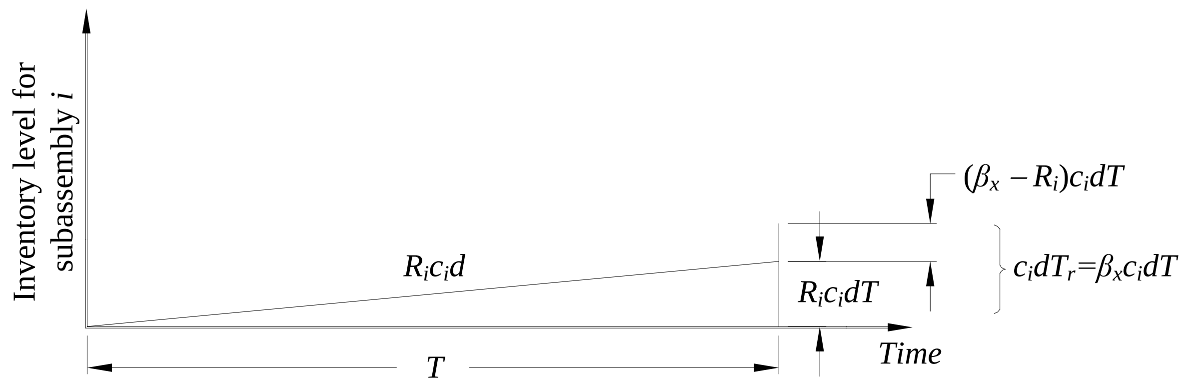


Figure 7.6. Inventory behaviour of the repairable stock for Case 3

From Figure 7.2, the holding cost function for the serviceable stock is determined as

$$H_1 = h_r \frac{dT_r^2}{2} + h_p \frac{dT_p^2}{2} = h_r \frac{d\beta_x^2 T^2}{2} + h_p \frac{d(1-\beta_x)^2 T^2}{2} = \frac{dT^2}{2}\left(h_r \beta_x^2 + h_p (1-\beta_2)^2\right) \qquad (7.1)$$

The holding cost functions for the repairable stock for the three cases discussed in Figures 7.4, 7.5, and 7.6, are given respectively as

Case 1: $R_i = \beta_x, H_2 = h_i R_i c_i dT_{r_i}^2 / 2 = h_i R_i c_i dT^2 / 2$

Case 2: $R_i > \beta_x, H_2 = h_i R_i c_i dT_{r_i}^2 / 2$

Case 3: $R_i < \beta_x, H_2 = h_i R_i c_i dT^2 / 2$ with an additional ordering cost = S_i

To differentiate between the three cases for each subassembly i, a binary decision variable, Y_i, is introduced, and the three cases are clustered in a single expression holding cost function,

$$H_2 = h_i \left[(1-Y_i) \frac{R_i c_i d}{2} T_{r_i}^2 + \frac{Y_i R_i c_i d}{2} T^2 \right] \qquad (7.2)$$

where Y_i = 0 for Case 1 and Case 2, Y_i = 1 for Case 3, and $T_{r_i} = \frac{\beta_x T}{R_i}$. The following two constraints are introduced to insure that Y_i is equal to 0 for Case 1 and Case 2, Y_i is equal to 1 for Case 3:

$$M(1-Y_i) + (1 - R_i / \beta_x) \geq 0$$

$$MY_i - (1 - R_i / \beta_x) \geq 0$$

where M is a large positive number. The total costs per interval T, including the production and remanufacturing setup costs, is then given as

$$\Psi = S_r + S_p + \sum_{i=1}^{u} Y_i S_i + \frac{dT^2}{2}\left[h_r \beta_x^2 + h_p (1-\beta_x)^2 + \sum_{i=1}^{u} h_i c_i \left[(1-Y_i)\frac{\beta_x^2}{R_i} + Y_i R_i \right]\right]$$

with its unit time cost form given as

$$\psi = \frac{S_r + S_p + \sum_{i=1}^{u} Y_i S_i}{T} + \frac{dT}{2}\left[h_r \beta_x^2 + h_p (1-\beta_x)^2 + \sum_{i=1}^{u} h_i c_i \left[(1-Y_i)\frac{\beta_x^2}{R_i} + Y_i R_i \right]\right] \qquad (7.3)$$

Letting $S = S_r + S_p + \sum_{i=1}^{u} Y_i S_i$ and

$H = h_r \beta_x^2 + h_p (1-\beta_x)^2 + \sum_{i=1}^{u} h_i c_i \left[(1-Y_i)\frac{\beta_x^2}{R_i} + Y_i R_i \right]$, then $\psi = \frac{S}{T} + \frac{dT}{2}[H]$. Differentiating ψ with respect to T, $\frac{\partial \psi}{\partial T} = -\frac{S}{T^2} + \frac{dH}{2} \rightarrow T = \sqrt{\frac{2S}{dH}} \rightarrow \psi = \sqrt{2SdH}$ or

$$\psi = \sqrt{2d\left(S_r + S_p + \sum_{i=1}^{u} Y_i S_i \right) \times \left(h_r \beta_x^2 + h_p (1-\beta_x)^2 + \sum_{i=1}^{u} h_i c_i \left[(1-Y_i)\frac{\beta_x^2}{R_i} + Y_i R_i \right]\right)} \qquad (7.4)$$

The minimum (optimum) of (7.4), ψ^*, is determined by solving the following model,

$$\psi^* = Minimize\ \psi, \qquad (7.5a)$$

Subject to,

$$M(1-Y_i) + (1 - R_i/\beta_x) \geq 0 \qquad (7.5b)$$

$$MY_i - (1 - R_i/\beta_x) \geq 0 \qquad (7.5c)$$

$$0 < \beta_x < 1, \qquad (7.5d)$$

$$Y_i \in \{0,1\} \qquad (7.5e)$$

The extreme polices of pure production (total disposal) and pure remanufacturing (no disposal) occur when $\beta_x = 0$ and $\beta_x = 1$, respectively. The pure production policy is presented as

$$\psi_p = \sqrt{2S_p d h_p} \qquad (7.6)$$

where $\beta_x = 0, Y_i = 0 \quad \forall i = 1, 2, 3 \ldots u$. The pure remanufacturing policy is presented as

$$\psi_R = \textbf{\textit{Minimize}}\sqrt{2d\left(S_r + \sum_{i=1}^{u} Y_i S_i\right) \times \left(h_r + \sum_{i=1}^{u} h_i R_i c_i\right)} \qquad (7.7a)$$

Subject to:

$$MY_i - (1 - R_i) \geq 0 \qquad (7.7b)$$

$$Y_i \in \{0,1\} \qquad (7.7c)$$

where $\beta_x = 1 \quad \forall i = 1, 2, 3 \ldots u$, and the optimum solution is given from (7.5a-7.5e), (7.6) and (7.7a-7.7c) as

$$\textbf{\textit{Minimize}}\ \{\psi^*, \psi_P, \psi_R\} \qquad (7.8)$$

7.3. Numerical Examples

Examples to illustrate the behaviour of the developed model and to draw some managerial conclusions are presented in this section. Example 7.1 is to study the effect of varying values of input parameters, while optimizing for Y_i and β_x. The change in the values of subassemblies' ordering costs is shown in Examples 7.2 and 7.3. The comparison between the assumption of treating returns as single units (as has been adopted in the literature) against the assumption adopted herein is discussed in Examples 7.4 and 7.5.

Example 7.1

Let d = 1000, u = 3 subassemblies, k_1 = 6, k_2 = 6, k_3 = 8, (i.e., c_1 = 0.3, c_2 = 0.3, c_3 = 0.4), R_1 = 0.3, R_2 = 0.6, R_3 = 0.85, S_1 = 50, S_2 = 50, S_3 = 50, S_p = 40, S_r = 32, h_p = 0.6, h_r = 0.12, and $h_1 = h_2 = h_3$ = 0.1. Substituting these values in Equation (7.8) and optimizing for Y_i and β_x, the optimum cost is ψ^* = 211.4 when β_x^* = 0.6, Y_1^* = 1, Y_2^* = 0, and Y_3^* = 0. The model's behaviour is examined by varying each cost input parameter over its range, and optimising for Y_i and β_x. The cost input parameters can be divided into two groups, production associated costs (h_p, S_p) and remanufacturing associated costs (h_r, h_i, S_r, S_i). For example, varying h_r over the range of (0.02-0.25) and optimizing the model for Y_i and β_x, the results are shown in Figure 7.7.

As shown in Figure 7.7, total costs increase by increasing h_r. Increasing h_r from 0.02 to 0.04, the pure production policy becomes more expensive than pure remanufacturing policy, and the optimum solution occurs at $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (1, 1, 1, 1). Increasing h_r from 0.04 to 0.05, increases the remanufacturing costs and makes production more economical, and the solution shifts from $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (1, 1, 1, 1) to $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (0.85, 1, 1, 0).

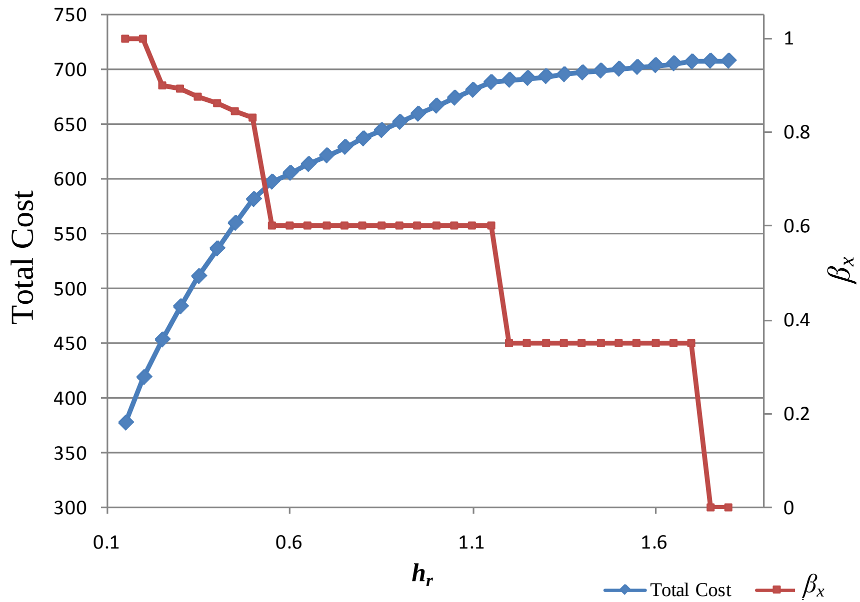


Figure 7.7. The behaviour of "β_x" and Total Cost for varying values of "h_r"

When h_r > 0.05 the solution shifts to $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (0.6, 1, 0, 0), and remains unchanged for values of h_r in the range 0.055−0.145. A similar behaviour was observed when h_r increases from 0.145 to 0.15, shifting the solution from $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (0.6, 1, 0, 0) to (0.3, 0, 0, 0), where the optimum solution remains unchanged. By increasing h_r beyond 0.235, the solution shifts from $(\beta_x^*, Y_1^*, Y_2^*, Y_3^*)$ = (0.3, 0, 0, 0) to (0, 0, 0, 0), which is the case of pure remanufacturing. Note that the optimum solution plateaus at R_1 = 0.3, R_2 = 0.6, and R_2 = 0.85, which are the cut-off values for the remanufacturing-production ratio β_x. To investigate the model further, the rest of the remanufacturing input cost parameters were increased, and the model behaved in a similar manner to that shown in Figure 7.7. For example, increasing the value of h_p produced a similar behaviour to total cost and β_x except for β_x being an increasing rather than a decreasing function.

Example 7.2

Let $d = 1000$, $u = 3$ subassemblies, $c_1 = 0.25$, $c_2 = 0.35$, $c_3 = 0.4$, $R_1 = 0.3$, $R_2 = 0.6$, $R_3 = 0.9$, $S_1 = 1$, $S_2 = 1$, $S_3 = 1$, $S_p = 200$, $S_r = 160$, $h_p = 0.6$, $h_r = 0.55$, and $h_1 = h_2 = h_3 = 0.1$. Substituting these values in Equation (7.8) and optimizing for Y_i and β_x, the optimum cost is $\psi^* = 447.6$ when $\beta_x^* = 1$, $Y_1^* = 1$, $Y_2^* = 1$, and $Y_3^* = 1$ (pure remanufacturing). The model is investigated by varying β_x over the range (0−1) while optimizing for Y_i and β_x. The results are shown in Figure 7.8.

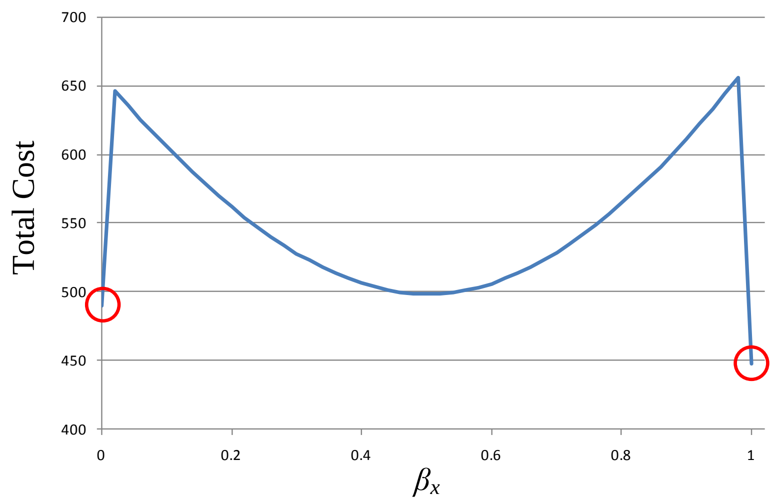


Figure 7.8. The behaviour of Total Cost of Example 7.2 for varying values of "β_x" with the solutions for the extreme cases ($\beta_x = 0, 1$) are circled

The values of S_i for i =1, 2, 3 in this example are reduced to eliminate their effects. The total cost drops sharply when $\beta_x = 0$ and $\beta_x = 1$ as shown in Figure 7.8. The reason for that is at the values of $\beta = 0$ or $\beta_x = 1$, only one setup cost is included in the cost function, either S_p or S_r, whereas for the case when $0 < \beta_x < 1$, which represent a mixed policy of remanufacturing and production, the total setup cost is $S_r + S_p$. The explains the dramatic shift in cost when β shifts from 0 or 1.

A minor change in the input parameter ($h_r = 0.8$) results in an optimum cost at $\psi^* = 489.9$ when $\beta_x^* = 0$, $Y_1^* = 0$, $Y_2^* = 0$, and $Y_3^* = 0$ (pure production). A similar behaviour to that shown in Figure 7.8 was observed, where the total cost drops sharply when $\beta_x = 0$ and 1, and a mixed production and remanufacturing policy is never optimum in this case, as shown in Figure 7.9.

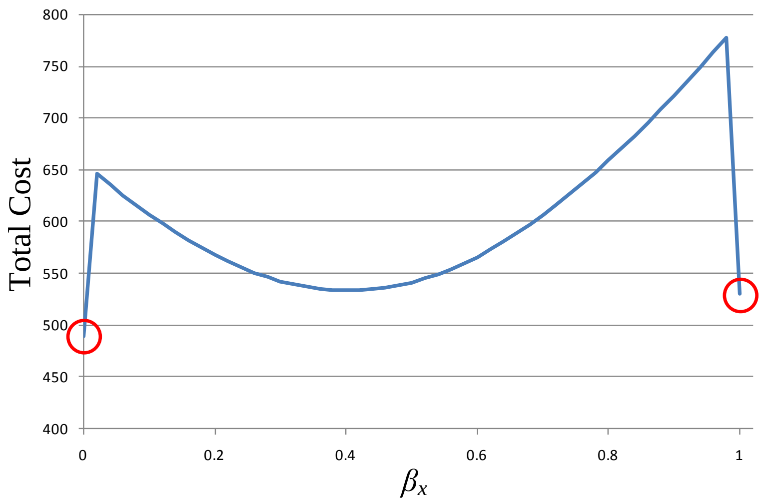


Figure 7.9. The behaviour of Total Cost of Example 7.2 (h_r = 0.8) for varying values of "β_x" with the solutions for the extreme cases (β_x = 0, 1) are circled

These results bring us to the finding of Dobos & Richter (2003, 2004), where either a pure production or a pure remanufacturing policy is always optimum (designated by circles) when compared to a mixed policy of production and remanufacturing.

Example 7.3

Let d = 1000, u = 3 subassemblies, c_1 = 0.25, c_2 = 0.35, c_3 = 0.4, R_1 = 0.3, R_2 = 0.6, R_3 = 0.9, S_1 = 80, S_2 = 80, S_3 = 80, S_p = 200, S_r = 160, h_p = 0.6, h_r = 0.2, and $h_1 = h_2 = h_3$ = 0.1.

Substituting these values in Equation (7.8) and optimizing for Y_i and β_x, the optimum cost is ψ^* = 432.4 when β_x^* = 0.6, Y_1^* = 1, Y_2^* = 0, and Y_3^* = 0 (mixed strategy of production and remanufacturing). Varying β_x over the range of (0−1) and optimizing the model for Y_i and β_x, the results are shown in Figure 7.10.

Values of S_i for i =1, 2, 3 are increased compared to those in Example 7.2 and results showed that total cost surges when $\beta_x = \{0, R_1, R_2, R_3\}$ and drops when β_x = 1. At the values of β_x = 0 or 1, setup costs are either S_p or $S_r + \sum_{i=1}^{u} S_i$. The reason for these results is as β_x increases, the return rates for different subassemblies reduce to values less than β_x, where additional ordering costs are incurred to match the higher remanufacturing-production ratio, β_x. These cost surges show an optimum solution other than the extreme cases of β_x (i.e., β_x = 0 or 1). The optimal solution occurs when β_x = 0.6.

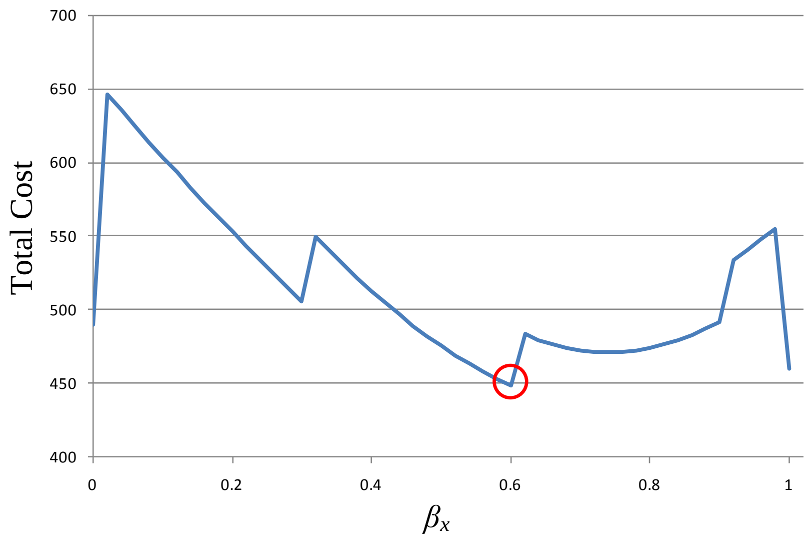


Figure 7.10. The behaviour of Total Cost of Example 7.3 for varying values of "β_x" with the optimal solution circled

Changing one input parameter : R_2 = 0.85, the optimum cost is found at ψ^* = 420.6 for β_x^* = 0.677, Y_1^* = 1, Y_2^* = 0, and Y_3^* = 0 as shown in Figure 7.11. The model behaves in a similar manner to the cases in Examples 7.2 and 7.3; however the optimum solution is attained when $0 < \beta_x = 0.677 < 1$, which is different from any of the R_i values, where a mixed policy is of less cost that pure policies.

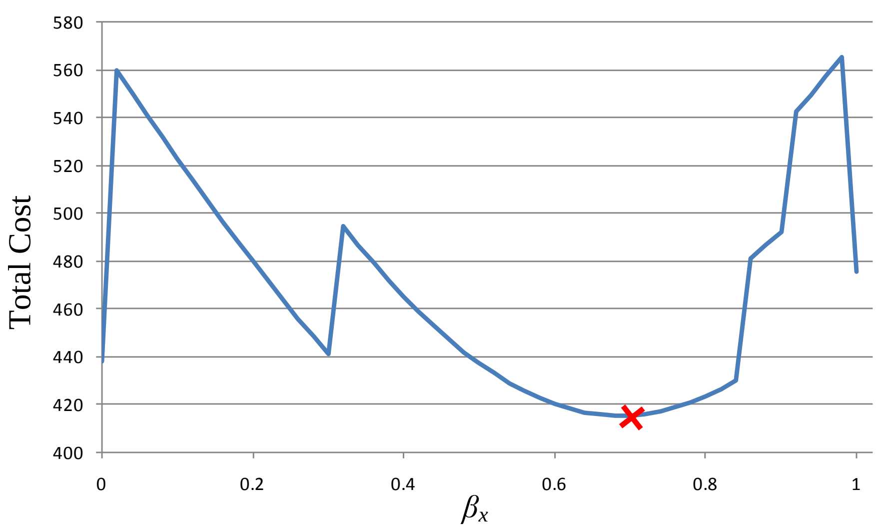


Figure 7.11. Varying "β_x" and its effect on Total Cost

These results suggest that a bang-bang policy of pure production or pure remanufacturing is not optimal when returns are disassembled for reuse. This suggests that recovering returns as whole units rather than disassembled components may not capture the benefits that product recovery programs are supposed to bring.

7.4. Summary and Conclusions

In the literature of reverse logistics lot sizing problems, the assumption that returns are treated as a whole and not disassembled results in missing the benefits reaped from product recovery programs. This chapter considered a single product EOQ model assuming that there is a single production cycle and a single remanufacturing cycle, per time interval. Returns of used items of the product are collected at a rate, disassembled and sorted into subassemblies, which are managed differently, so that each subassembly has its own percentage return rate different from the remanufacturing to production ratio. Therefore, each subassembly has its own inventory control policy. The results suggest that modelling disassembly of returns brings modelling closer to reality and captures the benefits that product recovery programs are supposed to bring. These results showed that the bang-bang (Richter, 1997; Dobos & Richter, 2003; 2004) policy of pure production or pure remanufacturing is not optimal when returns are disassembled.

In the literature, the return rate is assumed fixed or as a percentage of the forward flow. In the next chapter, the reverse flow of used products is treated as a function of price and quality.

CHAPTER 8: REVERSE LOGISTICS LOT SIZING MODEL WITH RETURNS' RATE VARIED BY PRICE AND QUALITY

8.1. Introduction

Reverse logistics and closed loop supply chain management literature considers a production environment that consists of two inventories: the serviceable and the repairable stocks. It was illustrated in several studies that the pure bang-bang policy of either no waste disposal (total remanufacturing) or no remanufacturing (pure production and total disposal) is the optimum strategy (e.g., Richter 1996a, b; Teunter 2001b). The pure policies are optimal under limited conditions; when there are only two purchasing prices and one quality, i.e., the purchasing price of a used/returned item is different from the (collective) purchasing price of raw materials used to produce a new unit, while used/returned and new units have the same quality. Dobos & Richter (2003) extended Richter (1996a, b) by assuming non-instantaneous production and remanufacturing rates, while Dobos & Richter (2004) generalized (Dobos & Richter, 2003) to the case of multiple repair and production cycles per time interval, and implied that their model has limitations since pure strategies are technologically infeasible. The authors recommended considering quality of returned items for an improved modeling of reverse logistics practise. In a similar finding, van der Laan et al. (1999a) concluded that it is unwise to remanufacture all remanufacture-able items even if return rate is less than demand (i.e., pure strategies are not recommended).

Most of the works surveyed in the literature review of Chapter 2 assumed a constant return rate and ignored the factors that govern this rate, which are the purchasing price of a collected used (returned) item with a certain quality that represent the value of the returned item and how useful would the production-remanufacturing process be. Remanufactured units are usually perceived to have a lower quality than the new products. In the tire industry, retreaded tires have to be marked to be distinguished from the new ones, and customers consider them of a lower quality (Debo et al., 2005). Several researchers discussed the importance of differentiating the returned units according to their quality (e.g., Bloemhof-Ruwaard et al., 1995; Smith et al., 1996; Reimer et al., 2000; Blackburn et al., 2004;

Grubbström & Tang, 2006; Behret & Korugan, 2009). However, there has not been any works that related the collection rate of used items with price and quality of returns.

Logically, when returns are less expensive to acquire than raw materials required for producing a new item, repair with no production is a better policy, and the opposite is true; however, pure remanufacturing is not feasible from a practical point of view. This corroborates the finding of Dobos & Richter (2004) who wrote (p.322): "*Probably these pure strategies are technologically not feasible and some used products will not return or even more as the sold ones will come back, some of them will be not recycled*". Practically, collected used/returned items are of varying price and quality.

Dobos & Richter (2006) is one of the works that approached reverse logistics by considering a quality issue. The authors extended their previous work, Dobos & Richter (2003, 2004), by considering two strategies to manage return rate according to its quality: either purchase all used items and reuse only a maximal proportion of them or buyback only a proportion of the used items and decide how much of them to reuse. The authors assumed that return flow is dependent on two inter-dependant decision variables, the buyback and use proportions. The product of these variables (0 ≤ (buyback proportion) × (use proportion) ≤ 1) is the return rate = (demand rate) × (buyback proportion) × (use proportion), which is assumed to be fixed. That is, both of the buyback proportion and the use proportion vary, but their product is fixed.

To further illustrate their limitation, let us assume a case where no ecological constraints are considered and decisions are only based on economical objectives, where recycling (i.e., recovery) is more expensive than production, then the strategy of pure production should be favoured (which Dobos & Richter (2006) did not consider). Furthermore, purchasing price of raw materials was ignored in Dobos & Richter's (2006) model, and the same for purchasing price of collected used items in the backward flow (returns). The authors limited their investigation to a fixed return rate, however, the practice considers the price of returns according to their quality, and accordingly, this governs the collection (or return) policy of used items.

One of the few research works that related demand with price and/or quality is Vörös (2002). The author presented product market demand (forward flow) as a function of price and quality, i.e., forward logistics. The author proposed an exponential function where demand increases as price (quality) decreases (increases), where the exponential curve represents price-demand and quality-demand relationships better than the linear curve (Currim & Sarin, 1984). Vörös's (2002) demand function relating price and quality describes a general and known behaviour that is well documented in the literature (e.g., Kalish, 1983; Smith, 1986; Teng & Thomson, 1996; Biglaisder & Ma, 2003). There has not been any works that addressed this concept in a reverse logistics context.

Therefore, this chapter's contribution is, first, the return rate of used items is variable (a decision variable), and second, the return rate is dependent on two decision variables, the purchasing price for returned items and its corresponding acceptance quality level. Vörös' (2002) perception will be introduced to the reverse direction by switching the logic of the model presented by Vörös (2002). That is, the reverse flow is assumed to increase as offered price increases, and to decrease as accepted quality increases. The models presented in this chapter are the first to model product reverse flow as a function of the quality and the corresponding price of a collected used item.

The presented models extend the models developed in Dobos & Richter (2003, 2004) by assuming that the collection rate is a function of purchasing price (decision variable 1) and the acceptance quality level (decision variable 2) of these returns. The reverse flow is modeled as a demand-like function, where the return rate increases as the purchasing price increases, and decreases as the corresponding acceptance quality level increases. In other words, when the purchasing price of returned items is fixed, an increase in the acceptance quality level of returns decreases flow in the reverse direction, allowing customers to return higher quality products only. Similarly, when the acceptance quality level of returns is fixed, an increase in the purchasing price for returns increases the flow in the reverse direction encouraging customers to return more products.

The models are decision tools that managers can use in determining the optimum acceptable acquisition quality level and the optimal corresponding price for used items. Three models are presented: Model I, a deterministic production and remanufacturing lot sizing model with return rate dependant on price and quality, with a singe production cycle and a single remanufacturing cycle per time interval. Model I is extended into two models:

Model II, which assumes m production cycles and n remanufacturing cycles per time interval, and Model III, which assumes production and remanufacturing processes satisfy a stochastic market demand, with backlogging allowing.

8.2. Returns' Rate Varied by Price and Quality: Mathematical Modelling

Models presented herein assume finite production and remanufacturing rates to satisfy demand, which is known, constant and independent. Remanufactured items are as good as new, lead-time is zero, and no shortages are allowed. Two decision variables are considered, P which is the purchasing price for a single returned item as a percentage of the cost of raw materials required to produce a new item of the product, and q which is acceptance quality level of returned (collected used) items; representing the percentage of useful parts in a remanufacturable item.

Returned items are usually of varying quality. Model I assumes that a returned item with a quality less than the acceptance (optimum) quality level q^*, will be rejected. Only returned items of quality better than or equal to q^* are accepted to flow in the reverse direction to be repaired. Returned items are purchased at an optimum price P^*. The multi-attribute q quality measure for a returned item may be determined using some judgmental and qualitative approaches, for example, ranking, rating (scaling), and paired comparison (e.g., Eckenrode, 1965; Hutton Barron & Barret, 1996; Jaccard et al., 1986; Ahn & Park, 2008).

The flow of materials and products in the forward and the reverse direction is described in Figure 8.1. Returns from the market to the system are screened to verify their quality and those not conforming to remanufacturing requirements are disposed outside the system.

Market demand $d > 0$ is satisfied from the serviceable stock, which stocks newly produced items, and remanufactured items where dT represents the total demand in an interval of length T.

$R(P, q)$, or R for simplicity, is the portion of demand which is returned to the system for either remanufacturing or disposal, as a function of price and acceptance quality level. This portion of returned demand R or $R(P, q)$ will be indicated as return rate. Accordingly, over an interval of length T, $R(P,q)\times T$ (or RT for simplicity) used/returned units are collected in the returned stock facility, where $0 < R/d < 1$. In this facility, disassembly and sorting are carried out, and waste disposal amount of the returned items is decided once the acceptance quality level is determined, i.e., disposal increases (decreases) as the acceptance quality level decreases (increases), with the number of used/returned items disposed per interval is $(1-q)RT$.

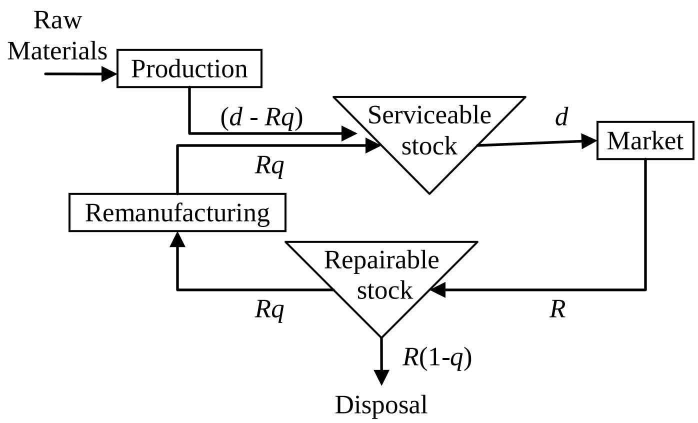


Figure 8.1. Material flow from the inventory system to the market and vice versa

The remaining collected used/returned units, qRT, are transferred to the remanufacturing facility in the serviceable stock. C_r is the cost to repair one unit (which includes cost components such as labour, energy, machinery, etc) excluding the cost to purchase a used item $P_M = P \times C_n$, where C_n is the cost of raw materials required to produce a newly produced unit. The remaining serviceable stock, $(d - qR)T$, is replenished by newly produced items.

8.3. Model I: A Single Remanufacturing Cycle and A Single Production Cycle

Model I assumes a single production and a single remanufacturing cycle per time interval T. Vörös' (2002) concept is modified to capture the backward flow and to relate it with price and acceptance quality level. A formula is presented, which is divided into two portions: the price factor and the quality factor. The price factor of the demand function is $f_P = \left(1 - ae^{-\theta P}\right)$, where $0 < a < 1$ and $\theta > 1$ are parameters. This price factor models the behaviour of returns for a fixed quality level. A similar approach was adopted by Debo et al.

(2005), where the willingness of customers to pay for a product is differentiated with a continuous variable. Figure 8.2 illustrates the behaviour of the price factor for the case when a = 0.3 and θ = 4, where f_P is a monotonically increasing function over P since $df_P/dP > 0$ and $d^2 f_P/dP^2 < 0 \;\forall\; P > 0$.

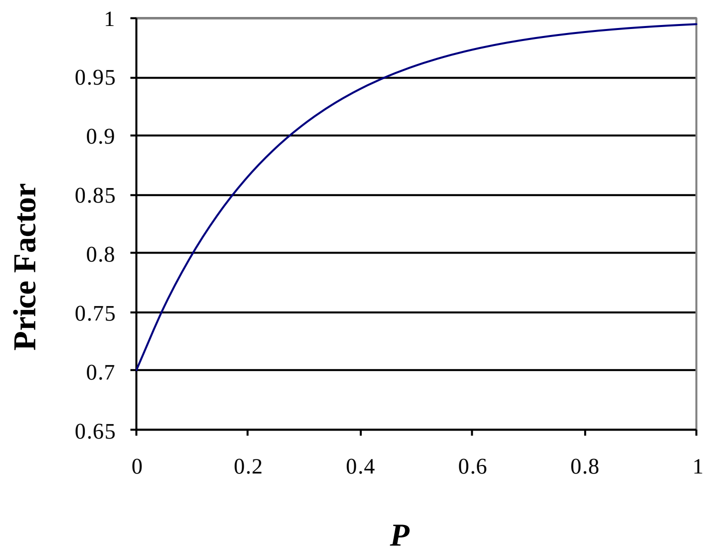


Figure 8.2. Price factor plotted against price "*P*"

The quality factor of the demand function is $f_q = be^{-\varphi q}$, where 0 < b <1 and φ > 1 are parameters. This quality factor models the behaviour of returns for a fixed price level. Figure 8.3 illustrates the behaviour of the quality factor for the case when b = 0.9 and φ = 5, where f_P is a monotonically decreasing function over q since $df_q/dq < 0$ and $d^2 f_q/dq^2 > 0 \;\forall\; q >$ 0. Therefore, the return rate of used/returned items (demand of the reverse flow) is modeled as a function of price and quality factors f_P and f_q, and is expressed as $R = R(P,q) = d\left(1 - ae^{-\theta P}\right)be^{-\varphi q}$.

The case of (R > 0 and q = 0) is technologically infeasible since it represents that all the returned/used items are non-remanufacturable and would be disposed. It is valid mathematically, but it is costly and therefore never optimal. On the other extreme, q = 1 means that a returned/used item must be of an identical quality to that of a newly produced one, for example, returns during trial periods or returns due to obsolete technology.

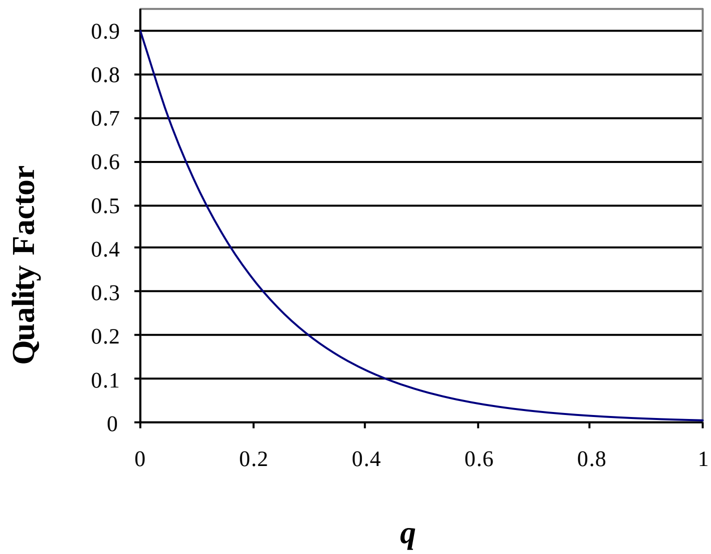


Figure 8.3. Quality factor plotted against accepted quality "q"

There is one remanufacturing cycle of length T_R and one production cycle of length T_P in the time interval T, where $T = T_R + T_P$. In the serviceable stock, inventory starts to build up at a rate of $(1/\delta - 1)d$ units per unit of time, and remanufacturing ceases when an inventory level of $I_{R,1} = (1-\delta)dT_R$ is attained. The production cycle commences once $I_{R,1}$ units are depleted, as shown in Figure 8.4.

Similarly, the inventory of newly produced items builds at a rate of $(1/\pi - 1)d$ units per unit of time with production ceasing when an inventory level of $I_{P,1} = (1-\pi)dT_P$ is attained. Once $I_{P,1}$ units are depleted, a new interval of length T is initiated.

In the repairable stock, inventory starts from the maximum level at $I_{r,1} = qRT(1 - qR\delta/d)$, and remanufacturing commences and depletes the repairable stock at a rate of $(qR - d/\delta)$. By the end of a remanufacturing cycle, $I_{r,1}$ units would have been depleted, bringing the inventory level to zero level, and a new collection cycle of used/returned items commences building up inventory at a rate of qR. It is assumed that the screening and sorting of collected used/returned items occur prior to storage, and items not conforming to quality standards are disposed, totalling $(1-q)RT$ units.

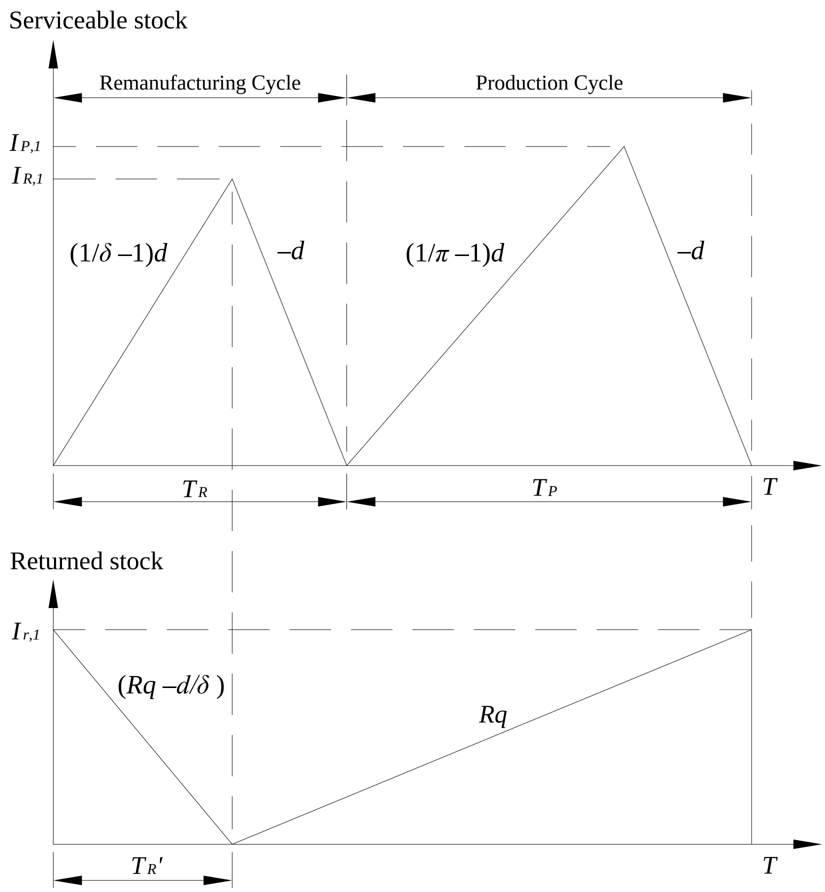


Figure 8.4. Inventory status of serviceable and returned stocks

Holding cost expressions

The inventory holding costs per unit per unit of time for serviceable and returned items is denoted by h_1 and h_r respectively. Also, denote H_P, H_r and H_u as the inventory costs for newly produced, remanufactured and returned items respectively. Let $\lambda = \lambda(R,q) = qR/d$, $0 < \lambda < 1$, to be the ratio of repairable items to total demand, where $T_R = qRT/d = \lambda T$ and $T_P = (1 - qR/d)T = (1-\lambda)T$. The inventory holding cost expressions for newly produced, remanufactured and returned items are given respectively as

$$H_{p,1} = h_1 \frac{I_{p,1}}{2} T_P = \frac{h_1}{2} T^2 (1-\lambda)^2 d(1-\pi) \tag{8.1}$$

$$H_{u,1} = h_1 \frac{I_{R,1}}{2} T_R = \frac{h_1}{2} T^2 \lambda^2 d(1-\delta) \tag{8.2}$$

$$H_{u,1} = h_u \frac{I_{r,1}}{2} T_r' = \frac{h_u}{2} T^2 d\lambda(1-\lambda\delta) \tag{8.3}$$

Appendix 6 illustrates the derivations of Equations (8.1), (8.2) and (8.3). Accordingly, the total inventory holding cost per unit of time is given as

$$H_{T,1} = \frac{H_p + H_r + H_u}{T} = Td\frac{\psi(\lambda)}{2} \tag{8.4}$$

where the term $\psi(\lambda)$ is given as

$$\begin{aligned}\psi(\lambda) &= h_1\left(\lambda^2(1-\delta) + (1-\lambda)^2(1-\pi)\right) + h_u\lambda(1-\lambda\delta) \\ &= \lambda^2\left(h_1(2-\delta-\pi) - h_r\delta\right) + \lambda\left(h_u - 2(1-\pi)h_1\right) + h_1(1-\pi)\end{aligned} \tag{8.5}$$

Lot size dependant cost expressions

The total setup cost is denoted by $S = S_r + S_p$, where S_r and S_p are the remanufacturing and production setup costs respectively. From S and (8.5), the lot-size related total cost per unit time is given as

$$C(\lambda,T) = \frac{S}{T} + Td\frac{\psi(\lambda)}{2} \tag{8.6}$$

where (8.6) is convex in T, i.e., $\partial^2 C(\lambda,T)/\partial T^2 > 0 \ \forall\ T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^* = \sqrt{\frac{2S}{d\psi(\lambda)}} \tag{8.7}$$

Substituting (8.7) in (8.6) to get

$$C(P,q) = C(\lambda) = \sqrt{2SD\psi(\lambda)} \tag{8.8}$$

where $\lambda(R,q) = qR(P,q)/D$, and to simplify the presentation of the mathematics it will be referred to from this point onwards by λ. The optimal remanufacturing and production cycle times are given respectively from (8.7) as

$$T_R^* = \lambda T^* = \lambda\sqrt{\frac{2S}{d\psi(\lambda)}} \tag{8.9}$$

$$T_P^* = (1-\lambda)T^* = (1-\lambda)\sqrt{\frac{2S}{d\psi(\lambda)}} \tag{8.10}$$

The corresponding remanufacturing and production lot sizes are determined from (8.9) and (8.10) respectively as $X_R^* = dT_R^*$ and $X_P^* = dT_P^*$.

Total cost expression

The total cost per unit of time is the sum of the following unit time costs

Setup cost per unit time: $(S_r + S_p)/T \ = S/T$

Holding costs per unit of time: $Td\,\frac{\psi(\lambda)}{2}$, where $\psi(\lambda)$ is given from (8.5)

Disposal costs per unit of time: $(1-q)RC_w$

Remanufacturing costs per unit time: qRC_r

Production costs per unit time: $(d-qR)C_p$

Purchasing costs per unit time: $RPC_n + (d-Rq)C_n$

Therefore, total cost per unit of time is expressed as

$$C(P,q,T) = \frac{S}{T} + Td\,\frac{\psi(\lambda)}{2} + (1-q)RC_w + qRC_r + (d-qR)C_p + RPC_n + (d-Rq)C_n$$

$$= \frac{S}{T} + Td\,\frac{\psi(\lambda)}{2} + R\left[q\left(C_r - C_w - C_p - C_n\right) + C_w + PC_n\right] + d\left(C_p + C_n\right) \quad (8.11)$$

where (8.11) is convex in T, i.e., $\partial^2 C(P,q,T)/\partial T^2 > 0 \ \forall \ T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T^* = \sqrt{\frac{2S}{d\psi(\lambda)}} \quad (8.12)$$

Substituting (8.12) in (8.11) to get

$$C(P,q) = \sqrt{2Sd\psi(\lambda)} + R\left[q\left(C_r - C_w - C_p - C_n\right) + C_w + PC_n\right] + d\left(C_p + C_n\right) \quad (8.13)$$

where $\psi(\lambda)$ is given in (8.5), and

$$\lambda = qR(P,q)/d \quad (8.14)$$

$$R = R(P,q) = d\left(1 - ae^{-\theta P}\right)be^{-\varphi q} \quad (8.15)$$

Numerical validation was conducted to prove the convexity of the cost function (8.13), by randomizing all the input parameters and the Hessian matrix was computed for each specific data set generated. Sample examples are provided in Appendix 7. The Hessian matrix held a positive value in all of these numerical examples (more than 10,000). Therefore, it is reasonable enough to conjecture that convexity of (8.13) holds. This approach was considered is several other works (e.g., Agrawal & Nahmias, 1997; Silver & Costa, 1998).

8.4. Model II: Multiple Remanufacturing and Production Cycles

Model II extends Model I by considering *m* remanufacturing cycles and *n* production cycles in an interval of length *T*, where $T = mT_R + nT_P$, where $m \geq 1$ and $n \geq 1$ are positive integers. Additional decision variables *m*, *n* are considered with *P*, *q*.

Holding cost expressions

The inventory status in the serviceable and repairable stock is shown in Figure 8.5.

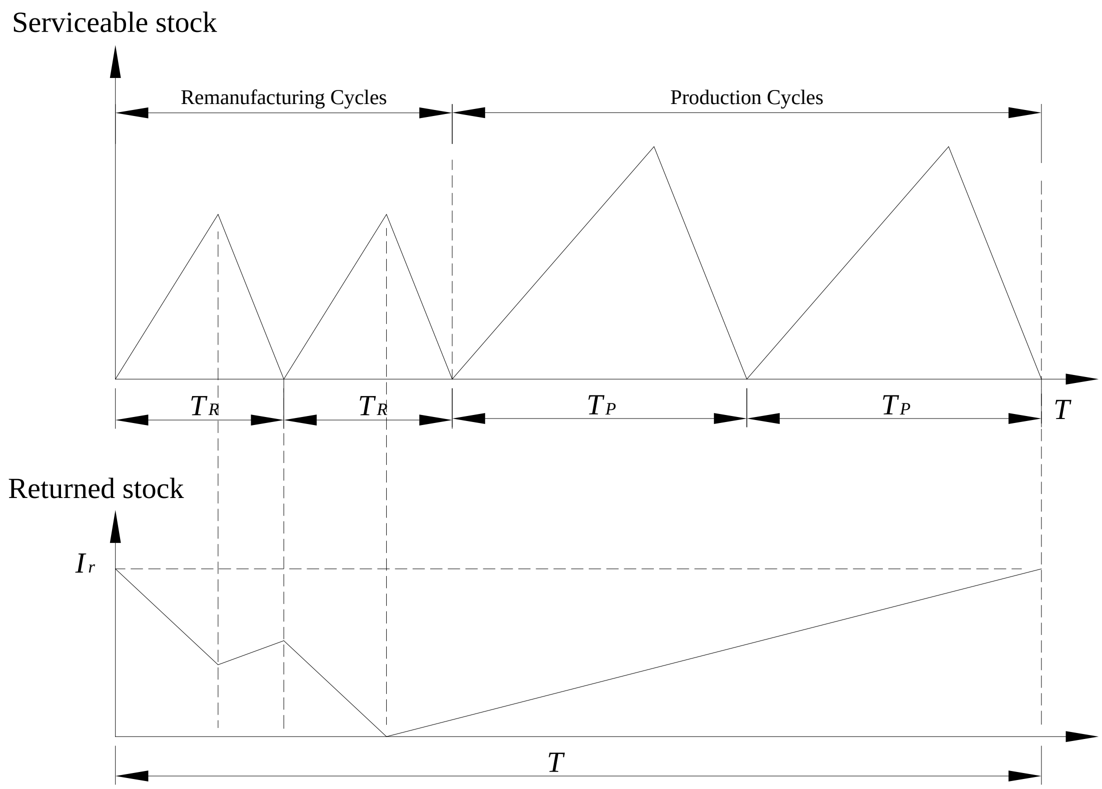


Figure 8.5. Inventory status for the case of *m* = 2, *n* = 2

In the serviceable stock, the maximum inventory levels attained for remanufacturing cycles and for production cycles are given respectively as $I_{R,m} = (1-\delta)dT_R = (1-\delta)dT\lambda/m$ and $I_{P,n} = (1-\pi)dT_P = (1-\pi)dT(1-\lambda)/n$. Similarly, the maximum inventory level of the repairable inventory is given as $I_{r,m} = T\lambda d\left(1+\lambda\left(1-\delta-m\right)/m\right)$. Accordingly, the inventory holding cost expressions for newly produced, remanufactured and returned items are given respectively as

$$H_{p,n} = h_1 \frac{I_{P,n}}{2} T_P = \frac{h_1}{2} T^2 \left(1-\lambda\right)^2 d\left(1-\pi\right)/n \tag{8.16}$$

$$H_{r,m} = h_1 \frac{I_{R,n}}{2} T_R = \frac{h_1}{2} T^2 \lambda^2 d\left(1-\delta\right)/m \tag{8.17}$$

$$H_{u,m} = h_u \frac{I_{r,n}}{2} T = \frac{h_u}{2} T^2 d\lambda\left(1+\lambda\left(1-\delta-m\right)/m\right) \tag{8.18}$$

Appendix 8 illustrates the derivations of Equations (8.16), (8.17), and (8.18). The total holding cost per unit of time unit is

$$H_{T,m,n} = \frac{H_{p,n} + H_{r,m} + H_{u.m}}{T} = \frac{Td\psi(m,n,\lambda)}{2} \tag{8.19}$$

$$\psi(m,n,\lambda) = h_1\left(\lambda^2\left(1-\delta\right)/m + \left(1-\lambda\right)^2\left(1-\pi\right)/n\right) + h_u\lambda\left(1+\lambda\left(1-\delta-m\right)/m\right) \tag{8.20}$$

Lot size dependant costs

The total setup cost for m remanufacturing and n production cycles in an interval of length T is denoted by $S_{m,n}$, where $S_{m,n} = mS_r + nS_p$. The cost per unit of time is given from (8.20) as

$$C\left(m,n,\lambda,T\right) = \frac{S_{m,n}}{T} + \frac{Td\psi}{2}\left(m,n,\lambda\right) \tag{8.21}$$

where (8.21) is convex in T, i.e., $\partial^2 C\left(m,n,\lambda,T\right)/\partial T^2 > 0 \ \forall \ T > 0$. Setting the first partial derivative to zero and solving for T we get

$$T_{m,n}{}^* = \sqrt{\frac{2S_{m,n}}{d\psi\left(m,n,\lambda\right)}} \tag{8.22}$$

Substituting (8.22) in (8.21) to get

$$C\left(m,n,P,q\right) = C\left(m,n,\lambda\right) = \sqrt{2S_{m,n} d\psi\left(m,n,\lambda\right)} \tag{8.23}$$

The optimal remanufacturing and production cycle times are given respectively from (8.22) as

$$T^*_{R,m,n} = \frac{\lambda T}{m} = \frac{\lambda}{m}\sqrt{\frac{2S_{m,n}}{d\psi(m,n,\lambda)}} \tag{8.24}$$

$$T^*_{P,m,n} = \frac{(1-\lambda)T}{n} = \frac{(1-\lambda)}{n}\sqrt{\frac{2S_{m,n}}{d\psi(m,n,\lambda)}} \tag{8.25}$$

The corresponding remanufacturing and production lot sizes are determined from (8.24) and (8.25) respectively as $X^*_{R,m,n,} = dT^*_{R,m,n}$ and $X^*_{P,m,n} = dT^*_{P,m,n}$.

Total cost expression

The holding costs per unit of time and is expressed as

$$H_{T,m,n} = \frac{Td}{2}\left[h_1\left(\lambda^2(1-\delta)/m + (1-\lambda)^2(1-\pi)/n\right) + h_u\lambda(1+\lambda(1-\delta-m)/m)\right]$$

Therefore, and similar to (8.11), the total cost per unit of time is expressed from (8.21) as

$$\begin{aligned} C(m,n,P,q,T) = \frac{S_{m,n}}{T} + \frac{Td}{2}\psi(m,n,\lambda) \\ + R\left(q\left(C_r - C_w - C_p - C_n\right) + C_w + PC_n\right) + d\left(C_p + C_n\right) \end{aligned} \tag{8.26}$$

where (8.26) is convex in T, i.e., $\partial^2 C(m,n,P,qT)/\partial T^2 > 0 \quad \forall\ T > 0$.

Setting the first partial derivative to zero and solving for T, the optimum interval of time is computed as

$$T^* = \sqrt{\frac{2S_{m,n}}{d\psi(m,n,\lambda)}} \tag{8.27}$$

Substituting (8.27) in (8.26) to get

$$\begin{aligned} C(m,n,P,q) = \sqrt{2S_{m,n}d\psi(m,n,\lambda)} + \\ + R\left(q\left(C_r - C_w - C_p - C_n\right) + C_w + PC_n\right) + d\left(C_p + C_n\right) \end{aligned} \tag{8.28}$$

where

$$\psi(m,n,\lambda) = h_1\left(\lambda^2(1-\delta)/m + (1-\lambda)^2(1-\pi)/n\right) + h_u\lambda(1+\lambda(1-\delta-m)/m) \tag{8.29}$$

while λ and R are given in (8.14) and (8.15), respectively.

Theorem A.1 in Appendix 9 illustrates that a policy with both m and n being even integers is never optimal, and accordingly, the following solution procedure is developed, which can be used to optimize Equation (8.28).

8.5. Solution Procedure

Step 1: For a set of input parameters d, C_n, a, θ, b, φ, h_1, h_u, C_r, C_p, C_w, δ, π, S_r and S_p. Set $i = n = 1$ and minimize $C(m,n,P,q)$ for $m = 1$ and $m = 2$ using an optimization tool (e.g., Solver from Microsoft Excel). With $C(m,n,P,q)$ being the objective function subject to $0 < P < 1$ and $0 < q < 1$, where P and q are decision variables. Record the minimum costs for the cases when $m = 1$ and $m = 2$ and their corresponding P and q values, where P and q are optimal for specific values of n and m.

Step 2: Compare $C(m,n,P,q)$ value for ($m = 1$) to that of ($m = 2$). If $C(1,1,P,q) < C(2,1,P,q)$, terminate the search for ($i = n = 1$) and record the value. If $C(1,1,P,q) > C(2,1,P,q)$, repeat for ($m = 3$), ($m = 4$), etc. Terminate once $C(m_1^* - 1,1,P,q) > C(m_1^*,1,P,q) < C(m_1^* + 1,1,P,q)$, where m_1^* is the optimal value for the number of remanufacturing cycles when there is 1 production cycle. Record the values of $C(m_1^*,1,P,q), m_1^*$, P and q for $n = 1$.

Step 3: For the same set of parameters in Step 1, set $i = n = 2$, however, optimize $C(m,n,P,q)$ for $m = 1$ and $m = 3$. (The case when m and n being even integers is never optimal. Refer to Theorem 1 in Appendix 9.

Step 4: Compare $C(m,n,P,q)$ values for ($m = 1$) to that of ($m = 3$). If $C(1,2,P,q) < C(3,2,P,q)$, terminate the search for ($i = n = 2$) and record the value. If $C(1,2,P,q) > C(3,2,P,q)$, repeat for ($m = 5$), ($m = 7$), etc. Terminate once $C(m_2^* - 2,2,P,q) > C(m_2^*,2,P,q) < C(m_2^* + 2,2,P,q)$, where m_2^* is the optimal value for the number of remanufacturing cycles when there are 2 production cycles. Record the values of $C(m_2^*,2,P,q)$, m_2^*, P and q, for $n = 2$.

Step 5: Compare $C(m,n,P,q)$ values for ($i = n = 1$) to that of ($i = n = 2$). If $C(m_1^*,1,P,q) < C(m_2^*,2,P,q)$, terminate the search and $C(m_1^*,1,P,q)$ is the minimum cost. If $C(m_1^*,1,P,q) > C(m_2^*,2,P,q)$, then drop the value of $C(m_1^*,1,P,q)$ and repeat steps 1 and 2 for $n = 3$. Compare $C(m,n,P,q)$ values for ($n = 2$) to that of ($n = 3$). If $C(m_2^*,2,P,q) < C(m_3^*,3,P,q)$, terminate the search and $C(m_2^*,2,P,q)$ is the minimum cost. If $C(m_2^*, 2, P, q) > C(m_3^*, 3, P, q)$, then drop the value of $C(m_2^*, 2, P, q)$ and repeat steps 3 and 4 for $n = 4$.

Step 6: Repeat steps 1 to 5. Terminate the search once $C(m_{i-1}^*, i-1, P, q) > C(m_i^*, i, P, q) < C(m_{i+1}^*, i+1, P, q)$, where i = 1, 2, 3, 4, …, etc.

For the case of Model I ($m = 1$, $n = 1$), the solution procedure will be reduced to Step 1 only.

8.6. Model III: Stochastic Production and Remanufacturing Lot Sizing Problem Considering Price and Quality of Returns

The main purpose of inventory models is to determine how much and when to order. Stochastic models are better to represent reality; however, assuming stochastic demand raises new modelling difficulties. In classical stochastic inventory models, a reorder point triggers a new order, and the order requires a "lead time" to replenish the inventory. An order is expected to arrive in a timely manner so the net inventory do not increase and the holding costs remain at a minimum, and to avoid stock-out costs. However, unexpected surges in demand may result in stock-out periods (Zipkin, 2000). Order-point order-quantity models, which are known as (s, Q) models, and order-point order-up-to-level models, which are known as (s, S) models, are examples of popular inventory policies. There is a significant amount of research that addresses stochastic inventory models (e.g., Nahmias, 1982; Silver et al., 1998). However, few have investigated stochastic inventory models in a reverse logistics context (e.g., Fleischmann, et al., 2002; Teunter, 2002; Ben-Daya & Hariga, 2003; van der Laan et al., 2004), and there is not any model investigating price and quality in a reverse logistics context. Readers may refer to Chapter 2 for a concise review of stochastic inventory models.

Hadley & Whitin (1963) presented a heuristics approximate treatment of Order-point, order-quantity model for both the backorders case and the lost sales case, which produced similar results to exact equations, in addition, being simple. Hadley & Whitin (1963) showed that the heuristics is applicable in a variety of situations and that the lost sales case differs very little from the backorders case. The objective was to minimize total costs and to determine the optimum Q, lot size quantity, and safety stock ss, where Q, $ss > 0$. There is never more than one order outstanding. In other words, at the time the reorder point is reached, there are no outstanding orders. Therefore, the inventory position (the amount on hand + one order – backorders) is equal to the net inventory (the amount on-hand – backorders), given that the net inventory is the on-hand inventory minus backorders. The expected value of net inventory at any time is the expected value of the on hand inventory minus the expected value of the backorders. After an order arrives, it is sufficient to fill all backorders and to raise the on hand inventory level to above the reorder point.

Define x as the lead-time demand, and $g(x)$ is the stochastic distribution of the lead-time demand, then $\mu = \int_0^\infty xg(x)dx$ is the expected (mean) lead time demand for a fixed lead time. Define r as the reorder point, and ss, the safety stock, is the reorder point minus the mean lead-time demand. The average inventory per unit time is the average order size plus safety stock $h\left(\frac{Q}{2} + ss\right) = h\left(\frac{Q}{2} + r - \mu\right)$, where h is the holding cost per unit per unit time. The expected number of backorders per cycle is $\eta(r) = \int_r^\infty (x-r)g(x)dx = \int_r^\infty xg(x)dx - rG(r)$, where $G(r) = \int_r^\infty g(x)dx$ is the complementary cumulative distribution of $g(x)$ and $\int_0^\infty g(x)dx = 1$.

To simplify the presented model, it is assumed that the demand d, is stochastic and follows a normal distribution, and the lead time demand x follows the normal distribution $g(x)$ with mean μ and standard deviation σ. Accordingly, and similar to Hadley & Whitin (1963), p. 167, the complementary cumulative distribution $G(r)$ = $\Phi\left(\frac{r-\mu}{\sigma}\right)$ and the average cost per unit time for a cycle is the summation of average setup, inventory and backorders costs, and is given as

$$\Psi(Q,r) = \frac{Sd}{Q} + h\left(\frac{Q}{2} + r - \mu\right) + \frac{C_B d}{Q}\left[(\mu - r)\Phi\left(\frac{r-\mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r-\mu}{\sigma}\right)\right] \qquad (8.30)$$

where S is the setup cost per cycle, and C_B is the backorder cost per unit backordered. The total cost function can be determined by using the normal tables.

$\Psi(Q, r)$ is differentiable with respect to Q and r, but a closed form to optimize total cost is not attainable. However, the differentiation resulted in two equations that can be solved simultaneously through an iteration process to determine the optimum Q and r within a predefined error window.

Model III presented herein extends Model I by considering a stochastic demand d to be satisfied by newly produced and remanufactured products. Return rate is a function of price P and acceptance quality level q, and after sorting and disposing, returns are collected in the repairable stock to be remanufactured to feed the serviceable stock, which is also replenished by newly produced units, as shown in Figure 8.6.

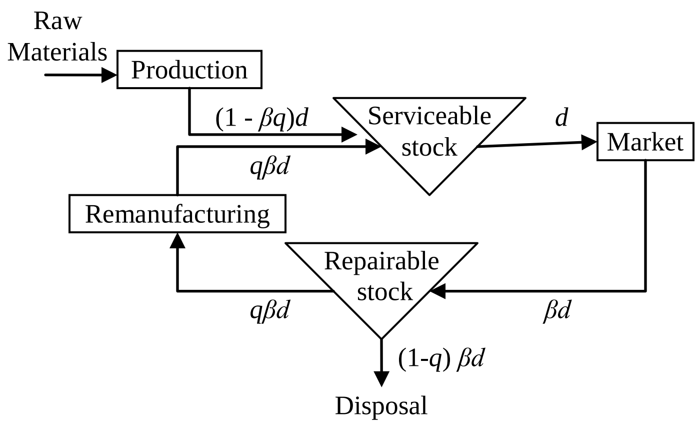


Figure 8.6. Material flow in an interval of length "T"

There is a single remanufacturing cycle and a single production cycle per time interval T. Production and remanufacturing rates are assumed to be infinite in line with the EOQ assumption, i.e., instantaneous replenishment. Additional decision variables are considered with P, q, which are Q, r_p, r_r. Q is the total demand per interval, which is equal to the remanufacturing lot size, Q_r, plus the production lot size, Q_p. r_p (r_r) is the reordering point for the production (remanufacturing) cycle, where r_p (r_r) triggers a remanufacturing (production) order.

There is one remanufacturing cycle of length T_r and one production cycle of length T_p in the time interval T, where $T = T_r + T_p$, as shown in Figure 8.7.

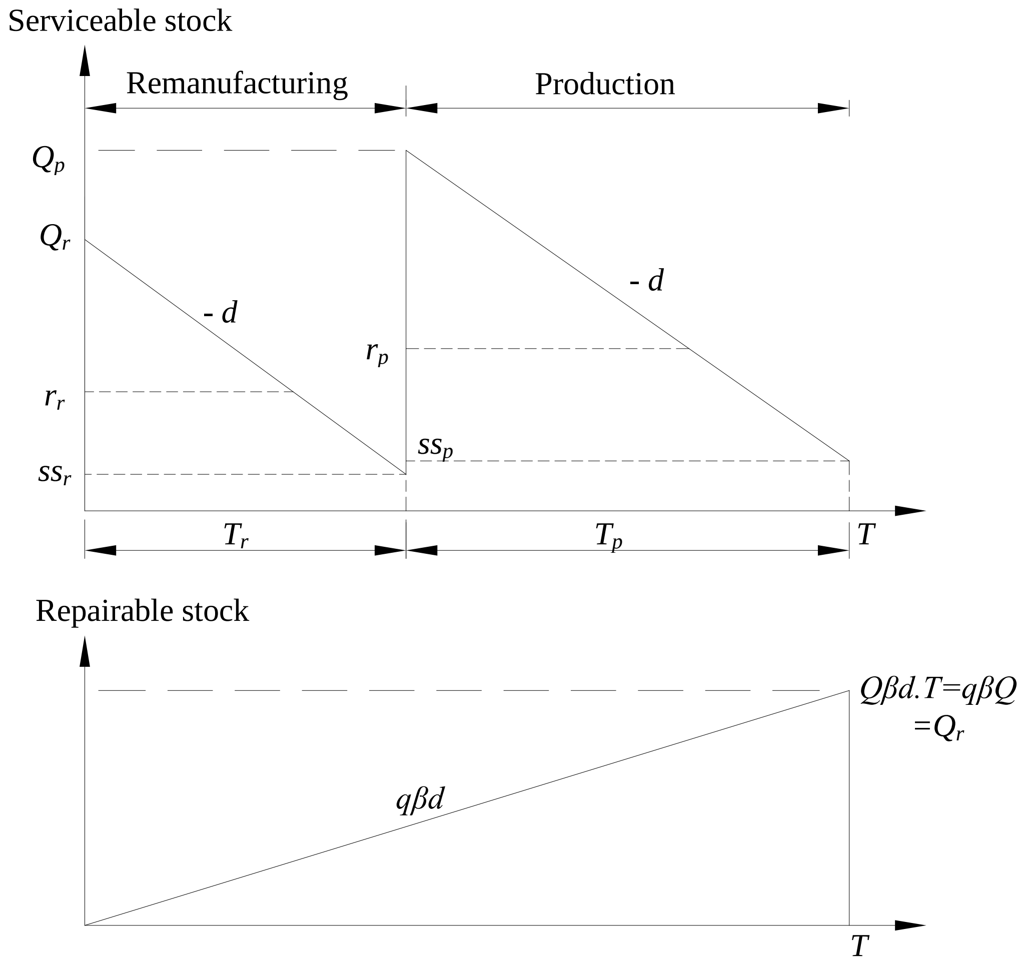


Figure 8.7. Inventory status of serviceable and repairable stocks

In the serviceable stock, inventory is replenished by a remanufactured lot size $Q_r = q\beta Td = q\beta Q$ which is depleted by average demand d, and when the inventory level hits the reorder point r_r, a production lot size $Q_p = (1-q\beta)Td = (1-q\beta)Q$ is ordered. From the time the production lot size is ordered (reorder point r_r) until it arrives, the mean lead time demand μ should be equal to the net inventory minus the safety stock (reorder point r_r – safety stock ss_r). The function of the safety stock is to cover for the unexpected increase in demand during the lead-time, as long as this increase in demand is less than or equal to ss_r., i.e., lead time demand is less than or equal r_r. In case the demand during the lead time, μ, is greater than (r_r), shortages occur, and this shortage is backordered at a cost, C_B per unit backordered.

Similarly, inventory is replenished by a production lot of size Q_p = $(1 - q\beta)Td$ = $(1-q\beta)Q$ which is depleted by average demand d, and when the inventory level hits the reorder point r_p, a remanufacturing lot size Q_r is ordered. From the time the remanufacturing lot size is ordered (reorder point r_p) until it arrives, the mean lead time demand μ should be equal to the net inventory minus safety stock (reorder point r_p – safety stock ss_p). The function of safety stock is to cover the unexpected increase in demand during the lead-time, as long as this increase in demand is less than or equal ss_p., i.e., lead time demand is less than or equal r_p. In case the demand during the lead time, μ, is greater than (r_p), shortages occur, and this shortage is backordered at a cost, C_B per unit backordered.

In the repairable stock, used items (returns) are collected at a rate $q\beta d$, where $\beta = \beta(P, q)$ is the return rate as a function of price P and acceptance quality level q, equivalent to R/d in Model I.

The inventory holding costs per unit per unit of time for produced (remanufactured) units in the serviceable stock is h_p (h_r), and for the reparable stock is h_u. Also, denote H_p and H_r as the inventory costs for newly produced and remanufactured units in the serviceable stock, and H_u as the inventory costs for the repairable items in the repairable stock, per interval. The inventory holding cost expressions for newly produced, remanufactured and returned items per interval are calculated based on the average inventory and are given respectively as

$$H_p = h_p(1-q\beta)T\left(\frac{(1-q\beta)Q}{2} + r_p - \mu\right) \tag{8.31}$$

$$H_r = h_r q\beta T\left(\frac{q\beta Q}{2} + r_r - \mu\right) \tag{8.32}$$

$$H_u = \frac{h_u q\beta QT}{2} \tag{8.33}$$

Now, denote η_p and η_r as the backorders costs for the production and remanufacturing cycles per interval, and these costs are given as

$$\eta_p(r_p) = C_B\left[(\mu - r_p)\Phi\left(\frac{r_p-\mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r_p-\mu}{\sigma}\right)\right] \tag{8.34}$$

$$\eta_r(r_r)=C_B\left[(\mu-r_r)\Phi\left(\frac{r_r-\mu}{\sigma}\right)+\sigma^2\phi\left(\frac{r_r-\mu}{\sigma}\right)\right] \tag{8.35}$$

The total setup cost is denoted by $S = S_r + S_p$, where S_r and S_p are the remanufacturing and production setup costs respectively. Additional costs are determined as follows,

Disposal costs per unit of time: $C_w(1-q)\beta Td$

Remanufacturing costs per unit time: $C_r q\beta Td$

Production costs per unit time: $C_p(1-q\beta)Td$

Purchasing costs of returns per unit time: $C_n P\beta Td$

Purchasing costs of returns per unit time: $C_n(1-q\beta)Td$

Similar to Equations (8.11) and (8.21), total cost per unit of time is expressed as

$$\Psi(Q,r_p,r_r,P,q)=\frac{(S+\eta_{pr})d}{Q}+\frac{Q}{2}H+h_p(1-q\beta)(r_p-\mu)+h_r q\beta(r_r-\mu)+C \tag{8.36}$$

where $\beta=\beta(P,q)=\left(1-ae^{-\theta P}\right)be^{-\varphi q}$,

$$\eta_{pr}=C_B\left[(\mu-r_p)\Phi\left(\frac{r_p-\mu}{\sigma}\right)+\sigma^2\phi\left(\frac{r_p-\mu}{\sigma}\right)+(\mu-r_r)\Phi\left(\frac{r_r-\mu}{\sigma}\right)+\sigma^2\phi\left(\frac{r_r-\mu}{\sigma}\right)\right],$$

$H=h_p(1-q\beta)^2+h_r(q\beta)^2+h_u q\beta$, and

$C=\beta(q(C_r-C_w-C_p-P_n)+C_w+C_nP)d+(C_p+C_n)d$.

Equation (8.36) is differentiated with respect to Q, r_p, r_r to get

$$Q=\sqrt{\frac{2d\left(S_p+S_r+C_B\left[(\mu-r_p)\Phi\left(\frac{r_p-\mu}{\sigma}\right)+\sigma^2\phi\left(\frac{r_p-\mu}{\sigma}\right)+(\mu-r_r)\Phi\left(\frac{r_r-\mu}{\sigma}\right)+\sigma^2\phi\left(\frac{r_r-\mu}{\sigma}\right)\right]\right)}{h_p(1-q\beta)^2+h_r(q\beta)^2+h_u q\beta}}$$

$$=\sqrt{\frac{2\left(S+\eta_{pr}(r_p,r_r)\right)d}{H}} \tag{8.37}$$

$$G(r_p)=\Phi\left(\frac{r_p-\mu}{\sigma}\right)=\sqrt{\frac{Qh_p(1-q\beta)}{C_Bd}} \tag{8.38}$$

$$G(r_r)=\Phi\left(\frac{r_r-\mu}{\sigma}\right)=\sqrt{\frac{Qh_r q\beta}{C_Bd}} \tag{8.39}$$

Equation (8.37) is a function of r_p and r_r, and Equation (8.38), similar to Equation (8.39), is a function of Q. Equations (8.37), (8.38) and (8.39) are solved simultaneously to optimize

$\Psi(Q, r_p, r_r, P, q)$, using a developed solution algorithm. The solution algorithm is lengthy, therefore, it is shown in Appendix 10.

8.7. Numerical Examples

Five numerical examples are provided to illustrate the behaviours of Models I, II and III, and to draw some conclusions. Example 8.1 illustrates earlier works and the basic contribution of this chapter. Example 8.2 addresses some managerial questions and it is shown how earlier works and the presented models respond to these questions. Dobos & Richter (2006) assumed that a pure recycling/reuse strategy (pure remanufacturing) to be more cost effective than a pure production strategy and did not consider the opposite, and assumed a fixed return rate. These limitations are addressed in Example 8.3. The convexity of the presented models is discussed in Example 8.4. Example 8.5 is a stochastic example and is solved by Model III.

Example 8.1

A comparison between Dobos & Richter (2003) and Model I is presented. Let d = 1000, h_1 = 1.6, h_u = 1.2, δ = 0.3, π = 0.6, S_p = 2400, S_r = 1600, C_r = 1.2, C_w = 0.1, C_p = 2, C_n = 5. First, these values are substituted in the model of Dobos & Richter (2003), where the buyback proportion is 0.231 and the use proportion is 0.829, which is equivalent to a reusable proportion = (buyback proportion) × (use proportion) × d = 0.231×0.829×1000 = 191.5 units or 19.15% of demand. In Dobos & Richter (2003), the cost for a mixed production and remanufacturing strategy is 9305, for a pure production strategy (buyback proportion = use proportion = 0) is 8752, and for a pure remanufacturing (buyback proportion = use proportion = 1) is 8704. This suggests that a pure remanufacturing strategy is the cheapest of the three strategies (8704 < 8752 < 9305).

Second, the values determined above are substituted in Model I, with the parameters of $R = R(P,q) = d\left(1 - ae^{-\theta P}\right)be^{-\varphi q}$ adjusted to a = 0.5, b = 0.95, θ = 8, φ = 1.5, so that buyback proportion = 0.231 and use proportion = 0.829, where $R(P,q)/d = \left(1 - ae^{-\theta P}\right)be^{-\varphi q}$ = 0.231. Solving Model I, a mixed production and remanufacturing/recycling policy has the lowest cost of 8386 of the other two strategies, where the cost of a pure production is 8752 and the cost of pure remanufacturing is 8704. The cost 8386 of a mixed production repair strategy is attained when $P^* = 0.146$ and $q^* = 0.829$. To illustrate, an optimum mixed strategy was found to be optimum because some of the returned items to be remanufactured/repaired are good quality items ($q^* = 0.829$ or more) that are purchased at a low price (Purchasing Price = $P^* \times C_n = 0.146 \times 5 = 0.73$). Such a case makes a mixed strategy advantageous to the pure strategies of either pure production or pure remanufacturing.

Example 8.2

This example illustrates Model I. Let $d = 1000$, $h_1 = 850$, $h_u = 80$, $\gamma = 2/3$, $\beta = 2/3$, $S_p = 1960$, $S_r = 440$ (Dobos & Richter, 2004). According to the calculations provided by Dobos & Richter (2004), the pure recycling costs 16,516 and pure production costs 33,326. Therefore, it is economical to recycle with the buyback and use all returned items. In reality, it is nearly unattainable to reach a 100% buyback and use rate (remanufacture/recycle every produced item), therefore, what if a decision maker cannot attain a 100% buyback and use rates, but he/she can only attain say 90 or 95 percent. Is such an acceptance percentage close to the optimal solution? Or, is it better to switch to a pure production policy?

The pure policy means either to include the setup cost for remanufacturing or the setup cost for production, but not both. The mixture of remanufacturing and production means the inclusion of both costs leading to higher costs than in a pure strategy (either pure production or pure remanufacturing). Figure 8.8 illustrates the behaviour of the total cost for Dobos & Richter's (2004) model when the marginal buyback and use rates are varied simultaneously (from 0% to 100%).

As shown in Figure 8.8, since a 100% buyback rate and 100% use rate cannot be attained, values for these rates close to 100% are far from being optimal. However, a mixed remanufacturing and production strategy suggesting at buyback and use rates of 70 % is the optimal case. Dobos & Richter (2003, 2004, and 2006) did not address this issue. To illustrate further, and for example, if the buyback rate is 99% and the decision maker decides to accept all returns, then such a policy would be 38% more expensive than the solution attained at 70% buyback and reuse rate.

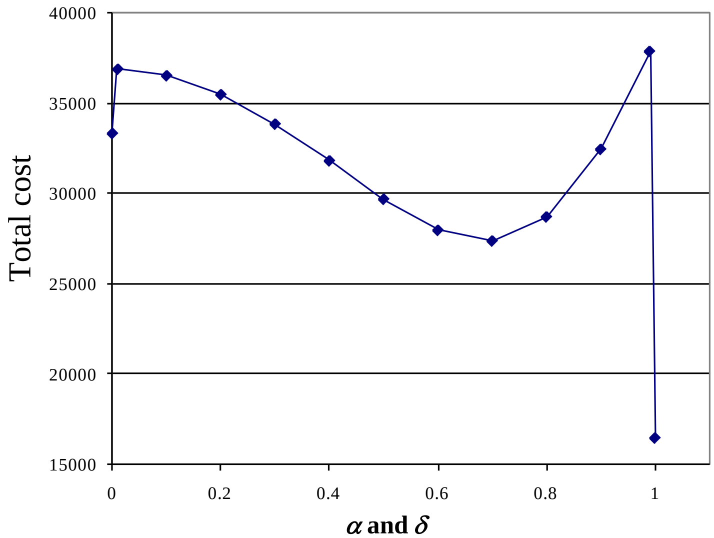


Figure 8.8. The behaviour Dobos & Richter's (2004) cost function for varying marginal buyback proportion and marginal use proportion

Substituting the same values of the parameters in Dobos & Richter (2004) example in Model I with a = 0.9, b = 0.9, θ = 6, φ = 2, while varying price and quality simultaneously, the cost and its corresponding return rate for are plotted in Figure 8.9. A mixed strategy of repair and production reaches its least cost (not optimal) at a price and quality of 0.6 corresponding to a return rate of 26 percent.

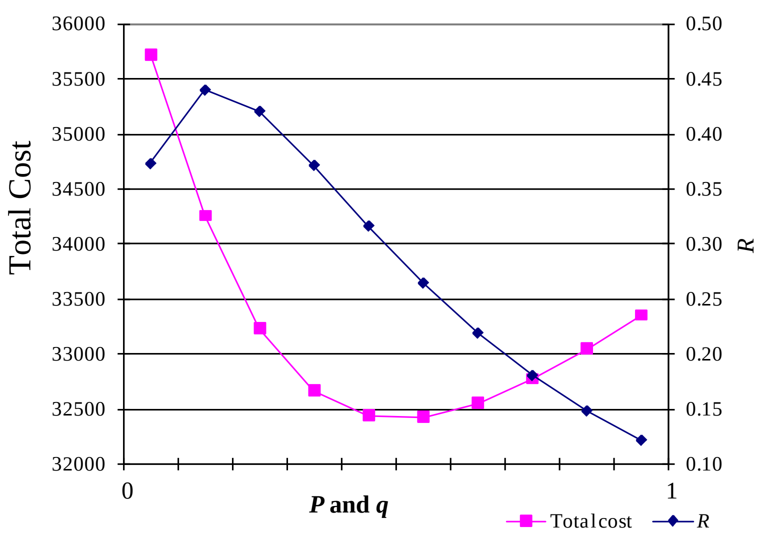


Figure 8.9. Behaviours of the Total Cost and the Return Rate "*R*" of Model I for varying price and quality values

Optimizing Model I by varying the values of purchasing price and acceptance quality level independently, the optimal solution was attained at a price of $0.69 \times C_n = 0.69 \times 5$ and a quality index of 0.53 corresponding to 30.7% returns. Note that the average values produced from the model of Dobos & Richter (2004) are less than the average solutions produced by the presented model, because the calculations in Dobos & Richter (2004) example did not consider the purchasing price of returned items, while price of raw materials C_n is considered equal in both examples. The second example is presented to demonstrate the infeasibility of the bang-bang policy.

Example 8.3

This example illustrates Model II with a case when pure remanufacturing is not as cost effective as pure production. Let $d = 1000$, $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, $h_1 = 4$, $h_u = 4$, $\delta = 0.8$, $\pi = 0.5$, $S_p = 6$, $S_r = 6$, $C_r = 2$, $C_w = 0.15$, $C_p = 2$, $C_n = 0.95$. Pure production costs 3105, while pure remanufacturing costs 3,155. Substituting the above parameters and optimizing Equation (8.28), the total cost is $C = 3085.5$, where $P^* = 0.21$ and $p^* = 0.87$. Although the case of pure remanufacturing is desirable, it is technologically unattainable. Therefore, the best practise is to accept high quality returns with low purchasing price, which will deliver a better solution than either pure production or pure remanufacturing. Even in the case when pure remanufacturing is more expensive than pure production, a pure production strategy is not optimal.

Example 8.4

This example illustrates Model II, and using similar data to Example 8.3, with $h_u = 3$, $S_r = 4$, $C_r = 0.1$, $C_n = 10$, the total cost is $C = 11160.7$. The model was varied over P and q and is shown in Figure A.5. The total cost is convex with respect to purchasing price, $P \times C_n$, and acceptance quality level, q. The Hessian matrix is positive and an optimum solution is obtained when $m = 1$, $n = 2$, $P = 0.71$ (corresponding to an optimal purchasing price $= P^* \times C_n = 0.71 \times 10 = 7.1$) and $q^* = 0.2365$. The optimal solution is obtained using the solution procedure described in Section 8.5. Table 8.1 illustrates this solution procedure numerically.

The convexity of expression (8.28) was demonstrated for varying values of m, n, P, and q to investigate the behaviour of the cost function in (8.28). Several plots of the cost function in (8.28) were generated and showed to have almost identical behaviours to that shown in Figure A.5.

Table 8.1. A numerical example illustrating the solution procedure

Step #	n	m	P	q	$C(m,n,P,q)$	Notes
1	1	1	0.237	0.709	11166	
2	1	2	0.238	0.708	11201	$m_1^* = 1$ and $C(m_1^*, 1, P, q) = 11166$
3	2	1	0.236	0.71	11161	
4	2	3	0.236	0.709	11202	$m_2^* = 1$ and $C(m_2^*, 2, P, q) = 11161$
5						Repeat step 1 and 2
1	3	1	0.235	0.711	11165	
2	3	2	0.235	0.711	11182	$m_3^* = 1$ and $C(m_3^*, 3, P, q) = 11165$
6	**2**	**1**	**0.236**	**0.71**	**11161**	Terminate. Optimum solution is attained when $C(m,n,P,q)$ is minimum

In addition, Example 8.4 was replicated for varying values of a, b, θ, φ, h_1, h_u, δ, C_r, S_r, S_p, C_w, π, and C_p, where expression (8.28) was optimized for 10,000 data sets. All of these data sets confirmed Theorem A.1, i.e., a solution is never optimal when m and n are even. Of the 10,000 replications two numerical examples generated optimal solutions when $m > 1$ and $n > 1$ and m and n are not even (e.g., $m = 3$, $n = 2$), with the remaining examples having either m or n equal to 1. To illustrate, since T is a decision variable dependent on m, n, P and q, the optimal solution tends to reside with smaller values of m and n that meets a specific return rate while minimizing the holding costs of serviceable and repairable inventories.

Example 8.5

This example illustrates Model III. Let $d = 1000$, $\mu = 20$, $\sigma = 5$, $a = 0.1$, $b = 0.9$, $\theta = 9.5$, $\varphi = 1$, $h_p = 5$, $h_r = 2$, $h_u = 0.5$, $C_B = 10$, $S_p = 60$, $S_r = 30$, $C_r = 0.1$, $C_w = 0.005$, $C_p = 0.1$, $C_n = 1$. Applying the solution algorithm in Appendix 10 and solving, the optimum cost $\Psi = 1544.36$, $P = 0.05$, $q = 0.998$, $r_p = 26.7$, $r_r = 30.7$, $Q = 260.48$, $Q_r = 81$ and $Q_p = 179.48$. It took the solver 51 seconds to reach the optimum solution with error window equal to 0.00001/269 = $3.7 \times 10^{-8} \approx 0$.

8.8. Summary and Conclusions

This chapter extended the works of Dobos & Richter (2003, 2004, and 2006) by assuming a variable return rate that follows a demand-like function of purchasing price and acceptance quality level of returns. Three mathematical models were developed. The first assumes a single remanufacturing cycle and a single production cycle, with the second being a generalized version of the first assuming multiple remanufacturing and production cycles. The third model extended the first model and assumed a stochastic demand with backorders are allowed. Two solution procedures were introduced, the first adopts an enhanced search technique that eliminates solution branches that do guarantee an optimal solution. This enhanced solution procedure was supported by a theorem, which shows that having even numbers of remanufacturing (m) and production (n) cycles in an interval never produces an optimal solution. The second solution algorithm used an iterative search technique, which is the golden section search method, and applied the technique in two dimensions.

Through the numerical analysis, it was shown that the pure (bang-bang) policy of either no waste disposal (total repair) or no repair (total waste disposal) as advocated in Dobos & Richter (2003, 2004) is not optimal. Dobos & Richter (2006) considered that a pure remanufacturing strategy should be more cost effective than pure production strategy. Results showed that a mixed (production + remanufacturing) strategy is optimal, when compared to either a pure strategy recycling (pure remanufacturing) or a pure strategy production. In addition, assuming fixed price and quality of returns may not capture the benefits of reverse logistics inventory models, and considering the return rate as a function of price and acceptance quality level was examined and was proven a valuable modelling approach, in both deterministic and stochastic approaches.

The stochastic model delivered similar results to that of the deterministic one, where the optimal solution is a mixture of produced and remanufactured products. A comparison between Model III and other stochastic reverse logistics inventory models in literature is advantageous, however, this is considered for future work.

CHAPTER 9: CONCLUSIONS AND RECOMMENDATIONS

Management of recycling grew in the 1970's and, within two decades, it was established as a new filed of research, which is Reverse Logistics. Many research gaps exist within reverse logistics, which either hindered the chance to capture the expected benefits from implementing remanufacturing/repair programs or misled reverse logistics practitioners from achieving better results.

In the reverse logistics inventory literature, several factors, which have significant effects on inventory optimization, were either ignored or forgotten. Examples are: the first time interval includes production activities without any remanufacturing/repair activity, costs due to switching between production and remanufacturing, the increase of production and remanufacturing rates due to learning effects, returns are disassembled and their inventories are managed differently, and the flow of returns in the reverse direction depends on price and quality.

Other assumptions were either unnecessarily made or mistakenly proven. For example, the quality of production and remanufacturing processes is perfect, the quality of remanufactured products are perceived similar to newly produced products, and the pure production and remanufacturing policies are optimal.

Given these limitations in the literature, the need to address these research gaps and to bring reverse logistics modelling efforts closer to reality became evident. Accordingly, in this dissertation, several models were developed to address these factors and assumptions in a production/remanufacturing context through a series of contributions, to bring research closer to reality and to benefit the practice of reverse logistics as it represents a valuable route towards a sustainable environment. The developed models assume unlimited storage capacity, infinite planning horizon and a single product case.

A first contribution is a deterministic reverse logistics inventory model accounting for switching costs which affects the behaviour of the model and the resulted optimal solutions. In addition, the first time interval was considered, which represents the system just before the remanufacturing process and resulted in an enhanced calculation of the inventory holding cost in the repairable stock, and, prevented an overestimated residual inventory amount that was estimated by some of the works in the literature. A publication related to this contribution is El Saadany & Jaber (2008b).

A second contribution is the discussion of the fact that the quality of production and remanufacturing processes is not perfect. Two models were developed, the first calculated the expected amount of imperfect products, which resulted from imperfect production and remanufacturing processes. The imperfect amount is withdrawn from the system, which affected the expected results of other works in the literature, as it showed and proved that the pure remanufacturing case is mathematically and practically infeasible. The second model discussed reworks issue. A practical situation was formulated in a deterministic approach where production processes are interrupted to restore quality. It was shown that there is optimal production and remanufacturing interruption periods. A publication related to this contribution is El Saadany & Jaber (2009b).

A third contribution is modelling inventory with learning effects in a reverse logistics context. Results showed that there is value for the learning rate below which investment in learning is not beneficial. However, a fast learning might not be optimal implying the existence of an optimal learning rate. A publication related to this contribution is Jaber & El Saadany (2008a).

A fourth contribution is represented in two deterministic models with newly produced and remanufactured products are perceived differently by customers. A production-remanufacturing system was presented where demand is satisfied by the two types of products, and accordingly, lost sales occur intermittently for each type. It was shown that it is favourable to compensate customers to settle for different types of products. A publication related to this contribution is Jaber & El Saadany (2009a).

A fifth contribution is representing the situation when returns are disassembled and the inventory of subassemblies is managed differently. A deterministic model considered the case when returns are disassembled and the subassemblies will have different collection rates due to usage and obsolescence factors. Results proved the invalidity of the assumption that pure remanufacturing and pure production policies are optimal. A mixture of production and remanufacturing is the optimal policy. A publication related to this contribution is El Saadany & Jaber (2008c).

A final contribution is presented in three models that consider the flow of returns in the reverse direction as a function of price and acceptance quality level. Two models are deterministic while the third considers the demand to be stochastic. Works in the literature are limited to the case of fixed return rate and fixed price and quality of returns. The presented models are the first to address return and collection rates as being variable. The formulated models proved that the pure policies are not optimal and presented the reverse flow in an applicable method which considers variable interdependent price and quality. Publications related to this contribution are El Saadany & Jaber (2007) and El Saadany & Jaber (2009a).

The presented models address reverse logistics in several directions. However, it is unreasonable to address all the presented issues and costs of reverse logistics in a single model as it will result in a completely impractical tool. Researchers and practitioners agreed that mathematical models not only have to be precise, to improve business practices and, to solve industrial problems, but also have to maintain an acceptable level of complexity, so as not to lose the chance to be applied and to be useful.

Reverse logistics is a new research area that still has many research gaps and offers numerous opportunities for future work. A recommended extension to the work presented herein, is to integrate the developed production/remanufacturing lot sizing models into a multistage supply chain coordination system. A second extension is to relax one of the assumptions of the developed models, which is assuming the case of multiple products, or the case of limited storage capacity, or the case of finite planning horizon. A third extension is to consider the relation between reverse logistics network design and any of the inventory models presented herein. An interesting extension is to consider modelling the remanufacturing of a short-life-cycle product, and to study its implications on one of the developed models.

APPENDICES

Appendix 1

Setup, holding and total costs are calculated for a single time interval, as shown in Figure A.1, as follows:

Setup costs = $m\, S_r + n\, S_p$

Holding costs in the serviceable stock $= \dfrac{h_1}{2}\left(\dfrac{(1-\beta)x}{n}.\dfrac{(1-\beta)T}{n}.n + \dfrac{\beta x}{m}.\dfrac{\beta T}{m}.m\right)$

Holding costs in the repairable stock $= h_u.\left[\dfrac{T.\beta x}{2} - \dfrac{\beta x}{m}.\dfrac{\beta T}{m}.\dfrac{(m-1)m}{2}\right]$

Notice that, $T = \dfrac{x}{d}$

Therefore, total cost = Setup costs + holding costs in the serviceable and repairable stocks,

$$K_2 = \left(mS_r + nS_p\right) + \frac{h_1}{2d}\left(\frac{(1-\beta)^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{h_u \beta T x}{2} - \frac{h_u \beta^2 T x (m-1)}{2m}$$

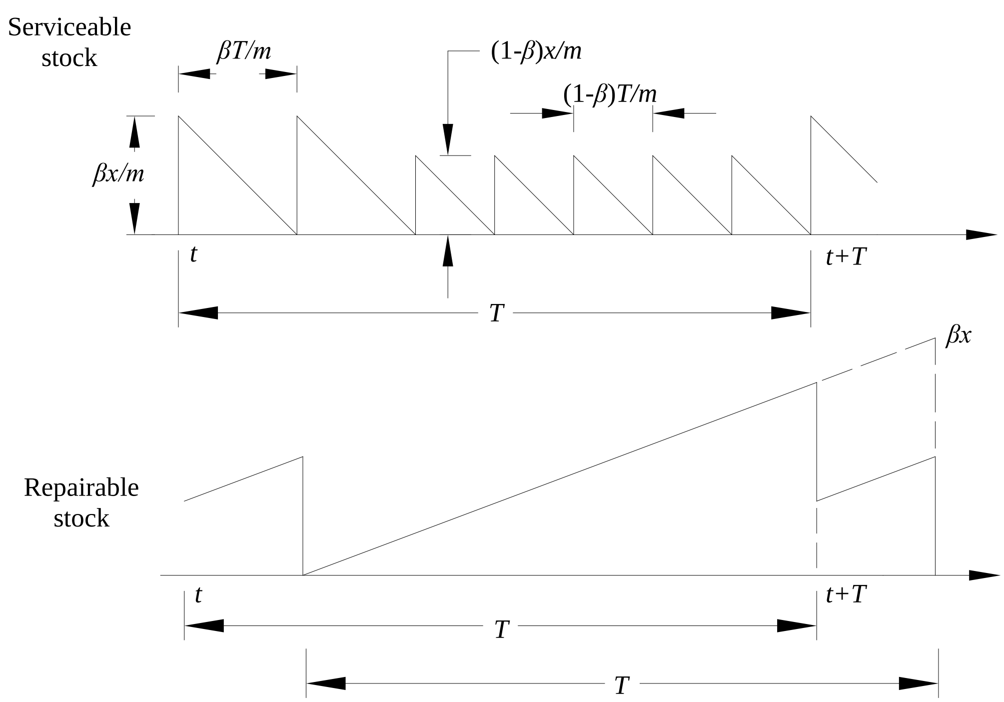


Figure A.1. Details of the derivation of Equation (3.6) in the case of m = 2 and n = 5

Appendix 2

The learning curve is a geometric progression that expresses the decreasing cost (or time) required to accomplish any repetitive task. As total quantity of units produced doubles, the cost per unit declines by some constant percentage (Jaber & El Saadany, 2008a). The learning curve is represented by

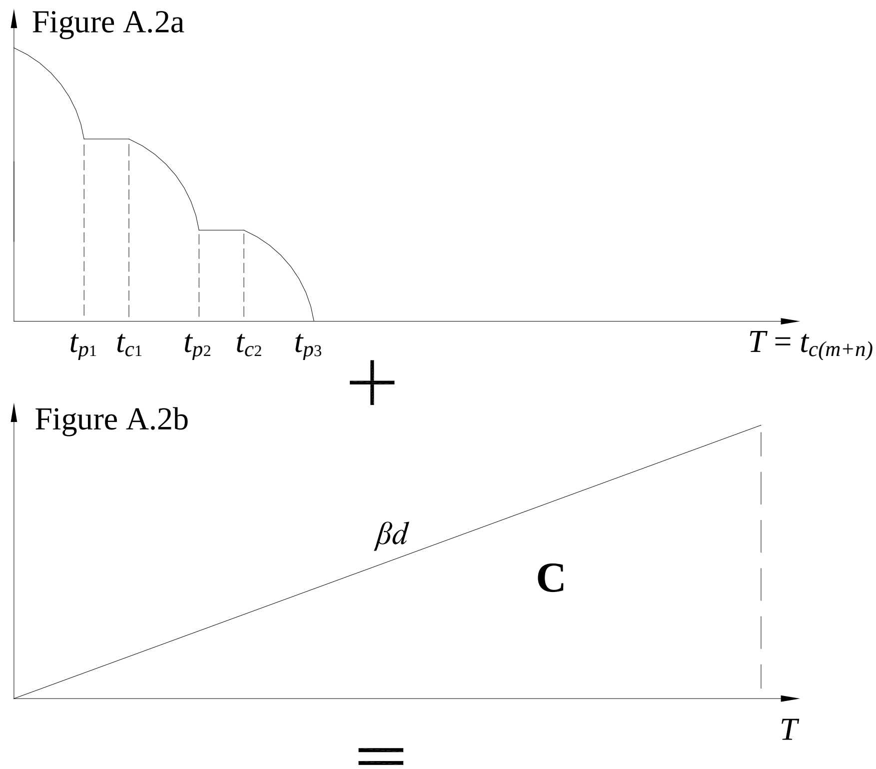
$$U_x = U_1 x^{-b} \tag{A.1}$$

where U_x is the time to produce the *x*th unit, U_1 is the time to produce the first unit, *x* is the production count, and *b* the learning curve exponent. In practice, the *b* parameter value is often replaced by another index number, which is referred to as the "learning rate". The learning rate occurs each time the production output is doubled, where $LR = U_{2x}/U_x = U_1(2x)^{-b}/U_1 x^{-b} = 2^{-b}$ and $b = -\log(LR)/\log(2)$. The time to produce *x* units by integrating Equation (A.1) over the proper limits is given as

$$t(x) = \sum_{n=1}^{x} U_1 n^{-b} \cong \int_0^x U_1 n^{-b} \mathrm{d}n \tag{A.2}$$

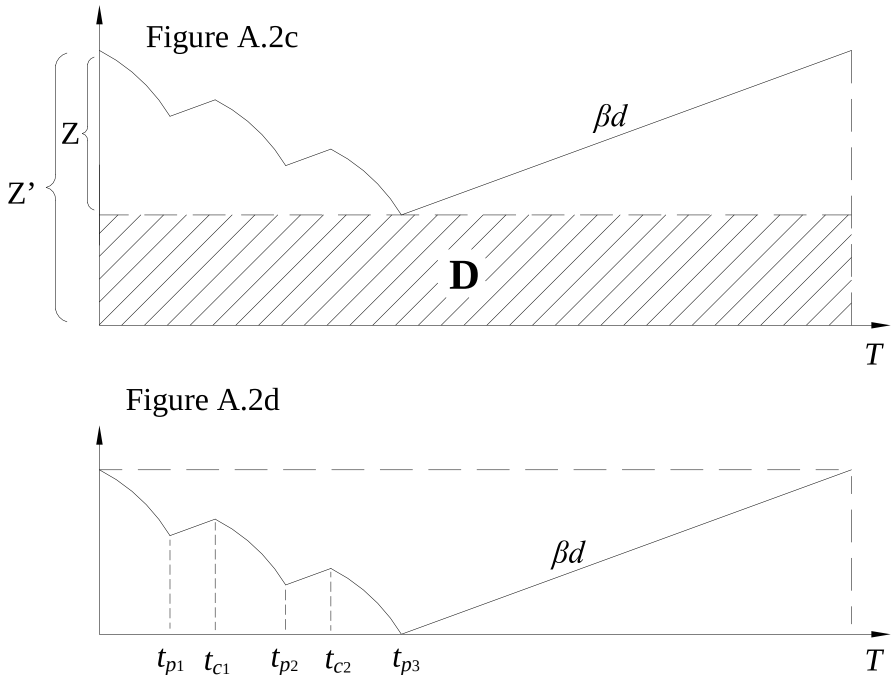
Appendix 3

To calculate the holding cost in the repairable stock, the inventory components are divided into two parts as shown in Figure A.2a and Figure A.2b.



Figures A.2a, A.2b. Depletion of repairable stock with reception of returned units

Figure A.2a represents the behaviour of inventory for the collected used items stored in the repairable stock that are later transferred to the serviceable stock. Figure A.2b represents the behaviour of collecting the used items in the repairable stock. The addition of these two charts results in Figure A.2c, where the residual inventory (rectangle D) needs to be eliminated to produce Figure A.2d. Otherwise, the maximum inventory will be overestimated having a value Z' instead of Z. The available studies in the literature assumed a general time interval and ignored the last time interval prior to remanufacturing where no repairs are performed. This assumption results in an overestimation of the average inventory level and subsequently the holding costs, which is represented by the area D in Figure A.2c.



Figures A.2c and A.2d. Breakdown of inventory in the repairable stock for m = 3 and n = 2

To calculate the area of Figure A.2a, the area will be divided into m areas named “A”, and several rectangles named “B” as shown in Figure A.3.

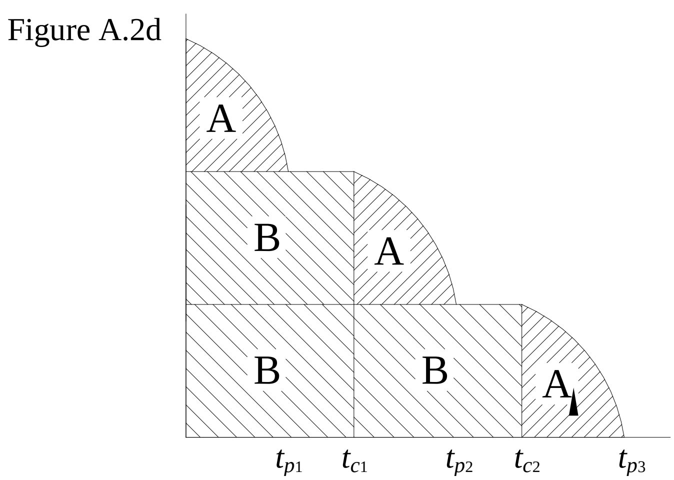


Figure A.3. Breakdown of Figure A.2a to A’s and B’s

The holding cost in the repairable stock is calculated as follows

$$H_2(n,m,Q,R)=h_u\left(m.A+\sum_{j=1}^{j=m-1} jB+C-D\right)$$

Area A is $A(m,R)=\int_{t=0}^{t=t_p} Y-Y(t)dt=r_1Y^{2-a}\left(\frac{m^{2-a}-(m-1)^{2-a}}{(1-a)(2-a)}-\frac{(m-1)^{1-a}}{(1-a)}\right)$

Area of a rectangle B is $B(Y)=Yt_c=Y.\frac{Y}{d}=\frac{Y^2}{d}$

Therefore, $\sum_{j=1}^{j=m-1} jB=\frac{Y^2}{d}\frac{(m-1)m}{2}=\frac{m(m-1)Y^2}{2d}$

Area of triangle C is $C(n,Q,\beta)=\frac{1}{2}\beta dT.T=\frac{\beta dT^2}{2}=\frac{\beta d}{2}\left(\frac{nQ}{(1-\beta)d}\right)^2=\frac{\beta}{2d}\left(\frac{nQ}{1-\beta}\right)^2$

Area of rectangle D is

$$D(m,Y)=\beta d.((m-1)t_c+t_p).T=\beta d\left((m-1)\frac{Y}{d}+\frac{r_1Y^{1-a}}{1-a}\left(m^{1-a}-(m-1)^{1-a}\right)\right).T$$

Therefore, $H_2(n,m,Q,Y)=h_u\left(m.A+\sum_{j=1}^{j=m-1} jB+C-D\right)$

$$=h_u\left(m.r_1Y^{2-a}\left(\frac{m^{2-a}-(m-1)^{2-a}}{(1-a)(2-a)}-\frac{(m-1)^{1-a}}{(1-a)}\right)+\frac{m(m-1)Y^2}{2d}+\frac{\beta}{2d}(mY+nQ)^2\right.$$

$$\left.-\beta d\left((m-1)\frac{Y}{d}+\frac{r_1Y^{1-a}}{1-a}\left(m^{1-a}-(m-1)^{1-a}\right)\right).\left(\frac{nQ}{(1-\beta)d}\right)\right)$$

Accordingly, the last time interval prior to remanufacturing is shorter than T and is equal to

$$T_1(m,n,Q,\beta)=T-(m-1)\left(\frac{Y}{d}\right)T-t_p=T-(m-1)\left(\frac{Y}{d}\right)T-\frac{r_1Y^{1-a}}{1-a}\left(m^{1-a}-(m-1)^{1-a}\right)$$

Appendix 4

From Figure 6.2, T_1 is calculated as follows, $\beta_p D_p T_1 = \frac{T_r}{m}(\gamma_r \beta_r D_r) + T_p(\gamma_p \beta_p D_p)$

$T_1 = \frac{x_2}{D_r.\beta_p D_p m}(\gamma_r \beta_r D_r) + \frac{x_1}{D_p.\beta_p D_p}(\gamma_p \beta_p D_p) = \frac{x_2}{\beta_p D_p m}(\gamma_r \beta_r) + \frac{x_1}{D_p}(\gamma_p)$. Substituting

$x_1 = \frac{x_2(1-\gamma_r \beta_r)}{\gamma_p \beta_p}$, then

$$T_1 = \frac{x_2}{\beta_p D_p}\left(1 - \frac{\gamma_r \beta_r (m-1)}{m}\right) \qquad (6.3)$$

Appendix 5

Holding costs in the second shop for the case of (m = 3, n = 4) are presented in Figure A.4:

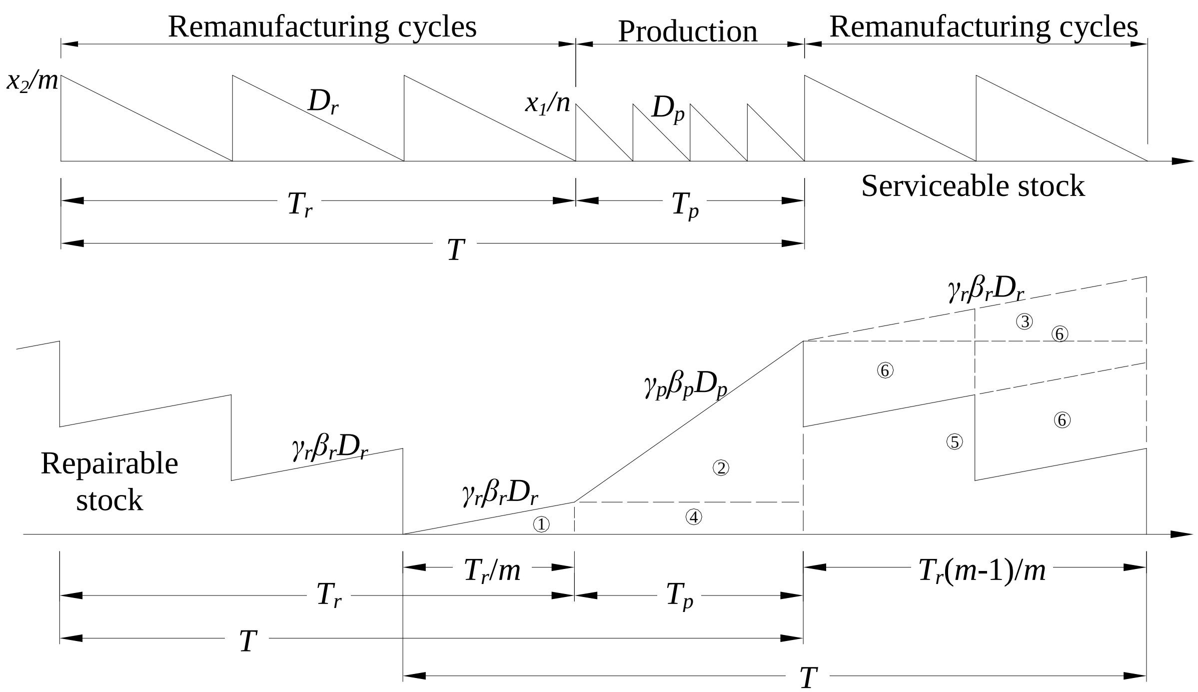


Figure A.4. The calculations of the repairable stock for the case of (m = 3, n = 4)

As shown in Figure A.4, the holding costs in the repairable stock are calculated as follows: The area of triangles 1, 2 and 3 plus the areas of rectangles 4 and 5 minus the area of $\frac{m(m-1)}{2}$ parallelograms 6. Therefore, the total inventory (I_u) in the repairable stock is computed as $h_u \times I_u$, where

$$I_u = \left[\tfrac{1}{2}\gamma_r\beta_r D_r\left(\frac{T_r}{m}\right)^2 + \tfrac{1}{2}\gamma_p\beta_p D_p T_p^2 + \tfrac{1}{2}\gamma_r\beta_r D_r\left(\frac{T_r}{m}(m-1)\right)^2 + \frac{T_r}{m}\gamma_r\beta_r D_r T_p + \left(\gamma_r\beta_r D_r\frac{T_r}{m} + \gamma_p\beta_p D_p T_p\right)\left(\frac{T_r}{m}(m-1)\right) - \frac{T_r}{m}\frac{x_2}{m}\frac{m(m-1)}{2}\right].$$

Appendix 6

The inventory holding cost for a production/remanufacturing cycle is computed by multiplying the holding cost per unit per unit of time by the average inventory level over a production/remanufacturing cycle length. The average inventory level in a cycle (production, remanufacturing or returned) is half the peak of inventory multiplied by the length of cycle time and given as

$$H_{p,1} = h_1 . \tfrac{1}{2} . I_{P,1} . T_P = h_1 . \tfrac{1}{2} . (1/\pi - 1) d(\pi T_P) . T_P = (1/2) h_1 T_P^{\,2} d(1-\pi) = \tfrac{1}{2} h_1 T^2 (1-\lambda)^2 d(1-\pi)$$

$$H_{r,1} = h_1 . \tfrac{1}{2} . I_{R,1} . T_R = h_1 . \tfrac{1}{2} . (1/\delta - 1) d(\delta T_R) . T_R = (1/2) h_1 T_R^{\,2} d(1-\delta) = \tfrac{1}{2} h_1 T^2 \lambda^2 d(1-\delta)$$

$$H_{u,1} = h_u . \left(\tfrac{1}{2} . I_{r,1} . \delta T_R + \tfrac{1}{2} . I_{r,1} . (T - \pi T_R) \right) = h_u . \tfrac{1}{2} \left(\left(\frac{d}{\delta} - Rq \right) \delta T_R . \delta T_R + Rq (T - \delta T_R) . (T - \delta T_R) \right)$$

$$H_{u,1} = h_u . \tfrac{1}{2} . TRq (1 - Rq\delta / d) . T = \tfrac{1}{2} h_u T^2 \lambda d (1 - \lambda\delta) . \ \blacksquare$$

Appendix 7

To prove that the total costs function in Equation (8.13), which is a function in two variables, is convex, the Hessian matrix has to be computed and to be proven positive. To compute the Hessian matrix for a general case (without substituting values other than P and q), the outcome turned out to be 'messy'.

The convexity of (8.13) was demonstrated numerically. More than 10,000 input parameter datasets were randomly generated and the Hessian matrix was computed for each dataset (numerical example). For all of these numerical examples, the Hessian matrix had positive values, suggesting that it is reasonable to conjuncture that Equation (8.13) is convex. A sample calculation of ten randomly selected numerical examples are represented in Table A.1, where $a, b, \theta, \varphi, h_1, h_u, \delta, C_r, S_r, S_P, C_w, \pi, C_P$ and P_n are randomly generated.

Table A.1. Sample calculations of randomly generated examples of Model I

a	b	θ	φ	h_1	h_u	δ	C_r	S_r	S_p	C_w	π	C_p	C_n	R	P	q	**Cost**	Hessian
0.506	0.967	9.282	5.535	0.159	0.427	0.683	0.537	1.435	6.703	0.138	0.755	10.9	0.531	173.6	0.033	0.226	**11109**	293989637
0.6	0.83	9.098	8.873	0.126	0.374	0.637	1.235	7.256	3.667	0.035	0.606	9.333	1.103	123.1	0.02	0.137	**10342**	208509397
0.449	0.932	8.004	3.702	0.66	0.251	0.813	1.956	5.659	1.126	0.128	0.871	9.257	0.53	183.7	0.031	0.322	**9427**	113145997
0.965	0.347	8.92	3.338	0.321	0.344	0.88	0.065	3.395	3.86	0.14	0.78	9.793	0.492	56.62	0.146	0.452	**10142**	13072114
0.659	0.923	7.047	7.916	0.578	0.34	0.7	1.473	7.468	4.525	0.191	0.774	2.938	1.991	87.38	0.046	0.216	**4945**	8671911
0.453	0.24	5.985	1.528	0.734	0.096	0.482	0.785	6.356	6.347	0.052	0.7	5.31	1.196	57.72	0.146	0.797	**6361**	1161911
0.731	0.398	4.997	2.431	0.543	0.338	0.532	0.069	3.377	2.351	0.196	0.516	1.724	1.528	73.92	0.261	0.602	**3204**	484774
0.81	0.639	3.462	9.953	0.159	0.553	0.618	1.559	3.213	4.855	0.132	0.578	1.141	1.464	13.71	0.021	0.245	**2636**	32177.06
0.695	0.023	5.894	5.229	0.333	0.503	0.636	1.614	3.257	6.164	0.16	0.854	1.995	1	1.576	0.038	0.358	**3024**	485.7093
0.707	0.001	8.422	3.114	0.439	0.247	0.448	0.317	7.523	5.497	0.184	0.671	1.307	0.553	0.157	0.14	0.532	**1921**	1.880676

Example A.1

Figure A.5 illustrates the behaviour of the cost function for the following input parameters: $d = 1000$, $a = 0.9$, $b = 0.9$, $\theta = 6$, $\varphi = 2$, $h_1 = 4$, $h_u = 3$, $\delta = 0.8$, $\pi = 0.5$, $S_r = 4$, $S_p = 6$, $C_r = 0.1$, $C_w = 0.15$, $C_p = 2$, $P_n = 1$. The total cost function is convex with respect to price, P, as well as quality, q, where optimal values are $P = 0.370929$ and $q = 0.668266$.

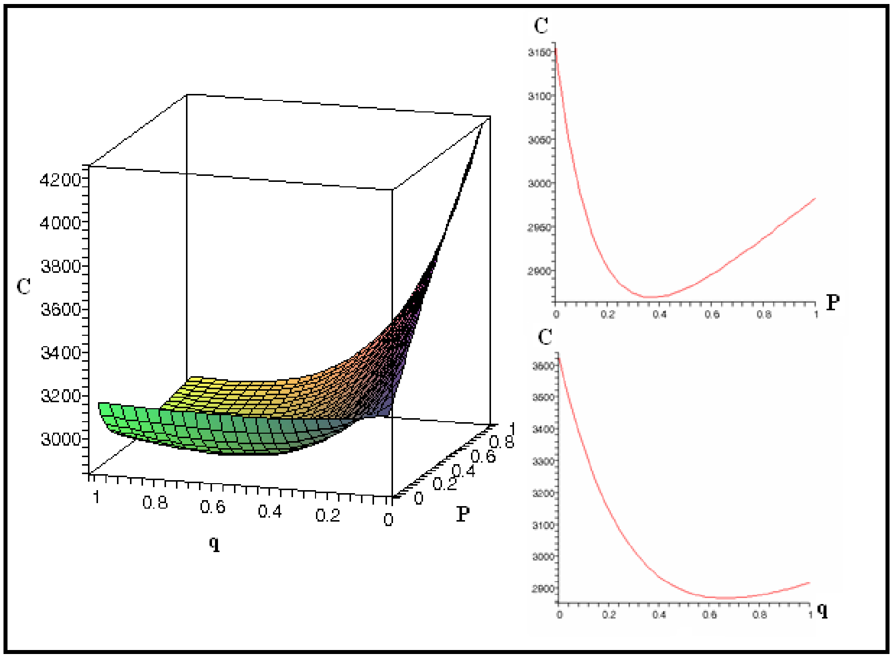


Figure A.5. Total Cost "*C*" versus price "*P*" and quality "*q*"

Appendix 8

The inventory holding cost for a production or manufacturing cycle is computed by multiplying the holding cost per unit per unit of time by the average inventory level over a production/remanufacturing cycle length. The average inventory level in a cycle is half the peak of inventory multiplied by the length of cycle time.

$$H_{p,n} = (1/2)h_1(1/\pi - 1)d(\pi T_P)T_P n = \frac{h_s}{2}T_P{}^2 d(1-\pi)n = \tfrac{1}{2}h_1 T^2(1-\lambda)^2 d(1-\pi)n/n^2$$

$$H_{p,n} = \tfrac{1}{2}h_1 T^2(1-\lambda)^2 d(1-\pi)/n$$

$$H_{r,m} = (1/2)h_1(1/\delta - 1)d(\delta T_R)T_R m = \tfrac{1}{2}h_1 T_R{}^2 d(1-\delta)m = \tfrac{1}{2}h_1 T^2\lambda^2 d(1-\delta)m/m^2$$

$$H_{r,m} = \tfrac{1}{2}h_1 T^2\lambda^2 d(1-\delta)/m$$

For $H_{u,m}$, the area is divided into m A triangles, B (m −1) triangles, a single C triangle and (m–1) D squares, as shown in Figure A.6.

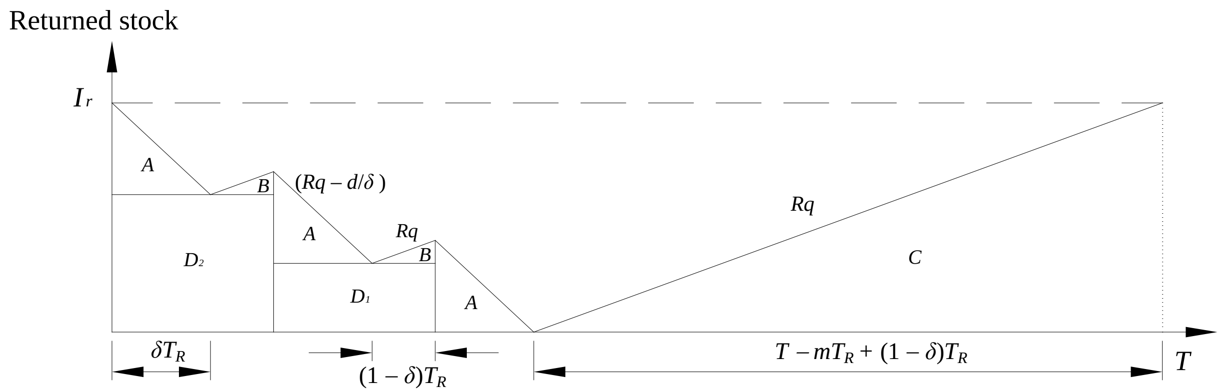


Figure A.6. Inventory calculations for "H_r"

Area of triangle A is $T_A = \delta\frac{T_R}{2}(d/\delta - Rq).\delta T_R = \frac{T_R^2}{2}\delta(d - Rq\delta)$

Area of triangle B is $T_B = \tfrac{1}{2}(1-\delta)T_R(Rq).(1-\delta)T_R = \tfrac{1}{2}RqT_R^2(1-\delta)^2$

Area of triangle C is $T_C = (1/2)(T - mT_R + (1-\delta)T_R).Rq(T - mT_R + (1-\delta)T_R)$, reducing to

$$T_C = \frac{Rq}{2d}d\left(T - m\frac{\lambda T}{m} + (1-\delta)\frac{\lambda T}{m}\right)^2 = \frac{T^2}{2}\lambda d(1 + \lambda(1-\delta-m)/m)^2$$

Area of square $T_{D_i} = ((d/\delta - Rq)\delta T_R - Rq(1-\delta)T_R).i.T_R = T_R^2(d - Rq)i$

$$H_{u,m}/h_u = mT_A + (m-1)T_B + T_C + \sum_{i=1}^{m-1} T_{D_i}$$

$$= (1/2)T_R^2 \left[m\delta(d - Rq\delta) + (m-1)Rq(1-\delta)^2 + \lambda d(1+\lambda(1-\delta-m)/m)^2 + (d-Rq)(m-1)m\right]$$

$$H_{r,m} = \tfrac{1}{2} h_u T^2 d\lambda(1+\lambda(1-\delta-m)/m).\ \blacksquare$$

As shown in Figure A.6, increasing *m* or *n* increases the average inventory in returned stock, while it has no effect on the average inventory in serviceable stock. This signifies the tendency of the presented models to produce optimal solutions with smaller *m* and *n*. This point is emphasised in Appendix 9.

Appendix 9

THEOREM A.1: A policy $C(m,n,\lambda,T)$[1] with both m and n being even integers can never be optimal, since the total cost rate associated with policy $C(m/2, n/2, \lambda, T/2)$ is smaller.

Proof: Figures A.7 and A.8 illustrate the inventories associated with policies $C(m,n,\lambda,T)$, *Case a*, and $C(m/2, n/2, \lambda, T/2)$, *Case b*, respectively.

In *Case a,* the parameters S_r, S_p, $S_{m,n}$, d, T, m, n, R, q, h_1, h_u, π, δ, correspond respectively to parameters S_r, S_p, $S_{m,n}/2$, d, $T/2$, $m/2$, $n/2$, R, q, h_1, h_u, π, δ, in *Case b*. For *Case a,* the cost per unit of time unit is $C_a(m,n,\lambda,T) = S_{m,n}/T + Td\,\psi_a(m,n,\lambda)/2$, where

$$\psi_a(m,n,\lambda) = h_1\left(\lambda^2(1-\delta)\frac{1}{m} + (1-\lambda)^2(1-\pi)\frac{1}{n}\right) + h_u\lambda\left(1+\lambda(1-\delta-m)\frac{1}{m}\right)$$

For *Case b*, the cost per unit of time unit is $C_b(m,n,\lambda,T) = \dfrac{S_{m,n}/2}{T/2} + \tfrac{1}{2}\dfrac{T}{2}d\,\psi_b(m,n,\lambda)$, where

$$\psi_b(m,n,\lambda) = h_1\left(\lambda^2(1-\delta)\frac{2}{m} + (1-\lambda)^2(1-\pi)\frac{2}{n}\right) + h_u\lambda\left(1+\lambda\left(1-\delta-\frac{m}{2}\right)\frac{2}{m}\right)$$

The difference in costs for *Case a* and *Case b* is given as

$$C_a(m,n,\lambda,T) - C_b(m,n,\lambda,T) = \frac{S_{m,n}}{T} + \frac{T}{2}d\,\psi_a(m,n,\lambda) - \frac{S_{m,n}/2}{T/2} - \frac{T}{2}d\,\psi_b(m,n,\lambda)$$

$$= \frac{T}{2}d\left(\psi_a(m,n,\lambda) - \tfrac{1}{2}\psi_b(m,n,\lambda)\right) = \frac{Td}{2}\left(h_u\lambda\left(1+\lambda(1-\delta-m)\frac{1}{m}\right) - \tfrac{1}{2}h_u\lambda\left(1+\lambda\left(1-\delta-\frac{m}{2}\right)\frac{2}{m}\right)\right)$$

$$= \frac{Td}{4}h_u\lambda(1-\lambda).$$

$\because 0 < \lambda < 1$, $\therefore (1-\lambda) > 0$ and

$$C_a(m,n,\lambda,T) - C_b(m,n,\lambda,T) = \frac{Td}{4}h_u\lambda(1-\lambda) > 0,\ \forall m,n,\lambda > 0.\ ■$$

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[1] $C(m,n,\lambda,T)$ is equivalent to $C(m,n,P,q)$

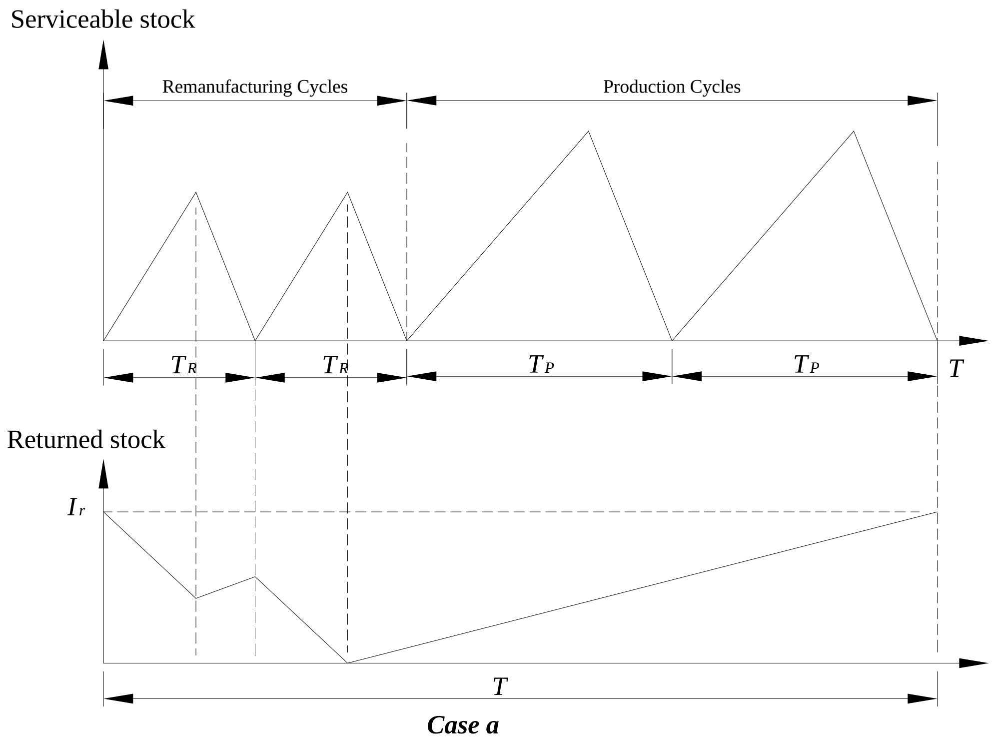

Figure A.7. The behaviour of inventory for Case *a*: *C* (*m, n, λ, T*)

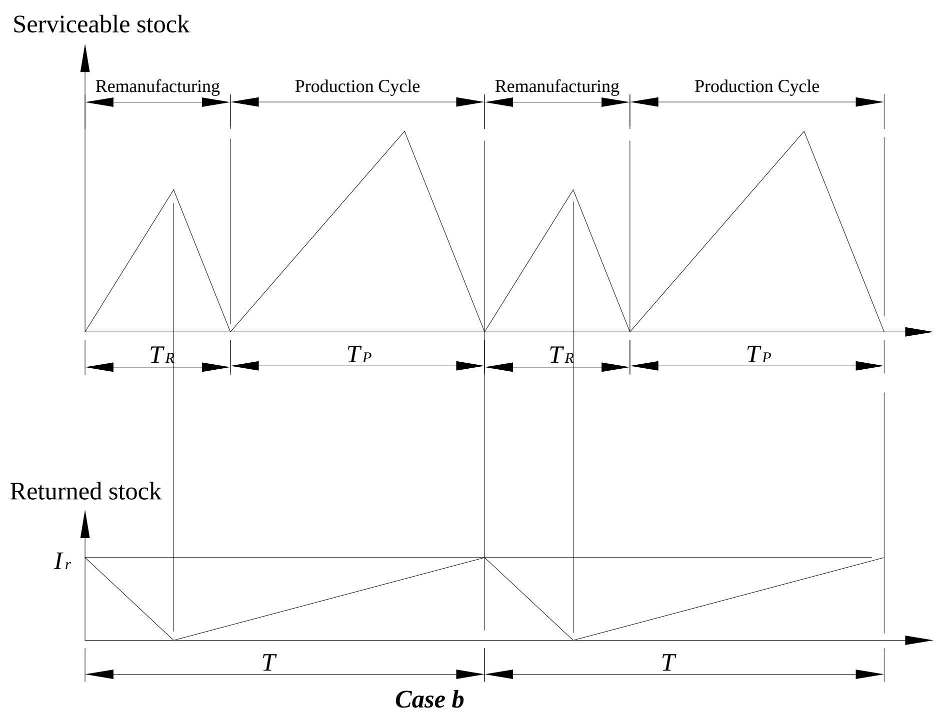

Figure A.8. The behaviour of inventory for Case *b*: *C* (*m*/2, *n*/2, *λ, T*/2)

Appendix 10

The developed solution algorithm optimizes total cost Ψ and is compromised of three subroutines that work together to find the optimum Q, r_p, r_r, P and q.

The golden section search optimization technique in used in Subroutine 2 and 3. It finds local maxim or minima of single variable functions and does not require gradient information. It is based on sectioning a function along a vector (e.g., a decision variable) and eliminating sections according to a minimization or maximization rule until the optimum solution is determined within an acceptable error window (e.g., Aarts & Lenstra, 2003; Huyer & Neumaier, 1999; Vogt & Cottrell, 1999). The golden section value is $= \frac{1}{2}\left(-1+\sqrt{5}\right) = 0.618$.

Subroutine 1 solves Q, r_p and r_r simultaneously to optimize total cost for given values of P and q. Subroutine 2 uses various sets of optimized Q, r_p and r_r values to find the optimum P at a given q. Each optimized set of Q, r_p and r_r values is determined using Subroutine 1. Accordingly, Subroutine 3 uses various sets of optimized Q, r_p, r_r and P values to find the optimum q. Each optimized set of Q, r_p, r_r and P values is determined using Subroutine 2.

Similar to Hadley & Whitin (1963), a starting point is required to initiate the algorithm. A deterministic optimum lot size is used as a starting value for Q, which can be easily calculated from Equation (8.37) as

$$Q_d = \sqrt{\frac{2d\left(S_p + S_r\right)}{h_p(1-q\beta)^2 + h_r(q\beta)^2 + h_u q\beta}}$$

In addition, initial values are used for P and q, which are chosen arbitrary at $P = 0.5$ and $q = 0.5$. An acceptable error, e, is determined (e.g., $e = \pm 0.0001$) at the beginning of the algorithm. The main subroutine of the algorithm is Subroutine 3, which calls Subroutine 2 numerous times, which in turn calls Subroutine 1 various times. The algorithm works successfully and solves the given examples in Chapter 8 in a mater of seconds.

Subroutine 1:

1- Find the input parameters of d, μ, σ, S_p, S_r, h_p, h_r, h_u and C_B

2- Calculate Q_d

3- $Q_1 = Q_d$

4- Calculate $G(r_p) = \Phi\left(\frac{r_p - \mu}{\sigma}\right) = \sqrt{\frac{Q_1 h_p (1 - q\beta)}{C_B d}}$

5- If $G(r_p) < 0$ or $G(r_p) > 1$, then $r_p = r_r = \mu$ and GoTo step 15

6- Calculate $G(r_r) = \Phi\left(\frac{r_r - \mu}{\sigma}\right) = \sqrt{\frac{Q h_r q\beta}{C_B d}}$

7- If $G(r_r) < 0$ or $G(r_r) > 1$, then $r_p = r_r = \mu$ and GoTo step 15

8- Calculate $G(r_p) = \Phi\left(\frac{r_p - \mu}{\sigma}\right) = \sqrt{\frac{Q h_p (1 - q\beta)}{C_B d}}$ and determine r_p from the tables

9- Calculate $G(r_r) = \Phi\left(\frac{r_r - \mu}{\sigma}\right) = \sqrt{\frac{Q h_r q\beta}{C_B d}}$ and determine r_r from the tables

10- Calculate

$$(S + \eta_{pr}(r_p, r_r)) =$$
$$\left(S_p + S_r + C_B\left[(\mu - r_p)\Phi\left(\frac{r_p - \mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r_p - \mu}{\sigma}\right) + (\mu - r_r)\Phi\left(\frac{r_r - \mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r_r - \mu}{\sigma}\right)\right]\right)$$

11- If $(S + \eta_{pr}(r_p, r_r)) < 0$, then $Q_2 = \frac{Q_1}{2}$, GoTo step 13

12- $$Q_2 = \sqrt{\frac{2d\left(S_p + S_r + C_B\left[(\mu - r_p)\Phi\left(\frac{r_p - \mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r_p - \mu}{\sigma}\right) + (\mu - r_r)\Phi\left(\frac{r_r - \mu}{\sigma}\right) + \sigma^2\phi\left(\frac{r_r - \mu}{\sigma}\right)\right]\right)}{h_p(1 - q\beta)^2 + h_r(q\beta)^2 + h_u q\beta}}$$

13- If $|Q_2 - Q_1| < e$, then GoTo step 15

14- $Q_1 = Q_2$, GoTo step 4

15- $Q = Q_1$, and r_p and r_r are as determined in the last execution of steps 8 and 9 and $\Psi(Q, r_p, r_r, P, q)$ is determined from Equation (8.36).

16- End

Subroutine 2:

1- a1 = 0.001
2- a2 = 0.999
3- T1 = a2 − a1
4- x1 = a2 − 0.618 * T1
5- *P* = x1
6- Run Subroutine 1, and z1 = cost resulted from Subroutine 1.
7- x2 = a1 + 0.618 * T1
8- *P* = x2
9- Run Subroutine 1, and z2 = cost resulted from Subroutine 1.
10- If z1 ≥ z2, then a1 = x1 Else GoTo step 17
11- T1 = a2 – a1
12- x1 = x2 and z1 = z2
13- x2 = a1 + 0.618 * T1
14- *P* = x2
15- Run Subroutine 1, and z2 = cost resulted from Subroutine 1.
16- GoTo step 23
17- If z2 > z1 Then a2 = x2
18- T1 = a2 − a1
19- x2 = x1 and z2 = z1
20- x1 = a2 − 0.618 * T1
21- *P* = x1
22- Run Subroutine 1, and z1 = cost resulted from Subroutine 1.
23- If $|x_2 - x_1| < e$, then GoTo step 25
24- GoTo step 10
25- End

Subroutine 3:

1- a3 = 0.001
2- a4 = 0.999
3- T3 = a4 – a3

4- x3 = a4 − 0.618 * T3

5- *q* = x3

6- Run Subroutine 2, and z3 = cost resulted from Subroutine 2.

7- x4 = a3 + 0.618 * T3

8- *q* = x4

9- Run Subroutine 2, and z4 = cost resulted from Subroutine 2.

10- If z3 ≥ z4, then a3 = x3 Else GoTo step 17

11- T3 = a4 – a3

12- x3 = x4 and z3 = z4

13- x4 = a3 + 0.618 * T3

14- *q* = x4

15- Run Subroutine 2, and z4 = cost resulted from Subroutine 2.

16- GoTo step 23

17- If z4 > z3 Then a4 = x4

18- T3 = a4 – a3

19- x4 = x3 and z4 = z3

20- x3 = a4 − 0.618 * T3

21- *q* = x3

22- Run Subroutine 2, and z3 = cost resulted from Subroutine 2.

23- If $|x_3 - x_4| < e$, then GoTo step 25

24- GoTo step 10

25- End

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