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PIECEWISE CONSTANT MODELING AND TRACKING OF SYSTEMATIC RISK IN FINANCIAL MARKET

by

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A thesis
presented to Ryerson University
in partial fulfillment of the
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Master of Applied Science
in the Program of
Electrical and Computer Engineering.

Toronto, Ontario, Canada, 2011

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PIECEWISE CONSTANT MODELING AND TRACKING OF SYSTEMATIC RISK IN FINANCIAL MARKET

Master of Applied Science, 2011

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Electrical and Computer Engineering

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Abstract

The objective of this thesis is to study the time-varying systematic risk in capital market represented by beta. By using statistical hypothesis testing, we show that beta changes in a piecewise constant pattern in which the changes are governed by triggering economic events. This pattern of beta is different from previously modeled time-varying patterns in literature, such as random walk and mean-reverting models and is consistent with the efficient market hypothesis.

We also present a new modeling technique based on Poisson process to represent piecewise constant beta. We develop a new tracking algorithm based on Kalman Filter in which Bayes' selection criteria is incorporated to track piecewise constant beta. Our simulation results show that our proposed tracking method outperforms the traditional random walk and mean-reverting model based Kalman Filter tracking. Our empirical case studies also show that our method is efficient in capturing the significant risk changes which are attributed to economic events.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation and Objectives

In finance theory, one of the most common measure of risk of an asset or a portfolio is the beta. This risk measure is quantified by comparing variability of the return on asset or portfolio with the return on market. Beta represents non-diversifiable part of the risk and is also known as systematic risk in capital market. Systematic risk is an asset's risk component correlated to the market and will be rewarded (priced) in the market. The other risk component is the idiosyncratic risk of an asset which can be eliminated by diversification and therefore is not rewarded or priced. Such risk return relationship, originally established in [1] and [2] is commonly known as Capital Asset Pricing Model (CAPM). As a widely used model in finance to measure how sensitive an asset or portfolio is with respect to the market, this model helps in calculation of cost of capital, mispricing of stocks and even evaluate the performance of asset managers. Beta, being a parameter in CAPM, needs to be estimated given the expected returns on an investment and market. The traditional method of its estimation employs Ordinary Least Squares (OLS) regression over a period of time with an assumption that it is constant over that period. Such OLS estimation is still dominant in Wall Street for firm valuation and Discount Cash Flow (DCF) analysis. However, the dynamics of the economy substantially leads one to believe that beta is not stable through time. Evidences of time varying nature of beta has already been well established in many literatures such as [3], [4], [5], [6] and so on.

Most of the literature on beta are focused on testing its stability for example [3], [5], [6], [4] and so on or testing the validity of CAPM within a certain period for example [7], [8], [9], [10], [11] and all that. However, very few literature have tried to justify the time varying characteristics of beta by realistically looking into economic phenomenon. The primary objective of this thesis is to study the time variation in beta and interpret it in terms of any economic event characterizing the movement. In order to track this movement we model beta as piecewise constant process and track this movement using state space and Kalman filtering algorithm.

1.2 Background

For any investment, the most important aspects of interest are the risk and return. Any investor would like to know what return he would get from the investment for the amount of risk he has to bear in it. In earlier studies, variance or standard deviation of the returns were recognized as a magnitude of risk of individual securities. Greater is the standard deviation, larger is the risk. In [12], this theory was contradicted by showing that standard deviations of two risky assets are not additive. Moreover in [12], it is emphasized that when the two assets are not absolutely positively correlated the standard deviation is less than the sum of the standard deviations of its constituents. Development of CAPM ([1] and [2]) provided a major contribution in defining and determining the risk of an investment. Since then CAPM has evolved as a favorite model in finance. The CAPM equation is given by (1.1).

$$E[R_e] = R_f + \beta(E[R_m] - R_f) \tag{1.1}$$

where, $E[\cdot]$ is the expectation operator. R_f is the risk free rate of return, R_e is the return on asset and R_m is the return on market portfolio. The OLS estimation utilizes the CAPM equation for beta estimation. The (1.1) states that the excess return on any asset or portfolio is linearly related to the market premium governed by the factor β.

As CAPM became more prominent in the financial world numerous studies were conducted on. These studies included empirically examining its performance for different kinds of data built up using manifold types of securities and portfolios. Predominantly studies are focused to investigate the validity of CAPM to explain the cross section of realized average returns of portfolios. These studies reported contradicting results that raised question on validity of the model. In the widely cited study [7], it is argued that static CAPM is not able to explain the cross-section of average returns when size and book-to-market ratio were considered to create portfolios. The stability and validity of beta, along with CAPM model itself, became questionable as more studies were conducted on it.

1.2.1 Literature Review

In one of the earlier studies [3], stationarity of beta coefficients were examined over time and it was observed that the risk coefficients estimates tend to regress towards their means. It is concluded in [3] that market risk estimated over one period would continue into next period but did not look stationary. The tendency of this happening is stronger for lower risk portfolios than for higher risk portfolios. Also in [13], stationarity of beta is analyzed and it is remarked that betas are obviously not constant. It is reported in this literature that beta is stationary for large portfolios, less stationary for smaller portfolios and unpredictable for individual securities. It is also pointed out that there existed a tendency for betas to regress towards their means in agreement with [3]. In [5], the behavior of beta was examined and it is concluded that beta may follow random coefficient model. The authors also pointed out that estimation using a single index model will be problematic because in the long run beta will experience changes. In [14], methodology is proposed to test stationarity of beta using random walk model. In [6], stochastic behavior of beta is analyzed considering it to follow first-order autoregressive process with a constant mean and found evidences for randomness of beta in the market model. Again in [4], variance of changes in beta for individual stocks and portfolios is presented under two hypothesis which were, beta followed a random walk process and it followed an autoregressive process.

Among other studies, various literature have also used these time series models applied to different data in order to examine beta process by considering fitness criteria such as

Root Mean Square Error (RMSE). In [15], German stock return data is analyzed and also confirmed instability of beta for given data. The authors used random walk to model the beta process and Kalman filter is used for estimation. Also in [16], random walk model is used to estimate time varying betas in Italian market. In [17], risk-return relationship in up and down market is investigated using random walk model. In [18], random walk and mean reverting model is used to analyze the Canadian sector portfolios. In [19], time varying betas in Australian industry portfolio is estimated using mean reverting and random walk process as well. In [20], random walk and mean reverting model is used to compare different modeling techniques considering time varying beta in eighteen pan-European industry portfolios.

The random walk model mentioned in these literature are simply a special case of autoregressive (AR(1)) model which is not weakly stationary. It can also be considered as a Markov process where at each step the state may change in either direction with equal probability. The random walk model is given by (1.2)

$$\beta_t = \beta_{t-1} + \zeta_t \tag{1.2}$$

where ζ_t is the noise parameter with normal distribution following $\zeta \sim N(0, \sigma_\zeta^2)$.

Another model mentioned in these literature is the mean reverting model for example in [21], [11], [20]. In the mean reverting model, it is assumed that beta is evolved through an autoregressive process consisting of two components in specific proportions. The first component is the mean beta and the second component is the previous period beta. The mean reverting model can be represented by (1.3). Here, B represents the speed at which beta attains its mean value. The next period's beta is the weighted average of this period's coefficient and its mean value. When value of B is allowed to be one, the mean reverting model represents the random walk model and when B is allowed to be zero the estimated value of beta follows a random coefficient model.

$$\beta_t = (1 - B)\overline{\beta} + B\beta_{t-1} + \zeta_t \tag{1.3}$$

In (1.3), ζ_t represents the disturbance following $\zeta \sim N(0, \sigma_\zeta^2)$.

In addition to examining stationarity of beta, plentiful research has also been conducted to test the validity of the model that determines the systematic risk. The estimation of beta is performed through one factor market model where the risk factor is only the market risk. In [22], CAPM is tested interpreting it as a two parameter model. The results in [22], showed that there is no other risk apart from the market portfolio risk that affects the expected returns and this relationship is linear and is positive. However, it is pointed out that estimation of the intercept term is significantly larger than the risk free rate in contrast to CAPM. In the widely cited paper [7], it is shown that simple linear relationship between beta and average returns shown in [22] do not exists for 1963-1990 period. They suggested that size of the firm and book-to-market ratio is capable of explaining the variation in cross section of expected returns associated with size, earning-to-price ratio, book-to-market ratio and leverage. In [9], it is assumed that CAPM holds in a conditional sense and included return on human capital to measure the return on aggregate wealth and showed that their model can explain the cross section of average returns. In [23], variation of securities and portfolio returns in up and down market is analyzed and inference is made that bear market beta coefficient would be more suitable to measure the portfolio risk. In [24], it is argued that when realized returns are used, the relation between beta and expected return is conditional on the excess market return. In more recent studies, [8] suggested that variation in betas and equity premium would have to be large enough to explain the asset pricing anomalies like momentum and value premium and also confirmed that betas very considerably over time. In [25], a three factor model is introduced in order to explain the cross section of expected returns. In [11], learning is introduced into standard conditional CAPM model, by estimating betas with Kalman Filter. Using this approach of unobserved long-run movements in beta and performing time series asset pricing tests on the size and book-to-market sorted portfolios they showed that pricing errors are substantially reduced. In [10], conditional CAPM is used with time varying betas to account for a book-to-market effect discussed in [7].

The event studies have also been conducted in order to examine the effect of an event on the systematic risk. Change in beta based on the information flow in the economy or the firm-specific events are found in these literature. These literature are generally focused on certain specific events in the firm. For example, in [26], time varying patterns in beta is analyzed by focusing on quarterly earnings announcements. In this study, the authors allowed stock's beta to vary at daily frequency and studied the behavior of beta by using the econometric of high frequency data around the dates of over 22,000 quarterly earnings. They decomposed the systematic risk into variance and covariance component. According to their findings, there is an increase in beta on the announcement days that declines on post-announcement days before reverting to its long-run average level. They also analyzed companies in different sectors and reported that High Tech sector experience large increase in beta. In [27], beta coefficient of stocks is examined around the split announcement dates, dates on which split becomes effective and dates following the stock split. The authors' findings from the experiment conducted on 1034 stock splits show that there is a temporary increase in average beta on both split-announcements date and the date split becomes effective. Also, as per their observations there is a permanent increase in beta following the ex-date. However, similar study in [28] on stock split ex-dates show that the shift in beta around stock-split announcement dates vanishes when longer measurement intervals are considered. In this study it is concluded that by using weekly returns and monthly returns on a larger sample than in [27], no statistically significant difference is found between pre- and post- split betas. The changes in beta around dividend announcement date is studied in [29].

The change in beta due to the availability of any firm-specific news provide a distinct conviction on the stability of systematic risk and the models that it is hypothesized to follow. Also, these literature show that studies for the stability of beta has not only been conducted in U.S. markets but also in European markets. However since different estimation and modeling techniques are available one cannot determine with certainty about the process that systematic risk follows and the model to be used for it.

1.2.2 Changes in Beta Based on Information Flow

The instability of beta over time has led to different challenges. If beta is unstable then how exactly can it be estimated so that economic significance can be reflected through it. The estimation of beta raises the question of modeling it in terms of time series models. Various methods dealing with the problem of estimation have been applied in literature and the most common and the simplest of them uses the rolling window regression on historical observations on a market model. Meanwhile, stochastic time series models provide an empirical justification of beta process using statistical criteria. However little or no economic significance can be availed using these techniques. The time series stochastic models do not connect beta changes with economic events and it is not certain which model is more justified in this case. Looking at the practical structures of these models, in random walk model next period's beta tend to deviate from previous period beta but with additional unpredictable noise. In random coefficient model, beta is assumed to be constant, however on each period, measured beta is affected by noise, thus causing the beta process to jump up and down over a constant level. In mean reverting model, beta always reaches its mean value and is made function of speed parameter with which it reverts back to its mean level.

From the efficient market point of view, beta, being an economic specification, the way it changes should be governed by certain economic events. The time series models discussed assume beta to change at every time instant through a pattern governed by unpredictable and random noise variable. However, in realization beta should change depending on the information flow in the economy. These time series models are based on statistical tests and do not take into account any significant economic event that could affect the systematic risk of an asset. In [6], it is iterated that micro-economic factors such as operational changes and business environment changes in a firm or macro-economic factors such as inflation rate and expectation of relevant future events can cause variation in the systematic risk. Event studies suggest that events such as stock splits [27] and earning announcements [26] and dividend announcements, [29], can cause changes in beta.

While these event studies support that economic events may cause changes in beta, the nature and pattern of changes in beta is still not clear, and the results of the event studies are sometimes subject to criticism. For example, in [30] and [27] a permanent increase in average beta subsequent to stock splits is suggested, i.e., a random walk procedure. In [28], this is criticized by arguing that it is because of the use of too short a return measurement interval to estimate beta and [28] found no permanent beta shift following ex-dates by employing a weekly or monthly return interval. In [26], betas around quarterly earnings announcements is studied, and suggested that beta of individual stocks increase on days of quarterly earnings announcements and revert to their average levels two to five days later. This implies a mean reverting process.

Another caution on the results of event studies is on the consistency of a certain event to cause changes in beta, i.e., a certain event may or may not be a triggering event that causes change in beta, depending on specific business environments/factors. For example, a new product announcement may not cause variation in beta for a stock when such announcement is in expectation, e.g. for a firm which constantly develops a new products. But, it may lead to change in beta when it is a game changing event, e.g., for a stock of a firm which has fallen off on competition without any recent new product development. A better than expected earning announcement may change stock price due to the change of expected future cash flow, but may not change the systematic risk of that firm, i.e., the systematic uncertainty of the future cash flow.

Thus, in real world, the change in beta should depend on the information flow in the economy since beta is a parameter governed by economic factors. A distinct beta change should be driven by suggestive information. A constantly changing unstable beta, as assumed by existing models, does not make much economic sense and is irrelevant for investment bankers in firm valuation and calculation of capital cost during merger and acquisitions. As a result, none of the existing time-varying beta models have been applied in practice as an alternative method to the dominant OLS method.

Also when evaluating different models, model fit measures such as RMSE are always used, e.g., [18], [20] and [19]. Usually, it represents the fit to the observed stock returns. However, as far as the risk is concerned, such error should represent idiosyncratic risk. Without right economic justifications, a model with smaller MSE may not be the best model but just a chance result that has the best fit to the specific data set.

Therefore, to model the economic significance in beta is of great challenge and contribution towards it has been extremely limited. In order to understand beta, its nature needs to be understood first before making any inferences. One way of modeling beta is to use state space approach. In state space modeling beta can be modeled as a hidden unobserved process of a market model. However, even in state modeling the question arises that which time series model is to be applied so that economic significance is revealed from the data such that the change in beta is justified. The modeling part is followed by the estimation part. Estimation also needs to adhere to economic significance and also needs to confirm with the model. So the main challenge lies in providing a proper justification based on economic significance which can be drawn from the change in beta process. Thus, the challenges are concentrated mainly in terms of modeling and estimating beta governed by explainable changes.

1.3 Contribution

In this thesis, we present a novel approach to model the time varying systmatic risk based on the triggering economic events. We employ various techniques to establish and verify this model and also use this model to track beta dynamically. The key contributions of this thesis are as follows:

1. Statistical hypothesis testing is employed to test equality of estimated beta coefficients between two adjacent time periods. In this, rejection of the hypothesis implies that beta estimated in these two time periods are significantly different. Then, we use dual rolling window search routine to scan the entire observation period and identify the transition time instants at which beta changes occur. Our results show that significant change in beta are infrequent and they do not occur at every time instants.

2. We perform use case studies and attribute significant change in beta to micro and macro economic events. This association is one of its kind to explain the time varying systematic risk. As a result, our results show that underlying unobserved beta has a tendency to be piecewise constant.

3. We apply a non linear model in which beta is considered to follow a piecewise constant process. In this, we model beta by assuming that triggering events, which cause significant beta change, follow Poisson process. Using Poisson process we specify a rate at which significant events relevant to a firm occur. This model we have derived is consistent with the efficient market hypothesis [31].

4. We modify the traditional Kalman filter algorithm to track the piecewise constant beta process. The traditional Kalman filter algorithm used in literature do not provide the feature of determining jump locations present in the time series representing a piecewise constant model. These jump locations are time instants at which beta change occurs. Thus, we introduce prior probabilities of beta jumps and use Bayes' criteria to choose between two estimation covariance matrices conditional on beta jumps at each time instant. Our simulation results show that, our method outperforms the traditional tracking based on random walk and mean reverting models to track piecewise constant process. Our empirical case studies also show that our method is effective enough to identify events that cause change in beta.

1.4 Organization of Thesis

In chapter 2, we briefly discuss the state space model and Kalman filter. In doing so, we discuss various steps involved in Kalman filter algorithm, the estimation technique and routines. We also discuss the application of Kalman filter in using the traditional models.

In chapter 3, we present a regression based estimation and statistical hypothesis testing to identify significant changes in beta. We present dual rolling window search routine in order to find transition points in beta process in the given observation period. Here, our results show that significant changes in beta are infrequent and has tendency to be piecewise constant. We also perform case studies to examine the relation between significant changes in beta and associated triggering economic events.

In chapter 4, we present the tracking of piecewise constant beta based on modified Kalman filter. We discuss the simulation results and also present empirical case studies and show that the tracking methodology employed is efficient in determining the transitions in beta due to triggering economic events.

In chapter 5, we conclude the thesis and give prospects for the future work.

Chapter 2

State Space Model and Kalman Filter Preliminaries

In this chapter, we review the state space modeling and Kalman filter algorithm. These methods are applied to track time varying systematic risk as shown in literature [20], [11], [18], [19] and so on. Here we list and explain the related equations and conditions involved in Kalman filter by specifying them using vector notation. We also give an example of its usage in tracking time varying beta using traditional models.

2.1 State Space Models

In control engineering, the system that varies through time can be described using the notion of state space. When representing a system using a state space form, two equations are generally specified. These equations comprises of input, output and state variables and are related by first-order differential equation. The variables can simply be a scalar quantity or also a vector. The first of the two equations that accounts the relationship between the output, which is a measurable quantity, input variables and corresponding state variables is called the measurement or observation equation. The second equation which explicitly describes the internal state of the system is known as the transition equation or state equation. These system of equations are collectively known as state-space equations. If state space is represented by a vector, it consists of all the possible internal states of the system. The states are unobserved component of the system whose effect can only be seen on the output through the measurement equation. The dynamics of the state variables is available in the transition equation which is based on the minimum set of information from the past. The analysis of time series using state space modeling is also explained in [32]. Here our focus is on general Kalman filter.

2.2 Kalman Filter Review

The Kalman filter method was originally developed by Rudolf E. Kalman (1960) for discrete linear systems. It is a recursive solution to a discrete signal linear filtering problem. The algorithm is designed to compute the forecasts and variances of the forecasts of states of a system in an iterative manner. A time series model which is represented in the form of state space system is also an ideal candidate for application of Kalman Filter. It is a very powerful tool when comes to noisy systems. It recursively estimates the instantaneous state of a system corrupted by noise by using measurements which is linearly related to the state and which is also corrupted by noise.

The recursive process of Kalman Filter involves forecast of the future state based on previous updated state estimate and the new observed data. That is, each consecutive forecast is computed by updating the previous forecast. Update of each forecast involves computation of weighted average of previous observation and the previous forecast error. The weights are chosen such that the forecast variances are minimized. Because of its recursive nature, Kalman Filter can be applied in real time. In terms of storage, only the previous estimate is required to be stored, thus, eliminating the need to store the entire past observed data. In terms of computational efficiency, it is more efficient than computing the estimate directly from the entire past observed data at each step of filtering process.

The state space system of equations is the basic representation of a system for which Kalman Filter recursion can be applied. The linear regression equation can be rewritten as time-varying regression equation by letting the regression coefficient to follow a given time process. This equation represents an observation equation in the state space model. The state space system can be represented by (2.1) and (2.2).

$$\mathbf{x}_t = \mathbf{F}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{a}_t + \mathbf{M}_t\mathbf{v}_t \tag{2.1}$$

$$\mathbf{y}_t = \mathbf{A}_t\mathbf{x}_t + \mathbf{w}_t \tag{2.2}$$

Where $\mathbf{x}$ is the state vector at a discrete time period t. $\mathbf{F}_t$ is the state transition matrix that is applied to the previous period state $\mathbf{x}_{t-1}$, $\mathbf{a}_t$ is the control input vector to which control input matrix $\mathbf{B}_t$ is applied. $\mathbf{v}_t$ is generally a serially uncorrelated and independent noise with $\mathbf{v}_t \sim N(0, \mathbf{R}_t)$. $\mathbf{M}_t$ is the matrix applied to this noise. $\mathbf{y}_t$ is the observation vector, $\mathbf{A}_t$ is the observation matrix applied to the state vector $\mathbf{x}_t$ at time period t. $\mathbf{w}_t$ is the observation noise independent and serially uncorrelated following distribution $\mathbf{w}_t \sim N(0, \mathbf{Q}_t)$. Furthermore, for all time periods the observation and state equation disturbances are uncorrelated with each other and also with the initial state vector $\mathbf{x}_0$. In this system of equation $\mathbf{a}_t$ is non-random input to the system. The matrices $\mathbf{F}_t$, $\mathbf{B}_t$, $\mathbf{A}_t$ and $\mathbf{M}_t$ depends on model specification. This representation of state space model with state noise matrix is also described in [33].

Along, with the random variables present in the above equations, the initial state $\mathbf{x}_0$ is also considered as a random variable with mean $\mathbf{x}$ and covariance matrix $\mathbf{P}_0$ [33], i.e.

$$E[\mathbf{x}_0] = \mathbf{x} \qquad and \qquad Var[\mathbf{x}_0] = \mathbf{P}_0$$

Additionally, the noises $\mathbf{w}_t$ and $\mathbf{v}_t$ are uncorrelated with each other at all time periods and also uncorrelated with the initial state, i.e.

$$\begin{aligned} E[\mathbf{w}\mathbf{v}^T] &= \mathbf{0} \qquad \text{for all } t = 1, 2, \ldots, T \\ E[\mathbf{x}\mathbf{v}^T] &= \mathbf{0} \qquad \text{for all } t = 1, 2, \ldots, T \end{aligned}$$

and

$$E[\mathbf{x}\mathbf{w}^T] = \mathbf{0} \qquad \text{for all } t = 1, 2, \ldots, T$$

2.2.1 Kalman Filter Recursion

The Kalman Filter algorithm gives an optimal estimation in terms of minimizing the mean square error. Kalman Filter algorithm consists of two basic phases: predict phase and update phase. Here, we only present and explain the relevant equations of these phases. Their detailed derivations can be found in literature such as [33] and [34].

In the predict step, the state at current time step is estimated using updated estimate of the previous state. The predicted estimate is also called the priori estimate because it is an estimate of the current time step and doesn't include information from the observation of the current time step. The state prediction is given by (2.3)

$$\hat{\mathbf{x}}_{t|t-1} = \mathbf{F}_t \hat{\mathbf{x}}_{t-1|t-1} + \mathbf{B}_t \mathbf{a}_t \tag{2.3}$$

The covariance of this prediction is $\mathbf{P}_{t|t-1} = E[(\mathbf{x}_t - \mathbf{x}_{t|t-1})(\mathbf{x}_t - \mathbf{x}_{t|t-1})^T]$, which is computed using (2.4)

$$\mathbf{P}_{t|t-1} = \mathbf{F}_t \mathbf{P}_{t-1|t-1} \mathbf{F}_t^T + \mathbf{M}_t \mathbf{Q}_t \mathbf{M}_t^T \tag{2.4}$$

The estimate of the observation at the current time step can be obtained using the predicted state variable. This is known as innovation and is given by (2.5)

$$\tilde{\mathbf{y}}_t = \mathbf{y}_t - \mathbf{A}_t \hat{\mathbf{x}}_{t|t-1} \tag{2.5}$$

The covariance of this innovation is given by (2.6)

$$\mathbf{S}_t = \mathbf{A}_t \mathbf{P}_{t|t-1} \mathbf{A}_t^{\mathrm{T}} + \mathbf{R}_t \tag{2.6}$$

The (2.5) representing the innovations indicates the new information. Several factors can contribute towards innovations. For example random fluctuations in observation values, changes in underlying states or error in previous error estimates. As this new information is available, the state and its covariance can be updated at a given time step t. This improves and refines the estimate with the knowledge of the current observation and thus is known and posteriori state estimate. The Kalman gain $\mathbf{K}_t$, optimally adjusts the state estimates in order to reflect the new information. Kalman gain is given by (2.7) which is calculated from covariance of innovations from (2.6) and predicted state error covariance $\mathbf{P}_{t|t-1}$ from (2.4).

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{A}_t^{\mathrm{T}} \mathbf{S}_t^{-1} \tag{2.7}$$

Thus, the state $\hat{\mathbf{x}}_{t|t}$ and its covariance $\mathbf{P}_{t|t}$ can be updated using (2.8) and (2.9) respectively.

$$\hat{\mathbf{x}}_{t|t} = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t \tilde{\mathbf{y}}_t \tag{2.8}$$

$$\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t \mathbf{A}_t)\mathbf{P}_{t|t-1} \tag{2.9}$$

Therefore, this recursive nature of Kalman filter allows the model to update the conditional mean and covariance estimates of the states at time t based on the sole estimate obtained at time $t-1$. Even though it takes into account the entire history, expanding memory is not required and hence, the algorithm is very efficient. The Kalman Filter algorithm is summarized in the flowchart depicted in the figure 2.1.

2.2.2 Maximum Likelihood Parameter estimation

The model matrices $\mathbf{F}_t$, $\mathbf{B}_t$, and $\mathbf{A}_t$ can be constant or vary over time along with the covariance matrices $\mathbf{Q}_t$ and $\mathbf{R}_t$. These parameters are known as the hyper-parameters of the model, which when not known can be estimated through Kalman recursion from the available observations. When these hyper-parameters are known, Kalman Filter derives the states and its covariance recursively. Therefore, the model gives the prediction of the observations and residuals can be computed conditional on hyper-parameter values. A maximization algorithm can be used to maximize the likelihood of the observed values as described in [34].

Here the maximum likelihood method implies the process of finding the estimate of the unknown parameter vector $\boldsymbol{\psi}$ in order to maximize the likelihood of generating the actual observed data. Given the sample of observations $\mathbf{y}_t$, finding a solution for $\boldsymbol{\psi}$ which maximizes the joint density probability function $\boldsymbol{L(y, \psi)}$ is the maximum likelihood estimation of $\boldsymbol{\psi}$.

If the disturbances and initial state vector in the state space model have proper multivariate normal distributions, the distribution of $\mathbf{y}_t$ conditional on $\mathbf{Y}_{t-1}$ is itself normal, where $\mathbf{Y}_{t-1} = \{\mathbf{y}_{t-1}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_1\}$. Furthermore, the mean and covariance matrix of this conditional distribution are given directly by Kalman Filter. Kalman filter provides an efficient way to evaluate the likelihood function of the data for estimation. The parameters of the model can be estimated by maximizing this likelihood function. Therefore the likelihood

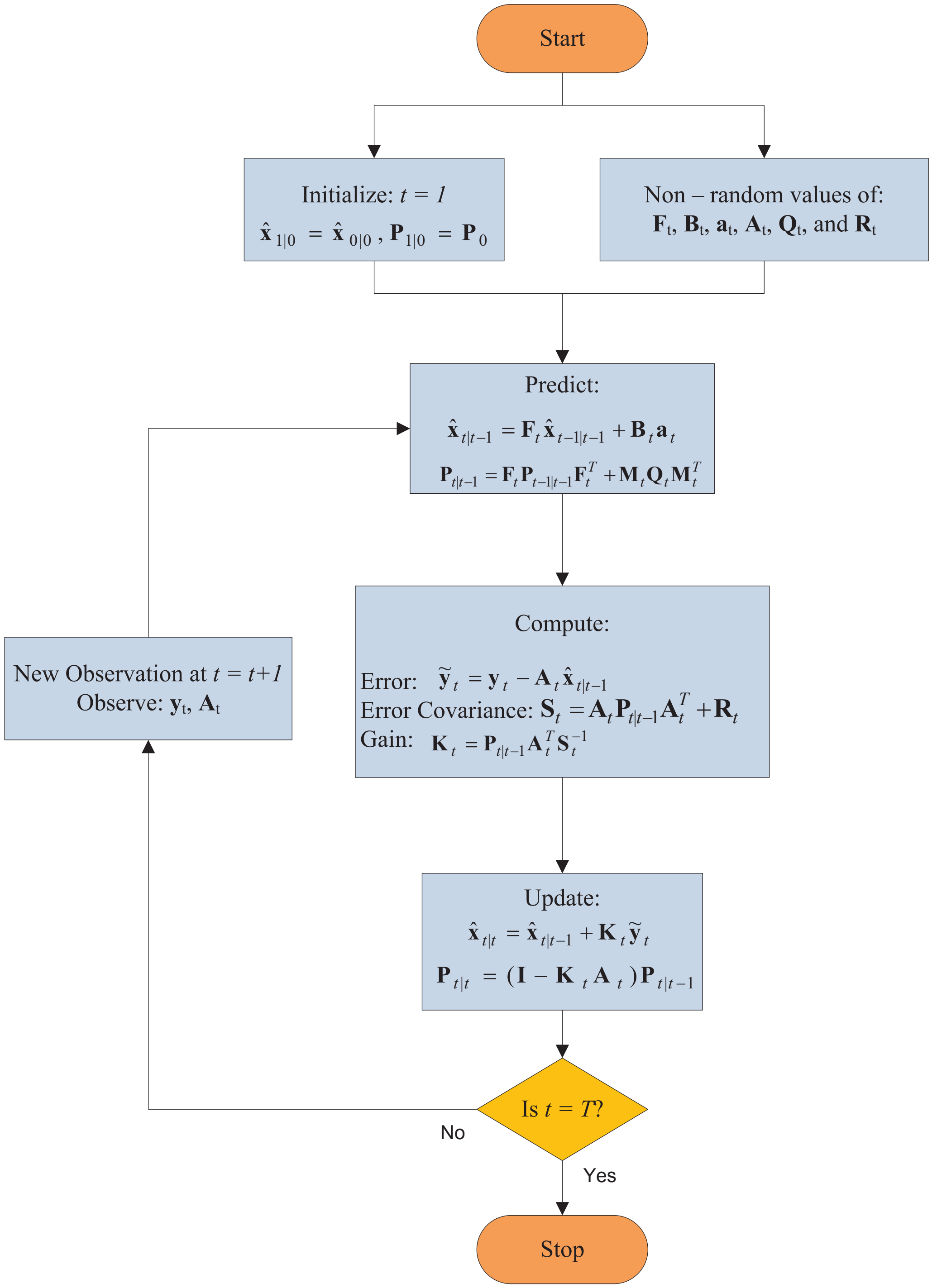

Figure 2.1: Flowchart for Kalman filter algorithm. This is used to estimate time-varying beta when beta follows traditional models like random walk and mean-reverting models

function under normality assumption is given by,

$$\boldsymbol{L}(\boldsymbol{y};\boldsymbol{\psi}) = \prod_{t=1}^{T} p(\mathbf{y}_t|\mathbf{Y}_{t-1}) \tag{2.10}$$

where, $\mathbf{y}_t|\mathbf{Y}_{t-1} \sim N(\mathbf{A}_t\hat{\mathbf{x}}_{t|t}, \mathbf{A}_t\mathbf{P}_{t|t-1}\mathbf{A}_t^{\mathrm{T}} + \mathbf{R}_t)$ and $\boldsymbol{\psi}$ represents the hyper parameters of the model.

Thus, the log likelihood function is given by:

$$\log \boldsymbol{L}(\boldsymbol{\psi}|\boldsymbol{Y_t}) = -\frac{T}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log \mathbf{S}_t - \frac{1}{2}\sum_{t=1}^{T}\tilde{\mathbf{y}}_t^T\mathbf{S}_t^{-1}\tilde{\mathbf{y}}_t \tag{2.11}$$

The maximum likelihood estimate of $\hat{\boldsymbol{\psi}}$ of the parameter vector $\boldsymbol{\psi}$ is the value that maximizes the log-likelihood function (2.11). The maximization equation is:

$$\hat{\boldsymbol{\psi}} = \arg\max_{\psi} \log \boldsymbol{L}(\boldsymbol{\psi}|\boldsymbol{Y_t}) \tag{2.12}$$

This results in the maximization of the log likelihood function (2.11) with respect to the given parameters $\boldsymbol{\psi}$. The equation (2.11) is a non linear function of the hyper parameters and it does not have analytical solution. Thus, numerical optimization procedure needs to be applied. One of the processes of maximization is to first make an initial guess to the numerical values of the unknown parameters. Using these initial numerical values, iteration can be done for the Kalman filter equations for $t = 1, 2, \ldots, T$. The resulting conditional state estimates and its conditional covariance from these iterations can then be used in (2.11) to calculate the value for the log-likelihood function that results from these initial parameter values. Then numerical optimization algorithms can be used to make better guesses such that these unknown parameters values are maximized. The hyper parameters estimation process can be summarized in the flowchart depicted in figure 2.2.

2.3 Kalman Filtered Conditional Betas

The familiarity of Kalman filter enables us to model the time varying systematic risk of an asset or a portfolio using time series models such that its movements will get reflected in

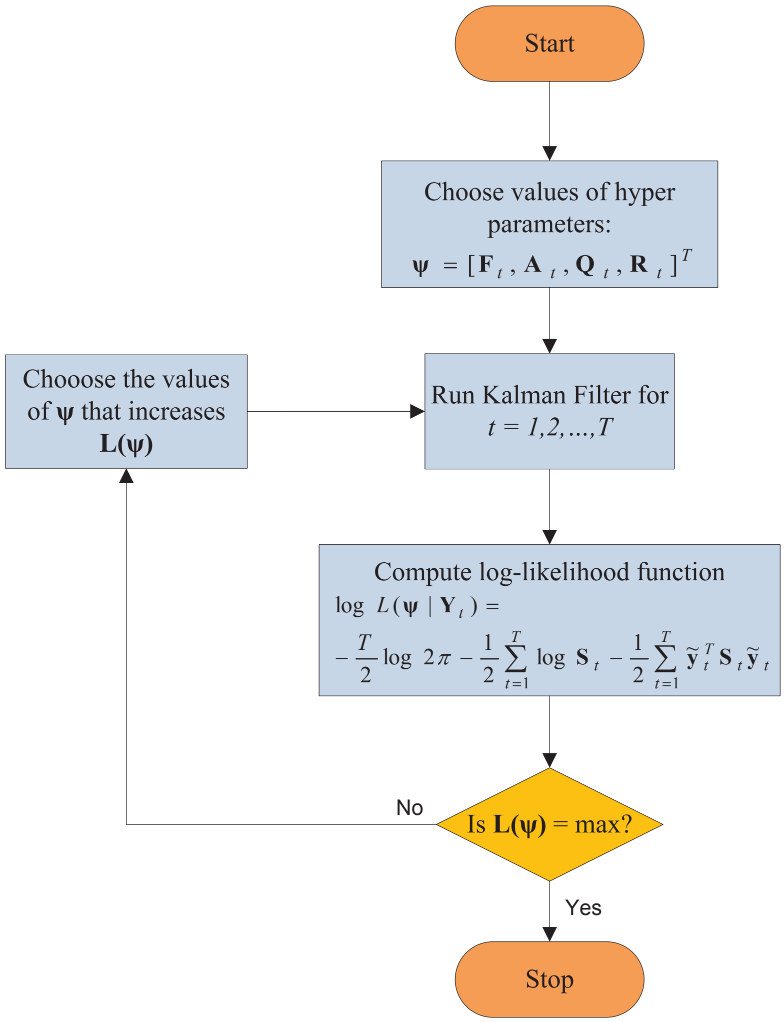

Figure 2.2: Flowchart for the estimation of hyper parameters of state space model using maximum likelihood

the returns on asset/portfolio. The state space models is used to incorporate unobserved variables and estimate them along with the observable model. Thus, time-varying structure of systematic risk can be achieved simply by letting CAPM beta to vary through time. So the unconditional betas measured using OLS becomes conditional on time when measured through Kalman Filter. Various assumption about the evolution of beta can be made when beta is allowed to vary with time and described using state space model for example [18], [20], [11], [19] and so on.

2.3.1 State Space Model Representation of Beta

The random walk model in (1.2) can be directly transformed into state space so that beta can be estimated using Kalman filter. The following assignments is used in general state equation (2.1) to cast it into (1.2):

$$\begin{aligned}
\mathbf{x}_t &= \beta_t \\
\mathbf{F}_t &= 1 \\
\mathbf{B}_t &= 0 \\
\mathbf{a}_t &= 0 \\
\mathbf{M}_t &= 1 \\
\mathbf{v}_t &= \zeta_t
\end{aligned}$$

In order to cast the mean reverting model represented by (1.3) into a general state equation in (2.1) the variables in (2.1) can be substituted as:

$$\begin{aligned}
\mathbf{x}_t &= \beta_t \\
\mathbf{F}_t &= B \\
\mathbf{B}_t &= (1 - B) \\
\mathbf{a}_t &= \overline{\beta} \\
\mathbf{M}_t &= 1 \\
\mathbf{v}_t &= \zeta_t
\end{aligned}$$

Therefore, when beta is represented in these state space forms, Kalman filter steps described in section 2.2 can be directly applied to estimate its value at each time step.

2.4 Chapter Summary

In this chapter, we reviewed the Kalman filter and also discussed an algorithm to estimate parameters of the stochastic state space model using maximum likelihood approach using Kalman filter. We also showed how general Kalman filter in its vector form can be transformed to represent conditional beta to apply Kalman filter recursion. Using the methods discussed in this chapter as preliminaries, we track the novel piecewise constant model for beta in chapter 4.

Chapter 3

Analysis of Piecewise Constancy in Beta

In this chapter, we apply OLS regression technique to identify significant changes in beta and examine if any events are associated with such changes. We develop a methodology to identify the number of significant events in the entire observation period. With this methodology, we show that length of observation sample used for estimation directly affects the number of significant events.

It is shown that, the adjacent period beta estimated at every time instant is not significantly different at all times. The significant changes in beta are triggered by significant events. As these events occur intermittently, they affect the investor's expectations about the future earnings of a company. Change in beta is the result of investors updating their expectation about the future earnings of the company. This expectation gets updated on the release of a significant news. In market, a news about one company can have partial impact on its competitors. It may also impact the companies in the same industry. So, in this chapter we not only look at beta of a single company but also its competitors and leading companies in the same industry. We also consider a single specific event and examine change in beta when that event occurs multiple times in a company at different periods. We consider several case studies in order to explain change in beta.

3.1 Pre and Post Time Instant Regression

We examine at any time instant, if there is a significant change in beta. This time instant is considered as a center instant. We require to estimate beta before and after this center instant and compare if these estimated betas are significantly different. This comparison is performed using statistical hypothesis test. Beta estimated before the center instant can be considered as the pre- instant beta and after this period as the post- instant beta. In this test, the null hypothesis is the beta between these periods are equal. Thus, when significant change in beta is encountered, we can examine if there are any firm specific information flows or occurrence of any economically compelling evidence that could have caused this change. Our method specifically involves statistical hypothesis test to compare pre- and post- instant beta estimates. With this method, we also approximate the number of such instants when pre and post beta estimates are significantly different when different period lengths are used for estimation. We also focus on specific event and how the variation in beta can be explained using economic information when this event occurs at different instants.

3.1.1 Estimation

Following [3], [13], [22], [25], we estimate the value of systematic risk, β, using OLS regression on the market model. The market model is the proxy version of CAPM given by (1.1). In realization, when β is to be estimated using regression, the expected values of return and risk premium are measured using their realized counterparts. Therefore, CAPM can be transformed into market model represented by (3.1).

$$R_{e,t} - R_f = \alpha_e + \beta_e(R_{m,t} - R_f) + \epsilon_t \tag{3.1}$$

where subscript t indicates time index for the time series regression, β_e is the systematic risk for an asset e, α_e represents any returns that are not explained by the model and is the constant term in regression, and ϵ is an error term in the regression and can be considered as the component of the excess return attributed to idiosyncrasies. The error term is assumed to have normal distribution with zero mean following $\epsilon \sim N(0, \sigma_\epsilon^2)$.

Let τ represent the date (or time instant) for which we need to determine if a significant change in beta occurs. We compare β, estimated through the market model (3.1), between two time intervals, the pre- and post-τ intervals. This τ can be located anywhere in the observation time series. So the entire observation series needs to be scanned, making the procedure a rolling regression. However, performing rolling regression produces averaging effect in the observations period and also introduces a time resolution problem. This problem is associated with the length of the observation window. Using shorter observation window increases the chance of measurement error while if the observation window is too long it will be difficult to distinguish the estimate from the average hence missing out the exact instant of change. Thus, to alleviate this problem, we use multiple observation window lengths ranging from 20 to 120 days as the pre- and post-τ intervals. Thus, performing estimation in multiscale manner. Another advantage of estimating beta using multiple observation period length is that it helps to interpret the change in beta as short lived or long term. If a significant change in beta is observed using shorter observation window we can interpret it to be short term change and if it is observed using longer observation window it can be interpreted as a long term change in beta.

3.1.2 Hypothesis Test

The estimated pre- and post- instant betas using different observation lengths need to be compared to examine if they are significantly different. The two intervals considered can be viewed as the sub-samples of the population and we can assume the homogeneity in the error distribution. Therefore, the regression model given by (3.1) is fitted across the two sub samples and it is required to be checked if the effect of the explanatory variable as expressed by the model (3.1) is same across the two sub samples. This can be performed by following hypothesis test:

$$\begin{aligned} H_0 &: \beta_{e,pre} = \beta_{e,post} \\ H_a &: \beta_{e,pre} \neq \beta_{e,post} \end{aligned} \tag{3.2}$$

where $\beta_{e,pre}$ and $\beta_{e,post}$ represent pre- and post-τ betas for security e, respectively. If null hypothesis is rejected in favor of the alternate hypothesis, it is considered that β between two intervals changes significantly. The date where a significant change in beta is identified is called the significant date.

The test such as F-test can be used to compare the equality of the regression coefficients. The methodology to collectively compare and test the equality of the regression coefficients is described in [35]. According to this procedure, the model under the null hypothesis, also known as the restricted model can be constructed. Then using sum of square errors for restricted and unrestricted model a test statistic can be obtained which follows an F-distribution. This test statistic is given by:

$$\frac{(SSE)_{H0} - SSE}{\frac{SSE}{N_1+N_2-(2k+2)}} \sim F(1, N_1 + N_2 - (2k + 2)) \tag{3.3}$$

where, SSE is the sum of square error of the model, SSE_{H0} is the sum of square error of the restricted model, N_1 and N_2 are the number of observations, k represents the number of explanatory variables in regression and $F(x, y)$ represents an F-distribution with parameters x and y. The p-value can then be obtained from the test statistic. Assuming that the null hypothesis is true, p-value indicates the probability of obtaining the test statistic at least as the one that was actually observed. When p-value is lesser than certain critical level, the hypothesis is rejected.

We consider 20 days as a unit interval as it is the minimum number of samples taken for estimation to determine a significant date. However this hypothesis is also tested with 40 to 120 days interval period, thus, resulting in multiscale hypothesis testing.

3.2 Multiscale Dual Window Search

As discussed in section 3.1, a thorough scan of the observation data series is required to determine significant dates. Here we describe the scanning procedure. The number of significant changes in firms' beta depends on the number of such dates identified in the given observation period.

As τ represent a date on which pre- and post-τ betas are measured, it is required to be determined whether τ is an event or significant date. If w_u is the unit period window size during which beta is assumed to remain unchanged and if W is any integer multiple of w_u such that $W = Mw_u$, where M is an integer, then in a time series samples with time range $1, 2, \ldots, T$ where T is the total number of samples, τ can only fall within $W+1 \leq \tau \leq T-W$ when the regression window with size W is used. The total number of significant dates can then be $n_c = T - 2W$.

At every time instant τ, we calculate $\beta_{e,pre}(\tau)$ and $\beta_{e,post}(\tau)$ over the windows { $(\tau - W)$ to $\tau - 1$ } and $\{(\tau+1)$ to $\tau + W$ } respectively. Therefore,

$$\beta_{e,pre}(\tau) = f_{regress}([r_{m,\tau-W}, ...r_{m,\tau-1}]^T, [r_{e,\tau-W}, ..., r_{e,\tau-1}]^T) \tag{3.4}$$

$$\beta_{e,post}(\tau) = f_{regress}([r_{m,\tau+1}, ...r_{m,\tau+W}]^T, [r_{e,\tau+1}, ..., r_{e,\tau+W}]^T) \tag{3.5}$$

Where, $f_{regress}(\mathbf{x}, \mathbf{y})$ is a function that performs regression on two vectors $\mathbf{x}$ and $\mathbf{y}$ to produce the coefficient β. Note that $r_{m,t}$ represents market premium at time t given by ($R_{m,t}$-R_f) and $r_{e,t}$ represents excess return of security e at time t given by $(R_{e,t} - R_f)$. Equations (3.4) and (3.5) give the pre- and post- τ beta estimation. The beta coefficients obtained from these equations are then compared using the hypothesis test in equation (3.6). The test function is defined as,

$$rej(\tau) = f_{test}(\beta_{e,pre}(\tau), \beta_{e,post}(\tau)) \tag{3.6}$$

Where, $f_{test}(x, y)$ is a function that uses values from equation (3.4) and (3.5) and produces value 0 if it fails to reject the null hypothesis and 1 if null hypothesis is rejected. The pre- and post-τ betas are separately estimated and compared for each values of W independently.

When $rej(\tau) = 1$ for a given time-scale (window size), τ is considered as significant date for that scale. However, because of the time resolution problem, multiple rejections can occur in the nearby region of the actual τ. In order to alleviate this, we apply a filter of length λ such that any τ rejected within this observation window is considered in a single τ. A distinct length for λ is selected for each window length. For our experiment, we adopt length of λ as the size of observation window length by taking into consideration the length of overlapping periods.

Thus, multiscale exhaustive search is performed within two non overlapping windows simultaneously by moving both these windows along the observation time series with step size one. The first window corresponds to the pre- instant interval and the second window corresponds to the post- instant interval. On each step we compare the two estimated beta coefficients using statistical hypothesis test. We use $w_u = 20$ and W is varied from 20 to 120 corresponding to the value of M varying from 1 to 6.

The use of filter in our dual rolling window exhaustive search is specifically for the purpose of counting the number of significant dates occurred in different scales by isolating these dates. The reason behind it is - when non overlapping windows are used and both of them are moved simultaneously along the time series there will be an overlapping observations between the two consecutive estimations. As a result of this overlap, if two pre- and post- instant betas are significantly different at one date, the betas estimated on the date immediately following this is/are also likely to be significantly different. This causes identification of event dates in the surrounding nearby regions as well. This is an inherent time resolution problem in rolling window estimations. The identified event date could be due to the collective effect of events within the window or due to a single event. Therefore, in the count of significant dates, we do not include any significant dates which have been identified within the size of the window from previously identified date. Thus, length of the observation window as a length of the filter is reasonable for both unit period window and its multiples since shorter window has smaller overlap and longer window has larger overlaps. Hence making the filter length dependent on observation window length is justifiable. The flowchart of dual rolling window exhaustive search is depicted in figure 3.1.

3.3 Data

Data used in this study employs daily stock returns from CRSP (Center for Research in Security Prices) database. Excess return is calculated from daily return by deducting risk free rate. To proxy the daily risk free rate, we use 1 month treasury bill return which is simple daily rate that over the number of trading days in a month compounds to 1-month

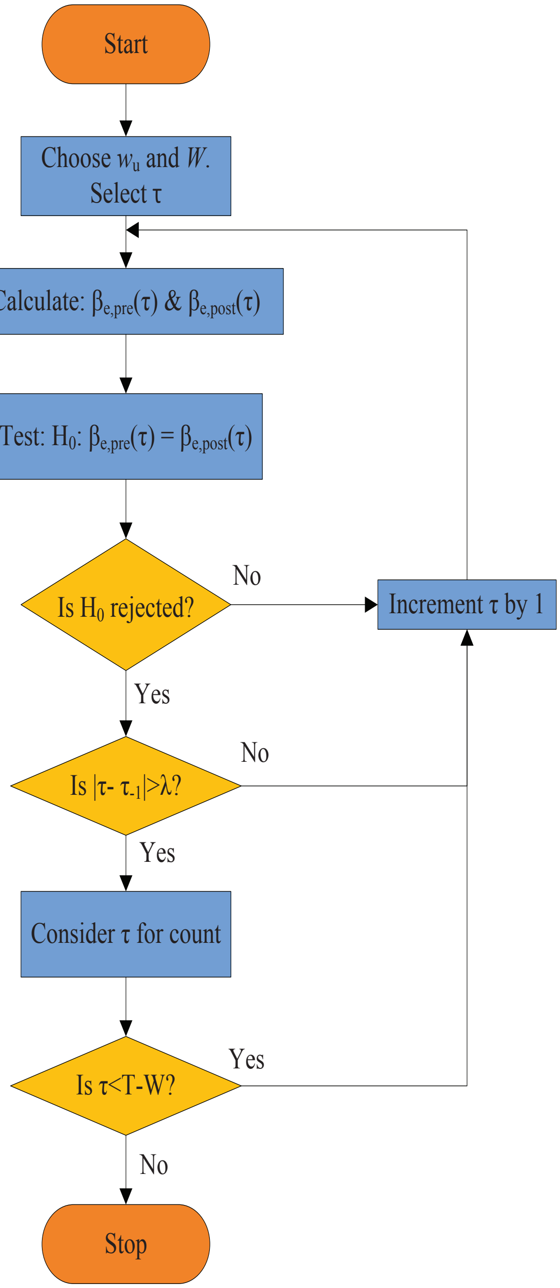

Figure 3.1: Flowchart for dual window exhaustive search

Table 3.1: Companies considered under technology sector

Company	Ticker	SIC Code	SIC Industry Category
Apple Inc.	AAPL	3571	Computers
IBM Corp.	IBM	3571	Cmputers
Hewlett Packard	HPQ	3571	Computers
Western Digital Corp.	WDC	3572	Computers
Micron Technology	MU	3674	Computers
Yahoo Inc.	YHOO	7375	Computer Software
Google Inc.	GOOG	7375	Computer Software
Microsoft Corp.	MSFT	7370	Computer Software
Research In Motion	RIMM	3661	Electronic Equipment

T-bill rate. The T-bill rates are obtained from K. French data library[1]. To proxy the market returns, the value weighted returns adjusted for dividend and splits for S&P 500 from CRSP database are used. S&P 500 to proxy the market index has also been used in [5], [13], [26] and so on. Daily risk premium is obtained by deducting the simple daily rate of 1-month T-bill rate which is also obtained from K.French data library.

Our study includes 9 technology stocks which belong to several SIC (Standard Industrial Classification) industries. These companies include Apple Inc., IBM Corp., Hewlett Packard, Western Digital Corp., Micron Technology, Yahoo Inc., Google Inc., Microsoft Corp., and Research in Motion. These stocks are selected as their companies are industry leaders and draw wide attention. Since we also look at industry betas in pre- and post-τ intervals, 4-digit SIC code are obtained from CRSP database for the respective stock and its corresponding industry index returns are obtained from K. French data library. Following [36], we use value weighted returns from all stock in NYSE (New York Stock Exchange), AMEX (American Stock Exchange) and NASDAQ (National Association of Securities Dealers Automated Quotations) and adjust it by deducting simple daily rate of 1-month T-bill rate as a market proxy to estimate industry beta.

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.4 Results and Discussion

We discuss our results in three parts. In section 3.4.1, we present the results of our exhaustive search technique in identifying significant dates for the technology companies listed in table 3.1. In this section, we also discuss the results from the application of filter discussed in section 3.2 to isolate the count of the number of significant events. In section 3.4.2, we compare beta around one type of firm-specific event for a stock in the given observation period history. In section 3.4.3, we discuss five significant dates as use cases and examine changes in beta around these significant dates by considering information and reactions from not only a firm but also its competitors and industry collectively.

3.4.1 Significant Dates

In this study we first use multiscale dual window search without filter and compare pre- and post-τ beta for the companies under study. We use time-varying p-values obtained from the statistical hypothesis test for this purpose. Then filter is applied to isolate and approximate the count of the number of significant events for the stock. The hypothesis is compared at 5 percent and 10 percent significance levels.

Figure 3.2 shows the time series of p-values from the multiscale dual window search without application of the filter for Apple Inc. The window we have used are from 20 to 120 days. The first point on each plot corresponds to the 121^{st} τ. We ignore the estimations before 121^{st} time index in order to align the time series on each scale. The horizontal line for 5% and 10% rejection criteria levels are also marked. It can be observed from the figure that p-values do not go below the rejection criteria levels frequently. Using 20 days window, the p-value change is frequent. The change is smoother when size of the observation window is increased. This shows, when longer observation window is used the difference in beta estimation is not prominent. This is due to the large number of overlapping data in the observation. It can also be observed that, there are certain points for which p-value is below the 5% level using all windows. Example of these points are at time index 526 to 528 and 823 to 834. These points occur intermittently in the observation time series. The existence of these point assures and emphasizes the beta change at this time instant. From these observation for Apple Inc. stock it is clear that significant changes in beta is not very frequent for any window size used for estimation which is an indication of piecewise constancy.

Figure 3.3 shows the time varying p-values for Google Inc. stock for the observation periods from January 1, 2005 to December 31, 2009. Here also we have ignored the estimations before 121^{st} time index in order to align the time series on each scale. The horizontal line for 5% and 10% rejection levels are also indicated. The pattern of results for Google Inc. is different from the pattern of results obtained for Apple Inc. in figure 3.2. Here we find that when the observation window used is 60 and higher, the p-value goes below rejection criteria and stay below this level for considerable number of time instants. This observation is more frequent when higher observation window size is used than lower observation window size. This is because the underlying beta has a large jump magnitude. Rolling window estimation with large observation window length is able to detect it as soon as this jump falls within the observation window. This causes the two adjacent sub-sample beta to be significantly different. Since the step size of the rolling window estimation is one, the two adjacent beta estimation is significantly different as long as the jump lies within the window. For example around time index 50 and with observation window period 60 and above, we can observe significant change in beta. The use of filter described in section 3.2 can isolate and approximate the count of significant time instants. For example, if the filter is applied for observation window 80 in case of Google Inc., the continuous significant dates around time index 100 could only be represented by one significant date.

The p-values for Hewlett Packard Inc. is shown in figure 3.4. In case of HPQ, we find that for time index 150 to 158, the p-values are below 5% level using 20 to 60 days window. The averaging effect of regression makes beta indistinguishable for these time indices when observation length of more than 60 days are used. Also, for time index 206 to 214 and 222, the p-values are below 5% level when 20 to 80 days window observation lengths are used. Such examples are also found at other time indices. It should be noted that regression averages the effect within the observation period. So if longer observation are used the effect due to a single point diminishes unless effect of a single point is large as compared to other points in the period like the case of Google Inc. But, in the case of HPQ, this does not seem to be the case.

The results for MSFT shown in figure 3.5 also conveys the same information. In this case, at time index 820 we find a significant difference in pre and post time instant beta estimation using all observation window lengths as indicated by the p-value lesser than 5% level. Also, it can be observed that at time index 360 the two adjacent beta are different when 20 to 80 days window are used.

Similary, the results for IBM Corp., Micron Technology, Research In Motion, Western Digital Corp. and Yahoo Inc. are depicted in figures 3.6 to 3.10 respectively. All these figures resemble similar pattern discussed. When we look at these figures vertically, we observe that there exists common time indices at which p-values are below significance levels which stresses significant changes in beta. If we look at these figures horizontally we can see the effect of averaging and the magnitude of underlying beta jump. From the time varying p-values, it can be interpreted that magnitude and location of the underlying beta jump plays a key role in its estimation. This occurs especially when the observation window based estimation is used. However, despite the size of the window used it can be observed that time varying p-values do not go below the significance levels frequently. Hence it can be remarked that beta tend to follow piecewise constancy.

The result of the application of filter to isolate the number of significant dates in dual rolling window exhaustive search for the companies listed in table 3.1 is summarized in table 3.2. There are total of 1259 days in our observation period from January 1, 2005 to December 31, 2009. As discussed in section 3.2, the maximum number of significant days can be identified by the dual rolling window exhaustive search is $n_c = T - 2w$, i.e., the maximum numbers of significant days that can be identified using 20 and 120 days windows are 1219 and 1019, respectively. Using a 20 days window, we identify an average of 12 significant dates (with a range of 9 to 21) at 5% significance level and average of 19 significant dates (with

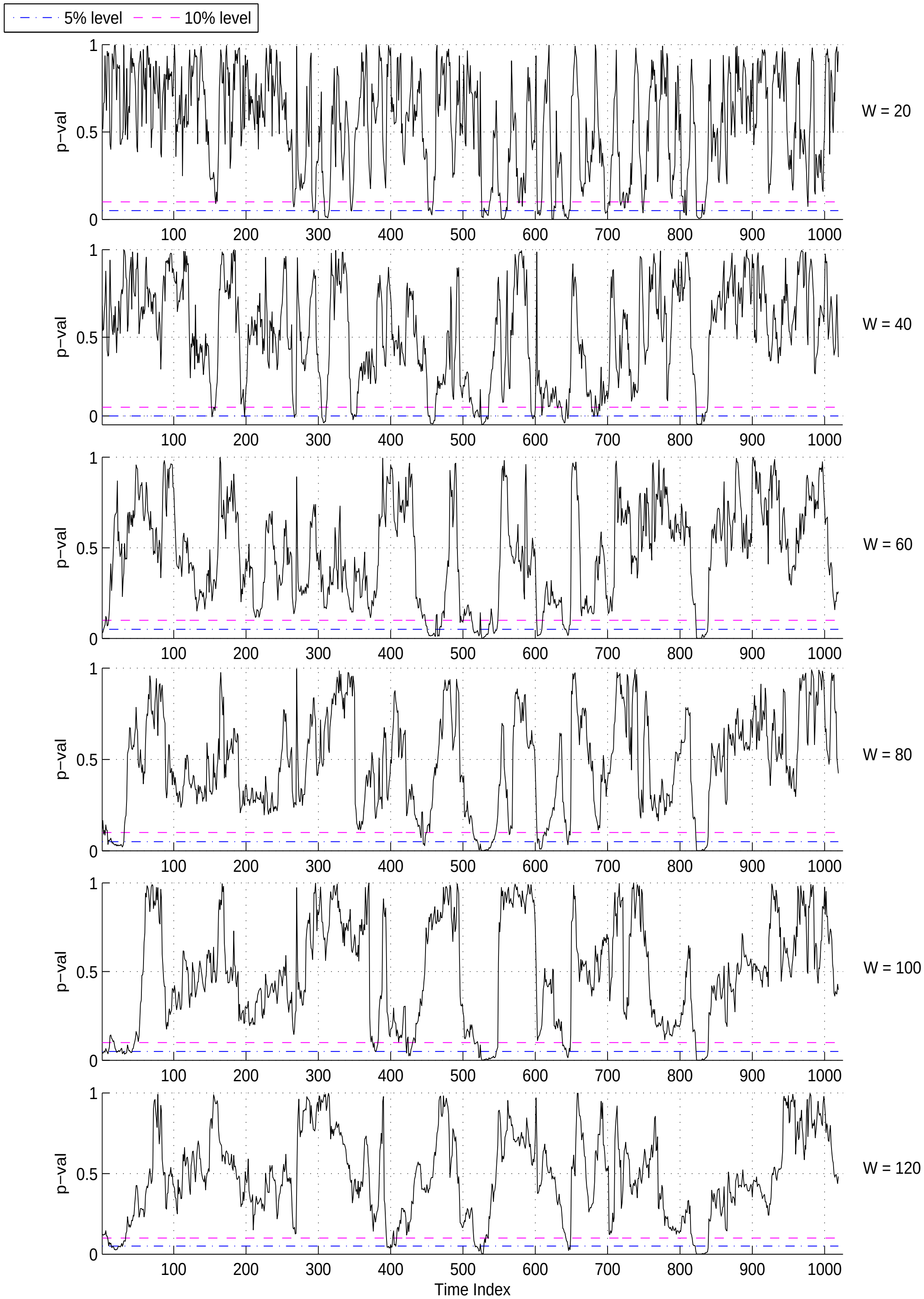

Figure 3.2: Time Series of p-values for AAPL (Apple Inc.)

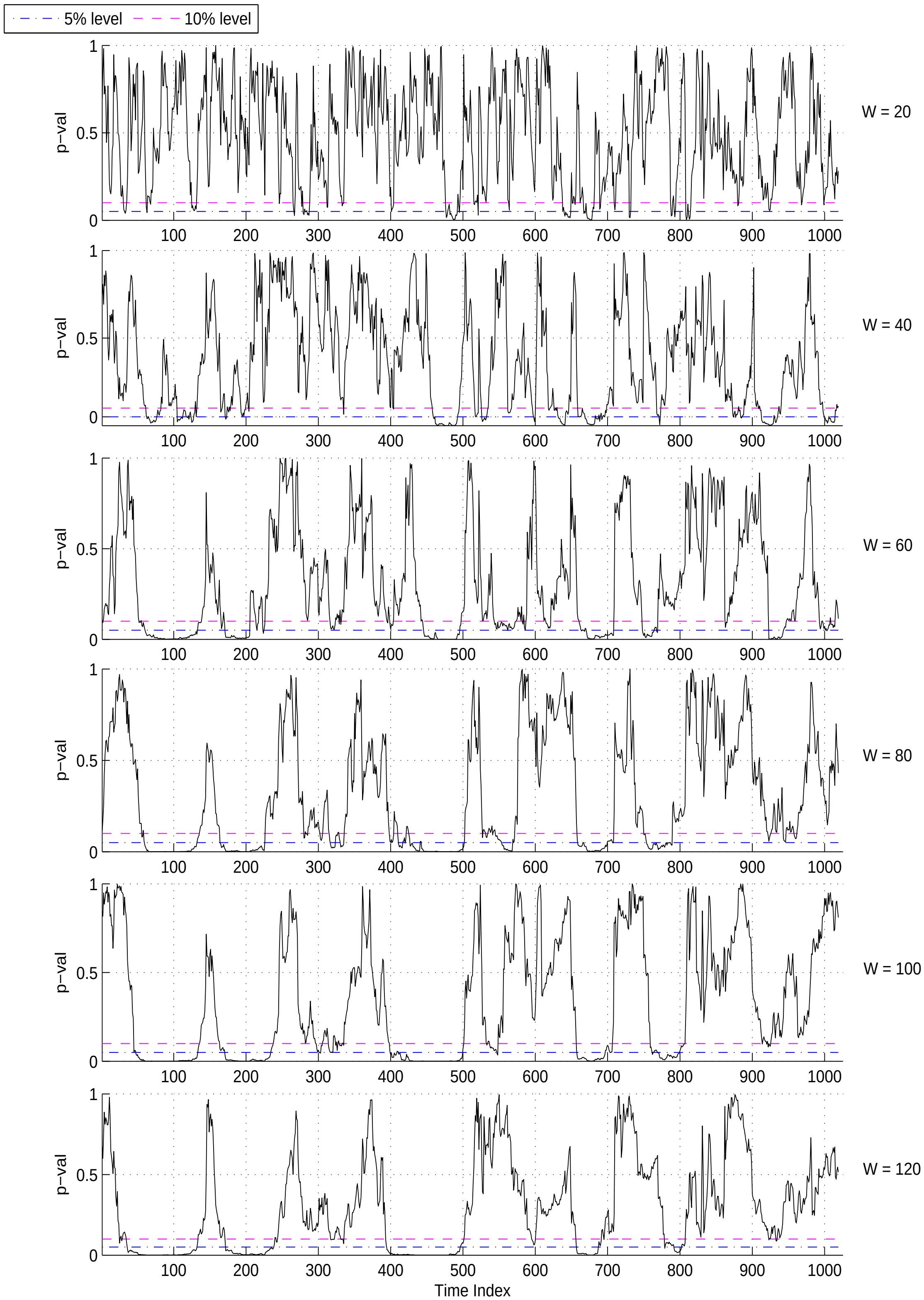

Figure 3.3: Time Series of p-values for GOOG (Google Inc.)

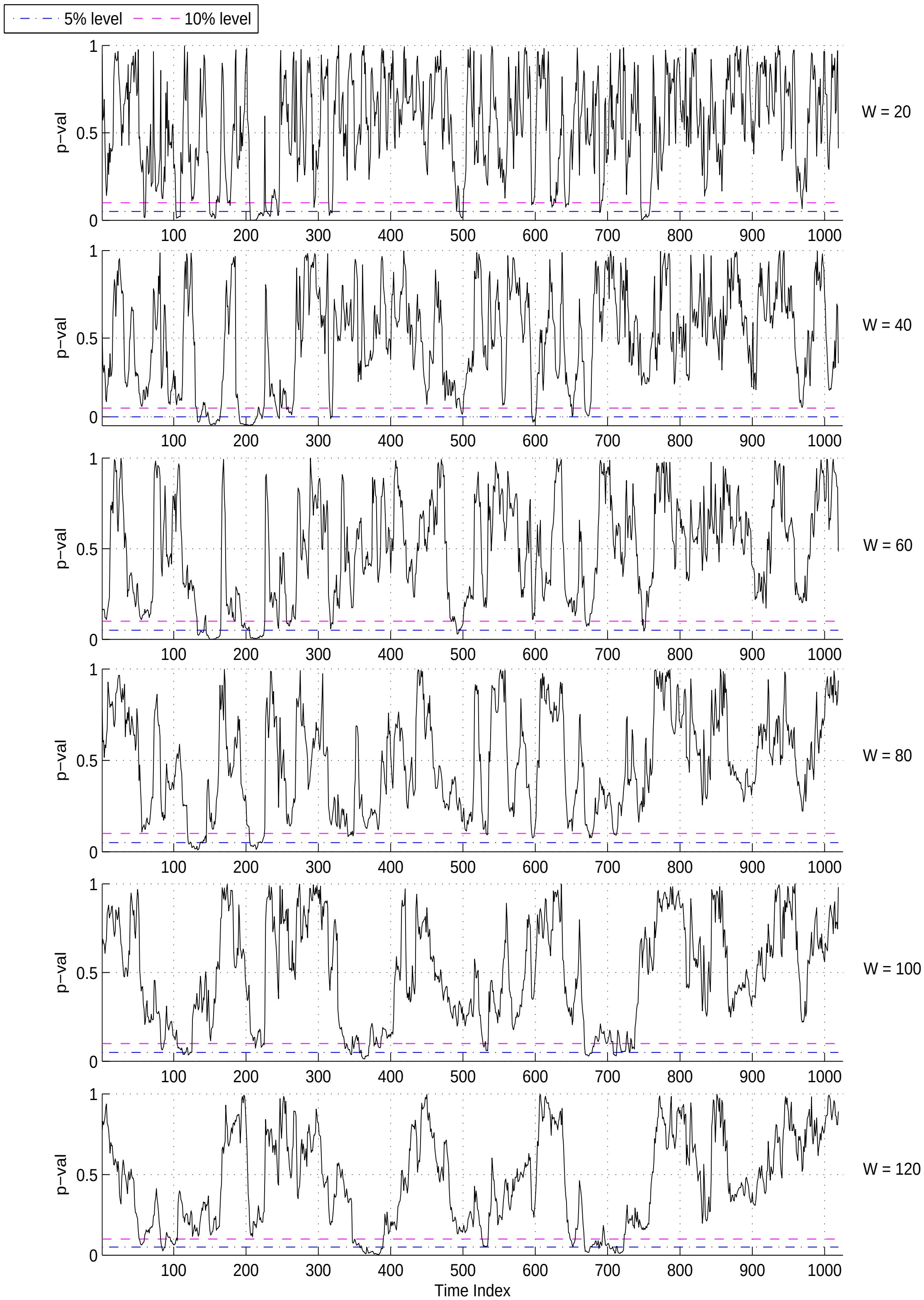

Figure 3.4: Time Series of p-values for HPQ (Hewlett Packard Inc.)

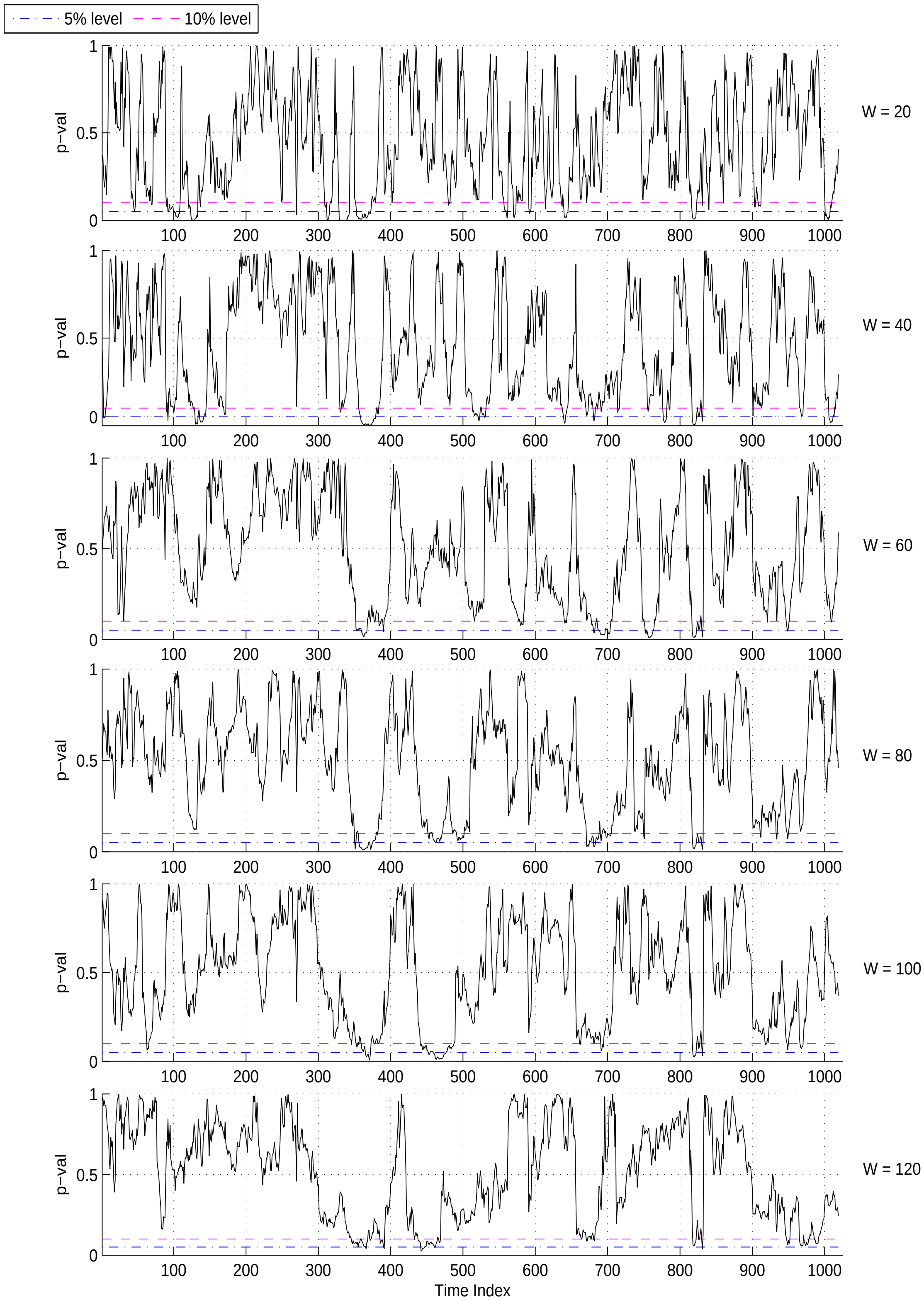

Figure 3.5: Time Series of p-values for MSFT (Microsoft Corp.)

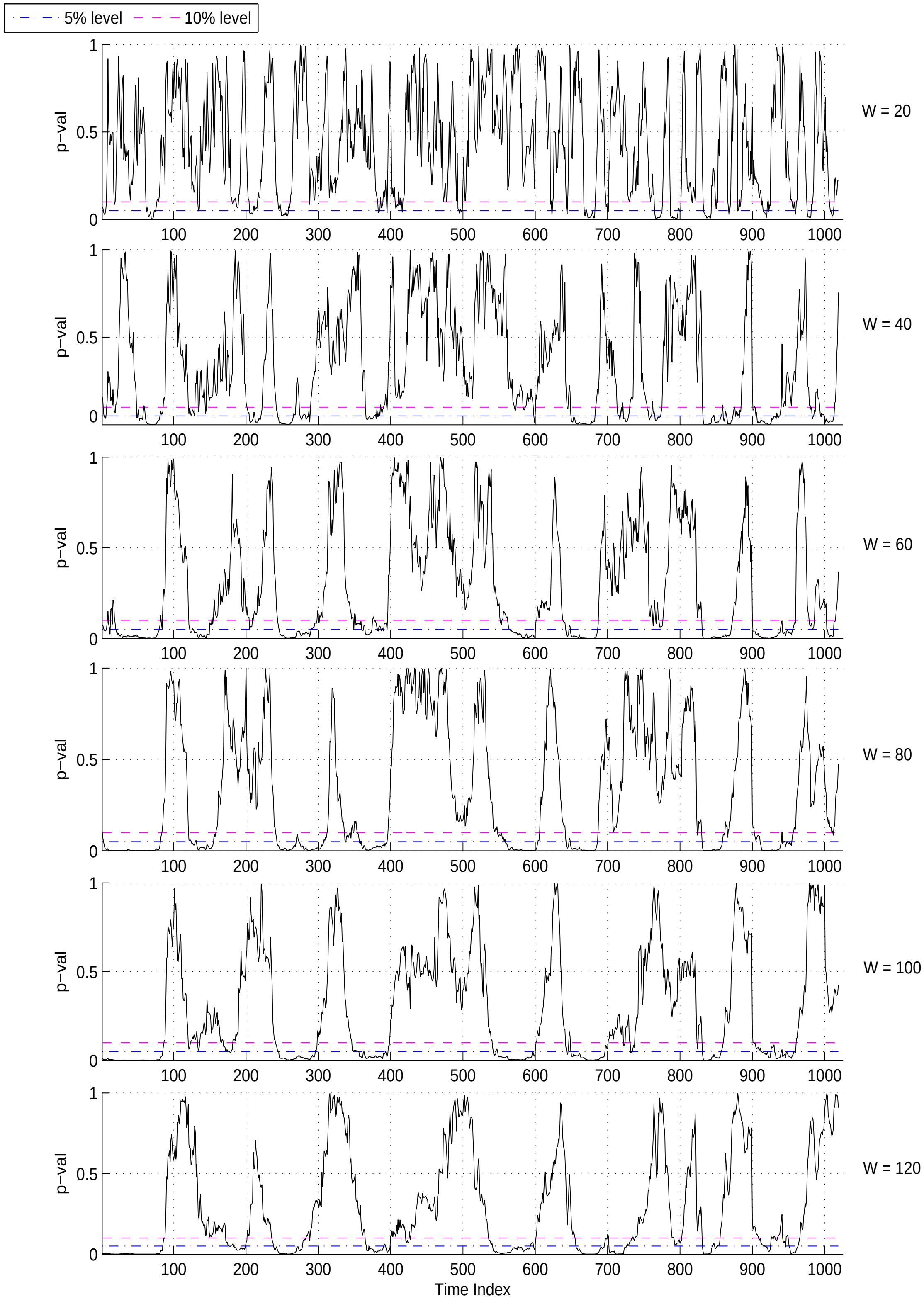

Figure 3.6: Time Series of p-values for IBM (International Business Machine)

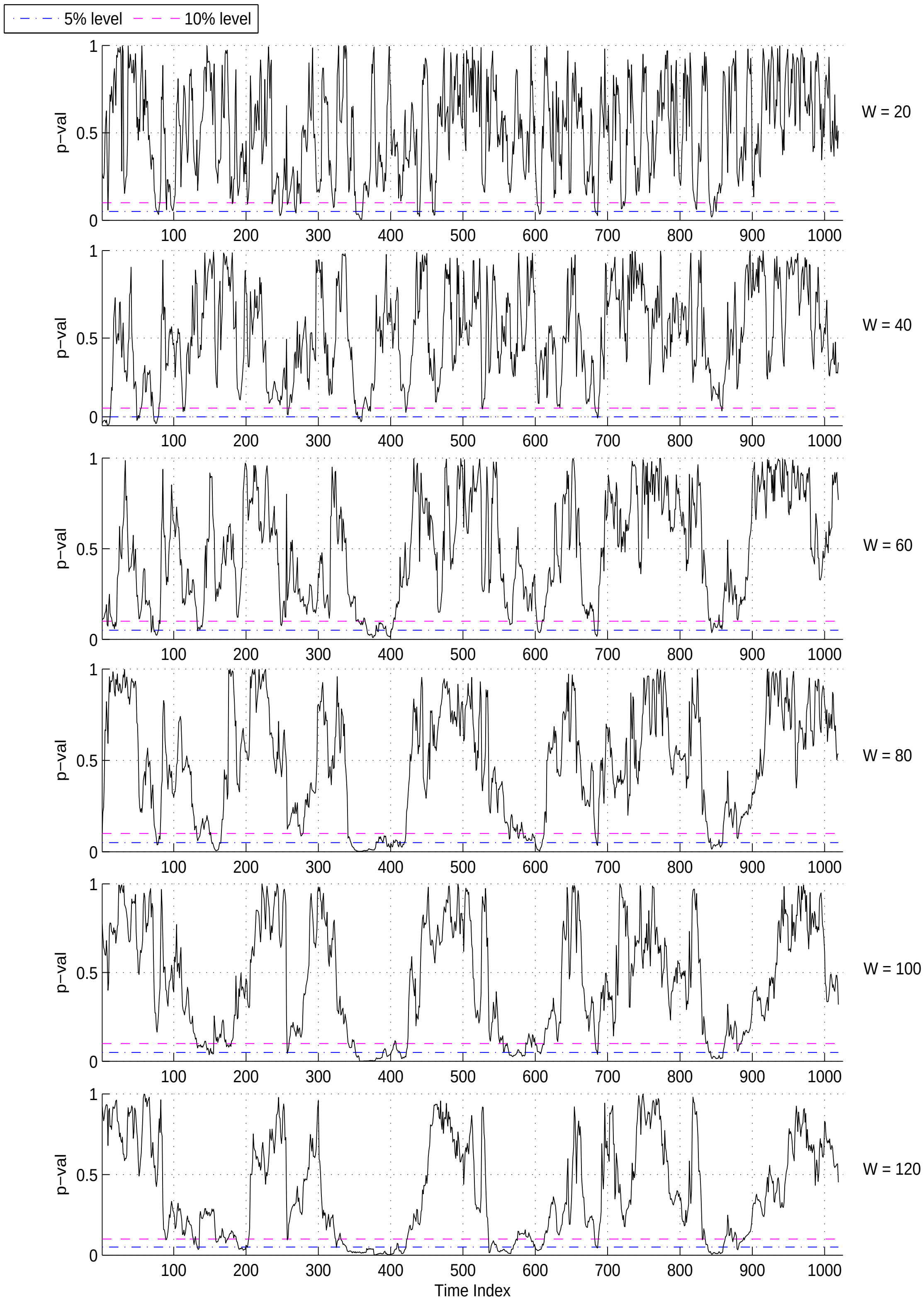

Figure 3.7: Time Series of p-values for MU (Micron Technology)

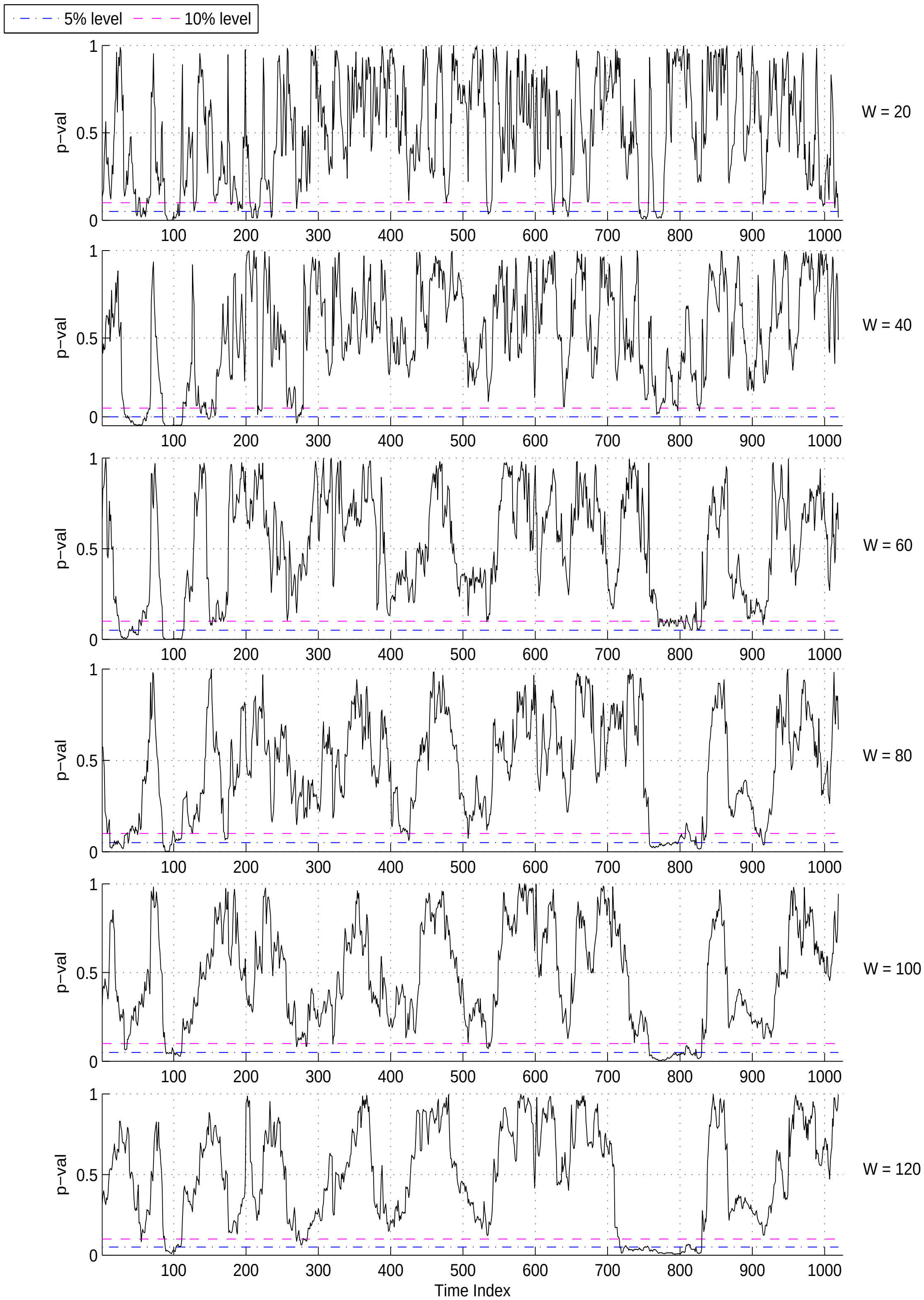

Figure 3.8: Time Series of p-values for RIMM (Research In Motion)

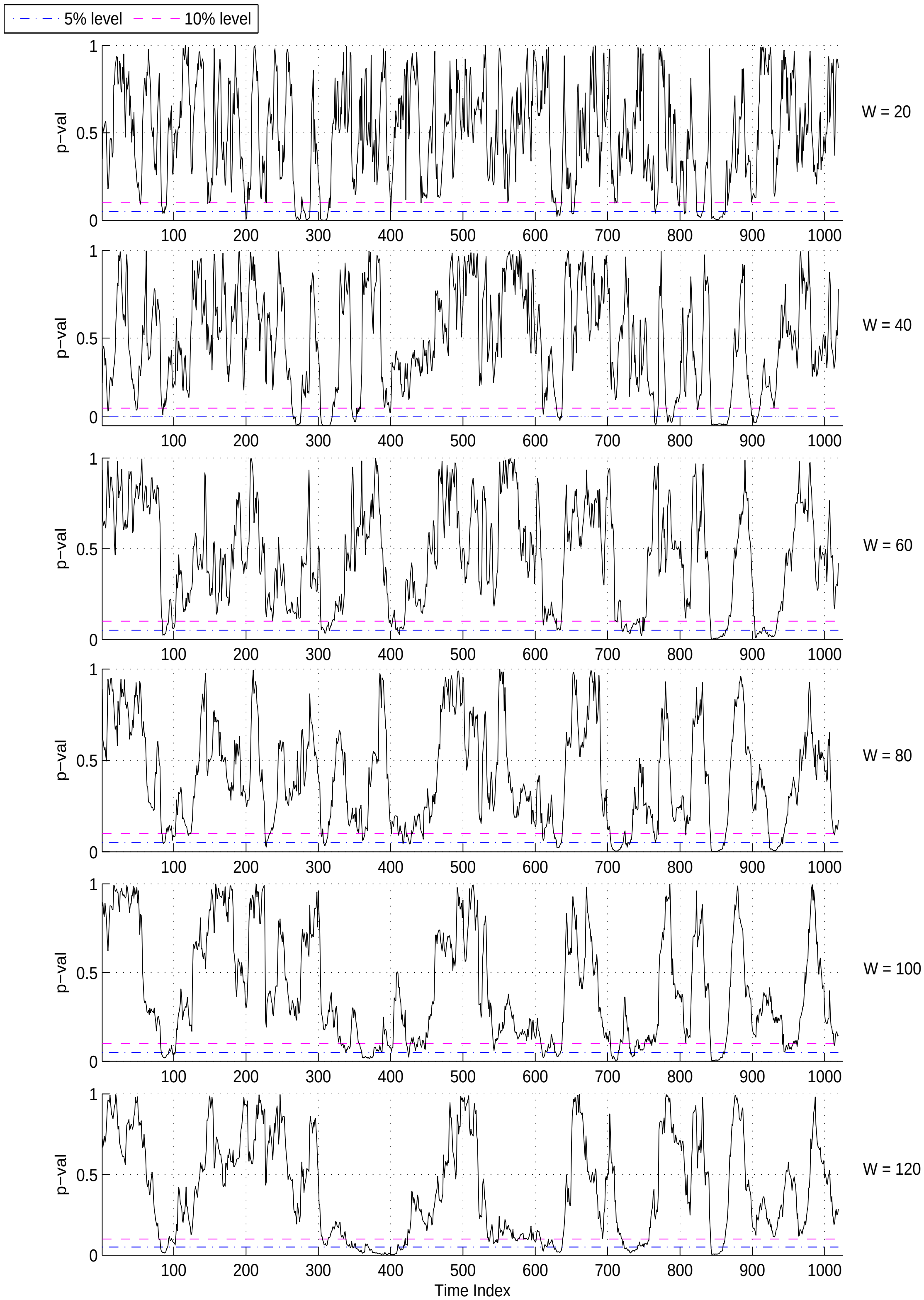

Figure 3.9: Time Series of p-values for WDC (Western Digital Corp.)

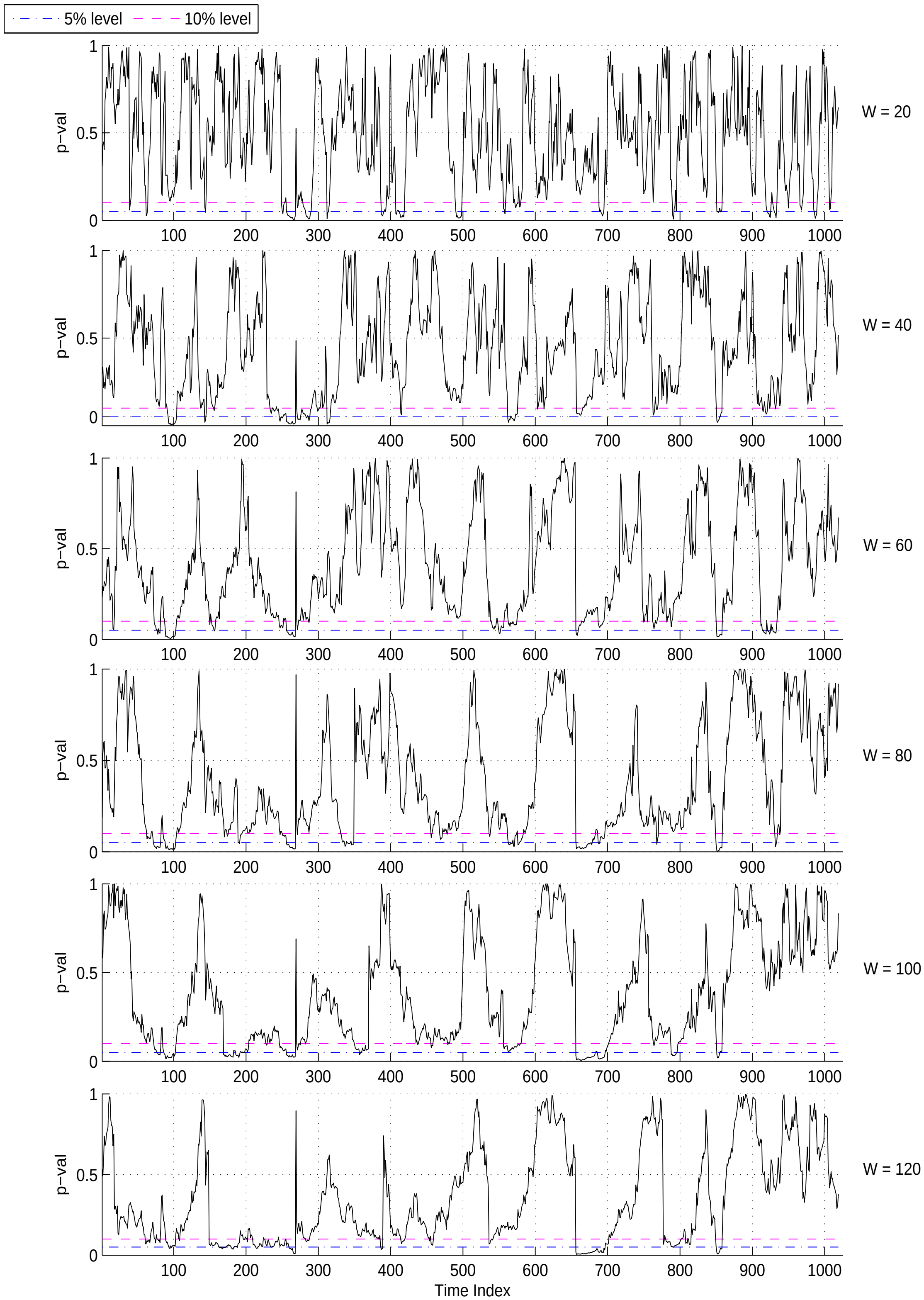

Figure 3.10: Time Series of p-values for YHOO (Yahoo Inc.)

a range of 16 to 24) at 10% significance level for the 9 stocks considered. The number of significant dates identified through the window of 120 days ranges from 2 to 7 with an average of 4 significant dates at 5% significance level and 3 to 7 with an average of 5 significant dates at 10% significant level. This clearly indicates that most of time, variations in beta are not significant and beta can be viewed as constant. Significant changes in beta do happen, but not at high frequency. Daily beta changes as modeled in time-varying models in literature, such as random walk and mean-reverting models does not confirm with this as beta change occurs at every time instant. Beta changes in a piecewise constant pattern. The daily insignificant variations in beta estimate may actually be stock idiosyncratic risk.

Our results also show the effect of window size, as suggested in previous literature [28]. The number of identified significant dates drops when the window size is increased and it is especially prominent for the range of window sizes from 20 to 60 days. Using a shorter window for beta estimation would find more significant dates than when a longer window size is used. It can also be noted that increasing the level of significance although increases the number of identified significant dates, the pattern across observation period lengths are same i.e. with an increase in the observation window length there is a decrease in the number of identified event dates.

The effect of window size partly reflects investors' short-term/immediate reactions. For example, [26] reveal that beta of individual stocks increase on days of quarterly earnings announcements change and revert to their average levels shortly.

3.4.2 Study of Specific Event on Beta Change

In the event based analysis, we consider specific event on which we observe the change in beta. Changes in beta are caused by triggering events and as discussed, the triggering events are of various types. Different from our study, event studies ([29], [27], [26]) focus on a specific type of events, such as stock splits, dividend payments, new product release, news about mergers and acquisition, and study the affect of a type of event to beta.

The observation results of dual rolling window test for the companies in table 3.1, showed

Table 3.2: Number of significant dates for companies listed in table 3.1

Stock Ticker	**Number of Significant Dates for observation window and rejection level**											
	20		**40**		**60**		**80**		**100**		**120**	
	5%	**10%**	**5%**	**10%**	**5%**	**10%**	**5%**	**10%**	**5%**	**10%**	**5%**	**10%**
AAPL	14	19	12	13	5	5	5	5	4	4	5	5
GOOG	11	24	13	15	9	14	7	9	7	8	5	7
HPQ	9	16	5	7	4	7	2	5	3	5	3	4
IBM	21	24	16	15	10	13	8	9	7	7	7	7
MSFT	11	21	11	15	6	10	5	7	3	7	2	5
MU	10	17	4	10	7	9	7	8	6	6	5	6
RIMM	10	17	4	7	3	6	4	7	2	4	2	3
WDC	12	19	7	11	7	8	9	9	6	7	5	5
YHOO	14	18	6	12	7	9	8	10	5	7	5	6
Average	12.44	19.44	8.67	11.67	6.44	9.00	6.11	7.67	4.78	6.11	4.33	5.33

that on an average there are 12 such events using 20-days window at 5% significance level which triggered change in beta. When examining the effect of certain type of events to beta by relating such events to identified significant dates, our results indicate that whether an event will trigger changes in beta depends on whether the event is important enough in changing investor's risk perception. A type of event, such as launch of new product, may or may not change stock systematic risk. These event can only act as a catalyst to the investor's ongoing perception about the company that would increase or decrease company's systematic risk.

We illustrate our result by presenting the relations between identified significant dates and Apple Inc.'s new product release events. Apple Inc. is of particular interest because of its recent popularity, market wide attention, and market leadership with continuous launch of new products. During our study period, January 1, 2005 to December 31, 2009, Apple Inc. has a total of 34 new product releases, which do not include software updates[2].

In our analysis, for each product release date we calculate and compare the pre- and post-τ betas. Significant dates identified in the test using 20, 40 and 60 days windows are reported in table 3.3. Among the 34 new release dates, 7 and 4 significant dates are identified

[2] The list of new releases was collected from Wikipedia (http://en.wikipedia.org/wiki/Timeline_of_Apple_Inc._products) and compared with information from Apple Inc. official website http://www.apple.com/pr/library/

using 20 and 60 days windows respectively. It is clear that not all new release of product launch will trigger changes in beta. Only those deemed important, given the firm's specific market situations and economic conditions, will trigger changes in beta. Also as shown in table 3.3, new product launch may increase or decrease beta. An example of decrease in beta is discussed in our case study section 3.4.3.

Here, we discuss the case when increase in beta is observed on the launch of new product. On August 7, 2007, it is observed from table 3.3 that beta of Apple Inc. is increased which is consistent with 20, 40 and 60 days window. Using 60 days window estimate, beta is increased from 0.61 to 1.48. Looking into the history, apart from the news about the launch of new products by Apple Inc., it is also revealed that on this particular day U.S. jury convicted the former chief executive of Brocade Communications Systems Inc. on trial for options backdating[3]. Along with this news it was also reported that researchers in the field speculated that criminal charges were possible to be laid down against Apple Inc., which has also been under scrutiny for options backdating. This also led to a critical view by the media on Apple Inc.'s executives during the period[4]. This kind of publicity is severe and hampers the reputation of the company in a negative way. Thus, the news about the launch of a product was overshadowed by this publicity which was deemed important during that period. So investors were more cynical about Apple's run thereby reflecting their sentiments as increase in beta. Hence, this result show that even for the same type of event, increase or decrease in beta depends on the position of the company on the particular period.

3.4.3 Case Studies

In this study, we pick specific dates in the company during which we find significant change in beta. We use historical news to relate these changes to any key event that has occurred on that day and examine if such news is a cause that directly or indirectly affected the beta change. On this day considered in a use case, we estimate beta using pre- and post-τ regression described in section 3.1 using 20, 40 and 60 days observation windows. The

[3] http://in.reuters.com/article/2007/08/07/idINIndia-28874620070807

[4] http://business.timesonline.co.uk/tol/business/law/article2221366.ece

Table 3.3: Pre and post instant beta estimates of Apple Inc. on new product launch. (* rejected at 5% level & ** rejected at 10% level)

Date	pre β	post β	p-val
20 days window			
August 7, 2007	0.68	2.00	0.03*
September 5, 2007	1.89	0.43	0.01*
January 8, 2008	1.66	0.38	0.08**
January 15, 2008	2.03	0.12	0.01*
September 9, 2008	0.60	1.51	0.03*
October 14, 2008	1.28	0.65	0.03*
July 30, 2009	0.73	1.38	0.04*
40 days window			
August 7, 2007	0.61	1.53	0.05**
January 8, 2008	1.69	0.60	0.02*
January 15, 2008	1.31	0.58	0.09**
March 17, 2008	0.53	1.34	0.05**
October 14, 2008	1.23	0.80	0.02*
July 30, 2009	0.64	1.21	0.02*
60 days window			
August 7, 2007	0.61	1.48	0.02*
January 8, 2008	1.52	0.92	0.08**
January 15, 2008	1.57	0.86	0.03*
October 14, 2008	1.24	0.80	0.01*

selection of these windows are based on our results from section 3.2 where we find that increasing the observation window greater than 60 does not decrease the number of identified significant changes by considerable amount.

While examining the historical news to identify triggering events of changes in beta, we examine not only firm specific events/news, but also news of competitors and the industry as these news also tend to relate to the firms' performance. For these significant dates found, we are able to identify triggering events that are deemed to change investors' long term risk perception of the firm/stock. These triggering events are of variety of types including events that have been examined in literature, such as stock splits, dividend announcement and quarterly earning announcements as well as events that have not been studied before, such as, new product release, mergers and acquisitions, change of management etc.

Here we discuss five significant dates to illustrate several different types of triggering events. For each significant dates, we provide beta comparison for all firms, and the industry of the key firm in discussion, so the relationships among competitors, partners, and industry can be comprehended.

Significant Date - January 8, 2008

January 8, 2008 is a significant date for AAPL (Apple Inc.) where a change in beta is identified. Table 3.4 summarizes the pre- and post-τ betas, estimated with 20, 40 and 60 days windows, of the 9 stocks in study and of the computer industry (SIC 3571). We also report the average price changes of the pre- and post-τ periods of the 20, 40 and 60 days windows. As shown in table 3.4, AAPL's beta decreased on January 8, 2008. Based on 60 days window, beta decreased from 1.52 to 0.92. It is observed that there is no significant difference between pre- and post-τ betas for the Computer Industry. MSFT and GOOG also show significant changes in beta when estimated with 20 and 40 days window but not with 60 days window. If we look at the average price change between the pre- and post-τ periods, we find that for all companies other than IBM, WDC and YHOO the average price is found to have decreased when 40 and 60 days observation window is used. Therefore, all companies being the competitors and in the same industry shows the similar pattern in prices. Since for all three observation windows AAPL shows a significant beta change, we focus on this firm. When we look into the historical news of Apple Inc., it is found that on this particular day Apple Inc. announced the launch a New Mac Pro which Apple claimed to be the fastest Mac ever with eight processor core standards[5]. Apple Inc. also announced the launch of New Xserve which it claimed to be the most powerful Apple Server ever[6]. These news were released one week before the Macworld show. This show is dedicated to the Apple platforms in which progress about the company and the new product launches are announced. With the launch of new products a week before this show, undoubtedly,

[5] http://www.apple.com/ca/pr/library/2008/01/08Apple-Introduces-New-Mac-Pro.html

[6] http://www.apple.com/ca/pr/library/2008/01/08Apple-Introduces-New-Xserve-Most-Powerful-Apple-Server-Ever.html

investors will expect bigger announcements during the Macworld. This drives the investors sentiments about the company in positive direction. Thus, decrease in systematic risk is observed. It is observed that Apple Inc.'s average price is decreased and similar trend is seen to have followed in the majority of the leading companies as well. The decrease in price is due to the decrease in expection of the future cash flow. However, the news have decreased the variance of the expected cash flow which is the systematic risk.

Significant Date – August 14, 2009

Table 3.5 summarizes beta and average price change data for August 14, 2009, which is identified as a significant date for MU (Micron Technology Inc.). Beta estimates using all 20, 40 and 60 days windows indicate that around this date, beta of MU has increased. For example, using the 60 days windows, beta of MU has increased from 1.45 of the pre-τ period to 2.44 of the post-τ period. This increase in systematic risk is caused by firm-specific news. On August 14, 2009, Rambus Inc. filed a lawsuit against Micron Technology in a antitrust case[7]. This news was perceived as highly positive for Rambus Inc. It was estimated that the trial would worth around $12 billion in damages. This kind of news in which a firm has to pay damages would certainly have negative impact on investor's perception. Also if the damage amount is large, there will be stronger reason for investors to lose trust on the firm. It is thus reflected on Micron Technology's beta as well, which has increased on this day. There is no significant difference in beta observed for the computers industry on this date. Hence, it is imminent that this significant changes in beta is due to the firm specific event. The observation from the table 3.5 also show that there is an increase in average price change from pre-τ to post-τ period for all companies except YHOO (20 days window) and RIMM (60 days window). The average price is observed to have increased for the S&P 500 base index as well. On this period, the market was recovering from the 2008 recession and investors' expectation about future earnings of the company were thus high as seen from the increase in average price. However, for Micron Technology, significant increase in beta

[7] http://www.reuters.com/article/2009/08/14/us-rambus-shares-idUSTRE57D4P320090814

Table 3.4: Pre and post instant beta estimates on Jan 8, 2008.
(* rejected at 5% level & ** rejected at 10% level)

Stock	**p-value**	**Pre β**	**Post β**	**Avg. Price Change %**
		20 days window		
AAPL	0.08**	1.66	0.38	-22.43
HPQ	0.60	1.17	0.99	-13.50
IBM	0.42	1.25	1.00	-3.35
WDC	0.61	1.18	1.54	-12.07
MU	0.60	1.11	1.59	-13.88
MSFT	0.09**	1.12	0.41	-7.88
GOOG	0.07**	1.09	-0.03	-16.27
RIMM	0.13	2.44	1.20	-14.81
YHOO	0.35	0.89	2.80	-3.86
Computers Ind.	0.38	1.18	0.91	
S&P 500				-7.14
		40 days window		
AAPL	0.02*	1.69	0.60	-25.28
HPQ	0.56	1.19	1.03	-10.55
IBM	0.53	1.16	1.02	0.88
WDC	0.47	1.18	1.49	0.28
MU	0.51	1.33	1.67	-15.41
MSFT	0.05**	1.04	0.55	-12.23
GOOG	0.01*	1.20	0.15	-21.26
RIMM	0.11	2.22	1.24	-12.19
YHOO	0.46	1.08	1.81	3.59
Computers Ind.	0.19	1.25	0.96	
S&P 500				-7.32
		60 days window		
AAPL	0.08**	1.52	0.92	-24.93
HPQ	0.51	1.06	0.93	-10.09
IBM	0.09**	1.01	0.72	1.28
WDC	0.38	1.13	1.46	4.75
MU	0.80	1.24	1.34	-24.54
MSFT	0.15	1.07	0.76	-12.39
GOOG	0.41	1.00	0.76	-25.47
RIMM	0.44	1.83	1.48	-11.25
YHOO	0.98	1.26	1.24	-0.07
Computers Ind.	0.21	1.22	1.03	
S&P 500				-9.30

on this news, means that investors were skeptic about its future earnings even though the expectation was high as shown by the increase in price.

Significant Date -Oct 14, 2008

October 14, 2008 is a significant date for both AAPL (Apple Inc.) and IBM Corp., where the beta of AAPL decreased and that of IBM increased. Table 3.6 summarizes the results which are consistent across the 20, 40, and 60 days windows for AAPL and IBM. For example, based on the 60 days window, the beta of AAPL decreased from 1.24 to 0.80 and that of IBM increased from 0.55 to 0.83. The decrease of the AAPL's beta is triggered by Apple's launch of Macbook products[8]. The increase of IBM's beta is triggered by IBM Corp. announcing the opening of tender offer[9]. Except for AAPL and IBM, no other consistent beta changes are identified for stocks under study and also for the computers industry. The average market price on this period is also observed to have decreased. This significant date falls on the period when 2008 financial crisis was at its peak. The launch of new products by Apple Inc. shows that the company is still progressing even during the crisis and heading forward positively. This certainly provides a boost on investor's positive sentiments about the company thus, causing systematic risk to decrease. However, in the case of IBM, the beta is observed to have increased. The tender offers are announced for the purpose of acquisition. This news did not reflect positively on investor's perspective during this period. Acquisition increases the cost for the acquiring company in terms of management change, resources and technology migration, employee relocations etc. During the financial crisis period, investors expect conservative strategies rather than aggressive ones, thus the news did not have positive effect as can be seen from the case of IBM. The decrease in average price, which are consistent across all the companies and the base index suggest the decrease in expcetation of future earnings due to recession. However, significant beta changes show the effect of the related news on the variance of the expected future earning.

[8] http://www.apple.com/pr/library/2008/10/14macbook.html
[9] http://www-03.ibm.com/press/us/en/pressrelease/25474.wss

Table 3.5: Pre and post instant beta estimates on August 14, 2009. (* rejected at 5% level & ** rejected at 10% level)

Stock	**p-value**	**Pre** β	**Post** β	**Avg. Price Change %**
		20 days window		
AAPL	0.08**	0.61	1.26	4.41
HPQ	0.98	0.90	0.89	5.54
IBM	0.56	0.70	0.53	0.46
WDC	0.14	0.37	1.20	8.90
MU	0.07**	0.51	1.97	13.19
MSFT	0.88	0.97	1.06	2.66
GOOG	0.66	1.10	0.95	3.63
RIMM	0.37	1.58	1.06	0.53
YHOO	0.45	0.45	1.13	-5.33
Computers Ind.	0.13	0.80	1.05	
S&P 500				3.37
		40 days window		
AAPL	0.15	0.76	1.09	17.45
HPQ	0.89	0.90	0.88	13.88
IBM	0.03*	0.87	0.47	8.17
WDC	0.53	1.09	1.31	23.04
MU	0.04*	1.41	2.42	34.21
MSFT	0.39	1.11	0.88	5.91
GOOG	0.58	0.97	0.86	11.29
RIMM	0.96	1.04	1.01	3.44
YHOO	0.95	1.00	0.97	3.55
Computers Ind.	0.36	0.88	0.98	
S&P 500				9.72
		60 days window		
AAPL	0.29	0.79	1.00	25.40
HPQ	0.91	0.86	0.84	20.17
IBM	0.11	0.83	0.59	10.74
WDC	0.23	1.06	1.41	29.86
MU	0.02*	1.45	2.44	37.73
MSFT	0.15	0.90	0.58	12.68
GOOG	0.60	0.84	0.76	17.35
RIMM	0.97	1.16	1.15	-5.13
YHOO	0.45	1.04	0.79	3.69
Computers Ind.	0.53	0.89	0.95	
S&P 500				11.79

Table 3.6: Pre and post instant beta estimates on Oct 14, 2008.
(* rejected at 5% level & ** rejected at 10% level)

Stock	**p-value**	**Pre** β	**Post** β	**Avg. Price Change %**
		20 days window		
AAPL	0.03*	1.28	0.65	-12.44
HPQ	0.65	0.85	0.93	-18.20
IBM	0.04*	0.53	0.78	-18.46
WDC	0.08**	1.25	0.70	-22.66
MU	0.77	1.25	1.13	0.32
MSFT	0.42	1.05	0.90	-10.49
GOOG	0.13	1.11	0.86	-12.55
RIMM	0.41	1.15	0.80	-34.14
YHOO	0.17	0.93	0.51	-25.86
Computers Ind.	0.52	1.00	0.93	
S&P 500				-16.54
		40 days window		
AAPL	0.02*	1.23	0.80	-31.26
HPQ	0.98	0.83	0.84	-22.78
IBM	0.00*	0.57	0.81	-26.26
WDC	0.80	1.21	1.15	-40.63
MU	0.68	1.30	1.44	-25.70
MSFT	0.72	1.00	0.95	-18.75
GOOG	0.25	1.00	0.82	-25.99
RIMM	0.22	1.16	0.79	-52.51
YHOO	0.83	0.95	0.90	-34.37
Computers Ind.	0.31	0.99	0.91	
S&P 500				-24.77
		60 days window		
AAPL	0.01*	1.24	0.80	-36.45
HPQ	0.84	0.82	0.84	-21.49
IBM	0.00*	0.55	0.83	-28.94
WDC	0.78	1.17	1.23	-48.00
MU	0.49	1.25	1.49	-33.68
MSFT	0.90	0.99	0.98	-21.08
GOOG	0.21	1.01	0.86	-29.51
RIMM	0.17	1.19	0.84	-57.73
YHOO	0.87	0.89	0.92	-35.49
Computers Ind.	0.23	0.99	0.90	
S&P 500				-26.59

Significant Date -May 16, 2007

Table 3.7 lists the beta estimates and the average price changes around May 16, 2007. This date is a significant date for GOOG (Google Inc.). Beta of GOOG decreased from 1.20 to 0.46 according to the estimates based on 60 days window. This decrease of systematic risk is due to Google's press release on a new search features and homepage design[10]. According to Google Inc., it is their critical first step towards universal search model. From this day onwards Google Inc., started providing integrated search results from videos, images, news, maps, books etc. into the websites. This was critical step in search technology development and provides a dominating competitions position for Google in the coming years, thus well perceived by investors. Meanwhile on May 10, 2007, Google Inc., proposed to buy Yell Group Plc[11]. This acquisition news was published within 20 days window of our estimation from the event date. This acquisition news along with the release of new search features seems to have played an important role in shifting company's beta during this period. Looking at the observed price change, it has increased in the post-τ period. This increase in price is again due to the increase in the expectation of the future cash flow since investors have positive sentiments about the company. Also looking at the industry beta, we do not find any consistent significant change in the industry's systematic risk. Therefore, Google's beta change is completely an isolated firm specific phenomenon.

Significant Date -Apr 18, 2006

Taking Apr 18, 2006 as an event date we estimate pre-and post-τ betas. These are shown in table 3.8. On this particular date we do not find any consistency across the companies when observing average price change or beta. However, for majority of companies, it is observed that average price have decreased using 40 and 60 days observation. This date is significant date for IBM. The beta for IBM is significantly different for pre- and post- τ period and is consistent across all windows. For example, beta for IBM changed from 0.95 to 0.58 as estimated using 60 days window. The computers industry does not show consistent

[10] http://www.google.com/intl/en/press/pressrel/universalsearch_20070516.html
[11] http://www.marketwatch.com/story/google-touted-as-favorite-to-buy-yell-group-report

Table 3.7: Pre and post instant beta estimates on May 16, 2007.
(* rejected at 5% level & ** rejected at 10% level)

Stock	**p-value**	**Pre** β	**Post** β	**Avg. Price Change %**
		20 days window		
AAPL	0.68	0.44	0.10	17.27
HPQ	0.85	0.58	0.65	5.86
IBM	0.21	0.39	1.08	3.90
WDC	0.59	0.43	0.81	4.65
MU	0.56	1.43	0.88	2.23
MSFT	0.36	0.74	1.11	1.95
GOOG	0.07**	1.18	0.38	5.05
RIMM	0.14	1.88	0.54	17.33
YHOO	0.85	1.51	1.25	-2.06
Software Ind.	0.31	0.90	1.11	
S&P 500				1.66
		40 days window		
AAPL	0.29	0.97	0.37	25.97
HPQ	0.18	0.60	0.98	8.94
IBM	0.62	0.59	0.75	7.94
WDC	0.54	0.65	0.96	10.65
MU	0.92	0.79	0.72	6.86
MSFT	0.39	0.90	1.12	4.04
GOOG	0.00*	1.25	0.17	9.49
RIMM	0.70	1.31	0.90	27.78
YHOO	0.24	1.50	0.67	-7.91
Software Ind.	0.63	1.03	0.96	
S&P 500				3.64
		60 days window		
AAPL	0.27	1.17	0.74	35.20
HPQ	0.54	0.82	0.94	12.27
IBM	0.84	0.74	0.69	12.10
WDC	0.75	0.94	0.82	12.23
MU	0.95	0.72	0.75	7.00
MSFT	0.44	1.07	0.94	5.20
GOOG	0.00*	1.20	0.46	11.18
RIMM	0.70	1.36	1.15	39.02
YHOO	0.21	1.13	0.64	-12.12
Software Ind.	0.04*	1.06	0.88	
S&P 500				4.42

significantly different beta across the three windows. The decrease in average price for IBM again show the decrease in investors expectation about future earnings. However, when reviewing the historical news, decrease in beta for IBM is triggered by the first quarter earning announcement which topped the forecasts[12]. The positive earning announcements certainly have positive impact on investors sentiments. As in the case of IBM it is observed that it caused beta to decrease. On this date, we also report about HPQ (Hewlett Packard Inc.) for which significantly different beta is observed using 40 and 60 days observation period. For HPQ, using 20 days window, beta change is on the verge of being significant. It can be observed that beta for HPQ has decreased. Looking into historical news, it is revealed that HPQ made an announcement about extending its businesses to emerging markets[13] [14]. A news about partnership between HPQ and BEA is also reported on this date[15]. Both these news shows growth of the company and shows a positive progress causing beta to decrease.

[12]http://money.cnn.com/2006/04/18/technology/ibm_earnings/index.htm
[13]http://www.hp.com/hpinfo/newsroom/press/2006/060418b.html
[14]http://www.hp.com/hpinfo/newsroom/press/2006/060418a.html
[15]http://www.hp.com/hpinfo/newsroom/press/2006/060418c.html

Table 3.8: Pre and post instant beta estimates on Apr 18, 2006.
(* rejected at 5% level & ** rejected at 10% level)

Stock	**p-value**	**Pre** β	**Post** β	**Avg. Price Change %**
		20 days window		
AAPL	0.74	2.70	2.18	7.60
HPQ	0.10	2.02	0.74	-0.83
IBM	0.04*	1.27	0.34	-0.20
WDC	0.18	2.71	0.86	3.05
MU	0.39	0.96	1.97	10.98
MSFT	0.85	0.55	0.80	-8.95
GOOG	0.74	2.60	2.17	4.75
RIMM	0.13	1.92	0.07	-8.79
YHOO	0.55	1.34	2.06	1.84
Computers Ind.	0.30	1.48	0.92	
S&P 500				0.84
		40 days window		
AAPL	0.46	2.16	1.54	-1.08
HPQ	0.00*	2.12	0.24	-2.87
IBM	0.02*	0.99	0.39	-0.98
WDC	0.87	1.97	1.80	-2.40
MU	0.39	1.45	1.99	6.69
MSFT	0.87	0.75	0.66	-12.45
GOOG	0.26	1.95	1.05	5.07
RIMM	0.90	0.87	0.74	-12.07
YHOO	0.55	1.20	1.56	-0.24
Computers Ind.	0.01*	1.56	0.83	
S&P 500				-0.64
		60 days window		
AAPL	0.34	2.18	1.59	-7.79
HPQ	0.04*	1.78	0.80	-0.82
IBM	0.05**	0.95	0.58	-2.02
WDC	0.70	1.92	1.66	-7.16
MU	0.77	1.36	1.51	4.77
MSFT	0.59	1.00	0.78	-13.14
GOOG	0.02	2.59	1.16	4.72
RIMM	0.61	1.28	0.91	-9.37
YHOO	0.41	1.24	1.58	-1.96
Computers Ind.	0.10	1.44	1.04	
S&P 500				-0.79

3.5 Chapter Summary

In this chapter, we studied dynamic nature of beta and its underlying changes based on triggering economic events. We employed regression for estimating beta in two adjacent time periods and used statistical hypothesis testing to compare them. It is demonstrated that, these beta estimates are not significantly different at all times. From our use case studies, it is emphasized that when there is a significant change in beta, a triggering event is associated with it. The evidences from this chapter show that systematic risk, which we have been modeling considering the uncertainty in the process has a tendency to be piecewise constant.

Chapter 4

Piecewise Constant Beta Tracking Using Kalman Filter

This chapter discusses the modeling of piecewise constant beta process and its tracking using modified Kalman filter. In this chapter first we model beta by considering occurrence of events in a firm as the Poisson process. Then we develop an adaptive estimation method by introducing prior probabilities of beta jumps and use Bayes criteria to choose between two estimation covariance matrices conditional on beta jumps at each time instant. We compare our simulation and empirical results with traditional random walk and mean reverting model based estimation methods and show that our method is superior in tracking the piecewise constant beta process.

4.1 Piecewise Constant Model Formulation

When significant triggering events are the cause of change in systematic risk, beta can be hypothesized to follow a piecewise constant process. This has also been discussed in chapter 3. We can thus model these changes from the perspective of time series analysis with an assumption that the events are unpredictable in firm's history. The beta process remains constant for a period of time and changes only at some time points where there are occurrences of significant triggering events. We assume that changes in beta at these time points are abrupt following the efficient market hypothesis[31].

With these considerations, piecewise constant time series can be modeled as a stochastic process of different regimes with regime changes at certain time instants. We consider the transition points where the regime changes occur as jumps that can be positive or negative. Thus, in this process, we have jumps at some time instants and after occurrence of each jump, beta remains at a certain regime until the next jump.

Events or a series of events can cause a significant increase or decrease in the systematic risk. Two factors are considered in modeling this process. The first factor is the time instant at which a jump occurs. The second is the magnitude of a jump. These two factors determine where a jump occurs and by how much.

We model the occurrence of significant events, which may be firm-specific or economic wide, as the Poisson process. The Poisson process is a counting process in which events arrive randomly at a given rate, λ. For Poisson processes, the probability of n events occurring during the interval θ [37] is:

$$P[(N(t+\theta) - N(\theta)) = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \tag{4.1}$$

where $\theta > 0$, and $N(\theta)$ denotes the total number of events occurring during time interval $[0, \theta]$. For financial time series, the observation time instants vary from as large as annual to as small as intra-day. For beta analysis, the observations are usually sampled return series of an asset. A large observation interval accounts for a low sampling rate. If a large observation interval is chosen, the probability of multiple events occurring within that interval will be higher. We choose an observation interval θ to be small enough relative to the arrival rate so that the probability of more than one event occurring during the observation interval becomes negligible. If $N(\Delta t)$ denotes the number of events occurring in one observation interval where Δt is the observation interval, it follows from the Poisson distribution [38] that,

$$P[N(\Delta t) = 0] = e^{-\lambda \Delta t} \tag{4.2}$$

$$P[N(\Delta t) = 1] = (\lambda \Delta t) e^{-\lambda \Delta t} \tag{4.3}$$

(4.2) represents the probability of no event occurring in interval Δt and (4.3) represents the probability of one event occurring in interval Δt.

Now we assume that $\lambda\Delta t << 1$ which results from our consideration that the occurring rate of significant triggering events is very low. And assume $P[N(\Delta t) > 1] \simeq 0$, i.e., the observation interval is chosen such that the probability of more than one triggering event occurring during that interval is almost zero during the interval. According to (4.2) and (4.3), $N(\Delta t)$ can be replaced by a Bernoulli random variable, which is equal to one with probability $\eta = (\lambda\Delta t)e^{-\lambda\Delta t}$. This is trivial since $e^{\lambda\Delta t} \simeq 1 + \lambda\Delta t$ using first order Taylor series expansion when $\lambda\Delta t$ is close to zero.

The mathematical expressions of this jump process have also been derived in [39]. Our main objective here is to apply the model to time-varying beta process and justify the assumptions from economic point of view by relating it to the arrival of significant triggering events that can cause the change in systematic risk of the firm.

With these assumptions and after representing observation time interval Δt by discrete time t, a Bernoulli Process z_t can be generated as,

$$z_t = \begin{cases} 1 & \text{w.p. } \eta, \\ 0 & \text{w.p. } 1-\eta \end{cases} \tag{4.4}$$

To model time-varying beta as a jump process, we include a random jump component. We use the binary variable z_t to denote whether a jump would occur at time instance t, and use a random variable u_t to specify the magnitude of a jump. We assume that the magnitude of a jump follows a normal distribution with zero mean. (4.5) models the piecewise constancy of the systematic risk,

$$\beta_t = \beta_{t-1} + z_t u_t \tag{4.5}$$

where, u_t is independently and identically distributed Gaussian process following $u_t \sim N(0, \sigma_u^2)$. This process can be interpreted in two ways. When the value of z_t is one, beta changes by variance associated with a normal random variable. When $z_t = 0$, the beta value remains unchanged as the previous value.

4.2 Kalman Filtering Based on Bayes Selection Criteria

As beta is modeled as a piecewise constant process in (4.5), we need to track and estimate beta at each time instant. Since the magnitude, direction and localization of beta are all modeled as random parameters, we use a Kalman filter technique based tracking method. A time series model represented in the state space form as in (4.5) is also an ideal candidate in application of the Kalman filter. The preliminaries of Kalman filter was introduced in chapter 2. The Kalman filtering for intermittent observations has been discussed in [40]. However, for our problem instead of observation being intermittent, the state change occurs only at certain intervals. The notations and equations discussed in the chapter 2 can be directly applied here in their scalar form. However, modification in algorithm is required to account for the piecewise constant formulation.

Our problem formulation for tracking involves (4.5). The systematic risk of an asset is an unobserved quantity and thus can be considered as the hidden state in system dynamics. Equation (4.5) is therefore a state equation. Our task is to effectively track β_t through data such as the excess stock return $r_{e,t}$. According to the CAPM, $r_{e,t}$ is a linear function of β_t. The beta observation equation is therefore given by the CAPM based equation (4.7). Along with the piecewise constant state equation, our system is as follows:

$$\beta_t = \beta_{t-1} + z_t u_t + \zeta_t \tag{4.6}$$

$$r_{e,t} = r_{m,t}\beta_t + \epsilon_t \tag{4.7}$$

In (4.6), ζ_t is introduced as the process noise parameter. It is assumed that ζ_t is uncorrelated with u_t and follows a distribution $\zeta_t \sim N(0, \sigma_\zeta^2)$. A noisy β_t is observed through an asset's excess return $r_{e,t}$, which can be calculated using the observed asset (stock) price Pr_t and dividend D_t and risk free rate $R_{f,t}$ as shown in (4.8).

$$r_{e,t} = \frac{Pr_t - Pr_{t-1} + D_t}{Pr_t - 1} - R_{f,t} \tag{4.8}$$

In (4.7), ϵ_t represents the noise, i.e., idiosyncratic risk of an asset, and follows a distribution $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Note that the economic meaning of ϵ_t is the firm-specific idiosyncratic risk, and $r_{m,t}$ is the market excess return, i.e., market risk premium, that is the difference between the market return $R_{m,t}$ and the risk-free rate $R_{f,t}$, i.e., $r_{m,t} = R_{m,t} - R_{f,t}$. The relationship (4.7) is a variant of CAPM in which expected returns are replaced by realized returns.

Equations (4.6) and (4.7) are not traditional Kalman filter state space equations due to the additional nonlinear term $z_t u_t$. Since z_t is a binary random variable, (4.6) can take two forms. When $z_t = 0$, (4.6) is a linear equation with Gaussian disturbance, and β_t takes its previous value. When $z_t = 1$, (4.6) is still a linear equation with Gaussian disturbance that is the sum of two Gaussian disturbances, u_t and ζ_t.

A one step prediction can be obtained using the Kalman filter prediction equation,

$$\hat{\beta}_{t|t-1} = \hat{\beta}_{t-1|t-1}, \tag{4.9}$$

where $\hat{\beta}_{t|t-1}$ is the conditional expectation at t, and $E_t[\beta_t|\beta_{t-1}]$ and $\beta_{0|0}$ are assumed to be normally distributed. As noted from [33], depending on the value of z_t, (4.6) can be viewed as the following probabilistic models,

$$p(\beta_t|\beta_{t-1}) \sim N(\beta_t; \beta_{t-1}, \sigma_\zeta^2) \tag{4.10}$$

$$p(\beta_t|\beta_{t-1}) \sim N(\beta_t; \beta_{t-1}, \sigma_u^2 + \sigma_\zeta^2) \tag{4.11}$$

where $p(\cdot|\cdot)$ represents conditional probability and $N(x; m, \sigma^2)$ represents normal distribution with parameter x, mean m and variance σ^2. Since z_t has a prior probability of its own given by η, the occurrence of (4.10) is with probability $(1-\eta)$ and (4.11) with probability η. Therefore, the covariance of estimation in (4.9) has two possibilities depending on the value of z_t. When $z_t = 0$ with probability $(1-\eta)$,

$$P^0_{t|t-1} = P_{t-1|t-1} + \sigma_\zeta^2 \tag{4.12}$$

and when $z_t = 1$ with probability η,

$$P^1_{t|t-1} = P_{t-1|t-1} + \sigma_u^2 + \sigma_\zeta^2 \tag{4.13}$$

In (4.12) and (4.13), $P_{t|t-1} = E_t[(\beta_t - \hat{\beta}_{t|t-1})^2]$, and $P_{t-1|t-1}$ is the covariance of the prior state estimate. Here, superscripts 0 and 1 are used for the purpose of indication of two possibilities. When observation of new $r_{e,t}$ becomes available at time t, we can obtain the innovations as

$$\tilde{r}_{e,t} = r_{e,t} - r_{m,t}\hat{\beta}_{t|t-1} \tag{4.14}$$

Also when new observation becomes available, we can obtain the likelihood of observing $r_{e,t}$ given the observations $r_{e,1:t-1}$ up to time $t-1$. This likelihood also follows a Gaussian distribution. Since we have two possible covariances, we obtain two probable likelihoods. This can be expressed as

$$p^0(r_{e,t}|r_{e,1:t-1}) \sim N(r_{e,t}; \hat{\beta}_{t|t-1}, P^0_{t|t-1} + \sigma^2_\epsilon) \tag{4.15}$$

$$p^1(r_{e,t}|r_{e,1:t-1}) \sim N(r_{e,t}; \hat{\beta}_{t|t-1}, P^1_{t|t-1} + \sigma^2_\epsilon) \tag{4.16}$$

where $p^0(\cdot|\cdot)$ and $p^1(\cdot|\cdot)$ represent the conditional likelihood distributions when $z_t = 0$ and $z_t = 1$ respectively.

A decision needs to be made on which one of (4.15) and (4.16) is more likely. Since there is a prior probability associated with each, according to the Bayes criteria, we weigh these values of conditional likelihood using the probabilities of z_t given by η, and the covariance is selected based on the larger value of the weighted likelihood. Therefore, the decision is made based on the following relationship,

$$(1-\eta)p^0(r_{e,t}|r_{e,1:t-1}) \geq \eta p^1(r_{e,t}|r_{e,1:t-1}), \tag{4.17}$$

which if true we select P^0 and assign value of $z_t = 0$, and otherwise we select P^1 and assign the value of $z_t = 1$. Then, the Kalman filter gain K_t at time instant t can be obtained,

$$K_t = P_{t|t-1}S_t^{-1}, \tag{4.18}$$

where,

$$S_t = P_{t|t-1} + \sigma^2_\epsilon, \tag{4.19}$$

where $P_{t|t-1}$ is selected using the relationship (4.17) and S_t is the covariance of the innovations given by (4.19). Thus, we can proceed with the regular Kalman updates as,

$$\beta_{t|t} = \beta_{t|t-1} + K_t \tilde{r}_{e,t}, \tag{4.20}$$

$$P_{t|t} = (1 - K_t r_{em,t}) P_{t|t-1}. \tag{4.21}$$

This process of comparing the two weighted probabilities is performed at every time instant in the observation time series. Thus, with these modifications in Kalman filter algorithm we can effectively track the systematic risk when it follows a piecewise constant process.

This algorithm is also depicted in the flowchart 4.1

4.3 Simulation Results and Discussion

Simulations are performed to illustrate the effectiveness of the beta tracking algorithm for the piecewise constant model. In simulations, we first simulate the piecewise constant process and then compare our tracking method to the tracking methods based on the random walk and mean reverting model. The use of Kalman filter in these models have been shown in chapter 2. The random walk model using the Kalman filtering algorithm has been used for systematic risk estimation in the finance literature, e.g., [15] and [20]. When beta follows the random walk, it is governed by equation

$$\beta_t = \beta_{t-1} + \zeta_t, \tag{4.22}$$

where ζ_t is the process noise following our usual notation. A mean reverting model with Kalman Filter is used, e.g., in [20] and [21]. The mean reverting beta can be described as:

$$\beta_t = (1 - B)\overline{\beta} + B\beta_{t-1} + \zeta_t, \tag{4.23}$$

where B is a constant which determines the rate at which β reverts back to its mean value, and $\overline{\beta}$ is the mean value of beta.

Again, it is to be noted that the ϵ_t, commonly treated as an observation noise in the state space model in control engineering and signal processing applications, should be interpreted

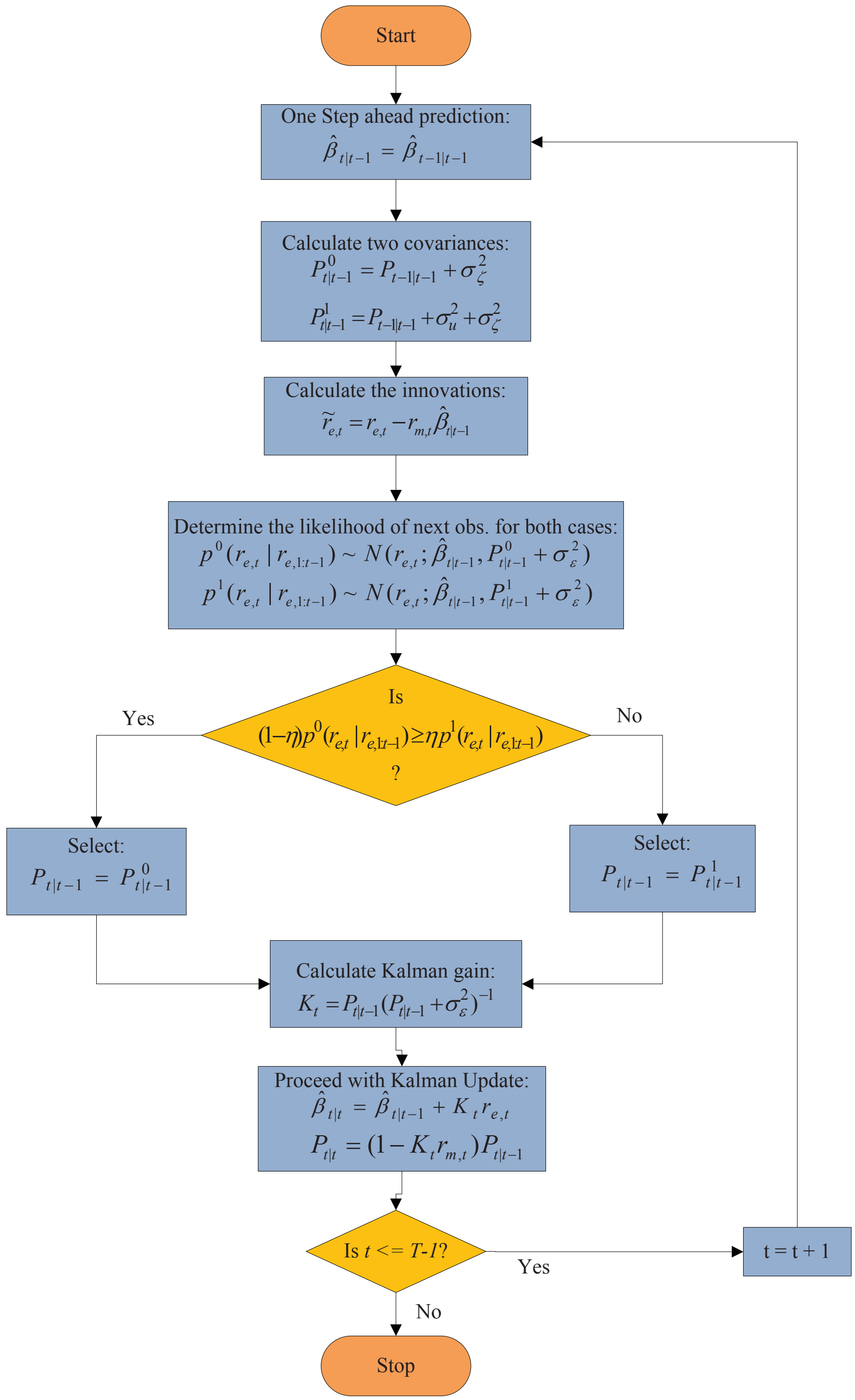

Figure 4.1: Flowchart for modified Kalman filter algorithm to track piecewise constant beta process

as an idiosyncratic shock in economic terms. We calculate the observation RMSE between the predicted (estimated) excess return and observed excess return in (4.7) as

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} \left(r_{e,t} - \hat{r}_{e,t}\right)^2}{T}} \tag{4.24}$$

where T is the number of observations and $\hat{r}_{e,t}$ is the estimation of the excess return at time t given by $\hat{r}_{e,t} = r_{m,t}\hat{\beta}_t$.

In a perfect model estimation, this observation RMSE should equal to σ_ϵ. That is to say, the beta only captures the systematic risk, not the idiosyncratic risk σ_ϵ. If the observation RMSE is smaller than σ_ϵ, the beta would capture the idiosyncratic risk rather than the systematic risk, i.e., the model is "overfit" to the observation noise.

To evaluate the tracking performance, we look at the different ranges of the model parameters. We compare the effects of changing the probability factor η, observation noise standard deviation σ_ϵ and process noise standard deviation σ_ζ in these experiments.

The simulated observation signals are generated using piecewise constant process (4.6) and CAPM based observation equation (4.7) with different model parameters. Monte-Carlo simulations with 50 times are performed when calculating average RMSEs of beta and observation errors of excess returns.

Figure 4.2 shows an example of tracking using piecewise constant model and random walk model along with true process. The signal is generated using the parameters $\sigma_\epsilon = 0.05$, $\sigma_\zeta = 0.0005$, $\sigma_u = 1$, $\eta = 0.04$ and $T = 500$. Since mean reverting model produces similar results as random walk model we do not include them for the clarity of figure. From this figure we can observe that using random walk results in a noisy tracking when a constant level has to be tracked however, our method produces smoother tracking which almost overlaps the true process. Moreover from the figure, it can be observed that even the quickest of transitions are tracked quite accurately using our methodology.

In the first experiment, we generate the piecewise constant signal with parameter values as $\sigma_\zeta = 0.0005$, $\sigma_u = 1$, $\eta = 0.04$ and $T = 500$. For each generated signals, we conduct model estimation using tracking algorithms based on the random walk model, the mean reverting

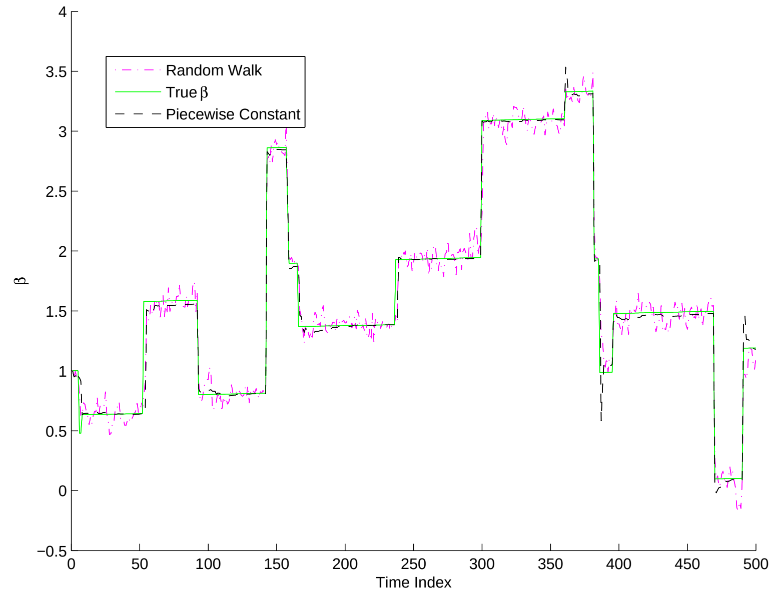

Figure 4.2: Example of tracking using piecewise constant method and random walk method compared to the true process.

model and the new tracking method based on the piecewise constant model.

Figure 4.3 shows the result of this experiment. In this experiment, the ideal model estimation should fit into a straight line of 45 degrees. The closer the plot to this straight line, the better is the estimation of the idiosyncratic risk. It can be observed that our new tracking method based on the piecewise constant model is the best in terms of representing the systematic risk and idiosyncratic risk components of the expected stock returns. Also note that random walk model and the mean reverting model based methods give similar results since the mean reverting model contains significant random walk component. Therefore, if the beta indeed follows a piecewise constant process, the new tracking method outperforms random walk model and mean reverting model in capturing the systematic risk as well as idiosyncratic risk components.

It is important to evaluate the effect of observation noise on the estimation method, because the observation noise is considered as idiosyncratic shocks in economics. A good model needs to be able to separate the systematic risk from the idiosyncratic risk. Figure 4.4 shows that the beta RMSE performance changes when the observation noise changes,

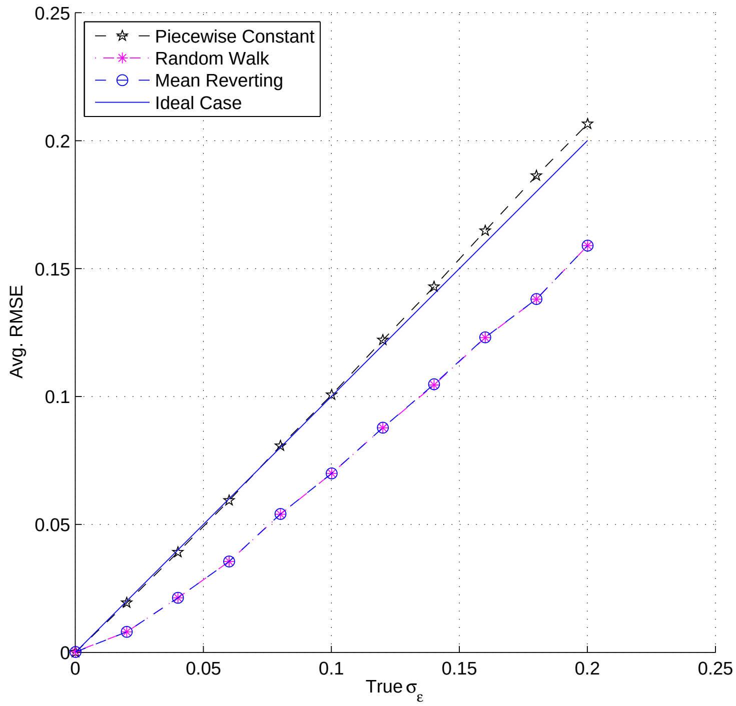

Figure 4.3: Average observation RMSEs vs. true observation noise standard deviation σ_ϵ (i.e., idiosyncratic risk). Note that the 45 degree line represents the ideal estimation.

with all other parameters fixed. In Figure 4.4, average beta RMSE is plotted against the observation noise standard deviation σ_ϵ. It can be seen that when the standard deviation of observation noise is below 0.18, our method consistently outperforms both the random walk and mean reverting models. Beyond this point the model performance is comparable to the other two models. This indicates that when the observation noise is too large, the systematic risk is buried and the algorithm is no longer able to discriminate between the systematic risk (state change) and idiosyncratic risk (observation noise), and the new method performed the same as the random walk model based method.

Figure 4.5 shows the performance of our tracking method for varying process noises. The fixed parameters of this experiment are $\sigma_\epsilon = 0.05$, $\sigma_u = 1$, $\eta = 0.04$ and $T = 500$. Generally the process noise in signal needs to be small such that the piecewise constant signal is generated in an appropriate manner. High process noises result in degradation of signals that divert away from the ideal piecewise constancy. Figure 4.5 demonstrates that

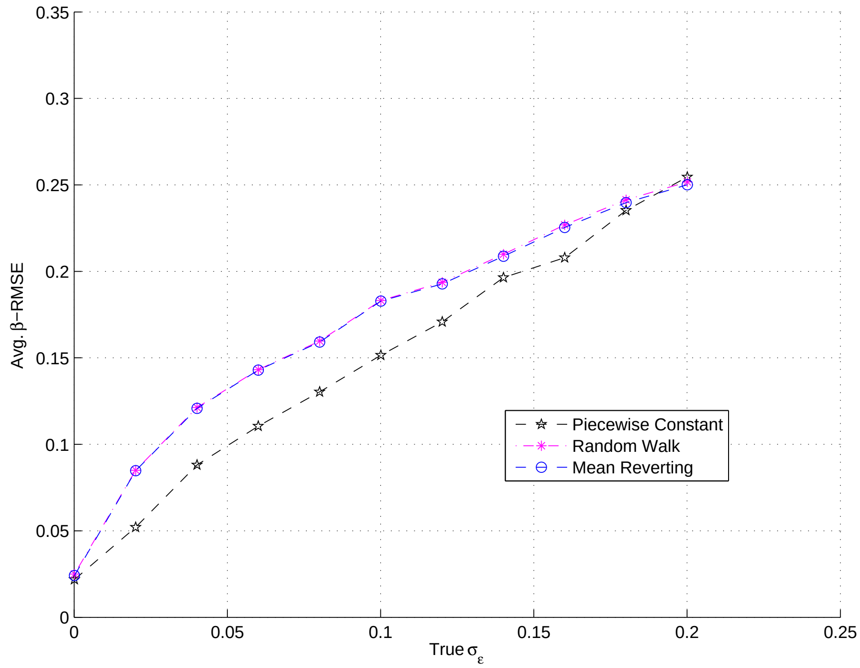

Figure 4.4: Average β RMSE vs. true observation noise standard deviation (σ_ϵ). Tracking is performed with three time varying techniques for signals for a range of σ_ϵ values

our method performs better than both the random walk and mean reverting models for varying process noises.

Figure 4.6 shows the performance of our method as compared to the random walk and mean reverting models in terms of varying η. In this figure, we have fixed the parameters of the signal generating process as $\sigma_\epsilon = 0.05$, $\sigma_u = 1$, $\sigma_\zeta = 0.0005$ and $T = 500$ and varied the value of η from 0.01 to 0.21. It can be observed that our method outperforms both random walk and mean reverting model consistently no matter how frequently the change occurs in beta. The piecewise constant model with very frequent change adheres closely to random walk model. But, for our problem beta does not change frequently and η is supposedly very low. However, even if η is as high as 0.21, our estimation method outperforms the random walk and mean reverting estimation techniques. Thus, RMSE of beta generated for the given range certainly becomes applicable for the problem.

Next, we examine the effectiveness of our model when the estimated probability η deviates from actual probability of the systematic risk. Figure 4.7 shows the experiment results. In

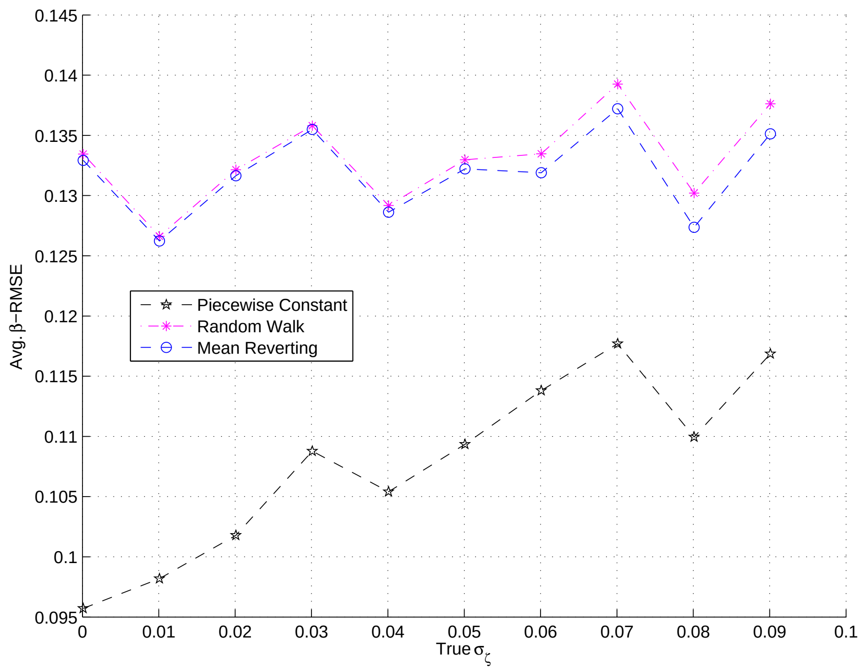

Figure 4.5: Average β RMSE vs. process noise (σ_ζ). Tracking is performed with three time varying techniques for signals for a range of σ_ζ values

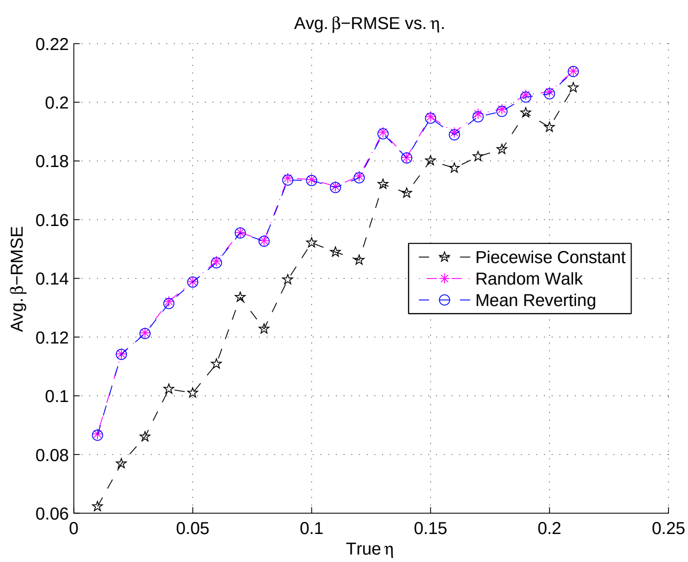

Figure 4.6: Average β RMSE vs. prior probability (η)- Tracking is performed with three time varying techniques for signal generated using range of η values keeping other parameters fixed. η is assumed known while tracking.

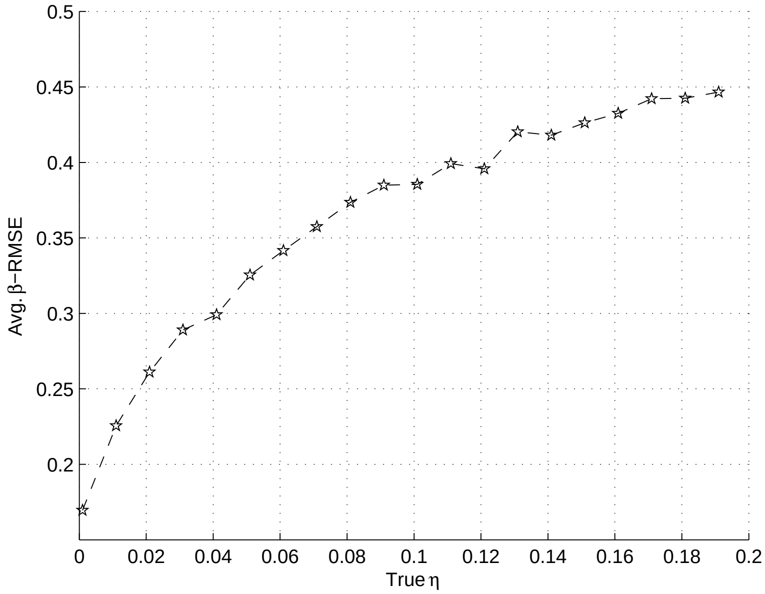

Figure 4.7: Average β-RMSE vs prior probability (η) - Signal is generated using different η specified in x-axis and estimated using $\eta = 0.04$ by piecewise const. method.

this experiment, we fix parameters $\sigma_\epsilon = 0.05$, $\sigma_u = 1$ and $\sigma_\zeta = 0.0005$, and generate signals with η ranging from 0.001 to 0.2. We track the signal by considering $\eta = 0.04$ for estimation. It can be observed that with the increase of true η, the RMSE also increases. When true η is small and the signal is estimated using assumed value of $\eta = 0.04$, the RMSE obtained is also small. This is due to the fact that small values of η result in infrequent beta changes, and these infrequent changes can still be tracked precisely with the assumption of high probability. When the true η is large, there are frequent β changes for which the estimated η of a low value will not be sufficient to generate those frequent changes and thus the beta RMSE is higher. Figure 4.7 also suggests the robustness of our method by showing that for infrequent beta changes, the difference of beta RMSEs is not significant.

4.4 Empirical Case Studies

Empirical tests are performed on five stocks in the technology sector. We retrieve stock's monthly value weighted returns from the CRSP (Center for Research in Securities Prices)

Table 4.1: Firms in the empirical tests

Firm	Ticker	CRSP Perm Number
Apple Inc.	AAPL	14593
Microsoft Corp.	MSFT	10107
Yahoo Inc.	YHOO	83435
Intel Corp.	INTC	59328
Dell Inc.	DELL	11081

database, a widely used stock price database in finance research. Monthly excess return of a stock is calculated from stock monthly return by deducting the monthly risk free rate. To proxy the monthly risk free rate, we use one-month Treasury Bill (T-bill) rate. The T-bill rates are obtained from K. French data library[1]. To proxy the market return, we use monthly value weighted returns of S&P 500 index from CRSP database adjusted for dividend and splits of component companies. The monthly market risk premium is obtained by deducting the monthly risk free rate, i.e., one-month T-bill rate, from the market return. We use the entire samples of data for each firm up to Dec 31, 2009 available from CRSP. The selected firms, their symbols and CRSP permanent number are listed in table 4.1.

For each firm we determine maximum likelihood estimation of each parameter of every model using the method discussed in section 2.2.2. Using these maximum likelihood parameters we estimate the monthly beta. For the piecewise constant model, we choose the value of $\eta = 0.047$ based on the empirical experience. We do not assume beta to change very frequently in this empirical examination. This empirical value of η represents that beta changes approximately once in every 20 months. Also, as can be seen in our previous simulations, the estimation results are not very sensitive to η.

Figures 4.8 to 4.12 show the visualization of estimation results of the piecewise constant model against the random walk model. In these figures, the vertical lines represent the instants with estimated $z_t = 1$, i.e., the time instants when the stock systematic risk has exactly changed as determined by the piecewise constant model. As shown, the piecewise constant model is very effective in capturing both the occurrence of beta changes and the

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

direction of the change. On the identified time instants, we examine historical news to identify the triggering economic event relevant to the firm. The results demonstrate and confirm the capability of our piecewise constant model to determine beta changes triggered or catalyzed by significant events.

Figure 4.8 shows a sudden fall of beta at month index 199, representing January 1998, as captured by the piecewise constant model. When examining the history of Apple Inc., it turns out that this fall of beta can be attributed to a surprising turnaround of Apple Inc. In January 1998, it was announced that the firm made a promising quarterly profit by earning $47 millions. It was much stronger than expected as Apple Inc. suffered a loss of $120 million in the previous year[2]. This turnaround led to the upgrade of this stock by many analysts[3]. Investors perceived that the risk to invest in the firm would reduce. The economic reason of beta decrease for Apple Inc. is clearly justified. Announcement of this good news not only lifted the stock price but also caused its beta to decrease. This is captured by our piecewise constant model and is not prominent in random walk model. (Note that not every event that moves stock price will change the perceived future risk of the firm. The stock price change may only reflect the change of earning expectations, but not the risks, i.e., the variance of futures earnings).

Again at month index 231 in Figure 4.8, i.e., September 2000, the beta of Apple Inc. stock increased suddenly as captured by our piecewise constant model. Increase of beta reflects the negative change of investors' perception of a firm/stock. In September 2000, Apple Inc. warned that its fourth-quarter earnings and revenue would fall below the expectations because of the lower than expected sales.[4]. This caused Apple's share price to drop by half as well[5]. This news have caused sudden rise in the Apple Inc.'s beta as it made a negative impact on the investors' sentiments on the future risk of the firm. This event, as a major event in Apple's history during that period, is captured accurately by our piecewise constant

[2]http://www.nytimes.com/1998/01/15/business/business-digest-091960.html

[3]http://www.nytimes.com/1998/01/15/business/company-reports-sales-of-g3-computer-lift-apple-back-to-profitability.html

[4]http://www.money.cnn.com/2000/09/29/markets/techwrap/

[5]http://www.money.cnn.com/2000/09/29/technology/apple/

model.

At month index 235, in Figure 4.8, we can see the decrease in beta for Apple Inc. This corresponds to the period January 2001. Looking into the history of Apple Inc., it is found that very crucial announcement was made by Apple Inc. CEO, Steve Jobs during this period. Acknowledging that past several months have been challenging for Apple Inc., Jobs showed positiveness for the company's future by announcing the launch of several new products which included first launch of iTunes and revamped Macintosh line of computers[6]. Many industry analysts also agreed that Job's strategy gives a chance to restart company's growth and would help set Apple Inc. apart again. This news was certainly positive enough for investors to increase their trust on Apple Inc., thus decreasing the beta. This is accurately captured by piecewise constant model. In addition to the effectiveness of our tracking method, the case study of Apple Inc. also justifies/supports the economic logic of our piecewise constant model, i.e., beta does demonstrate piecewise constant characteristics.

Figure 4.9 shows results of Microsoft Corp. The significant beta increase identified by the piecewise constant model at month index 163 corresponds to April 2000. Looking back, we find that a federal judge ruled that Microsoft had committed monopolization and the Justice Department would request court to break Microsoft into two separate companies. This news caused a plunge in Microsoft share value and analysts had negative remarks about the firm[7]. This event was definitely a major event in the history of Microsoft. The prospect of breaking up the firm brought massive risk to the firm. The systematic risk thus increased on this particular event and is captured by our piecewise constant estimation very effectively.

Figure 4.10 shows event detection by the piecewise constant method for Yahoo! Inc. One significant event identified is at the month index 38, corresponding to December 1999. We find that Yahoo! Inc. was listed in the Standard & Poor's top 500 list during the period, thus making it one of the 500 large-cap common stocks[8]. In that period, it was also reported that

[6]http://www.nytimes.com/2001/01/10/business/technology-apple-putting-hopes-on-new-macintosh-line.html?

[7]http://money.cnn.com/2000/04/24/companies/microsoft/

[8]http://www.nytimes.com/1999/12/02/business/the-markets-stocks-bonds-broad-rally-led-by-blue-

Yahoo Inc. was allying with Kmart Corporation to create a new company that provides free access to the Internet[9]. Note that Yahoo! Inc. was only three years old and was a growing company before then. These events definitely signaled the growing-up of Yahoo! Inc. The alliance would help Yahoo! to share Kmart's customers and thus reduced the systematic risk of the firm. Thus, the decrease in beta as depicted in the plot is expected.

Figure 4.11 shows the beta tracking for Intel Corp. A significant event is detected on month index 327, i.e., September 2000. In September 2000, Intel warned that its third-quarter revenue would fall below the company's previous expectation because of weaker demand of microprocessor chips in Europe[10]. This event not only caused a decrease in Intel Corp.'s stock price but was proved to be significant enough to cause the change of its systematic risk. This event is very important for Intel Corp. and has also been studied in [41]. However, [41]'s study focuses only on the drop in stock price and analysts recommendations. Our piecewise constant model suggests a clear regime shift of systematic risk caused by this event. Such identification of beta change is very valuable for finance/business professionals.

In Figure 4.12, beta tracking for Dell Inc. is shown. On month index 50 which corresponds to February 1993, an event is identified. This month was indeed a significant month for Dell Inc. as can be seen by the historical news which reported that Dell Inc. had reduced its long-term profit goal[11]. This news also caused the plunge in the stock price of Dell Inc. Additionally, another news reported that Dell Inc. was withdrawing an offering of four million new common shares[12]. Both these news are not positive and created an adverse effect on investors perception causing an increase in Dell Inc.'s beta. This event is captured by our piecewise constant estimation effectively. Again at time index 145 corresponding to January 2001, it can be observed that beta has decreased. It is reported by two research companies that Dell increased its share of market with strong sales growth despite the slowness of

chips-lifts-the-dow-120-points.html

[9] http://www.nytimes.com/1999/12/16/business/kmart-is-joining-yahoo-in-deal-to-create-new-internet-shopping-site.html

[10] http://www.nytimes.com/2000/09/22/business/intel-expects-its-revenue-to-be-below-expectations.html

[11] http://www.nytimes.com/1993/02/25/business/company-news-dell-s-stock-price-plunges-after-forecast.html

[12] http://www.nytimes.com/1993/02/25/business/more-trouble-for-dell.html

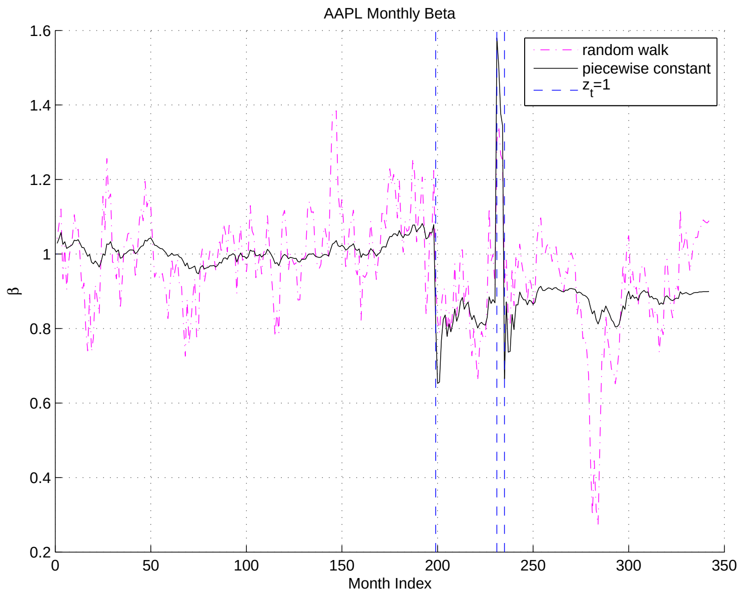

Figure 4.8: Apple Inc. β tracking

global sales of personal computers. With this move Dell gained the most[13]. Also growth of Dell is supported by the news such as the alliance between Dell and Unisys[14], which was expected to generate \$1 billion in added sales. Along with these, news that reported Dell opening a second factory in Malaysia to double the production[15] was certainly positive and idicated Dell's progress during the period which was also perceived positively by investors. These events which caused beta of Dell Inc. to decrease was thus effectively captured by our piecewise constant estimation method.

We also show the RMSEs of the three competing methods. It is observed from table 4.2 that the piecewise constant model has less RMSE values than the random walk model for stocks YHOO, INTC and DELL. The RMSEs obtained by the mean-reverting model are the least for all four cases. However, as we explained, the RMSE should be comprehended as the captured idiosyncratic risk. Thus, the RMSE does not signify the model capability to

[13] http://www.nytimes.com/2001/01/22/business/dell-increases-its-market-share-as-pc-sales-slow.html

[14] http://www.nytimes.com/2001/01/25/business/technology-briefing-e-commerce-dell-and-unisys-expand-alliance.html

[15] http://www.zdnetasia.com/dell-to-open-2nd-msian-factory-double-production-10033761.htm

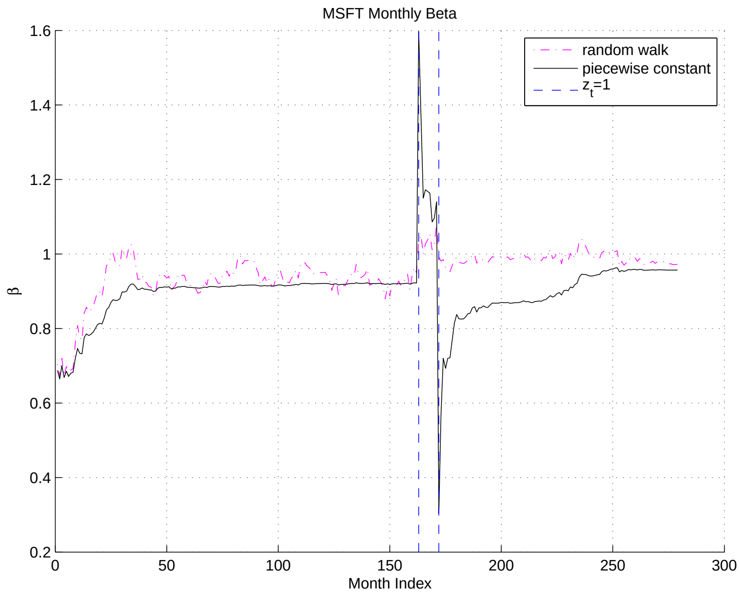

Figure 4.9: Microsoft Corp. β tracking

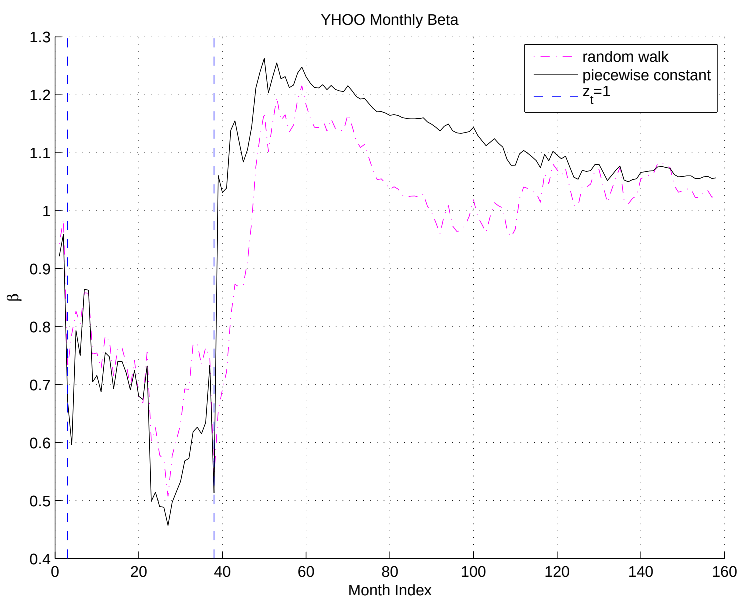

Figure 4.10: Yahoo Inc. β tracking

capture the major events that change the systematic risk.

From these empirical results, we demonstrate that our piecewise constant model helps

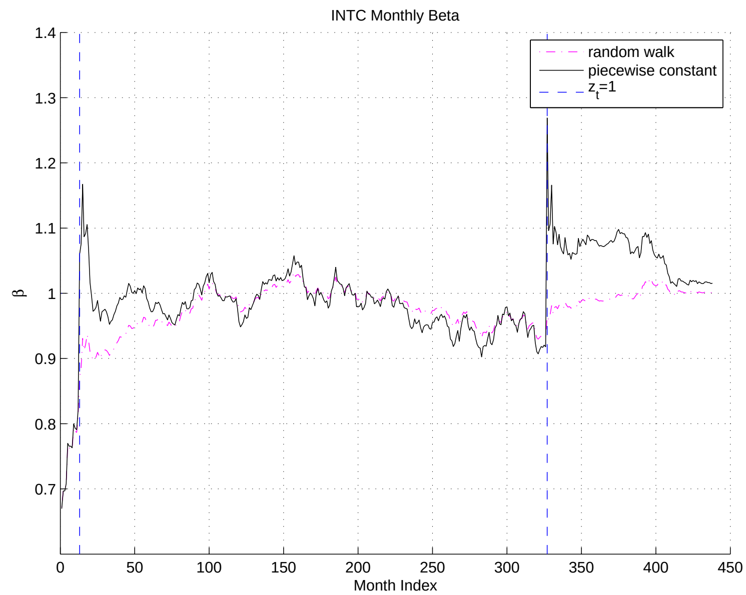

Figure 4.11: Intel Corp. β tracking

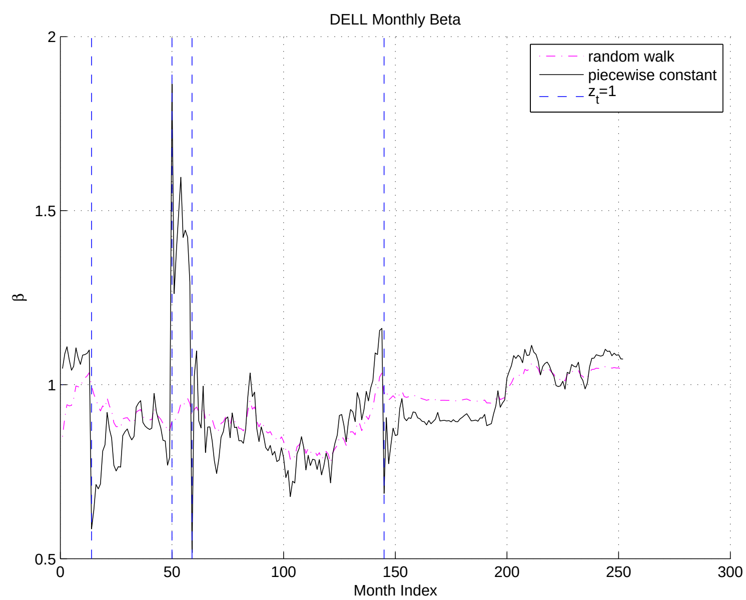

Figure 4.12: Dell Inc. β tracking

track changes in beta. The shift in systematic risk signals that one or multiple events act as a catalyst in the ongoing perception or skepticism of investors about the firm. The events

Table 4.2: Root Mean Square Error to determine the performance of empirical results of the three competing models

Stock	Piecewise - constant	Random Walk	Mean-Reverting
MSFT	0.0874	0.0844	0.0336
YHOO	0.1785	0.1826	0.1004
AAPL	0.1221	0.1036	0.1030
INTC	0.0975	0.1022	0.0956
DELL	0.1117	0.1276	0.0613

identified by our model proved to be significant to cause beta to change.

4.5 Chapter Summary

In this chapter, we presented the modeling and tracking performance of piecewise constant model. We developed a new methodology to estimate the piecewise constant process using Kalman Filter. We compared our method with random walk and mean reverting models. Through controlled simulation we found that our method is superior to both random walk and mean reverting model based estimation to track piecewise constant process. We also considered five empirical use cases from which we showed that our method is able to identify the change in beta effectively.

Chapter 5

Conclusion and Future Work

In finance, beta, a measure of systematic risk, is used to value the cost of capital, to determine if a stock is overvalued or undervalued and also evaluate the performance of portfolio managers. Therefore, it is essential to accurately estimate stock or portfolio beta. The modes of measurement or estimation of beta is however disputable as its true nature cannot be observed. CAPM is used as the fundamental model for the estimation of beta. This model is ex ante in nature. However in realization, historical values are required to estimate beta using this model. In literature, it has been shown that beta has a time varying characteristic, which is in line with dynamic nature of economy. This characteristic of beta has added a complexity in its estimation. Thus, estimating beta has become a challenging topic in finance.

This thesis has explored the time varying nature of beta. In this thesis, we showed that statistically significant change in beta is not frequent. We estimated beta for the leading companies in the technology sector using regression and compared the adjacent time period betas using statistical hypothesis test. It is found that the number of significantly different adjacent estimations in the observed time series depends on the estimation interval used. It is found that there is an inverse relationship between the estimation interval length and the number of significantly different adjacent estimations. The significant beta changes using longer observation interval are rare in comparison to the shorter observation interval.

Our study also examined the existence of any event occurring around the significant change in adjacent period beta estimations. Our results showed that these events do exist. These events are due to either firm specific or economic wide information flows. These information changes the investor's perceptions about the firm that in turn are expected to affect the systematic risk. Thus, our results showed that changes in beta are infrequent and the change in beta is accompanied by a triggering event. As a result, underlying beta, which is unobservable, has a tendency to be piecewise constant. This model for beta is different from the ones discussed in the existing literature. It is also consistent with efficient market hypothesis as this model associates the real world information flow with the change in beta.

In this thesis, we also model the piecewise constant beta process using Poisson process by considering the random occurrence of the major events. Based on this new piecewise beta model, we presented a methodology for tracking the piecewise constant process using modified Kalman filter algorithm. We take into consideration the prior probabilities of the events and follow a Bayes criteria to weight the likelihood of observations, and choose the covariance of the prediction between two choices of beta process at any given time instant. At any given time instant the two choices for beta are whether it would change or not.

We performed simulations and compared our method with the traditional random walk and mean reverting models using Kalman filter technique. The simulation results show that our beta tracking method outperforms the traditional models based methods when tracking the piecewise constant process in capturing both systematic risk and idiosyncratic risk. We also conducted the empirical case studies from five technology firms. This study is based on historical stock prices and economic events. It determined the effectiveness of our model and tracking technique in analyzing the real world data. We showed that our method is very effective in identifying beta changes caused by significant events. The shift in systematic risk indicated that the events during the period become significant enough to act as a catalyst in the ongoing perception of investor to cause these shift.

The new model and its estimation algorithm are expected to help investors, bankers and financial professionals to better estimate the cost of capital and make better investment judgment.

In the future, we would like to explore the following:

1. The empirical study performed in this thesis constitutes individual stocks. In future, we can extend this study to examine the piecewise constancy in beta for industries or sectors and other types of portfolios. We can also examine the piecewise constancy of beta in small cap stocks whose capital structure changes very often. Also, the rating change of a company can be examined when its beta changes.

2. Using Kalman filter based tracking discussed in chapter 4, we can identify and verify the significant beta changes identified using regression and statistical hypothesis testing in chapter 3. Thus, comparing the results of two methodologies. Additionally, we can use regression based method to calibrate the parameters estimated in Kalman filter tracking.

3. In this thesis, when modeling piecewise constant beta, we assumed the magnitude of beta change, u_t, follows a Gaussian distribution. However, for future work, we can perform multiple tests to confirm this assumption.

4. In this thesis, the estimation method used for the piecewise constant model is Kalman Filter based on Bayes' criteria. Our piecewise constant model is a linear Gaussian model. However, this model can have two Gaussian disturbances with prior probabilities associated with each. The online estimation of this kind of model can also be performed by alternate algorithms like particle filters ([42], [43], [44]) by considering the disturbance as bi-modal Gaussian as in [39]. In future, we can also compare and evaluate the performace of our modified Kalman Filter based algorithm with algorithms based on particle filters.

5. Based on the methodology and concept in this thesis we can also extend the new beta tracking technique to multifactor models such as Fama-French three factor model. The three factor model was proposed in [25] after the invalidation claim of CAPM in [7] by the same authors. The three factor model includes HML (High-Minus-Low) indicating

book-to-market ratio and SMB (Small-Minus-Big) indicating market capitalization as two more factors in addition to market premium that describes the return on stock or portfolio. Thus, using three factor model for estimation we can examine how the piecewise constancy in systematic risk gets affected.

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