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Optimization of Culvert Dimensions and Reliability

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OPTIMIZATION OF CULVERT DIMENSIONS

AND RELIABILITY

by

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Abstract

A culvert is a hydraulic structure constructed to increase the water carrying capacity away from highway and buildings in the environment. Culverts have received less attention over the years because they are not highly visible, even if they have sufficient performance. Culverts offer much smaller investment options compared with bridges and in many cases they have replaced small bridges. Culverts are also less hazardous in the case of failure.

This study brings together results about several variables of culverts including optimum dimensions, shapes, materials and inlet configurations. During culvert design, hydraulic testing was required for sizing of structures, where risk analysis calculation has been performed regarding the probability of inadequate capacity of culvert design to pass floods. The failure probability is estimated using the advanced first order second moment (AFOSM) method. Therefore, the methodology ensures the design of the hydraulic structure fulfills the required role, while minimizing its future effects in the environment.

Acknowledgement

First I want to express a word of gratitude to Dr. S. Easa for his great contribution to this research where the results of this thesis would not be the same without his support. Dr. S. Easa is an excellent advisor who has given me the time and dedication by passing on his knowledge in many ways. He has shown me ways on how to eliminate the iterative processes which are related to culvert dimensions, normal depth, and unity factor in the Solver software. His suggestions and requests for including different shapes of culverts, as well as optimizing the different shapes in a single process, have helped me to integrate reliability analysis within the optimization process. The inclusion of cost optimization besides culvert dimensions has led to much more accurate results than having the integration to the end of the process.

For further improvements in the quality of this research, Dr. Easa, has suggested ways to include the probability of failure as a constraint which has involved the optimization of the different shapes in a single process. In addition to efforts of Dr. Easa, Dr.Shehata has also advised me on my thesis and gave me financial support for both my studies and presentations; they introduced the new technologies to me and never doubted about my capabilities in this field.

I also want to express my gratitude to the professors who served as the committee members for this thesis they assisted me and gave me feedbacks to improve this study. I am grateful to Dr. Joksimovic for his help by sharing his knowledge with me. During this work I have collaborated with many colleagues for whom I have great regard, and I wish to extend my warmest thanks to all those in the Department of Civil Engineering in Ryerson University, who have helped me with my thesis.

Also, I would like to thank my daughters for their help and support during my study. Finally, I would like to dedicate all this work to my deceased husband who has believed in me in every work I have done in my career.

Table of Content

List of Tables

List of Figures

List of Appendices

List of Abbreviations

A = Cross-sectional area of flow (m^2)

a to f = Regression coefficients for each type of culvert

A^a = Area, (ha)

AASHTO = American Association of State Highway and Transportation Officials

AF = Full area of cross section of culvert

AFOSM = Advanced First-Order Second Moment Method

AHD = American Heritage Dictionary

A_L = spread wheel load area at the outside top of the pipe, m^2

AR = Flow area for given depth

ASCE = American Society of Civil Engineers

B = Width of the box culvert (m)

BPR = Division of Hydraulic Research, Bureau of Public Roads

C_a = Capacity of any system

CA = Corrugated aluminum

C^a = Runoff coefficient

calc - dc = calculated critical depth flow (m)

C_d = Overtopping discharge coefficient

CDOT = Colorado Department of Transportation

CFR = Code Federal Regulation

CHBDC = Canadian Highway Bridge Design Code

C_i *and* C_j = Values of the partial derivatives

C_m = Antecedent precipitation factors for the rational formula

CM = Corrugated metal

CMP = Corrugated metal pipe

CS = Corrugated steel

D = Interior height of culvert barrel, (m)

dc = Critical depth (m)

D_E = Height of earth cover over the top of the pipe, (m)

D_{ia} = Diameter of circular culvert (m)

dn = Normal depth (m)

d_o = Initial depth (m)

d_{out} = Depth of flow at outlet (m)

F = Function of average outflow discharge through a culvert

F_e = Soil–structure interaction factor

FERIC = Forest Engineering Research Institute of Canada

FHWA = Federal Highway Administration

F_r = Froude number (dimensionless)

g = Acceleration of gravity, $9.81 m/s^2$

G = Vector

G^T = Transposed matrix

GUI = Graphical User Interface

h = Depth of flow (m)

H = Total head

H/B = Ratio of height over base length of culvert

Hc = Specific head at critical depth ($dc + Vc^2/2g$), (m)

H_e = Entrance loss head (m)

H_f = Friction losses head (m)

H_h = Average depth between headwater and low roadway elevation (m)

Hi = Height of arch culvert (m)

h_o = Adopted tailwater depth, (m)

H_o = Exit loss head (m)

H_t = Depth between tailwater and low roadway elevation (m)

H_v = Velocity head

HW = Depth of headwater (m)

HW_i = Headwater depth above inlet control section invert (m)

HWL_i = Allowable headwater elevation at Inlet (m)

HWLmax = Maximum allowable headwater elevation (m)

HWL_o = Allowable headwater at outlet (m)

HW_o = Headwater under outlet control (m)

I^a = Intensity, (mm/hr)

IM = Dynamic load allowance

$K, M, C,$ *and* Y = Constants from as shown in Table 4.2

K_e = Entrance coefficient

k_t = Over-embankment flow adjustment factor

K_u = Constant, (1.811)

K_U = Constant, (19.63)

L = Length of the culvert barrel,(m)

L_e= Effective supporting length of pipe, (m)

L_i = Horizontal length of overflow m

L_o = Load on the system,

L_p = Dimension of A parallel to the longitudinal axis of pipe

LP = Linear Programming Models

LTEC = Least total expected cost

MPP = Most probable value

MTO = Ministry of Transportation, Ontario

MVFOSM = Mean Value First-Order Second Moment Method

n = Manning's roughness coefficient, dimensionless

NBS = National Bureau of Standards

NRCS = Natural Resources Conservation Service

P = Probability of the function

PF = Full wetted perimeter of water surface

P_f = Probability of failure

P_L = total live wheel load applied at the surface, kN

P_w = Wetted perimeter (m)

Q = Discharge or flow rate (m^3/s)

$Q_{3.0}$ = Discharge m^3/s at which HW_i/D = 3.

Q_c = Capacity function of the culvert

Q_L = Peak runoff or Load function of the culvert

Q_o = Overtop discharge

R = Hydraulic radius of the full culvert barrel (m)

RC = Reinforced concrete

RI = Recurrence interval

R_L = Reliability of System

R_o= Outside vertical rise of pipe, (m)

R_s = Risk system

S = Safety margin of the system

S_f = Friction gradient, m/m

S_L= Outside horizontal span of pipe (m)

S_o = Slope of the Culvert

S_p = Span of the arch culvert (m)

T = Top width of channel (m)

TEC = Total Expected Cost

T_r = Return period

TW = Depth of tailwater (m)

TxDOT = Texas Department of Transportation

V = Velocity (m/s)

Vc = Velocity at critical depth, (m/s)

Vd = Downstream velocity of the culvert, (m/s)

Vo = Velocity at outlet of culvert, (m/s)

W = Width or span of culvert, (m).

$W.S$ = Water surface

W_L = Wheel load average pressure intensity, (kN/m^2)

W_{LP}= Live load on top of pipe, kN per linearmeter

W_T = Total live load, kN

x^*_i = Initial design point

x_i = The mean values of the basic variables

Z = Performance function of the system

α = Unit conversion constant, 1.0 (SI)

α_i = Sensitivity factors

β = Reliability index

γ = Central angle

δ = Central angle

μ = Expected value

σ = Standard deviation

Φ = Standard cumulative normal probability function

Chapter 1: Introduction

1.1 Overview

Highway drainage is the process of preventing collection of and removing water from the top, under and within the structure of the pavement. Highway drainage is one of the main factors which have an effect on road design and construction. If the design and construction of a highway is done well, but it has poor drainage, the road will quickly fail due to the infiltration of water into the pavement structure. Improper drainage of roads can cause failures in their usage due to a loss in strength of the pavement materials caused by stripping bitumen from the pavement, while pumping the fine materials from the rigid pavement, hydroplaning, and other detrimental effects. Highway damage due to the water in the pavement can be prevented by controlling the water flowing out of the pavement structure, which is done by several drainage methods. There are two types of drainage useful to highways: subsurface and surface drainage (Fwa, 2006).

Subsurface drainage removes the infiltrated water from the pavement. Some sources of subsurface water are: capillary rise from the ground water table, infiltration through surface cracks, and water seepage from the sides of the pavement. Installing drainage beds in the pavement, using transverse drains, and the application of side slopes on the road surface are proper methods for preventing subsurface drainage. Surface drainage deals with preparations made for taking the water collected on the surface of the pavement, shoulders of the highway, slopes of embankments, cuts sections, and the land adjoining the highway quickly and efficiently away. The collected water is a guide to natural or artificial channels and can be used to prevent water affecting the function of the highway (Flaherty, 1988).

Small open channels are called roadside channels that are constructed as part of a highway drainage system. Roadside channels collect water from the highway pavement and convey runoff to larger channels or culverts within the drainage system (FHWA, 2005). A culvert is one of the drainage systems in the highway and hydraulic structures which convey the flow of water from streams or channels through a road or railroad embankment without overflowing the embankment (PDH, 2009).

1.2 Background

For many years, culverts have been given less attention by engineers than bridges because they are less visible for drivers, especially when performing adequately (Calderon, 2009). In most instances, culvert failure happens suddenly and without warning. The cost of replacing a culvert is high, as it is in any highway related structure. Construction activities cause more economic impact where higher traffic levels are present; therefore a culvert replacement is not recommended in urban areas where there is a high traffic load. Perrin (2006) explains that the most visible and recognizable problems on roadways are roads and bridges because they are easily assessable for inspection and management. This leads to the consideration that culverts are forgotten assets and that thousands of aging culverts under roadways are often ignored (Calderon, 2009).

However, the Ontario Provincial highways have not focused specific interest on the selection of the material type and design of culvert pipes or storm sewer pipes. Early in 1980s, the MTO Drainage Manual was developed to assist designers in the design of hydraulic pipes, and it was updated in 1997 (MTO, 1997). Also, this document indicated a lack of attention among designers about the technological advances in the pipe manufacturing industry, as well as misconceptions concerning performance of pipes and the selection of materials. In the mid-1990s, an attempt was made by the manufacturing industry, supported by the Ministry of Transportation Ontario (MTO), to develop guidelines for selecting the pipes and materials for culverts. Then in 2003, the MTO, with the support of the pipe industry, developed gravity pipe design guidelines and an associated software design package. However, the detailed hydraulic design of a pipe is outside of the scope of this guideline. The objective of the guideline for hydraulic evaluation is to identify a series of alternative pipes with different materials and different inlet structures to determine the minimum design inside diameter from the available pipes meeting the criteria. In other words, this guideline uses a trial and error method in selection of the design pipes based on the nomographs in the design steps (MTO, 2005). Almost all hydraulic and hydrologic designs require an evaluation of peak discharge.

Hydraulic structure design is based on the frequency of flow for a specific return period. A 50-year flood, for example, has a two percent chance of occurring in any given year. Culverts and bridges must pass the peak discharge to prevent flooding or failure of the structure. A prediction of the return period of flows enables designers to impose the most economical structure that is dependable for public safety (Cordes, 1993).

The most inclusive publication available in the literature is the FHWA HDS-5, "*Hydraulic Design of Highway Culverts*", which is a combination of culvert research that includes the classic studies done for the Bureau of Public Roads by the National Bureau of Standards during the 1950s and 1960s. HDS-5 covers conventional culvert design considerations such as, tapered inlets for various types of culverts, storage routing, and special considerations. The HDS-5 developed design methods and equations where optimization is the design principle being used in performance curves, design charts, tables, and forms. HDS-5 defines culvert hydraulics in terms of inlet and outlet control, depending on the factors affecting the head elevation. Regression equations have been developed for each inlet shape to directly represent the headwater as a function of discharge intensity or to compute a loss component that can be added to the critical head to yield headwater. These regression equations apply for a range of discharge intensities which include low flows (Brunner, 2010).

During the late 1980s, there was an increased interest in the hydraulics of long-span culverts, which were commonly proposed as low-cost alternatives to short bridges. Laboratory experiments at the FHWA Hydraulics Laboratory were conducted to investigate the effects of some of the characteristic features of long-span culverts, namely, culvert shape, the span-to-rise ratio, and the contraction ratio. Typically, long-span culverts are nearly the full width of water flow; therefore the contraction ratios are small, where the approach flow velocity is almost as high as the velocity in the culvert. According to these standards, it was found there was no logical explanation for the visible advantage of the high-profile arch at low flows. GKY & Associates, Inc. consolidated design coefficients, including the fifth-order polynomials that were used to calculate the inlet headwater and were used to code computer programs such as HY-8 (GKY, 1998).

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1.3 Objective

This thesis focuses on optimizing the culvert dimensions which meet all the hydraulic requirements. In order to achieve this objective, the following steps were conducted:

1. A literature review was conducted which covered the extensive research undertaken surrounding culvert design in order to identify the important aspects in the design stage of culverts.
2. The optimization of the culvert design was conducted, which focused on the dimensions of the culvert. This stage was performed exclusively without using the earlier conventional trial-error methods.
3. The optimization methodology is developed for two main objectives: minimum cost and minimum probability of failure.

1.4 Thesis Organization

The thesis covers the following chapters organized by this approach:

Chapter 1 covers the detailed background and objective of this research study. Chapter 2 presents a review of hydrology and hydraulics principles, as well as the relation of peak discharge versus frequency, fundamentals of hydrodynamics, and types of flows. Chapter 3 covers detailed information about culverts, the factors which affect culvert design considerations, the hydraulics in culverts, as well as the types of flow controls like inlet and outlet control.

Chapter 4 describes the different design procedures and their respective steps. This chapter also covers the conditions, required formulas, and additional constraints in details. Chapter 5 highlights the factors which affect culvert failure and the interrelation of their importance to economics. This chapter also covers reliability, a significant topic in culvert design. Chapter 6 contains two main parts. The first section explains the methodology for selecting the optimum dimensions for culverts of certain shape, material, and inlet configurations, and the second part covers the performance function which is used in reliability index calculation. The optimum dimensions for the designed culvert will obtain using MATLAB 2011 to minimize the cost and the reliability index is illustrated in Chapter 7. Also this chapter covers the probability of failure, and the reliability of the existing culverts. This analysis was achieved by calculating the reliability index using the advanced first order second moment method.

Chapter 2: Hydrology

2.1 Introduction

Hydrology is the scientific study of the properties, distribution, and effects of water on earth's surface, in the soil, underlying rocks, and in the atmosphere (AHD, 2000). Hydrologic analysis is needed in designing highway drainage in order to determine the amount and frequency of flows. Drainage facilities must not only be hydraulically efficient, but must be taken into consideration in relation to the importance of safety, roads, cost, environmental impacts, maintenance, aesthetics, and legal responsibilities (CDOT, 2004). In the design of highway drainage, floods are usually considered in terms of peak runoff or discharge in cubic meters per second (m^3/s). The peak discharge is used to design storm drain systems, culverts, and bridges. The analysis of peak rate of runoff, volume of runoff, and time distribution of flow are important for highway drainage design.

Any errors in the estimation of these data would result in related errors in the design of the structure, which could be either undersized, causing drainage problems, or oversized, resulting in an unnecessarily large cost for the structure. Because hydrology is not a precise science, different hydrologic methods were developed for determining flood runoff. Engineering judgment and certain federal or state agencies may require (or local agencies may recommend) different techniques to select the proper method for computing the runoff.

2.2 The Hydrologic Cycle

The hydrologic cycle describes the occurrence, distribution, and continuous movement of water, as well as the interrelationship between these factors. The hydrologic cycle also includes various processes associated with the movement of water in nature. Understanding these processes is important for the application of hydrologic models and their improvement. Figure 2-1 shows the precipitation event, and illustrates how some of the precipitation returns to the atmosphere by evaporation and transpiration. A portion of the precipitation infiltrates the soil and eventually reaches a close-by stream or infiltrates the groundwater supply. The remaining water flows overland to streams and lakes (MTO, 1986).

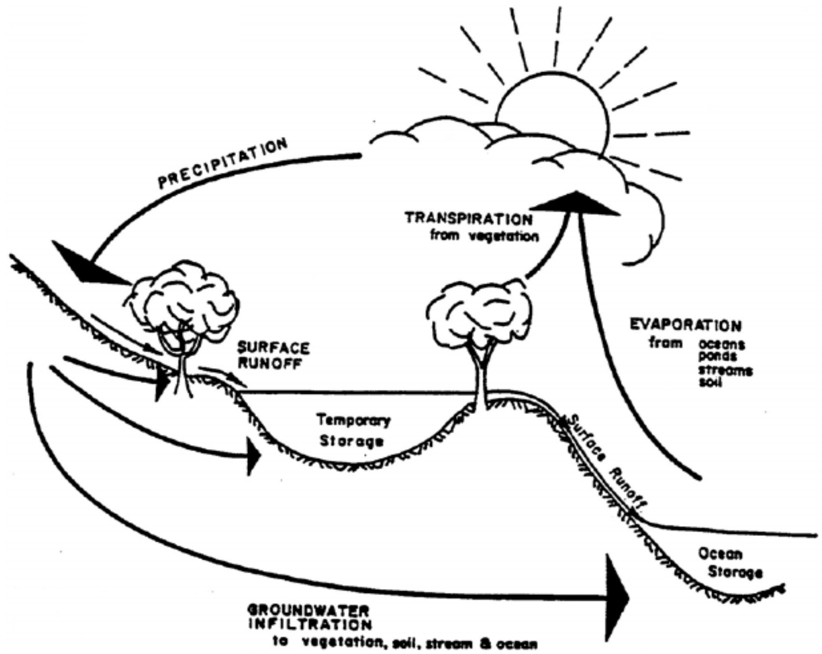


Figure 2-1 Hydrologic Cycle [Source: MTO 1986].

Interception, infiltration and evapotranspiration are factors influencing the discharge of surface runoff into streams. When rain falls on the surface, some of the water evaporates, some infiltrates the soil moving downward to the groundwater, and some is intercepted by vegetation. The amount of water which remains on the surface that can flow into streams is called runoff. Generally, these steps (Nelson, 2010) can be represented as:

$$\text{Runoff} = \text{Precipitation} - \text{Infiltration} - \text{Interception} - \text{Evapotranspiration} \tag{2.1}$$

2.3 The Design Flood

Drainage of highway projects, like all other drainage infrastructures, is designed for a real or theoretical storm event that may or may not happen during the lifetime of the structure. A design flood for a highway is based on some measure of acceptable risk associated with failure and, importantly, the safety of the public. Failure or damage may result when the storms are larger than the design storm. It might be cost-prohibitive and impractical to establish an absolute zero probability failure or damage during a project lifetime. Design criteria will change with the type of project in recognition of the impacts of failure.

2.3.1 Return Period and Probability of Occurrence

A highway drainage designer has to select predetermined discharges in order to avoid significant flood hazards. Flood discharges are referred to as peak discharges where the discharge magnitudes are functions of their average frequency of occurrence (TxDOT, 2009). In flood estimation, the term frequency, also referred to as the return period, *Tr*, is the average number of years between occurrences of a discharge equal to or greater than a given rate. The probability of occurrence is the reciprocal of the return period. For example, a 25-year flood, or a flood with a return period or frequency of 25 years, has a probability of occurrence, in any given year, of 1/25 or 4% (MTO, 1986).

Table 2-1 represents various flood frequencies in relation to the life of the structure. As an example, from the table it can be seen that during the 50-year life of a culvert, there is a 99% probability of flood in 10-years and there is a 40% probability of flood in 100-years.

Table 2-1 Probability of Occurrence for Design Frequencies [Source: MTO, 1986]

Average Return Period (Years)	Probability of Exceedance During: n-years of life of structure (percent)					
	2.3	5	10	25	50	100
2.33	73	94	100	100	100	100
5	41	67	89	100	100	100
10	22	41	64	93	99	100
25	9	18	34	64	87	98
50	5	9	18	41	64	87
100	2	5	9	22	40	64
1000	0	1	1	3	5	10

In most cases, it is not economical to design a structure for the maximum runoff, therefore, a design frequency must be determined in advance. This predetermined frequency will illustrate the probability that the chance of flood will be present or exceed, to symbolize the runoff in a given year. The design of highway features requires a hydrologic analysis to determine the magnitude and frequency of flows, as well as hydraulic analysis to locate the drainage size facilities. Drainage facilities must not only be hydraulically efficient, but must also be dependent on factors such as, safety, initial cost, aesthetics, environmental considerations, maintenance and legal responsibilities.

The flood frequency relation is typically represented by a flood frequency curve. In Figure 2-2 the ordinate represents the discharge and the abscissa provides the probability of occurrence that is expressed as the return interval (Years).

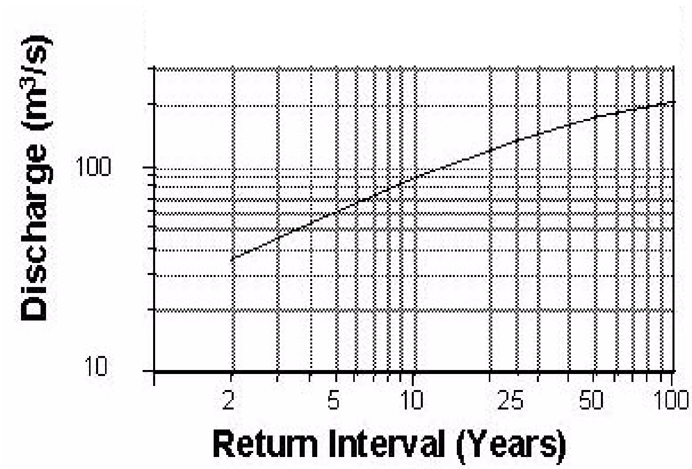


Figure 2-2 Typical Flood Frequency Curve [Source: TxDOT, 2009].

In addition to peak discharges, the flood volume and time distribution of runoff are covered in the design. Flood hydrographs can be used to transmit floods through culverts, flood storage structures, and other highway facilities (TxDOT, 2009).

There are many factors that affect the selection of design return period, such as the importance and the level of services of the appropriate highway, future development, possible hazards to nearby property, and fund limitations.

Structures with high design return period usually have higher capital costs, but lower operational costs and this applies vice versa. In other words, the higher the design return period, the higher the capital cost, lower operational cost (maintenance cost), and the structure can last for a long life time. Figure 2-3 shows a graph of the cost for design alternatives of varying design return periods. The best design balances the capital costs with the operational costs to find the lowest total cost as shown in Figure 2-3. As a result, the objective of cost optimization is to select a design return period that will result in a feature meeting all the design requirements with the total lowest cost (TxDOT, 2009).

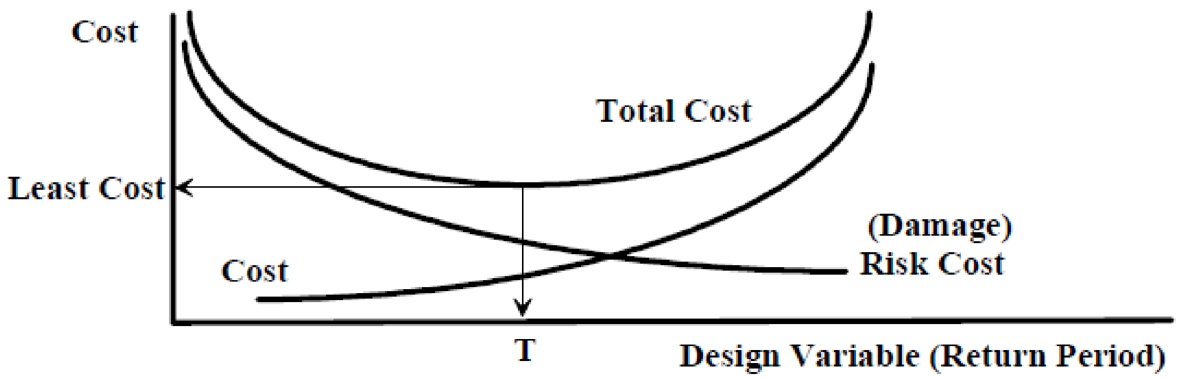


Figure 2-3 Optimization of Total Cost for Drainage Structure [Source: Guo, 1998].

2.4 Factors Affecting Floods

There are many factors that affect floods in the hydrologic analysis for a drainage structure design. Factors which need to be recognized and considered on a site are:

- Characteristics of the drainage basin, including: size, shape, slope, length, land use, orientation, vegetation, geology, soil type, surface infiltration, watershed development, elevation and storage.
- Characteristics of the stream channel, including: geometry and configuration, roughness, slope, natural and artificial controls, and channel modification.
- Characteristics of the floodplain, including: vegetal cover and channel storage.
- Meteorological characteristics, including: amount of precipitation, type of precipitation (rain, snow, hail, or combinations), and storm direction (CDOT, 2004).

In addition to these factors, other variables must be considered in determining a design flood, such as cost, the importance of the facility, the hydraulic integrity of the structure, and the human consequences of failure. Each of these are factored in establishing the risk, and hence the design flood magnitude (MTO, 1986).

2. 5 Design Flood Estimation Methods

Runoff can be calculated from the design rainfall which is obtained from rainfall-runoff transformation methods by taking the hydrologic losses into account, such as infiltration and the frequency of occurrence of floods. Normally, rainfall data are more easily achieved than runoff measurements in streams, and therefore design rainfall are frequently used. However, the results of the rainfall-runoff approach are considered less accurate than those involving methods that utilize streamflow records.

Several methods are available which can be grouped into two wide categories:

1. rainfall-runoff transformation methods; and
2. analysis of streamflow records.

Figure 2-4 provides a listing of the different methods. The peak flow can be calculated with one of these hydrological methods:

• The Rational Method

• Regional Frequency Analysis Methods

  - Modified Index Flood Method

  - Northern Ontario Hydrology Method

• Single Station Frequency Analysis Method

• Hydrograph Methods

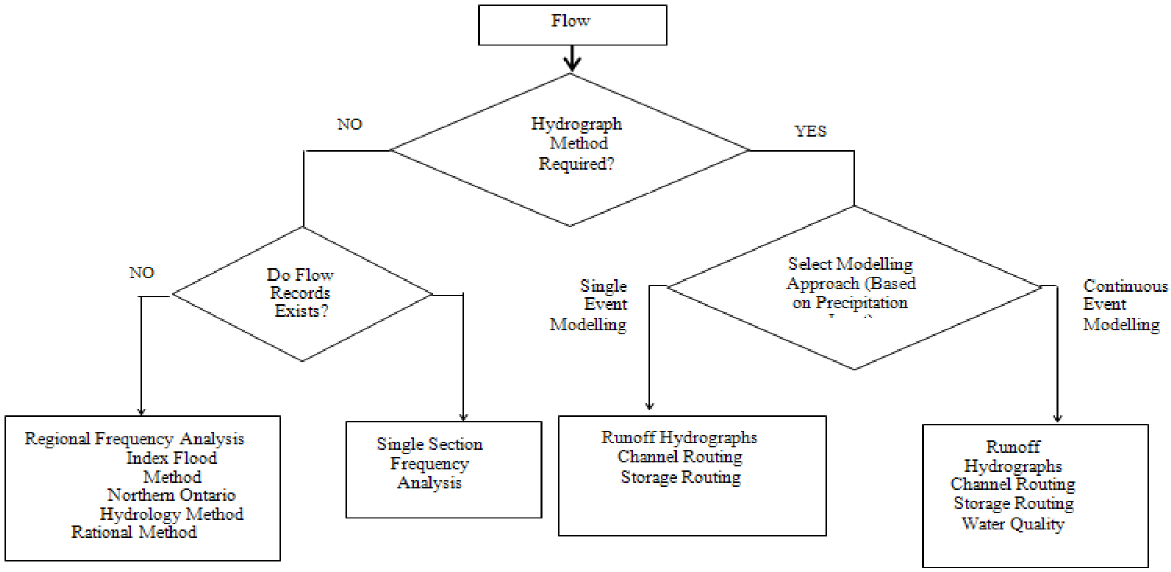


Figure 2-4 Methods for Estimating Design Flow Rates [Source: MTO, 1986].

2.5.1 Rational Method

The Rational method is developed early in the process to calculate peak flows. Regardless of the availability of advanced techniques using computers, it remains a valid method for estimation of the peak flow for small drainage areas.

The method is expressed as follows:

$$Q_L = (C^a * I^a * A^a) / 360 \tag{2.2}$$

where;

Q_L = peak runoff rate, m^3/s;

C^a = weighted runoff coefficient for the catchment area;

I^a = rainfall intensity, mm/h;

A^a = drainage area, ha.

One of the applications of the Rational Method is to determine the peak flows to size channels, sewers, ditches and culverts (MTO, 1986).

2.5.2 Regional Frequency Analysis

This method uses regional watershed and climatic characteristics to compute peak flows. It is easy to apply, requires limited data, and is widely used for unmeasured watersheds. It is one of the most accurate methods for analysis of medium to large rural watersheds with design flow return periods up to 100 years.

2.5.2.1 The Modified Index Flood Method

The Index Flood Method was developed by the U.S. Geological Service and modified for MTO in 1986 to reflect Ontario conditions (MTO, 1986). The development of the index flood method required developing a regression equation expressing an index flood, of a specific return period, in terms of watershed characteristics (slope, shape, detention) and climate, within a homogeneous region. The modification of the Index Flood Method (MTO, 1986), modifies the USGS method of using the 2-year index flood, and uses instead the 25-year flood, as the peak flood with an annual probability of being equaled or exceeded of 4%. The modification is applied because the 25-year event is the most widely used return period for bridges and culverts.

2.5.2.2 The Northern Ontario Hydrology Method

This method, developed by the Civil Engineering Department of Queen's University in Kingston, Ontario for the MTO fulfills the need to provide realistic flow rates for the design of water crossings across unmeasured streams with small to medium watershed areas (1 km2 < A^a < 100 km2) in northern Ontario. The previous method based on rainfall data only, is of questionable applicability to northern Ontario watersheds where inland lakes have a pronounced effect on the rainfall-runoff relationship. Therefore, this method is based on flood quantities estimated using probabilistic/statistical methods with data from 15 stream gauge stations across northern Ontario (MTO, 1994).

2.5.3 Single Station Frequency Analysis

The basic approach in this method is the statistical frequency analysis used to determine the magnitude of a design flood. Estimation of the design discharge relies on the annual floods recorded at a stream gauging station, and is statistically correlated to provide a reasonably accurate estimate for the design discharge. The method involves interpretation of past stream flow data and derivation of a probability of occurrence by fitting a data series into a theoretical probability distribution. The discharge corresponding to the required design frequency may then be read from the distribution function curve. The major limitations in this method are the availability of a suitable length of stream flow records and the quality of data available. In this way, a short period may represent a non-typical wet or dry period that may not include any major floods. The accuracy of this method increases with the number of years of records (MTO 1997).

2.5.4 Hydrograph Method

Figure 2-5 shows a hydrograph graph which represents discharge versus time at a given point in a stream. This graph is a result of calculating the runoff processes containing overland flow, interflow and baseflow. The hydrograph shape is a reflection of the physical and meteorological conditions in a watershed. A hydrograph contains three parts, the rising limb, the crest and the recession limb. The shape of the rising limb represents the basin physiographic features, and the rainfall characteristics, such as duration of rainfall, uniformity of distribution over the basin, and intensity. The crest expands between the inflection point of the rising curve and recession limbs. The peak, which occurs within the crest portion, usually shows that all areas of the watershed are contributing flow.

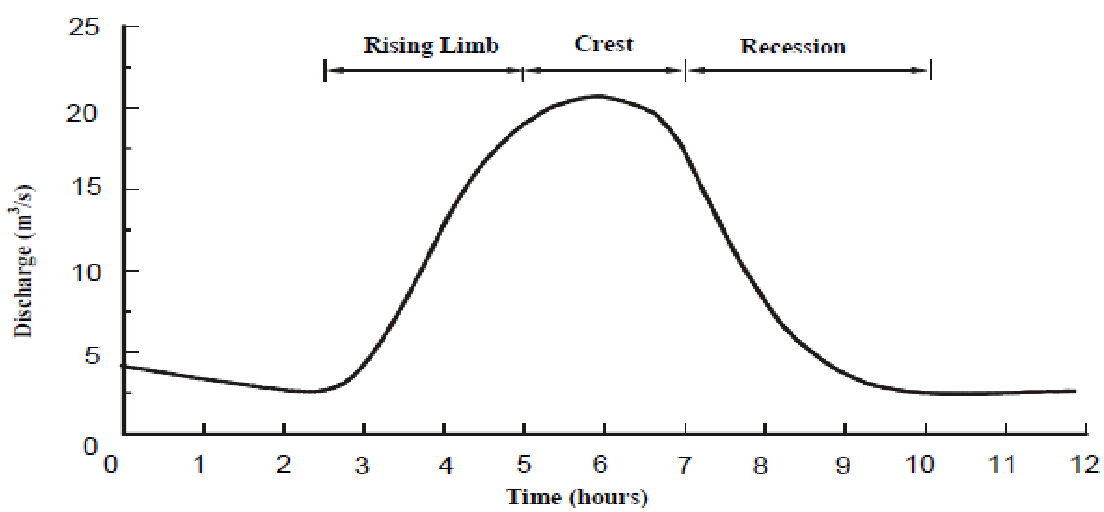


Figure 2-5 A Typical Hydrograph [Source: MTO, 1986].

2.6 Hydraulic Principles of Drainage Systems Design

Water can move in all directions or it can be channeled in a direction, such as when water is flowing in the pipe, the particles move uniformly ahead, some move sideways and the others move at different speeds. Two basic parameters are used to define the water motion in a pipe, velocity V *(m/s^2)*, and discharge Q_L *(m^3/s)* (Gribbin, 2007) .

2.6.1 Types of Flow

The flow velocities within the channel cross section are not uniformly distributed. This is due to friction along the channel bed, banks and free surface. The velocity of water near the bed and banks is the slowest because of the highest friction against the bed and banks, while the highest velocity is typically found just below the free surface (DOTMR, 2010). Therefore, the flow of water is classified in different ways to help in the study of hydraulic problems and determining both the critical and normal depth which would occur in the culvert is necessary.

The basic of the flow types are as follows:

1. Laminar flow and turbulent flow: laminar flows illustrate the smooth flow and the low velocity flow of the water. The water movements are parallel and there is no cross current. As the velocity increases, the flow becomes cross current and the flow becomes rougher and is associated with energy loss due to the interactions between the water and the wall of the pipe.
2. Uniform and non- uniform flow: this is the flow when the depth is the same at every cross section along the channel. This is possible when the slope and roughness remain constant along the channel, while non-uniform flow is the depth of flow changes along the length of channel, including a change in pipe size or flow from a reservoir into a channel.
3. Steady flow and unsteady flow: in steady flow, the flow occurs when the discharge is not changed over time, while unsteady flow occurs during rapid change of the discharge (Gribbin, 2007).

2.6.2 Continuity Equation

The continuity principle is usually assumed that discharge is constant at any cross section within a reach under uniform and steady flow conditions. The basic equation for analyzing the flow in the channel describes the relationship between flow rate, velocity and the cross sectional area of flow (MTO, 1986). As shown in Equation 2.3:

$$Q = V \, x \, A \tag{2.3}$$

where;

Q = flow rate (m^3/s);

V = flow velocity (m/s);

A = cross-sectional area of flow (m^2).

Also this equation forms the basis of Continuity Equation theory. This theory illustrates simple analysis over the changes in the channel without considering the cross section, roughness or slope and, as well, the theory assumes that the discharges are equal between sections. (i.e., Q_1 = Q_2).

The Continuity Equation is:

$$Q_1 = A_1 \, x \, V_1 = Q_2 = A_2 \, x \, V_2 \tag{2.4}$$

Subscripts 1 and 2 in Equation 2.4 denote cross sections 1 and 2 as illustrated in Figure 2-6 (MTO, 1986).

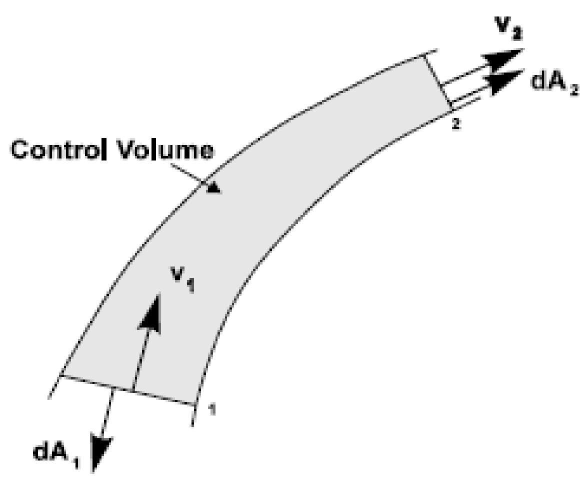


Figure 2-6 Continuity Equation at Cross Section 1 and 2 [Source: MTO, 1986].

2.6.3 Energy Head

Energy head concepts are used widely in solving hydraulic problems. They describe the energy of the water per unit weight of water and units of length used. The most common form of energy head is described as follows:

1. Position head: defined as the energy per unit weight of mass of water due to its height above specified datum.
2. Pressure head: defined as the energy per unit of mass of water due to the pressure exerted from above.
3. Velocity head: defined as the kinetic energy per unit weight of mass of water due to kinetic energy resulting from its motion.
4. Head loss is the loss of energy due to friction and turbulence (Gribbin, 2007).

2.6.4 Critical depth *(dc)*

Critical flow takes place when the summation of the kinetic energy (velocity head) and the potential energy (static or depth head equal to the depth of the flow) for a given discharge is at a minimum. On the other hand, the discharge through a channel with a given total energy head will be maximum at critical flow. The depth of the flow is defined as critical depth, and the slope required for producing critical flow is defined as critical slope (FHWA, 2005). A numerical method to calculate actual critical depth is developed when using the Froude number as an index. The Froude number is a simple and convenient way of classifying flow for purposes of channel design, and other uses.

The Froude number is the ratio of inertial to gravitational forces and is expressed as follows:

$$F_r = Q/\sqrt{g(A^3/T)} \tag{2.5}$$

where;

T = top width of channel (m);

g = acceleration of gravity, 9.81m/s^2;

F_r = Froude number (dimensionless).

The Froude number is used to classify the state of the flow, as follows: subcritical flow is distinguished from supercritical flow by Froude number (F_r), representing the ratio of inertial forces to gravitational forces. When $(F_r < 1.0)$ is defined as the subcritical flow, it has a deeper and slower velocity flow. Supercritical flow $(F_r > 1.0)$ is described as rapid, shallow, high velocity flow, and with flow depth less than critical depth. Whereas the critical flow occurs when $F_r = 1$, which can be defined as the low velocity to be reached at maximum depth, whereas the water flows in a uniform channel as it reaches constant velocity and depth is defined as a normal depth (FHWA,2005).

2.6.5 Normal depth *(dn)*

Normal depth is defined as the depth of flow in a channel when both the slope of the water surface and the bottom of the channel remain constant. Normal depth takes place when the gravitational force of the water is equal to the friction along the culvert and there is no acceleration in flow, as well as when it is outside the influence of the inlet and outlet tailwater.

Several conditions can occur in the channel which can change the flow of the water, such as a change in the channel slope, change in channel cross section or an obstruction build up in the channel like bridge or culvert. The water surface profile is classified according to the slope of the channel bed. The slope of the channel bed can be mild, horizontal, steep, critical, or adverse flow. Subcritical flow occurs in mild slope, where normal depth is above critical depth. Flat slope channel bed results in horizontal flow. A steep slope results in a supercritical flow, where critical depth is above normal depth. Critical slope is a slope that results in critical flow, where normal depth concurs with critical depth. Adverse slope is opposite to the direction of the flow. Figure 2-7 is illustrating the classification of the channel slopes for varied flow.

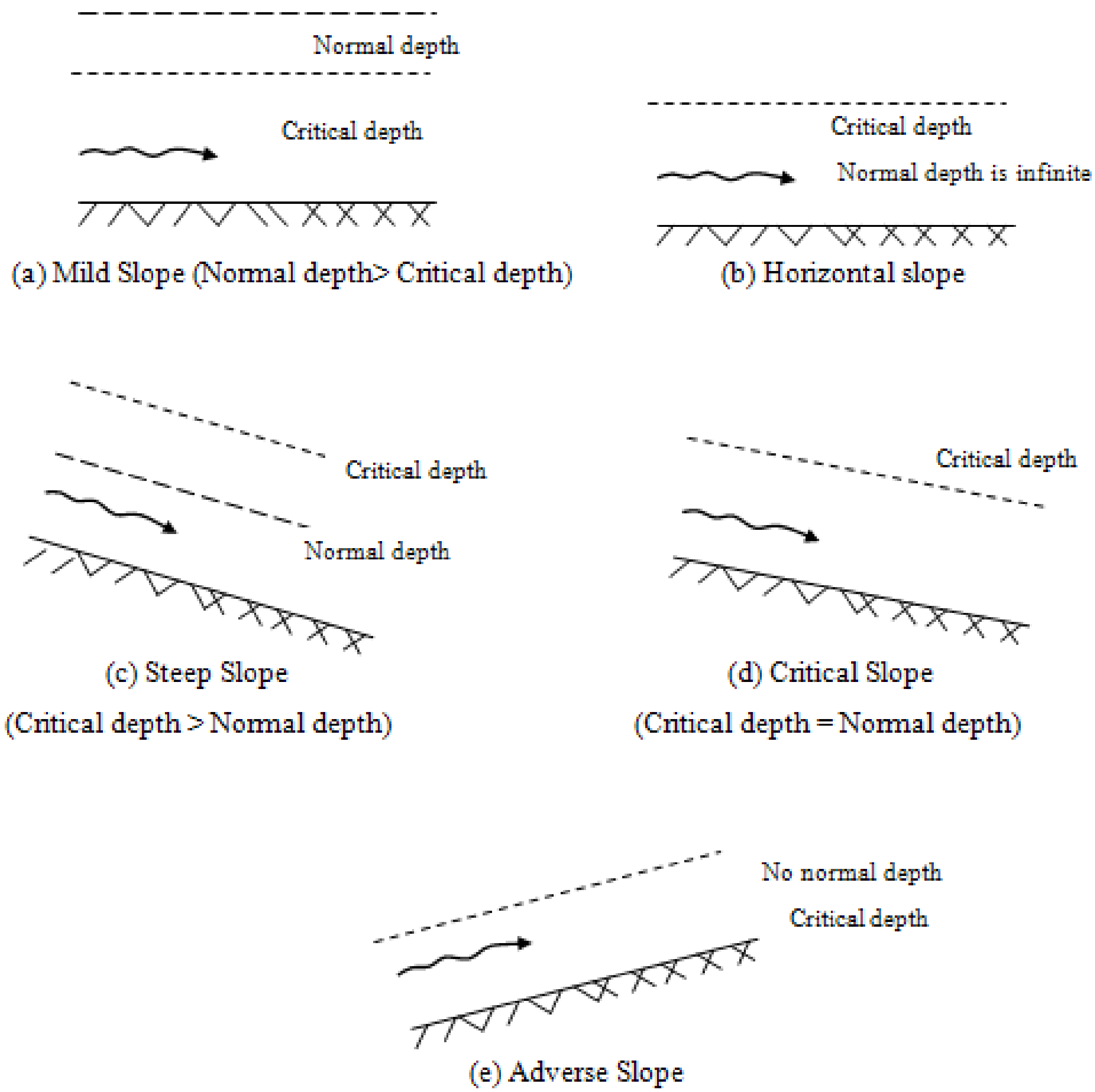


Figure 2-7 Classification of Channel Slopes for Varied Flow [Source: Gribbin, 2007].

2.6.6 Manning's Equation

There are different empirical formulas which have been developed to calculate the normal depth in the uniform channels. The first uniform formula was created by the French engineer Antoine Chezy around 1775, while another formula was developed by the Irish engineer Robert Manning in 1889. Manning's equation is widely used in the United States to calculate the steady, uniform flow in channels (ARMCO, 1958).

$$V = \frac{\propto}{n} R^{2/3} S_f^{1/2} \tag{2.6}$$

This equation combined with the continuity equation determines the velocity of flow at a known discharge

$$Q = \frac{\propto}{n} AR^{2/3} S_f^{1/2} \tag{2.7}$$

where;

Q = discharge, (m^3/s);

n = Manning's roughness coefficient, dimensionless;

A = cross-sectional area of flow, (m^2);

R = hydraulic radius, (m);

S_f = friction gradient, for uniform flow conditions equals the channel bed gradient, S_o (m/m);

α = unit conversion constant, 1.0 (SI).

The hydraulic radius is given by:

$$R = A/P_w \tag{2.8}$$

where;

P_w = wetted perimeter (m).

The wetted perimeter is defined as the length of line where the water touches the surface of the bottom of the channel (TxDOT, 2009).

While the Manning equation is used for determining the normal depth, critical depth *(dc)* in the channel can be calculated by Equation 2.5 to cross section A and T and for given discharge Q by setting the equation equal to $F_r = 1$ (Osman, 2006).

Chapter 3: Culverts

3.1 Overview

Culverts have been used for thousands of years as a way to convey water underneath any construction. Normally, a culvert is open and is not connected by both ends to a drainage structure. Sometimes, a culvert is connected to a drainage system located in the ditch, or median (NYDOT, 2011).

AASHTO gives the following definition for a culvert: "A culvert is a structure 20 feet or less in centerline length between the extreme ends of openings for multiple boxes, usually covered with embankment and composed of structural material around the entire perimeter, which is usually designed hydraulically to take advantage of submergence to increase hydraulic capacity [for conveying] surface runoff through the embankment" (Cordes, 1993).

National Bridge Inspection Standards defined a bridge as “supports erected over a depression or an obstruction, such as water, highway, or railway, and having a track or passageway for carrying traffic or other moving loads, and having an opening measured along the center of the roadway of more than 20 feet [6.1 meters] between under copings of abutments or spring lines of arches.” Traditional bridges will have distinct decks, superstructures, and substructures, whereas culverts are structures “designed hydraulically to take advantage of submergence to increase hydraulic capacity. Culverts, as distinguished from bridges, are usually covered with embankment and are composed of structural material around the entire perimeter, although some are supported on spread footings with the streambed serving as the bottom of the culvert. Culverts may qualify to be considered ‘bridge’ length.” (CFR, 2008).

The hydraulic design of culverts is mainly affected by headwater depth. If the headwater depth is not sufficient to allow for flow through the culvert, the embankment may overtop or the flow may back up causing upstream flooding. Other factors such as pipe size, pipe length, pipe roughness, pipe slope, inlet geometry, and tailwater conditions also influence the discharge through a culvert (Clara, 2007). In most cases, a culvert is simply set up without an extensive consideration of how much water it needs to transmit under severe conditions. Therefore, the proper number of culverts and appropriate culvert diameter must be determined before the installation procedure in order to transmit expected water through the pipe(s) (Edwards, 2011).

A method for designing culverts is to take into the account the suggested critical storm duration. This duration is based on the estimated design floods and a hydraulic design approach proposed to optimize the dimensions and hydraulic variables of the culverts (Kang, et al., 2009).

Culverts are constructed from different materials resulting in many shapes and configurations. Many factors are involved in designing a culvert, such as roadway profiles, flood damage evaluations, channel characteristics, construction and maintenance costs and estimates of service life. A typical culvert is composed of the components shown in Figure 3-1.

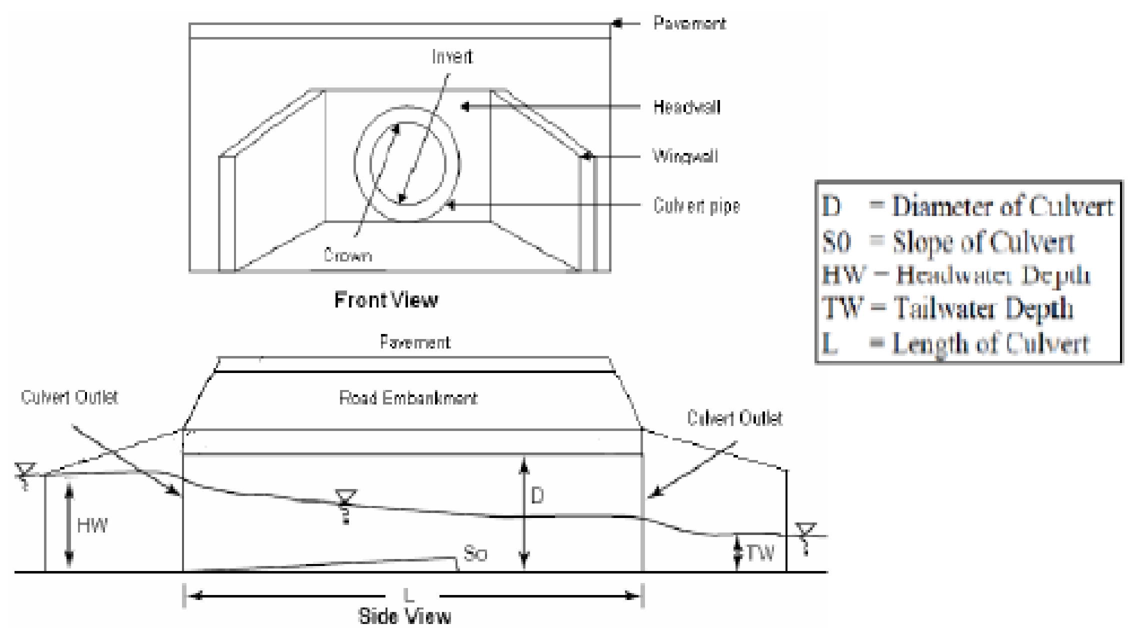


Figure 3-1 Typical Culvert Components [Source: Najafi, et al., 2008].

3.2 Culvert Design Considerations

In culvert design processes, many engineering and non-engineering principles are considered, including factors such as the shape and size of the culvert, materials, inlet configuration, hydrology, hydraulics, location, structure, geotechnical, roadway data, and economics (Normann, et al., 2001).

3.2.1 Shape and size

Culverts come in several cross-sectional shapes and, as shown in Figure 3-2, box, circular, elliptical, pipe-arch, and arch are the most used (Normann, et al., 2001). However, other configurations also include multiple barrels of the illustrated shapes (Najafi, et al., 2008).
Typically, several shapes provide hydraulically adequate design alternatives:

- Box (or rectangular): This type of culvert has the lowest allowable headwater compared to other shapes. Increasing the span dimension can satisfy hydraulic capacity with a low headwater. In addition, multiple barrel box culverts can accommodate large flow rates with low profiles.
- Circular: This is the most common shape used for culverts, and is available in various sizes, strengths, and lower cost than other shapes.
- Pipe-arch and elliptical: Generally used instead of circular pipe, but are more expensive than circular shapes for equal hydraulic capacity.
- Arch: The span of the arch culvert is used as the bottom of a natural streambed. Therefore this type of culvert is good to maintain the natural stream bottom, to accommodate fish passage and natural biodiversity. Scour potential and the structural stability of the streambed must be evaluated. Although, structural plate metal arches are limited in use for low cover situations, they have the advantage of rapid construction with low transportation and handling costs (TxDOT, 2009)..

Selection of the shape of a culvert is a function of several factors, such as construction cost, the limitations of headwater elevation, roadway embankment height, structural performance, potential for clogging by waste, and hydraulic performance (Mahmood, 2004).

The size of the culvert is described by the entering rise and span. The span refers to the highest inside width, while the rise represents the maximum inside height of the culvert. The inside rise of the culvert is important for determining whether the headwater and tailwater elevations are sufficient to submerge the inlet or outlet of the culvert, as well as determining the total flow area of the culvert (Brunner, 2010).

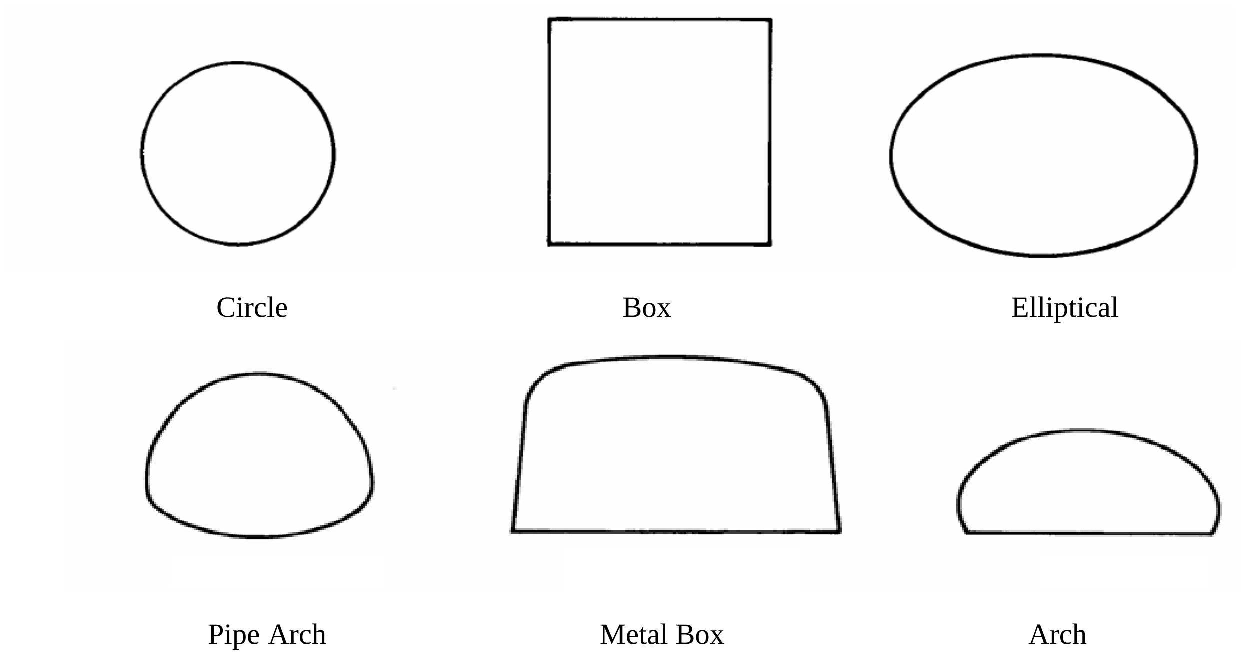


Figure 3-2 Commonly Used Culvert Shapes [Source: FHWA, 2005].

3.2.2 Materials

Culverts are constructed from many different materials. Each material has its own characteristics, advantages, and weaknesses. The three materials most commonly used in culvert construction are: concrete, corrugated metal pipe *(CMP)* and smooth steel. The selection of culvert material depends on hydraulic roughness, structural strength, durability, bedding conditions, and corrosion and abrasion resistance (AASHTO, 2007). In additional to these materials, culverts can be made of vitrified clay, masonry, stone, plastic, wood, fiber, cast iron, and bituminous.

Culverts may also be mixed with other materials to reduce corrosion and abrasion and extend the service life of the culvert. For example, protection of metal culverts from corrosion usually consists of bituminous fiber-bonded coating or mill-applied thermoplastic coating (Calderon, 2009).

3.2.3 Inlets

Water flow entering a culvert is usually subjected to a severe contraction. The contraction is less severe with rounded and smooth inlet section culverts, where a large number of different inlet configurations are developed on culvert barrels. As shown in Figure 3-3, the main types of culvert entrances are: projecting entrance, wing wall entrance and mitered entrance which are set flush having a sloping embankment on a vertical wall. There are some factors also considered in the selection of inlet configurations, such as structural stability, erosion control, aesthetics, and fill retention.

One other important factor which affects the hydraulic capacity of a culvert is the inlet geometry. The hydraulic capacity of a culvert may be improved by appropriate inlet selection. Since culvert barrels are narrower than the natural channel, the culvert inlet edge represents a contraction flow. The shape of the entrance is one of the factors which has an effect on the hydraulic property of flow as shown in Figure 3-4. An ideal design has an inlet such that no flow contraction occurs allowing full flow through the entrance of the barrel downstream (Driss, 1988).

In situations of a more steady flow, change will reduce the energy loss and thus create a more hydraulically efficient inlet condition so that the beveled edges are more efficient than square edges. Side-tapered and slope tapered inlets are usually addressed as improved inlets because their geometry can reduce the flow contraction and increase the culvert efficiency as represented in Figures 3-5(a) and 3-5 (b).

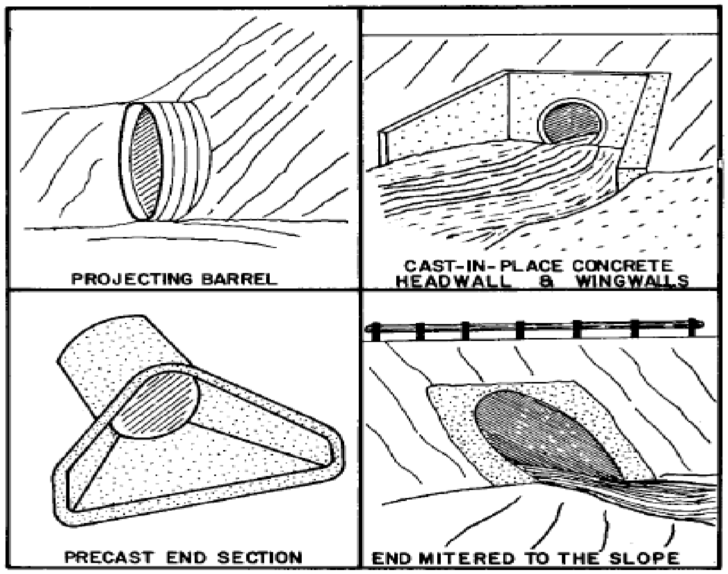


Figure 3-3 Four Standard Inlet Types [Source FHWA, 2005].

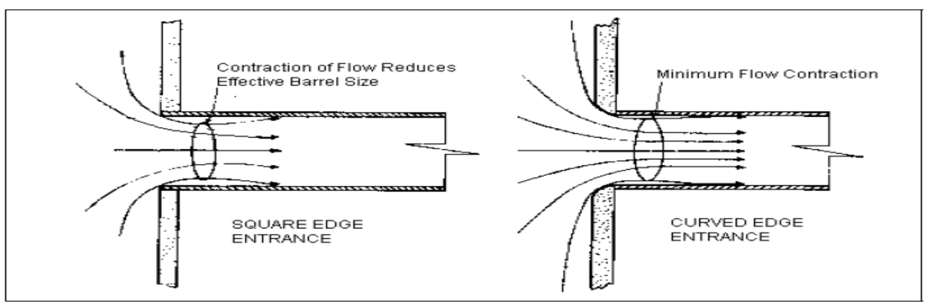
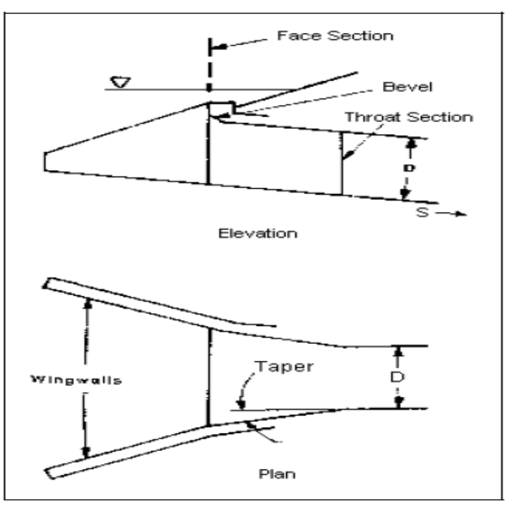


Figure 3-4 Entrance Contraction [Source: FHWA, 2005].



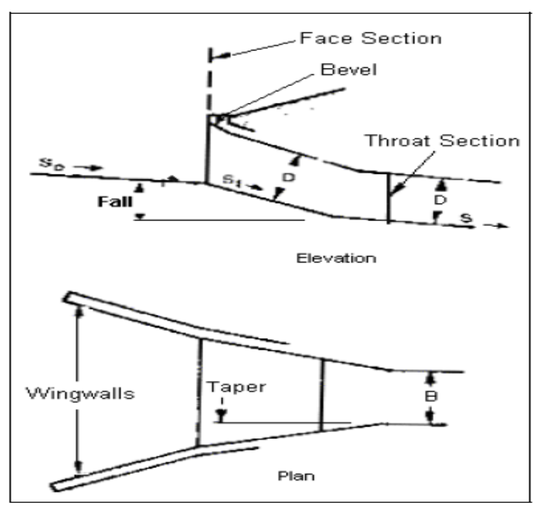


Figure 3-5 (a) Side-tapered inlet Figure 3-5 (b) Slope-tapered inlet

[Source FHWA, 2005].

3.2.4 Hydrology

Water and water resources are very important factors when designing highway locations. Effects of the drainage system, river characteristics, potential flood hazards, and the general environment on any road or structure should be considered in the overall design (AASHTO, 2007). For this reason, hydrology or hydrologic analysis for culvert design should involve estimation of a design flow rate based on climate and watershed characteristics. Hydrologic analyses often contain statistics to estimate the return period concept, which is the frequency of occurrence of rare events such as floods, peak design flow, hydrographs and hydraulic cycle. Therefore, a wide hydrologic analysis may be needed in culvert designs (Normann, et al., 2001).

3.2.5 Hydraulics

Hydraulic design may decide whether the barrel of a culvert may fully or partially flow over its length. Water surface profile calculations are used to determine how much of the culvert barrel flows full or under pressure. A high downstream water surface elevation or a high upstream water surface elevation causes a pressurized flow. A free surface flow, open channel flow or partly full flow in the culvert barrel can be grouped as subcritical, critical or supercritical, determined by calculating the Froude number. In addition to the flow conditions, culvert hydraulic design is based on how the flow can be controlled, which is divided into inlet and outlet control (Normann, et al., 2001). This topic will be explained in detail in section 3.3.2.

3.2.6 Location

A culvert location compromises the horizontal and vertical alignment of a culvert with respect to the stream and the road (AASHTO, 2007). The best way to minimize the costs associated with excavation and channel work is to locate the culvert in the existing channel bed (Normann, et al., 2001). However, this is not always possible. Some streambeds are twisting and cannot accommodate a straight culvert (HDS 5, 1985). In any case, culvert barrels should be placed in the existing channel bed because this is the cheapest location requiring the least earthwork and re-routing of the water.

Exemptions may include mountainous areas where the channel needs a long culvert, or winding channels that need a straight culvert (Fwa, 2006). Figure 3-6 displays two examples of culvert location procedures. In case (a), the culvert follows the natural channel alignment. In case (b), the channel has been relocated to reduce the culvert length. Brice (1981) concluded that minor channel relocations for culvert alignments have been successful unless the natural channel was already unstable.

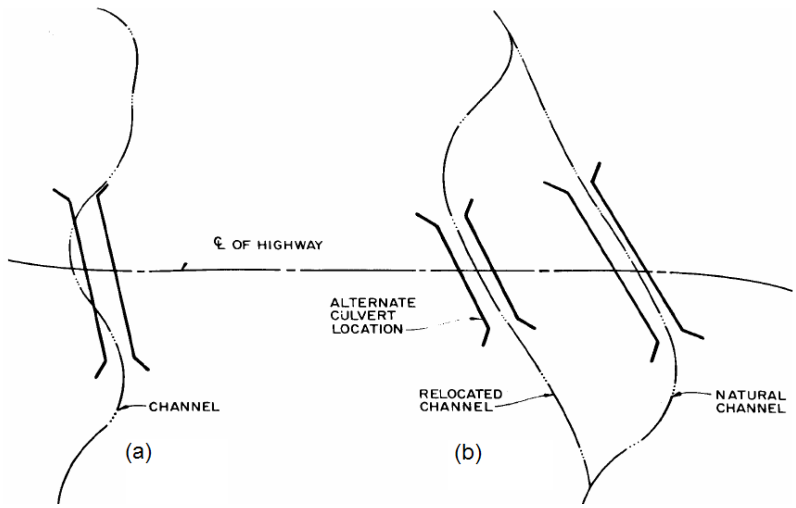


Figure 3-6 Culvert Location Methods [Source: HDS 5, 1985].

3.2.7 Structure

The proper structural design of a culvert must be carried out to ensure the culvert is strong enough to resist the imposed traffic or earth loads that act upon it (Ballinger, et al., 1995). The structural design of the culvert is concerned with the analysis of moments, thrusts, and shears caused by embankment and traffic loads, and by hydrostatic and hydrodynamic forces. The structural strengths of a culvert barrel can be divided as flexible behavior or rigid behavior.

Flexible behavior is built on the assumption that the culvert barrel and the soil act as one structure. According to this assumption the vertical loads are transmitted from the culvert to the surrounding soil, allowing the reduction of the vertical diameter and the expansion of the horizontal diameter. The rigid behavior theory assumes that the barrel is so rigid that it can carry vertical loads without relation to the surrounding soil. In this case, when vertical loads are applied to the culvert, zones of tension and compression are created in the culvert structure (Najafi, et al., 2008).

3.2.8 Geotechnical

The conditions of a culvert's foundation must be investigated before the design procedure. In particular, factors such as soil classification, allowable bearing pressure, groundwater table, slope stability, bedding requirements, erosion control and rock excavation require attention (Calderon, 2009).

3.2.9 Roadway Data

The proposed or existing roadway effects on hydraulic capacity, culvert alignment, and culvert cost also require consideration. Roadway data are the first information which may be provided to the culvert designer because they are easily obtained from preliminary roadway drawings or from standard details on roadway sections. The culvert cross section, length and the longitudinal roadway profile may be subproducts of the roadway data evaluation (Normann, et al., 2001).

3.2.10 Traffic Safety

Generally, drainage is one of the most important aspects in a highway design project. Cross- and longitudinal drainage are necessary to relieve drainage from the natural phenomenon of runoff to the highway drainage system. However, due to their fixed nature, they can also cause a safety risk to drivers and passengers. Therefore, culvert ends must be a smooth, clean way of mitigating unsafe conditions, in addition to its original purpose as a drainage structure. Culverts must be designed in accordance with the AASHTO Roadside Design Guide to minimize hazards for vehicles that run off the travelled way. Accordingly the placement of culvert headwalls and endwalls must be at an appropriate safe distance from the travelled way (CIRIA, 2010). For example, using mitered end sections will increase the hydraulic head loss; on the other hand, a non-reinforced mitered end may affect the structural integrity of the culvert.

Also, protection of the culvert end by metal fence is a conventional method and has proven very effective for safety. However, a metal fence can also be more expensive than a safety end treatment. Generally, if clear zone requirements can be met, neither safety end treatment nor protection, such as a guard fence, are necessary (TxDOT, 2009).

3.2.11 Ice Buildup

Ice buildup should be considered during the design procedure of a culvert by evaluating the potential of flood damage from a blocked culvert. Increasing the height and width of the designed culvert may become necessary in order to prevent property damage.

3.2.12 Debris Control

The designer should search for information related to the type and the amount of debris expected during a major flow. Since it is not easy to calculate the volume by visual observation of the basin, only the historical data from previous flows of the site would be reliable. The designer should be much more concerned about this problem, when the culvert is located in mountainous or steep regions, where culverts are under high fills, and where clean-out access is limited.

3.2.13 Economy

In the design of culverts, several economic aspects should also be considered both for newly constructed or for major repairs to existing culverts, such as construction costs, maintenance costs, replacement costs, estimation of service life, risk of failure and risk of property damage (Ballinger, et al., 1995). Less attention to the economic aspects will lead to an increase in the initial construction cost of the culvert to accommodate floods along its design life and also prevent failure of culvert (Normann, et al., 2001). However, it is important to consider that the most economical culvert installation is the one with less total cost over the design life period (Ballinger, et al., 1995).

3.3 Culvert Hydraulic

Each culvert shape has different hydraulic properties and each material has a different wall roughness while both factors influence hydraulic operations (HDS 5, 1985). The whole analysis of the hydraulics of a culvert is time-consuming and difficult. Flow conditions change from one culvert to another and also change over time for any given culvert. When the flow in a stream runs into a culvert, it contracts, thereby causing a change in flow depth. Water backs up as it is waiting to go into the culvert barrel, yet once it enters the barrel, it speeds up (Equation 2.4). The slow moving water upstream of the culvert has an increased depth, and the faster water is shallower. The barrel of the culvert may flow full or partly full according to several factors.

These factors are the size of the opening (cross sectional area), length of the culvert, slope of the culvert, roughness of the culvert, entrance geometry, and the downstream depth of flow (tailwater) (ARMCO, 1958).

3.3.1 Flow Conditions

A culvert barrel may flow full over or partly full along its length. In rare cases, the barrel flows in full flow where at least part of the barrel flows partly full.

a. Full Flow: Pressure flow is called the full flow hydraulic condition in the culvert barrel. If the cross-sectional area of the culvert in pressure flow were increased, the flow area would expand. However, the back pressure caused by a high downstream water surface elevation can create pressure flow in a culvert, where the high upstream water surface elevation can produce full flow. Regardless of the cause, the capacity of a culvert under pressure flow is affected by the hydraulic characteristics of the culvert, by upstream and downstream conditions.

b. Partly Full (Free Surface) Flow: Free surface flow or open channel flow may be grouped as subcritical, critical, or supercritical. A determination of the suitable flow is done by evaluating the dimensionless number, Fr, called the Froude number as it has been mentioned previously in the types of flow section 2.6.1 (HDS 5, 1985).

3.3.2 Types of Flow Control

Inlet and outlet control are the two basic types of flow control in the culvert:

a) Inlet Control: occurs when the culvert barrel is able to convey more flow than the inlet will accept. In other words, the inlet control occurs when the flow capacity of the culvert entrance is less than the flow capacity of the culvert barrel, where the control section is located just inside the entrance in the case of inlet control flow.

   1. Different examples of Inlet Control are shown in Figures 3-7 to 3-10. The type of flow depends on the submergence of the inlet and outlet ends of the culvert. For all these examples, the control section is at the inlet of the culvert. Depending on the tailwater, a hydraulic jump may occur downstream of the inlet. Figure 3-7 represents a condition where both of the inlet and the outlet end of the culvert are unsubmerged. The flow in

the barrel is supercritical and the critical depth just passes downstream of the culvert entrance. The barrel flows partly full over its length, and the flow approaches normal depth at the outlet end. This flow can be seen during low flow condition.

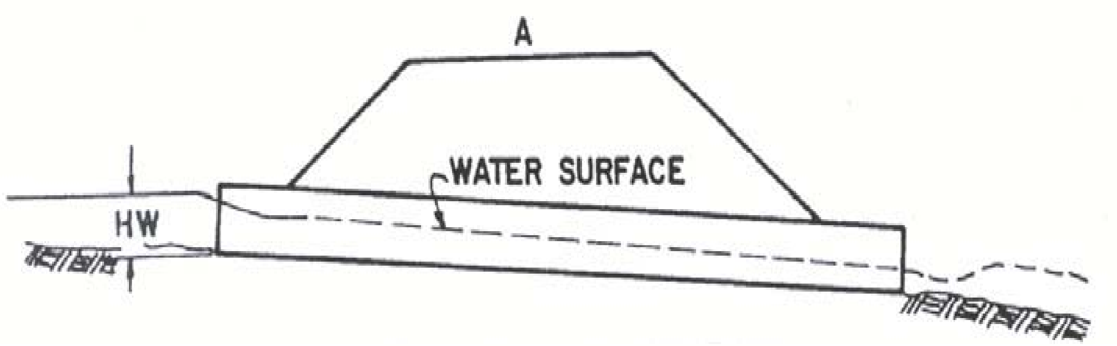


Figure 3-7 Inlet and Outlet Unsubmerged [Source: FHWA, 2005].

Figure 3-8 shows that the outlet end is submerged while the inlet end is not. In this case, the flow just downstream of the inlet is supercritical and a hydraulic jump forms in the culvert barrel.

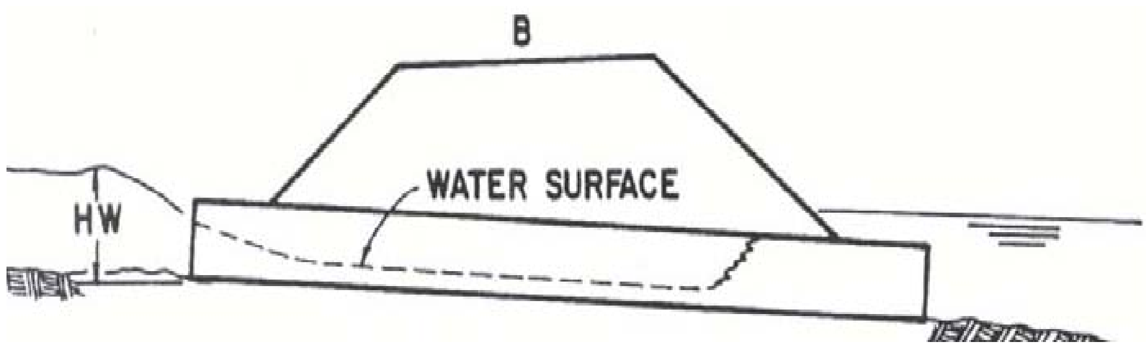


Figure 3-8 Inlet Unsubmerged, Outlet Submerged [Source: FHWA, 2005].

Figure 3-9 represents a more classic design condition. The inlet end is submerged and the flow at the outlet end is freely flow. Again, as in Figure 3-7, the flow is supercritical and the barrel flows partly full over its length. Critical depth occurs just downstream of the culvert entrance, and the normal flow occurs at the downstream end of the culvert.

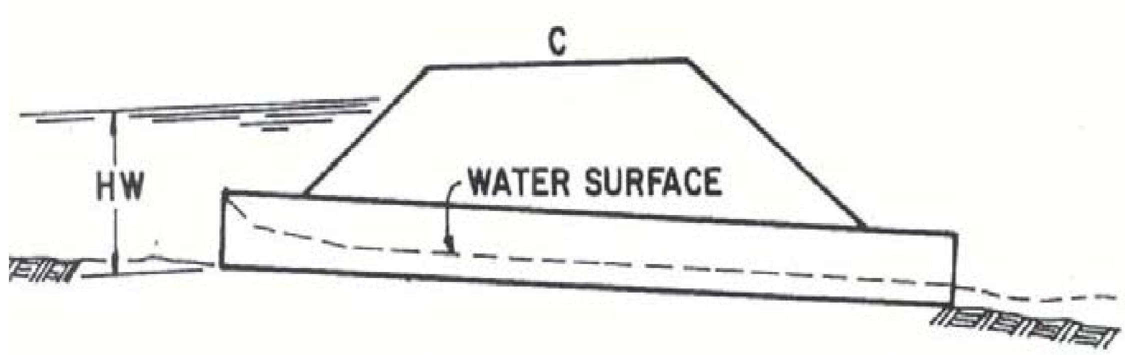


Figure 3-9 Inlet Submerged [Source: FHWA, 2005].

Figure 3-10 is an abnormal situation showing that even submergence of both the inlet and the outlet ends of the culvert does not assure full flow. In this case, the median inlet provides ventilation of the culvert barrel to prevent hydraulic jump in the barrel. This procedure will help the ventilation of the culvert barrel. However, if the barrel was not ventilated, sub-atmospheric pressures could occur which might create an insecure situation during which the barrel would exchange between full flow and partly full flow (FHWA, 2005).

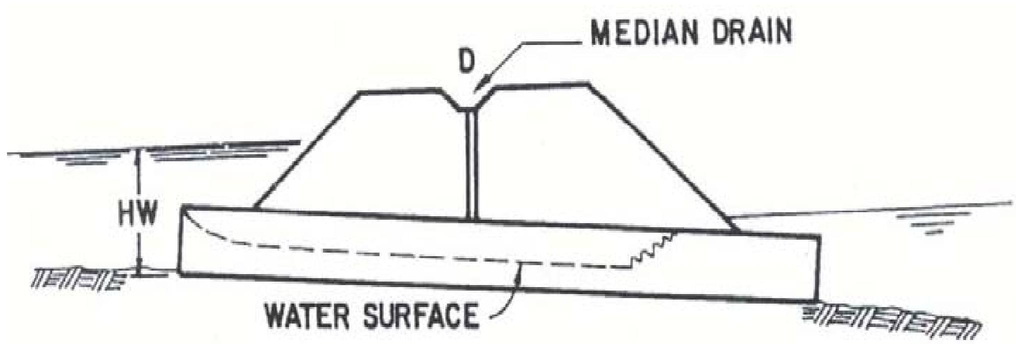


Figure 3-10 Inlet and Outlet Submerged [Source: FHWA, 2005].

2. Factors affecting Inlet Control: For inlet control design, culvert capacity is determined by the entrance opening area where the culvert never flows full (HDS 5, 1985). To calculate the headwater elevation for a culvert operation under inlet control only the rate of flow Q and the shape and the size of the entrance must be taken into

consideration. Barrel length, slope, roughness, and tailwater depth are not important for the inlet control flow as shown in Table 3-1 (ARMCO, 1958).

Entrance geometry also has a significant effect on a culvert operating under inlet control. The inlet area is the cross-sectional area of the face of the culvert. Typical inlet edge configurations are thin edge projecting, mitered, square edges in a headwall, and beveled edge. The inlet shape is usually the same as the shape of the culvert barrel, but for tapered inlets, the face area is enlarged and the control section is at the throat. Entrance loss is a function of entrance geometry, so the smaller the entrance loss, the greater becomes the overall culvert capacity (AISI, 1984). A method of increasing inlet performance is the use of beveled edges at the entrance of the culvert. Beveled edges reduce the contraction of the flow by enlarging the face of the culvert (FHWA, 2005).

b) Outlet Control: occurs when the culvert barrel is not able to transmit as much flow as the inlet opening will allow. The control section for the culvert barrel is located on the downstream where only subcritical or pressure flows can exist in this condition.

1. Figures 3-11 to 3-15 represent different conditions for outlet control flow. For all cases, the control section is the outlet end of the culvert or further downstream. For the partly full flow situations, the flow in the barrel is subcritical. Figure 3-11 illustrates the typical full flow condition, having both inlet and outlet ends submerged. The barrel is in pressure flow throughout its length.

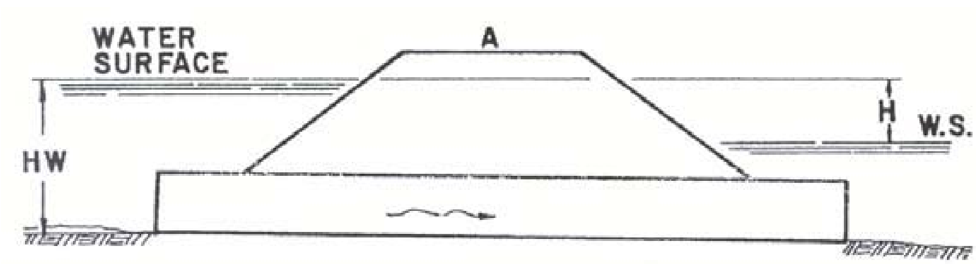


Figure 3-11 Inlet and Outlet Submerged [Source: FHWA, 2005].

Figure 3-12 represents the inlet unsubmerged and the outlet submerged leading the headwater to be shallow so that the inlet crown is exposed when the flow contracts into the culvert.

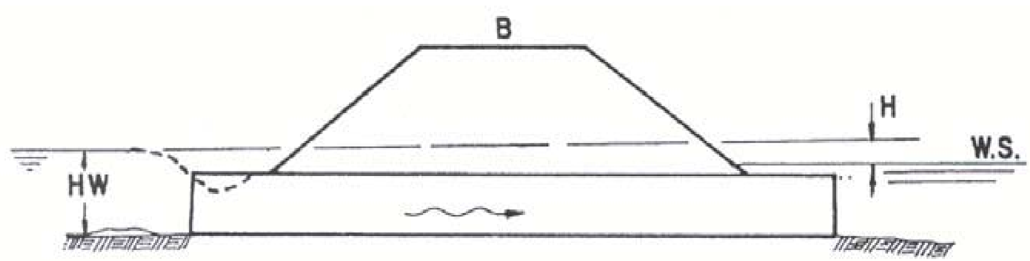


Figure 3-12 Outlet Submerged, Inlet Unsubmerged [Source: FHWA, 2005].

Figure 3-13 illustrates the entrance as submerged and the outlet end is unsubmerged. The flow is full throughout its total length. However, this condition is not very common, as it requires a very high headwater to maintain full barrel flow with no tailwater. Therefore, higher outlet velocities are most likely to occur under this condition.

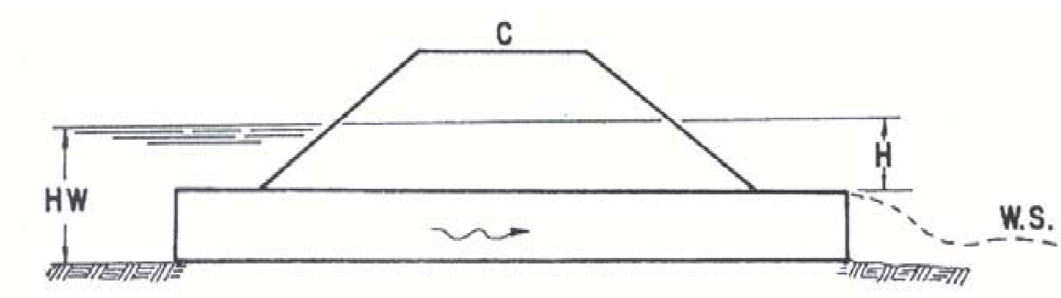


Figure 3-13 Inlet Submerged, Outlet Unsubmerged [Source: FHWA, 2005].

Figure 3-14 is a more classical type. The culvert inlet end is submerged with headwater and the flow at the outlet end flows freely with a low tailwater elevation. Therefore the barrel flows partly full (subcritical flow) over at least part of its length, and the flow passes through critical depth just upstream of the outlet.

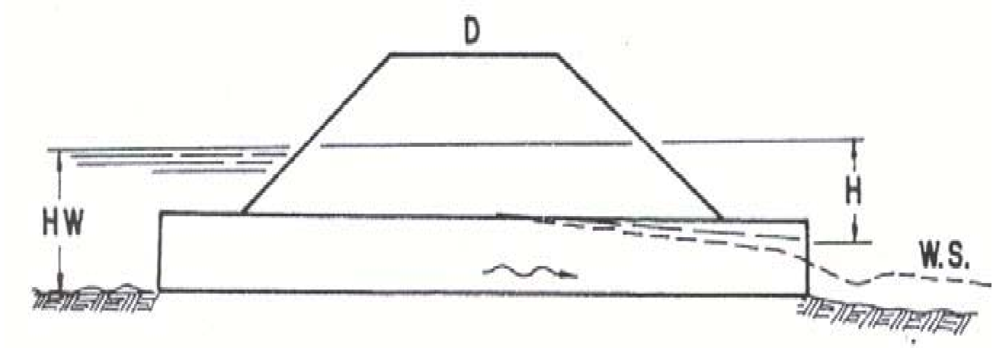


Figure 3-14 Inlet Submerged, Outlet Partially Submerged [Source: FHWA, 2005].

Figure 3-15 represents another classical type, with an unsubmerged culvert for both ends. The barrel flows partly full over its total length, and the flow profile is subcritical flow (FHWA, 2005).

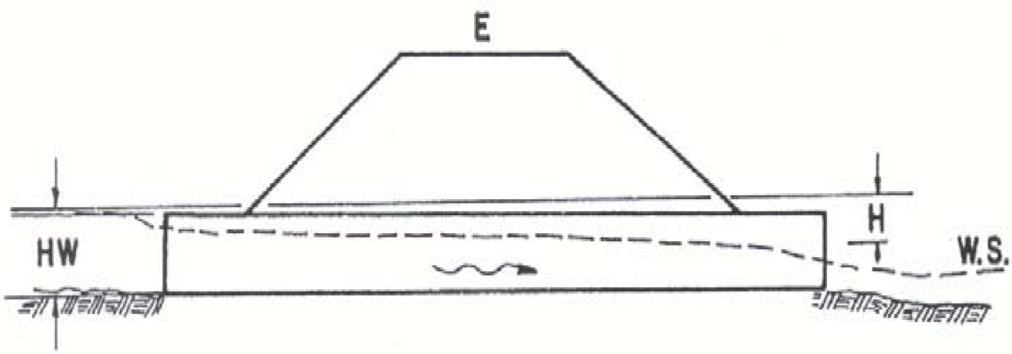


Figure 3-15 Inlet Unsubmerged, Outlet Unsubmerged [Source: FHWA, 2005].

2. Factors Influencing Outlet Control. Unlike inlet control, culvert capacity is determined by the geometry and hydraulic characteristics (Normann, et al., 2001). There are several factors having effects on the outlet control flow, such as the barrel characteristics (roughness, area, shape, length, and slope) and the tailwater elevation, in addition to the same factors which affect inlet control. All the factors are shown in Table 3-1 (FHWA, 2005).

Table 3-1 Factors Influencing Culvert Performance [Source: FHWA, 2005].

Factor	Inlet Control	Outlet Control
Headwater Elevation	X	X
Inlet Area	X	X
Inlet Edge Configuration	X	X
Inlet Shape	X	X
Barrel Roughness		X
Barrel Area		X
Barrel Shape		X
Barrel Length		X
Barrel Slope *		X
Tailwater Elevation		
* Barrel slope affects inlet control performance to a small degree, but may be neglected		

3.4 Headwater (HW)

Headwater depth is defined as the depth of the upstream water surface measured from the invert elevation at the culvert entrance. Energy is required to drive the water through the culvert as shown in Figure 3-1 (HDS 5, 1985). The geometric configuration has the primary effect on upstream headwater. This configuration involves the entrance characteristics, the length and number of the barrels, dimensions, roughness characteristics, and the culvert slope (TxDOT, 2009).

Headwater elevations should be established to define possible flood zones, so that the elevation above the allowable headwater will cause damages to the road as well as to the adjacent property. Allowable headwater depth is the primary basis for selecting the size of the culvert determined from an evaluation of land use upstream of the culvert and the proposed or existing roadway elevation. The allowable head water will be limited by one or more conditions, such as the allowable headwater must not damage upstream property, and ponding depth is to be no greater than the low point in the road grade (Tennessee, 2002).

3.5 Tailwater (TW)

Tailwater is defined as the culvert depth of water downstream measured from the outlet invert elevation of the culvert as shown in Figure 3-1 (HDS 5, 1985). Tailwater depth depends on the characteristics of the stream, which are the main factors in determining culvert capacity under outlet control conditions. The downstream hydraulic conditions of the culvert must be evaluated by calculating the tailwater depth for a range of discharges. Tailwater depth is generally determined by calculating the normal depth of the stream (Tennessee, 2002).

Tailwater may be caused by an obstruction in the downstream channel or by the hydraulic resistance of the channel. In each case, backwater calculations are required to calculate tailwater; however, when appropriate, normal depth approximations may be used instead of backwater calculations (HDS 5, 1985).

3.6 Outlet Velocity

Flow velocities in the culvert are expected to be higher than in the channel because culverts are usually narrower than channel area. Increasing velocities can cause streambed erosion in the culvert outlet area. Occasionally, this problem can be avoided by increasing the roughness of the barrel. Also, outlet velocities can be reduced by adding a roughened section or by flattening the barrel slope, especially in the case of a culvert operated by inlet control not under full capacity (HDS 5, 1985).

3.7 Performance Curves

A performance curve is a graphical interpretation of headwater depth versus flow rate. It is a useful tool in determining the hydraulic capacity of a culvert for different headwaters. Plotting both inlet and outlet control curves is necessary when developing culvert performance curves, because it is hard to predict the dominant control at a certain headwater. Also, the dominant control can be changed from the outlet to inlet or vice-versa over a range of flow rates. Figure 3-16 represents a performance curve for a typical culvert (HDS 5, 1985).

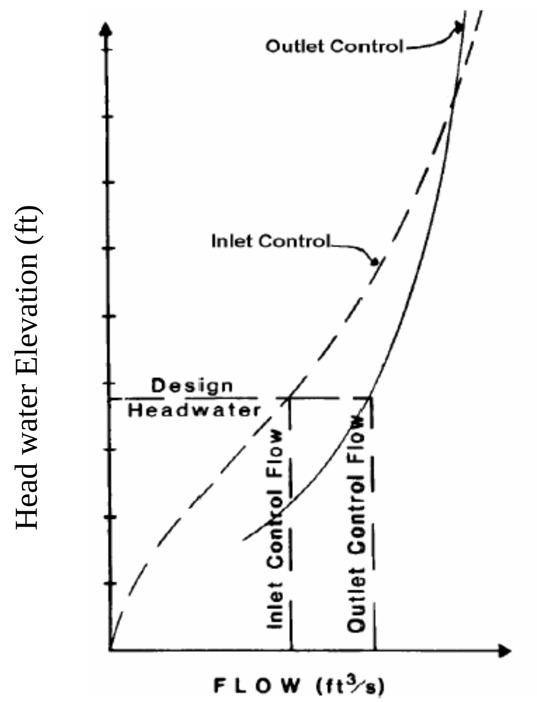


Figure 3-16 Culvert Performance Curve [Source: HDS 5.1985].

Chapter 4: Culvert Design

4.1 Overview

The culvert design process has not been set up as a standard design protocol where two or more individuals will always get the same answer. This is because the engineer's explanation of field data and hydrology is often influenced by personal decisions based on their previous experiences. The culvert design process will lead to a balanced result, achievable under the actual field conditions; when fluid mechanics are combined with practical considerations to reach a satisfactory performance. Currently, the design procedure consists of collecting field data, facts about the roadway, and making a rational estimate of flood flow. In other words, the culvert design process can be defined as a step to designing an economical culvert to handle the flow with minimum damage to the highway, street, and adjacent structures (AISI, 1984).

4.2 What Makes a Good Culvert?

The features of a good highway culvert are defined by ASCE as:

1. The culvert, entrance and the outlet structures should properly take care of water, bed-load, and floating waste at all stages of flow. Additionally, it should transfer the materials normally without any excessive damage to the structure.
2. Entrance structures should be designed to screen out materials which will not pass through the culvert. This design should make use of velocity to approach the practical range, then reduce entrance loss to the minimum value, and subsequently use transitions and increased slopes as required.
3. Entrance design should be completed to avoid excessive ponding which can cause property damage, accumulate drifts and wastes, and culvert clogging.
4. Outlet design should resist undermining and erosion.
5. The culvert should be designed to accommodate any increase in the runoff.
6. The culvert should be designed to handle future improvement in the channel or the highway with less loss and difficulties.
7. The culvert should also be hydraulically adequate to handle design discharge, while remaining structurally durable, easy to maintain, and economical to build.

8. Culvert dissipaters should be simple, easy to construct, economical, and reasonably self-cleaning during low flow periods.

The permissible height of water at the inlet control should be stipulated for each site as well as the allowed ponding according to the following risk conditions:

1. Traffic interruption.
2. Roadway damage due to the saturation of the embankment and pavement disruption due to freeze and thaw.
3. Risk of overtopping the embankment and causing risk to the human life.
4. Damage to adjacent property.
5. Damages to the bed-load by clogging waste or debris and causing recession of flow.
6. Improper outlet velocities which will cause erosion or sediment to the channel bed (AISI, 1984).

4.3 Data Needs for Culvert Design

The culvert design method provides the best suitable procedure for designing culverts, while considering inlet and outlet control. Following the design method without a sound understanding of culvert hydraulics is not recommended. The result could lead to an inadequate and possibly unsafe structure, which may cause a dangerous set of consequences. Table 4-1 represents all the data required that in culvert design.

An exact analysis of culvert flow is very complex because the flow is mostly a non-uniform, varying flow. Flow is usually used as the concept of “inlet control” and “outlet control” to simplify the analysis. Generally, design charts were used to determine the headwater in order to avoid the mathematic difficulty involved in more complex calculations (HDS 5, 1985). Most culvert design is empirical and relies on nomographs and standard procedures (Tennessee, 2002). Nomographs are the method required for a trial-and-error solution. The culvert design method is based on the use of design charts and nomographs (CDOT, 2004)

Table 4-1 Data Requirements for Culvert Design [Source: HDS 5, 1985].

Hydrology Data	Peak Flow Hydrographs
Waterway Data	Cross Sections Longitudinal Slope Resistance Tailwater Field Upstream storage
Site Data	Culvert Location
Roadway Data	Cross Section Profile Culvert Length
Design Headwater Critical points on roadway	Surrounding buildings or structures Regulatory Constraints Arbitrary Constraints

Headwater-discharge relationships for the various types of circular and pipe-arch culverts flowing with inlet control are based on laboratory research with models. This research is reported in the National Bureau of Standards Report No. 4444 entitled "Hydraulic Characteristics of Commonly Used Pipe Entrances", by John L. French and "Hydraulics of Conventional Highway Culverts", by H. G. Bossy (Presented at the Tenth National Conference, Hydraulics Division, ASCE, August 1961). Experimental data for box culverts with headwalls and wingwalls were obtained from an unpublished report of the U.S. Geological Survey. Division of Hydraulic Research, Bureau of Public Roads (BPR 1965) analyzed and developed a series of nomographs charts and scale numbers for determining culvert capacity for inlet control.(FHWA, 1965). These charts and nomographs are based on data from many hydraulic tests and depend on theoretical calculations.

These charts may contain some minor errors because the graphs were based on the best fit data which involves a scatter in the test data. In addition, the relationship between the design equations and the design nomographs is not exact. Additional errors are developed from the reproduction of the design charts. Therefore, it should be assumed that the results of the procedure are accurate to within the range of ± 10 percent of the equation values in terms of headwater (inlet control) or head loss (outlet control) (HDS 5, 1985). In relation to the disadvantages of the nomographs method, extensive laboratory tests were conducted by the National Bureau of Standards (NBS) under the sponsorship of the Bureau of Public Roads (now the Federal Highway Administration, FHWA).

4.4 Headwater under Inlet Control

The series of equations which describe the inlet control headwater under various conditions were consequently developed. The two basic conditions of inlet control depend upon whether or not the inlet end of the culvert is submerged by the upstream headwater. If the inlet is not submerged, the inlet performs as a weir. If the inlet is submerged, the inlet performs as an orifice. For the unsubmerged case, two cases can be used as shown in Equations 4.1 and 4.2. Equation 4.1 is theoretically more accurate and it is based on the specific head at critical depth, adjusted with two correction factors.

Equation 4.2 is an exponential equation similar to a weir equation. Equation 4.1 is preferable from a theoretical standpoint, while Equation 4.2 is easier to use. A direct relationship between HWi/D and $Q/AD^{0.5}$ may be obtained for the submerged condition. For the unsubmerged condition, it is necessary to obtain the flow rate and equivalent specific head at critical depth.

<u>Unsubmerged</u>

Case 1

$$\frac{HW_i}{D} = \frac{H_c}{D} + K\left[\frac{K_u Q}{AD^{0.5}}\right]^M - 0.5S_o^2 \qquad (4.1)$$

Case 2

$$\frac{HW_i}{D} = K\left[\frac{K_u Q}{AD^{0.5}}\right]^M \qquad (4.2)$$

Submerged

$$\frac{HW_i}{D} = K\left[\frac{K_u Q}{AD^{0.5}}\right] + Y - 0.5S_o^2 \qquad (4.3)$$

where;

HW_i = Headwater depth above inlet control section invert (m);

D = Interior height of culvert barrel, (m);

Hc = Specific head at critical depth $(dc + Vc^2/2g)$, (m);

Q = Discharge, m^3/s;

A = Full cross sectional area of culvert barrel, m^2;

S_o = Culvert barrel slope, m/m;

K, M, C, and Y = Constants, shown in Table 4.2;

$Ku = 1.811$.

Notes: Equation 4.1 and 4.2 unsubmerged and apply up to $Q/AD^{0.5} = 1.93$

For mitered inlet use $+0.7S_o$ instead of $-0.5S_o$ as a slope correction factor

Equation 4.3 for submerged, applies above about $Q/AD^{0.5} = 2.21$

Table 4-2 contains coefficients for each shape, material, and edge configuration arranged in the same order as the design nomographs, and provides the unsubmerged and submerged equations.

By using the formula above, the submerged and unsubmerged curves were plotted. The hydraulics of Inlet Control performance is defined by the three regions of flow as shown in Figure 4-1 There is a transition zone between the unsubmerged and the submerged states; this zone is defined empirically by drawing a curve between and tangent to the curves. NBS research provided only limited information about the transition zone whereas, in most cases, the transition zone is short and easily constructed.

Table 4-2 Constants for Inlet Control Design Equations [Source: HDS 5 1985].

Chart No.	Shape and Material	Nomo-graph Scale	Inlet Edge Description	Eq. form	Unsubmerged		Submerged		Ref.
					K	M	C	Y	
1	Circular Concrete	1	Square edge w/headwall	1	.0098	2.0	.0398	.67	56/57
		2	Groove w/headwall		.0018	2.0	.0292	.74	
		3	Groove end projecting		.0045	2.0	.0317	.69	
2	Circular CMP	1	Headwall	1	.0078	2.0	.0379	.69	56/57
		2	Mitered to slope		.0210	1.33	.0463	.75	
		3	Projecting		.0340	1.50	.0553	.54	
3	Circular	A	Beveled ring 45^0 bevels	1	.0018	2.50	.0300	.74	57
		B	Beveled ring 45^0 bevels		.0018	2.50	.0243	.83	
8	Rectangular Box	1	30^0 to 75^0 wingwall flares	1	.026	1.0	.0347	.81	56
		2	90^0 and 15^0 wingwall flares		.061	.75	.0400	.80	56
		3	0^0 wingwall flares		.061	.75	.0423	.82	8
9	Rectangular Box	1	45^0 wingwall flare d = .043D	2	.510	.667	.0309	.80	8
		2	18^0 to 33.7^0 wingwall flare d =.083		.486	.667	.0249	.83	
10	Rectangular Box	1	90^0 headwall w/3/4” chamfers	2	.515	.667	.0375	.79	8
		2	90^0 headwall w/45^0 bevels		.495	.667	.0314	.82	
		3	90^0 headwall w/33.7^0 bevels		.486	.667	.0252	.865	
11	Rectangular Box	1	3/4” chamfers 45^0 skewed headwall	2	.545	.667	.04505	.73	8
		2	3/4” chamfers 30^0 skewed headwall		.533	.667	.0425	.705	
		3	3/4” chamfers 15^0 skewed headwall		.522	.667	.0402	.68	
		4	45^0 bevels 10^0-45^0 skewed headwall		.498	.667	.0327	.75	
12	Rectangular Box 3/4” chamfers	1	45^0 non-offset wingwall flares	2	.497	.667	.0339	.803	8
		2	18.4^0 non-offset wingwall flares		.493	.667	.0361	.806	
		3	18.4^0 non-offset wingwall flares 30^0 skewed barrel		.495	.667	.0386	.71	
13	Rectangular Box Top Bevels	1	45^0 wingwall flares – offset	2	.497	.667	.0302	.835	8
		2	33.7^0 wingwall flares – offset		.495	.667	.0252	.881	
		3	18.4^0 wingwall flares – offset		.493	.667	.0227	.887	
16-19	C M Boxes	2	90^0 headwall	1	.0083	2.0	.0379	.69	57
		3	Thick wall projecting		.0145	1.75	.0419	.64	
		5	Thin wall projecting.		.0340	1.5	.0496	.57	

Continued Constants for Inlet Control Design Equations [Source: HDS 5 1985].

Chart No	Shape and Material	Nomo-graph Scale	Inlet Edge Description	Eq. Form	Unsubmerged		Submerged		Ref.
					K	M	C	Y	
29	Horizontal	1	Square edge w/headwall	1	.0100	2.0	.0398	.67	57
	Eclipse	2	Groove end w/headwall		.0018	2.5	.0292	.74	
	Concrete	3	Groove end projecting		.0045	1.0	.0317	.69	
30	Vertical	1	Square edge w/headwall	1	.0100	2.0	.0398	.67	57
	Eclipse	2	Groove end w/headwall		.0018	2.5	.0292	.74	
	Concrete	3	Groove end projecting		.0095	2.0	.0317	.69	
34	Pipe Arch	1	90^0 headwall	1	.0083	2.0	.0379	.69	57
	18” Corner	2	Mitered to slope		.0300	1.0	.0463	.75	
	Radius CM	3	Projecting		.0340	1.5	.0496	.57	
35	Pipe Arch	1	Projecting	1	.0300	1.5	.0496	.57	56
	18” Corner	2	No Bevels		.0088	2.0	.0368	.68	
	Radius CM	3	33.7^0 bevels		.0030	2.0	.0269	.77	
36	Pipe Arch	1	Projecting	1	.0300	1.5	.0496	.57	56
	31” Corner	2	No Bevels		.0088	2.0	.0368	.68	
	Radius CM	3	33.7^0 Bevels		.0030	3.0	.0269	.77	
41-43	Arch CM	1	90^0headwall	1	.0083	2.0	.0379	.69	57
		2	mitered to slope		.0300	1.0	.0463	.75	
		3	Thin wall projecting		.0340	1.5	.0496	.57	
55	Circular	1	Smooth tapered inlet throat	2	.534	.555	.0196	.90	3
		2	rough tapered inlet throat		.519	.64	.0210	.90	
56	Eliptical	1	Tapered inlet-beveled edges	2	.536	.622	.0368	.83	3
	Inlet face	2	Tapered inlet-square edges		.5035	.719	.0478	.80	
		3	Tapered inlet-thin edge protecting		.547	.80	.0598	.75	
57	Rectangular	1	Tapered inlet throat	2	.475	.667	.0179	.97	3
58	Rectangular	1	Side tapered-less favourable edges	2	.56	.667	.0446	.85	3
	Concrete	2	side tapered-more favourable edges		.56	.667	.0378	.87	
59	Rectangular	1	Slope tapered-less favourable edges	2	.50	.667	.0446	.65	3
	Concrete		Slope tapered-more favourable edges		.50	.667	.0378	.71	

Both axes in Figure 4-1 are dimensionless but the figures could be used to develop dimensional curves for any selected size of conduit by multiplying the term $Q/AD^{0.5}$ by $AD^{0.5}$ and HW_i /D by D. In order to derive overall inlet control equations, the designer can combine the three zones by plotting the unsubmerged and submerged curves from these equations and draw the connecting transition line. Then, perform the best fit curve whereas a polynomial curve in the following form has been found to provide an adequate fit.

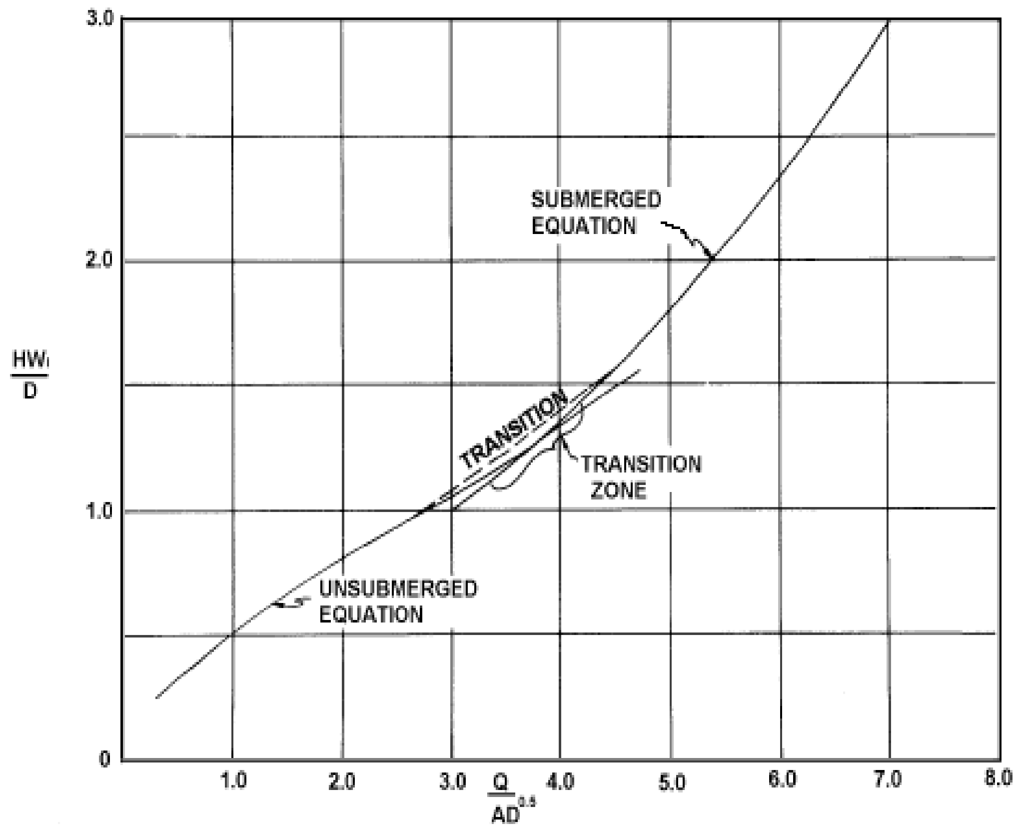


Figure 4-1 Inlet Control Curves [Source: FHWA, 2005].

The FHWA performance equations, Equation 4.1 to 4.2, remain the most commonly used for culvert design through hand calculation, and by using nomographs (Herr and Bossy, 1965; Normann et al., 1985). The equations use four parameters (*K*, *M*, *C*, and *Y)*. Estimation of flow conditions near the transition between unsubmerged and submerged performance curves usually requires manual or other adjustment (Normann et al., 1985); The parameters for different culvert shapes, materials, and configurations are provided in Table 4-2 (Charbeneau, et al., 2006).

In the process of developing computer programs to automate culvert hydraulic computations, the Federal Highway Administration (FHWA) faced the problem of implementing the procedure for developing inlet control performance curves. For the purposes of the program, fifth-degree polynomial curves were fitted. The polynomials obtained all three zones of flow: unsubmerged (weir flow), transition, and submerged (orifice flow). One fifth-degree polynomial with its corresponding coefficients was obtained for every combination of culvert shape and inlet edge configurations, as this shows the general form of the fifth degree polynomial (FHWA, 2005).

A fifth-degree polynomial equation based on regression analysis was used to model the inlet control of headwater for a given flow. The range of the regression equations are from one-half to three times the culvert rise. For range $0.5 \leq HW_i/D \leq 3.0$ as it is represented in the Equation 4.4.

$$HW_i = [a + bF + cF^2 + dF^3 + eF^4 + fF^5]D - 0.5DS_o \quad (4.4)$$

where;

HW_i = inlet control headwater, m;

D = rise of the culvert barrel, m;

a to f = regression coefficients for each type of culvert as shown in Table 4-3;

S_o= culvert slope, m/m;

F = function of average outflow discharge through a culvert; culvert barrel rise; and for box and pipe-arch culverts, width of the barrel, B, shown in Equation 4.5;

W = width or span of culvert m.

$$F = 1.8113 \frac{Q}{WD^{3/2}} \quad (4.5)$$

For $HW_i/D > 3.0$, orifice equation, shown in Equation 4.6, is used.

$$k = 0.6325 \frac{Q_{3.0}}{D^{1/2}} \quad (4.6)$$

$$HW_i = \left[\frac{Q}{k}\right]^2 + \frac{D}{2} \quad (4.7)$$

k = orifice equation constant;

$Q_{3.0}$ = discharge m^3/s at which HW_i/D = 3.

Charbeneau et al., (2006), stated a flexible two-parameter model describing the hydraulic performance of highway culverts operating under inlet control for both unsubmerged and submerged conditions instead of the four parameters (K, M, C, and Y) provided by FHWA.

Thiele et al. (2006) conclude that the new model developed by Charbeneau et al. (2006) to define performance curves that smoothly span the transition zone appears more efficient when using the polynomial equations, rather than further developing a process that repeats what has already been done. It is recommended that the existing polynomial equations and coefficients can be used rather than conducting additional studies in order to obtain values for coefficients through a similar curve-fitting process.

Table 4-3 Regression Coefficients for Inlet Control Equations [Source: FHWA, 2005].

Shape	Material	Entrance type	a	b	c	d	e	f
BOX	RCP	Square	0.122117	0.505435	−0.10856	0.0207809	−0.00136757	0.00003456
		1.5 bevel	0.0967588	0.4551575	−0.08128951	0.01215577	−0.00067794	0.0000148
		1.1 bevel	0.1566086	0.3989353	−0.6403921	0.01120135	−0.0006449	0.000014566
		30°–50° square	0.0724927	0.507087	−0.117474	0.0221702	−0.00148958	0.000038
		90°–15° square	0.122117	0.505435	−0.10856	0.0207809	−0.00136757	0.00003456
		0° square	0.144133	0.461363	−0.921507	0.0200028	−0.00136449	0.0000358
		1.5 bevel	0.0895633	0.4412465	−0.07434981	0.01273183	−0.0007588	0.00001774
		Bevel	0.0895633	0.4412465	−0.07434981	0.01273183	−0.0007588	0.00001774
		Taperd inlet throat	0.1295033	0.3789944	−0.0437778	0.00426329	−0.000106358	0.000000
CIRCULAR	RCP	Square projecting	0.167287	0.558766	−0.159813	0.0420069	−0.00369252	0.000125169
		Headwall	0.087483	0.706578	−0.253295	0.0667001	−0.00661651	0.000250619
		Groove	0.108786	0.662381	−0.233801	0.0579585	−0.0055789	0.000205052
		Groove with headwall	0.114099	0.653562	−0.233615	0.0597723	−0.00616338	0.000242832
		1.1 bebel	0.063343	0.766512	−0.316097	0.0876701	−0.009836951	0.00041676
		1.5 bevel	0.08173	0.698353	−0.253683	0.065125	−0.0071975	0.000312451
	CSP/CAP	Thin	0.187321	0.56771	−0.156544	0.0447052	−0.00343602	0.000089661
		Mitered	0.107137	0.757789	−0.361462	0.1233932	−0.01606422	0.00076739
		Headwall	0.167433	0.538595	−0.149374	0.0391543	−0.00343974	0.000115882
		1.1 bevel	0.063343	0.766512	−0.316097	0.0876701	−0.009836951	0.00041676
		1.5 bevel	0.08173	0.698353	−0.253683	0.065125	−0.0071975	0.000312451
ELLIPSE	CSPE	Headwall	0.01267	0.79435	−0.2944	0.07114	−0.00612	0.00015
	CAPE	Mitered	−0.14029	1.437	−0.92636	0.32502	−0.04865	0.0027
		Bevel	−0.00321	0.92178	−0.43903	0.12551	−0.01553	0.00073
		Thin	0.0851	0.70623	−0.18025	0.01963	0.00402	−0.00052
	RCPE	Square	0.13432	0.55951	−0.1578	0.03967	−0.0034	0.00011
		Groove with headwall	0.15067	0.50311	−0.12068	0.02566	−0.00189	0.00005
		Groove end projecting	−0.03817	0.84684	−0.32139	0.0755	−0.00729	0.00027
PIPE ARCH	CMPA (Span 17–83in)	projecting	0.08905	0.71255	−0.27092	0.07925	−0.00798	0.00029
		Mitered	0.0833	0.79514	−0.43408	0.16377	−0.2491	0.00141
		Headwall	0.11128	0.61058	−0.19494	0.05129	−0.00481	0.00017
	CMPA (Span 60–142in)	projecting	0.08905	0.71255	−0.27092	0.07925	−0.00798	0.00029
		Mitered	0.0833	0.79514	−0.43408	0.16377	−0.2491	0.00141
		Headwall	0.11128	0.61058	−0.19494	0.05129	−0.00481	0.00017
	CSPA (Span 73–199in)	projecting	0.08905	0.71255	−0.27092	0.07925	−0.00798	0.00029
		Mitered	0.0833	0.79514	−0.43408	0.16377	−0.2491	0.00141
		Headwall	0.11128	0.61058	−0.19494	0.05129	−0.00481	0.00017
	CSPA (Span 159–247in)	projecting	0.12263	0.4825	−0.00002	−0.04287	0.01454	−0.00117
		Mitered	0.1062	0.7037	−0.3531	0.1374	−0.02076	0.00117
		Headwall	0.12346	0.50432	−0.13261	0.0402	−0.00448	0.00021
	CSPA (Span 240–364in)	projecting	0.14168	0.49323	−0.03235	−0.02098	0.00989	−0.00086
		Mitered	0.23645	0.37198	−0.0401	0.03058	−0.00576	0.00045
		Headwall	0.099728	0.57515	−0.15977	0.04223	−0.00374	0.00012
	CAPA	projecting	0.09219	0.65732	−0.19423	0.04476	−0.00176	−0.00012
		Mitered	0.10212	0.70503	−0.34558	0.12454	−0.01676	0.00081
		Headwall	0.09455	0.61669	−0.22431	0.07407	−0.01002	0.00054
	RCPA (Span 18–169in)	Headwall	0.16884	0.38783	−0.03679	0.01173	−0.00066	0.00002
		Groove with headwall	0.1301	0.43477	−0.07911	0.01764	−0.00114	0.00002
		Groove end projecting	0.09618	0.52593	−0.13504	0.03394	−0.00325	0.00013

Note: RC=reinforced concrete; CS=corrugated steel; CA=corrugated aluminum; and CM=corrugated metal.

4.5 Headwater under Outlet Control

For outlet control flow, the necessary upstream energy passing the given flow must be computed considering several conditions. The headwater flow in outlet control is a function of discharge, therefore, the slope, cross-sectional area, roughness and length of the culvert barrel have to be considered, as well as entrance geometry, and tailwater level (Road Drainage, 2010). Culverts flowing with outlet control can flow with full or with partly-full flow. Full flows occur under outlet control and both the inlet and outlet are submerged. The culvert can also flow full flow over part of its length and with partly full flow at the outlet. Flow under outlet control can be calculated based on energy balance. The total energy *(H)* required for passing the flow through the culvert barrel is made up of the velocity head *(H_v)*, entrance loss *(H_e)*, the friction losses through the barrel *(H_f)*, and the exit loss *(H_o)*.

$$H = H_v + H_e + H_f + H_o \tag{4.8}$$

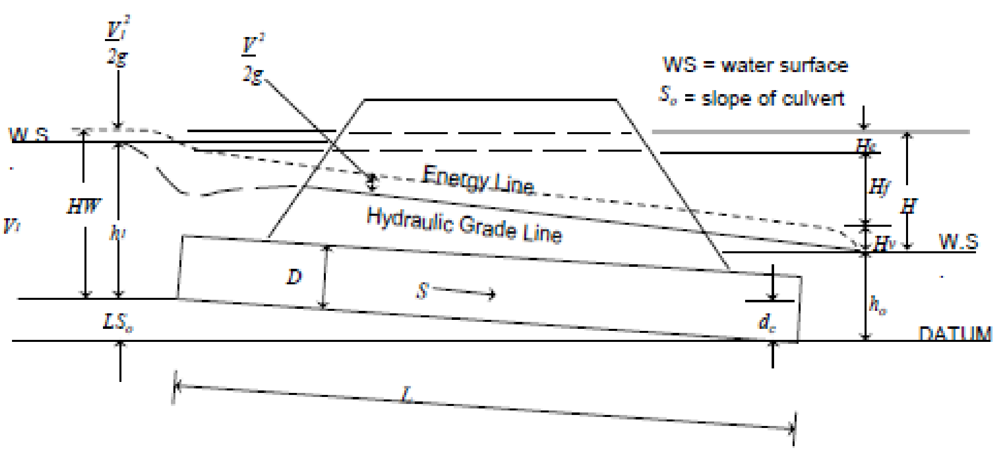


Figure 4-2 Hydraulics of Culvert Flowing Full under Outlet Control [Source: Road Drainage, 2010].

Figure 4-2 illustrates the parameters which are used in the Equation 4.8 in order to calculate the flow under outlet control and also represents the energy grade line and the hydraulic grade line for full flow in a culvert barrel. The energy grade line shows the total energy at any point along the barrel. The hydraulic grade line is the depth of the water as it rises in the tube, which is connected to the side of the barrel. In full flow, the energy and the hydraulic grade lines are aligned parallel and, as straight lines separated by the velocity head, except in the area surrounding the inlet of the culvert barrel when the flow is under contraction.

Velocity in the barrel is calculated as follows:

$$V = \frac{Q}{A} \tag{4.9}$$

where:

V = average velocity in the culvert barrel, m/s;

Q= flow rate, m^3/s;

A= full cross section area of the flow, m^2.

The velocity head is:

$$H_v = \frac{V^2}{2g} \tag{4.10}$$

g = acceleration due to gravity, 9.8 m/s.

Both the entrance and friction loss can be defined as a function of the velocity head in the barrel. The entrance loss can be stated as an entrance coefficient multiplied by the velocity head,

$$H_e = K_e \frac{V^2}{2g} \tag{4.11}$$

Entrance coefficient K_e changed depending on various inlet configurations as given in Table 4-4.

The friction loss will be based on the Manning equation.

$$H_f = \left[\frac{K_U n^2 L}{R^{1.33}}\right] \frac{V^2}{2g} \tag{4.12}$$

where:

K_U = constant = 19.63;

n = the Manning roughness coefficient (Table 4-5);

L = the length of the culvert barrel, m;

R = the hydraulic radius of the full culvert barrel = A/P_w, m;

A = the cross-sectional area of the barrel, m^2;

P_w =the perimeter of the barrel, m;

V = the velocity in the barrel, m/s.

The exit loss is a function of the change in velocity at the outlet of the culvert barrel. For a sudden expansion, such as an endwall, the exit loss is:

$$H_o = 1.0 \left[\frac{V^2}{2g} - \frac{Vd^2}{2g}\right] \tag{4.13}$$

Vd = downstream velocity of the culvert, m/s

Equation 4.13 may lead to an overestimated calculation, in which case a multiplier of less than 1.0 can be used. If the exit loss is equal to the full flow, then the downstream velocity can be neglected, as shown in Equation 4.14.

$$H_o = H_v = \left[\frac{V^2}{2g}\right] \tag{4.14}$$

Inserting the above relationships for entrance, friction loss, and exit loss (Equation 4.14) into Equation (4.8), the following loss equation is obtained:

$$H = 1 + K_e \left[\frac{K_u n^2 L}{R^{1.33}}\right] \frac{V^2}{2g} \tag{4.15}$$

Table 4-4 Entrance Loss Coefficient [Source: FHWA, 2005].

Outlet Control, Full or Partly Full Entrance Head Loss

$$H_e = K_e\left[\frac{V^2}{2g}\right]$$

Type of Structure and Design of Entrance	Coefficient K_e
• Pipe, Concrete	
Projecting from fill, socket end (groove-end)	0.2
Projecting from fill, sq. cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove-end	0.2
Square-edge	0.5
Rounded (radius = D/12	0.2
Mitered to conform to fill slope	0.7
*End-Section conforming to fill slope	0.5
Beveled edges, 33.7^0 or 45^0 bevels	0.2
Side- or slope-tapered inlet	0.2
• Pipe. or Pipe-Arch. Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls square-edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
*End-Section conforming to fill slope	0.5
Beveled edges, 33.7^0 or 45^0 bevels	0.2
Side- or slope-tapered inlet	0.2
• Box, Reinforced Concrete	
Headwall parallel to embankment (no wingwalls)	
Square-edged on 3 edges	0.5
Rounded on 3 edges to radius of D/12 or B/12 or beveled edges on 3 sides	0.2
Wingwalls at 30^0 to 75^0 to barrel	
Square-edged at crown	0.4
Crown edge rounded to radius of D/12 or beveled top edge	0.2
Wingwall at 10^0 to 25^0 to barrel	
Square-edged at crown	0.5
Wingwalls parallel (extension of sides)	
Square-edged at crown	0.7
Side- or slope-tapered inlet	0.2

*Note: "End Sections conforming to fill slope," made of either metal or concrete, are the sections commonly available from manufacturers. From limited hydraulic tests they are equivalent in operation to a headwall in both inlet and outlet control. Some end sections, incorporating a closed taper in their design have a superior hydraulic performance. These latter sections can be designed using the information given for the beveled inlet.

Table 4-5 Manning's (n) Values for Culverts.* [Source: FHWA, 2005].

Type of culvert	**Roughness or corrugation**	**Manning's n**	**Reference**
Concrete Pipe	Smooth	0.010-0.011	(64, 66, 67, 70)
Concrete Boxes	Smooth	0.012-0.015	(23)
Spiral Rib Metal Pipe	Smooth	0.012-0.013	(65, 69)
Corrugated Metal Pipe, Pipe-Arch and Box (Annular and Helical corrugations manning's n varies with barrel size)	68 by 13 mm 2-2/3 by 1/2 in Annular	0.022-0.027	(25)
	68 by 13 mm 2-2/3 by 1/2 in Helical	0.011-0.023	(25, 68)
	150 by 25 mm 6 by 1 in Helical	0.022-0.025	(25)
	125 by 25 mm 5 by 1 in	0.025-0.026	(25)
	75 by 25 mm 3 by 1 in	0.027-0.028	(25)
	150 by 50 mm 6 by 2 in Structural Plate	0.033-0.035	(25)
	230 by 64 mm 9 by 2-1/2 in Structural Plate	0.033-0.037	(25)
Corrugated Polyethylene	Smooth	0.009-0.015	(71,72)
Corrugated Polyethylene	Corrugated	0.018-0.025	(73, 74)
Polyvinyl chloride (PVC)	Smooth	0.009-0.011	(75, 76)
*NOTE: the manning's n values indicated in this table were obtained in the laboratory and are supported by the provided reference. Actual field values for culverts may vary depending on the effect of abrasion, corrosion, deflection, and joint conditions.			

From the energy balance and by using Figure 4-3 *H* can be defined as the difference between the energy line at the inlet and the elevation of the hydraulic grade line at the outlet. As the velocity head in the entrance is usually small due to the ponding condition of the water at the entrance ($V^2/2g \approx 0$), there will be no differences between the water surface of headwater and the energy line elevation.

Equation 4.16 is used to calculate the headwater depth under outlet control

$$HW_o = H + h_o - LS_o \tag{4.16}$$

where;

h_o = adopted tailwater depth, m;

HW_o = Headwater under outlet control.

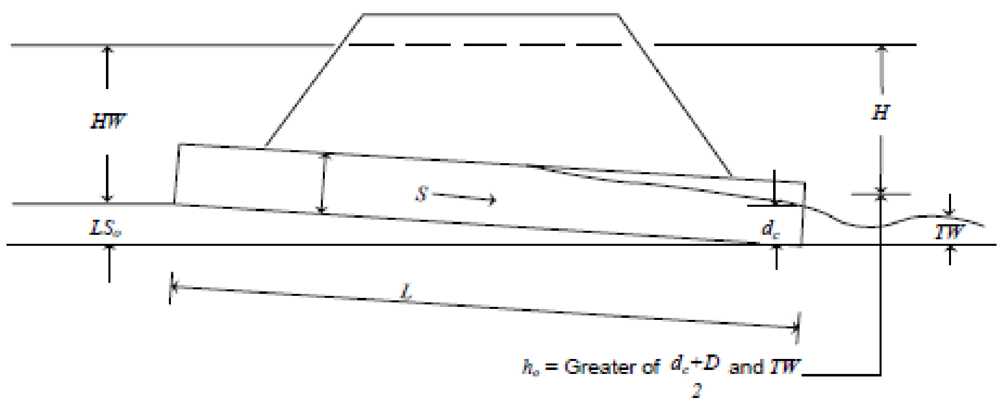


Figure 4-3 Determination of Adopted Tailwater (h_o) [Source: Road Drainage, 2010].

The adopted tailwater level (*ho*) can be determined due to conditions such

$h_o = TW$ if $TW > D$, and

if $TW < D$, h_o = the greater of: *TW*, or $((dc+D)/2)$.

Compare the values of *HWi* and *HWo*. The higher headwater indicates the flow control type so if *HWi* > *HWo* the culvert is under inlet control and the culvert will be under outlet control if the *HWi* < *HWo*.

4.6 Roadway Overtopping

Overtopping will start when the elevation of the headwater is higher than the elevation of the roadway. This phenomenon usually occurs when the headwater exceeds the low point of the sag vertical curve on the roadway, causing part of the water to flow over the roadway embankment. In this case using the calculation for the roadway overtopping discharge is recommended. Equation 4.17 represents the overtop discharge.

$$Q_o = k_t C_d H_h^{1.5} \quad (4.17)$$

where;

Q_o = Overtopping discharge m^3/s;

k_t = Over-embankment flow adjustment factor as shown in Figure 4-5;

C_d = Overtopping discharge coefficient = 1.66 in metric for roadway overtopping;

L_i = Horizontal length of overflow m;

H_t = Depth between tailwater and low roadway elevation (m);

H_h = Average depth between headwater and low roadway elevation m.

For practical purposes, H_t/H_h may approach *0.8* without any correction coefficient. For Ht/H_h values above *0.8*, Figure 4-5 is used to determine k_t.

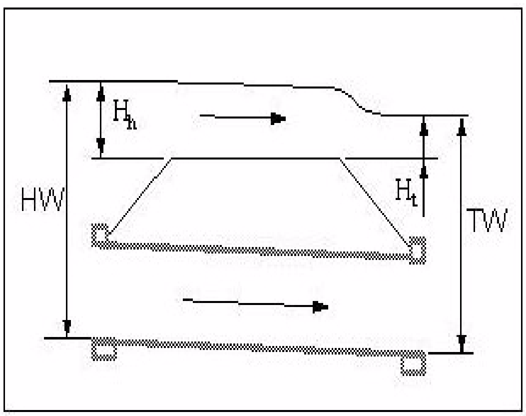


Figure 4-4 Roadway Overtopping with High Tailwater

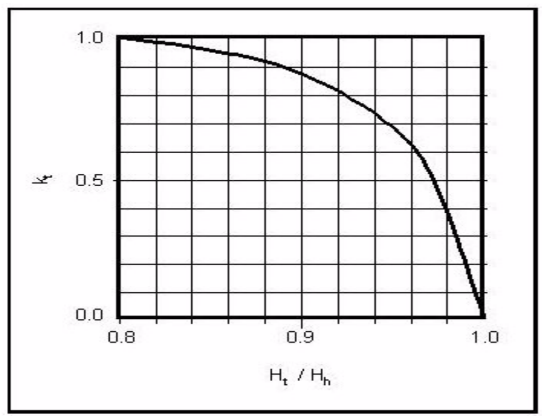


Figure 4-5 Over Embankment Flow Adjustment Factor

[Source: TxDOT, 2009].

Commonly the flow over highway embankments is parabolic or otherwise irregular. In some cases, it is required to divide the section into segments and then calculate the individual flows for the segments as shown in Figure 4-6. At the end, the total overtop flow is based on the summation of segment flows (TxDOT, 2009).

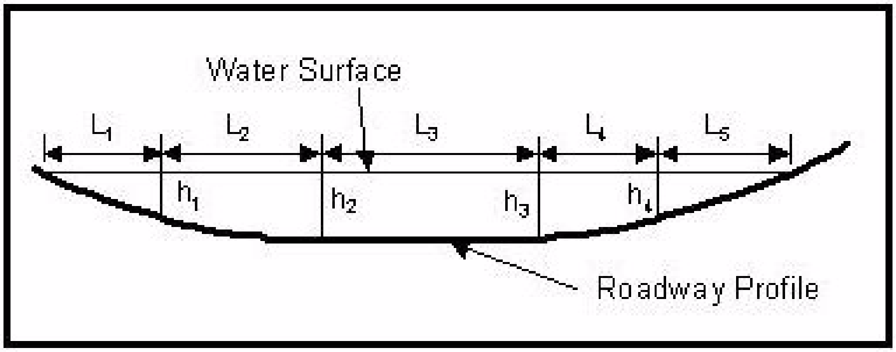


Figure 4-6 Cross Section of Flow over Embankment [Source: TxDOT, 2009].

4.7 Outlet Velocity

Culvert outlet velocities are important to determine the need for erosion protection at the culvert exit. Generally, outlet velocities are higher than the natural stream velocities; therefore, outlet velocities may need flow readjustment to prevent erosion downstream of the culvert.

The average outlet velocity for all culvert types can be calculated using:

$$V_o = \frac{Q}{A} \tag{4.18}$$

where;

Q = design discharge per culvert barrel (m^3/s);

A = cross sectional area of flow from culvert barrel (m^2).

The cross-sectional area of flow (A) depends on the flow depth at the outlet as illustrated in Figure 4-7. The normal depth flow is used for determining the outlet velocity of culverts operating in inlet control. Normal depth in common culvert shapes may be calculated using a trial and error solution of the Manning equation.

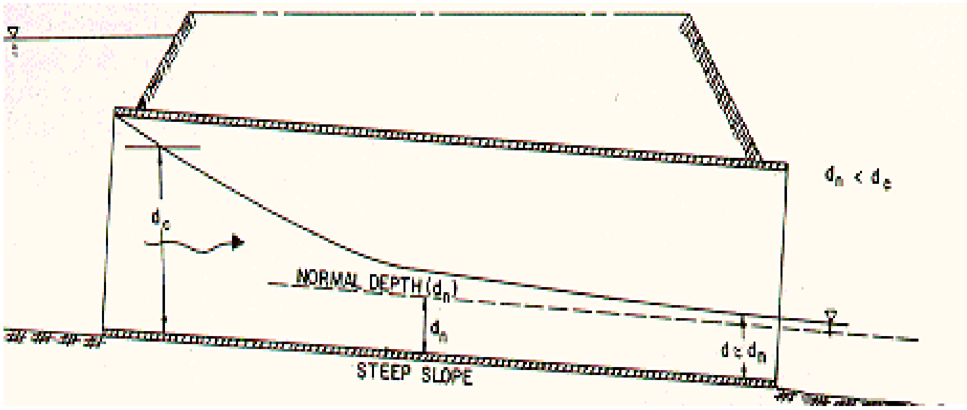


Figure 4-7 Outlet Velocity with Inlet Control [Source: HDS 5, 1985]

In the case of a culvert under outlet control, the cross sectional area of the flow is classified by the geometry of the outlet and one of the following: critical depth, tailwater depth, or the height of the conduit. Critical depth *(dc)* is used when the tailwater *(TW)* is less than critical depth, or tailwater depth (*TW*) if the tailwater is between critical depth, while the top of the barrel or the total barrel depth *(D)* is used when the tailwater exceeds the top of the barrel, as shown in Figure 4-8 (HDS 5, 1985).

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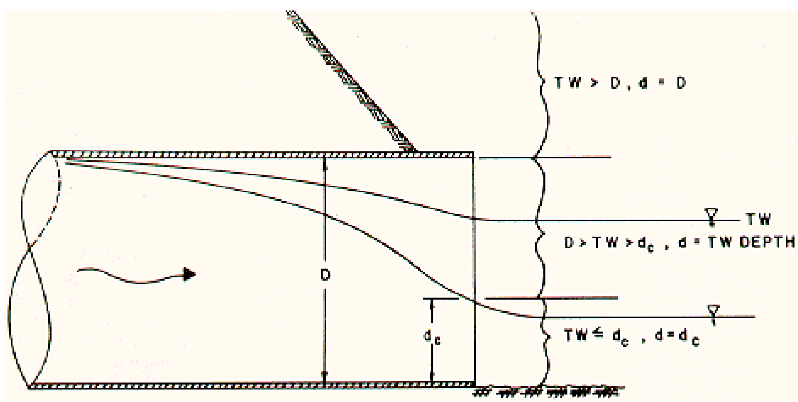

Figure 4-8 Outlet Velocity with Outlet Control [Source: HDS 5, 1985].

In addition to the hydraulic criteria, the types and characteristics of soil also influence outlet velocities. For example, culvert channels with rock or shale bottoms typically tolerate high velocities *(4.5 to 6.0 m/s).* On the other hand, channels with silt or sand bottoms may erode at low velocities. Extreme caution must be taken when considering culvert designs with outlet velocities of greater than *(4.5 m/s).* Velocities of less than approximately *(0.5 m/s)* usually cause deposition of sediments. Therefore, *(0.5 m/s)* is recommended as a minimum outlet velocity for culvert design and operation (TxDOT, 2009).

4.8 Economics

The best selection for culvert design is minimizing the total annual cost. The annual cost includes capital, maintenance, and costs that are associated with flooding. It is necessary to set up an initial analysis between a culvert and a bridge. If a culvert is chosen, a comparison of the available materials and shapes would follow. Many factors must be taken into account in engaging the process of construction cost analysis, such as durability, maintenance, and replacement costs. These results are used to evaluate the design flood based on total annual cost. This procedure is referred to as a risk analysis.

4.8.1 Service Life

The service life of a culvert should be considered in the selection process as its service life must be equal to the highway's service life

4.8.2 Comparisons between Culverts and Bridges

Economic factors are the most important in making the decision between use of a bridge or culvert, where either will satisfy hydraulic and structural requirements. The preliminary cost of a culvert is generally less compared to a bridge. This advantage must be balanced with possible flood damage due to increasing headwater, especially at higher discharges. Maintenance costs for culverts may result from erosion at the inlet and outlet, sedimentation and debris buildup, and embankment repair in case of overtopping. However, bridge maintenance is significantly more costly and, therefore, safety, environmental, and aesthetic factors are included when selecting between a bridge or culvert. Safety considerations for culverts include the use of safety grates.

Bridge decks which often involve narrow shoulders and median widths are subject to icing, which can be a traffic safety problem. Fish and wildlife passage may affect the selection of the bridge over a culvert in terms of environmental issues. Also, a bridge may be considered more aesthetically pleasing than a culvert.

4.8.3 Comparisons between Materials and Shapes

Cost comparisons between various materials and shapes change according to time and region. Detailed economic analysis of culvert material selection requires site-specific considerations, especially where structural strength is a concern under high fills. There are many factors that have an impact on the annual cost of the culvert based on the selected material, such as steep channel slopes which produce high exit velocities, which are further accelerated by using smooth pipes.

Acidic drainage will promote corrosion of some materials. Certain materials cannot withstand the attack of abrasive bed-loads. Also many shapes can be manufactured from different materials. Still, circular culverts are the most common shape. They are generally reasonably priced, hydraulically efficient, and can support high structural loads. However, limited height may require the use of a pipe-arch, ellipse, or an arch, because it is less expensive than a pipe-arch, while ellipses are more expensive than circular pipes.

Additional cost is associated with making inlet improvements for the culvert. By improving the inlet configuration, a decrease in barrel size will decrease the overall culvert cost. The savings on the reduced barrel size usually outweigh the construction costs of the improved inlet. However, the cost of excavation through rock or difficult material for enlarged slope-tapered inlets, as well as the depressed side-tapered inlets should be taken into consideration (FHWA, 2005).

4.8.4 Economic Risk Analysis

Risk analysis is a process of evaluating the economic behavior of different design alternatives. Each design can be used to estimate an annual capital cost and economic risk (cost), the sum of which is called the total expected cost *(TEC)*. Optimization of engineering and economic analyses will give the least total expected cost *(LTEC)*. The basic part of the risk analysis procedure is to establish suitable design alternatives. Engineering, legislative, and policy constraints may limit the total range of alternatives (FHWA, 2005).

Examples of such constraints include:

- Set minimum design flood criteria.
- Limitations due to geometry of the roadway such as maximum or minimum grade lines, site distance, and vertical curvature.
- Channel stability considerations which limit culvert velocity or the amount of limitation (FHWA, 2005).

4.9 Method of Culvert Design

The culvert design procedures for both inlet and outlet control culverts must take some important factors into consideration (AASHTO, 1990):

- Establishment of hydrology,
- Design of downstream channel,
- Assumption of a trial configuration,
- Computation of inlet control headwater,
- Computation of outlet control headwater at inlet,
- Evaluation of the controlling headwater,
- Computation of discharge over the roadway, and then the total discharge,
- Computation of outlet velocity and normal depth,
- Comparison of headwater and velocity to limiting values,
- Adjustment of configuration (if necessary),
- Recomputation of hydraulic characteristics (if necessary),

The computation of headwater for inlet or outlet control is based either on nomographs or design equations.

4.9.1 Nomographs Method

The nomograph method has the following acceptable procedures for standard culverts:

1. Identify the culvert size and flow rate to be used. It is important to note that for box culverts, the flow rate according to barrel width is used.

2. Connect the culvert size and discharge and then extend a straight line to cut the *HW*/*D* axis. In the case where *HW*/*D* is required in another scale, extend the value at the original intersection horizontally to cut across the appropriate scale.
3. Once the *HW*/*D* ratio is computed either by the design equations or the nomograph, compute the inlet control headwater depth, *HWi*, by multiplying the barrel diameter by the ratio *HW*/*D*.

The nomographs can also be used in the following procedure to determine the headwater at outlet control:
1. Identify the culvert size, *D*, and length, *L*,
2. Use the appropriate scale for K_e to connect *D* and *L* with a straight line and identify the intersection of this line with the turning line.
Classic design procedure relies on the trial and error and by using several nomographs and charts for specific types of culvert shapes (Tuncok, et al., 2004). The standard culvert design procedure is illustrated by the flow chart in Figure 4-9.

4.9.2 Design equations

The design equations are used to determine the condition of the inlet control. A culvert performs as an orifice in the case when the inlet is submerged. However, it performs as a weir when it is unsubmerged as described in Chapter Two.

Unlike the inlet control culverts, the headwater required for an outlet culvert cannot be computed using a single equation as it is represented in this chapter (Tuncok, et al., 2004).

Culvert hydraulic analysis can also be accomplished using computer software (i.e. HY-8). After creating a file, the user will be prompted for the discharge range, site data, and culvert shape, size, material, and inlet type. As an initial size estimate, the size and the type of the culvert would be entered. The output of the program is resolved if the designed culvert satisfies the criteria. This method may be fast and have fewer errors than the nomographs method, but it also relies on the trial and error approach.

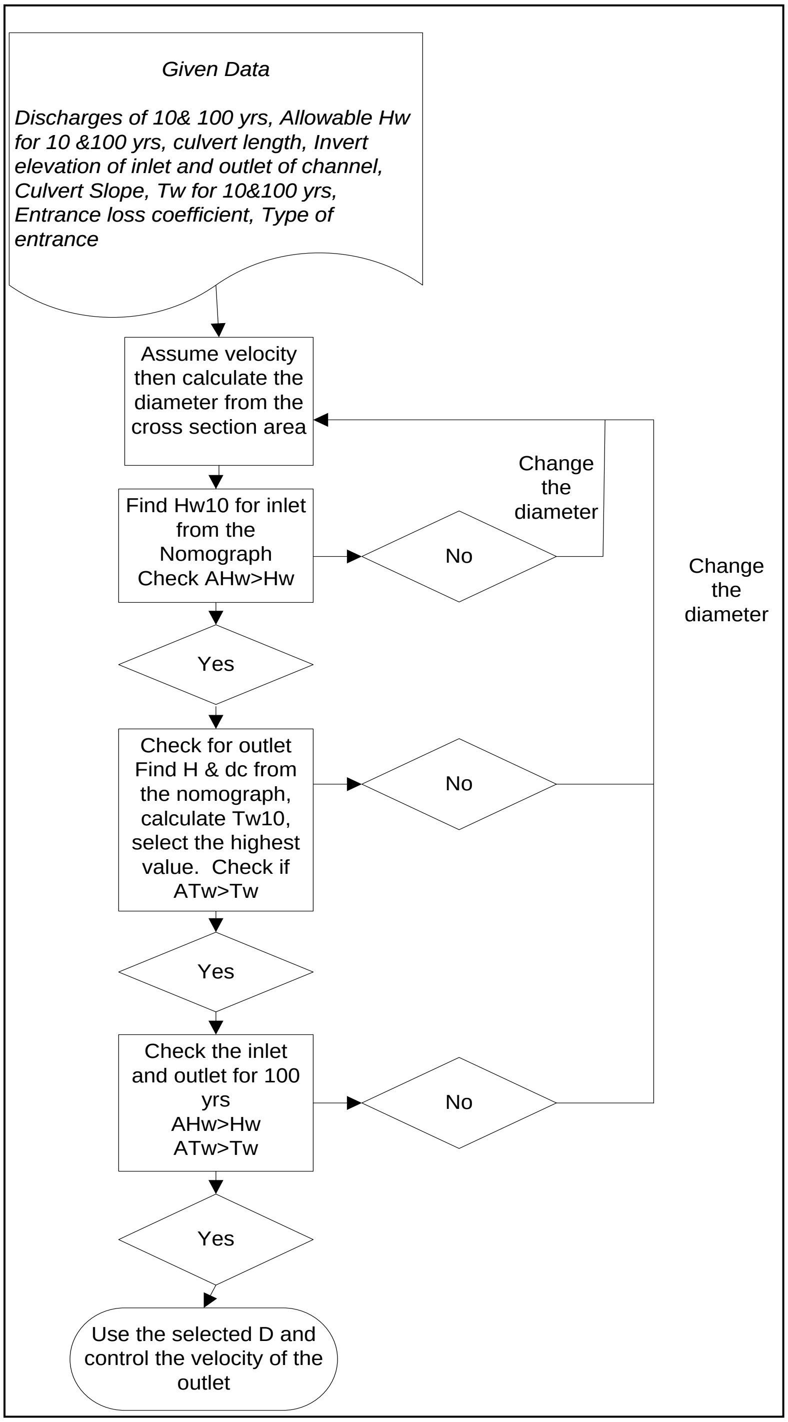

Figure 4-9 Flow Charts for the Hydraulic Design Procedure by Using the Nomograph.

Chapter 5: Reliability and Risk Assessment

Engineering projects are always subject to the possibility of failure. A failure of the system can be defined as an event in which the system no longer properly functions in respect to its desired objectives. Basically, there are two types of failures in engineering projects: performance failure and structural failure. Performance failure takes place when the system is unable to perform as expected and unwanted results occur. For example, a flood control structure may not be able to protect the area from extreme flood, or a canal cannot convey excess water from the structure , etc. Structural failures involve damages or changes in the actual structure that cause a decrease in the capacity of the structure. Buckling of a beam, breaking of a bridge, and failure of a pump are some of the examples for structural failures.

Reliability can be defined as the probability of non-failure. There are two applications for types of reliability analysis to engineering problems: evaluation of the reliability of an existing system and design of a new system based on reliability. Reliability in engineering projects entails considering tolerance within design parameters, as well as uncertainties in applications within an environment.

5.1 Approach of Reliability Evaluation

Traditional and probabilistic are the two approaches considered for safety evaluations of engineering systems. In the traditional approach to safety analysis, the worst case scenario is considered to determine the load and capacity of a system while tolerance is stacked up in terms of safety factors and margins, as shown in Figure 5-1. In most cases, such safety margins and factors are rarely based on any mathematical rigor or true knowledge of the underlying risk and often results in over design of a structure. This leads to designs that are heavier and costlier than needed, and in some cases these designs do not even result in greater safety or reliability (Modarres et al., 1999)

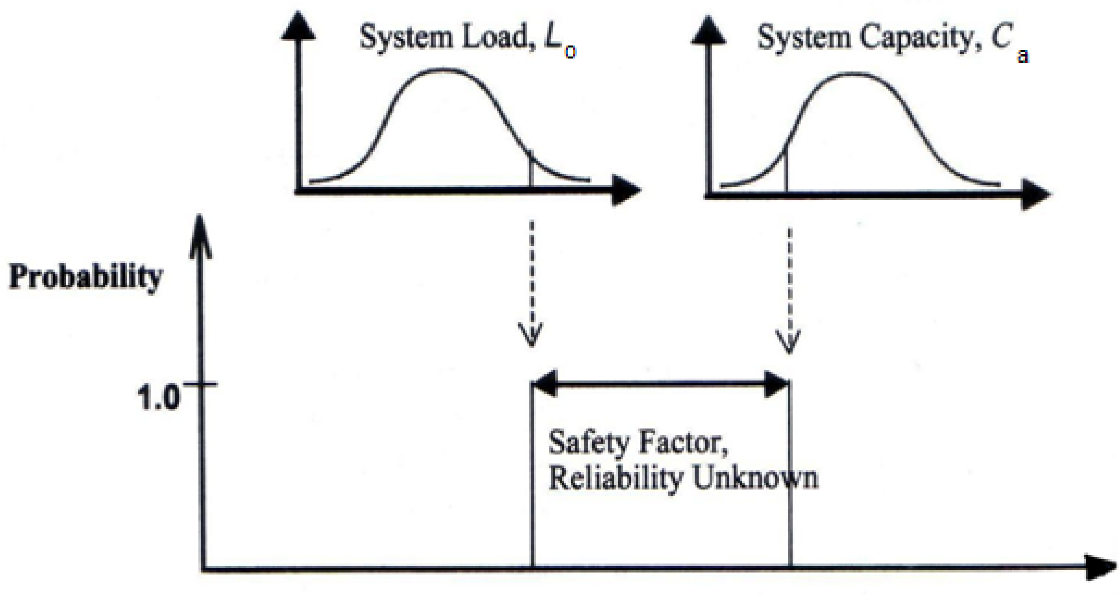

Figure 5-1 Traditional Approach of Safety Analysis Considered in Engineering System

[Source: Singh, et al., 2007].

The probabilistic design approach is a logical extension of the traditional safety method. Probabilistic calculation techniques are more laborious and complicated compared with deterministic ones. However, they correspond better with the aim of producing sophisticated designs and yield insight into actual risks. In a probabilistic approach, both the system capacity and loading can take on a wide range of values by explicitly incorporating uncertainty into system parameters. A probabilistic reliability analysis is used as a technique for identifying, characterizing, quantifying and evaluating the probability of pre-identified hazard. In other words, in a probabilistic reliability analysis, an engineer could generate not just single performance predictions, but a distribution of performance predictions with associated probabilities of occurrence, as is shown graphically in Figure 5-2.

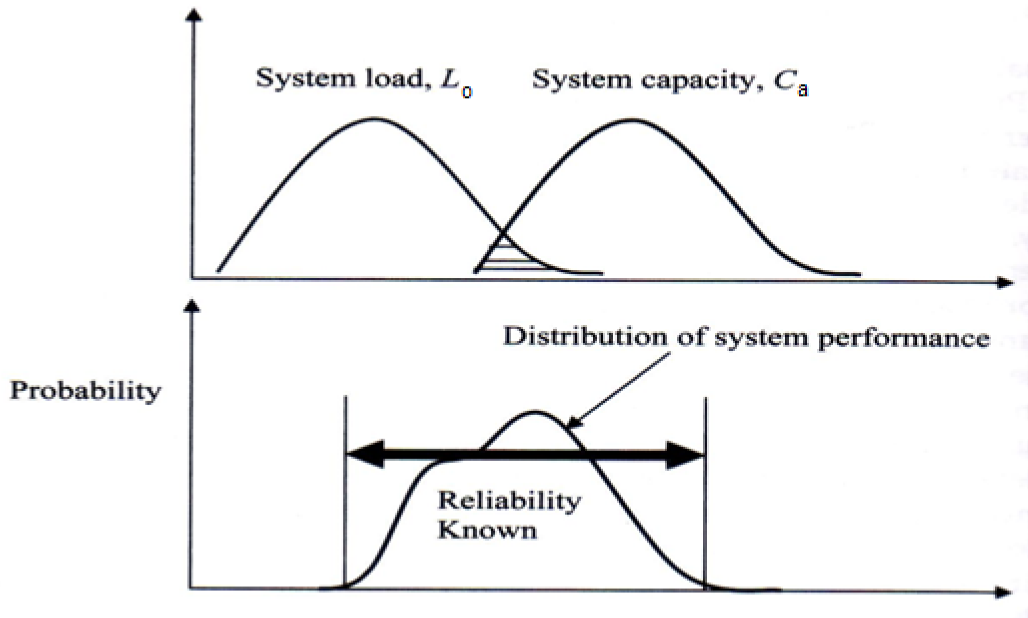

Figure 5-2 Probabilistic Approach of Safety Analysis of Engineering Systems

[Source: Singh, et al., 2007].

In most hydrologic, hydraulic, and environmental engineering projects, empirically developed or theoretically derived, mathematical models are used to evaluate systems performance. The reliability of a system can be most realistically measured in terms of probability. The failure of a system can be considered as an event in which the demand, or loading (L_o) on the system exceeds the capacity, or resistance (C_a), of the system, so that the system fails to perform satisfactorily for its intended use. The objective of reliability analysis is to ensure that the probability of the event $(C_a<L_o)$ throughout the specified useful life is acceptably small. The risk P_f defined as the probability of failure can be expressed by (Ang and Tang, 1984; Yen et al., 1986) by this formula:

$$P_f = P\,(L_o > C_a) \tag{5.1}$$

Equation 5.1 can be rewritten in terms of performance function Z

$$P_f = P\,(Z < 0) \tag{5.2}$$

Z can be expressed as:

$$Z = C_a - L_o \tag{5.3}$$

$$Z = (C_a - L_o) - 1 \tag{5.4}$$

$$Z = \ln (C_a / L_o) \tag{5.5}$$

Then the reliability R_L of the system can be defined as:

$$R_L = P\,(Z > 0) = 1 - P_f \tag{5.6}$$

where;

P= Probability of the function;

P_f = Probability of failure;

C_a= Capacity of the system;

L_o = Load on the system;

R_L = Reliability of the system;

Z = Performance function of the system.

5.2 Reliability Measures

The reliability index used to measure the reliability of engineering system reflects the mechanics of the problem and the uncertainty in the input variables and is defined as β. This index was developed in structural engineering to provide a measure of reliability without having to calculate an exact value of the probability of failure. It is defined by the expected value and standard deviation of the performance function of the system. Calculating the probability index requires two conditions:

1. The performance function Z must be defined and $Z= 0$, and the expected value $\mu(Z)$ and standard deviation $\sigma(Z)$ must be evaluated. So $Z< 0$ is unsafe zone, $Z > 0$ safe zone and limit state when $Z = 0$.
2. Assumption that the Z distribution is normally distributed

According to these requirements, the probability of failure P_f is given as

$$P_f = P(Z < 0 = \int_{-\infty}^{0} f_Z\ (Z)dZ = \int_{-\infty}^{0} \frac{1}{\sigma(Z)\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{z-\mu(Z)}{\sigma(Z)}\right)^2\right] \qquad (5.7)$$

The probability of failure is the area of the curve when *Z* below zero *(Z< 0)*, as shown in Figure 5-3. Equation 5.7 can be rewritten by substituting *Z=0*, and replacing the term *[μ(Z)/σ(Z)]* by *(β)* (Singh, et al., 2007):

$$P_f = \int_{-\infty}^{0} \frac{1}{\sigma(Z)\sqrt{2\pi}} exp\left[-\frac{1}{2}(-\beta)^2\right] = \Phi(-\beta) = 1 - \Phi(\beta) \qquad (5.8)$$

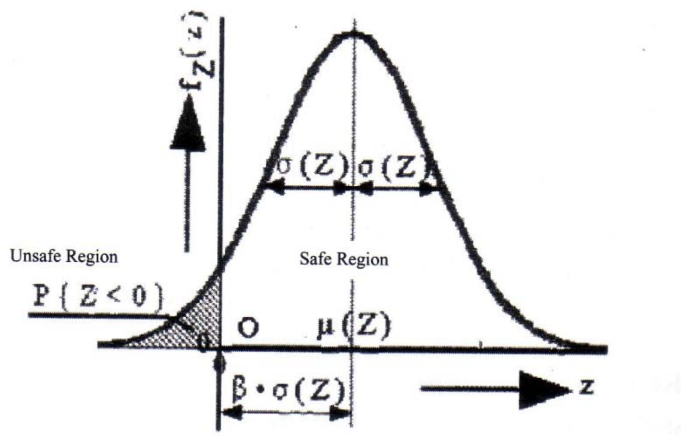

Figure 5-3 Interpretation of Reliability Index [Source: Singh, et al., 2007].

Further, reliability index β can be defined as

$$\beta = -\Phi^{-1}(P_f) \qquad (5.9)$$

The reliability index concept has gained considerable popularity. However it is not an absolute measure of probability. The assessment of reliability is made by comparing the calculated reliability index with that found adequate on the basis of previous experience with the structure under consideration (Galambos et al., 1982). The process begins with a mathematical model that relates the capacity and the loading of the system. This relation is called limit state function and the safety margin of the system is defined as (S), and $S = C – L$. The safe state is represented by $S > 0$, and the failure state represent by $S < 0$. The limit state is specified as $S = 0$ as shown in Figure 5.4.

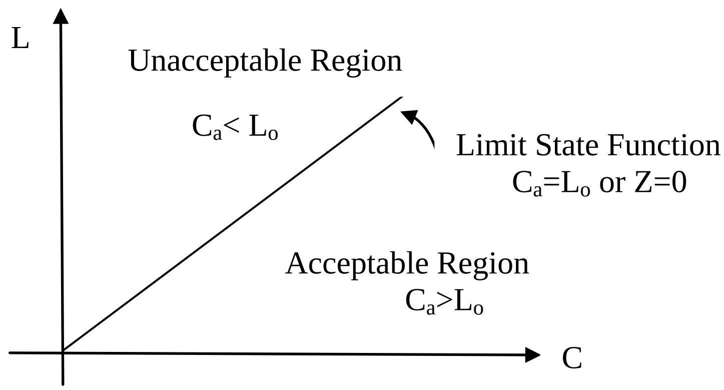

Figure 5-4 Limit State Function [Source: Singh, et al., 2007]

Further, defining reduced variables as

$$\mu_{L_o} = \frac{L_o - \mu(L_o)}{\sigma(L_o)} \quad and \quad \mu_{C_a} = \frac{C_a - \mu(C_a)}{\sigma(C_a)} \tag{5.10}$$

By substituting the values of C_a and L_o in the formulae above will perform the performance function in the limit state as

$$Z = C_a - L_o = 0, \quad \sigma (C_a)\, \mu\, C_a - \sigma (L_o)\, \mu_L + \mu(C_a) - \mu (L_o) = 0 \tag{5.11}$$

By plotting the equation 5.11 in the reduced coordinates system, the point chosen for the linearization is one which has the minimum distance from the origin in the space of transformed standard random variables. The point is known as the design point or most probable point *(MPP)* since it has the highest likelihood among all points in the failure domain. This is illustrated by Figure 5-5.

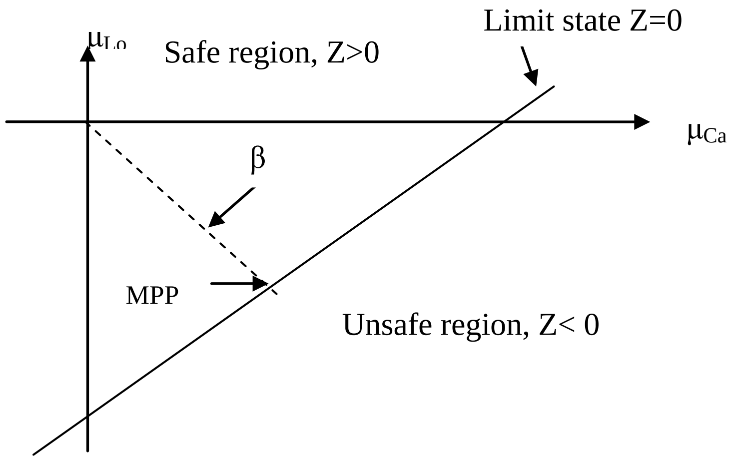

Figure 5-5 Limit State Function in Reduced Coordinate System, [Source: Singh, et al., 2007].

The straight line generated by this expression is equal to the reliability index *β*. The shortest distance of this line from the origin is equal to the perpendicular line from the origin, which is represented as

$$\beta = \frac{\mu(C_a)\text{-}\ \mu(L_o)}{\sqrt{\sigma^2\ (C_a) + \sigma^2\ (L_o)}} \tag{5.12}$$

And the probability of failure becomes

$$P_f = 1 - \Phi\ (\beta) \tag{5.13}$$

5. 3 Reliability Analysis Methods

The reliability analysis of a given system would be done by the assessment of a probability distribution function. The evaluation of this function should cover all the basic and significant factors of uncertainties affecting the output of a system. This would require several assumptions and the combination of input variable distribution, which is a challenging task. Furthermore, in real life problems, the aggregation of uncertainties as the basic variable of a model is completed using assumptions as well. Therefore, to overcome these assumptions there are various analytical methods in order to determine the uncertainties in engineering projects.

The most commonly used methods in hydraulics, hydrology, and environmental engineering are as follows (Singh et al., 2007):

1. Direct Integration Method
2. First Order Approximation Method
3. Monte Carlo Simulation
4. Second Order Approximation Method
5. First Order Reliability Method
    a) Rackwitz's Numerical Algorithm
    b) Lagrange Multiplier Method
    c) Ellipsoid Approach
6. Second Order Reliability Method

5.4 Risk and Reliability in Hydrology

Uncertainty analysis of the parameters related to the hydraulic design provides a means to assess the relative uncertainty contribution of these parameters to the entire design. Accordingly, measures to reduce uncertainties or to minimize the effects of uncertainties can be contemplated. A simple technique to assess the relative uncertainty contribution of parameters is through using a first order variance estimation technique. Yen (1979) gave an example by applying this first order technique in assessing the relative uncertainty of the contributing factors for the design of a culvert. For simplicity in illustration, the flood is estimated using the rational method and the culvert capacity is computed by using Manning's formula.

The risk is the probability of the failure of a system to perform the function for which it was designed. Generally, failure in an engineering system is defined as the load, L_o, exceeding the systems' capacity to resist. In hydrologic engineering, and especially in flood warning systems, the load is the peak discharge, and the resistance (capacity) may be considered as the critical flood stage. So the risk system (R_s) can be expressed as:

$$R_s = 1\text{-}R_L = P_f\,(L>R) \tag{5.14}$$

where;

R_L= is the reliability of the system.

As it has mentioned before, a convenient way to evaluate the risk is to use the performance function *(Z)* (Singh et al., 2007).

5.4.1 The Direct Integration Method

The direct integration method is based on the direct integration of the joint probability density function of which basic random variables are involved in the design. The standard integration methods such as analytical integration and advanced numerical integration methods can be used to simplify this equation for the direct integration method. Equation 5.15 illustrates the probability of failure (Singh et al., 2007).

$$R_s = \int_0^{\infty}\int_0^{l} F_{C_a,L_o}\,(c_a,l_o)dc_a\;dl_o \tag{5.15}$$

This equation can be further simplified in the case in which the capacity is statistically independent of the load, L_o, as shown in equation 5.16.

$$R_s = \int_0^{\infty} F_{L_o}\,(l_o)[\int_0^{l} F_{C_a}\,(c_a c)dc_a\;]dl_o \tag{5.16}$$

In the case of finding the appropriate distribution functions which correctly describe the load and capacity, the result would be an exact risk evaluated by this method. Since the evaluated risk is affected by the distribution functions, the greatest difficulty with this method is the selection of the proper distribution functions. This selection can be further complicated by the nature of the functions relating the basic variables to the load and capacity. For uncertainty analysis of hydrologic models, which are used for to real-time flood forecasting, similar conclusions may be made. Direct integration can only be used for very simple hydrologic models. Thus, for realistic flood warning cases, direct integration methods are not practical.

Therefore, given the nature and severity of the practical problems in applying the direct integration method, it is used primarily for very simple systems or for the analysis of a portion of the total system reliability, to check the validity and accuracy of simplified reliability methods for specific cases, and for systems which require highly accurate risk determination (Melching, et al., 1987).

5.4.2 Monte Carlo Simulation Method

Monte Carlo simulation is a process of using a particular set of values of random variables generated in accordance with the corresponding basic variable probability distribution. For each simulation, the performance function is calculated using the appropriate basic variable values, and the risk is estimated as the ratio of the number of failures versus the number of simulations. The Monte Carlo simulation method is an extremely flexible and widely used method and as such, it is a very useful method. The major application of the Monte Carlo technique is as an estimation of the probability distribution of a function of one or more random variables. Therefore it may be the only method which can estimate risk for cases with highly nonlinear and/or complex system relationships. Despite its flexibility, the Monte Carlo simulation is not a highly recommended way to analyze system risk. The risk estimated using this method is not unique because it depends on the size of the samples and the number of trials. To combat this error, large numbers of trials must be performed and thus the computer time required can become prohibitively expensive. These high computation costs tend to cancel out the flexibility of the Monte Carlo simulation methods. Furthermore, Monte Carlo simulation methods are also quite sensitive to the assumed distributions for the basic variables. Hence, Monte Carlo simulation methods are generally used as a last choice (Garen and Burges, 1981).

5.4.3 Mean Value First-Order Second Moment *(MVFOSM)* Method

The idea behind the first-order second moment reliability analysis methods was initially proposed a long time ago. Mayer (1926) suggested the use of the mean and variance of the random variables in the analysis of structural safety. In 1959, Su stated that the physical side of many structural problems is now well investigated, but the conventional method of structural design is still far from satisfactory. He developed a *MVFOSM* formulation based on the normal distribution and recommended its use for more rational determinations of structural safety factors. But it was not until Cornell (1967) elaborated on a formulation very similar to Su's that the *MVFOSM* method established a foothold in structural engineering.

Originally, Cornell (1969) used the simple two variable approaches. On the basic assumption that the resulting probability of *Z* is a normal distribution, Cornell (1969) defined the reliability index *(β)* as the ratio of the expected value of Z over its standard deviation. The *MVFOSM* method was first adopted for a hydraulic system risk evaluation by Tang and Yen (1972).

In the first-order methods, a Taylor series expansion of the performance function is reduced after the first-order term

$$Z = g(\underline{x}) + \sum_{i=1}^{P} (x_i - \overline{x}_i)\ \frac{dg}{dx_i} \qquad (5.17)$$

In the *MVFOSM* method, the expansion point is at the mean values of the basic variables. Thus, the performance function's expected value and variance are:

$$E\,(Z) \approx g\,(\,\underline{\overline{x}}\,) \qquad (5.18)$$

$$VAR\ (Z) \approx \sum_{i=1}^{P} C_i^2\ VAR(x_i) + \sum_{i=1}^{P} \sum_{j=1}^{P} C_i C_j\ COV\ (x_i\ ,\ x_j\ ) \qquad (5.19)$$

where;

x_i = the mean values of the basic variables;

C_i and C_j = the values of the partial derivatives dg/dx_i and dg/dx_j respectively, evaluated at $\overline{x}_1$, $\overline{x}_2, \ldots \overline{x}_P$.

If the variables are statistically independent, the covariance terms in equation (5.19) will eliminate and Equation 5.20 will develop as

$$VAR\ (Z) = \sigma_z^2 \approx \sum_{i=1}^{P} C_i^2\ VAR\ (x_i) \tag{5.20}$$

This is a reasonable approximation if the coefficients of variation of the basic variables are not large and the system performance function, *Z*, is approximately linear.

In the *MVFOSM* method, no distributional assumptions are made regarding the basic variables. Hence, the distribution of *Z* remains undefined, and the probability information contained in β is poor. Typically, it is assumed that *Z* is normally distributed, and thus the system risk (or the probability of failure P_f as in Equation 5.13) is:

$$R_s = P_f = 1 - \Phi\ (\beta) \tag{5.21}$$

If performance function *(Z)* is linear, and the load and capacity are normally distributed, then the Equation (5.21) gives the exact risk. If the system performance function is nonlinear such that $Z = ln(C/L)$ is appropriate and the load and capacity are lognormally distributed, Equation (5.21) gives a very close approximation of the exact risk as long as the coefficients of variation of *L* and *C* are relatively small. Therefore, the selection of the normal distribution for *Z* is quite reasonable and effective because many natural systems and / or variables can be shown to be normally or lognormally distributed.

The greatest advantage of the *MVFOSM* method is its simplicity; no higher order moments or distributional information on the system's basic variables are necessary. Only the mean and variance of the variables are needed to obtain a reasonable estimate of the system risk. The simplicity and practicality of the *MVFOSM* method has made it popular for a variety of water resources systems uncertainty analyses. The *MVFOSM* method has been directly applied to hydraulic structure reliability analysis for storm sewers (Tang and Yen, 1972) and culverts (Yen et al., 1980).

5.4.4 Advanced First-Order Second Moment *(AFOSM)* Method

Recently, researchers have wanted to maintain some of the simplicity of the *MVFOSM* method and yet reduce its errors. The basic concept behind the *AFOSM* method was first proposed by Hasofer and Lind (1974), but Rackwitz (1976) was the first to tie the entire *AFOSM* method together. Rackwitz's version of the *AFOSM* method will be outlined in the following paragraphs.

"*The essence of this method is to linearize the performance function via Taylor series expansion at a likely failure point (x_1*, x_2*, . ., x_p*) on the failure surface, i.e., when the performance function, g($\underline{x}$*), equals zero. The expected value and variance of the performance function as approximated by a first-order Taylor series at this point for the case of statistically independent basic variables are*

$$E(Z) \approx g\left(\underline{X}^*\right) + \sum_{i=1}^{P} C_i \; (\overline{x}_i - x_i^*) \tag{5.22}$$

$$VAR\ (Z) = \sigma_z^2 \; \approx \; \sum_{i=1}^{P} C_i^2 \; VAR(x_i) \tag{5.23}$$

$$\sigma_z \; \approx \; [\sum_{i=1}^{P} (C_i \sigma_i)^2]^{1/2} \tag{5.24}$$

where;

Ci, in this case, is dg/dx_i evaluated at (x_1 , x_2 , . . . , x_p*).

The expression for σ_z may be rewritten in a linearized form as below

$$\sigma_z \; \approx \; \sum_{i=1}^{P} \alpha_i \; C_i \; \sigma_i \tag{5.25}$$

In which the α_i's are sensitivity factors and are evaluated from

$$\propto_i = \frac{C_i \, \sigma_i}{\left[\sum_{j=1}^{P} (C_j \, \sigma_j)^2\right]^{1/2}} \tag{5.26}$$

Substituting equations 5.22, and 5.25 into the equation of forward difference method, the reliability index for the *(AFOSM)* method will be as below (Melching et al., 1987).

$$\beta = \frac{g(\underline{x}^*) + \sum_{i=1}^{m} C_i(\overline{x}_i - x_i^*)}{\sum_{i=1}^{P} \alpha_i\, C_i\, \sigma_i} \tag{5.27}$$

5.5 Reliability and Risk Analysis on Culvert

The Process of Culvert Risk Analysis is mainly based on the visual evaluation of the culvert during the site inspection and assessment. In general, there is insufficient information where the visual assessment is not enough. Therefore the risk analysis procedure uses a series of rating scales which can reflect the conditional probabilities that differentiate the chain of events that might occur for culvert failure which results in a risk to life and economic impact (RTA, 2010). In other words, not only is the inspection of culverts and drainage facilities of the most importance, but a risk assessment plan is also necessary for the analysis (Dunmire, 2011).

5.5.1 Factors Affecting Culvert Failure

Failure may occur due to insufficient capacity to pass floods. There are one or more reasons which may cause a culvert failure, such as: geotechnical, structural, flooding, erosion, and other causes such as earthquake, as shown in the Figure 5-6 (Henley, et al., 1981).

Figure 5-6 describes the qualitative analysis of the events, or combination of events that can lead to a failure of a system, which also illustrates the possible events that may result in the structural failure of a culvert.

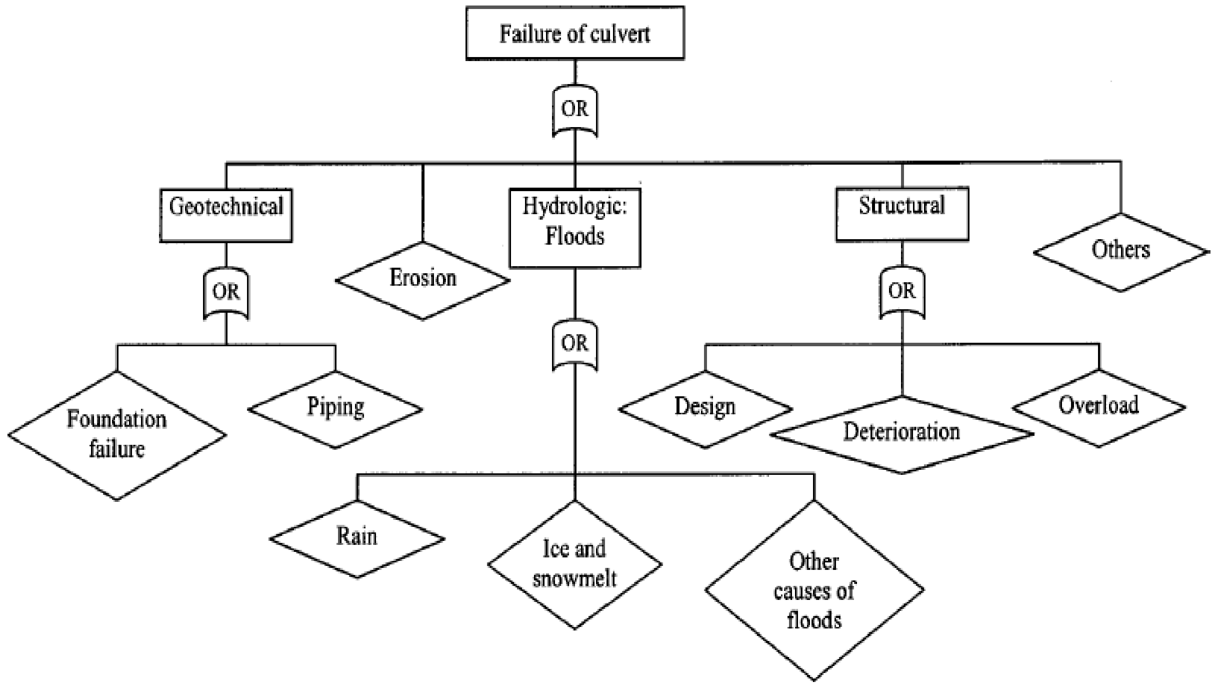

Figure 5-6 Logic Tree of Culvert Failure [Source: Lian, et al., 2003].

However, the performance failure possibly will occur due to inadequate capacity of the culvert to pass floods. This condition may not and often does not involve structural failure. Figure 5-7 illustrates hydrologic failure of a culvert due to flooding from rainfall. Ice blockage or melting of snow also can be also reasons for flooding.

Culvert Failure occurs when the loading (Q_L), produced from run off or rainstorms exceeds the carrying capacity or the resistance (Q_C) of the culvert. The risk of failure is then the probability (P) of the functions event Q_L and Q_C. Therefore, the performance function (Z) which represents the relationship between the capacity and the loading can be expressed as Equation 5.5.

$$Z = \ln (Q_C / Q_L) \tag{5.28}$$

In addition to hydraulic functions, culverts must carry the weight of the embankment and any load on the embankment (Lian, et al., 2003).

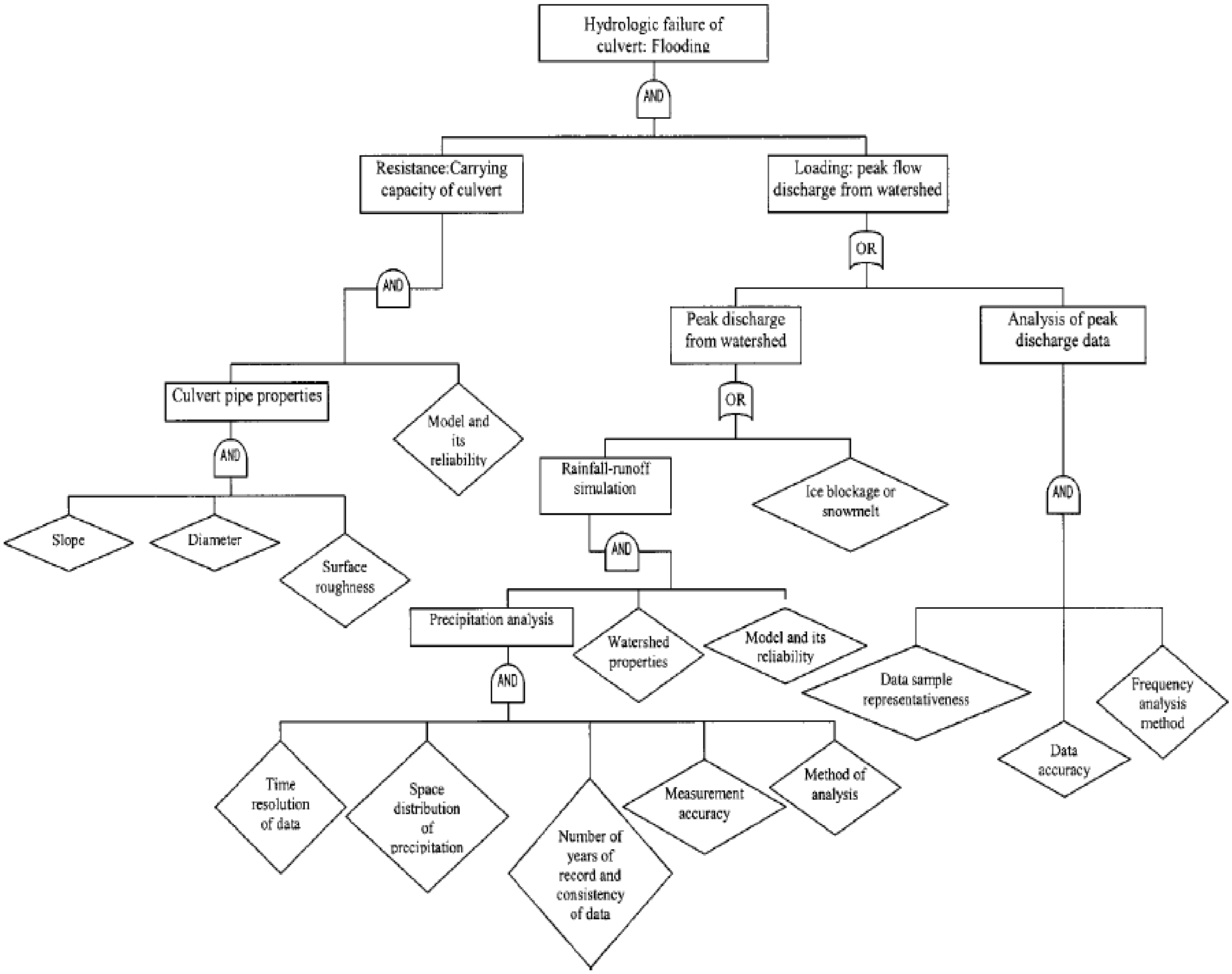

Figure 5-7 Hydrologic Causes for Culvert Failure [Source: Lian, et al., 2003].

5.5.2 Loads on the culvert

The evaluation of the effect of live loads, highway truck loads, in addition to dead loads are imposed by the soil and surcharge loads. It is also necessary to determine the required supporting strength of culvert barrel installed under intermediate and thin thicknesses of pavements or relatively shallow earth cover (ACPA, 2009). Generally there are two types of loads which must be carried by the culvert: dead loads and live loads.

5.5.2.1 Dead Load

Dead loads include the weight of the soil over the culvert with any surcharge loads, such as buildings (Rossow, 2009). The discharge can be uniformly distributed and converted to an equivalent height of fill, which is then evaluated as an additional soil load. When a concrete pipe has been installed underground, the soil-structure system will continually show an increase in load capacity (ACPA, 2009). The design pressure is a function of the projection condition, different soil properties, and the ratio of height over base length of the culvert *H*/*B*. Clarke (1967) developed typical formulae to determine the value of the soil–structure interaction factor, *Fe*.

$$F_e = \frac{e^{0.38\left(\frac{H}{B}\right)}}{0.38\left(\frac{H}{B}\right)} \qquad H/B \leq 2.42 \qquad (5.29)$$

$$F_e = 1.69 - \frac{0.12}{HB} \qquad H/B > 2.42 \qquad (5.30)$$

where;

F_e = Soil–structure interaction factor;

H/B = Ratio of height over base length of culvert.

The vertical pressure on the culvert is defined as the pressure due to the weight of the soil column above the culvert multiplied by the soil–structure interaction factor F_e (Bennett et al., 2005). The factor F_e is used to adjust the vertical earth load carried by the culvert because it is numerically equal to the factor by which the soil unit weight can be multiplied to obtain the equivalent soil unit weight (LRFD, 2009). If the actual unit weight of earth is unknown, a constant of 1920 kg/m^3 is generally assumed (Rossow, 2009).

5.5.2.2 Live Loads

Live loads include the loads and forces which act upon the culvert due to vehicular or pedestrian traffic. The surface load is assumed to be uniformly spread on any horizontal subsoil surface. The spread load area is developed by increasing the length and width of the wheel with the contact area for a load configuration, as illustrated in Figure 5-8. Figure 5-9 illustrates dual wheels of two trucks in passing mode; while the example for two dual wheels of axles 2 and 3 in passing mode is shown in Figure 5-10 respectively (ACPA-M, 2009).

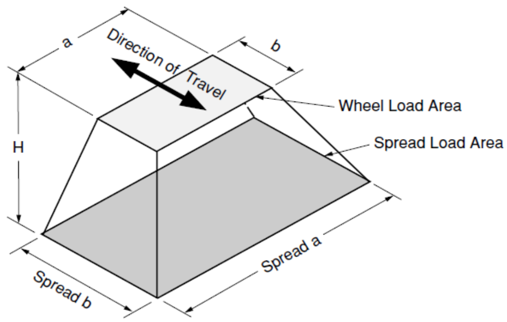

Figure 5-8 Spread Load Area-Single Dual Wheel.

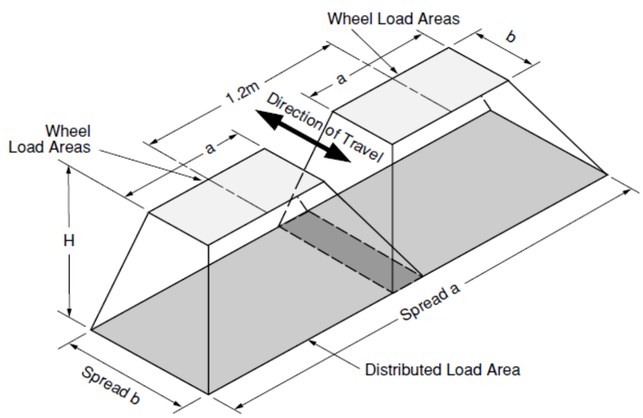

Figure 5-9 Spread Load Area-Two Single Dual Wheels of Trucks in Passing Mode.

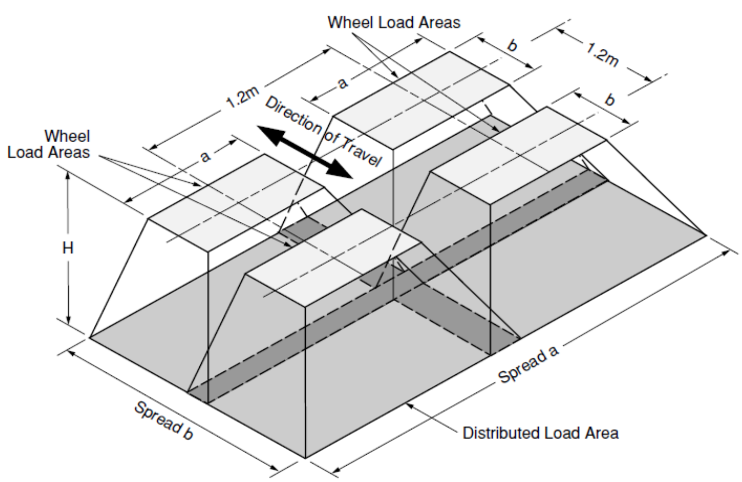

Figure 5-10 Spread Load Area-Two Single Dual Wheels of Axles 2 and 3 in Passing Mode.

[Source: ACPA-M. 2009].

The thickness of the pavement will have an effect on load transmission. Pavement is designed for heavy traffic with high thickness and strength, which reduces the pressure transmitted through a wheel of subgrade and, therefore, to the underlying pipe. The pressure reduction is so great that generally the live load can be neglected. Also, when a sufficient buffer between the pipe and pavement is provided, then the live load is transmitted to the buried pipe and is usually negligible at any depth. If any culvert is within a heavy duty traffic highway right-of-way, but not under the pavement structure, then such a pipe should be analyzed for the effect of live load transmission from an unsurfaced roadway. This is due to the possibility of trucks leaving the pavement. However, intermediate and slight thicknesses of pavements do not cause a reduction in the pressure transmitted from a wheel to the pavement subgrade, and therefore, these types of pavements should be considered for unsurfaced roadways (ACPA-M, 2009).

The effect of live loads decreases as the height of the embankment increases. As the thickness of the embankment increases, the loads may be considered as being spread uniformly over a square with sides *1.75* times the depth of cover, as illustrated in Figures 5-8 to 5-10. In fact, for single spans, if the height of embankment is more than 2.4 meters, exceeding the span length, all effects of live loads can be ignored (Rossow, 2009).

AASHTO has commonly used *HS 20* with a *32,000* pound axle load in the Normal Truck Configuration, and a *24,000* pound axle load in the Alternate Load Configuration. The Canadian Highway Bridge Design Code *(CHBDC)* has been used as the design loads, which applies on CL-W Truck and CL-625 ONT Truck as in Figure 5-11. These design truck axles are carried on dual wheels where the contact area of the dual wheels with the ground is assumed to be a rectangle as shown in Figure 5-12 with dimensions; $a = 0,60m$ as width and $b = 0.25m$ as length (CHBDC, 2000).

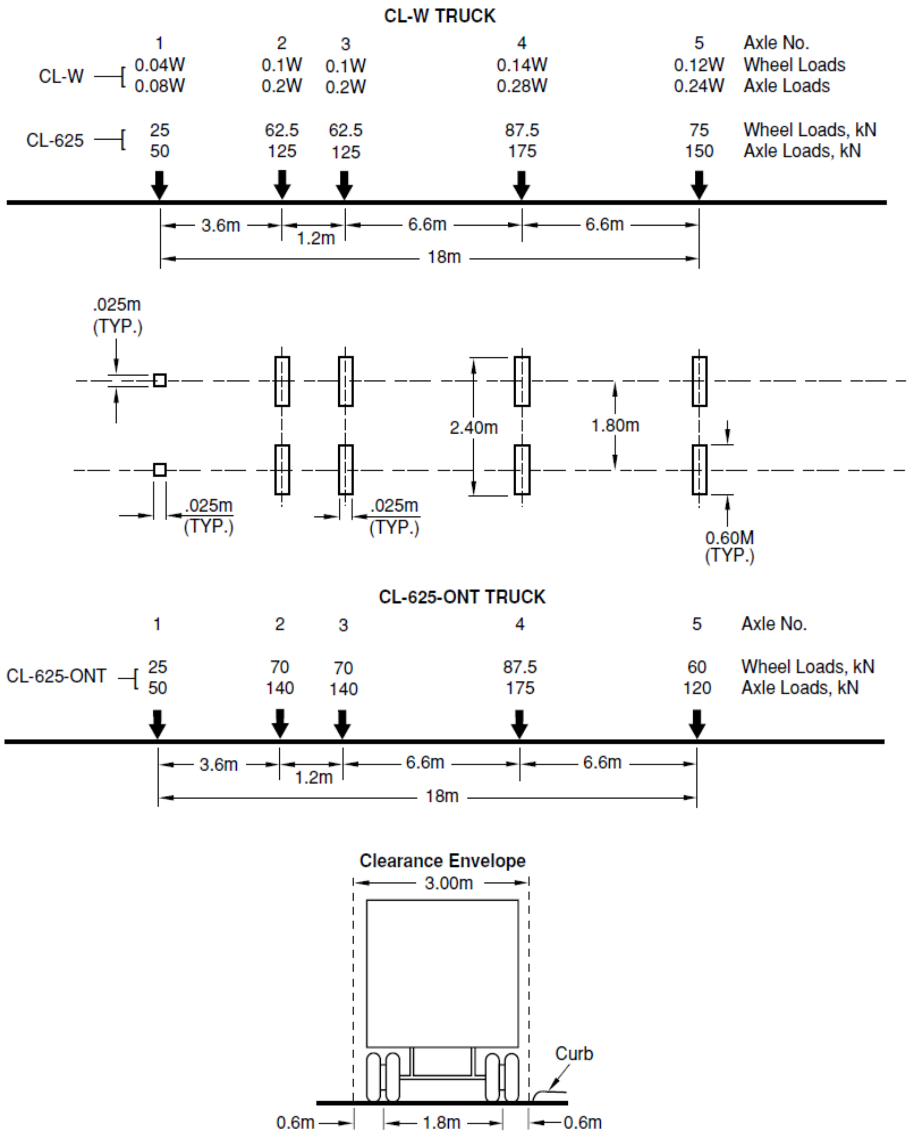

Figure 5-11 CHBDC Wheel Loads and Wheel Spacings [Source: ACPA-M, 2009].

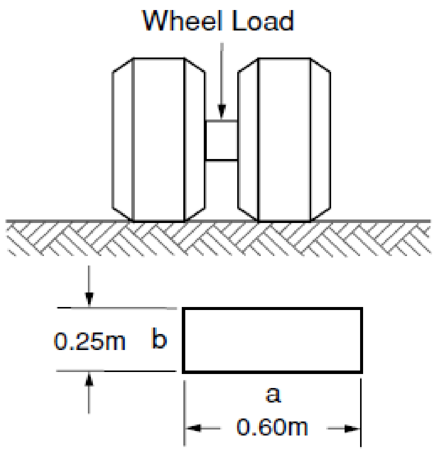

Figure 5-12 Illustrates the CHBDC Wheel Load Surfaces Contact Area (Foot Print)

[Source: ACPA-M, 2009].

The average pressure intensity caused by a wheel load is calculated by Equation 5.31.

$$W_L = P_L\,(1 + IM)\,/A_L \tag{5.31}$$

where;

W_L = wheel load average pressure intensity, kN/m^2;

P_L = total live wheel load applied at the surface, kN;

A_L = spread wheel load area at the outside top of the pipe, m^2;

IM = dynamic load allowance.

The *CHBDC* considers that the truck load is non-static, which applies a dynamic load allowance *IM*, from Equation 5.32

$$IM = 0.40\,(1.0 - 0.5D_E) \geq 0.10 \tag{5.32}$$

where;

D_E = height of earth cover over the top of the pipe, m.

The maximum possible load and its distribution over the culvert barrel must be considered in the design stage. The amount of the load depends on the pipe size and height of cover, where the most critical loading orientation would occur either when the truck travels transverse or parallel to the centerline of the pipe. The spread load area will change in relation to whether the truck travel is transverse or parallel to the centerline of the pipe as illustrated in Figure 5-13.

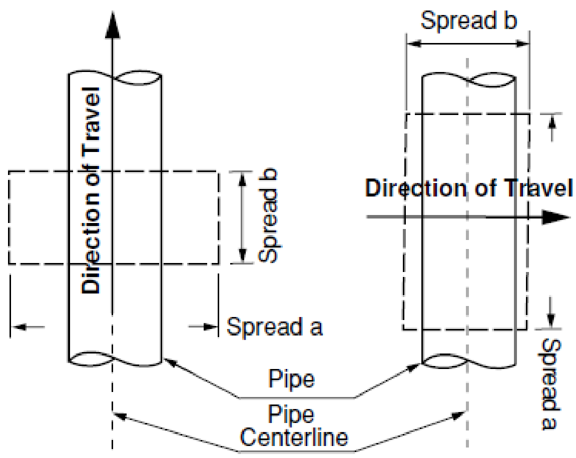

Figure 5-13 Spread Load Area Dimensions verses Direction of Truck [Source: ACPA-M, 2009].

Total live load, W_T, must be calculated for each travel orientation from Equation 5.33, and the maximum calculated value must be used in Equation 5.34 to determine the live load on the pipe in kN/m.

$$W_T = W_L * L_P * S_L \tag{5.33}$$

where;

W_T = total live load, kN;

W_L = wheel load average pressure intensity, kN/m^2 (at the top of the pipe);

L_P = dimension of A parallel to the longitudinal axis of pipe, (m);

S_L= outside horizontal span of pipe.

$$W_{LP} = W_T / L_e \tag{5.34}$$

And the effective supporting length of pipe is calculated by Equation 5.35.

$$L_e = L_p + 1.75(3/4R_o) \quad (5.35)$$

where,

W_{LP} = live load on top of pipe, kN per linearmeter;

L_e = effective supporting length of pipe, (m);

R_o = outside vertical Rise of pipe, (m), as shown in Figure 5-14 (ACPA-M, 2009).

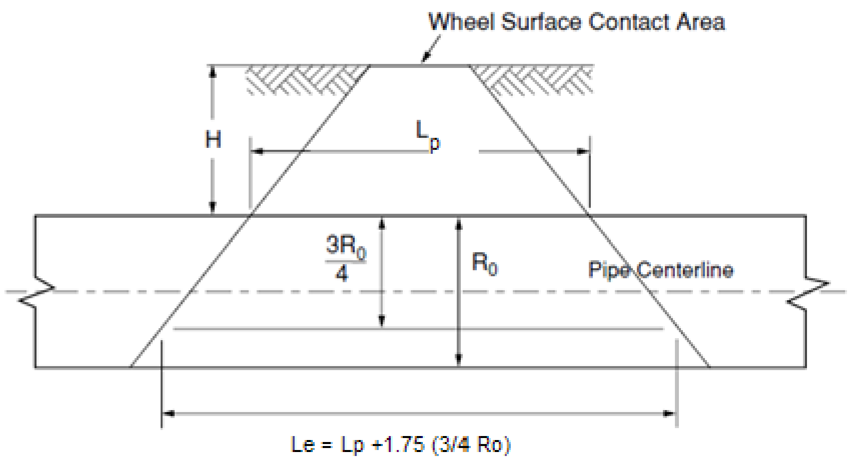

Figure 5-14 Effective Supporting Length of Pipe [Source: ACPA-M, 2009].

Therefore, culverts must be designed to carry the dead load of the soil over the culvert even in the addition of live loads of traffic. Although live loads on culverts are insignificant when the embankment is thick, the type of culvert, depth and type of embankment, as well as the amount of live load are important factors in the calculation the capacity of the culvert (Rossow, 2009).

Chapter 6: Developing Optimization and Reliability Model

6.1 Overview

Mathematical programming is a general technique used to solve resource allocation problems using optimization. Linear and Non-Linear Optimization are designed to calculate an optimized set of decision variables that either minimize or maximize a given objective function, while also satisfying a set of arbitrary constraints defined by the user. The reliability of any system is the probability that the system performs a specified function under specified operational and environmental conditions at, and throughout a specified time (Trani and Rakha, 2000).

More than a half century ago, industrial engineers applied some reliability techniques for quality control of manufactured product. In civil engineering considerable advances have been made by structural engineers on earthquakes and the high wind risks of structures. Some progress has been made in geotechnical and water resources engineering as well. In the design of hydraulic structures, analyses of flow processes in hydrology and hydraulic are involved (Yen and Tung, 1993).

The optimization model is defined by algebraic formulas, as in an algebraic modeling system. There can be a wide range of mathematical relationships for distinguishing decision variables, and including other variables, objectives, or constraints from other formulas.

6.2 Optimization by MATLAB

The MATLAB optimization program offers broadly used algorithms for simple, standard and large-scale optimization models. These algorithms solve constrained and unconstrained as well as continuous and discrete problems. The toolbox incorporates functions for linear programming, quadratic programming, binary integer programming, nonlinear optimization, nonlinear least squares, systems of nonlinear equations, and multi-objective optimization. It can be used to find optimal solutions, perform tradeoff analysis, balance multiple design alternatives, and incorporate optimization methods into algorithms and models (MATLAB Getting Started Guide, 1984).

Suppose a watershed needs to have the excess water drained away from the structures. Many constraints will affect the design procedure in selecting the proper culvert type. In this study, it is required to determine the decision variables which include the dimensions of different types of culvert [B (width of box culvert), D (depth of box culvert), D_{ia} (Diameter of circular culvert) and span and the height of arch culvert), the critical depth (dc), and the normal depth (d_n) for these culverts. The objective function is to minimize the cost of the designed culvert. The cost analysis includes the cost of the culvert, according to the size (dimensions) of the culvert, type of the inlet configuration, and the type of the culvert material. The cost analysis is contained installation costs and the maintenance cost within the service life of the culvert (see Appendix A). In addition to the cost analysis, minimizing the reliability index was one of the main constraints in optimizing the dimension of the design culvert.

This model provided effective solutions to overcome these conventional methods:

1. The nomographs which contain some errors since they are a graphical method,
2. The trial and error method when calculating critical and normal depth,
3. The trial and error method when selecting the size of the culvert by checking the velocity and other hydraulic criteria. If it does not meet the criteria another trial culvert size must be selected.

So in this model, the best diameter, material, inlet configuration and the minimum cost for the culvert will be determined, without any trials leading to time consumption or any errors.

6.3 Procedures

As mentioned earlier, the objective function is designed to minimize the total cost of the culvert. Total cost of the culvert is the function of the culvert dimension which optimized by the MATLAB program, material of the culvert, and the culvert inlet configuration, so therefore it can be written as:

Minimize the Total Cost

$$Subject to \;\; B_i, D_i, Dia_i, Sp_i, Hi_i, dc_i, dn_i \qquad \forall i \tag{6.1}$$

6.3.1 Calculating Area of Flow and Wetted Perimeter for Different Culvert Types

Calculation of the area and wetted perimeter are used in calculation of the critical depth and the normal depth or for any given depth of water.

6.3.1.1 Box Culvert

Calculation of the flow area for any given depth h is given by

$$AR = B*h \tag{6.2}$$

where;

AR = Flow area at given depth (m^2),

h = depth of the flow (m),

B= Width of the box culvert (m),

D= Depth of the box culvert (m).

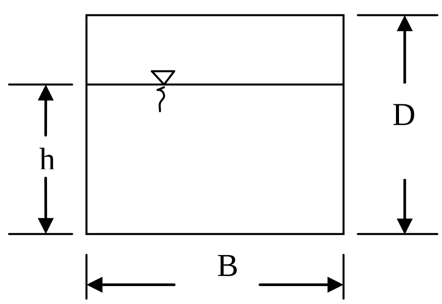

Figure 6-1 Box Culvert

The wetted perimeter P_w is calculated as

$$Pw = B + 2h \tag{6.3}$$

P_w = Wetted perimeter (m)

6.3.1.2 Circular Culvert

Calculating the depth of the water is by using

$$h = Dia/2 + Dia/2 * \cos\delta \tag{6.4}$$

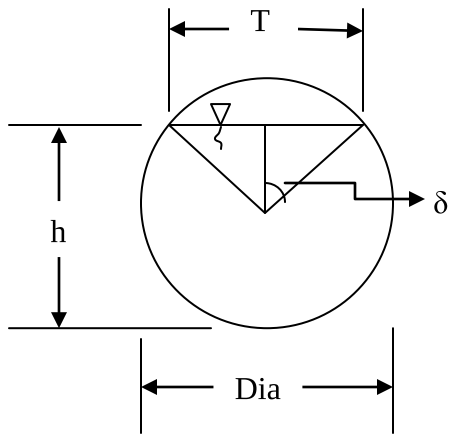

Figure 6-2 Circular Culvert

The corresponding flow area *AR* will be:

$$AR = (\pi - \delta + \sin(2\delta) / 2) * Dia^2 / 4 \tag{6.5}$$

The wetted perimeter P_w:

$$P_w = Dia * (\pi - |\delta|) \tag{6.6}$$

The top width T:

$$T = 2(h - Dia/2)\tan(\delta) \tag{6.7}$$

Where;

δ = Central angle

Dia = Diameter of the circle

T = Top width

6.3.1.3 Arch Culvert

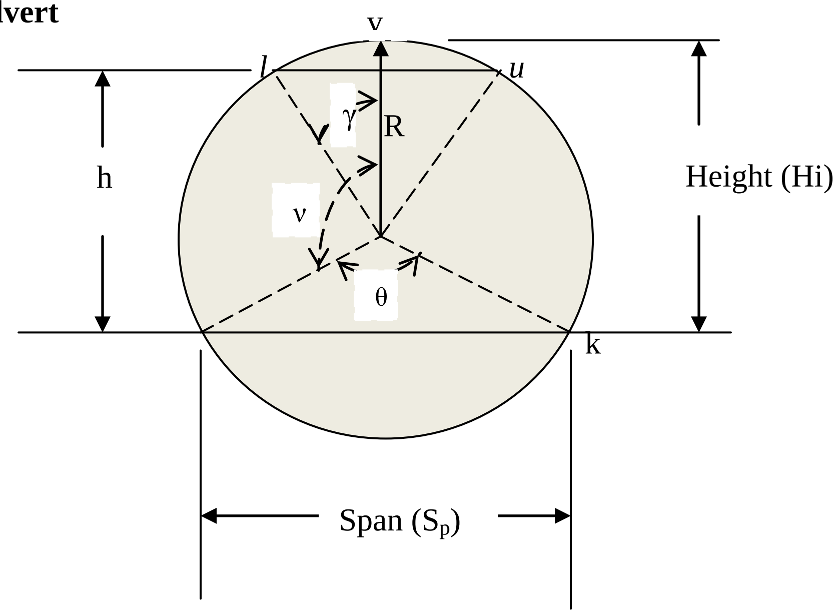

Figure 6-3 Arch Culvert

Height of arch culvert Hi:

$$Hi = R\,(1+\cos(\theta/2)) \tag{6.8}$$

Span of arch culvert S_p:

$$S_p = 2R \sin(\theta/2) \tag{6.9}$$

The full flow area *AF*:

$$AF = \pi R^2 - Area\ (nwk) \tag{6.10}$$

$$Area\ (nwk) = Area\ (cnwk) - Area\ (cnk) = \pi R^2 * \theta/ (2\pi) - S_p*(Hi - R)/2 \tag{6.11}$$

then *AF*:

$$AF = \pi R^2 - R^2\, \theta/2 + S_p * (Hi - R)/2 \tag{6.12}$$

The full wetted perimeter P_wF is given by:

$$P_wF = 2Rv + S_p \tag{6.13}$$

where;

$$\nu = (2\pi - \theta)/2 \qquad (6.14)$$

The flow area *AR* for any given depth *h:*

$$h = Hi - (R - R\cos\gamma) \qquad (6.15)$$

$$AR = AF - [Area\ (clvu) - Area\ (clu)] = AF - \pi R^2(2\gamma/2\pi) + 0.5T\,R\cos\gamma \qquad (6.16)$$

$$AT = AF - R^2\,\gamma + T\,R\cos\gamma/2 \qquad (6.17)$$

Where *T* represents the top width of flow:

$$T = 2R\sin\gamma \qquad (6.18)$$

$$\cos\gamma = (h - R\cos(\theta/2))/R \qquad (6.19)$$

The wetted perimeter P_h for any given depth *h*:

$$P_h = P_wF - 2R\gamma \qquad (6.20)$$

6.3.2 Inlet Headwater Calculation

$$HWi = [a + bF_i + cF_i^2 + dF_i^3 + eF_i^4 + fF_i^5]\,D_i - 0.5D_i\,S_o \qquad \forall_i \qquad (6.21)$$

where,

$$F = 1.8113\,\frac{Q}{W_i D_i^{3/2}} \qquad \forall_i \qquad (6.22)$$

IF $0.5 \leq (HW_i/D_i) \leq 3.0$ IF not, then use Equation (6.23)

$$HW_i = \left[\frac{Q}{k}\right]^2 + \frac{D}{2} \qquad \forall_i \qquad (6.23)$$

where,

$$k = 0.6325\,\frac{Q_{3.0}}{D^{1/2}} \qquad \forall_i \qquad (6.24)$$

$$HWi < HW_{max} \qquad \forall_i \qquad (6.25$$

where,

$$HW_{max} = HWL_{max} - IEI \qquad \forall_i \qquad (6.26)$$

$$HWL_i = HW_i + IEI \qquad \forall_i \qquad (6.27)$$

6.3.3 Critical depth calculation

d_{ci} will be calculated according to the Froude Number:

$$F_r = \frac{Q^2 T_i}{g A_{ci}^3} \qquad \forall i \qquad (6.28)$$

Calculate the velocity at selected critical depths for all types of culverts from the equation:

$$V_{ci} = Q / A_{ci} \qquad \forall i \qquad (6.29)$$

and V_{ci} must be within the range

$$0.5 \le V_{ci} \le 4.5 \ m/s \qquad \forall i \qquad (6.30)$$

6.3.4 Normal depth calculation

The d_{ni} will be calculated according to the Manning equation:

$$Q = (A_{ni} * R_{ni}^{2/3} * S_o^{0.5}) / n \qquad \forall i \qquad (6.31)$$

6.3.5 Outlet headwater calculation

The outlet headwater elevation would be calculated using the initial depth *(d_o)*. The *(d_o)* variable is the maximum value of the tailwater *(TW)* or the maximum of the calculated critical depth *(cal_d_c)*. The value of the calculated critical depth can be found from the comparison between TW and D_i in the case of box culvert, and it can be compared with diameter of the circle pipe *(D_{iai})*, and in the case of the arch culvert, compared with the height of the arch *(H_i)*.

IF $\quad TW > D_i \quad cal\_d_{ci} = TW \quad \forall i \quad (6.32)$

IF $\quad TW < D_i \quad cal\_d_{ci} = (d_{ci} + D_i) / 2 \quad \forall i \quad (6.33)$

Height of the water at the outlet (H_o) will be calculated from

$$H_{oi} = (1 + K_e + (K_u * n^2 * L) / R_{oi}^{1.33}) * V_{oi}^2) / 2g \qquad \forall i \qquad (6.34)$$

Substitute d_{oi} as the depth of the water in all calculations for H_o,

The headwater at outlet *(HW_o)* will be calculated from

$$(HW_{oi}) = H_{oi} + d_{oi} - LS_o \qquad \forall i \qquad (6.35)$$

Allowable headwater at outlet *(HWL_o)* of the culvert

$$HWL_{oi} = HW_{oi} + IEO \qquad \forall i \qquad (6.36)$$

The Inlet and outlet control of the culverts can be found by comparing the allowable head water elevation at inlet *(HWL_i)* with the allowable headwater at outlet *(HWL_o)* as shown in the condition below

IF $\quad HWL_i > HWL_o \quad$ *Inlet Control* $\quad \forall i \quad (6.37)$

IF $\quad HWL_i < HWL_o \quad$ *Outlet Control* $\quad \forall i \quad (6.38)$

Check the velocity at the culvert outlet (V_{oi}) by calculating the d_{out} under these conditions

IF $\quad TW < d_{ci}, \quad d_{outi} = d_{ci} \quad \forall i \quad (6.39)$

IF $\quad d_{ci} < TW < D_i, \quad d_{outi} = TW \quad \forall i \quad (6.40)$

IF $\quad TW > D_i, \quad d_{outi} = D_i \quad \forall i \quad (6.41)$

V_{oi} must be within the range $\quad 0.5 < V_{oi} < 4.5\ m/s \quad \forall i \quad (6.42)$

Check for $\quad B_i, D_i, D_{iai}, S_{pi}, \text{ and } Hi_i > 0.3\ m \quad \forall i \quad (6.43)$

Check for $\quad 0.3 \leq (D_i / B_i) \leq 1 \quad \forall i \quad (6.44)$

Check for $\quad 1.35 \leq (S_{pi} / Hi_i) \leq 3 \quad \forall i \quad (6.45)$

Check for maximum of $\quad (HWL_i \text{ and } HWL_o) < HWL_{max} \quad \forall i \quad (6.46)$

6.3.6 Reliability Analysis

As explained in chapter 5, the hydraulic failure of the culvert is occur when the capacity of the culvert cannot convey the water during the flood. This is mainly considered as a major factor and can be caused by structural failure due to the total load which is more than the structural strength of the culvert. This factor will be in effect when the thickness of embankment is less than 2.4 meters. Therefore, in this case, the effects of live loads can be ignored.

6.3.6.1 Performance Function

Performance function which has been used in the optimization model includes two major functions. These functions are capacity of the culvert and the hydraulic loading on the culvert. The hydraulic load on the culvert is calculated according to the rational method. The rational method for estimating peak flows is widely used by designer engineers. It is based on intensity–runoff relationship.

$$Q_L = C^a I^a A^a / 360 \tag{6.47}$$

where:

Q_L = Runoff, m^3/s

C^a = Runoff coefficient

I^a = Intensity, mm/hr

A^a = Area, ha

The runoff coefficient (C^a) is the only manipulative factor in the rational formula, which must include most of the hydrological abstraction, soil types, and antecedent conditions, among others.

The values of C^a coefficient are listed in Table 6-1

Table 6-1 Values of Runoff Coefficient C^a [Source: AISI, 1984]

Types of Surface	Factor C^a
For all watertight roof surfaces	0.75 to 0.95
For asphalt runway pavements	0.80 to 0.95
For concrete runway pavements	0.70 to 0.90
For gravel and macadam pavements	0.35 to 0.70
* For impervious soil (heavy)	0.40 to 0.65
* For impervious soil, with turf	0.30 to 0.55
* For slightly pervious soils	0.15 to 0.40
* For slightly pervious soils, with turf	0.10 to 0.30
* For moderately pervious soils	0.05 to 0.20
* For moderately pervious soils, with turf	0.00 to 0.10

* For slopes from 1% to 2%

In this study, the modified rational formula was used in the calculation of the hydraulic load on the culvert, as illustrated in Equation (6.48).

$$Q_L = (C^a C_m I^a A^a)/360 \tag{6.48}$$

Table 6-2 lists the recommended antecedent precipitation factors, which the value (C_m*C^a) should not exceed 1. In this study C_m=1.0 was chosen to calculate the hydraulic load of the culvert.

Table 6-2 Recommended Antecedent Precipitation Factors for the Rational Formula [Source: AISI, 1984]

Recurrence Interval (Years)	C_m
2 to 10	1.0
25	1.1
50	1.20
100	1.25

Check for the reliability of the culvert by the performance function Z

$$Z = \ln(\sigma_C / \sigma_L) > 0 \tag{6.49}$$

where:

σ_C capacity of the culvert according to the Manning equation,

σ_L the magnitude of load on the culvert according rational equation

The Manning equation was used to calculate the capacity for determining the reliability and risk of failure of the pipes (Lian and Yen, 2003) and the same formula was used for open channels (Easa, 1994).

6.3.6.2 Reliability Index

The advanced first order second moment (AFOSM) method was used for calculating the reliability index which determines the reliability and probability of failure for the designed culvert. In this study (AFOSM) is applied with Rackwitz's numerical algorithm.

The following algorithm is used to calculate the Reliability Index:

The matrix procedure:

1. Formulate the limit state function and appropriate parameters for all random $X_i,\ i = 1,...,n$ variables.
2. Obtain an initial design point { X^*_i} assuming values for n-1 of the random variables (mean values are often a reasonable initial choice). Solve the limit state equation g=0 for the remaining random variable. This ensures that the design point is on the failure boundary.
3. Determine the reduced variables { Z^*_i} corresponding to the design point { X^*_i}

using $$Z^*_i = \frac{X^*_i - \mu_{xi}}{\sigma_{xi}} \qquad \forall i \qquad (6.50)$$

4. Determine the partial derivatives of the limit state function with respect to the reduced variables. For convenience, define a column vector {G} as the vector whose elements are those partial derivatives multiplied by –1:

$$G = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{bmatrix},\ G_i = \frac{\delta g}{\delta Z_i}\Big| \quad \textit{evaluaeted at deign po}\text{int}, \qquad \forall i \qquad (6.51)$$

Where; $$\frac{\delta g}{\delta Z_i} = \frac{\delta g}{\delta X_i}\frac{\delta X_i}{\delta Z_i} = \frac{\delta g}{\delta X_i}\sigma_{Xi} \qquad \forall i \qquad (6.52)$$

5. Calculate an estimate of β using the following formula:

$$\beta = \frac{G^T Z^*}{\sqrt{G^T G}} \qquad \forall i \qquad (6.53)$$

where:

$$Z^* = \begin{bmatrix} Z^*{}_1 \\ Z^*{}_2 \\ \vdots \\ Z^*{}_n \end{bmatrix} \qquad \forall i \qquad (6.54)$$

6. Calculate a column vector containing the sensitivity factor using

$$\alpha = \frac{G^T}{\sqrt{G^T G}} \qquad \forall i \qquad (6.55)$$

7. Determine a new design point in reduced variables for n-1 of the variables using

$$Z^*{}_i = \beta\alpha_i \qquad \forall i \qquad (6.56)$$

8. Determine the corresponding design point values in original coordinates for the n-1 values in Step 7 in form

$$X^*{}_i = \sigma_{xi} Z^*{}_i + \mu_{xi} \qquad \forall i \qquad (6.57)$$

9. Determine the value of the remaining random variable (i.e., the one not found in Steps 7 and 8) by solving the limit state function g = 0.

10. Repeat Steps 3 to 9 until β and the design point $\{ X^*{}_i \}$ converge.

Assume that the variables are uncorrelated and are normally distributed. First, the performance function will simplify, then initial values for the design points will select (usually, they will assume the exact value as initial value) then calculate the reduced variables and calculate the partial derivatives of the limit state function with respect to the reduced variables at the designed point. Then steps (5) – (10) are solved using the MATLAB software.

6.3.7 Constraints

During the calculation stages for the optimum diameter of a culvert, there are several constraints which must meet the criteria. These conditions are illustrated in Table 6-3 and the hydraulic constrains are described from the section 6.3.1 to section 6.3.5.

Table 6-3 Summary of the Constraints Used in the Program

Q in manning formula must be equal to Q_{design}	
F = 1 or Q = Q_{design} in calculation of critical depth	
Normal depth, dn > zero	
Critical depth, dc > zero	
Headwater for inlet, (HW_i) is smaller than the maximum headwater, (HW_{max})	$HW_i < HW_{max}$
The ratio of width of box culvert to the height of culvert must be	0.35 < ratio < 1
The ratio of arch span to the height of span must be	1.35 < ratio < 3
Outlet velocity, (V_o) must be	$0.5\ m/s^2 < Vo < 4.5\ m/s^2$
Risk of failure > 0	
Minimum diameter of any pipe culvert or box dimension,	$di \geq 0.3$ m
Check for HW_i/D_i to select required HW_i formula	$0.5 \leq (HW_i/D_i) \leq 3.0$
Defined the flow control type in the culvert by	$HWL_i > HWL_o$ or $HWL_i < HWL_o$
Calculate the depth of water at outlet under these conditions	
	IF $TW < d_{ci}$, $d_{outi} = d_{ci}$
	IF $d_{ci} < TW < D_i$, $d_{outi} = TW$
	IF $TW > D_i$, $d_{outi} = D_i$
Performance function =0 by calculate the minimum β (reliability index by Advanced first order second moment method in the same optimization program)	

Chapter 7: Application of Model

7.1 Overview

The optimized model can be applied for new designed culvert to determine the optimum dimensions of the culvert with minimum cost and reliability. This model can also be used for determine the reliability of an exist culvert. In this case of application the dimensions of the culvert will be as input data rather than an objective function to be optimized.

7.2 Input Data

There are three types of culverts with different materials and inlet configurations. The objective of this problem is finding the minimum cost for these optimum dimensions of culvert with the minimum probability of failure.

The input parameters are used in this optimization model: the design flow or runoff (Q *or* Q_L), which is calculated from watershed area in hectares ($A^a = 200\ ha$), runoff coefficient ($C^a = 0.09$), and rainfall intensity for the estimated time of concentration in millimeter per hour ($I^a = 50\ mm/h$).

Maximum allowable headwater in meters ($HWL_{max} = 33.528m$), invert elevation of outlet in meters ($IEO=28.737m$), invert elevation at inlet in meters ($IEI = 30.48m$), longitudinal slope of the culvert in meters per meter ($So=0.003m/m$), length of the culvert in meters ($L = 15m$), acceleration of gravity ($g = 9.81\ m/s$), regression coefficients (*a to f*) for calculating the headwater at inlet control for each type of culvert are given in Table 4-3.

Inlet coefficient for different culvert types (K_e), and roughness coefficient of the bed for different type of materials of culvert (n) as shown in Table 7-1. Table 7-2 illustrates the mean and the coefficient of variance for the input data used in the calculation of the reliability index.

Table 7-1 Input Data

Shape	material	Type of Entrance	n	K_e
Box	Concrete	Parallel to 15 wingwall	0.015	0.5
		Straight wingwall		0.2
		30-70 Flared wingwall		0.2
Circular	Concrete	Groove end Projecting	0.011	0.3
		Square Wingwall		0.3
		Improved flared		0.3
	Metal Steel	Projecting	0.022	0.9
		Mitered		0.7
		Improved flared		0.5
Arch	Concrete	Groove end Projecting	0.011	0.3
		Groove with Headwall		0.3
		Headwall		0.3
	Metal Steel	Thin wall Projecting	0.022	0.6
		Mitered		0.7
		Parallel Headwall		0.5

Table 7-2 Input Data for Calculating the Reliability Index

Variables	Mean	Coefficient of variance
n	See Table 6-1	0.00007
S_o	0.003	0.0008
C[a]	0.9	0.15
I[a]	50	0.015
A[a]	200	0.05
Dimension of the design culvert	Optimized by MATLAB	0.05

7.3 Result Analysis

Output results of the optimization program are summarized in Table 7-3. The values of the variable at limit state when the performance function = 0 (surface of failure) are illustrated in Table 7-4.

Table 7-3 Summary of the Program Results

Optimum Dimensions								
Box Concrete	**Base (m)**	**Depth (m)**	**HWi (m)**	**HWo (m)**	**Vo (m/s^2)**	**HW Control at**	**β**	**Total Cost ($)**
Parallel to 15 wingwall	3.492	2.333	3.047	3.019	3.51	Inlet	1.42	100,000
Sstraight wingwall	3.787	2.138	3.029	2.855	3.51	Inlet	1.40	102,710
30-70 Flared wingwall	3.030	2.647	3.032	3.343	3.03	Outlet	1.07	96,082
Circle Concrete		**Diameter (m)**						
Groove end Projecting		3.552	3.029	3.870	3.11	Outlet	2.77	52,667
Square Wingwall		3.831	3.031	4.114	2.56	Outlet	3.24	61,757
Improved flared		3.382	3.023	3.753	3.45	Outlet	2.45	48,514
Circle Metal Steel								
Projecting		4.278	3.040	4.480	2.24	Outlet	2.17	225,950
Mitered		3.875	3.022	4.155	3.06	Outlet	1.10	192,480
Improved flared		3.576	2.944	3.912	4.47	Outlet	0.97	169,310
Ach Concrete	**Height (m)**	**Span (m)**						
Groove end Projecting	2.330	3.995	2.808	3.280	3.48	Outlet	3.23	102,530
Groove with Headwall	2.330	3.995	2.727	3.280	3.48	Outlet	3.23	102,530
Headwall	2.264	4.442	2.842	3.206	3.42	Outlet	3.98	114,380
Arch Metal Steel								
Thin wall Projecting	2.996	5.057	2.552	3.947	3.15	Outlet	1.14	218,510
Mitered	2.996	5.057	2.409	3.955	3.15	Outlet	1.14	218,510
Parallel Headwall	2.996	5.057	2.402	3.940	3.15	Outlet	1.14	218,510

Table 7-4 Data of the variables at limit state

Performance function variables at failure surface (Z=0)							
Box Concrete	**Base (m)**	**Depth (m)**	**A[a] (ha)**	**I[a] (mm/hr)**	**C[a]**	**n**	**So (m/m)**
Parallel to 15 wingwall	3.492	2.333	203.052	50.069	1.0116	0.015	0.003
Sstraight wingwall	3.787	2.138	203.052	50.069	1.0116	0.015	0.003
30-70 Flared wingwall	3.030	2.647	202.052	51.069	1.0116	0.015	0.003
Circle Concrete		**Diameter (m)**					
Groove end Projecting		3.552	206.680	67.833	1.1708	0.011	0.003
Square Wingwall		3.831	207.833	80.464	1.2172	0.011	0.003
Improved flared		3.382	205.932	62.554	1.1402	0.011	0.003
Circle Metal Steel							
Projecting		4.278	205.246	59.016	1.1125	0.022	0.003
Mitered		3.875	202.340	51.418	0.9948	0.022	0.003
Improved flared		3.576	198.365	50.522	0.8338	0.022	0.003
Ach Concrete	**Height (m)**	**Span (m)**					
Groove end Projecting	2.330	3.995	209.410	50.221	1.1994	0.011	0.003
Groove with Headwall	2.330	3.995	209.410	50.221	1.1994	0.011	0.003
Headwall	2.264	4.442	211.801	50.280	1.2612	0.011	0.003
Arch Metal Steel							
Thin wall Projecting	2.996	5.057	203.131	50.072	1.0143	0.022	0.003
Mitered	2.996	5.057	203.131	50.072	1.0143	0.022	0.003
Parallel Headwall	2.996	5.057	203.131	50.072	1.0143	0.022	0.003

7.4 Effect of Materials on the Cost and Performance of the Culvert

The initial costs of reinforced concrete culverts are generally higher than metal steel culverts as shown in the Appendix A. However, this phenomenon will change by considering the maintenance cost in the service of the life of the culvert. The horizon life (service life of the culvert) is taken as 100 years. According to the site study done by Perrin and Jhaveri in 2004, on different types of culverts in USA and Canada, they conclude that the metal steel culvert need maintenance or replacement every 25 years, whereas concrete culverts need maintenance every 75 years. Therefore, by taking the maintenance and replacement cost in addition to the initial cost in the cost analysis, an equation was developed for estimating the cost of three shapes of culverts with two different materials. The estimated cost equation covered the cost of the culvert for 100 years, as shown in Appendix A. Using the developed cost estimation equation in the MATLAB program optimizes the culvert dimension by minimizing the cost and the reliability index. Figure 7-1 shows the effect of material type on the cost. Metal steel culverts have higher costs than the concrete culverts within 100 years of service life for all three shapes used in the optimization design program.

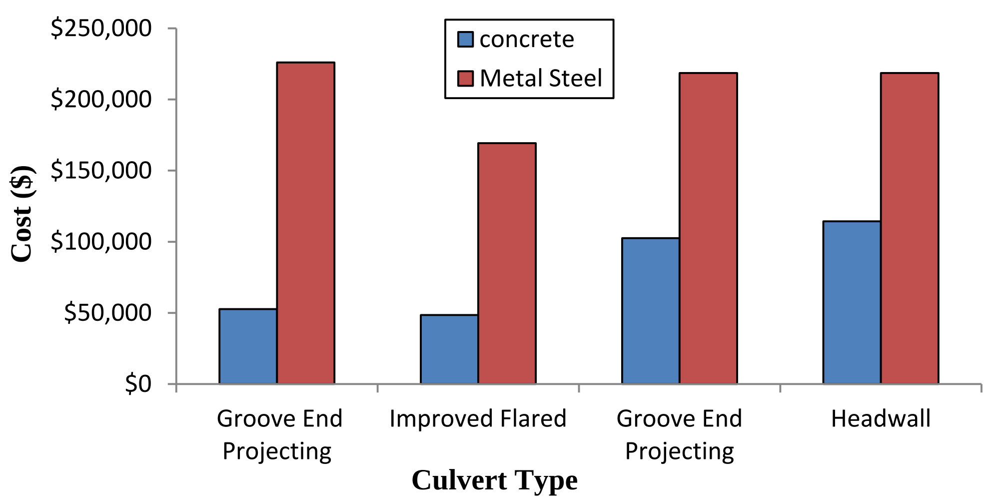

Figure 7-1 Relation between the Materials and the Cost

In Figure 7-2, the reliability of different of culverts was illustrated. The minimum reliability index was one of the constraints in the design procedure; therefore, all types of culverts with different inlet configurations have a high degree of reliability, except culverts with improved flared inlet configuration which have a higher risk of failure due to its generally low reliability, although it has low cost. On the other hand, a reinforced concrete culvert has higher reliability than the metal steel culvert; therefore, the performance of concrete will be better than the steel culvert during the service life, as shown in Figures 7-1 and 7-2.

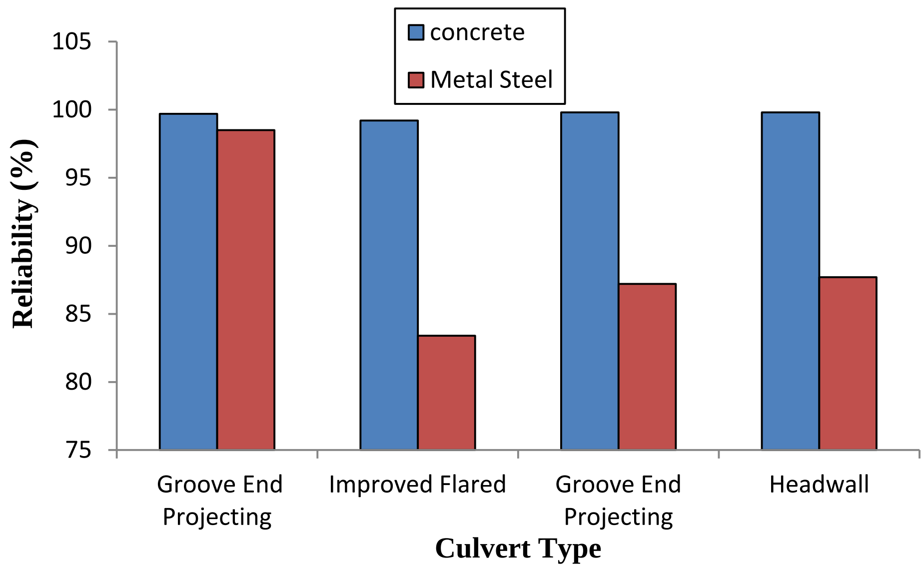

Figure 7-2 Relation between the Materials and the Performance of the Culvert

7.5 Effect of Entrance Configuration on the Cost and Performance of the Culvert

As shown in Figure 7-3, the improved flared entrance configuration of the metal circle culvert has a minimum cost in culvert design. This is because the optimum diameter for this type of culvert will be smaller than the other shapes of culvert. So the cost of the material will be less; however, its risk will be higher compared with other types. Although the inlet headwater elevation (HW_i) in this type will be lower because the contraction of water at the entrance of this type is less, which allows it to convey more water. Therefore, this culvert shows good performance in conveying water, but due to its smaller diameter, the culvert may cause more concern of risk because of flooding compared with other inlets. Any increase in the inlet headwater will cause overtopping on the highway or on any other structure.

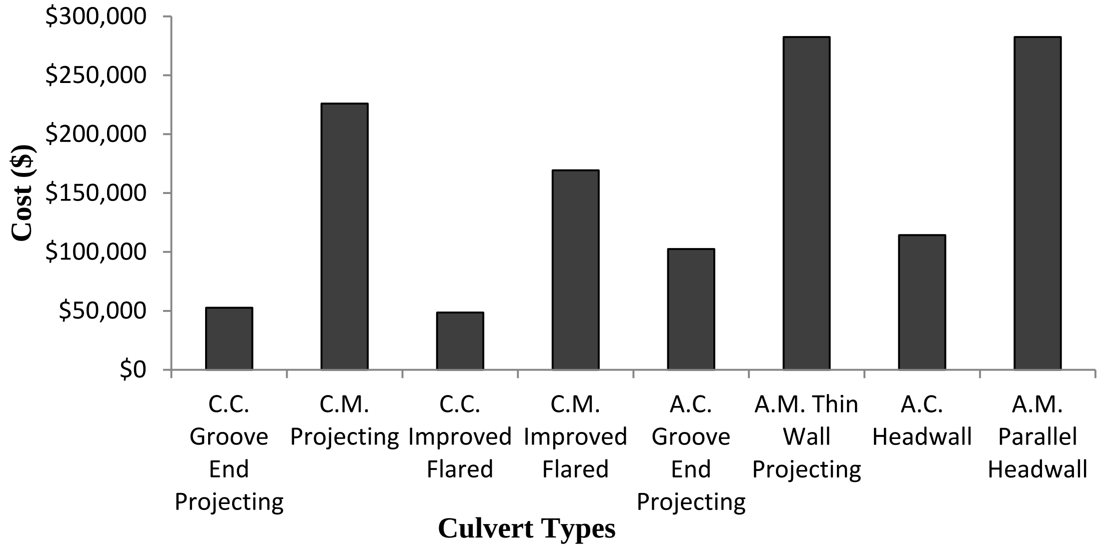

Figure 7-3 Relation between the Entrance Configuration and the Cost

The functional elements of the arch culverts represent better performances due to less risk of failure as shown in Figure 7-4. Higher performances are caused due to the shape of these culverts. This is mainly because their spans are wider than the circular pipes leading them to have low inlet headwater elevations. Also, the heights of arch culverts are higher than circular pipes leaving more space in the case of over flooding, which eventually reduces the risk of failures.

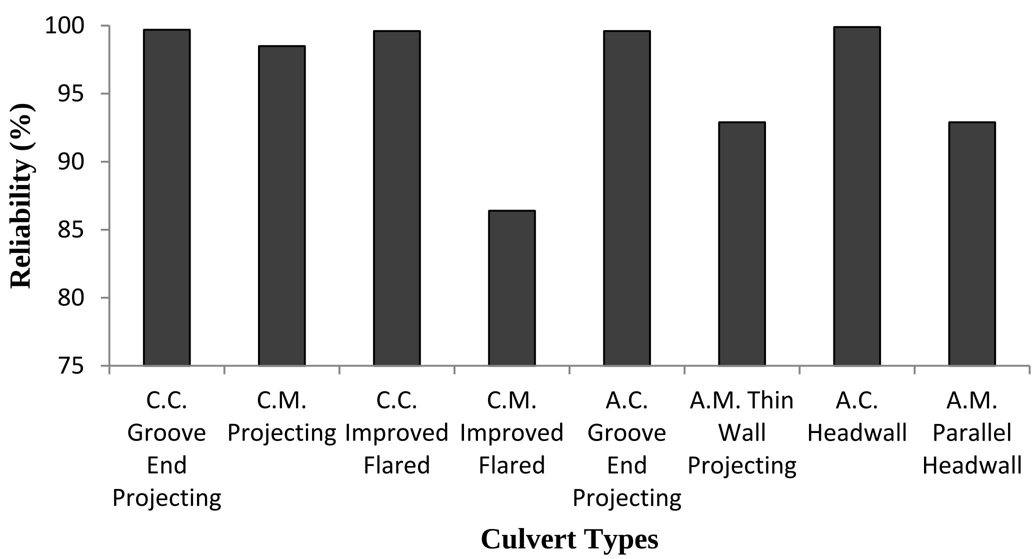

Figure 7-4 Relation between the Entrance Configuration and the Performance

7.6 Probability of Failure and Reliability for Existing Culvert

The probability of failure is very important in the design stage of any structure. The level of reliability depends on the function of this structure during its design life. To measure the reliability of any existing structure, it is also important to know the capacity of the structure to resist any failure. Reliability indexes for different types of culverts with different sizes were calculated by AFOSM method by using the MATLAB.

Table 7-5 Reliability for Various Shapes and Sizes of Culverts

Culvert Shape	Culvert Size (m)	Probability of Failure (P_f)	Reliability (R)
Box	(Base x Depth)		
	3x3	0.9995	0.0005
	4x4	0.9998	0.1922
	5x5	0.0256	0.9744
	6x6	0.0000	1.0000
	4x3	0.9874	0.0126
	4x5	0.3372	0.6628
	4x6	0.0495	0.9505
	4x7	0.0025	0.9975
	4x8	0.0000	1.0000
Circle	(Diameter)		
	3	0.9973	0.0027
	4	0.3745	0.6255
	5	0.0001	0.9999
	6	0.0000	1.0000
Arch	(Height x Span)		
	3x1.5	0.9913	0.0087
	3.5x2	0.7356	0.2644
	4x2	0.1685	0.8315
	4.5x2.5	0.0017	0.9983
	5x2	0.0000	1.0000
	5.5x3.5	0.0000	1.0000

All the input data are the same; in addition the diameters of the existing culvert are also part of input data when calculating the reliability of the existing culvert for any risk of flooding as shown in Table 7-5.

The program was used until it found the dimension of different types of culvert that has 90% reliability. So the dimension can show a good performance coupled with a 10% probability of failure during the service life. The probability of failure in box culverts is affected by the dimensional measures for the width and depth of the box culvert, which are illustrated in Table 7-5 and are shown in Figures 7-5 and 7-6.

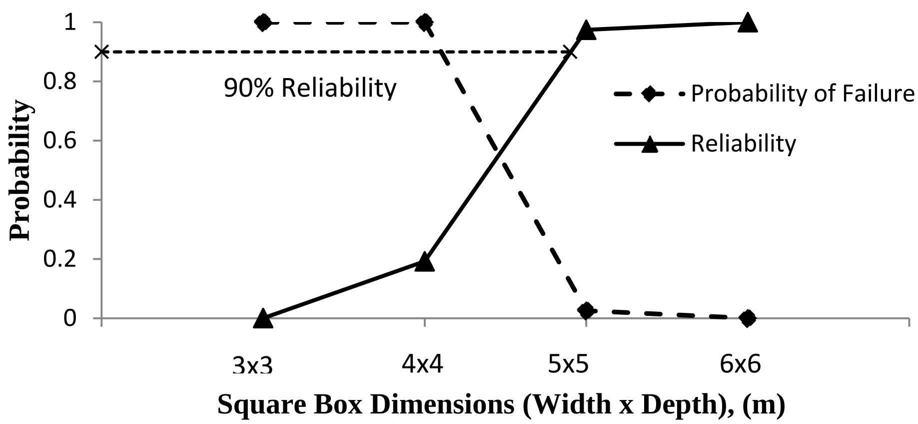

Figure 7-5 Reliability and Probability of Failure of Box Culvert (Square Shape)

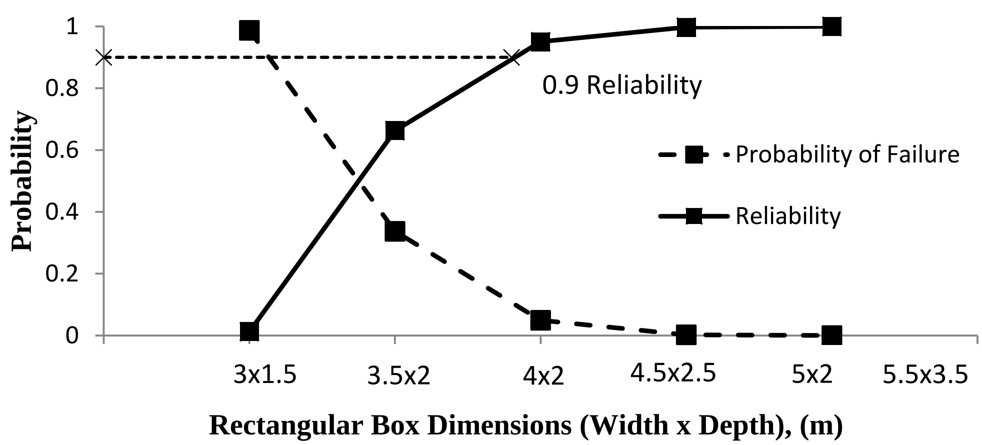

Figure 7-6 Reliability and Probability of failure of Box Culvert (Rectangular Shape)

For circle culverts, the culvert obtained from the optimization which has a diameter of 5 meters and shows high reliability (almost equal to 99%). The 90% reliability level is reached if a pipe with 4.75 m diameter was use which is illustrated in Figure 7-7.

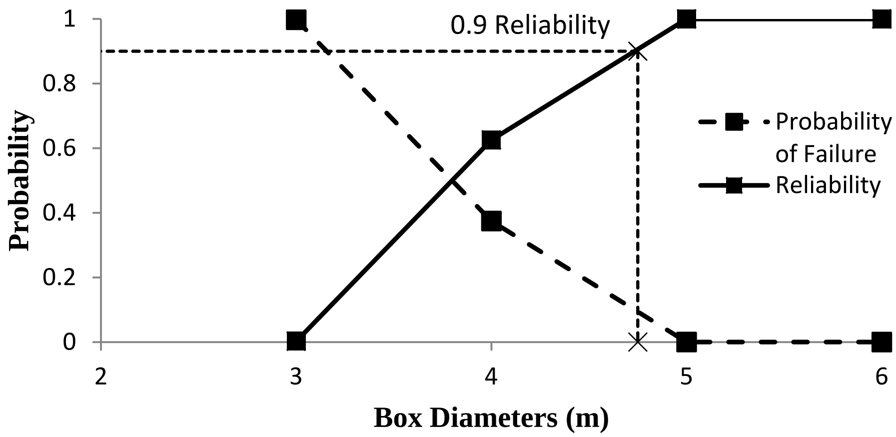

Figure 7-7 Reliability and Probability of Failure of Circular Culvert

As shown in Figure 7-8, the same procedure applied for the arch culvert is the optimum arch culvert having a minimum dimension which lead to higher failure risks, whereas the increase in span and height will cause an increase in the reliability for the culvert. For instance, an arch culvert with a span = 4m and a height = 2.5m has 90% reliability.

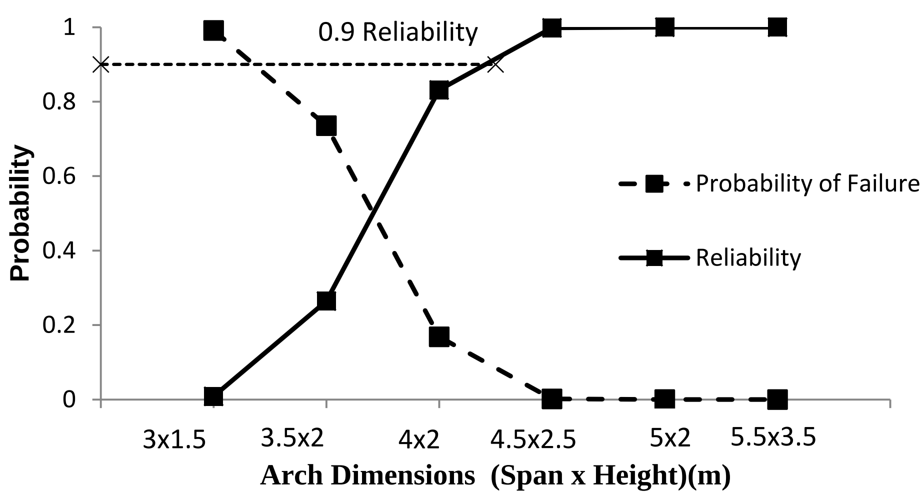

Figure 7-8 Reliability and Probability of Failure of Arch Culvert

Chapter 8: Summary and Conclusions

Culvert design can be expensive and complex due to the consideration of several factors in the overall design. This thesis covers culvert design, mainly by focusing on hydraulic analysis since it plays a major role in helping to determine the smallest and, hence, the most economical structure to carry the design discharge. The hydraulic design of culverts is quite complex since many factors have impacts on the overall design. Generally, the most efficient designs are the ones based on the flow of the culvert, represented by the computer program which has the analytical approach being illustrated in this thesis for the hydraulic analysis of culverts. Using this approach and the procedure in this thesis, a designer has an opportunity to compute culvert capacity. The newly developed method in this study provides the opportunity to replace the usage of standard nomographs and charts. Therefore, this will help the designer save time and avoid the limitations, in the case of using culvert capacity charts. For example, these charts do not include curves for all the culvert dimensions.

Since culvert capacity can be modified by several factors, inlets play a significant role in this regard. It is important to treat the inlet and outlet of the pipe culvert, whereas inlets must be designed to prevent improper flows which can result in flooding, excessive erosion, and scouring at the ends. Therefore, this thesis covers various inlet designs which affect the performance capacity of the culvert. For example, it is common with small culverts to run the culvert out of the roadway. This is called a projecting pipe. Another type of inlet configuration is called mitered. It has been developed by cutting the end of the culvert to fit the slope of the ditch side of the road which results in better hydraulics. These mitered ends can handle a higher flow of water. Moreover, the higher flow of water can be achieved by having improved flared ends at the inlet to a culvert. This will improve the flow by guiding the water into the culvert, minimizing turbulence due to lower contraction of water at the inlet region. Improved flared ends at the outlet region of a culvert distribute the flow over a wider opening, reducing the discharge velocity which helps prevent scouring. Due to the good hydraulic performances of these types of culvert inlet, the optimum dimensions were calculated by the program and the results indicated that a smaller culvert size, rather than other types of culverts which have a higher risk of failure than others.

As the design specifications play an important role in the performance and cost of designing culverts, the effect of materials also has an influence on performance. The relative research and cost analysis done during this study shows that reinforced concrete culverts have higher material costs (not including service life, or installation costs) than metal steel culverts. But on the other hand, reinforced concrete has a lower total cost than the metal steel culverts. Total cost includes the initial construction cost, plus maintenance cost, in additional to replacement cost during the culvert's service life. Generally, this applies to all different shapes of culverts which have various inlet configurations. On the other hand, a reinforced concrete culvert will be more durable when compared with metal steel culverts, which leads to a lower risk of failure.

In the case study, the optimum dimensions obtained using the optimization program for different types of culverts represents a reliability of over 90%. This shows these optimum dimensions are reliable and represent optimum and effective solutions to the problems of culvert design.

Finally, the developed optimization program can be used for designing new culverts with minimum cost and minimum probability of failure. In addition, it can be used to calculate the reliability of an existing culvert. This study mainly covers the factors involved in culvert design and their contributions to the cost, hydraulic performance and reliability. The main focus of this thesis is to design culvert using an optimization model that has achieved a significant improvement in the method. The developed model should displace the conventional methods for culvert design based on nomographs, design charts, as well as trial and error methods.

Appendix A

Cost Estimation

Total cost estimates for three types of culvert were obtained by gathering the list price of the installation cost and maintenance cost of the culverts from companies. The total costs were calculated with two service lives. The maintenance cost was calculated for 25 and 50 years.

A.1 Initial Costs

A.1.1 Initial Cost of Box Culvert

The initial prices were taken from Munro (2011) and Hanson (Hanson pipe & precast, 2011) as shown in Table (A-1).

Table A-1 Initial Prices of Box Culvert [Source: Munro and Hanson Companies, 2011]

Width x Height (B x D) (mm)	Wall Thickness (mm)	Approximate Mass (Kg/m)	Price / Meter	
			OPSS 1821 0.6m - 3.5m	CHBDC
1800 x 900	200	3380	$1,374.60	$1,729.70
1800 x 1200	200	3690	1,500.40	1,881.70
2400 x 1200	200	4560	1,914.40	2,406.60
2400 x 1500	200	4870	2,051.00	2,598.40
2400 x 1800	200	5170	2,187.90	2,738.30
3000 x 1500	250	6860	2,995.10	3,683.60
3000 x 1800	250	7250	3,167.00	3,838.10
3000 x 2100	250	7630	3,337.10	4,038.20
3000 x 2400	250	8020	3,507.00	4,240.20

A.1.2 Initial Cost of Circular Culvert

Table A- 2Initial Prices of Circular Concrete Culvert [Source: Hanson, 2011]

Diameter (m)	Price/Meter ($)	Diameter (m)	Price/Meter ($)
0.300	64.0	2.100	2282.1
0.375	79.0	2.250	2595.8
0.450	101.8	2.400	3035.6
0.525	122.4	2.550	3419.9
0.600	172.5	2.700	3796.9
0.675	261.0	3.000	4651.5
0.750	344.7	3.300	5618.5
0.825	399.9	3.600	6669.7
0.900	479.4	3.900	7810.9
0.975	552.1	4.200	9042.1
1.050	631.8	4.500	10363.3
1.200	791.7	4.800	11774.5
1.350	968.7	5.100	13275.7
1.500	1185.7	5.400	14866.8
1.650	1420.1	5.700	16547.9
1.800	1716.8	6.000	18319.0
1.950	1990.7	6.600	22131.2

Table A-3 Initial Cost of Circular Steel Culvert [Source: Vemax, 2009]

Diameter (m)	Price/Meter ($)	Diameter (m)	Price/Meter ($)
0.7	595.8	3.0	4374.2
0.8	682.6	3.3	5152.6
0.9	707.4	3.6	5634.1
1.0	803.8	4.0	6816.7
1.2	1385.6	4.5	8259.2
1.4	1559.6	5.0	9832.9
1.6	1775.5	5.5	11537.7
1.8	1992.8	6.0	13373.8
2.0	2335.7	6.5	15341.1
2.2	2556.1	7.0	17439.6
2.4	2787.1	7.5	19669.3
2.7	3930.1	8.0	22030.2

A.1.3 Initial Cost of Arch Culvert

Table A- 4 Initial Cost of Concrete Arch Culvert [Source: Hanson, 2011]

Rise (m)	Span (m)	Length (m)	Price /meter ($)
0.735	1.145	2.44	622.7
0.865	1.345	2.44	766.3
0.965	1.525	2.44	943.9
1.090	1.725	2.44	1187.5
1.220	1.930	2.44	1416.8
1.345	2.110	2.44	1678.2
1.475	2.310	2.44	2015.7
1.600	2.490	2.44	2322.6
1.725	2.690	2.44	2648.0
1.955	3.075	2.44	3505.9
2.210	3.455	2.44	4148.1

Table A- 5 Initial Cost of Steel Arch Culvert [Source: FERIC, 2003]

Rise (m)	Span (m)	Length (m)	Price /Meter ($)
3.10	1.98	14.0	46900
2.06	1.52	14.9	49176
2.59	1.88	14.9	54187
3.10	1.98	19.5	60693
3.73	2.10	22.6	70262
1.39	0.97	10.0	13131

A.2 Maintenance Cost

Any structure has its service life during which it can normally perform well. Various types of culvert material have different life expectancies. As these culverts reach the end of their useful life, agencies should replace them. If not, these pipes are destined to fail and create a traffic dangers and congestion points. The number of agencies actually inspecting and tracking age/condition of culverts and performing maintenance/replacement as needed should also be explored.

The United States Army Corps of Engineers (1998), identified recommendations on pipe design life by material in a report of March 1998. The following are quotes from that report:

1. Service Life: “For major infrastructure projects, designers should use a minimum project service life of 100 years when considering life cycle design.”
2. Concrete: “Most studies estimated product service life for concrete pipe to be between 70 and 100 years. Of nine state highway departments, three listed the life as100 years, five states stated between 70 and 100 years, and one state gave 50 years.”
3. Steel: “Corrugated steel pipe usually fails due to corrosion of the invert or the exterior of the pipe. Properly applied coatings can extend the product life to at least 50 years for most environments.”
4. Aluminum: “Aluminum pipe is usually affected more by soil-side corrosion than by corrosion of the invert. Long-term performance is difficult to predict because of a relatively short history of use, but the designer should not expect a product service life of greater than 50 years.”
5. Plastic: “Many different materials fall under the general category of plastic. Each of these materials may have some unique applications where it is suitable or unsuitable. Performance history of plastic pipe is limited. A designer should not expect a product service life of greater than 50 years.”(Perrin and Jhaveri, 2004).

A.2.1 Calculation of the Maintenance and Replacement Cost of Culvert

Properly installed and regularly maintained culverts will protect the surrounded area from flooding. The capacity of culverts can be increased by repeated maintenance to remove the silt, debris, or ice that cause blocking and damaging to the culvert (Highway Maintenance Guidelines and Level of Service Manual, 2000). To calculate the maintenance cost of the culvert, assume total 3 hours needed to clean a plugged culvert by using excavator ($90/h) and float truck ($75/h) therefore the estimated cost would be approximately $500/year (FERIC, 2002). In this study the maintenance cost was added to the initial cost and it used to calculate the replacement cost of the during the horizon time.

An explanation for installation/replacement costs is given below and shown in equation (a.1). The installation/replacement cost (I_H) is computed from the initial installation cost based on the present value, and then projected at a discount rate (r) for any replacements during the time horizon (H_r) depending on the assumed life of the culvert (L_f). Note that the discount rate is the differential between inflation and interest rates (Perrin and Jhaveri, 2004). Therefore,

$$I_H(L_f) = \sum_{k=0}^{n} I_i \,(1+r)^{kL_f} \tag{a.1}$$

where,

$$n = (H_r/L_f)-1 \tag{a.2}$$

I_H = Installation cost in horizon time

I_i = Initial installation cost

H_r = Horizon time of the culvert

L_f = Assumed life of the culvert

r = Discount rate

In our cost calculation, the assumed lives of the culvert were 25, and 50 years, while the horizon time =100 years and a 4% discount rate were used. Further, the initial installation costs for each types of culvert are obtained from the Table A.1 to Table A.5. Therefore, the total costs of the culvert were calculated and used to develop an equation for each type of culvert within its service life. Figure A-1 to Figure A.5 represents the total cost for different types of culverts with 25 and 50 year service lives. The developed equation form of the graph is used in MATLAB for designing the dimensions of the culvert at minimum cost and minimum reliability index.

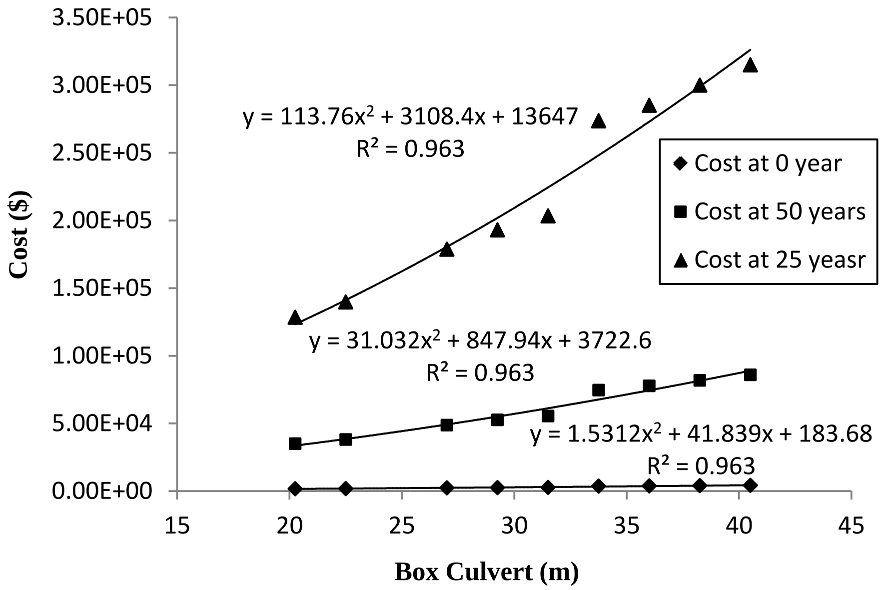

Figure A-1 Total Cost of Box Culvert (Price /Meter)

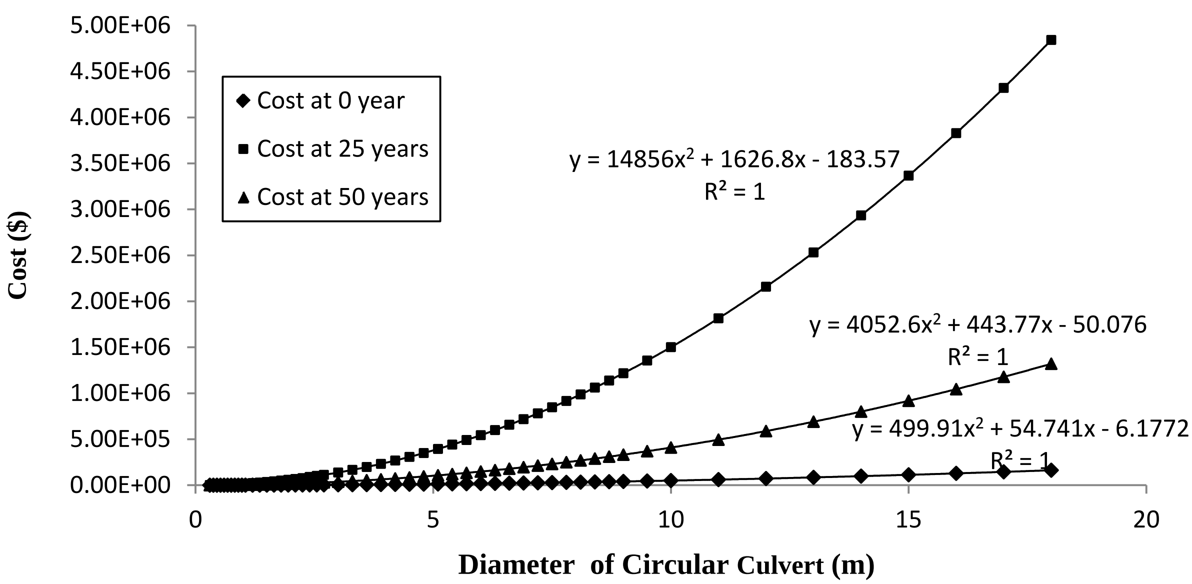

Figure A-2 Total Cost of Concrete Circular Culvert

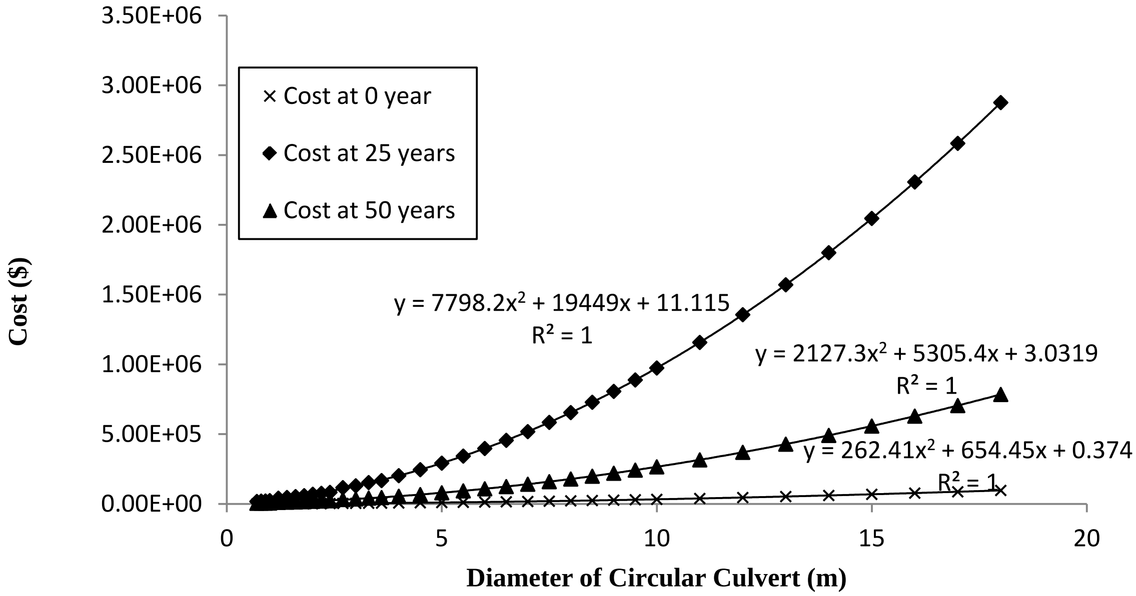

Figure A-3 Total Cost of Steel Circular Culvert

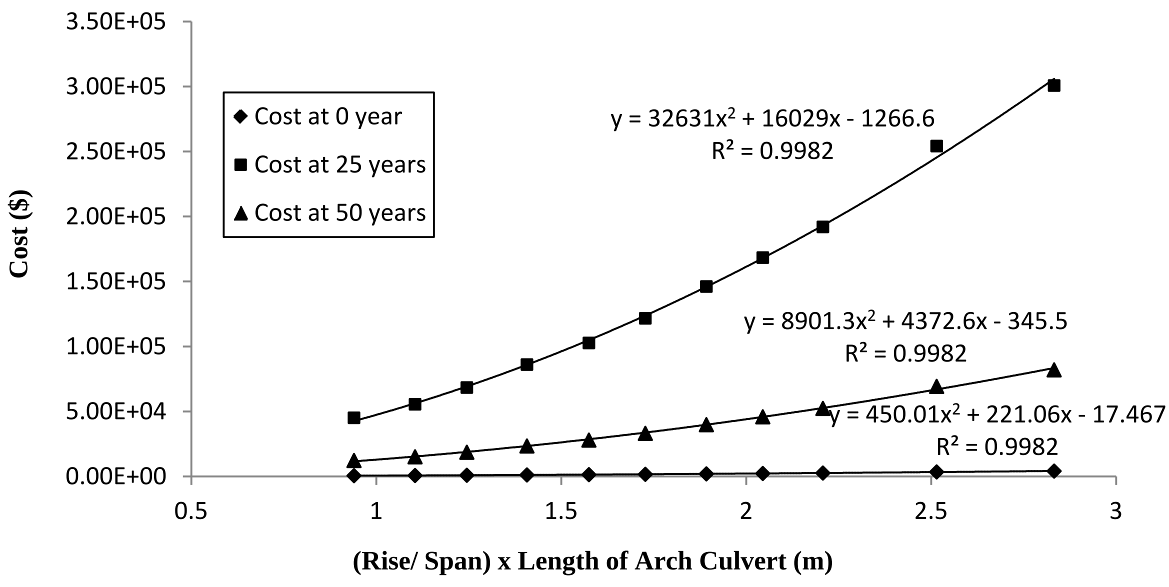

Figure A-4 Total Cost of Concrete Arch Culvert

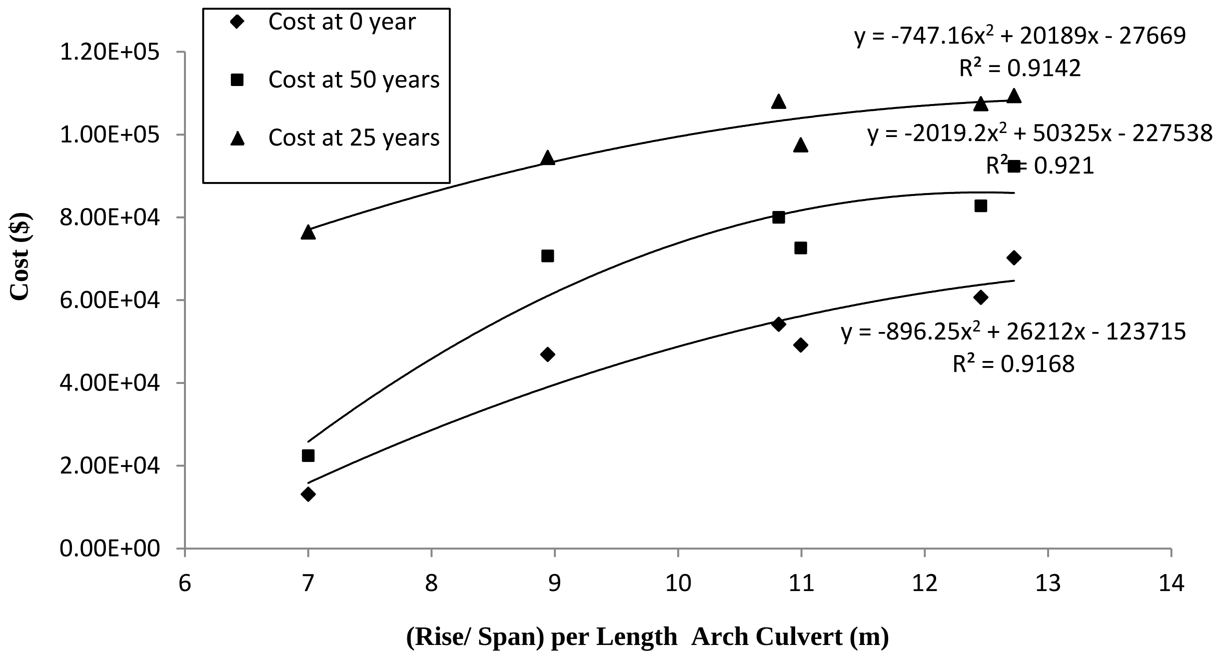

Figure A-5 Total Cost of Steel Arch Culvert

Appendix B

Formulas of Critical and Normal Depth Calculation

Calculation of the dimensions that meet the hydraulic criteria. Then the price for the designed culvert is calculated.

Table B-1 Calculation Steps

For the Box Culvert		
Critical Depth Calculation	$A_c = d_c * B$	(b.1)
	$T_c = B$	(b.2)
Normal Depth Calculation	$A_n = d_n * B$	(b.3)
	$R_n = A_n / P_n$	(b.4)
	$P_n = 2d_n + B$	(b.5)
For the Circle Pipe		
Critical Depth Calculation	$A_c = Dia^2 * (\theta_c - \cos\theta_c * \sin\theta_c)$	(b.6)
	$\theta_c = \arccos(1 - (d_c / Dia))$	(b.7)
	$T_c = 2Dia * (\sin\theta_c)$	(b.8)
Normal Depth Calculation	$A_n = Dia^2 * (1 - \cos\theta_n)$	(b.9)
	$\theta_n = \arccos(1 - (d_n / Dia)$	(b.10)
	$P_n = 2Dia\ (\theta_n)$	(b.11)
	$R_n = A_n / P_n$	(b.12)
For Arch		
Critical Depth Calculation	$A_c = AF_c - R_c^2 * \gamma_c + (T_c * R_c * \cos\gamma_c) / 2$	(b.13)
	$T_c = 2 * R_c * \sin\gamma_c$	(b.14)
	$AF_c = \pi R_c^2 - (R^2_c * (\theta_c/2)) + (S_c * (H_c - R_c))/2$	(b.15)
	$\gamma_c = \arccos(d_c - R_c \cos(\theta_c/2))/R_c$	(b.16)
	$\theta_c = 4\arctan(S_c/2 * H_c)$	(b.17)
	$R_c = (H_c / 1 + \cos(\theta_c/2)$	(b.18)

Normal Depth Calculation

$$A_n = AF_n - R_n^2 * \gamma_n + (T_n * R_n * \cos\gamma_n) / 2 \quad \text{(b.19)}$$

$$T_n = 2*R_n*\sin\gamma_n \quad \text{(b.20)}$$

$$AF_n = \pi R_n^2 - (R^2{}_n*(\theta_n/2)) + (S_n*(H_n - R_n))/2 \quad \text{(b.21)}$$

$$\gamma_n = \arccos(d_n - R_n \cos(\theta_n/2)) / R_n \quad \text{(b.22)}$$

$$\theta_n = 4\arctan(S_n / 2H_n) \quad \text{(b.23)}$$

$$R_n = (H_n / 1 + \cos(\theta_n/2) \quad \text{(b.24)}$$

$$PF_n = 2R_n\alpha_i + S_n \quad \text{(b.25)}$$

$$\alpha_i = (2\pi - \theta_n) / 2 \quad \text{(b.26)}$$

$$P_n = PF_n - 2R\gamma_n \quad \text{(b.27)}$$

Appendix C

MATLAB Source Program

(Box Type)

```
% For Box

% Select the type of box

clear; clc;

reply = input('Concrete Box: CB, Concrete PIPE: CP, Metal PIPE: MP ', 's');

if reply=='CB';n=0.015;end

if reply=='CP';n=0.011;end

if reply=='MP';n=0.022;end

reply = input('Parallel to 15 wingwall: P, Straight wingwall: S, Flared
wingwall: F ', 's');

if reply=='P';a=0.122117;b=0.505435;c=-0.10856;d=0.0207809;e=-
0.00136757;f=0.00003456;end

if reply=='S';a=0.144138;b= 0.461363;c= -0.09215;d= 0.020003;e= -0.00136;f=
0.000036;end

if reply=='F';a=0.0724927;b=0.507087;c= -0.117474;d=0.0221702;e=-
0.00148958;f=0.000038;end

reply = input('Parallel: P, Straight: S, Groove: G, Thin: T, T Mitered: M,
Projecting: PJ ', 's');

if reply=='P';Ke=0.5;end

if reply=='S';Ke=0.2;end

if reply=='G';Ke=0.3;end

if reply=='T';Ke=0.6;end

if reply=='M';Ke=0.7;end

if reply=='PJ';Ke=0.9;end

% Intialize acceptable values for B and D and RI

B0=[];
```

```
D0=[];

RI0=[];

Cost0=[];


% Intialize the minimum values for Realiability Index and cost (select  large
values for these two variables

Cost_min=10000000000;

RI_min=4000;

% given data

HWL_max=33.528; TWL=30; IEO=28.737; IEI=30.48;

C=0.9; I=50; A=200;  S0=0.003; g=9.81; K0=1; Ku=19.63; L=15;

% change B and D to find the values of B and D for minimum cost and RI

j=1;

mI=50; mA=A;mn=n;mS=S0; mC=C;


for B=0.1:.1:10;

        for  D=0.1:.1:10;


            % calculate the reliability index

            % Box type of culvert

 % Data of mean and standard deviations of different variable

 % Means

 % For boxes with diffrent dimensions just change the following value

 md=D; mB=B;

 % Standard deviations

 sigC=C*0.15; sigI=I*0.015; sigA=A*0.05 ;sign=n*0.0008; sigS=S0*0.00007;

 % For boxes with diffrent dimensions just change the following value
```

```
sigd=0.05*D; sigB=0.05*B;

 % Intial guess for the design point which is mean values of differnt

  S=mS; N=mn; C=mC; A=mA;

 % parameters except the variable I which is

  R=B*D/(B+2*D); An=B*D;

  II=360*An*R^(2/3)*S^(1/2)/(n*C*A);

% Intialize values of the varaibale I

  I0(1)=II;

% This loop updates and calculates the design point and beta iteratively.

BB=B; DD=D;SS=S;N=n;CC=C;AA=A;

for k=2:1:50,

    % Calculate the standardized variabels

    B_bar=(BB-mB)/sigB; d_bar=(DD-md)/sigd; n_bar=(N-mn)/sign; S_bar=(SS-
     mS)/sigS; A_bar=(AA-mA)/sigA;C_bar=(CC-mC)/sigC; I_bar=(II-mI)/sigI;

    % Derivative of the performance function with respect to each variabel

    G=-[(5/3)*sigB/BB-(2/3)*(sigB/(BB+2*DD));(5/3)*sigd/(DD)-
       (4/3)*(sigd/(BB+2*DD)); 0.5*sigS/SS; -sign/N;-sigC/CC; -sigI/II; -
       sigA/AA];

    % The  standardized design point

    Z=[B_bar;d_bar;S_bar;n_bar;C_bar;I_bar;A_bar];

    % Calculate new beta based on the algorithm

    betaa=G'*Z/sqrt(G'*G);

    % Calculate alpha the sensitivity coefficient

    alpha=G/sqrt(G'*G);

    % New standaridized design point

    Z=betaa*alpha;

    % New design point
```

```
    BB=sigB*Z(1)+mB;

    DD=sigd*Z(2)+md;

    SS=sigS*Z(3)+mS;

    N=sign*Z(4)+mn;

    CC=sigC*Z(5)+mC;

    AA=sigA*Z(7)+mA;

    II=sigI*Z(6)+mI;

    %%%%%%%%%

    R=BB*DD/(BB+2*DD); An=BB*DD;

   II=360*An*R^(2/3)*SS^(1/2)/(N*CC*AA);

  %%%%%%%%%%

    % calulate the variable I such that the function is equal to zero

    Q=25;


%    R=BB*DD/(BB+2*DD); An=BB*DD;
%
%    II=360*An*R^(2/3)*SS^(1/2)/(N*CC*AA);

% Store new value of beta in the 'bet' vector

    bet(k)=betaa;

% Store new value of I in the 'I0' vector

    I0(k)=II;
    A0(k)=AA;
    C0(k)=C;
    B0(k)=BB;
    D0(k)=DD;
    N0(k)=N;

    % checK convergenc eof the algorithm by checking convergence of the
design point

    if abs(I0(k)-I0(k-1))<1e-5; break;end

end;

if imag(betaa)==0
```

```
 % This "if condition" applies the constraint of 0.35 <=D/B<= 1
            if  0.35*BB<=DD & DD<=BB;
% The prcedure starts here
            HW_max=HWL_max-IEI;
 % considering equation Q=(An*Rn^(2/3)*S0^(1/2))/n we have
 % dn^5-n*Q/S0^(1/2)*(2*dn+B)^2
 % dn^5-(n*Q/(S0^(1/2))(4dn^2+4dn*B+B);
        M=N*Q/(BB^(5/3)*sqrt(SS));
        Dn=roots([1 0 0 -M^3*[4 4*BB BB^2]]);
        for i=1:1:5;
        if imag(Dn(i))==0 & real(Dn(i))>0;
             dn=Dn(i);
        end
        end
        TW=dn;
  % W=B in box
        Q=25;
        W=BB;
        F=1.8113*Q/(W*DD^(3/2));
        Hwi=(a+b*F+c*F^2+d*F^3+e*F^4+f*F^5)*DD-0.5*DD*SS;
       if 0.5<=(Hwi/DD) & (Hwi/DD)<=3;
        Hwi=Hwi;
        F=1.8113*Q/(W*DD^(3/2));
       end
        if Hwi/DD>3
```

```
         K=0.6325*Q/(DD^.5);

         Hwi=(Q/K)^2+DD/2;


        end

         HWLi=IEI+Hwi;

        if Hwi<= HW_max;

 % for box TC=B; So, Froud formula: Q^2B/(g*B*dc^3)=1

        dc=(Q^2*BB/(g*BB^3))^(1/3);

 % velocity of critical depth

         Ac=BB*dc;

         Vc=Q/Ac;

        if TW>DD; calc_dc=TW; end;

        if TW<DD; calc_dc=(dc+DD)/2; end

% calculate intial depth: di

        di=max(TW,calc_dc);

% calculate headwater at outlet (HW0)

% for box

         Vi=Q/(BB*di);

         Pi=BB+2*di;

         Ku=19.63;

         Ai=BB*di;

         Ri=Ai/Pi;

         H=1+Ke*(Ku*N^2*L/(Ri^(1.33)))*Vi^2/(2*g);

           L=15;

         HWo=H+di-(L*SS);

         HWLo=IEO+HWo;
```

```
        HWL=max(HWLi, HWLo);

        if HWL < HWL_max

        if TW<=dc; dout=dc;end;

        if dc<=TW & TW<=DD; dout=TW; end;

        if DD<=TW; dout=DD; end;

        Vout=Q/(BB*dout);

 %if 0.5<=Vout & Vout<=4.5;

        if 0.5<=Vout & Vout<=4.5;


% save the results

B0(j)=BB;

D0(j)=DD;

RI0(j)=betaa;

cost=6982.1*((BB+DD)/2)^2+12719*((BB+DD)/2)+3722.6;

Cost0(j)=cost;

j=j+1;

% check for the minimum values of cost and RI

if betaa>1;

if betaa<=RI_min & cost<=Cost_min;

% save the results of minimum values

   B_design=BB; D_design=DD ;S_design=SS; n_design=N; C_design=CC;
I_design=II; A_design=AA;



%H=360*B_design*D_design*((B_design*D_design)/(B_design+2*D_design))^(2/3)*(S
_design)^.5/(n_design*C_design*I_design*A_design);

   RI_min=betaa;

   Cost_min=cost;
```

```
    Vout_final=Vout;

    Hwi_final=Hwi;

    HWo_final=HWo;

    dout_final=dout;

    dc_final=dc;

    Tw_final=TW;

    dn_final=dn;



end

    end

        end

        end

        end

        end

        end

        end

end

QL=C_design*I_design*A_design/360;

QC=B_design*D_design*(B_design*D_design/(B_design+2*D_design))^(2/3)*sqrt(S_d
esign)/n_design;

QL-QC;
```

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