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HYDRAULIC ANALYSIS OF OIL SPILL CONTROL SYSTEMS IN TRANSFORMER STATIONS

by

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A thesis

presented to Ryerson University

in partial fulfillment of the
requirement for the degree of
Master of Applied Science
in the Program of
Civil Engineering

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(iii)

ABSTRACT

Title: Hydraulic Analysis of Oil Spill Control Systems in Transformer Stations

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Electrical transformer stations use transformer oil to increase the efficiency of the electrical voltage transfer and to reduce the moisture and air in an electrical transformer. Each year, there is a high probability of spilling the transformer oil accidentally into the environment. Some spill events contain large volume of transformer oil. The objective of this thesis is to investigate oil spill control systems for spilled transformer oil during all operating and weathering conditions at a *Hydro One*'s transformer station near the city of Burlington. This thesis examines the designs of (1) oil trap systems which trap the spilled transformer oil and (2) the oil back-up systems which back up the transformer oil spills to the transformer station. This research focuses primarily on *Hydro One*'s transformer stations and the normal operation conditions in Ontario.

ACKNOWLEDGEMENTS

TABLE OF CONTENTS

(viii)

LIST OF TABLES

LIST OF FIGURES

(xi)

LIST OF FIGURES

(xii)

LIST OF FIGURES

(xiii)

Chapter 1   INTRODUCTION

1.1   Background

Oils are complex compounds composed of carbon and hydrogen. They are known as hydrocarbons and are found under the earth's surface. In most circumstances, oils are fossils of prehistoric vegetation and marine life, where these plants and animals were buried under layers of sediment before decomposition. Over the past millions of years, the pressure and slow bacterial degradation converted these organisms into crude oil (Sit, 1999).

Oils are an essential part in today's society because it is a relatively inexpensive source of energy. This natural resource can be extracted and processed for various uses such as fuel, lubricants and composite materials for industrial uses (Truong, 2002). Since crude oil by-products are being used in a wide variety of ways, there may be a high probability of spilling the oil by-products accidentally into the environment.

Spilled oil can harm the environment in several ways. First, spilled oil has a direct impact on freshwater because the toxicity of the oil can poison the organisms living within the contaminated water. Also, spilled oil can have catastrophic effects on larger animals such as birds being coated with a harmful layer of crude oil by-products. Finally, spilled oil that enters the sanitary sewers can cause deleterious effects, turbidity, filtration difficulty, objectionable tastes and odour (Zhen, 1998).

In 1972, Canadian and American governments signed the Great Lakes Water Quality Agreement in an attempt to control pollutant discharges into the Great Lakes (Zhen, 1998). Hydrocarbons are major pollutant discharges that have drawn both the attention of the public and politicians. With increased public awareness and stringent environmental regulations on oil spills, *Hydro One*, the largest electrical transmission company in Ontario, has used electrically-based oil spill control systems to control and retain spilled transformer oil at its electrical transformer stations. The advantages and disadvantages of the existing system will be discussed in Section 2.1 in detail. *Hydro One* is currently developing a hydraulically-based oil spill control system that can replace their existing electrical oil spill control system (Truong, 2002).

1.2 Problem Identification

Electrical transformers are generally located at several areas of a power system. Although they may vary in sizes, their functions are to increase the electrical voltage from about 100 to 500 kilovolts for transmission purposes and to decrease the electrical voltage for industrial (480 volts) or residential (240 or 120 volts) use once the electricity has reached its destinations. Transformer oil is applied to increase the transfer efficiency and to reduce the moisture and air in an electrical transformer. The general characteristics of transformer oil are discussed in Appendix J.

Each year, there is a high frequency of spilling the transformer oil accidentally into the environment. According to *Hydro One Spill Study* report provided by Weslake Inc. in 2002, over *860 000* litres of spilled transformer oil were reported between 1993 and 1999. Some transformer oil spill events were large in volume. The largest transformer oil spill event volume between 1993 and 1999 was *200,090* litres (Weslake Inc., 2002).

Most of the transformer oil contains PCBs. The *Sewer Use Bylaw* of the city of Toronto states that PCBs for sanitary/combined sewers discharges, and storm sewer discharges must be less than *0.001 mg/L* and *0.0004 mg/L respectively.*[1] *Hydro One* is currently seeking for a more reliable oil spill control system, other than the electrically-based oil spill control system, to solve the transformer oil spill problems at their transformer stations.

Transformer oil spills are primarily caused by leakage in the system, oil aging and fire or explosion at transformer stations. For a small transformer oil spill, the entire volume will be trapped within the concrete pad underneath the transformer (Figure 1.1). If a rain event occurs after the spill, runoff will carry small amount of spilled transformer oil into the underground oil spill control system. Large spill events will go directly into the oil spill control system. The function of an oil spill control system is to prevent the transformer oil spills from entering the sewer system (Figure 1.2).

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[1] Reference from website *http://www.city.toronto.on.ca/legdocs/municode/1184_681.pdf*
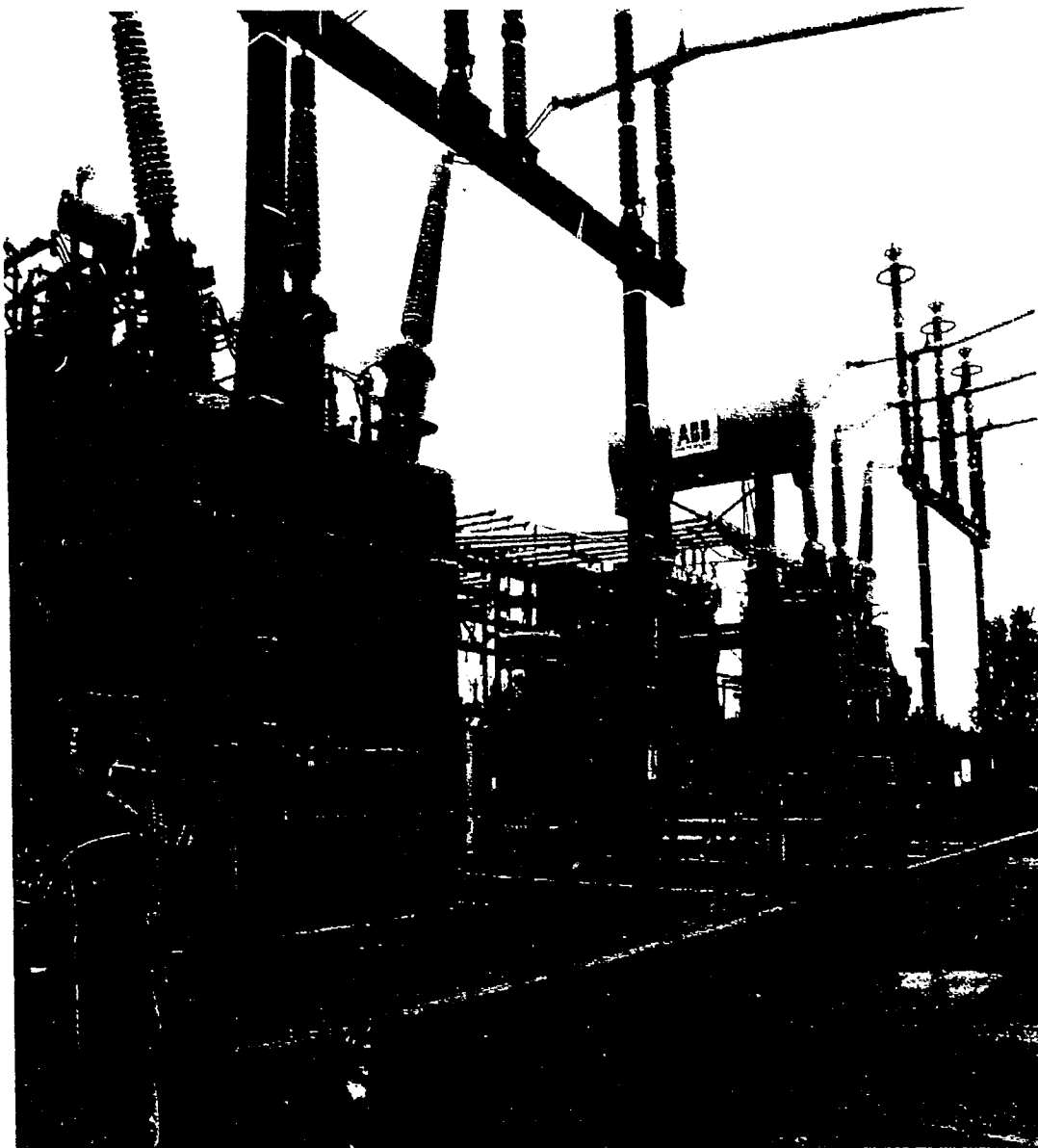
Figure 1.1: The transformer station near the city of Burlington.
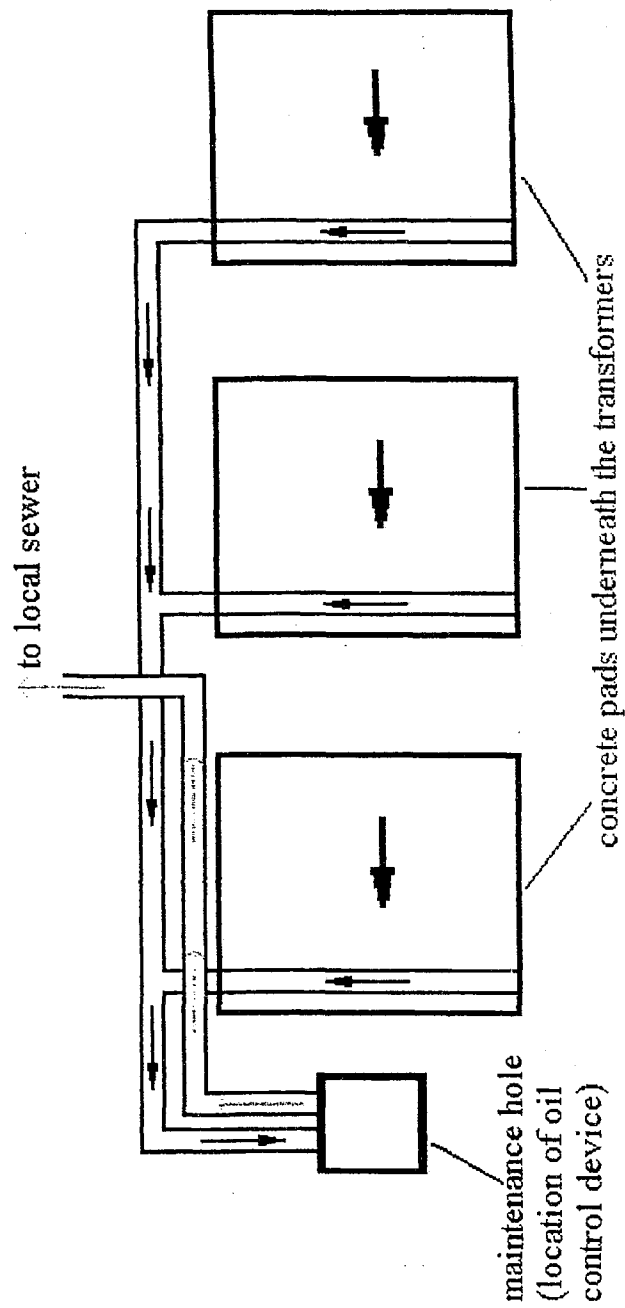
Figure 1.2: General storm water drainage for a typical transformer station (Truong, 2002).

to local sewer

concrete pads underneath the transformers

maintenance hole
(location of oil
control device)

All oil spill control systems, either electrically- or hydraulically-based, use the concept of different specific gravities between oil and water for oil containment and separation. It is also assumed that the trapped transformer oil globules in the oil spill control system be held for a short period of time until cleanup mechanism is activated. At a non-staffed transformer station, a sudden surge of spilled transformer oil can be trapped by the oil spill control system. Since the trapped transformer oil may not be cleaned up right away, subsequent rain storms may flush out the trapped oil. Although the design recommendations of American Petroleum Institute (API) for oil-water separators are sufficient for the separation of transformer oil and water, additional design criteria should be developed to prevent the previously trapped transformer oil spills from escaping the separator during a rain storm.

1.3 Objectives and Scope

The objective of this thesis is to investigate the hydraulic behaviour of an oil spill control systems under all operating and weathering conditions at a *Hydro One*'s transformer station near the city of Burlington (Figure 1.3). This thesis examines two types of oil spill control systems: (1) oil trap systems which retain spilled transformer oil and (2) oil back-up systems which back up transformer oil spills at the transformer.. For the oil trap systems, dimensionless analyses and physical model studies were conducted. For the oil back-up systems,the mass balance and hydrostatic concept of oil

and water were applied to develop a mathematical model which simulates the hydraulic characteristics.

1.4   Organization of the Thesis

This thesis is organized in the following manner. Chapter 2 reviews the functions and disadvantages of electrical oil spill control systems, transformer oil characteristics and the operating conditions of the system, and previous research works in oil/water separation. Chapter 3 presents the physical model study for the oil trap system. Chapter 4 presents the numerical model study of the oil back-up system. Finally, Chapter 5 concludes the research with recommendations.

transformer station   transformer station   transformer station   transformer station   transformer station   transformer station

3.40

4.48   4.45

200 mm diameter pipe

1.70

34.75

14.24

- - - - hydraulic flow length

location of oil trap system
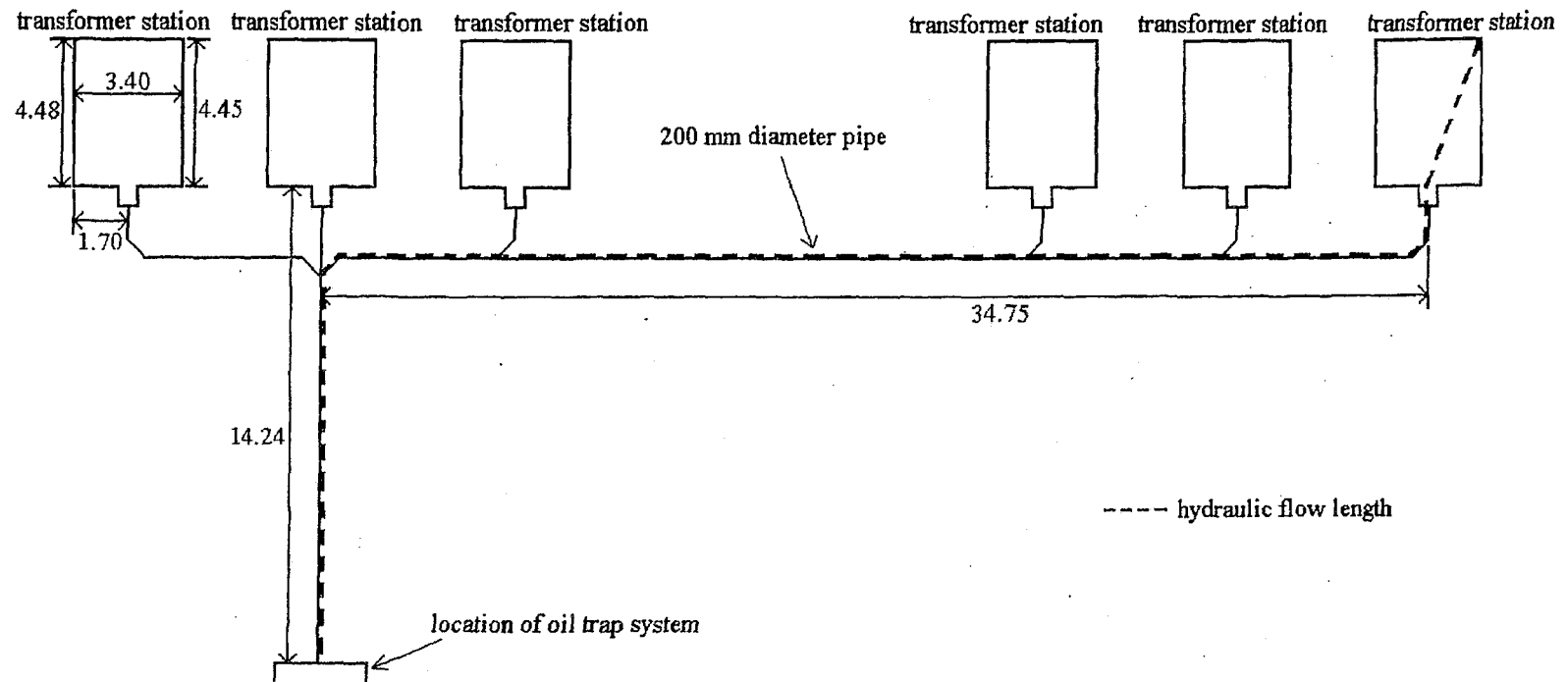
Figure 1.3: Hydro One's transformer station drawing near the city of Burlington.

Chapter 2   LITERATURE REVIEW

The first part of this chapter is to review the electrically-based and the hydraulically-based oil spill control systems and their limitations. Literatures related to the characteristics of the transformer oil spills at power stations, the different operating conditions of an oil spill control system, the oil-water separation concepts and the dynamics of oil slicks on water surface are then discussed.

2.1   Electrically-Based Oil Spill Control System

As indicated in Section 1.1, the electrically-based oil spill control system is currently used by some electrical companies in Ontario, such as *Hydro One*, to prevent transformer oil spills from entering the sewer system. This system uses several float devices (Figure 2.1) to detect the elevations of water and oil at the maintenance hole. Some float devices will float in the presence of water while other float devices will float in the presence of oil. Figures 2.2 to 2.4 show the locations of float devices (red circles in Figure 2.3) and the sump pump (blue dotted circle in Figures 2.3 and 2.4) at a typical electrically-based oil spill control system. The float devices are connected to switches which control the sump pump. When the float devices reach point B (Figure 2.3), the sump pump will start activating. The function of sump pump is to draw out water from the maintenance hole. When the float devices reach point E (Figure 2.3), the sump pump will be stopped in order to prevent drawing out the spilled transformer oil from the maintenance hole. The major advantage of the electrically-based oil spill control

system is its relatively compact size, compared to the enormous size of the tank in the hydraulically-based oil trap system or the long pipe columns of the hydraulically-based oil back-up system. In areas where bedrock is closed to the surface elevation, an electrically-based oil spill control system is cost-effective as the rock excavation costs are expensive (Truong, 2002).

The effectiveness of the electrically-based oil spill control system is better than that of the hydraulically-based oil spill control system. If all components of the electrically-based oil spill control system perform perfectly, no transformer oil spills should escape from the system. The effectiveness of hydraulically-based oil spill control systems must rely on the design diameter of oil globules,

Electrically-based oil spill control systems have two major problems. First, when the float devices are surrounded by grits, they may not float properly on the surface. Thus, the inaccurate measurement of different fluid levels may unintentionally allow the transformer oil spills to escape from the system. In order to minimize this problem, electrical companies must spend an enormous amount of money on labours every year to check and maintain the performance of equipment. Secondly, the sump pump in the system is initiated and terminated electronically. If the electricity is out of service due to thunderstorms, fire or other problems, runoff will not be drawn out from the maintenance hole properly. This will cause the transformer oil spills to escape from the system.
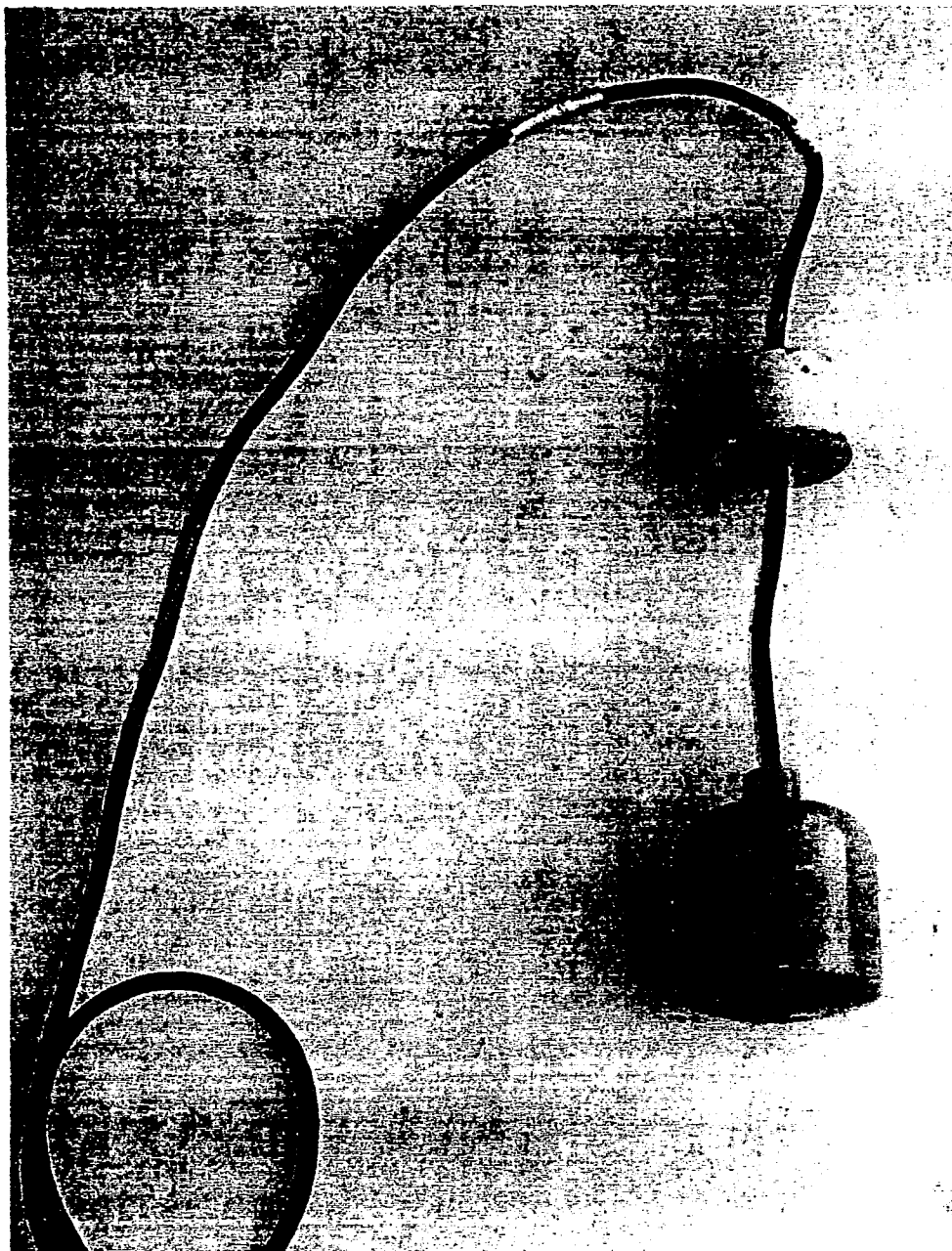
Figure 2.1: A typical float device.
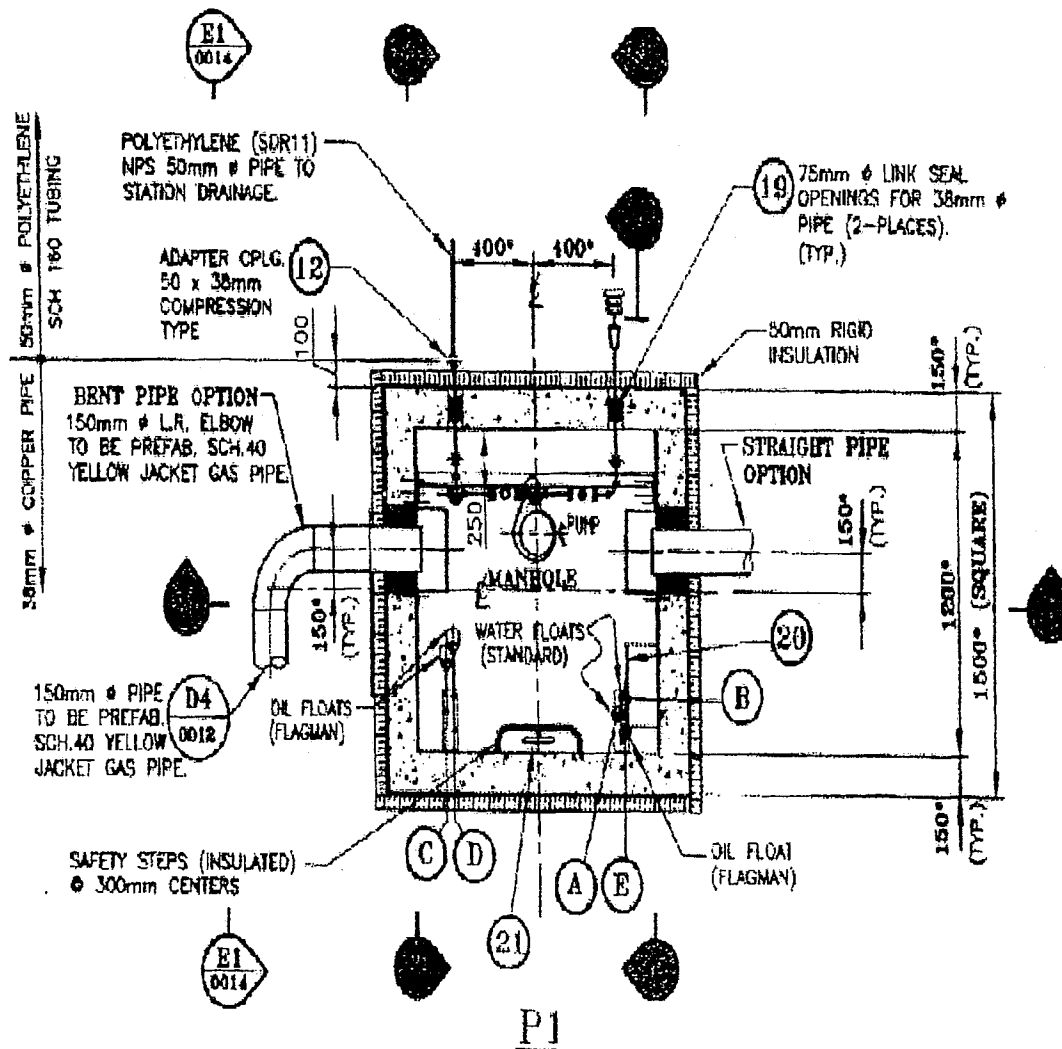
Figure 2.2: Top view of a typical electrical oil spill control system (Troung, 2002).
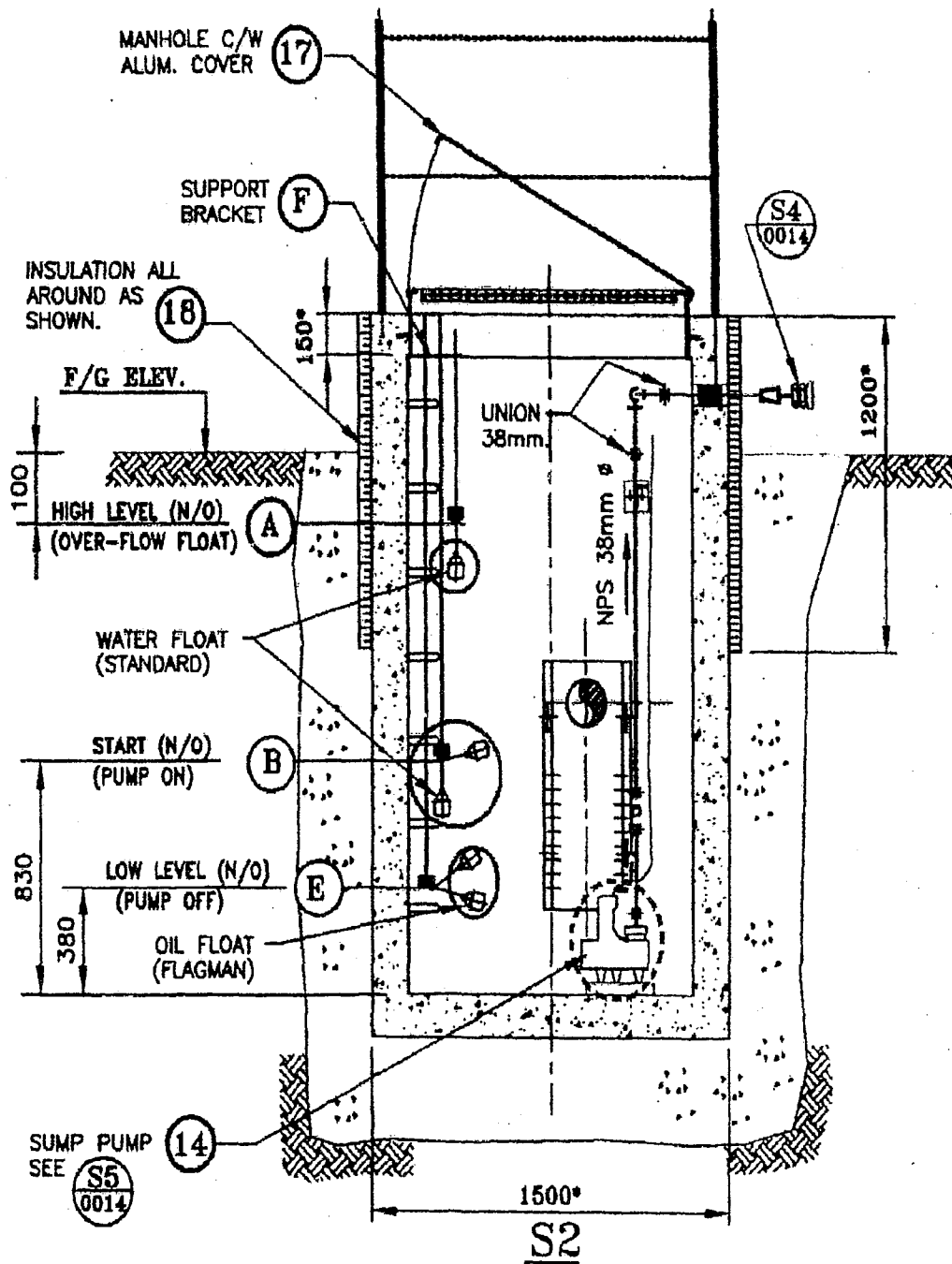
Figure 2.3: Side view #1 of a typical electrical oil spill control system (Truong, 2002).
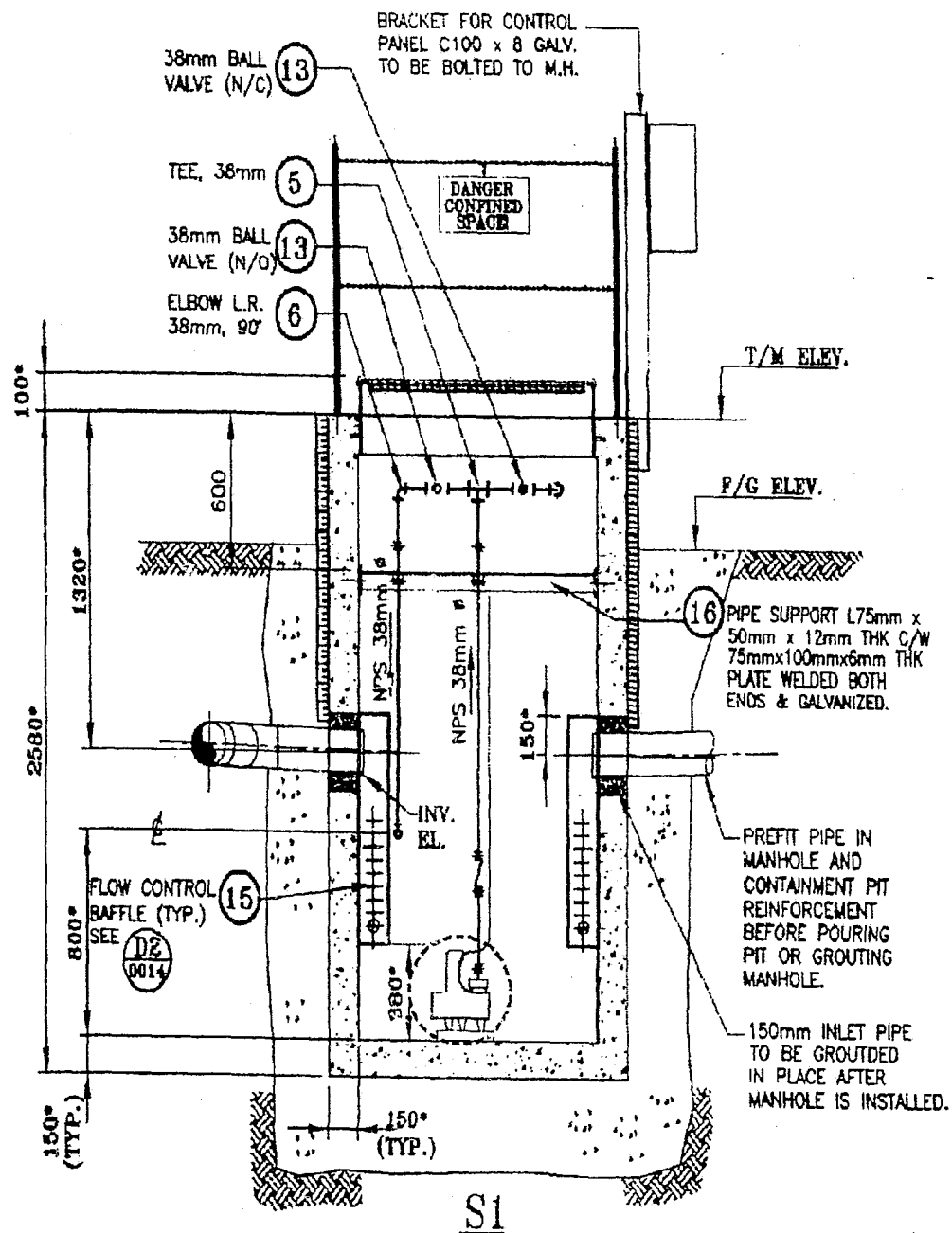
Figure 2.4: Side view #2 of a typical electrical oil spill control system (Troung, 2002).

2.2   Hydraulically-Based Oil Trap Systems

A hydraulically-based oil trap systems is designed to trap transformer oil spills and prevent the spilled oil from entering the storm sewer system until cleanup mechanism is activated. Thus, an oil trap system must have adequate storage capacity to hold the spilled transformer oil especially for the major oil spill operation condition discussed in Section 2.5. When spilled transformer oil enters into the system, it floats to the water surface due to the density difference between oil and water and is trapped in the system (Figure 2.5). If the inflow is a combination of spilled transformer oil and stormwater, the oil-water separation process will still occur between the two vertical pipes (red dotted lines in Figure 2.5). The purpose of the long inflow and outflow pipes is to trap and prevent the spilled transformer oil from escaping from the system. If a rain occurs after a transformer oil spill, runoff will pass under the oil layer in the oil trap system and exit to local sewers.

The hydraulically-based oil spill back-up system has two advantages when compared to the electrically-based oil spill control system. First, since the hydraulically-based control system does not need any electricity, it will not be prone to electric surges during lightening storms. Thus, the reliability of a hydraulically-based control system is greater than that of an electrical control system. Secondly, the hydraulically-based control system requires less maintenance than that of the electrically-based control system. The required maintenance activities of a hydraulic control system are the removal of the transformer oil spills and grit, cleaning the maintenance hole, and the

occasional inspection of the two pipe columns and maintenance hole for cracks and clogs to the structure. The electrical oil spill control system requires similar maintenances as the hydraulic control system, but it also requires frequent checks and calibration on its parts, such as the float devices and the sump pumps, due to its proximity to power surges and storm water (Truong, 2002).

The oil trap system is bigger in size than the hydraulically-based oil back-up system discussed in Section 2.3. Thus, the oil trap system should be installed at transformer stations where underground space is available. On the other hand, it may not appropriate to apply the oil trap system in transformer stations near the downtown area where underground utility lines are concentrated.
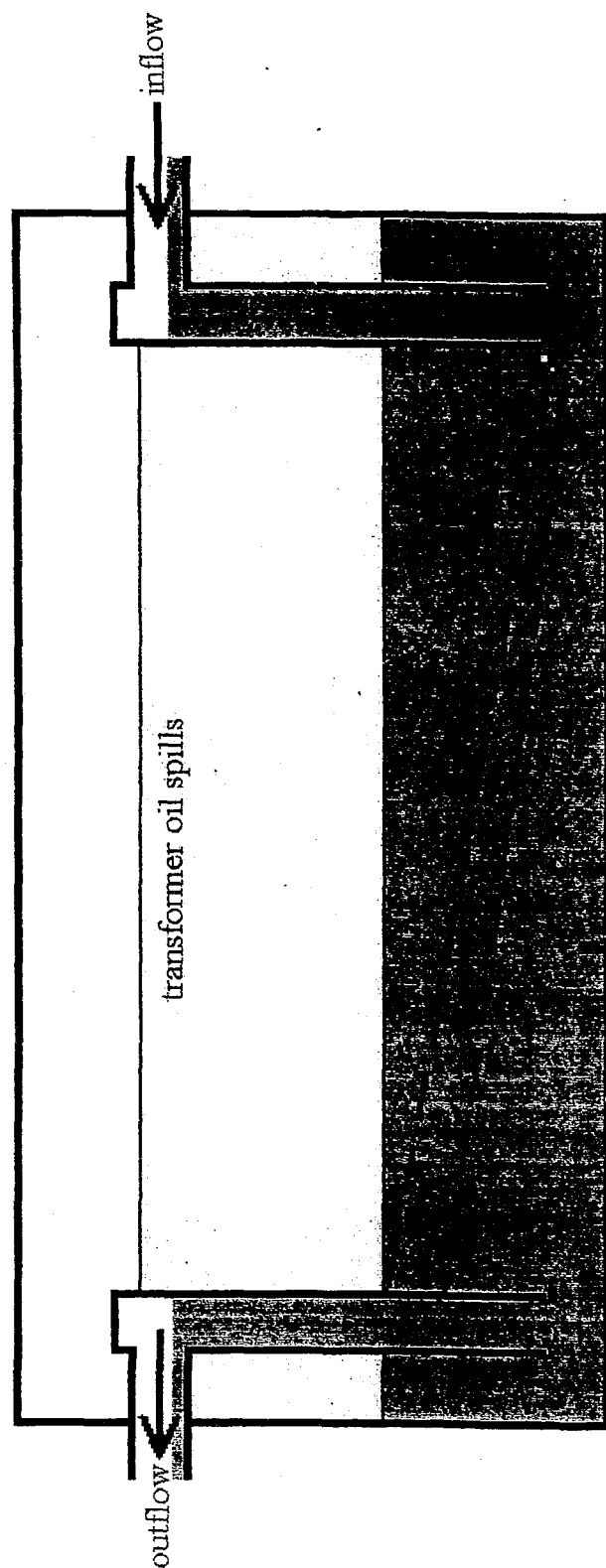
Figure 2.5: An oil trap system design.

2.3   Hydraulically-Based Oil Back-up System

The hydraulically-based oil back-up system discussed in this section was designed by Wilkinson Concrete Precast Ltd. and investigated by Patrick Truong at Ryerson University in 2002 (Figure 2.6). Figure 2.7 demonstrates the initial condition of the oil back-up system. The separation of water and oil occurs inside two long, vertical pipe columns. After the spilled transformer oil has entered the oil back-up system through the horizontal pipe *hp1* (Figure 2.8), the transformer oil stays on the water surface and pushes the water downward to the external pipe column. In order to exit the system, the water travels through the external and the internal pipe columns. At the same time, the external pipe column holds the spilled transformer oil back to the concrete pad underneath the transformer.

The base area of the oil back-up system is small. Thus, it should be applied at transformer stations where underground area is limited. The only disadvantage of the hydraulically-based oil spill back-up system is its long, vertical structure. If there is any damage on the bottom part of the pipe columns, the spilled transformer oil may pollute the ground water underneath the transformer station. Truong (2002) conducted only a conceptual experiment on an oil back-up system model (Figure 2.6). This thesis will develop a mathematical model using the Matlab 6.1 program to analyze the hydraulic characteristics of the oil back-up system.

Figure 2.6: Preliminary experiments on the oil back-up system (Truong 2002).
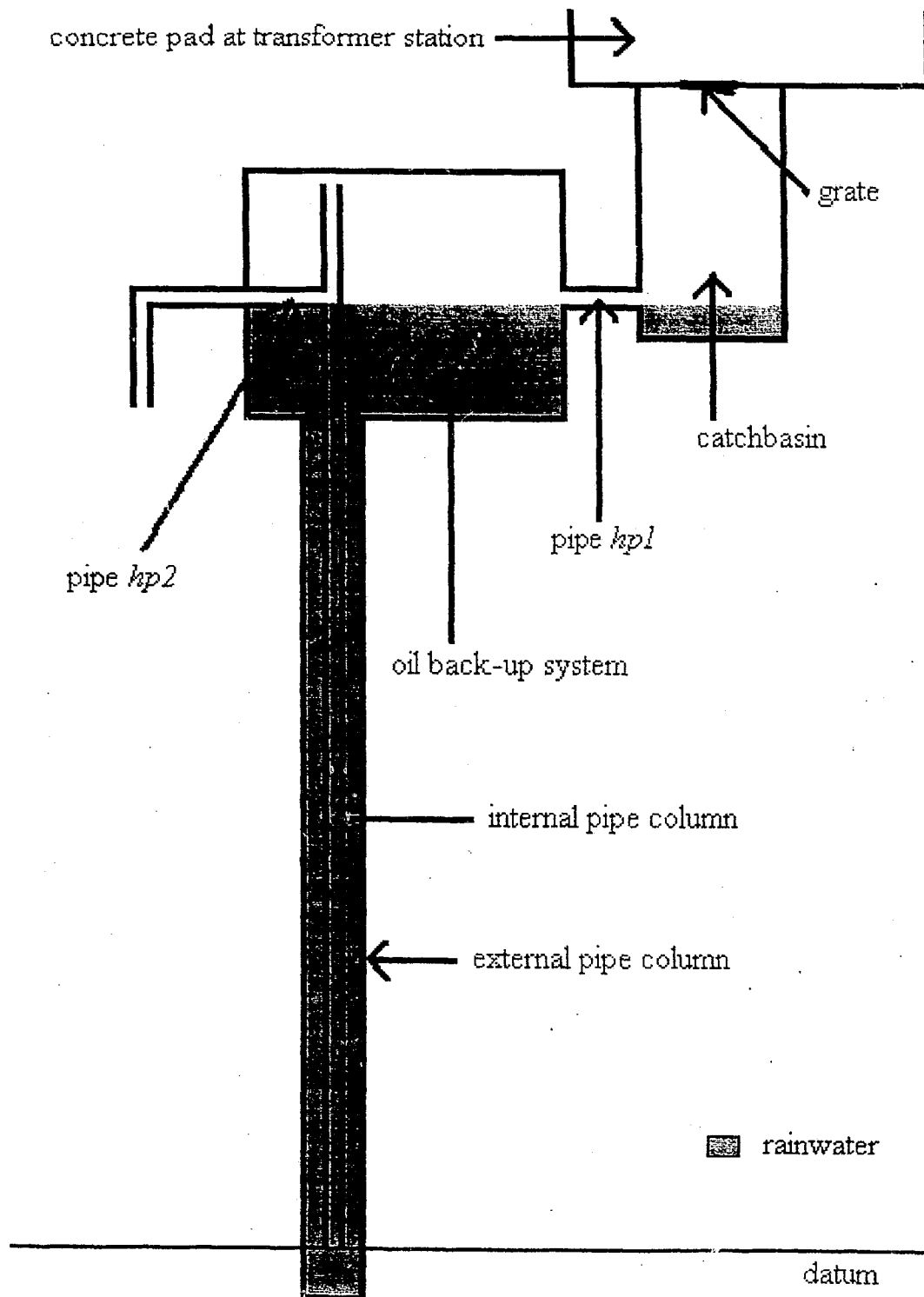
Figure 2.7: Oil back-up system.
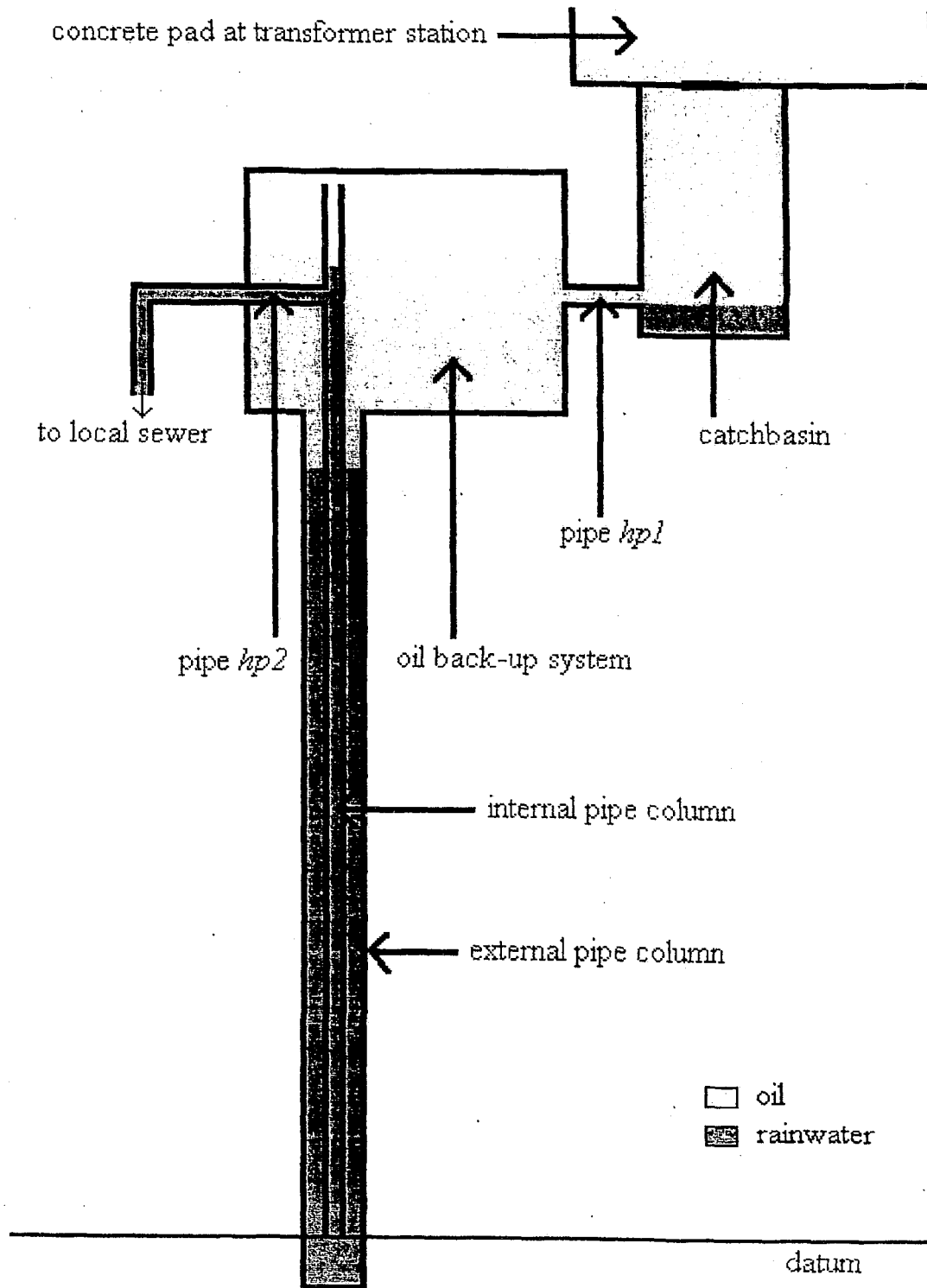
Figure 2.8: Transformer oil enters into the oil back-up system.

2.4   Oil Spill Characteristics

Between 1993 and 1999, 2,852,931 litres of materials were reported lost at transformer stations due to spillage, of which over 860,000 litres have been identified as transformer oil (Weslake 2002). Assuming a cost of $3.00 per litre of transformer oil, Hydro One loses on average over $368, 000 per year due to spillage. This is only the material cost. Not included is the manpower and equipment cost associated with clean up, disposal cost of contaminated spilled oil, of contaminated soil material, and monitoring of environmental impacts.   Of greater significance is the impact on the surrounding environment and its incalculable cost. These include impacts on the ground water and sources of drinking water, contamination of substrates in spawning area, destruction of micro-invertebrates source of food supply, contamination of aquatic vegetations, and contamination of agricultural lands.

Generally, oil spills may contain oil in three major forms: free oil, emulsified oil and dissolved oil. Free oil has the largest discrete oil globules. The oil globules rise as a result of buoyant forces and form an oil layer on top of the water. Under proper quiescent flow condition, free oil can be removed by gravity separation. This removal is a function of residence time, temperature, stability of the emulsion and the differences in specific gravity between oil and water. Some coarse oily solids with a specific gravity greater than 1.0 will settle to the bottom of the oil control system. Most of the oil and the associated fine solids with a specific gravity less than 1.0 will rise to the water surface.   Emulsified oil contains smaller oil globules with a diameter of less

than 20 microns. Most of these oil globules are between 1 to 10 microns. These globules form a stable suspension in the water as a result of the predominance of interparticle forces over buoyant forces. The presence of particulates also contributes to emulsion formation. Emulsified oil cannot be removed by gravity separation alone; it may be removed by flotation or chemical addition and coalescing. Dissolved oil is the petroleum fraction that forms a true molecular solution with water. Further treatment, such as biological treatment is necessary if removal of dissolved oil is required (API, 1990). The transformer oil spills on transformer stations are mostly in free oil form. Therefore, the conventional oil-water separation concept can still be applied to the design of the oil trap system.

2.5   Operating Conditions

All transformer oil spill control systems, either electrically- or hydraulically-based, must meet three general operating conditions: normal, normal oil spill and major oil spill. The normal operating condition assumes that the inflow to the oil spill control system contain rainwater only. Therefore, the function of the oil spill control system is to provide an adequate flow capacity for proper drainage. The normal oil spill operating condition assumes that the inflow to the oil spill control system is a combination of small transformer oil spills and rainwater. Minor transformer oil spills (less than 100 L) may come from minor leakages at the transformer station resulting in small quantities. The function of the oil spill control system under minor spill

conditions is trap the small quantity of spilled transformer oil and pass the rainwater to the sewer system. The major oil spill operating condition assumes that a large sudden surge of oil (larger than 1000 L) enters into the oil spill control system. This condition is caused by an explosion and a fire at the transformer station due to lightening or by transformer oil oxidation. Thus, the oil spill control system should have adequate storage capacities to hold the sudden surge of spilled transformer oil. It should prevent any trapped transformer oil from being flushed out from the system by subsequent rain storms until cleanup mechanism is activated (Truong, 2002).

2.6   Basic Theories of Oil-Water Separation

Since this research applies the oil-water separation concept to the design of the oil trap system, it is important to understand the basic theories of oil-water separation. Generally, an oil-water separator is a chamber designed to provide laminar flow conditions where globules of free oil can rise to the surface of water and coalesce into a separate oil layer (Figure 2.9). The theory of the oil-water separation is based on the rising rate of the oil globules and its relationship to the surface-loading rate of the separator. The rising rate (V_S) is the vertical velocity at which oil particles move upward to the surface of water due to the differential density between oil and water. The surface-loading rate (V_H) is the flow rate to the separator divided by the surface area of the separator. Ideally, any oil globule with a rising rate greater than or equal to surface-loading rate will reach the surface of water and has to be removed manually.

The required surface-loading rate for removal of a specified size of oil droplet can be determined from the following equation of the rising rate (API, 1990).

$$V_S = \frac{g(\rho_w - \rho_o)d^2}{18\mu}$$

[1]

where V_S = vertical velocity of the design oil globule, in m/sec

g = acceleration due to gravity, in m/sec^2 ($g = 9.81$ m/sec^2)

μ = dynamic viscosity, in Pa $\cdot$ sec

ρ_w = density of water at the design temperature, in kg/m^3

ρ_o = density of oil at the design temperature, in kg/m^3

d = diameter of the oil globule to be removed, in metres

The rising velocity of an oil globule is highly dependent on the globule's diameter. Small oil globules rise slower than the larger ones. Normally, oil globules with a diameter greater than or equal to *0.15* millimetres can be removed effectively in any gravity separation device. The performance of oil-water separators varies with changes in the characteristics between oil and water, including flow rate, viscosity, oil-globule size, specific gravity and temperature. The closer the specific gravity between oil and water, the slower the oil globules will rise. Furthermore, oil globules will rise slower at lower temperatures (API, 1990).

Page 25
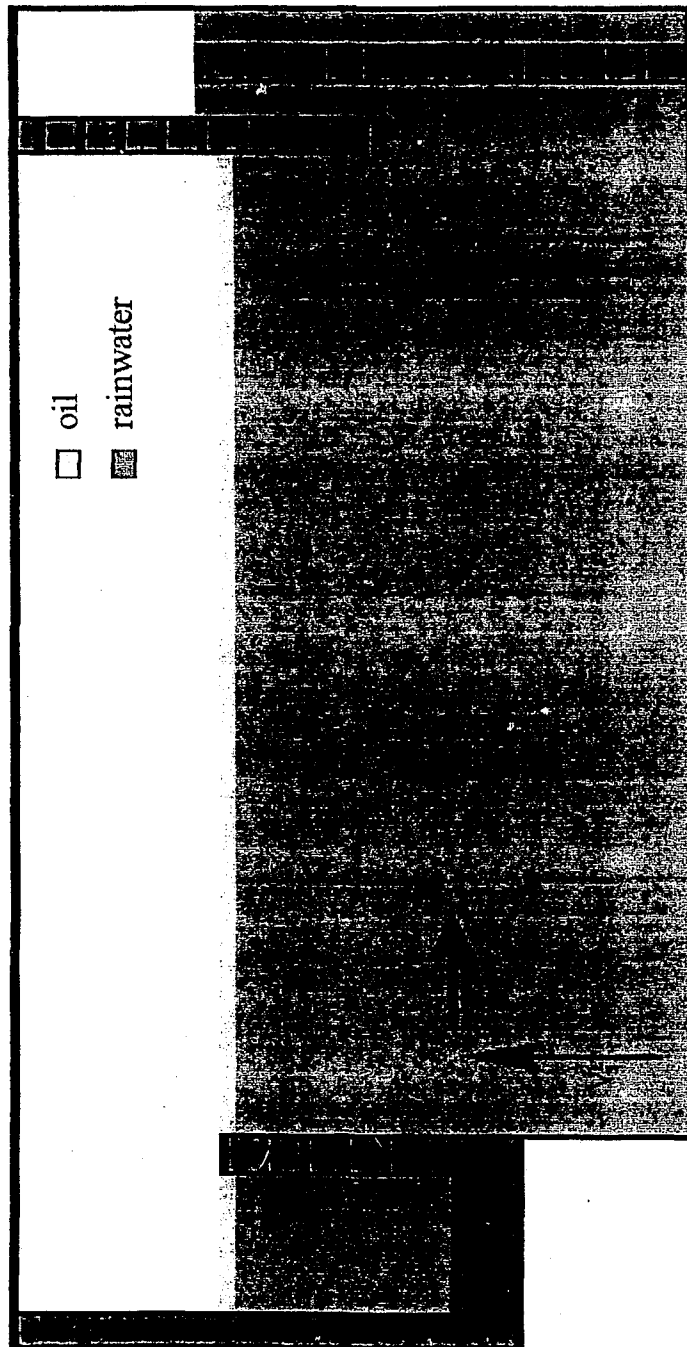
Figure 2.9: Side view of a typical oil-water separator (API, 1990).

2.7   Literature Review on Dynamics of Oil Slicks on Water Surface

There are only a few published papers, Wilkinson (1972, 1973), Wicks (1969), Cross and Hoult (1971), Jones (1972) and Moir and Lau (1975), which dealt with the dynamics of contained oil slicks on the water surface using floating booms. No literature can be found which address oil spill control systems for transformer stations. This section will review these theories which are similar to the oil and water layers situation in the oil trap system.

All the papers discussed below assumed one-dimensional flows which were uniform throughout the depth, except Wilkinson (1973) considered the effects of finite depths of flow. This effect is very important because it causes the change in momentum of the water flowing under the slick which in turn leads to instability of the slick (Moir and Lau, 1975).

Both Wilkinson (1973) and Wicks (1969) recognized that an oil slick on the water surface, such as rivers, can be divided into two regions: the frontal zone and the viscous zone (Figure 2.10). The frontal zone is the upstream region of an oil slick where viscous forces can be neglected in comparison with the dynamic forces. The viscous zone is the downstream of the frontal zone where the viscous shear at the oil-water interface and along the bottom of the river becomes the same order of magnitude as the dynamic forces and can no longer be neglected. The oil and water layers in the oil trap system are similar to the viscous zone discussed by Wilkinson and Wicks (Moir and

Lau, 1975). The only difference is that the viscous zone is an open system, whereas the oil trap system is a closed system. Thus, more forces components should be involved in the oil trap system.

Wilkinson's theory of the viscous zone is based on three main assumptions: (1) the momentum flux due to circulation in the oil slick is small compared with interfacial shear forces, (2) the flow beneath the oil slick is steady and uniform over the depth and (3) a change in inertia and pressure force must be balanced by boundary shear stresses for equilibrium of the contained oil slick (Wilkinson, 1973). He developed an equation (Equation 2) for the equilibrium of forces acting on a combined oil-water layer at length δx (Figure 2.10):

$$\frac{\partial}{\partial x}\left[\frac{\rho_w\,g\,(1-\Delta)\,d^2}{2}\right]+\frac{\rho_w g\,\Delta\,(d-h)^2}{2}+\frac{\rho_w\,U_o^{\,2}\,d_o^{\,2}}{d-h}=-\tau_b \qquad [2]$$

where x = downstream coordinate measured from the beginning of the viscous zone

ρ_w = density of water

g = gravitational acceleration

$\Delta = (1-\rho_o)/\rho_w$

d = total thickness of the oil-water layers

h = thickness of the slick in the viscous zone

U_o = mean velocity of the flow upstream of the oil slick

d_o = flow depth upstream of the oil slick

τ_b = boundary shear stress at the bottom of the channel
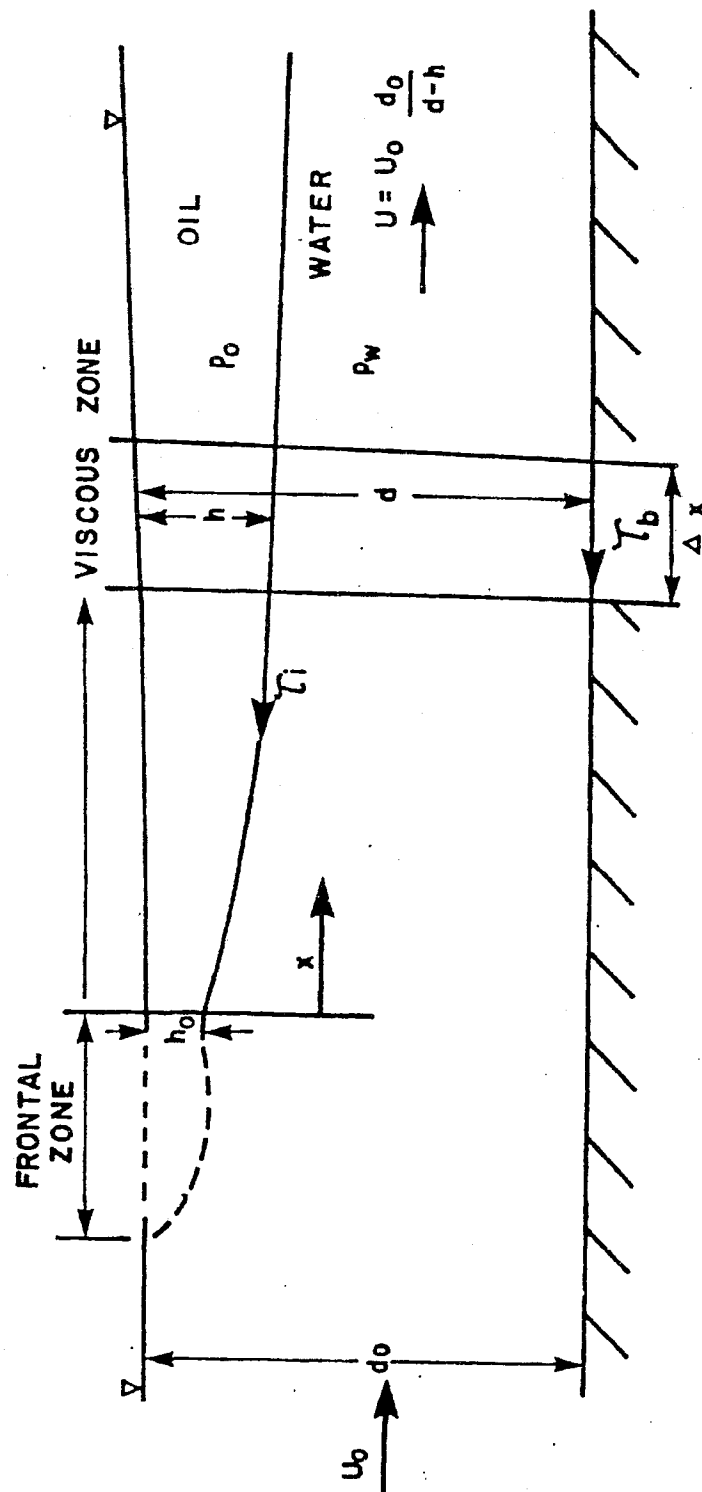
Figure 2.10: Equilibrium of forces acting on a combined oil-water layer (Moir and Lau, 1975).

He also developed another equation (Equation 3) for the forces acting on the oil layer only. This equation demonstrates that equilibrium is achieved by the balance of the pressure forces, the shear at the interface and the weight component of the oil acting along the interface.

$$\frac{\partial}{\partial x}\left(\frac{\rho_o g h^2}{2}\right) = \tau_i - \rho_o g h \frac{\partial (d-h)}{\partial x} \qquad [3]$$

where τ_i = shear stress at the oil-water interface

Equations [2] and [3] can be solved simultaneously for the two unknowns d and h, given the upstream velocity U_o and depth d_o, the oil density and values of τ_b and τ_i. Wilkinson expressed the shear stress terms as,

$$\tau_i = \frac{f_i}{4} \frac{\rho_w U^2}{2} \qquad [4]$$
$$\tau_b = \frac{f_b}{4} \frac{\rho_w U^2}{2}$$

where f_i = interfacial friction coefficient

f_b = boundary friction coefficient

Wilkinson showed that for a given set of flow conditions, there was a limiting thickness of the slick which failure would occur. This limiting thickness is independent of the viscous forces. However, the length of the slick and the slick profile are dependent upon the interfacial and boundary shear stresses (Wilkinson, 1973).

Page 30

Wicks used the balance of pressure and shear forces to determine the slick profile in the viscous zone. He also introduced the effect of circulation velocity in the oil (Wicks, 1969). However, he used wrong expressions for both the frontal thickness and the oil velocity according to Lau's analysis. Therefore, his calculated slick profiles could not be correct.

Cross and Hoult (1971) do not divide the oil slick into different regions, but treats it as one unit starting from the leading edge and growing with the distance downstream. The authors considered the equilibrium of the oil layer to be maintained by a balance of pressure forces and shear stresses at the interface (Cross and Hoult, 1971). However, the authors left out a term representing the weight component of the slick along the interface. This leads to unreasonable results of this research (Moir and Lau, 1975).

Jones (1972) attempted to explain certain types of boom failures in terms of hydrodynamic instability at the interface. The concept is that as instability develops at an oil-water interface, waves would form and grow. When a wave becomes steep enough, interfacial tension causes the crest of the wave to be broken off into droplets which are then entrained into the flow.

Jones considered the case of a slick of constant thickness h at rest on top of an infinitely deep water layer flowing at velocity U. Both layers were considered to be inviscid. An infinitesimal disturbance was then superimposed on the steady flow pattern. Classical methods of stability analysis were used to determine the neutral stability curve. From a

series of these stability curves, Jones obtained curves of the non-dimensionalized critical velocity $U / (gh)^{1/2}$ versus the parameter $\sigma/(\rho_o gh^2)$. Finally, Jones' analysis showed that thin slicks become unstable at lower velocities than thick slicks.

Part of Wilkinson and Jones' analyses can be applied to the oil trap system. First, since both the oil trap system and Wilkinson's research deal with uniform water flows underneath an oil slick in one-dimensional direction with finite depth, the two shear stresses components in Wilkinson's calculation can be used in the oil trap system. Thus, modifications must be done in developing the equilibrium equations for the oil trap system due to its closed system design. Secondly, Jones analysis on the conditions of the occurrence of a Kelvin-Helmholtz type instability at the oil-water interface can be also applied to the oil trap system. It may be too difficult to observe visually if any transformer oil has escaped from the oil trap system due to the small diameter of the outlet pipe and high water flow rate. The formation of waves at the interface can provide some signals of when the failure of the oil trap system will occur.

Chapter 3   PHYSICAL MODEL STUDY OF THE OIL TRAP SYSTEM

This chapter describes the physical model study of the oil trap system. It also states the experimental procedures and observations.

3.1   Dimensionless Analysis Method of the Oil Trap System

In order to apply the Buckingham's method for the dimensionless analysis of the oil trap system, variables in Table 3.1 are used to derive the Π-terms. The visual meanings of these variables are shown in Figure 3.1 below.
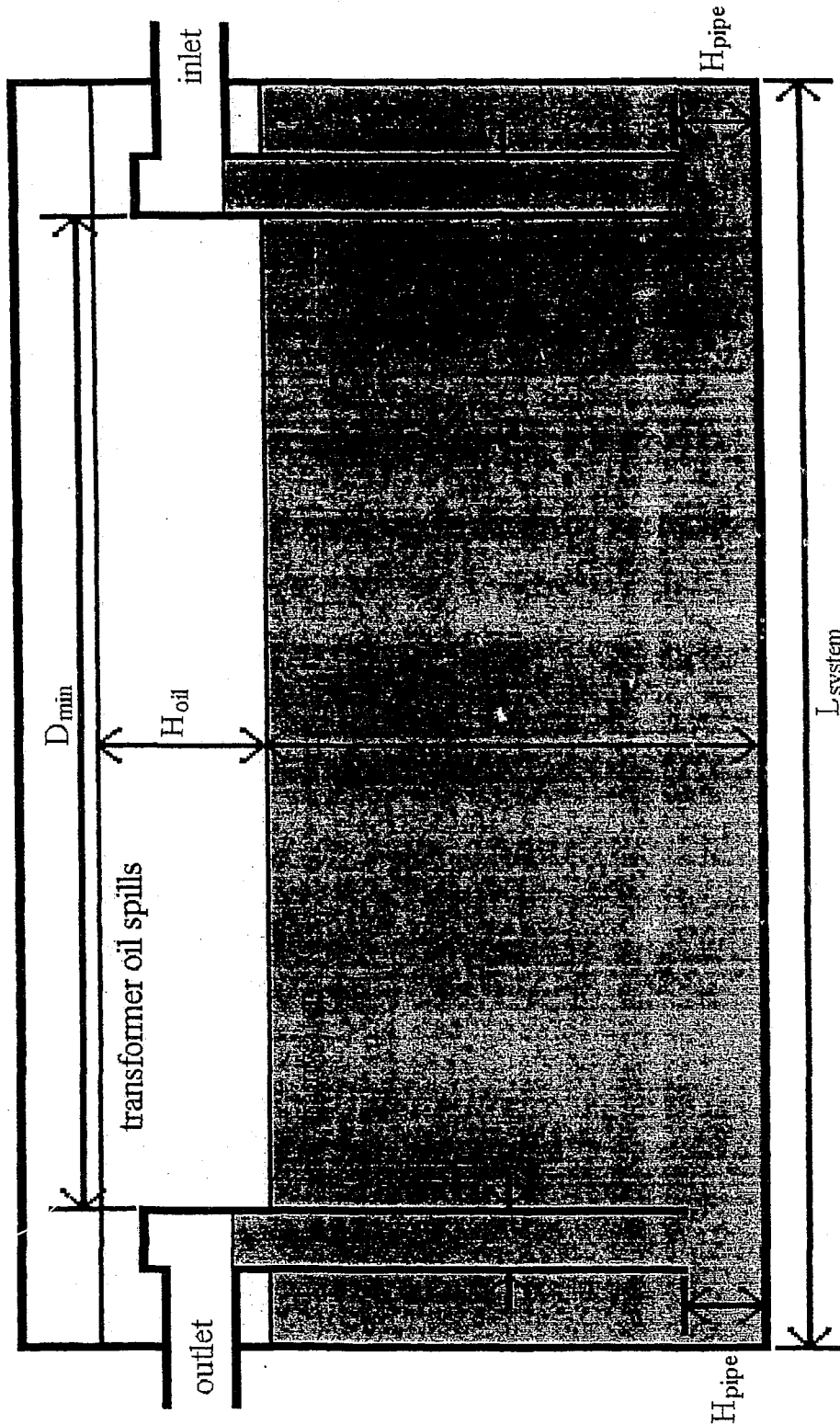
Figure 3.1: Variables for the determination of Π-terms in the oil trap system.

Table 3.1: Variables involved in the Buckingham's method.

Variables	Definition of Variable	Dimension
X	volume of oil containment in dimensionless form	-----
B	width of the oil trap system	L
d_{pipe}	diameter of the vertical pipe	L
D_{min}	minimum distance between the vertical pipes	L
H_{pipe}	height between the bottom part of the vertical pipe and the oil trap system	L
H_{water}	head of water at failure situation	L
H_{oil}	head of oil	L
ρ_{water}	density of water	ML^{-3}
ρ_{oil}	density of oil	ML^{-3}
Q	flow of water at failure situation	$L^3 T^{-1}$
μ_{water}	dynamic viscosity of water	$ML^{-1} T^{-1}$
μ_{oil}	dynamic viscosity of oil	$ML^{-1} T^{-1}$
σ	oil-water interfacial tension	MT^{-2}
g	gravitational acceleration	LT^{-2}
t	time for oil containment stays in the oil trap system	T

Since there are fourteen variables involved in the oil trap system, eleven Π-terms can be formed. Q, H_{water} and ρ_{water} are selected as the three relevant variables for the Buckingham's method. The Π-terms are derived as follows:

$$\Pi_1 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(B^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([L]^d)$$

[M]: $c = 0$
[T]: $-a = 0$
[L]: $3a + b - 3c + d = 0$
$\qquad\qquad b = -d$

Therefore, $\Pi_1 = (B)(H_{water}{}^{-1})$ [5]

$$\Pi_2 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(d_{pipe}{}^d)$$

Since dimension of d_{pipe} is similar to dimension of B,

$$\Pi_2 = (d_{pipe})(H_{water}{}^{-1})$$ [6]

$$\Pi_3 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(D_{min}{}^d)$$

Since dimension of D_{min} is similar to dimension of B,

$$\Pi_3 = (D_{min})(H_{water}{}^{-1})$$ [7]

$$\Pi_4 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(H_{pipe}{}^d)$$

Since dimension of H_{pipe} is similar to dimension of B,

$$\Pi_4 = (H_{pipe})(H_{water}{}^{-1})$$ [8]

Page 36

$$\Pi_5 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(H_{oil}{}^d)$$

Since dimension of H_{oil} is similar to dimension of B,

$$\Pi_5 = (H_{oil})(d_{pipe}{}^{-1}) \qquad [9]$$

$$\Pi_6 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(\rho_{oil}{}^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([M]^d[L]^{-3d})$$

[M]: $c + d = 0$
         $d = -c$
[T]: $-a = 0$
[L]: $3a + b - 3c - 3d = 0$
         $b = 3c + 3(-c) = 0$

Therefore, $\Pi_6 = (\rho_{water})(\rho_{oil}{}^{-1}) \qquad [10]$

$$\Pi_7 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(\mu_{water}{}^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([M]^d[L]^{-d}[T]^{-d})$$

[M]: $c + d = 0$
         $c = -d$
[T]: $-a - d = 0$
         $a = -d$
[L]: $3a + b - 3c - d = 0$
         $b = -3(-d) + 3(-d) + d$
         $b = d$

Therefore, $\Pi_7 = (\mu_{water})(H_{water})(Q^{-1})(\rho_{water}{}^{-1}) \qquad [11]$

$$\Pi_8 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(\mu_{oil}{}^d)$$

Since dimension of μ_{oil} is similar to dimension of μ_{water},

$$\Pi_8 = (\mu_{oil})(H_{water})(Q^{-1})(\rho_{water}{}^{-1}) \qquad [12]$$

Page 37

$$\Pi_9 = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(\sigma^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([M]^d[T]^{-2d})$$

[M]: $c + d = 0$
$$c = -d$$

[T]: $-a - 2d = 0$
$$a = -2d$$

[L]: $3a + b - 3c = 0$
$$b = -3(-2d) + 3(-d)$$
$$b = 3d$$

Therefore, $\Pi_9 = (Q^{-2})(H_{water}{}^3)(\rho_{water}{}^{-1})(\sigma)$ [13]

$$\Pi_{10} = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(g^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([L]^d[T]^{-2d})$$

[M]: $c = 0$

[T]: $-a - 2d = 0$
$$a = -2d$$

[L]: $3a + b - 3c + d = 0$
$$b = -3(-2d) - d$$
$$b = 5d$$

Therefore, $\Pi_{10} = (Q^{-2})(H_{water}{}^5)(g)$ [14]

$$\Pi_{11} = (Q^a)(H_{water}{}^b)(\rho_{water}{}^c)(t^d)$$

$$([M][L][T])^0 = ([L]^{3a}[T]^{-a})([L]^b)([M]^c[L]^{-3c})([T]^d)$$

[M]: $c = 0$

[T]: $-a + d = 0$
$$a = d$$

[L]: $3a + b - 3c = 0$
$$b = -3a = -3d$$

Therefore, $\Pi_{11} = (Q)(t)(H_{water}{}^{-3})$ [15]

Page 38

Equation of oil containment, X, in dimensionless form can be expressed as:

$$X = \varphi \left(\frac{B}{H_{water}} ; \frac{d_{pipe}}{H_{water}} ; \frac{D_{min}}{H_{water}} ; \frac{H_{pipe}}{H_{water}} ; \frac{H_{oil}}{H_{water}} ; \frac{\rho_{water}}{\rho_{oil}} ; \frac{H_{water} \mu_{water}}{Q \rho_{water}} ; \frac{H_{water} \mu_{oil}}{Q \rho_{water}} ; \right.$$

$$\left. \frac{H_{water}^{3} \sigma}{Q^{2} \rho_{water}} ; \frac{H_{water}^{5} g}{Q^{2}} ; \frac{Q t}{H_{water}^{3}} \right) \qquad [16]$$

Variable X in Equation [16] is the volume of oil inside the oil trap system in dimensionless form where

$$X = V_{oil} / (B * H_{oil} * L_{system}) \qquad [17]$$

Since transformer oil spills will be retained in the system until cleanup mechanism is activated, $\Pi_{11} = (Q)(t)(H_{water}^{-3})$ can be eliminated from Equation [16] as $t \to \infty$. In other word, X is eventually in steady-state or independent of time. Also, $\Pi_{6} = (\rho_{water})(\rho_{oil}^{-1})$ can be replaced by $1/\Delta$ where Δ is the relative density difference.

$$\Delta = (\rho_{water} - \rho_{oil}) / \rho_{water} \qquad [18]$$

The effect of Δ is important only when considered in conjunction with the gravity force, Therefore, Δ can be combined with $\Pi_{10} = (Q^{-2})(H_{water}^{5})(g)$ to form $\Pi_{10}' = (Q^{-2})(H_{water}^{5})(g)(\Delta)$.

3.2   Similitude Analysis Method of the Oil Trap System

Several standard dimensionless products can be found from the Π-terms in Section 3.1. For instance, if the formula of the Froude number (Table B.1 in Appendix B), Fr, is converted in terms of Q:

$$
\begin{aligned}
Fr &= (V)(g^{-1/2})(L^{-1/2}) \\
&= (Q*A^{-2})(g^{-1/2})(L^{-1/2}) \quad \text{where dimension of } A \text{ is same as dimension of } L. \\
&= (Q*L^{-2})(g^{-1/2})(L^{-1/2}) \\
&= (Q)(g^{-1/2})(L^{-5/2})
\end{aligned}
$$

Fr must be squared in order to eliminate the rational exponent.

$$
Fr^2 = (Q^2)(g^{-1})(L^{-5}) \tag{19}
$$

Thus, $\Pi_{10}' = (Q^{-2})(H_{water}{}^5)(g)(\Delta)$ is equivalent to the reciprocal of the Froude number shown in Equation [19].

Secondly, if the formula of the Reynolds number (Table B.1 in Appendix B), Re, is converted in terms of Q:

$$
\begin{aligned}
Re &= (V)(L)(v^{-1}) \\
&= (Q*A^{-2})(L)[(\rho_{water})(\mu_{water}{}^{-1})] \\
&= (Q)(L^{-1})(\rho_{water})(\mu_{water}{}^{-1})
\end{aligned} \tag{20}
$$

Hence, $\Pi_7 = (\mu_{water})(H_{water})(Q^{-1})(\rho_{water}{}^{-1})$ is the same as the reciprocal of the Reynolds number in Equation [20]. Also, $\Pi_8 = (\mu_{oil})(H_{water})(Q^{-1})(\rho_{water}{}^{-1})$ can be divided with the reciprocal of the Reynolds number in Equation [20] to form the ratio μ_{oil}/μ_{water}.

Page 40

Finally, if the formula of the Weber number (Table B.1 in Appendix B), *We*, is converted in terms of Q:

$$We = (V)(\rho_{water}^{1/2})(L^{1/2})(\sigma^{-1/2})$$
$$= (Q*A^{-2})(\rho_{water}^{1/2})(L^{1/2})(\sigma^{-1/2})$$

Both sides are squared for eliminating the rational exponent.

$$We^2 = (Q*A^{-2})^2(\rho_{water})(L)(\sigma^{-1})$$
$$= (Q^2)(\rho_{water})(L^{-3})(\sigma^{-1})$$
[21]

Thus, $\Pi_9 = (Q^{-2})(H_{water}^3)(\rho_{water}^{-1})(\sigma)$ equals to the reciprocal of the Weber number in Equation [21].

After rearrange the Π-terms, Equation [16] can be rewritten as:

$$X = \varphi \left(\frac{B}{H_{water}} ; \frac{d_{pipe}}{H_{water}} ; \frac{D_{min}}{H_{water}} ; \frac{H_{pipe}}{H_{water}} ; \frac{H_{oil}}{H_{water}} ; \frac{1}{\Delta} ; \frac{1}{Re} ; \frac{\mu_{oil}}{\mu_{water}} ; \frac{1}{We} ; \frac{1}{Fr} \right)$$
[22]

The next step is to determine which Π-terms in Equation [22] dominate the physical model of the oil trap system. Unfortunately, all the dimensions of the model are fixed due to practical reasons. Therefore, it is impossible to determine any relationships between the first four Π-terms and the standard dimensionless products in Equation [22]. However, according to the oil-water separation theories discussed in Section 2.6, Froude number will be selected for scaling the model results back to the prototype.

3.3   Scales of the Physical Model

The oil trap system was constructed by Wilkinson Heavy Precast Limited where the base area of the prototype follows the dimensions of the precast concrete tank product (Figure 3.2). Table 3.2 shows the dimensions and the ratios of the physical model and its prototype. The slight difference between the ratios of length and width in Table 3.2 is due to the convenience of the model construction. The percentage error between these two ratios is within 1%.

Table 3.2: Dimensions and ratios between the physical model and the prototype.

	Dimension, in mm		Ratio	
	Model	Prototype	Model	Prototype
Length	690	8540	1	12.3768
Width	260	3190	1	12.2692
Height	265	4100	1	15.4717

PRECAST CONCRETE TANK   114 CUBIC METRES

Standard Construction

Concrete: 35 MPA at 28 Days, 5 to 8% Air Entrainment.

Reinforcing: 20M Bars at 200 mm centres each way in roof, walls and floor.
              Eight extra 15M bars around each access opening.

Weight: 196,000 Lbs. (98,000 Lbs. Per Half Section)

Actual Capacity: 27,832 Litres per Vertical Metre.
                  114 Cubic Metres to underside of roof slab.

Notes:

1. Standard Access opening is 685 mm Diameter.
2. Four Heavy Lift hooks are provided in the Roof and Floor.
3. Close tolerance of Tongue and Groove Joint and a Fibrous Mastic Sealant ensure a solid structural and water tight seal. It is recommended that this joint be grouted both inside and outside approximately 24 hours after the tank has been assembled. Grout will be supplied upon request.
4. 32 mm Galvanized Steel Brackets and Turnbuckles in 4 Places to prevent separation of seal. (Optional)
5. 150 mm diameter Galvanized Steel Pipe Cross Beams. (Also available in Stainless Steel)
6. Precast Concrete AC 400 Access Extension with Anti-Frost Heave System and itegrally cast Aluminum Frame and Drip Proof Cover. (Optional)

- This tank is available with Aluminum Ladder Rungs to the floor in six different locations Consult with the factory as to how this will effect the size and location of the access opening.
- This product can be supplied with an Internal Polyethylene or P.V.C. Lining cast as part of the tank at the factory.
- The Unit shown is a Typical Configuration. Please call the factory to discuss your requirements for Flexible Water Tight Pipe/Wall Connectors, Frost Resistant Water Tight Grade Extensions, Galvanized Steel or Aluminum Hinged Covers, Flanged Roof Vents and many other Custom Appurtenances that can be supplied.

Recommended Burial Depth - 1.5 Metres over the Top Slab in Firm soil away from any area of vehicular traffic.
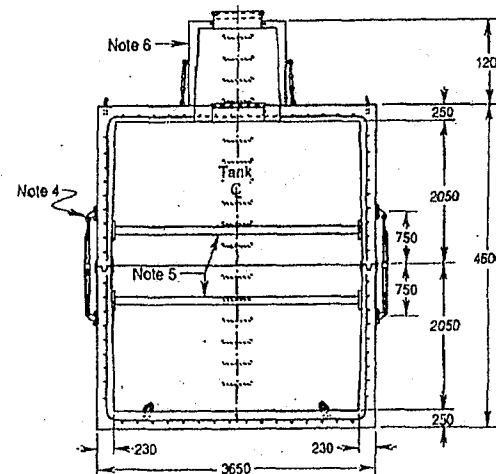
For recommended installation procedures refer to Wilkinson Installation Guidelines and Lifting & Assembly Instructions.

Figure 3.2: Precast concrete tank drawing for the dimensions design of the oil trap system.

3.4   Experimental Procedures

The objective of the physical model study is to examine whether the oil trap system can trap the transformer oil spills during a rain storm. The physical model of the oil trap system is shown in Figure 3.4. Two smaller tanks, the inlet tank and the outlet tank, are installed before and after the oil trap system in order to create a steady flow entering into the system. In order to achieve this objective, two sets of experiments were conducted. The first set of the experiments is to determine the relationship between the volumes of transformer oil in the system and their failure situations. The failure situation is defined as the flow rate that causes the formation of waves at the interface of transformer oil and water (Figure 3.3). Since before the transformer oil escapes from the oil trap system, waves should be formed between the interface of oil and water (Moir and Lau. 1975). The purpose of this set of experiments is to determine at what flow rate will transformer oil escape from the system.

The procedures for the first set of the experiment are discussed below. First, the pump used in the experiment must be calibrated before adding the transformer oil into the system. Appendix A shows the determination of the flow rate at each dial of the switch of the pump (Figure 3.5). The pump is located in the outlet tank (Figure 3.4). Once the outlet tank is filled up with water, flow rate of a typical dial at the pump's switch can be measured by dividing the volume of water entering into the inlet tank from the outlet tank through the plastic pipe over time.
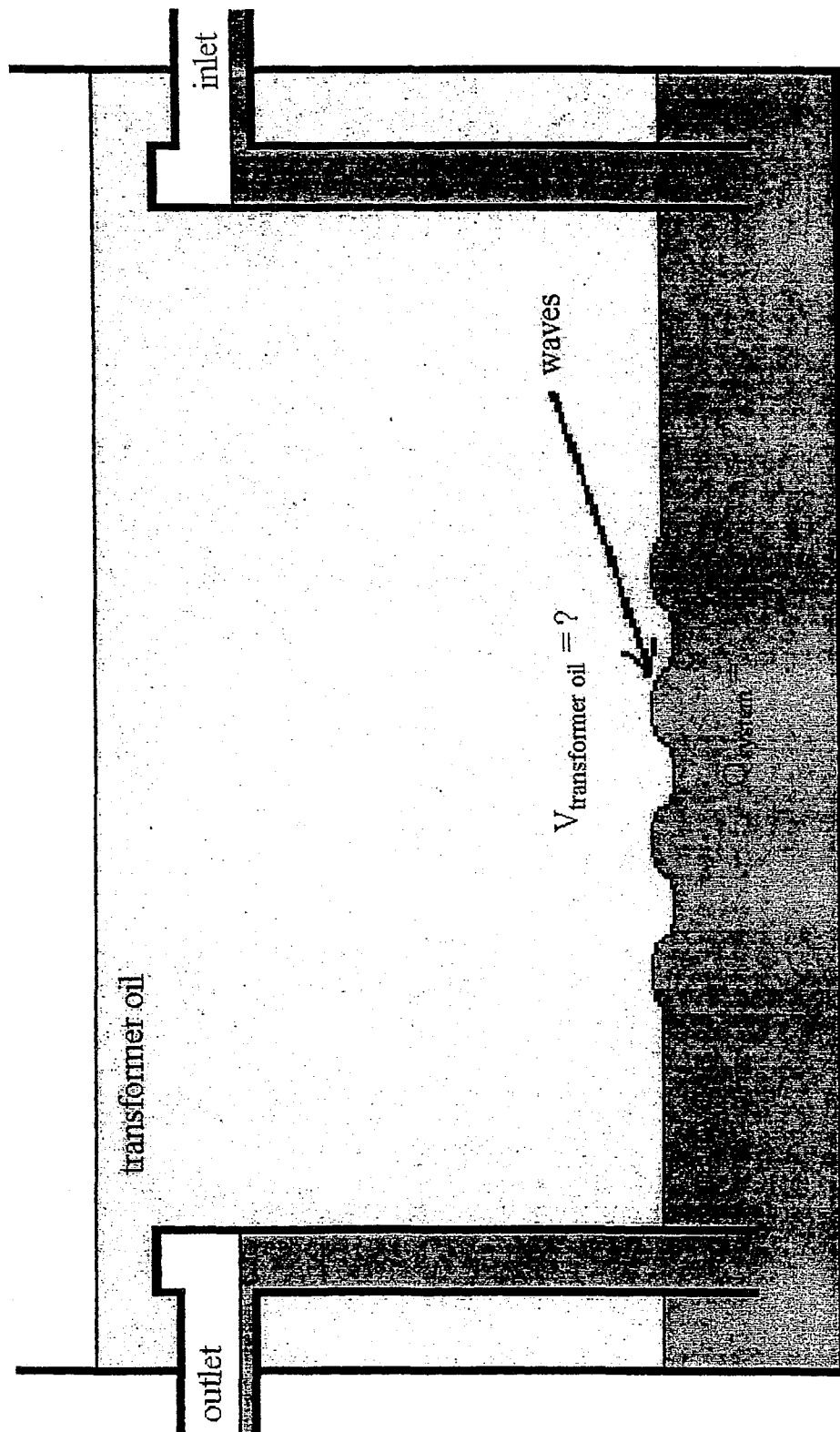
Figure 3.3: Failure situation of the oil trap system.
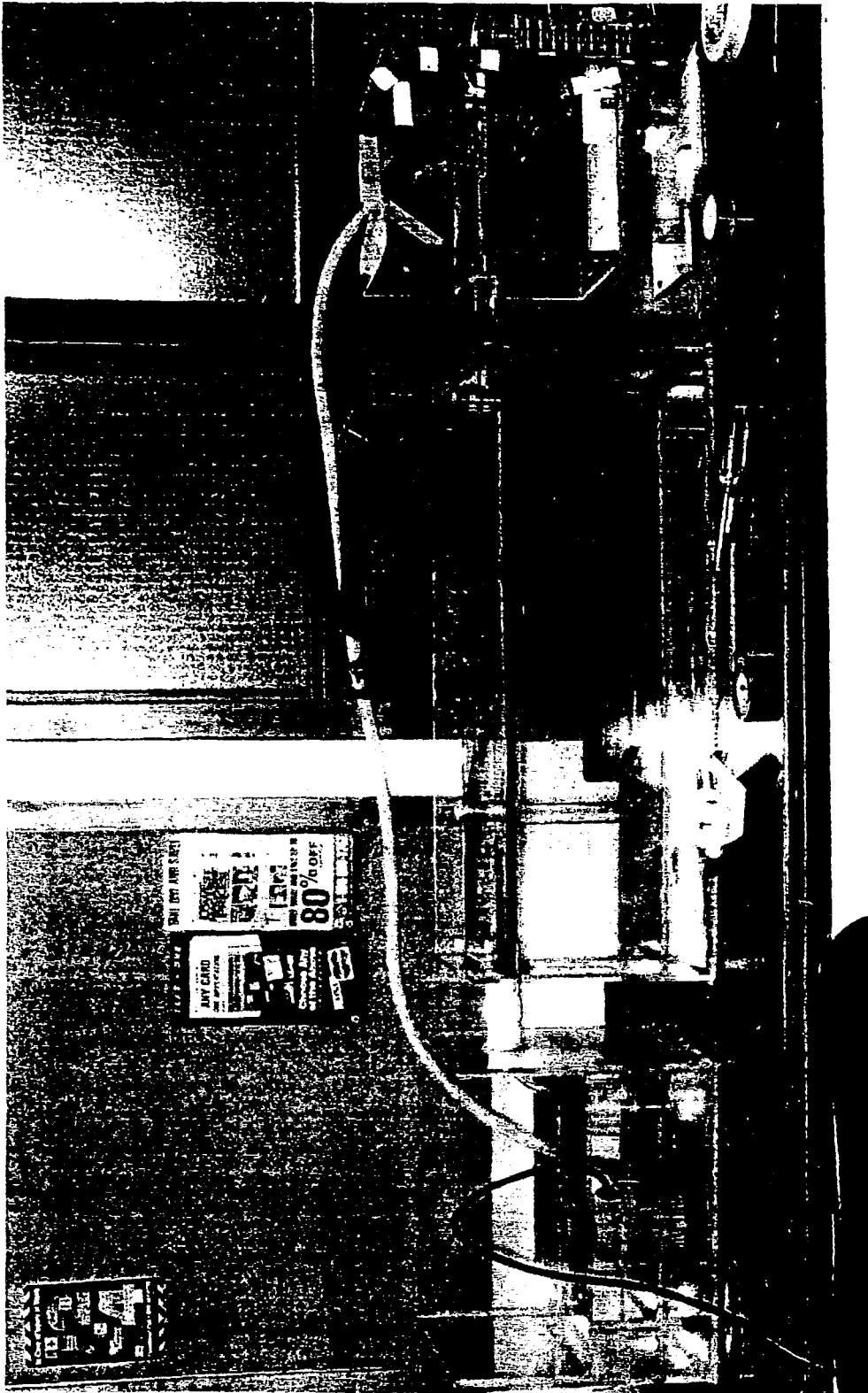
Figure 3.4: Physical model of the oil trap system
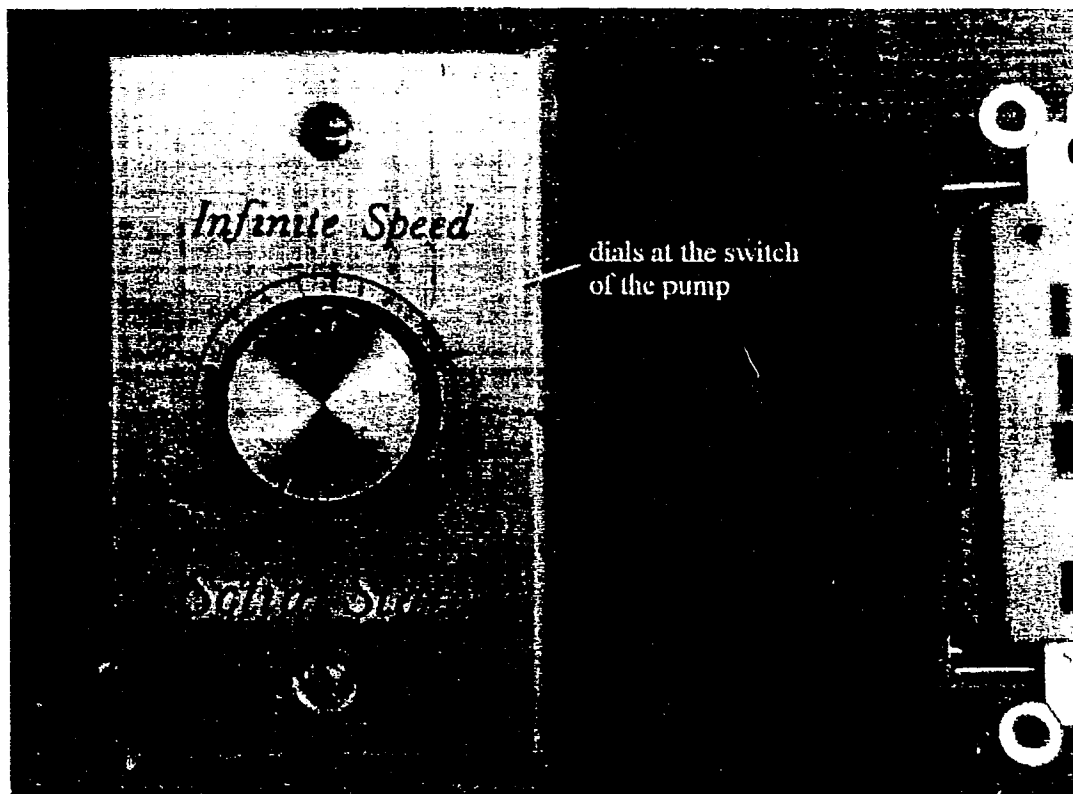
Figure 3.5: Switch of the pump used in the oil trap system experiment.

After the calibration of the pump, the oil trap system is filled in with water (Figure 3.6). Then, transformer oil is added into the oil trap system slowly in order to avoid any escape of oil through the vertical pipes (Figure 3.7). Once the excessive water in the oil trap system has escaped through the vertical outlet pipe (Figure 3.7), pump will be turned on and switch of the pump will be adjusted until the system reaches two different situations: (1) the water level in the inlet tank is stable and (2) the maximum flow entering into the system. The first situation is the equilibrium state of the system. After observations are taken, more transformer oil will be added into the oil trap system and the above steps (Figure 3.7) are repeated until the amount of transformer oil in the system reaches its maximum capacity (Figure 3.8).
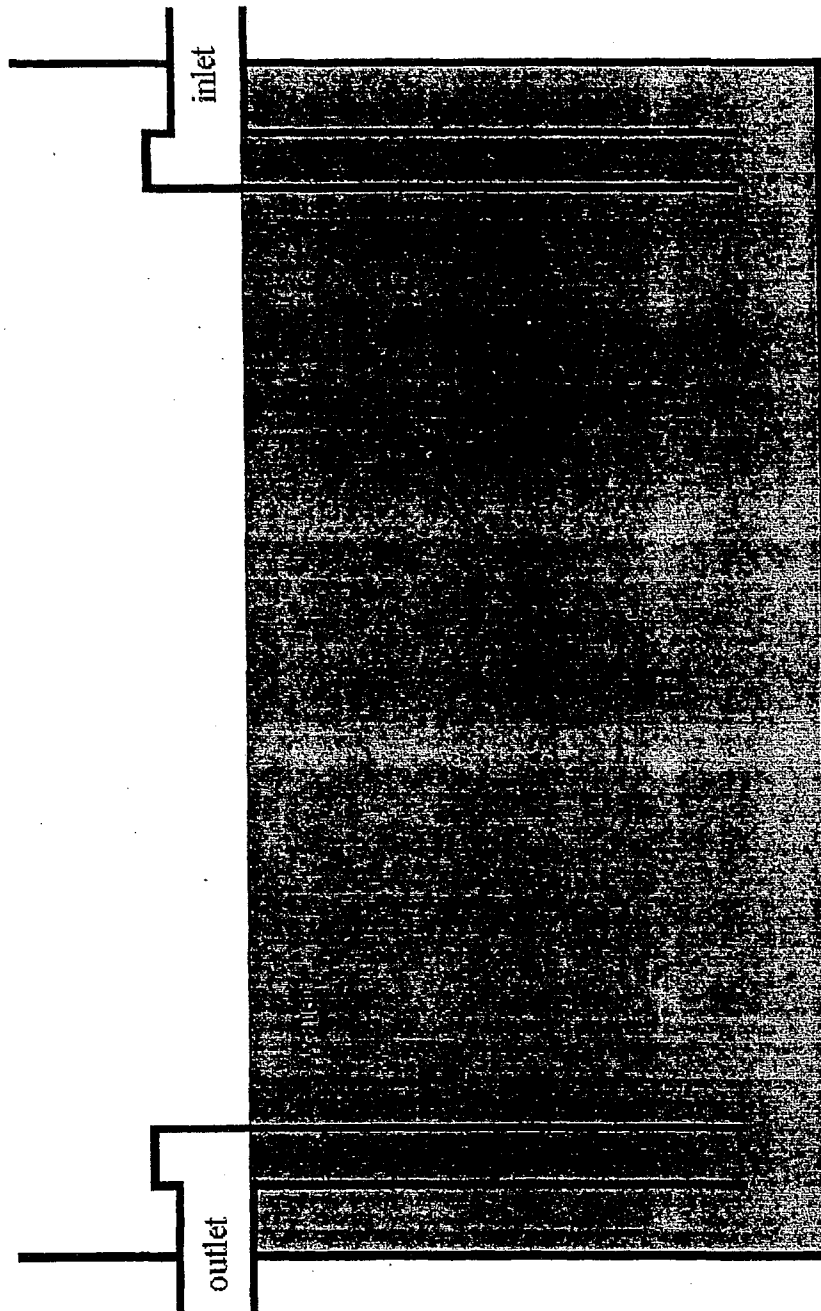
Figure 3.6: Preliminary setup of the oil trap experiment.
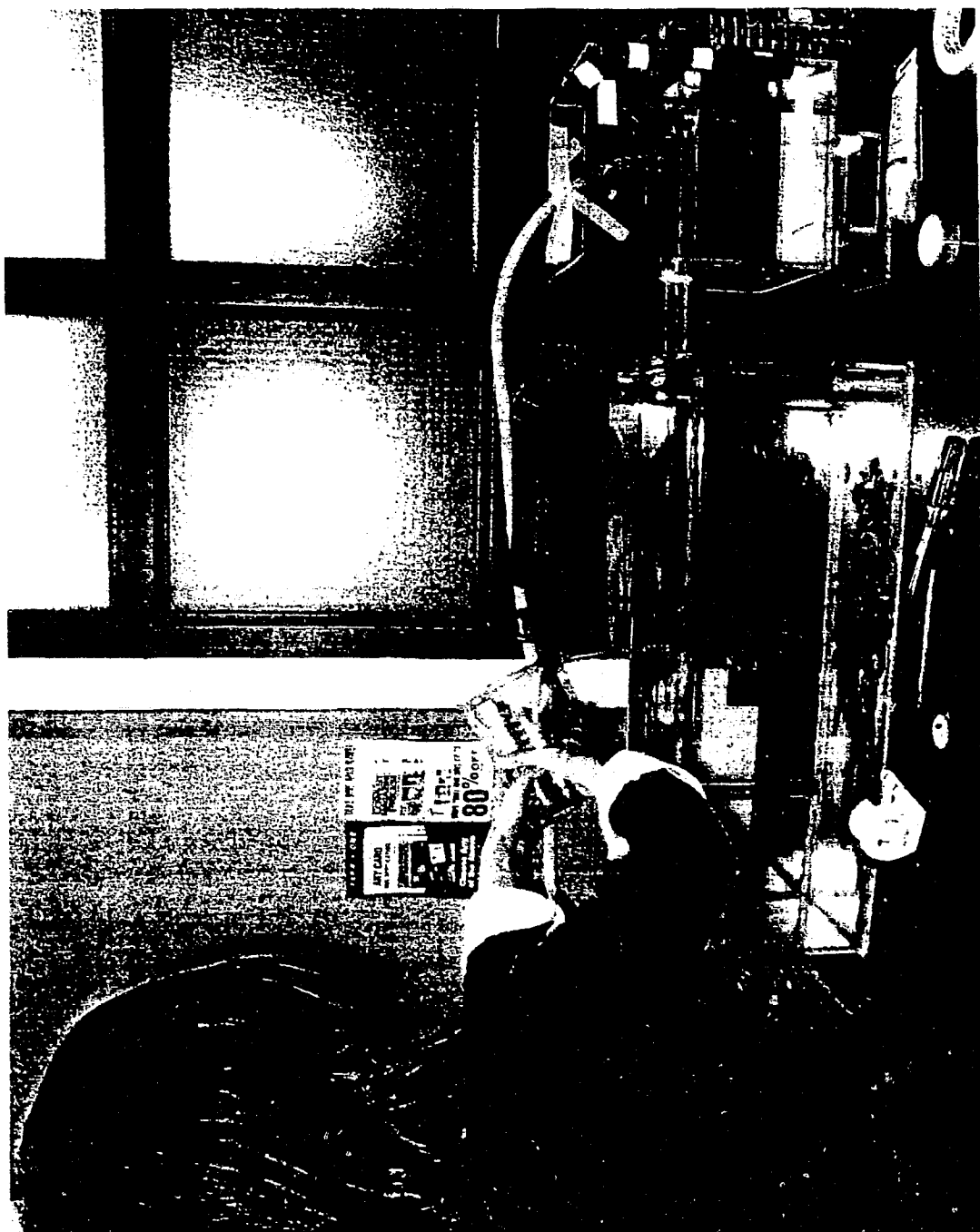
Figure 3.7: Transformer oil is added into the oil trap system experiment.
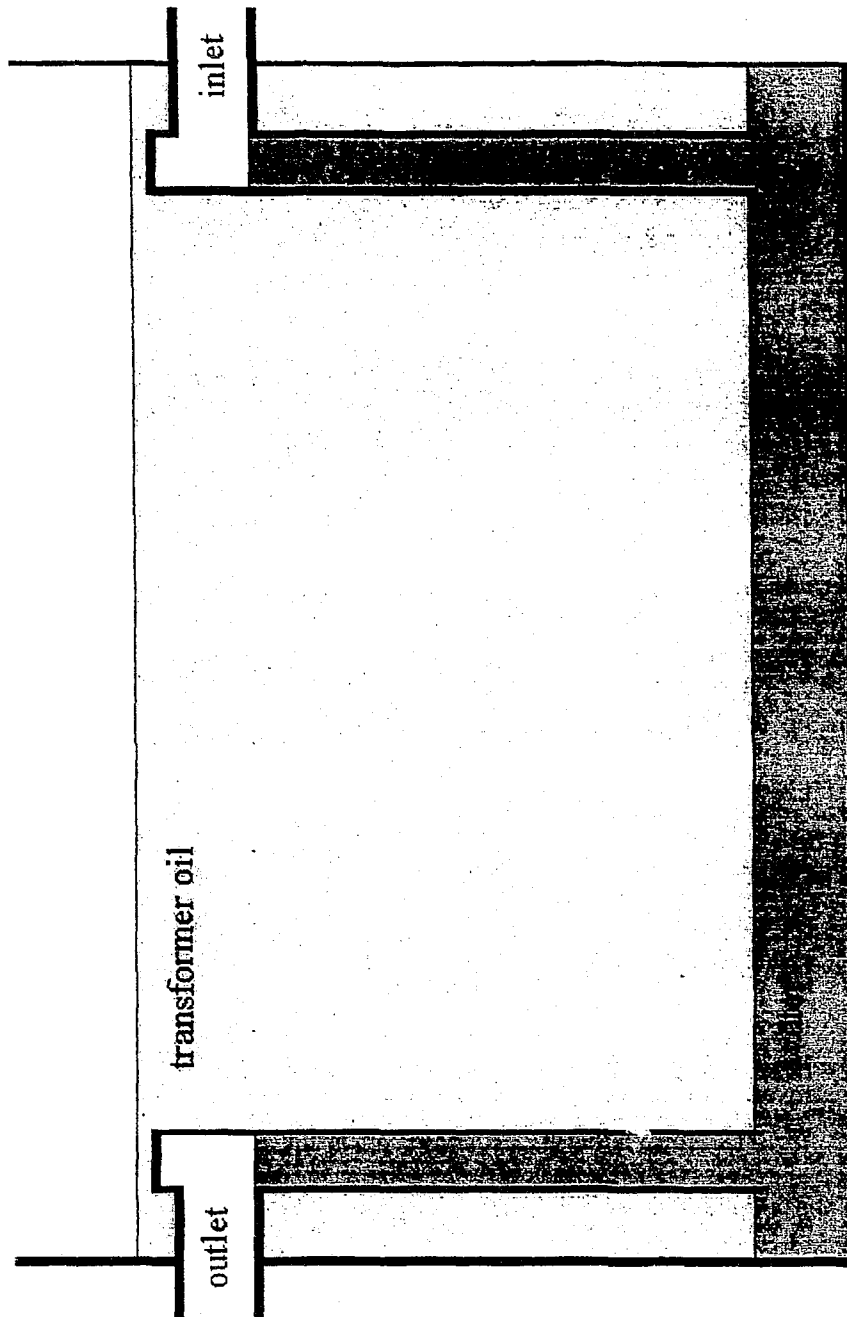
Figure 3.8: Maximum capacity of transformer oil the oil trap system can trap.

The second set of the experiment is to examine the oil-water separation process within the oil trap system. During a major oil spill, a sudden surge of transformer oil enters into the system. This is different from the inflow characteristics of a typical oil-water separator described in API design manual. Therefore, it is important to observe the oil-water separation process in the oil trap system by conducting a series of experiments. Four experiments are designed to examine the oil-water separation process: (1) small amount of transformer oil entering into the system without turn on the pump, (2) large amount of transformer oil entering into the system without turn on the pump, (3) small amount of transformer oil entering into the system with pump turns on and (4) large amount of transformer oil entering into the system with pump turns on.

It is important to recognize that the physical model was designed specifically for the first set of experiments where the main focus is on the instability of the interface between oil and water. Thus, the second set of experiments only provide some insight to the oil/water separation.

Preliminary setup (Figure 3.6) must be done before adding different amounts of transformer oil in the inlet tank. Transformer oil in the second set of the experiment is added into the inlet tank instead of the oil trap system because observation of oil-water separation process will be taken around the vertical inlet pipe area of the system. The first two experiments, without the operation of the pump, demonstrate the spilled transformer oil entering into the oil trap system without rain storms. The last two

experiments show the transformer oil spills entering into the system during rain storms period.

3.5   Observation and Evaluation of the Trapping Oil Experiments

In the first set of experiments, no waves were formed at the interface of oil and water for the two situations even when the oil trap system reached its maximum trapping capacity (Figures 3.9 and 3.10). This indicates that the system will not fail even though the maximum flow rate has reached. The attached CD contains video clips of the experiment for the extreme case, the maximum trapping capacity and the maximum flow situation. The total volume of transformer oil for the maximum trapping capacity of the physical model is *36 L* (Figures 3.9 and 3.10). This is equivalent to *84,580 L* in the prototype since

$$V_{prototype} = (36\ L)(12.3768)(12.2692)(15.4717)$$
$$\approx 84580\ L \qquad [23]$$

where the last three numbers are the dimension ratios between the model and the prototype (Table 3.2).
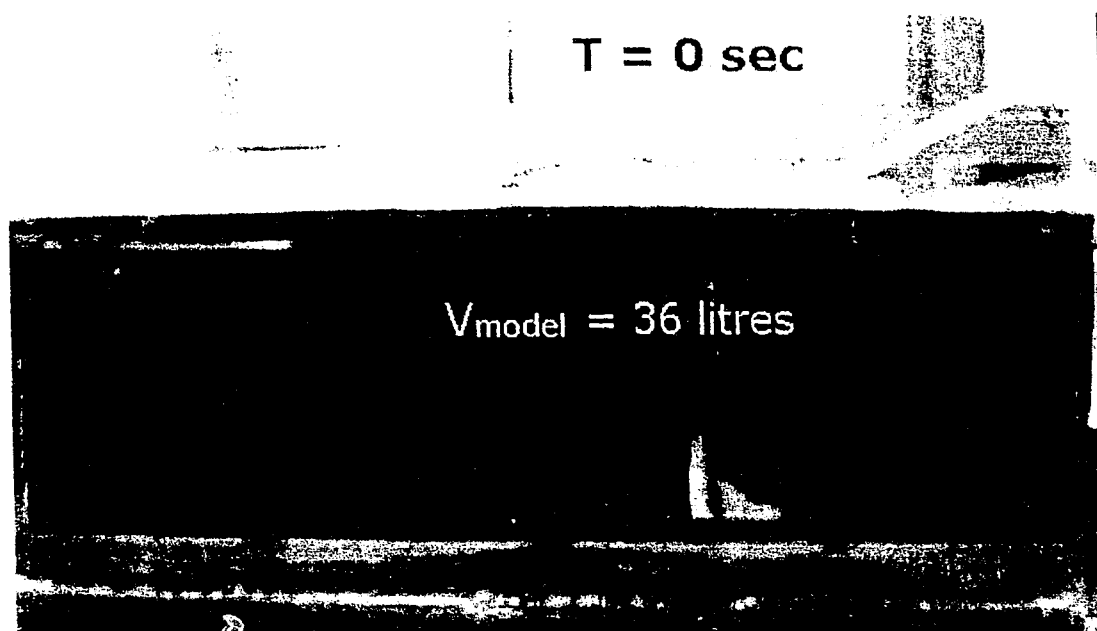
Figure 3.9: Experiment for the maximum flow situation and maximum trapping capacity at time = 0 seconds.
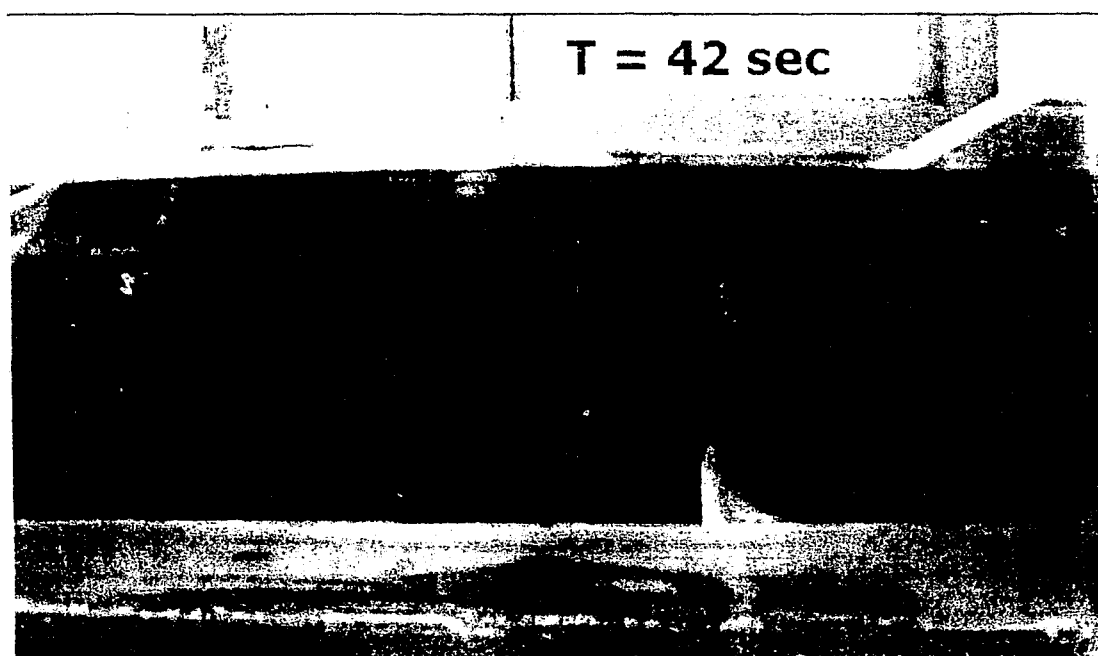
Figure 3.10: Experiment for the maximum flow situation and maximum trapping capacity at time = 42 seconds.

The total volume of transformer oil in *Hydro One's* transformer station near the city of Burlington is *256000 L*. Therefore, three oil trap systems will be needed to capture all the transformer oil. However, this extreme case rarely happens. There are six individual transformers (six concrete pads in Figure 1.3) at *Hydro One's* transformer station near the city of Burlington and is unusual to have all the transformers exploded at the same time.

The following calculation shows the conversion of the maximum flow rate entering into the physical model (Appendix F) to the flow rate in the prototype using the Froude number (Equation [19]) and the length dimensions between the model and the prototype (Table 3.2).

$$\left(\frac{L^5 g \Delta}{Q^2}\right)_{model} = \left(\frac{L^5 g \Delta}{Q^2}\right)_{prototype} \tag{24}$$

$$Q_{prototype} = \sqrt{\frac{(L_{prototype})^5 (Q_{model})^2}{(L_{model})^5}}$$

$$= \sqrt{\frac{(8540\ mm)^5 (0.17983\ L/sec)^2}{(690\ mm)^5}}$$

$$= 96.9135\ L/sec$$

where g and Δ are the same for both model and prototype.

The calculation in Equation [25] is exactly the same as in Equation [24] except width dimensions (Table 3.2) is used.

$$\left(\frac{L^5 g \Delta}{Q^2}\right)_{model} = \left(\frac{L^5 g \Delta}{Q^2}\right)_{prototype} \qquad [25]$$

$$Q_{prototype} = \sqrt{\frac{(L_{prototype})^5 (Q_{model})^2}{(L_{model})^5}}$$

$$= \sqrt{\frac{(3190\,mm)^5 (0.17983\,L/sec)^2}{(260\,mm)^5}}$$

$$= 94.8 \quad L/sec$$

The flow rate of prototype in Equation [24] is used since it is the worst scenario. This flow rate is compared to the runoff rate of a 100-year design storm in the city of Burlington. The calculation of the flow rate for a 100-year design storm using the rational method is shown below. First, the total area of *Hydro One*'s transformer station near the city of Burlington (Figure 1.3) is determined as:

$$A = (base\ area\ of\ concrete\ pad)(number\ of\ concrete\ pad\ at$$
$$\quad\ transformer\ station)$$
$$= [0.5 * (2.2 + 2.28 + 4.45) * 3.4] * 6$$
$$= 91.086\ m^2 \qquad\qquad\qquad\qquad\qquad\qquad [26]$$

Since transformer station was built above a concrete pad in order to avoid the grounding effect, the *"paved area (sheet flow) and small upland gullies"* line in Figure 3.11 was selected. Assuming the slope of the transformer station (the y-axis in Figure 3.11), is 2%, the velocity using the Upland Method, V_{upland} is *2.8 ft/sec* (red circle in Figure 3.11). Secondly, the hydraulic flow length, L_{flow}, of the transformer station is calculated below (pink dotted line in Figure 1.3).

$$L_{flow} = \textit{horizontal traveling distance} + \textit{vertical traveling distance}$$
$$+ \textit{traveling distance at the transformer station} \qquad [27]$$
$$= (14.33 + 20.42) + (11 + 3.24) + [(2.2 + 2.28)^2 + 1.7^2]^{1/2}$$
$$= 53.782 \, m \quad or \quad 176.45 \, ft$$

The time of concentration, t_c , can be determined using the Upland Method (Mays, 2001).

$$t_c = L_{flow} / (3600 * V_{upland}) \qquad [28]$$
$$= 176.45 / (3600 * 2.8)$$
$$= 0.01750 \, hr \quad or \quad 1.050 \, min$$

The rainfall intensity can be calculated from i_{100} in Figure 3.12 where

$$i_{100} = 1114.20 / (t_d + 5)^{-0.761} \qquad [29]$$
$$= 1114.20 / (1.011 + 5)^{-0.761}$$
$$= 284.57 \, mm/hr \quad or \quad 7.904722 \times 10^{-5} \, m/sec$$

where t_d is equivalent to t_c.

By applying the Rational Method (Lewis and Viessman, 1996),

$$Q = C \, i \, A \qquad [30]$$
$$= (1.0)(7.904722 \times 10^{-5} \, m/sec)(91.086 \, m^2)$$
$$= 7.2 \times 10^{-3} \, m^3/sec \quad or \quad 7.2 \, L/sec$$

where C is the runoff coefficient in dimensionless form. i is equivalent to i_{100} (Equation [29]). A is the total area of the transformer station (Equation [26]).
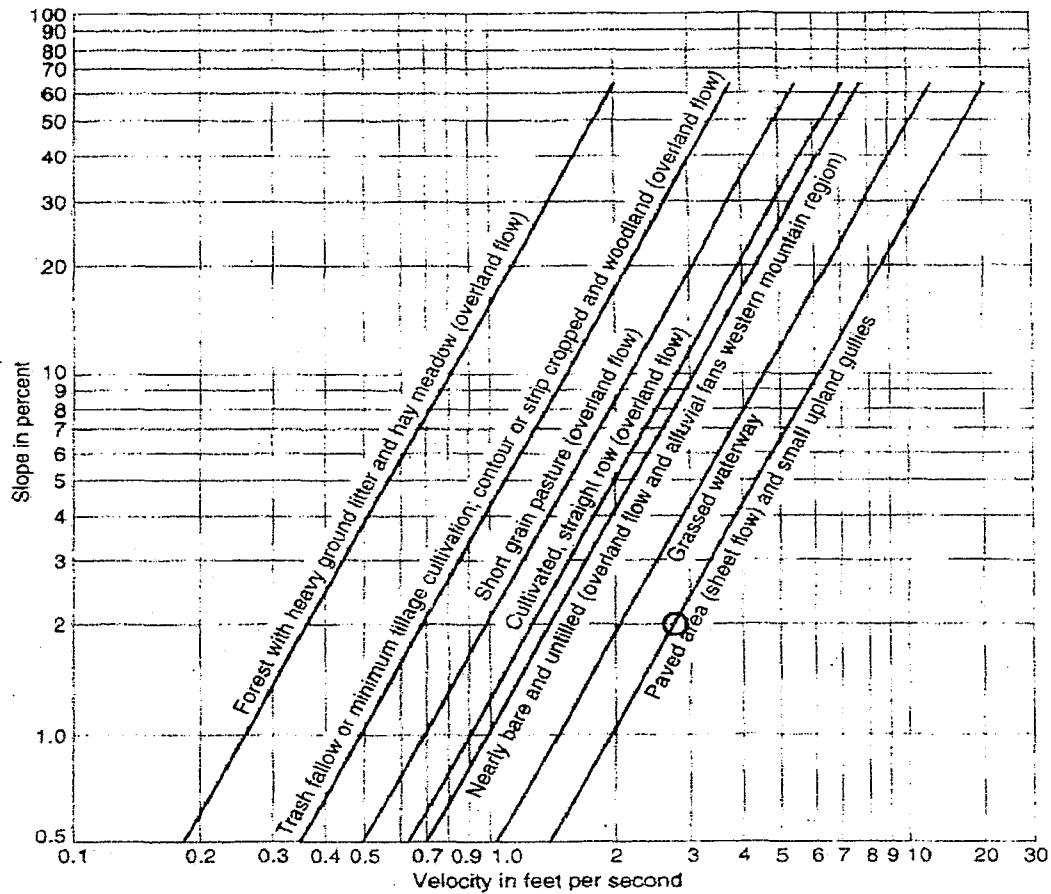
Figure 3.11: Velocities for velocity upland method of estimating t_c (U.S. Department of Agriculture Soil Conservation Service, 1986).
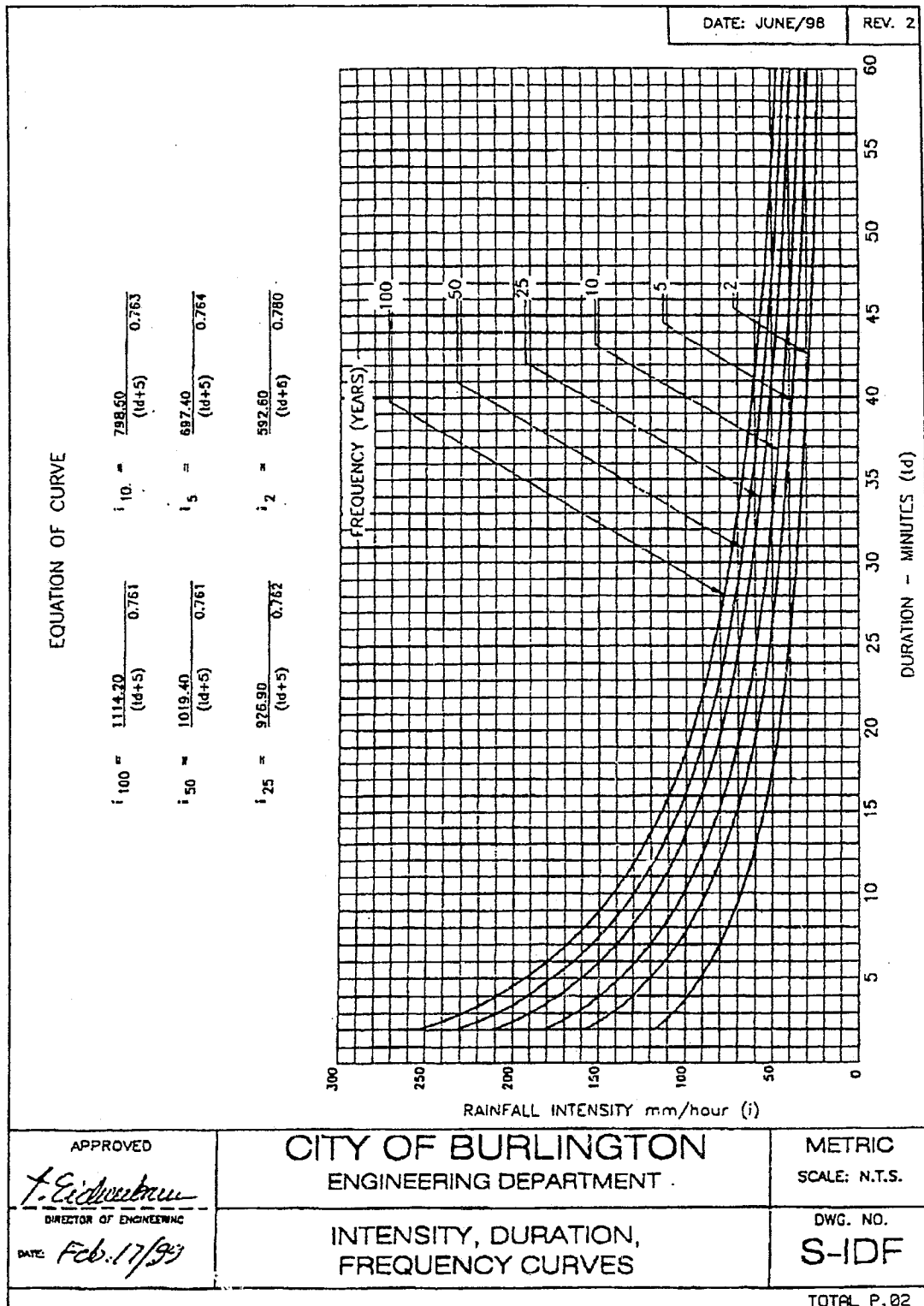
Figure 3.12: IDF curve for the city of Burlington.

The maximum prototype flow (Equation [25]) are much larger than the flow rate for a 100-year design storm (Equation [30]). Since there is no interfacial waves observed in the experiment, the oil trap system should trap the spilled transformer oil successfully for a typical 100-year design storm. However, there are two uncertainties in this experiment. First, the oil layer was pushed up by the water flow rate entering into the system during the experiments of the maximum flow situation (Figure 3.10). This is caused by: (1) the hydraulic grade line difference between the inlet tank and the oil trap system and (2) the design of the physical model as an open system, with opening on the top surface of the oil trap system, instead of a pressurized system. The chance of forming waves at the interface may be increased if a fixed upper boundary is set up for the oil layer and the oil layer moves upward. Secondly, this experiment has not applied the whole theory of dimensionless analysis. As mentioned in Section 3.2, the Π-terms in this experiment have not be verified yet. Other Π-terms may also dominate this experiment.

3.6 Observation and Evalution of Oil-Water Separation Experiments

The following discussion. are based on the four experiments of the oil-water separation. The video clips of the four experiments can be found in the attached CD.

- **First experiment**

When small amount (*500 mL*) of transformer oil entered into the system slowly without pump operation (Figure 3.13), small oil globules were formed and rose up along the vertical inlet pipe region due to the density difference between oil and water. The diameters of the oil globules are around *5* to *8 mm* by visual observation and the density of the transformer oil equals to *0.86 kg/L* (Section 4.3). Oil globules traveled upward around the vertical inlet pipe region because there was no water flow which might force the oil globules to travel horizontally along the oil trap system. Thus, transformer oil did not escape from the system in this experiment.
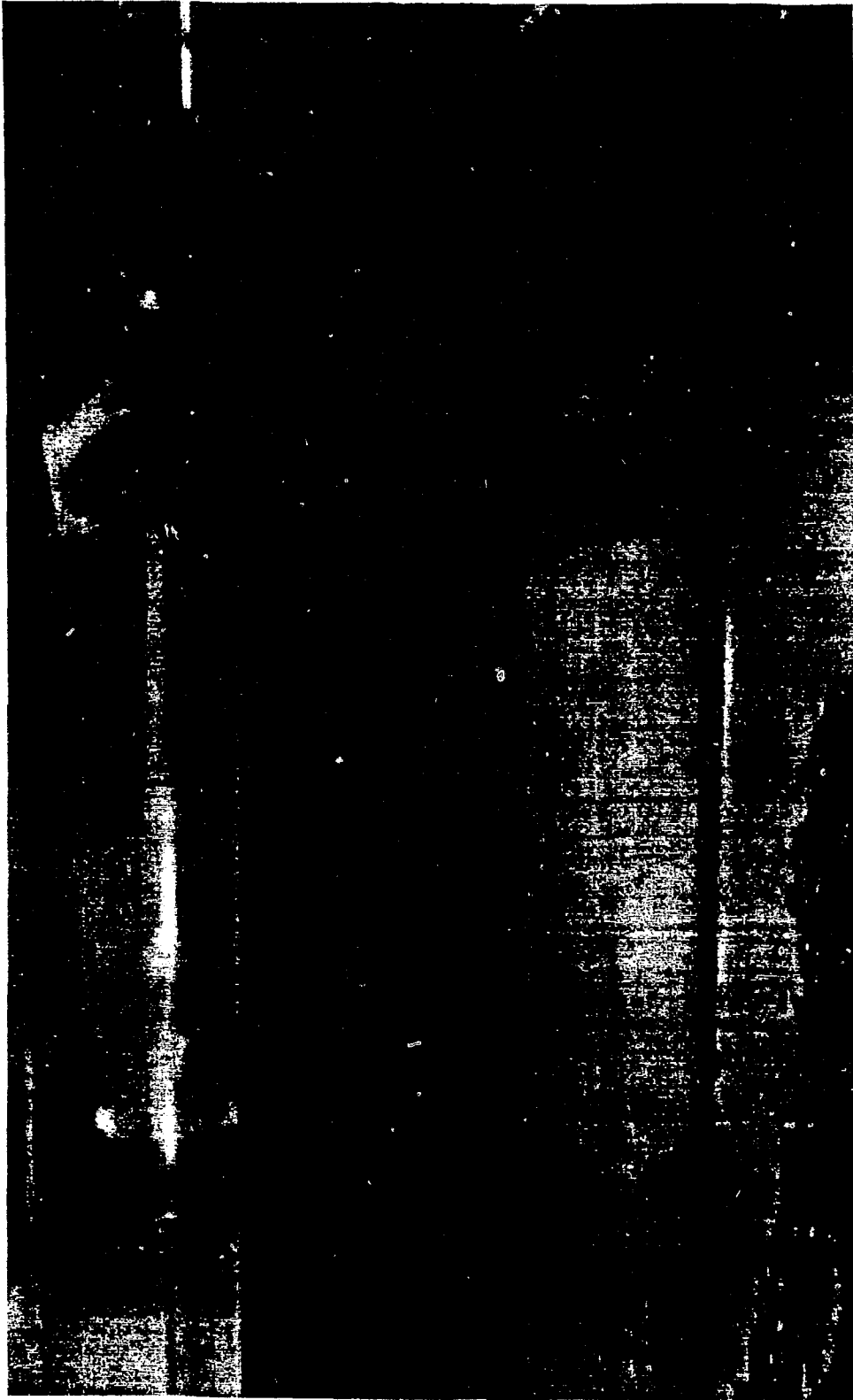
Figure 3.13: Small amount of transformer oil entered into the system without pump operation.

- **Second experiment**

When large amount (*15 L*) of transformer oil entered into the system rapidly without pump operation (Figures 3.14 and 3.15), the experimental results are exactly the same as the first experiment except the oil globules were much larger in size. Most of the oil globules were in irregular elliptical shape and traveled closely along the surface of the vertical inlet pipe. The average maximum diameters of these irregular elliptical shapes are around *15 mm*. The formation of large oil globules in this experiment is due to large amount of transformer oil entering into the oil trap system through the vertical inlet pipe and the coalescence of small oil globules. Since the oil globules are larger in size in this experiment, they rose faster and traveled closer to the vertical inlet pipe. Therefore, the transformer oil in this experiment did not escape from the system.
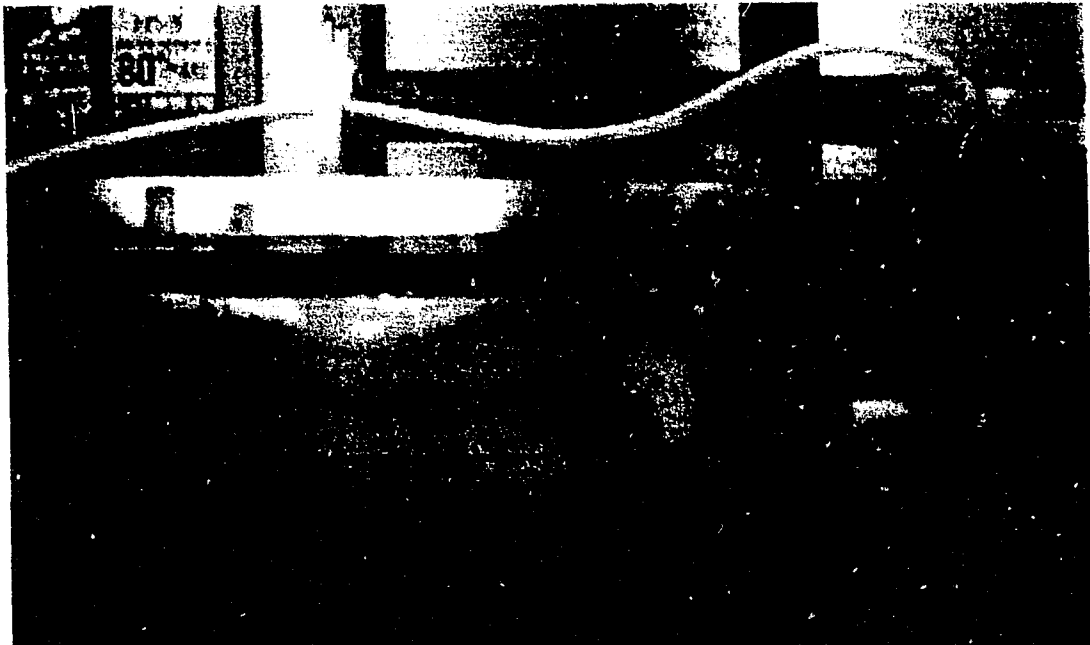
Figure 3.14: Large amount of transformer oil entered into the system without pump operation.
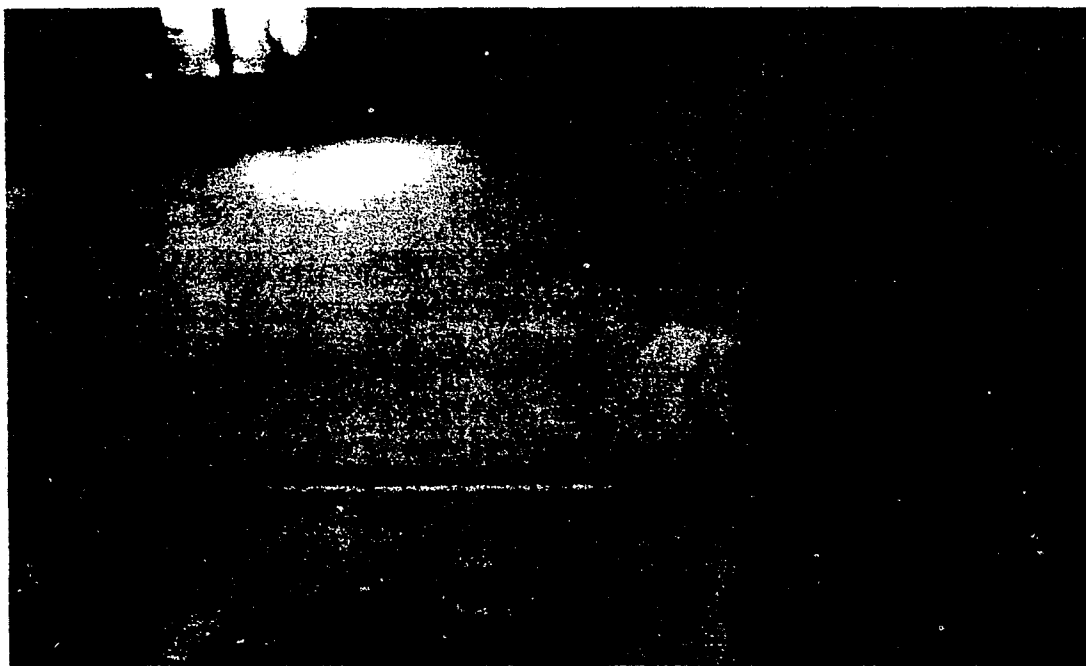
Figure 3.15: A closeup for the large amount of transformer oil entered into the system without pump operation.

- **Third experiment**

When small amount (*500 mL*) of transformer oil entered into the system slowly with pump operation (Figures 3.16 and 3.17), oil globules were much smaller in size compared to those in the first two experiments. Most of the oil globules were around 1 mm in diameter by visual observation. The oil globules spread out to about one-third of the oil trap system length before they rose up. Thus, the transformer oil still did not escape from the system.

- **Fourth experiment**

When large amount (*15 L*) of transformer oil entered into the system rapidly with pump operation, the shapes of the oil globules were similar to the second experiment. Their sizes were around 10 mm and were smaller than those in the second experiment. The oil globules traveled along the outside of the inlet pipe instead of the surface of the inlet pipe in the second experimental results. This observation may be caused by the water inflow to the oil trap system which forced the oil globules to move further away from the inlet pipe. Since the oil globules were larger sizes in this experiment, they could not travel as far as the oil globules in the third experiment. Thus, the transformer oil still did not escape from the system.
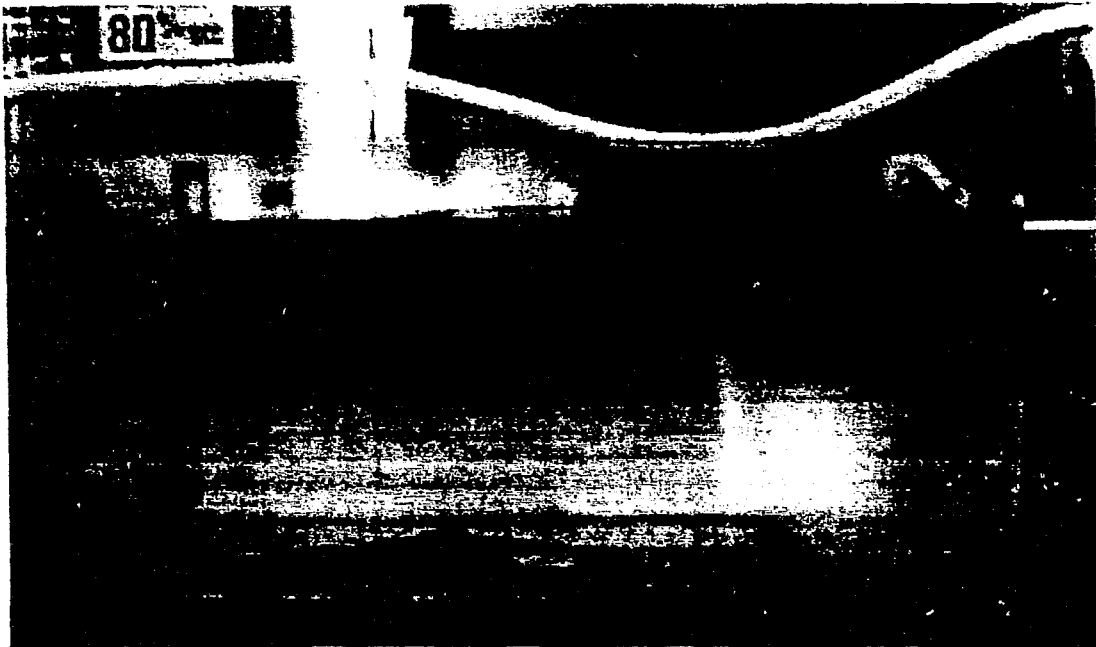
Figure 3.16 : Small amount of transformer oil entered into the system with pump operation.

Figure 3.17: A closeup for the small amount of transformer oil entered into the system with pump operation.
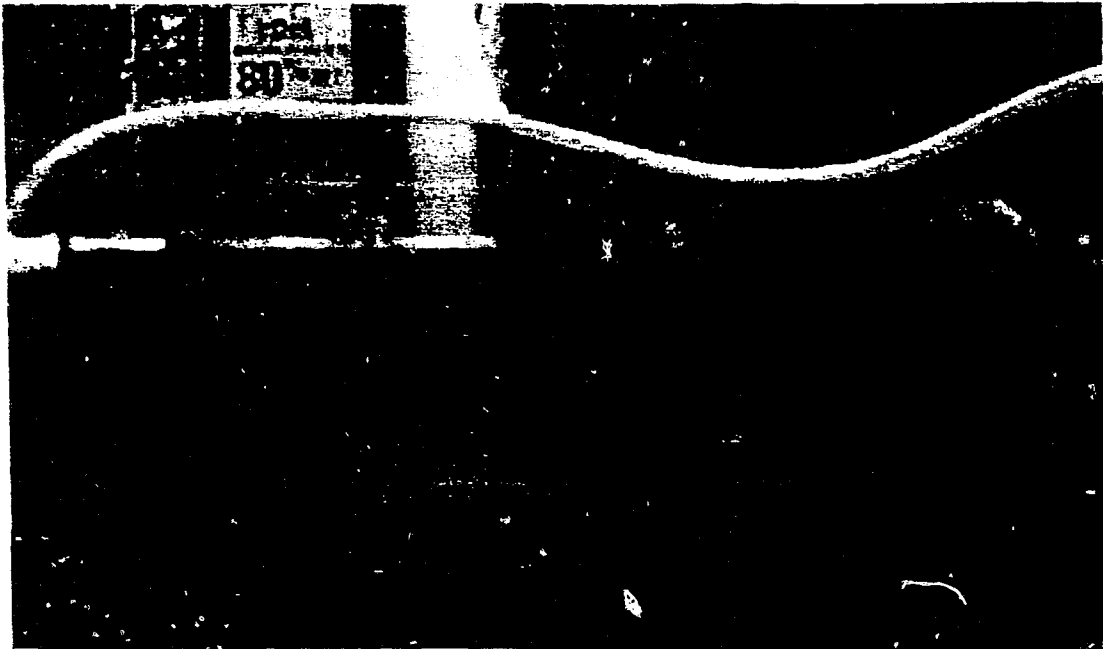
Figure 3.18 : Large amount of transformer oil Entered into the system with pump operation.

Figure 3.19: Close shot for the large amount of transformer oil entered into the system with pump operation.

Page 67

Chapter 4 NUMERICAL EXPERIMENTS ON THE OIL BACK-UP SYSTEM

An oil back-up system could be installed underneath each concrete pad of *Hydro One's* transformer station in Burlington (Figure 1.3). The first section of this chapter reviews the mass balance concept of oil and water used in the development of a mathematical model using the MATLAB 6.1 program. The subsequent sections present the development of the mathematical model, the determination of the input information, the MATLAB programs and the numerical analysis and results. The purpose of the mathematical model is to determine the heights of the oil and water layers over certain period of time (Figures 4.2 and 4.3). The parameters and variables involved in the oil back-up system model are shown in Tables 4.1 and 4.2. Figures 4.1 and 4.3 demonstrate the definition of the parameters and variables.

4.1 The Mass Balance Concepts of Transformer Oil and Water for the Oil Back-up System

The mass balance concept is applied in two parts, the catchbasin and the oil back-up system, for analyzing the hydraulic conditions during a transformer oil spill.

Table 4.1: Parameters involved in the oil back-up system.

Parameters	Corresponding Parameters Used in MATLAB	Definition of Parameters
A_{hp1}	area_hp1	cross-sectional area of horizontal pipe $hp1$ between the catchbasin and the oil back-up system in m^2
D_{hp1}	diameter_hp1	diameter of horizontal pipe $hp1$ between the catchbasin and the oil back-up system in metre
A_{hp2}	area_hp2	cross-sectional area of horizontal pipe $hp2$ inside the oil back-up system in m^2
D_{hp2}	diameter_hp2	diameter of horizontal pipe $hp2$ inside the oil back-up system in metre
L_{hp2}	length_hp2	length of horizontal pipe $hp2$ inside the oil back-up system in metre
A_{int}	area_internal_pipe	cross-sectional area of internal pipe column in m^2
D_{int}	diameter_internal_pipe	diameter of internal pipe column in metre
A_{ext}	area_external_pipe	cross-sectional area of external pipe column in m^2
D_{ext}	diameter_external_pipe	diameter of external pipe column in metre
$A_{catchbasin}$	area_catchbasin	base area of the catchbasin in m^2
A_{system}	area_system	base area of the oil back-up system in m^2
h_{column}	h_column	height between datum and the bottom of the oil back-up system in metre
h	h	height between the bottom of horizontal pipe inside the oil back-up system and the bottom of the oil back-up system in metre

Page 69

Table 4.1: Parameters involved in the oil back-up system *(cont.)*.

Parameters	Corresponding Parameters Used in MATLAB	Definition of Parameters
g	gravity	gravitational acceleration, *9.806 m/sec²*
c	c	roughness coefficient of the construction material used for the oil back-up system, $c_{concrete} = 0.6$
Δt	delta_time	time step used for determination of the variables in the mathematical model in seconds
ρ_{oil}	density_oil	density of transformer oil in kg/m^3
ρ_{water}	density_water	density of water in kg/m^3
Q_{in}	inflow	inflow rate of transformer oil to the oil back-up system from the concrete pad of the transformer station in m^3/sec
A_{pad}	area_concrete_pad	area of concrete area in m^2

Table 4.2: Variables involved in the oil back-up system.*

Variables	Definition of Variables
$H1[n]$	height between the datum and the surface of fluid at the catchbasin at the beginning of a time step
$H1[n+1]$	height between the datum and the surface of fluid at the catchbasin at the end of a time step
$H2[n]$	height of the oil layer in the oil back-up system at the beginning of a time step
$H2[n+1]$	height of the oil layer in the oil back-up system at the end of a time step
$H3[n]$	height of water layer between the datum and the interface of oil and water in the oil back-up system at the beginning of a time step
$H3[n+1]$	height of water layer between the datum and the interface of oil and water in the oil back-up system at the end of a time step
$H4[n]$	height of water in the internal pipe column at the beginning of a time step
$H4[n+1]$	height of water in the internal pipe column at the end of a time step

* The units of all the variables in Tables 4.2 are in metres.

:
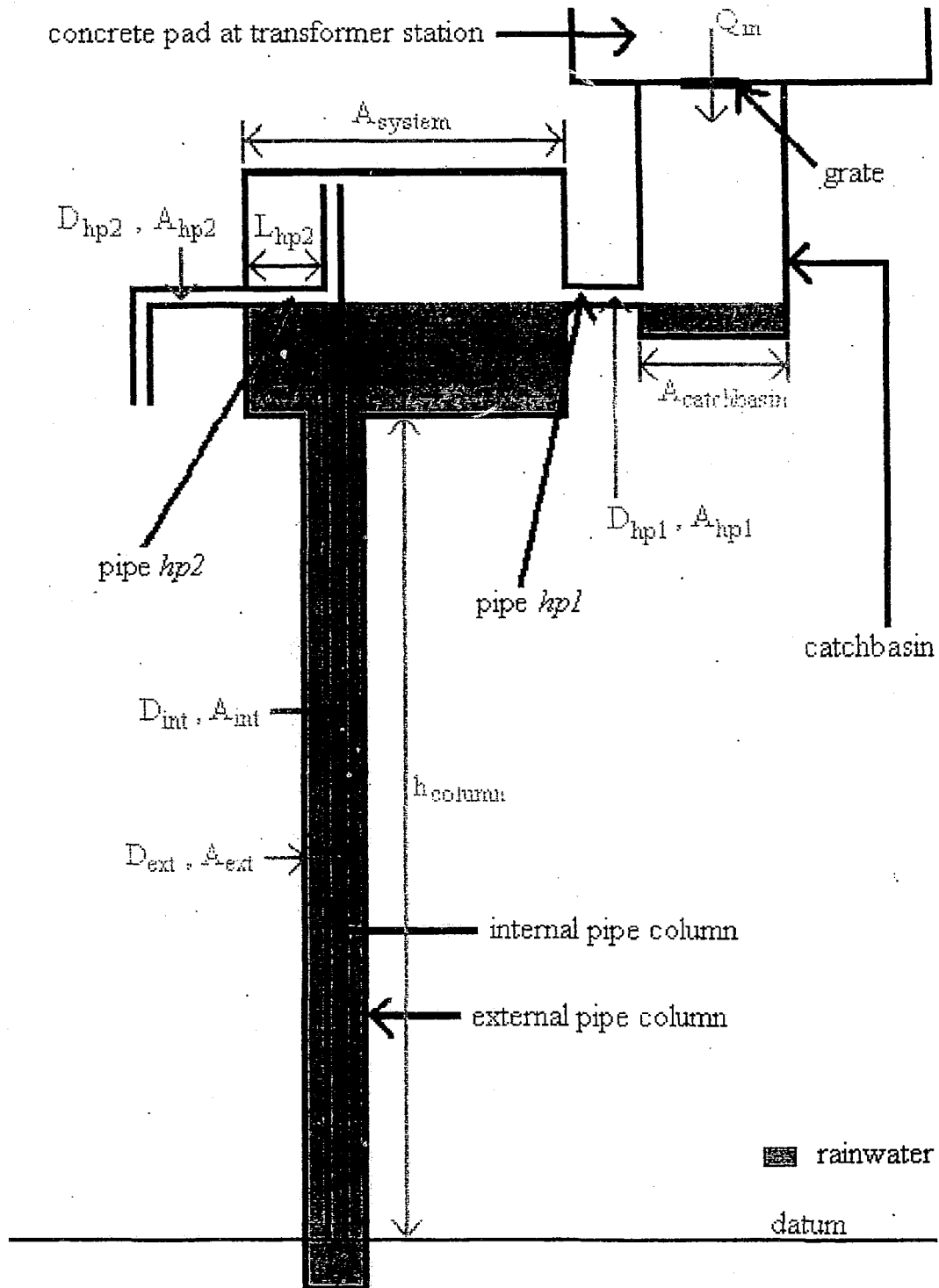
Page 71

Figure 4.1: Definition of parameters in the oil back-up system.

Figure 4.2: Case 1 *($h_{column} \leq H3[n]$)* of the oil back-up system.
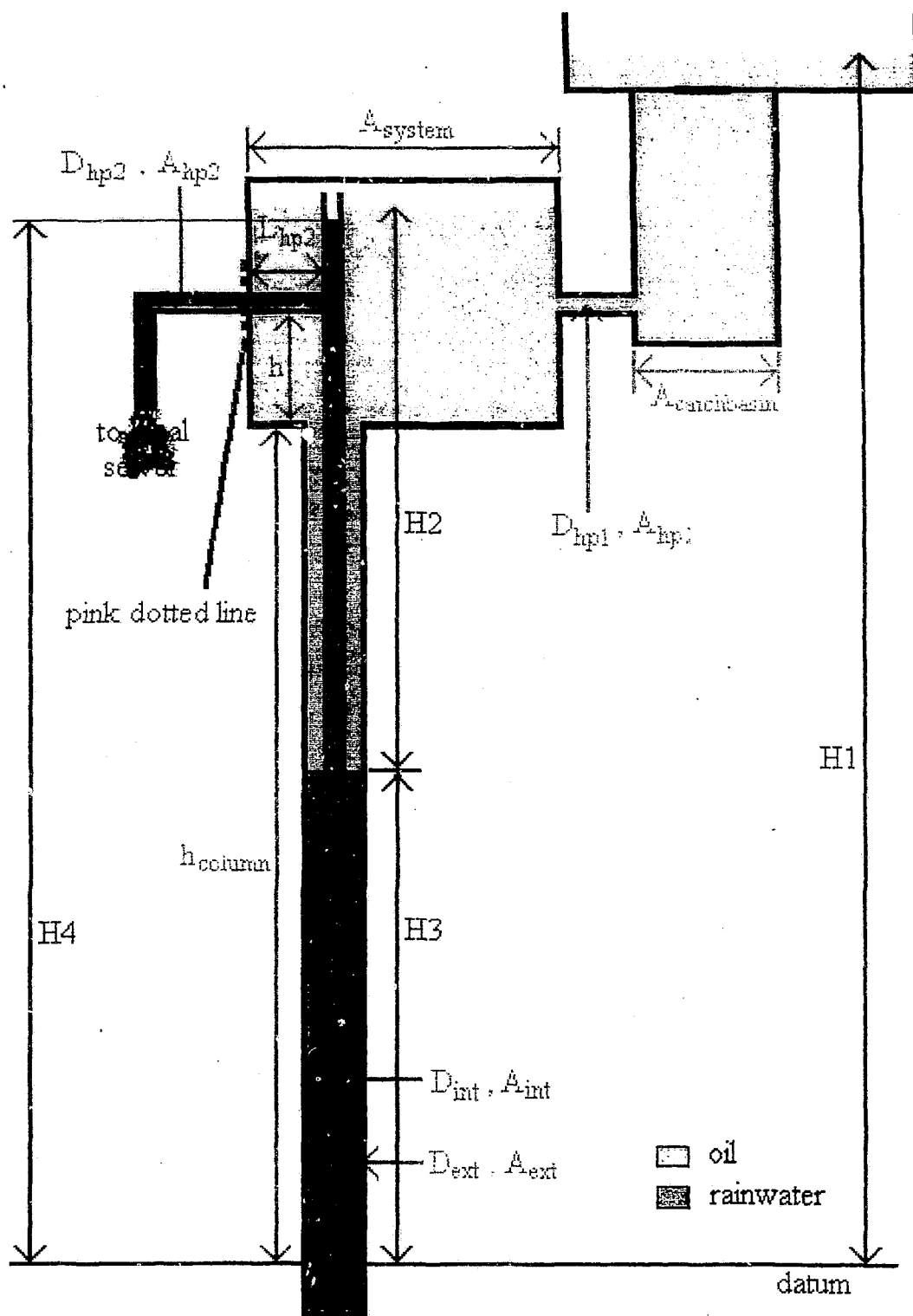
Figure 4.3: Case 2 $(h_{column} > H3[n])$ of the Oil Back-up System.

The initial condition of the oil back-up system is shown in Figure 4.1, where the system contains only rainwater. Also, horizontal pipes *hp1* and *hp2* are at the same elevation. Generally, the spilled transformer oil will first enter into the catchbasin from the concrete pad (variable Q_{in} in Figure 4.1). Then, it will enter the oil back-up system through the horizontal pipe *hp1*. The purpose of the oil back-up system is to prevent the spilled transformer oil from reaching the datum (Figures 4.2 and 4.3). The datum is defined as the horizontal elevation at the bottom of the internal pipe column. Once the transformer oil reaches the datum, the oil back-up system will fail and the spilled transformer oil will enter into the local sewer system. Therefore, it is important to find out the changes of heights of oil and water layers at different time intervals in order to determine whether the design dimensions of the oil back-up system (Figure 4.4) can prevent the failure of the system.

The general conservation of mass flow balance equation is shown below.

$$\Delta Q_{in} * \Delta t = \Delta Q_{out} * \Delta t + \Delta S$$
$$\Delta S = [\Delta Q_{in} - \Delta Q_{out}] \Delta t \qquad [31]$$

where ΔQ_{in} is the average inflow entering into the system at one time step in m^3/s; ΔQ_{out} is the average outflow exiting from the system at one time step in m^3/s; ΔS is the changes of storage in the system at one time step in m^3; and Δt is the time step in *sec*.

The general mass balance equation (Equation [31]) discussed above is applied to: (1) the transformer oil at catchbasin, (2) the transformer oil at the oil back-up system and (3) the water at the oil back-up system.

Page 75

oil back-up system = 1.3 m X 1.3 m   concrete pad = 15.181 m^2

1.35 m

D_{hp2} = 100 mm

to local sewer

D_{hp1} = 250 mm

catchbasin = 0.61 m X 0.61 m

D_{int} = 100 mm

D_{ext} = 300 mm

3.225 m

rainwater

datum
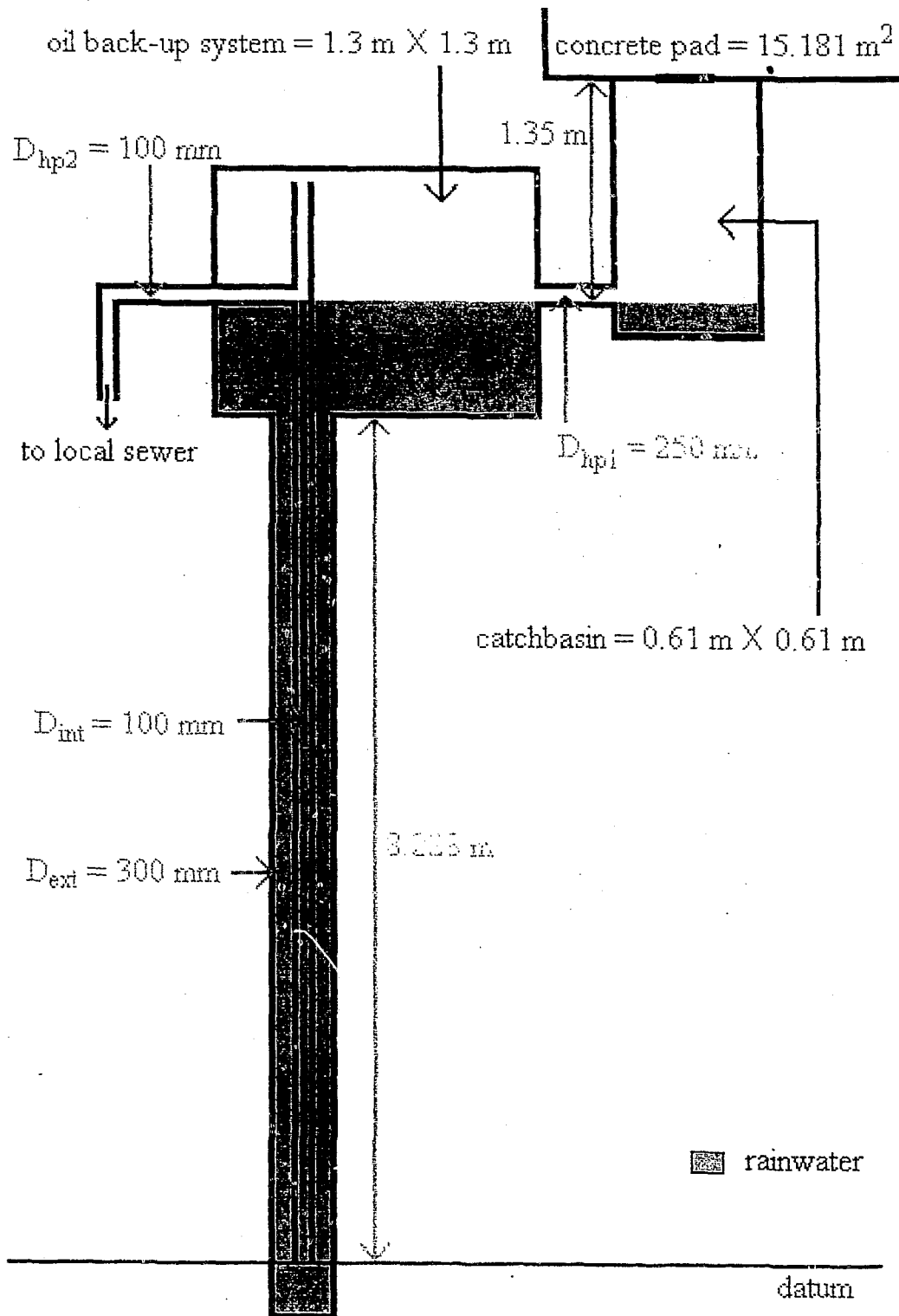
Figure 4.4: Design dimensions of the oil back-up system.

4.1.1 Mass Balance of Transformer Oil at the Catchbasin

For the mass balance of transformer oil at catchbasin, the change of storage of transformer oil, ΔS, at catchbasin is:

$$\Delta S = A_{catchbasin} \left(Hl[n+1] - Hl[n] \right) \qquad [32]$$

where $A_{catchbasin}$ is the base area of the catchbasin in m^2 (Table 4.1); $Hl[n+1]$ is height between the datum and the surface of fluid at the catchbasin at the end of a time step in *metres* (Table 4.2); and $Hl[n]$ is the height between the datum and the surface of fluid at the catchbasin at the beginning of a time step in *metres* (Table 4.2).

The average inflow of transformer oil enters into the catchbasin from the concrete pad of the transformer station is calculated as:

$$\Delta Q_{in} = 1/2 \left(Q_{in}[n+1] + Q_{in}[n] \right) \qquad [33]$$

where $Q_{in}[n]$ is the inflow rate entering into the catchbasin at the beginning of a time step in m^3/sec. $Q_{in}[n+1]$ is the inflow rate entering into the catchbasin at the end of a time step in m^3/sec.

The outflow rate is determined using the orifice equation.

$$Q = c A \sqrt{2 \rho g H} \qquad [34]$$

Page 77

where Q is the flow coming out from the pipe in m^3/sec; c is the orifice coefficient of the pipe in dimensionless form; A is the cross-sectional area of the pipe in m^2; ρ is the density of fluid in kg/m^3; g is the gravitational acceleration, which is 9.81 m/sec^2; and H is the head or the oil surface elevation above the centre of the pipe in *metres*.

Thus, the average outflow rate of transformer oil from the catchbasin to the oil back-up system through the horizontal pipe *hp1* is:

$$\Delta Q_{out} = \frac{1}{2} \left(Q_{out}[n+1] + Q_{out}[n] \right) \qquad [35]$$

$$= \frac{1}{2} \left\{ c\, A_{hp1} \sqrt{2\, \rho_{oil}\, g\, (H1[n+1] - H2[n+1] - H3[n+1])} \right.$$
$$\left. + c\, A_{hp1} \sqrt{2\, \rho_{oil}\, g\, (H1[n] - H2[n] - H3[n])} \right\}$$

$$= \frac{c\, A_{hp1} \sqrt{2\, \rho_{oil}\, g}}{2} \left\{ \sqrt{H1[n+1] - H2[n+1] - H3[n+1]} \right.$$
$$\left. + \sqrt{H1[n] - H2[n] - H3[n]} \right\}$$

where $Q_{out}[n+1]$ is the outflow rate exiting the catchbasin at the end of a time step in m^3/s; and $Q_{out}[n]$ is the outflow rate exiting the catchbasin at the beginning of a time step in m^3/s. Both $Q_{out}[n+1]$ and $Q_{out}[n]$ are derived from Equation [34]. The terms, $(H1[n] - H2[n] - H3[n])$ and $(H1[n+1] - H2[n+1] - H3[n+1])$, represent the transformer oil surface elevations in the catchbasin at the beginning and the end of a time step in *metres* respectively.

Therefore, the conservation of mass flow balance equation for the transformer oil at the catchbasin is defined as:

$$\Delta S = [\Delta Q_{in} - \Delta Q_{out}]\Delta t$$
$$A_{catchbasin}(H1[n+1] - H1[n]) = \Delta t \left\{ \frac{1}{2}(Q_{in}[n+1] + Q_{in}[n]) \right.$$
$$- \frac{c\, A_{hp1}\sqrt{2\,\rho_{oil}\, g}}{2}(\sqrt{H1[n+1] - H2[n+1] - H3[n+1]}$$
$$\left. + \sqrt{H1[n] - H2[n] - H3[n]}) \right\}$$

[36]

4.1.2 Mass Balance of Transformer Oil at Oil Back-up System

The average outflow rate, ΔQ_{out}, in this scenario is equal to zero since transformer oil should not escape from the oil back-up system. The average inflow rate, ΔQ_{in}, equals to the average outflow rate of transformer oil from the catchbasin (Equation [35]) because the catchbasin and the oil back-up system are connected by the horizontal pipe *hp1*. There are two cases for calculating the change of storage at the oil back-up system: $h_{column} \le H3[n]$ and $h_{column} > H3[n]$. Figures 4.2 and 4.3 illustrate the details of these two cases. The change of the storage for the Case 1, $\Delta S_{(1)}$, is given by:

$$\Delta S_{(1)} = (A_{system} - A_{int})(H2[n+1] - H2[n])$$

[37]

where A_{system} is the base area of the oil back-up system in m^2; A_{int} is the cross-sectional area of internal pipe column in m^2; $H2[n+1]$ is the height of the oil layer in the oil back-up system at the end of a time step in *metres*; $H2[n]$ is the height of the oil layer in the oil back-up system at the beginning of a time step in *metres* (Tables 4.1 and 4.2).

The change of storage in Case 2, $\Delta S_{(2)}$, is given by:

$$\Delta S_{(2)} = \{ ((A_{system} - A_{int})(H2[n+1] + H3[n+1] - h_{column})$$
$$- (A_{hp2})(L_{hp2})) + (A_{ext} - A_{int})(h_{column} - H3[n+1])\}$$
$$- \{ ((A_{system} - A_{int})(H2[n] + H3[n] - h_{column})$$
$$- (A_{hp2})(L_{hp2})) + (A_{ext} - A_{int})(h_{column} - H3[n])\} \qquad [38]$$

where h_{column} is the height between datum and the bottom of the oil back-up system in metres; A_{hp2} is the cross-sectional area of horizontal pipe $hp2$ inside the oil back-up system in m^2; L_{hp2} is the length of horizontal pipe $hp2$ inside the oil back-up system in metres; A_{ext} is the cross-sectional area of external pipe column in m^2; $H3[n]$ is the height of water layer between the datum and the interface of oil and water in the oil back-up system at the beginning of a time step in metres; and $H3[n+1]$ is the height of water layer between the datum and the interface of oil and water in the oil back-up system at the end of a time step in metres (Tables 4.1 and 4.2).

The conservation of mass flow balance equations for the transformer oil at the oil back-up system for the two cases are given by

$$\Delta S_{(1)} = [\Delta Q_{in} - \Delta Q_{out}]\Delta t \qquad [39]$$
$$\Delta S_{(1)} = (\Delta Q_{in})\Delta t$$
$$(A_{system} - A_{int})(H2[n+1] - H2[n]) =$$
$$\Delta t \frac{c\, A_{hp1}\sqrt{2\,\rho_{oil}\,g}}{2}\, (\sqrt{H1[n+1] - H2[n+1] - H3[n+1]}$$
$$+ \sqrt{H1[n] - H2[n] - H3[n]}\,)$$

Page 80

$$\Delta S_{(2)} = [\Delta Q_{in} - \Delta Q_{out}] \Delta t \tag{40}$$

$$\Delta S_{(2)} = (\Delta Q_{in}) \Delta t$$

$$\{ ((A_{system} - A_{int})(H2[n+1] + H3[n+1] - h_{column})$$
$$- (A_{hp2})(L_{hp2})) + (A_{ext} - A_{int})(h_{column} - H3[n+1]) \}$$
$$- \{ ((A_{system} - A_{int})(H2[n] + H3[n] - h_{column})$$
$$- (A_{hp2})(L_{hp2})) + (A_{ext} - A_{int})(h_{column} - H3[n]) \} =$$

$$\Delta t \frac{c \, A_{hp1} \sqrt{2 \, \rho_{oil} \, g}}{2} \left(\sqrt{H1[n+1] - H2[n+1] - H3[n+1]} \right.$$
$$\left. + \sqrt{H1[n] - H2[n] - H3[n]} \right)$$

4.1.3 Mass Balance of Water at Oil Back-up System

The mathematical model discussed in this chapter is designed for transformer oil spills without the occurrence of rain storms, where Q_{in} is defined as the inflow rate of transformer oil to the oil back-up system (Table 4.1). Therefore, the average inflow rate, ΔQ_{in}, for the mass balance of water at the oil back-up system is equal to zero.

The average outflow rate of water exiting the oil back-up system is measured at horizontal pipe *hp2* (pink dotted line in Figure 4.3). Orifice equation (Equation [34]) is used to determine the average outflow rate such that

$$\Delta Q_{out} = \frac{1}{2} (Q_{out}[n+1] + Q_{out}[n]) \tag{41}$$

$$= \frac{1}{2} \left\{ c \, A_{hp2} \sqrt{2 \, \rho_{water} \, g \, (H4[n+1] - h_{column} - h)} \right.$$
$$\left. + c \, A_{hp2} \sqrt{2 \, \rho_{water} \, g \, (H4[n] - h_{column} - h)} \right\}$$

$$= \frac{c \, A_{hp2} \sqrt{2 \, \rho_{water} \, g}}{2} \left\{ \sqrt{H4[n+1] - h_{column} - h} \right.$$
$$\left. + \sqrt{H4[n] - h_{column} - h} \right\}$$

There are two different equations for the change of storage in this scenario. The first equation (Equation [42]) can be applied for Case 1 in Section 4.1.2.

$$
\begin{aligned}
\Delta S_{(1)} = &\{(A_{system})(H3[n+1] - h_{column}) + (A_{int})(H4[n+1] \\
&- H3[n+1]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\} \\
&- \{(A_{system})(H3[n] - h_{column}) + (A_{int})(H4[n] \\
&- H3[n]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\}
\end{aligned}
\qquad [42]
$$

The second equation is used for Case 2 in Section 4.1.2 given by:

$$
\begin{aligned}
\Delta S_{(2)} = &\{(A_{ext})(H3[n+1]) + (A_{int})(H4[n+1] - H3[n+1]) \\
&+ (A_{hp2})(L_{hp2})\} - \{(A_{ext})(H3[n]) + (A_{int})(H4[n] \\
&- H3[n]) + (A_{hp2})(L_{hp2})\}
\end{aligned}
\qquad [43]
$$

Therefore, the mass balance of water at the oil back-up system is determined as:

$$
\begin{aligned}
\Delta S_{(1)} &= [\Delta Q_{in} - \Delta Q_{out}]\Delta t \qquad [44] \\
\Delta S_{(1)} &= (-\Delta Q_{out})\Delta t \\
&\{(A_{system})(H3[n+1] - h_{column}) + (A_{int})(H4[n+1] \\
&- H3[n+1]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\} \\
&- \{(A_{system})(H3[n] - h_{column}) + (A_{int})(H4[n] \\
&- H3[n]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\} = \\
&- \Delta t \frac{c\, A_{hp2}\sqrt{2\, \rho_{water}\, g}}{2} \{\sqrt{H4[n+1] - h_{column} - h} \\
&+ \sqrt{H4[n] - h_{column} - h}\}
\end{aligned}
$$

Page 82

$$\Delta S_{(2)} = [\Delta Q_{in} - \Delta Q_{out}]\Delta t \qquad [45]$$

$$\Delta S_{(2)} = (-\Delta Q_{out})\Delta t$$

$$\{(A_{ext})(H3[n+1]) + (A_{int})(H4[n+1] - H3[n+1]) + (A_{hp2})(L_{hp2})\}$$

$$- \{(A_{ext})(H3[n]) + (A_{int})(H4[n] - H3[n]) + (A_{hp2})(L_{hp2})\} =$$

$$-\Delta t \ \frac{c \ A_{hp2}\sqrt{2 \ \rho_{water} \ g}}{2} \ \{\sqrt{H4[n+1] - h_{column} - h} $$

$$+ \sqrt{H4[n] - h_{column} - h}\}$$

4.2   Development of the Mathematical Model

The purpose of the mathematical model discussed below is to determine the heights of oil and water layers, ($H1[n+1]$, $H2[n+1]$, $H3[n+1]$, $H4[n+1]$), at different time intervals. Since there are four unknowns in the mathematical model and only three sets of mass balance equations were developed in the previous section, one more equation is needed in order to solve all the unknowns. The last equation involved in the model is the hydrostatic pressure at the interface of transformer oil and water.

$$(\rho_{oil})(H2[n+1]) = (\rho_{water})(H4[n+1] - H3[n+1]) \qquad [46]$$

$$(\rho_{oil})(H2[n]) = (\rho_{water})(H4[n] - H3[n]) \qquad [47]$$

The Newton-Raphson method will be used to solve the four algebraic equations simultaneously. Since most of the unknowns in the mass balance equations are in rational exponent forms, substitution method must be applied to simplify the unknowns before solving the system of algebraic equations. By assuming

Page 83

$$X1 = \sqrt{H1 - H2 - H3}$$
$$X2 = H2$$
$$X3 = H3$$
$$X4 = \sqrt{H4 - h_{column} - h}$$

[48]

H1 and *H4* can be rewritten as

$$H1 = X1^2 + X2 + X3$$

[49]

$$H4 = X4^2 + h_{column} + h$$

[50]

The mass balance equation for the transformer oil at the catchbasin (Equation [36]) can be modified as:

$$\Delta S = [\Delta Q_{in} - \Delta Q_{out}]\Delta t$$

[51]

$$A_{catchbasin}(X1^2[n+1] + X2[n+1] + X3[n+1] - X1^2[n]$$
$$-X2[n] - X3[n]) = \Delta t \left\{ \frac{1}{2}(Q_{in}[n+1] + Q_{in}[n]) \right.$$
$$\left. - \frac{c\, A_{hpl}\sqrt{2\, \rho_{oil}\, g}}{2}(X1[n+1] + X1[n]) \right\}$$

The calculation for the average inflow rate of transformer oil, $(\Delta Q_{in})_{oil}$, to the catchbasin is demonstrated in Section 4.3 where

$$(\Delta Q_{in})_{oil} = 1/2\, (Q_{in}[n+1] + Q_{in}[n])$$

[52]

Page 84

Further expanding of Equation [51] gives,

$$(A_{catchbasin})(X1^2[n+1]) + (A_{catchbasin})(X2[n+1]) + (A_{catchbasin})(X3[n+1])$$
$$- (A_{catchbasin})(X1^2[n]) - (A_{catchbasin})(X2[n]) - (A_{catchbasin})(X3[n])$$
$$- \Delta t \, (\Delta Q_{in})_{oil} + \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n+1])$$
$$+ \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n]) = 0$$

$$(A_{catchbasin})(X1^2[n+1]) + \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n+1])$$
$$+ (A_{catchbasin})(X2[n+1]) + (A_{catchbasin})(X3[n+1]) - (A_{catchbasin})(X1^2[n])$$
$$+ \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n]) - (A_{catchbasin})(X2[n]) - (A_{catchbasin})(X3[n])$$
$$- \Delta t \, (\Delta Q_{in})_{oil} = 0 \tag{53}$$

The mass balance of transformer oil at the oil back-up system for Case 1 (Equation [39]) can be modified to:

$$\Delta S_{(1)} = [\, \Delta Q_{in} - \Delta Q_{out} \,] \Delta t$$
$$\Delta S_{(1)} = (\Delta Q_{in}) \Delta t$$

$$(A_{system} - A_{int})(X2[n+1]) - (A_{system} - A_{int})(X2[n])$$
$$= \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n+1] + X1[n])$$

$$\frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n+1]) - (A_{system} - A_{int})(X2[n+1])$$
$$+ \frac{c \, A_{tpi} \, \Delta t \, \sqrt{2 \, \rho_{oil} \, g}}{2} \, (X1[n]) + (A_{system} - A_{int})(X2[n]) = 0 \tag{54}$$

Page 85

For Case 2, the mass balance of transformer oil at the oil back-up system (Equation 40) can be rewritten as:

$$\Delta S_{(2)} = [\Delta Q_{in} - \Delta Q_{out}]\Delta t$$
$$\Delta S_{(2)} = (\Delta Q_{in})\Delta t$$

$$(A_{system} - A_{int})(X2[n+1]) + (A_{system} - A_{int})(X3[n+1])$$
$$- (A_{system})(h_{column}) + (A_{int})(h_{column}) - (A_{int})(L_{tp2})$$
$$+ (A_{ext})(h_{column}) - (A_{int})(h_{column}) - (A_{ext} - A_{int})(X3[n+1])$$
$$- (A_{system} - A_{int})(X2[n]) - (A_{system} - A_{int})(X3[n])$$
$$+ (A_{system})(h_{column}) - (A_{int})(h_{column}) + (A_{tp2})(L_{tp2})$$
$$- (A_{ext})(h_{column}) + (A_{int})(h_{column}) + (A_{ext} - A_{int})(X3[n])$$
$$= \frac{c\, A_{tp1}\, \Delta t\, \sqrt{2\, \rho_{oil}\, g}}{2} (X1[n+1] + X1[n])$$

$$(A_{system} - A_{int})(X2[n+1]) + (A_{system})(X3[n+1]) - (A_{int})(X3[n+1])$$
$$- (A_{ext})(X3[n+1]) + (A_{int})(X3[n+1]) - (A_{system} - A_{int})(X2[n])$$
$$- (A_{system})(X3[n]) + (A_{int})(X3[n]) + (A_{ext})(X3[n])$$
$$- (A_{int})(X3[n]) = \frac{c\, A_{tp1}\, \Delta t\, \sqrt{2\, \rho_{oil}\, g}}{2} (X1[n+1])$$
$$+ \frac{c\, A_{tp1}\, \Delta t\, \sqrt{2\, \rho_{oil}\, g}}{2} (X1[n])$$

$$\frac{c\, A_{tp1}\, \Delta t\, \sqrt{2\, \rho_{oil}\, g}}{2} (X1[n+1]) - (A_{system} - A_{int})(X2[n+1])$$
$$+ (A_{ext} - A_{system})(X3[n+1]) + \frac{c\, A_{tp1}\, \Delta t\, \sqrt{2\, \rho_{oil}\, g}}{2} (X1[n])$$
$$+ (A_{system} - A_{int})(X2[n]) + (A_{system} - A_{ext})(X3[n]) = 0 \qquad [55]$$

By substituting Equations [49] and [50] into Equation [44], the modification of Case 1 for the mass balance of water at the oil back-up system scenario becomes:

$$\Delta S_{(t)} = [\Delta Q_{in} - \Delta Q_{out}]\Delta t$$
$$\Delta S_{(t)} = (-\Delta Q_{out})\Delta t$$

$$\{(A_{system})(H3[n+1] - h_{column}) + (A_{int})(H4[n+1]$$
$$- H3[n+1]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\}$$
$$- \{(A_{system})(H3[n] - h_{column}) + (A_{int})(H4[n]$$
$$- H3[n]) + (A_{ext})(h_{column}) + (A_{hp2})(L_{hp2})\} =$$
$$-\Delta t \frac{c A_{hp2}\sqrt{2\rho_{water}g}}{2} \{\sqrt{H4[n+1] - h_{column} - h}$$
$$+ \sqrt{H4[n] - h_{column} - h}\}$$

$$(A_{system})(X3[n+1]) - \cancel{(A_{system})(h_{column})} + (A_{int})(X4^2[n+1])$$
$$+ \cancel{(A_{int})(h_{column})} + \cancel{(A_{int})(h)} - (A_{int})(X3[n+1]) + (A_{ext})(h_{column})$$
$$+ \cancel{(A_{hp2})(L_{hp2})} - (A_{system})(X3[n]) + \cancel{(A_{system})(h_{column})}$$
$$- (A_{int})(X4^2[n]) - \cancel{(A_{int})(h_{column})} - \cancel{(A_{int})(h)} + (A_{int})(X3[n])$$
$$- (A_{ext})(h_{column}) - \cancel{(A_{hp2})(L_{hp2})} = - \frac{c A_{hp2}\Delta t\sqrt{2\rho_{water}g}}{2}(X4[n+1])$$
$$- \frac{c A_{hp2}\Delta t\sqrt{2\rho_{water}g}}{2}(X4[n])$$

$$(A_{system} - A_{int})(X3[n+1]) + (A_{int})(X4^2[n+1])$$
$$+ \frac{c A_{hp2}\Delta t\sqrt{2\rho_{water}g}}{2}(X4[n+1]) + (A_{int} - A_{system})(X3[n])$$
$$- (A_{int})(X4^2[n]) + \frac{c A_{hp2}\Delta t\sqrt{2\rho_{water}g}}{2}(X4[n]) = 0 \qquad [56]$$

For Case 2, the mass balance of water at the oil back-up system scenario (Equation [45]) can be modified using the similar procedure in Equation [56] such that:

$$\Delta S_{(2)} = [\Delta Q_{in} - \Delta Q_{out}]\Delta t$$
$$\Delta S_{(2)} = (-\Delta Q_{out})\Delta t$$

$$\{(A_{ext})(H3[n+1]) + (A_{int})(H4[n+1] - H3[n+1]) + (A_{hp2})(L_{hp2})\}$$
$$-\{(A_{ext})(H3[n]) + (A_{int})(H4[n] - H3[n]) + (A_{hp2})(L_{hp2})\} =$$
$$-\Delta t \frac{c\, A_{hp2}\sqrt{2\,\rho_{water}\, g}}{2}\{\sqrt{H4[n+1] - h_{column} - h}$$
$$+\sqrt{H4[n] - h_{column} - h}\}$$

$$(A_{ext})(X3[n+1]) + (A_{int})(X4^2[n+1]) + (A_{int})(h_{column})$$
$$+(A_{int})(h) - (A_{int})(X3[n+1]) + (A_{hp2})(L_{hp2})$$
$$-(A_{ext})(X3[n]) - (A_{int})(X4^2[n]) - (A_{int})(h_{column})$$
$$-(A_{int})(h) + (A_{int})(X3[n]) - (A_{hp2})(L_{hp2}) =$$
$$-\frac{c\, A_{hp2}\Delta t\sqrt{2\,\rho_{water}\, g}}{2}(X4[n+1]) - \frac{c\, A_{hp2}\Delta t\sqrt{2\,\rho_{water}\, g}}{2}(X4[n])$$

$$(A_{ext} - A_{int})(X3[n+1]) + (A_{int})(X4^2[n+1])$$
$$+\frac{c\, A_{hp2}\Delta t\sqrt{2\,\rho_{water}\, g}}{2}(X4[n+1]) + (A_{int} - A_{ext})(X3[n])$$
$$-(A_{int})(X4^2[n]) + \frac{c\, A_{hp2}\Delta t\sqrt{2\,\rho_{water}\, g}}{2}(X4[n]) = 0 \qquad [57]$$

Since one of the hydrostatic pressure equations at the interface of transformer oil and water does not contain any unknowns, only Equation [46] will be used for the nonlinear system. Equation [46] can be rewritten as:

$$(\rho_{oil})(X2[n+1]) = (\rho_{water})(X4^2[n+1] + h_{column} + h - X3[n+1])$$

$$(\rho_{oil})(X2[n+1]) + (\rho_{water})(X3[n+1]) - (\rho_{water})(X4^2[n+1])$$
$$-(\rho_{water})(h_{column}) - (\rho_{water})(h) = 0 \qquad [58]$$

Equations [53] to [58] are the fundamental nonlinear system of algebraic equations for the mathematical model. Since there are four sets of nonlinear equations, the Jacobian matrix in Newton-Raphson method will contain sixteen elements. The elements of the Jacobian matrix for Case 1 are demonstrated below. These elements are derived from Equations [53], [54], [56] and [58].

$$(a_{11})_{case\ 1} = \frac{\partial F_1}{\partial X1[n+1]} = (2)(A_{catchbasin})(X1[n+1]) + \frac{c\,A_{hp1}\,\Delta t\,\sqrt{2\,\rho_{oil}\,g}}{2}$$

$$(a_{12})_{case\ 1} = \frac{\partial F_1}{\partial X2[n+1]} = A_{catchbasin}$$

$$(a_{13})_{case\ 1} = \frac{\partial F_1}{\partial X3[n+1]} = A_{catchbasin}$$

$$(a_{14})_{case\ 1} = \frac{\partial F_1}{\partial X4[n+1]} = 0$$

$$(a_{21})_{case\ 1} = \frac{\partial F_2}{\partial X1[n+1]} = \frac{c\,A_{hp1}\,\Delta t\,\sqrt{2\,\rho_{oil}\,g}}{2}$$

$$(a_{22})_{case\ 1} = \frac{\partial F_2}{\partial X2[n+1]} = A_{int} - A_{system}$$

$$(a_{23})_{case\ 1} = \frac{\partial F_2}{\partial X3[n+1]} = 0$$

$$(a_{24})_{case\ 1} = \frac{\partial F_2}{\partial X4[n+1]} = 0$$

$$(a_{31})_{case\ 1} = \frac{\partial F_3}{\partial X1[n+1]} = 0$$

$$(a_{32})_{case\ 1} = \frac{\partial F_3}{\partial X2[n+1]} = 0$$

$$(a_{33})_{case\ 1} = \frac{\partial F_3}{\partial X3[n+1]} = A_{system} - A_{int}$$

$$(a_{34})_{case\ 1} = \frac{\partial F_3}{\partial X4[n+1]} = (2)(A_{int})(X4[n+1]) + \frac{c\,A_{hp2}\,\Delta t\,\sqrt{2\,\rho_{water}\,g}}{2}$$

$$(a_{41})_{case\ 1} = \frac{\partial F_4}{\partial X1[n+1]} = 0$$

$$(a_{42})_{case\ 1} = \frac{\partial F_4}{\partial X2[n+1]} = \rho_{oil}$$

$$(a_{43})_{case\ 1} = \frac{\partial F_4}{\partial X3[n+1]} = \rho_{water}$$

$$(a_{44})_{case\ 1} = \frac{\partial F_4}{\partial X4[n+1]} = (-2)(\rho_{water})(X4[n+1]) \qquad [59]$$

Page 89

The elements of Jacobian matrix for Case 2 are shown below.

$$(a_{11})_{case\ 2} = \frac{\partial F_1}{\partial X1[n+1]} = (2)(A_{catchbasin})(X1[n+1]) + \frac{c\ A_{hp1}\ \Delta t \sqrt{2\ \rho_{oil}\ g}}{2}$$

$$(a_{12})_{case\ 2} = \frac{\partial F_1}{\partial X2[n+1]} = A_{catchbasin}$$

$$(a_{13})_{case\ 2} = \frac{\partial F_1}{\partial X3[n+1]} = A_{catchbasin}$$

$$(a_{14})_{case\ 2} = \frac{\partial F_1}{\partial X4[n+1]} = 0$$

$$(a_{21})_{case\ 2} = \frac{\partial F_2}{\partial X1[n+1]} = \frac{c\ A_{hp1}\ \Delta t \sqrt{2\ \rho_{oil}\ g}}{2}$$

$$(a_{22})_{case\ 2} = \frac{\partial F_2}{\partial X2[n+1]} = A_{int} - A_{system}$$

$$(a_{23})_{case\ 2} = \frac{\partial F_2}{\partial X3[n+1]} = A_{ext} - A_{system}$$

$$(a_{24})_{case\ 2} = \frac{\partial F_2}{\partial X4[n+1]} = 0$$

$$(a_{31})_{case\ 2} = \frac{\partial F_3}{\partial X1[n+1]} = 0$$

$$(a_{32})_{case\ 2} = \frac{\partial F_3}{\partial X2[n+1]} = 0$$

$$(a_{33})_{case\ 2} = \frac{\partial F_3}{\partial X3[n+1]} = A_{ext} - A_{int}$$

$$(a_{34})_{case\ 2} = \frac{\partial F_3}{\partial X4[n+1]} = (2)(A_{int})(X4[n+1]) + \frac{c\ A_{hp2}\ \Delta t \sqrt{2\ \rho_{water}\ g}}{2}$$

$$(a_{41})_{case\ 2} = \frac{\partial F_4}{\partial X1[n+1]} = 0$$

$$(a_{42})_{case\ 2} = \frac{\partial F_4}{\partial X2[n+1]} = \rho_{oil}$$

$$(a_{43})_{case\ 2} = \frac{\partial F_4}{\partial X3[n+1]} = \rho_{water}$$

$$(a_{44})_{case\ 2} = \frac{\partial F_4}{\partial X4[n+1]} = (-2)(\rho_{water})(X4[n+1]) \qquad [60]$$

4.3   Determination of the Input Data

The density of transformer oil, ρ_{oil}, and the inflow rate of oil, Q_{in}, are the two input data for the mathematical model. The density of transformer oil can be determined by conducting an experiment described below. First, the mass of an empty 100 mL graduated cylinder is measured on an electrical balance (row 2 of Table 4.3). Then, (100 ± 0.5) mL of transformer oil is filled in the graduated cylinder and its weight is measured again (row 3 of Table 4.3). Finally, the density of oil is calculated by dividing the net weight of transformer oil over its volume (row 4 of Table 4.3).

Table 4.3: Experimental results for the Density of the Transformer Oil.

Description	Trial #1	Trial #2
mass of 100 mL graduated cylinder, in grams	67.38 ± 0.01	67.51 ± 0.01
mass of 100 mL transformer oil in the 100 mL graduated cylinder, in grams	153.49 ± 0.01	153.61 ± 0.01
net weight of 100 mL transformer oil, in grams	86.11 ± 0.02	86.1 ± 0.02
density of transformer oil, in kg/L	0.8611 ± 0.00451	0.861 ± 0.00451

After taking the average of the two trials in Table 4.3, the density of transformer oil used in the mathematical model is *0.86 kg/L*.

The determination of the inflow rate, Q_{in}, is shown in Appendix G. The concrete pad and containment wall at the transformer station is acted as a big container with $15.18\ m^2$ (Equation [26]) in base area and $1\ m$ in height. A transformer oil spill must first reach the concrete pad before it gets into the catchbasin and the oil back-up system. Any spilled transformer oil that cannot go to the concrete pad is assumed to be spilled out to the ground. This is only happened on the spill events with enormous amount of transformer oil. First, a certain height of spilled transformer oil at the concrete pad at $t = 0$ (row 2 column 3 on Appendix G) is assumed in order to determine the inflow rate. Thus, the total volume of transformer oil remains on the concrete pad (row 2 column 4 on Appendix G) and the inflow rate (row 2 column 5 on Appendix G) at that time interval can be determined. The inflow rate is calculated from the orifice equation (Equation [34]) by letting the cross-sectional area be the total area of grate (Figure 4.5) that transformer oil spills exiting from the concrete pad (horizontal red line and vertical Q_{in} arrow above the concrete pad in Figure 4.2). The calculation of the total area of grate is demonstrated below.

$$A_{grate} = 4 * (area\ M) + 4 * (area\ N) + 4 * (area\ P) + 8 * (area\ Q)$$
$$= 4\ [(0.025 + 0.06)(0.025)(0.5)] + 4\ [(0.13 + 0.16)(0.025)(0.5)$$
$$+ 4\ [(0.235 + 0.275)(0.025)(0.5)] + 8\ (0.35)(0.025)$$
$$= 0.11\ m^2 \tag{61}$$
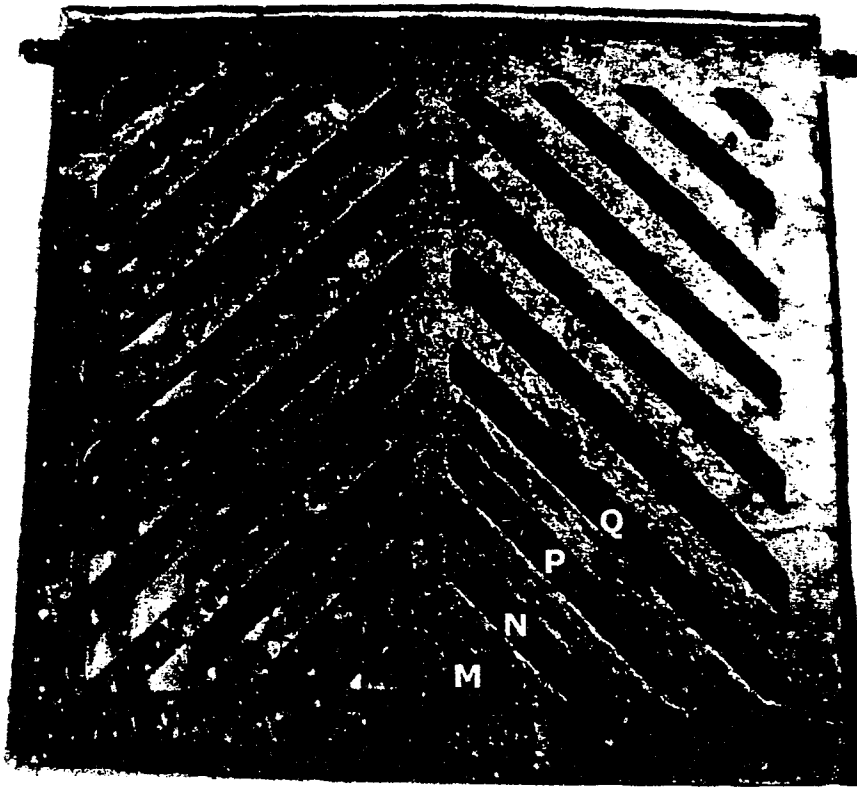
Figure 4.5: Typical Grate Located at the Concrete Pad.

Secondly, the volume of transformer oil goes into the catchbasin within the time step (row 3 column 2 on Appendix G) is calculated by multiplying the inflow rate with the length of the time step. The new height of spilled transformer oil at the concrete pad (row 3 column 3 on Appendix G) is found by dividing the volume of transformer oil remains on the concrete pad at the previous time step with the base area of the concrete pad. The data in columns 4 and 5 always follow the same calculation procedures that were discussed above. Finally, the average inflow rate entering into the catchbasin within the first time step (row 3 column 6 on Appendix G) is determined by taking the average inflow of the first time interval (column 5 rows 2 and 3 on Appendix G).

The determination of inflow rate is a function of the surface elevation of the transformer oil layer at catchbasin. A typical catchbasin's sump is *1.8 m* in height. The height between the bottom part of the catchbasin and the bottom part of the outlet pipe is usually *0.45 m*. Therefore, the inflow rate equation is used when the surface elevation of the oil layer is within

1.8 m – 0.45 m = 1.35 m

Once the surface elevation exceeds this height, the head of the transformer oil at the concrete pad will dominate the flow entering the oil back-up system.

Appendix H demonstrates the four MATLAB programs involved for the implementation of the mathematical model of the oil back-up system.

4.4   Numerical Analysis and Results

The numerical modeling results can be found in Appendix I. Figures 4.6 to 4.9 illustrate the heights of the layers at different time intervals. Figure 4.10 indicates the variation of water and oil levels over time during a transformation oil spill.

In the numerical experiment of *0.5 m* depth of spilled transformer oil on the concrete pad at *t = 0* seconds (Figure 4.6), *H1* increases rapidly between *0* to *6* seconds (column 6 rows 2 to 5 in Appendix I) and the transformer oil levels inside the catchbasin and on the concrete pad merge together (Figure 4.7). After *H1* reaches its highest elevation at

48 seconds (row 6 column 6 in Appendix G), it decreases slowly and stops at *146* seconds where *H1* is equal to *10.92 m* (Figure 4.9). The spilled transformer oil is eventually backed up at a depth of *0.12 m* by the oil back-up.

The total height of oil and water layers (*H2 + H3*) at the oil back-up system peaks at *48* seconds (Figure 4.8). The mathematical model of the oil back-up system is designed without defining the height of the tank because the height of the tank can always be determined from the model results. For instance, (*H2 + H3*) is around *11 m* at *48* seconds (Figure 4.8). Therefore, the minimum height requirement for the oil back-up system is *11 m* from datum. Since the height between the datum and the bottom part of the tank is *8.23 m*, the minimum height of the tank is *2.77 m*. Once (*H2+H3*) reaches its highest elevation, it decreases slowly and stops at the height of *10.92 m* at *146* seconds.

When the variations of *H2* and *H3* are observed separately, the rate of change in the first half of the numerical experiment, *t = 0* to *100* seconds, are much slower than the second half, *t = 100* to *146* seconds (Figure 4.10). Also, the rate of change of each height is opposite to each other, where *H2* (oil layer height) increases and *H3* (water layer height) decreases throughout the experiment. These results seem reasonable and satisfy the hydrostatic pressure properties between transformer oil and water (Equation [58]). Thus, the numerical model of oil back-up system perform logically.

The height of *H3* at *146* seconds equals *0.31 m* (Figure 4.9). Since it is greater than zero, the oil back-up system with the design dimensions (Figure 4.4) will not fail for an initial 0.5 m depth of spilled transformer oil on the concrete pad. Finally, this numerical experiment stops at *t* = *146* seconds since the solutions of *H1* and *H4* are in negative after this time interval (Appendix G). The reason for the negative results after *t* = *146* seconds is because the heights of the layers become stable and the numerical scheme become unstable.
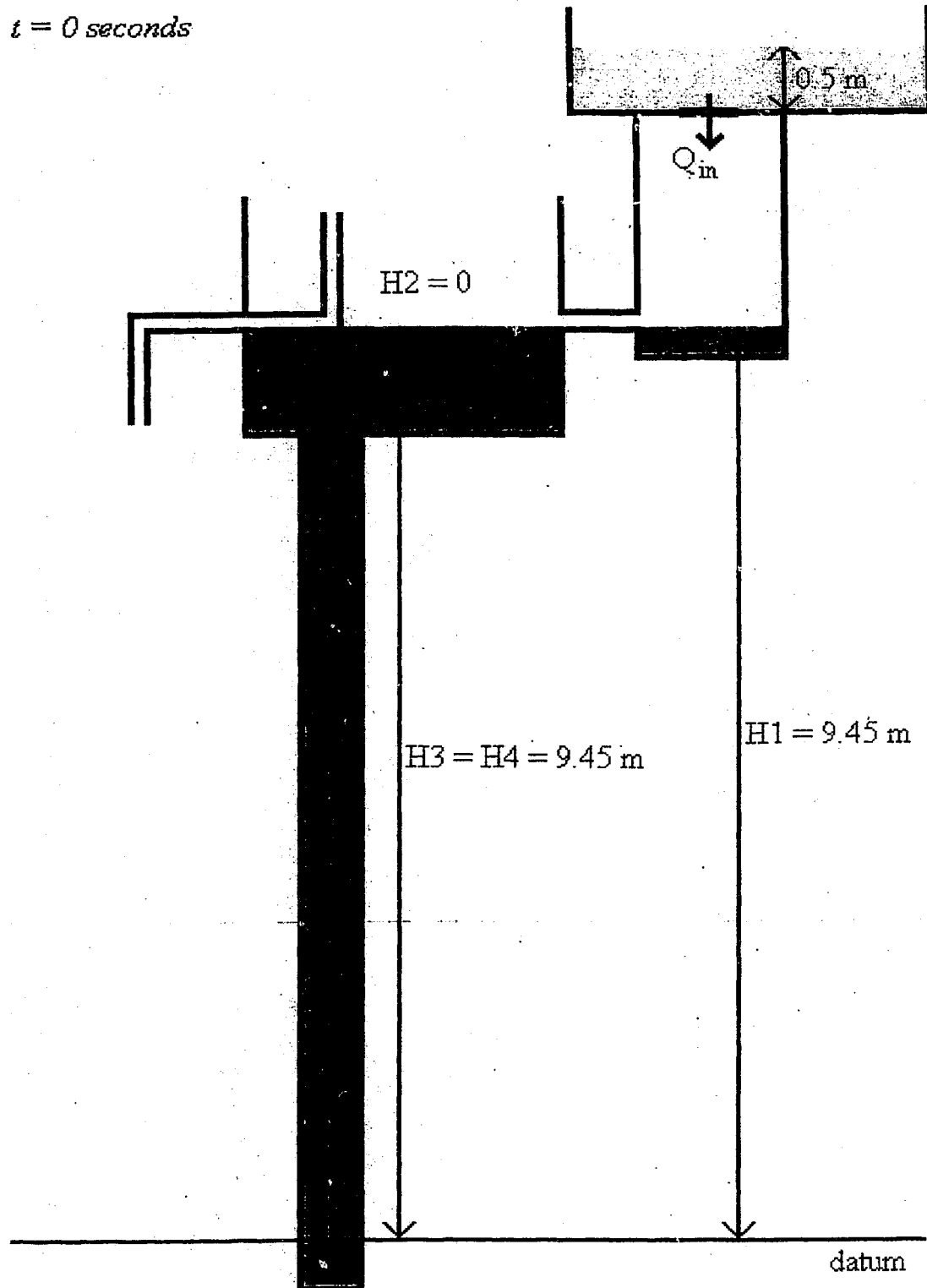
Figure 4.6: Numerical result of the oil back-up system at $t = 0$ seconds.
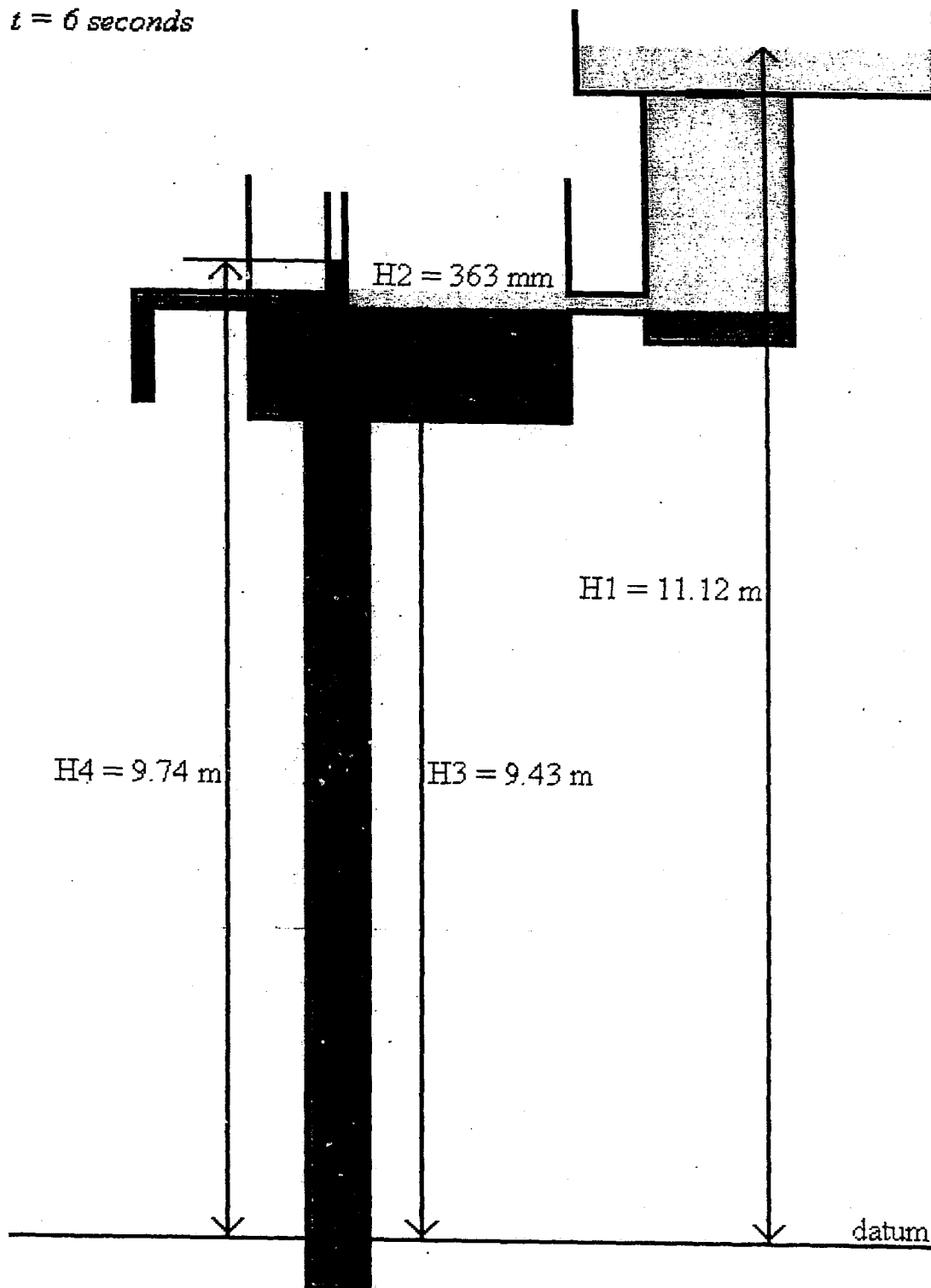
Figure 4.7: Numerical result of the oil back-up system at $t = 6$ seconds.
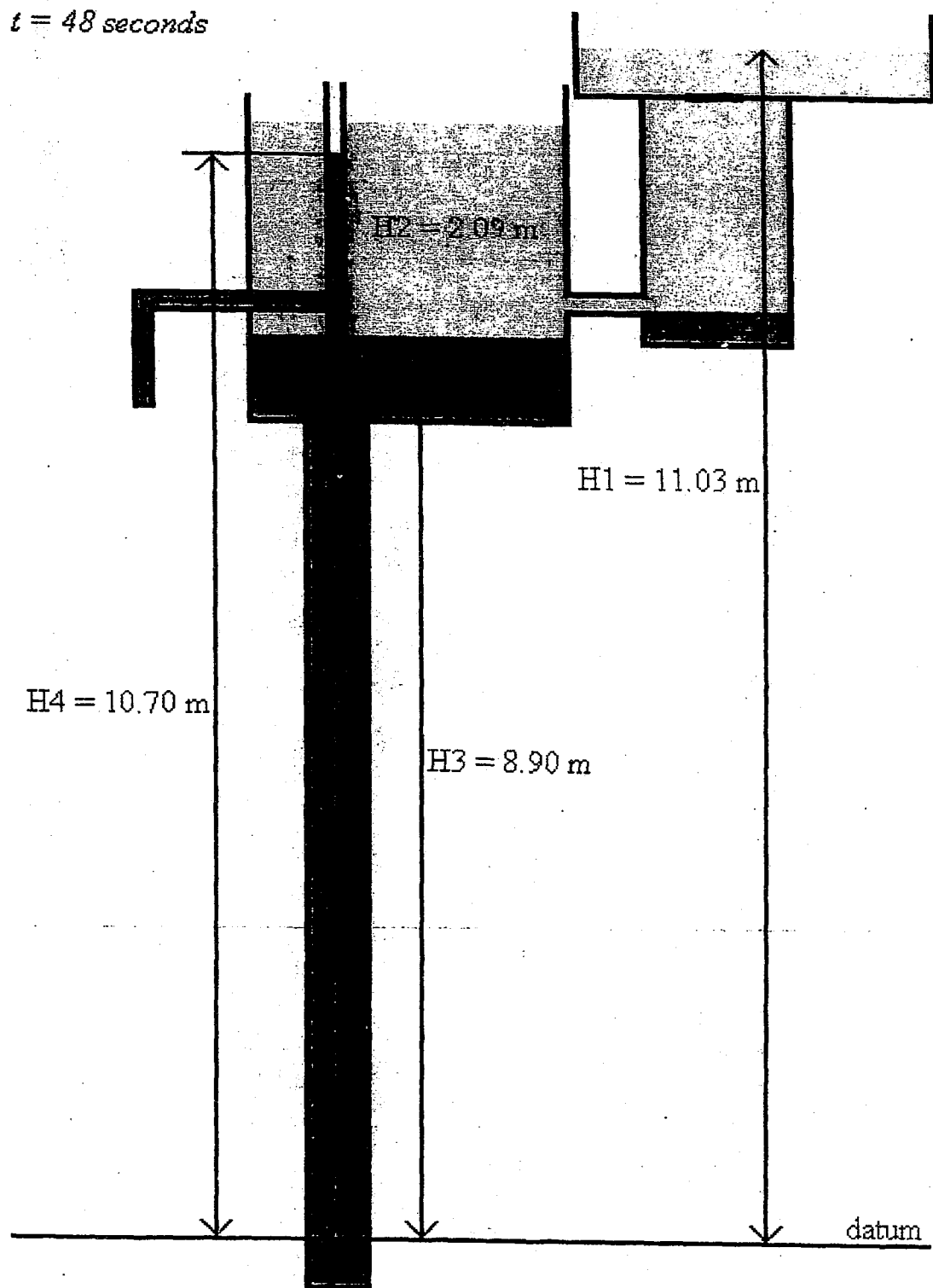
Figure 4.8: Numerical result of the oil back-up system at $t = 48$ seconds.

t = 146 seconds

124 mm transformer oil remains on concrete pad

H1 = 10.92 m

H4 = 9.45 m

H2 = 10.61 m

H3 = 313 mm

datum
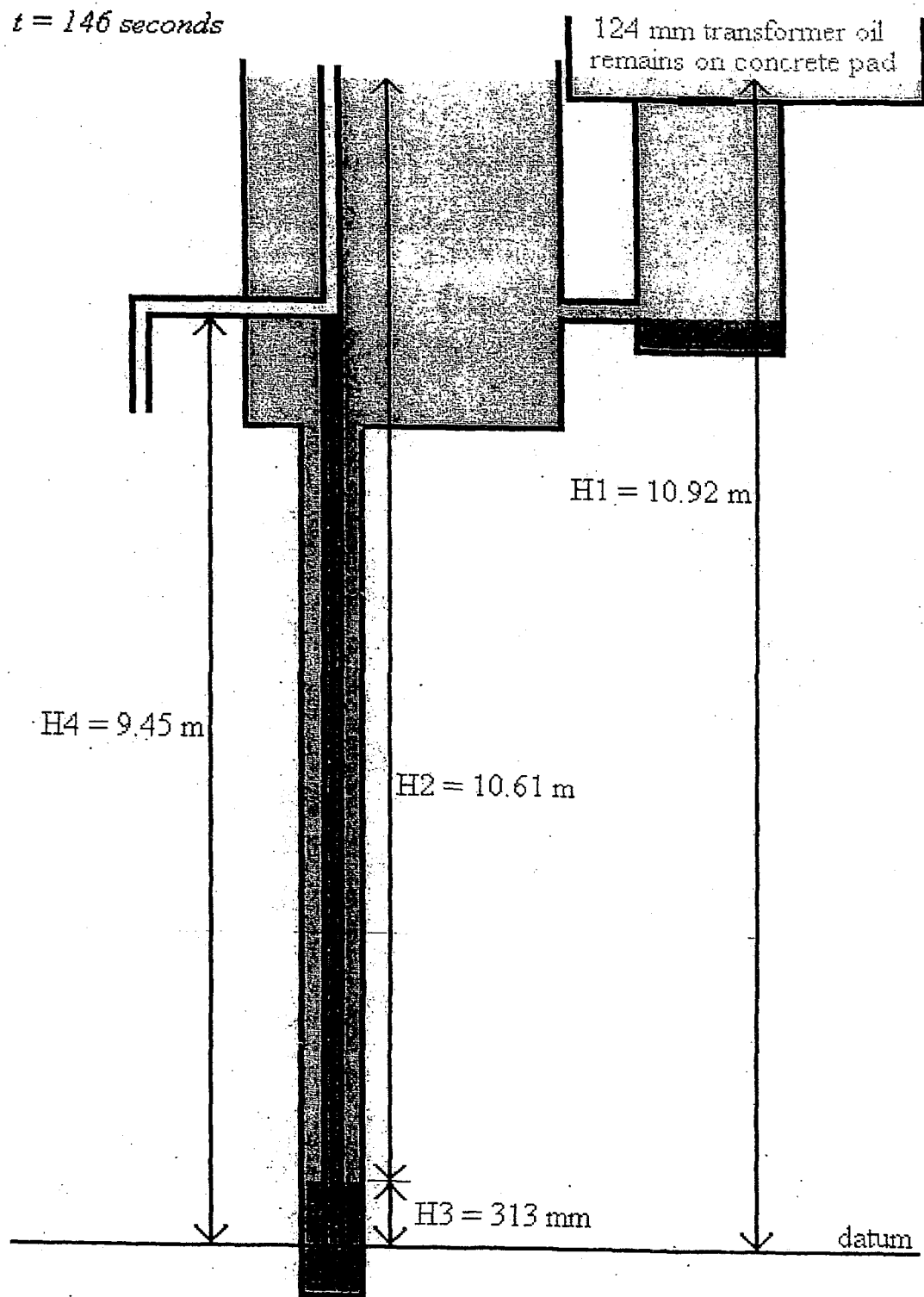
Figure 4.9: Numerical result of the oil back-up system at *t = 146* seconds.

Page 100
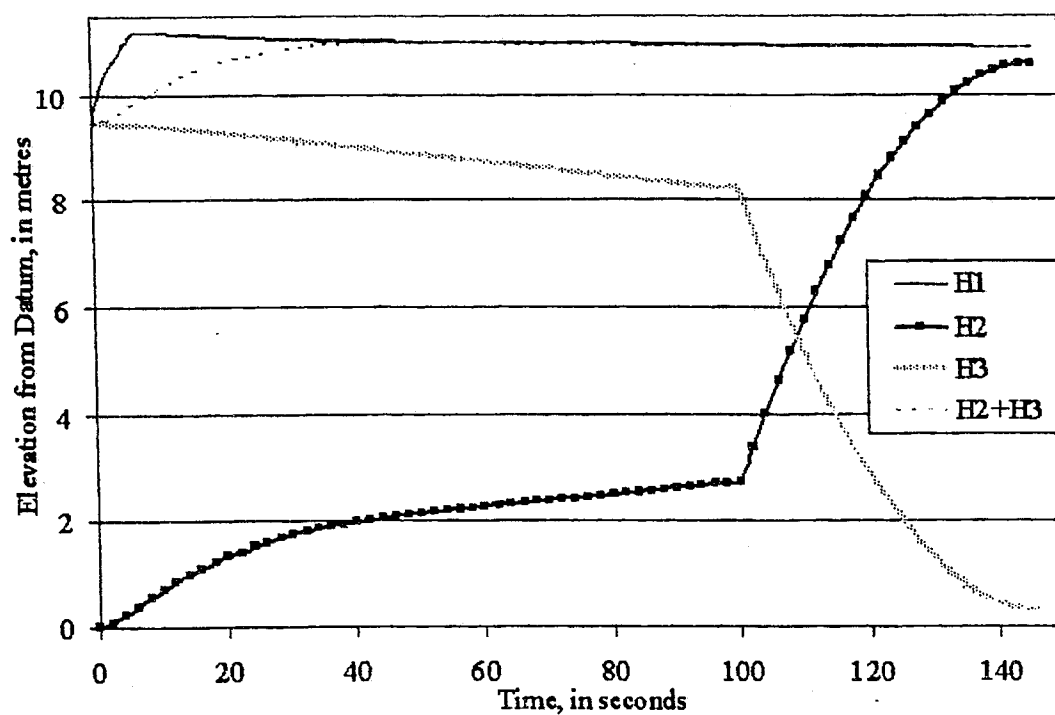
Figure 4.10: Heights' variables versus time.

Chapter 5   CONCLUSIONS AND RECOMMENDATIONS

5.1   Conclusions

Based on the physical and mathematical model analyses, the two hydraulically-based oil spill control systems can prevent spilled transformer oil entering into the local sewers. For the oil trap system, the physical model experiments indicate that the model can trap a quantity of *36 L* transformer oil without being flush out at the maximum water flow of *0.18 L/sec*. These model results are equivalent to approximately *85,000 L* transformer oil in the prototype and a flow greater than the 100 year storm. This amount of transformer oil is about one-third of the total volume of transformer oil used in *Hydro One's* transformer station near the city of Burlington. Therefore, three oil trap systems will be needed to capture the most severe transformer oil spill. However, it is very unusual to have six individual transformer stations to spill out its total volumes of transformer oil at the same time.

The oil-water separation experiments conducted in the physical model of the oil trap system demonstrate that the spilled transformer oil cannot escape from the system for both dry and wet weather conditions. Without pump operation in the experiments, it represents the occurrence of a dry weather transformer oil spill. The oil-water separation in this situation is mainly dependent on the diameters of oil globules (Equation [1]). The diameter of the oil globule and the rising velocity are in direct proportional to each other. Without pump operation (i.e. dry weather condition), the oil

Page 102

globules of large spills rise faster than the oil globules of small spills. With pump operation, it represents the occurrence of a transformer oil spill during a rainstorm. The oil globules of small spills spread out to about one-third of the oil trap system length before they rose up. The oil globules of large spills are much larger than that of the previous case (without pump operation). They also rose along the outside of the inlet pipe area instead of spreading out to the oil trap system.

The mathematical model in Chapter 4 demonstrates that the design dimensions of the oil back-up system shown in Figure 4.4 can back up approximately *7600 L* of spilled transformer oil to the concrete pad of *Hydro One's* transformer station near the city of Burlington. The final depth of spilled transformer oil on the concrete pad is *0.12 m* (Section 4.4). There is a gap of *0.31 m* (Section 4.4) between the bottom of the internal pipe column and the interface of oil and water. This indicates that the spilled transformer oil will not escape from the system. The height of the oil back-up system is determined to be *11 m* above datum, which is *0.2 m* above the bottom of the concrete pad. Since the height of the concrete pad is about *1 m* and it is constructed *0.5 m* below the ground, the oil back-up system should be constructed *0.3 m* below ground level. This mathematical model can be used to determine the designed volume of spilled transformer oil for dry weather condition.

Finally, the practical contribution of this research is to provide an estimate of the performance of the two hydraulically-based oil spill control systems to *Hydro One* in order to solve their existing problems in the transformer station near the city of

Burlington. The scientific contribution of this research is to develop the mathematical model for the hydraulically-based oil back-up system. This mathematical model can simulate the levels of the transformer oil and water layers within the system at different time step and provide a design tool for this type of devices.

5.2 Recommendations

As indicated in Section 3.5, a new physical model must be constructed to minimize the uncertainties in the oil trap system experiments. This new model must be in closed system form in order to prevent the rising up of the transformer oil layer. Thus, a stronger construction material, such as steel, should be chosen instead of plexi-glass. Since steel is non-transparent, a window with sufficient strength to stand the internal pressure forces of a closed system should be built on one side of the physical model for observation purpose. Valve should be also installed on the top of the model in order to insert the transformer oil into the system.

Once the new physical model is ready, experiments for the determination of the failure flow situation versus different volumes of transformer oil within the oil trap system can be conducted. Different amounts of transformer oil should be added into the system through the valve. After inserting each amount of transformer oil, water can be pumped to the system from the vertical inlet pipe until waves are formed at the interface of oil and water. A powerful pump may be needed in order to provide a larger flow rate for the formation of the waves. Furthermore, different sizes of physical models for the oil

trap system should be constructed for verification of the significance of the Π-terms derived in Equation [22].

Another set of experiments can be developed in order to determine the relationship between the diameter of the outlet pipe and the width of the oil trap system. The diameter of the outlet pipe is a major factor to cause transformer oil escaping from the system. If the diameter of the outlet pipe is small, the exiting velocity of water in the system will be larger. This may increase the possibility of dragging out the transformer oil from the system. On the other hand, the chance for transformer oil escapes from the system with a small diameter of the outlet pipe is smaller than a large diameter of the outlet pipe. Therefore, experiments must be developed in order to find out the best ratio between the diameter of the outlet pipe and the width of the oil trap system to prevent transformer oil from escaping from the system.

The major oil spill operating condition (Section 2.5) assumes that a large amount of transformer oil spill enters into the system rapidly. Hence, the length of the vertical inlet pipe may create a back-up situation for the spilled transformer oil at this situation. The purpose of the oil trap system is to force spilled transformer oil entering into the system and trap in the system. There is a good chance that the spilled transformation oil be back up if the vertical inlet pipe is long. Thus, experiments should be developed to determine the effect on the creation of a back-up situation with different lengths of the vertical inlet pipe.

Finally, the results for the oil-water separation experiments of the oil trap system are for observation purpose only. In order to determine whether the distance between the two vertical pipes are suitable for oil-water separation, the rising velocity ratio between the model and the prototype must be maintained (Appendix C). Therefore, a new physical model and a new set of experiments must be developed in order to determine the oil-water separation process within the oil trap system.

For the oil back-up system, it is important to verify the numerical result by conducting a physical model experiment in a future study. Also, this research only did one numerical experiment to demonstrate the operation of the mathematical model and to solve the problem at *Hydro One's* transformer station in the city of Burlington due to the lack of time. More numerical experiments should be conducted in order to determine the relationship between different amounts of spilled transformer oil at the concrete pad and the base area of the tank in the oil back-up system. Another relationship between different volumes of spilled transformer oil and the height from datum to the bottom of the tank in the oil back-up system (variable h_{column}) should be determined in future study as well. These two relationships are the major factors to cause the failure of the system. Once the sensitivity between different amounts of spilled transformer oil and the above two dimensions are developed, the oil back-up system design may be done without running the numerical model.

It is also interesting to find out whether the diameters for the internal and external pipe columns are a major factor to cause the failure of the oil back-up system. The volume

between the internal and external pipe columns is small. Therefore, the size of the tank in the oil back-up system should dominate the failure of the system. The design of the pipe columns are to slow down the increasing rate of the oil layer due to hydrostatic pressure. More numerical models should be conducted to determine whether the change of the diameters for the internal and external pipe columns affect the final elevation of the interface between oil and water.

Finally, the numerical experiment is designed for the occurrence of a major transformer oil spill under dry weather condition. It does not consider the situation of a transformer oil spill occurring during a rainstorm. Modification for the numerical model is needed for the wet weather condition. Furthermore, when a major transformer oil spill occurs during rainstorm, the rainwater should sink to the catchbasin due to the density difference between oil and water. Since the catchbasin and the concrete pad have limited space, it may cause the transformer oil to spill out to the ground depending on the duration and the intensity of the rainstorm. Thus, more considerations must be included in the modification of the numerical model.

APPENDIX   A

DIMENSIONAL ANALYSIS METHOD

Since the experimental studies were conducted on a scaled-down model of the oil trap system, it is important to determine the possible relationships of all the system using the dimensional analysis, such relationships can be used variables to convert the experimental results back to its prototype.

Dimensional analysis is a method of partial analysis which is based on the work of Fourier who developed the principle of dimensional homogeneity (Sharp, 1981). His works stated that any equation is correct only if all terms of the equation have the same dimensions. More specifically, the dimensions of any term can be represented as multiples of mass $[M]$, length $[L]$ and time $[T]$. Therefore the exponents of the mass, length and time dimensions must be the same in each term of the equation. For example, the equation for distance travelled, d, by an object moving from rest through a time, t_1, with acceleration, a, and through a time, t_2, with constant velocity, v, is given by:

$$d = \tfrac{1}{2}\, a\, t_1^{\,2} + v\, t_2 \qquad\qquad [A.1]$$

Dimensionally this can be rewritten as

$$[L] = \tfrac{1}{2}\, (\, [L]\,/\,[T]^{\,2}\,)\,(\,[T]^{\,2}\,) + (\,[L]\,/\,[T]\,)\,(\,[T]\,) \qquad\qquad [A.2]$$

where $\tfrac{1}{2}$ at the first term on the right side is a constant (Sharp, 1981).

Inspection of equation [A.2] shows that the dimension in each term of any equation must be the same. This principle provides immediate insight into the variables which may be relevant in any fluid system. However, one important concept about dimensional analysis is that any correct equation must be dimensionally homogeneous but any

dimensionally homogeneous equation is not necessarily complete or corrects (Sharp, 1981).

The application of the Fourier's principle has received considerable attention since it was proposed in the early nineteenth century. In particular, Rayleigh and Buckingham developed methods that are widely used today (Sharp, 1981). This research uses the Buckingham's method for the dimensional analysis study of hydraulic oil spill control systems.

BUCKINGHAM'S METHOD

Buckingham developed the arguments at the previous section into a theorem which related the number of parameters in a correct functional equation to the number of variables needed to specify the phenomenon and to the number of dimensions involved. Generally, he stated that if m variables and n dimensions are involved, a correct partial analysis will result in *(m − n)* dimensionless parameters. Buckingham called these parameters Π-terms. Thus, his theorem is usually called the Π-theorem (Sharp, 1981).

More specifically, with m variables and n dimensions there will be *(m − 1)* exponents to be determined from n simultaneous equations. Therefore, it is possible to solve for n exponents in terms of the remaining *(m − 1 − n)* to obtain an equation of the form below.

$$A = C \, \Pi_1{}^a \, \Pi_2{}^b \, \Pi_3{}^c \, ... \, \Pi^k{}_{m-1-n} \qquad \text{[A.3]}$$

where each Π-term is a dimensionless parameter and *(A/C)* is another Π-term. Altogether, there are *(m − n)* Π-terms.

Buckingham also suggested a method of approach in which each Π-term is derived independently from the others. The procedure is to select three relevant variables involving all dimensions present and to combine each other variable in turn with these three relevant variables to form a dimensionless parameter. A practical example will be discussed below to demonstrate Buckingham's method (Sharp, 1981).

Consider the flow of fluid, Q, over a variety of vee-notches, B and H, with the dynamic viscosity of the fluid, μ, and the density, ρ.

$$Q = \varphi \, (\, g \, , \, H \, , \, B \, , \, \mu \, , \, \rho \,) \qquad \text{[A.4]}$$

From the above starting equation, six variables *(m=6)* and three dimensions *(n=3)* are presented. So, three Π-terms *(m − n = 6 − 3 = 3)* are necessary in a complete functional equation. Taking μ , ρ and g as the three relevant variables, since they contain all dimensions (*[M]* , *[L]* , *[T]*) involved in the problem (Equation [A.4]) and cannot form Π-term themselves. The Π-terms are obtained by combining H , B and Q with the three relevant variables one at a time so that each combination is dimensionless. Thus,.

$$\Pi_1 = \mu^a \, \rho^b \, g^c \, H^d \qquad \text{[A.5]}$$

where a , b , c and d are constants. Writing equation [A.5] in dimensional form:

$$([M][L][T])^0 = (\frac{[M]}{[L][T]})^a \, (\frac{[M]}{[L]^3})^b \, (\frac{[L]}{[T]^2})^c \, [L]^d \qquad \text{[A.6]}$$

where [M] : $a + b = 0$

   [L] : $-a - 3b + c + d = 0$

   [T] : $a + 2c = 0$

Solving for a , b and d in terms of c so that equation [A.6] can be rewritten as:

$$\Pi_1 = \mu^{-2c} \, \rho^{2c} \, g^c \, H^{3c} = (\frac{g \, \rho^2 H^3}{\mu^2})^c \qquad \text{[A.7]}$$

The value of c is unimportant because the power of a dimensionless number is irrelevant when included in a functional equation. The two Π-terms may be formed in the same manner above (Sharp, 1981).

$$\Pi_2 = \mu^a \rho^b \, g^c \, B^d = \mu^{-2c} \, \rho^{2c} \, g^c \, B^{3c} = (\frac{g \, \rho^2 \, B^3}{\mu^2})^c \qquad \text{[A.8]}$$

$$\Pi_3 = \mu^a \rho^b \, g^c \, Q^d = \mu^{-5c} \, \rho^{5c} \, g^c \, Q^{3c} = (\frac{g \, \rho^5 \, Q^3}{\mu^5})^c \qquad \text{[A.9]}$$

Thus, the solution for this example is:

$$\frac{g \, \rho^5 Q^3}{\mu^5} = \varphi \, (\frac{g \, \rho^2 H^3}{\mu^2} \, , \, \frac{g \, \rho^2 B^3}{\mu^2}) \qquad \text{[A.10]}$$

APPENDIX   B

SIMILITUDE ANALYSIS

Similitude and dimensional analyses are both methods of partial analysis. Each provides an incomplete answer. Generally, the final result of both methods is a dimensionless, functional equation suitable for planning experimental studies or designing hydraulic models. However, dimensional analysis is directed towards experimentation, the model laws being obtained by inspection of the dimensionless equation. Similitude analysis concentrates primarily on developing model laws from which a dimensionless, functional equation may be derived (Sharp, 1981).

The three general types of similarity are geometric, kinematic and dynamic. Geometric similarity requires the shape of the model to be the same as the prototype. This is probably the most obvious requirement for a model. It is achieved by making sure that each length of the prototype is reduced by a constant factor called the *scale*. Kinematic similarity requires that the shape of the streamlines at any particular time is the same in both model and prototype. In order to achieve the same boundaries of the fluid system, the boundaries in the model and prototype must be similar. Therefore, geometric similarity is a prerequisite of kinematic similarity. The basic requirement for dynamic similarity is that the ratio of any two forces acting in the model must be equal to the corresponding force ratio in the prototype. Furthermore, the forces acting on a fluid system includes: (1) forces external to the fluid, such as gravity forces or forces resulting from pressure differences; (2) forces related to the physical properties of the fluid, such as viscous or surface tension forces; (3) resultant forces, such as drag force on a submerged object or the force of the fluid on a hydraulic structure and (4) hypothetical inertia force, such as inertia forces experienced by accelerative fluid systems (Kline,

1965). Table B.1 contains the standard dimensionless products relevant to fluid mechanics.

The application of the above similarity theory to the derivation of functional equations is known as similitude analysis. There are two basic approaches for similitude analysis. The first method uses the Buckingham Π-theorem to predict the necessary number of dimensionless parameters. These Π-terms are then developed using similarity theory. The second method is generally less applicable but valuable in certain specific circumstances. In cases where the dependent variable is a force, it is possible to write a dimensionally homogeneous equation in which every term has dimensions of force. This thesis will use the first method for similitude analysis (Sharp, 1981).

Table B.1: Standard Dimensionless Products Relevant to Fluid Mechanics (Sharp, 1981).

Name	Format	Comment
Froude number	$V/(gL)^{1/2}$	May also be defined as the square of this quantity. Relevant to **gravity forces**.
Reynolds number	VL/ν	Relevant to **viscous force** action.
Weber number	$V(\rho L/\sigma)^{1/2}$	May also be defined as the square of this quantity. Refers to the action of **surface tension forces**.
Euler number	$\Delta P/(\rho V^2)$	Also expressed as $V/(2\Delta P/\rho)^{1/2}$. Important when **pressure forces** exist. ΔP = pressure difference.
Cauchy number	$(\rho V^2)/K$	Relevant to system where compressibility is important. K = bulk modulus of fluid.
Mach number	V/C	Refers to compressibility effects, high-speed flow. C = local velocity of sound wave in fluid.
Richardson number	$(g\,\rho')/[\rho(V')^2]$	Relevant in cases where fluids mix or interact. ρ' and V' represent vertical density and velocity gradients, i.e. $\partial\rho/\partial H$ and $\partial V/\partial H$.
Froude-Reynolds number	$(g^{1/2}H^{3/2})/\nu$	Combined gravity and viscous effects.
Thoma number	$(P_1-P_v)/(P_2-P_1)$	A form of cavitation parameter. P_1 and P_2 are absolute pressures on the low- and high-pressure sides of a hydraulic machine. P_v is the vapour pressure of the liquid.

APPENDIX   C

DETERMINATION OF MODEL SCALES

The fundamental scale for any hydraulic model is the geometric scale. That is, the ratio of some length in the model to the corresponding length in the prototype. Choice of a suitable geometric scale depends on the type of fluid system to be studied and on the space available to build the model. However, once the scale of the model has been fixed, the requirements for dynamic similarity may be used to determine other model scales. These are needed in order to operate the model so that it will be dynamically similar to the prototype. Thus, measurements made in the model can be then used to determine prototype values (Pankhurst, 1964).

Although scaling criteria vary depending on the particular model laws to be followed, the procedure used to generate these scales does not change. Therefore, it is sufficient to demonstrate this procedure using the following model law, gravitational forces, which is the most appropriate to hydraulic models.

When gravity forces are important it has been shown that the Froude number (Table B.1), in some form or another, must be the same in model and prototype. For instance,

$$\left[\frac{V}{(gL)^{1/2}}\right]_m = \left[\frac{V}{(gL)^{1/2}}\right]_p \qquad \text{[C.1]}$$

where subscripts m and p refer to model and prototype.

Assuming that gravitational acceleration, g, is constant over the surface of the earth, rearrangement of Equation [C.1] gives the following velocity scale.

$$\frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2}$$

[C.2]

The ratio L_m/L_p on equation [C.2] is the geometric scale. This is often a source of unnecessary confusion because the geometric scale may be quoted as L_m/L_p or as L_p/L_m. However, a model would be referred to as, for example, a *1/20* scale model in which the scale is obviously L_m/L_p.

Equation [C.2] may be employed in two different ways. First, it can be used to determine the velocity at which the model should be run to ensure that it will accurately simulate prototype velocities. For instance,

$$V_m = V_p \left(\frac{L_m}{L_p}\right)^{1/2}$$

[C.3]

Alternatively, it may be used to predict prototype velocities from measured model velocities:

$$V_p = V_m \left(\frac{L_p}{L_m}\right)^{1/2}$$

[C.4]

Other scales may be derived by transforming the Froudian requirement into different forms. For example, discharge, Q, is proportional to the product of velocity and area so that

$$V \propto Q / L^2$$

[C.5]

Substituting equation [C.5] into equation [C.2] gives

$$\frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$ [C.6]

This is the discharge scale, commonly employed to determine the discharge at which the model should be run to simulate a known prototype discharge (Sharp, 1981).

APPENDIX   D

NEWTON-RAPHSON METHOD

Oil back-up system discussed in Chapter 4 involves simultaneous nonlinear equations with several variables. The Newton-Raphson method will be applied in order to solve these nonlinear equations. This method is derived from the Newton-Raphson method for solving equation of one variable. Its concept is discussed below.

NEWTON'S METHOD FOR SOLVING EQUATION WITH ONE VARIABLE

The Newton-Raphson method is calculus based and is one of the most popular root-solving methods for equations in the form of

$$f(x) = 0 \qquad\qquad [D.1]$$

In this method, information about a function $f(x)$ and its first derivative $f'(x)$ are used to improve estimates of a root of Equation [D.1] above. Let x_o be an initial estimate or guess of the root. A change of h will be determined such that $f(x_o + h)$ is zero.

The Taylor series expansion for a function about a point x_o is the infinite series.

$$f(x_o + h) = f(x_o) + h\frac{df(x_o)}{dx} + \frac{h^2}{2!}\frac{d^2 f(x_o)}{dx^2} + \cdots + \frac{h^n}{n!}\frac{d^n f(x_o)}{dx^n} + \cdots \qquad [D.2]$$

where $n!$ is the factorial product given by

$$n! = \begin{cases} (n)(n-1)(n-2)\ldots(2)(1); & n > 0 \\ 1; & n = 0 \end{cases} \qquad [D.3]$$

If it is assumed that the function $f(x)$ can be approximated locally by a straight line so that the second and higher derivatives of the Taylor series expansion (Equation [D.2]) are taken to be zero, the equation can be truncated immediately after the first derivative term. The local straight line approximation is then equivalent to the tangent line at x_o. The value of h for which $f(x_o + h)$ is zero can be approximated by

$$h = -\frac{f(x_o)}{f'(x_o)} \qquad \text{where} \quad f'(x_o) = \frac{df(x_o)}{dx} \qquad \text{[D.4]}$$

The improved estimate, $X_{improved}$, of the root is therefore given by

$$X_{improved} = x_o - \frac{f(x_o)}{f'(x_o)} \qquad \text{[D.5]}$$

After $X_{improved}$ is found from Equation [D.5], it is replaced by x_o and Equation [D.5] is applied again. This iterative process continues until convergence is achieved as indicated by the condition $|f(x_o)| \leq \varepsilon$, where ε is some small quantity for the stopping conditions (Fausett, 1999).

The reason of why the Newton-Raphson method so popular is that although the determination of the derivative may be tedious for complicated equations, the implementation of the method is simple and convergence is fairly rapid. Nevertheless, the method may fail for a variety of reasons. One obvious difficulty occurs when $f'(x_o)$ in the denominator of Equation [D.5] is zero. This situation may occur at the start of the procedure because of an unfortunate choice for the starting value x_o. The corrective

action is to reset x_o to a different value whenever $|f(x_o)|$ exceeds $A|f'(x_o)|$, where A is some large value based on the overflow characteristics of the computer (Akai, 1994).

Oscillations of $X_{improved}$ values may indicate that there is no real root of an equation as shown in Figure D.1. They may also occur when there is a root as shown in Figure D.2. For the case in Figure D.2, round-off errors will break the cycle; but then divergence is just as likely to occur as convergence. Divergence is shown for a function with two real roots in Figure D.3. For the functions in Figures D.2 and D.3, appropriate care in choosing the starting value x_o is necessary to guarantee convergence (Akai, 1994).
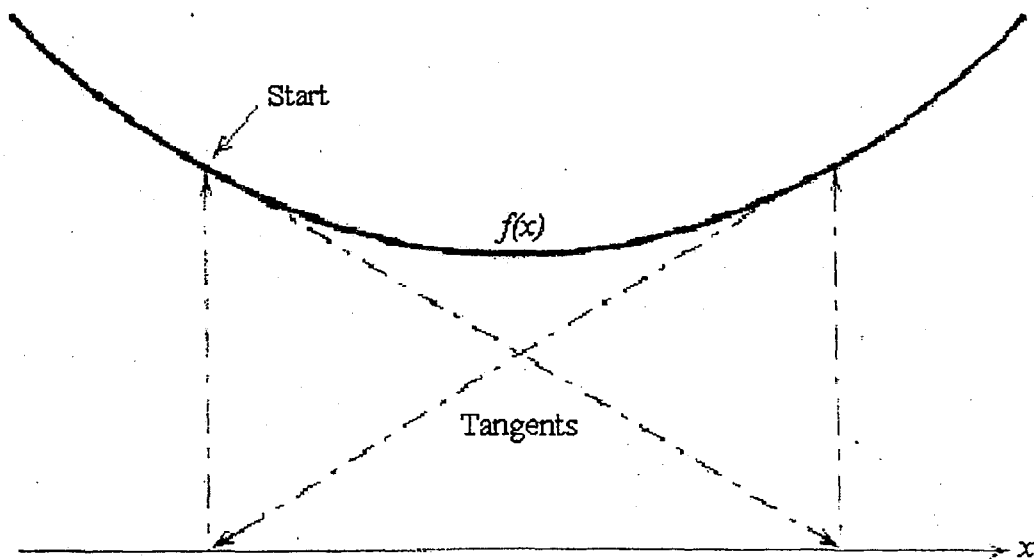
Figure D.1: Oscillations for a function with no real root (Akai, 1994).
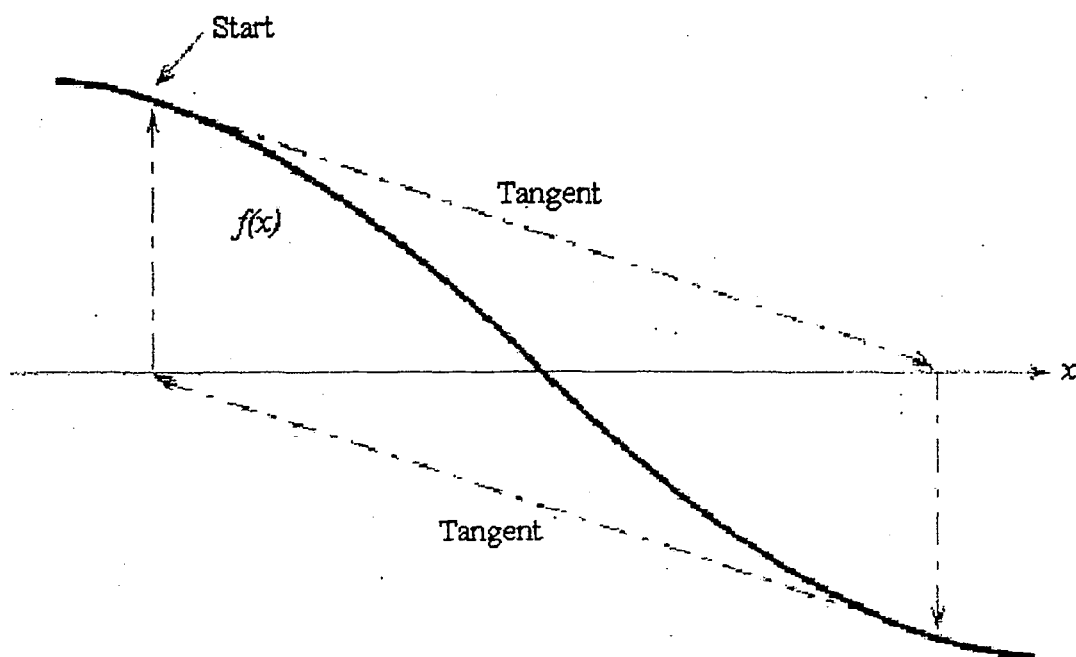
Figure D.2: Oscillations for a function with a real root (Akai, 1994).
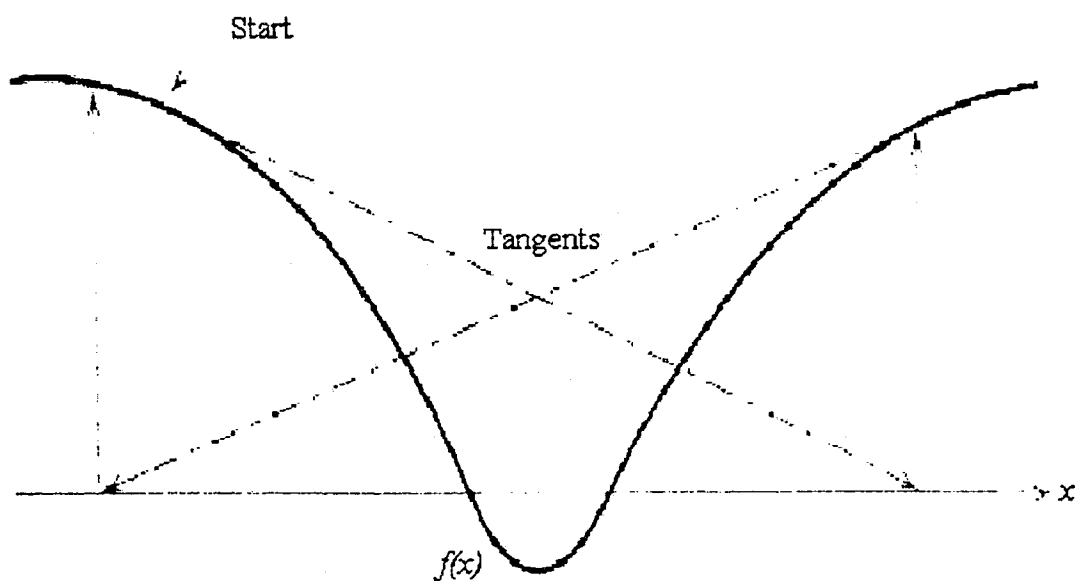
Figure D-3: Divergence for a function with two real roots (Akai, 1994).

It is also noticed that the Newton-Raphson method will not necessarily converge to a root that is nearest to the starting point. For example, the equation

$$\sin x = 0 \qquad where\ x\ is\ in\ radians \qquad [D.6]$$

has an infinite number of roots $n\Pi$, where n is an integer. If the starting value of x_o is *1.5*, the method converges to the root *(-4Π)*. It is therefore prudent to verify that the root found by the Newton-Raphson method is the desired one (Akai, 1994).

NEWTON'S METHOD FOR SOLVING SIMULTANEOUS NONLINEAR EQUATIONS

Solving simultaneous nonlinear equations using Newton-Raphson method is derived from the above method. For systems consist of n equations in n variables *(x_1, x_2, ... , x_n)* with its general form

$$f_1(x_1, x_2, x_3, \ldots, x_n) = 0 ;$$

$$f_2(x_1, x_2, x_3, \ldots, x_n) = 0 ;$$

$$f_3(x_1, x_2, x_3, \ldots, x_n) = 0 ;$$

$$\vdots$$

$$f_n(x_1, x_2, x_3, \ldots, x_n) = 0 \qquad [D.7]$$

The development of the procedure for solving systems of nonlinear equations requires the concepts of matrices, vectors, norms and solutions of linear systems. First, a column vector **X** is defined as

$$\mathbf{X}^T = [\; x_1 \quad x_2 \quad x_3 \; \ldots \; x_n \;] \tag{D.8}$$

Then, the system of equations can be rewritten in a more compactly form as

$$f_k(\mathbf{X}) = 0 \; ; \qquad \textit{where } k = 1, 2, \ldots, n \tag{D.9}$$

As in the Newton-Raphson method for solving equation with one variable, the solution process is started with an initial estimate $\mathbf{X}_0$ of the solution vector. The initial vector is written as

$$(\mathbf{X}_0)^T = [\; (x_1)_0 \quad (x_2)_0 \; \ldots \; (x_n)_0 \;] \tag{D.10}$$

and the improved estimate is

$$\mathbf{X}_{improved} = \mathbf{X}_0 + \mathbf{h} \tag{D.11}$$

where

$$(\mathbf{X}_{improved})^T = [(x_1)_{improved} \quad (x_2)_{improved} \; \ldots \; (x_n)_{improved}\;] \tag{D.12}$$

$$\mathbf{h}^T = [\; h_1 \quad h_2 \; \ldots \; h_n \;] \tag{D.13}$$

$$(x_k)_{improved} = (x_k)_0 + h_k \;; \qquad \textit{where } k = 1, 2, \ldots, n \tag{D.14}$$

The procedure for determining $\mathbf{h}$ is based on the truncated Taylor series for a function of several variables. If the truncation takes place immediately following the first derivative terms of the series, the equation of $f_k(\mathbf{x}_0 + \mathbf{h})$ becomes

$$f_k(\mathbf{x}_0 + \mathbf{h}) = f_k(\mathbf{x}_0) + \sum_{j=1}^{n} [\; h_j \;\; \partial f_k(\mathbf{x}_0)/\partial x_j \;] \; ; \tag{D.15}$$

$$\textit{where } k = 1, 2, \ldots, n$$

The matrix form of Equation [D.15] is

$$\mathbf{J} \cdot \mathbf{h} = -\mathbf{f} \qquad \text{[D.16]}$$

in which $\mathbf{f}$ is the column vector given by

$$\mathbf{f}^{\mathbf{T}} = [\, f_1(x_o) \quad f_2(x_o) \quad \cdots \quad f_n(x_o) \,] \qquad \text{[D.17]}$$

and $\mathbf{J}$ is the partial derivative matrix known as the *Jacobian* and is given by

$$\mathbf{J} = \begin{bmatrix} \partial f_1(\mathbf{x}_o)/\partial x_1 & \partial f_1(\mathbf{x}_o)/\partial x_2 & \cdots & \partial f_1(\mathbf{x}_o)/\partial x_n \\ \partial f_2(\mathbf{x}_o)/\partial x_1 & \partial f_2(\mathbf{x}_o)/\partial x_2 & \cdots & \partial f_2(\mathbf{x}_o)/\partial x_n \\ \vdots & \vdots & & \vdots \\ \partial f_n(\mathbf{x}_o)/\partial x_1 & \partial f_n(\mathbf{x}_o)/\partial x_2 & \cdots & \partial f_n(\mathbf{x}_o)/\partial x_n \end{bmatrix} \qquad \text{[D.18]}$$

The vector $\mathbf{h}$ in Equation [D.16] may be easily found using Equation [D.11] when n is small.

$$\mathbf{h} = \mathbf{X}_{\text{improved}} - \mathbf{X}_o \qquad \text{[D.19]}$$

The procedure for the multivariable Newton-Raphson method is as follows:

1. Choose a starting vector $\mathbf{X}_0$.

2. Compute $f_k(\mathbf{X}_0)$ for $k = 1, 2, \ldots, n$

3. If $\|\mathbf{f}\| \le \varepsilon$, $\mathbf{X}_0$ is the estimated solution. Otherwise, go on to step 4.

4. Obtain $\mathbf{h}$ from equation [D.16] and compute $\mathbf{X}_{\text{improved}}$ from equation [D.11].

5. Set $\mathbf{X}_0$ equal to $\mathbf{X}_{\text{improved}}$ and return to step 2.

The convergence criterion used in step 3 of the above procedure is similar to the one used in the previous section, except that the absolute value of a single function is now replaced by the norm of the vector whose components are the n function values $f_k(\mathbf{X_o})$. The ∞-norm, which is the component with the largest magnitude, provides an upper bound on the magnitudes of all components of $\mathbf{f}$. The corrective actions in the event of failure for multivariable Newton-Raphson method are the same as the single-variable version. However, the presence of several variables instead of only one may make corrective steps more difficult to accomplish (Akai, 1994).

APPENDIX   E

MATLAB FUNCTION FOR SOLVING NON-LINEAR

EQUATIONS

The development of the mathematical model for the oil back-up system is basically applying the MATLAB function for solving nonlinear equations using Newton's method below (Fausett, 1999).

```
function x = Newton_sys(F, JF, x0, tol, max_it)
% Solve the nonlinear system F(x) = 0 using Newton's method
% vectors x and x0 are row vectors (for display purposes)
% function F returns a column vector, [f1(x), .. fn(x)] '
% stop if norm of change in solution vector is less than tol
% solve JF(x) y = - F(x) using Matlab's "backslash operator"
% y = - feval(JF, x_old) \ feval(F, x_old);
% the next approximate solution is     x_new = x_old + y';
    x_old = x0;
    disp([0  x_old ]);
    iter = 1;
    while (iter <= max_it)
        y = - feval(JF, x_old) \ feval(F, x_old);
        x_new = x_old + y';
        dif = norm(x_new - x_old);
        disp([iter    x_new    dif]);
        if dif <= tol
            x = x_new;
            disp('Newton method has converged')
            return;
        else
            x_old = x_new;
        end
        iter = iter + 1;
    end
    disp('Newton method did not converge')
    x = x_new;
```

Lines 2 to 8 are explanations of the purpose of the MATLAB function. Line 9 is equivalent to step 1 of the procedure discussed on Appendix D. Line 13 is the determination of vector **h** (Appendix D). Line 14 is same as Equation [D.11]. Line 15

equals to step 3 of the procedure discussed on Appendix D. Variable *tol* at line 17 is the

stopping condition, ε, set by the programmer (Appendix D).

APPENDIX   F

DETERMINATION OF THE FLOW RATE AT EACH DIAL

Dial	Base Area, in m^2	Height, in metres	Volume, in litres	Time, in seconds	Flow Rate for Each Trial, in L/sec	Average Flow Rate for Each Dial, in L/sec
1	0.0952±0.00031	0.077±0.0005	7.330±0.07147	40.28±0.05	0.1820±0.002000	
		0.085±0.0005	8.092±0.07395	45.50±0.05	0.1778±0.001821	
		0.095±0.0005	9.044±0.07705	50.56±0.05	0.1789±0.001701	
		0.09±0.0005	8.568±0.0755	47.44±0.05	0.1806±0.001782	0.1798±0.001826
2	0.0952±0.00031	0.0725±0.0005	6.902±0.07008	40.40±0.05	0.1708±0.001946	
		0.082±0.0005	7.806±0.07302	45.46±0.05	0.1717±0.001795	
		0.092±0.0005	8.758±0.07612	50.48±0.05	0.1735±0.001680	
		0.1±0.0005	9.52±0.0786	55.73±0.05	0.1708±0.001564	0.1717±0.001746
3	0.0952±0.00031	0.063±0.0005	5.998±0.06713	40.55±0.05	0.1479±0.001838	
		0.081±0.0005	7.711±0.07271	50.58±0.05	0.1525±0.001588	
		0.0875±0.0005	8.33±0.07473	55.55±0.05	0.1500±0.001480	
		0.096±0.0005	9.139±0.07736	60.58±0.05	0.1509±0.001402	0.1503±0.001577
4	0.0952±0.00031	0.0705±0.0005	6.712±0.06946	50.58±0.05	0.1327±0.001504	
		0.086±0.0005	8.187±0.07426	55.76±0.05	0.1468±0.001463	
		0.088±0.0005	8.378±0.07488	60.62±0.05	0.1382±0.001349	
		0.102±0.0005	9.710±0.07922	70.53±0.05	0.1377±0.001221	0.1389±0.001384

Dial	Base Area, in m^2	Height, in metres	Volume, in litres	Time, in seconds	Flow Rate for Each Trial, in L/sec	Average Flow Rate for Each Dial, in L/sec
5	0.0952±0.00031	0.062±0.0005	5.902±0.06682	60.44±0.05	0.09766±0.001186	
		0.085±0.0005	8.092±0.07395	70.55±0.05	0.1147±0.001129	
		0.0855±0.0005	8.140±0.07411	75.53±0.05	0.1078±0.001053	
		0.0925±0.0005	8.806±0.07628	80.48±0.05	0.1094±0.001016	0.1074±0.001096
6	0.0484±0.00022	0.044±0.0005	2.130±0.03388	30.57±0.05	0.06966±0.001222	
		0.052±0.0005	2.517±0.03564	35.58±0.05	0.07074±0.001101	
		0.061±0.0005	2.952±0.03762	40.60±0.05	0.07272±0.001016	
		0.049±0.0005	2.372±0.03498	32.57±0.05	0.07282±0.001186	0.07148±0.001132
7	0.0484±0.00022	0.041±0.0005	1.984±0.03322	30.57±0.05	0.06491±0.001193	
		0.051±0.0005	2.468±0.03542	40.48±0.05	0.06098±0.0009503	
		0.058±0.0005	2.807±0.03696	50.67±0.05	0.05540±0.0007841	
		0.053±0.0005	2.565±0.03586	45.60±0.05	0.05625±0.0008481	0.05939±0.0009439
8	0.0484±0.00022	0.049±0.0005	2.372±0.03498	60.41±0.05	0.03926±0.0006115	
		0.0675±0.0005	3.267±0.03905	80.44±0.05	0.04061±0.0005107	
		0.051±0.0005	2.468±0.03542	70.30±0.05	0.03511±0.0005288	
		0.054±0.0005	2.614±0.03608	75.51±0.05	0.03461±0.0005007	0.03740±0.0005379

APPENDIX   G

DETERMINATION OF THE INFLOW RATE OF THE

TRANSFORMER OIL

Time, in seconds	Volume of Transformer Oil Goes into Catchbasin for One Time Step, in m^3	Height of Transformer Oil at Concrete Pad, in metres	Total Volume of Transformer Oil Remains at Concrete Pad, in m^3	Flow Enters to Catchbasin, in m^3/sec	Average Flow Enters to Catchbasin at Each Time Step, in m^3/sec
0	-----	0.5	7.5905	0.2147	-----
2	0.4293	0.4717	7.161	0.2085	0.2116
4	0.4170	0.4443	6.744	0.2023	0.2054
6	0.4047	0.4176	6.339	0.1962	0.1993
8	0.3924	0.3917	5.947	0.1900	0.1931
10	0.3800	0.3667	5.567	0.1838	0.1869
12	0.3677	0.3425	5.199	0.1777	0.1808
14	0.3553	0.3191	4.844	0.1715	0.1746
16	0.3430	0.2965	4.501	0.1653	0.1684
18	0.3306	0.2747	4.171	0.1591	0.1622
20	0.3182	0.2538	3.852	0.1529	0.1560
22	0.3059	0.2336	3.546	0.1467	0.1498
24	0.2935	0.2143	3.253	0.1405	0.1436
26	0.2811	0.1958	2.972	0.1343	0.1374
28	0.2686	0.1781	2.703	0.1281	0.1312
30	0.2562	0.1612	2.447	0.1219	0.1250

Time, in seconds	Volume of Transformer Oil Goes into Catchbasin for One Time Step, in m^3	Height of Transformer Oil at Concrete Pad, in metres	Total Volume of Transformer Oil Remains at Concrete Pad, in m^3	Flow Enters to Catchbasin, in m^3/sec	Average Flow Enters to Catchbasin at Each Time Step, in m^3/sec
32	0.2438	0.1451	2.203	0.1157	0.1188
34	0.2313	0.1299	1.972	0.1094	0.1125
36	0.2188	0.1155	1.753	0.1032	0.1063
38	0.2063	0.1019	1.547	0.09691	0.1000
40	0.1938	0.08913	1.353	0.09063	0.09377
42	0.1813	0.07719	1.172	0.08434	0.08749
44	0.1687	0.06608	1.003	0.07804	0.08119
46	0.1561	0.05580	0.8470	0.07171	0.07487
48	0.1434	0.04635	0.7036	0.06536	0.06853
50	0.1307	0.03774	0.5729	0.05897	0.06217
52	0.1179	0.02997	0.4550	0.05255	0.05576
54	0.1051	0.02305	0.3499	0.04609	0.04932
56	0.0922	0.01697	0.2577	0.03955	0.04282
58	0.07910	0.01176	0.1786	0.03293	0.03624
60	0.06585	0.007426	0.1127	0.02616	0.02954
62	0.05232	0.003979	0.06041	0.01915	0.02266
64	0.03830	0.001456	0.02211	0.01159	0.01537

APPENDIX   H

MATLAB PROGRAMS FOR THE MATHEMATICAL MODEL

There are four MATLAB programs used for the mathematical model of the oil back-up system. Each program represents different stages and conditions of the time steps. The mathematical model starts with the *initial.m* file (Figure H.1), where boundary conditions are applied to the heights at $t = 0$ (lines 27 to 31 in Figure H.1). Equations discussed in Section 4.2 are the nonlinear equations and the elements of the Jacobian matrix used in this program. In Figure H.1, lines 3 to 25 are the design dimensions of the oil back-up system assuming the initial height of spilled transformer oil at the concrete pad at $t = 0$ is 0.5 m (Appendix G). Lines 33 to 37 are the transformation of the variables. The numbers shown in lines 39 to 43 are the initial guesses for the Newton-Raphson method (Appendix D). Lines 44 to 65 are the preliminary setup of the variables and matrices involving in the Newton-Raphson method, where lines 56 to 59 are the function matrix of the four simultaneous nonlinear equations and lines 62 to 65 demonstrates the Jacobian matrix. MATLAB program uses space to separate the elements within a row. Thus, the yellow indications in lines 62 to 65 represent the starting point of each element in the Jacobian matrix. Lines 67 to 83 show the code from Appendix E for solving simultaneous nonlinear equations using Newton's method. Figure H.2 illustrates the commands between lines 43 to 83 of the *initial.m* file inside the MATLAB program. In reality, each row of the equation matrix (lines 56 to 59 in Figure H.2) and the Jacobian matrix (lines 62 to 65 in Figure H.2) in MATLAB program must be displayed within one line. This is different than the illustrations in Figure H.1 due to the lack of space.

```
Line
1     % Solve nonlinear system F(x)=0 using Newton's method
2
3     % input data
4     inflow = 0.211582;
5     delta_time = 2;
6     gravity = 9.806;
7     c = 0.6;
8     density_oil = 0.8611;
9     density_water = 1;
10    area_catchbasin = 0.61^2;
11    area_system = 1.3^2;
12    diameter_internal_pipe = 0.1;
13    diameter_external_pipe = 0.3;
14    diameter_hp1 = 0.25;
15    diameter_hp2 = diameter_internal_pipe;
16    area_subtracted = 0;
17    length_hp2 = 0.6;
18    h = 1.222;
19    h_column = 8.228;
20
21    % cross-sectional areas of different pipes in the system
22    area_internal_pipe = pi*((diameter_internal_pipe/2)^2);
23    area_external_pipe = pi*((diameter_external_pipe/2)^2);
24    area_hp1 = pi*((diameter_hp1/2)^2);
25    area_hp2 = area_internal_pipe;
26
27    % heights at t=0, boundary situation
28    H1_n = h_column+h;
29    H2_n = 0;
30    H3_n = h_column+h;
31    H4_n = h_column+h;
32
33    % substitute height variables into x variables in order to eliminate square root
      expression in orifice equation
34    X1_n = sqrt(H1_n-H2_n-H3_n);
35    X2_n = H2_n;
36    X3_n = H3_n;
37    X4_n = sqrt(H4_n-h-h_column)
38
39    % set initial iteration for the next time step, variables X_n1
40    X1_n1 = 0.88754914631796;
```

```
41    X2_n1 = 0.06386127788409;
42    X3_n1 = 9.44693059030206;
43    X4_n1 = 0.22786907583763;
44    X_n1_old = [X1_n1
45                X2_n1
46                X3_n1
47                X4_n1];
48
49    % set stopping conditions and maximum iteration runs
50    error = 1*10^-5;
51    max_iteration_runs = 10;
52    iteration = 1;
53    disp([0    X_n1_old']);
54
55    % function F for case #1
56    F1 = [area_catchbasin*(X1_n1^2)+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)*X1_n1+area_catchbasin*X2_n1
            +area_catchbasin*X3_n1-area_catchbasin*(X1_n^2)+0.5*delta_time*c
            *area_hp1*sqrt(2*density_oil*gravity)*X1_n-area_catchbasin*X2_n
            -area_catchbasin*X3_n-delta_time*inflow
57          0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n1
            -(area_system-area_internal_pipe)*X2_n1+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)*X1_n+(area_system-area_internal_pipe)
            *X2_n
58          (area_system-area_internal_pipe)*X3_n1+area_internal_pipe*(X4_n1^2)
            +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n1
            +(area_internal_pipe-area_system)*X3_n-area_internal_pipe*(X4_n^2)
            +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n
59          density_oil*X2_n1+density_water*X3_n1-density_water*(X4_n1^2)
            -density_water*h_column-density_water*h];
60
61    % Jacobian matrix for F1
62    J1 = [2*area_catchbasin*X1_n1+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)  area_catchbasin  area_catchbasin  0
63          0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)
            area_internal_pipe-area_system  0  0
64          0  0  area_system-area_internal_pipe  2*area_internal_pipe*X4_n1
            +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)
65          0  density_oil  density_water  -2*density_water*X4_n1];
66
67    while (iteration<=max_iteration_runs)
68      X_n1_new=X_n1_old-(inv(J1)*F1);
```

```
69      difference=norm(X_n1_new-X_n1_old);
70      disp([iteration    X_n1_new'    difference]);
71      if difference<=error
72         disp('Newton method has converged')
73         disp(' ')
74         H1=X_n1_new(2,1)+X_n1_new(3,1)+X_n1_new(1,1)^2
75         H2=X_n1_new(2,1)
76         H3=X_n1_new(3,1)
77         H4=h_column+h+X_n1_new(4,1)^2
78         return
79      else
80         X_n1_old=X_n1_new;
81      end
82      iteration=iteration+1;
83    end
```

Figure H.1: Commands in *initial.m* file.

File Edit Text Window Help

```matlab
43  X4_nl = 0.2278690758763;
44  X_nl_old = [X1_nl
45              X2_nl
46              X3_nl
47              X4_nl];
48
49  % set stopping conditions and maximum iteration runs
50  error = 1*10^-5;
51  max_iteration_runs = 10;
52  iteration = 1;
53  disp([0    X_nl_old']);
54
55  % function F for case #1
56  F1 = [area_catchbasin*(X1_nl^2)+0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_nl+area_catchbasin*X2_nl+area_catchbasin*X3_nl-are
57        0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_nl-(area_system-area_internal_pipe)*X2_nl+0.5*delta_time*c*area_hp1*sqrt(2*c
58        (area_system-area_internal_pipe)*X3_nl+area_internal_pipe*(X4_nl^2)+0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_nl+(ar
59        density_oil*X2_nl+density_water*X3_nl-density_water*(X4_nl^2)-density_water*h_column-density_water*h];
60
61  % Jacobian matrix for F1
62  J1 = [2*area_catchbasin*X1_nl+0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)    area_catchbasin    area_catchbasin    0
63        0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)    area_internal_pipe-area_system    0    0
64        0    0    area_system-area_internal_pipe    2*area_internal_pipe*X4_nl+0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)
65        0    density_oil    density_water    -2*density_water*X4_nl];
66
67  while (iteration<max_iteration_runs)
68      X_nl_new=X_nl_old-(inv(J1)*F1);
69      difference=norm(X_nl_new-X_nl_old);
70      disp([iteration    X_nl_new'    difference]);
71      if difference<error
72          disp('Newton method has converged')
73          disp(' ')
74          H1=X_nl_new(2,1)+X_nl_new(3,1)+X_nl_new(1,1)^2
75          H2=X_nl_new(2,1)
76          H3=X_nl_new(3,1)
77          H4=h_column+h*X_nl_new(4,1)^2
78          return
79      else
80          X_nl_old=X_nl_new;
81      end
82      iteration=iteration+1;
83  end
```

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Figure C.2: *initial.m* file in MATLAB program

A new program in *case1.m* file (Figure H.3) will be applied on the mathematical model for calculating the heights at $t = 4$. This program is used a similar set of commands as the *initial.m* file. As the name of this file has indicated, this program represents Case 1 of the mathematical model (Sections 4.1 and 4.2). Simultaneous nonlinear equations and Jacobian matrix in this program are exactly the same as the last program in the *initial.m* file. The reason for applying another program at $t = 4$ is because boundary conditions are no longer dominate the heights. Instead, the solutions of $X1[n+1]$, $X2[n+1]$, $X3[n+1]$, $X4[n+1]$ found at $t = 2$ (row 3 columns 2 to 4 in Appendix I) will be substituted into $X1[n]$, $X2[n]$, $X3[n]$, $X4[n]$ at $t = 4$ (Lines 27 to 31 in Figure H.3). The first line of the *case1.m* file states the restrictions of the purpose of this program. This program is only applied when

$$(h_{column} + h) < (H2[n] + H3[n]) < (h_{column} + h + D_{hp2})$$
$$H1[n] < (h_{column} + h + 1.35)$$
$$H3[n] \geq h_{column} \qquad\qquad\qquad\qquad [H.1]$$

Line	
1	% H1 has not exceed the catchbasin yet and H3 is higher than h_column
2	
3	%input data
4	inflow = 0.205421;
5	delta_time = 2;
6	gravity = 9.806;
7	c = 0.6;
8	density_oil = 0.8611;
9	density_water = 1;
10	area_catchbasin = 0.61^2;
11	area_system = 1.3^2;
12	diameter_internal_pipe = 0.1;
13	diameter_external_pipe = 0.3;
14	diameter_hp1 = 0.25;
15	diameter_hp2 = diameter_internal_pipe;
16	radius_hp2 = 0.5*diameter_hp2;
17	length_hp2 = 0.6;
18	h = 1.222;
19	h_column = 8.228;
20	
21	% cross-sectional areas of different pipes in the system
22	area_internal_pipe = pi*((diameter_internal_pipe/2)^2);
23	area_external_pipe = pi*((diameter_external_pipe/2)^2);
24	area_hp1 = pi*((diameter_hp1/2)^2);
25	area_hp2 = area_internal_pipe;
26	
27	% substitute x variables, which is the solutions of previous time step calculation
28	X1_n = 0.88754911249787;
29	X2_n = 0.06386127545065;
30	X3_n = 9.44693066368018;
31	X4_n = 0.22786313437244;
32	
33	% set initial iteration for the next time step, variables X_n1
34	X1_n1 = 1.06026848508006;
35	X2_n1 = 0.20639738602528;
36	X3_n1 = 9.43851166778697;
37	X4_n1 = 0.40772596887888;
38	X_n1_old = [X1_n1
39	X2_n1
40	X3_n1
41	X4_n1];

```
42
43    % set stopping conditions and maximum iteration runs
44    error = 1*10^-5;
45    max_iteration_runs = 10;
46    iteration = 1;
47    disp([0    X_n1_old']);
48
49    % function F for Case 1a
50    F1 = [area_catchbasin*(X1_n1^2)+0.5*delta_time*c*area_hp1
      *sqrt(2*density_oil*gravity)*X1_n1+area_catchbasin*X2_n1+area_catchbasin
      *X3_n1-area_catchbasin*(X1_n^2)+0.5*delta_time*c*area_hp1
      *sqrt(2*density_oil*gravity)*X1_n-area_catchbasin*X2_n-area_catchbasin
      *X3_n-delta_time*inflow
51    0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n1-(area_system
      -area_internal_pipe)*X2_n1+0.5*delta_time*c*area_hp1
      *sqrt(2*density_oil*gravity)*X1_n+(area_system-area_internal_pipe)*X2_n
52    (area_system-area_internal_pipe)*X3_n1+area_internal_pipe*(X4_n1^2)
      +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n1
      +(area_internal_pipe-area_system)*X3_n-area_internal_pipe*(X4_n^2)
      +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n
53    density_oil*X2_n1+density_water*X3_n1-density_water*(X4_n1^2)
      -density_water*h_column-density_water*h];
54
55    % Jacobian matrix for F1
56    J1 = [2*area_catchbasin*X1_n1+0.5*delta_time*c*area_hp1
      *sqrt(2*density_oil*gravity)  area_catchbasin  area_catchbasin  0
57    0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)
      area_internal_pipe-area_system  0  0
58    0  0  area_system-area_internal_pipe  2*area_internal_pipe*X4_n1
      +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)
59    0  density_oil  density_water  2*density_water*X4_n1];
60
61    while (iteration<=max_iteration_runs)
62       X_n1_new=X_n1_old-(inv(J1)*F1);
63       difference=norm(X_n1_new-X_n1_old);
64       disp([iteration    X_n1_new'    difference]);
65       if difference<=error
66          disp('Newton method has converged')
67          disp(' ')
68          H1=X_n1_new(2,1)+X_n1_new(3,1)+X_n1_new(1,1)^2
69          H2=X_n1_new(2,1)
70          H3=X_n1_new(3,1)
```

```
71        H4=h_column+h+X_n1_new(4,1)^2
72        return
73    else
74        X_n1_old=X_n1_new;
75    end
76    iteration=iteration+1;
77  end
```

Figure H.3: Commands in *case1.m* file.

Since the second statement in Equation [H.1], $H1 < (h_{column} + h + 1.35)$, failed at $t = 8$, the mathematical model is shifted to another program (Figure H.4) in the *case1A.m* file. Equations involved in this program are exactly the same as the first two programs. There are two adjustments in this program. First, variable Q_{in} will be equal to zero, since the flow of transformer oil goes into the oil back-up system through horizontal pipe *hp1* is now depending by its head. Thus, $X1[n]$ at $t = 8$ (line 28 in Figure H.4) is calculated using the height of transformer oil at concrete pad at the previous time step (row 5 column 3 in Appendix G) where

$$
\begin{aligned}
(X1[n])_{t=8} &= \sqrt{H1[n] - H2[n] - H3[n]} \\
&= \sqrt{(height\ of\ transformer\ oil\ at\ concrete\ pad)_{t=6} - H2[n] - H3[n]} \\
&= \sqrt{(0.417594 + 9.45 + 1.35) - 0.363408 - 9.426267} \\
&= 1.1949555
\end{aligned}
\qquad [H.2]
$$

Secondly, the change of $H1$ is occurred at the concrete pad instead of the catchbasin once the second statement in equation [H.1] failed. Therefore, the parameter, $A_{catchbasin}$, in and in the first row of the Jacobian matrix (Line 51 in Figure H.4) will be replaced by A_{pad}.

Line

1   *%H1 exceeds the catchbasin and H3 is higher than h_column*

2

3   *%input data*

4   *inflow = 0;*

5   *delta_time = 2;*

6   *gravity = 9.806;*

7   *c = 0.6;*

8   *density_oil = 0.8611;*

9   *density_water = 1;*

10   *area_catchbasin = 0.61^2;*

11   *area_system = 1.3^2;*

12   *diameter_internal_pipe = 0.1;*

13   *diameter_external_pipe = 0.3;*

14   *diameter_hp1 = 0.25;*

15   *diameter_hp2 = diameter_internal_pipe;*

16   *radius_hp2 = 0.5*diameter_hp2;*

17   *length_hp2 = 0.6;*

18   *h = 1.222;*

19   *h_column = 8.228;*

20   *area_concrete_pad = 15.181;*

21

22   *% cross-sectional areas of different pipes in the system*

23   *area_internal_pipe = pi*((diameter_internal_pipe/2)^2);*

24   *area_external_pipe = pi*((diameter_external_pipe/2)^2);*

25   *area_hp1 = pi*((diameter_hp1/2)^2);*

26   *area_hp2 = area_internal_pipe;*

27

28   *% substitute x variables, which is the solutions of previous time step calculation*

29   *X1_n = 1.32950888859256;*

30   *X2_n = 0.36340804118698;*

31   *X3_n = 9.42626730751471;*

32   *X4_n = 0.53777130062957;*

33

34   *% set initial iteration for the next time step, variables X_n1*

35   *X1_n1 = 0.93900277273695;*

36   *X2_n1 = 0.52663286562454;*

37   *X3_n1 = 9.41102316871508;*

38   *X4_n1 = 0.64382197204378;*

39   *X_n1_old = [X1_n1*

40   *X2_n1*

41   *X3_n1*

```
42              X4_n1];

43

44    % set stopping conditions and maximum iteration runs
45    error = 1*10^-5;
46    max_iteration_runs = 10;
47    iteration = 1;
48    disp([0    X_n1_old']);

49

50    % function F for Case 1a
51    F1 = [area_concrete_pad *(X1_n1^2)+0.5*delta_time*c*area_hp1
              *sqrt(2*density_oil*gravity)*X1_n1+ area_concrete_pad *X2_n1
              + area_concrete_pad *X3_n1- area_concrete_pad *(X1_n^2)
              +0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n
              - area_concrete_pad *X2_n- area_concrete_pad *X3_n-delta_time*inflow
52            0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n1
              -(area_system-area_internal_pipe)*X2_n1+0.5*delta_time*c
              *area_hp1*sqrt(2*density_oil*gravity)*X1_n+(area_system
              -area_internal_pipe)*X2_n
53            (area_system-area_internal_pipe)*X3_n1+area_internal_pipe*(X4_n1^2)
              +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n1
              +(area_internal_pipe-area_system)*X3_n-area_internal_pipe*(X4_n^2)
              +0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)*X4_n
54            density_oil*X2_n1+density_water*X3_n1-density_water*(X4_n1^2)
              -density_water*h_column-density_water*h];

55

56    % Jacobian matrix for F1
57    J1 = [2* area_concrete_pad *X1_n1+0.5*delta_time*c*area_hp1
              *sqrt(2*density_oil*gravity)  area_concrete_pad  area_concrete_pad  0
58            0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)
              area_internal_pipe-area_system  0  0
59            0  0  area_system-area_internal_pipe
              2*area_internal_pipe*X4_n1+0.5*delta_time*c*area_hp2
              *sqrt(2*density_water*gravity)
60            0  density_oil  density_water  -2*density_water*X4_n1];

61

62    while (iteration<=max_iteration_runs)
63      X_n1_new=X_n1_old-(inv(J1)*F1);
64      difference=norm(X_n1_new-X_n1_old);
65      disp([iteration    X_n1_new'    difference]);
66      if difference<=error
67        disp('Newton method has converged')
68        disp(' ')
```

```
69          H1=X_n1_new(2,1)+X_n1_new(3,1)+X_n1_new(1,1)^2
70          H2=X_n1_new(2,1)
71          H3=X_n1_new(3,1)
72          H4=h_column+h+X_n1_new(4,1)^2
73          return
74      else
75         X_n1_old=X_n1_new;
76      end
77      iteration=iteration+1;
78  end
```

Figure H.4: Commands in *case1A.m* file.

Finally, the mathematical model changes to another program in *case2A.m* file (Figure H.5) at $t = 102$ since $H3$ is lower than h at $t = 100$. This means that the interface of oil and water is eventually reached the pipe column of the oil back-up system at $t = 100$. Equations in Case 2 of Section 4.2 are the nonlinear equations and the elements of the Jacobian matrix involved in this program. The structure of this program is similar to the one in *case1A.m* file.

```
Line
1      %H1 exceeds catchbasin and H3 is lower than h_column
2
3      %input data
4      inflow = 0;
5      delta_time = 2;
6      gravity = 9.806;
7      c = 0.6;
8      density_oil = 0.8611;
9      density_water = 1;
10     area_catchbasin = 0.61^2;
11     area_system = 1.3^2;
12     diameter_internal_pipe = 0.1;
13     diameter_external_pipe = 0.3;
14     diameter_hp1 = 0.25;
15     diameter_hp2 = diameter_internal_pipe;
16     radius_hp2 = 0.5*diameter_hp2;
17     length_hp2 = 0.6;
18     h = 1.222;
19     h_column = 8.228;
20     area_concrete_pad = 15.181;
21
22     % cross-sectional areas of different pipes in the system
23     area_internal_pipe = pi*((diameter_internal_pipe/2)^2);
24     area_external_pipe = pi*((diameter_external_pipe/2)^2);
25     area_hp1 = pi*((diameter_hp1/2)^2);
26     area_hp2 = area_internal_pipe;
27
28     % substitute x variables, which is the solutions of previous time step calculation
29     X1_n = 0.16354985861792;
30     X2_n = 2.72515762854350;
31     X3_n = 8.20399165473126;
32     X4_n = 1.04910670986060;
33
34     % set initial iteration for the next time step, variables X_n1
35     X1_n1 = 0.1631;
36     X2_n1 = 2.75;
37     X3_n1 = 8.17;
38     X4_n1 = 1.046;
39     X_n1_old = [X1_n1
40                 X2_n1
41                 X3_n1
```

```
42          X4_n1];
43
44   % set stopping conditions and maximum iteration runs
45   error = 1*10^-5;
46   max_iteration_runs = 10;
47   iteration = 1;
48   disp([0    X_n1_old']);
49
50   % function F for Case 1a
51   F1 = [area_concrete_pad*(X1_n1^2)+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)*X1_n1+area_concrete_pad*X2_n1
            +area_concrete_pad*X3_n1-area_concrete_pad*(X1_n^2)
            +0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n
            -area_concrete_pad*X2_n-area_concrete_pad*X3_n-delta_time*inflow
52          0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)*X1_n1
            -(area_system-area_internal_pipe)*X2_n1+(area_external_pipe
            -area_system)*X3_n1+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)*X1_n+(area_system
            -area_internal_pipe)*X2_n+(area_system-area_external_pipe)*X3_n
53          (area_external_pipe-area_internal_pipe)*X3_n1+area_internal_pipe
            *(X4_n1^2)+0.5*delta_time*c*area_hp2*sqrt(2*density_water
            *gravity)*X4_n1+(area_internal_pipe-area_external_pipe)*X3_n
            -area_internal_pipe*(X4_n^2)+0.5*delta_time*c*area_hp2
            *sqrt(2*density_water*gravity)*X4_n
54          density_oil*X2_n1+density_water*X3_n1-density_water*(X4_n1^2)
            -density_water*h_column-density_water*h];
55
56   % Jacobian matrix for F1
57   J1 = [2*area_concrete_pad*X1_n1+0.5*delta_time*c*area_hp1
            *sqrt(2*density_oil*gravity)  area_concrete_pad  area_concrete_pad  0
58          0.5*delta_time*c*area_hp1*sqrt(2*density_oil*gravity)
            area_internal_pipe-area_system  area_external_pipe-area_system  0
59          0  0  area_external_pipe-area_internal_pipe  2*area_internal_pipe
            *X4_n1+0.5*delta_time*c*area_hp2*sqrt(2*density_water*gravity)
60          0  density_oil  density_water  -2*density_water*X4_n1];
61
62   while (iteration<=max_iteration_runs)
63      X_n1_new=X_n1_old-(inv(J1)*F1);
64      difference=norm(X_n1_new-X_n1_old);
65      disp([iteration    X_n1_new'    difference]);
66      if difference<=error
67          disp('Newton method has converged')
```

```
68          disp(' ')
69          H1=X_n1_new(2,1)+X_n1_new(3,1)+X_n1_new(1,1)^2
70          H2=X_n1_new(2,1)
71          H3=X_n1_new(3,1)
72          H4=h_column+h+X_n1_new(4,1)^2
73          return
74       else
75          X_n1_old=X_n1_new;
76       end
77       iteration=iteration+1;
78    end
```

Figure H.5: Commands in *case2A.m* file.

APPENDIX   I

NUMERICAL RESULTS OF THE MATHEMATICAL MODEL

Time in sec	X1[n+1]	X2[n+1]	X3[n+1]	X4[n+1]	H1, in metres	H2, in metres	H3, in metres	H4, in metres	X1[n]	X2[n]	X3[n]	X4[n]	H2+H3
0	0	0	9.45	0	9.45	0	9.45	9.45	-----	-----	-----	-----	9.45
2	0.8876	0.06386	9.447	0.2279	10.30	0.06386	9.447	9.502	0	0	9.450	0	9.511
4	1.061	0.2041	9.439	0.4053	10.77	0.2041	9.439	9.614	0.8876	0.06386	9.447	0.2279	9.643
6	1.153	0.3634	9.426	0.5378	11.12	0.3634	9.426	9.739	1.061	0.2041	9.439	0.4053	9.790
8	1.122	0.5301	9.411	0.6461	11.20	0.5301	9.411	9.867	1.195	0.3634	9.426	0.5378	9.941
10	1.050	0.6863	9.393	0.7310	11.18	0.6863	9.393	9.984	1.123	0.5301	9.411	0.6461	10.08
12	0.9795	0.8324	9.374	0.8004	11.17	0.8324	9.374	10.09	1.050	0.6863	9.393	0.7310	10.21
14	0.9107	0.9684	9.353	0.8583	11.15	0.9684	9.353	10.19	0.9795	0.8324	9.374	0.8004	10.32
16	0.8435	1.095	9.331	0.9072	11.14	1.095	9.331	10.27	0.9107	0.9684	9.353	0.8583	10.43
18	0.7780	1.211	9.307	0.9488	11.12	1.211	9.307	10.35	0.8435	1.095	9.331	0.9072	10.52
20	0.7144	1.319	9.283	0.9840	11.11	1.319	9.283	10.42	0.7780	1.211	9.307	0.9488	10.61
22	0.6528	1.417	9.258	1.014	11.10	1.417	9.258	10.48	0.7144	1.319	9.283	0.9840	10.67
24	0.5935	1.507	9.232	1.039	11.09	1.507	9.232	10.53	0.6528	1.417	9.258	1.014	10.74
26	0.5368	1.588	9.206	1.060	11.08	1.588	9.206	10.57	0.5935	1.507	9.233	1.039	10.79
28	0.4830	1.661	9.179	1.077	11.07	1.661	9.179	10.61	0.5368	1.588	9.206	1.060	10.84
30	0.4326	1.727	9.152	1.091	11.07	1.727	9.152	10.64	0.4830	1.661	9.179	1.077	10.88
32	0.3860	1.786	9.125	1.101	11.06	1.786	9.125	10.66	0.4326	1.727	9.152	1.091	10.91
34	0.3438	1.839	9.097	1.109	11.05	1.839	9.097	10.68	0.3860	1.786	9.125	1.101	10.94

Time in sec	X1[n+1]	X2[n+1]	X3[n+1]	X4[n+1]	H1, in metres	H2, in metres	H3, in metres	H4, in metres	X1[n]	X2[n]	X3[n]	X4[n]	H2+H3
36	0.3064	1.885	9.070	1.115	11.05	1.885	9.070	10.69	0.3438	1.839	9.097	1.109	10.96
38	0.2743	1.927	9.042	1.119	11.04	1.927	9.042	10.70	0.3064	1.885	9.070	1.115	10.97
40	0.2476	1.965	9.014	1.121	11.04	1.965	9.014	10.71	0.2743	1.927	9.042	1.119	10.98
42	0.2264	1.999	8.986	1.121	11.04	1.999	8.986	■	0.2476	1.965	9.014	1.121	10.99
44	0.2102	2.030	8.958	1.121	11.03	2.030	8.958	10.71	0.2264	1.999	8.986	1.121	10.99
46	0.1983	2.060	8.931	1.120	11.03	2.060	8.931	10.70	0.2102	2.030	8.958	1.121	10.99
48	0.1899	2.088	8.903	1.118	11.03	2.088	8.903	10.70	0.1983	2.060	8.931	1.120	■
50	0.1840	2.114	8.875	1.116	11.02	2.114	8.875	10.70	0.1899	2.088	8.903	1.118	10.99
52	0.1800	2.141	8.848	1.114	11.02	2.141	8.848	10.69	0.1840	2.114	8.875	1.116	10.99
54	0.1773	2.166	8.820	1.112	11.02	2.166	8.820	10.69	0.1800	2.141	8.848	1.114	10.99
56	0.1754	2.192	8.792	1.109	11.02	2.192	8.792	10.68	0.1773	2.166	8.820	1.112	10.98
58	0.1740	2.217	8.765	1.106	11.01	2.217	8.765	10.67	0.1754	2.192	8.792	1.109	10.98
60	0.1730	2.242	8.738	1.104	11.01	2.242	8.738	10.67	0.1740	2.217	8.765	1.106	10.98
62	0.1722	2.267	8.710	1.101	11.01	2.267	8.710	10.66	0.1730	2.242	8.738	1.104	10.98
64	0.1716	2.291	8.683	1.098	11.00	2.291	8.683	10.66	0.1722	2.267	8.710	1.101	10.97
66	0.1710	2.316	8.656	1.096	11.00	2.316	8.656	10.65	0.1716	2.291	8.683	1.098	10.97
68	0.1705	2.341	8.629	1.093	11.00	2.341	8.629	10.64	0.1710	2.316	8.656	1.096	10.97
70	0.1700	2.365	8.602	1.090	11.00	2.365	8.602	10.64	0.1705	2.341	8.629	1.093	10.97

Time in sec	X1[n+1]	X2[n+1]	X3[n+1]	X4[n+1]	H1, in metres	H2, in metres	H3, in metres	H4, in metres	X1[n]	X2[n]	X3[n]	X4[n]	H2+H3
72	0.1696	2.390	8.575	1.087	10.99	2.390	8.575	10.63	0.1700	2.365	8.602	1.090	10.96
74	0.1691	2.414	8.548	1.085	10.99	2.414	8.548	10.63	0.1696	2.390	8.575	1.087	10.96
76	0.1687	2.438	8.521	1.082	10.99	2.438	8.521	10.62	0.1691	2.414	8.548	1.085	10.96
78	0.1683	2.463	8.494	1.079	10.98	2.463	8.494	10.61	0.1687	2.438	8.521	1.082	10.96
80	0.1678	2.487	8.467	1.076	10.98	2.487	8.467	10.61	0.1683	2.463	8.494	1.079	10.95
82	0.1674	2.511	8.441	1.074	10.98	2.511	8.441	10.60	0.1678	2.487	8.467	1.076	10.95
84	0.1670	2.535	8.414	1.071	10.98	2.535	8.414	10.60	0.1674	2.511	8.441	1.074	10.95
86	0.1665	2.559	8.388	1.068	10.97	2.559	8.388	10.59	0.1670	2.535	8.414	1.071	10.95
88	0.1661	2.583	8.361	1.066	10.97	2.583	8.361	10.59	0.1665	2.559	8.388	1.068	10.94
90	0.1657	2.607	8.335	1.063	10.97	2.607	8.335	10.58	0.1661	2.583	8.361	1.066	10.94
92	0.1653	2.631	8.309	1.060	10.97	2.631	8.309	10.57	0.1657	2.607	8.335	1.063	10.94
94	0.1648	2.654	8.282	1.057	10.96	2.654	8.282	10.57	0.1653	2.631	8.309	1.060	10.94
96	0.1644	2.678	8.256	1.055	10.96	2.678	8.256	10.56	0.1648	2.654	8.282	1.057	10.93
98	0.1640	2.702	8.230	1.052	10.96	2.702	8.230	10.56	0.1644	2.678	8.256	1.055	10.93
100	0.1635	2.725	8.204	1.049	10.96	2.725	8.204	10.55	0.1640	2.702	8.230	1.052	10.93
102	0.1608	3.393	7.534	1.003	10.95	3.393	7.534	10.46	0.1635	2.725	8.204	1.049	10.93
104	0.1562	4.032	6.894	0.957	10.95	4.032	6.894	10.37	0.1608	3.393	7.534	1.003	10.93
106	0.1506	4.641	6.284	0.912	10.95	4.641	6.284	10.28	0.1562	4.032	6.894	0.957	10.93

Time in sec	X1[n+1]	X2[n+1]	X3[n+1]	X4[n+1]	H1, in metres	H2, in metres	H3, in metres	H4, in metres	X1[n]	X2[n]	X3[n]	X4[n]	H2+H3
108	0.1442	5.221	5.704	0.8660	10.95	5.221	5.704	10.20	0.1506	4.641	6.284	0.9116	10.93
110	0.1373	5.771	5.153	0.8205	10.94	5.771	5.153	10.12	0.1442	5.221	5.704	0.8660	10.92
112	0.1301	6.292	4.633	0.7750	10.94	6.292	4.633	10.05	0.1373	5.771	5.153	0.8205	10.92
114	0.1228	6.783	4.141	0.7296	10.94	6.783	4.141	9.982	0.1301	6.292	4.633	0.7750	10.92
116	0.1153	7.245	3.680	0.6842	10.94	7.245	3.680	9.918	0.1228	6.783	4.141	0.7296	10.92
118	0.1077	7.676	3.248	0.6388	10.94	7.676	3.248	9.858	0.1153	7.245	3.680	0.6842	10.92
120	0.1001	8.079	2.846	0.5934	10.93	8.079	2.846	9.802	0.1077	7.676	3.248	0.6388	10.92
122	0.09245	8.451	2.473	0.5480	10.93	8.451	2.473	9.750	0.1001	8.079	2.846	0.5934	10.92
124	0.08480	8.794	2.130	0.5026	10.93	8.794	2.130	9.703	0.09245	8.451	2.473	0.5480	10.92
126	0.07715	9.107	1.817	0.4572	10.93	9.107	1.817	9.659	0.08480	8.794	2.130	0.5026	10.92
128	0.06949	9.391	1.533	0.4118	10.93	9.391	1.533	9.620	0.07715	9.107	1.817	0.4572	10.92
130	0.06183	9.645	1.279	0.3664	10.93	9.645	1.279	9.584	0.06949	9.391	1.533	0.4118	10.92
132	0.05417	9.870	1.054	0.3210	10.93	9.870	1.054	9.553	0.06183	9.645	1.279	0.3664	10.92
134	0.04651	10.06	0.8596	0.2756	10.93	10.06	0.8596	9.526	0.05417	9.870	1.054	0.3210	10.92
136	0.03885	10.23	0.6944	0.2303	10.93	10.23	0.6944	9.503	0.04651	10.06	0.8596	0.2756	10.92
138	0.03119	10.36	0.5589	0.1849	10.92	10.36	0.5589	9.484	0.03885	10.23	0.6944	0.2303	10.92
140	0.02354	10.47	0.4530	0.1395	10.92	10.47	0.4530	9.469	0.03119	10.36	0.5589	0.1849	10.92
142	0.01588	10.55	0.3768	0.0941	10.92	10.55	0.3768	9.459	0.02354	10.47	0.4530	0.1395	10.92

Time in sec	X1[n+1]	X2[n+1]	X3[n+1]	X4[n+1]	H1, in metres	H2, in metres	H3, in metres	H4, in metres	X1[n]	X2[n]	X3[n]	X4[n]	H2+H3
144	0.00822	10.59	0.3302	0.04869	10.92	10.59	0.3302	9.452	0.01588	10.55	0.3768	0.09408	10.92
146	0.00056	10.61	0.3132	0.00330	10.92	10.61	0.3132	9.450	0.00822	10.59	0.3302	0.04869	10.92
148		10.61	0.3130						0.00056	10.61	0.3132	0.00330	

APPENDIX   J

CHARACTERISTICS OF TRANSFORMER OIL

Transformer oils are obtained by processing petroleum. Petroleum differs in chemical composition according to its source. This also applies to its products, especially to transformer oils. The hydrocarbon compounds, which constitute the major part of the oil, can be divided into three main groups: paraffins, naphthenes and aromatic compounds.

Paraffins are saturated hydrocarbons with a straight chain (normal paraffins) or a branched chain (isoparaffins) without any cyclic structures (Figure J.1). Cycloparaffins or alicyclic compounds (popularly called naphthenes) are saturated hydrocarbons containing one or more five- or six-membered rings. Each of these rings may have one or several straight or branched bicyclic, tricyclic (Figure J.2). Aromatic hydrocarbons contain one or several aromatic rings which may be combined with alicyclic rings. The rings may or may not have alkyl chains and/or paraffin side chains. The aromatic nuclei may be condensed as in naphthalene and phenanthrene or isolated (Figure J.3). A mixed alicyclic-aromatic hydrocarbon is also shown in Figure J.3.

Generally, the solubility of water in transformer oil is negligible. From the point of view of the molecular theory, the very low solubility of water in petroleum products is explained by the great differences in sizes between the hydrocarbon and water molecules. The overall field of the intramolecular forces prevents mixing of the liquids.

$$H_3C - (CH_2)_n - CH_3$$

normal paraffins

$$H_3C - \underset{\underset{\underset{\displaystyle H_3C \quad CH_3}{\diagdown}}{\overset{\displaystyle CH_3}{|}}{\overset{\displaystyle CH_3}{|}}}{C} - CH_2 - \underset{\underset{\displaystyle CH_3}{|}}{\overset{\displaystyle CH_3}{|}}{C} - (CH_2)_n - CH_3$$
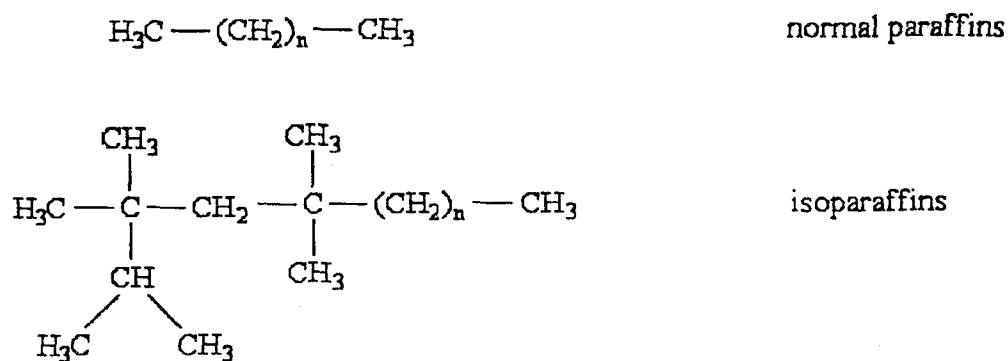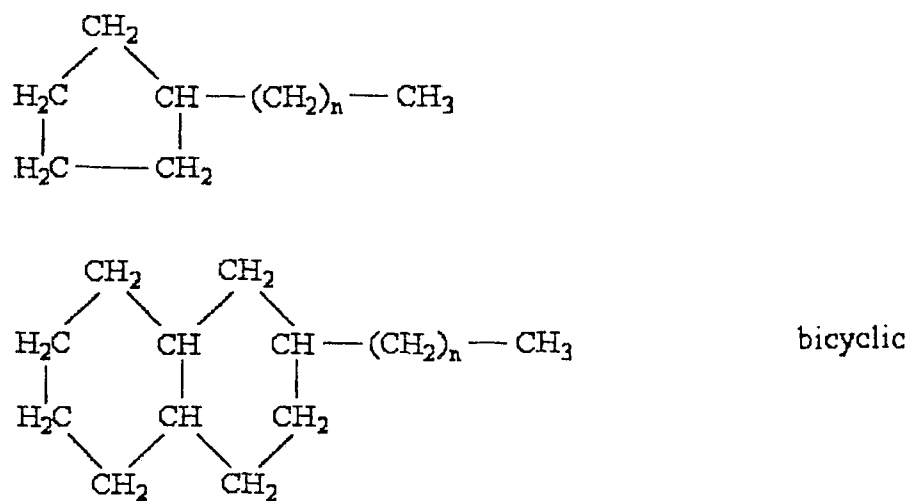
isoparaffins

Figure J.1: Paraffins group of hydrocarbon compounds.

$$H_2C \overset{\displaystyle CH_2}{\diagup \diagdown} CH - (CH_2)_n - CH_3$$

$$H_2C ---- CH_2$$
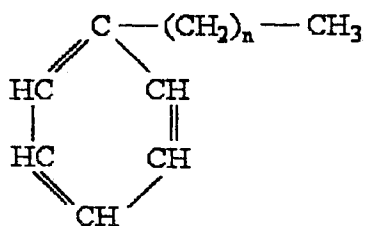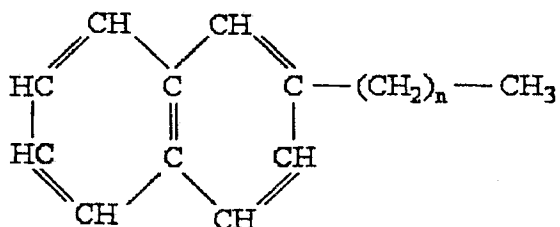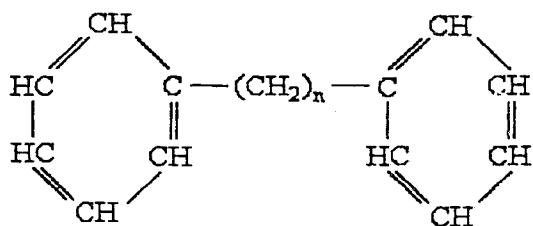
$$CH_2 \qquad CH_2$$

bicyclic

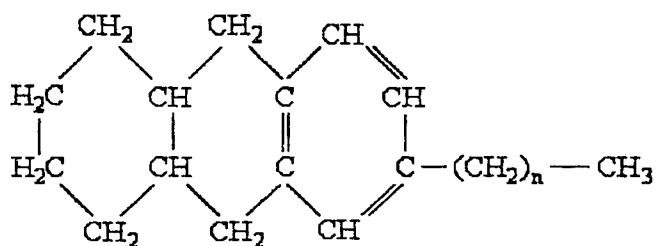Figure J.2: Naphthenes group of hydrocarbon compounds.

aromatic hydrocarbon with
one aromatic ring

aromatic nuclei condensed
as phenanthrene

aromatic nuclei are
isolated

mixed alicyclic-aromatic
hydrocarbon

Figure J.3: Aromatic group of hydrocarbon compounds.

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