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Free Vibration Analysis Of Horizontally Curved Composite Concrete Deck Over Steel I-Girder Bridges

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by

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ABSTRACT

Curved composite I-girder bridges provide an excellent solution to problems of urban congestion, traffic, and pollution, but their behavior is quite complex due to the coupled bending and torsion response of the bridges. Moreover, dynamic behavior of curved bridges further complicates the problem. The majority of curved bridges today are designed using complex analytical methods; therefore, a clear need exists for simplified design methods in the form of empirical equations for the structural design parameters. In this thesis paper, a sensitivity study is conducted to examine the effect of various design parameters on the free-vibration response of curved composite I-girder bridges. To determine their fundamental frequency and corresponding mode shape an extensive parametric study is conducted on 336 straight and curved bridges. From the results of the parametric study, simple-to-use equations are developed to predict the fundamental frequency of curved composite I-girder bridges. It is shown that the developed equations are equally applicable to curved simply supported and composite multi-span bridges with equal span lengths.

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TO STACEY

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NOTATIONS

B	bridge width
b_f	flange width
c	damping coefficient
D	depth of steel girder
$[c]$	damping matrix
DLA	dynamic load allowance
E	modulus of elasticity
$F(t)$	forcing function with-respect-to-time
F_y	steel yield stress
f	natural frequency of vibration
f_n	natural frequency of the n^{th} mode of vibration
f_1	fundamental frequency of vibration
I	moment of inertia of the composite girder
k	spring stiffness constant
$[K]$	global stiffness matrix
L	centerline span length
L_{as}	arc span length
m	rigid body mass
$[m]$	mass matrix
M	mass of bridge per kilogram
MPC	multi-point constaint
n	vibration mode number

N	number of girders
[P]	nodal load vector
R	radius of curvature of the centre span of the curved bridge
S	girder spacing
S_{max}	maximum cross-brace spacing
T	natural period of vibration
[U]	nodal displacement vector
U	displacement (degree of freedom)
W_c	deck width
W_e	width of design lane
α	span-to-depth modifier
κ	curvature factor
Φ	rotation (degree of freedom)
ω	natural circular frequency of vibration

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CHAPTER I
INTRODUCTION

1.1 General

In the past, curved alignments in bridges were achieved using straight chord segments supported on a large number of piers. With increasing curvatures more straight segments are required to form the curve and consequently more piers are required to support these segments resulting in a lot of used up space. With urban congestion on the rise in nearly all cities in North America, curved bridges have become very appealing as they take up much less space. Other benefits of curved bridges include: aesthetically pleasing designs, smooth transitions, economical benefits, less pollution due to idle car emissions, less traffic jams, shallower sections, and generally more efficient designs. However, the behaviour of curved bridges is quite complex due to the coupled bending and torsion response of the bridges. From the many material types and geometric cross-section shapes available for curved bridge design, the composite slab-on-girder bridge is the most popular choice due to its practicality, economy, and simplicity in construction. But, open-sections of the I-girder offer very little torsional resistance thereby making cross-braces a primary design component which further complicates the overall design process for the engineer. This puts pressure on researchers to study the behaviour of these bridges in order to simplify the process. Figure 1.1 shows a typical cross section of a composite slab-on-girder bridge with four girders.

There are two processes used to fabricate curved girders: *cut curving*, and *heat curving*. Cut curving is a process where the flanges of the girders are cut from plates and welded to a web which is most often bent into shape using the *cold-rolling* – a process where the web is deformed by passing it through rollers at temperatures below recrystallization temperature. Heat curving, the more common of the two processes, is carried out by continuously heating the flanges along their length to produce the desired curvature.

Erection of curved I-girder bridges is also of great importance as the girders have a tendency to roll-over when lifted leading to combined bending and torsional stresses. Generally, three different lifting schemes are used to alleviate the problem, the two-girder vertical lift, the single girder vertical lift, and the single girder inclined lift. Figure 1.2 shows diagrams for the three erection schemes. To determine the stresses in the girder due to each lifting scheme a small finite elements study was conducted (Penrose and Davidson.). The results showed that the two-girder vertical lift method resulted in the lowest maximum stresses. Also, the method hastens erection as more girders are lifted at once.

1.2 The Problem

Although the popularity of curved bridges has increased dramatically over the past few decades, the complexities associated with the behaviour of these bridges have not been fully worked out. Furthermore, the design of these bridges requires analysis by methods that are complex and inefficient. North American bridge design codes prescribe that as a part of the design process the serviceability states dealing with excessive deflections or vibrations must be checked. Currently, the American Association of State Highway and Transportation Officials (AASHTO) Bridge Design Specifications put a limit on deflections as a function of the span length (AASHTO, 2004). In a study conducted by Roeder, et al. (2002), it was shown that the current AASHTO deflection limits are inadequate in controlling vibration and may even lead to structural damage as a result of deformation. In the concluding chapter of that study recommendations were made to link maximum deflections to the fundamental frequency similar to the current practice in the Canadian Highway Bridge Design Code, CHBDC, (CSA, 2006). As it stands, the CHBDC method of controlling excessive deformations and vibration is the most accurate in the world. However, no equation is given in the CHBDC to accurately determine the fundamental frequency of curved bridges. Much research has been conducted on methods to predict the natural frequencies of curved bridges. However, the methods lead to either closed-form solutions that are too complex for engineers to use in practice, or to algebraic equations that are too simple to accurately capture the effects of all the design parameters. Consequently, an empirical, but more reliable, equation that captures the effects of all the important design parameters on the fundamental frequency of curved bridges is much needed.

1.3 Objectives

The objectives of this study are:

1. To identifying the key parameters that influence the free-vibration response of curved composite concrete slab-on-steel I-girder bridges,
2. To outline a database of curved bridges that includes all of the identified key parameters,
3. To develop an empirical equation that will enable engineers to accurately predict the fundamental frequency of single span curved composite concrete-steel I-girder bridges,

4. To evaluate the applicability of the developed equation to continuous span composite concrete-steel I-girder bridges.

1.4 Scope

The scope of this study includes the following:

1. A literature review of all research studies, textbooks, and codes of practice pertaining to curved composite steel girder bridges,
2. A sensitivity study to determine the effects of key design parameters on the fundamental frequency. The range of parameters studied includes: span length, radius-of-curvature, span-to-depth ratio, boundary conditions, girder spacing, number of girders, number of cross-braces, cross-brace axial and flexural stiffness, end diaphragm plate thickness, flange thickness, and horizontal bracing,
3. A parametric study on selected bridge configurations to establish a database for the bridge fundamental frequency for simple span curved composite concrete-steel I-girder bridges,
4. A nonlinear regression analysis of the data to develop a simple equation to predict the first flexural frequency of the studied bridges,
5. An additional database and parametric study of two-span continuous bridges with equal spans to examine the applicability of the developed equation on continuous bridge systems,

1.5 Contents and Arrangement of This Study

Following the Introduction Chapter, Chapter II contains a thorough literature review pertaining to this research study as well as a brief summary of the basic concepts of dynamic analysis, and an overview of the current bridge design codes being used in North America. Chapter III introduces the fundamentals of the finite element method and the finite element software "ABAQUS" used in this study. In Chapter IV, the results of the sensitivity study conducted on various bridges are presented along with the conclusions with regards to the key parameters to be used in the parametric study. In Chapter V, a full overview of the bridges contained in the parametric study for both simple span and continuous span bridges is given along with the outcomes of the parametric study and the proposed equations. Chapter VI contains the summary, conclusions, and recommendations for future research.

CHAPTER II
LITERATURE REVIEW

2.1 Fundamental Behaviour of Curved Bridges

2.1.1 Review of Linear Elastic Behaviour

In general, the behaviour of curved bridges is similar to that of straight bridges but with a few differences which make curved bridge design much more complex. Due to the curvature of the bridge, the bridge bends and rotates inducing combined bending and torsional moments in the girders. While box girders are better at resisting this torsion, open cross-sections such as those of I-girders have very little torsional stiffness. For this reason cross-braces become a significant component of curved bridges and the overall stability of the bridge depends on them. The cross-braces apply restoring torques to the girders resulting in equal but opposite lateral bending moments in the top and bottom flanges. The lateral flange bending moment multiplied by the distance between the top and bottom flanges of the girder is commonly referred to as the *bimoment*. The bimoment causes twisting along the longitudinal axis of the girder. Ignoring the web, the net effect of the non-uniform torsion due to curvature always increases curvature of the compression flange; however, the net effect on the tension flange can be either to increase or decrease the curvature depending on the flange stiffness and curvature. The web has a restraining effect which helps to prevent bowing out of the flanges. This causes the lateral flange moment to vary such that the stress at the extreme fibre of the flange on the outside of the curve is the largest at the brace points, but at the extreme fibre of the flange on the inside of the curve it is largest midway between the brace points (Hall, 1996).

The complexities associated with curved bridge design have turned engineers away for many years, thus there is a clear need for simplified analysis equations and design methods. Over the years many different approaches have been documented. Zureick and Naqib (1999), in their State-of-the-art report, provide a summary of some of the most commonly used analysis methods available. The following section is a reproduction of the summary of those methods. For a more detailed description of the methods, the reader is directed to the above mentioned report which contains a list of references that focus on the methods directly.

2.1.2 Method of Analysis

Methods of analysis of horizontally curved bridges fall under two categories: approximate methods and refined methods. Approximate methods can be used for preliminary analysis and design while refined methods are more detailed and thus should be used for the final analysis or design. The following methods fall under the approximate methods category: Plane-Grid Method, Space-Frame Method, and the V-Load Method. Under the refined category the following methods are available: Finite-Element Method, Finite-Strip Method, Finite-Difference Method, Direct solutions to Differential Equations, and the Slope-Deflection Method.

2.1.2.1 Plane-Grid Method (Grillage)

In this method flat plate structures are modelled as a grid of two-dimensional beam members. Each beam elements is assigned a bending and torsional stiffness equivalent to the portion of the structure it replaces. Equivalent loads are applied to the joints and a solution is obtained by the stiffness method for displacements. Usually the grillages are complex enough that a computer-aided finite-element method is employed.

2.1.2.2 Space-Frame Method

Similar to the grillage method, this is a three-dimensional method developed for the analysis of open and closed curved members. Straight members are used to model the curved members. Diaphragms and cross-braces are modelled using truss elements with stiffness in the axial direction only. The method does not take into account warping.

2.1.2.3 V-Load Method

The V-Load method accounts for curvature with the application of external loads at diaphragm locations. Curved members are treated as straight members of equivalent arc length and the application of the externally applied loads results in internal force equilibrium similar to that which is expected in curved members subjected to vertical loads only. The loads are dependant on the radius of curvature, bridge width, and diaphragm spacing.

2.1.2.4 Finite-Element Method

This is one of the most robust and accurate methods available. The structure is divided or *discretized* into small elements connected at their nodes. Each element is defined by a specific number of nodes and degrees-of-freedom. The material stiffness of the elements is compiled in matrix form into one global matrix equation which describes the behaviour of the entire system. Once the appropriate boundary conditions and loads are specified, a solution for the displacements and stresses is obtained by solving the matrix equation.

2.1.2.5 Finite-Strip Method

The curved bridge is divided into narrow strips in the circumferential direction that are supported in the radial direction. Bending, membrane, warping, and distortion are all included in this method. The method has more limitations than the finite element method, but it is simpler and requires less computational time.

2.1.2.6 Finite-Difference Method

The governing differential equations describing the structure are replaced by algebraic difference equations which are solved for every grid point on a grid that is superimposed on the structure.

2.1.2.7 Solutions to Differential Equations

A solution is obtained to the differential equations which describe the structural model. The solution may be closed form or in the form of a series such as the Fourier series. The complexities of the mathematical equations make this method of solution impractical to the everyday engineer.

2.1.2.8 Slope-Deflection Method

Partial differential equations in terms of slope-deflection equations are developed. The solution is usually in the form of a Fourier series. Once again, this method is accurate but impractical due to its complexity.

2.2 Fundamentals of Dynamic Analysis

The simple definition of dynamic loading is; a load that varies with time. No loading is truly static; time is an inherent property. The time at which the load was applied, the time at which the load was taken away, the duration of the load, and the sequence of loading are all time quantities that help to distinguish the problem between a static one and a dynamic one. One way to describe this distinction is to compare the duration of the load to the *natural period of vibration* of the structure which can be defined as the time required for the structure to go through one cycle of free vibration. If the duration of the load is similar to the natural period of vibration the structure, the structure will respond dynamically (Tedesco, 1999). Generally, the response of structures to dynamic loading is more severe than the response to static loading. Another important reason to study the dynamic response of structures is to avoid a phenomenon known as *resonance* which occurs when similar frequencies amplify. Vibrations in real life structures can be induced in many different ways such as: the response of bridges to moving vehicles, or the response of structures to wind or earthquakes. If the frequency of the excitation matches the natural frequency of the structure, the amplitude of vibration increases and resonance occurs; therefore, it is very important to accurately determine the natural frequencies of structures.

2.2.1 Single-degree-of-freedom systems (SDOF)

In general, structures possess infinitely many degrees-of-freedom (DOF), but for practical reasons, dynamic analysis is performed on models with a finite number of DOF. Much information can be obtained from a rather simple SDOF model, but more importantly, one must understand the principles of vibration of SDOF systems in order to solve more complex dynamic problems.

Dynamic systems are defined by the *equation of motion* and to study dynamic analysis is to learn how to formulate and solve this equation. The basic components that make up a vibration system are: mass, stiffness, damping, and forcing. The summation of these components forms the simple SDOF second-order differential equation of motion:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2.1}$$

Where,

F(t) : is the forcing function with-respect-to-time,

$m\ddot{x}$: is the inertia of the system,

$c\dot{x}$: is the damping of the system,

kx : is the stiffness of the system,

If the forcing function is set to zero the analysis becomes that of *free vibration*, otherwise it is a *forced vibration* analysis. If the equation above is divided by the mass, an equation for the *natural circular frequency* of the system is obtained, ω, in units of radians per second,

$$\omega = \sqrt{\frac{k}{m}} \tag{2.2}$$

The *natural frequency*, f, of the system in Hertz units is obtained by simply dividing the above circular frequency by 2π. The natural period of vibration, T, is inversely proportional to the natural frequency.

2.2.2 Multi-degree-of-freedom systems (MDOF)

SDOF systems result in only one differential equation of motion describing the entire system, but for most structures there are many degrees of freedom for which there exist an equal number of equations of motion. For this reason, matrix formulation is required. The equation of motion in matrix form for a MDOF system can be written as:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F(t)\} \tag{2.3}$$

Where,

$\{F(t)\}$: is the force vector,

$[m]$: is the mass matrix,

$[c]$: is the damping matrix,

$[k]$: is the stiffness matrix.

Similar to SDOF system, without the damping term and with the force vector set to zero, the analysis becomes that of undamped free vibration with a solution obtained by standard *eigenvalue* analysis.

2.3 Overview of North American Bridge Design Codes

This section presents an overview of the U.S. and Canadian approach to curved bridges, dynamic analysis, and serviceability. In Canada, bridge design is governed by the Canadian Highway Bridge Design Code which covers all straight bridges, but no standalone code governing the analysis and design of curved bridges is available. In the United States, the current bridge design specifications include the AASHTO LRFD Bridge Design Specifications (AASHTO, 2004) which govern all straight bridges and the AASHTO Guide Specifications for Horizontally Curved Steel Girder Highway Bridges (AASHTO, 2003), from here on in referred to as just Guide Specifications.

2.3.1 Canadian Highway Bridge Design Code (CHBDC)

The treatment of dynamic response in the CHBDC is somewhat different from AASHTO Bridge Specifications as shown later. To account for dynamic effects, a dynamic load allowance or impact factor is applied; the usage of the latter term is being phased out as it implies an impact load which is misleading. The exact definition of the *Dynamic Load Allowance* (DLA) is – an equivalent static load expressed as a fraction of the traffic load, which is considered to be equivalent to the dynamic and vibratory effects of the interaction of the moving vehicle and the bridge, including the vehicle response to irregularity in the riding surface. Previously, the DLA was linked to the first flexural frequency, but in the current code the factors are specified based on the number of axles and the component being designed as shown in Table 2.1. For superstructure vibration the code requires that a single truck load be positioned in such a way as to produce the largest deflection at the centre of the sidewalk or at the inside face of the barrier. This truck load must be multiplied by

(1+DLA) to obtain the total deflection. The allowable deflection limit for vibration control is directly linked to the first flexural frequency of the bridge, as depicted in Fig 2.1. However, CHBDC does not provide any specific equations to calculate the first flexural frequency of a bridge. A much bigger gap in the code is the lack of explicit coverage of curved bridge design. The code provides designers with a simple equation of questionable validity to determine if the curved bridge can be treated as a straight one,

$$\frac{L^2}{BR} \leq 0.5 \tag{2.4}$$

Where,

L : is the arc length of the bridge,

R : is the radius of curvature,

B : is the width of the bridge.

2.3.2 AASHTO Guide Specifications

Similar to the CHBDC, the AASHTO Guide Specifications account for dynamic effects through static load multipliers called impact factors, as shown in Table 2.2 for I-girder bridges and Table 2.3 for closed box and tub girder bridges. Superstructure vibrations are controlled by limiting the span-to-depth ratio to not more than 25 when the specified yield stress is 345 MPa (50 ksi) or less. The following formula is given to calculate the span-to-depth ratio of girders with specified yield strengths greater than 345 MPa (50 ksi),

$$\frac{L_{as}}{d} = 25\sqrt{\frac{50}{F_y}} \tag{2.5}$$

Where,

L_{as} : is the arc span for simple spans and 0.9 or 0.8 times the arc span for continuous end spans and interior spans, respectively,

d : is the depth of steel girder,

F_y : is the steel yield stress in ksi,

The maximum deflections in any girder, including impact factors, are limited to L/800. The maximum deflections under pedestrian sidewalks, including impact factors, are limited to L/1000. L is defined as the arc length between bearings. Although the Guide Specifications deal with curved bridge design explicitly, the following criteria which enable designers to neglect curvature are provided. For I-girder bridges, the following three conditions must be satisfied:

1) girders are concentric,
2) bearing lines are not skewed more than 10 degrees from radial,
3) the arc span, L_{as} , divided by the girder radius is less than 0.06 radians, where the arc span is defined similar to equation 2.5.

For closed box and tub girders, similar three conditions must be met:

1) girders are concentric,
2) bearings are not skewed,
3) the arc span divided by the girder radius is less than 0.3 radians and the girder depth is less than the width of the box at mid-depth, where the arc span is defined similar to equation 2.5.

Clearly, the limits for open sections are much more stringent than for closed box sections; however, as was mentioned earlier, no such distinction is noted in the formula given in the CHBDC.

2.3.3 AASHTO-LRFD Bridge Design Specifications

For curved bridge design, the LRFD bridge design specifications should be used in conjunction with the Guide Specifications. Supplementing the deflection criteria specified in the Guide Specifications additional deflection criteria are specified. However, these criteria are optional (except the provisions for orthotropic decks and reinforced concrete three-sided structures) and thus they have been left out of this section. Impact factors, shown in Table 2.4 as a percentage of the static load, should be applied to account for dynamic load effects. Similar to the CHBDC, curvature limits are given that enable designers to treat a curved bridge as a straight bridge. For curved bridges with torsionally stiff closed sections, the limit on central angle subtended by a curved span is 12˚. For cast-in-place multi-cell box girders the limit on the central angle is 34˚. For open cross-sections, the limits on the central angle, as shown in Table 2.5, are much stricter due to the relatively low torsional stiffness of open cross-sections.

2.4 Historical Overview of Curved Bridge Design

The behaviour of curved or bent beams is not a relatively new concept; in fact, Saint-Venant studied the bending of curved bars over 150 years ago (Timoshenko, 1953). However, it was not until roughly 60 years ago when the wheels were really set in motion in the field of curved bridge design. The reason for this emerging interest can most likely be attributed to urban congestion, but other reasons such as aesthetics, the emergence of stronger materials, or simply the drive to propel engineering forward more than likely also played a part.

McManus, et al. (1969) compiled a thorough state-of-the-art report regarding the design of horizontally curved bridges; the report contained 202 references. A few months later, Tan, et al. (1969) supplemented the list with an additional 30 references of which the majority originated from Japan. In the same year, the Federal Highway Administration formed the Consortium of University Research Teams (CURT) to scrutinize the behaviour of horizontally curved bridges. The team consisted of the University of Pennsylvania, Carnegie-Mellon University, Syracuse University, and the University of Rhode Island. The consortium reviewed all existing publications on curved bridges, conducted analytical and experimental research related to curved bridges, developed simple analysis and design methods, and correlated these analysis and design methods with existing experimental data (Linzell, et al., 2004). This research ultimately led to the release of the 1980 AASHTO Guide Specifications for Horizontally Curved Highway Bridges (AASHTO, 1980). Since then, AASHTO has updated the specifications and two more editions were released in 1993 (AASHTO, 1993) and in 2003 (AASHTO, 2003). It is expected that the next release of the specifications will include load-resistance-factor-design (LRFD) criteria. For more recent summaries relating to the research conducted on the design of curved bridges the reader is referred to works by Zureick and Naqib (1999) and Linzell, et al. (2004) which cover horizontally curved I-girder design and the State-of-the-art report by Sennah and Kennedy (2001) on horizontally curved box girder design.

2.5 Free-Vibration Studies

Prior to the widespread of personal computers, the majority of researchers focused on analytical solutions to dynamic analysis problems of straight and curved bridges. In 1967, an exact solution to the governing differential equations for free vibration of doubly-symmetric horizontally curved beams on simple supports was presented by (Culver, 1967). The equation was equally applicable to straight bridges by letting the radius of curvature approach infinity. An approximate solution for the two lowest frequencies, utilizing the Rayleigh-Ritz method, was also obtained for the free vibration of horizontally curved beams with either fixed-simple or fixed-fixed support conditions. It was noted that the effect of restraints on the natural frequency was similar to that of a straight beam in that the frequencies increase with an increase in fixity. Two years later, Culver and Oestel (1969) expanded on their previous research and developed equations based on the Rayleigh-Ritz method and Lagrange multiplier concept to predict the natural frequencies of curved multi-span girders of equal or unequal spans. It was shown that for the case of beams with equal spans, the mode shapes were either symmetric or anti-symmetric and they were independent; thus, they can be considered separately. This independence is important because it makes it possible to relate the frequencies of multi-span curved bridges to those of simple span curved bridges of equal length. In 1968, Tan and Shore (1968a) obtained the exact solutions to differential equations governing free vibration of simply supported curved bridges idealized as a single prismatic beam. The effects of shearing deformation, flexural rotary inertia, and axial forces were neglected. Christiano and Culver (1969) formulated the differential equations governing the free vibration of a single span curved beam which resulted in a cubic equation. The first two roots of the equation, representing the coupled natural frequencies of a doubly-symmetric curved member, were the same as those predicted by Culver's equation in his earlier study (Culver, 1967); however, the third root which was not considered in the previous study represented the fundamental lateral frequency. The frequencies predicted for the doubly-symmetric curved member are analogous to the vibration of straight beams of unsymmetrical cross section. Chaudhuri and Shore (1977) modified the analytical approach to the dynamic analysis of curved bridges problem to include the effects of shear deformation and rotary inertia. Billing (1984) studied the dynamic response of 27 straight simple and continuous bridges in the Southern Ontario region of which 14 were steel bridges. In a follow up article, Billing and Green (1984) developed a simple equation to predict the fundamental frequency,

$$f = \frac{110}{L_{\max}} \tag{2.6}$$

Where,

f: is the fundamental vertical frequency in Hz,

L_{max} : is the maximum span in metres.

It was noted that equation 2.6 is too simple to cover the wide range of variables in bridge construction and lacking sufficient accuracy to be codified, but it was useful for preliminary predictions of fundamental frequency. Snyder and Wilson (1992) obtained a closed form solution to an 8th order differential equation governing the dynamic behaviour of continuous curved thin-walled beams. The equation was applicable to multiple equal and unequal spans and varying boundary conditions.

With the influx of personal computers in the 1990's, the trend has been to steer away from complex analytical methods and to guide research more towards computer-aided methods of solutions and experimental testing. More importantly the need for simpler solution methods has become quite clear. Memory, et al. (1995) studied the free vibration of straight, continuous, and skewed bridges with the intent to propose an accurate method of predicting natural frequencies of such bridges. The fundamental frequency was calculated and compared using three methods: simple beam theory, Raleigh Energy method, and Eigenvalue analysis, with solutions to the latter obtained with a computer-aided grillage analysis. A modified procedure combining the Rayleigh method and a grillage analysis was found to yield theoretically accurate results. It was suggested that the calculated frequency using single beam analogy does not account for the energy dissipated in the transverse direction, which means that using a single beam analogy should always result in an overestimation for the natural frequency; however, as the aspect ratio of the bridge increases, it is expected that the frequency predicted by single beam idealization will approach the true frequency. Therefore, the fundamental frequencies calculated for straight bridges, idealized as a single beam, are relatively accurate. In 1998, as part of the National Earthquake Hazards Program in the United States, a research study was conducted on steel and concrete bridges along the I-55 interstate in the southeastern Missouri region (Dusseau, 1998). 166 of these bridges were steel girder bridges. The following two simple empirical equations for the vertical and lateral fundamental frequency were developed based on the experimental results from 25 of these bridges,

$$f_v = 588.2 L_s^{-1.45} D^{-0.4} \tag{2.7}$$

Where,

f_v : is the fundamental vertical frequency in cycles per second,

L_s : is the span length in metres,

D: is the steel girder depth in metres,

$$f_l = 597.4 L_b^{-1.6} W^{0.8} H^{-0.05} \tag{2.8}$$

Where,

f_l: is the fundamental lateral frequency in cycles per second,

L_b : is the overall bridge length in metres,

W: is the deck width in metres,

H: is the maximum support height in metres.

The vertical frequencies predicted by this equation did not correlate well with experimental results as continuity effects were not taken into account; however, the results for the lateral frequency were within 5%. Lee et. al. (2003) derived differential equations governing the free vibration of horizontally curved beams with unsymmetrical axes in Cartesian coordinates. The validity of the equations was confirmed using SAP2000 finite element software with very accurate results. A relatively recent study by Yoon, et al. (2005) was conducted to examine free vibration of horizontally curved steel I-girder bridges. As opposed to previous analytical studies which were based on Vlasov's theory, a curved beam element was derived based on Kang and Yoo's (1994) theory of thin-walled curved beams. Next, a software program based on the finite element method was developed to carry out free vibration analysis using the curved element. The results were verified with ABAQUS finite element software resulting in an error of less than 3%. Yoon et. al. (2006) compared the analytical solution of the equations of motion derived in the previous study to the frequencies predicted in previous studies by Culver (1967) and Shore and Chaudhuri (1972) . In the latter two studies, the curved beam elements used were formulated based on Vlasov's theory. It was shown that the results were in good agreement for predicting dynamic response. Barth and Wu (2007) conducted an extensive finite element study, consisting of 202 bridge models to determine the fundamental frequencies of straight single span, two-span, and three-span I-girder bridges. The range of parameters covered by their study included: bridge lengths varying between 30 m and 100 m, span-to-depth ratios of 20, 25, and 30, and four different 4-girder bridge cross-section configurations. Based on a nonlinear regression analysis of the finite element data, the following simple equation was developed,

$$f = \lambda^2 \cdot f_{sb} \qquad (2.9)$$

Where,

λ^2 : is equal to $a \cdot \left(\frac{I^c}{L^b_{max}}\right)$

L_{max} : is the maximum span length in metres,

I: is the average moment of inertia of the composite girder section m^4,

a = 1.44; b = 0.046 and c = 0.032 for two span bridges,

a = 1.49; b = -0.033 and c = 0.033 for three or more span bridges,

The proposed equation predicted the fundamental frequencies very well compared to the finite element results and agreed well with the results of experimental testing of three existing bridges. The effects of parapets and material strength on the fundamental frequency were also considered. It was determined that the effect of parapets increased the frequency of the bridges by up to 10%, but for bridges with lengths of 45 m or greater the difference was only 3%. Of importance was the confirmation that for simply supported single span bridges, the simple beam idealization, see equation 3.2, predicts the fundamental frequency with less than 5% error. For two-span and three-span bridges, the simple beam equation results in erroneous predictions with predicted frequencies that are between 5% and 10% lower for two-span bridges, and up to 90% lower for three-span bridges. For the three-span case, the bridges were modeled with unequal spans which explains the error. However, for the two-span case the bridge models were of equal span length and it is expected that the simple beam equation can be used to predict the frequency accurately as will be shown later in this study.

2.6 Forced-Vibration Studies

Forced vibration studies followed similar research trends to those of free vibration presented in the previous section. Earlier studies of the dynamic response of curved bridges were of the analytical form beginning with studies of curved beams, progressing through to bridge-vehicle interaction models of increasing complexity, and finally to computer-aided methods and solutions. Tan and Shore (1968a) studied the dynamic response of horizontally curved bridges subjected to a constant moving force. Equations of motion for the curved bridge idealized as a prismatic curved beam were derived. The effects of several factors on the dynamic response of curved bridges were studied and it was shown that the radius of curvature had the greatest effect. A few months later, a similar study was conducted (Tan and Shore, 1968b) to solve the dynamic response of curved bridges, but this time the effect of the vehicle modeled as a sprung mass was taken into account. The vehicle model displacements were restrained in all but the vertical deflections. For practical purposes, torsional inertia was ignored in the equations. Furthermore, it was assumed that the centrifugal force is counteracted by the centripetal force resulting from the super-elevation of the bridge road surface. Damping effects of the bridge were neglected. Christiano and Culver (1969) formulated differential equations describing the response of curved bridges subjected to a moving mass load consisting of sprung and unsprung masses. The vehicle model considered both vertical as well as torsional springing of the mass but the horizontal springing was neglected. The infinite series solution of these equations correlated well with experimental test results obtained using a scaled down bridge and a sprung-mass load-carriage simulating the vehicle loading. The results of the study show that the effect of vehicle mass has a significant influence over dynamic response. In 1977, as part of an investigation conducted by the Consortium of University Research Teams (CURT), Chaudhuri and Shore (1977) conducted a finite element study to determine the dynamic response of horizontally curved I-girder bridges subjected to a simulated truck load. The vehicle was modeled as a two-axle sprung-unsprung mass with three degrees of freedom enabling bouncing, rolling and pitching motions, but translational movement between the sprung and unsprung mass portion of the model was restrained. Based on thin-walled curved beam elements and the linear-acceleration method, the dynamic problem for curved bridges of varying parameters was solved with the aid of a software program developed at the University of Pennsylvania called "Dynamic Analysis of Curved Bridges/I-Girders" (DYNCRB/IG). Some of the parameters considered in the study were the span length, radius of curvature, number of girders, number or spans, and the weight-of-vehicle to weight-of-bridge ratio. Impact factors to account for the dynamic effects were developed for deflections, deck slab stresses, curved girder stresses, and support reactions. Wang, et al. (1992), studied the dynamic response of straight simple and continuous multi-girder bridges subjected to one truck or two truck loadings with varying degrees of road-surface roughness. Two nonlinear sprung mass models were considered to represent standard AASHTO H20-44 and HS20-44 trucks. The H20-44 truck was modeled with two axles, three masses, and a total of seven degrees of freedom corresponding to bouncing, rolling, and pitching motions. The HS20-44 truck was modeled with three axles, five masses, and a total of twelve degrees of freedom. The bridge was idealized as a grillage beam system. Dynamic response of the bridge was analyzed with the finite element method. It was shown that intermediate diaphragms significantly affect the lateral distribution of load to the girders and that the road-surface roughness has a great influence on the impact factors. Huang, et al. (1995) repeated the methodology used in their previous article to study the dynamic behaviour of horizontally curved I-girder bridges subjected to additional parameters which have not been previously examined such as the effects of bridge damping, number of trucks, and loading position. In all cases the dominant mode was bending and torsion combined with the first mode dominated by the bending of the outside girder and the second mode by the bending of the inside girder. Transverse slope had an almost negligible effect on the dynamic response. Michaltsos, et al. (1996) compared the dynamic response of a simple beam subjected to a moving load with and without inertia effects. The conclusions resulting from this study confirmed the results obtained in previous studies by Christiano and Culver (1969) and Chaudhuri, et al. (1977). Yang, et al. (2001) studied the dynamic response of curved beams subjected to both vertical and horizontal constant moving forces. A closed form solution for the lateral frequency of the bridge was obtained. The analytical results were in agreement with a solution obtained by finite element method. The centrifugal force was directly linked to the lateral vibration response. Michaltsos, et al. (2002) compared the results of three analytical models for the following cases: 1) moving concentrated load with varying velocity; 2) moving 2-axle vehicle model with varying velocity and; 3) the second case including light damping. The objective of the study was to determine the effects of acceleration and de-acceleration on dynamic response. The results showed at minimum a 7% dynamic response increase when considering acceleration vs. constant velocity and a maximum increase of up to 33%.

In the years to come, it is expected that advances in computer technology will enable researchers to consider more complex and accurate bridge and vehicle models when studying the dynamic behaviour of bridges; but the real challenge will be to make the results practical for everyday engineering use.

2.7 Parametric Studies

The majority of research studies on the behaviour of straight and curved bridges include parametric studies. The most influential parameters, such as the curvature ratio or length, are usually the ones that are included in the studies; however, many research studies have been conducted to determine the effects of certain parameters specifically. In 1996, a study conducted by Davidson, et al. (1996) dealt primarily with the effects of cross-frame spacing on the warping-to-bending stress ratio. Other parameters such as: length, radius of curvature, depth of girder, number of girders, girder spacing, and flange width were also considered. The following equation was proposed to determine the maximum spacing of cross-frames to limit the warping-to-bending stress ratio to 0.25,

$$S_{max} = L \left[-ln \left(\frac{Rb_f}{2000L^2} \right) \right]^{-1.52} \tag{2.10}$$

Where,

S_{max}: is the maximum cross-braces spacing in metres,

L : is the span length in metres,

R: is the radius of curvature in metres,

b_f: is the flange width in millimeters.

Maneetes and Linzell (2003), conducted experimental research on the effect of cross-braces and lateral bracing on the free vibration response of curved I-girder bridges during the construction phase (non-composite). The results were verified with a finite element model that was calibrated for the parametric study. The range of parameters studied included: the type of cross-braces used (K-type vs. X-Type), the effect of axial and flexural stiffness of the cross-braces, cross-brace spacing, and the effects of several arrangements of lateral bracing. Significant increases in natural frequencies were observed for non-composite sections reinforced with lateral bracing. Sennah, et al. (2001) studied the effect of horizontal bracing systems on the moment distribution factors for non-composite curved steel I-girder bridges. Four different arrangements were studied: vertical braces only, vertical braces with torsion box, vertical braces with horizontal bracing in outer bays, and vertical braces with horizontal bracing in all bays. It was concluded that the system with vertical braces and horizontal bracing in outer bays provides optimum results. Samaan, et al. (2002), studied the effects of six different arrangements of support bearings on the maximum stress and reaction distributions as well as on the natural frequencies. An optimal arrangement based on the tangential method was selected.

CHAPTER III

FINITE ELEMENT ANALYSIS

3.1 General Background

Development of the finite element method dates back to the early 1940's but widespread usage of the method parallels the advancement of computer technology. This is evident, for example, in the advancement of research pertaining to curved bridge design where the majority of early studies present solutions to complex differential equations that only partially describe the physical problem, but a majority of the more recent studies utilize the finite element method. In fact, due to the many unknowns and variables that are required to fully describe the physical problem, it may not be possible to obtain analytical solutions altogether. For this reason, numerical methods such as the finite element method are invaluable to researchers today. However; as was mentioned before, the method has gained much popularity due to the availability of powerful computers which make possible the solution of large systems of simultaneous algebraic equations associated with the finite element method. One of the greatest advantages to the method is that it is very versatile. The method can be used to solve problems in many different areas of interest with various arrangements of elements, material properties, boundary conditions, and loading. This makes the method an ideal analysis tool for problems in curved bridge design. Many commercially available finite element analysis software programs are available with a windows based interface that makes it possible to interact with the model, visualize the response, and to obtain solutions very quickly. The software packages enable users to model complex engineering problems with greater simplicity and increased efficiency. This makes the finite element method very appealing to practicing engineers.

The following sections of this chapter contain: a brief summary of the finite element method approach to structural engineering problems, a description of the commercially available finite element software ABAQUS (Hibbit, et al. 2004) with focus on the relevant elements used to model the curved bridge components in this study, and a detailed description of the models and procedures used to conduct the parametric study.

3.2 Finite Element Approach

The finite element method is a method by which a structure is divided into a system of smaller bodies or *finite elements* connected together at common points called *nodes* to form a large *mesh.* A solution to a structural problem is obtained by determining the displacements at the nodes and the stresses in the elements. The process used to separate the structure into an equivalent system of smaller elements is called *discretization.* Similar to most approximate methods of analysis, the solution to the problem improves with an increase in terms or in this case elements. Often, a convergence study must be carried out to ensure that a sufficient number of elements have been used to define the structure. It is also important to fully understand the capabilities of the elements used to define the structure or the solution to the problem may yield meaningless results.

There are two approaches to the finite element problem. One approach is called the flexibility method where the internal forces are the problem unknowns. The other approach is called the displacement or stiffness method where the unknowns are the displacements of the nodes. The majority of finite element software is based on the second method therefore the displacement method will be the focus of this section. As was mentioned earlier, the accuracy of the solution is dependent on the experience and knowledge of the user. It cannot substitute sound engineering judgment. But, in the hands of an experienced user it becomes a very powerful tool. The solution process begins with a choice of the type, size, and number of elements that appropriately describe the physical behaviour of the structure. The basic idea is to strike a balance between solution accuracy and computational effort. There are many different types of elements available. The most common elements are line elements, plane or area elements, and three-dimensional solid elements. Line elements are most commonly used for modeling trusses and frames. Area elements can be used to model slabs, plates, and shear walls. Solid elements are used when a three-dimensional stress analysis is required. There are also many other special purpose elements available, for example, to model nonlinear behaviour or dampers. Once the elements are selected the stiffness matrix for each element is formulated using force-equilibrium and force-deformation relationships. Finally, the stiffness matrices defined in the previous step are assembled into one *global* stiffness matrix which relates all the forces and displacements in the structure with the following matrix equation,

$$[P] = [K][U] \tag{3.1}$$

Where:

[P] = nodal load vector;

[K] = the global stiffness matrix;

[U] = the nodal displacement vector;

3.3 ABAQUS – Finite Element Software

ABAQUS is a general all around finite element software suite capable of handling engineering problems of varying degrees of complexity ranging between simple linear analyses to complex nonlinear simulations. Originally developed for the nuclear industry to model concrete containment structures, the suite is now being used across nearly all engineering industries. The ABAQUS suite consists of three core software packages: ABAQUS/Standard, ABAQUS/Explicit, and ABAQUS/CAE and a number of smaller special-purpose add-on packages. The ABAQUS/Standard package is the general finite element package used to solve static and dynamic, linear and nonlinear, and thermal analyses. ABAQUS/Explicit is used for dynamic blast and impact problems and nonlinear problems with changing contact condition. ABAQUS/CAE is the interactive windows-based portion of the suite used to quickly and efficiently create models and to view results using the ABAQUS/Viewer – a built in Visualization module. ABAQUS/Standard was used to create the bridge models in this study; therefore, the rest of this section deals specifically with the modelling methods, elements, and results of this package.

Finite element modelling using ABAQUS can be separated into three stages: pre-processing, simulation, and post-processing. In the preprocessing stage the finite element model is defined using either ABAQUS/CAE or for simpler geometries, an input file. Next the numerical problem is solved internally in the simulation stage. Finally, in the post-processing stage, the results can be reviewed directly in the data file or using the ABAQUS/Viewer. The transition between the physical problem and the finite element model follows a logical sequence of parts. First, the geometry of the structure is defined, followed by the element section properties, the material data, the loading, the boundary conditions, the type of analysis to be conducted, and lastly the outputs requests. For further clarity, the parts are grouped into two sections, the *model data* which includes all data lines required to completely define the problem, and the *history data* which include the loading and analysis parts. The history data can include any number of analysis *steps* which can be used to separate loading conditions or analysis cases. The entire input file is organized into *data lines* that are grouped into *option blocks.* Every option block begins with a keyword followed by parameters linked to the keyword. Another very useful feature of the program is the ability to group nodes and elements into *sets.* The sets enable the user to reference a group of nodes or elements in the program; therefore, it is important to use meaningful names to define the sets. After the input file is created a *data* check can be run to ensure the model definition is error free and that there is enough memory available to run the analysis. The results of the analysis can be printed in table format to the data file using output variables and/or viewed using the ABAQUS/Viewer. This structured system of problem definition makes it easy for the user to define the problem, adjust the model, and to correct errors.

3.4 Finite-Element Modeling of Curved Composite I-Girder Bridges

3.4.1 Element Selection and Convergence Study

The three-dimensional curved composite slab-on-girder bridges in this study were modelled using ABAQUS/Standard package. Frame elements are ideal for modeling slender member such as beams where one dimension is significantly larger than the other two dimensions. Shell elements are ideal for modeling thin-walled open sections with one dimension much smaller than the others. Therefore, shell and frame elements were used to model the different parts of the bridges. The concrete slab, the steel web, and the flanges were all modeled with shell elements. The cross-braces and the chords were modeled using frame elements. Figure 3.1 shows a typical finite element model bridge cross-section.

Following the standard approach to a finite element problem, a convergence study was conducted to determine the number of elements that would yield accurate results while keeping computational time to a minimum. The webs and flanges were modelled using four and two elements across the depth and width respectively; any increase in the number of elements beyond this resulted in less than 1% change in the solution. In the longitudinal direction, the slabs, webs, and flanges were all modeled using 72 elements for all bridges in the study except for the 10 m span bridges. For the 10 m span bridges, 36 elements were used. This was done to minimize the *aspect ratio* for the concrete deck slab. The aspect ratio can be defined as the ratio of the longest-to-shortest dimension of an element. It was shown by Logan (2007) that keeping the aspect ratio below 4 will limit the error in the solution to below 15%; therefore, an aspect ratio of 4, was chosen as the upper limit. It is important to note that the graph presented in this textbook relates the error of the *vertical displacement* of a cantilever beam to the aspect ratio. However, for axially loaded plates it is expected that the aspect ratio would play less of a role. To examine this, a simple 100 x 50 x 10 flat plate fixed at one end and loaded in tension at the other was modeled using 480 elements resulting in an aspect ratio of 6.67. The solution accuracy was within 2% of the theoretical value. For this reason it can be expected that even for aspect ratios of 4 the accuracy of the solution should be much lower than 15%. One of the bridges in this study was selected to further verify the above assumption. The concrete slab was modeled with aspect ratios of 1.1 and 3.6 and the difference in solution between the two bridges was only 4%. In all the bridges considered in this study, the aspect ratio for the web was maintained at 1.1. For all bridge models, the number of elements between girders and across the concrete slab overhangs was 4 and 2 respectively; therefore, the total number of elements across the bridge width varied with the number of girders. Figures 3.2 and 3.3 show the finite element meshed models for straight and curved bridges respectively.

3.4.2 Finite-Element Models - Calibration

To calibrate the finite element models, a simply supported, 25 m long, three-girder straight bridge was modeled with varying degrees of shear interaction between the concrete deck slab and the top flanges of the steel girders. The width of the bridge was 7.5 m, the girders were spaced 2.5 m apart, and the slab overhang was 1.25 m. The number and type of elements used was as specified in the section above. Based on the length-to-width ratio and the longitudinal-to-lateral stiffness ratio of the bridge, it is reasonable to assume that the behaviour of the bridge will be similar to that of a simply supported beam of equal length and stiffness. To assess the accuracy of the bridge models, the stresses and deflections of the middle girder bottom flange were compared to those calculated using simple beam theory. A comparison of the fundamental natural frequency of the bridge was made using the following equation:

$$f_1 = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}} \qquad (3.2)$$

Where,

f_1 : is the first flexural frequency, L: is the length of the bridge,

EI: is the flexural stiffness of the composite section,

m: is the weight of the bridge per metre,

The models were calibrated by varying the percentage of shear interaction between the concrete slab and steel top flanges from 33% to 100%, and by varying the equivalent area representing the number of shear studs used for frame and shell elements. As was mentioned in the previous section, the number of elements in the longitudinal direction was equal for the concrete slab and the steel flanges to ensure that all the nodes lined up. The shear studs between the elements were modeled using frame elements, shell elements, and multi-point constraints (MPC). A detailed explanation of the behaviour of the elements and constraints is presented in the next section. The results of the calibration study are summarized in Table 3.1. In Table 3.2, the percent error of the solution compared to the solution calculated using beam theory is presented. As can be seen, using MPC constraints with 100% shear interaction yields very accurate results with an average percent error of only 1.7%. Therefore, all the bridges in the parametric and sensitivity studies were modeled using MPC and 100% shear interaction.

3.4.3 Geometric and Material Modeling Using ABAQUS

The concrete deck slab, girder flanges, and the web, were all modeled using linear shell elements. The ABAQUS shell elements used were of type S4R which is a linear 4-node reduced integration element with 6-degrees-of-freedom at each node, three displacements (U1, U2, U3) and three rotations (Φ_1, Φ_2, Φ_3). The shell elements are formulated as true curved shell elements which is ideal for modeling curved bridges. Figure 3.4 shows a diagram of the shell elements used in this study. The cross braces and chords were modeled using beam elements of type B31H. These elements are three-dimensional linear interpolation elements with 6 degrees-of-freedom similar the shell elements described above. The elements are shear deformable Timoshenko beams; thus they can be used to model both slender and non-slender beams. Figure 3.5 shows a diagram of the beam elements used in this study. As was mentioned in the previous section, shear interaction was modeled using the standard ABAQUS multi-point constraint (MPC) type BEAM. The MPC type BEAM provides a rigid beam between the two constrained nodes so that the displacement and rotation of the first node is equal to the displacement and rotation of the second node.

CHAPTER IV

SENSITIVITY STUDY

A sensitivity study was conducted to determine the effects of a broad range of parameters on the fundamental frequency of curved slab-on-girder bridges (from here on in referred to as just curved bridges). The objective of the study was to determine the key parameters affecting the free vibrations response so that they can be included in the parametric study and in the development of the proposed equation. The following parameters were considered in this study:

- Boundary Conditions
- Number of Cross-braces
- Span length
- Radius of Curvature
- Girder Spacing
- Number of Girders
- Depth of Girders
- Flange Thickness
- Cross-brace Stiffness
- Solid End Plate Diaphragms
- Horizontal Bracing Systems

Detailed discussions of the effects of all the parameters listed above are summarized in the following sections of this chapter. The key parameters affecting free vibration response are identified.

4.1 Effect of Boundary Conditions

To study the effects of boundary conditions on the fundamental frequency of curved bridges six different arrangements of support bearings were used as shown in Figures 4.1 to 4.6, where in all cases the bridge is restrained in the gravity direction at all support bearings. The bearings were arranged based on two different approaches: the secant or chord method and the tangential method. As all the bridges considered were simply supported, the bearings for the secant method were arranged in the global x, y, and z directions of the bridge. For the tangential method, all bearings were aligned with the curvature of the bridge or in the local directions. To distinguish between the different arrangements the six cases were labelled as: SA, SB, and SC for bearings in the global directions, and TA, TB, and TC for bearings in the local directions. Table 4.1 and Figure 4.7 show the effects of boundary support conditions on the first five natural frequencies. It can be observed that the effects of the support conditions on the fundamental frequency are negligible with a maximum difference of 9.7%; however, for higher modes the arrangements SB and TB result in lower frequencies with maximum differences of 25% for the second mode, 47% for the third mode, 25% for the fourth mode, and 30% for the fifth mode. Furthermore, these bearing arrangements result in coupled bending, torsion, and lateral flexure modes. This is consistent with the results of the study by Samaan, et al. (2002) . Therefore, for seismic design, arrangements SB and TB should be avoided. For the rest of the sensitivity study and the parametric study, all support bearings for simply supported bridges were oriented in the global directions based on bearing arrangement type SA. For simplicity, the continuous two-span bridge support bearings were oriented in the tangential directions following bearing arrangement type TA.

4.2 Effect of Number of Cross-Braces

The importance of cross-braces as primary load-carrying members in curved bridge design was explained in section 2.1.1 of the literature review. This makes it important to study the effects that cross-braces have on the natural frequencies. Figures 4.8 to 4.11 show the effect of number of cross braces on the first five frequencies of 25 m long curved bridges with different curvature ratios. The number of cross-braces suggested using Davidson, et al.'s equation is also shown. Clearly, there exists a relationship between the natural frequencies of curved bridges and the warping-to-bending stress ratio. In almost all cases, ignoring the maximum cross-brace spacing limit of equation 2.10 can result in a dramatic decrease in natural frequencies. Figures 4.12 to 4.16 show a similar trend for the fundamental frequency of curved bridges with varying span lengths and curvature ratios. Therefore, for all of the bridges modeled in the sensitivity and parametric studies, equation 2.10 was used to determine the number of cross-braces required for further free-vibration analysis.

4.3 Effect of Span Length

It has been long established that span length has a significant effect on the dynamic response of curved bridges. In the case of straight simple span bridges, or more specifically, simply-supported beams, length squared is inversely proportional to the frequency as can be seen in Eq. 3.2. Therefore, it is expected that span length should have a similar effect on the dynamic response of curved bridges. Figures 4.17 and 4.18 show the effect of span length on the fundamental frequency of a simply-supported curved bridges with varying girder spacing and number of girders respectively. Figure 4.19 shows the effect of span length on two-span continuous bridges with varying girder spacing. As is expected, in both cases the frequency decreases with the square of the length. For bridges with the same span length, increasing the number of girders does not have any considerable effect on the fundamental frequency; however, the effect of girder spacing has an increasing effect with decreasing span length with a percent difference up to 18% for 10m long span bridges. This can be attributed to the change in load path for shorter spans. Thus, it can be concluded that span length has a significant impact on the dynamic response therefore, it should be included in the parametric study.

4.4 Effect of Radius of Curvature

Similar to span length, the radius of curvature has a significant impact on the dynamic response of curved bridges. The curvature ratio can be defined herein as the span length divided by the radius of curvature and the aspect ratio is defined as the span length divided by the bridge width. Figures 4.20 to 4.24 show the effects of curvature ratio on the first five natural frequencies of three-girder curved bridges of spans 10, 15, 25, 35, and 45 m. Figures 4.25 to 4.54 show the effect of curvature ratio on the fundamental frequencies of simply supported bridges with different span length, number of girders, and girder spacing. Figures 4.55 to 4.58 show the effect of curvature ratio on the fundamental frequencies of three-girder two-span continuous bridges with different span lengths and girder spacing. Figures 4.59 to 4.68 show views of ABAQUS output for the first five modes of vibration of simply-supported and two-span continuous curved composite I-girder bridges. As can be seen in the figures, the frequencies decrease significantly with an increase in curvature ratio and an increase in aspect ratio. Also, it can be observed that the frequencies associated with modes exhibiting a coupled bending-torsion response have a tendency to decrease; however, the frequencies associated with purely torsional modes have a tendency to increase with curvature. The increase in frequency can be attributed to at least some parts of the bridge twisting in a direction opposite to which the bridge naturally wants to respond under gravity. It is interesting to note that with an increase in length and curvature, the *difference* between fundamental frequencies for bridges with varying girder spacing or with varying number of girders has a tendency to decrease as will be shown in the following sections. In general, the fundamental frequency of curved bridges decreases with an increase in curvature ratio. Clearly, the radius of curvature has a considerable effect on the natural frequencies of curved bridges and should be included in the parametric study.

4.5 Effect of Girder Spacing

In general, the fundamental frequency decreases with an increase in girder spacing. This trend is apparent in Figures 4.69 to 4.99 for simply-supported bridges and Figures 4.100 to 4.103 for two-span continuous bridges, where the fundamental frequency is plotted against the girder spacing for bridges with varying span lengths, curvature ratios, and number of girders. It can be concluded that, for shorter bridges, the fundamental frequency *decreases* with an increase in spacing, but as the span length and curvature ratio increases, there comes a point where increasing the girder spacing results in very little change or even a slight *increase* in fundamental frequency. The latter can be depicted in Figure 4.99 for bridge span of 45 m and curvature ratio of 0.9.

4.6 Effect of Number of Girders

Increasing the number of girders does not seem to have a significant impact on the fundamental frequency of curved bridges. This is expected because in Eq. 3.2, the square root of the flexural stiffness divided by the mass per metre is proportional to the fundamental frequency but an increase in number of girders results in an increase in overall weight so the ratio of flexural stiffness to the bridge weight per metre does not change considerably. Figure 4.104 shows the effect of number of girders on the first five frequencies of 25 m span bridge width 2.0 m girder spacing and curvature ratio of 0.5. Figures 4.105 to 4.112 show the effect of increase of the number of girders on the fundamental frequency of curved bridges with different span lengths and different girder spacing. For shorter bridges, the frequencies tend to decrease very slightly with an increase in number of girders. However, with increases in span length and curvature ratio, a point is reached where increase the number of girders results in an *increase* in fundamental frequency. This increase can be attributed to a decrease in the effects of curvature as the bridge aspect ratio decreases. Although the changes in frequency due to a change in number of girders are not very significant, the coupled effects of both the number of girders and the girder spacing, or simply the bridge width, are significant enough that they should be included in the parametric study.

4.7 Effect of Depth of Girders

As outlined in section 2.3.2 of the literature review, the AASHTO Guide Specifications limit the span-to-depth ratio to a maximum of 25 as a measure of controlling vibrations. For this reason, a practical limit of 20 was selected for *all* the bridges in this thesis study. But, with increasing material strengths, one may see more and more bridge designs that push the limits on the span-to-depth ratios to the maximum allowable or beyond. Therefore, to study the effects of girder depths on the fundamental frequency the following span-to-depth (L/D) ratios were selected: 20, 25, and 30. From Figures 4.113 to 4.116, it can be concluded that span-to-depth ratios have a considerable impact on the fundamental frequency of curved bridges. Therefore, span-to-depth ratio should be included in the parametric study. The increase or decrease in stiffness of the bridge, as a result of varying the span-to-depth ratios, is accounted for in equation 3.2. However, a small deviation in the frequencies can be seen with an increase in curvature ratio.

4.8 Effect of Flange Thickness

Figure 4.117 shows the effects of increasing the bottom flange thickness, and consequently the stiffness of the bridge, on the fundamental frequency. An increase in bridge stiffness by approximately 10% results in an increase in fundamental frequency of less than 4%. From a practical standpoint, the increase does not impact the frequency enough to be justified. Also, as mentioned in the previous section, the change in flexural stiffness is accounted for in equation 3.2. As will be explained in the next chapter, the results from the parametric study are normalized to the predicted frequency using equation 3.1; therefore, there is no reason to exclusively include the flange thickness in the parametric study.

4.9 Effect of Cross-Brace Stiffness

To study the effect of cross brace axial stiffness on the fundamental frequency of curved bridges, several structural angle sizes were modeled as circular rods of equivalent cross-section. The following angle sizes were considered: L150x150x19, L200x150x19, L150x150x25, L200x150x25, and L200x200x25. Figure 4.118 shows the relationship between the cross brace size used and the fundamental frequency. It can be clearly observed that the axial stiffness has a negligible effect on the fundamental frequency.

4.10 Effect of End Plate Diaphragms

Two, 25 m span, simply supported bridge prototypes were used to study the effect of solid plate diaphragms on the fundamental frequency. One was a straight bridge and the other was a curved bridge with a curvature ratio of 0.5. The end diaphragms were first modeled as cross-braces and then as solid plates of varying thickness ranging from 5 mm to 25 mm in increments of 5 mm. It should be noted that the CHBDC recommends a minimum thickness of 10 mm for end diaphragms. Figures 4.119 and 4.120 show the results of the study for the straight and curved bridges respectively. As shown in the figures, the stiffness of the bridges with cross-braces only was equivalent to the stiffness of bridges with end diaphragms of at least 25 mm thick. Using thinner end plates results in a significant decrease in fundamental frequency and should be avoided; however, for seismic design, stiffer diaphragms can lead to brittle connection failures as lateral forces are transferred to the supports through the end diaphragms. Since the scope of this thesis research is the vertical free vibration response of curved bridges, the end diaphragms were modeled as cross-braces for all of the bridges considered in this study.

4.11 Effect of Horizontal Bracing Systems

Horizontal bracing systems are used to stabilize slab-on-girder bridges during construction before the concrete has reached its full strength. Naturally, these systems are permanent in that they stay in place even after the concrete has hardened. The purpose of this section was to determine the effect of these systems on the natural frequencies of curved bridges once full composite action has been reached. Figure 4.121 shows a typical framing plan of a curved girder bridge with vertical x-braces only. Three different types of horizontal bracing arrangements were considered as shown in Figures 4.122 to 4.124. In the first arrangement, the last bays adjacent to the end of the bridge are braced. This type of system is referred to as a "torsion box". In the second arrangement, horizontal bracing is placed along the outer bays of the bridge. Lastly, the third arrangement has horizontal braces placed in all the bays. In reality, both the top and bottom flanges of the girders are braced, but for simplicity in modeling, only the bottom flange braces were included because the top flanges are restrained by the slab diaphragm. Figure 4.125 shows the effects of these horizontal bracing systems on the first five frequencies of a 25 m span bridge with 4 girders, 2.5 m girder spacing, and 0.5 curvature ratio. Clearly, horizontal bracing has negligible effects on the fundamental frequency of curved bridges once the concrete deck slab has cured. However, it enhances the higher frequencies.

Based on the results of the sensitivity study, the key parameters affecting free vibration of curved bridges are identified. These key parameters are included in the parametric study of the next chapter.

CHAPTER V

PARAMETRIC STUDY

In the previous chapter, the key parameters affecting the fundamental frequency of curved composite concrete slab on I-girder bridges were identified as follows: Span Length (L), Curvature Ratio (L/R), Girder Spacing (S), Number of Girders (N), Span-to-Depth Ratio (L/D), and Number of Cross Braces. The last parameter has been excluded from the parametric study as the number of cross braces for each bridge was calculated based on equation 2.10. In 2005, Al-Hashimy (2005) conducted a research study to determine the load distribution factors for curved composite slab-on-girder bridges. In his study, 320 straight and curved slab-on-girder bridges were analyzed using SAP2000 finite element software. Due to the similarity of the parameters considered in Al-hashimy's research to the key parameters listed above, the database of straight and curved bridges from that study has been used as a basis for the database of bridges considered in the parametric study of this chapter.

As outlined in section 1.3, the objectives of this research study are: 1) to examine the effects of various parameters on the fundamental frequency of curved bridges; 2) to generate a database of curved bridges that includes all of the key parameters affecting fundamental frequency; 3) to develop a simple equation that enables engineers to accurately predict the fundamental frequency of single span curved composite concrete-steel I-girder bridges and; 4) to determine the applicability of the equation to continuous two-span bridges. The first of the four objectives was achieved as outlined in Chapter 4. In this chapter, the methodology and results are presented that show how the last three objectives were achieved. As a side objective, the validity of the CHBDC limit (equation 2.4), that enables designers to treat curved bridges that fall below this limit as equivalent straight bridges, is examined with respect to the fundamental frequency.

5.1 Outline of the Parametric Study

In total, 336 straight and curved concrete slab-on-I-girder bridges were included in the parametric study consisting of 288 simple span bridges and 48 two-span continuous bridges. Out of the 288 simple span bridges, 48 were used specifically to study the effects of span-to-depth ratio. The following sections provide an outline of the general design parameters used in the finite element models as well as the range in values of the key parameters included in the parametric study. Figure 5.1 shows a typical bridge cross-section of bridges considered in the parametric study while, Figures 5.2 and 5.3 show the schematic framing plan of a straight and a curved simply-supported bridge, respectively.

5.1.1 General Design Parameters

Two design materials were defined for all the bridges modeled in the sensitivity study and the parametric study. The concrete bridge deck was modeled with a modulus of elasticity of 28,000 MPa, a Poisson's ratio of 0.2, and a density of 24 kN/m^3. For the flanges, webs, and cross-braces, a steel material was defined with a modulus of elasticity of 200,000 MPa, a Poisson's ratio of 0.3, and a density of 76.5 kN/m^3. The concrete deck thickness was 225 mm and the width (W_c) was taken equal to the total bridge width minus 1.0 m to account for parapet thickness. The slab overhangs were taken as half the girder spacing. The girder flanges were 300 x 20 mm thick and the thickness of the webs was taken as 16 mm. The cross-braces and chords were modeled as circular rods with a diameter of 0.04886 m or a cross-sectional area of 0.0075 m^2. This is equivalent to the cross-sectional area of a single L150x150x25 angle. The cross-braces were spaced equally along the length of the bridge. The design parameters above were kept constant for the concrete deck and steel girders for all models except for models used to study the effect of flange thickness and cross-brace stiffness on the fundamental frequency as described in sections 4.8 and 4.9, respectively. A summary of all the general design parameters is presented in Table 5.1. Table 5.2 shows the number of design lanes as a function of bridge width taken directly from the CHBDC.

5.1.2 Key Parameters

The key parameters as identified in the sensitivity study were listed at the beginning of the chapter. The following values have been used to conduct the parametric study:

- Span Lengths (L): 10 m, 15 m, 25 m, 35 m, and 45 m;
- Curvature Ratio (L/R):
    - For 10 m spans: 0.0, 0.1, 0.2, and 0.3;
    - For 15 m spans: 0.0, 0.1, 0.2, and 0.3;
    - For 25 m spans: 0.0, 0.1, 0.3, and 0.5;
    - For 35 m spans: 0.0, 0.1, 0.4, and 0.7;
    - For 45 m spans: 0.0, 0.1, 0.5, and 0.9;
- Girder Spacing (S): 2 m, 2.5 m and 3 m
- Number of Girders (N):

- For 2 m girder spacing: 2, 3, 4, 5, 6, and 7;
- For 2.5 m girder spacing: 2, 3, 4, 5, and 6;
- For 3 m girder spacing: 2, 3, 4, and 5;

- Span-to-Depth Ratio (L/D): 20, 25, and 30;
- Number of Cross-Braces: 3 braces to 13 braces depending on equation 2.10.

As was mentioned earlier, 48 additional bridges were modeled exclusively to study the effect of span-to-depth ratio on the fundamental frequencies of curved bridges. For these bridges, the number of girders and girder spacing were kept constant at 3 and 2.5 m respectively, and the 10 m span length was excluded. All other parameters remained as described above. Similarly, for the 48 continuous two-span bridges, the 10 m span length was excluded and only 3 girders across the bridge width were considered. The span-to-depth ratio was kept constant at L/20 and the girder spacing was 2 m, 2.5 m, and 3 m. Table 5.3 shows a summary of all the bridge configurations considered in the parametric study.

5.2 Research Assumptions

For all bridges modeled in the parametric study, the following assumptions were made:

1.) Full shear interaction between the concrete deck slab and the top flanges of the steel girders was assumed. The validity of this assumption was checked as described in chapter 3, section 3.4.2;

2.) All materials were elastic and homogenous;

3.) The effects of superelevation, parapets, and cross-brace flexural stiffness were not considered in the study;

4.) Simple supports (pins and rollers) were assumed for all bridges;

5.) Constant radius of curvature was maintained between bridge supports.

5.3 Proposed Fundamental Frequency Equation For Simple Span Bridges

Data for 240 simple span bridges with constant span-to-depth ratio were collected and normalized to the fundamental frequency of simple span beams calculated using equation 3.2 to account for the stiffness of the bridge. The equation is reproduced in this chapter as equation 5.1 shown below:

$$f_{sb} = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}} \tag{5.1}$$

Where,

f_{sb} : is the first flexural frequency,

L: is the length of the bridge,

EI: is the flexural stiffness of the composite section,

m: is the weight of the bridge per metre.

Based on the data obtained from the parametric study, a multi-variable nonlinear regression analysis was conducted to develop the following equation for a curvature parameter, κ, which represents the ratio between the curved bridge fundamental frequency and that of an equivalent straight simple beam.

$$\kappa = 0.97\left[1 - 1.1\left(\frac{L}{R}\right)^{1.33} (S \cdot N)^{-0.33}\right] \quad (5.2)$$

Where,

κ: is the curvature factor,

$\frac{L}{R}$: is the curvature ratio,

S: is the girder spacing in metres,

N: is the number of girders across the bridge width.

Multiplying equations 5.1 and 5.2 above results in the following proposed equation for the fundamental frequency of curved composite I-girder bridges:

$$f_1 = \kappa \cdot f_{sb} \quad (5.3)$$

Where,

f_1 : is the fundamental flexural frequency of curved bridges,

f_{sb} : is the fundamental frequency of a simple beam,

κ: is the curvature factor.

Section B.1 of Appendix B summarizes the data used to generate the above mentioned curvature parameter, κ. Figure 5.4 depicts the fundamental frequency obtained for all studied bridges using the finite element modeling as a function of span length. Figure 5.5 shows a correlation between the FEA fundamental frequencies of the bridges and those obtained for an equivalent straight beam using equation 5.1 and Figure 5.6 shows a correlation between the FEA fundamental frequencies of the bridges and those obtained from the developed equation 5.3. It can be observed that equation 5.3 predicts well the fundamental frequency of simply-supported curved composite I-girder bridges with 66% of all the bridges in the parametric study falling within 3% of the value from finite element analysis, 85% of all the bridges falling within 5% of the value from finite element analysis, 95% of all bridges falling within 8% of the value from finite element analysis, and 97% of all bridges falling within 10% of the value from finite element analysis. Also, of the 3% of all the bridges with errors greater than 10%, no bridges resulted in an error greater than 18%. It is important to mention that the greatest scatter in frequencies occurs between 7 and 10 Hz as shown in Figure 5.6. These higher frequencies are characterized for shorter span bridges with lower curvature ratios. Considering the CHBDC superstructure deflection limitations shown in Figure 2.1, the difference in static deflection limits for the 7 to 10 Hz range is very small. Therefore, the proposed equation works very well with Figure 2.1 and can be used with good accuracy to control vibrations.

5.4 Proposed Fundamental Frequency Equation For Continuous Two-Span Bridges

As mentioned in section 2.5 of the literature review, Culver and Oestel (1969) show that for the case of beams with equal spans, the mode shapes were either symmetric or anti-symmetric and independent; thus, they can be considered separately. This makes it possible to relate the fundamental frequency of simple span bridges to the fundamental frequency of multi-span bridges. Considering equation 5.4, the first circular frequency of a simple span bridge of span length L is *equal* to the nth circular frequency of a simple span bridge with span length, xL, where x is equal to n,

$$\omega_n = \frac{(n\pi)^2}{(xL)^2}\sqrt{\frac{EI}{m}} \tag{5.4}$$

Where,

ω_n : is the natural circular frequency for the nth mode,

L: is the length of the bridge,

x: is the span length multiplier (x = n),

EI: is the flexural stiffness of the composite section,

m: is the weight of the bridge per metre,

n: is the mode number.

The inflection points of the anti-symmetric modes of simple span bridges of length xL can be related to the supports of multi-span bridges of span length L and number of span x. Figure 5.7 shows this relationship. Thus, the first frequency of a multi-span bridge with length xL is also equal to the *first* frequency of a bridge with length L.

To verify if the proposed equation 5.3 can be used to predict the natural frequency of multi-span bridges, 48 straight and curved two-span bridges with equal span lengths were modeled. Figure 5.8 shows the correlation between the fundamental frequency for simple and continuous span bridges. Also, Section B.2 of Appendix B summarizes the results for continuous span bridges considered in this study. As expected, the difference between the fundamental frequency of the multi-span bridges was within 1% of the frequency obtained for simple span bridges of equal span length L. Therefore, equation 5.3 is equally applicable to multi-span bridges with *equal* span lengths.

5.5 Modification Factor for Span-to-Depth Ratio

As was mentioned in Chapter 4, the span-to-depth ratio has a significant impact on the fundamental frequencies of bridges. In the case of straight bridges, equation 5.3 written as a function of the simple beam equation, which includes the flexural stiffness EI, accounts for the changes in span-to-depth ratio. This is expected as the first mode response of straight bridges is vertical bending only. However, in the case of curved bridges, the dynamic response is a coupled bending and torsion and the flexural stiffness parameters EI in equation 5.1 may not fully capture the effects of span-to-depth ratio on the fundamental frequency. To study the effect of span-to-depth ratio, 48 additional simply-supported single span straight and curved bridges were modeled. The bridges were modeled with constant girder spacing and number of girders, and with span-to-depth ratios of 20, 25, and 30 as summarized in Table 5.3. The results of the finite element analysis were compared to fundamental frequencies predicted using equation 5.3. Out of the 48 bridges, 31% were within 3%, 60% were within 5%, 77% were within 8%, and 85% were within 10%, with the largest error in values of 16%. Therefore, the flexural stiffness EI in the simple beam equation 5.1 does not fully capture the effect of span-to-depth ratio. The following equation modifier was developed to account for span-to-depth ratios:

$$\alpha = 0.33\left[1 + \left(\frac{L}{D}\right)^{0.238}\right] \qquad (5.5)$$

Where,

α : is the span-to-depth modifier,

L: is the span length in metres,

D: is the girder depth in metres.

Equation 5.3 multiplied by the modifier in equation 5.5 results in fundamental frequencies that are much closer to those obtained from finite element analysis of the 48 bridges. Out of the 48 bridges considered: 60% of the modified values fall within 3% of the value from finite element analysis, 81% of the modified values fall within 5% of the value from finite element analysis, and 96% of the modified values fall within 10% of the values from finite element analysis. No bridges result in an error greater than 13%. Figure 5.9 shows the correlation between the values obtained from FEA analysis and equation 5.6 for span-to-depth ratios of 20, 25, and 30. To further verify the accuracy of the modifier equation, ten arbitrary bridges were selected and modeled with span-to-depth ratios ranging from 20 to 30 as shown in Table 5.4. The results show that the modifier equation works well at reducing the error associated with varying span-to-depth ratios for curved bridges. The final modified equation to predict the fundamental frequency of curved composite I-girder bridges is:

$$f_1 = \alpha \cdot \kappa \cdot f_{sb} \tag{5.6}$$

Where,

f_1 : is the fundamental flexural frequency of curved bridges,

f_{sb} : is the fundamental frequency of a simple beam,

κ : is the curvature factor,

α : is the span-to-depth modifier (if span-to-depth ratio is not 20).

In summary, equation 5.3 may be used to predict the fundamental frequency of straight and curved simple and continuous span bridges with *equal* spans where the span-to-depth ratios are equal to or relatively close to 20. For span-to-depth ratios greater than 20, the modifier presented in equation 5.5 can be used.

5.6 Validity of the CHBDC L^2/BR Curved Bridge Limit

The Canadian Highway Bridge Design Code (CHBDC) provides a simple equation (Eq. 2.4) to determine if a curved bridge can be treated as a straight one. Out of 240 simple span bridges considered in the parametric study, 180 are curved bridges of which 123 fall under the CHBDC limit and therefore according to the code can be treated as straight. The calculated frequency using the simple beam equation tends to over-estimate the frequency predicted using finite element analysis. Out of 123 bridges, 86 of the bridges result in a calculated frequency that is higher then the finite element result by more than 5%, and 62 of the bridges result in a calculated frequency that is higher then the finite element result by more than 10%. Furthermore, the percent difference can be as high as 33% and if the CHBDC limit is exceeded the percent difference increases dramatically. Based on these results the current limits in the CHBDC should be examined further for both open and closed sections with respect to static and dynamic response. It is expected that the response of curved bridges with open cross sections will differ from the response of curved bridges with closed sections and therefore separate limits should be used similar to AASHTO Guide Specifications. In general, the current limit of 0.5 in equation 2.4 should be decreased and the CHBDC needs to be updated to include a section that specifically deals with curved bridge design.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

To eliminate discomfort and structural damage as a result of superstructure vibrations the Canadian Highway Bridge Design Code limits deflections as a function of the fundamental frequency of bridges. AASHTO codes limit superstructure vibrations with a deflection limit that is a function of length. In the literature review of this study it was observed that limits based on length are insufficient and that limits based on fundamental frequency are much more adequate at controlling superstructure vibrations. However, the codes do not provide any equations to calculate the fundamental frequency of curved bridges and the equations that exist in research are either to simple to account for the effects of all the key parameters, or too complex for practical engineering use.

Using a finite element analysis, 336 straight and curved concrete slab-on-I-girder bridges were included in a parametric study consisting of 288 simple span bridges and 48 two-span continuous bridges. The key parameters included were as follows:

- Span Lengths (L): 10 m, 15 m, 25 m, 35 m, and 45 m;
- Curvature Ratio (L/R):
    - For 10 m spans: 0.0, 0.1, 0.2, and 0.3;
    - For 15 m spans: 0.0, 0.1, 0.2, and 0.3;
    - For 25 m spans: 0.0, 0.1, 0.3, and 0.5;
    - For 35 m spans: 0.0, 0.1, 0.4, and 0.7;
    - For 45m spans: 0.0, 0.1, 0.5, and 0.9;

- Girder Spacing (S): 2 m, 2.5 m and 3 m
- Number of Girders (N):
    - For 2 m girder spacing: 2, 3, 4, 5, 6, and 7;
    - For 2.5 m girder spacing: 2, 3, 4, 5, and 6;
    - For 3 m girder spacing: 2, 3, 4, and 5;
- Span-to-Depth Ratio (L/D): 20, 25, and 30;
- Number of Cross-Braces: 3 braces to 13 braces depending on equation 2.10.

The key parameters listed above were identified in a sensitivity study conducted to determine the effects of several design parameters on the fundamental frequency of curved composite I-girder bridges. The parameters included in the sensitivity study were as follows:

- Boundary Conditions
- Number of Cross-braces
- Span length
- Radius of Curvature
- Girder Spacing
- Number of Girders
- Depth of Girders
- Flange Thickness
- Cross-brace Stiffness
- Solid End Plate Diaphragms
- Horizontal Bracing Systems

6.2 Conclusions

From the results of the sensitivity and parametric studies, the following conclusions are drawn:

1.) Curvature ratio (L/R) and span length (L) have a significant impact on the fundamental frequency of curved composite I-girder bridges. The fundamental frequency decreases substantially with an increase in curvature ratio or with an increase in span length.

2.) The girder spacing and the number of girders have a less pronounced effect on the fundamental frequency but in general the fundamental frequency decreases with an increase in spacing and number of girders. However; with an increase in span and curvature ratio a point is reached where an increase in number of girders results in an increase in fundamental frequency.

3.) The effect of number of cross-braces on the fundamental frequency stabilizes if the bracing spacing used is less than the maximum calculated using Davidson et al.'s equation. If the spacing is increased beyond the limit specified in Davidson et al.'s equation the fundamental frequencies begin to decrease dramatically.

4.) The span-to-depth ratio has a significant effect on the fundamental frequency of both straight and curved bridges. The fundamental frequency calculated using the simple beam equation is a function of the flexural stiffness EI; thus, the effect of span-to-depth is accounted for; however, for curved bridges the flexural stiffness EI does not appear to account for the full effect. A modifier to account for this is provided.

5.) From a nonlinear regression analysis of the parametric study results, a simple to use equation is developed to obtain the fundamental frequency of curved bridges as a function of the equivalent straight simple beam frequency, curvature ratio, span length, girder spacing, number of girders, and span-to-depth ratio. The fundamental frequencies predicted using this equation correlate well with the fundamental frequencies calculated using the finite element method.

6.) The current limit in the CHBDC which enables designers to treat a curved bridge as a straight one is inaccurate for some curved bridge configurations and needs to be examined.

6.3 Recommendations for Future Research

The author recommends future research in the following areas:

1- Experimental verification of bridges with similar geometry to those included in this study to provide more confidence in the proposed equations.

2- The study of free vibration response of multi-span curved bridges with unequal spans.

3- The study of human perception to the superstructure vibration of curved bridges.

4- Effect of boundary condition restraints and sub-structure on the fundamental frequency

5- Investigation on the safe limiting curvature ratio in the CHBDC to treat a curved bridge as an equivalent straight bridge.

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Table 2.1 CHBDC Dynamic Load Allowance (CSA, 2006)

Component	DLA
Deck Joints	0.5
CL-W Truck where only one axle is used, except for deck joints	0.4
CL-W Truck where any two axles, or axles 1,2, and 3 are used	0.3
CL-W Truck where three axles, except for axles 1,2, and 3 are used, or more than three axles are used	0.25

Table 2.2 AASHTO Impact Factor for I-Girders (AASHTO, 2003)

Load Effect	Impact Factor	
	Vehicle	Lane
Girder bending moment, torsion and deflections	0.25	0.20
Reactions, shear, cross frame and diaphragm actions	0.30	0.25

Table 2.3 AASHTO Impact Factor for Closed Box and Tub Girders (AASHTO, 2003)

Load Effect	Impact Factor	
	Vehicle	Lane
Girder bending moment, torsion and deflections	0.35	0.30
Reactions, shear, cross frame and diaphragm actions	0.40	0.35

Table 2.4 AASHTO LRFD Impact Factor, IM (AASHTO, 2004)

Component	IM
Deck Joints – All Limit States	0.75
All Other Components • Fatigue and Fracture Limit State • All Other Limit States	 0.15 0.33

Table 2.5 Limiting Central Angle for Neglecting Curvature in Determining Primary Bending Moments (Open Cross-Sections) (AASHTO, 2003)

Number of Beams	Angle for One Span	Angle for Two or More Spans
2	2°	3°
3 or 4	3°	4°
5 or more	4°	5°

Table 3.1 Effect of the Type of Shear Connector on the Fundamental Frequency of Straight Bridges (Bridge: L=25m, S=2.5m, N=3)

		Shell Elements (% of Nodes Connected)				Frame Elements (% of Nodes Connected)				MPC		
Beam Theory		**33%**	**50%**	**66%**	**100%**	**33%**	**50%**	**66%**	**100%**	**100%**	**50%**	**33%**
$\Delta(x10^{-2})$ mm	2.20	2.29	2.26	2.24	2.23	2.90	2.57	2.43	2.32	2.20	2.40	2.26
σ (MPa)	78.0	77.0	76.7	76.6	76.4	83.0	79.7	78.4	77.3	76.2	78.1	76.8
f (Hz)	3.78	3.61	3.64	3.65	3.66	3.23	3.42	3.51	3.59	3.68	3.52	3.64

Table 3.2 Effect of the Type of Shear Connector as a Percentage Difference to the Value Calculated Using Beam Theory

		Shell Elements (% of Nodes Connected)				Frame Elements (% of Nodes Connected)				MPC		
Beam Theory		**33%**	**50%**	**66%**	**100%**	**33%**	**50%**	**66%**	**100%**	**100%**	**50%**	**33%**
$\Delta(x10^{-2})$ mm	0.0	4.1	2.7	1.8	1.4	31.8	16.8	10.5	5.5	0.0	9.1	2.7
σ (MPa)	0.0	1.3	1.7	1.8	2.1	6.4	2.2	0.5	0.9	2.3	0.1	1.5
f (Hz.)	0.0	4.5	3.7	3.4	3.2	14.6	9.5	7.1	5.0	2.6	6.9	3.7
Avg.	0.0	3.3	2.7	2.4	2.2	17.6	9.5	6.0	3.8	1.7	5.4	2.7

Table 4.1 Effect of the Type of Boundary Conditions on the Fundamental Frequency of Curved Bridges (Bridge: L=25m, S=2.5m, N=4, L/R = 0.5)

Method	Bearing Type	f_1 (Hz) / Mode Shape	f_2 (Hz) / Mode Shape	f_3 (Hz) / Mode Shape	f_4 (Hz) / Mode Shape	f_5 (Hz) / Mode Shape
Secant	A (SA)	2.89 / FST	5.44 / ST	10.68 / AT	11.93 / L	14.81 / AT
	B (SB)	2.67 / FST	4.58 / ST	7.72 / AT	11.29 / AT	11.85 / L
	C (SC)	2.93 / FST	5.65 / ST	10.75 / AT	12.60 / L	15.00 / AT
Tangent	A (TA)	2.84 / FST	5.59 / ST	10.03 / AT	11.59 / AT	14.99 / AT
	B (TB)	2.67 FST	4.53 / ST	7.31 / ST	11.28 / AT	11.54 / L
	C (TC)	2.85 / FST	5.63 / ST	10.64 / ST	14.13 / L	15.06 / AT

FST = Flexural & Symmetric Torsion, ST = Symmetric Torsion, AT = Anti-symmetric Torsion, L = Lateral

Table 4.2 Number of Braces Required by Davidson et al.'s Equation

Bridge Family	Length (m)	Radius of Curvature	Bridge Radius (m)	Flange Width (mm)	S_{max}	# of Braces (Simple)	# of Braces (2-Span)
L10R1	10	0.1	100.0	300	3.78	4	n/a
L10R2	10	0.2	50.0	300	2.35	5	n/a
L10R3	10	0.3	33.3	300	1.89	7	n/a
L15R1	15	0.1	150.0	300	4.22	5	9
L15R2	15	0.2	75.0	300	2.83	7	13
L15R3	15	0.3	50.0	300	2.33	7	13
L25R1	25	0.1	250.0	300	5.19	7	10
L25R3	25	0.3	83.3	300	3.14	9	13
L25R5	25	0.5	50.0	300	2.61	13	19
L35R1	35	0.1	350.0	300	6.12	7	13
L35R4	35	0.4	87.5	300	3.51	13	19
L35R7	35	0.7	50.0	300	2.95	13	25
L45R1	45	0.1	450.0	300	7.00	7	13
L45R5	45	0.5	90.0	300	3.88	13	25
L45R9	45	0.9	50.0	300	3.28	13	25

Table 5.1 Bridge Configurations Considered in the Parametric Study

Bridge Width (m)	Deck Width W_c (m)	Number of Girders	Number Spacing	Number of Design Lanes n
6	5	3	2	1-lane
7.5	6.5	3	2.5	2-lane
9	8	3	3	2-lane
8	7	4	2	2-lane
10	9	4	2.5	2-lane
12	11	4	3	2-lane & 3-lane
10	9	5	2	2-lane
12.5	11.5	5	2.5	2-lane & 3-lane
15	14	5	3	4-lane
12	11	6	2	2-lane & 3-lane
15	14	6	2.5	4-lane
14	13	7	2	2-lane & 3-lane

Table 5.2 Number of Design Lanes

W_c	n
6.0 m or less	1
Over 6.0 m to 10.0 m incl.	2
Over 10.0 m to 13.5 m incl.	2 or 3
Over 13.5 m to 17.0 m incl.	4
Over 17.0 m to 20.5 m incl.	5
Over 20.5 m to 24.0 m incl.	6
Over 24.0 m to 27.5 m incl.	7
Over 27.5 m	8

Table 5.3 Parametric Study – Simple Span Bridge Configurations

Span Length (L)	Girder Spacing (S)	Number of Girders (N)	Span-to-Depth Ratio (L/D)	Curvature Ratio (L/R)
SIMPLE SPAN BRIDGES				
10	2, 2.5, 3	3-7	20	0.0, 0.1, 0.2, 0.3
15	2, 2.5, 3	3-7	20	0.0, 0.1, 0.2, 0.3
25	2, 2.5, 3	3-7	20	0.0, 0.1, 0.3, 0.5
35	2, 2.5, 3	3-7	20	0.0, 0.1, 0.4, 0.7
45	2, 2.5, 3	3-7	20	0.0, 0.1, 0.5, 0.9
15	2.5	3	20, 25, 30	0.0, 0.1, 0.2, 0.3
25	2.5	3	20, 25, 30	0.0, 0.1, 0.3, 0.5
35	2.5	3	20, 25, 30	0.0, 0.1, 0.4, 0.7
45	2.5	3	20, 25, 30	0.0, 0.1, 0.5, 0.9
TWO-SPAN CONTINUOUS BRIDGES				
15	2, 2.5, 3	3	20	0.0, 0.1, 0.2, 0.3
25	2, 2.5, 3	3	20	0.0, 0.1, 0.3, 0.5
35	2, 2.5, 3	3	20	0.0, 0.1, 0.4, 0.7
45	2, 2.5, 3	3	20	0.0, 0.1, 0.5, 0.9

Table 5.4 Span-to-Depth Ratio – Simple Span Bridge Configurations

Span Length (L)	Girder Spacing (S)	Number of Girders (N)	Span-to-Depth Ratio (L/D)	(L/R)	f_{FEA}	Unmodified Frequency (Eq. 5.3) (7)	Modified Frequency (Eq.5.6) (8)	% Change (8) / (9)
10	2.5	3	20	0.1	8.74	9.05	9.08	0.33%
35	2.0	7	21	0.7	1.86	1.87	1.89	1.07%
35	3.0	5	22	0.1	2.27	2.21	2.25	1.81%
15	2.0	6	23	0.3	5.07	4.95	5.08	2.63%
35	2.5	4	24	0.1	2.21	2.13	2.20	3.29%
25	2.5	5	25	0	2.97	2.88	3.00	4.17%
45	2.5	5	26	0.5	1.27	1.27	1.33	4.72%
15	3.0	3	27	0.3	4.04	3.76	3.96	5.32%
25	2.0	3	28	0.5	2.30	2.03	2.15	5.91%
25	3.0	4	29	0.3	2.21	2.06	2.20	6.80%
45	3.0	5	30	0.9	0.81	0.77	0.83	7.79%

Figure 1.1 Typical Bridge Cross-Section

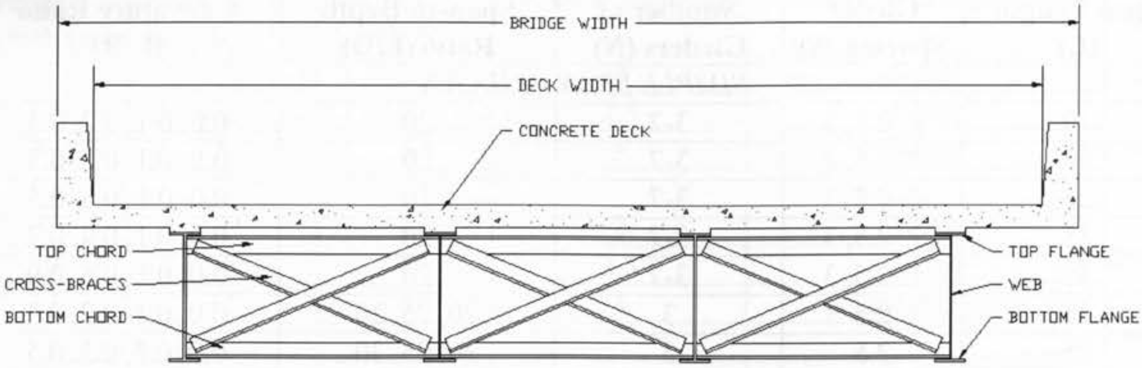


Figure 1.2 Typical Girder Erection Schemes

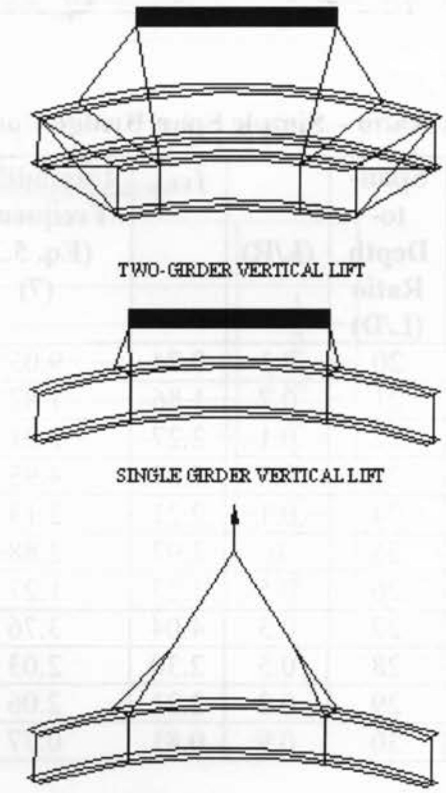


Figure 2.1 **Deflection Limitations for Highway Bridge Superstructure Vibration (CHBDC, 2006)**

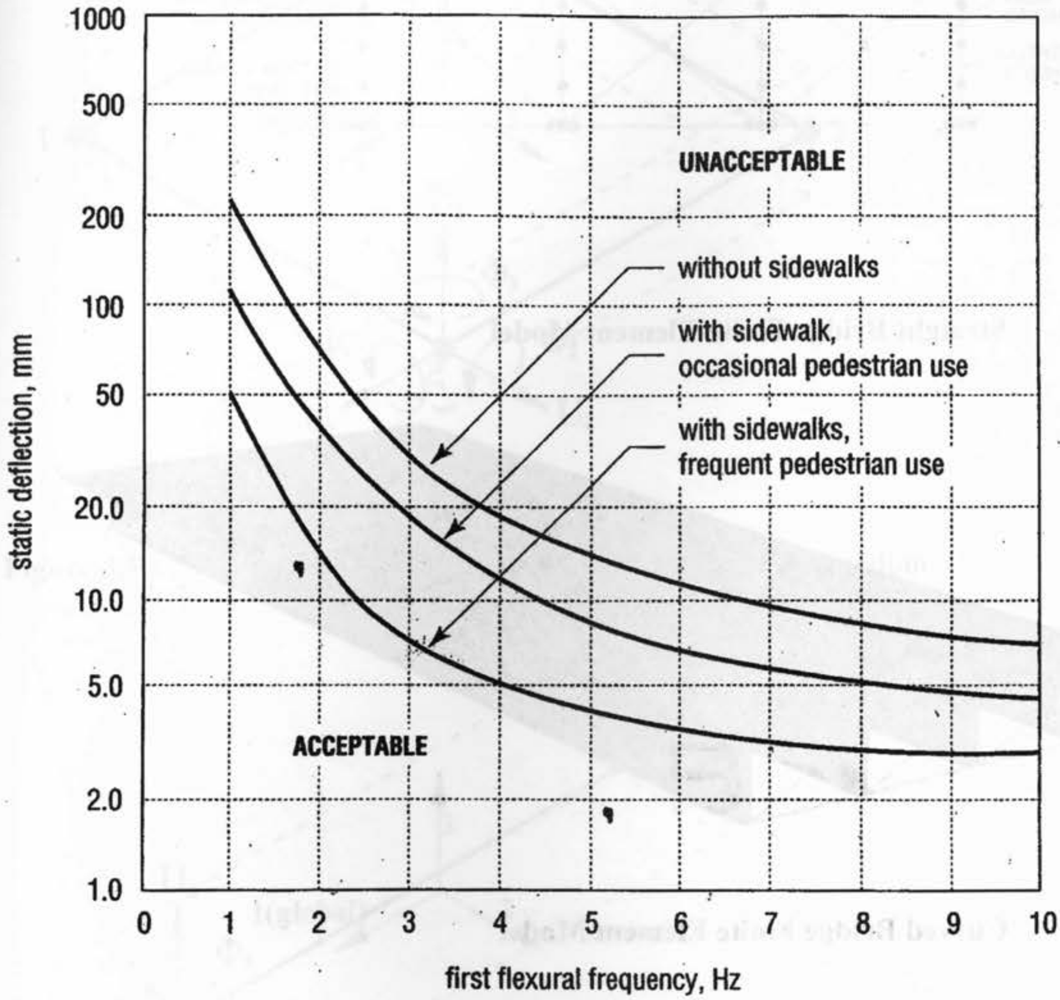


Figure 3.1 Typical Finite Element Model Bridge Cross-Section

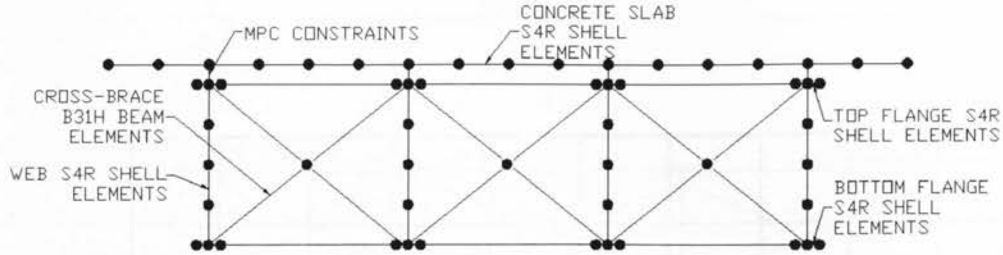


Figure 3.2 Straight Bridge Finite Element Model

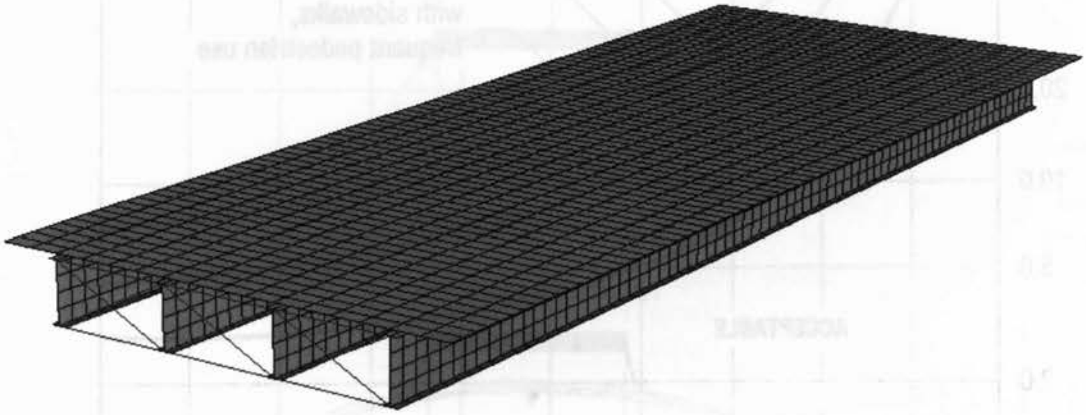


Figure 3.3 Curved Bridge Finite Element Model

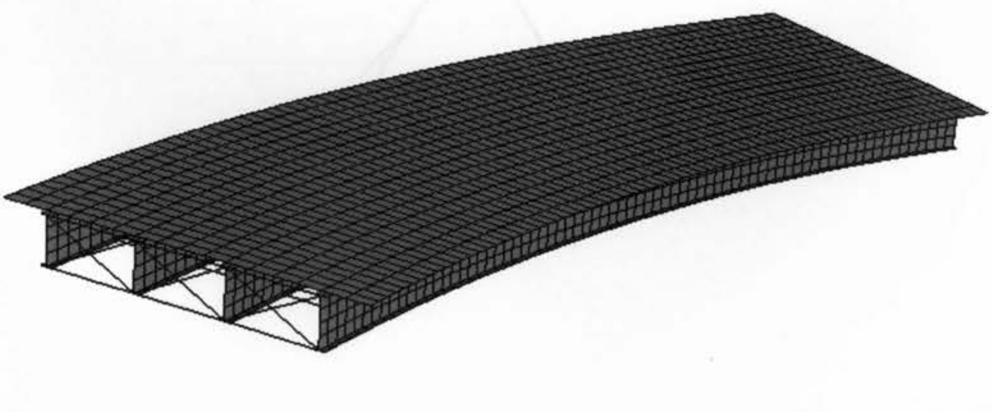


Figure 3.4 ABAQUS Shell Element (S4R) Degrees-of-Freedom

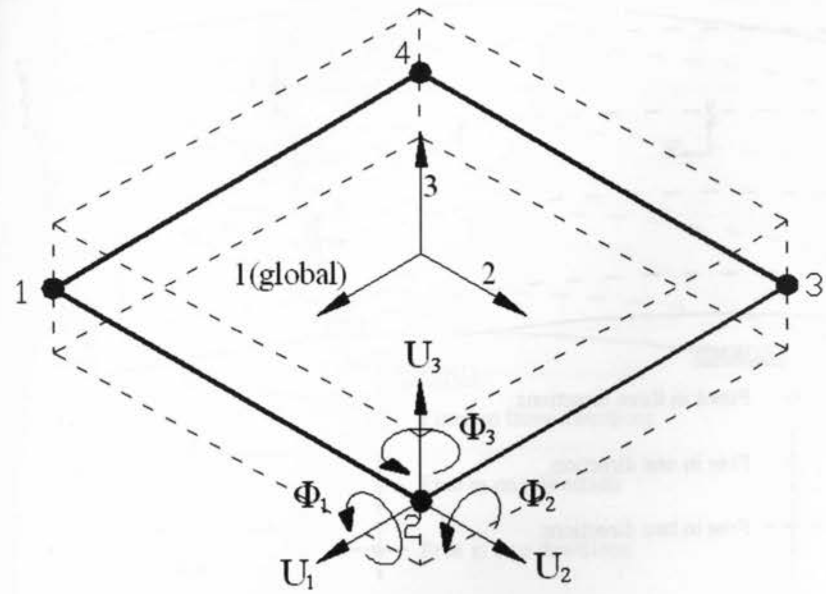


Figure 3.5 ABAQUS Beam Element (B31H) Degrees-of-Freedom

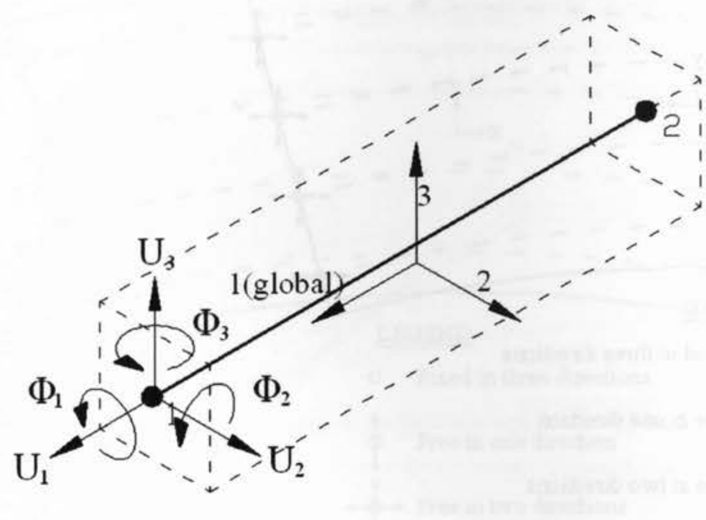


Figure 4.1 Bridge Bearing Support Arrangement Type SA

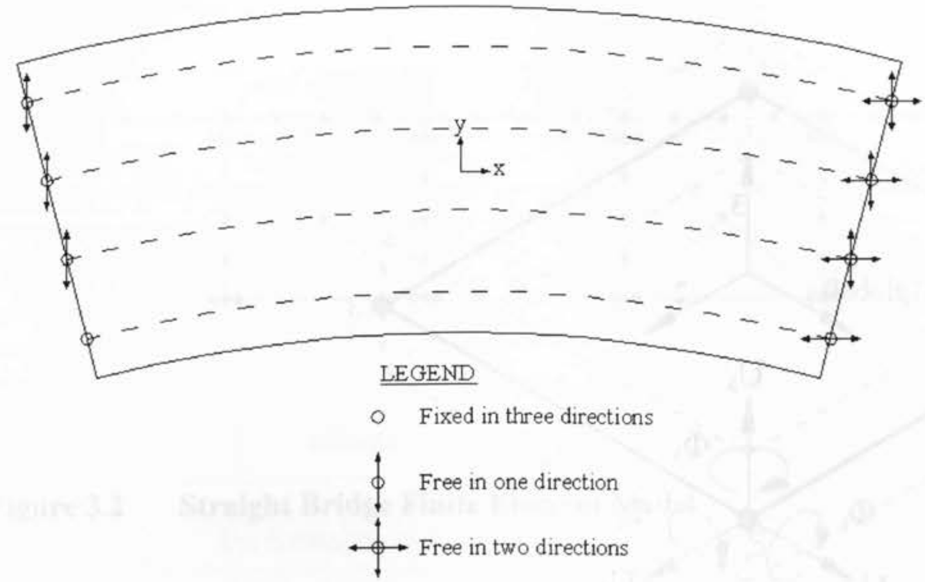


Figure 4.2 Bridge Bearing Support Arrangement Type SB

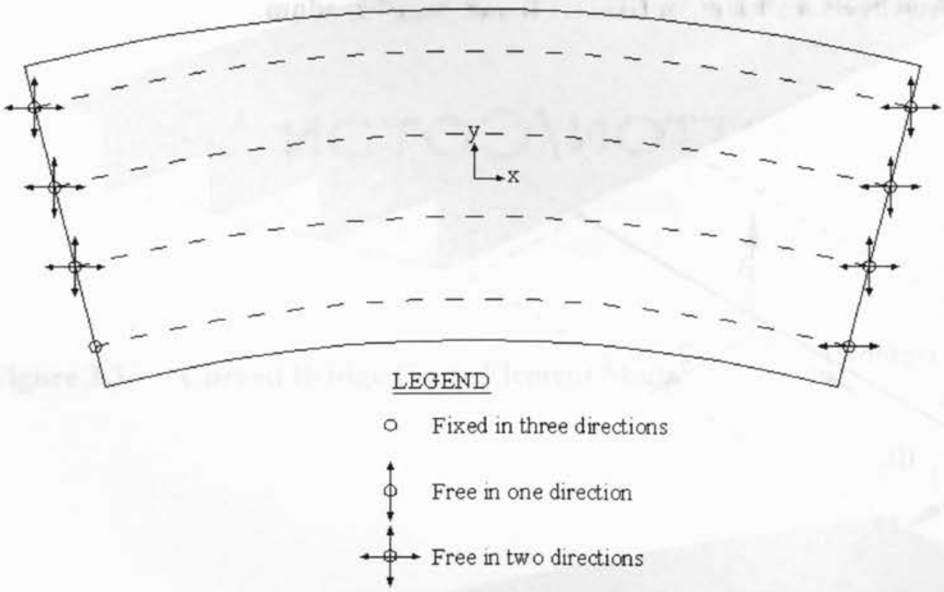


Figure 4.3 Bridge Bearing Support Arrangement Type SC

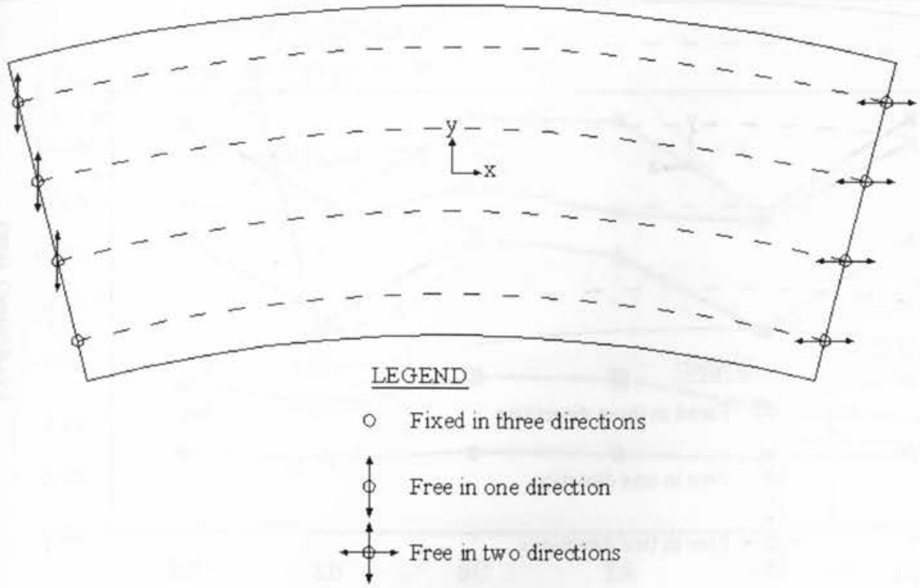


Figure 4.4 Bridge Bearing Support Arrangement Type TA

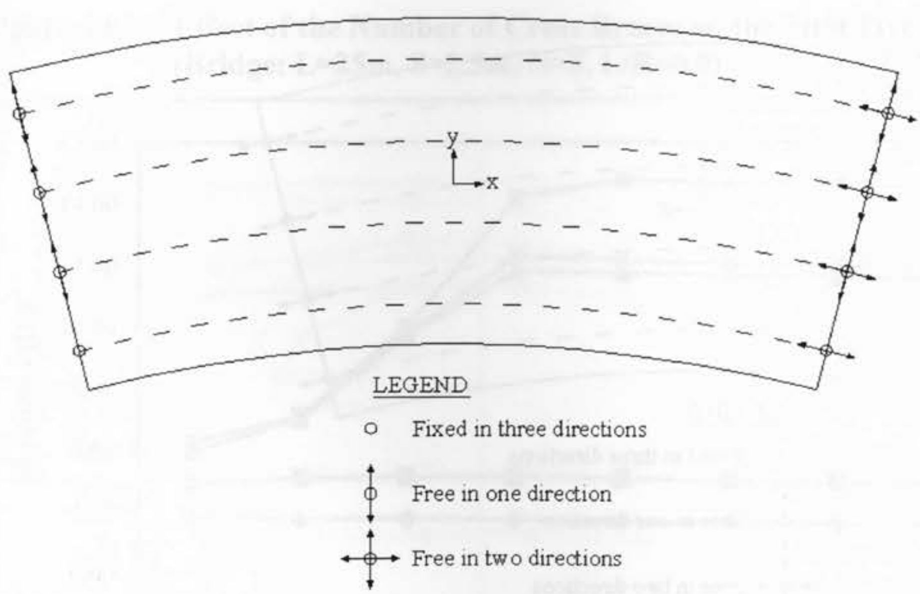


Figure 4.5 Bridge Bearing Support Arrangement Type TB

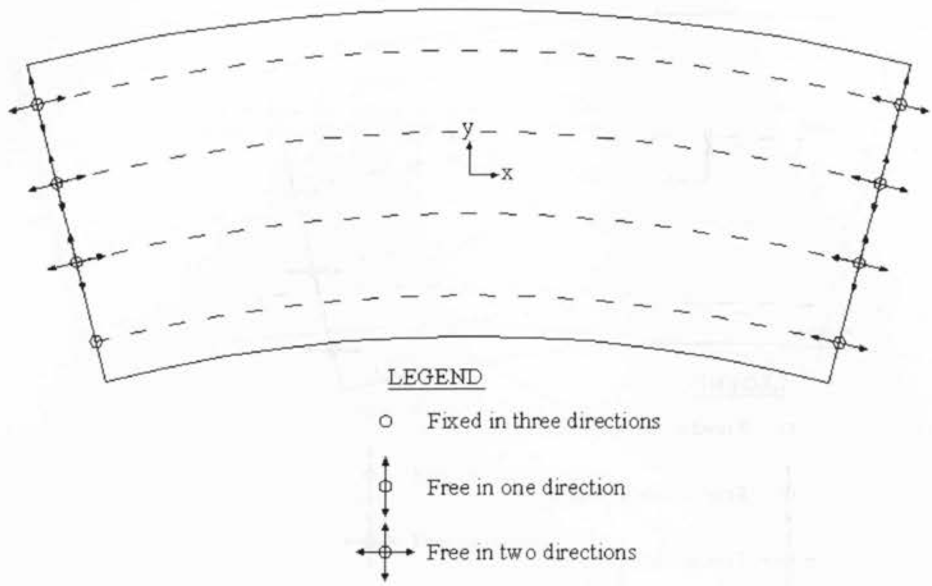


Figure 4.6 Bridge Bearing Support Arrangement Type TC

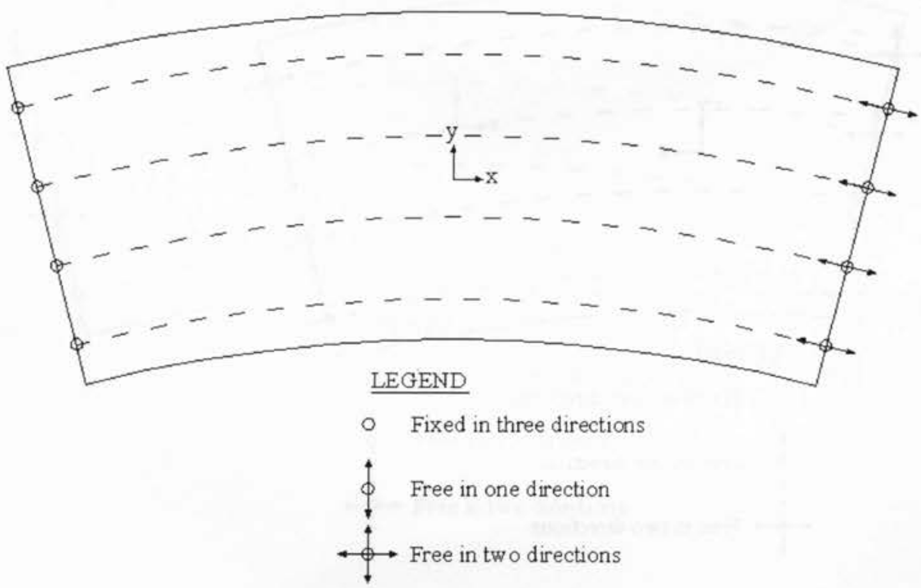


Figure 4.7 Effect of the Type of Boundary Conditions on the First Five Frequencies of Curved Bridges (Bridge: L=25m, S=2.5m, N=4, L/R = 0.5)

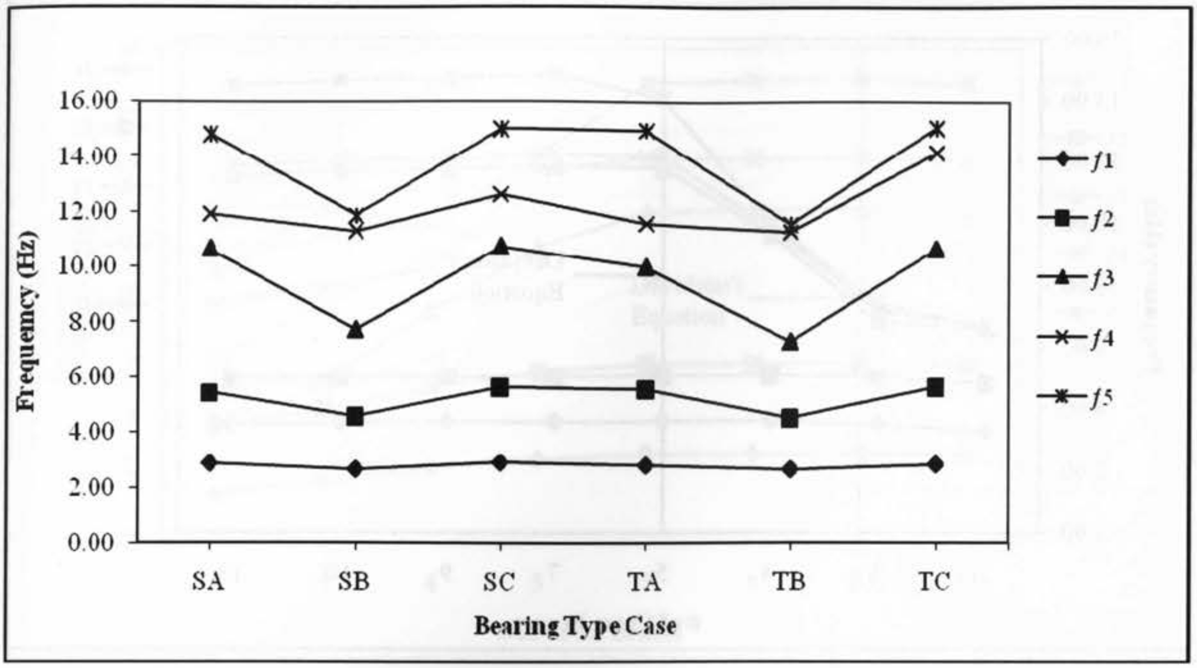


Figure 4.8 Effect of the Number of Cross Braces on the First Five Frequencies (Bridge: L=25m, S=2.5m, N=3, L/R=0.0)

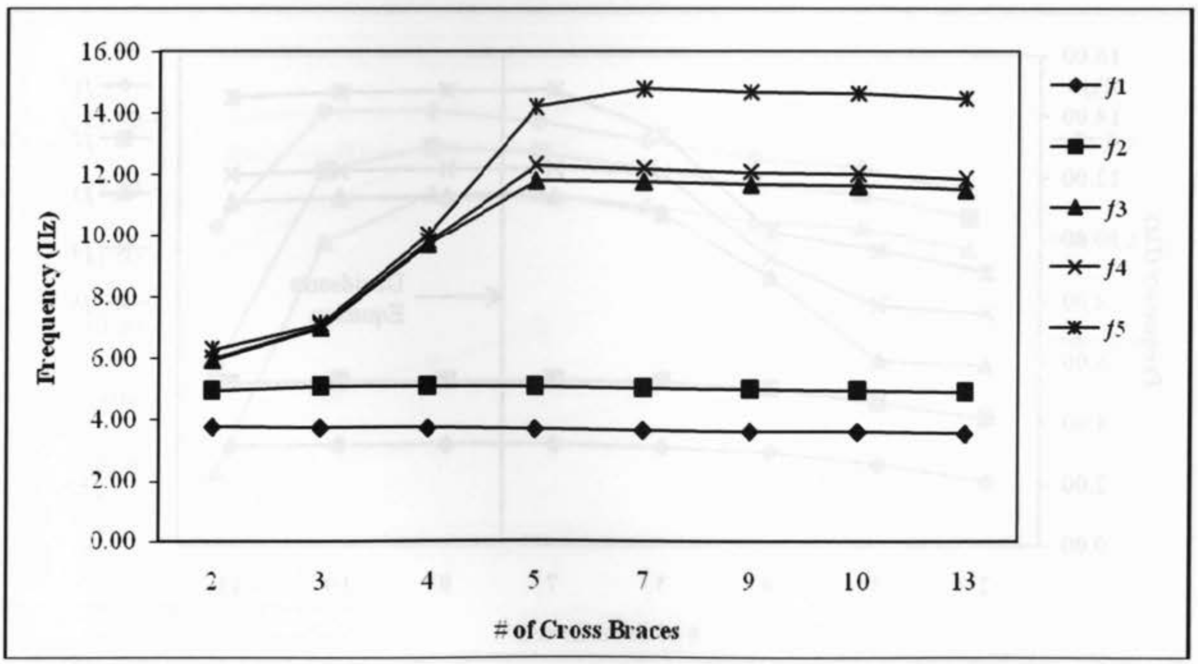


Figure 4.9 **Effect of the Number of Cross Braces on the First Five Frequencies (Bridge: L=25m, S=2.5m, N=3, L/R=0.1)**

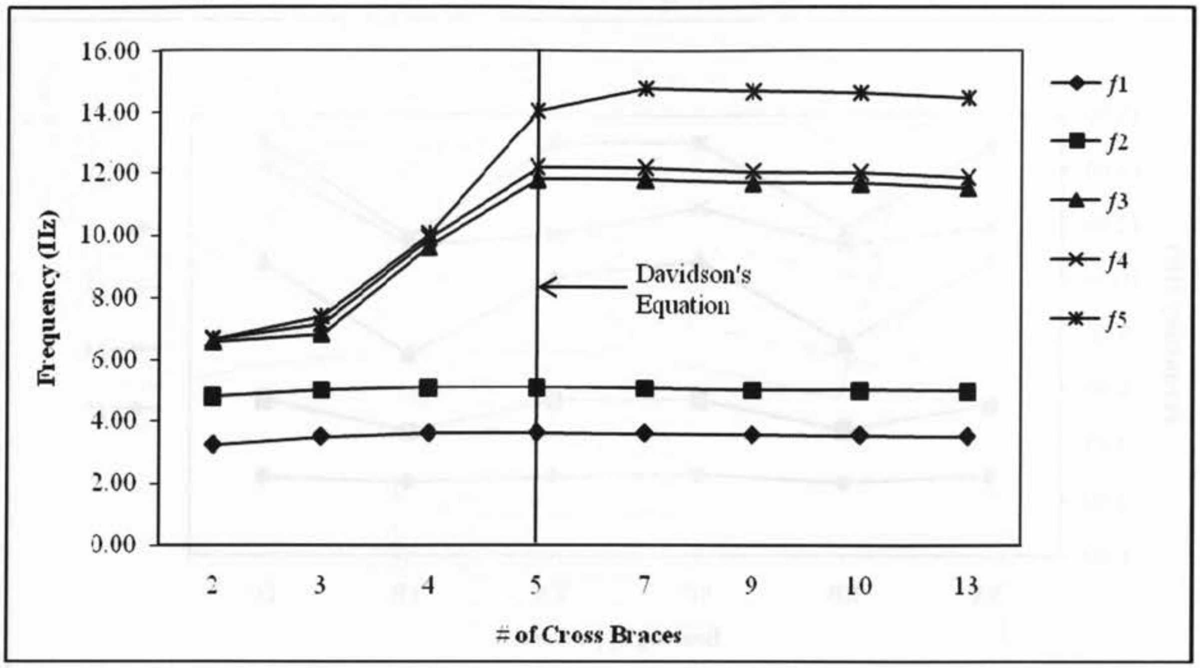


Figure 4.10 **Effect of the Number of Cross Braces on the First Five Frequencies (Bridge: L=25m, S=2.5m, N=3, L/R=0.3)**

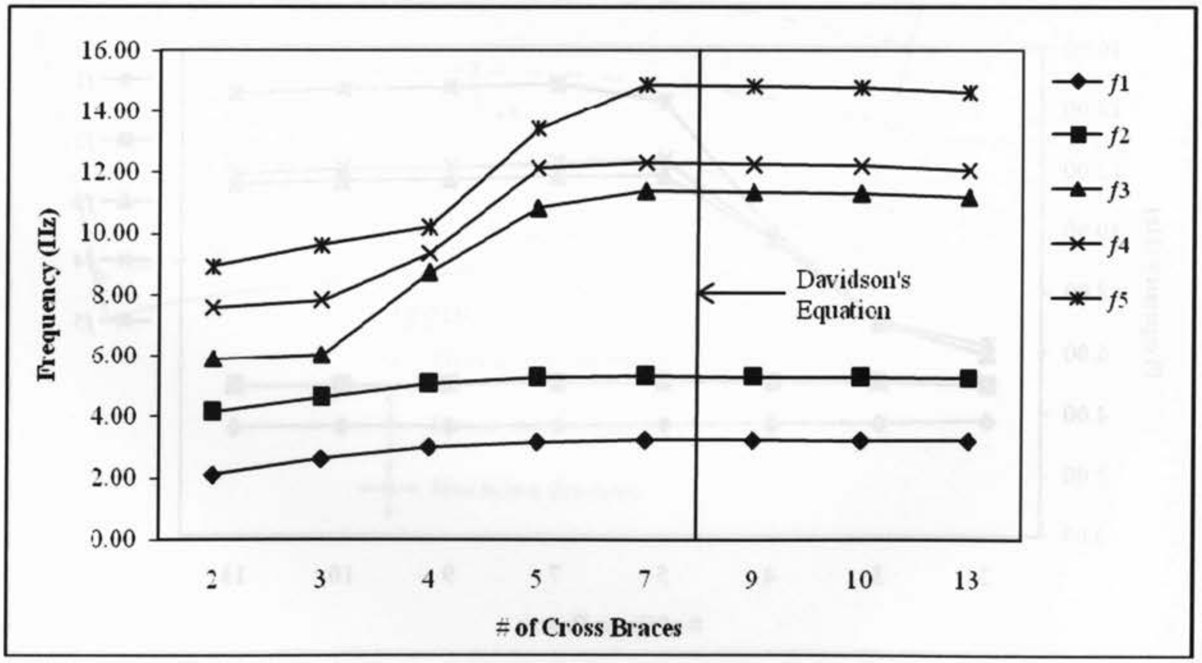


Figure 4.11 Effect of the Number of Cross Braces on the First Five Frequencies (Bridge: L=25m, S=2.5m, N=3, L/R=0.5)

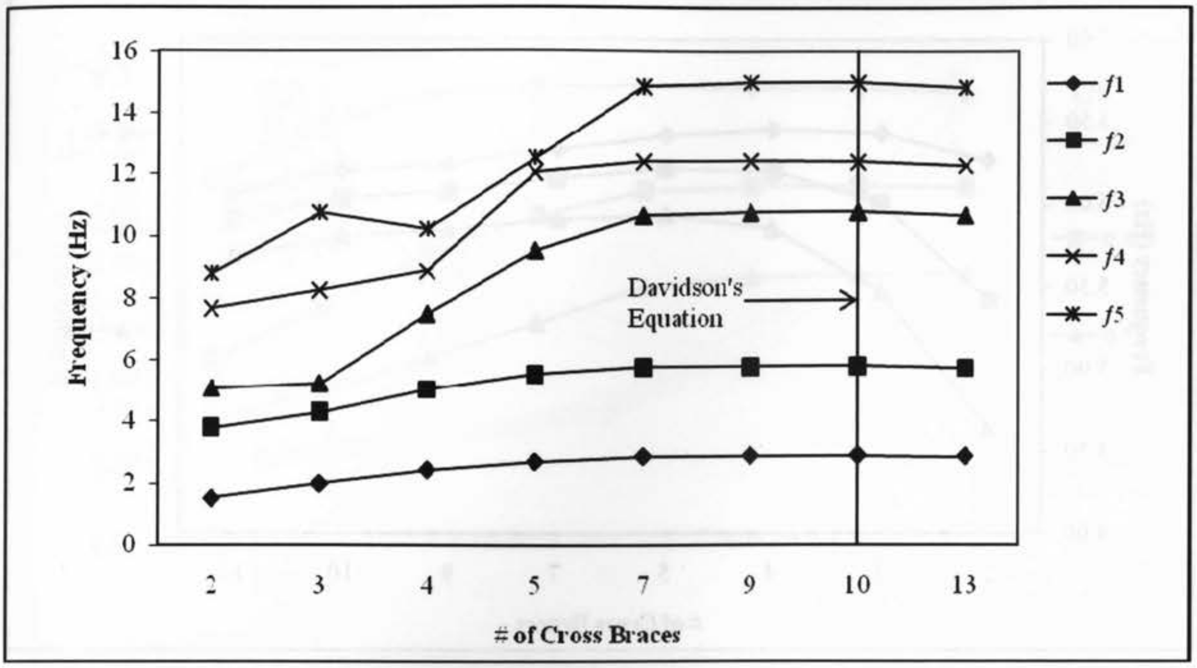


Figure 4.12 Effect of the Number of Cross Braces on the Fundamental Frequencies with Differing Curvature Ratios L/R (Bridge: L=10m, S=2.0m, N=3)

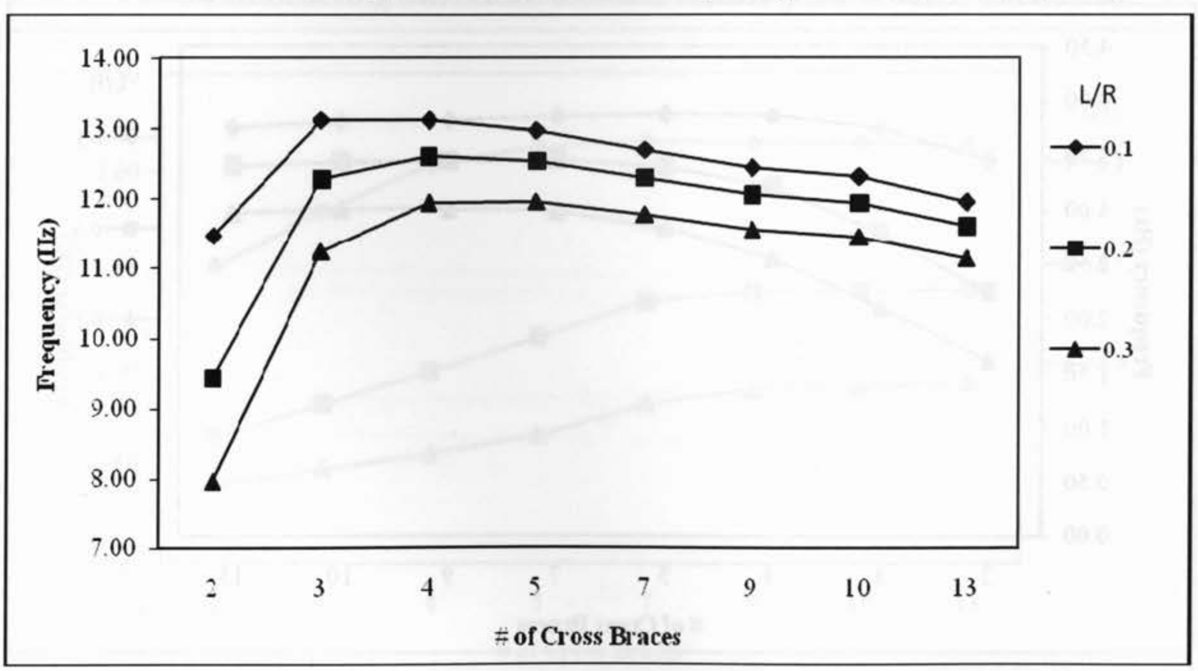


Figure 4.13 Effect of the Number of Cross Braces on the Fundamental Frequencies with Differing Curvature Ratios L/R (Bridge: L=15m, S=2.0m, N=3)

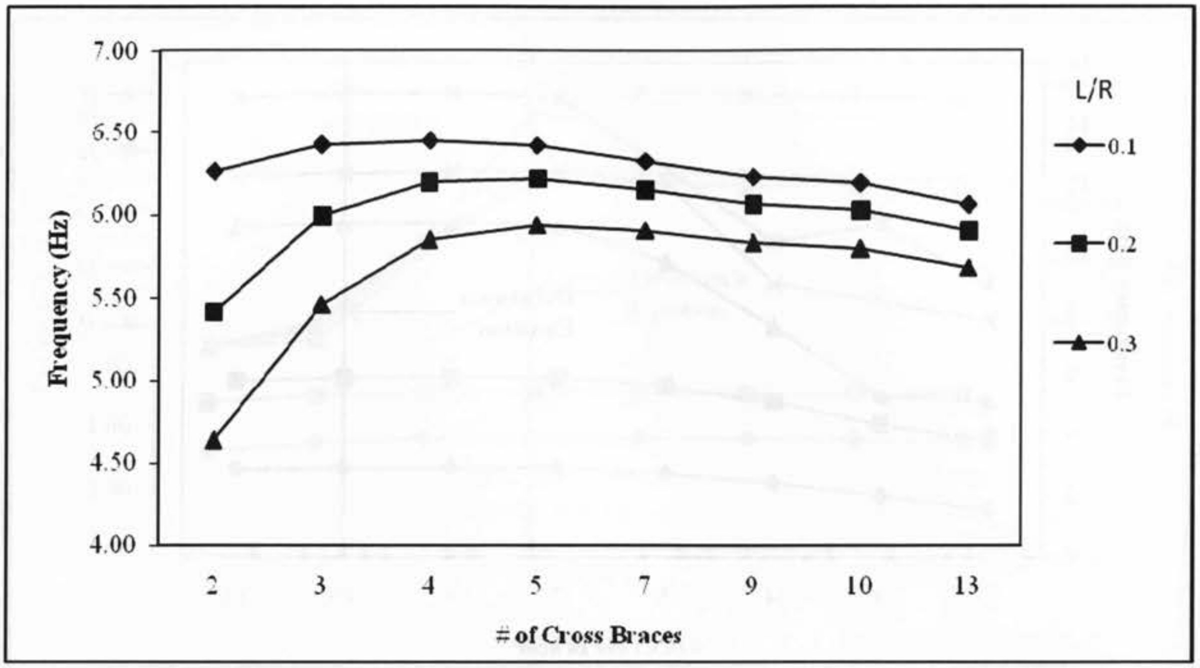


Figure 4.14 Effect of the Number of Cross Braces on the Fundamental Frequencies with Differing Curvature Ratios L/R (Bridge: L=25m, S=2.0m, N=3)

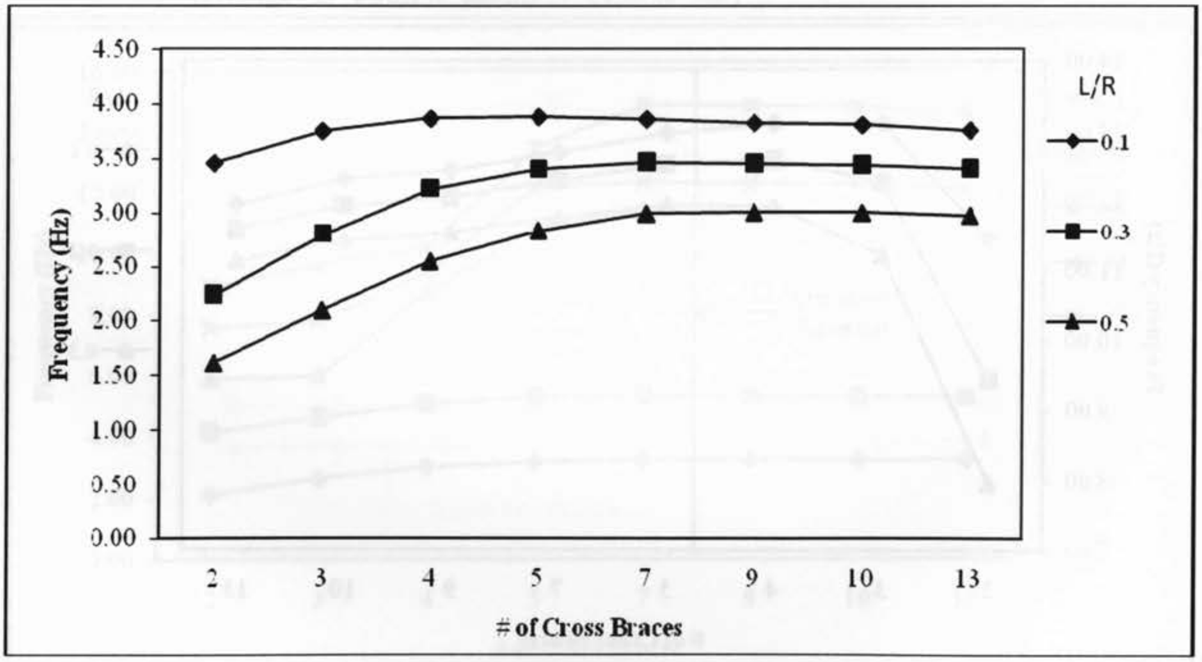


Figure 4.15 Effect of the Number of Cross Braces on the Fundamental Frequencies with Differing Curvature Ratios L/R (Bridge: L=35m, S=2.0m, N=3)

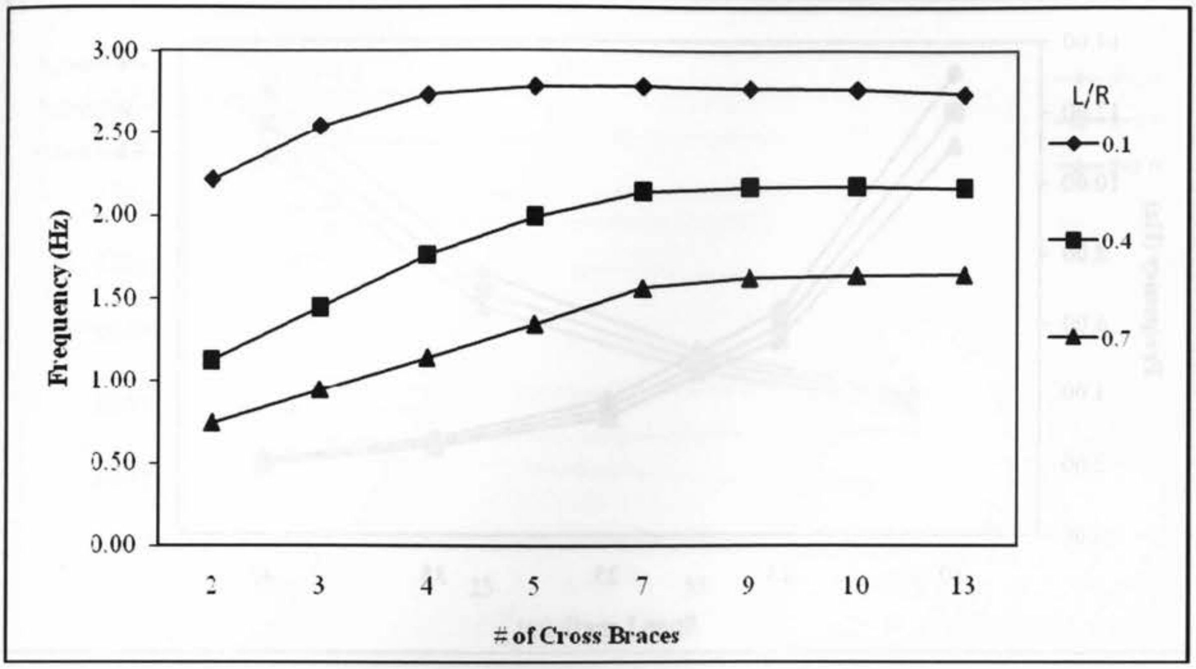


Figure 4.16 Effect of the Number of Cross Braces on the Fundamental Frequencies with Differing Curvature Ratios L/R (Bridge: L=45m, S=2.0m, N=3)

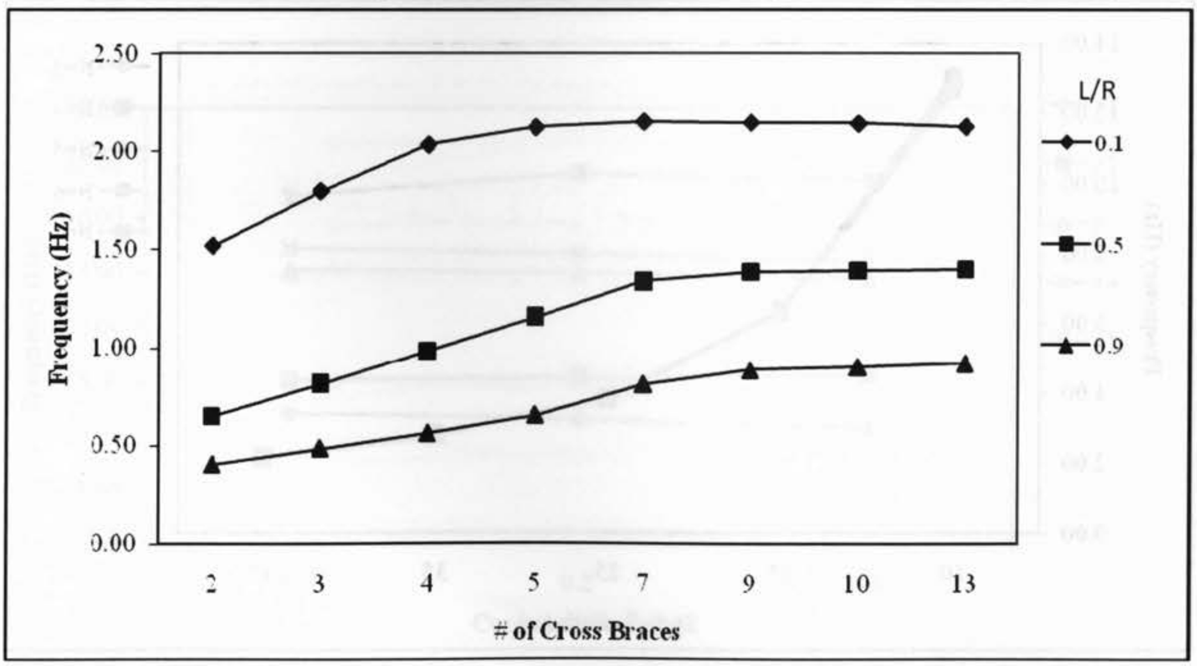


Figure 4.17 Effect of Span Length on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (Bridge: N=3, L/R=0.1)

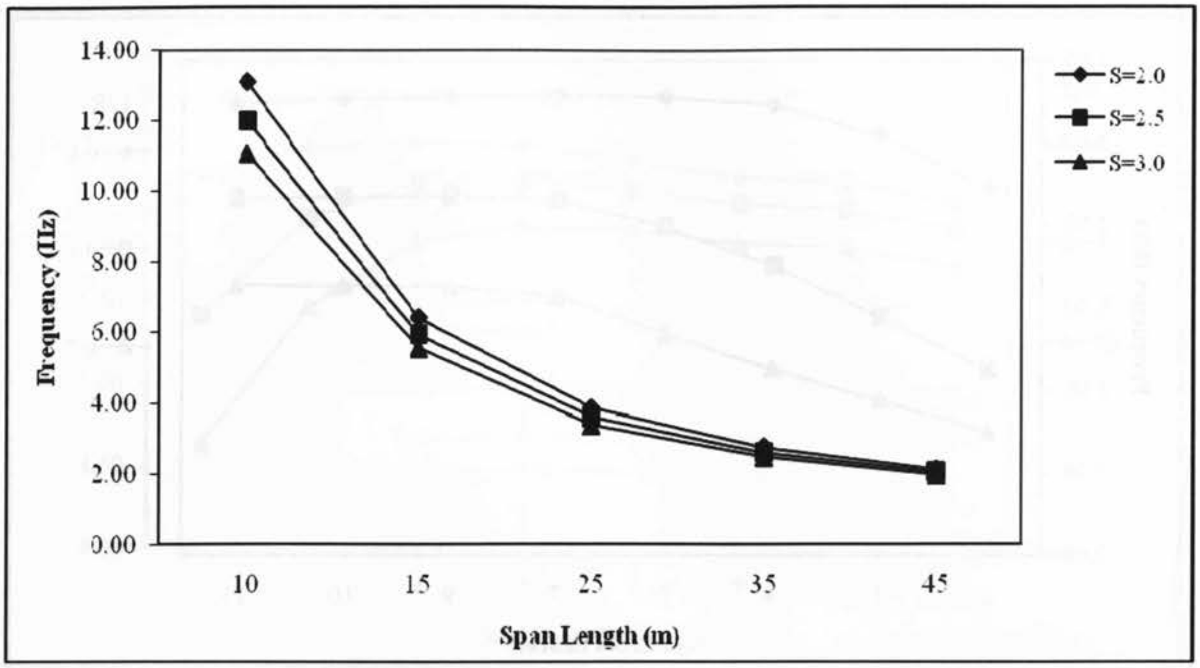


Figure 4.18 Effect of Span Length on the Fundamental Frequencies of Curved Bridges with Differing Number of Girders (Bridge: S=2.0, L/R=0.1)

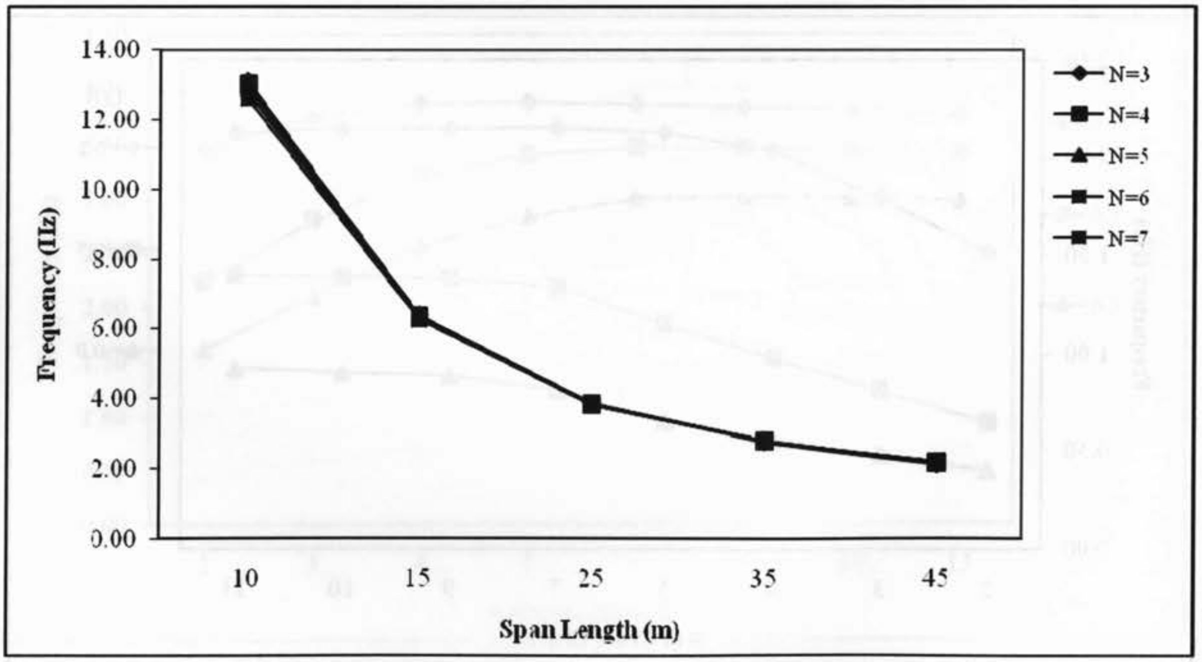


Figure 4.19 Effect of Span Length on the Fundamental Frequencies of Curved 2-Span Bridges with Differing Girder Spacing (Bridge: N=3, L/R=0.1)

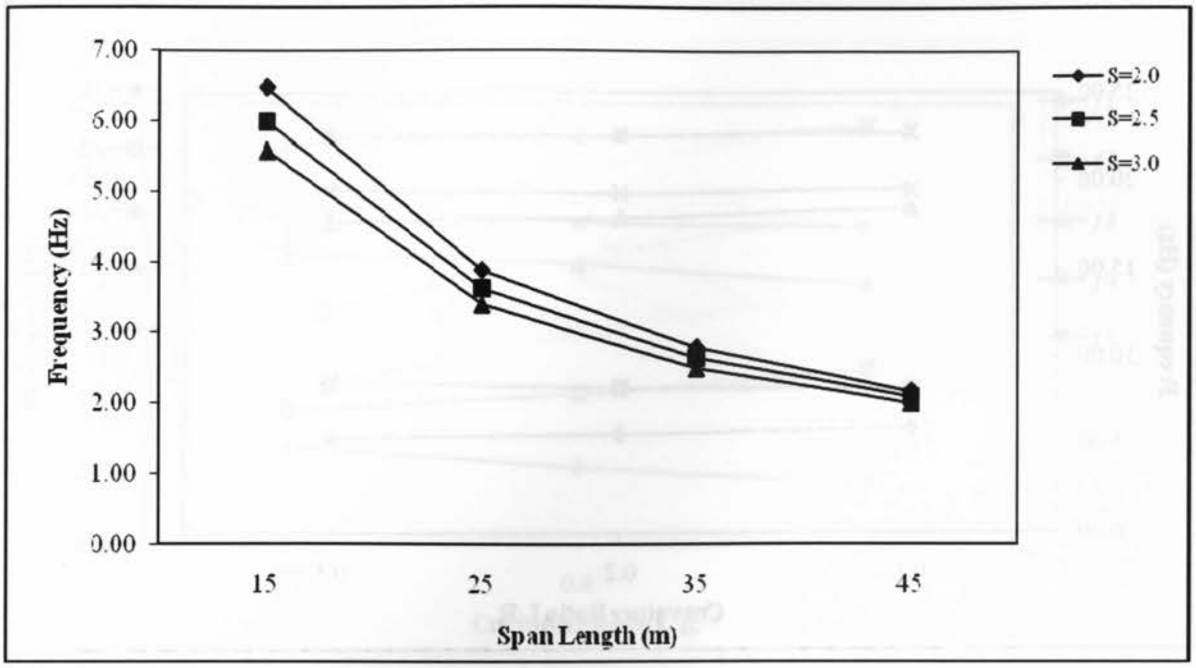


Figure 4.20 Effect of Curvature on the First Five Frequencies of Curved Bridges (Bridge: L=10m, S=2.5m, N=3)

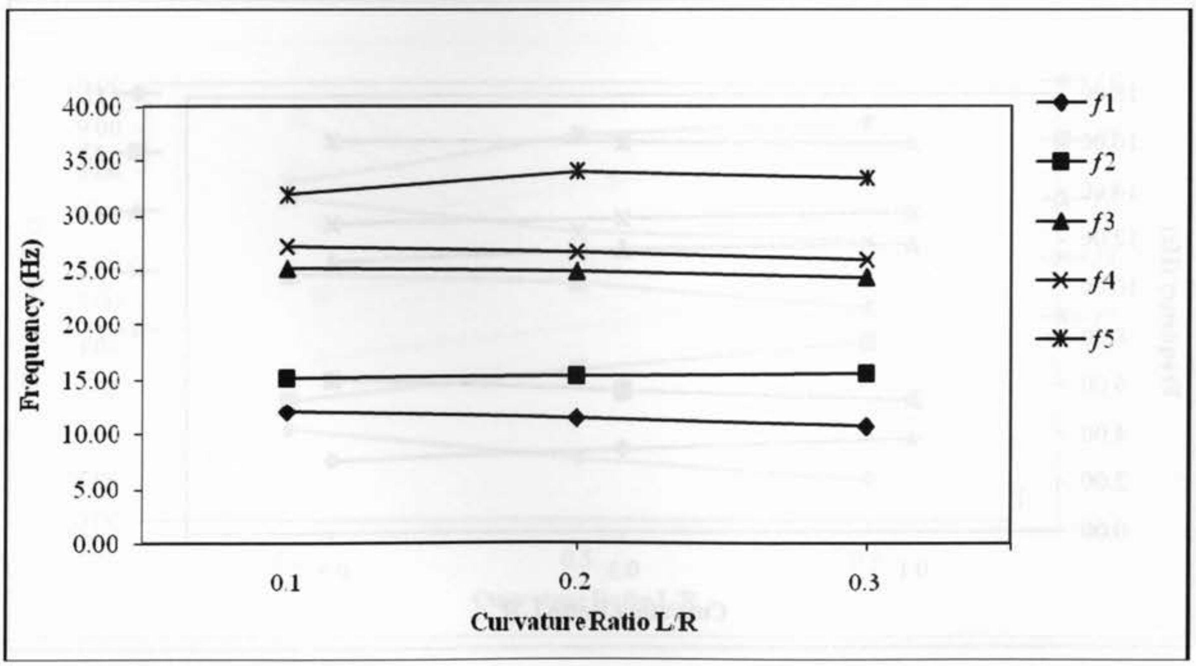


Figure 4.21 Effect of Curvature on the First Five Frequencies of Curved Bridges (Bridge: L=15m, S=2.5m, N=3)

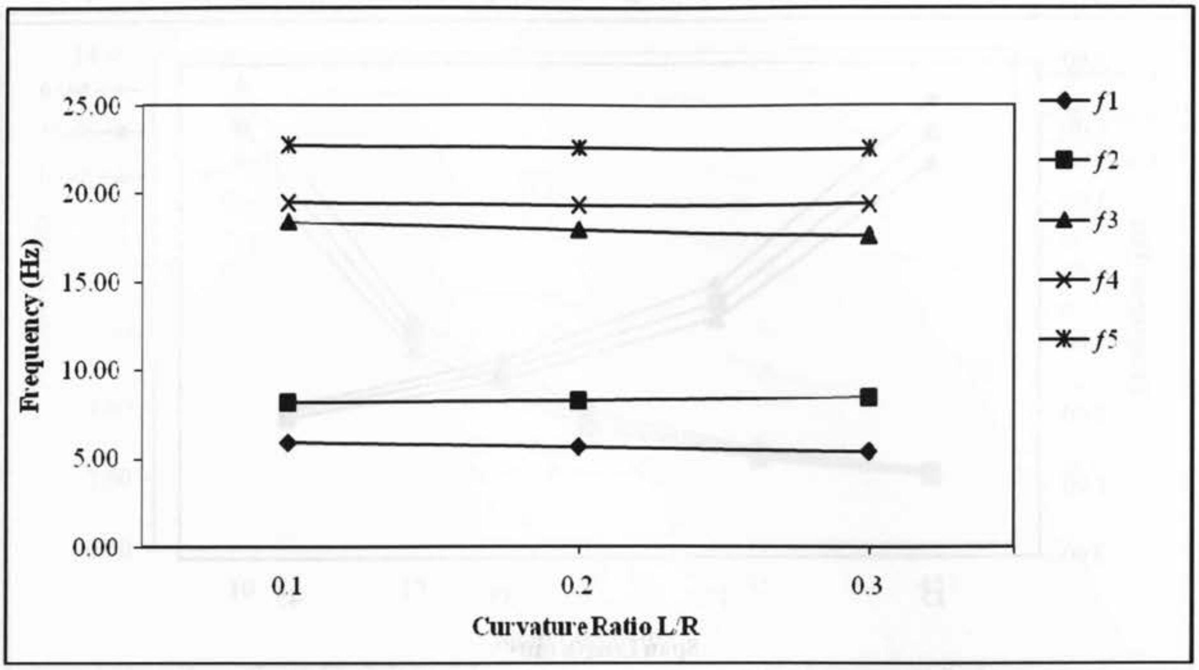


Figure 4.22 Effect of Curvature on the First Five Frequencies of Curved Bridges (Bridge: L=25m, S=2.5m, N=3)

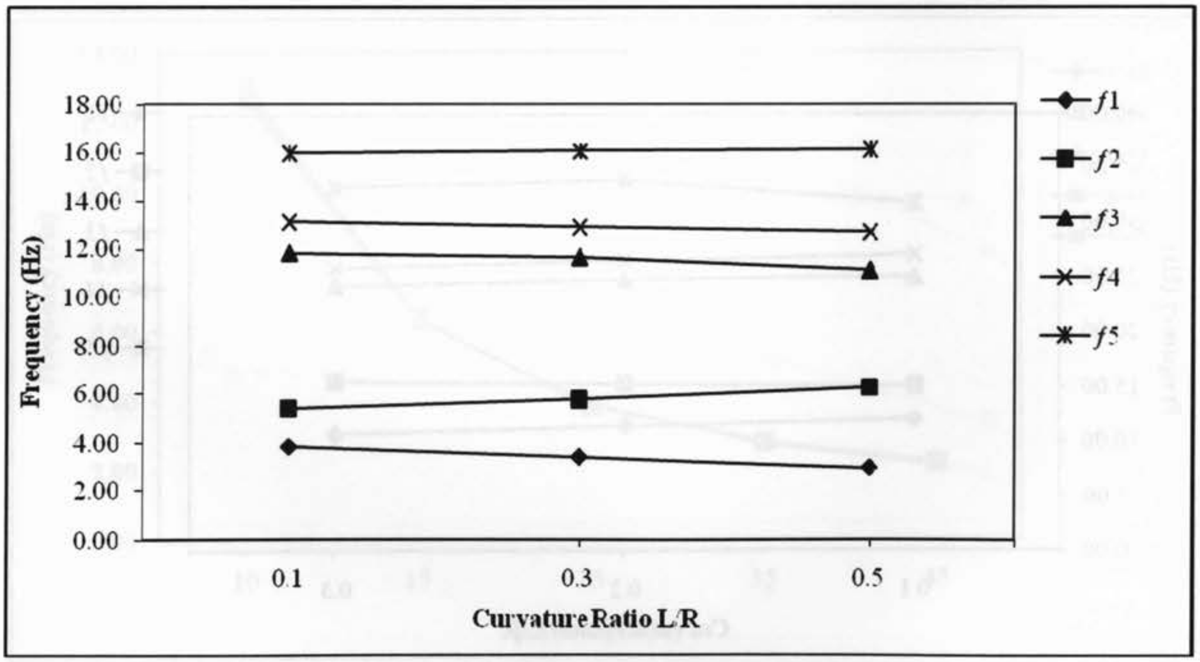


Figure 4.23 Effect of Curvature on the First Five Frequencies of Curved Bridges (Bridge: L=35m, S=2.5m, N=3)

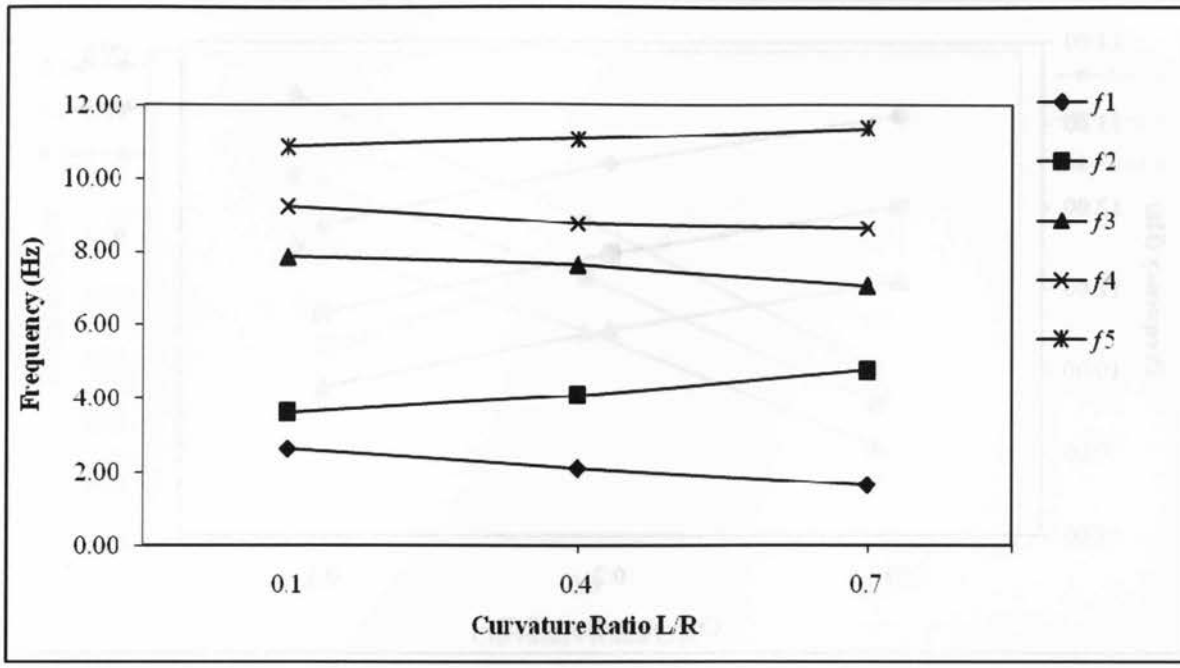


Figure 4.24 Effect of Curvature on the First Five Frequencies of Curved Bridges (Bridge: L=45m, S=2.5m, N=3)

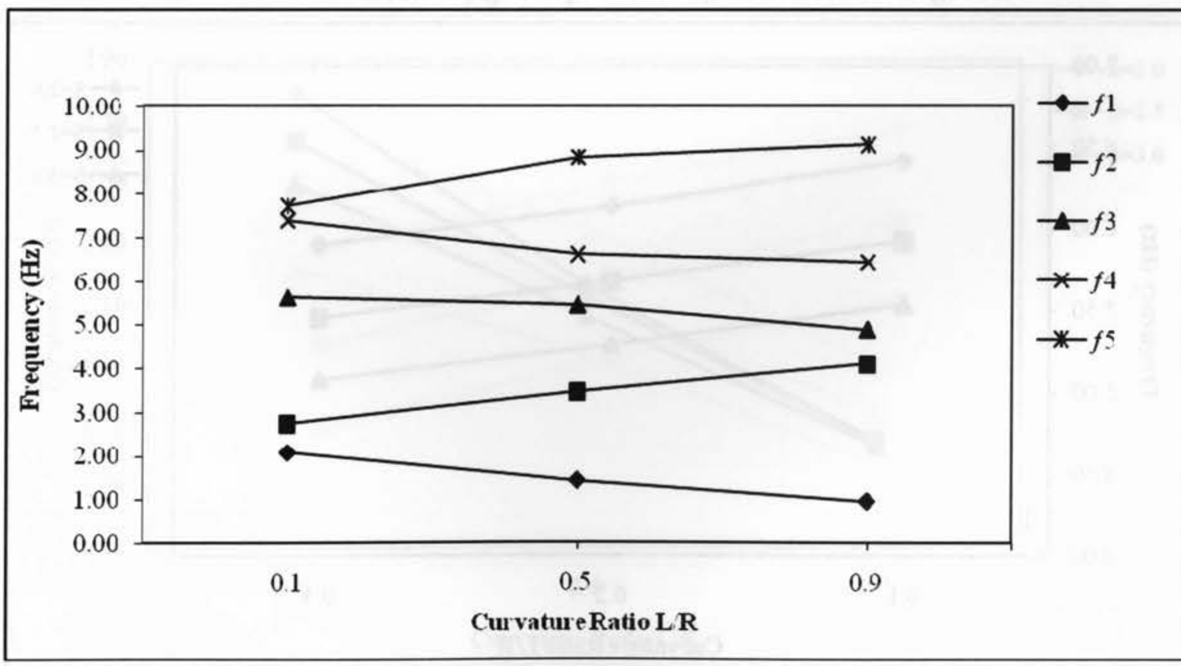


Figure 4.25 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Girder Spacing (L=10, N=3)

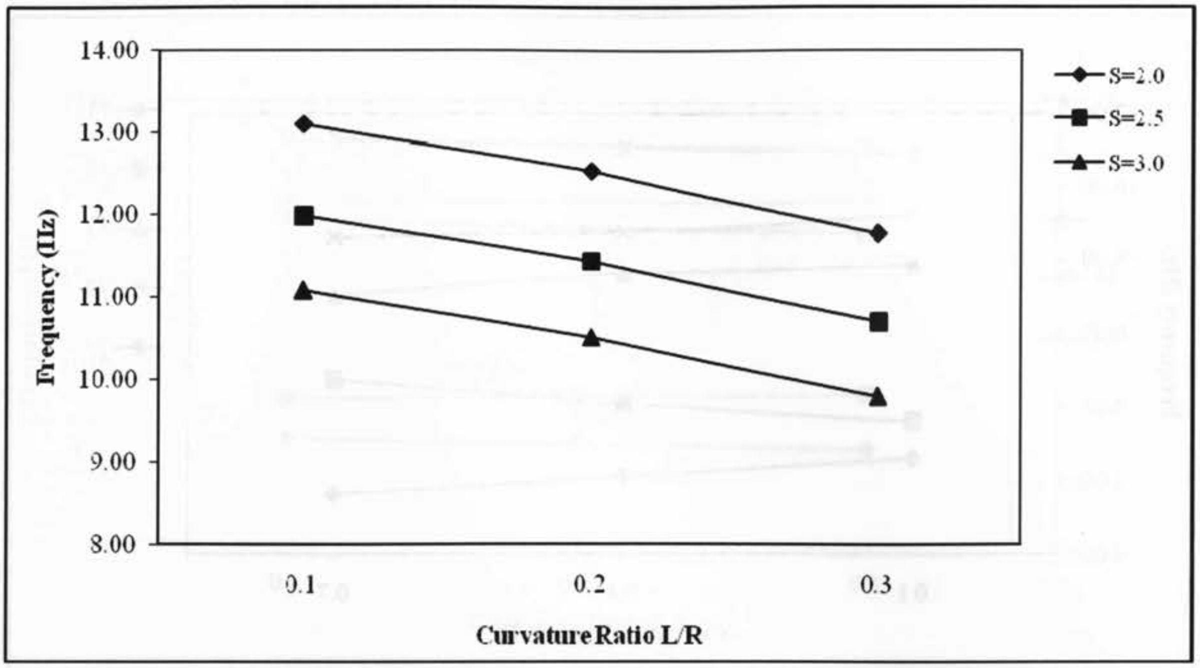


Figure 4.26 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Girder Spacing (L=15, N=3)

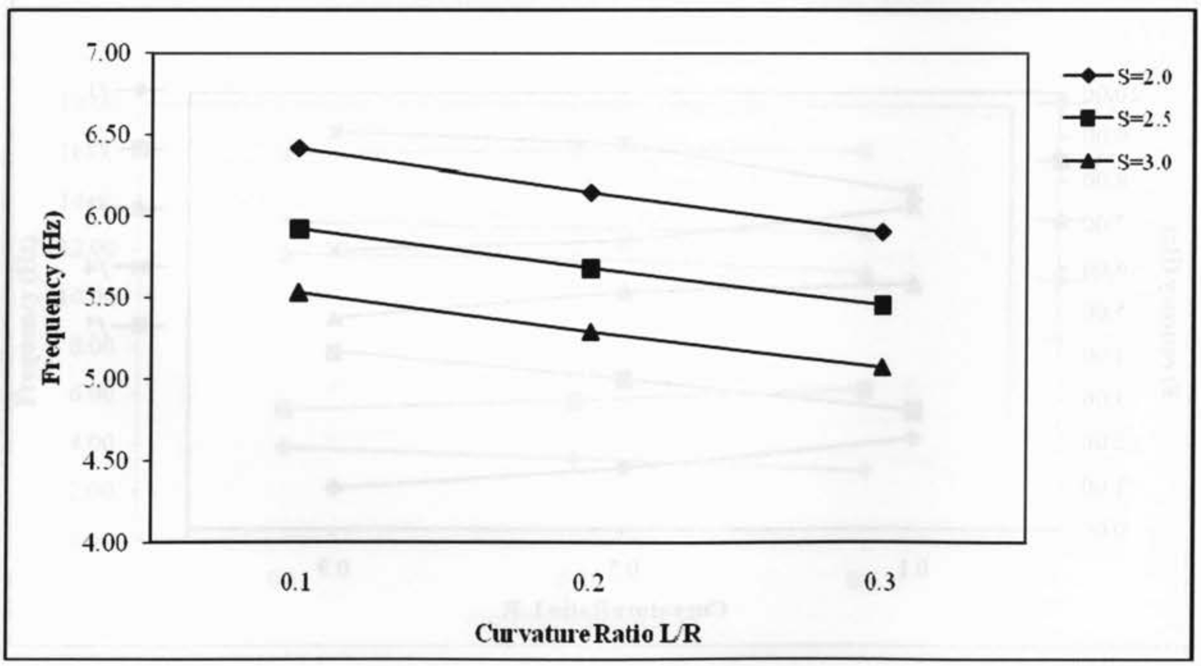


Figure 4.27 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Girder Spacing (L=25, N=3)

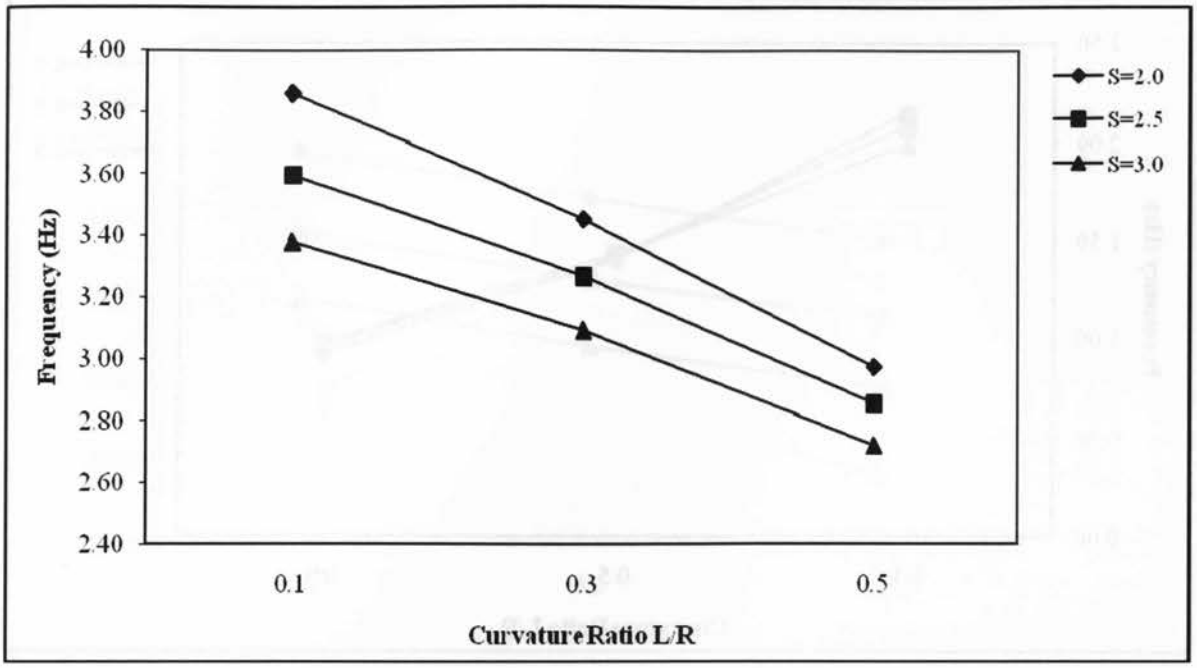


Figure 4.28 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Girder Spacing (L=35, N=3)

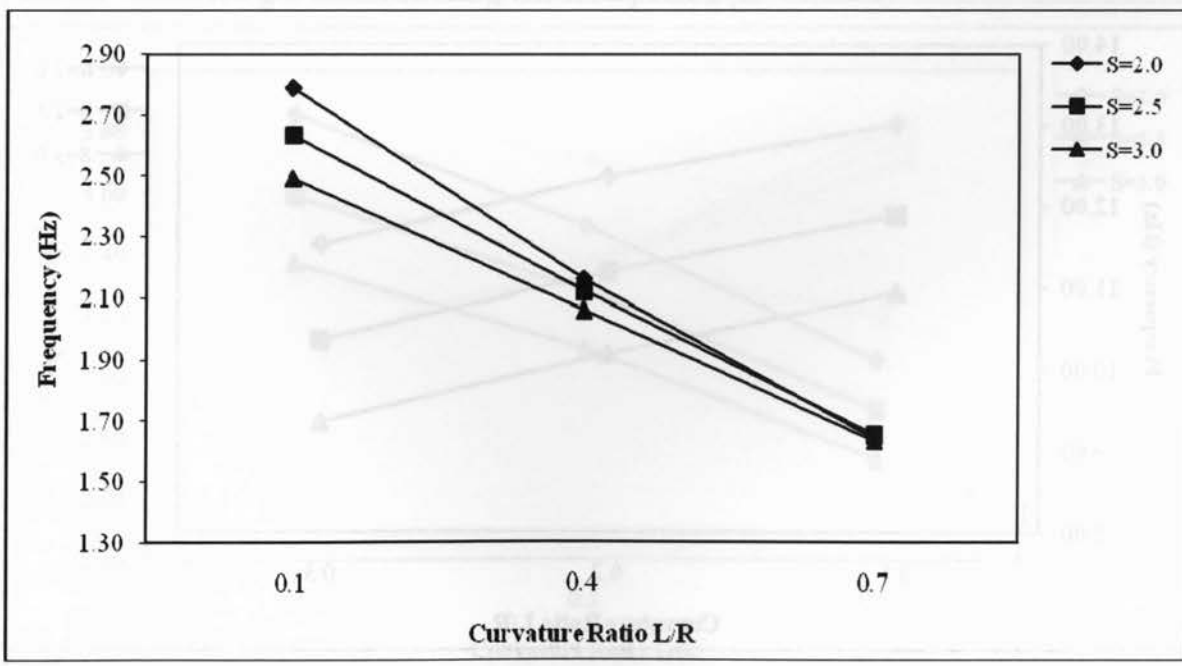


Figure 4.29 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Girder Spacing (L=45, N=3)

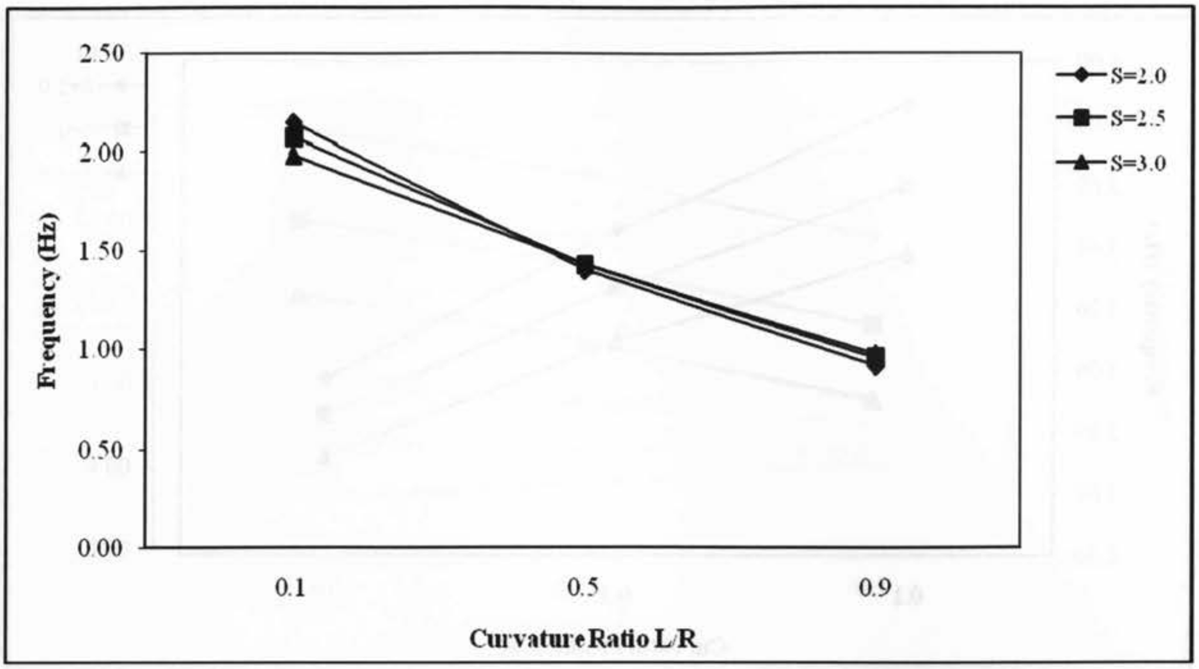


Figure 4.30 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Girder Spacing (L=10, N=4)

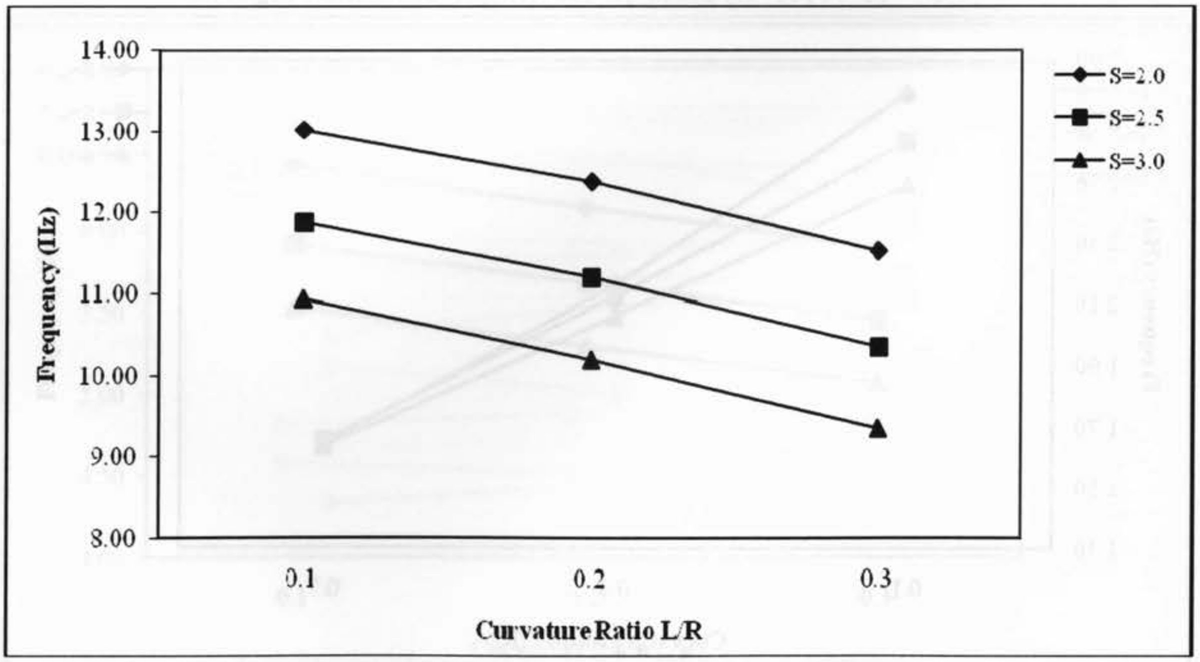


Figure 4.31 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Girder Spacing (L=15, N=4)

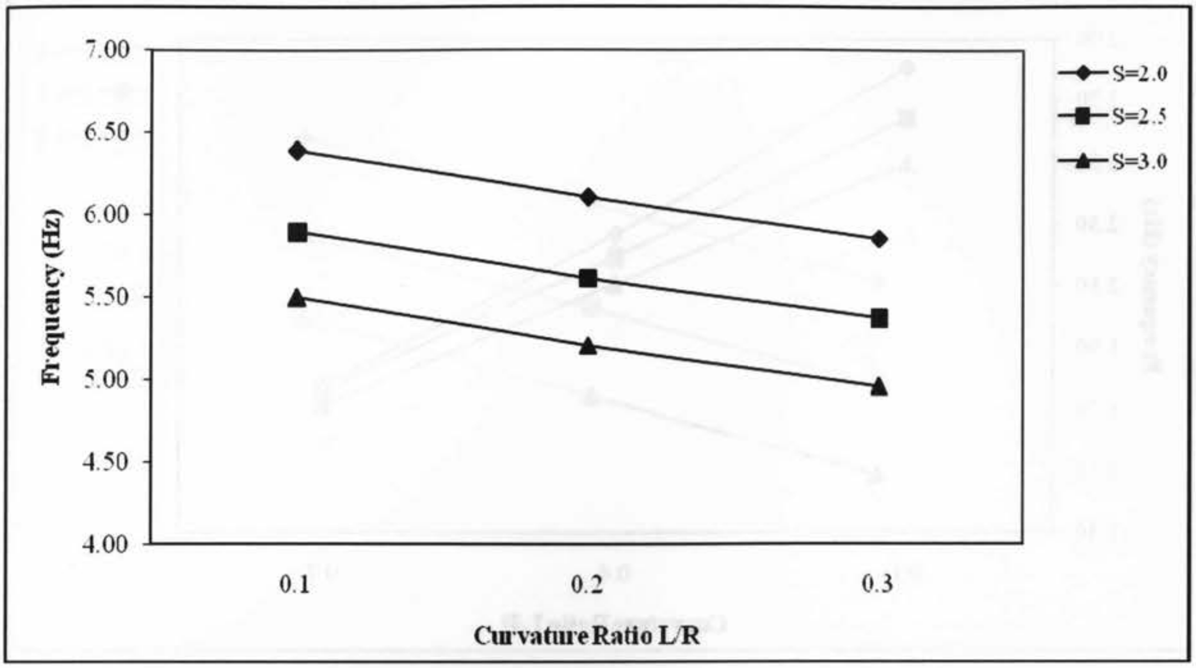


Figure 4.32 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Girder Spacing (L=25, N=4)

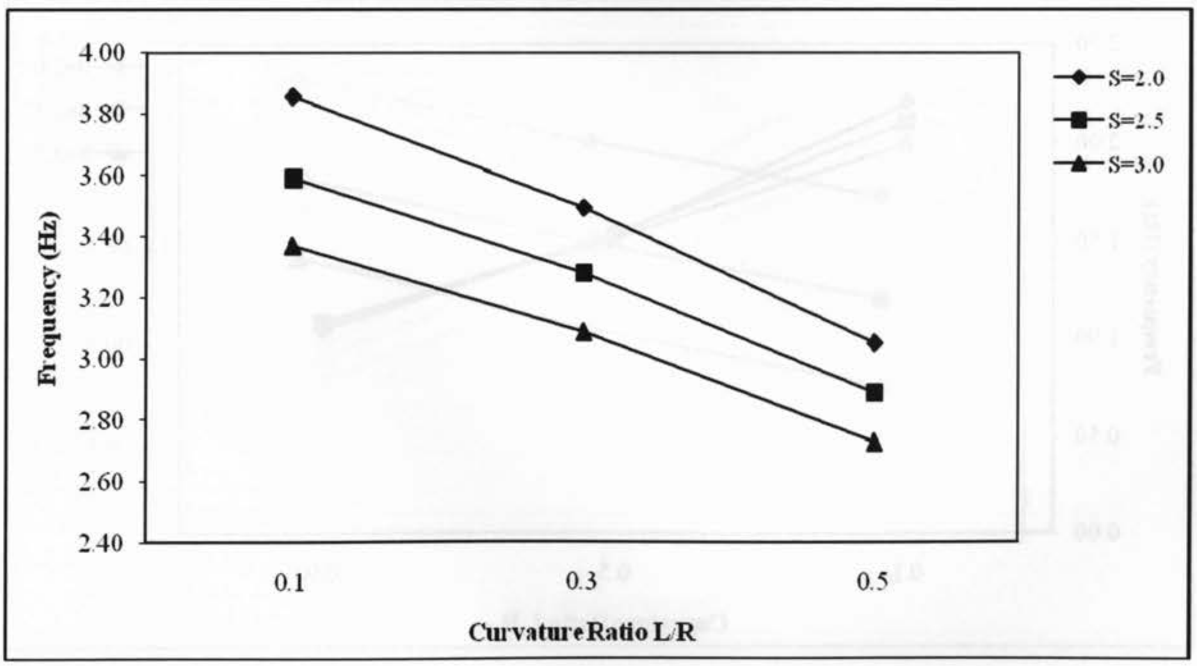


Figure 4.33 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Girder Spacing (L=35, N=4)

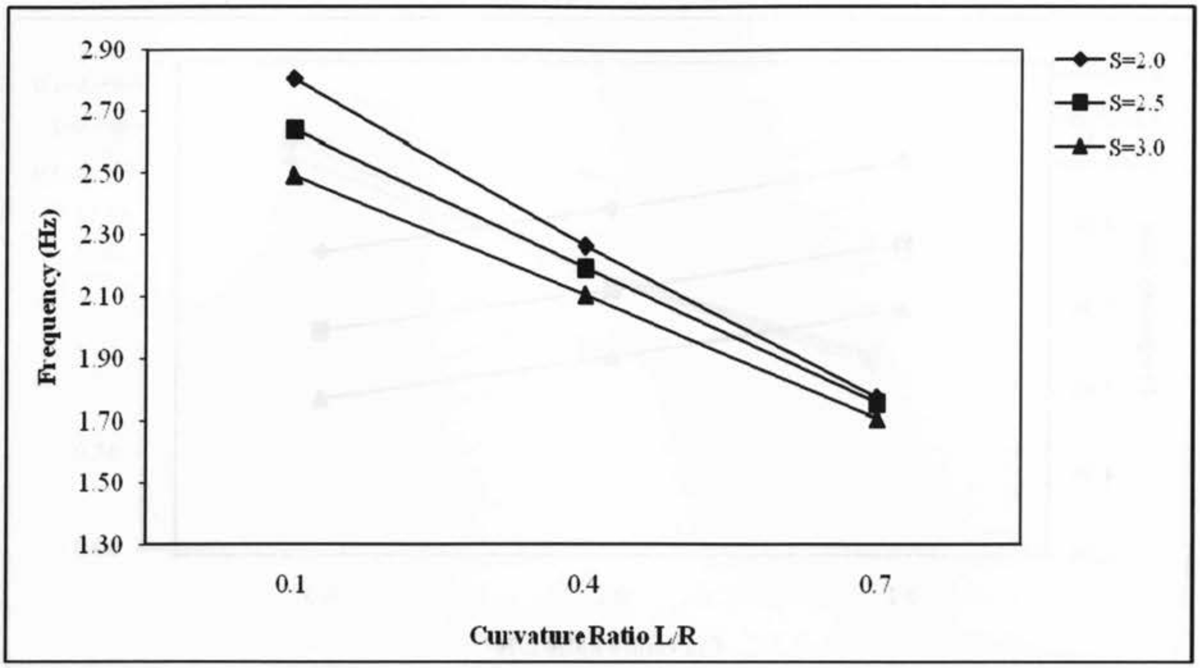


Figure 4.34 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Girder Spacing (L=45, N=4)

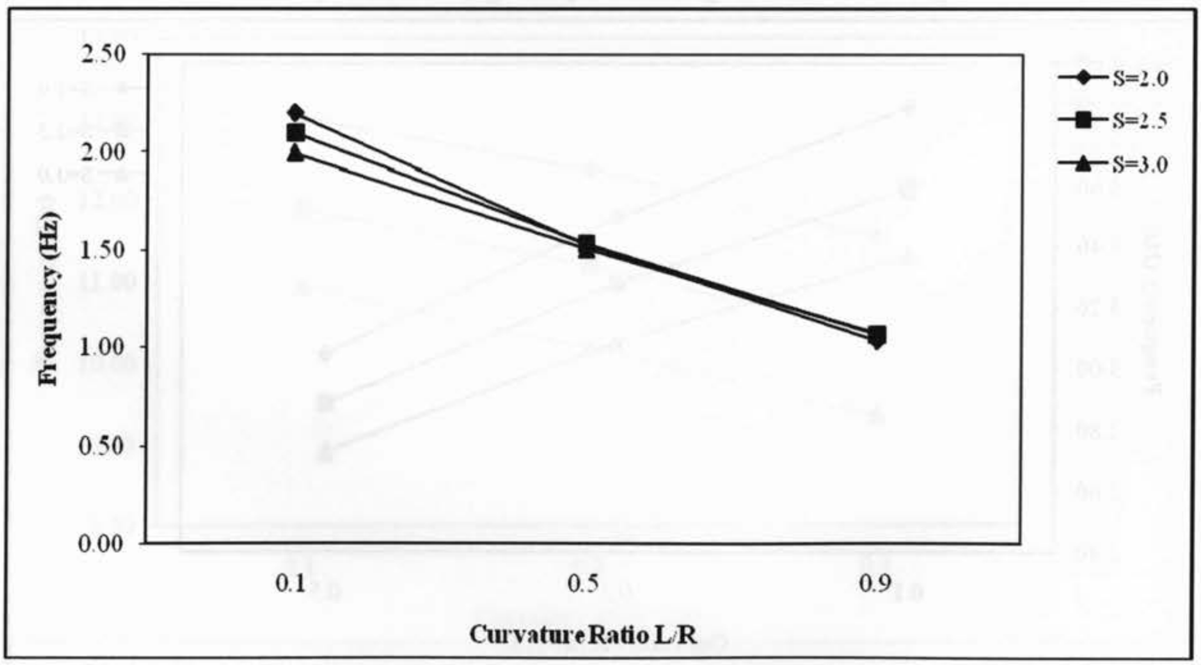


Figure 4.35 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Girder Spacing (L=10, N=5)

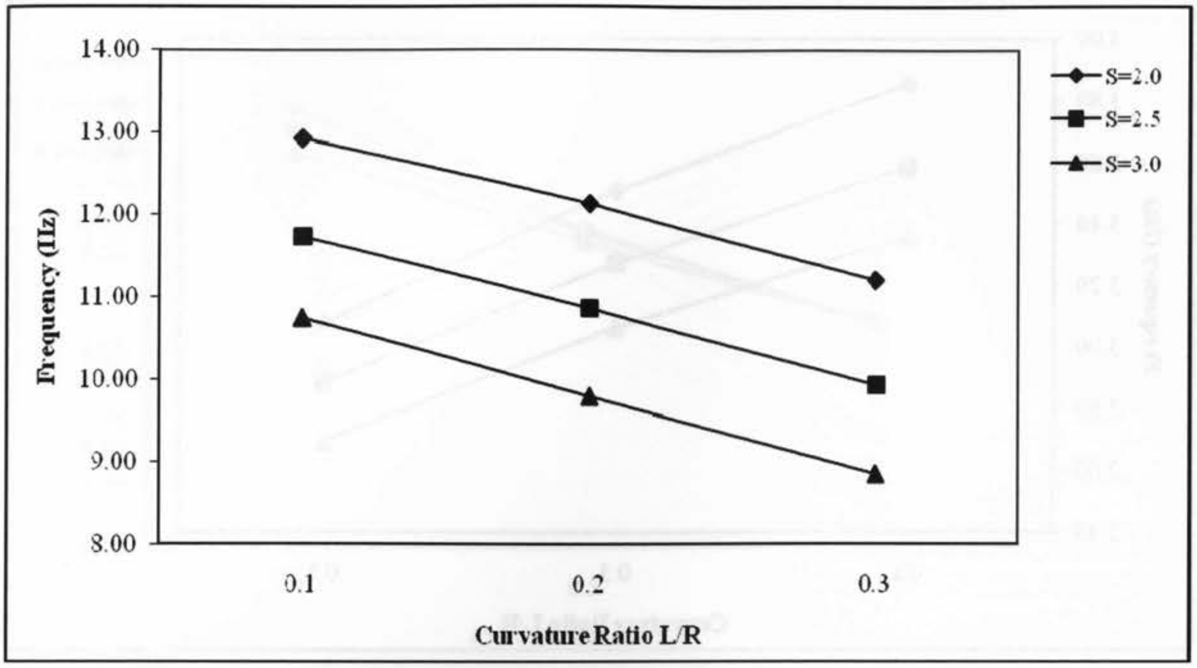


Figure 4.36 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Girder Spacing (L=15, N=5)

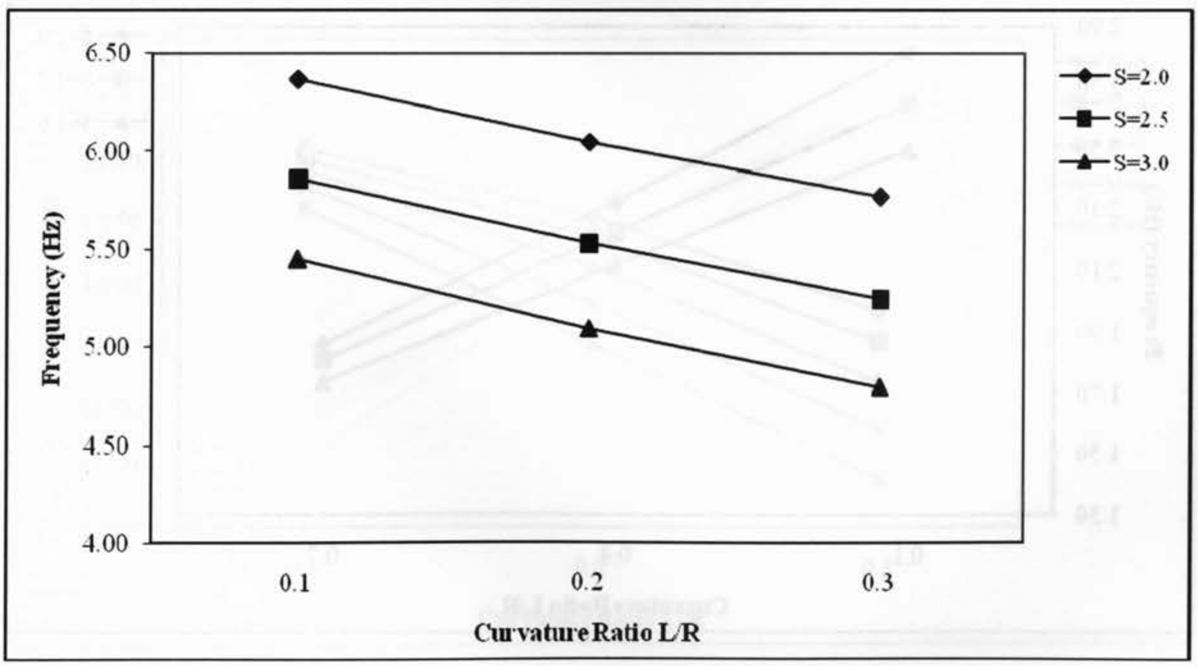


Figure 4.37 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Girder Spacing (L=25, N=5)

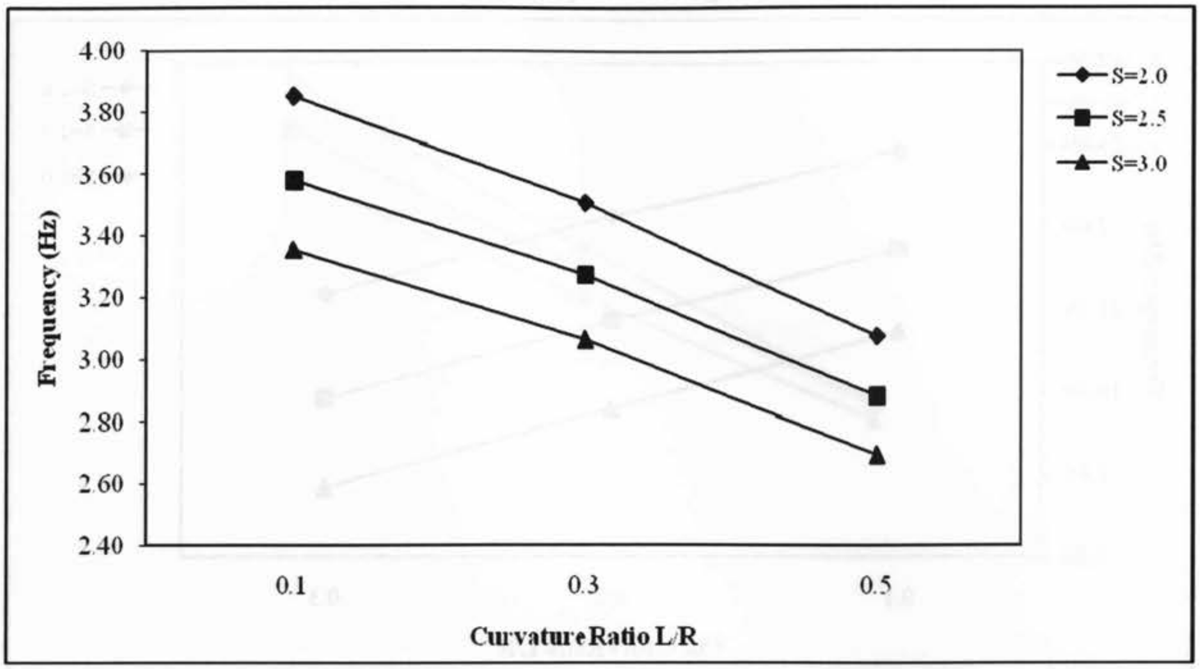


Figure 4.38 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Girder Spacing (L=35, N=5)

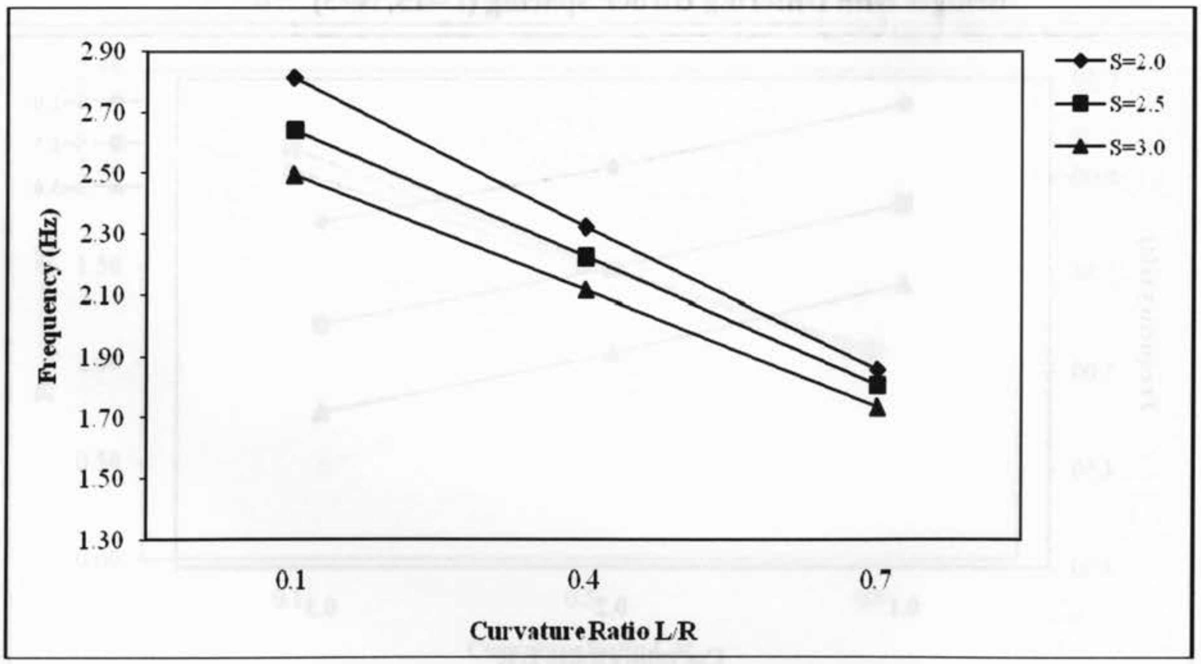


Figure 4.39 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Girder Spacing (L=45, N=5)

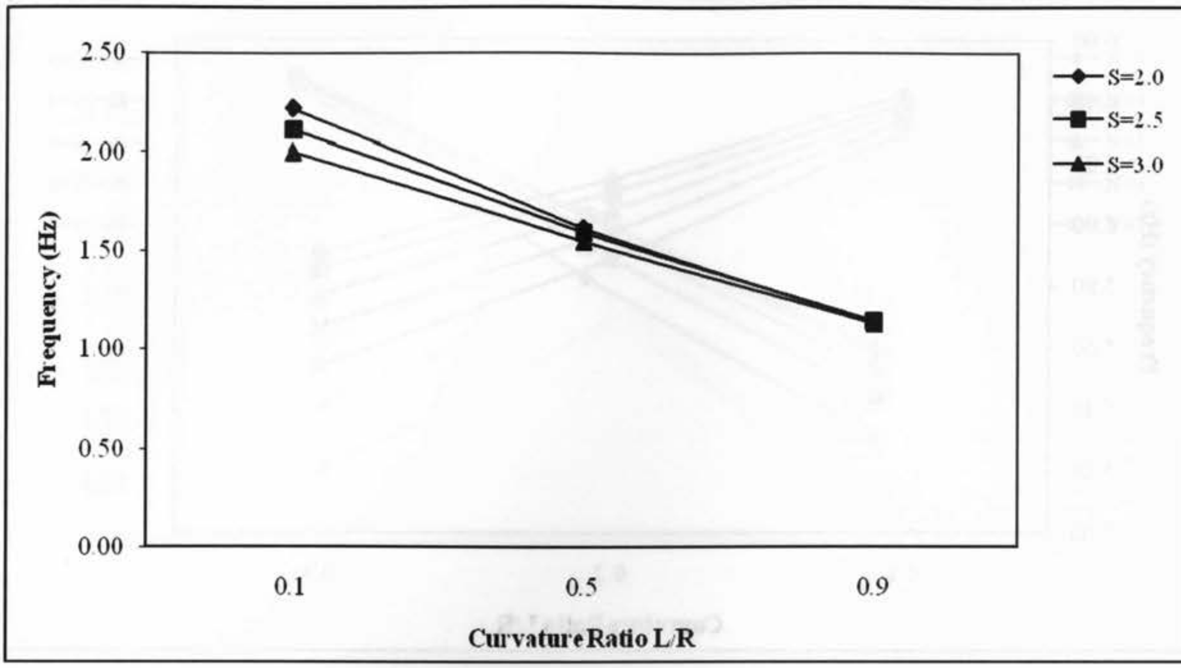


Figure 4.40 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, S=2.0)

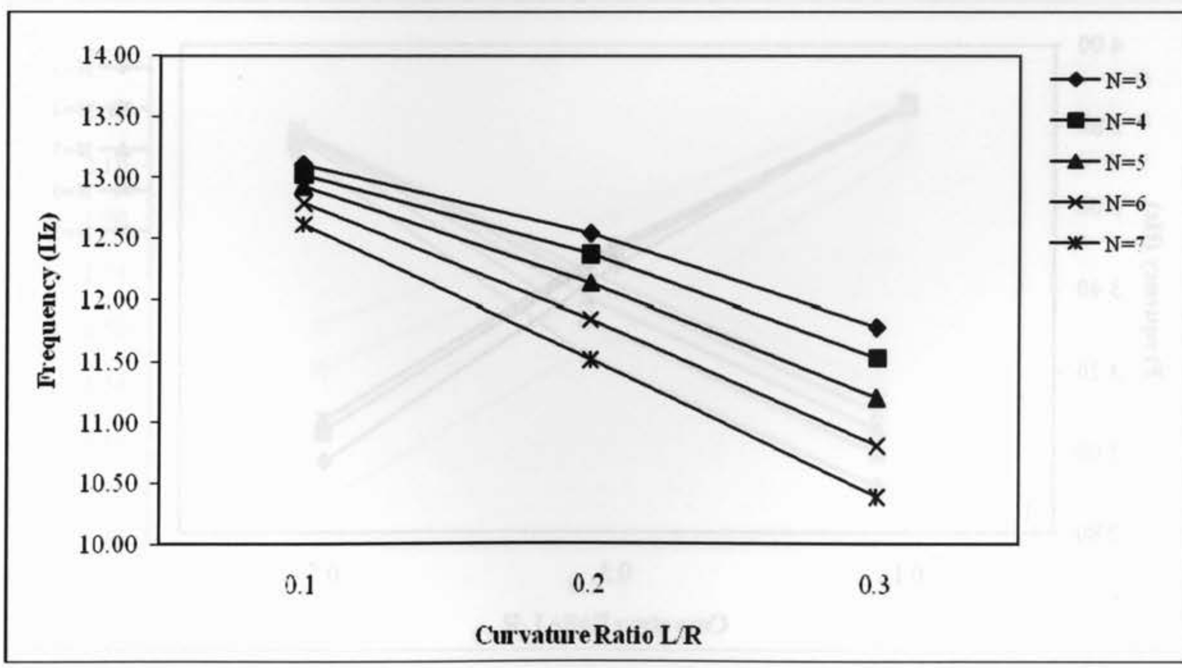


Figure 4.41 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, S=2.0)

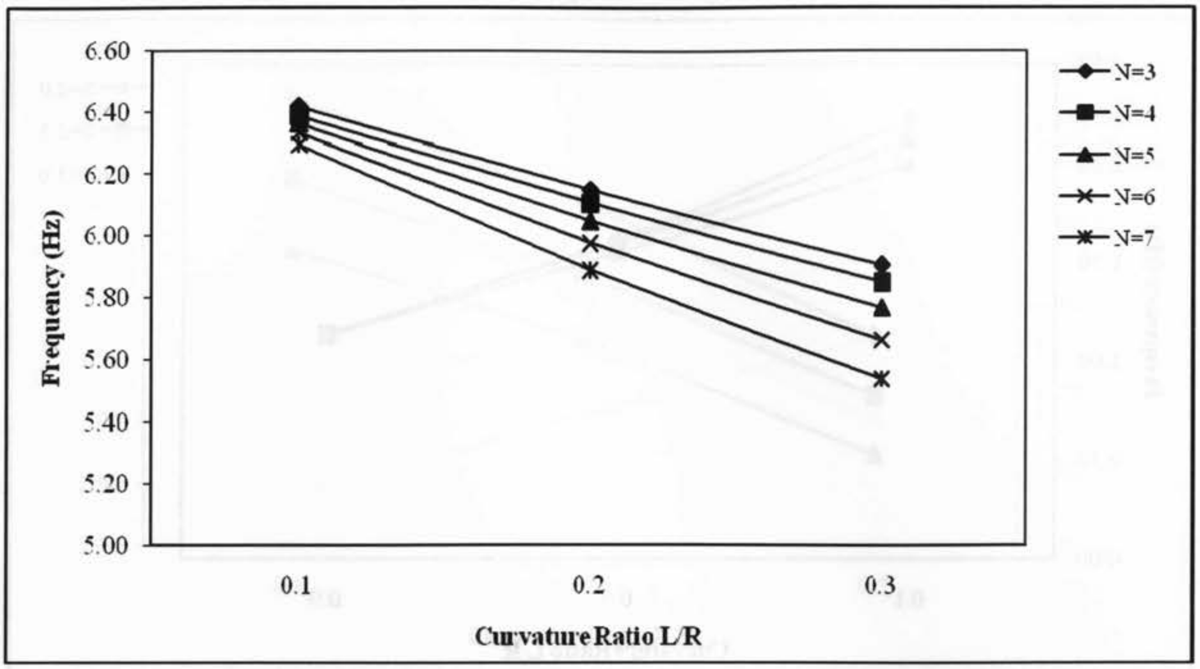


Figure 4.42 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, S=2.0)

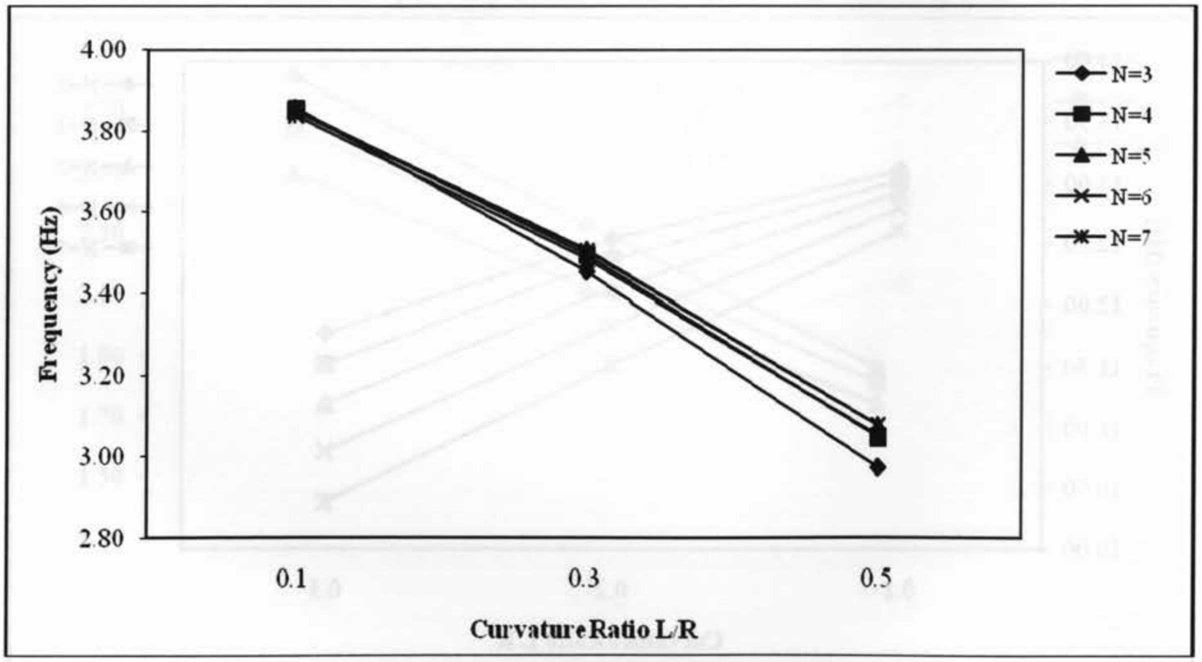


Figure 4.43 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, S=2.0)

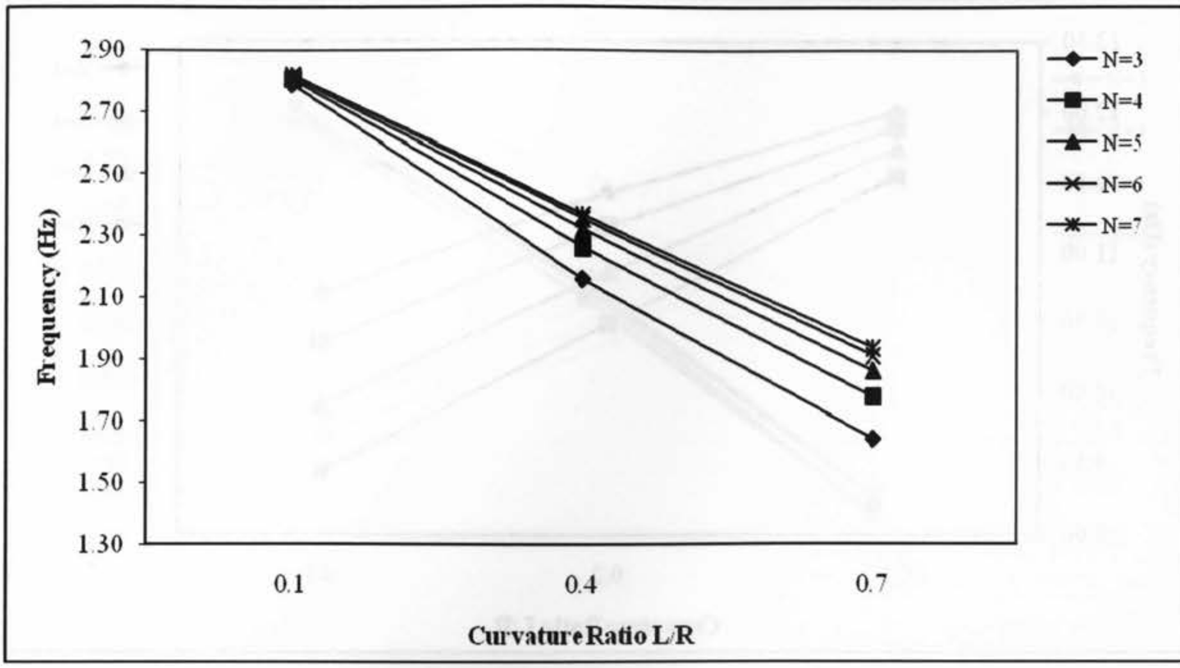


Figure 4.44 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, S=2.0)

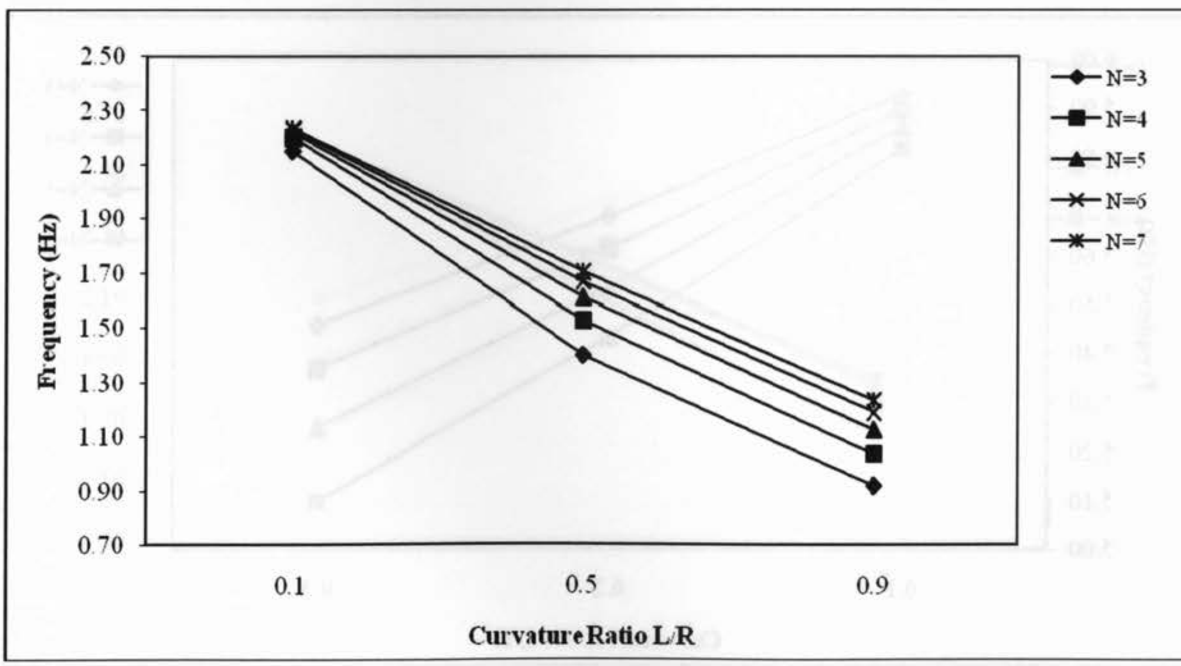


Figure 4.45 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, S=2.5)

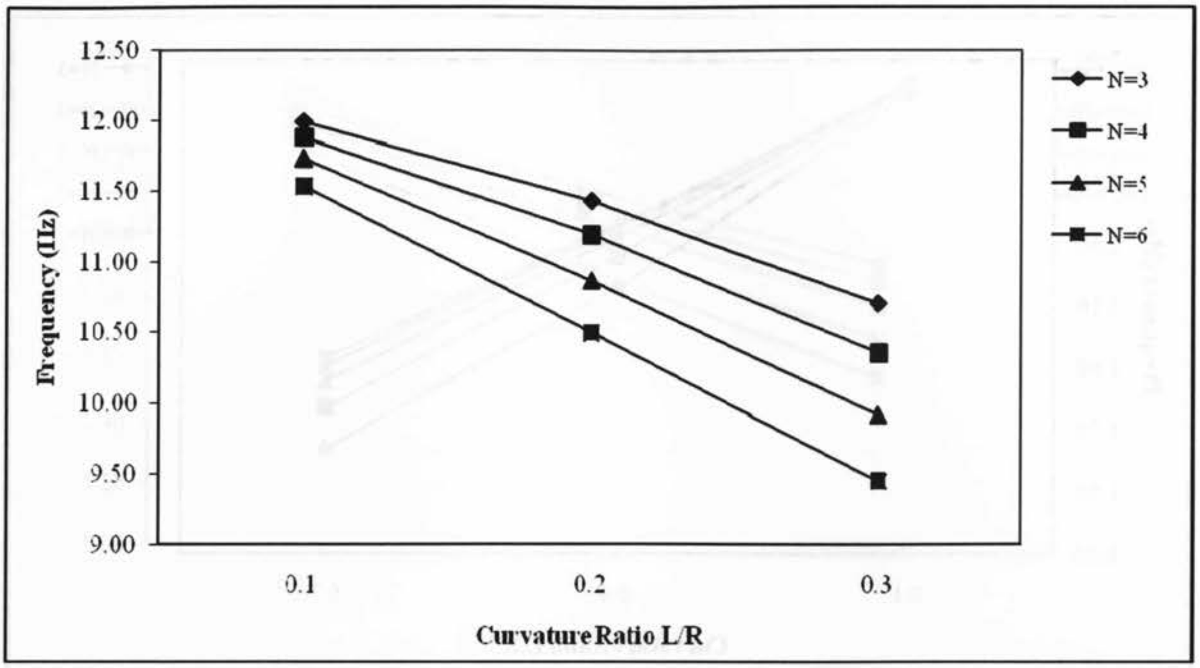


Figure 4.46 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, S=2.5)

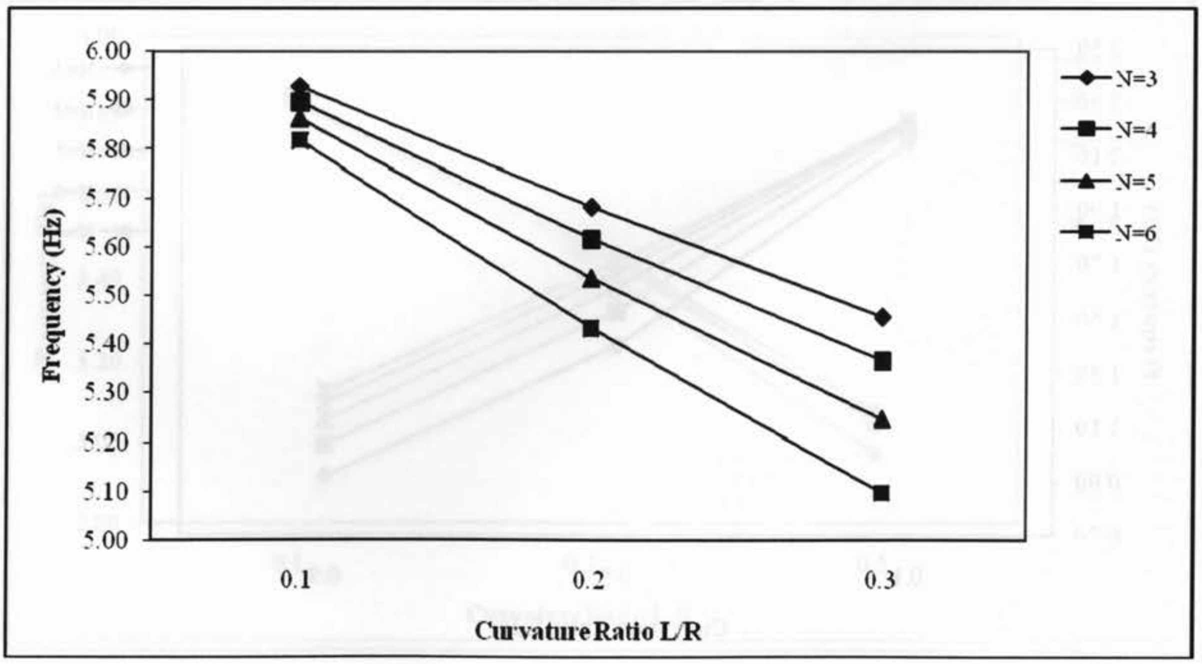


Figure 4.47 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, S=2.5)

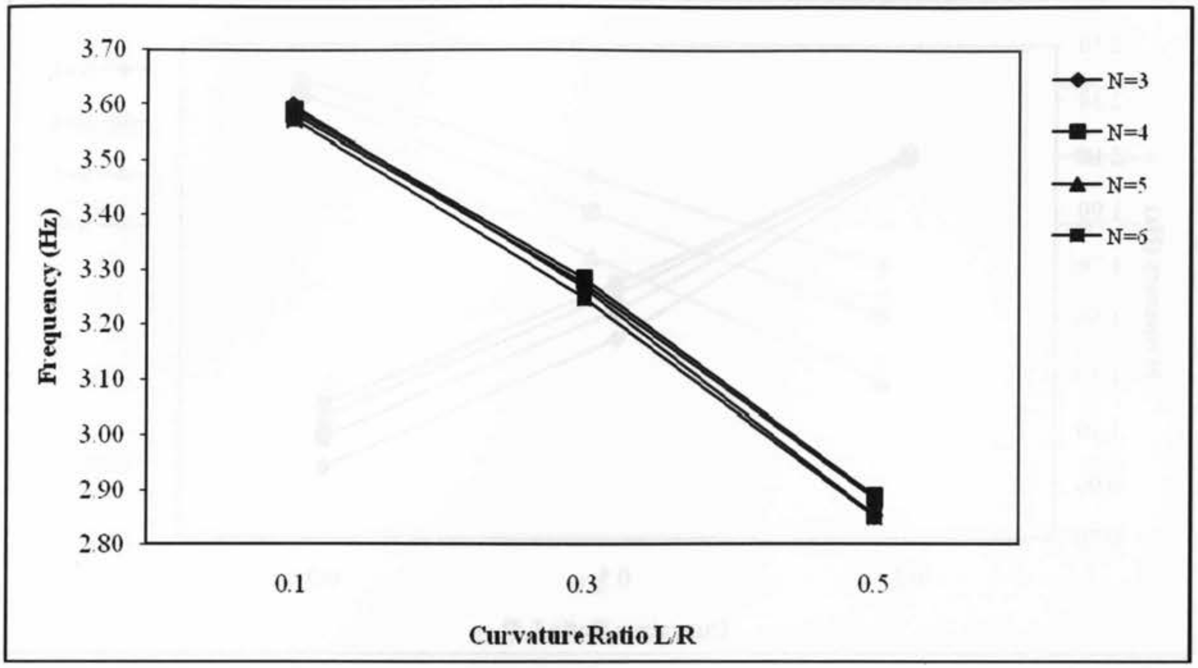


Figure 4.48 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, S=2.5)

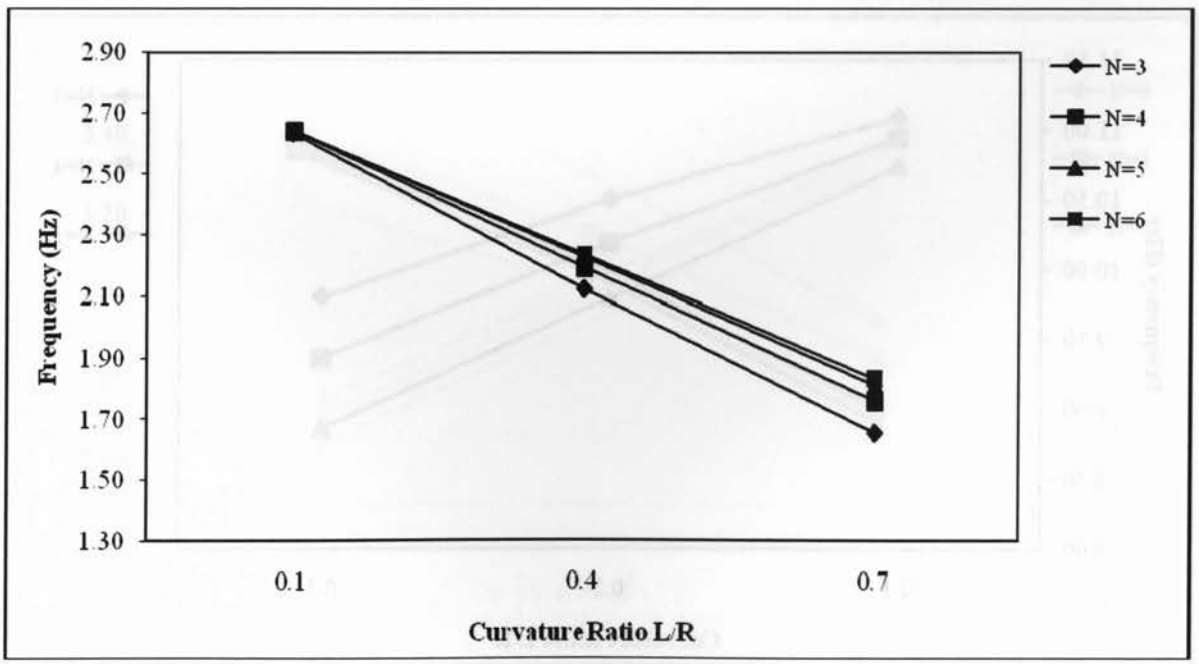


Figure 4.49 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, S=2.5)

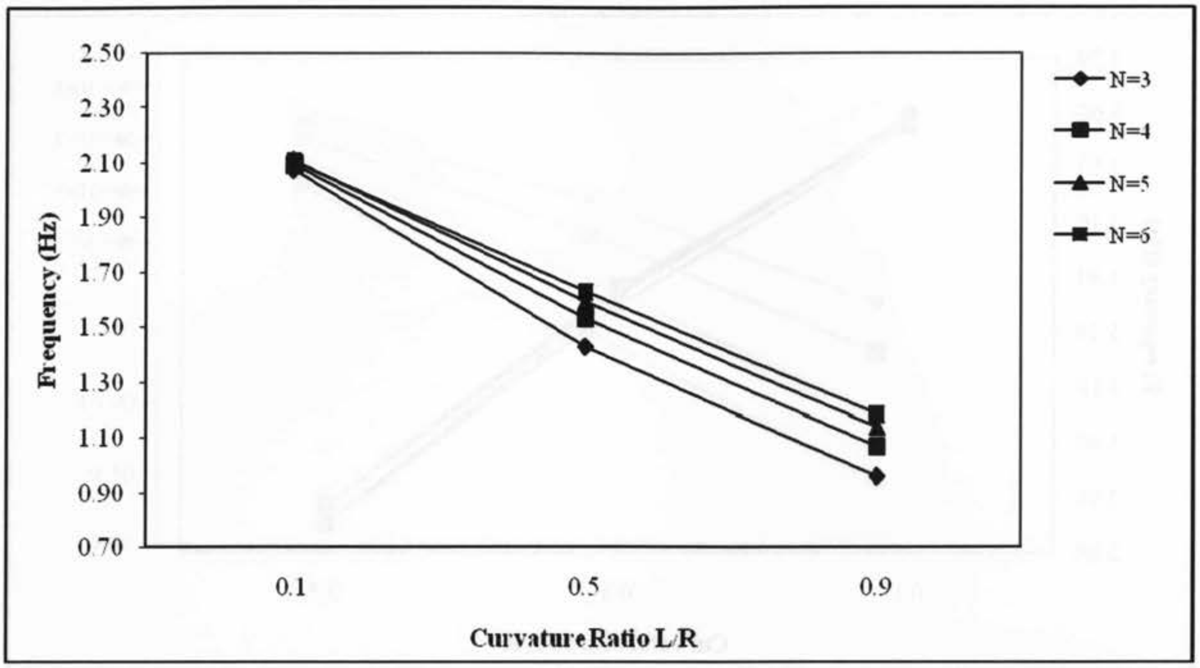


Figure 4.50 Effect of Curvature on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, S=3.0)

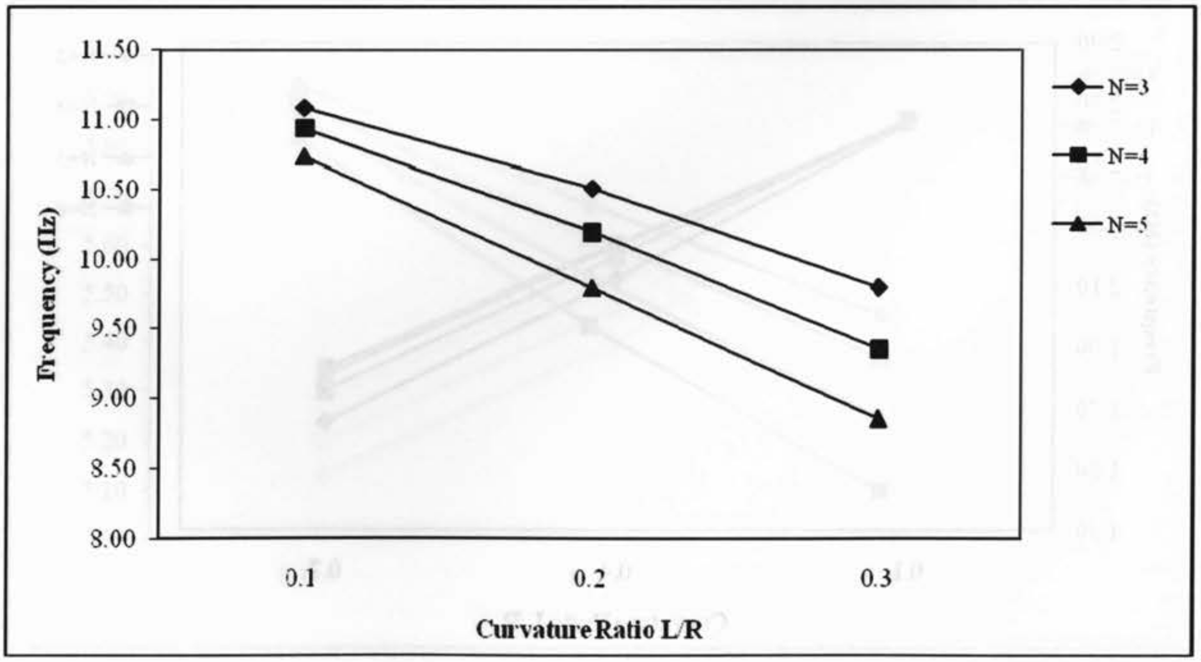


Figure 4.51 Effect of Curvature on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, S=3.0)

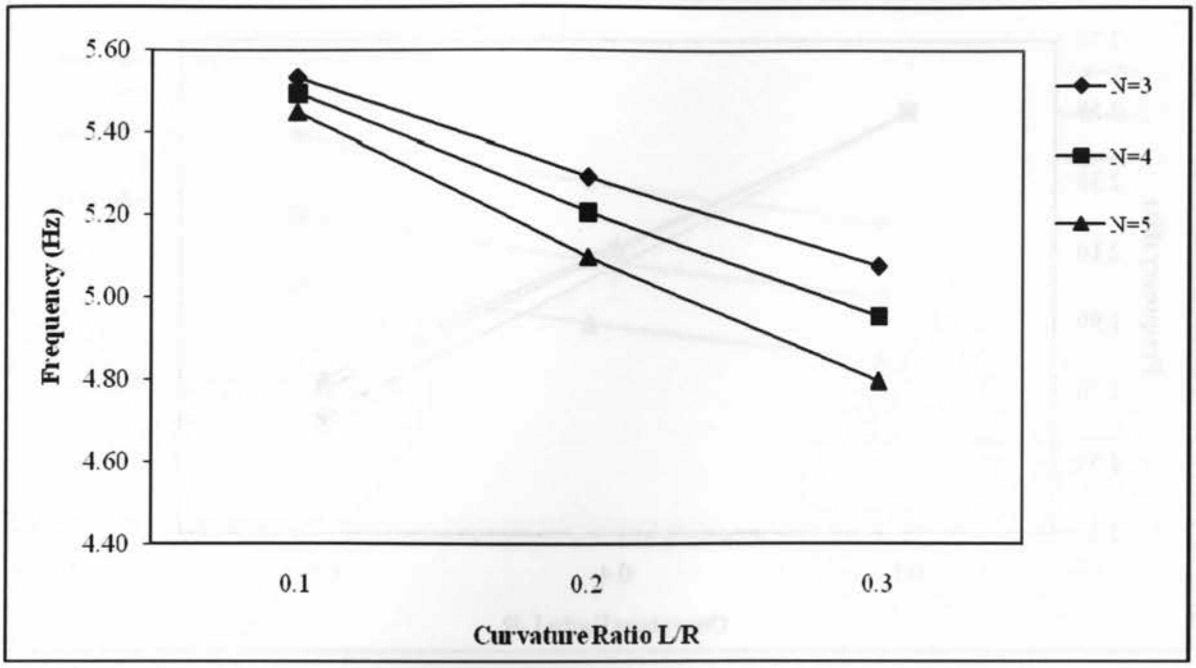


Figure 4.52 Effect of Curvature on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, S=3.0)

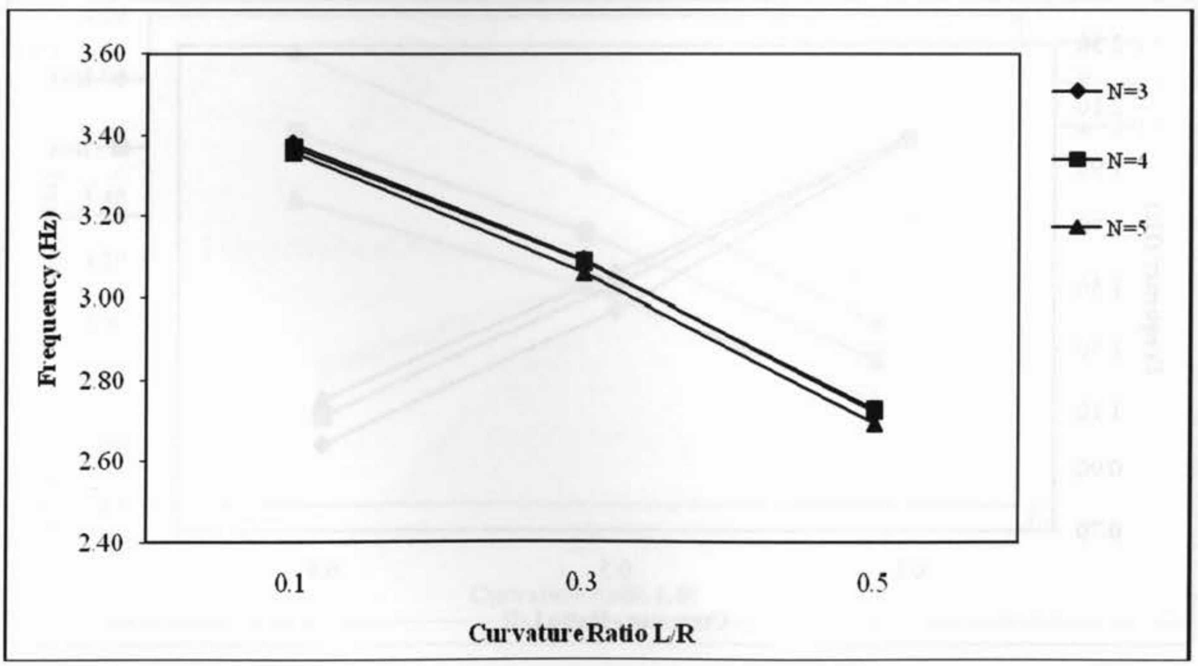


Figure 4.53 Effect of Curvature on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, S=3.0)

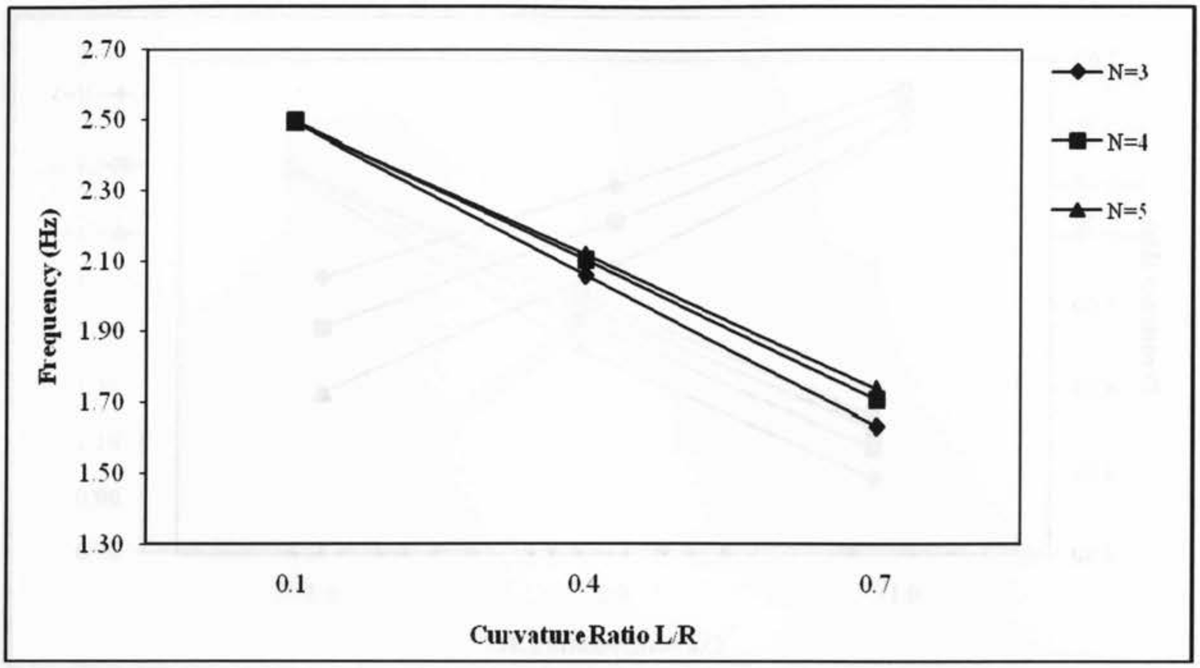


Figure 4.54 Effect of Curvature on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, S=3.0)

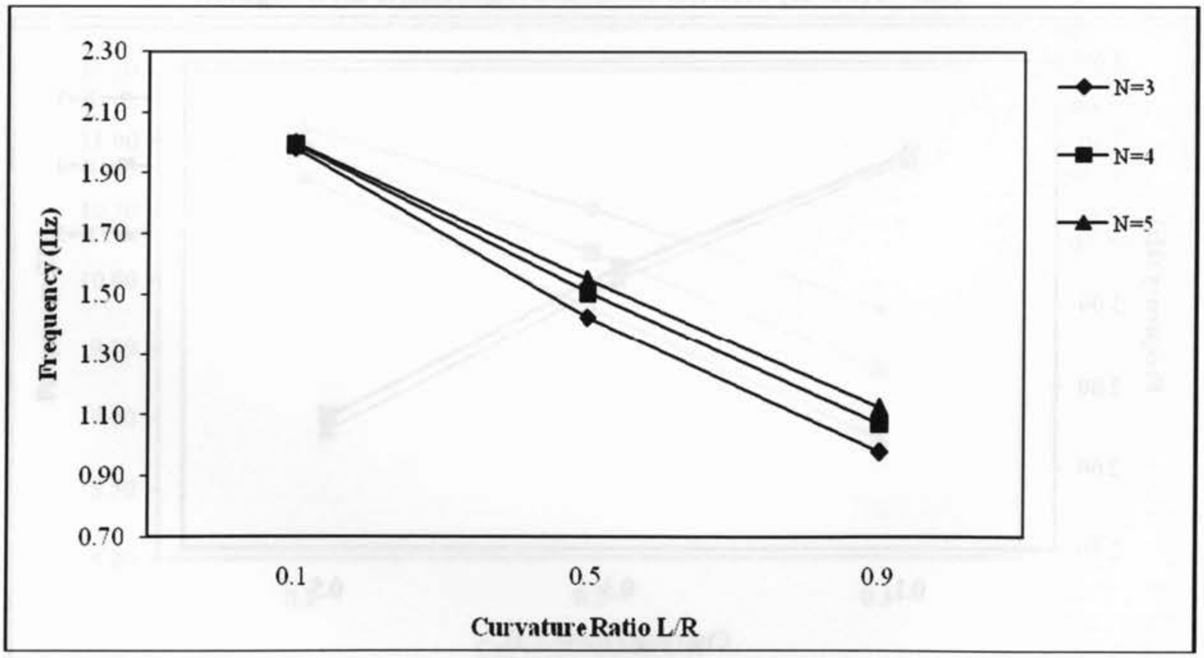


Figure 4.55 Effect of Curvature on the Fundamental Frequency of Curved 2-Span Bridges with Differing Girder Spacing (L=15, N=3)

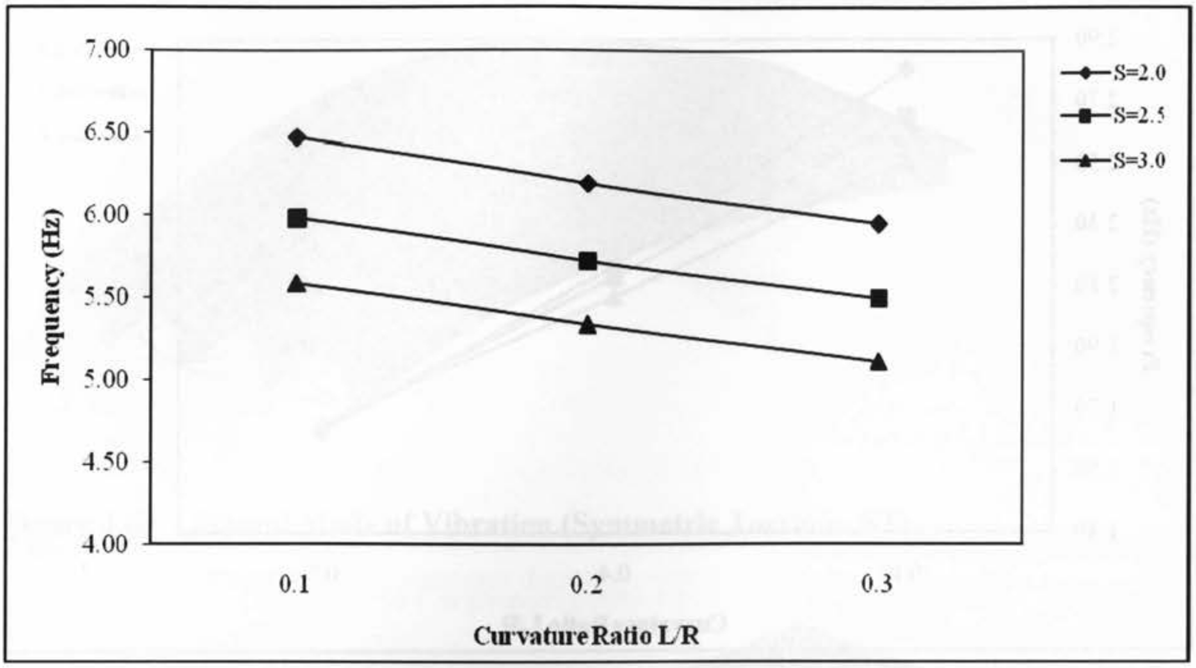


Figure 4.56 Effect of Curvature on the Fundamental Frequency of Curved 2-Span Bridges with Differing Girder Spacing (L=25, N=3)

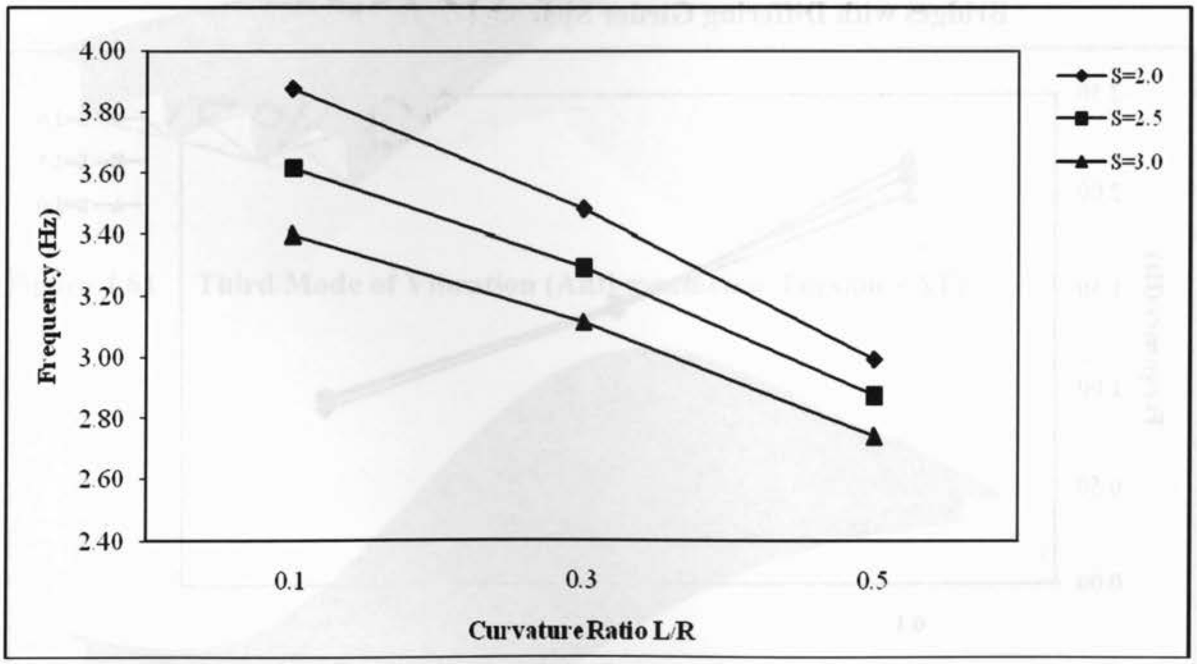


Figure 4.57 Effect of Curvature on the Fundamental Frequency of Curved 2-Span Bridges with Differing Girder Spacing (L=35, N=3)

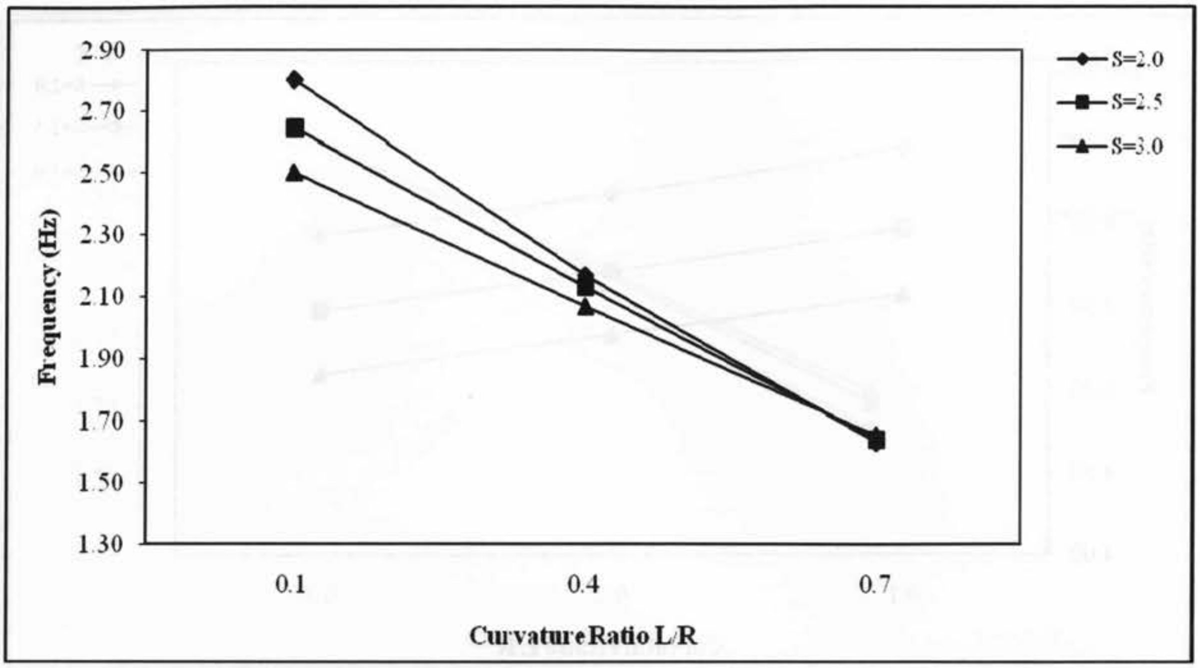


Figure 4.58 Effect of Curvature on the Fundamental Frequency of Curved 2-Span Bridges with Differing Girder Spacing (L=45, N=3)

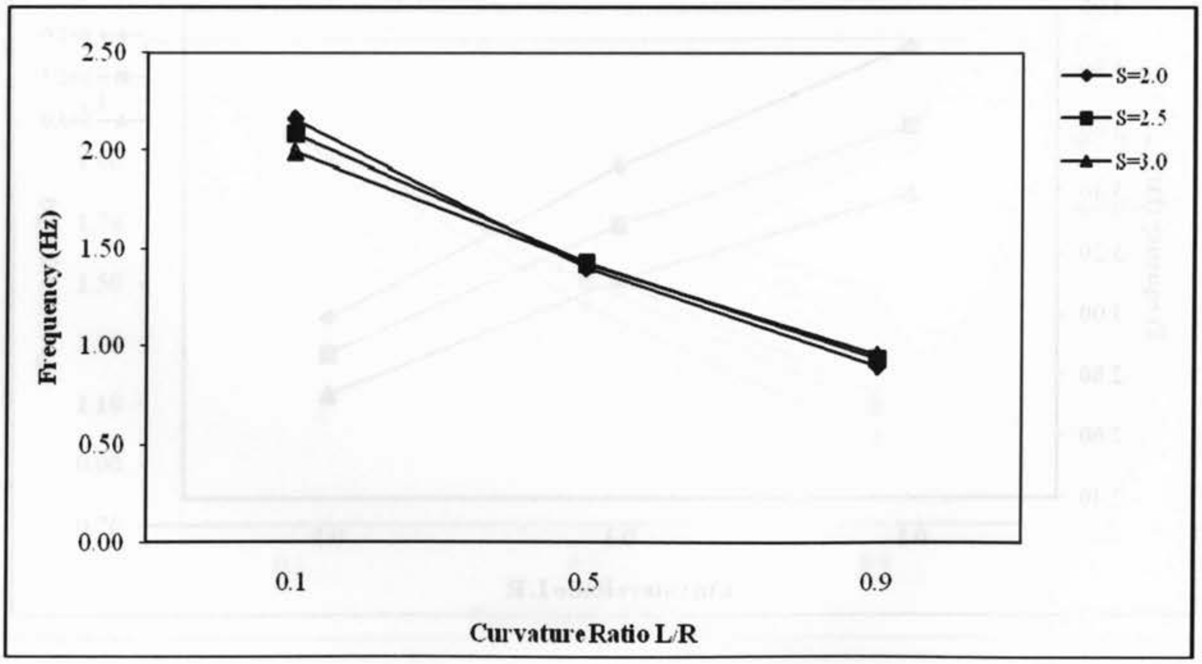


Figure 4.59 First Mode of Vibration (Flexural & Symmetric Torsion - FST)

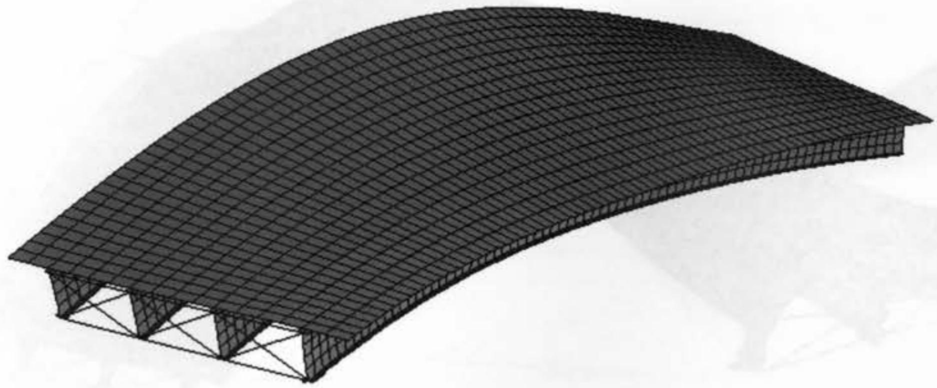


Figure 4.60 Second Mode of Vibration (Symmetric Torsion - ST)

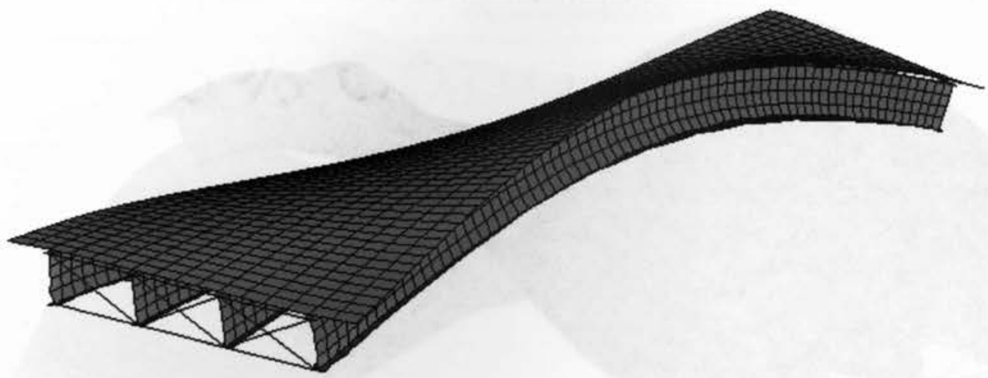


Figure 4.61 Third Mode of Vibration (Anti-symmetric Torsion - AT)

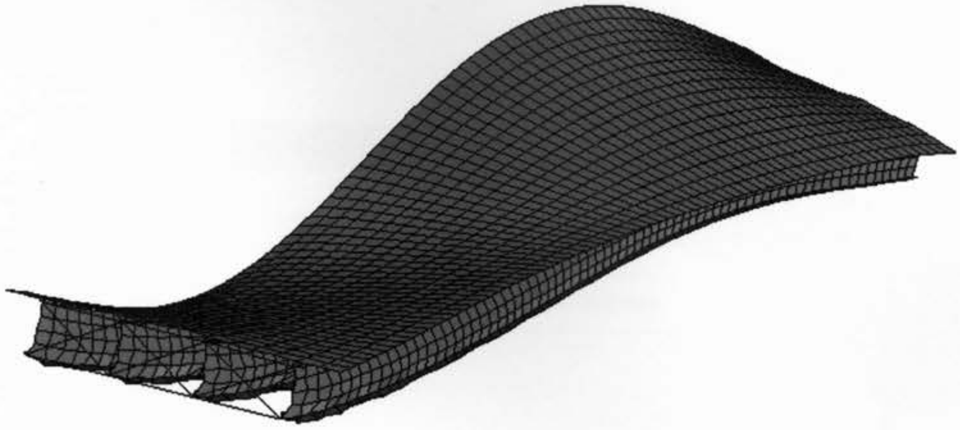


Figure 4.62 Fourth Mode of Vibration (Lateral - L)

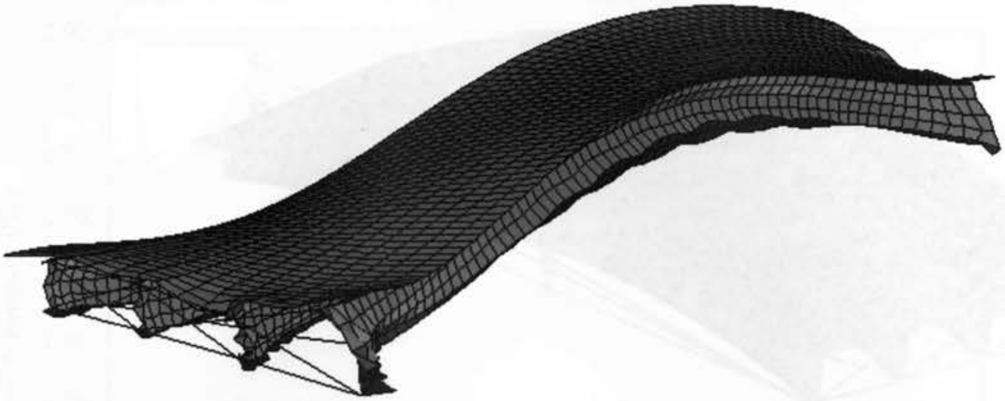


Figure 4.63 Fifth Mode of Vibration (Anti-symmetric Torsion - AT)

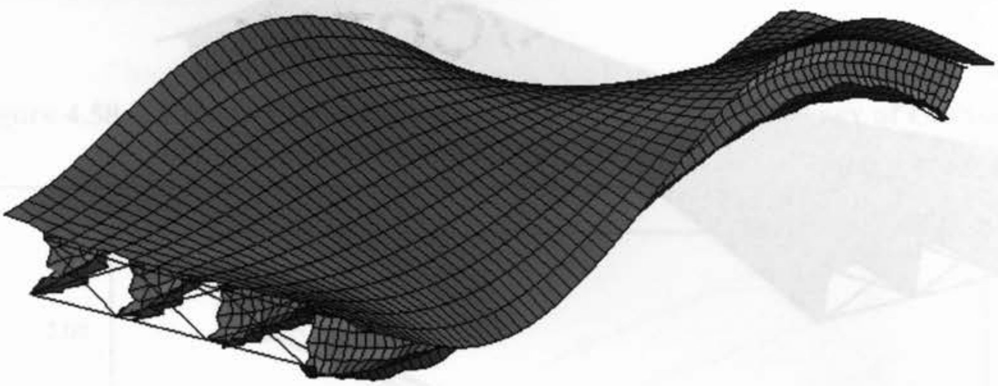


Figure 4.64 First Mode of Vibration for Two-Span Continuous Curved Bridge (Flexural & Anti-Symmetric Torsion - FAT)

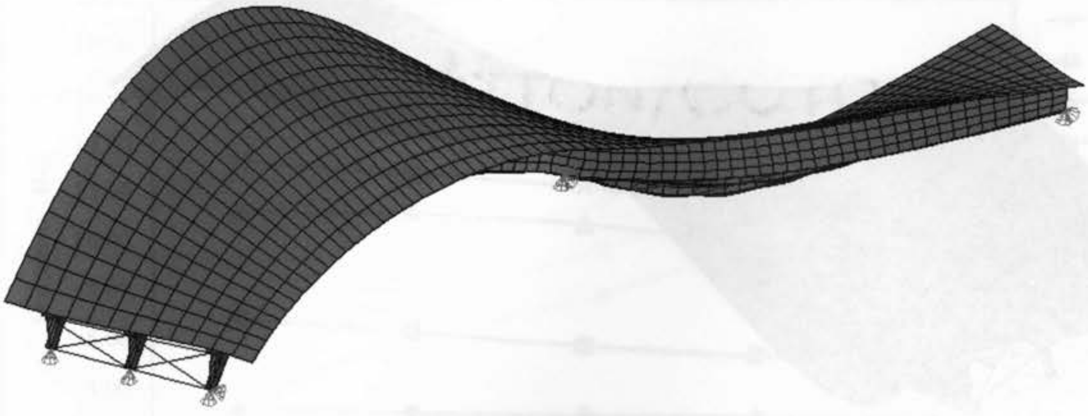


Figure 4.65 Second Mode of Vibration for Two-Span Continuous Curved Bridge (Flexural & Symmetric Torsion - FST)

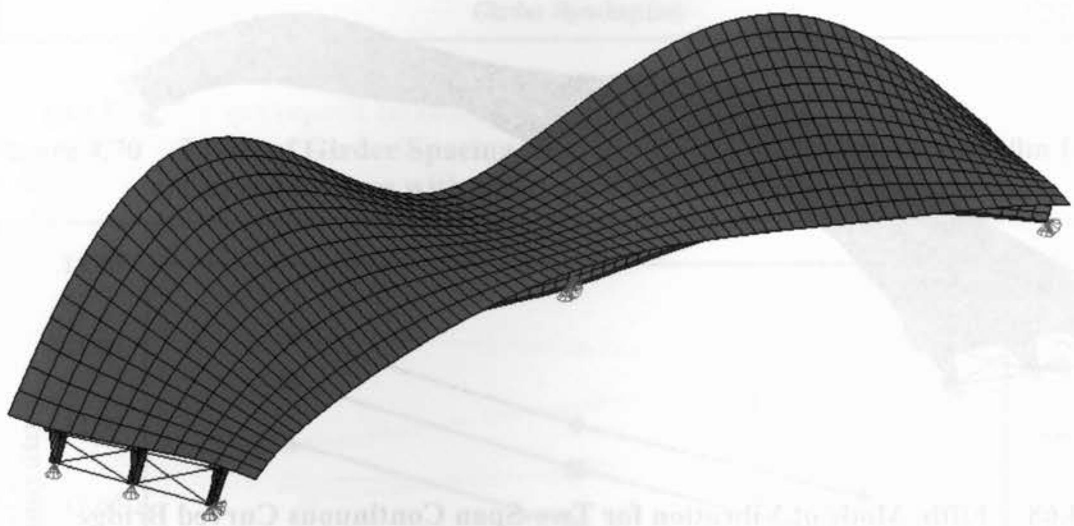


Figure 4.66 Third Mode of Vibration for Two-Span Continuous Curved Bridge (Anti-symmetric Torsion - AT)

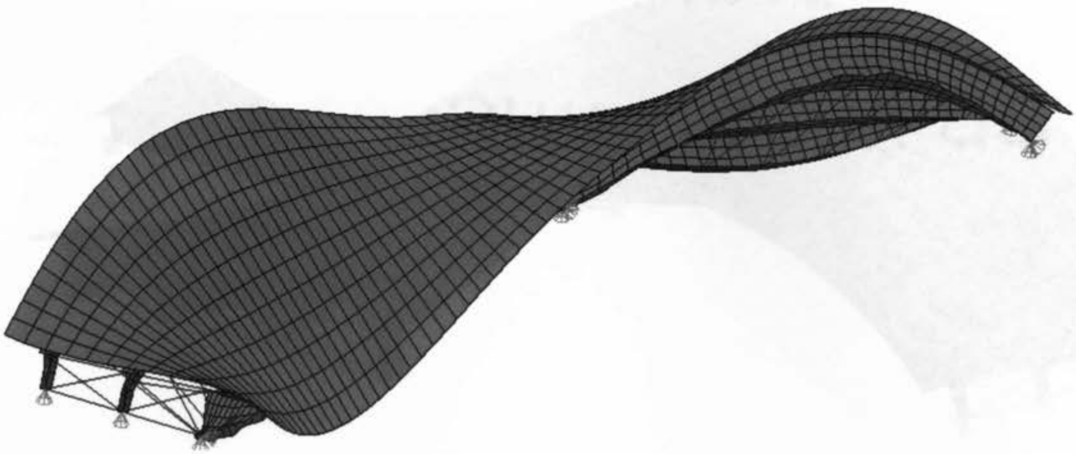


Figure 4.67 Fourth Mode of Vibration for Two-Span Continuous Curved Bridge (Symmetric Torsion - ST)

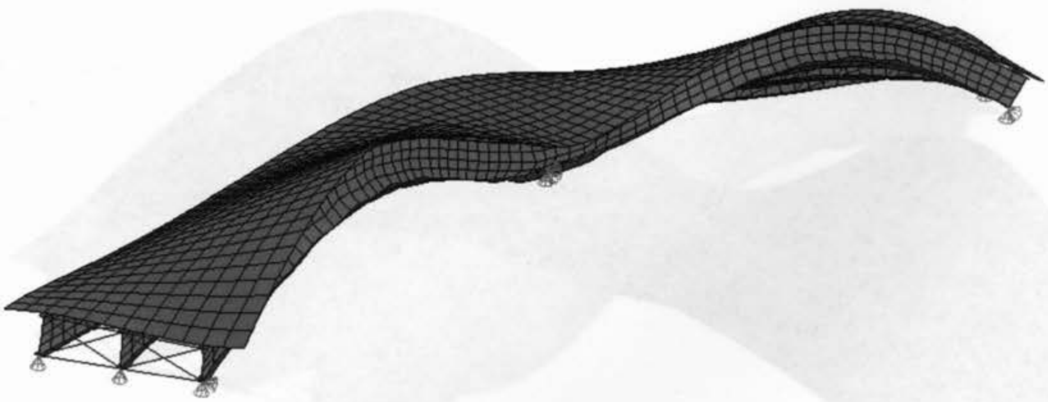


Figure 4.68 Fifth Mode of Vibration for Two-Span Continuous Curved Bridge (Lateral - L)

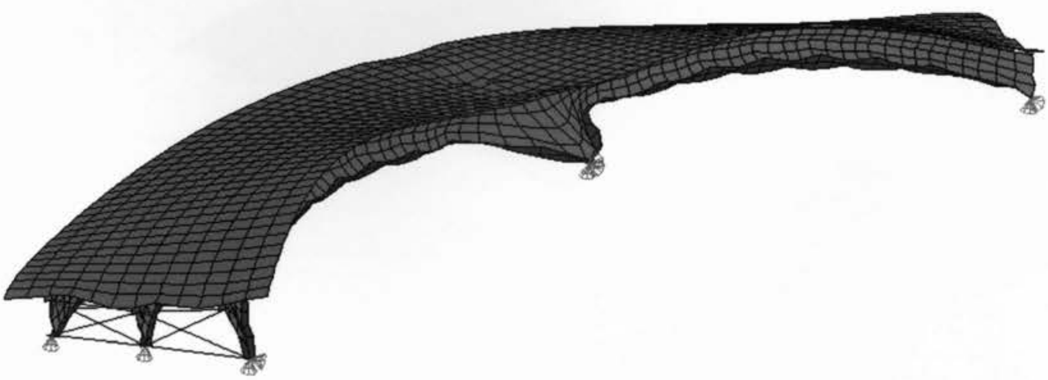


Figure 4.69 Effect of Girder Spacing on the First Five Frequencies of 25m Long Curved Bridges (L=25, N=3, L/R=0.5)

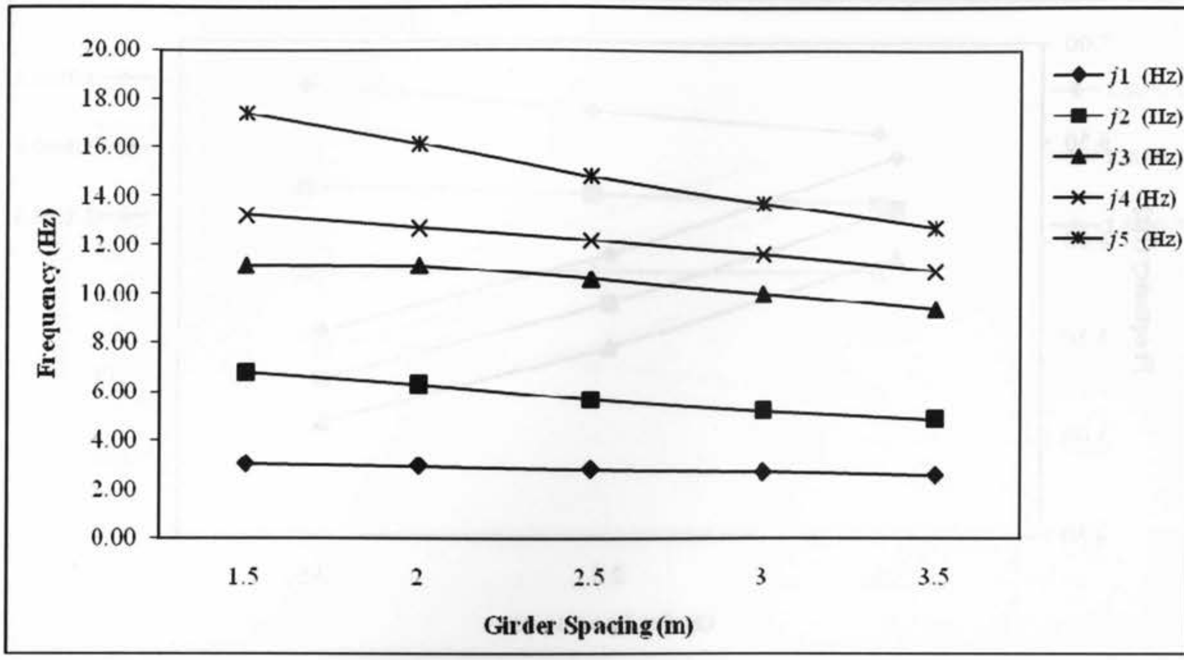


Figure 4.70 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Curvature Ratios (L=10, N=3)

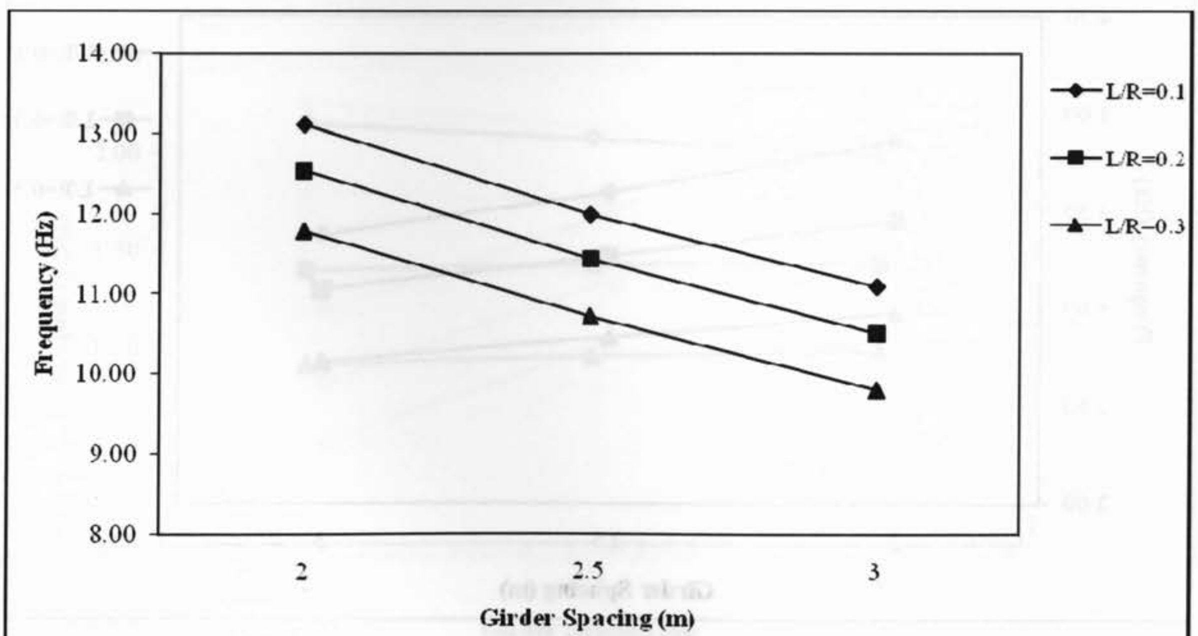


Figure 4.71 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Curvature Ratios (L=15, N=3)

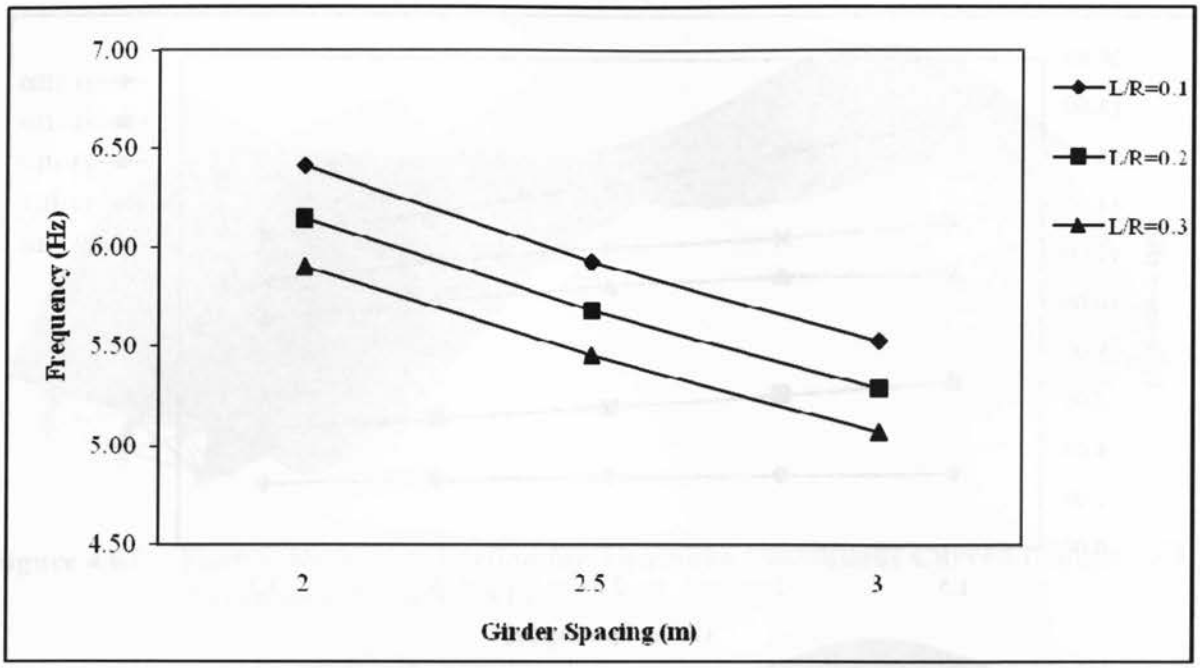


Figure 4.72 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Curvature Ratios (L=25, N=3)

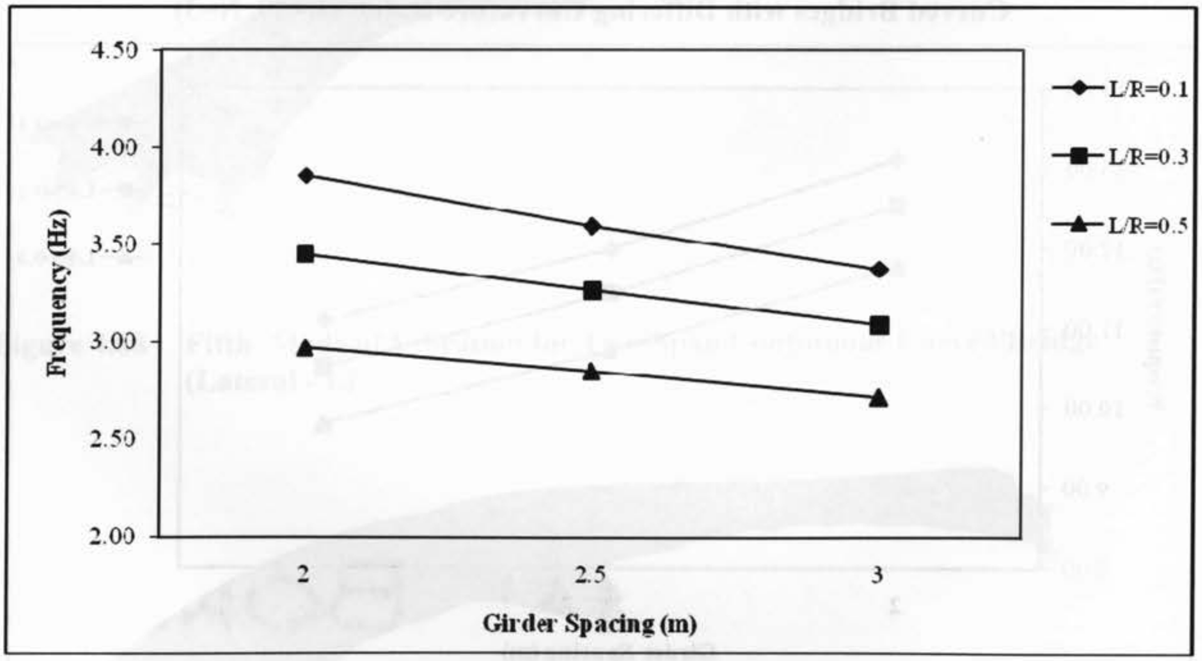


Figure 4.73 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Curvature Ratios (L=35, N=3)

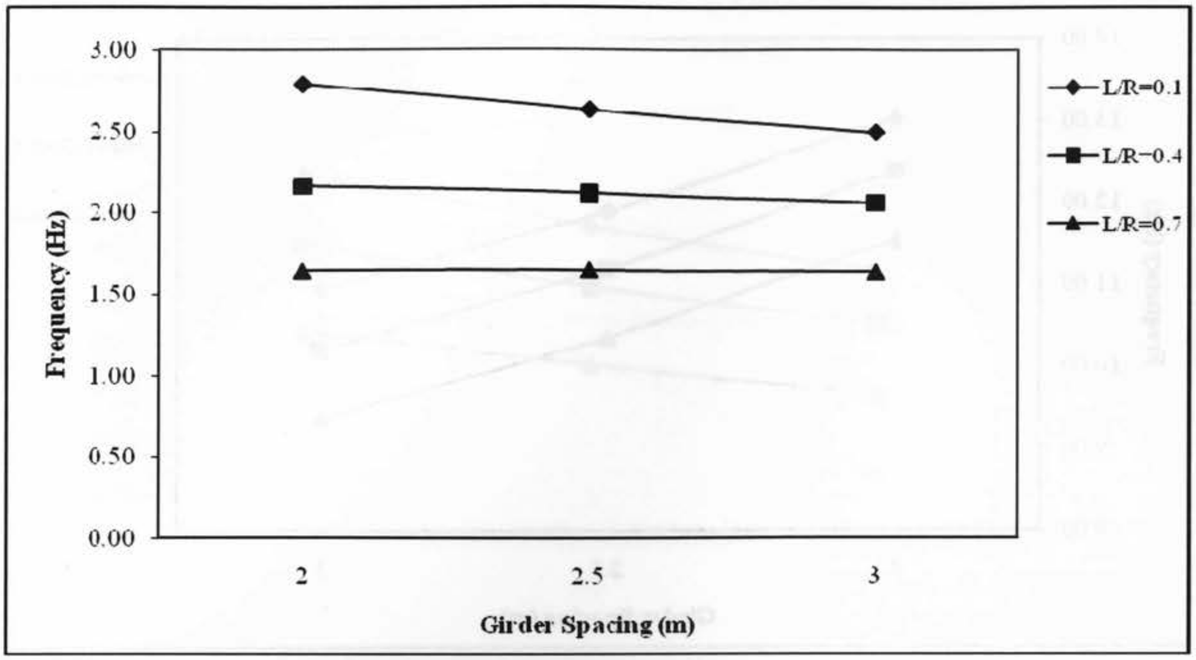


Figure 4.74 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Curvature Ratios (L=45, N=3)

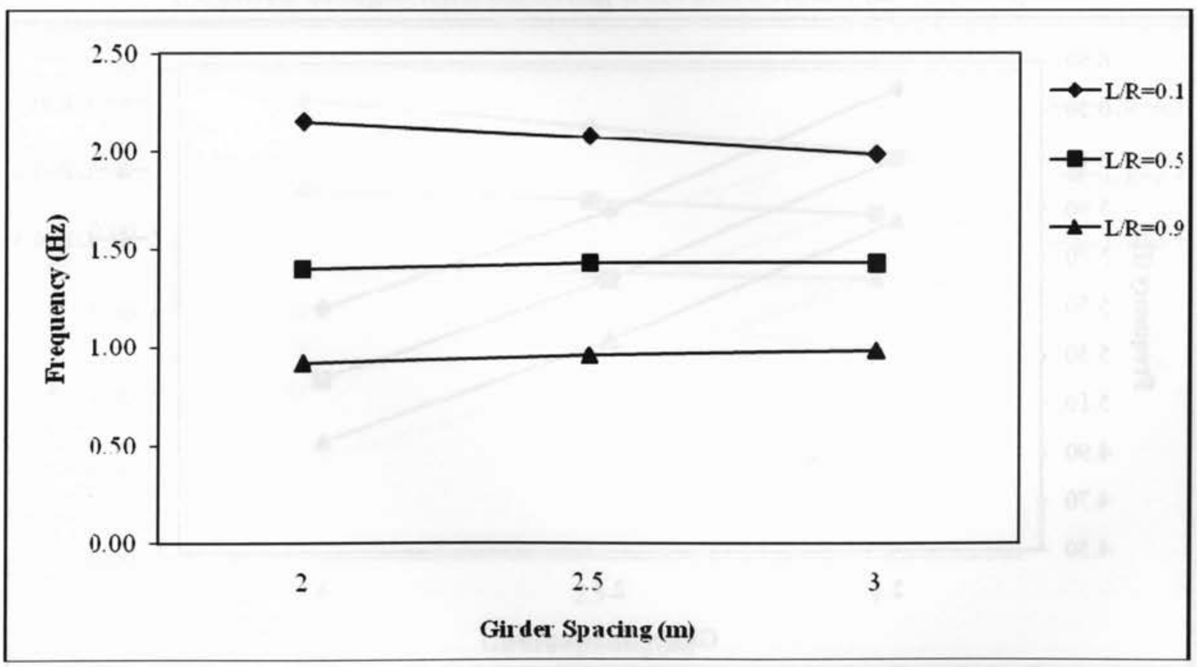


Figure 4.75 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Curvature Ratios (L=10, N=4)

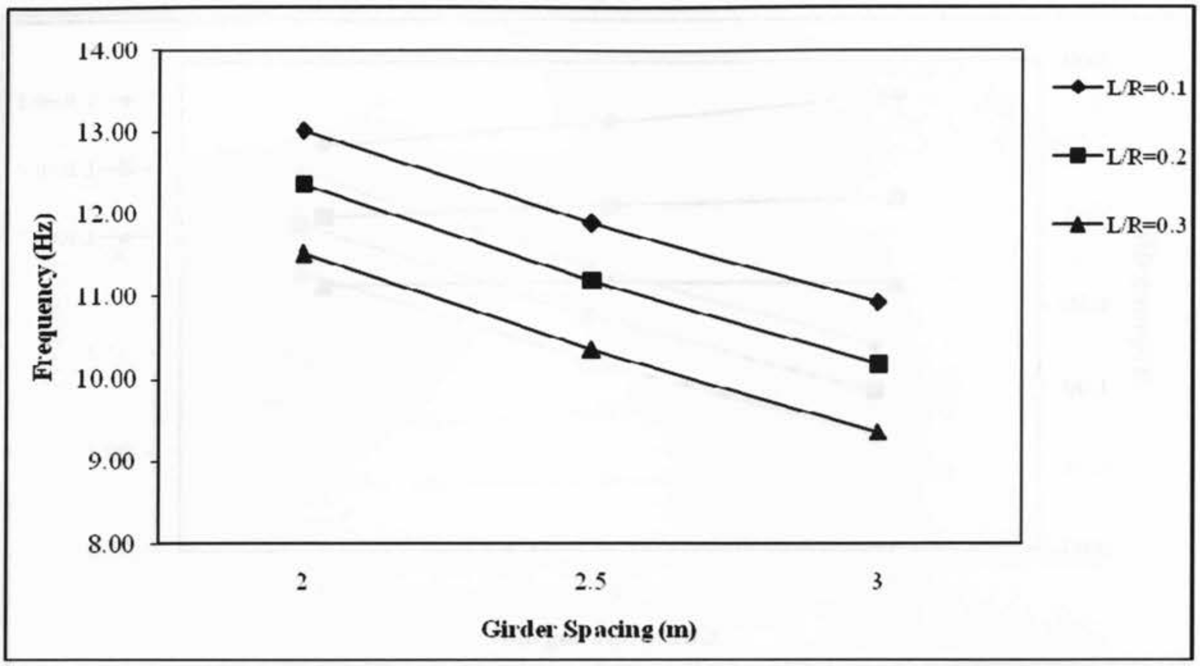


Figure 4.76 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Curvature Ratios (L=15, N=4)

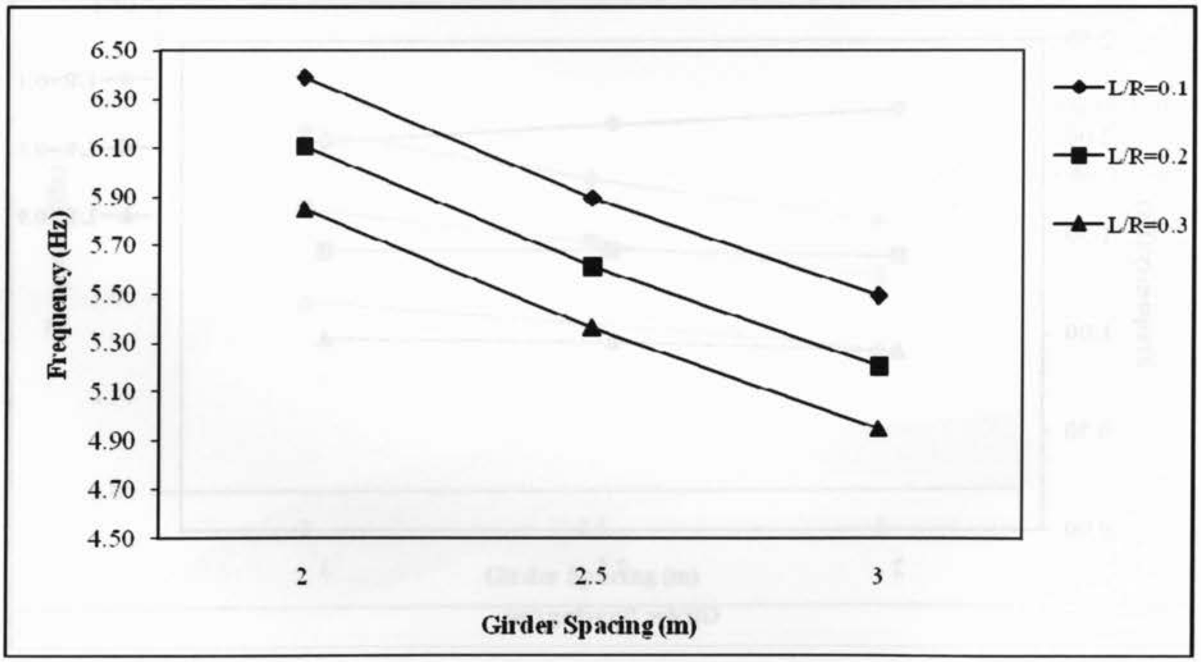


Figure 4.77 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Curvature Ratios (L=25, N=4)

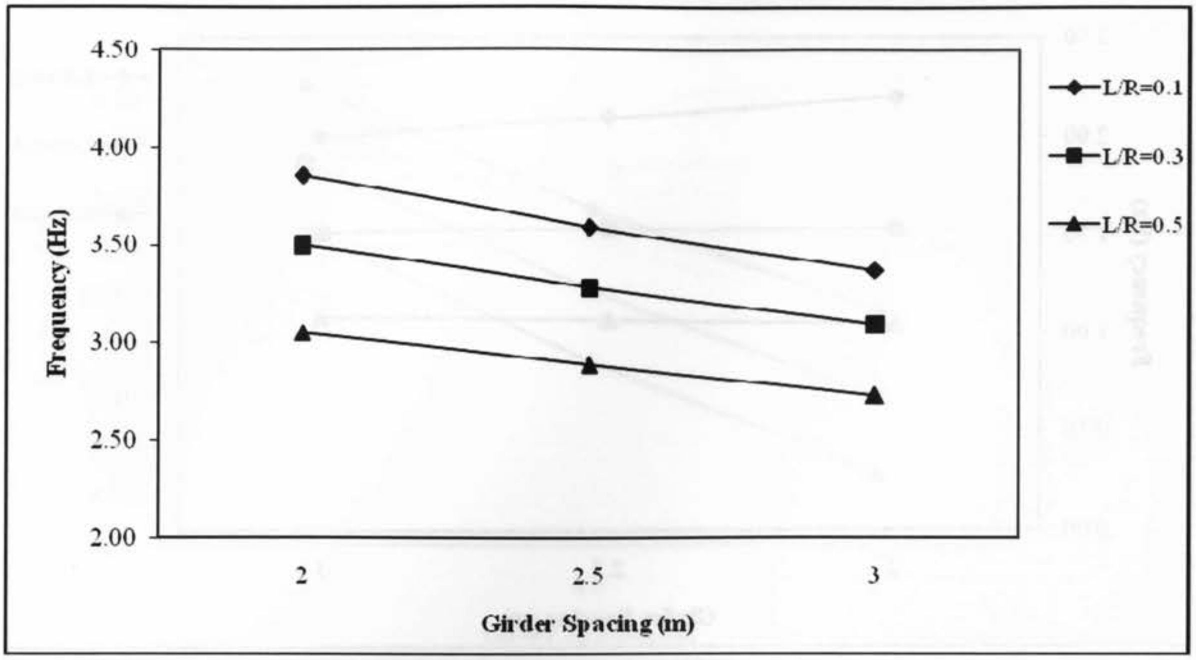


Figure 4.78 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Curvature Ratios (L=35, N=4)

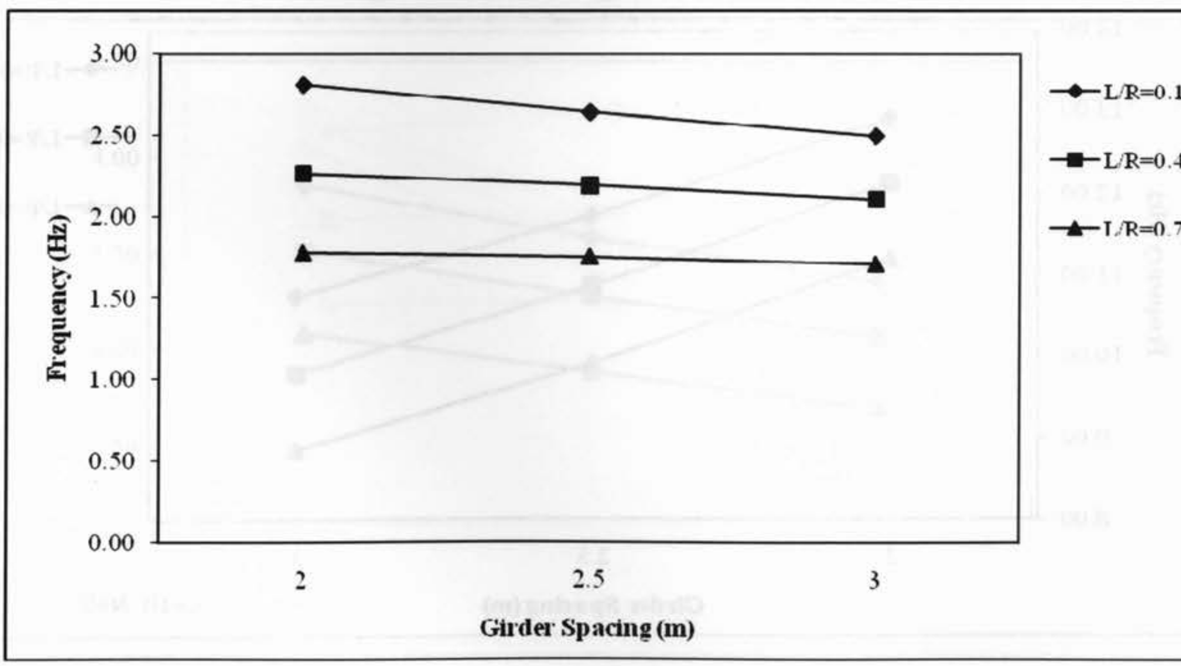


Figure 4.79 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Curvature Ratios (L=45, N=4)

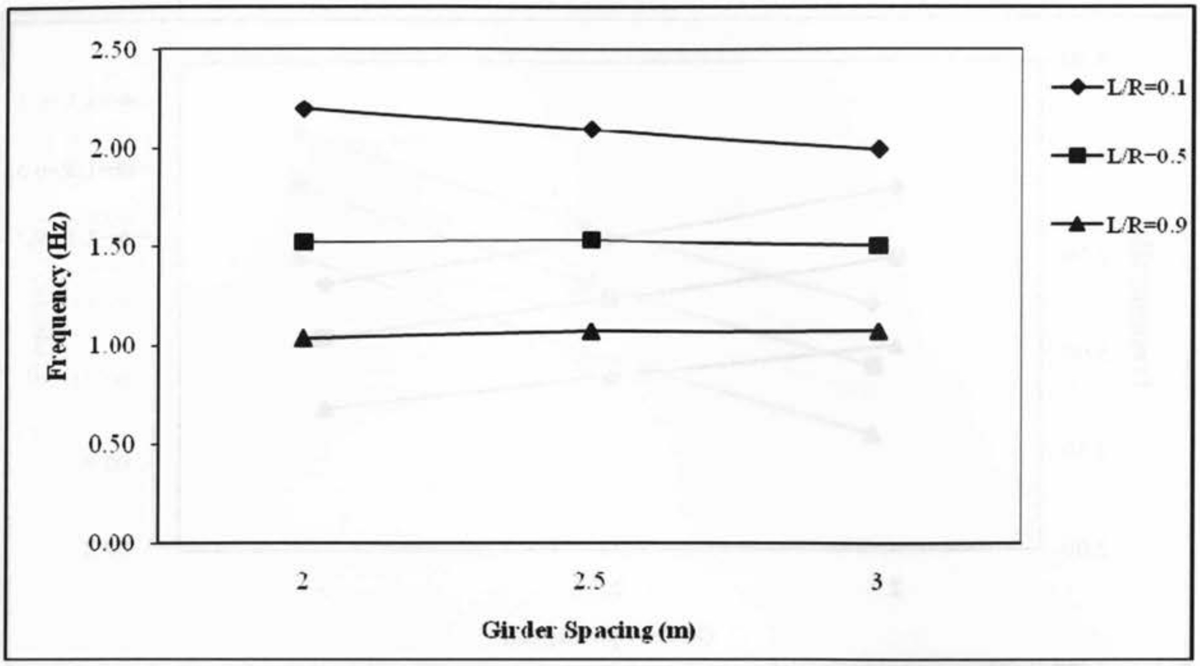


Figure 4.80 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Curvature Ratios (L=10, N=5)

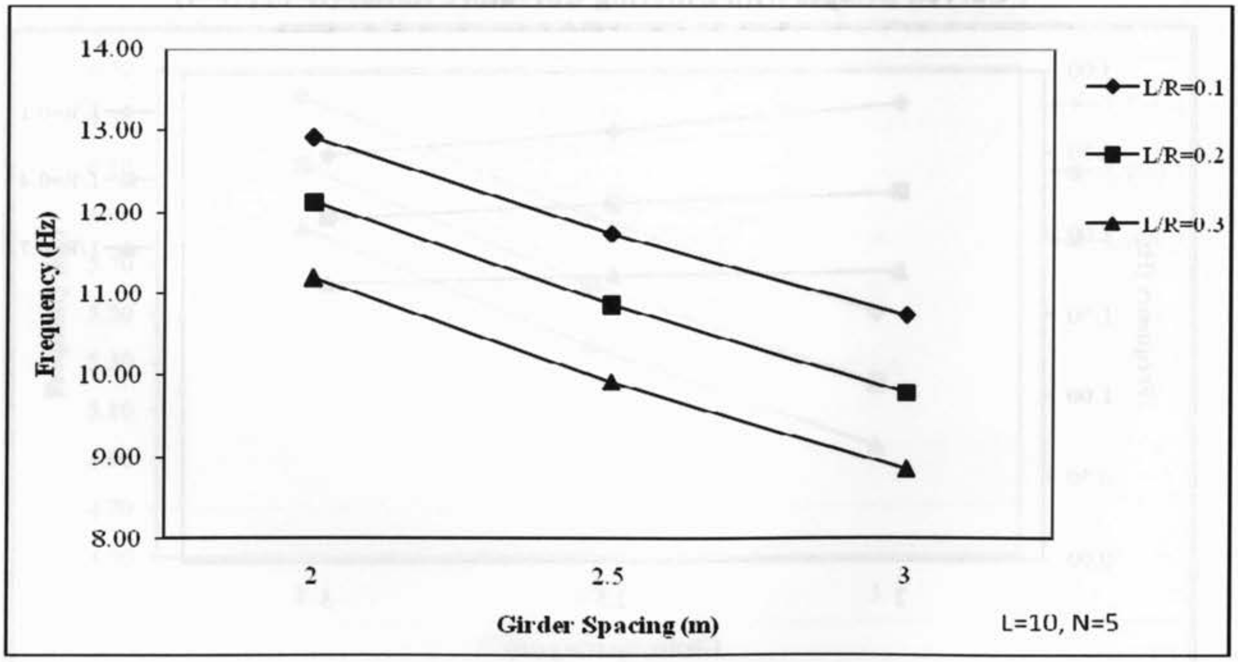


Figure 4.81 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Curvature Ratios (L=15, N=5)

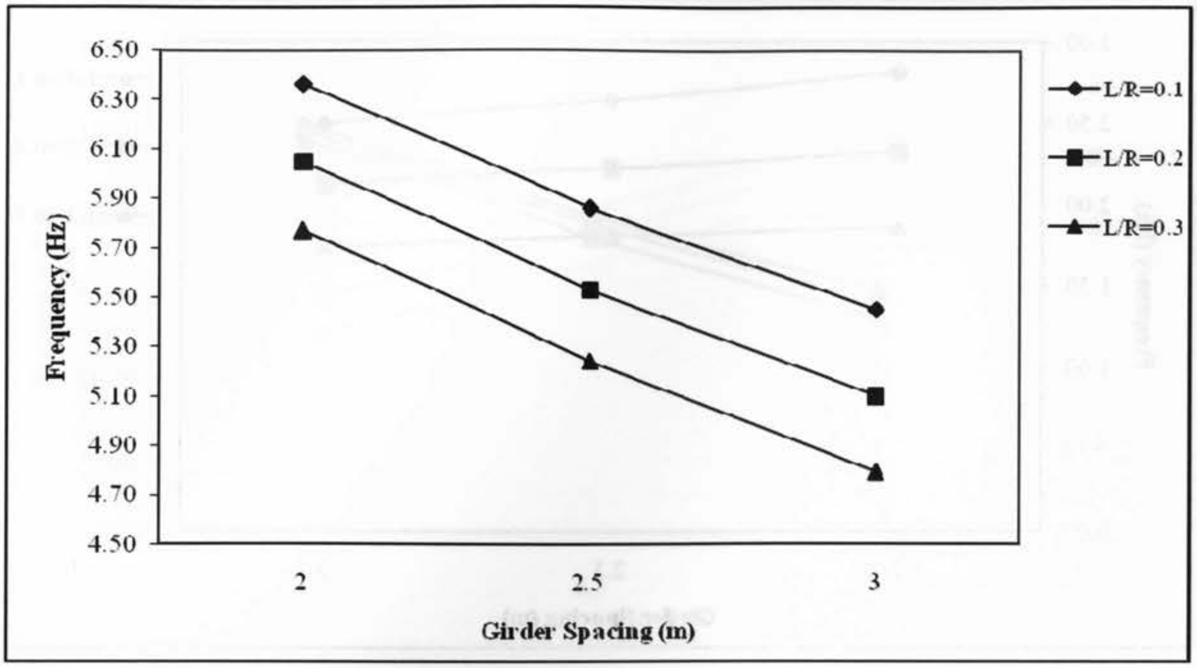


Figure 4.82 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Curvature Ratios (L=25, N=5)

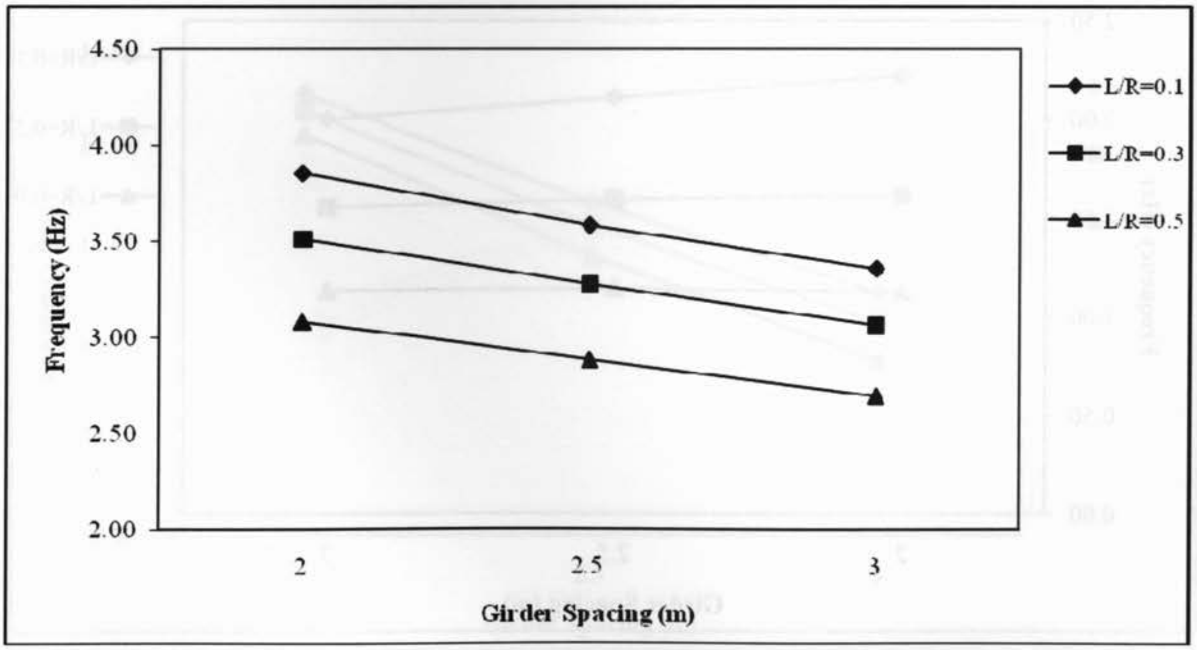


Figure 4.83 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Curvature Ratios (L=35, N=5)

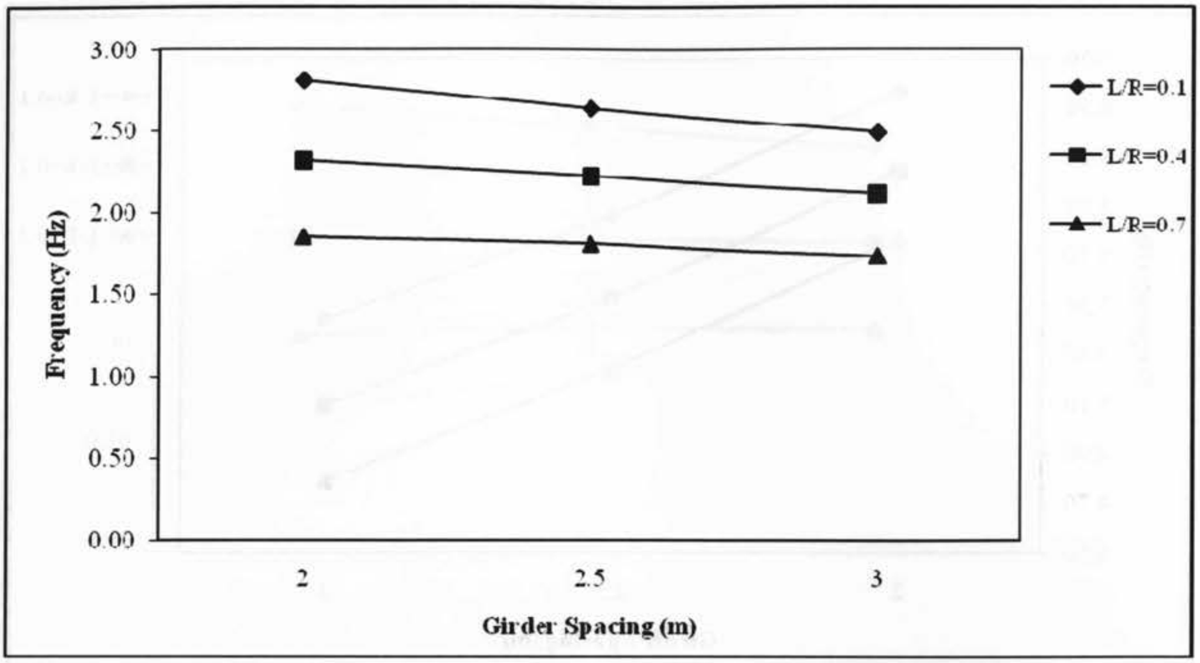


Figure 4.84 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Curvature Ratios (L=45, N=5)

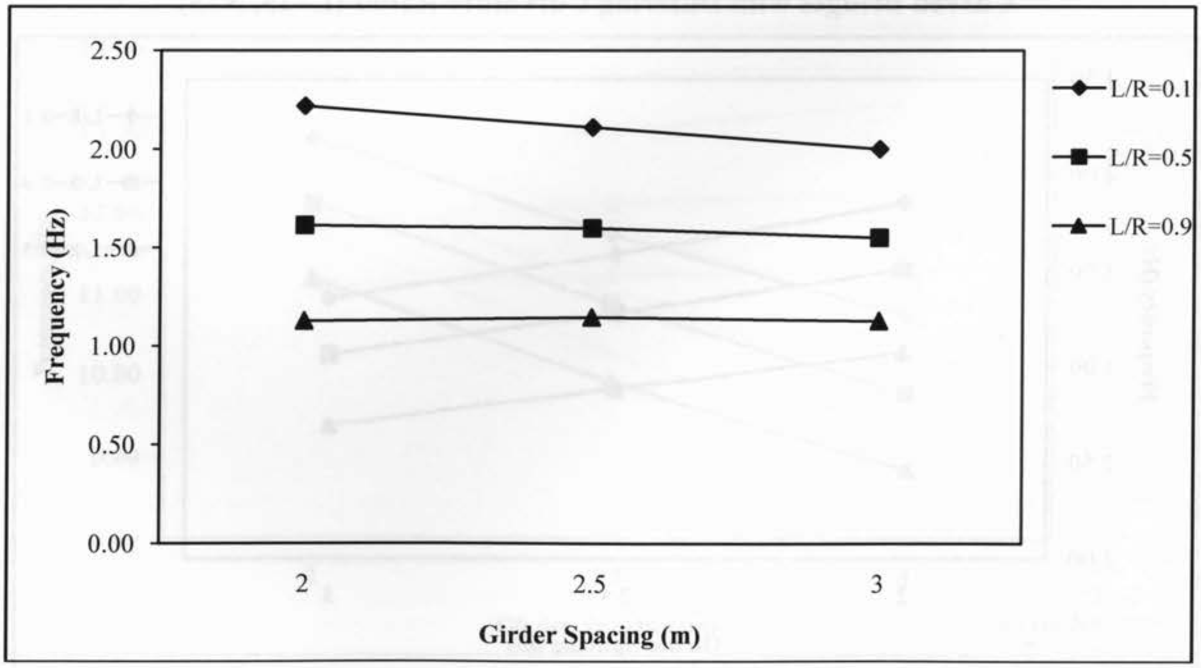


Figure 4.85 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, L/R=0.1)

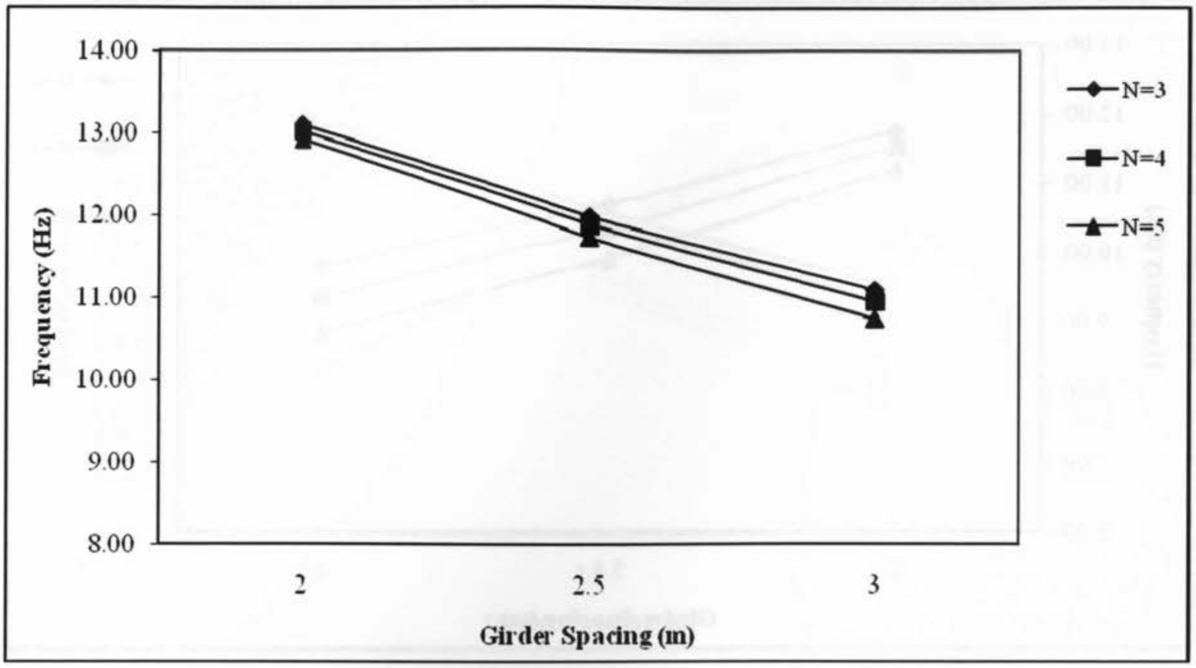


Figure 4.86 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, L/R=0.2)

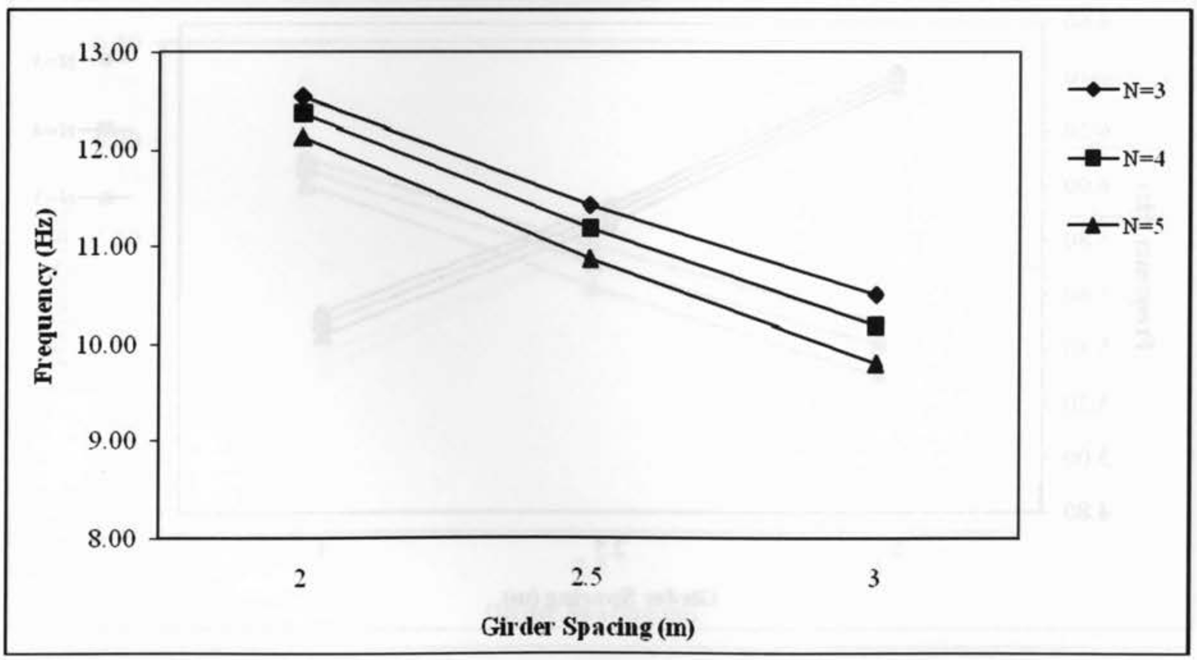


Figure 4.87 Effect of Girder Spacing on the Fundamental Frequency of 10m Long Curved Bridges with Differing Number of Girders (L=10, L/R=0.3)

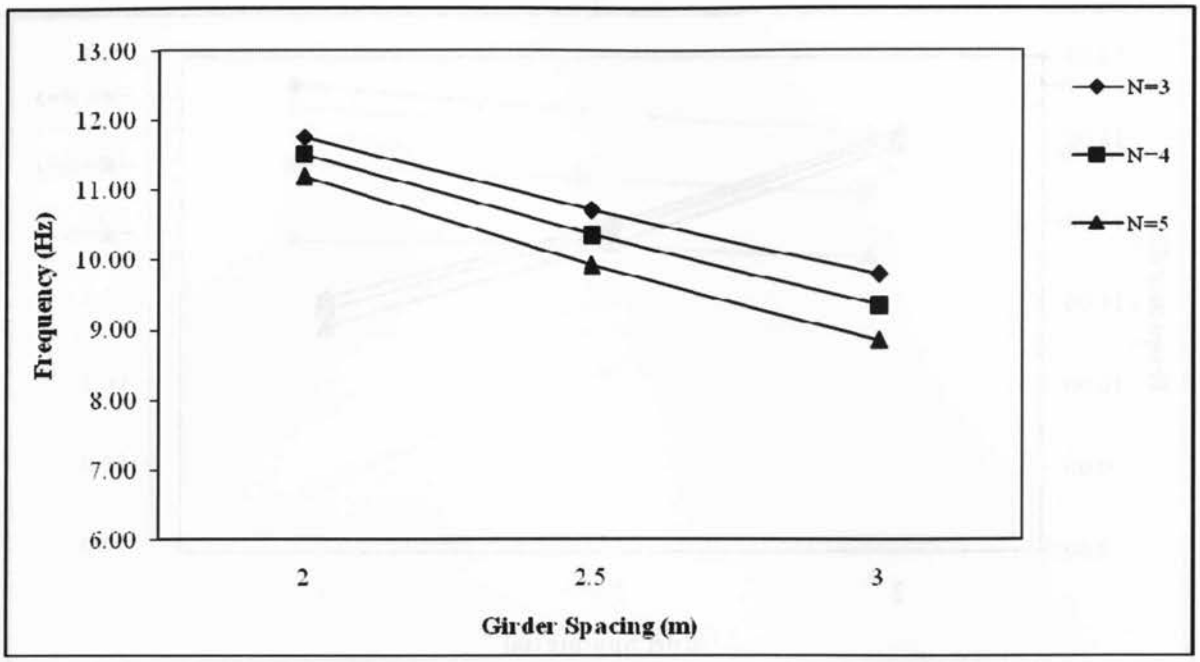


Figure 4.88 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, L/R=0.1)

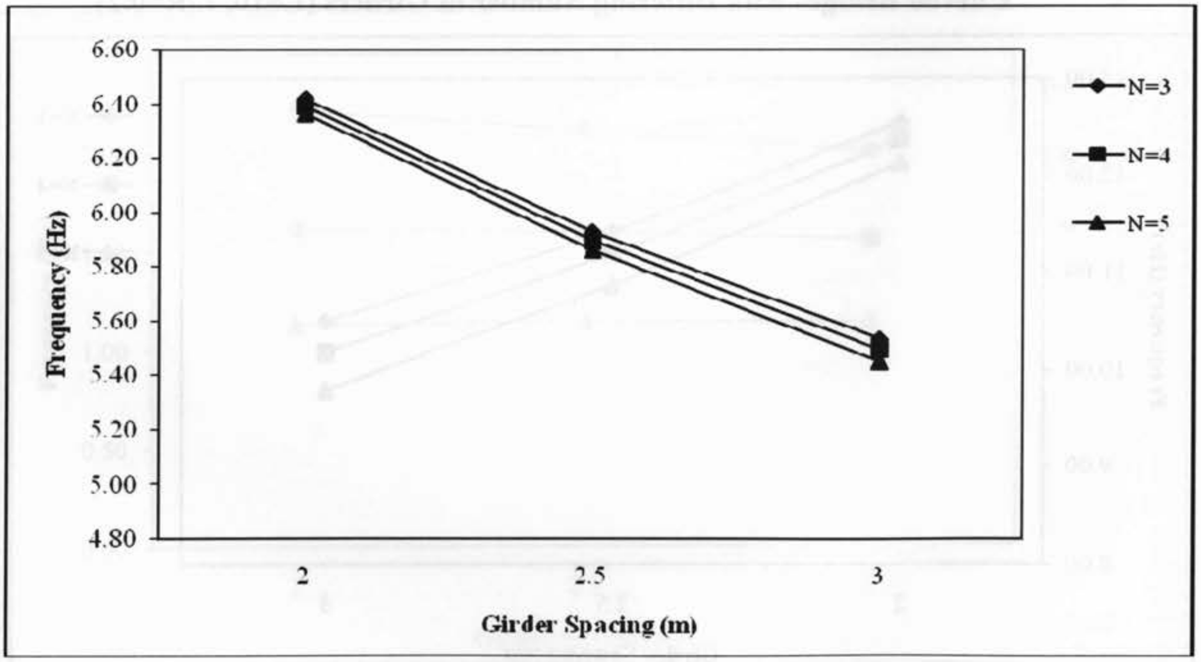


Figure 4.89 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, L/R=0.2)

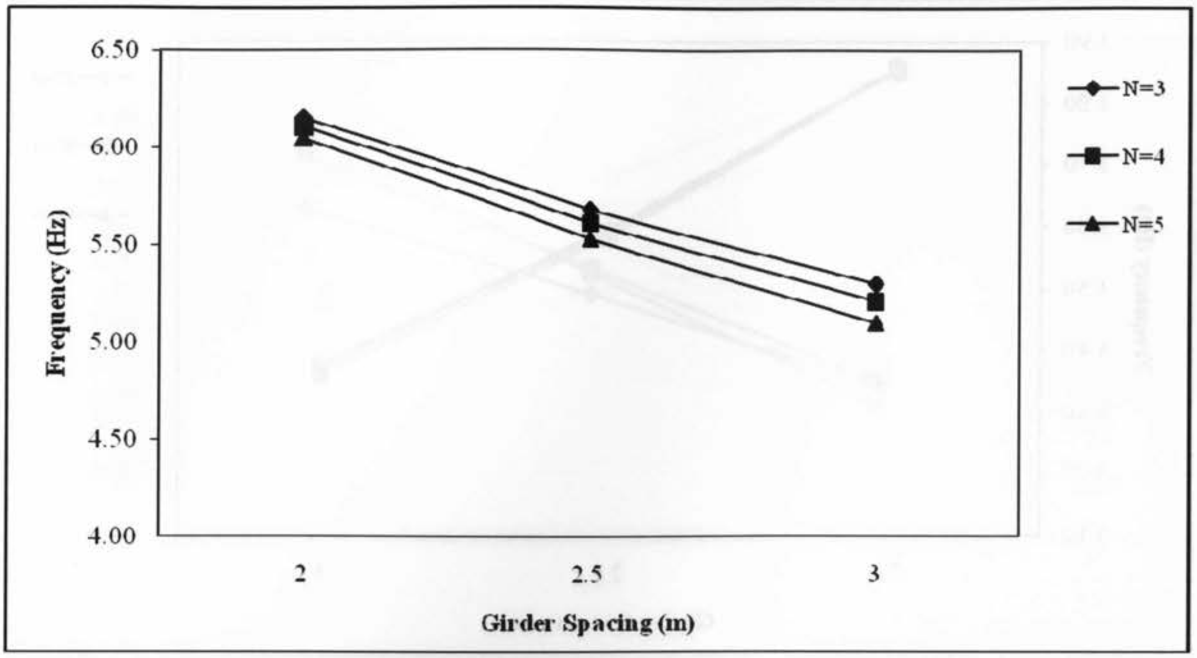


Figure 4.90 Effect of Girder Spacing on the Fundamental Frequency of 15m Long Curved Bridges with Differing Number of Girders (L=15, L/R=0.3)

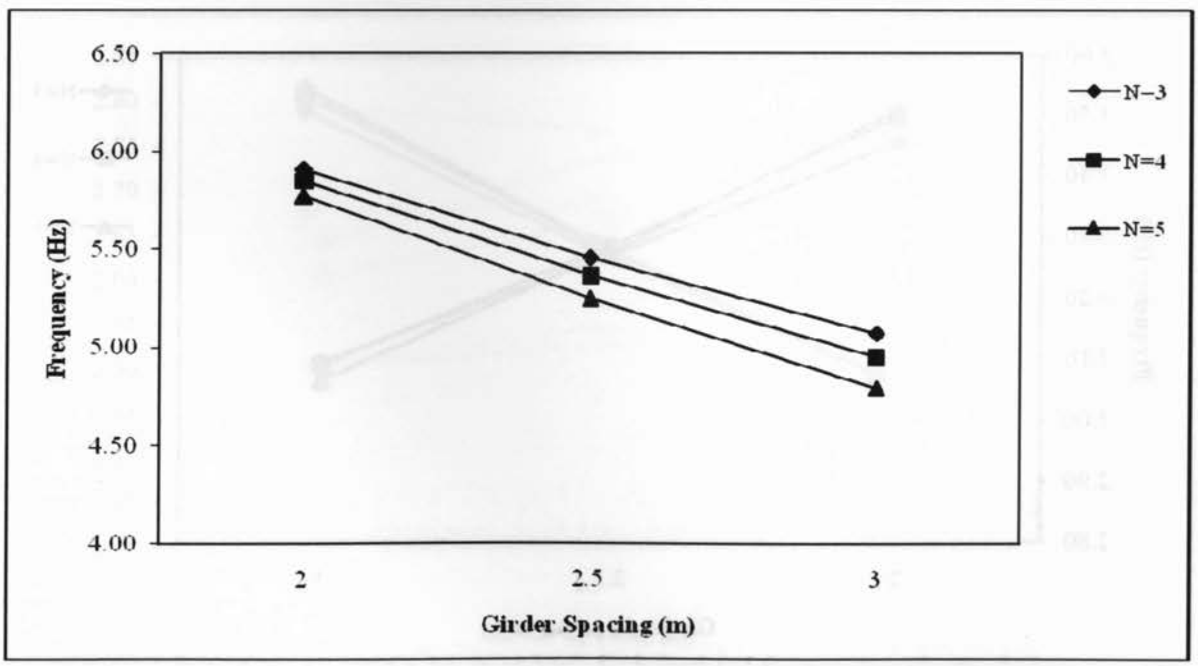


Figure 4.91 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, L/R=0.1)

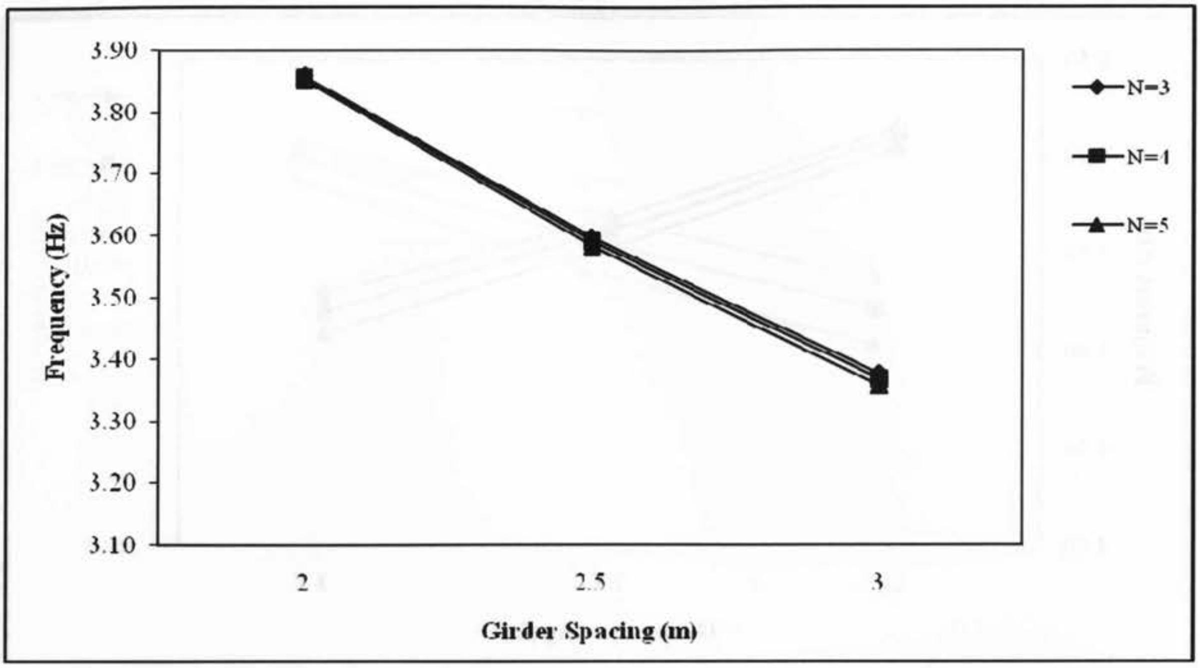


Figure 4.92 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, L/R=0.3)

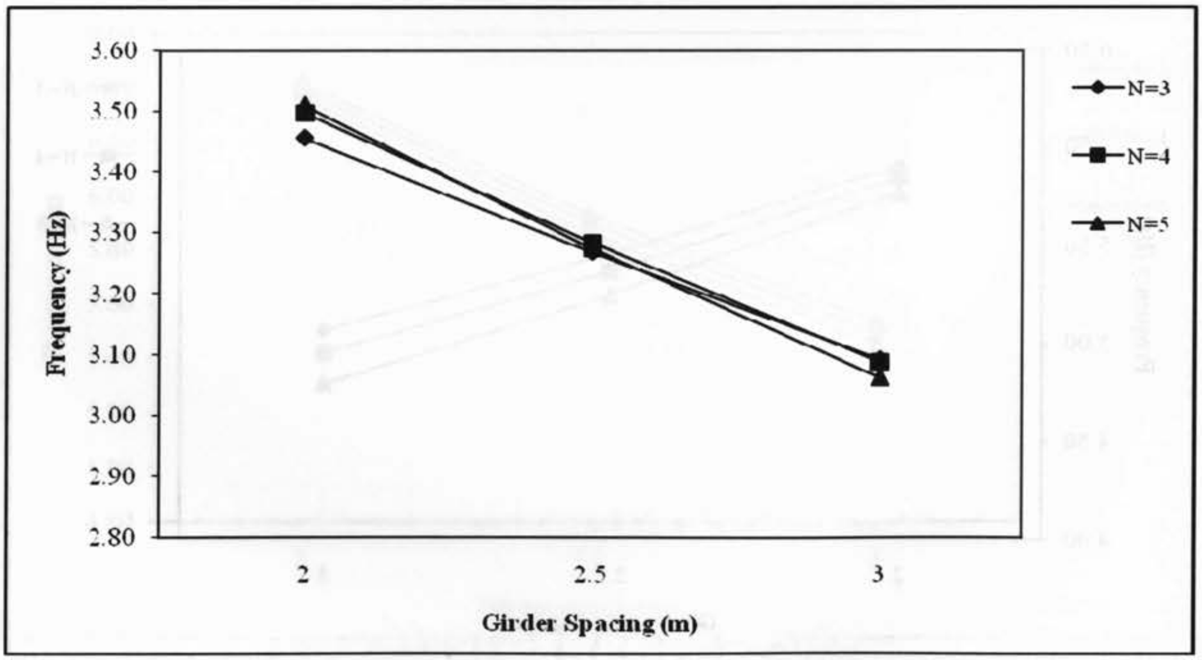


Figure 4.93 Effect of Girder Spacing on the Fundamental Frequency of 25m Long Curved Bridges with Differing Number of Girders (L=25, L/R=0.5)

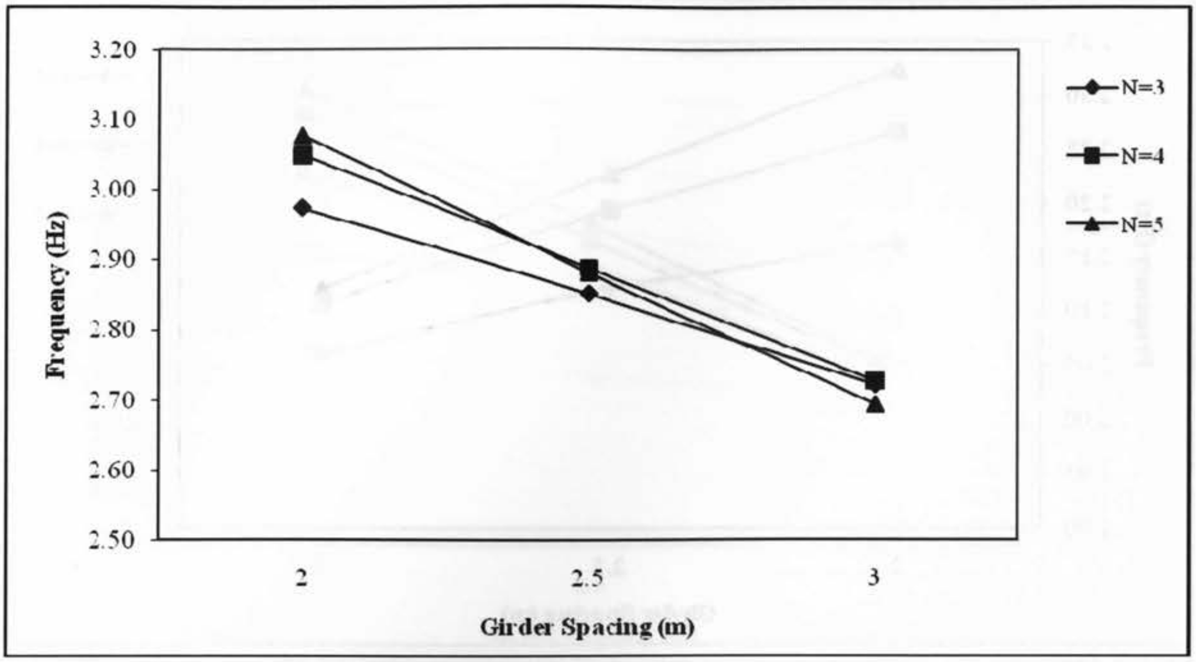


Figure 4.94 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, L/R=0.1)

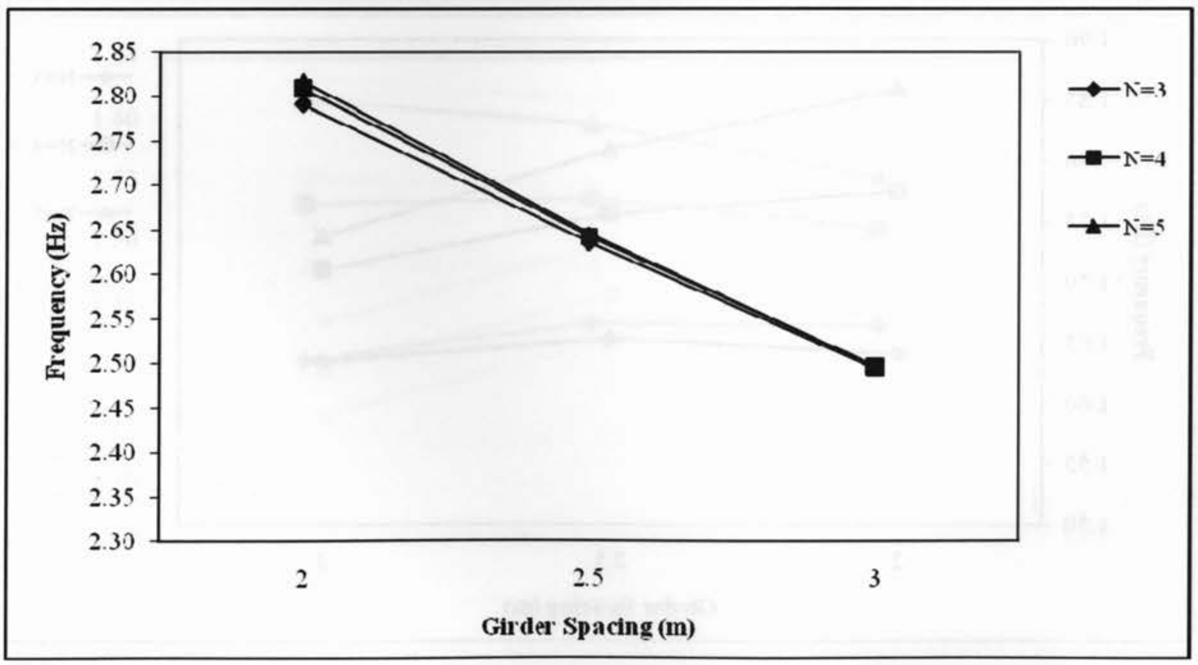


Figure 4.95 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, L/R=0.4)

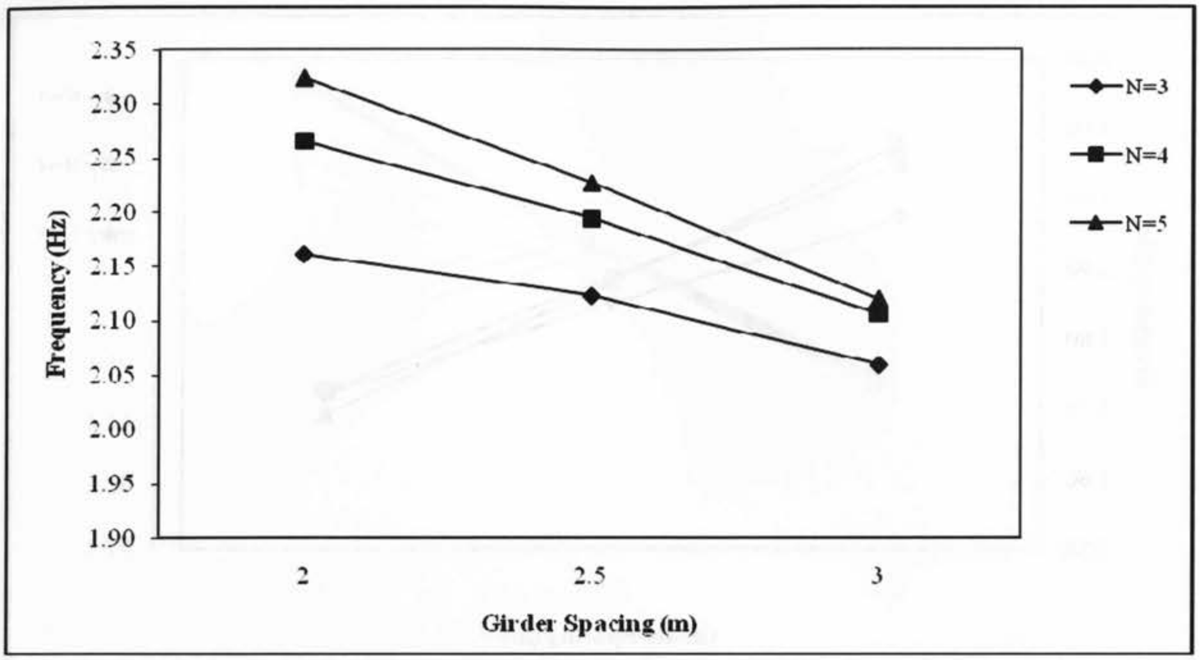


Figure 4.96 Effect of Girder Spacing on the Fundamental Frequency of 35m Long Curved Bridges with Differing Number of Girders (L=35, L/R=0.7)

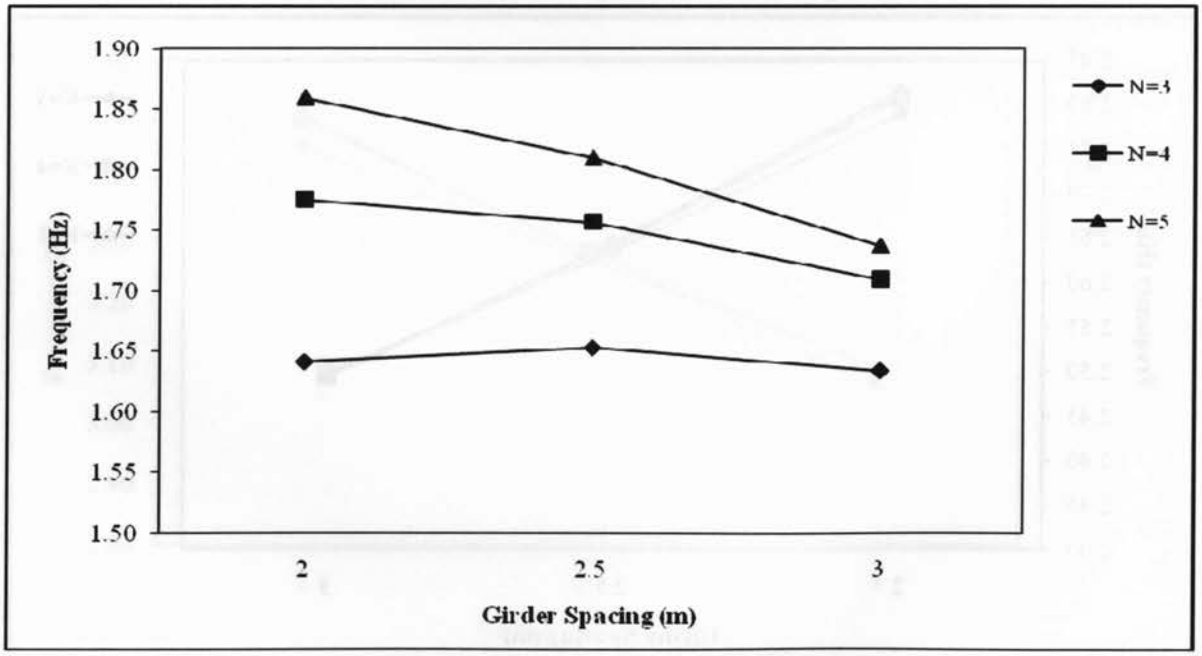


Figure 4.97 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, L/R=0.1)

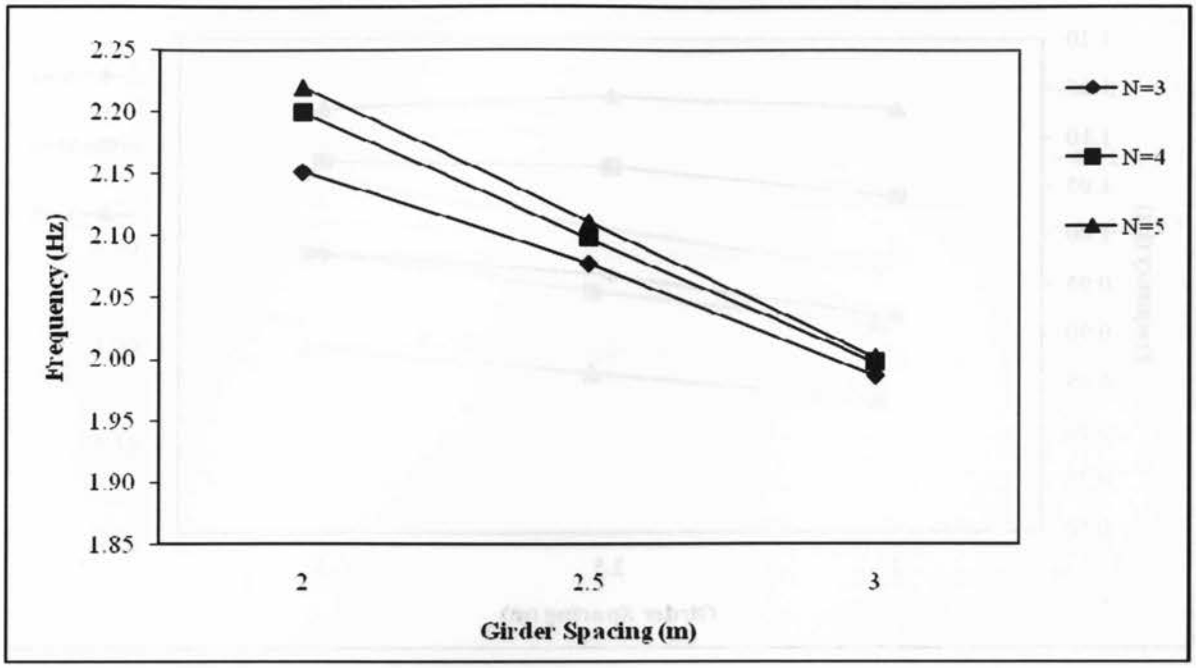


Figure 4.98 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, L/R=0.5)

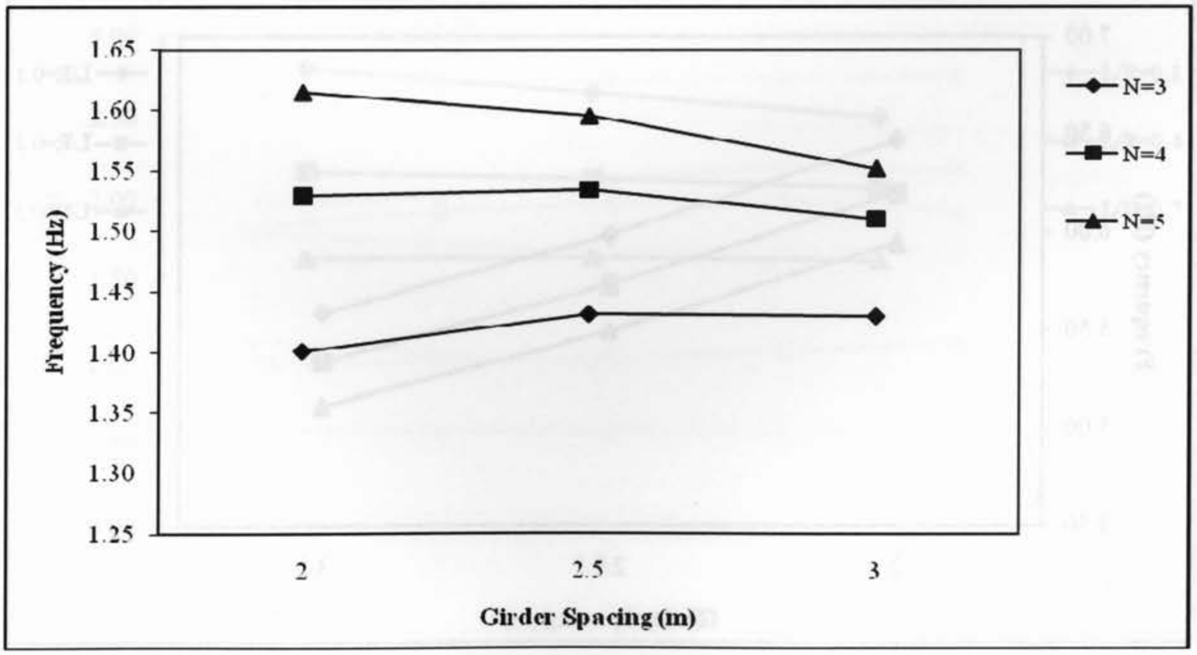


Figure 4.99 Effect of Girder Spacing on the Fundamental Frequency of 45m Long Curved Bridges with Differing Number of Girders (L=45, L/R=0.9)

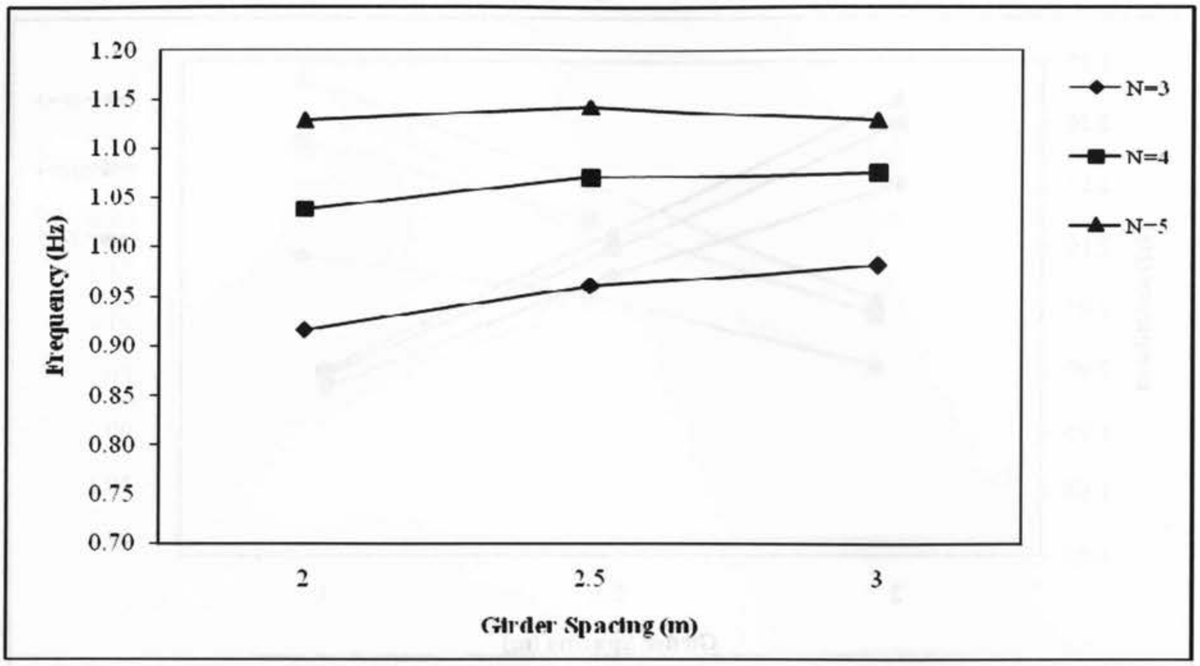


Figure 4.100 Effect of Girder Spacing on the Fundamental Frequency of Curved 2-Span Bridges with Differing Curvature Ratios (L=15, N=3)

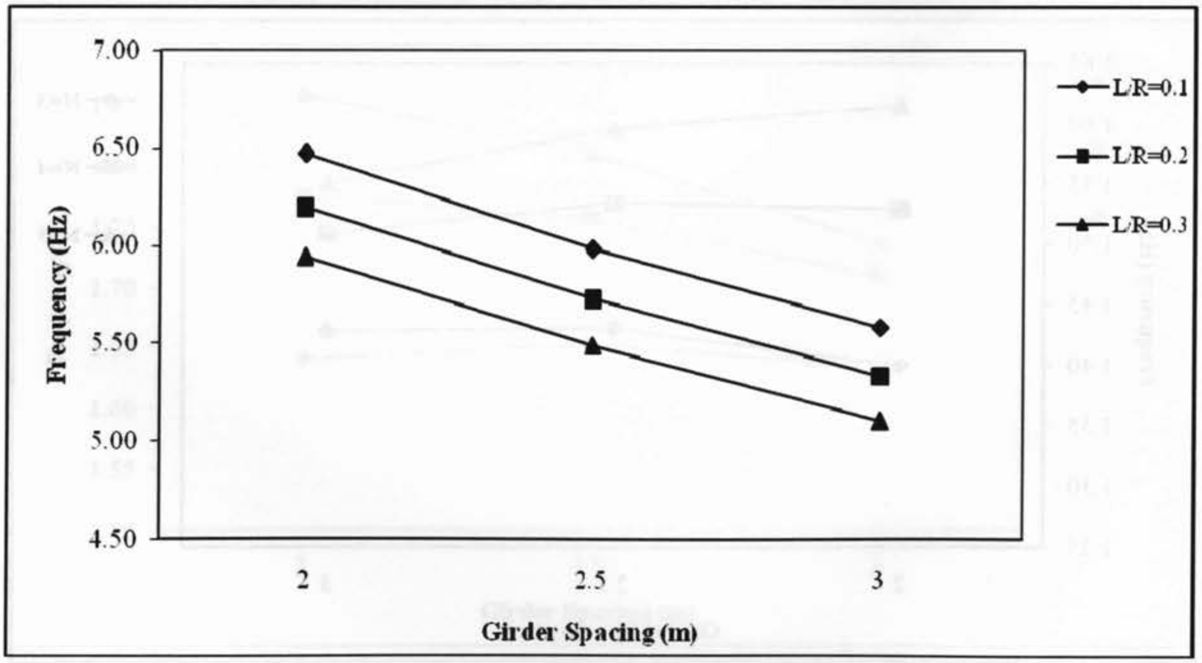


Figure 4.101 Effect of Girder Spacing on the Fundamental Frequency of Curved 2- Span Bridges with Differing Curvature Ratios (L=25, N=3)

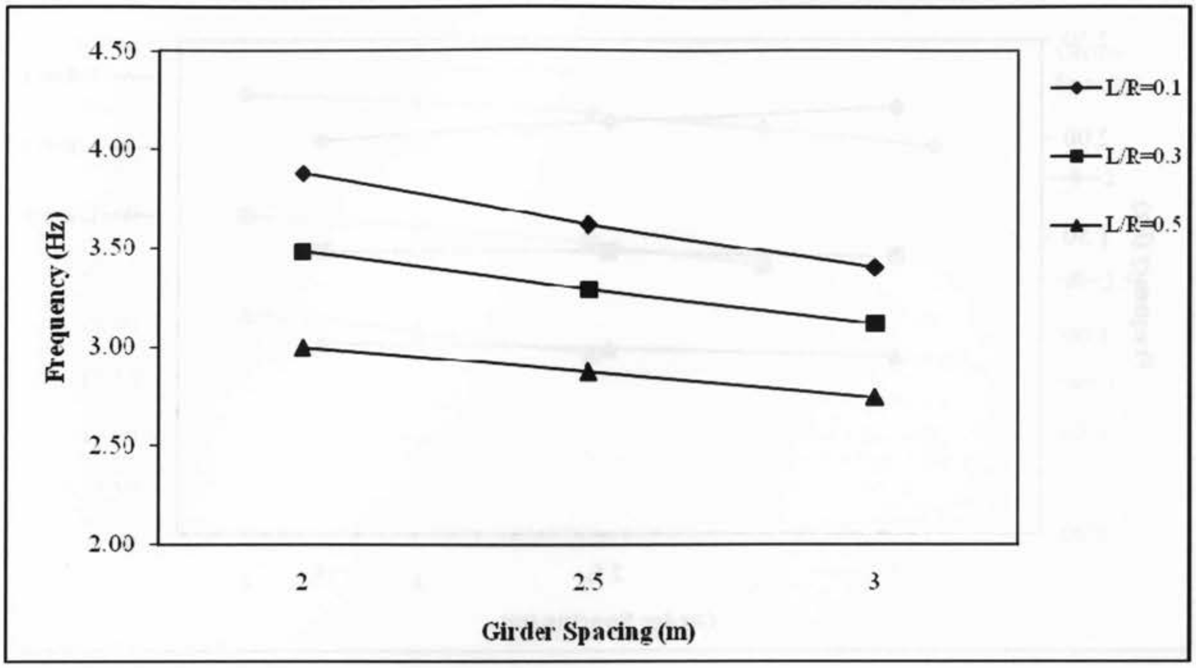


Figure 4.102 Effect of Girder Spacing on the Fundamental Frequency of Curved 2-Span Bridges with Differing Curvature Ratios (L=35, N=3)

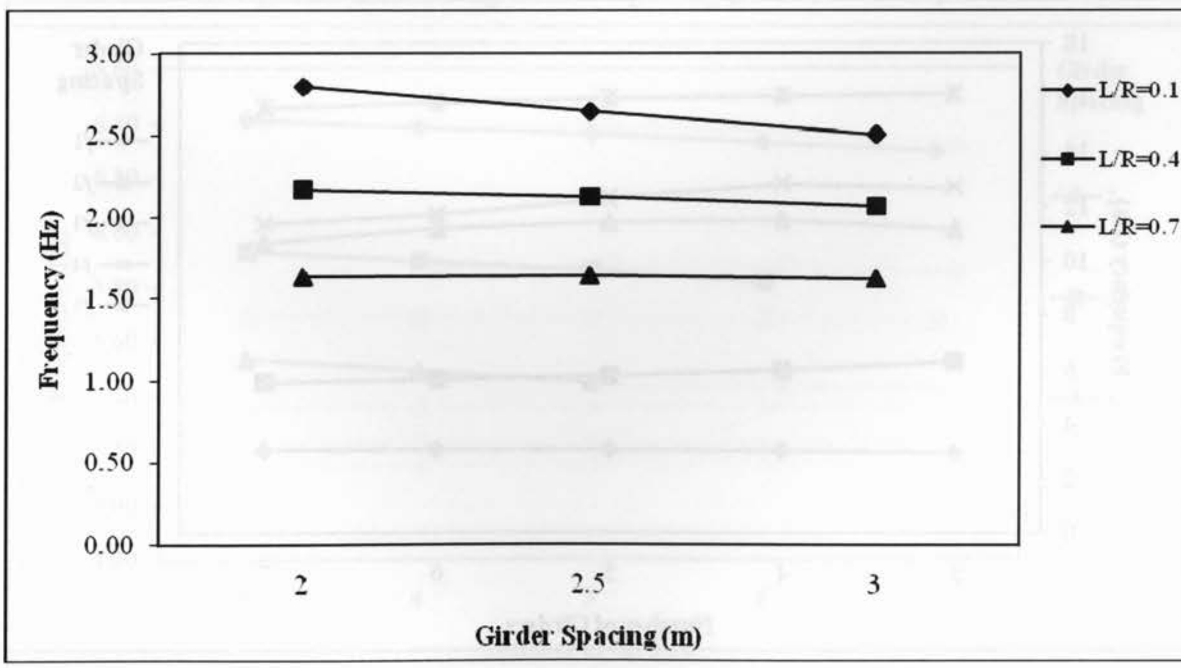


Figure 4.103 Effect of Girder Spacing on the Fundamental Frequency of Curved 2-Span Bridges with Differing Curvature Ratios (L=45, N=3)

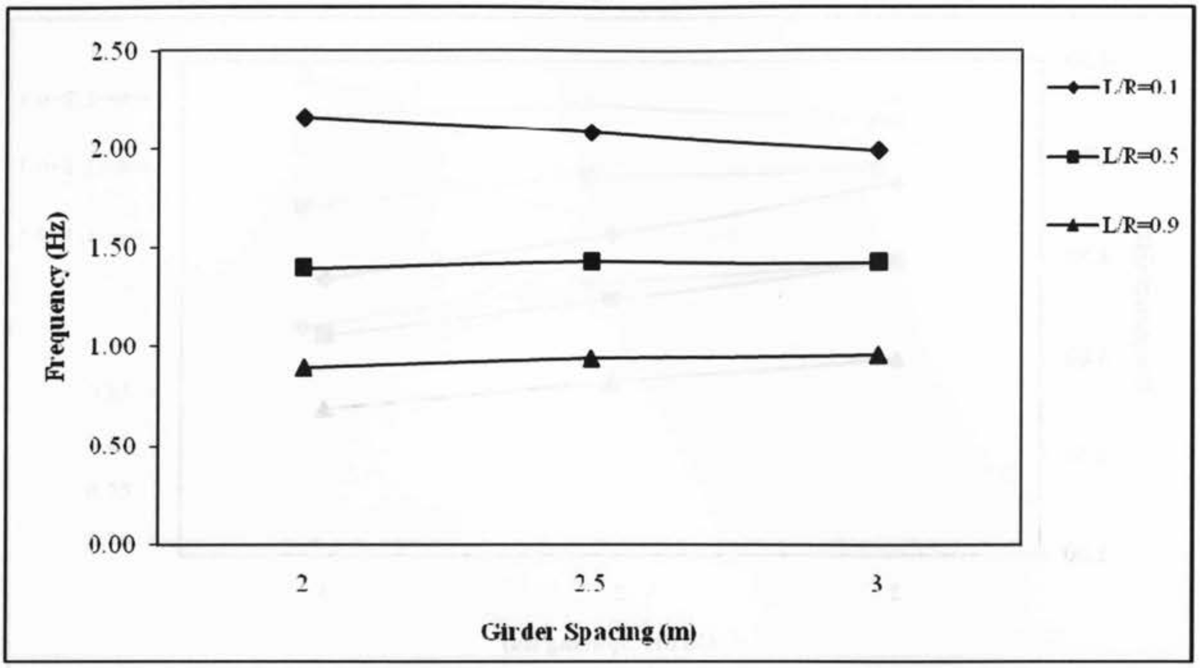


Figure 4.104 Effect of Number of Girders on the First Five Frequencies of 25m Long Curved Bridges (L=25, S=2.0, L/R=0.5)

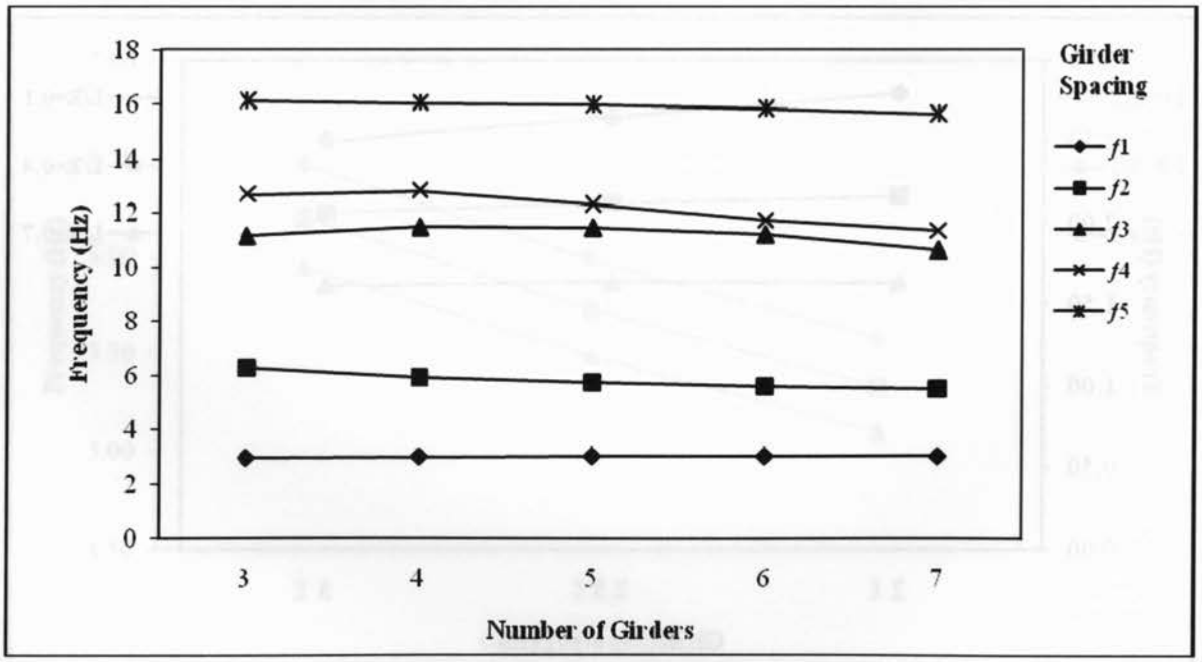


Figure 4.105 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (L=10, L/R=0.1)

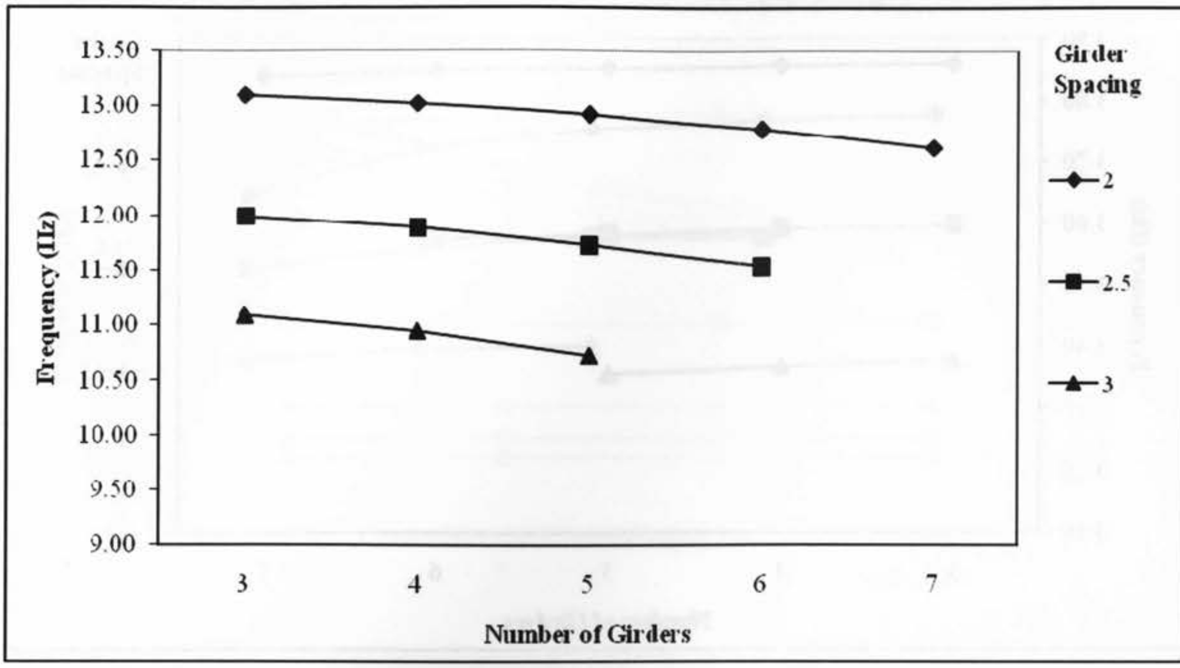


Figure 4.106 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (L=15, L/R=0.1)

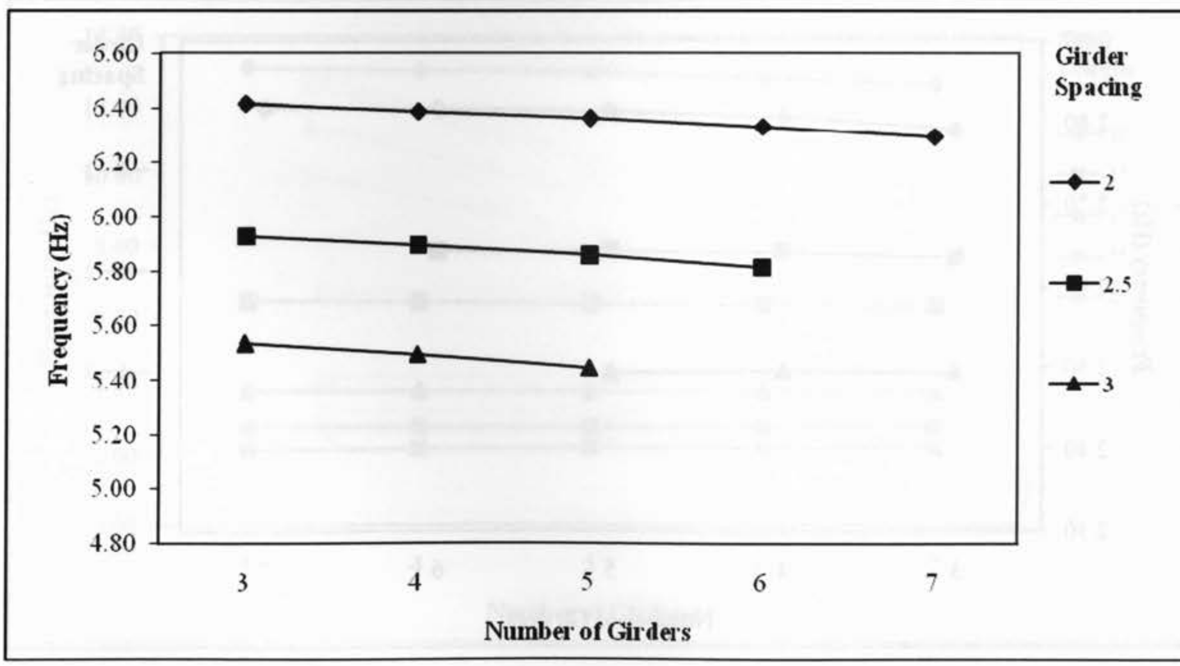


Figure 4.107 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (L=25, L/R=0.1)

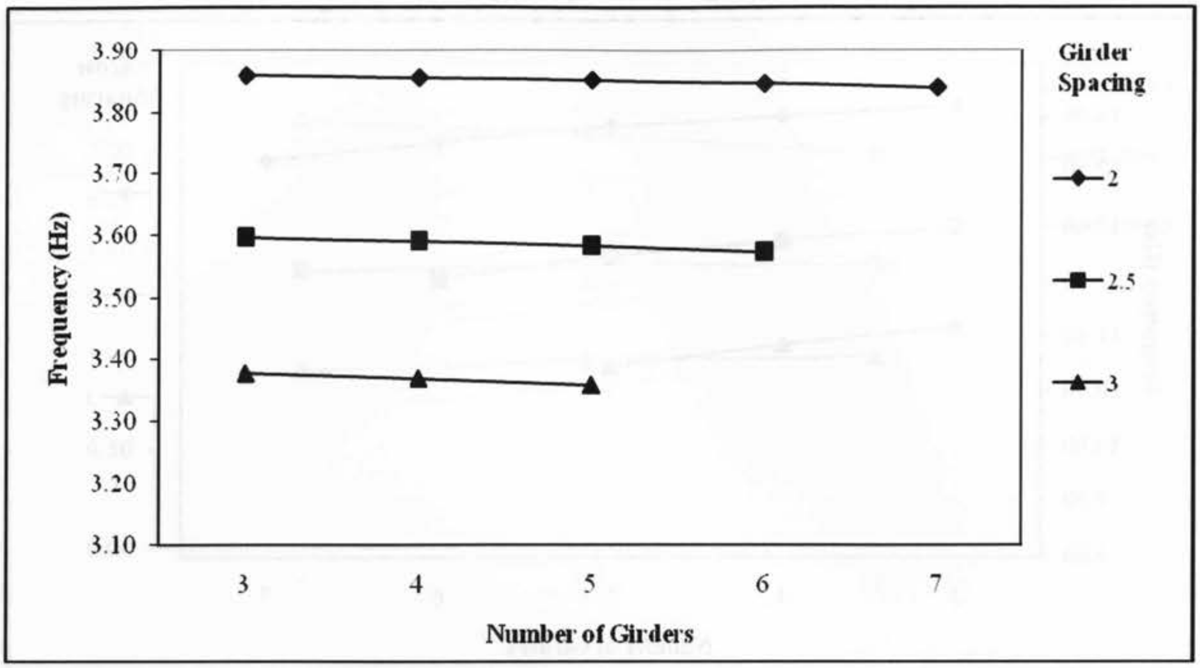


Figure 4.108 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (L=35, L/R=0.1)

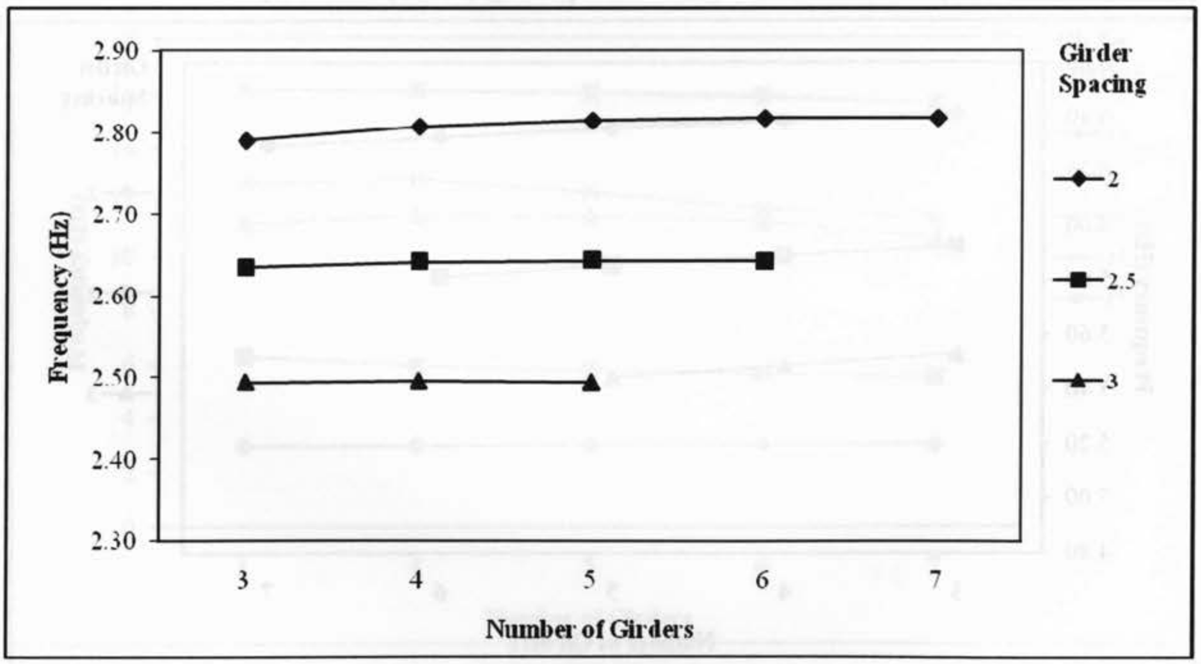


Figure 4.109 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Girder Spacing (L=45, L/R=0.1)

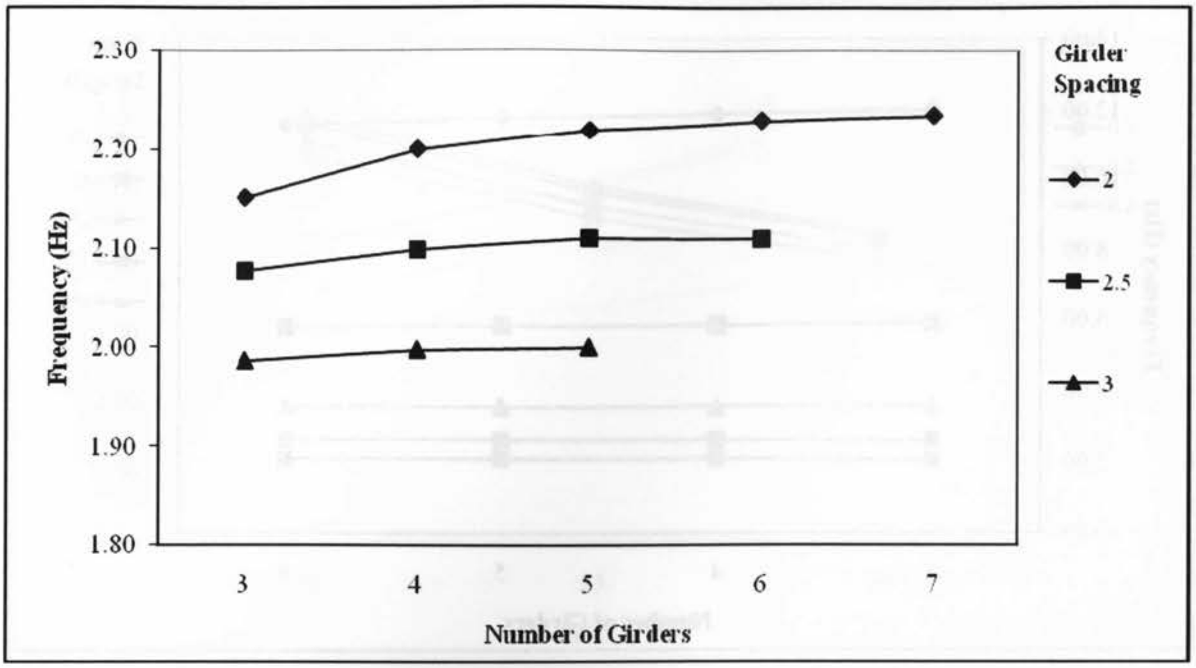


Figure 4.110 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Span Lengths (S=2.0, L/R=0.1)

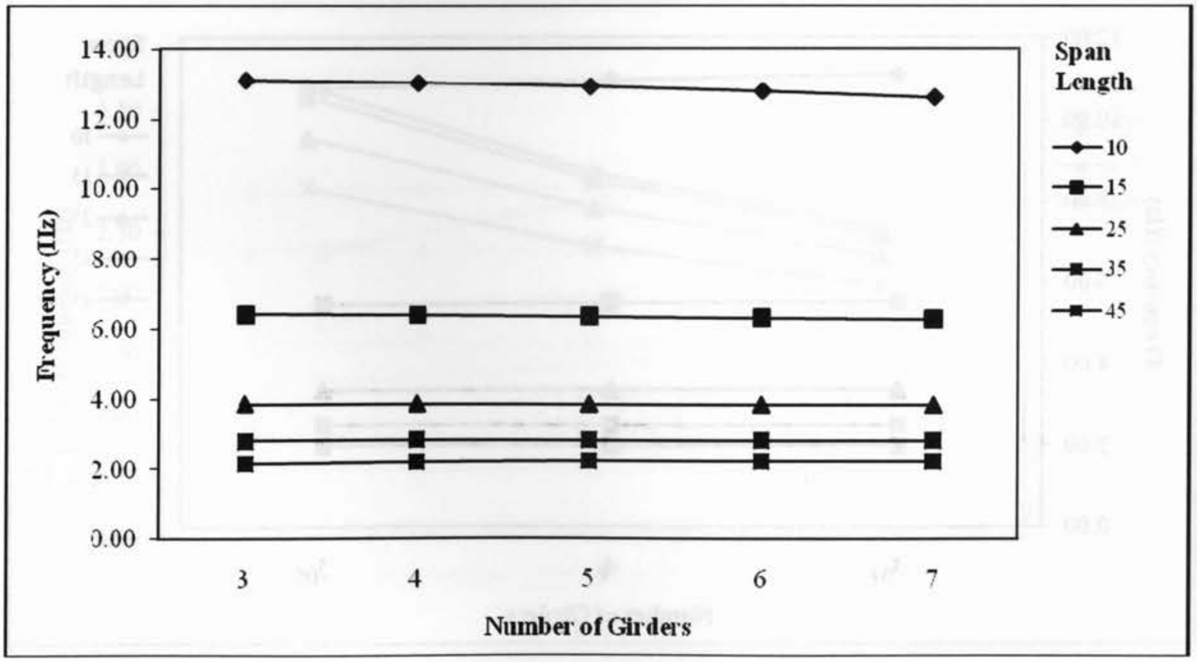


Figure 4.111 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Span Lengths (S=2.5 L/R=0.1)

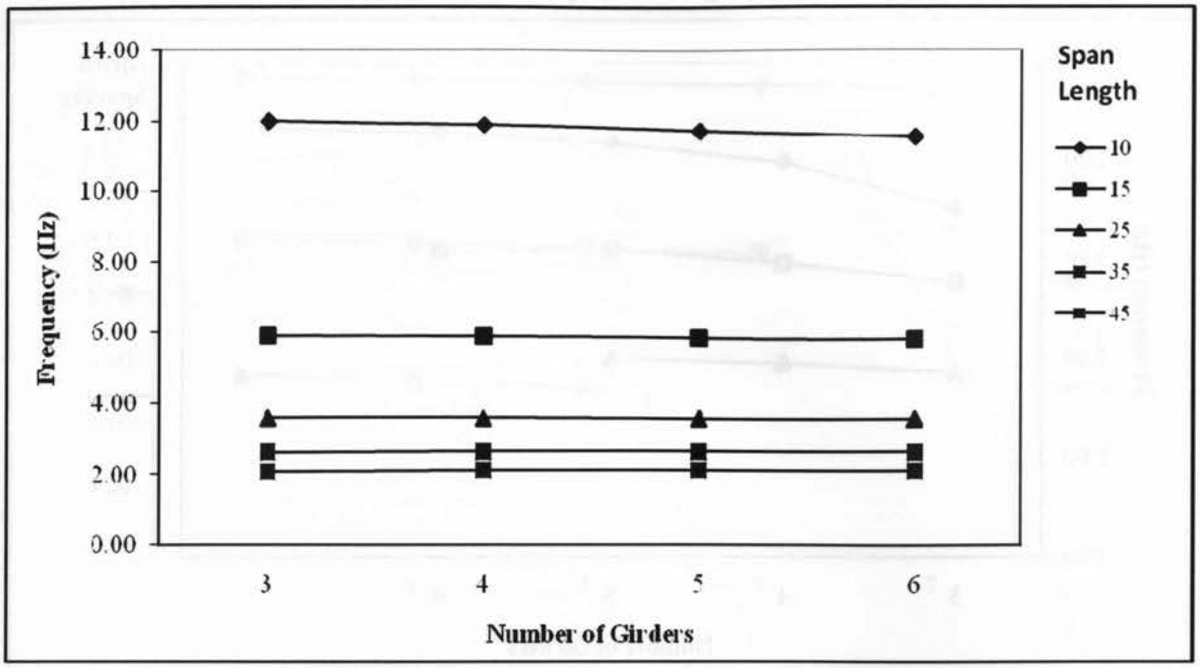


Figure 4.112 Effect of Number of Girders on the Fundamental Frequencies of Curved Bridges with Differing Span Lengths (S=3.0 L/R=0.1)

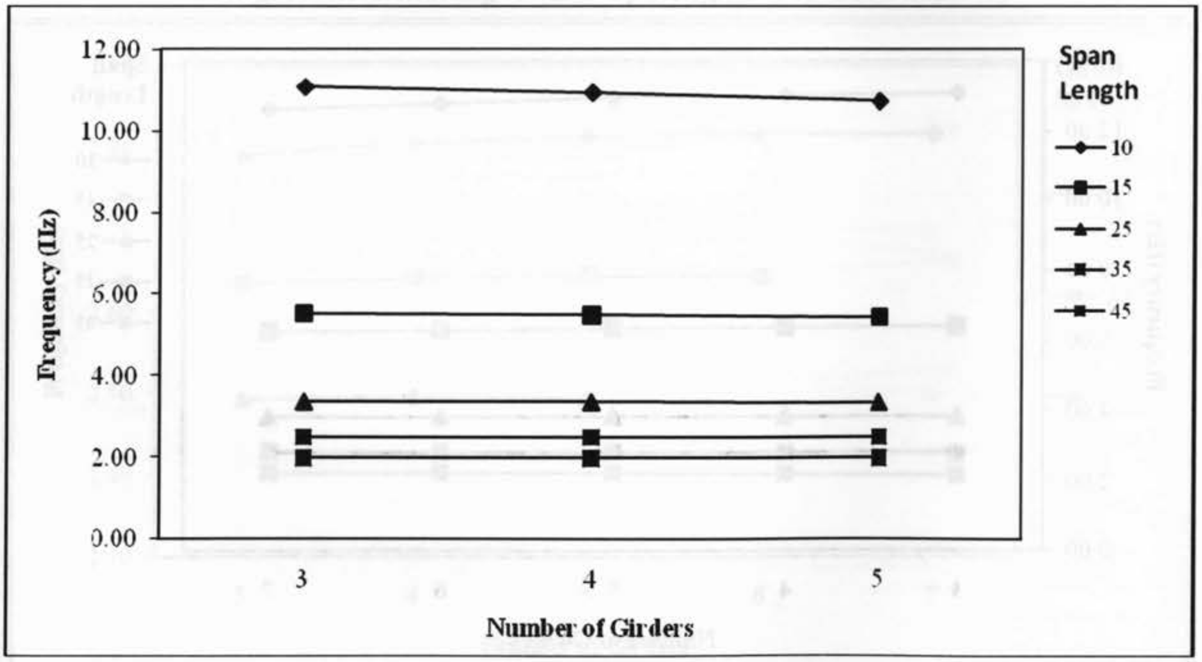


Figure 4.113 Effect of Girder Depth on the Fundamental Frequencies of Curved Bridges with Differing Curvature Ratios (L=15, S=2.5, N=3)

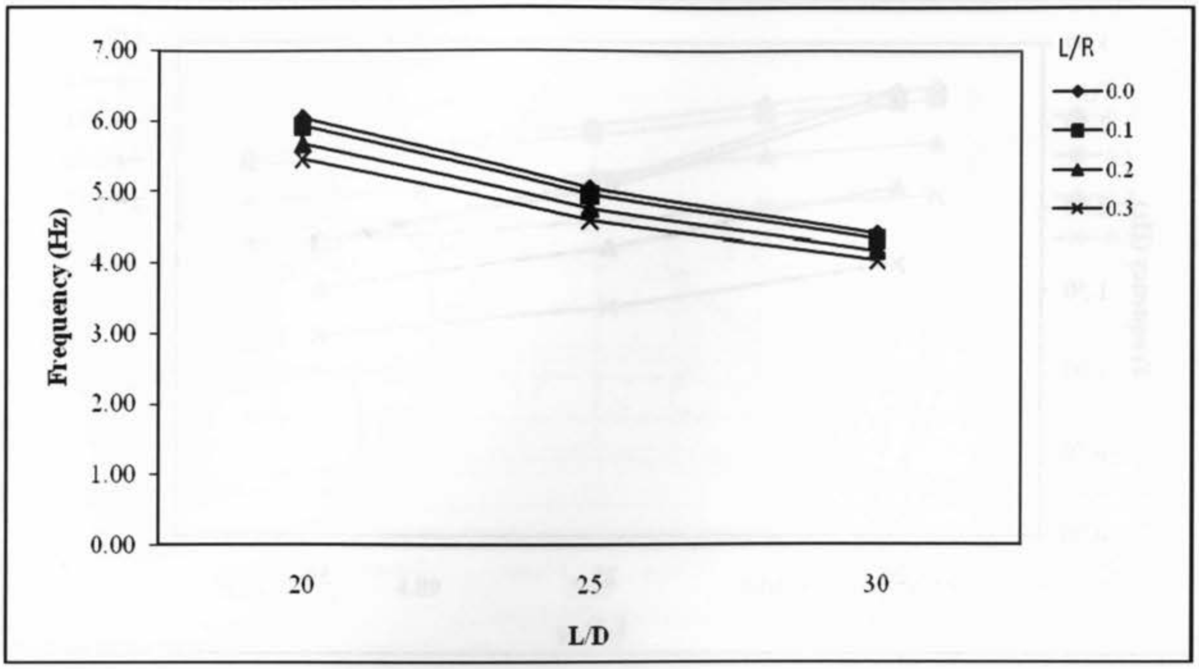


Figure 4.114 Effect of Girder Depth on the Fundamental Frequencies of Curved Bridges with Differing Curvature Ratios (L=25, S=2.5, N=3)

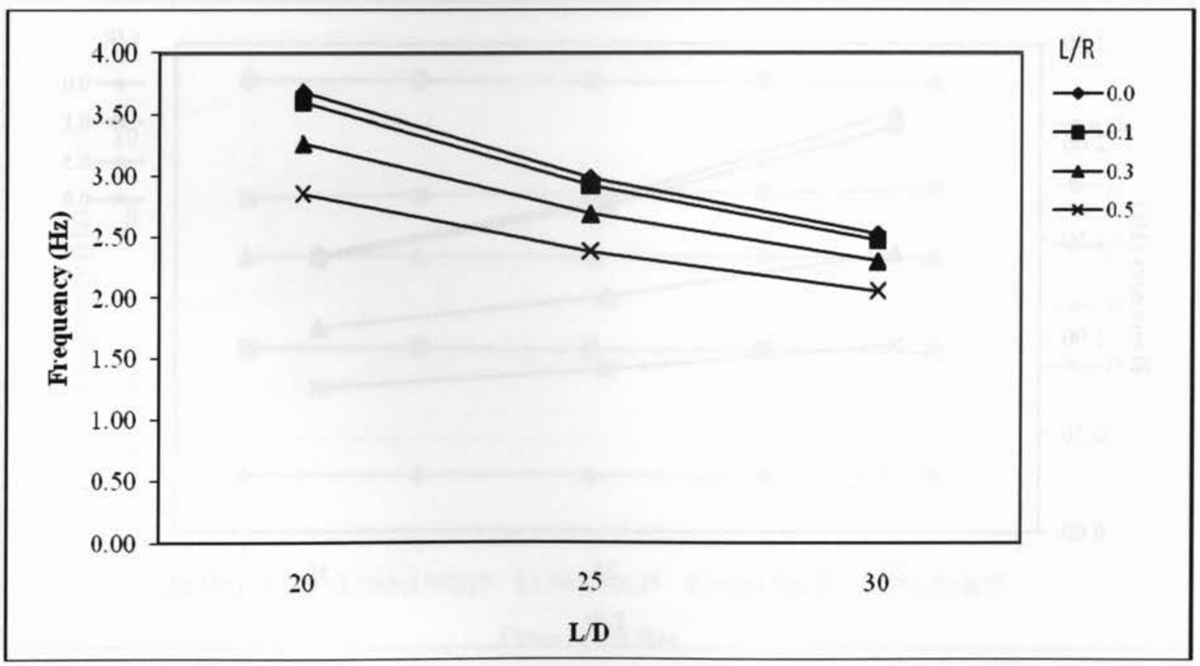


Figure 4.115 Effect of Girder Depth on the Fundamental Frequencies of Curved Bridges with Differing Curvature Ratios (L=35, S=2.5, N=3)

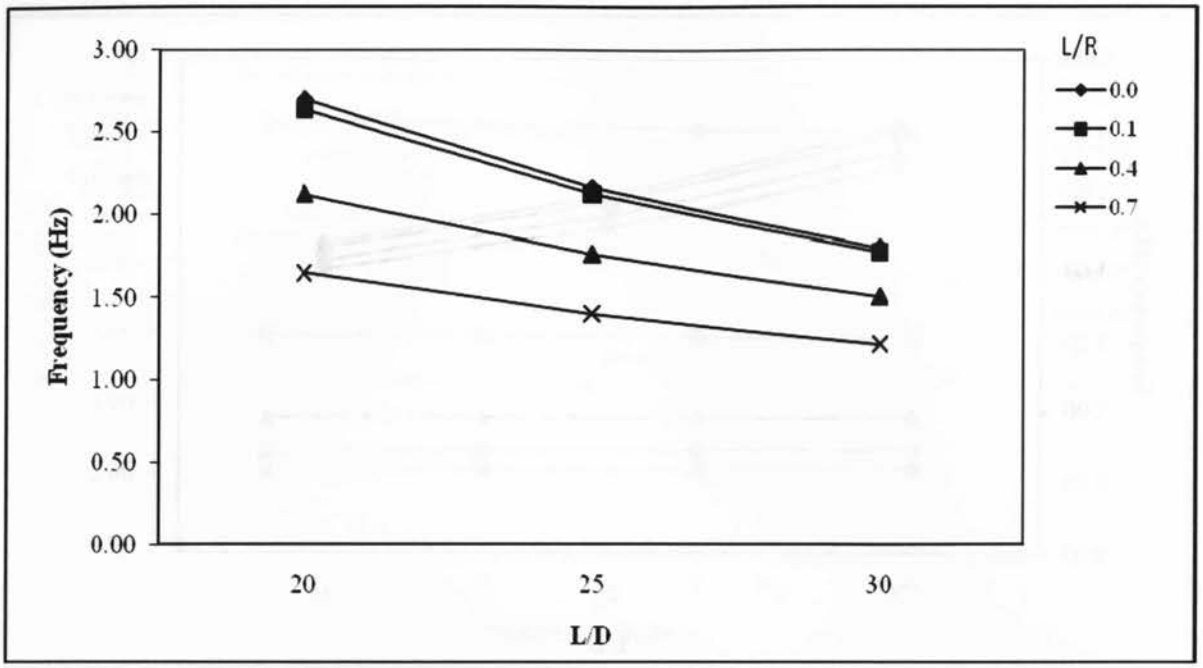


Figure 4.116 Effect of Girder Depth on the Fundamental Frequencies of Curved Bridges with Differing Curvature Ratios (L=45, S=2.5, N=3)

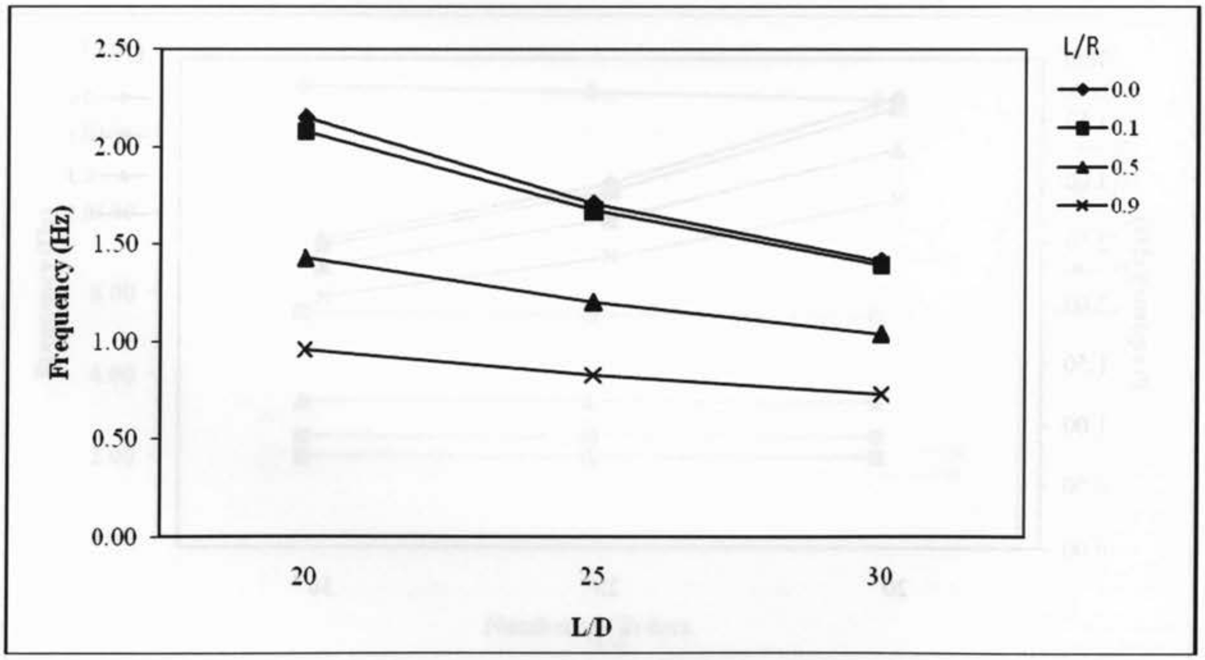


Figure 4.117 Effect of Bottom Flange Thickness on the Fundamental Frequency of Curved Bridges (L=25, S=2.5, N=3)

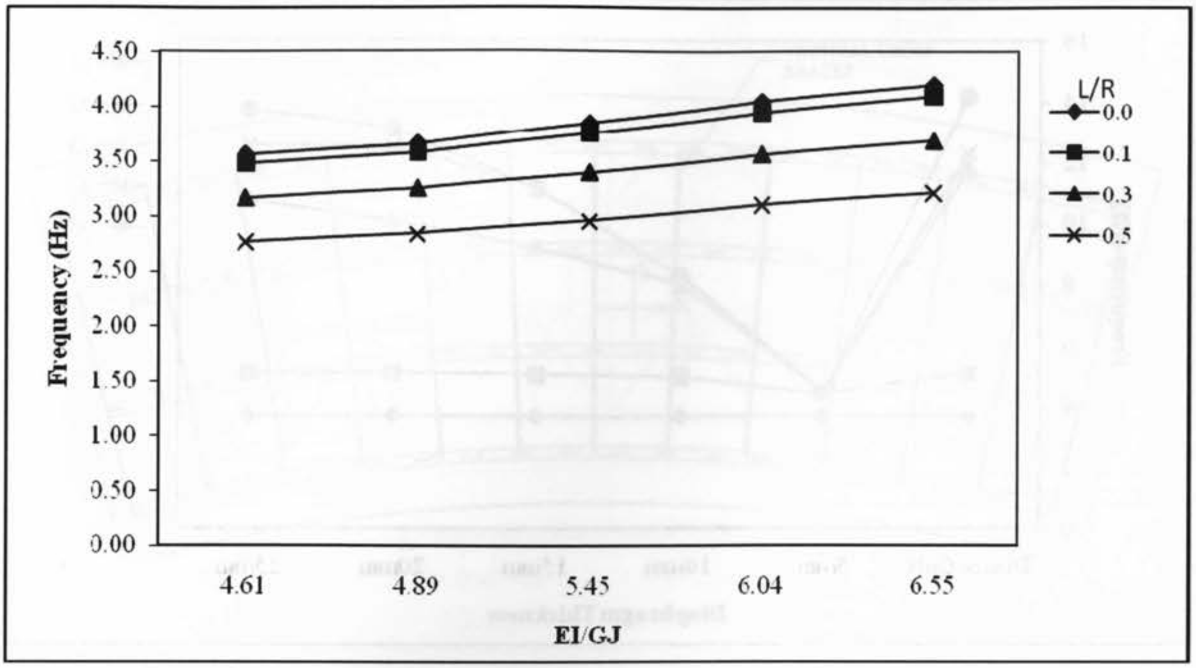


Figure 4.118 Effect of Cross-Bracing Cross-Sectional Area on the Fundamental Frequency of Curved Bridges (L=35, S=2.5, N=3, L/R=0.7)

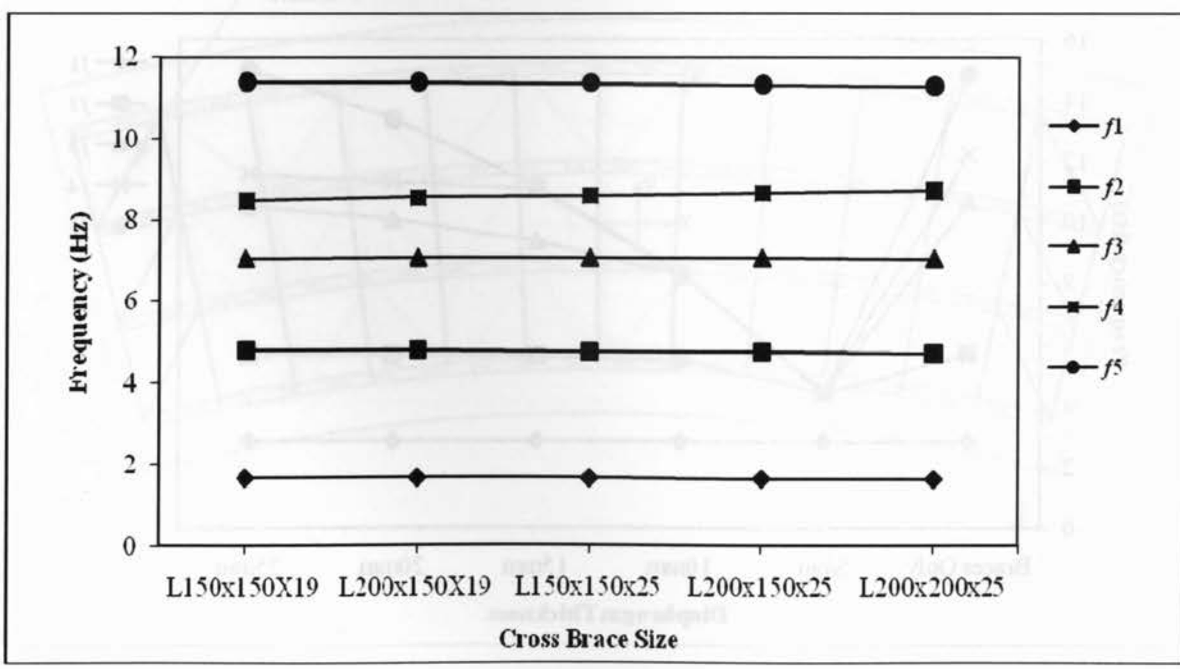


Figure 4.119 Effect of Solid-Plate End Diaphragm Thickness on the Natural Frequencies of Straight Bridges (L=25, S=2.5, N=3, L/R=0)

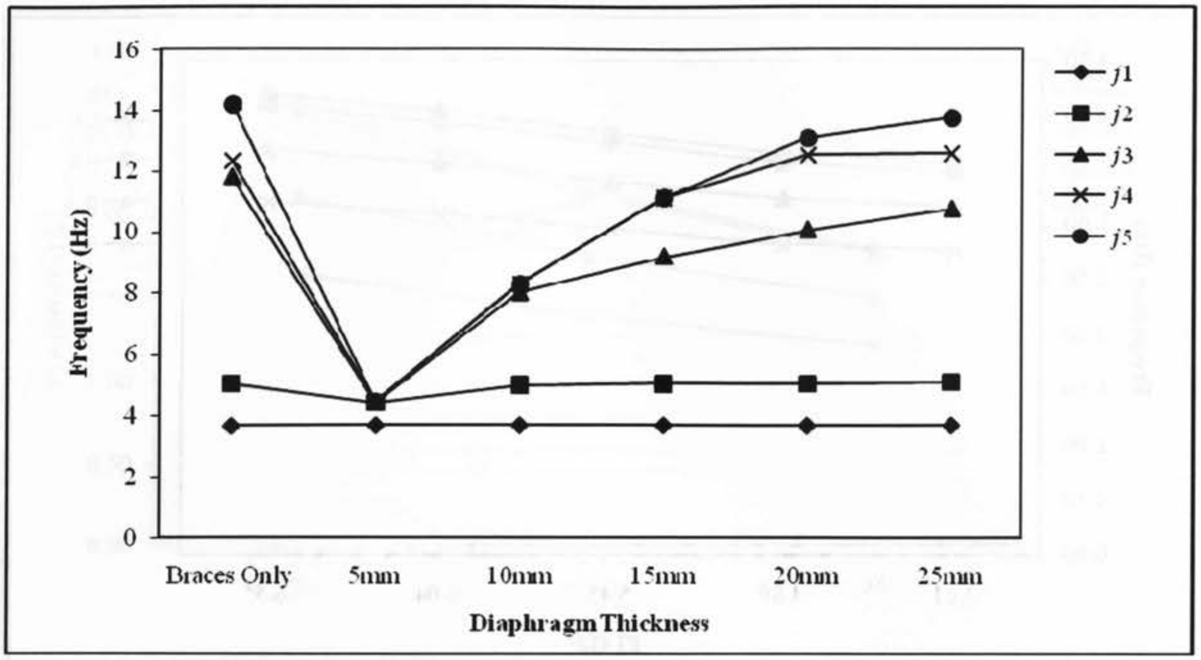


Figure 4.120 Effect of Solid-Plate End Diaphragm Thickness on the Natural Frequencies of Curved Bridges (L=25, S=2.5, N=3, L/R=0.5)

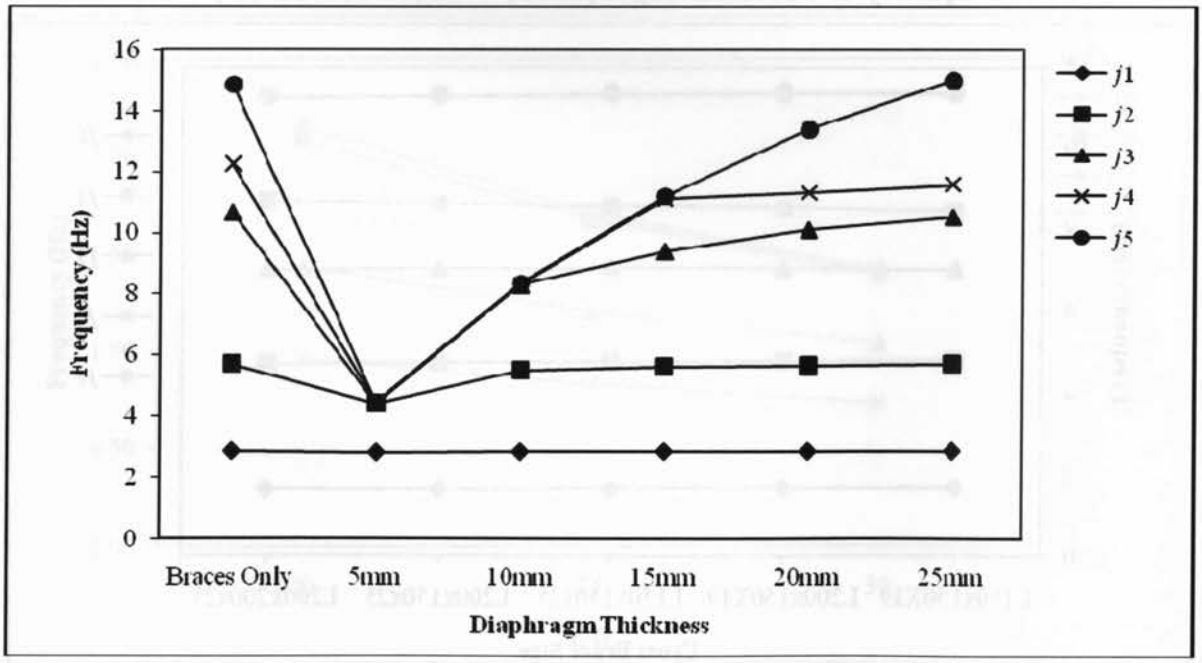


Figure 4.121 Horizontal Bracing - Vertical Bracing Only

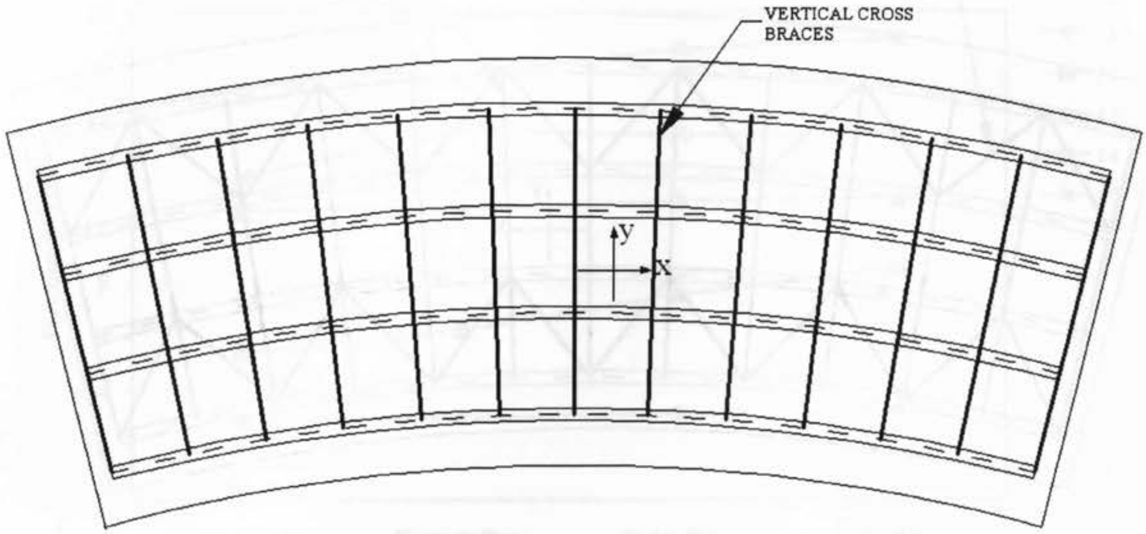


Figure 4.122 Horizontal Bracing - Torsion Box

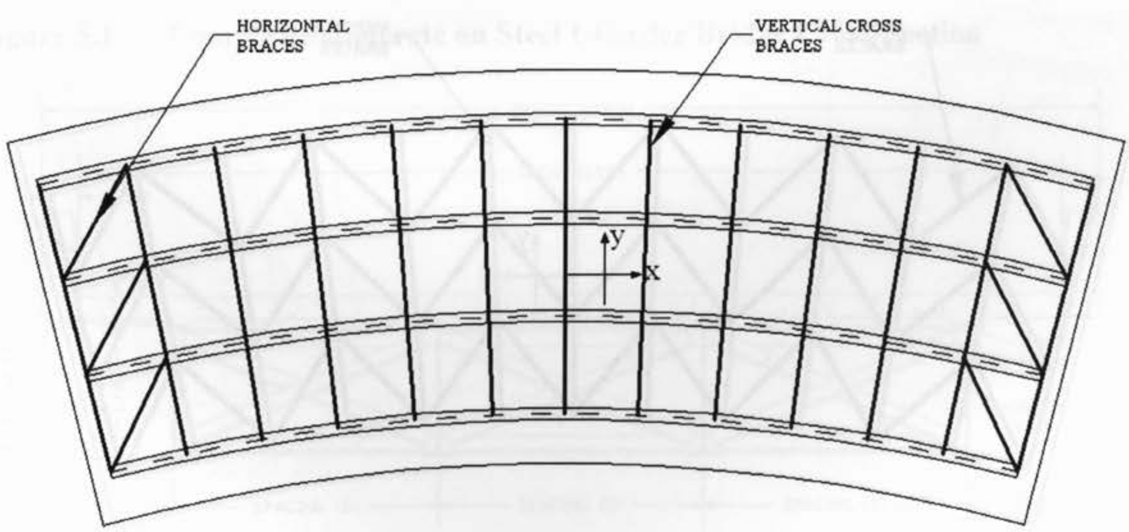


Figure 4.123 Horizontal Bracing - Outer Bays

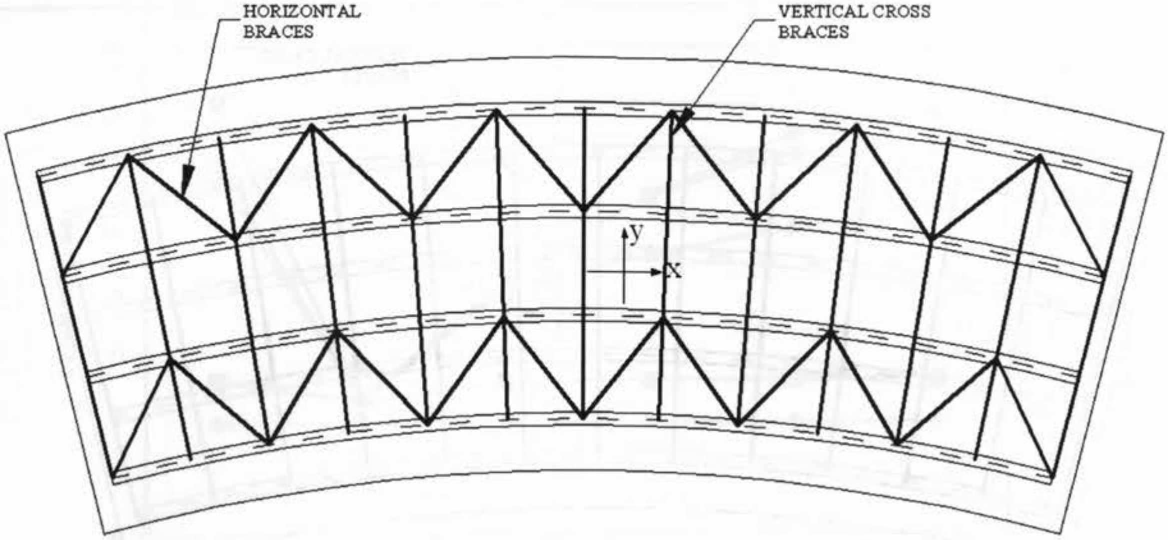


Figure 4.124 Horizontal Bracing - All Bays

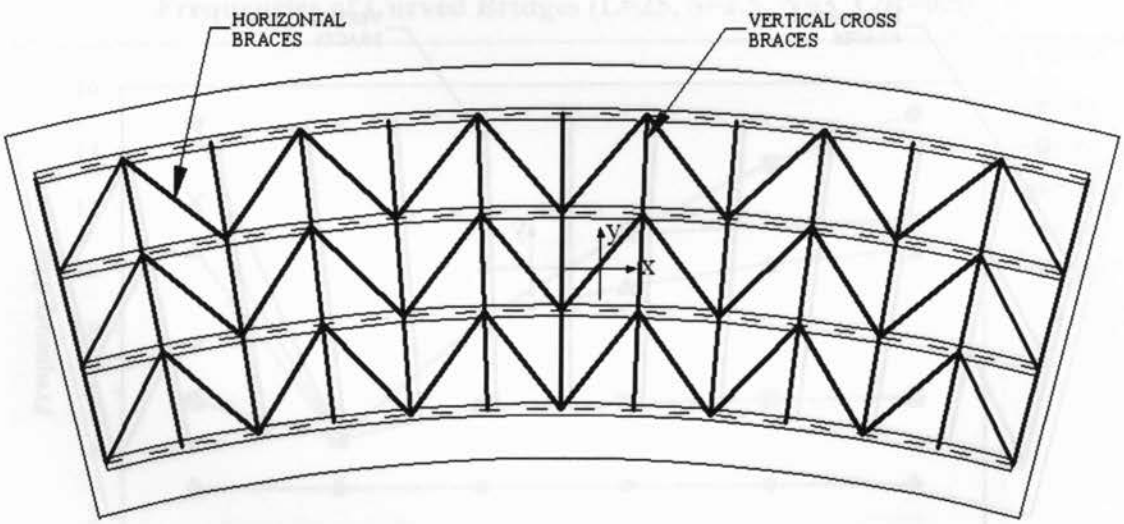


Figure 4.125 Effect of Horizontal Bracing on the First Five Frequencies of Curved Bridges (L=25, S=2.5, N=4, L/R=0.5)

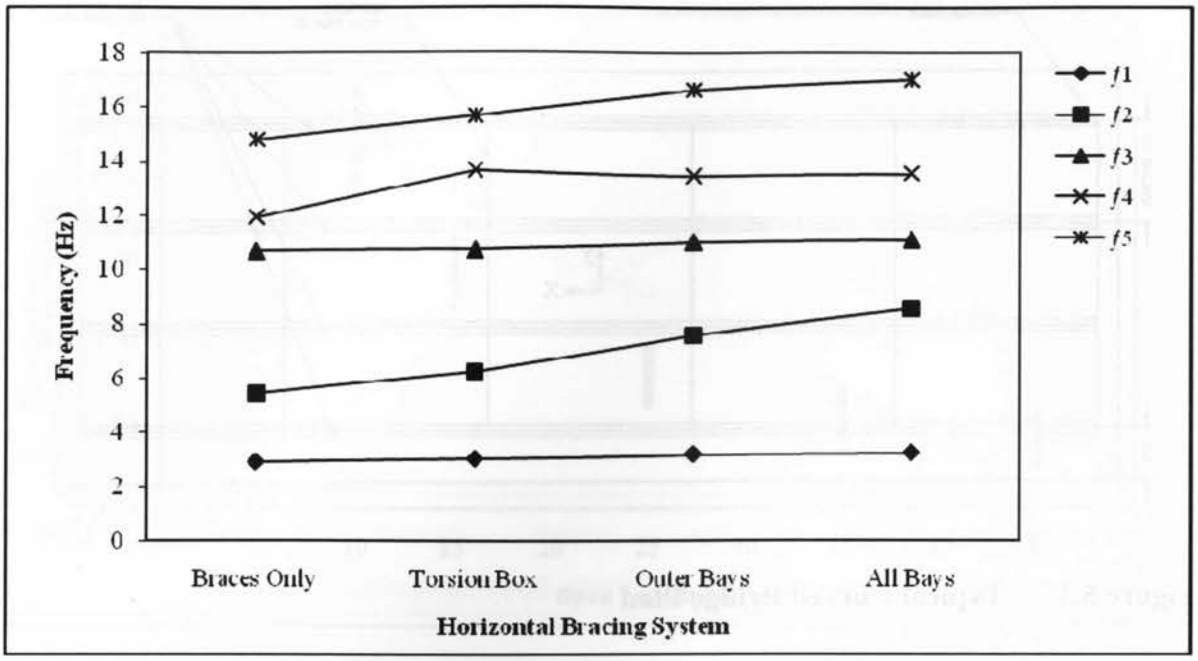


Figure 5.1 Composite Concrete on Steel I-Girder Bridge Cross-Section

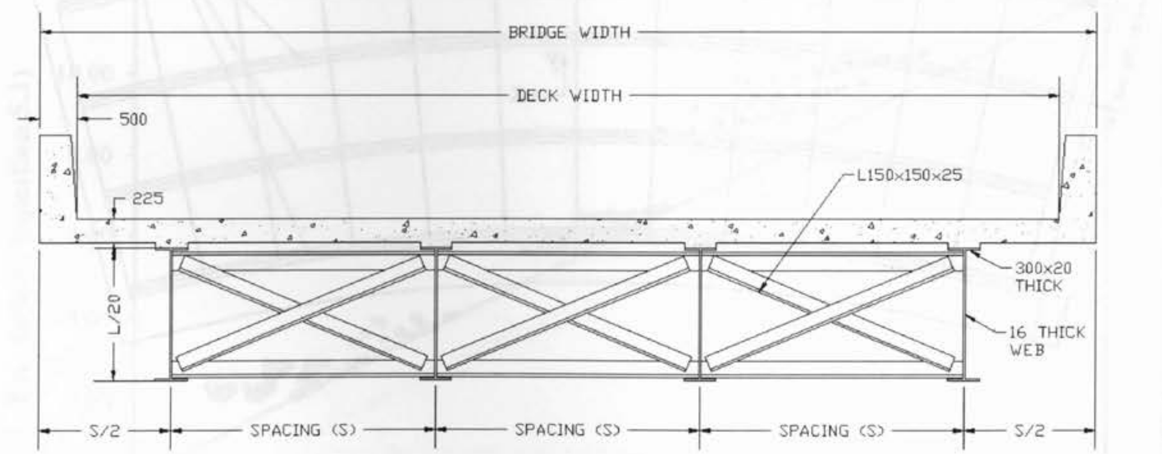


Figure 5.2 Typical Straight Bridge Plan

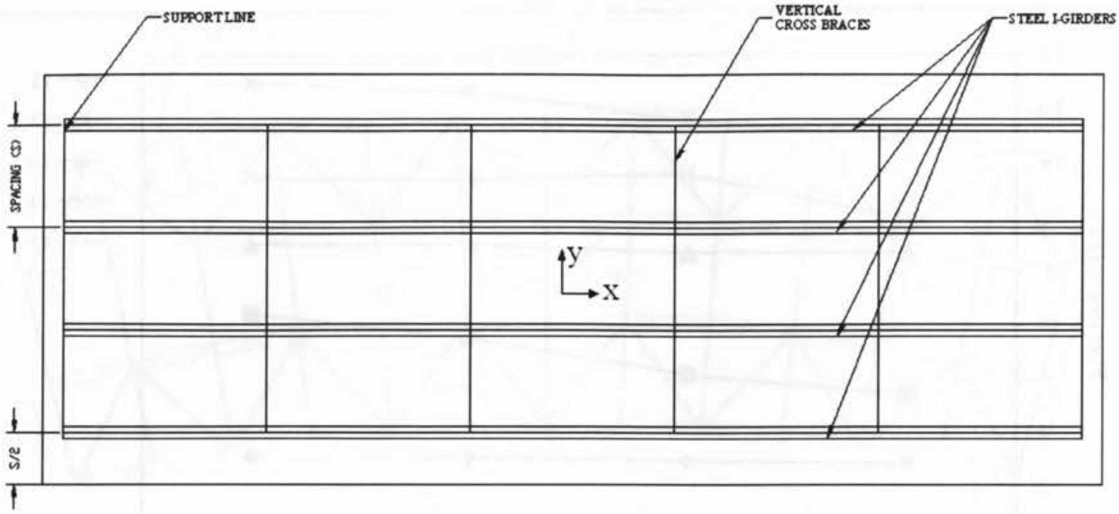


Figure 5.3 Typical Curved Bridge Plan

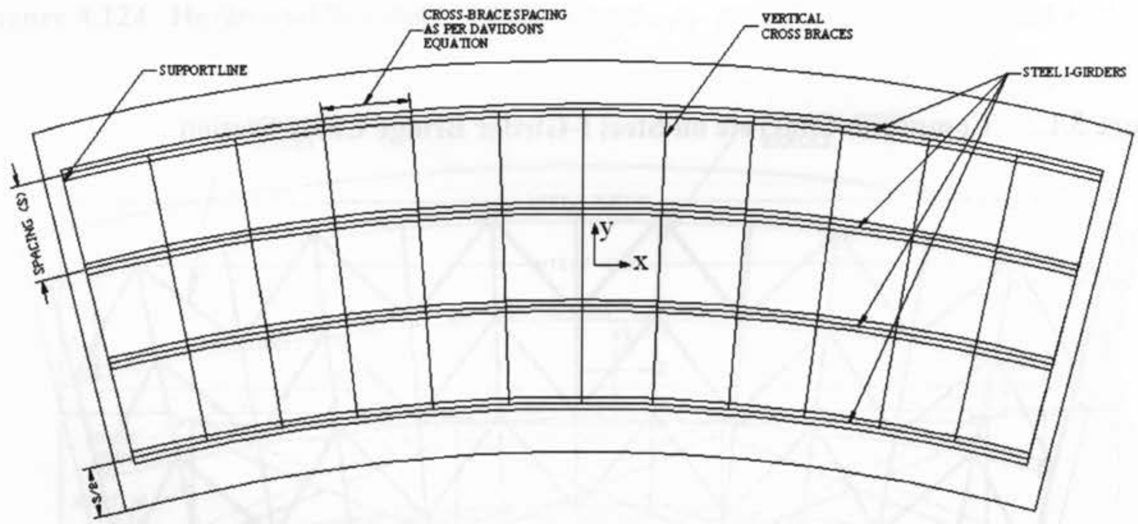


Figure 5.4 Finite Element Analysis Fundamental Frequency vs. Span Length

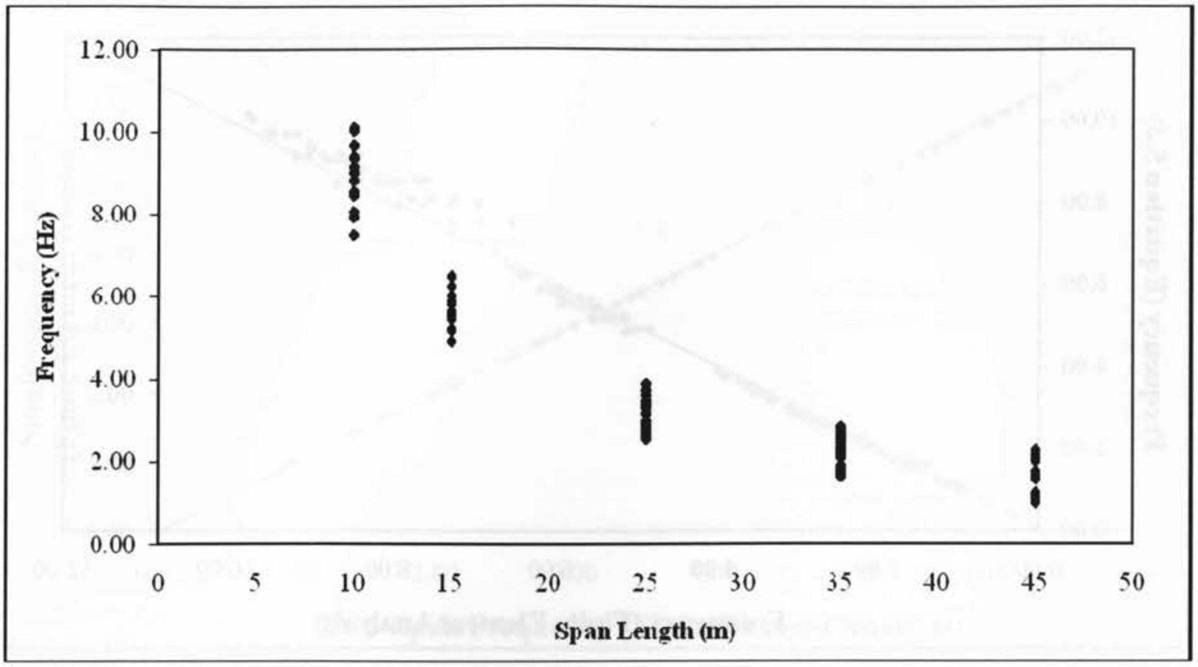


Figure 5.5 Fundamental Frequency (Simple Straight Beam - Equation 5.1) vs. Frequency (FEA)

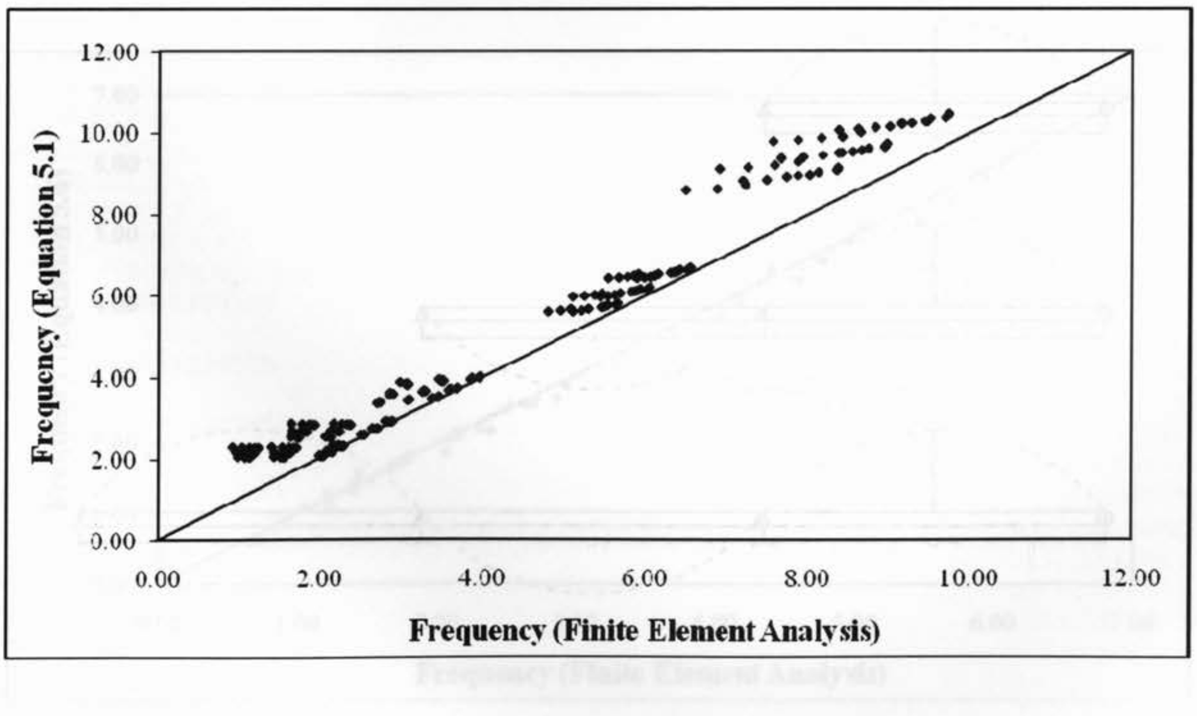


Figure 5.6 Fundamental Frequency (Equation 5.3) vs. Frequency (FEA)

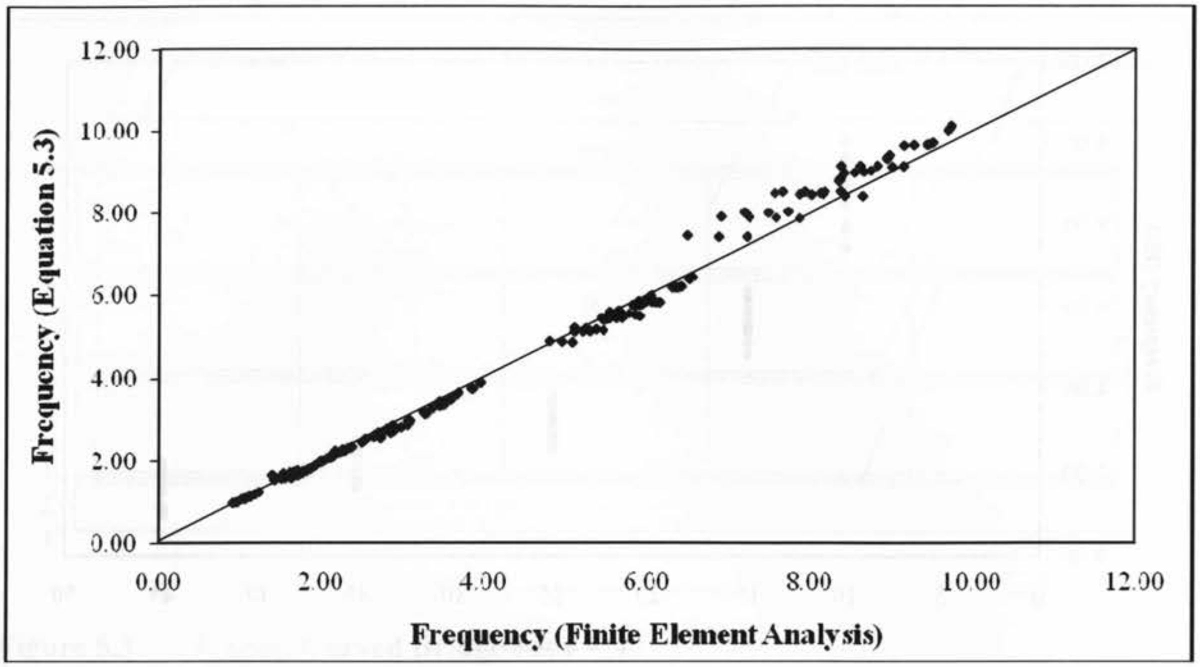


Figure 5.7 Fundamental Mode Shapes of Multi-Span Bridges

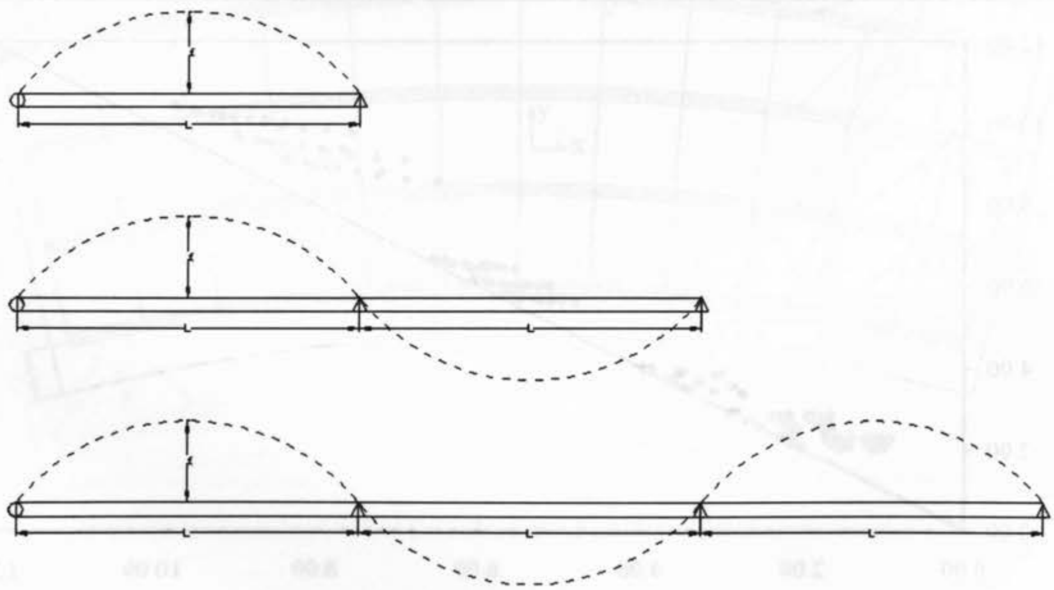


Figure 5.8 Single-Span Frequency (FEA) vs. Two-Span Frequency (FEA)

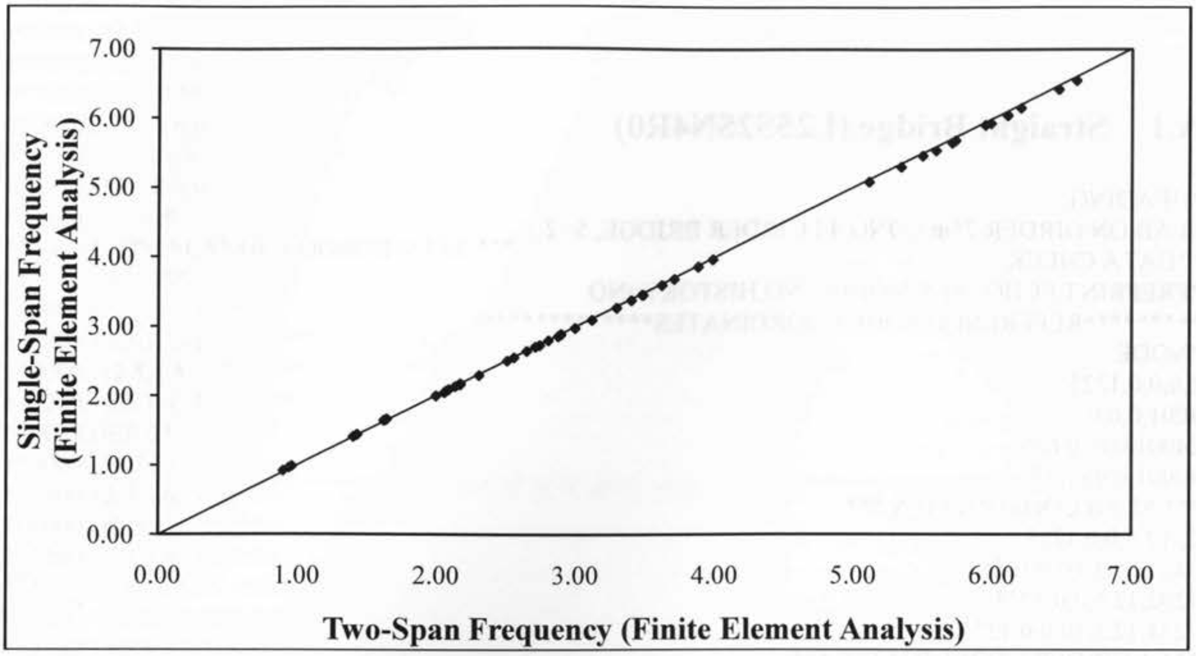
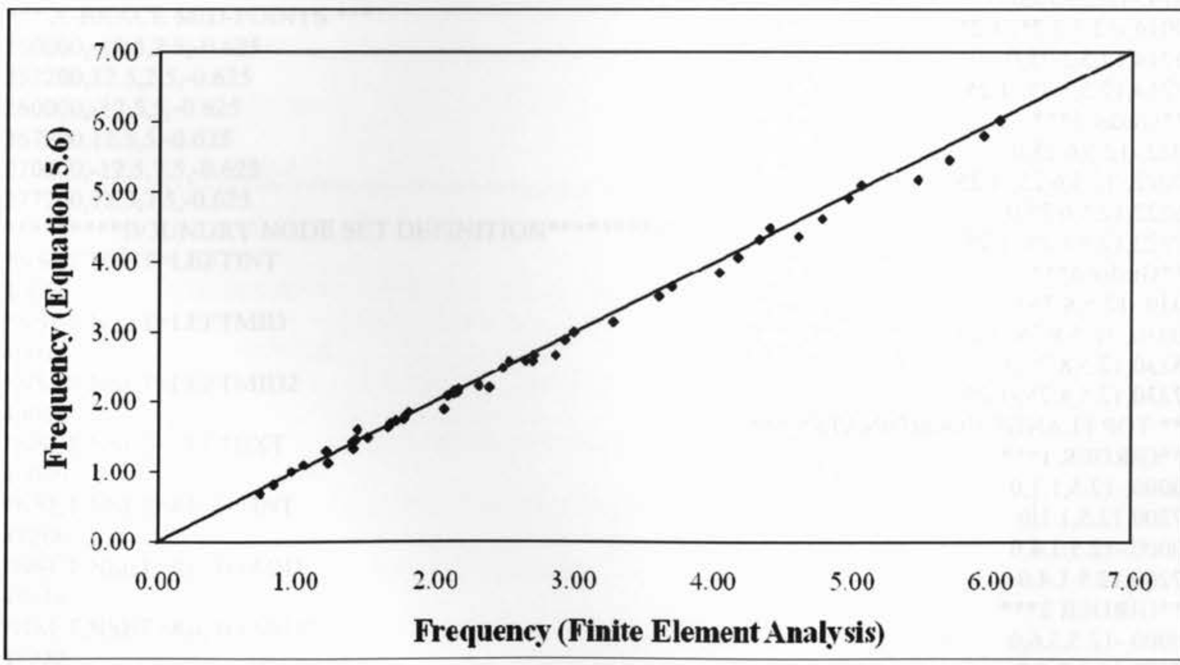


Figure 5.9 Fundamental Frequency (Equation 5.6) vs. Frequency (FEA)



APPENDEX (A): ABAQUS INPUT FILES

A.1 Straight Bridge (L25S25N4R0)

```
*HEADING
SLAB ON GIRDER 25m LONG 4 I GIRDER BRIDGE, S=2.5
**DATA CHECK
*PREPRINT,ECHO=YES,MODEL=NO,HISTORY=NO
*********REFERENCE NODE COORDINATES*************
*NODE
1,0,0,0.1225
8001,0,0,0
24001,0,0,-0.625
40001,0,0,-1.25
*** SLAB COORDINATES ***
2,-12.5,0,0.1225
34,-12.5,10.0,0.1225
7202,12.5,0,0.1225
7234,12.5,10.0,0.1225
*** 4-GIRDER COORDINATES ***
***Girder 1***
8006,-12.5,1.25,0
40006,-12.5,1.25,-1.25
15206,12.5,1.25,0
47206,12.5,1.25,-1.25
***Girder 2***
8014,-12.5,3.75,0
40014,-12.5,3.75,-1.25
15214,12.5,3.75,0
47214,12.5,3.75,-1.25
***Girder 3***
8022,-12.5,6.25,0
40022,-12.5,6.25,-1.25
15222,12.5,6.25,0
47222,12.5,6.25,-1.25
***Girder 4***
8030,-12.5,8.75,0
40030,-12.5,8.75,-1.25
15230,12.5,8.75,0
47230,12.5,8.75,-1.25
*** TOP FLANGE COORDINATES ***
***GIRDER 1***
50000,-12.5,1.1,0
57200,12.5,1.1,0
60000,-12.5,1.4,0
67200,12.5,1.4,0
***GIRDER 2***
70000,-12.5,3.6,0
77200,12.5,3.6,0
80000,-12.5,3.9,0
87200,12.5,3.9,0
***GIRDER 3***
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90000,-12.5,6.1,0
97200,12.5,6.1,0
100000,-12.5,6.4,0
107200,12.5,6.4,0
GIRDER 4
110000,-12.5,8.6,0
117200,12.5,8.6,0
120000,-12.5,8.9,0
127200,12.5,8.9,0
*** BOTTOM FLANGE COORDINATES ***
GIRDER 1
130000,-12.5,1.1,-1.25
137200,12.5,1.1,-1.25
140000,-12.5,1.4,-1.25
147200,12.5,1.4,-1.25
GIRDER 2
150000,-12.5,3.6,-1.25
157200,12.5,3.6,-1.25
160000,-12.5,3.9,-1.25
167200,12.5,3.9,-1.25
GIRDER 3
170000,-12.5,6.1,-1.25
177200,12.5,6.1,-1.25
180000,-12.5,6.4,-1.25
187200,12.5,6.4,-1.25
GIRDER 4
190000,-12.5,8.6,-1.25
197200,12.5,8.6,-1.25
200000,-12.5,8.9,-1.25
207200,12.5,8.9,-1.25
*** X-BRACE MID-POINTS ***
250000,-12.5,2.5,-0.625
257200,12.5,2.5,-0.625
260000,-12.5,5,-0.625
267200,12.5,5,-0.625
270000,-12.5,7.5,-0.625
277200,12.5,7.5,-0.625
*********BOUNDRY NODE SET DEFINITION************
*NSET,NSET=LEFTINT
40006
*NSET,NSET=LEFTMID
40014
*NSET,NSET=LEFTMID2
40022
*NSET,NSET=LEFTEXT
40030
*NSET,NSET=RIGHTINT
47206
*NSET,NSET=RIGHTMID
47214
*NSET,NSET=RIGHTMID2
47222
*NSET,NSET=RIGHTEXT
47230
*NSET,NSET=REACT
LEFTINT,LEFTMID,LEFTMID2,LEFTEXT,RIGHTINT,RIGHTMID,RIGHTMID2,RIGHTEXT

```
*********BASIC GEOMETRY NODE GENERATION************
*NGEN,NSET=ORGIN
8001,40001,8000
***LEFT END***
*NGEN,NSET=LEND
2,34,2
8006,40006,8000
8014,40014,8000
8022,40022,8000
8030,40030,8000
***RIGHT END***
*NGEN,NSET=REND
7202,7234,2
15206,47206,8000
15214,47214,8000
15222,47222,8000
15230,47230,8000
********************************NODE GEN. FOR TOP SLAB**********************
*NGEN,NSET=NSLAB
2,7202,100,1
4,7204,100,1
6,7206,100,1
8,7208,100,1
10,7210,100,1
12,7212,100,1
14,7214,100,1
16,7216,100,1
18,7218,100,1
20,7220,100,1
22,7222,100,1
24,7224,100,1
26,7226,100,1
28,7228,100,1
30,7230,100,1
32,7232,100,1
34,7234,100,1
*******************************NODE GEN. FOR WEBS***************************
*NGEN,NSET=NWEB
8006,15206,100,8001
8014,15214,100,8001
8022,15222,100,8001
8030,15230,100,8001
16006,23206,100,16001
16014,23214,100,16001
16022,23222,100,16001
16030,23230,100,16001
24006,31206,100,24001
24014,31214,100,24001
24022,31222,100,24001
24030,31230,100,24001
32006,39206,100,32001
32014,39214,100,32001
32022,39222,100,32001
32030,39230,100,32001
40006,47206,100,40001
40014,47214,100,40001
```

```
40022,47222,100,40001
40030,47230,100,40001
*****************************NODE GEN. FOR TOP FLANGE**************************
*NGEN,NSET=NTFLANGE
50000,57200,100,8001
60000,67200,100,8001
70000,77200,100,8001
80000,87200,100,8001
90000,97200,100,8001
100000,107200,100,8001
110000,117200,100,8001
120000,127200,100,8001
*****************************NODE GEN. FOR BOTTOM
FLANGE**************************
*NGEN,NSET=NBFLANGE
130000,137200,100,40001
140000,147200,100,40001
150000,157200,100,40001
160000,167200,100,40001
170000,177200,100,40001
180000,187200,100,40001
190000,197200,100,40001
200000,207200,100,40001
*****************************NODE GEN. FOR X-BRACE MID-
POINTS**************************
*NGEN,NSET=XBRACE
250000,257200,100,24001
260000,267200,100,24001
270000,277200,100,24001
*****************************MPC NODE
SET**********************************************
*NGEN,NSET=SLABN6
6,7206,100,1
*NGEN,NSET=FLANGEN6
8006,15206,100,8001
*NGEN,NSET=SLABN14
14,7214,100,1
*NGEN,NSET=FLANGEN14
8014,15214,100,8001
*NGEN,NSET=SLABN22
22,7222,100,1
*NGEN,NSET=FLANGEN22
8022,15222,100,8001
*NGEN,NSET=SLABN30
30,7230,100,1
*NGEN,NSET=FLANGEN30
8030,15230,100,8001
*MPC
BEAM,SLABN6,FLANGEN6
BEAM,SLABN14,FLANGEN14
BEAM,SLABN22,FLANGEN22
BEAM,SLABN30,FLANGEN30
*****************************ELEMENT GEN FOR TOP SLAB (2016 elements max.- 7
girders)********************
*ELEMENT,TYPE=S4R
1,2,102,104,4
```

```
*ELGEN,ELSET=ESLAB
1,72,100,16,16,2,1
*************************ELEMENT GEN FOR WEBS ********************(288 elements)
*ELEMENT,TYPE=S4R
2500,16006,16106,8106,8006
2788,16014,16114,8114,8014
3076,16022,16122,8122,8022
3364,16030,16130,8130,8030
*ELGEN,ELSET=WEB
2500,72,100,4,4,8000,1
2788,72,100,4,4,8000,1
3076,72,100,4,4,8000,1
3364,72,100,4,4,8000,1
*************************ELEMENT GEN FOR TOP FLANGE*****************(72 elements/half
flange)
*ELEMENT,TYPE=S4R
4500,50000,8006,8106,50100
4572,8006,60000,60100,8106
4644,70000,8014,8114,70100
4716,8014,80000,80100,8114
4788,90000,8022,8122,90100
4860,8022,100000,100100,8122
4932,110000,8030,8130,110100
5004,8030,120000,120100,8130
*ELGEN, ELSET=TFLANGE
4500,72,100,1,1,1,1
4572,72,100,1,1,1,1
4644,72,100,1,1,1,1
4716,72,100,1,1,1,1
4788,72,100,1,1,1,1
4860,72,100,1,1,1,1
4932,72,100,1,1,1,1
5004,72,100,1,1,1,1
*************************ELEMENT GEN FOR BOTTOM FLANGE*****************(72
elements/half flange)
*ELEMENT,TYPE=S4R
5076,130000,40006,40106,130100
5148,40006,140000,140100,40106
5220,150000,40014,40114,150100
5292,40014,160000,160100,40114
5364,170000,40022,40122,170100
5436,40022,180000,180100,40122
5508,190000,40030,40130,190100
5580,40030,200000,200100,40130
*ELGEN, ELSET=BFLANGE
5076,72,100,1,1,1,1
5148,72,100,1,1,1,1
5220,72,100,1,1,1,1
5292,72,100,1,1,1,1
5364,72,100,1,1,1,1
5436,72,100,1,1,1,1
5508,72,100,1,1,1,1
5580,72,100,1,1,1,1
*************************ELEMENT GEN FOR SHEAR STUD CONNECTOR 0.5m
SPACING*****************(72 shell elements, )
**ELEMENT,TYPE=S4R
```

```
**6000,8006,6,106,8106
**6072,8014,14,114,8114
**6144,8022,22,122,8122
**6216,8030,30,130,8130
**ELGEN, ELSET=STUDS
**6000,72,100,1,1,1,1
**6072,72,100,1,1,1,1
**6144,72,100,1,1,1,1
**6216,72,100,1,1,1,1
************************ELEMENT GEN FOR X-BRACE*****************(4 elements/BRACE)
*ELEMENT,TYPE=B31H
***BRACE1***
8000,8006,8014
8010,40006,40014
8020,8006,40014
8030,40006,8014
***BRACE2***
8040,8014,8022
8050,40014,40022
8060,8014,40022
8070,40014,8022
***BRACE3***
8080,8022,8030
8090,40022,40030
8100,8022,40030
8110,40022,8030
*ELGEN, ELSET=XBRACES
8000,5,1800,1,1,1,1
8010,5,1800,1,1,1,1
8020,5,1800,1,1,1,1
8030,5,1800,1,1,1,1
8040,5,1800,1,1,1,1
8050,5,1800,1,1,1,1
8060,5,1800,1,1,1,1
8070,5,1800,1,1,1,1
8080,5,1800,1,1,1,1
8090,5,1800,1,1,1,1
8100,5,1800,1,1,1,1
8110,5,1800,1,1,1,1
********************************MATERIAL PROPERTIES ****************
**SOLID SECTION,ELSET=XBRACES,MATERIAL=STEEL
**.0075
*BEAM SECTION,SECTION=CIRC,ELSET=XBRACES,MATERIAL=STEEL
.04886
*SHELL SECTION,ELSET=WEB,MATERIAL=STEEL
.016,5
*SHELL SECTION,ELSET=TFLANGE,MATERIAL=STEEL
.02,5
*SHELL SECTION,ELSET=BFLANGE,MATERIAL=STEEL
.02,5
**SHELL SECTION,ELSET=STUDS,MATERIAL=STEEL
**7.9E-4,5
***********************************************
*MATERIAL,NAME=STEEL
*DENSITY
8004.72
```

```
*ELASTIC
200000E6,.3
*SHELL SECTION,ELSET=ESLAB,MATERIAL=CONCRETE
.225,5
*MATERIAL,NAME=CONCRETE
*DENSITY
2447.32
*ELASTIC
28000E6,.2
**********************************************************
*BOUNDARY
LEFTINT,PINNED
LEFTMID,1
LEFTMID,3
LEFTMID2,1
LEFTMID2,3
LEFTEXT,1
LEFTEXT,3
RIGHTINT,2
RIGHTINT,3
RIGHTMID,3
RIGHTMID2,3
RIGHTEXT,3
************************HISTORY DATA*******************************************
*******************SELF WEIGHT LOADING***************************************
*STEP, PERTURBATION
*STATIC
*DLOAD
ESLAB,GRAV,9.81,0,0,-1
TFLANGE,GRAV,9.81,0,0,-1
BFLANGE,GRAV,9.81,0,0,-1
WEB,GRAV,9.81,0,0,-1
XBRACES,GRAV,9.81,0,0,-1
**STUDS,GRAV,9.81,0,0,-1
******************************OUTPUTS********************************************
*NODE PRINT, TOTALS=YES
RF,
*END STEP
************************NATURAL FREQUENCY STEP**********************************
*STEP
*FREQUENCY
5,
*END STEP
```

A.2 Curved Bridge (L25S25N4R5)

```
*HEADING
SLAB ON GIRDER 25m LONG 4 I GIRDER BRIDGE, L/R=0.5, S=2.5
**DATA CHECK
*PREPRINT,ECHO=YES,MODEL=NO,HISTORY=NO
*********REFERENCE NODE COORDINATES*************
*NODE
1,0,0,0.1225
8001,0,0,0
24001,0,0,-0.625
40001,0,0,-1.25
*** SLAB COORDINATES ***
2,-11.1332,43.6011,0.1225
34,-13.6072,53.2902,0.1225
7202,11.1332,43.6011,0.1225
7234,13.6072,53.2902,0.1225
*** 4-GIRDER COORDINATES ***
***Girder 1***
8006,-11.4424,44.8122,0
40006,-11.4424,44.8122,-1.25
15206,11.4424,44.8122,0
47206,11.4424,44.8122,-1.25
***Girder 2***
8014,-12.0609,47.2345,0
40014,-12.0609,47.2345,-1.25
15214,12.0609,47.2345,0
47214,12.0609,47.2345,-1.25
***Girder 3***
8022,-12.6795,49.6568,0
40022,-12.6795,49.6568,-1.25
15222,12.6795,49.6568,0
47222,12.6795,49.6568,-1.25
***Girder 4***
8030,-13.2980,52.0790,0
40030,-13.2980,52.0790,-1.25
15230,13.2980,52.0790,0
47230,13.2980,52.0790,-1.25
*** TOP FLANGE COORDINATES ***
***GIRDER 1***
50000,-11.4053,44.6669,0
57200,11.4053,44.6669,0
60000,-11.4795,44.9575,0
67200,11.4795,44.9575,0
***GIRDER 2***
70000,-12.0238,47.0891,0
77200,12.0238,47.0891,0
80000,-12.0891,47.3798,0
87200,12.0891,47.3798,0
***GIRDER 3***
90000,-12.6423,49.5114,0
97200,12.6423,49.5114,0
100000,-12.7166,49.8021,0
107200,12.7166,49.8021,0
***GIRDER 4***
```

```
110000,-13.2609,51.9337,0
117200,13.2609,51.9337,0
120000,-13.3351,52.2244,0
127200,13.3351,52.2244,0
*** BOTTOM FLANGE COORDINATES ***
***GIRDER 1***
130000,-11.4053,44.6669,-1.25
137200,11.4053,44.6669,-1.25
140000,-11.4795,44.9575,-1.25
147200,11.4795,44.9575,-1.25
***GIRDER 2***
150000,-12.0238,47.0891,-1.25
157200,12.0238,47.0891,-1.25
160000,-12.0891,47.3798,-1.25
167200,12.0891,47.3798,-1.25
***GIRDER 3***
170000,-12.6423,49.5114,-1.25
177200,12.6423,49.5114,-1.25
180000,-12.7166,49.8021,-1.25
187200,12.7166,49.8021,-1.25
***GIRDER 4***
190000,-13.2609,51.9337,-1.25
197200,13.2609,51.9337,-1.25
200000,-13.3351,52.2244,-1.25
207200,13.3351,52.2244,-1.25
*********BOUNDRY NODE SET DEFINITION************
*NSET,NSET=LEFTINT
40006
*NSET,NSET=LEFTMID
40014
*NSET,NSET=LEFTMID2
40022
*NSET,NSET=LEFTEXT
40030
*NSET,NSET=RIGHTINT
47206
*NSET,NSET=RIGHTMID
47214
*NSET,NSET=RIGHTMID2
47222
*NSET,NSET=RIGHTEXT
47230
*NSET,NSET=REACT
LEFTINT,LEFTMID,LEFTMID2,LEFTEXT,RIGHTINT,RIGHTMID,RIGHTMID2,RIGHTEXT
*********BASIC GEOMETRY NODE GENERATION************
*NGEN,NSET=ORGIN
8001,40001,8000
***LEFT END***
*NGEN,NSET=LEND
2,34,2
8006,40006,8000
8014,40014,8000
8022,40022,8000
8030,40030,8000
***RIGHT END***
*NGEN,NSET=REND
```

```
7202,7234,2
15206,47206,8000
15214,47214,8000
15222,47222,8000
15230,47230,8000
*******************************NODE GEN. FOR TOP SLAB**********************
*NGEN,NSET=NSLAB, LINE=C
2,7202,100,1
4,7204,100,1
6,7206,100,1
8,7208,100,1
10,7210,100,1
12,7212,100,1
14,7214,100,1
16,7216,100,1
18,7218,100,1
20,7220,100,1
22,7222,100,1
24,7224,100,1
26,7226,100,1
28,7228,100,1
30,7230,100,1
32,7232,100,1
34,7234,100,1
******************************NODE GEN. FOR WEBS*****************************
*NGEN,NSET=NWEB, LINE=C
8006,15206,100,8001
8014,15214,100,8001
8022,15222,100,8001
8030,15230,100,8001
16006,23206,100,16001
16014,23214,100,16001
16022,23222,100,16001
16030,23230,100,16001
24006,31206,100,24001
24014,31214,100,24001
24022,31222,100,24001
24030,31230,100,24001
32006,39206,100,32001
32014,39214,100,32001
32022,39222,100,32001
32030,39230,100,32001
40006,47206,100,40001
40014,47214,100,40001
40022,47222,100,40001
40030,47230,100,40001
******************************NODE GEN. FOR TOP FLANGE***************************
*NGEN,NSET=NTFLANGE, LINE=C
50000,57200,100,8001
60000,67200,100,8001
70000,77200,100,8001
80000,87200,100,8001
90000,97200,100,8001
100000,107200,100,8001
110000,117200,100,8001
120000,127200,100,8001
```

```
******************************NODE GEN. FOR BOTTOM FLANGE****************************
*NGEN,NSET=NBFLANGE, LINE=C
130000,137200,100,40001
140000,147200,100,40001
150000,157200,100,40001
160000,167200,100,40001
170000,177200,100,40001
180000,187200,100,40001
190000,197200,100,40001
200000,207200,100,40001
*****************************MPC NODE SET***********************************************
*NGEN,NSET=SLABN6, LINE=C
6,7206,100,1
*NGEN,NSET=FLANGEN6, LINE=C
8006,15206,100,8001
*NGEN,NSET=SLABN14, LINE=C
14,7214,100,1
*NGEN,NSET=FLANGEN14, LINE=C
8014,15214,100,8001
*NGEN,NSET=SLABN22, LINE=C
22,7222,100,1
*NGEN,NSET=FLANGEN22, LINE=C
8022,15222,100,8001
*NGEN,NSET=SLABN30, LINE=C
30,7230,100,1
*NGEN,NSET=FLANGEN30, LINE=C
8030,15230,100,8001
*MPC
BEAM,SLABN6,FLANGEN6
BEAM,SLABN14,FLANGEN14
BEAM,SLABN22,FLANGEN22
BEAM,SLABN30,FLANGEN30
*****************************ELEMENT GEN FOR TOP SLAB (2016 elements max.- 7 girders)*********************
*ELEMENT,TYPE=S4R
1,2,102,104,4
*ELGEN,ELSET=ESLAB
1,72,100,16,16,2,1
*************************ELEMENT GEN FOR WEBS ********************(288 elements)
*ELEMENT,TYPE=S4R
2500,16006,16106,8106,8006
2788,16014,16114,8114,8014
3076,16022,16122,8122,8022
3364,16030,16130,8130,8030
*ELGEN,ELSET=WEB
2500,72,100,4,4,8000,1
2788,72,100,4,4,8000,1
3076,72,100,4,4,8000,1
3364,72,100,4,4,8000,1
*************************ELEMENT GEN FOR TOP FLANGE*****************(72 elements/half flange)
*ELEMENT,TYPE=S4R
4500,50000,8006,8106,50100
4572,8006,60000,60100,8106
```

```
4644,70000,8014,8114,70100
4716,8014,80000,80100,8114
4788,90000,8022,8122,90100
4860,8022,100000,100100,8122
4932,110000,8030,8130,110100
5004,8030,120000,120100,8130
*ELGEN, ELSET=TFLANGE
4500,72,100,1,1,1,1
4572,72,100,1,1,1,1
4644,72,100,1,1,1,1
4716,72,100,1,1,1,1
4788,72,100,1,1,1,1
4860,72,100,1,1,1,1
4932,72,100,1,1,1,1
5004,72,100,1,1,1,1
*************************ELEMENT GEN FOR BOTTOM FLANGE*****************(72
elements/half flange)
*ELEMENT,TYPE=S4R
5076,130000,40006,40106,130100
5148,40006,140000,140100,40106
5220,150000,40014,40114,150100
5292,40014,160000,160100,40114
5364,170000,40022,40122,170100
5436,40022,180000,180100,40122
5508,190000,40030,40130,190100
5580,40030,200000,200100,40130
*ELGEN, ELSET=BFLANGE
5076,72,100,1,1,1,1
5148,72,100,1,1,1,1
5220,72,100,1,1,1,1
5292,72,100,1,1,1,1
5364,72,100,1,1,1,1
5436,72,100,1,1,1,1
5508,72,100,1,1,1,1
5580,72,100,1,1,1,1
*************************ELEMENT GEN FOR SHEAR STUD CONNECTOR 0.5m
SPACING*****************(72 shell elements, )
**ELEMENT,TYPE=S4R
**6000,8006,6,106,8106
**6072,8014,14,114,8114
**6144,8022,22,122,8122
**6216,8030,30,130,8130
**ELGEN, ELSET=STUDS
**6000,72,100,1,1,1,1
**6072,72,100,1,1,1,1
**6144,72,100,1,1,1,1
**6216,72,100,1,1,1,1
*************************ELEMENT GEN FOR X-BRACE*****************(4 elements/BRACE)
*ELEMENT,TYPE=B31H
***BRACE1***
8000,8006,8014
8020,40006,40014
8040,8006,40014
8060,40006,8014
***BRACE2***
8080,8014,8022
```

```
8100,40014,40022
8120,8014,40022
8140,40014,8022
***BRACE3***
8160,8022,8030
8180,40022,40030
8200,8022,40030
8220,40022,8030
*ELGEN, ELSET=XBRACES
8000,13,600,1,1,1,1
8020,13,600,1,1,1,1
8040,13,600,1,1,1,1
8060,13,600,1,1,1,1
8080,13,600,1,1,1,1
8100,13,600,1,1,1,1
8120,13,600,1,1,1,1
8140,13,600,1,1,1,1
8160,13,600,1,1,1,1
8180,13,600,1,1,1,1
8200,13,600,1,1,1,1
8220,13,600,1,1,1,1
********************************MATERIAL PROPERTIES ****************
**SOLID SECTION,ELSET=XBRACES,MATERIAL=STEEL
**.0075
*BEAM SECTION,SECTION=CIRC,ELSET=XBRACES,MATERIAL=STEEL
.04886
*SHELL SECTION,ELSET=WEB,MATERIAL=STEEL
.016,5
*SHELL SECTION,ELSET=TFLANGE,MATERIAL=STEEL
.02,5
*SHELL SECTION,ELSET=BFLANGE,MATERIAL=STEEL
.02,5
**SHELL SECTION,ELSET=STUDS,MATERIAL=STEEL
**7.9E-4,5
**********************************************
*MATERIAL,NAME=STEEL
*DENSITY
8004.72
*ELASTIC
200000E6,.3
*SHELL SECTION,ELSET=ESLAB,MATERIAL=CONCRETE
.225,5
*MATERIAL,NAME=CONCRETE
*DENSITY
2447.32
*ELASTIC
28000E6,.2
*******************************************************
*BOUNDARY
LEFTINT,PINNED
LEFTMID,1
LEFTMID,3
LEFTMID2,1
LEFTMID2,3
LEFTEXT,1
LEFTEXT,3
```

```
RIGHTINT,2
RIGHTINT,3
RIGHTMID,3
RIGHTMID2,3
RIGHTEXT,3
************************HISTORY DATA******************************************
*******************SELF WEIGHT LOADING***************************************
*STEP, PERTURBATION
*STATIC
*DLOAD
ESLAB,GRAV,9.81,0,0,-1
TFLANGE,GRAV,9.81,0,0,-1
BFLANGE,GRAV,9.81,0,0,-1
WEB,GRAV,9.81,0,0,-1
XBRACES,GRAV,9.81,0,0,-1
**STUDS,GRAV,9.81,0,0,-1
*****************************OUTPUTS*****************************************
*NODE PRINT, TOTALS=YES
RF,
*END STEP
*************************NATURAL FREQUENCY STEP*******************************
*STEP
*FREQUENCY
5,
*END STEP
```

A.3 Curved Two-Span Bridge (C2L25S25N3R5)

```
*HEADING
SLAB ON GIRDER CONTINUOUS 2 25m LONG 3 I GIRDER BRIDGE L/R=0.5, S=2.5
**DATA CHECK
*PREPRINT,ECHO=YES,MODEL=NO,HISTORY=NO
*********REFERENCE NODE COORDINATES*************
*NODE
1,0,0,0.1225
8001,0,0,0
24001,0,0,-0.625
40001,0,0,-1.25
*** SLAB COORDINATES ***
2,-22.1734,40.5882,0.1225
26,-25.7691,47.1701,0.1225
7202,22.1734,40.5882,0.1225
7226,25.7691,47.1701,0.1225
*** 3-GIRDER COORDINATES ***
***Girder 1***
8006,-22.7727,41.6852,0
40006,-22.7727,41.6852,-1.25
15206,22.7727,41.6852,0
47206,22.7727,41.6852,-1.25
***Girder 2***
8014,-23.9713,43.8791,0
40014,-23.9713,43.8791,-1.25
15214,23.9713,43.8791,0
47214,23.9713,43.8791,-1.25
***Girder 3***
8022,-25.1698,46.0731,0
40022,-25.1698,46.0731,-1.25
15222,25.1698,46.0731,0
47222,25.1698,46.0731,-1.25
*** TOP FLANGE COORDINATES ***
***GIRDER 1***
50000,-22.7008,41.5535,0
57200,22.7008,41.5535,0
60000,-22.8446,41.8168,0
67200,22.8446,41.8168,0
***GIRDER 2***
70000,-23.8994,43.7475,0
77200,23.8994,43.7475,0
80000,-24.0432,44.0108,0
87200,24.0432,44.0108,0
***GIRDER 3***
90000,-25.0979,45.9414,0
97200,25.0979,45.9414,0
100000,-25.2418,46.2047,0
107200,25.2418,46.2047,0
*** BOTTOM FLANGE COORDINATES ***
***GIRDER 1***
110000,-22.7008,41.5535,-1.25
117200,22.7008,41.5535,-1.25
120000,-22.8446,41.8168,-1.25
```

```
127200,22.8446,41.8168,-1.25
***GIRDER 2***
130000,-23.8994,43.7475,-1.25
137200,23.8994,43.7475,-1.25
140000,-24.0432,44.0108,-1.25
147200,24.0432,44.0108,-1.25
***GIRDER 3***
150000,-25.0979,45.9414,-1.25
157200,25.0979,45.9414,-1.25
160000,-25.2418,46.2047,-1.25
167200,25.2418,46.2047,-1.25
*** X-BRACE MID-POINTS ***
**250000,-12.5,2.5,-0.625
**257200,12.5,2.5,-0.625
**260000,-12.5,5,-0.625
**267200,12.5,5,-0.625
*********BOUNDRY NODE SET DEFINITION************
*NSET,NSET=LEFTINT
40006
*NSET,NSET=LEFTMID
40014
*NSET,NSET=LEFTEXT
40022
*NSET,NSET=MIDINT
43606
*NSET,NSET=MIDMID
43614
*NSET,NSET=MIDEXT
43622
*NSET,NSET=RIGHTINT
47206
*NSET,NSET=RIGHTMID
47214
*NSET,NSET=RIGHTEXT
47222
*NSET,NSET=REACT
LEFTINT,LEFTMID,LEFTEXT,MIDINT,MIDMID,MIDEXT,RIGHTINT,RIGHTMID,RIGHTEXT
*********BASIC GEOMETRY NODE GENERATION************
*NGEN,NSET=ORGIN
8001,40001,8000
***LEFT END***
*NGEN,NSET=LEND
2,26,2
8006,40006,8000
8014,40014,8000
8022,40022,8000
***RIGHT END***
*NGEN,NSET=REND
7202,7226,2
15206,47206,8000
15214,47214,8000
15222,47222,8000
********************************NODE GEN. FOR TOP SLAB*********************
*NGEN,NSET=NSLAB, LINE=C
2,7202,100,1
4,7204,100,1
```

```
6,7206,100,1
8,7208,100,1
10,7210,100,1
12,7212,100,1
14,7214,100,1
16,7216,100,1
18,7218,100,1
20,7220,100,1
22,7222,100,1
24,7224,100,1
26,7226,100,1
******************************NODE GEN. FOR WEBS****************************
*NGEN,NSET=NWEB, LINE=C
8006,15206,100,8001
8014,15214,100,8001
8022,15222,100,8001
16006,23206,100,16001
16014,23214,100,16001
16022,23222,100,16001
24006,31206,100,24001
24014,31214,100,24001
24022,31222,100,24001
32006,39206,100,32001
32014,39214,100,32001
32022,39222,100,32001
40006,47206,100,40001
40014,47214,100,40001
40022,47222,100,40001
******************************NODE GEN. FOR TOP FLANGE****************************
*NGEN,NSET=NTFLANGE, LINE=C
50000,57200,100,8001
60000,67200,100,8001
70000,77200,100,8001
80000,87200,100,8001
90000,97200,100,8001
100000,107200,100,8001
******************************NODE GEN. FOR BOTTOM
FLANGE****************************
*NGEN,NSET=NBFLANGE, LINE=C
110000,117200,100,40001
120000,127200,100,40001
130000,137200,100,40001
140000,147200,100,40001
150000,157200,100,40001
160000,167200,100,40001
******************************NODE GEN. FOR X-BRACE MID-
POINTS****************************
**NGEN,NSET=XBRACE
**250000,257200,100,24001
**260000,267200,100,24001
******************************ELEMENT GEN FOR TOP SLAB (2016 elements max.- 7
girders)*********************
*ELEMENT,TYPE=S4R
1,2,102,104,4
*ELGEN,ELSET=ESLAB
1,72,100,12,12,2,1
```

```
*************************ELEMENT GEN FOR WEBS ********************(288 elements)
*ELEMENT,TYPE=S4R
2500,16006,16106,8106,8006
2788,16014,16114,8114,8014
3076,16022,16122,8122,8022
*ELGEN,ELSET=WEB
2500,72,100,4,4,8000,1
2788,72,100,4,4,8000,1
3076,72,100,4,4,8000,1
*************************ELEMENT GEN FOR TOP FLANGE******************(72 elements/half
flange)
*ELEMENT,TYPE=S4R
4500,50000,8006,8106,50100
4572,8006,60000,60100,8106
4644,70000,8014,8114,70100
4716,8014,80000,80100,8114
4788,90000,8022,8122,90100
4860,8022,100000,100100,8122
*ELGEN, ELSET=TFLANGE
4500,72,100,1,1,1,1
4572,72,100,1,1,1,1
4644,72,100,1,1,1,1
4716,72,100,1,1,1,1
4788,72,100,1,1,1,1
4860,72,100,1,1,1,1
*************************ELEMENT GEN FOR BOTTOM FLANGE*****************(72
elements/half flange)
*ELEMENT,TYPE=S4R
4932,110000,40006,40106,110100
5004,40006,120000,120100,40106
5076,130000,40014,40114,130100
5148,40014,140000,140100,40114
5220,150000,40022,40122,150100
5292,40022,160000,160100,40122
*ELGEN, ELSET=BFLANGE
4932,72,100,1,1,1,1
5004,72,100,1,1,1,1
5076,72,100,1,1,1,1
5148,72,100,1,1,1,1
5220,72,100,1,1,1,1
5292,72,100,1,1,1,1
*************************ELEMENT GEN FOR SHEAR STUD CONNECTOR 0.5m
SPACING*****************(72 shell elements, )
**ELEMENT,TYPE=S4R
**6000,8006,6,106,8106
**6072,8014,14,114,8114
**6144,8022,22,122,8122
**ELGEN, ELSET=STUDS
**6000,72,100,1,1,1,1
**6072,72,100,1,1,1,1
**6144,72,100,1,1,1,1
*************************ELEMENT GEN FOR X-BRACE*****************(4 elements/BRACE)
*ELEMENT,TYPE=B31H
***BRACE1***
8000,8006,8014
8020,40006,40014
```

```
8040,8006,40014
8060,40006,8014
***BRACE2***
8080,8014,8022
8100,40014,40022
8120,8014,40022
8140,40014,8022
*ELGEN, ELSET=XBRACES
8000,19,400,1,1,1,1
8020,19,400,1,1,1,1
8040,19,400,1,1,1,1
8060,19,400,1,1,1,1
8080,19,400,1,1,1,1
8100,19,400,1,1,1,1
8120,19,400,1,1,1,1
8140,19,400,1,1,1,1
*********************************MATERIAL PROPERTIES ****************
**SOLID SECTION,ELSET=XBRACES,MATERIAL=STEEL
**.0075
*BEAM SECTION,SECTION=CIRC,ELSET=XBRACES,MATERIAL=STEEL
.04886
*SHELL SECTION,ELSET=WEB,MATERIAL=STEEL
.016,5
*SHELL SECTION,ELSET=TFLANGE,MATERIAL=STEEL
.02,5
*SHELL SECTION,ELSET=BFLANGE,MATERIAL=STEEL
.02,5
**SHELL SECTION,ELSET=STUDS,MATERIAL=STEEL
**7.9E-4,5
**************************************************
*MATERIAL,NAME=STEEL
*DENSITY
8004.72
*ELASTIC
200000E6,.3
*SHELL SECTION,ELSET=ESLAB,MATERIAL=CONCRETE
.225,5
*MATERIAL,NAME=CONCRETE
*DENSITY
2447.32
*ELASTIC
28000E6,.2
******************************MPC NODE
SET************************************************
*NGEN,NSET=SLABN6,LINE=C
6,7206,100,1
*NGEN,NSET=FLANGEN6,LINE=C
8006,15206,100,8001
*NGEN,NSET=SLABN14,LINE=C
14,7214,100,1
*NGEN,NSET=FLANGEN14,LINE=C
8014,15214,100,8001
*NGEN,NSET=SLABN22,LINE=C
22,7222,100,1
*NGEN,NSET=FLANGEN22,LINE=C
8022,15222,100,8001
```

```
*MPC
BEAM,SLABN6,FLANGEN6
BEAM,SLABN14,FLANGEN14
BEAM,SLABN22,FLANGEN22
************************************************************
*NSET, NSET=LEFTREACT
LEFTINT,LEFTMID,LEFTEXT
*NSET, NSET=MIDREACT
MIDINT,MIDMID, MIDEXT
*NSET, NSET=RIGHTREACT
RIGHTINT,RIGHTMID,RIGHTEXT
*TRANSFORM, NSET=LEFTREACT, TYPE=C
0,0,-0.625,0,0,0
*TRANSFORM, NSET=MIDREACT, TYPE=C
0,0,-0.625,0,0,0
*TRANSFORM, NSET=RIGHTREACT, TYPE=C
0,0,-0.625,0,0,0
*BOUNDARY
LEFTINT,1
LEFTINT,3
LEFTMID,3
LEFTEXT,3
MIDINT,PINNED
MIDMID,2
MIDMID,3
MIDEXT,2
MIDEXT,3
RIGHTINT,1
RIGHTINT,3
RIGHTMID,3
RIGHTEXT,3
************************HISTORY DATA*******************************************
*******************SELF WEIGHT LOADING*****************************************
*STEP, PERTURBATION
*STATIC
*DLOAD
ESLAB,GRAV,9.81,0,0,-1
TFLANGE,GRAV,9.81,0,0,-1
BFLANGE,GRAV,9.81,0,0,-1
WEB,GRAV,9.81,0,0,-1
XBRACES,GRAV,9.81,0,0,-1
**STUDS,GRAV,9.81,0,0,-1
******************************OUTPUTS*****************************************
*NODE PRINT, TOTALS=YES
RF,
*END STEP
************************NATURAL FREQUENCY STEP*********************************
*STEP
*FREQUENCY
5,
*END STEP
```

APPENDEX (B): ABAQUS DATA USED TO DEVELOP THE PROPOSED EQUATIONS

B.1 DATA FOR SIMPLE SPAN BRIDGES

(L)	(S)	(N_G)	(B)	(R)	(k)	L^2/BR	f_{FEA}	f_{BEAM}	$f_{EQN.\ 5.3}$
10	2	3	6	100000.0	0	0.00	9.73	10.50	10.13
10	2	4	8	100000.0	0	0.00	9.71	10.43	10.07
10	2	5	10	100000.0	0	0.00	9.70	10.43	10.07
10	2	6	12	100000.0	0	0.00	9.69	10.40	10.04
10	2	7	14	100000.0	0	0.00	9.69	10.38	10.02
10	2	3	6	100.0	0.1	0.17	9.50	10.37	9.71
10	2	4	8	100.0	0.1	0.13	9.43	10.30	9.66
10	2	5	10	100.0	0.1	0.10	9.46	10.29	9.67
10	2	6	12	100.0	0.1	0.08	9.27	10.25	9.65
10	2	7	14	100.0	0.1	0.07	9.15	10.23	9.64
10	2	3	6	50.0	0.2	0.33	9.15	10.25	9.15
10	2	4	8	50.0	0.2	0.25	9.00	10.17	9.13
10	2	5	10	50.0	0.2	0.20	8.82	10.15	9.16
10	2	6	12	50.0	0.2	0.17	8.61	10.11	9.16
10	2	7	14	50.0	0.2	0.14	8.37	10.08	9.16
10	2	3	6	33.3	0.3	0.50	8.64	10.03	8.42
10	2	4	8	33.3	0.3	0.38	8.41	9.92	8.43
10	2	5	10	33.3	0.3	0.30	8.16	9.89	8.48
10	2	6	12	33.3	0.3	0.25	7.87	9.84	8.50
10	2	7	14	33.3	0.3	0.21	7.56	9.81	8.51
10	2.5	3	7.5	100000.0	0	0.00	8.96	9.75	9.41
10	2.5	4	10	100000.0	0	0.00	8.95	9.72	9.38
10	2.5	5	12.5	100000.0	0	0.00	8.94	9.69	9.35
10	2.5	6	15	100000.0	0	0.00	8.94	9.67	9.33
10	2.5	3	7.5	100.0	0.1	0.13	8.74	9.64	9.04
10	2.5	4	10	100.0	0.1	0.10	8.65	9.59	9.01
10	2.5	5	12.5	100.0	0.1	0.08	8.55	9.55	9.00
10	2.5	6	15	100.0	0.1	0.07	8.41	9.53	8.99
10	2.5	3	7.5	50.0	0.2	0.27	8.37	9.52	8.54
10	2.5	4	10	50.0	0.2	0.20	8.17	9.46	8.54
10	2.5	5	12.5	50.0	0.2	0.16	7.93	9.42	8.54
10	2.5	6	15	50.0	0.2	0.13	7.66	9.39	8.55
10	2.5	3	7.5	33.3	0.3	0.40	7.87	9.31	7.90
10	2.5	4	10	33.3	0.3	0.30	7.58	9.23	7.91
10	2.5	5	12.5	33.3	0.3	0.24	7.25	9.17	7.93

10	2.5	6	15	33.3	0.3	0.20	6.90	9.14	7.95
10	3	3	9	100000.0	0	0.00	8.36	9.17	8.85
10	3	4	12	100000.0	0	0.00	8.35	9.11	8.80
10	3	5	15	100000.0	0	0.00	8.35	9.11	8.79
10	3	3	9	100.0	0.1	0.11	8.12	9.06	8.51
10	3	4	12	100.0	0.1	0.08	8.01	8.99	8.46
10	3	5	15	100.0	0.1	0.07	7.87	8.98	8.47
10	3	3	9	50.0	0.2	0.22	7.73	8.95	8.06
10	3	4	12	50.0	0.2	0.17	7.48	8.87	8.04
10	3	5	15	50.0	0.2	0.13	7.18	8.85	8.05
10	3	3	9	33.3	0.3	0.33	7.22	8.74	7.47
10	3	4	12	33.3	0.3	0.25	6.87	8.65	7.47
10	3	5	15	33.3	0.3	0.20	6.49	8.62	7.50
15	2	3	6	100000.0	0	0.00	6.54	6.72	6.48
15	2	4	8	100000.0	0	0.00	6.53	6.69	6.46
15	2	5	10	100000.0	0	0.00	6.52	6.68	6.44
15	2	6	12	100000.0	0	0.00	6.51	6.67	6.43
15	2	7	14	100000.0	0	0.00	6.51	6.66	6.43
15	2	3	6	150.0	0.1	0.25	6.42	6.67	6.24
15	2	4	8	150.0	0.1	0.19	6.39	6.63	6.23
15	2	5	10	150.0	0.1	0.15	6.37	6.61	6.22
15	2	6	12	150.0	0.1	0.13	6.34	6.60	6.22
15	2	7	14	150.0	0.1	0.11	6.30	6.59	6.21
15	2	3	6	75.0	0.2	0.50	6.15	6.56	5.86
15	2	4	8	75.0	0.2	0.38	6.11	6.52	5.86
15	2	5	10	75.0	0.2	0.30	6.05	6.50	5.86
15	2	6	12	75.0	0.2	0.25	5.98	6.48	5.87
15	2	7	14	75.0	0.2	0.21	5.89	6.47	5.87
15	2	3	6	50.0	0.3	0.75	5.91	6.56	5.51
15	2	4	8	50.0	0.3	0.56	5.85	6.52	5.54
15	2	5	10	50.0	0.3	0.45	5.77	6.50	5.57
15	2	6	12	50.0	0.3	0.38	5.67	6.48	5.59
15	2	7	14	50.0	0.3	0.32	5.54	6.47	5.61
15	2.5	3	7.5	100000.0	0	0.00	6.05	6.24	6.02
15	2.5	4	10	100000.0	0	0.00	6.03	6.21	6.00
15	2.5	5	12.5	100000.0	0	0.00	6.02	6.20	5.98
15	2.5	6	15	100000.0	0	0.00	6.02	6.19	5.97
15	2.5	3	7.5	150.0	0.1	0.20	5.93	6.19	5.80
15	2.5	4	10	150.0	0.1	0.15	5.90	6.16	5.79
15	2.5	5	12.5	150.0	0.1	0.12	5.86	6.14	5.78
15	2.5	6	15	150.0	0.1	0.10	5.82	6.13	5.78

15	2.5	3	7.5	75.0	0.2	0.40	5.68	6.09	5.46
15	2.5	4	10	75.0	0.2	0.30	5.62	6.05	5.46
15	2.5	5	12.5	75.0	0.2	0.24	5.53	6.03	5.46
15	2.5	6	15	75.0	0.2	0.20	5.43	6.01	5.47
15	2.5	3	7.5	50.0	0.3	0.60	5.46	6.09	5.17
15	2.5	4	10	50.0	0.3	0.45	5.37	6.05	5.19
15	2.5	5	12.5	50.0	0.3	0.36	5.25	6.03	5.21
15	2.5	6	15	50.0	0.3	0.30	5.10	6.01	5.23
15	3	3	9	100000.0	0	0.00	5.65	5.85	5.65
15	3	4	12	100000.0	0	0.00	5.64	5.82	5.62
15	3	5	15	100000.0	0	0.00	5.63	5.81	5.60
15	3	3	9	150.0	0.1	0.17	5.53	5.80	5.45
15	3	4	12	150.0	0.1	0.13	5.49	5.77	5.43
15	3	5	15	150.0	0.1	0.10	5.45	5.75	5.42
15	3	3	9	75.0	0.2	0.33	5.29	5.71	5.14
15	3	4	12	75.0	0.2	0.25	5.21	5.67	5.14
15	3	5	15	75.0	0.2	0.20	5.10	5.64	5.13
15	3	3	9	50.0	0.3	0.50	5.07	5.71	4.88
15	3	4	12	50.0	0.3	0.38	4.95	5.67	4.90
15	3	5	15	50.0	0.3	0.30	4.79	5.64	4.91
25	2	3	6	100000.0	0	0.00	3.96	4.05	3.91
25	2	4	8	100000.0	0	0.00	3.96	4.04	3.90
25	2	5	10	100000.0	0	0.00	3.95	4.03	3.89
25	2	6	12	100000.0	0	0.00	3.95	4.03	3.89
25	2	7	14	100000.0	0	0.00	3.95	4.02	3.88
25	2	3	6	250.0	0.1	0.42	3.86	4.02	3.76
25	2	4	8	250.0	0.1	0.31	3.86	4.00	3.75
25	2	5	10	250.0	0.1	0.25	3.85	3.99	3.75
25	2	6	12	250.0	0.1	0.21	3.85	3.98	3.75
25	2	7	14	250.0	0.1	0.18	3.84	3.98	3.75
25	2	3	6	83.3	0.3	1.25	3.46	3.98	3.34
25	2	4	8	83.3	0.3	0.94	3.50	3.95	3.36
25	2	5	10	83.3	0.3	0.75	3.51	3.94	3.38
25	2	6	12	83.3	0.3	0.63	3.50	3.93	3.40
25	2	7	14	83.3	0.3	0.54	3.49	3.93	3.41
25	2	3	6	50.0	0.5	2.08	2.97	3.90	2.80
25	2	4	8	50.0	0.5	1.56	3.05	3.87	2.86
25	2	5	10	50.0	0.5	1.25	3.08	3.86	2.91
25	2	6	12	50.0	0.5	1.04	3.08	3.85	2.94
25	2	7	14	50.0	0.5	0.89	3.05	3.84	2.97
25	2.5	3	7.5	100000.0	0	0.00	3.68	3.78	3.65

25	2.5	4	10	100000.0	0	0.00	3.68	3.77	3.64
25	2.5	5	12.5	100000.0	0	0.00	3.67	3.76	3.63
25	2.5	6	15	100000.0	0	0.00	3.67	3.76	3.63
25	2.5	3	7.5	250.0	0.1	0.33	3.60	3.74	3.51
25	2.5	4	10	250.0	0.1	0.25	3.59	3.73	3.50
25	2.5	5	12.5	250.0	0.1	0.20	3.58	3.72	3.50
25	2.5	6	15	250.0	0.1	0.17	3.57	3.71	3.50
25	2.5	3	7.5	83.3	0.3	1.00	3.27	3.71	3.14
25	2.5	4	10	83.3	0.3	0.75	3.28	3.69	3.16
25	2.5	5	12.5	83.3	0.3	0.60	3.27	3.68	3.18
25	2.5	6	15	83.3	0.3	0.50	3.25	3.67	3.19
25	2.5	3	7.5	50.0	0.5	1.67	2.85	3.64	2.67
25	2.5	4	10	50.0	0.5	1.25	2.89	3.61	2.72
25	2.5	5	12.5	50.0	0.5	1.00	2.88	3.60	2.76
25	2.5	6	15	50.0	0.5	0.83	2.85	3.59	2.79
25	3	3	9	100000.0	0	0.00	3.46	3.56	3.43
25	3	4	12	100000.0	0	0.00	3.45	3.54	3.42
25	3	5	15	100000.0	0	0.00	3.45	3.54	3.41
25	3	3	9	250.0	0.1	0.28	3.38	3.52	3.31
25	3	4	12	250.0	0.1	0.21	3.37	3.51	3.30
25	3	5	15	250.0	0.1	0.17	3.36	3.50	3.30
25	3	3	9	83.3	0.3	0.83	3.09	3.49	2.98
25	3	4	12	83.3	0.3	0.63	3.09	3.47	2.99
25	3	5	15	83.3	0.3	0.50	3.06	3.46	3.01
25	3	3	9	50.0	0.5	1.39	2.72	3.42	2.55
25	3	4	12	50.0	0.5	1.04	2.73	3.40	2.60
25	3	5	15	50.0	0.5	0.83	2.69	3.38	2.63
35	2	3	6	100000.0	0	0.00	2.88	2.95	2.84
35	2	4	8	100000.0	0	0.00	2.88	2.94	2.84
35	2	5	10	100000.0	0	0.00	2.87	2.93	2.83
35	2	6	12	100000.0	0	0.00	2.87	2.93	2.83
35	2	7	14	100000.0	0	0.00	2.87	2.93	2.82
35	2	3	6	350.0	0.1	0.58	2.79	2.95	2.76
35	2	4	8	350.0	0.1	0.44	2.81	2.94	2.76
35	2	5	10	350.0	0.1	0.35	2.82	2.93	2.76
35	2	6	12	350.0	0.1	0.29	2.82	2.93	2.76
35	2	7	14	350.0	0.1	0.25	2.82	2.93	2.76
35	2	3	6	87.5	0.4	2.33	2.16	2.89	2.26
35	2	4	8	87.5	0.4	1.75	2.27	2.87	2.29
35	2	5	10	87.5	0.4	1.40	2.32	2.86	2.31
35	2	6	12	87.5	0.4	1.17	2.36	2.86	2.33

35	2	7	14	87.5	0.4	1.00	2.37	2.85	2.35
35	2	3	6	50.0	0.7	4.08	1.64	2.89	1.67
35	2	4	8	50.0	0.7	3.06	1.78	2.87	1.75
35	2	5	10	50.0	0.7	2.45	1.86	2.86	1.81
35	2	6	12	50.0	0.7	2.04	1.91	2.86	1.86
35	2	7	14	50.0	0.7	1.75	1.93	2.85	1.90
35	2.5	3	7.5	100000.0	0	0.00	2.70	2.77	2.67
35	2.5	4	10	100000.0	0	0.00	2.69	2.76	2.66
35	2.5	5	12.5	100000.0	0	0.00	2.69	2.75	2.66
35	2.5	6	15	100000.0	0	0.00	2.69	2.75	2.65
35	2.5	3	7.5	350.0	0.1	0.47	2.64	2.77	2.59
35	2.5	4	10	350.0	0.1	0.35	2.64	2.76	2.59
35	2.5	5	12.5	350.0	0.1	0.28	2.64	2.75	2.59
35	2.5	6	15	350.0	0.1	0.23	2.64	2.75	2.59
35	2.5	3	7.5	87.5	0.4	1.87	2.12	2.71	2.15
35	2.5	4	10	87.5	0.4	1.40	2.19	2.69	2.18
35	2.5	5	12.5	87.5	0.4	1.12	2.23	2.69	2.20
35	2.5	6	15	87.5	0.4	0.93	2.24	2.68	2.21
35	2.5	3	7.5	50.0	0.7	3.27	1.65	2.71	1.64
35	2.5	4	10	50.0	0.7	2.45	1.76	2.69	1.71
35	2.5	5	12.5	50.0	0.7	1.96	1.81	2.69	1.76
35	2.5	6	15	50.0	0.7	1.63	1.83	2.68	1.80
35	3	3	9	100000.0	0	0.00	2.54	2.61	2.52
35	3	4	12	100000.0	0	0.00	2.54	2.61	2.52
35	3	5	15	100000.0	0	0.00	2.53	2.60	2.51
35	3	3	9	350.0	0.1	0.39	2.49	2.61	2.46
35	3	4	12	350.0	0.1	0.29	2.50	2.61	2.45
35	3	5	15	350.0	0.1	0.23	2.50	2.60	2.45
35	3	3	9	87.5	0.4	1.56	2.06	2.56	2.05
35	3	4	12	87.5	0.4	1.17	2.11	2.55	2.08
35	3	5	15	87.5	0.4	0.93	2.12	2.54	2.10
35	3	3	9	50.0	0.7	2.72	1.63	2.56	1.60
35	3	4	12	50.0	0.7	2.04	1.71	2.55	1.66
35	3	5	15	50.0	0.7	1.63	1.74	2.54	1.70
45	2	3	6	100000.0	0	0.00	2.28	2.34	2.25
45	2	4	8	100000.0	0	0.00	2.28	2.33	2.25
45	2	5	10	100000.0	0	0.00	2.27	2.32	2.24
45	2	6	12	100000.0	0	0.00	2.27	2.32	2.24
45	2	7	14	100000.0	0	0.00	2.27	2.32	2.24
45	2	3	6	450.0	0.1	0.75	2.15	2.35	2.20
45	2	4	8	450.0	0.1	0.56	2.20	2.34	2.20

45	2	5	10	450.0	0.1	0.45	2.22	2.34	2.20
45	2	6	12	450.0	0.1	0.38	2.23	2.34	2.20
45	2	7	14	450.0	0.1	0.32	2.23	2.34	2.20
45	2	3	6	90.0	0.5	3.75	1.40	2.31	1.66
45	2	4	8	90.0	0.5	2.81	1.53	2.30	1.70
45	2	5	10	90.0	0.5	2.25	1.61	2.29	1.73
45	2	6	12	90.0	0.5	1.88	1.67	2.29	1.75
45	2	7	14	90.0	0.5	1.61	1.71	2.29	1.77
45	2	3	6	50.0	0.9	6.75	0.92	2.31	0.98
45	2	4	8	50.0	0.9	5.06	1.04	2.30	1.08
45	2	5	10	50.0	0.9	4.05	1.13	2.29	1.15
45	2	6	12	50.0	0.9	3.38	1.19	2.29	1.21
45	2	7	14	50.0	0.9	2.89	1.24	2.29	1.25
45	2.5	3	7.5	100000.0	0	0.00	2.15	2.21	2.13
45	2.5	4	10	100000.0	0	0.00	2.14	2.20	2.12
45	2.5	5	12.5	100000.0	0	0.00	2.14	2.19	2.12
45	2.5	6	15	100000.0	0	0.00	2.14	2.19	2.11
45	2.5	3	7.5	450.0	0.1	0.60	2.08	2.22	2.08
45	2.5	4	10	450.0	0.1	0.45	2.10	2.21	2.08
45	2.5	5	12.5	450.0	0.1	0.36	2.11	2.20	2.07
45	2.5	6	15	450.0	0.1	0.30	2.11	2.21	2.08
45	2.5	3	7.5	90.0	0.5	3.00	1.43	2.18	1.60
45	2.5	4	10	90.0	0.5	2.25	1.53	2.17	1.64
45	2.5	5	12.5	90.0	0.5	1.80	1.60	2.15	1.65
45	2.5	6	15	90.0	0.5	1.50	1.63	2.16	1.68
45	2.5	3	7.5	50.0	0.9	5.40	0.96	2.18	1.00
45	2.5	4	10	50.0	0.9	4.05	1.07	2.17	1.09
45	2.5	5	12.5	50.0	0.9	3.24	1.14	2.15	1.15
45	2.5	6	15	50.0	0.9	2.70	1.19	2.16	1.20
45	3	3	9	100000.0	0	0.00	2.04	2.09	2.02
45	3	4	12	100000.0	0	0.00	2.03	2.09	2.01
45	3	5	15	100000.0	0	0.00	2.03	2.08	2.01
45	3	3	9	450.0	0.1	0.50	1.99	2.10	1.98
45	3	4	12	450.0	0.1	0.38	2.00	2.10	1.98
45	3	5	15	450.0	0.1	0.30	2.00	2.10	1.98
45	3	3	9	90.0	0.5	2.50	1.43	2.07	1.55
45	3	4	12	90.0	0.5	1.88	1.51	2.06	1.58
45	3	5	15	90.0	0.5	1.50	1.55	2.05	1.60
45	3	3	9	50.0	0.9	4.50	0.98	2.07	1.01
45	3	4	12	50.0	0.9	3.38	1.08	2.06	1.09
45	3	5	15	50.0	0.9	2.70	1.13	2.05	1.14

B.2 DATA FOR CONTINUOUS TWO-SPAN BRIDGES

(L)	(S)	(N_G)	(B)	(R)	(k)	L^2/BR	$f_{FEA\text{-}CONT.}$	$f_{FEA\text{-}SIMPLE\ SPAN}$
15	2	3	6	100000.0	0	0.00	6.60	6.54
15	2	3	6	150.0	0.1	0.25	6.47	6.42
15	2	3	6	75.0	0.2	0.50	6.20	6.15
15	2	3	6	50.0	0.3	0.75	5.94	5.91
15	2.5	3	7.5	100000.0	0	0.00	6.10	6.05
15	2.5	3	7.5	150.0	0.1	0.20	5.98	5.93
15	2.5	3	7.5	75.0	0.2	0.40	5.73	5.68
15	2.5	3	7.5	50.0	0.3	0.60	5.49	5.46
15	3	3	9	100000.0	0	0.00	5.70	5.65
15	3	3	9	150.0	0.1	0.17	5.59	5.53
15	3	3	9	75.0	0.2	0.33	5.34	5.29
15	3	3	9	50.0	0.3	0.50	5.11	5.07
25	2	3	6	100000.0	0	0.00	3.98	3.96
25	2	3	6	250.0	0.1	0.42	3.88	3.86
25	2	3	6	83.3	0.3	1.25	3.48	3.46
25	2	3	6	50.0	0.5	2.08	3.00	2.97
25	2.5	3	7.5	100000.0	0	0.00	3.71	3.68
25	2.5	3	7.5	250.0	0.1	0.33	3.62	3.60
25	2.5	3	7.5	83.3	0.3	1.00	3.29	3.27
25	2.5	3	7.5	50.0	0.5	1.67	2.87	2.85
25	3	3	9	100000.0	0	0.00	3.48	3.46
25	3	3	9	250.0	0.1	0.28	3.40	3.38
25	3	3	9	83.3	0.3	0.83	3.12	3.09
25	3	3	9	50.0	0.5	1.39	2.74	2.72
35	2	3	6	100000.0	0	0.00	2.89	2.88
35	2	3	6	350.0	0.1	0.58	2.80	2.79
35	2	3	6	87.5	0.4	2.33	2.17	2.16
35	2	3	6	50.0	0.7	4.08	1.63	1.64
35	2.5	3	7.5	100000.0	0	0.00	2.71	2.70
35	2.5	3	7.5	350.0	0.1	0.47	2.65	2.64
35	2.5	3	7.5	87.5	0.4	1.87	2.13	2.12
35	2.5	3	7.5	50.0	0.7	3.27	1.64	1.65
35	3	3	9	100000.0	0	0.00	2.56	2.54
35	3	3	9	350.0	0.1	0.39	2.51	2.49
35	3	3	9	87.5	0.4	1.56	2.07	2.06
35	3	3	9	50.0	0.7	2.72	1.62	1.63
45	2	3	6	100000.0	0	0.00	2.30	2.28
45	2	3	6	450.0	0.1	0.75	2.16	2.15

45	2	3	6	90.0	0.5	3.75	1.40	1.40
45	2	3	6	50.0	0.9	6.75	0.90	0.92
45	2.5	3	7.5	100000.0	0	0.00	2.17	2.15
45	2.5	3	7.5	450.0	0.1	0.60	2.09	2.08
45	2.5	3	7.5	90.0	0.5	3.00	1.43	1.43
45	2.5	3	7.5	50.0	0.9	5.40	0.94	0.96
45	3	3	9	100000.0	0	0.00	2.06	2.04
45	3	3	9	450.0	0.1	0.50	1.99	1.99
45	3	3	9	90.0	0.5	2.50	1.43	1.43
45	3	3	9	50.0	0.9	4.50	0.96	0.98

B.3 DATA FOR SPAN-TO-DEPTH RATIO BRIDGES

(L)	(S)	(N_G)	L/D	(R)	(k)	L^2/BR	f_{FEA}	$f_{EQN.\ 5.3}$	$f_{EQN.\ 5.6}$
15	2.5	3	20	100000.0	0	0.00	6.05	6.02	6.04
15	2.5	3	20	150.0	0.1	0.20	5.93	5.80	5.82
15	2.5	3	20	75.0	0.2	0.40	5.68	5.46	5.48
15	2.5	3	20	50.0	0.3	0.60	5.46	5.17	5.18
15	2.5	3	25	100000.0	0	0.00	5.05	4.91	5.10
15	2.5	3	25	150.0	0.1	0.20	4.96	4.73	4.92
15	2.5	3	25	75.0	0.2	0.40	4.77	4.45	4.63
15	2.5	3	25	50.0	0.3	0.60	4.60	4.21	4.38
15	2.5	3	30	100000.0	0	0.00	4.39	4.20	4.50
15	2.5	3	30	150.0	0.1	0.20	4.32	4.05	4.34
15	2.5	3	30	75.0	0.2	0.40	4.16	3.81	4.08
15	2.5	3	30	50.0	0.3	0.60	4.03	3.60	3.86
25	2.5	3	20	100000.0	0	0.00	3.68	3.65	3.66
25	2.5	3	20	250.0	0.1	0.33	3.60	3.51	3.52
25	2.5	3	20	83.3	0.3	1.00	3.27	3.14	3.15
25	2.5	3	20	50.0	0.5	1.67	2.85	2.67	2.68
25	2.5	3	25	100000.0	0	0.00	2.98	2.90	3.01
25	2.5	3	25	250.0	0.1	0.33	2.92	2.79	2.90
25	2.5	3	25	83.3	0.3	1.00	2.69	2.50	2.60
25	2.5	3	25	50.0	0.5	1.67	2.38	2.12	2.21
25	2.5	3	30	100000.0	0	0.00	2.52	2.41	2.59
25	2.5	3	30	250.0	0.1	0.33	2.47	2.32	2.49
25	2.5	3	30	83.3	0.3	1.00	2.30	2.08	2.23
25	2.5	3	30	50.0	0.5	1.67	2.05	1.77	1.89
35	2.5	3	20	100000.0	0	0.00	2.70	2.67	2.68
35	2.5	3	20	350.0	0.1	0.47	2.64	2.59	2.60
35	2.5	3	20	87.5	0.4	1.87	2.12	2.15	2.16

35	2.5	3	20	50.0	0.7	3.27	1.65	1.64	1.64
35	2.5	3	25	100000.0	0	0.00	2.16	2.10	2.18
35	2.5	3	25	350.0	0.1	0.47	2.12	2.04	2.12
35	2.5	3	25	87.5	0.4	1.87	1.76	1.69	1.76
35	2.5	3	25	50.0	0.7	3.27	1.40	1.29	1.34
35	2.5	3	30	100000.0	0	0.00	1.80	1.73	1.85
35	2.5	3	30	350.0	0.1	0.47	1.77	1.68	1.80
35	2.5	3	30	87.5	0.4	1.87	1.51	1.39	1.49
35	2.5	3	30	50.0	0.7	3.27	1.22	1.06	1.14
45	2.5	3	20	100000.0	0	0.00	2.15	2.13	2.13
45	2.5	3	20	450.0	0.1	0.60	2.08	2.08	2.09
45	2.5	3	20	90.0	0.5	3.00	1.43	1.60	1.61
45	2.5	3	20	50.0	0.9	5.40	0.96	1.00	1.01
45	2.5	3	25	100000.0	0	0.00	1.71	1.67	1.73
45	2.5	3	25	450.0	0.1	0.60	1.67	1.63	1.69
45	2.5	3	25	90.0	0.5	3.00	1.21	1.25	1.30
45	2.5	3	25	50.0	0.9	5.40	0.83	0.79	0.82
45	2.5	3	30	100000.0	0	0.00	1.42	1.37	1.46
45	2.5	3	30	450.0	0.1	0.60	1.39	1.34	1.43
45	2.5	3	30	90.0	0.5	3.00	1.04	1.03	1.10
45	2.5	3	30	50.0	0.9	5.40	0.73	0.65	0.69