EVALUATION OF FINITE ELEMENT SOFTWARE FOR PAVEMENT STRESS ANALYSIS

By

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EVALUATION OF FINITE ELEMENT SOFTWARE FOR PAVEMENT STRESS ANALYSIS

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Abstract

Different approaches are usually taken when designing flexible and rigid pavement: the rigid concrete slab carries major portion of the traffic load; while for flexible pavement, external loads are distributed to the subgrade because of the relative low modulus of elasticity of asphalt layer comparing to concrete in the case of rigid pavement.

Pavement engineering has gone through major developments; the transition from Empirical Design Method to Mechanistic-Empirical Methods is becoming a near-future trend. The Mechanistic-Empirical Method has two components: (1) stress, strain and deflection are calculated based on analyzing mechanical characteristics of materials; (2) critical pavement distresses are quantitatively predicted by experimental calibrated equations. Hence, stress analysis has become an important role in pavement engineering.

The most practical and widely used stress analysis method for flexible pavement is Burmister's Elastic Layered Theory; and for analyzing rigid pavement is Finite Element Method. KENSLABS and STAAD-III are both Finite Element software; KENSLABS is designed specifically for concrete pavement stress analysis, therefore it is more user-friendly for pavement design; STAAD-III is more suitable for general plane and space structures. The project compares the use of both software for stress analysis in rigid pavement in term of simplicity and precision.

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I am very grateful for my husband Jerry, for his love and patience along this journey. The birth of our twin boys Braydon and Dylan is the most joyful experience we lived through in this period.

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Executive Summary

The first chapter of this report briefly introduced the pavement types, available pavement design methods and objectives of this project. There are two major types pavement: flexible or asphalt pavement and rigid or concrete pavement. As an important component of Mechanistic-Empirical Method, the most practical stress analysis method for flexible pavement is Burmister's Elastic Layered Theory and for rigid pavement is Finite Element Method.

The second chapter of this report reviewed critical stress, strain and deflection, which would induce major types of distress in flexible pavement and rigid pavement. Horizontal tensile strain at the bottom of the asphalt and the vertical compressive strain at the top of the subgrade are two critical measurements for predicting flexible pavement distresses such as fatigue cracking and rutting. Flexural stress in concrete slab is a major factor controlling rigid pavement distress such as transverse cracking. The application of Burmister's Elastic Layered Theory in flexible pavement design and Finite Element Method in rigid pavement design will also be elaborated in this Chapter.

Two available Finite Element software, *KENSLABS* and *STAAD-III* are compared in Chapter 3 by solving two pavement design examples. As a software designed specifically for rigid pavement stress analysis, *KENSLABS* has several advantages in terms of simplicity; as a powerful while complicated finite element software, *STAAD-III* is more suitable for analyzing general plane and space structures. The last chapter presents the conclusions of this project.

Chapter 1: Introduction

1.1 Definition of pavement types

There are mainly two types of pavement structure currently in use around the world: flexible pavement and rigid pavement. The major difference between these two types of pavement is: flexible pavement has an asphalt concrete surface layer at the very top, base and/or subbase layer in the middle, and natural subgrade layer at the very bottom; while for rigid pavement, regardless as continuously reinforced concrete or lean concrete pavement, it has a concrete slab sitting right above the soil, a layer of base course may sit in-between. Compositions of two types of pavement are simply illustrated as below in Fig. 1 and Fig. 2:

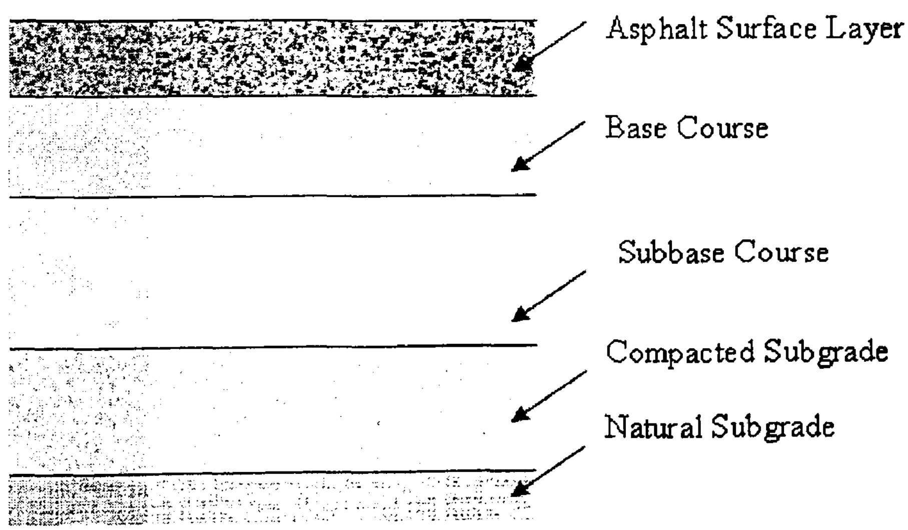


Fig. 1 Flexible Pavement Composition[2]

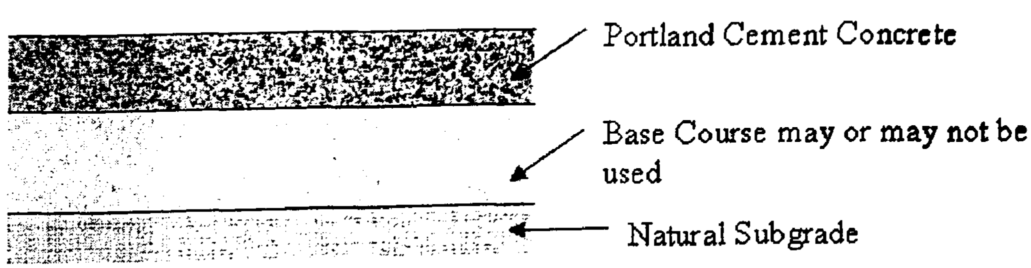


Fig. 2 Rigid Pavement Composition[2]

1.2 Pavement design methods

Pavement design has evolved from art to science during the past two centuries. The first asphalt-paved roadway in the United States was constructed in 1870 at Newark by a Belgian Chemist named Edmund J. DeSmedt;[1] and the first Portland cement concrete pavement in the United States was believed to be laid in 1893 in Bellefontaine, Ohio by J. Y. McClintock.[1]

As a simple design approach, the Empirical Design Method has been adopted by many transportation agencies nowadays. By inputting environmental, material and traffic data, Empirical Method is able to provide design engineers the pavement design thickness by using calibrated equations and Nomographs. Unlike the Empirical Design Method, Mechanistic-Empirical Method is mainly based on analyzing mechanical characteristics of materials. Stress, strain and deflection can be calculated by using Elastic Layered Theory (flexible pavement) and Finite Element Method (rigid pavement); then critical pavement distresses can be quantitatively predicted by experimental calibrated models.[3] With the development of high-speed calculation with computer software, critical stresses and deflections of pavement under traffic and environmental loadings is much more convenient to be predicted than the past.

1.3 Objectives

The objectives of this project are firstly, to review critical stress, strain and deflection that would induce major types of distress in flexible pavement and rigid pavement. Distress is an important consideration in terms of pavement performance. The magnitude/frequency of loading and number of load repetitions during the design period are major factors contribute to the damage effects of pavement structures. For flexible pavement, primary distresses are rutting, fatigue cracking and thermal cracking;[3] for concrete pavement, major distresses are transverse cracking and faulting.[3]

Secondly, available stress analysis methods will be reviewed. As a realistic approach for analyzing flexible pavement responses, Burmister's Elastic Layered Theory will be briefly introduced; and as a common structural analyzing tool, Finite Element Method and how the method is applied in solving rigid pavement problem will be elaborated.

Finally, two available Finite Element software: *KENSLABS* and *STAAD-III* will be compared by solving two pavement design examples. The results from running two software will be explained and analyzed. A brief introduction on design procedures and reports generated by both software are included in Appendices.

Chapter 2: Review of available stress analysis methodology

For flexible and rigid pavement

From the standpoint of stress analysis, the essential differences between flexible and rigid pavement is: because of the large modulus of elasticity of the rigid concrete slab, major portion of the load transferred from the tires of the vehicles will be carried by the concrete slab itself, thus the main goal of the rigid pavement design is to build a strong concrete slab. While for flexible pavement, external loads will be distributed to the subgrade layer because of the relative low modulus of elasticity of asphalt layer and other upper layers, the design process consists of choosing optimum layer combinations and structural design of each layer component.

2.1 Stress analysis for flexible pavement

The simplest way to solve flexible pavement problem is to consider the pavement system as a homogeneous half-space with infinite area and depth, the original theory developed by Boussinesq (1885)[1] was based on a concentrated load applied on an elastic half space, however, this assumption is not realistic. First of all, the subgrade soils are not elastic even though under the moving traffic load some of the deformation is recoverable; secondly, flexible pavement is a layered system with better materials at the top; each layer has different material characteristics, such as modulus of elasticity E and Poisson's ratio ν.

The Elastic Layered Theory developed by Burmister (1943) is a realistic solution to treat the flexible pavement as a multi-layered system. Some basic assumptions when applying this theory are:[2]

1. Pavement layers extend infinitely in the horizontal direction.

2. The bottom layer (usually the subgrade) extends infinitely downward.

3. Materials are not stressed beyond their elastic ranges.

4. Continuity conditions are satisfied at the layer interfaces.

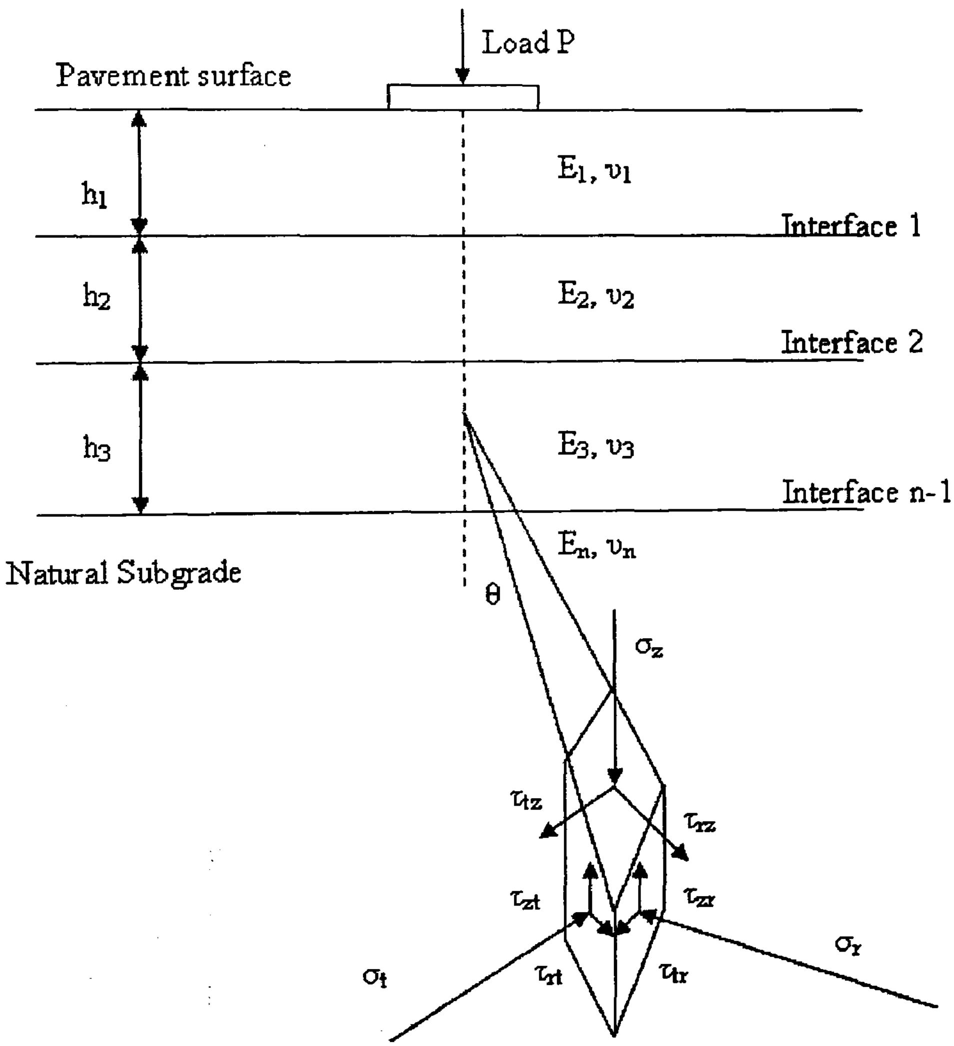


Fig. 3 Flexible pavement stress analysis[(2)]

From the above illustration, according to the mechanistic theories, at any point within the layered system, there are 9 stresses: 3 normal stresses σ_z, σ_r, σ_t and 6 shear stresses, τ_{tz}, τ_{zt}, τ_{tr}, τ_{rt}, τ_{rz}, τ_{zr}, and the following relationships exist:

$\tau_{tz} = \tau_{zt}$

$\tau_{tr} = \tau_{rt}$

$\tau_{rz} = \tau_{zr}$

Typically, flexible pavement is designed to put materials with higher modulus of elasticity at the upper layers in order to reduce stresses and deflection transferred to the subgrade layer. Basing on the elastic body theory, the basic stress and strain relationship should be satisfied within this axially symmetrical system.

2.1.1 Vertical compressive stress

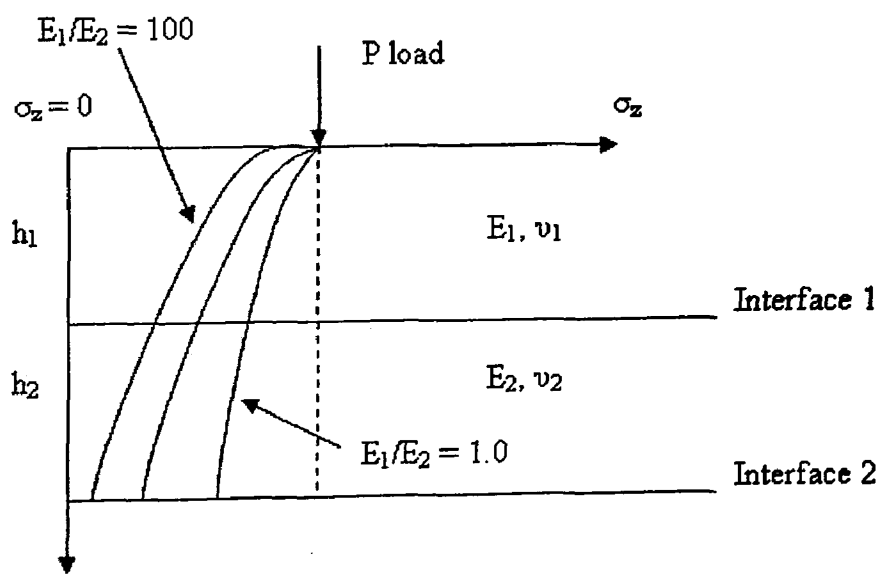


Fig. 4 Distribution of vertical compressive stress with Modulus of Elasticity[2]

From above Fig. 4, vertical compressive stress at the top surface $\sigma_z = -P$, where P stands for the external load applied on the unit circular area. According to the graph, it is self-explanatory that to increase the modulus of elasticity, or in other words, to increase the rigidity of the surface layer can reduce vertical compressive stress transferred to lower layers. This explains why we design the flexible pavement as materials with higher modulus of elasticity at the upper layers. On the other hand, under same traffic loading conditions, soft asphalt mixtures or excessive air voids or inadequate compaction in any pavement layer are tend to be more susceptible to rutting damages.[3]

From the Fig. 5 below it could also be seen that σ_z will be reduced when h_1/h_2 increase or $1/h_2$ decrease. That means if h_2, the thickness of the base course layer should remain constant, to increase h_1, the thickness of surface layer will bring the vertical compressive stress down.

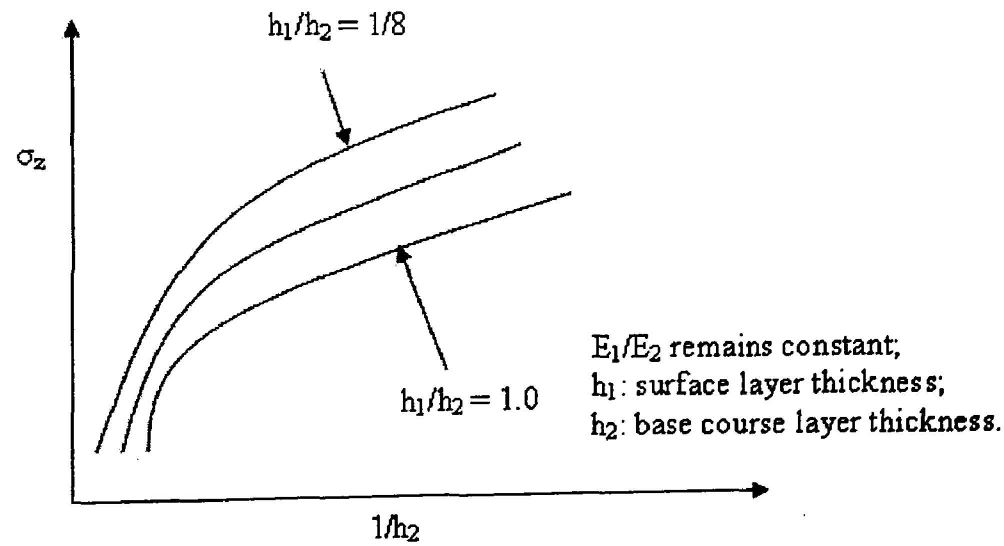


Fig. 5 Distribution of vertical compressive stress with layer thickness[2]

2.1.2 Horizontal shearing stress

Fig. 6 simply illustrates the distribution of shearing stress over layers. From the figure, the maximum τ_{rz} occurs approximately at the midpoint of the surface layer; and at the top surface horizontal shear stress $\tau_{rz} = 0$. To increase the modulus of elasticity of the surface layer will cause the shearing stress to be increased. Inadequate shear strength of the asphalt mixture will induce two-dimensional movement under heavy traffic,[3] and this will induce two-dimensional rutting damage. As a major type of distress in asphalt pavement, rutting appears as a surface depression in the wheel paths. Rutting damage relates to material properties and loading, major portion of the rutting occur within the asphalt layer.

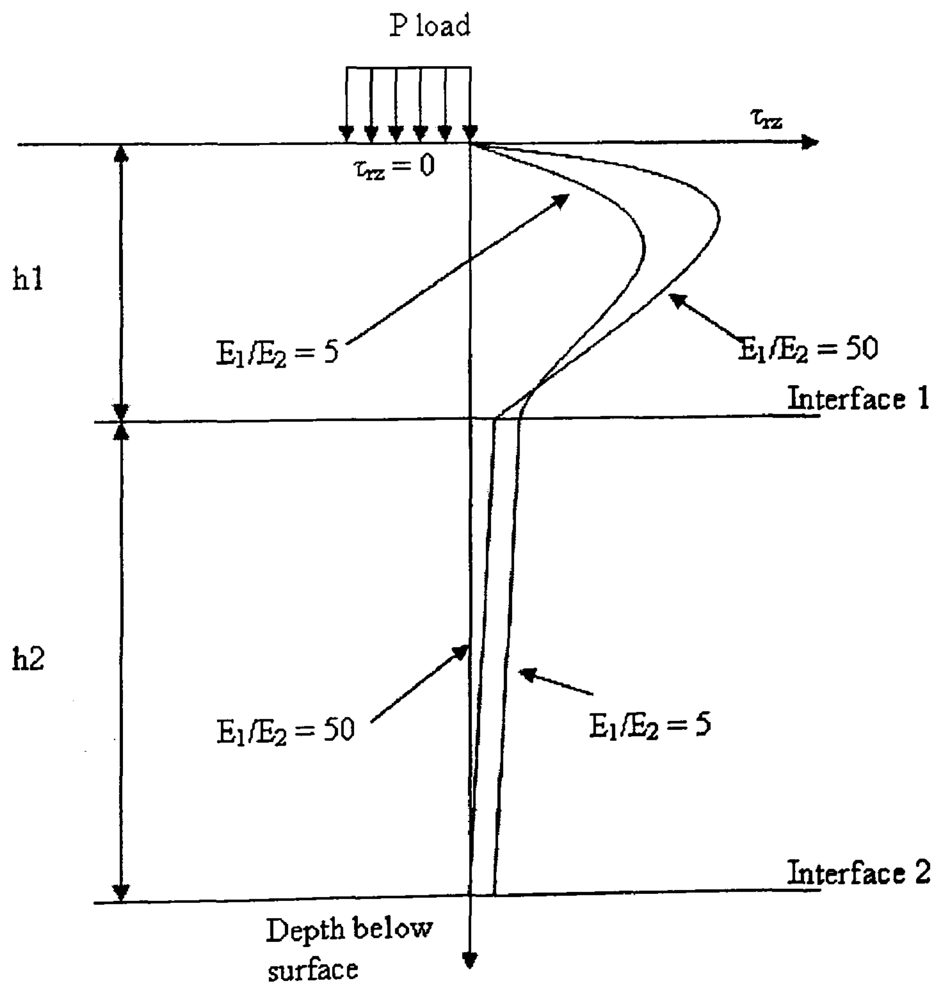


Fig. 6 Distribution of horizontal shear stress with Modulus of Elasticity[2]

2.1.3 Horizontal tensile stress

Horizontal tensile stress would normally be developed at first interface, right underneath the asphalt layer. The simplified distribution pattern is as shown below Fig. 7.

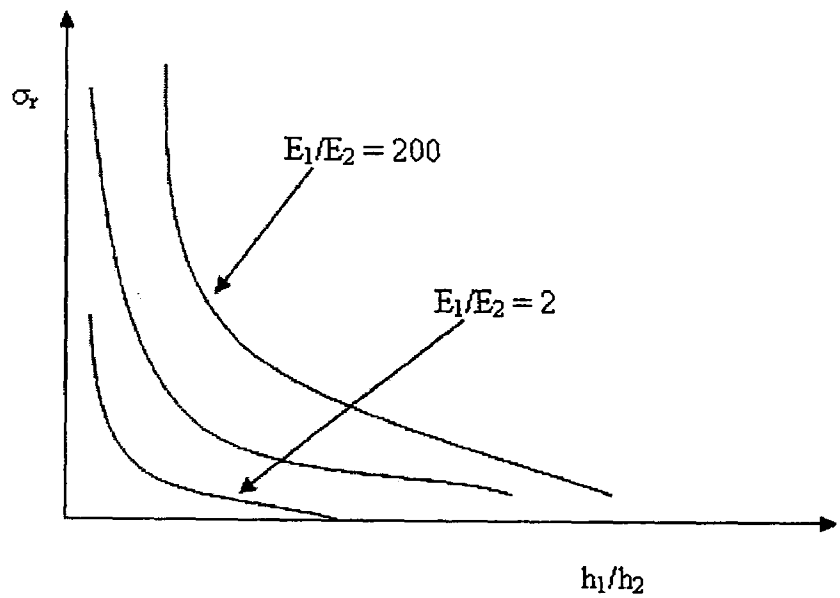


Fig. 7 Distribution of horizontal tensile stress with Modulus of Elasticity and layer thickness[2]

The information we can get from Fig. 7 is: to increase E_1/E_2 or reduce h_1/h_2, σ_r will significantly increase; but when asphalt layer thickness approaches zero, there would be no horizontal tensile stress exists.

When horizontal tensile stress at the bottom of the asphalt layer exceed its tensile strength under repeated loading applications, cracks will form and propagate to the surface, this type of distress is called fatigue cracking which is the result of repeated bending of asphalt layer under traffic.[3]

Many factors contribute the forming of fatigue cracking, material properties and layer thickness, traffic load and number of load repetitions, temperature and environmental conditions. Studies have shown that 3 to 5 inches thick asphalt is most susceptible to fatigue cracking damage.[3] The proper thickness of asphalt layer must be either as thin as practical or as thick as possible. The flexible pavement with very thin asphalt layer tend to have less problem with fatigue cracking since compressive effect tend to be more significant.

2.1.4 Vertical deflection

Studies have shown that about 70 to 95% of the surface deflection is the function of the elastic compression on the subgrade layer.[3] Based on mechanical theories about stress/strain relationship, in order to minimize surface deflections, vertical compressive stress should be kept low. From the above analysis about vertical compressive stress, to increase E_1 or h_1 at the surface layer when thickness of base course layer remains the same can all help to reach this goal.

Excessive vertical deflection on subgrade layer will cause rutting damage. Rutting is an incremental plastic damage throughout the layered system. National Cooperative Highway Research Program conducted by State of Illinois suggested that the total rutting in the pavement structure is to sum up the permanent deformation at asphalt layer, granular base layer and subgrade layer. Currently there are several models available for estimating plastic strain within asphalt layer such as Ohio State Model,[3] $\epsilon_p = a_1 \times (\epsilon_r)^{a2} \times N^{a3}$ and Illinois Model,[3] $\epsilon_p = a_1 \times \epsilon_r \times T^{a2} \times N^{a3}$. In both models, plastic strain ϵ_p is a function of ϵ_r, the resilient strain (also called recoverable strain) and N, number of repetitions of loading. In Illinois model, T, the temperature effect is also considered when estimating plastic strain in asphalt layer, since as a viscoelastic material, strain in asphalt is a time- and temperature-dependent value. 'a_n' in above equations represent calibration factors.

2.2 Stress analysis for rigid pavement

Stresses in rigid pavement are resulted from a variety of reasons: loading from the wheels, temperature or moisture changes, and deflection or deformation of the base course layer or subgrade layer.

Two factors will affect the decision on whether to use plain concrete or reinforced concrete pavement: (1) spacing of joints and (2) whether a base course is used over natural subgrade layer. In most cases, if the slab length is less than 20 feet and a cement-treated base course is used; plain concrete pavement would be a cost-saving choice.[2] When the joint spacing increased to more than 40 feet, wire mesh reinforcement should be used for crack control purpose only.[2]

Fig. 8 as shown below illustrates the general layout of rigid concrete pavement. Dowel bars are used at transverse joints to act as a load-transferring media; dowel bars normally are heavy steels and will be placed at relative close intervals; they should also be smooth and lubricated at one end to allow slab movements.

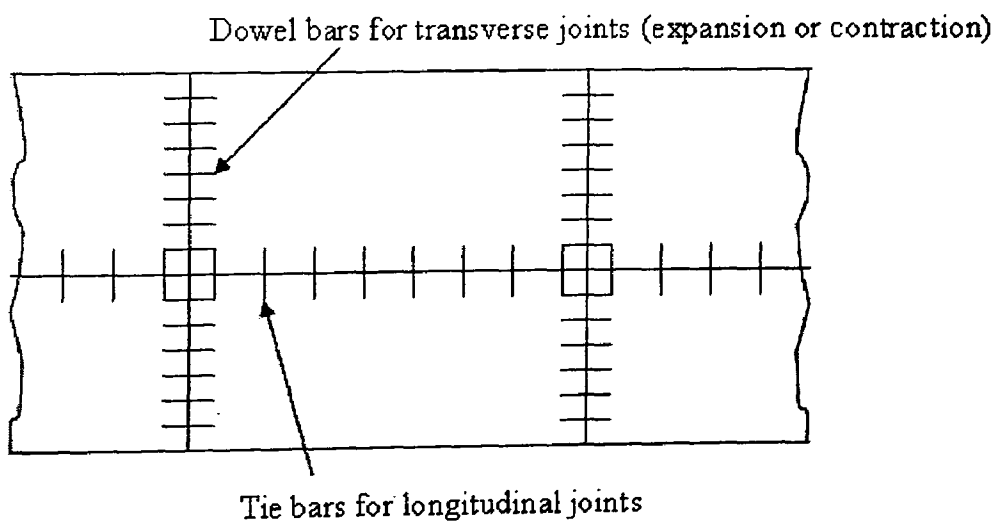
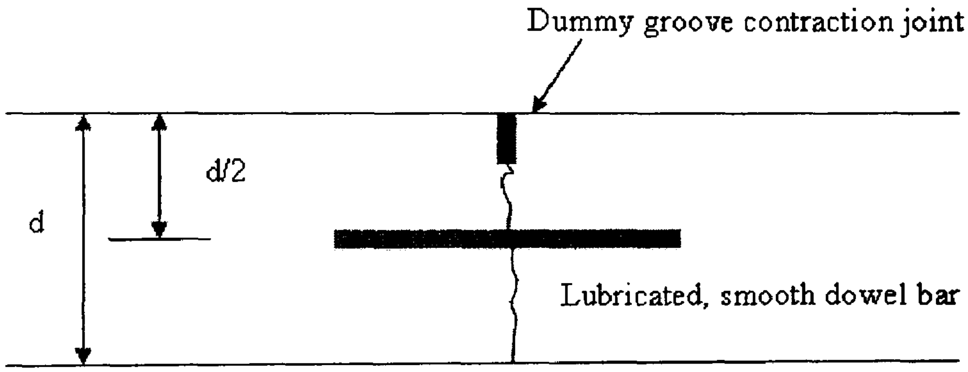


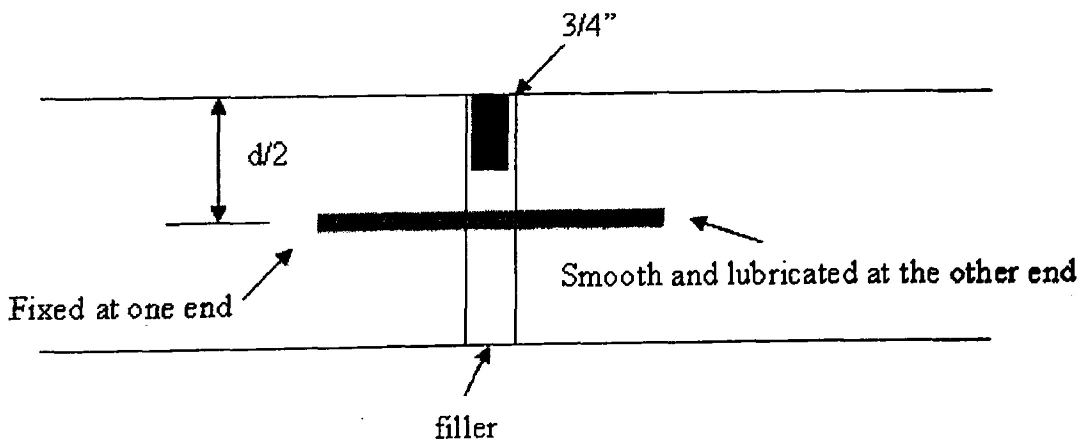
Fig. 8 General layout of rigid concrete pavement[1]

Tie bars are used along longitudinal joints, the function of the tie bars is tying two slabs together to improve load transfer and prevent joints from opening and/or faulting.[8] As recommended in the design manual of Illinois Department of Transportation, tie bar must be deformed and hooked and must be firmly anchored into the concrete; tie bars should be placed at relatively large intervals.

Basically there are four groups of joints for rigid concrete pavements: contraction joints, expansion joints, construction joints and warping joints.[2] They are illustrated in Fig. 9 and 10.

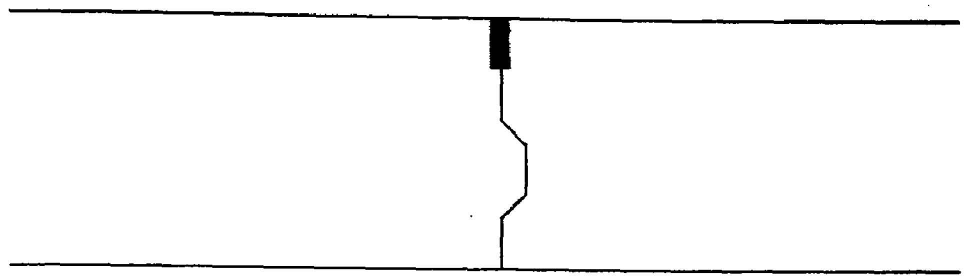


(a) Contraction Joint

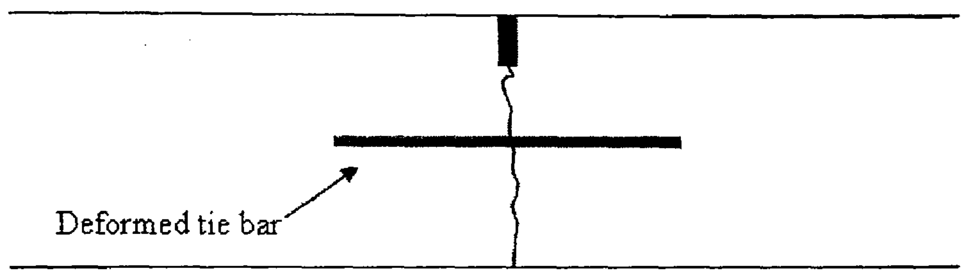


(b) Expansion Joint

Fig. 9 Contraction joint (a) and Expansion joint (b)[2]



(c) Construction Joint



(d) Warping Joint

Fig. 10 Construction joint (c) and Warping joint (d)[2]

Each kind of joint has its specific functions. The intention of applying contraction joints is to relieve the horizontal tensile stress due to contraction and curling/warping of concrete slab. Normal joint opening is about 0.25 inch. Dowel bar may or may not be used at the contraction joints location.[2] When dowel bars are not used, load transfer can be achieved by grain interlock of the lower portion of the slab.

Expansion joints should allow concrete to expand, joint opening is about 0.75 inch. Since the gap is relatively big, infiltrations of subgrade materials into the expansion joints may cause the joints to expand, and sometimes, inadequate load transfer may cause the pumping of material underneath the concrete slab, thus expansion joints are used more often for airport pavements.

Construction joints are also about 0.25 inch wide, the function is to transit from old concrete to the construction of new concrete. It can be keyed construction joints as shown in Fig. 10 or butt type construction joints which are more commonly used.

Warping joints are used to control longitudinal cracks to occur along the centerline of the pavement. Tie bars should be used to connect two pieces of slab, load transfer can be achieved by the grain interlock of the lower portion of the slab.

2.2.1 Curling stress

When the concrete pavement is exposed to the sun during the day, the top of the slab warms faster than the subgrade, if the temperature gradient occurs through the depth of the slab, curling stress will be induced.

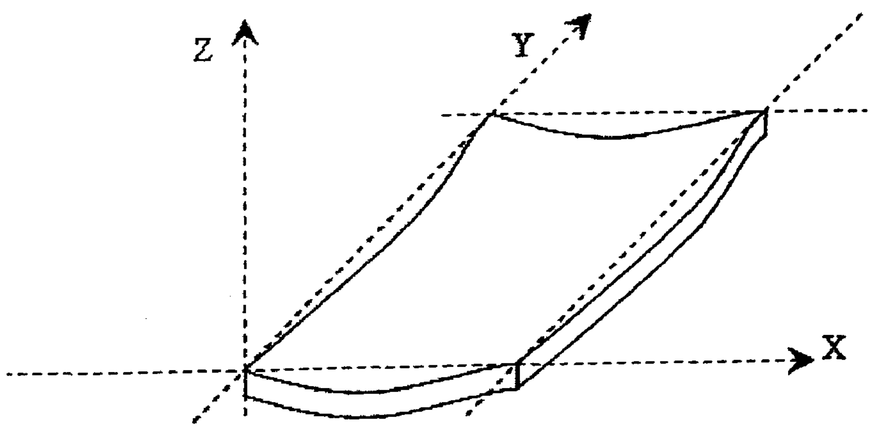


Fig. 11 Curling effect due to temperature gradient[2]

Regardless the slab to curl upward or downward, the weight of the slab will always tend to hold the slab in its original position, thus the stresses are induced. Based on the work by Westergaard and Bradbury, curling stress at the edge and any point inside the slab can be calculated by the following equations.[1]

For edge stresses:

$$\sigma = (C \times E \times \alpha_t \times \Delta t) / 2 \qquad \textbf{(Eq. 2.1)}$$

For interior stresses:

$$\sigma_y = [(E \times \alpha_t \times \Delta t) / 2\,] \times [(C_y + \upsilon \times C_x) / (1 - \upsilon^2)] \qquad \textbf{(Eq. 2.2)}$$

$$\sigma_x = [(E \times \alpha_t \times \Delta t) / 2\,] \times [(C_x + \upsilon \times C_y) / (1 - \upsilon^2)] \qquad \textbf{(Eq. 2.3)}$$

Within the above interior curling stresses equations, C_x and C_y are curling stress coefficients. They are fixed values based on the chart provided by Bradbury.[1] The first equation measures interior curling stress due to bending in y direction, and the second equation measures interior curling stress due to bending in x direction; E is the modulus of elasticity of concrete (lbs/sq. inch); υ, the Poisson's ratio; α_t, coefficient of thermal expansion (inch/inch/°F) and Δt is the temperature differential (°F).

As a major distress in rigid pavement, transverse cracking is the combined effect of external load and temperature. When a heavy load is near the longitudinal edge of the slab, midway between the transverse joints, a severe tensile bending stress will occur at the bottom of the slab. The situation will become worse when the top of the slab is warmer than the bottom of the slab. Repeated heavy loading under this condition would result fatigue damage along the bottom edge of the slab, which will induce a transverse crack and propagate to the surface of the pavement. When top of the slab is cooler than the bottom, repeated heavy loading would result in fatigue damage at the top of the slab, which eventually will induce transverse cracks initiated from the surface of the pavement.[3]

Many factors contribute to the forming of fatigue cracking, not only slab thickness and concrete strength, joint spacing also play a big role in the formation of transverse cracks. The effective way to minimize transverse cracking is to increase slab thickness, reduce joint spacing or use a widened slab, and if possible, to provide a stabilized base layer.

2.2.2 Frictional stress

From the above discussion, we understand that curling stresses would occur with temperature gradient through the depth of the concrete slab; while uniform temperature changes will cause the concrete slab to contract or expand, and friction stresses are resulted from frictional resistance in-between the slab and subgrade. Frictional resistance is critical for long slabs (normally refer to the slabs more than 100 feet long),[2] while for short and average length slabs (less than 40 feet),[2] frictional resistance value is not considered as significant.

The purpose of analyzing frictional stress is to determine the spacing between contraction joints and the number of tie bars. For pavement with long joint spacing, steel wire mesh must be used to take care of the tensile stress induced by friction.

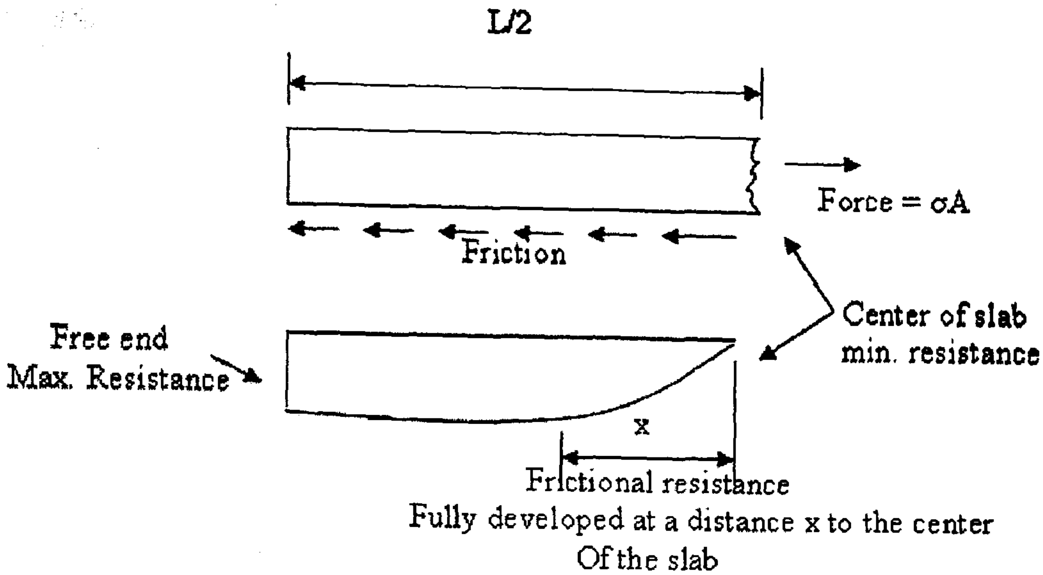


Fig. 12 Distribution of frictional resistance[1]

According to the above illustration, Kelly has suggested an equation to measure the frictional stress:[1]

$\sigma_c = \gamma_c \times L \times f_a / 2$ (Eq. 2.4)

Where σ_c is the frictional stress (lbs/sq. inch); γ_c is unit weight of concrete (lbs/cubic inch); L: the length of slab and f_a is the average coefficient of friction between slab and subgrade layer, in most occasions $f_a = 1.5$.

2.2.3 External loading stress

Westergaard had extraordinary contributions on the pavement thickness design.[2] He developed a set of equations for determining stresses and deflections in concrete pavements under different loads conditions: at the interior of the slab, at free edges and at the corners. These equations allow the prediction of critical stresses under extreme conditions of loading cases.[2]

For corner case,

$$\sigma_c = (3 \times P/h^2) \times [1 - (a_1 \times 1.414/\ell)^{0.6}] \qquad \textbf{(Eq. 2.5)}$$

For interior case,

$$\sigma_i = (0.316 \times P/h^2) \times [4 \times \log_{10}(\ell/b) + 1.069],\ b = (1.6 \times a^2 + h^2) - 0.675h \qquad \textbf{(Eq. 2.6)}$$

For edge case,

$$\sigma_e = (0.572 \times P/h^2) \times [4 \times \log_{10}(\ell/b) + 0.359],\ b = (1.6 \times a^2 + h^2) - 0.675h \qquad \text{(Eq. 2.7)}$$

Where P is the external load; ℓ is the radius of relative stiffness of concrete to subgrade,(1) $\ell = [(E \times h^3) / (12 \times (1 - \upsilon^2) \times k)]^{0.25}$. ℓ is a function of E, the modulus of elasticity of concrete; υ, the Poisson's ratio of concrete; k: the modulus of subgrade reaction and h: the thickness of concrete slab; a_1 is called contact radius, it is the distance from the point where load applies to corner of slab; a is the distance from the point where load applies to the point tangent to slab edge.

Basing on Westergaard theory, in 1950's, Pickett and Ray developed a set of charts called 'influence charts' which allow analyzing stress and deflection more conveniently.(1) The charts were developed basing on two assumptions: (1) pavement built on dense liquid foundation (2) pavement itself is an elastic solid. Liquid foundation is also called Winkler foundation, liquid foundation assumes under vertical force, vertical deflection at node i is independent of deflections at neighboring nodes.

The procedure to calculate stress and deflection is relative simple by using influence chart. The first step is to find the size of tire imprint by the given tire pressure; second step is to trace the tire imprints on the influence chart with using appropriate radius of relative stiffness as a scale factor and the third step is to count number of blocks within the imprint area and finally solve a set of equations to find deflection and stress.

2.2.4 Dowel bar stress

Upon above discussion, we understood that dowel bars are used at transverse joints to transfer loads when "grain interlock" is hardly achieved or heavy loads shall be applied, dowel joints are normally applied for joint openings from 0.04 to 0.25 inch wide.

The design of dowel bars are based on the analysis of concrete bearing stresses under dowel bars. Following assumptions should be applied in order to simplify the case: (1) it is assumed that dowel bars are perfectly aligned and free to move and (2) characteristics of subgrade materials are overlooked; dowel bar diameter and length design is based on pavement thickness only.

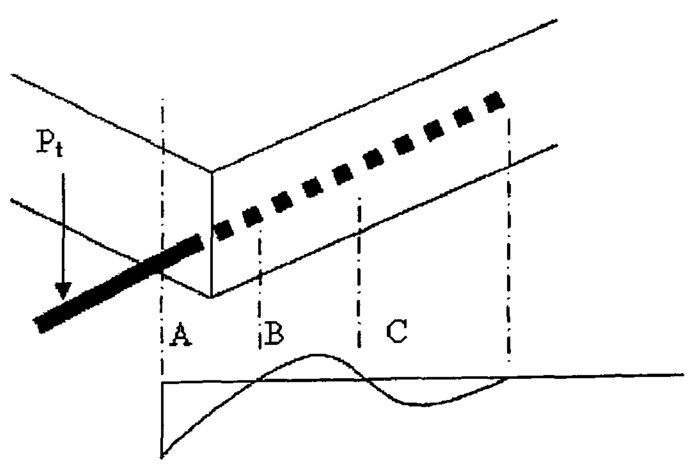


Fig. 13 Deflected shape of dowel bar under load[2]

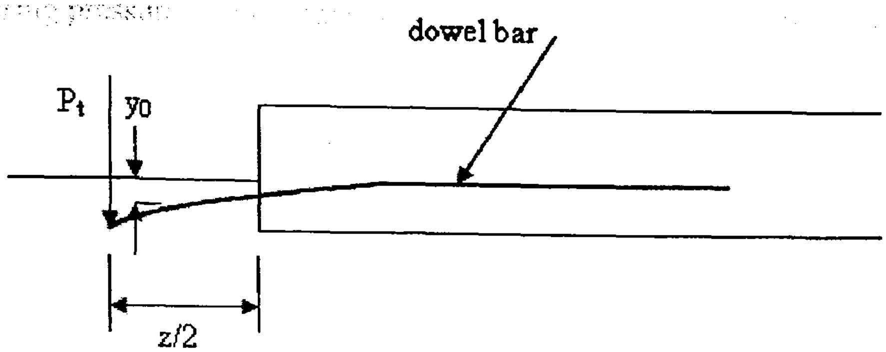


Fig. 14 Dowel bar deformation under load[1]

Based on the above assumptions, Timoshenko, Friberg (1940)[1] developed following equations to estimate bearing stress on concrete underneath the dowel. For a joint opening z, if the dowel bar is subjected to external load P_t, the moment at the dowel-concrete interface is $M_0 = -P_t \times z/2$; the deflection of the dowel at the joint can be estimated by:[1]

$$y_0 = P_t \times (2 + \beta \times z)/ (4 \times \beta^3 \times E_d \times I_d) \qquad \text{(Eq. 2.8)}$$

Where β is the relative stiffness of dowel bar versus concrete,

$$\beta = [(K \times d)/(4 \times E_d \times I_d)]^{1/4} \qquad \text{(Eq. 2.9)}$$

At above equation, where K is the modulus of dowel support, K value is also called steel-concrete K value, it expresses the bearing stress in the concrete developed under a unit deflection of dowel bar.[9] Due to the difficulty of establishing the value of K theoretically, K value is normally selected from 3×10^5 to 1.5×10^6 pci.[1] 'd' is the diameter of dowel bar; and E_d, I_d are modulus of elasticity of the dowel bar and the moment of inertia of the dowel bar respectively.

The actual bearing pressure on the concrete at the joint face $\sigma = K \times y_0$ has to be compared with allowable bearing stress of concrete since the concrete is much weaker than the steel. The allowable bearing stress of concrete can be estimated by $f_b = [(4-d) \times f_c']/3$ (American Concrete Institute, 1956),[1] d is the dowel diameter in inches and f_c' is the ultimate compressive strength of concrete. The load transferring capacity of a single dowel is determined by allowable bearing stress of concrete, the load transferring capacity equals to the actual bearing pressure σ divided by allowable bearing stress f_b and times 100.[2] Load transferring capacity also depends upon the length of embedment. Tests indicated that for a 0.75-inch diameter dowel, the length of embedment should be about 8 times of diameter.[2] For bigger size dowels, the length of embedment decreases.

Since dowel bars work as a group, as per the study by Friberg (1940),[1] when the load is applied at a joint, the dowel bars right under the applied load carry a major portion of load. Other dowel bars carry a progressively reduced amount of load, this result a portion of load can be transferred into the next slab.

The Fig. 15 below shows that the load carrying capacity of a group of dowels is equal to the sum of loads carried by each dowel. And in the case of superimposing effect of two wheels, similar as shown above, the load carried by the dowel at point A should be the load carried by the dowel at A due to load at A plus the load carried by the dowel at A due to load at B. From the point where load applies, load transferring capacity will reduce to zero at a distance equals to 1.8 times radius of relative stiffness of concrete versus subgrade.

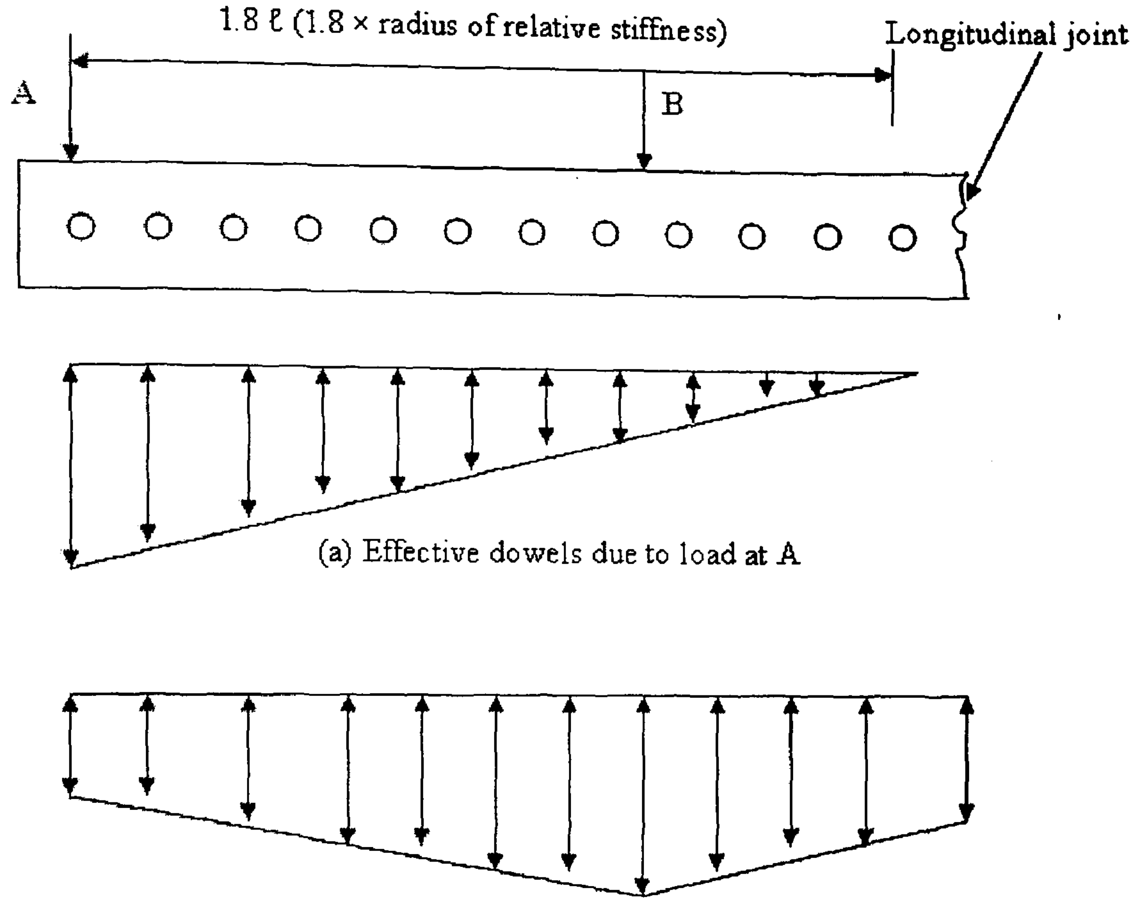


Fig. 15 Loads distribution on dowel group[1]

Joint faulting is mainly caused by repeated heavy loading crossing transverse joints; it appears as base material build-up at one slab corner, and loss of support at adjacent corner.[3] It has adverse impact on riding quality. For pavements with dowel bars as load transferring device, faulting is assumed mainly caused by erosion of the concrete around dowel bars under repeated traffic loading. Since the concrete bearing stress under dowel can be calculated, faulting can be predicted by calibrated equation.[3] There are many ways to reduce the formation of joint faulting, either to increase the load transfer efficiency by increase the diameter of the dowel bars or to reduce the joint spacing, or move the heavy load from slab corners by widening the slab or provide stabilized base course.

2.2.5 Reinforcing steel and tie bar stress

As per above mentioned, the design of dowel bars is conducted by analyzing the concrete bearing stress and actual load transferring capacity of dowel; while for reinforcing steel wire mesh and tie bar, the design is mainly based on the analysis of frictional stress induced by uniform temperature changes.

Wire mesh reinforcement will not add structural capacity to the pavement slab, it can tie cracked concrete pavement together and control crack developments in order to allow effective load transferring efficiency through grain interlock. Basing on the assumption that tensile stress along the slab length induced by the frictional stress is carried by wire mesh reinforcement, the area of steel can be approximately estimated.

Similar to the design of wire mesh reinforcement, tie bars are used along longitudinal joints to tie pavement slabs together, the approximate area of tie bars required for connecting longitudinal joints can be estimated by the same concept that tensile stress along the slab lane width induced by the frictional stress is carried by the tie bar steels.

The amount of steel required depends upon the size of the slab. For short slabs, steel could be reduced or omitted. When the pavement is designed without transverse joint, as it is called continuously reinforced slab, adequate steels must be provided to ensure cracks to be tightly closed.

2.2.6 Thickness design criteria

From the above discussion we could see that stress-inducing factors in rigid concrete pavement are quite complex, while some factors could be ignored for thickness design. Besides subgrade type, curling stress will not be taken into consideration when deciding thickness of pavement. Herewith we applied the conclusive statement by E. J. Yoder:[2] "Joints and steel are used to relieve and/or take care of warping (curling) stresses, and the design, then, is based upon load alone when considering thickness. This principle is so important that it must be clearly understood by the designer. Recall that a joint is nothing more than a designed crack".

Department of Transportation of individual states has developed their own minimum pavement thickness criteria. For instance, the Colorado Department of Transportation has stated in its Pavement Design Manual that the minimum thickness of Portland Cement concrete pavement is 8 inches for traffic greater than 1 million 18K ESAL and 6 inches for traffic less than or equal to 1 million 18K ESAL.[4]

2.3 Application of Elastic Layered Theory in flexible pavement design

Flexible pavements are layered systems with better materials on the top and being represented by a homogeneous mass is not realistic. Burmister's layered theory is more appropriate when we try to simulate the actual stress state of flexible pavements.[1] Burmister first developed solutions for a two layered system which is more suitable for a full-depth asphalt construction, and then he extended the solutions to a three-layered system. With the advent of computer software, the elastic layered theory can be applied to a multilayered system with any number of layers. The application of elastic layered theory on solving flexible pavement problem will be briefly introduced as below.

In order to use elastic layered theory to solve flexible pavement problem, a stress function Φ should be defined for each layer in the system. Φ is a function of r, z, H and vertical load, where r is the radial distance from the study point to the external load; z is the depth from the study point to the surface; H is the distance from the surface to the upper boundary of the lowest layer. Φ also contains integration constants; each layer has a stress function Φ with different integration constants.

Once the stress function Φ is defined, stress and strain within the layered system can all be expressed by the stress function Φ; the stress function can also allow stress and strain to satisfy all elastic stress/strain relationships. By the classical theory of elasticity, stress function should satisfy the governing differential equation: $\blacktriangledown^4 \Phi = 0$, where $\blacktriangledown$ is called Laplace operator, $\blacktriangledown^2 = \partial^2/\partial r^2 + (1/r)\,\partial/\partial r + \partial^2/\partial z^2$.

To apply the continuity conditions basing on the assumption that all layers are fully bonded with the same vertical compressive stress, horizontal shear stress, vertical displacement and radial displacement at every point along the layer interface, the following relationship should be satisfied: $(\sigma_z)_i = (\sigma_z)_{i+1}$; $(\tau_{rz})_i = (\tau_{rz})_{i+1}$; $(\omega)_i = (\omega)_{i+1}$; $(u)_i = (u)_{i+1}$. And if the ith layer interface is unbonded or frictionless, the continuation of shear stress and radial displacement must be zero, $(\sigma_z)_i = (\sigma_z)_{i+1}$; $(\tau_{rz})_i = 0$; $(\omega)_i = (\omega)_{i+1}$; $(\tau_{rz})_{i+1} = 0$.

At the upper surface, as we have discussed before under Fig. 4 and Fig. 6, vertical compressive stress under the external load $(\sigma_z)_1$ = -P, where P is the vertical load applied on a unit area; horizontal shear stress $(\tau_{rz})_1$ = 0. With the application of these boundary and continuity conditions, unknown integration constants for each layer can be found, and stress and displacement can be determined.

Comparing with Boussinesq's elastic homogeneous half-space theory, Burmister's elastic layered theory is more practical for analyzing flexible pavement problems. Based on two-layered and three-layered system, various charts were developed for determining pavement responses conveniently. With the application of elastic layered theory in multilayered pavement system, available flexible pavement stress analysis software such as *KENLAYER* developed by Hung (1985) [1] and his colleagues makes multilayered pavement system stress analysis problems much easier to solve.

2.4 Application of Finite Element Method in rigid pavement design

Finite element method is a numerical technique for solving problems. By finite element method, a continuous physical problem will be transformed into a discretized finite element problem. The following paragraphs elaborate how finite element method can be applied in the case of concrete pavement design.

2.4.1 Discretize the continuum

For rigid concrete slabs, the shape of rectangular is selected to discretize the whole concrete slab into numerous finite elements.

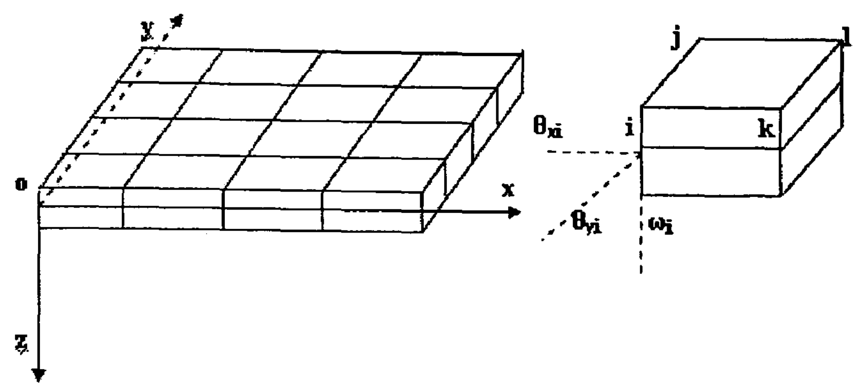


Fig. 16 Discretize rectangular concrete slab(1)

Each nodal point (i, j, k, l) at each rectangular element has three degrees of freedom ($\theta_x, \theta_y, \omega$): θ_x, the rotation about x axis; θ_y, the rotation about y axis and ω is the deflection along z axis. Nodal displacements for each rectangular element can be represented by the matrix as shown below which contains 12 items.

$$\{\delta\}^e = [\delta_i \ \ \delta_j \ \delta_k \ \delta_l] = [\omega_i \ \theta_{xi} \ \theta_{yi} \ \ \omega_j \ \theta_{xj} \ \theta_{yj} \ \omega_l \ \theta_{xl} \ \theta_{yl} \ \omega_k \ \theta_{xk} \ \theta_{yk}] \qquad \text{(Eq. 2.10)}$$

2.4.2 Select interpolation functions

A polynomial containing x and y coordinate will be chosen, the degree of the polynomial will be based on the number of nodes assigned to the element. Since each elements contains four nodes, a forth degree polynomial is assigned to represent vertical deflection ω, since for slender slabs under flexural bending, displacements, internal forces and stresses can all be expressed in term of ω (θ_x = $-d\omega/dy$ and θ_y = $d\omega/dx$).

$$\omega = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3 \quad \text{(Eq. 2.11)}$$

We randomly pick one element of size a by b, if the center of the rectangular element is defined as (0, 0) of the xyz coordinate system, the coordinate reading of each node within the element can be found; and by inputting (x, y) coordinate of each node, deflection ω at each node can all be expressed by above polynomial with a_n as unknown coefficients. For instance, (–a/2) and (–b/2) are the x and y coordinate for the nodal point i, deflection ω, rotation θ_x and θ_y at point i can be expressed as three functions as shown below:

$$\omega i = a_1 + a_2 \times (-a/2) + a_3 \times (-b/2) + a_4 \times (a^2/4) + a_5 \times (ab/4) + a_6 \times (b^2/4) + a_7 \times (-a^3/8) + a_8 \times (-a^2b/8) + a_9 \times (-ab^2/8) + a_{10} \times (-b^3/8) + a_{11} \times (a^3b/16) + a_{12} \times (ab^3/16) \quad \text{(Eq. 2.12)}$$

$$(\theta_x)i = -\,d\omega/dy = -\,[a_3 - a_5 \times (a/2) - a_6 \times b + a_8 \times (a^2/4) + a_9 \times (ab/2) + a_{10} \times (3b^2/4) - a_{11} \times (a^3/8) - a_{12} \times (3ab^2/8)] \quad \text{(Eq. 2.13)}$$

$$(\theta_y)i = d\omega/dx = a_2 - a_4 \times a - a_5 \times (b/2) + a_7 \times (3a^2/4) + a_8 \times (ab/2) + a_9 \times (b^2/4) - a_{11} \times (a^2b/8) - a_{12} \times (b^3/8) \quad \text{(Eq. 2.14)}$$

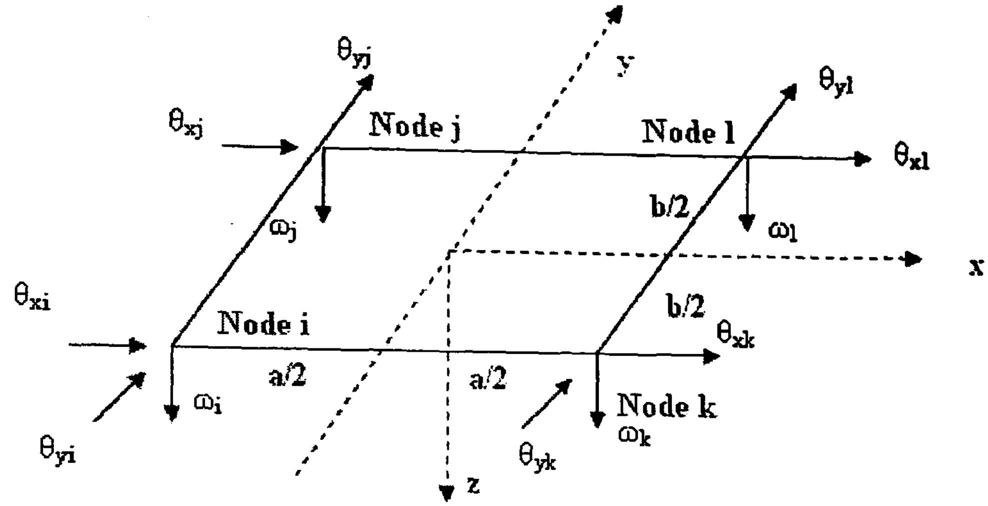


Fig. 17 Nodal displacement[6]

And for the rest three nodes j(-a/2, b/2), k(a/2, -b/2) and l(a/2, b/2), similarly, there are three polynomials to represent (θ_x, θ_y, ω) for each node respectively. For such a rectangular element containing four nodes, there are totally 12 equations to represent the displacement matrix, in order to simplify, the displacement matrix can be expressed by $[\delta] = [C]\,[a]$, where [C] is a 12 by 12 matrix containing a and b (size of the element), and [a] contains a_1 to a_{12} twelve unknown coefficients.

2.4.3 Define the element property

Stiffness matrix for the element shall be established at this step. Stiffness is the resistance of an elastic body to the deflection induced by an external load. Stiffness can be measured by $k = P/\delta$, where P is the external force and δ represent the displacement. When both force and deflection are vectors, stiffness matrix can be expressed as a function of two major characteristics of material properties: E, the modulus of elasticity and υ, the Poisson's ratio.

For elastic body, the following stress/strain relationships exist:

$\sigma_x = E \times (\epsilon_x + \upsilon \times \epsilon_y) / (1-\upsilon^2)$ (Eq. 2.15)

$\sigma_y = E \times (\epsilon_y + \upsilon \times \epsilon_x) / (1-\upsilon^2)$ (Eq. 2.16)

$\tau_{xy} = E \times \gamma_{xy} / [2 \times (1+\upsilon)]$ (Eq. 2.17)

And

$\epsilon_x = du/dx = -z \times d^2\omega / dx^2$ (Eq. 2.18)

$\epsilon_y = d\nu/dy = -z \times d^2\omega / dy^2$ (Eq. 2.19)

$\gamma_{xy} = du/dx + d\nu/dy = -2z \times d^2\omega / dxdy$ (Eq. 2.20)

Where ϵ_x, elastic strain along x direction equals du/dx, the deformation per unit length of the object, du represents the size changes along x direction and dx represent the size of the object along x direction. $d\nu$ and dy represent size changes and size of the object along y direction respectively.

2.4.3.1 Stiffness of concrete slab

For slender slabs subject to flexural bending as shown below in Fig. 18, bending moment can be expressed in the following form:

$M_x = \int_{(-t/2,\, t/2)} z\sigma_x dz$ (Eq. 2.21)

$M_y = \int_{(-t/2,\, t/2)} z\sigma_y dz$ (Eq. 2.22)

$M_{xy} = \int_{(-t/2,\, t/2)} z\tau_{xy} dz$ (Eq. 2.23)

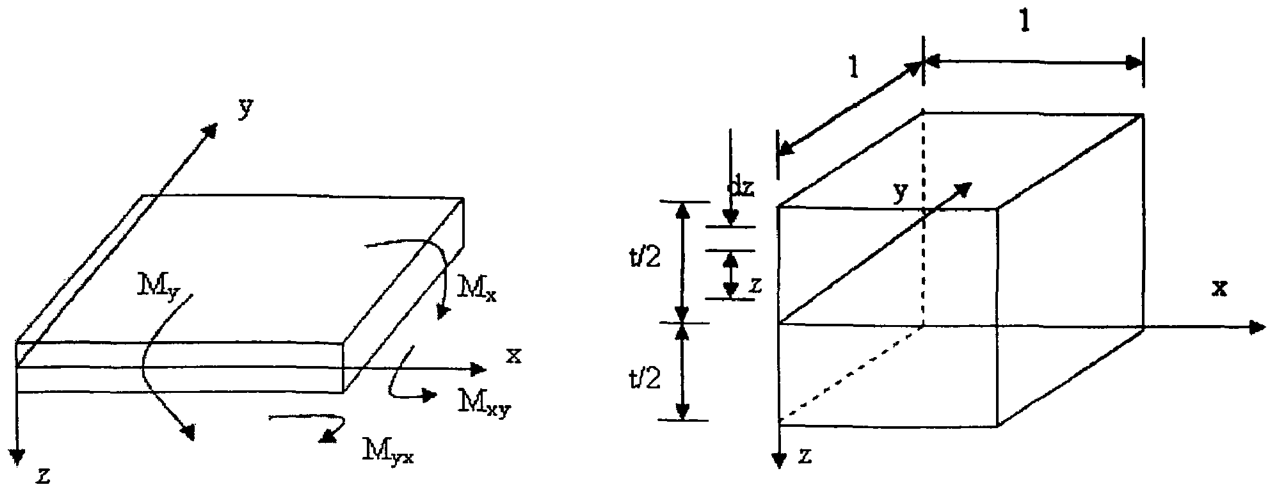


Fig. 18 Slender slab subjected to bending moment

Integrating the above moment equations, the moment matrix can be expressed as [M] = [D] [ψ],[6] where [D] is called modulus of rigidity of the slab,[1] it is comprised of E, modulus of elasticity, υ Poisson's ratio and t, thickness of the slab.

$$[D] = [(E \times t^3) / (12 \times (1 - \upsilon^2))] \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \qquad \textbf{(Eq. 2.24)}$$

$$[\psi] = \begin{bmatrix} -d^2\omega/dx^2 \\ -d^2\omega/dy^2 \\ -d^2\omega/dxdy \end{bmatrix} \qquad \textbf{(Eq. 2.25)}$$

Element stiffness matrix can always be expressed in the following form:[6]

$$[K_1]^e = \iint [B]^T [D] [B]\, dxdy \qquad \textbf{(Eq. 2.26)}$$

Where, [B] is called displacement differentiation matrix. It is obtained by differentiation of displacement expressed through shape functions and nodal displacements. For better understanding of the form of [B], $[B] = [Q]\,[C]^{-1}$ and $[B]^T = [Q]^T\,[C]^{-1T}$, as explained before, [C] is a 12 by 12 matrix containing a and b (size of the element), [Q] is a 3 by 12 matrix, it equals inputting coordinate reading at one node into $[\psi]$. Thus [K] is a matrix containing only material properties, i.e. E and υ.

In order to simulate real world situation, the stiffness of pavement should include stiffness of concrete slab, foundation and the joint: $[K]^e = [K_I]^e + [K_{II}]^e + [K_{III}]^e$ [(1)]

2.4.3.2 Stiffness of foundation

Two different types of foundation can normally be considered: Winkler foundation and solid foundation. Winkler foundation was also called liquid foundation which assumes vertical force at node i depends only on the vertical deflection at node i and is independent of all other nodes.

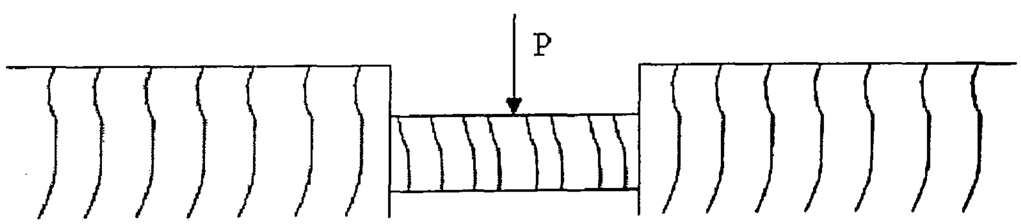


Fig 19 Winkler foundation[(1)]

The stiffness of the Winkler foundation can be expressed as: $k = P/\omega$, where k is the modulus of subgrade reaction; P is the unit force and ω stands for the vertical deflection.

Assume an element of size a by b, nodal points right locate at the corner of the slab, total resisting force from the foundation can be expressed as $F = a \times b \times k \times \omega$.[(1)] Since from the assumption, every nodal point is independent of deflections from other nodal point, resisting force from each nodal point can be expressed as $F = a \times b \times k \times \omega/4$. Transforming into matrix form $[F] = [K] [\delta]$, we can easily get foundation stiffness matrix $[K_{II}]^{e} = (k \times a \times b/4)$ [1 0 0].[1]

Winkler foundation oversimplified the actual characteristics of soil foundation. A solid foundation model is believed to be more realistic: the deflection at any nodal point depends not only on the force at the node itself but also on the forces at all other nodes. The following equation is used to determine the stiffness matrix: $\omega_{i,j} = P_j (1 - \upsilon_f)/ (\pi \times E_f \times d_{i,j})$.[1] Where $\omega_{i,j}$ is the deflection at nodal point i due to force at nodal point j; P_j is the force at nodal point j; υ_f is the Poisson's ratio for foundation and $d_{i,j}$ is distance between nodal point i and j. The stiffness matrix of the foundation element is defined as deflection at one node due to forces at all other nodes. In order to determine the stiffness matrix, a unit concentrated force can be transferred as distributed load over a "$4 \times a \times b$" area as a uniform pressure $1 / (4 \times a \times b)$, then integrate over the whole area.

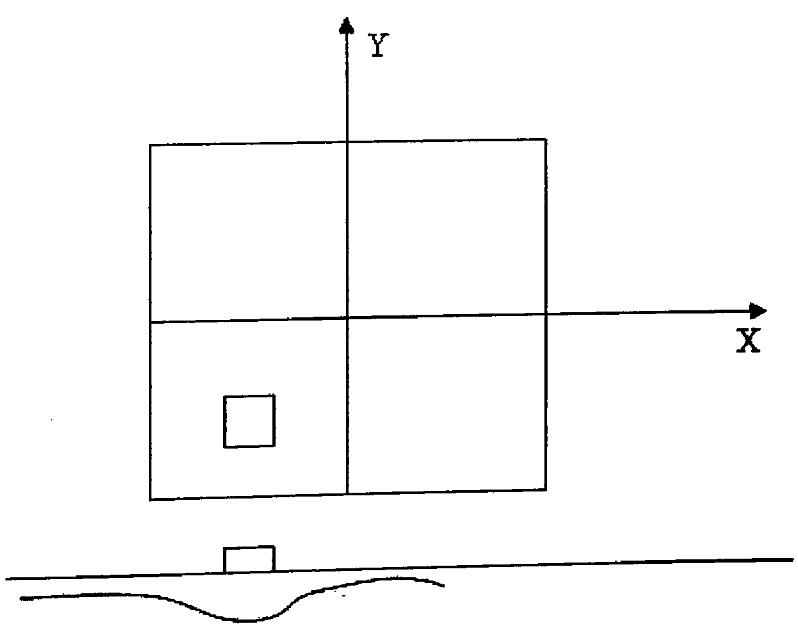


Fig. 20 Solid foundation[1]

2.4.3.3 Stiffness of joints

The stiffness of joint is represented by shear spring constant C_ω, which can be expressed by:

C_ω = shear force per unit length of joint / difference in deflection between two slabs.

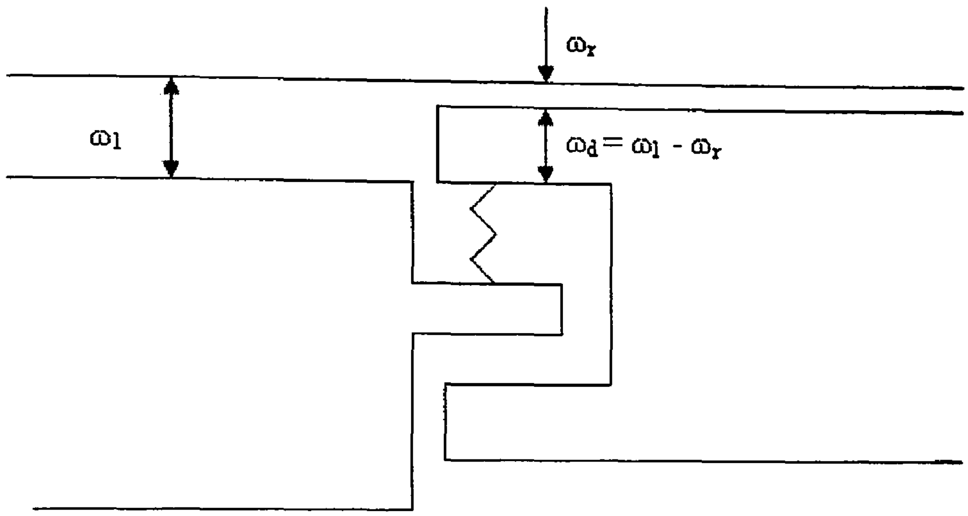


Fig. 21 Shear transfers through joint by grain interlock[1]

When we use grain interlock to transfer the load over joint as shown above in Fig. 21, the shear force can be expressed as $F = L \times C_\omega \times \omega_d$, where F is the nodal force applied to both slabs; L is the average spacing between nodes at the joint and ω_d is the deflection between two slabs.

When dowel bars are used to transfer loads, the situation can be simulated by Fig. 22, and the deflection between two slab can be expressed as $\omega_d = \Delta S + 2 \times y_o$. Where ΔS is the deformation of dowel under load and y_o is deformation of concrete under the dowel. Both items can be expressed as a function of P_t which is the force applied on each dowel bar and equals to $F \times S_b/L$, where F is nodal force applied at the slab, S_b is dowel spacing and L is the average nodal spacing at joint. By knowing ω_d, C_ω, shear spring constant at the joint can be defined.

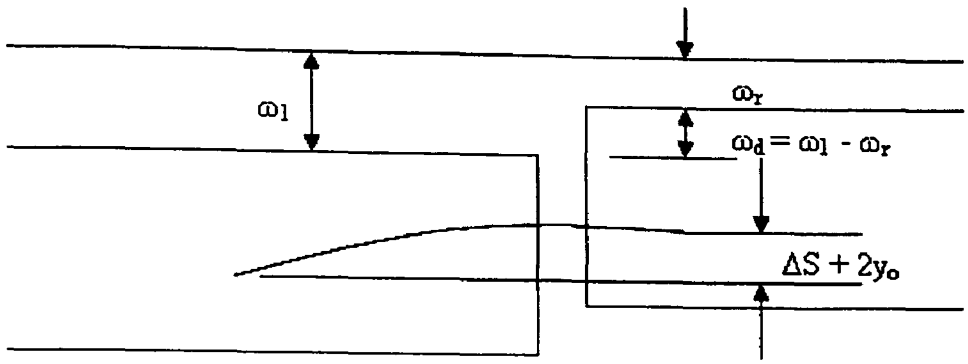


Fig. 22 Shear transfers through joint by dowel bar[1]

2.4.4 Assemble the element equations

The core equation of the finite element method is [F] = [K] [δ],[6] where [F] is the force vector; [K] is the stiffness matrix and [K] = [K_I] + [K_{II}] + [K_{III}]; [δ] is the displacement vector. [F] could contain external force and thermal stress due to temperature changes. Since [F] and [K] contain all known information, [δ], the displacement vector can be found by solving a group of equations.

In order to keep the consistency, external load P should be converted into 12 nodal loads (at 3 directions at each nodal point and totally 4 nodal points for one element).

External load vector can be expressed by the following form:

$$\{F\}^e = [Z_i \ \ Tx_i \ \ T_{yi} \ \ Z_j \ \ Tx_j \ \ T_{yj} \ \ Z_l \ \ Tx_l \ \ T_{yl} \ \ Z_k \ \ Tx_k \ \ T_{yk}] \qquad \textbf{(Eq. 2.27)}$$

From the Fig. 23 below we can see that, since the slab only subject to vertical loading, the 'T' item at the above matrix should be equal zero.

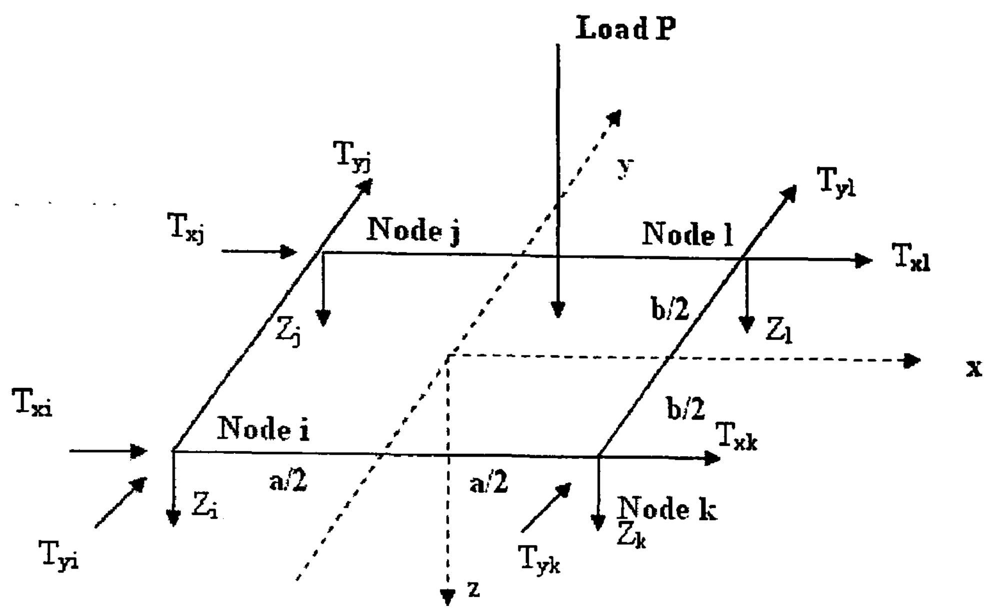


Fig. 23 Finite Element under external load[6]

If the curling effect considered, the above matrix can be expressed by another format: $[F] = [K_p]\,[\delta] - [K_f]\,[\delta']$.[1] Where $[K_p]$ is the stiffness matrix of the slab including the joint; $[\delta]$ is the nodal displacement of the slab; $[K_f]$ is the stiffness matrix of the foundation; $[\delta']$ is the nodal displacement of the foundation. The items in the $[\delta']$ should equal to the numbers in $[\delta]$ subtracted by the curling deformation, and the rotation items should equal zero in $[\delta']$, since foundation will have no rotations.

2.4.5 Solve the global equation system

Once the element equation $[F]^e = [K]^e\,[\delta]^e$ [6] be set up, the global equation system will be established in the similar format: $[F]^T = [K]^T\,[\delta]^T$ [6] basing on the element equation. The global stiffness matrix and the global load vector can normally be expressed in the following form:[6]

$$[K]^T = [A]^T\,[K]^e\,[A] \qquad \text{(Eq. 2.28)}$$

$[F]^T = [A]^T [F]^e$ **(Eq. 2.29)**

Where $[A]^T$ and $[A]$ are the matrix providing transformation from local to global system, they contain all known numbers.

2.4.6 Compute additional results

By solving the equation system $[F] = [K] [\delta]$, nodal displacements can be found. By following the similar procedure as mentioned above, internal force matrix [f] can also be found. [f] can be expressed as $[f] = [D] [A]$ (6), where [D] is a known member within stiffness matrix and [A] is a transformation matrix containing all known numbers. By solving [f], basing on the relationship $[M] = [f] [\delta]$, [M], the moments can be determined. And finally stress and strain can be solved.

Chapter 3: Comparison of KENSLABS and STAAD-III

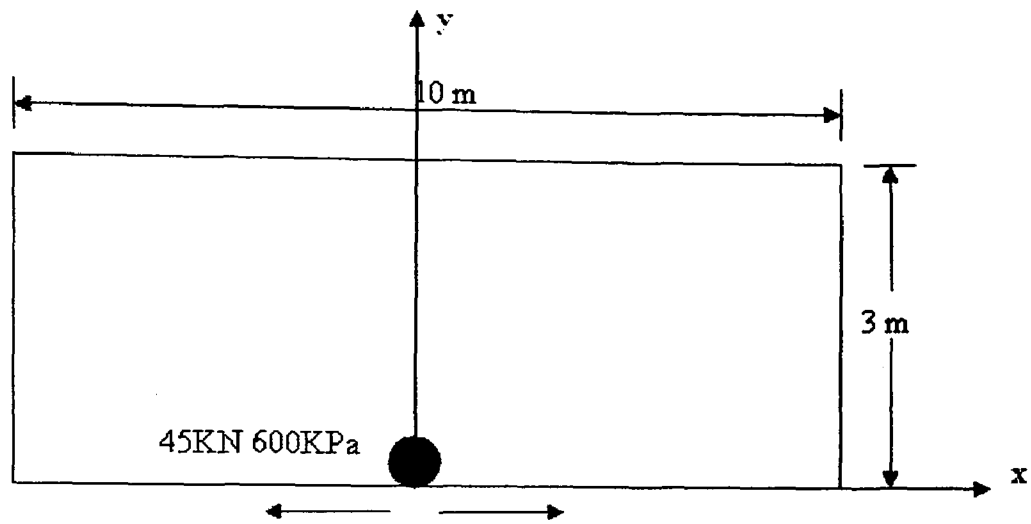
3.1 Background on Finite element software

Various computer software has been developed with the application of Finite Element Method; the high-speed computation of PC makes the prediction of pavement distress and stress analysis much more convenient today than the past. *EverFE* delivered by Washington State Department of Transportation and University of Washington (1997);(5) *KENSLABS* developed by Huang (1985) are both finite element program to investigate concrete pavement performance and rehabilitation alternatives. *SAP2000* and *STAAD* are both the most commonly used finite element program for structural engineers to simulate plane and space structures responses.

KENSLABS developed by Huang (1985) and his colleagues is a concrete pavement stress analysis program basing on finite element method. The program allows the study of a maximum of 6 slabs, 7 joints and 420 nodes, and each slab can have a maximum of 15 nodes in x direction and 15 nodes in y direction.(1) The following example will elaborate how a combined warping and loading stress analysis case be solved by using *KENSLABS* program.

3.2 Design example 1 – the study on combined effect of warping and loading

A concrete slab, 10 meter (32.8 feet) long, 3 meter (9.8 feet) wide and 20 centimeter (7.9 inch) thick, is placed on a modulus of subgrade reaction of 55 MN/m^3 (202.7 pci). The pavement is subjected to a temperature differential of 0.5°C per centimeter (2.28°F per inch) at night when a 45 KN (10100 lbs) single axle load is applied on the edge of the slab over a circular area with a contact pressure of 600 KPa (87 psi) as shown below in Fig. 24. Determine the combined pressure due to curling and loading at the edge beneath the load.



Determine σ_c due to combined effect of warping and loading

Fig. 24 Pavement design example 1

3.2.1 Analysis using *KENSLABS*

In order to save computer storage and running time of computation, since the slab and loading condition is symmetrical about Y-axis, only half of the slab needs to be discretized into rectangular finite elements. We usually number the nodes by 'from bottom to top' and 'from left to right' sequence. The general rule of numbering the nodes is trying to keep the maximum difference between nodal numbers on the opposite corners of the element low. The following Fig. 25 and Fig. 26 show how half of the slab was discretized and nodes were numbered. Half slab has been discretized into $5 \times 7 = 35$ elements, and half slab structure contains $6 \times 8 = 48$ nodes.

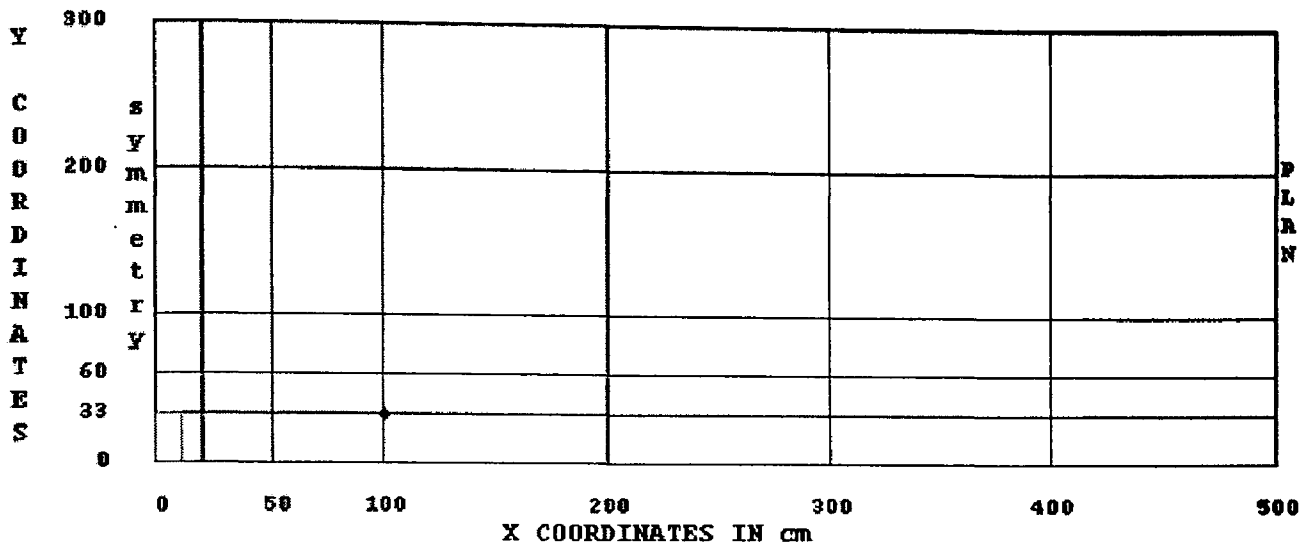


Fig. 25 Discretize half slab for design example 1 (KENSLABS)

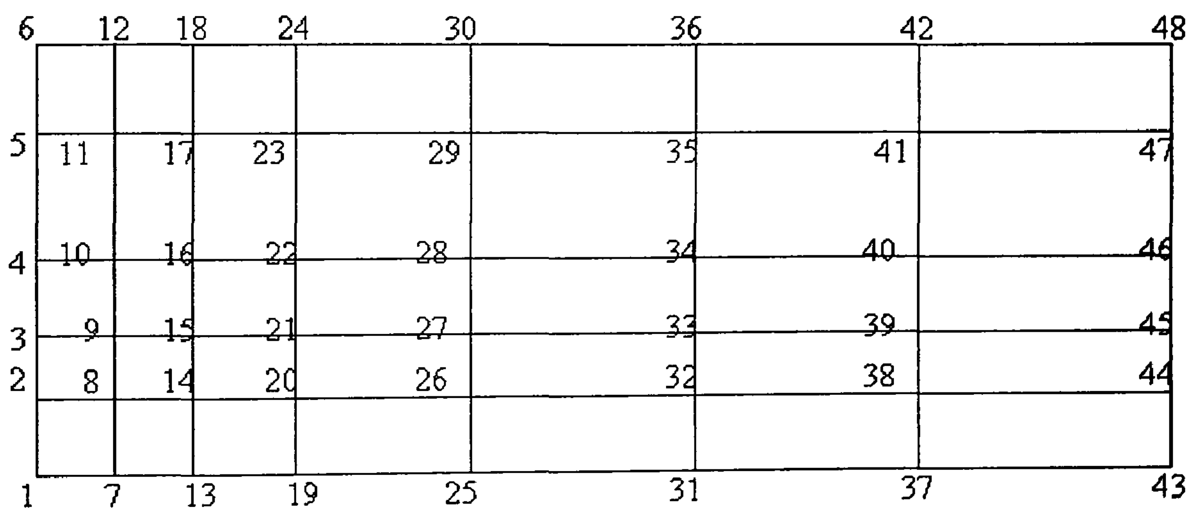
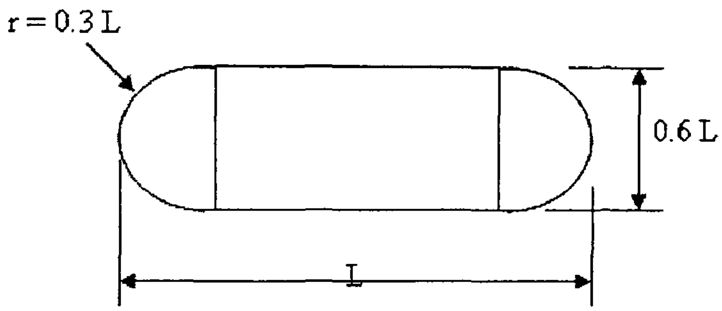
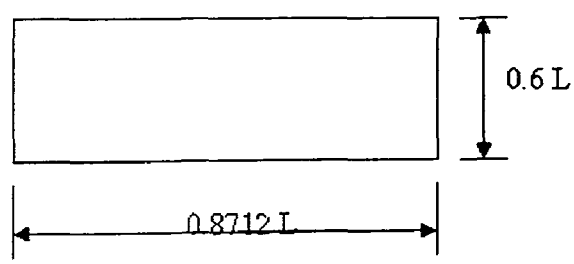


Fig. 26 Numbering of nodes for design example 1 (KENSLABS)

KENSLABS can analyze the pavements under the combined effects of temperature loading and physical loading. Since the pavement is subjected to a temperature differential 0.5°C per centimeter (2.28°F per inch) throughout the thickness of the slab, Δt, the temperature differential between the top and the bottom of the slab is $0.5 \times 20 = 10$°C (18°F). With knowing 45 KN (10100 lbs) load is applied uniformly on the circular area with a contact pressure 600 KPa (87 psi), we need to convert the actual circular tire contact area to equivalent rectangular area (as shown in Fig. 27 below) which equals to $45 \times 10^4/600 = 750\ cm^2$ (116 sq. inch). On half of the slab, 22.5 KN (5050 lbs) load is uniformly applied on a approximate 11.5 cm × 33 cm (4.5 inch × 13 inch) rectangular area. When discretize the slab into finite elements, for concentrated loads, we always try to locate the external loads right on or close to the nodes. For uniformly distributed loads, we always try to locate the load boundaries right on or close to the boundaries of elements.



Actual area = $0.5227\ L^2$, when actual area = 750 cm^2 (116 sq. inch), L ≈38 cm (15 inch)



Equivalent area = 0.6L × 0.8712L ≈23 cm × 33 cm (9 inch × 13 inch)

Fig. 27 Conversion of actual tire contact area into equivalent rectangular area[1]

Other necessary input information is: coefficient of thermal expansion of concrete slab is 9×10^{-6} mm/mm/°C (5×10^{-6} in/in/°F); υ, Poisson's ratio of concrete is 0.15; E, the modulus of elasticity of concrete is 27.6×10^6 KPa (4×10^6 psi). The following Fig. 28 shows the stress contour generated by *KENSLABS* for the studied slab under combined effect of temperature loading and uniform loading. Maximum tensile stress at the bottom of the slab occurs at node '1', which is the corner of the slab where the uniform load applies; the maximum compressive stress at the bottom of slab occurs along the edge of the slab due to the warping effect.

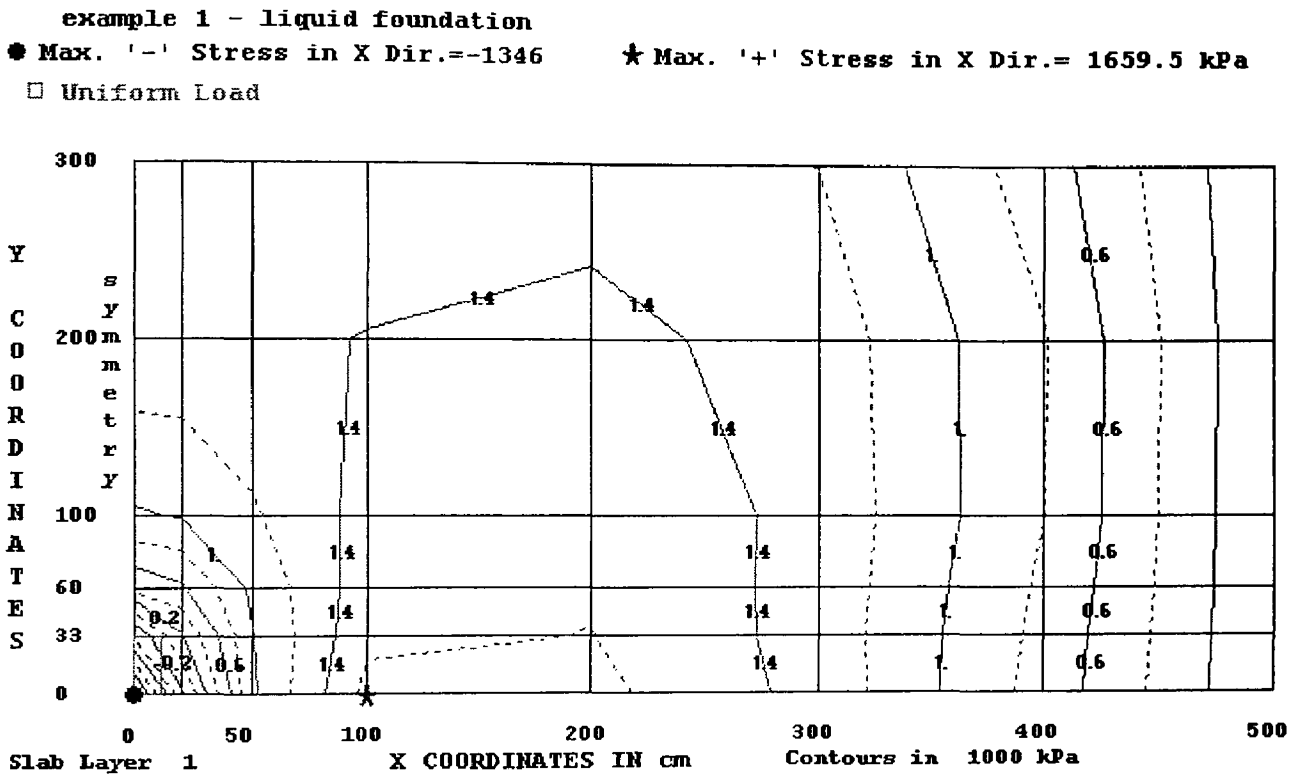


Fig. 28 Stress contour due to combined effect of warping and loading (liquid foundation)

Under the combined effect of warping and loading, stress level at node '1' at the bottom of the slab is 1346 KPa as a tensile stress. Combined stresses at other locations can refer to Appendix A as attached with this report.

NODE	LAYER	STRESS X	STRESS Y	STRESS XY	MAX.SHEAR	MAJOR	MINOR
1	1	-1346.006	0.000	0.000	673.003	0.000	-1346.006
2	1	-401.183	-20.808	0.000	190.187	-20.808	-401.183
3	1	427.547	793.160	0.000	182.807	793.160	427.547

Fig. 29 Stresses at node '1' due to combined effects of warping and loading

As shown above in Fig. 29, 'Stress x' represents the stress in x direction, negative when bottom of the slab is in tension; 'Stress y' is the stress in y direction, negative when bottom of the slab is in tension; 'Stress xy' represents shear stress in xy plane. 'Major' and 'Minor' are major and minor principle stresses in slab when shear stress equals zero, which is used to compare with flexural stress of concrete, and 'Max Shear' represents maximum shear stress in slab. At node '1' 'Stress y' equals zero, since node '1' locates at the edge of slab.

Warping effect is a significant factor when designing concrete pavement joints; for the same concrete slab if without the warping effect, maximum tensile stress occurs at node '1' at the bottom of the slab is 2608 KPa as a tensile stress as shown in Fig. 30 and Fig. 31.

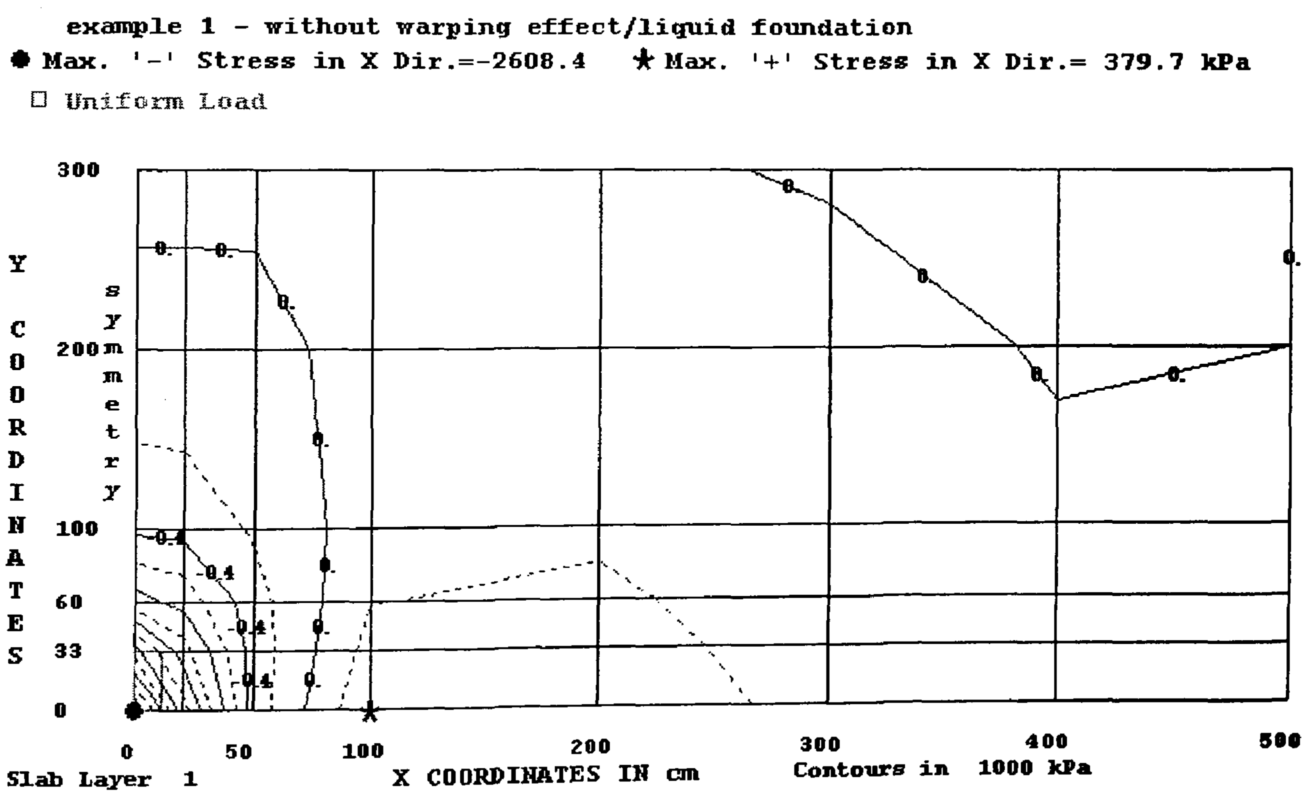


Fig. 30 Stress contour due to uniform load alone (liquid foundation)

NODE	LAYER	STRESS X	STRESS Y	STRESS XY	MAX.SHEAR	MAJOR	MINOR
1	1	-2608.392	0.000	0.000	1304.196	0.000	-2608.392
2	1	-1681.987	-146.780	0.000	767.603	-146.780	-1681.987
3	1	-879.051	490.968	0.000	685.009	490.968	-879.051

Fig. 31 Stresses at node '1' due to uniform load

The result is quite reasonable: as shown in Fig. 32 and Fig. 33 below, the studied concrete slab is experiencing a temperature differential of 0.5°C per centimeter (2.28°F per inch) at night; the top of the concrete slab is cooler than bottom of the slab. The top tends to contract and the bottom tends to expand, however, the weight of the slab restrains it from expansion and contraction, tensile stress will be induced at the top and compressive stress at the bottom. This pair of the stress can balance part of the stress induced by uniform loading at the edge which is in opposite direction, thus compare with 2608 KPa when uniform load alone applied on the slab, the tensile stress at node '1' become 1346 KPa when warping effect is also considered.

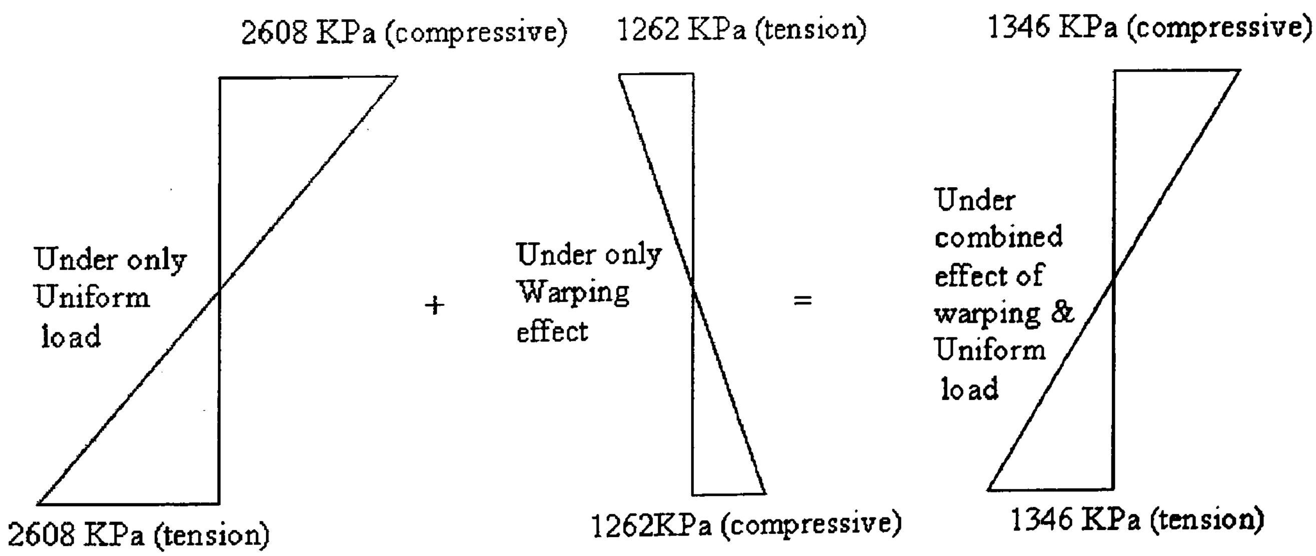


Fig. 32 Combined effect of warping and loading at node '1'

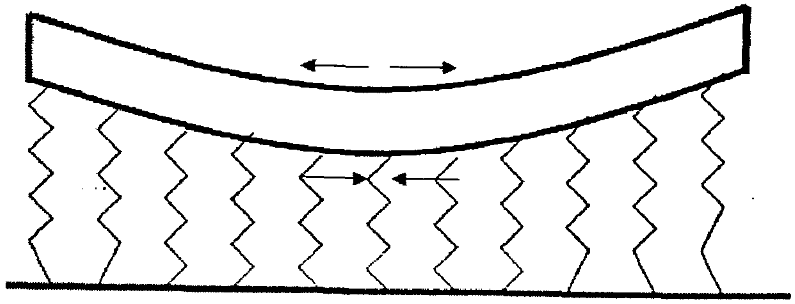


Fig. 33 Upward curling[1]

Above calculation was based on the assumption that concrete slab was built on liquid foundation; since *KENSLAB* can offer three different types of foundation for analysis: liquid, soild or layer; even though liquid foundation is a simple approach as it requires less computer time and storage, solid foundation is still a more realistic solution to treat concrete pavement problems.

In stead of providing modulus of subgrade reaction k for liquid foundation, resilient modulus M_R and Poisson's ratio υ of subgrade should be provided for solid foundation. Resilient modulus M_R is the elastic modulus based on the recoverable strain under repeated loads, the value of M_R for granular material and fine-grained soil is normally determined by repeated triaxial test.[1] A calibrated equation is recommended to determine M_R with knowing subgrade reaction k,[1] $M_R = 18.8 \times k$, where k is in pci and M_R in psi. In this design example, $M_R = 18.8 \times 202.7 = 3811$ psi (26276 KPa), Poission's ratio of subgrade soil υ is assumed to be 0.45.

Solid foundation is able to provide more realistic result since subgrade reaction k used in liquid foundation is not a true characteristic of soil behaviors.[1] Comparing with calculated tensile stress at node '1' on liquid foundation under the combined effect of warping and loading as 1346 KPa, stress level at node '1' become 1509 KPa on solid foundation as shown in Fig. 34.

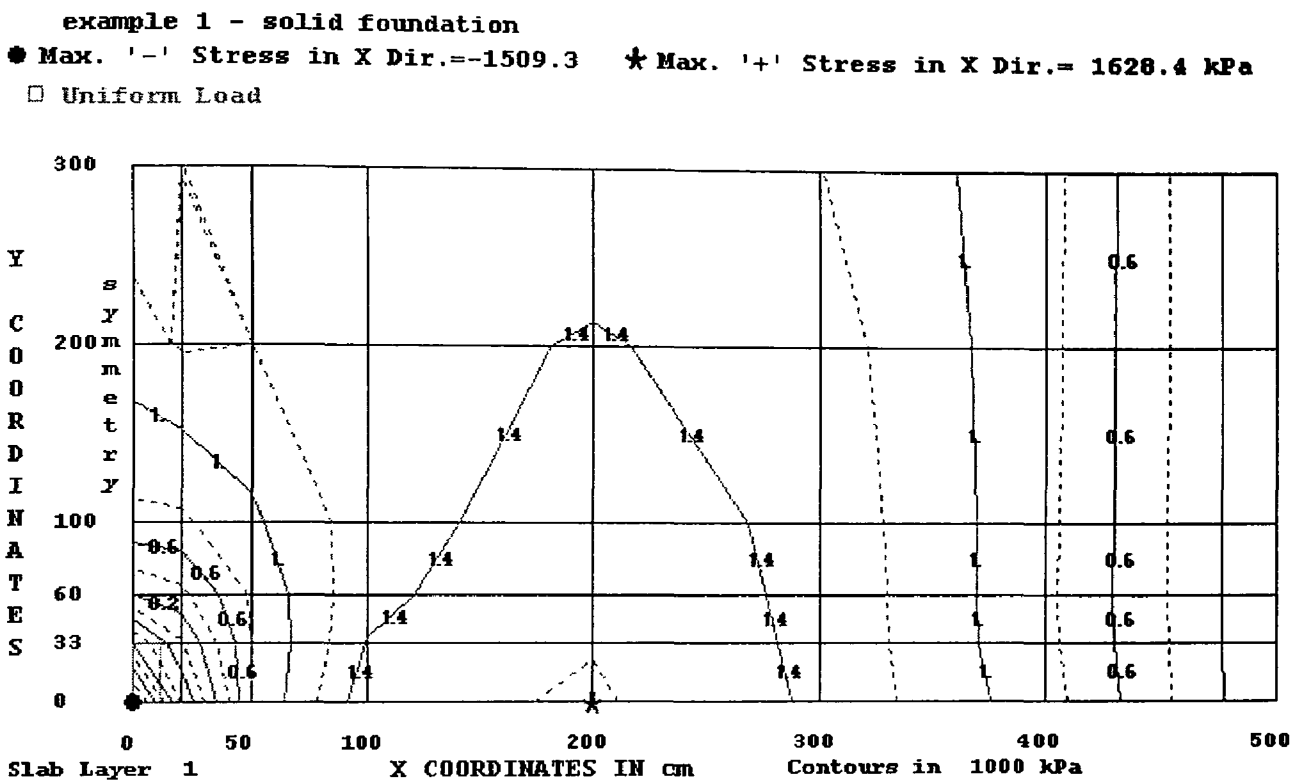


Fig. 34 Stress contour due to combined effect of warping and loading (solid foundation)

Another important feature of *KENSLABS* is to evaluate the contact condition between concrete pavement and subgrade foundation. The above design example 1 was solved basing on the assumption that slab and foundation are always in full contact. Under the full contact condition, the precompression due to the weight of the slab is more than the deflection due to temperature curling effect; the spring supports of subgrade are always able to contact the slab within their elastic range.

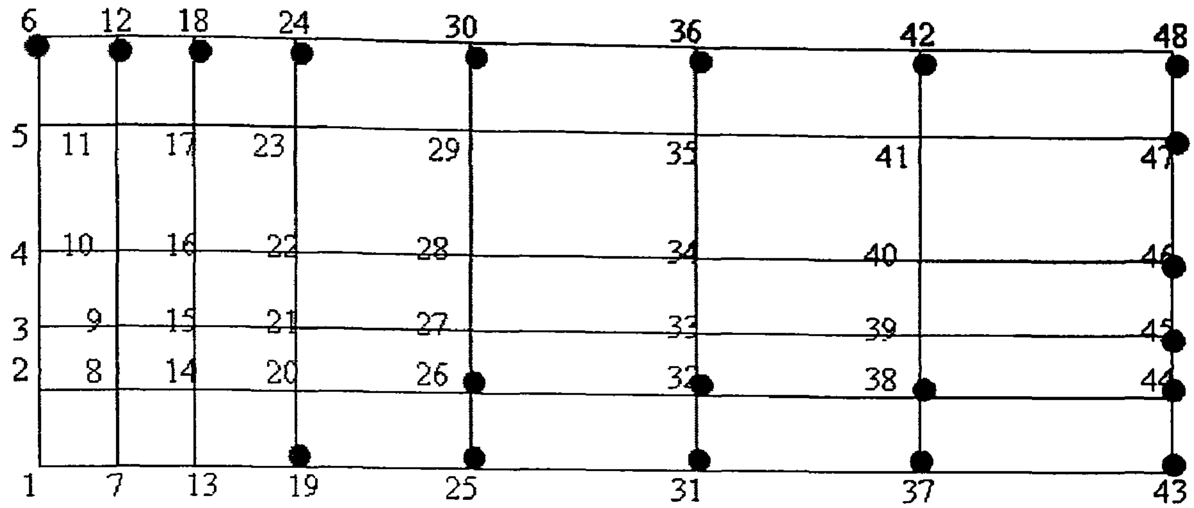


Fig. 35 Negative reactive pressure at nodes

According to the report generated by *KENSLABS* as attached Appendix A (liquid foundation), as shown in Fig. 35 above, support reactive pressures at highlighted nodes are negative in sign (compression positive). That means under temperature curling effect, the slab at these nodes will curl up, since the slab and subgrade are always in full contact within the elastic range of spring support of subgrade, the springs of subgrade will pull the slab back into position, tensile stresses are induced at these nodes as support reactions.

Besides the condition of full contact, the slab and foundation can be in partial contact when the slab is subjected to curling or pumping before any load applications. Under the partial contact condition, initial gaps between slab and subgrade may or may not exist depending on whether there is pumping or plastic deformation of subgrade induced by repeated heavy traffic. Under a high intensity of traffic, some supporting spring of subgrade will fail to function elastically; gaps will form at these locations.

KENSLABS is able to evaluate the contact condition for partial contact with applied load for both liquid and solid foundations, and a two-step analysis is recommended when analyzing such cases. Most importantly, in the case of partial contact, the weight of the slab must be considered in order to counterbalance the positive reactive forces.

In the case of liquid foundation, firstly, the gaps and precompression due to temperature curling, weight of the slab and initial gaps are pre-determined (gaps are positive in sign and precompression is negative in sign); secondly, using the gaps and precompression obtained from the first step to calculate the stresses and displacements under the applied load. In the case of solid foundation, the contact condition is determined by the reactive forces and precompression due to temperature curling, weight of the slab and initial gaps. Compressive reactive forces are positive in sign, which means slab and subgrade are in contact. Negative (tensile) reactive force means slab and subgrade are not in contact. All tensile reactive forces are assigned to zero and the program will automatically run iteration cycles until there is no negative (tensile) reactive force of subgrade. The procedure of two-step analysis will be briefly introduced in Appendix A.

3.2.2 Analysis using *STAAD-III*

STAAD-III, developed by REI, Research Engineers Inc., is another powerful finite element program for analyzing linear and nonlinear, static and dynamic three-dimensional concrete, aluminum, timber and steel structures. The above stress analysis example will be solved once again by *STAAD-III* program.

STAAD-III grouped all types of structures into four categories:[7] a *SPACE* structure is a three-dimensional framed structure with loads applied on any plane; a *PLANE* structure is a two-dimensional structure bound by a global X-Y coordinate system with loads in the same plane; a *TRUSS* structure is a structure consists of truss members which can have only axial forces and no bending in members; a *FLOOR* structure is a two-dimensional or three-dimensional structure with no horizontal movement. In this case, we define the concrete pavement slab as a *SPACE* structure.

In order to specify the structure geometry, *STAAD-III* uses Cartesian coordinate system as global coordinate system to define joint locations and loading directions. The Cartesian coordinate system as shown in Fig. 36 follows the orthogonal right hand rule; the translational degrees of freedom are denoted by u_1, u_2, u_3 and the rotational degrees of freedom are denoted by u_4, u_5 and u_6.

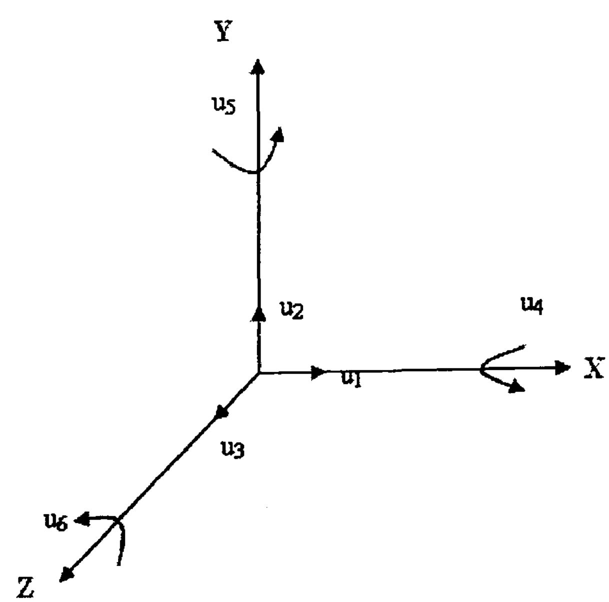


Fig. 36 Cartesian coordinate system[7]

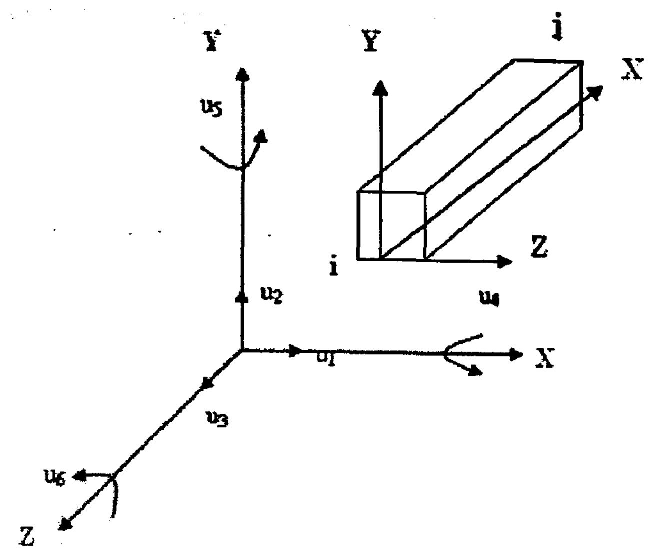


Fig. 37 Association of global and local coordinate system (7)

The local coordinate system also follows the right hand rule, but it associates with each member. As shown in Fig. 37 above, the beam member with starting node "i" and end node "j", the positive direction of local X axis is joining node "i" to "j" and projecting in the same direction. All element force output is in the local coordinate system.

Similar to other finite element software, in order to save computing time, similar elements should be numbered sequentially in *STAAD-III;* and when assigning nodes to elements, nodes should be specified either clockwise or counter clockwise. The program also automatically generate a node at element center; after running the program, element force output is available at the center of the element.

STAAD-III builds the finite element model through text input file. The text file contains a series of commands such as 'joint coordinates' and 'repeat', 'joint coordinate' command specifies joint coordinates for specific elements in the global Cartesian coordinate system; 'repeat' command is used to define joint locations by same size increments. The following Fig. 38 and Fig. 39 show how the joint coordinates are specified and elements are numbered for design example 1.

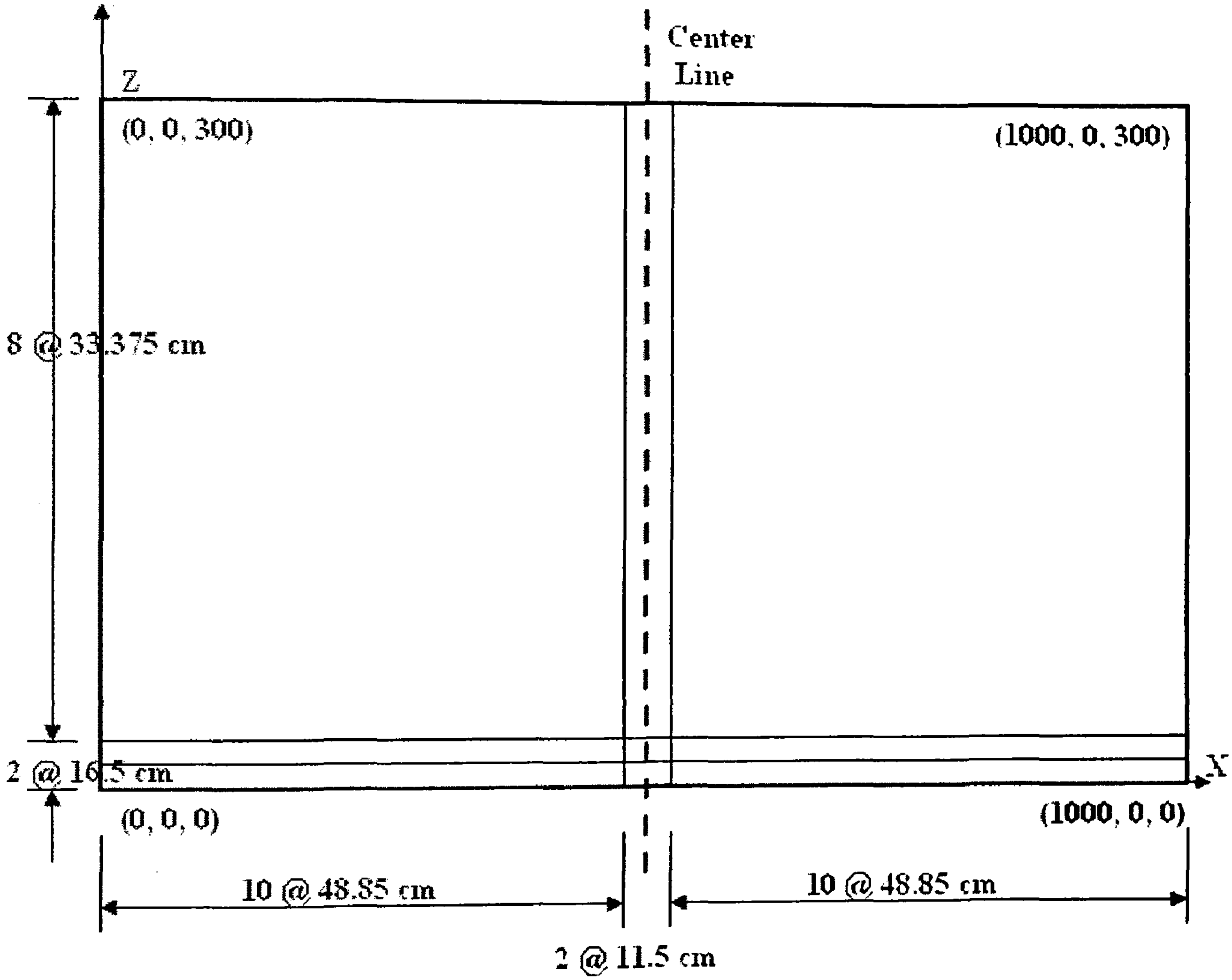


Fig. 38 Discretize the slab for design example 1 (STAAD-III)

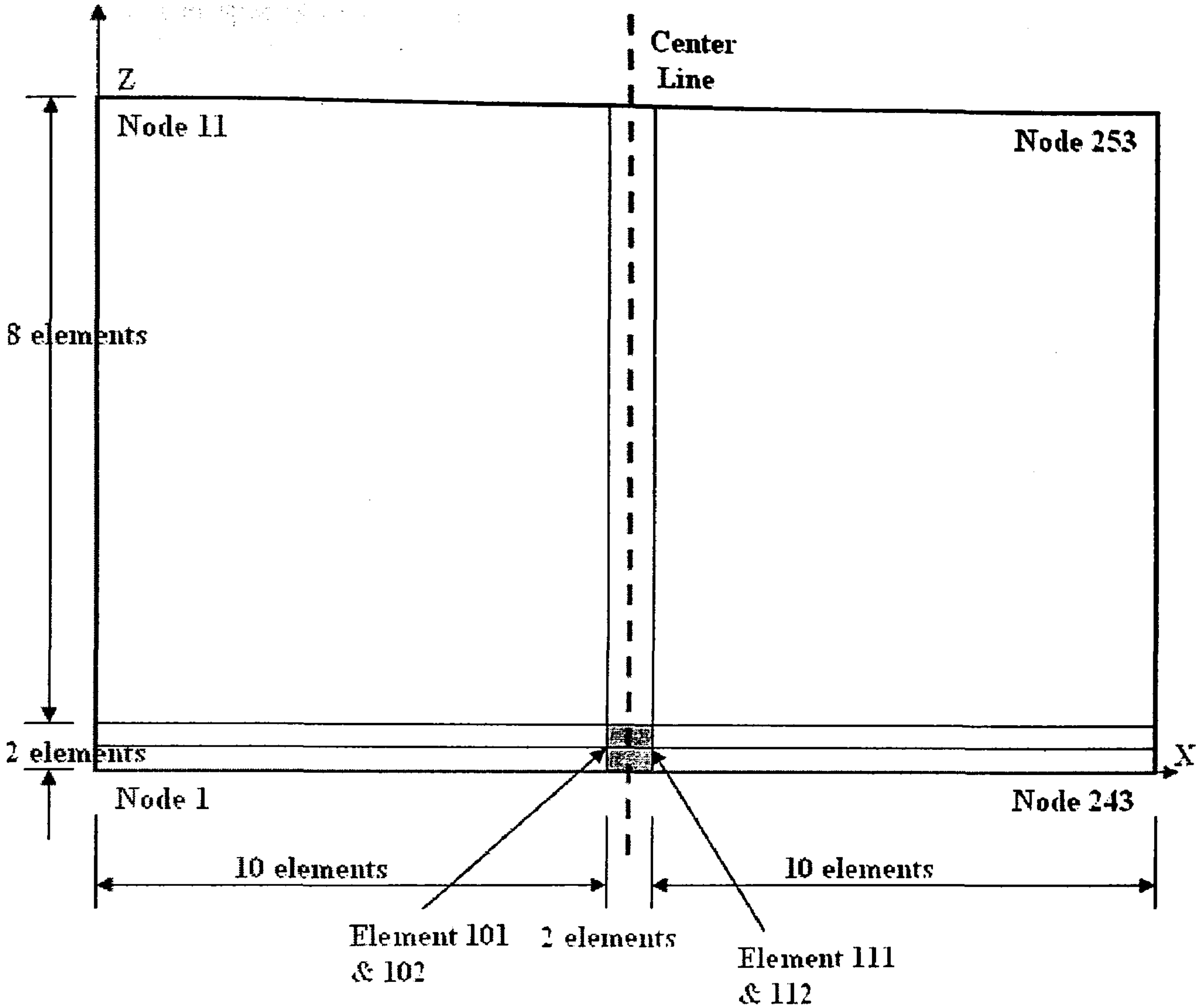


Fig. 39 Numbering the nodes for design example 1 (STAAD-III)

The whole slab has been discretized into 10 × 22 = 220 elements; and the whole structure contains 11 × 23 = 253 nodes as shown above. The 45 KN (10100 lbs) uniformly distributed load is applied on a 23 cm × 33 cm ≈750 cm^2 (116 $inch^2$) area where element 101, 102 and element 111, 112 are located. Similar as *KENSLABS*, when discretize the slab into finite elements, for concentrated loads, we always try to locate the external loads right on or close to the nodes; and for uniform distributed loads, we always try to locate the boundary of area load right on boundary of the elements.

STAAD-III can specify temperature load on members and elements by applying 'temp load' command. Within the 'temp load' command line, both f_1 and f_2 should be specified [7]; where f_1 is the change in temperature which will cause axial elongation in the members or uniform expansion in element; f_2 is the temperature differential from the top to the bottom of the member or element ($T_{top} - T_{bottom}$), it is the f_2 that cause the member to bend. In this case, $f_1 = 0$ and $f_2 = -10°C$ (18°F).

For analysis purpose, *STAAD-III* can separately calculate the stress under different loading cases with the using of 'load combination' command. In this case, four group of load cases are specified: load case 1: uniformly distributed external load alone; load case 2: temperature load alone (upward curling); load case 3: uniformly distributed load with upward curling ($T_{top} - T_{bottom} = -10°C$); load case 4: uniformly distributed load with downward curling ($T_{top} - T_{bottom} = 10°C$).

The other necessary information to be provided is modulus of elasticity of concrete slab $E = 27.6 \times 10^6$ KPa (4×10^6 psi) and Poisson's ratio $\upsilon = 0.15$. *STAAD-III* is also capable of modeling elastic spring support for subgrade foundation by using 'support' and 'elastic mat' command, in this case, subgrade reaction $k = 55$ MN/m^3 (202.7 pci), this means the analysis is basing on the assumption that the slab is built on liquid foundation.

After running the *STAAD-III* input file, a report containing joint displacements, support reactions, member end forces, element principle stress, shear force Q_x, Q_y, membrane force F_x,

F_y, F_{xy}, M_x, M_y and M_{xy} will be printed. The positive directions of element forces are as shown below in Fig. 40.

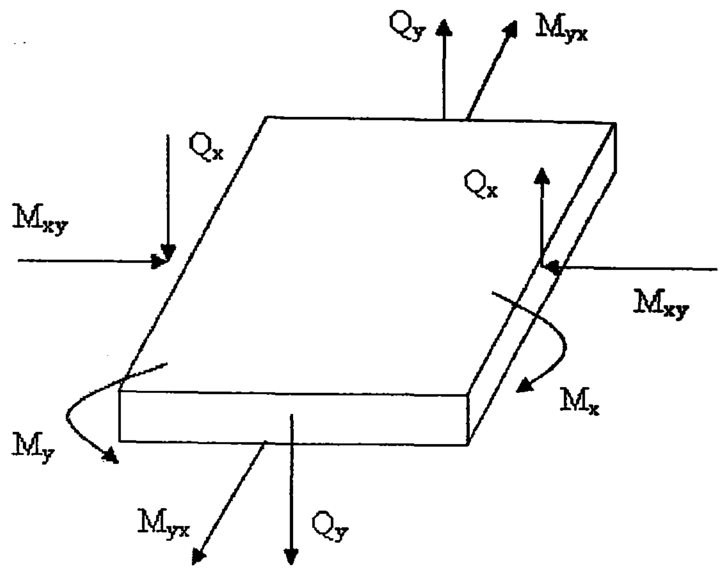


Fig. 40 Sign convention of element forces [7]

Refer to attached Appendix B for detailed report of results; the critical stresses at the centre of element '101' under uniformly distributed load are listed as below.

<u>Element forces at centre of element '101':</u>

Load case 1 (uniformly distributed load alone):

Bottom of slab: 2229 KPa (tension)

Load case 2 (upward curling):

Bottom of slab: -1246 KPa (compression)

Load case 3 (uniformly distributed load with upward curling):

Bottom of slab: 988 KPa (tension)

Load case 4 (uniformed distributed load with downward curling):

Bottom of slab: 3474 KPa (tension)

Comparing the result from *KENSLABS* and *STAAD-III*, based on the assumption that the slab is built on liquid foundation, when only external load applies, 2608 KPa at node '1' was calculated by *KENSLABS* and 2229 KPa at the center of element '101' was calculated by *STAAD-III*. When considering external load combined with upward curling effect, 1346 KPa at node '1' was calculated by *KENSLABS* and 988 KPa at the center of element '101' was calculated by *STAAD-III*.

Many factors may affect the precision of the calculation. First of all, the size of finite elements can be one of the important factors. Normally the coarser the finite element mesh, the smaller stresses and deflections we can get, thus the coarser mesh may lead to unsafe design. Secondly, *KENSLABS* program provides the stresses at nodes, while *STAAD-III* provides the stresses at the center of the elements.

As a finite element program designed specifically for pavement stress analysis, *KENSLABS* has several advantages in term of simplicity. For *KENSLABS*, When the slab and loading exhibit symmetry, only one-half or one-quarter of the slab need to be considered. While for *STAAD-III*, the finite element program for general plane and space structures, the program was not tailored to recognize 'free edge', 'joint' or 'centerline' of the slab like *KENSLABS*. Especially for the design example 1, the slab is subjected to combined effect of edge loading at center of slab and temperature loading, the whole pavement slab need to be studied since center portion of slab experience critical stresses when the whole slab curling upward or downward. Besides, *KENSLABS* is able to provide straightforward answers as far as we are concerned in the pavement design problems, for instance, stress at the corner of the slab. Furthermore, *KENSLABS* has three foundation options to choose, and solid foundation is believed to be the solution close to real conditions. The ability to evaluate contact condition between slab and subgrade is another advantage of *KENSLABS* program. In reality, the slab and subgrade are not always in full contact due to temperature curling or pumping. For old pavements under high intensity of traffic, plastic deformation will form at some locations of subgrade at where initial gaps exist. *KENSLABS* is able to determine the contact condition for partial contact cases with applied load; the stresses and displacements of slab can be analyzed basing on pre-determined gaps and precompression.

3.3 Design example 2 – the study on dowel bars at transverse joint

We will now use both *KENSLABS* and *STAAD-III* to solve another stress analysis example dealing with dowel bars at transverse joint as load transferring devices.

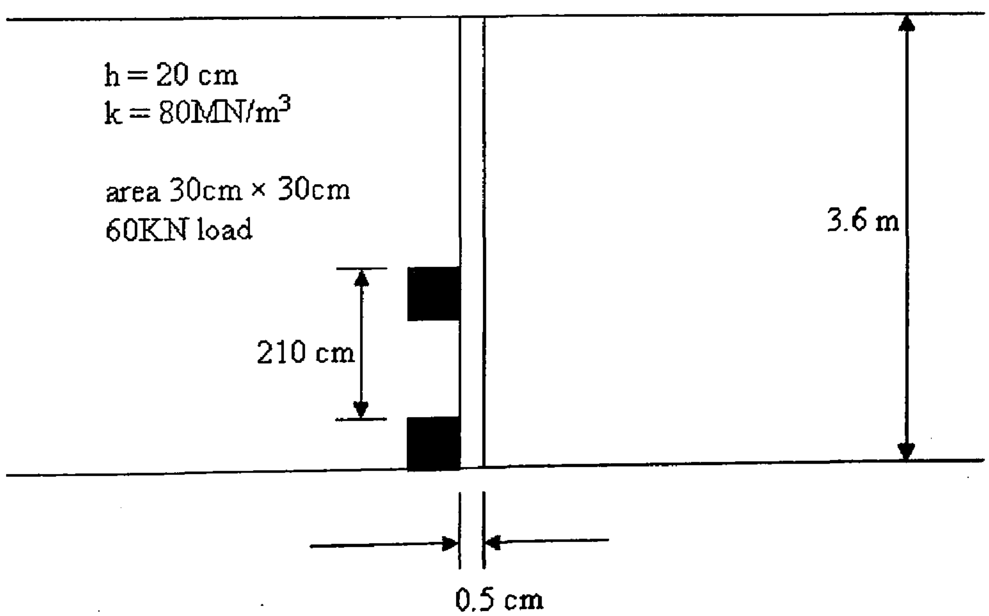


Fig. 41 Pavement design example 2

As shown in Fig. 41 above, a concrete slab has a width of 3.6 meter (141.7 inch) wide, a thickness of 20 centimeter (7.9 inches) and a modulus of subgrade reaction of 80 MN/m^3 (294.9 pci). Two 30 centimeter (11.8 inch) by 30 centimeter (11.8 inch) loaded area, each weighing 60 KN (13470 lbs) and spaced at 210 centimeter (82.7 inch) apart, are applied at the joint with the outside loaded area adjacent to the pavement edge as shown below. Determine the maximum bearing stress between concrete and dowel. The joint has an opening of 0.5 centimeter (0.2 inch) and the dowels are 2.5 centimeter (0.98 inch) in diameter and 30 centimeter (11.8 inch) on centers.

3.3.1 Analysis using *STAAD-III*

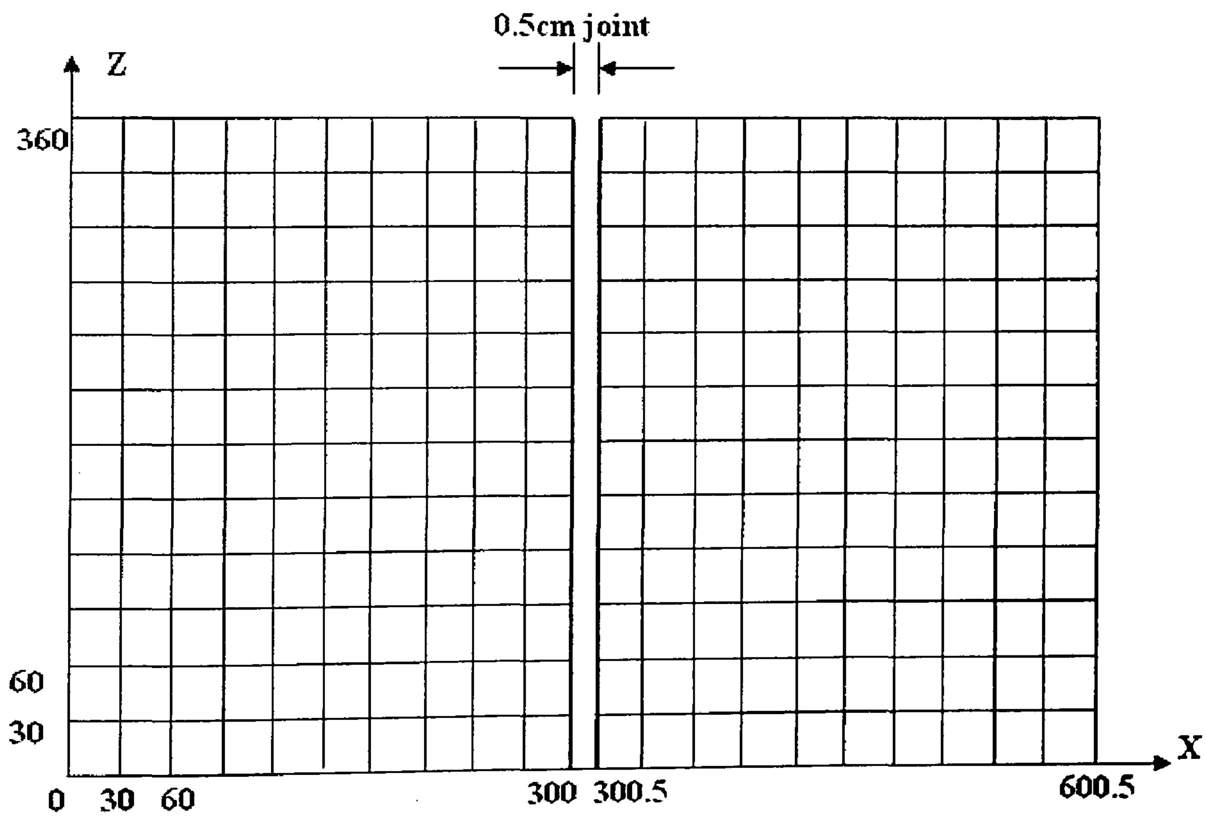


Fig. 42 Discretize the slab for design example 2 (STAAD-III)

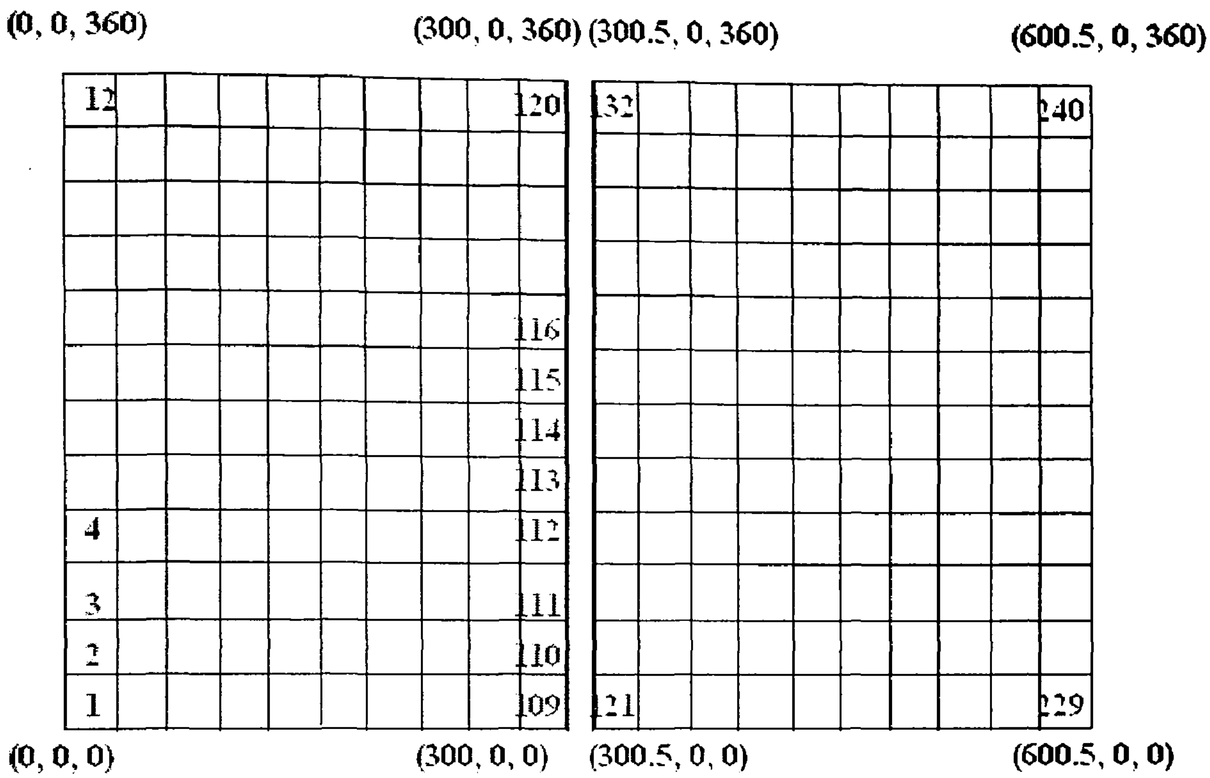


Fig. 43 Numbering of nodes for design example 2 (STAAD-III)

Firstly, we use *STAAD-III* program to solve the problem. In this case, 3 m (118 inches) of the pavement slab on both sides of the transverse joint is studied. 30 cm (11.8 inch) by 30 cm (11.8 inch) square elements are defined; there are totally 12 × 20 = 240 finite elements. Fig. 42 and Fig. 43 above showed how the joint coordinates are specified and elements are numbered.

In order to properly simulate the dowel bars, we have to fully understand how dowel bars function at transverse joint. Dowel bars are fixed at one end and free to move at other end, they are used across transverse joints to transfer only vertical loads. Thus dowel bars can be simulated as a group of steel beams members fixed at one end and hinged at other end between two concrete slabs. From the given information, 2.5 cm (1 inch) diameter dowel bars are arranged at

30 cm (11.8 inch) center to center, there are totally 13 steel beam members are defined, for instance, the steel beam member connecting node 131 and node 144 represent one dowel bar.

Since dowel bars do not transfer any moment or horizontal movement, *STAAD-III* allows specification of release of degrees of freedom for specific members by using "release" command. In this case, since dowel bars transfer only vertical load along Y-axis, member force F_x, F_z and moment M_x, M_y and M_z has to be released.

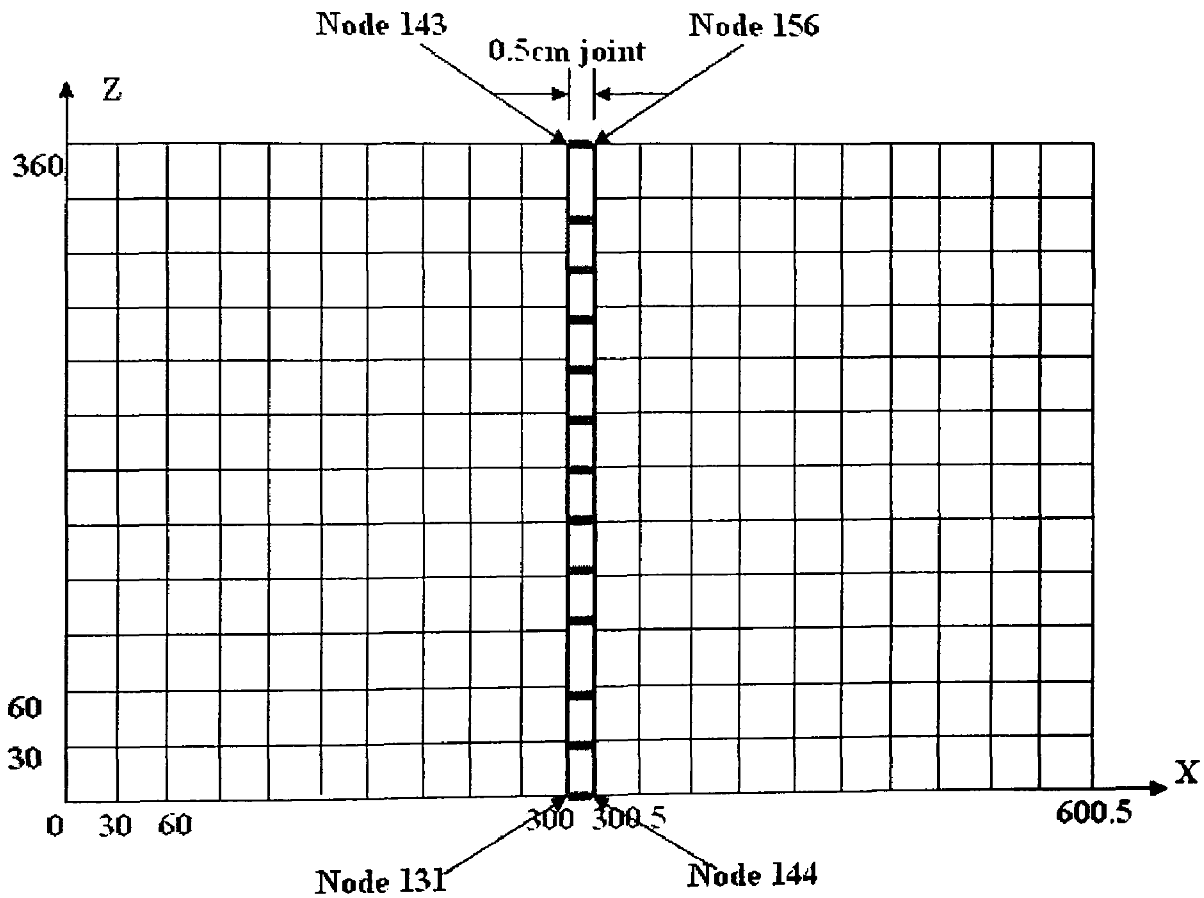


Fig. 44 Simulate dowel bars as steel beam members (STAAD-III)

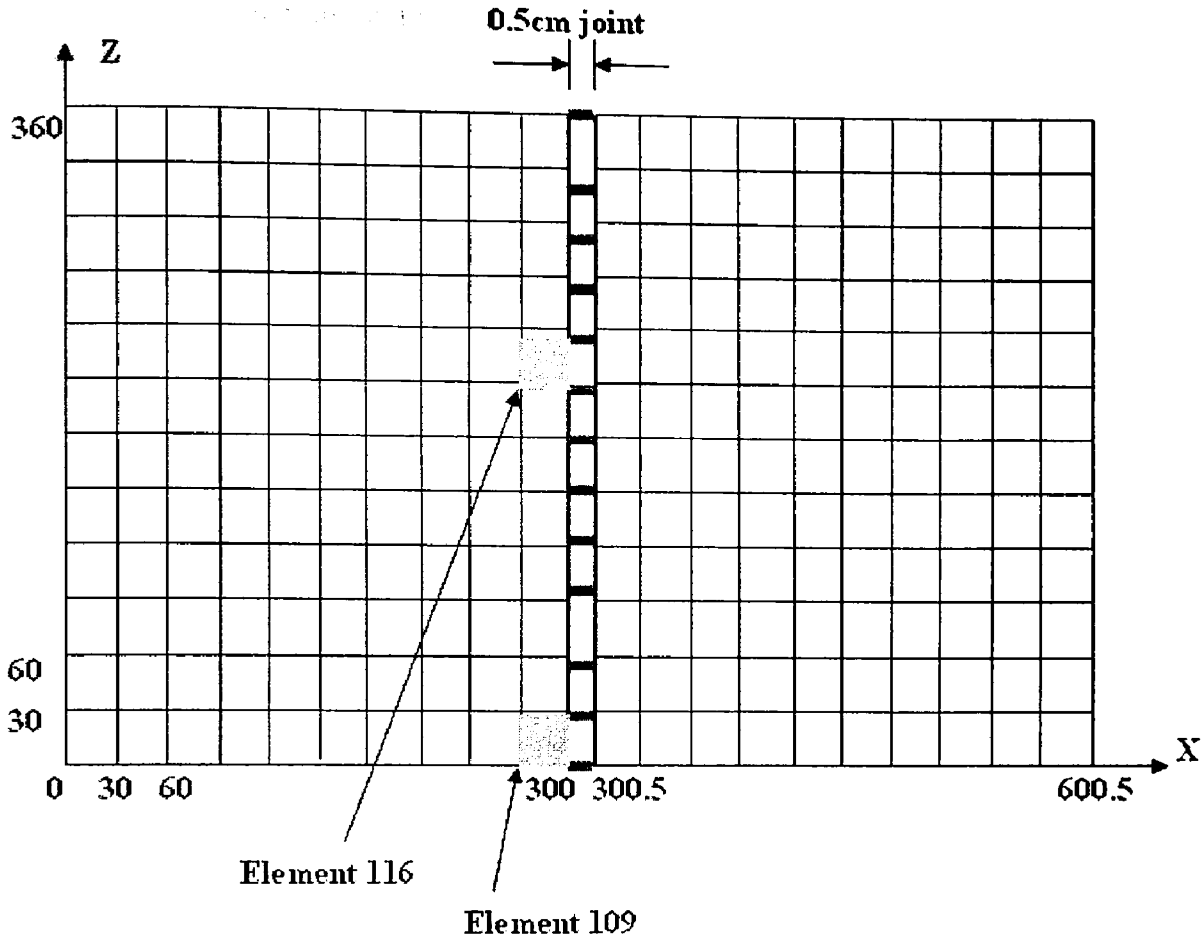


Fig. 45 Uniformly distributed load on element 109 & 116 (STAAD-III)

Finally, 60 KN (13470 lbs) uniformly distributed loads are applied on two 30 cm (11.8 inch) by 30 cm (11.8 inch) areas, which are 210 cm (82.7 inch) apart, *STAAD-III* can specify uniformly distributed load applied directly on elements by the 'pressure' command. As show in Fig. 45 below, area loads are applied on element 109 and 116, uniform pressure equals to $60 \times 10^4 / 900$ cm^2 = 667 KPa (96.7 psi).

Other necessary information shall be provided including modulus of elasticity of concrete slab E = 27.6×10^6 KPa (4×10^6 psi); Poisson's ratio $\upsilon = 0.15$; modulus of elasticity of steel dowel beam $E_d = 2 \times 10^8$ KPa (29×10^6 psi); cross sectional area of steel bar $A = \pi \times d^2/4 = 4.9$ cm^2

(0.76 square inch); moment of inertia of steel dowel beam $I_y = I_z = \pi \times d^4 / 64 = 1.92$ cm^4 (0.045 inch^4) and torsional constant $I_x = \pi \times r^4 / 2 = 3.83$ cm^4 (0.09 inch^4). *STAAD-III* is also capable of modeling elastic spring support for subgrade foundation by using 'support' and 'elastic mat' command, in this case, subgrade reaction k = 80 MN/m^3 (294.9 pci). This means the analysis is based on the assumption that the slab is built on liquid foundation.

The steel member stress are listed as below, a detail report of results is attached in **Appendix C.** Maximum shear force occur at steel members under area loads are as shown below.

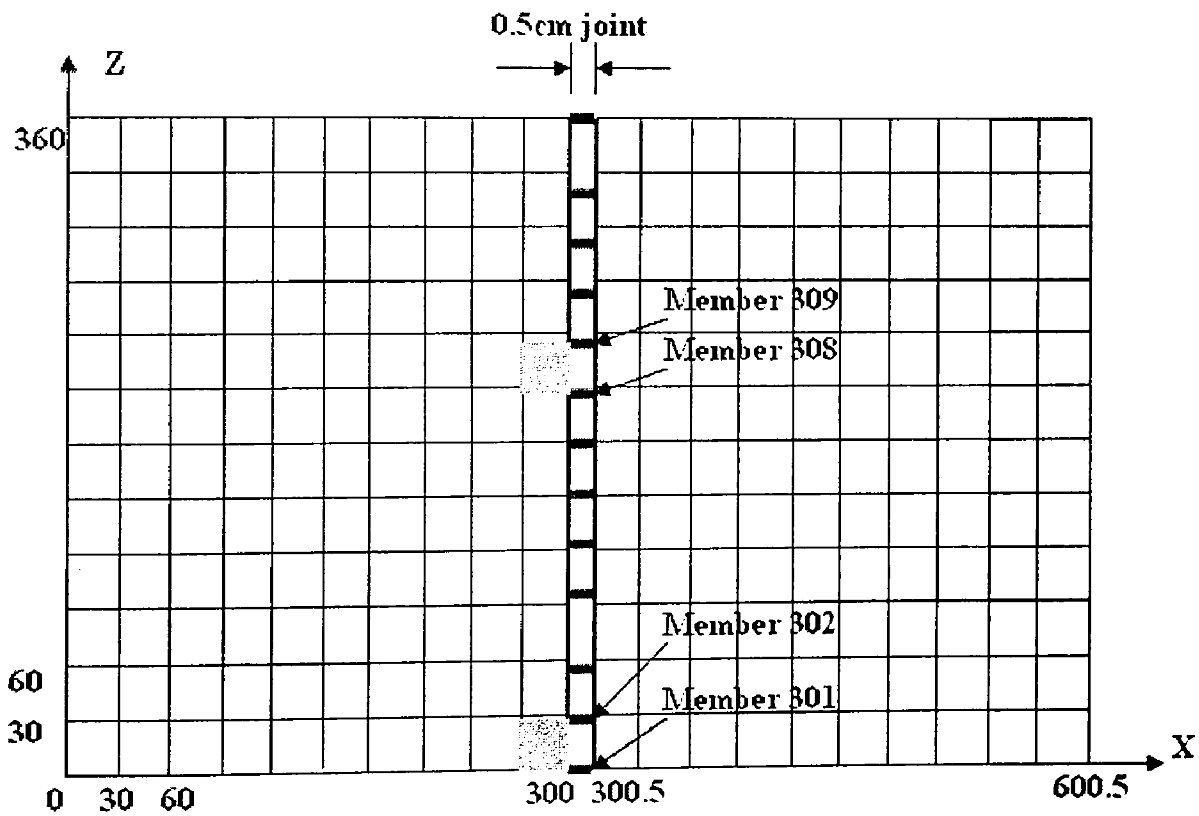


Fig. 46 Maximum stress and deflection at critical locations

Member end forces:

Member 301, shear-Y = -13.11 KN (-2937 pounds)

Member 302, shear-Y = -11.88 KN (-2661 pounds)

Member 308, shear-Y = -10.78 KN (-2415 pounds)

Member 309, shear-Y = -10.82 KN (-2424 pounds)

The relative stiffness of dowel bars embedded in concrete, β, can be defined by $\beta = [(K \times d) / (4 \times E_d \times I_d)]^{1/4}$.[(1)] Where K is the modulus of dowel support, in this case we assume K = 406950 KN/m^3 [(1)] (1.5×10^6 pci); the modulus of elasticity of steel dowel $E_d = 2 \times 10^8$ KPa (29×10^6 psi); moment of inertia of dowel bar $I_d = \pi \times d^4 / 64 = 1.92$ cm^4 (0.045 $inch^4$); diameter of dowel bars d = 2.5 cm (0.98 inch). Thus $\beta = 1.85$ cm (0.73 inch).

Since the maximum shear force on the steel member '301' P_t is 13.11 KN (2937 pounds), concrete bearing stress σ_b can be determined by $\sigma_b = K \times P_t \times (2 + \beta \times z) / (4 \times \beta^3 \times E_d \times I_d)$, where the joint width z = 0.5 cm (0.2 inch). σ_b = 4655 psi (32 Mpa).

Relative stiffness of dowel bar $\ell = [(E \times h^3) / (12 \times (1 - \upsilon^2) \times k]^{1/4}$,[(1)] where modulus of elasticity of concrete $E = 27.6 \times 10^6$ KPa (4×10^6 psi); thickness of the slab h = 20 cm (7.9 inches); Poisson's ratio of concrete $\upsilon = 0.15$ and modulus of subgrade reaction k = 80 MN/m^3 (294.9 pci). ℓ = 27.5 inch (70 cm). From the above discussion about dowel group reaction, we normally assume that the shear in each dowel decreases with the distance of the dowel from the point of loading. Being maximum for the dowel under or nearest to the point of loading and zero at a distance of 1.8ℓ, $1.8 \times \ell = 126$ cm; since two uniformly distributed area loads are 210 cm apart, the shear force on steel member 308 will not affect the concrete bearing stress under steel member 301. Thus, the maximum bearing stress between concrete and dowel is 32 Mpa (4655 psi).

3.3.2 Analysis using *KENSLABS*:

Now we use *KENSLABS* to solve the same problem, a different set of finite element arrangement is laid out as shown in Fig. 47 and Fig. 48 as below. 4 m (157.5 inch) length of the pavement slab on both sides of transverse joint is studied. On one slab, there are total 8 × 5 = 40 elements and 9 × 6 = 54 nodes. The system of units for this example is English (length in inch, force in pound, stress in psi and dowel K value and subgrade reaction in pci).

KENSLABS can define area loads by specifying the boundaries of the load, in this case, the boundary along X-axis of two area loads is the same, from 145.7 inch (370 cm) to 157.5 inch (400 cm); the Y boundaries are from 0 to 11.8 inch (30 cm) and from 82.7 inch (210 cm) to 94.5 inch (240 cm).

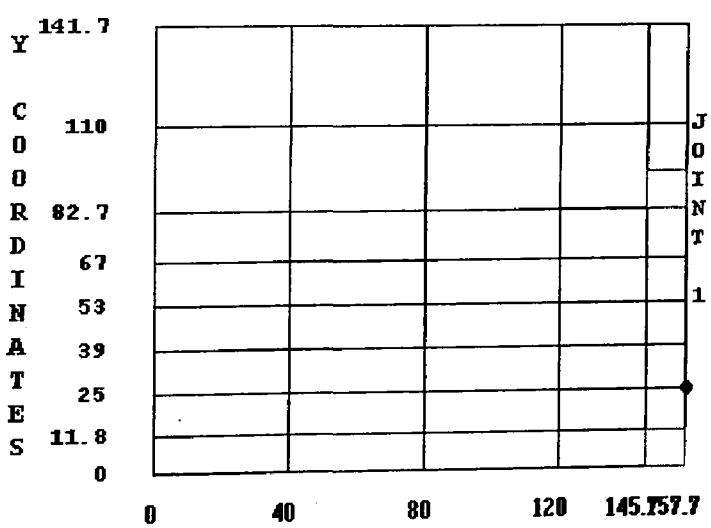


Fig. 47 Discretize the slab for design example 2 (KENSLABS)

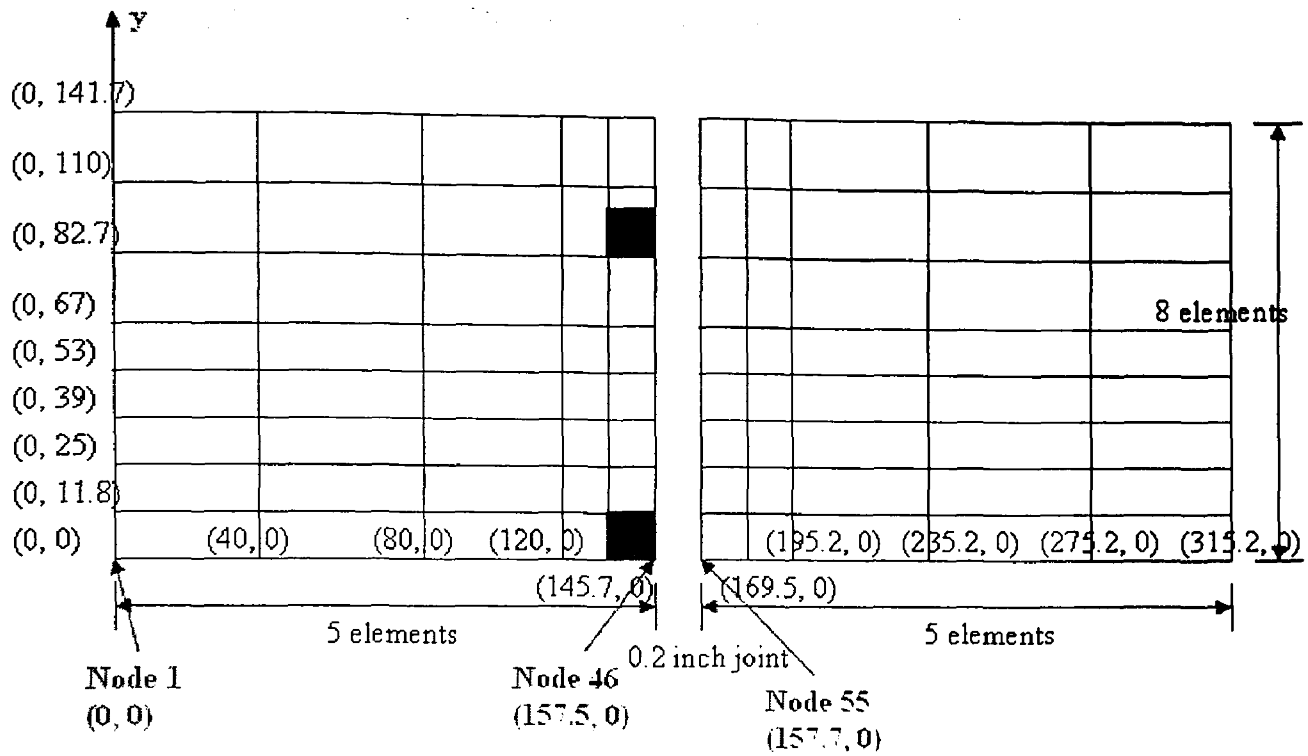


Fig. 48 Numbering of nodes for design example 2 (KENSLABS)

Other necessary information is modulus of elasticity of concrete slab $E = 4 \times 10^6$ psi (27.6×10^6 KPa); Poisson's ratio $\upsilon = 0.15$; modulus of subgrade reaction k = 295 pci (80 MN/m^3); dowel bars are 0.98 inch (2.5 cm) in diameter and 11.8 inch (30 cm) on centers, thus totally 13 dowel bars are used at the transverse joint; joint is 0.2 inch (0.5 cm) wide. Two uniformly distributed area loads 13470 pounds (60 KN) were applied on two 11.8 inch (30 cm) by 11.8 inch (30 cm) areas, load pressure equals to $13470 / (11.8 \times 11.8) = 96.7$ psi (667 KPa). Assume modulus of dowel support $K = 1.5 \times 10^6$ pci (406950 KN/m^3).

Since *KENSLABS* is a computer program designed specifically for concrete pavement stress analysis, after running the program, the report contains the critical stresses such as bearing stress of concrete and shear stress of dowel along joints. Refer to attached Appendix D for more detailed report; maximum shear force in dowel bars and maximum concrete bearing stress is listed as below.

Maximum shear force at the dowel bars occur at the first dowel which connects node '46' to node '55', the force value equals to 2653 pounds (11.8 KN). Maximum bearing stress of concrete occurs at node '46'; the stress value equals 4226 psi (29 Mpa). Stress contour is as shown in Fig. 49 below.

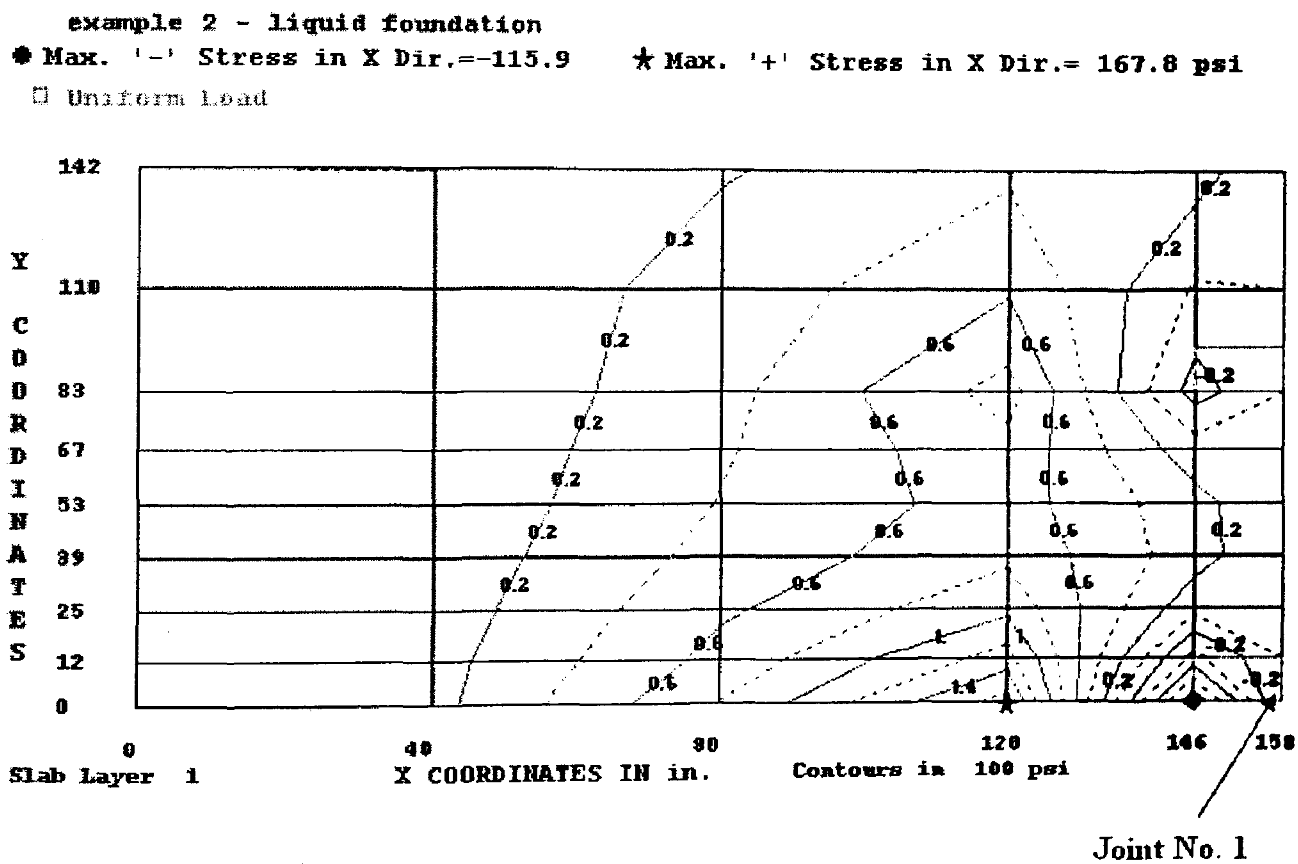


Fig. 49 Stress contour (liquid foundation)

```
FOR JOINT NO. 1 SHEAR IN ONE DOWEL BAR (FAJPD) AT THE NODES IS:
   46    -2652.8    47    -1778.6    48    -813.9    49    -288.3    50    -357.4
   51     -962.3    52    -1851.8    53   -1367.7    54    -160.3

FOR JOINT NO. 1 BEARING STRESS (BEARS) OF CONCRETE AND SHEAR STRESS
(SHEARS) OF DOWELS AT THE NODES ARE:
     46  -4226.2  -3516.9    47  -2833.5  -2357.9    48  -1296.6  -1079.0
     49   -459.4   -382.3    50   -569.4   -473.8    51  -1533.0  -1275.7
     52  -2950.2  -2455.1    53  -2179.0  -1813.2    54   -255.4   -212.5
```

Fig. 50 Maximum shear in one dowel and maximum bearing stress

Now if we change the foundation as solid foundation while remain rest of the information. In stead of providing modulus of subgrade reaction k for liquid foundation, resilient modulus M_R and Poisson's ratio υ of subgrade should be provided for solid foundation. A calibrated equation is recommended to determine M_R with knowing subgrade reaction k,[1] $M_R = 18.8 \times k$, where k is in pci and M_R in psi. In this design example, $M_R = 18.8 \times 295 = 5546$ psi (38238 KPa), Poission's ratio of subgrade soil υ is assumed to be 0.45.

Comparing with the results basing on liquid foundation, maximum shear force occurs at dowel bar connecting node '46' and '55' as 2653 pounds (11.8 KN) and maximum bearing stress of concrete occurs at node '46' as 4226 psi (29 Mpa); when the analysis is based on solid foundation as shown in Fig. 51 and Fig. 52 below, maximum shear force occurs at the dowel bar connecting node '46' and node '55' as 2760 pounds (12.3 KN) and maximum bearing stress of concrete occurs at node '46' as 4397 psi (30 Mpa). Solid foundation is able to provide more realistic results since subgrade reaction k used in liquid foundation is not a true characteristic of soil behavior.[1]

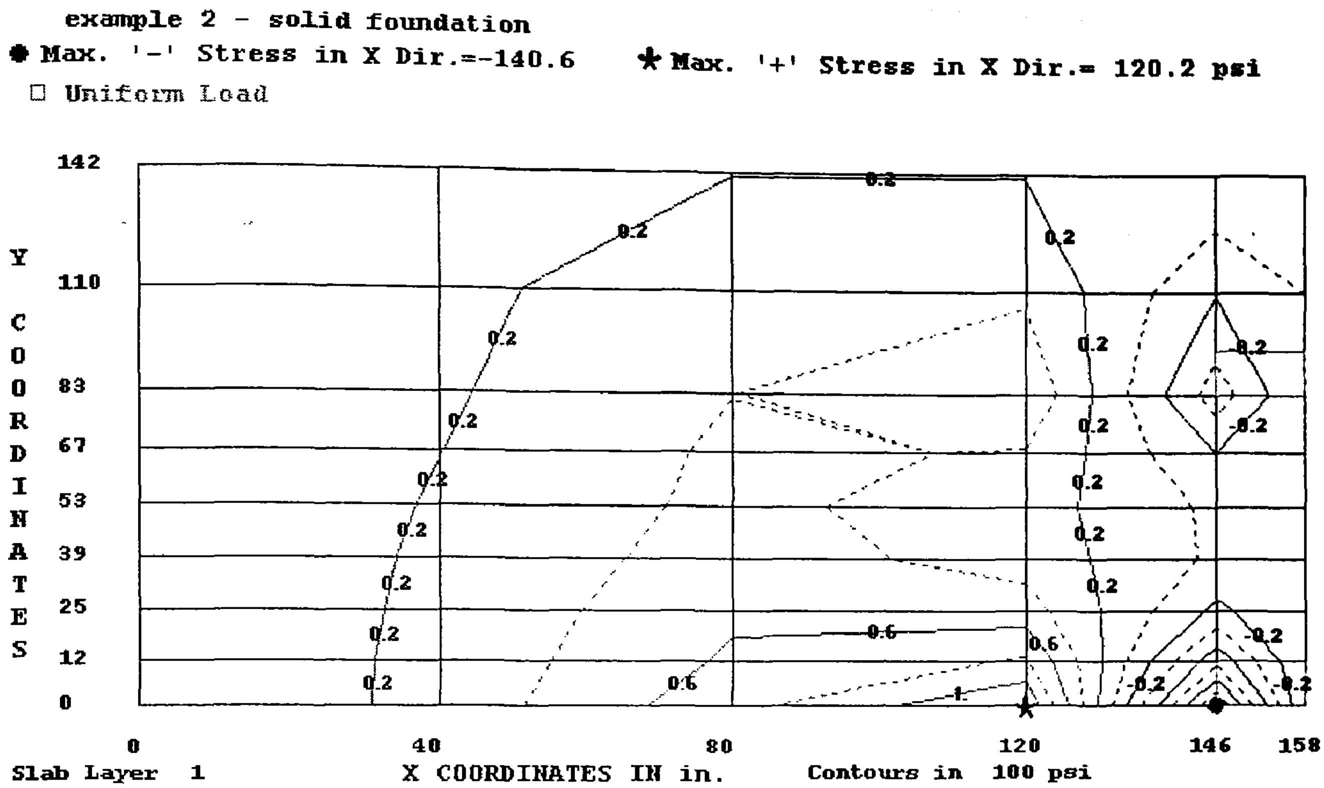


Fig. 51 Stress contour (solid foundation)

```
FOR JOINT NO. 1 SHEAR IN ONE DOWEL BAR (FAJPD) AT THE NODES IS:
   46    -2760.1    47    -1953.2    48    -1043.2    49     -542.6    50     -615.0
   51    -1222.4    52    -2127.5    53    -1650.2    54     -353.6

FOR JOINT NO. 1 BEARING STRESS (BEARS) OF CONCRETE AND SHEAR STRESS
(SHEARS) OF DOWELS AT THE NODES ARE:
    46  -4397.2  -3659.2     47  -3111.6  -2589.4     48  -1661.9  -1383.0
    49   -864.4   -719.3     50   -979.7   -815.3     51  -1947.4  -1620.5
    52  -3389.4  -2820.5     53  -2629.0  -2187.8     54   -563.4   -468.8
```

Fig. 52 Maximum shear in one dowel and maximum bearing stress

Comparing the result of two programs, when we use *STAAD-III*, maximum member shear force occur at member '301' which is the first dowel located at the edge of the slab, shear-Y = -13.11 KN (-2937 pounds); maximum bearing stress between concrete and that dowel is 32 Mpa (4655 psi). For *KENSLABS*, if under the assumption that the slab was built on liquid foundation, maximum shear in one dowel bar occurs under node '46' where locates the first dowel at the edge of the slab, shear-Y = -2653 pounds (-11.8 KN); maximum bearing stress of concrete between concrete and that dowel is 4226 psi (29 Mpa).

As we discussed before, many factors may affect the accuracy of the calculation, the size of elements can be one important factor. Normally the coarser the finite element mesh, the smaller stresses and deflections we can get, thus the coarser mesh may lead to unsafe design. Secondly, *KENSLABS* program provides the stresses at nodes, while *STAAD-III* provides the stresses at the center of the elements.

As a finite element program designed specifically for pavement stress analysis, *KENSLABS* has several advantages in term of simplicity. For the design example 2, *KENSLABS* is able to provide straightforward answers that we concern in the pavement design, for instance, concrete bearing stress under the dowel bar. Furthermore, *KENSLABS* has three foundation options to choose, and solid foundation is believed to be the solution close to real conditions.

Chapter 4: Conclusions

Over the past few decades, pavement engineering has gone through considerable developments. Even though Empirical Method is a simple design approach which has been adopted by many agencies, there are still some drawbacks. One of the drawbacks might be that the set of calibrated equations or Nomograph can only be applied to common environmental, material and traffic conditions; when these conditions change for specific jobs, the design outputs will no longer be reliable. As a design method combining engineering experience and the analysis of mechanical characteristics of materials, the Mechanistic-Empirical method has been widely accepted and the transition from Empirical Design Method to Mechanistic-Empirical Method is becoming a near-future trend.

The most practical and widely used stress analysis method for flexible pavement is Burmister's Elastic Layered Theory. Horizontal tensile strain at the bottom of the asphalt layer and the vertical compressive strain at the top of the subgrade layer are two critical measurements for quantitatively predicting fatigue and rutting damage under repeated traffic load. The most practical method for analyzing rigid pavement is Finite Element Method. Transverse cracking and joint faulting are two major types of distress that adversely affect the concrete pavement performance. Transverse cracking is the effect of fatigue damage, and joint faulting is the result of a combination of many factors: repeated heavy axle load, poor load transfer across the joint; erosion of the supporting base material and upward curling of the slab, etc. Joint faulting has adverse impact on riding quality.

KENSLABS and *STAAD-III* are both commercial Finite Element software. They are both capable of analyzing stress state and deflection of concrete pavement under applied loads and temperature loads. For *KENSLABS*, when the slab and loading exhibit symmetry, only one-half or one-quarter of the slab need to be considered. While for *STAAD-III*, the finite element program for general plane and space structures, the program is not tailored to recognize 'free edge', 'joint' or 'centerline' of the slab, the whole pavement slab information need to be input into the system.

KENSLABS is designed specifically for studying concrete pavement; it is able to provide straightforward solutions for pavement design purpose. For instance, the bearing stress of concrete under dowel bar and stress at corner of the slab under load. Moreover, *KENSLABS* has three types of foundation, liquid, solid and layered. Since the modulus of subgrade reaction k is not a true characteristic of subgrade soil behaviors, solid foundation is a more realistic solution to treat pavement problems. The ability to evaluate contact condition between slab and subgrade is another advantage of *KENSLABS* program. In reality, the slab and subgrade are not always in full contact due to temperature curling or pumping. For old pavements under high intensity of traffic, plastic deformation will form at some locations of subgrade at where initial gaps exist. *KENSLABS* is able to determine the contact condition for partial contact cases with applied load; the stresses and displacements of slab can be analyzed basing on pre-determined gaps and precompression.

As one of the Finite Element software commonly used by many structure engineers, *STAAD-III* is more suitable for studying general plane and space structures. Comparing with *KENSLABS*, *STAAD-III* is a relatively complicated Finite Element software. Training and experience are necessary for writing the text input file properly. Even though *STAAD-III* can provide precise calculations for general plane and space structures containing more elements and nodes, *KENSLABS* is more suitable for pavement design purposes.

References

1. Yang H. Huang, *Pavement Analysis and Design*, Second Edition, August 2003
2. E. J. Yoder, M. W. Witczak, *Principles of pavement design*, Second Edition, October 1975
3. *The Mechanistic-Empirical Pavement Design Guide*, State Departments of Transportation and Federal Highway Administration, USA, March 2004
4. *Pavement design manual of CDOT*, the Colorado Department of Transportation, July 2004
5. *Pavement structural design manual of WSDOT*, Washington State Department of Transportation, July 2003
6. Thomas J. R. Hughes, *The Finite Element Method: Linear Static & Dynamic Finite Element Analysis,* 1987
7. *STAAD-III Structural Analysis and Design Software Reference Manual,* 2001
8. *Bureau of Design & Environment Manual*, Illinois Department of Transportation, 2002 Edition
9. Scott Murison, Department of Civil Engineering, University of Manitoba, Winnipeg, Manitoba, *Modeling of concrete pavement dowel-slab interaction*, 4th Transportation Specialty Conference of Canadian Society for Civil Engineering, June 2002

Appendix A: Use KENSLABS to solve pavement design example 1

1. General Information screen.

General Information of SLABSINP

TITLE: design example 1

Type of foundation (0=liquid, 1=solid, 2=layer)	(NFOUND)	0
Damage analysis (0=no, 1=PCA criteria, 2=user specified)	(NDAMA)	0
Number of periods per year	(NPY)	1
Number of load groups	(NLG)	1
Number of slab layers	(NLAYER)	1
Bond between two slab layers (0=unbonded, 1=bonded)	(NBOND)	0
Number of slabs	(NSLAB)	1
Number of joints	(NJOINT)	0
Nodal number for checking convergence	(NNCK)	1
Number of nodes for stress printout	(NPRINT)	0
Number of nodes on X axis of symmetry	(NSX)	0
Number of nodes on Y axis of symmetry	(NSY)	6
More detailed printout (0=no, 1=yes)	(MDPO)	1
Number of nodes with different thicknesses of slab layer 1	(NAT1)	0
Number of nodes with different thicknesses of slab layer 2	(NAT2)	0
System of units (0=English, 1=SI)	(NUNIT)	1

Default options are shown by black dots. If not true, please click the other button.

- (•) with uniform load
- ( ) without uniform load

- (•) with temperature gradient and/or checking contact
- ( ) without temperature gradient and/or checking contact

- ( ) with concentrated load
- (•) without concentrated load

Print | *Data Set 1* OI

- NFOUND – type of foundation: '0' for liquid foundation, '1' for solid foundation and '2' for layered foundation. Modulus of subgrade reaction k (pci or KN/m^3) in the case of liquid foundation, resilient modulus M_R (psi or KPa) and Poisson's ration υ in the case of solid foundation should be provided at the foundation information screen as shown below. Layered foundation (up to six layers) is recommended when the slab is placed on one or more layers of granular materials. Thickness, elastic modulus and Poisson's ratio of each foundation layer shall be specified.

Liquid Foundation

Double click (or press the E

Unit MN/m^3

	SUBMOD	NAS
	55	0

- SUBMOD – modulus of subgrade reaction k (pci or KN/m^3). Provide one modulus of subgrade reaction k if the foundation is uniform and assign NAS = '0' (refer to 'NAS' below).

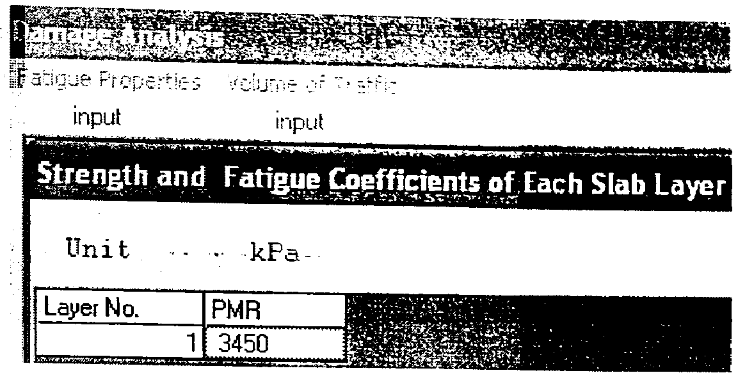
- NAS – numbers of additional modulus of subgrade reaction if the foundation is not uniform. Maximum 120 additional k and nodal numbers at where subgrade modulus is different can be assigned.

Solid Foundation

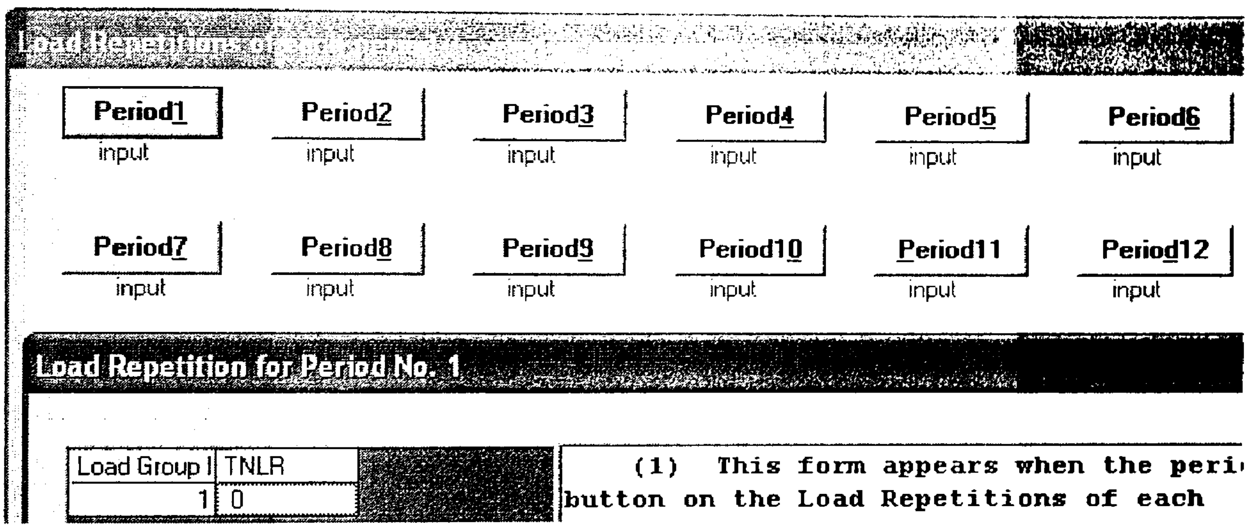
Young's modulus of subgrade in kPa	(YMS)	26276
Poisson's ratio of subgrade	(PRS)	0.45

- YMS – Young's modulus (resilient modulus) of subgrade in the case of rigid foundation (psi or KPa). Resilient modulus M_R is the elastic modulus based on recoverable strain under repeated loads, the value of M_R for granular material and fine-grained soil is normally determined by repeated triaxial test. A calibrated equation is recommended to determine M_R with knowing subgrade reaction k, $M_R = 18.8 \times k$, where k is in pci and M_R in psi.
- PRS – Poisson's ratio of subgrade for rigid foundation.

- NDAMA – damage analysis, '0' for no damage analysis is required; '1' for damage analysis based on fatigue equations recommended by Portland Cement Association and '2' for fatigue damage analysis based on user specified fatigue coefficients. Damage analysis is based on fatigue cracking only, the allowable number of repetitions of loading is expressed as $\log N_f = f_1 - f_2 \times (\sigma/S_c)$, where σ is the flexural stress in concrete slab and S_c is the modulus of rupture of concrete, f_1 and f_2 are coefficients. When the program is asked to perform damage analysis, a damage analysis screen as shown below will ask for information such as modulus of rupture of concrete pavement slab S_c (psi or KPa) and total number of load repetitions for each load group during each period (refer to NPY and NLG below).



- PMR – modulus of rupture of concrete pavement slab S_c (psi or KPa).



- TNLR - total predicted number of load repetitions for each load group in each period.

- NPY – number of periods per year. Maximum 12 periods can be specified for each year for damage analysis. Modulus of subgrade reaction k varies with the season of the year, after the allowable number of load repetitions is determined based on predicted number of load repetitions for each load group in each period, pavement design life can be estimated.
- NLG – number of load groups. Axle loads can be divided into maximum of 12 groups for damage analysis. In the case of no applied load and only temperature curling effect is considered, NLG = '1'.

- NLAYER – number of slab layers, maximum 2 layers of slab can be specified in the case of asphalt overlay on top of concrete slab as a rehabilitation method or concrete slab over a cement-treated base.
- NBOND – bond between two slab layers, '0' for unbonded and '1' for bonded slab layers. In the case of bonded slab layers, flexural stress in the concrete is reduced by a composite moment of inertia about neutral axis; while in the case of unbonded slab layers, each layer works as an independent slab with the same displacements at nodes.
- NSLAB – number of slabs, maximum 6 slabs can be studied.
- NJOINT – number of joints, maximum 7 joints can be specified. In the case of single slab, NJOINT should be assigned '0'.
- NNCK – nodal number for checking convergence. The program recommends using nodal number under the heaviest load to check convergence. In design example 1, node '1' under uniform load is assigned for checking convergence.
- NPRINT – number of nodes for stress printout, stresses at maximum 420 nodes are printed. When NPRINT = '0', the stresses at every node will be computed and printed.
- NSX – number of nodes on X-axis of symmetry, maximum 50 nodes can be defined. When NSX = '0', it represents X-axis is not an axis of symmetry; when NSX = '1', it represents X-axis of symmetry is along a joint. In design example 1, X-axis is not an axis of symmetry, thus NSX = '0'.
- NSY - number of nodes on Y-axis of symmetry, maximum 50 nodes can be defined. When NSY = '0', it represents Y-axis is not an axis of symmetry; when NSY = '1', it represents Y-axis of symmetry is along a joint. In design example 1, there are 6 nodes on Y-axis of symmetry, thus NSY = '6'.
- MDPO – more detailed printout, '1' for yes and '0' for no.
- NAT1 – number of nodes with different thickness of pavement slab layer 1, maximum 120 nodes can be specified with different thickness. When NAT1 = '0', the slab layer 1 is of uniform thickness. In design example 1, NAT1 = '0'.
- NAT2 – number of nodes with different thickness of pavement slab layer 2, maximum 120 nodes can be specified with different thickness. When NAT2 = '0', the slab layer 2 is of uniform thickness. In design example 1, NAT2 = '0'.

- NUNIT – system of units, '0' for English (length in inch, force in pounds, stress in psi, subgrade reaction and dowel support K value in pci and temperature in °F) and '1' for SI (length in cm, force in KN, stress in KPa, subgrade reaction and dowel support K value in KN/m^3 and temperature in °C).

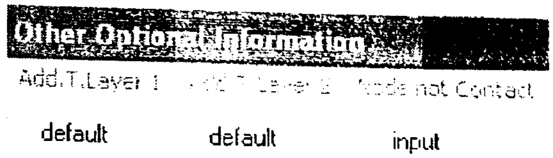
2. Temperature load screen.

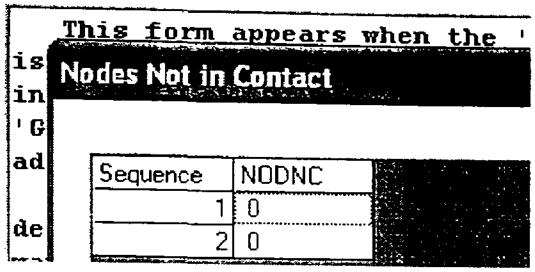
Curling and Contact Conditions

Number of nodes not in contact	(NOTCON)	0
Number of nodes with initial gaps	(NGAP)	0
Input of gaps from previous problem (1=yes, 0=no)	(INPUT)	0
Temperature curling (1=yes, 0=no)	(NTEMP)	1
Weight of slabs (1=yes, 0=no)	(NWT)	0
Maximum number of cycles for checking contact	(NCYCLE)	1
Temperature differential between top and bottom in C	(TEMP)	10
Coefficient of thermal expansion per C	(CT)	0.000009
Tolerance for iteration	(DEL)	0.001
Maximum allowable deflection in cm	(FMAX)	2.54

- NOTCON – number of nodes not in contact, maximum 120 nodes can be specified where slab and subgrade are assumed not in contact. The concrete slab and subgrade foundation can be in full contact or partial contact depending on whether there is separation due to curling or pumping before any load applications. Nodal numbers at which the slab and subgrade are not in contact have to be specified, and at those nodes subgrade reactive forces are assumed to be 0.

  In the case of liquid foundation, at these nodes initially not in contact, the slab and subgrade may or may not be in contact with load applications depending on how many iteration cycles (refer to NCYCLE below) is specified. If only one iteration cycle is specified, the slab and foundation at these nodes will never be in contact with the load applications. In the case of solid foundation, the nodes specified as not in contact will never be in contact, thus in the case of solid foundation NOTCON should be assigned '0' unless users are very sure that these nodes will definitely be out of contact.



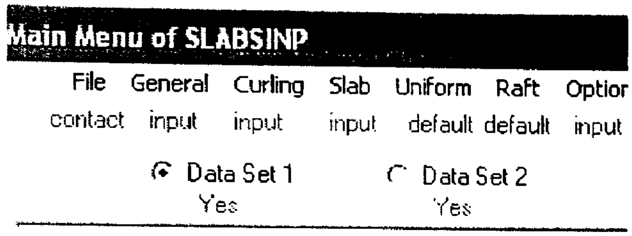


- NODNC – nodal numbers at where the slab and subgrade are not in contact (nodal numbers do not have to be in sequence).

- NGAP – number of nodes with initial gaps, maximum 120 nodes can be specified. When the pavement slab and subgrade foundation are assumed to be in partial contact, initial gap may or may not exist depending on whether there is pumping or plastic deformation of subgrade induced by repeated heavy traffic. For old pavements under high intensity of traffic, plastic deformation will form at the subgrade; elastic support of subgrade will fail to function at some nodes. For new pavements, each elastic spring of foundation is in good condition thus no initial gap exists. In the case of partial contact will no initial gaps, NGAP = '0'; otherwise, nodal numbers at where initial gaps exist and the sizes of the gap shall be specified.

  Initial gaps, temperature curling and the weight of the slab are used to determine contact condition at nodes before any load applications for partially contacted problems; a two-step analysis is recommended when using *KENSLABS* to solve such problems (refer to INPUT below).

- INPUT – input of gaps from previous problem, '1' for yes and '0' for no. *KENSLABS* studies partial contact condition with applied load by two-step analysis: In the case of liquid foundation, (1) determine the gap and precompression between slab and subgrade due to temperature curling, self-weight and initial gaps; (2) use the gap and precompression obtained from first step to determine the stresses and displacements under applied load. As shown below, in 'data set 1', INPUT = '0', NCYCLE = 10 and NWT = '1' (refer to NCYCLE and NWT below); in 'data set 2', INPUT = '1', NCYCLE = 10 and NWT = '0'. These two data sets are established in one study case, the first data set studies the gaps and precompression of slab under temperature curling, self-weight and initial gaps; the second data set determines the stresses and displacement of slab under applied load. These two steps are executed in the same run and one immediately after the other.



In the case of solid foundation, similar to the two-step analysis for liquid foundation, the contact condition of slab and subgrade is determined by analyzing reactive forces and precompression due to temperature curling, initial gaps and self-weight of the slab. When the reactive forces are compressive forces that mean slab and subgrade are in contact, when the reactive forces are negative (tensile) forces, they are assigned to zero and the program runs iteration cycles until no negative (tensile) reactive force is calculated at any node.

- NTEMP – temperature curling, '1' for yes and '0' for no.
- NWT – weight of slabs, assign '0' if weight is not considered and assign '1' if weight is considered. In the case of full contact of slab and foundation, it is not necessary to consider the weight of the slab; while in the case of partial contact, the weight of the slab must be considered.

  Under the full contact condition, the weight of the slab will cause uniform precompression before temperature effect and load application, the slab and foundation will remain in contact under temperature gradient because the deflections due to curling are smaller than the precompression. The stresses and deflections due to curling and loading can be determined separately and each independently of the other.

  Under the partial contact condition, in order to determine the stresses and deflection due to applied load, the deformed shape of the slab before any load applications must be determined first. Thus, a two-step analysis is recommended for partial contact condition with applied load. As stated above for INPUT, in dataset 1, NWT = '1', the gaps and precompression due to self-weight of the slab, initial gap and temperature curling is firstly determined; in dataset 2, NWT = '0', the stresses and deflection due to applied load is calculated basing on the gaps and precompression obtained from the first step.

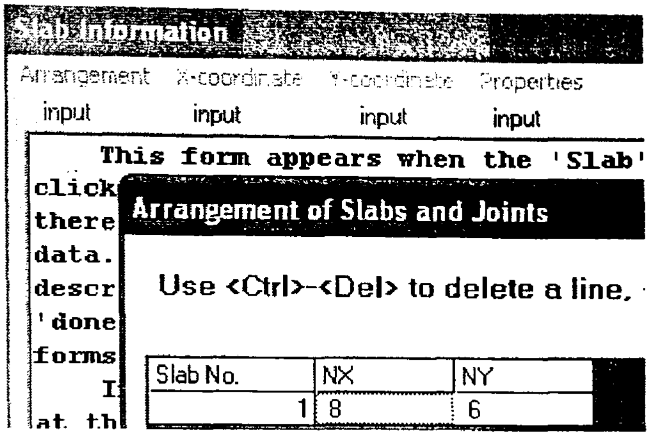
- NCYCLE – maximum number of cycles for checking contact condition. In the case of full contact, NCYCLE = '0' and in the case of partial contact, NCYCLE = '10'.

  In the case of liquid foundation, the program initially assumes the slab and foundation are in full contact for all partially contacted problems and calculate the gaps and precompression due to temperature curling, initial gaps and self-weight of the slab. If it turn out there is no gap between slab and subgrade due to temperature curling, initial gap and self-weight, no iteration cycle is necessary; otherwise up to 10 iterations cycles will be performed until the gaps at nodes calculated are same in sign and within tolerance of iteration (as per DEL below) as that in the previous cycle.

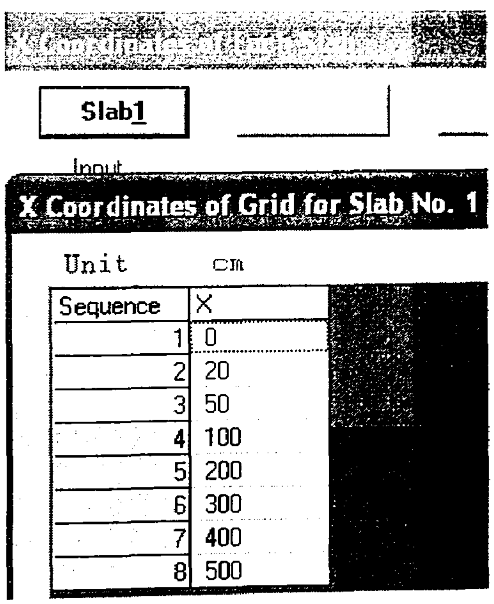
  In the case of solid foundation, reactive forces and precompression is used to determine the contact condition for slab under temperature curling, self-weight and initial gaps. If the reactive forces at nodes calculated are all positive in sign (compressive forces), that means the slab and subgrade are in full contact, no iteration cycle is needed; otherwise, all negative reactive forces are assigned to zero and the program will run iteration cycles until there is no negative reactive force calculated.

- TEMP – temperature differential between top and bottom, TEMP = temperature at bottom of slab - temperature at top of slab. Positive for upward curling and negative for downward curling.
- CT – coefficient of thermal expansion of concrete, recommended value is 5×10^{-6} in/in/°F or 9×10^{-6}/mm/mm/°C.
- DEL – tolerance for iteration, 0.001 is suggested by the program.
- FMAX – maximum allowable vertical deflection at node NNCK (as per above). 1.0 inch (2.54 cm) is recommended by the program, if the vertical displacement calculated by the program is greater than FMAX, the program will stop.

3. Slab information screen.



- NX - number of nodes in X direction for each slab, maximum 15 nodes can be defined. In design example 1, NX = '8'.
- NY – number of nodes in Y direction for each slab, maximum 15 nodes can be defined. In design example 1, NY = '6'.



- X - X coordinate of each node on each slab starting from 0 and increase from left to right.

Y Coordinates of Each Slab

Slab1

Input

Y Coordinates of Grid for Slab No. 1

Unit cm

Sequence	Y
1	0
2	33
3	60
4	100
5	200
6	300

- Y - Y coordinate of each node on each slab starting from 0 and increase from bottom to top.

Thickness, Poisson's Ratio, Young's Modulus, and Unit Weight of Concrete

Unit cm kPa

Layer No.	T	PR	YM
1	20	.15	2.76E+07

- T - thickness of slabs when uniform (cm or inch).
- PR – Poisson's ratio of the pavement slab.
- YM – Young's modulus of concrete pavement slab (psi or KPa).

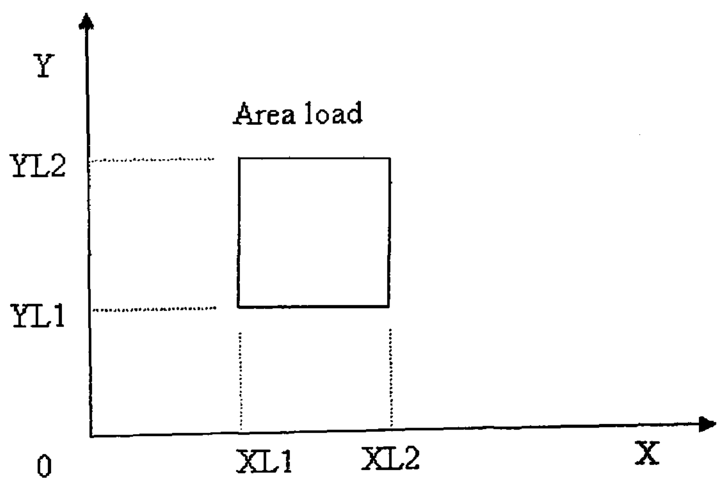
4. Uniform load screen.

Loaded Areas for Load Group No. 1

Use <Ctrl>-<Del> to delete a line, <Ctrl>-<Ins> to insert a line, and <Del

Unit		cm	cm	cm	cm	kPa
Load Sequence	LS	XL1	XL2	YL1	YL2	QQ
1	1	0	11.5	0	33	600

- LS - slab number on which load is applied.
- XL1 - left limits of loaded area in X local coordinate system.
- XL2 – right limits of loaded area in X local coordinate system.
- YL1 - lower limits of loaded area in Y local coordinate system.
- YL2 – upper limits of loaded area in Y local coordinate system.



- QQ – tire contact pressure of each loaded area (psi or KPa).

```
INPUT FILE NAME  -C:\KENPAVE\design example 1.TXT

NUMBER OF PROBLEMS TO BE SOLVED =  1

TITLE -example 1 - liquid foundation

TYPE OF FOUNDATION (NFOUND)                    =            0
TYPE OF DAMAGE ANALYSIS (NDAMA)                =            0
NUMBER OF PERIODS PER YEAR (NPY)               =            1
NUMBER OF LOAD GROUPS (NLG)                    =            1
TOTAL NUMBER OF SLABS (NSLAB)                  =            1
TOTAL NUMBER OF JOINTS (NJOINT)                =            0

ARRANGEMENT OF SLABS
SLAB   NO. NODES (NX)   NO. NODES (NY)    JOINT NO. AT FOUR SIDES (JONO)
 NO.   IN X DIRECTION   IN Y DIRECTION        LEFT  RIGHT  BOTTOM   TOP
  1          8                6                 0      0      0      0

NUMBER OF LAYERS (NLAYER)-------------------------------------=  1
NODAL NUMBER USED TO CHECK CONVERGENCE (NNCK)-----------------=  1
NUMBER OF NODES NOT IN CONTACT (NOTCON)-----------------------=  0
NUMBER OF GAPS (NGAP)-----------------------------------------=  0
NUMBER OF POINTS FOR PRINTOUT (NPRINT)------------------------=  0
CODE FOR INPUT OF GAPS OR PRECOMPRESSIONS (INPUT)- -----------=  0
BOND BETWEEN TWO LAYERS (NBOND)-------------------------------=  0
CONDITION OF WARPING (NTEMP)----------------------------------=  1
CODE INDICATING WHETHER SLAB WEIGHT IS CONSIDERED (NWT)-------=  0
MAX NO. OF CYCLES FOR CHECKING CONTACT (NCYCLE)---------------=  1
NUMBER OF ADDITIONAL THICKNESSES FOR SLAB LAYER 1 (NAT1)------=  0
NUMBER OF ADDITIONAL THICKNESSES FOR SLAB LAYER 2 (NAT2)------=  0
NUMBER OF POINTS ON X AXIS OF SYMMETRY (NSX)------------------=  0
NUMBER OF POINTS ON Y AXIS OF SYMMETRY (NSY)------------------=  6
MORE DETAILED PRINTOUT FOR EACH CONTACT CYCLE (MDPO)----------=  1
TOLERANCE FOR ITERATIONS (DEL)--------------------------------=  0.001
MAXIMUM ALLOWABLE VERTICAL DISPLACEMENT (FMAX)----------------=  2.54
DIFFERENCE IN TEMP. BETWEEN TOP AND BOTTOM OF SLAB (TEMP)-----=  10
COEFFICIENT OF THERMAL EXPANSION (CT)-------------------------=  0.000009

SYSTEM OF UNITS (NUNIT)---------------------------------------=  1
Length in cm, force in kN, stress in kPa, unit weight in kN/m^3
subgrade and dowel K value in MN/m^3, and temperature in C

FOR SLAB NO. 1 COORDINATES OF FINITE ELEMENT GRID ARE:
X =  0  20  50  100  200  300  400  500
Y =  0  33  60  100  200  300

LAYER        THICKNESS (T)        POISSON'S        YOUNG'S
 NO.                              RATIO (PR)     MODULUS (YM)
 1             20.00000            0.15000        2.760E+07

No. OF LOADED AREAS (NUDL) FOR EACH LOAD GROUP ARE:  1
NO. OF NODAL FORCES (NCNF) AND MOMENTS (NCMX AND NCMY) ARE: 0  0  0

FOR LOAD GROUP NO. 1 LOADS ARE APPLIED AS FOLLOWS:
SLAB NO.    X COORDINATES          Y COORDINATES        INTENSITY
  (LS)    (XL1)       (XL2)       (YL1)      (YL2)         (QQ)
   1     0.00000    11.50000    0.00000    33.00000     600.00000
```

NODES ON Y AXIS OF SYMMETRY (NODSY) ARE: 1 2 3 4 5 6

FOUNDATION ADJUSTMENT FACTOR (FSAF) FOR EACH PERIOD ARE: 1

NUMBER OF ADDITIONAL SUBGRADE MODULI (NAS) TO BE READ IN-----= 0
SUBGRADE MODULUS (SUBMOD)------------------------------------= 55

SLAB NO., INITIAL NODAL NUMBER(INITNP), LAST NODAL NUMBER(LASTNP),
INITIAL ELEMENT NO.(INITEN), AND LAST ELEMENT NO.(LASTEN) ARE:
1 1 48 1 35

NODAL COORDINATES (XN AND YN) OF INDIVIDUAL SLAB ARE:

1	0.000	0.000	2	0.000	33.000	3	0.000	60.000
4	0.000	100.000	5	0.000	200.000	6	0.000	300.000
7	20.000	0.000	8	20.000	33.000	9	20.000	60.000
10	20.000	100.000	11	20.000	200.000	12	20.000	300.000
13	50.000	0.000	14	50.000	33.000	15	50.000	60.000
16	50.000	100.000	17	50.000	200.000	18	50.000	300.000
19	100.000	0.000	20	100.000	33.000	21	100.000	60.000
22	100.000	100.000	23	100.000	200.000	24	100.000	300.000
25	200.000	0.000	26	200.000	33.000	27	200.000	60.000
28	200.000	100.000	29	200.000	200.000	30	200.000	300.000
31	300.000	0.000	32	300.000	33.000	33	300.000	60.000
34	300.000	100.000	35	300.000	200.000	36	300.000	300.000
37	400.000	0.000	38	400.000	33.000	39	400.000	60.000
40	400.000	100.000	41	400.000	200.000	42	400.000	300.000
43	500.000	0.000	44	500.000	33.000	45	500.000	60.000
46	500.000	100.000	47	500.000	200.000	48	500.000	300.000

AMOUNT OF INITIAL CURLING AND GAP (CURL) AT THE NODES IS

1	0.05063	2	0.03080	3	0.01823	4	0.00563	5	0.00563
6	0.05063	7	0.05153	8	0.03170	9	0.01913	10	0.00653
11	0.00653	12	0.05153	13	0.05625	14	0.03643	15	0.02385
16	0.01125	17	0.01125	18	0.05625	19	0.07313	20	0.05330
21	0.04073	22	0.02813	23	0.02813	24	0.07313	25	0.14063
26	0.12080	27	0.10823	28	0.09563	29	0.09563	30	0.14063
31	0.25313	32	0.23330	33	0.22073	34	0.20813	35	0.20813
36	0.25313	37	0.41063	38	0.39080	39	0.37823	40	0.36563
41	0.36563	42	0.41063	43	0.61313	44	0.59330	45	0.58073
46	0.56813	47	0.56813	48	0.61313				

LOADS ARE APPLIED ON THE ELEMENT NO.(NE) WITH COORDINATES(XDA AND YDA)
1 -1.000 0.150 -1.000 1.000 0.060

HALF BAND WIDTH (NB) = 24

PERIOD 1 LOAD GROUP 1 AND CYCLE NO. 1

DEFLECTIONS OF SLABS (F) ARE: (DOWNWARD POSITIVE)

1	0.01871	2	0.02365	3	0.02452	4	0.02318	5	0.01164
6	-0.02154	7	0.01711	8	0.02259	9	0.02388	10	0.02287
11	0.01161	12	-0.02151	13	0.01096	14	0.01831	15	0.02099
16	0.02139	17	0.01146	18	-0.02136	19	-0.00132	20	0.00921
21	0.01431	22	0.01761	23	0.01108	24	-0.02080	25	-0.01741
26	-0.00292	27	0.00522	28	0.01239	29	0.01143	30	-0.01835
31	-0.02103	32	-0.00516	33	0.00411	34	0.01275	35	0.01413
36	-0.01419	37	-0.02305	38	-0.00715	39	0.00227	40	0.01122

41	0.01322	42	-0.01402	43	-0.04439	44	-0.02885	45	-0.01958
46	-0.01076	47	-0.00872	48	-0.03506				

NODAL NUMBER AND REACTIVE PRESSURE (SUBR) ARE: (COMPRESSION POSITIVE)

1	10.290	2	13.005	3	13.488	4	12.749	5	6.401
6	-11.848	7	9.409	8	12.426	9	13.134	10	12.579
11	6.385	12	-11.831	13	6.026	14	10.068	15	11.544
16	11.767	17	6.303	18	-11.745	19	-0.726	20	5.068
21	7.872	22	9.684	23	6.096	24	-11.441	25	-9.576
26	-1.605	27	2.872	28	6.815	29	6.288	30	-10.095
31	-11.567	32	-2.836	33	2.258	34	7.010	35	7.770
36	-7.803	37	-12.675	38	-3.932	39	1.251	40	6.174
41	7.272	42	-7.710	43	-24.417	44	-15.869	45	-10.770
46	-5.917	47	-4.797	48	-19.284				

NODE	ROTAT.X	ROTAT.Y	NODE	ROTAT.X	ROTAT.Y	NODE	RORAT.X	ROTAT.Y
1	4.575E-04	-1.219E-18	2	4.469E-04	-8.919E-19	3	4.081E-04	9.852E-19
4	2.808E-04	4.748E-18	5	-3.505E-05	8.498E-18	6	-1.410E-04	4.349E-18
7	4.374E-04	-5.681E-05	8	4.310E-04	-8.537E-06	9	3.957E-04	2.742E-05
10	2.752E-04	5.958E-05	11	-3.641E-05	8.703E-05	12	-1.414E-04	9.300E-05
13	3.744E-04	-1.529E-05	14	3.770E-04	5.167E-05	15	3.496E-04	1.034E-04
16	2.498E-04	1.601E-04	17	-4.295E-05	2.184E-04	18	-1.432E-04	2.324E-04
19	2.731E-04	2.244E-04	20	2.841E-04	2.764E-04	21	2.653E-04	3.191E-04
22	1.916E-04	3.727E-04	23	-6.054E-05	4.431E-04	24	-1.492E-04	4.649E-04
25	1.515E-04	8.078E-04	26	1.671E-04	8.341E-04	27	1.578E-04	8.540E-04
28	1.068E-04	8.793E-04	29	-9.276E-05	9.176E-04	30	-1.645E-04	9.357E-04
31	1.112E-04	1.347E-03	32	1.247E-04	1.354E-03	33	1.177E-04	1.359E-03
34	7.374E-05	1.366E-03	35	-1.088E-04	1.378E-03	36	-1.802E-04	1.390E-03
37	1.151E-04	1.723E-03	38	1.204E-04	1.722E-03	39	1.104E-04	1.721E-03
40	6.626E-05	1.722E-03	41	-1.147E-04	1.724E-03	42	-2.001E-04	1.733E-03
43	1.308E-04	1.854E-03	44	1.276E-04	1.850E-03	45	1.149E-04	1.846E-03
46	6.808E-05	1.841E-03	47	-1.179E-04	1.839E-03	48	-2.190E-04	1.854E-03

SUM OF FORCES (FOSUM) = 22.8 SUM OF REACTIONS (SUBSUM) = 22.8

NODE	LAYER	STRESS X	STRESS Y	STRESS XY	MAX.SHEAR	MAJOR	MINOR
1	1	-1346.006	0.000	0.000	673.003	0.000	-1346.006
2	1	-401.183	-20.808	0.000	190.187	-20.808	-401.183
3	1	427.547	793.160	0.000	182.807	793.160	427.547
4	1	978.423	1168.842	0.000	95.210	1168.842	978.423
5	1	1345.533	971.715	0.000	186.909	1345.533	971.715
6	1	1281.328	0.000	0.000	640.664	1281.328	0.000
7	1	-240.984	0.000	0.000	120.492	0.000	-240.984
8	1	180.014	97.598	-334.787	337.314	476.120	-198.508
9	1	564.321	726.355	-270.310	282.190	927.527	363.148
10	1	1013.099	1153.134	-134.177	151.347	1234.463	931.770
11	1	1347.078	968.468	-31.578	191.921	1349.694	965.852
12	1	1280.926	0.000	0.000	640.463	1280.926	0.000
13	1	961.863	0.000	0.000	480.932	961.863	0.000
14	1	988.311	243.326	-472.843	601.940	1217.758	13.879
15	1	1036.229	625.643	-418.405	466.056	1296.992	364.880
16	1	1175.383	1065.669	-256.341	262.145	1382.670	858.381
17	1	1357.332	950.168	-70.455	215.429	1369.179	938.321
18	1	1280.764	0.000	0.000	640.382	1280.764	0.000
19	1	1659.486	0.000	0.000	829.743	1659.486	0.000
20	1	1558.555	289.802	-379.799	739.379	1663.557	184.800
21	1	1517.934	559.267	-362.324	600.866	1639.466	437.735

22	1	1473.186	912.781	-267.521	387.403	1580.386	805.581
23	1	1407.287	896.665	-93.467	271.882	1423.858	880.094
24	1	1286.094	0.000	0.000	643.047	1286.094	0.000
25	1	1659.202	0.000	0.000	829.601	1659.202	0.000
26	1	1604.426	225.567	-185.677	713.995	1628.991	201.001
27	1	1581.998	444.005	-168.691	593.476	1606.478	419.526
28	1	1555.137	735.256	-135.669	431.807	1577.004	713.390
29	1	1460.786	809.275	-56.793	330.669	1465.700	804.362
30	1	1314.837	0.000	0.000	657.419	1314.837	0.000
31	1	1332.498	0.000	0.000	666.249	1332.498	0.000
32	1	1322.657	173.268	-48.927	576.774	1324.736	171.189
33	1	1328.335	367.243	-45.249	482.672	1330.461	365.117
34	1	1341.526	641.312	-39.637	352.343	1343.762	639.076
35	1	1316.622	763.934	-22.375	277.248	1317.526	763.029
36	1	1196.054	0.000	0.000	598.027	1196.054	0.000
37	1	727.058	0.000	0.000	363.529	727.058	0.000
38	1	746.666	133.304	5.165	306.724	746.709	133.261
39	1	769.148	298.912	-0.527	235.119	769.149	298.911
40	1	802.532	547.364	-4.489	127.663	802.611	547.285
41	1	815.072	692.182	-8.183	61.988	815.614	691.639
42	1	690.082	0.000	0.000	345.041	690.082	0.000
43	1	0.000	0.000	0.000	0.000	0.000	0.000
44	1	0.000	55.105	0.000	27.553	55.105	0.000
45	1	0.000	201.547	0.000	100.773	201.547	0.000
46	1	0.000	442.107	0.000	221.054	442.107	0.000
47	1	0.000	585.670	0.000	292.835	585.670	0.000
48	1	0.000	0.000	0.000	0.000	0.000	0.000

MAXIMUM STRESS (SMAX) IN LAYER 1 IS 1663.557 (NODE 20)

MAXIMUM NEGATIVE STRESS IN X DIRECTION = -1346.0 (NODE 1)
MAXIMUM POSITIVE STRESS IN X DIRECTION = 1659.5 (NODE 19)
MAXIMUM NEGATIVE STRESS IN Y DIRECTION = -20.8 (NODE 2)
MAXIMUM POSITIVE STRESS IN Y DIRECTION = 1168.8 (NODE 4)

Appendix B: Use STAAD-III to solve pavement design example 1

```
**************************************************
*                                                *
*              S T A A D - III                   *
*              Revision 22.3b                    *
*              Proprietary Program of            *
*              Research Engineers, Inc.          *
*              Date=    AUG 15, 2005             *
*              Time=    11:40:52                 *
*              Build No.     2500                *
*        USER ID: Kawneer Company Canada Limited *
**************************************************
```

```
STAAD SPACE WARPING-1
UNIT METER KN
JOINT COORDINATES
1 0 0 0 3 0 0 0.33 1
4 0 0 0.6 11 0 0 3 1
REPEAT ALL 10 0.4885 0 0
122 5 0 0   124 5 0 0.33 1
125 5 0 0.6 132 5 0 3    1
REPEAT ALL 1 0.115 0 0
144 5.6035 0 0 146 5.6035 0 0.33 1
147 5.6035 0 0.6 154 5.6035 0 3 1
REPEAT ALL 9 0.4885 0 0
*FINISH
ELEMENT INCIDENCES
1 1 2 13 12 TO 10 1 1
REPEAT 21 10 11
ELEMENT PROPERTIES
1 TO 220 TH 0.2
UNIT KN METER
CONSTANTS
E 27.6E6 ELEM 1 TO 220
POISSON 0.15 ELEM 1 TO 220
ALPHA 9E-6 ELEM 1 TO 220
SUPPORT
1 TO 253 ELASTIC MAT DIRECTION Y SUB 55000
LOAD 1
ELEMENT LOAD
101 102 111 112 PR GY -600
LOAD 2
TEMP LOAD
1 TO 220 TEMP 0 -10
LOAD COMB 3
1 1 2 1
LOAD COMB 4
1 1 2 -1
PERFORM ANALYSIS
```

37. PRINT JOINT DISPL LIST 100 TO 132

JOINT DISPLACEMENT (CM RADIANS) STRUCTURE TYPE = SPACE

JOINT	LOAD	X-TRANS	Y-TRANS	Z-TRANS	X-ROTAN	Y-ROTAN	Z-ROTAN
100	1	.0000	-.0399	.0000	-.0003	.0000	-.0003
	2	.0000	.0274	.0000	.0006	.0000	.0000
	3	.0000	-.0125	.0000	.0003	.0000	-.0002
	4	.0000	-.0673	.0000	-.0009	.0000	-.0003
101	1	.0000	-.0351	.0000	-.0003	.0000	-.0002
	2	.0000	.0183	.0000	.0005	.0000	.0000
	3	.0000	-.0168	.0000	.0002	.0000	-.0002
	4	.0000	-.0533	.0000	-.0008	.0000	-.0003
102	1	.0000	-.0303	.0000	-.0003	.0000	-.0002
	2	.0000	.0105	.0000	.0004	.0000	.0000
	3	.0000	-.0198	.0000	.0001	.0000	-.0002
	4	.0000	-.0408	.0000	-.0007	.0000	-.0002
103	1	.0000	-.0229	.0000	-.0003	.0000	-.0001
	2	.0000	.0006	.0000	.0003	.0000	.0000
	3	.0000	-.0223	.0000	.0000	.0000	-.0001
	4	.0000	-.0235	.0000	-.0006	.0000	-.0002
104	1	.0000	-.0148	.0000	-.0002	.0000	-.0001
	2	.0000	-.0076	.0000	.0002	.0000	.0000
	3	.0000	-.0224	.0000	.0000	.0000	-.0001
	4	.0000	-.0072	.0000	-.0004	.0000	-.0001
105	1	.0000	-.0086	.0000	-.0002	.0000	-.0001
	2	.0000	-.0116	.0000	.0001	.0000	.0000
	3	.0000	-.0202	.0000	-.0001	.0000	.0000
	4	.0000	.0031	.0000	-.0002	.0000	-.0001
106	1	.0000	-.0041	.0000	-.0001	.0000	.0000
	2	.0000	-.0121	.0000	.0000	.0000	.0000
	3	.0000	-.0162	.0000	-.0001	.0000	.0000
	4	.0000	.0080	.0000	-.0001	.0000	.0000
107	1	.0000	-.0011	.0000	-.0001	.0000	.0000
	2	.0000	-.0091	.0000	-.0001	.0000	.0000
	3	.0000	-.0102	.0000	-.0002	.0000	.0000
	4	.0000	.0080	.0000	.0001	.0000	.0000
108	1	.0000	.0010	.0000	-.0001	.0000	.0000
	2	.0000	-.0022	.0000	-.0003	.0000	.0000
	3	.0000	-.0011	.0000	-.0003	.0000	.0000
	4	.0000	.0032	.0000	.0002	.0000	.0000
109	1	.0000	.0026	.0000	.0000	.0000	.0000
	2	.0000	.0094	.0000	-.0004	.0000	.0000
	3	.0000	.0120	.0000	-.0005	.0000	.0000
	4	.0000	-.0068	.0000	.0004	.0000	.0000
110	1	.0000	.0040	.0000	.0000	.0000	.0000
	2	.0000	.0266	.0000	-.0006	.0000	.0000
	3	.0000	.0306	.0000	-.0006	.0000	.0000
	4	.0000	-.0225	.0000	.0005	.0000	.0000
111	1	.0000	-.0507	.0000	-.0004	.0000	-.0001
	2	.0000	.0276	.0000	.0006	.0000	.0000
	3	.0000	-.0231	.0000	.0002	.0000	-.0001
	4	.0000	-.0783	.0000	-.0010	.0000	-.0001

JOINT DISPLACEMENT (CM RADIANS) **STRUCTURE TYPE = SPACE**

JOINT	LOAD	X-TRANS	Y-TRANS	Z-TRANS	X-ROTAN	Y-ROTAN	Z-ROTAN
112	1	.0000	-.0441	.0000	-.0004	.0000	-.0001
	2	.0000	.0184	.0000	.0005	.0000	.0000
	3	.0000	-.0257	.0000	.0001	.0000	-.0001
	4	.0000	-.0625	.0000	-.0009	.0000	-.0001
113	1	.0000	-.0377	.0000	-.0004	.0000	-.0001
	2	.0000	.0107	.0000	.0004	.0000	.0000
	3	.0000	-.0270	.0000	.0001	.0000	-.0001
	4	.0000	-.0483	.0000	-.0008	.0000	-.0001
114	1	.0000	-.0278	.0000	-.0003	.0000	.0000
	2	.0000	.0008	.0000	.0003	.0000	.0000
	3	.0000	-.0271	.0000	.0000	.0000	.0000
	4	.0000	-.0286	.0000	-.0006	.0000	.0000
115	1	.0000	-.0177	.0000	-.0003	.0000	.0000
	2	.0000	-.0074	.0000	.0002	.0000	.0000
	3	.0000	-.0251	.0000	-.0001	.0000	.0000
	4	.0000	-.0103	.0000	-.0004	.0000	.0000
116	1	.0000	-.0102	.0000	-.0002	.0000	.0000
	2	.0000	-.0115	.0000	.0001	.0000	.0000
	3	.0000	-.0217	.0000	-.0001	.0000	.0000
	4	.0000	.0013	.0000	-.0002	.0000	.0000
117	1	.0000	-.0050	.0000	-.0001	.0000	.0000
	2	.0000	-.0119	.0000	.0000	.0000	.0000
	3	.0000	-.0170	.0000	-.0002	.0000	.0000
	4	.0000	.0069	.0000	-.0001	.0000	.0000
118	1	.0000	-.0015	.0000	-.0001	.0000	.0000
	2	.0000	-.0089	.0000	-.0001	.0000	.0000
	3	.0000	-.0105	.0000	-.0002	.0000	.0000
	4	.0000	.0074	.0000	.0001	.0000	.0000
119	1	.0000	.0008	.0000	-.0001	.0000	.0000
	2	.0000	-.0020	.0000	-.0003	.0000	.0000
	3	.0000	-.0012	.0000	-.0003	.0000	.0000
	4	.0000	.0028	.0000	.0002	.0000	.0000
120	1	.0000	.0026	.0000	.0000	.0000	.0000
	2	.0000	.0096	.0000	-.0004	.0000	.0000
	3	.0000	.0122	.0000	-.0005	.0000	.0000
	4	.0000	-.0070	.0000	.0004	.0000	.0000
121	1	.0000	.0041	.0000	.0000	.0000	.0000
	2	.0000	.0267	.0000	-.0006	.0000	.0000
	3	.0000	.0308	.0000	-.0006	.0000	.0000
	4	.0000	-.0226	.0000	.0005	.0000	.0000
122	1	.0000	-.0515	.0000	-.0004	.0000	.0000
	2	.0000	.0276	.0000	.0006	.0000	.0000
	3	.0000	-.0239	.0000	.0002	.0000	.0000
	4	.0000	-.0790	.0000	-.0010	.0000	.0000
123	1	.0000	-.0447	.0000	-.0004	.0000	.0000
	2	.0000	.0184	.0000	.0005	.0000	.0000
	3	.0000	-.0263	.0000	.0001	.0000	.0000
	4	.0000	-.0632	.0000	-.0009	.0000	.0000

JOINT DISPLACEMENT (CM RADIANS) **STRUCTURE TYPE = SPACE**

JOINT	LOAD	X-TRANS	Y-TRANS	Z-TRANS	X-ROTAN	Y-ROTAN	Z-ROTAN
124	1	.0000	-.0381	.0000	-.0004	.0000	.0000
	2	.0000	.0107	.0000	.0004	.0000	.0000
	3	.0000	-.0274	.0000	.0000	.0000	.0000
	4	.0000	-.0488	.0000	-.0008	.0000	.0000
125	1	.0000	-.0281	.0000	-.0003	.0000	.0000
	2	.0000	.0008	.0000	.0003	.0000	.0000
	3	.0000	-.0273	.0000	.0000	.0000	.0000
	4	.0000	-.0288	.0000	-.0006	.0000	.0000
126	1	.0000	-.0178	.0000	-.0003	.0000	.0000
	2	.0000	-.0074	.0000	.0002	.0000	.0000
	3	.0000	-.0252	.0000	-.0001	.0000	.0000
	4	.0000	-.0104	.0000	-.0004	.0000	.0000
127	1	.0000	-.0103	.0000	-.0002	.0000	.0000
	2	.0000	-.0115	.0000	.0001	.0000	.0000
	3	.0000	-.0217	.0000	-.0001	.0000	.0000
	4	.0000	.0012	.0000	-.0002	.0000	.0000
128	1	.0000	-.0051	.0000	-.0001	.0000	.0000
	2	.0000	-.0119	.0000	.0000	.0000	.0000
	3	.0000	-.0170	.0000	-.0002	.0000	.0000
	4	.0000	.0069	.0000	-.0001	.0000	.0000
129	1	.0000	-.0016	.0000	-.0001	.0000	.0000
	2	.0000	-.0089	.0000	-.0001	.0000	.0000
	3	.0000	-.0105	.0000	-.0002	.0000	.0000
	4	.0000	.0074	.0000	.0001	.0000	.0000
130	1	.0000	.0008	.0000	-.0001	.0000	.0000
	2	.0000	-.0020	.0000	-.0003	.0000	.0000
	3	.0000	-.0012	.0000	-.0003	.0000	.0000
	4	.0000	.0028	.0000	.0002	.0000	.0000
131	1	.0000	.0026	.0000	.0000	.0000	.0000
	2	.0000	.0096	.0000	-.0004	.0000	.0000
	3	.0000	.0122	.0000	-.0005	.0000	.0000
	4	.0000	-.0070	.0000	.0004	.0000	.0000
132	1	.0000	.0041	.0000	.0000	.0000	.0000
	2	.0000	.0267	.0000	-.0006	.0000	.0000
	3	.0000	.0308	.0000	-.0006	.0000	.0000
	4	.0000	-.0226	.0000	.0005	.0000	.0000

************** END OF LATEST ANALYSIS RESULT **************

38. PRINT ELEMENT FORCE LIST 100 TO 110

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ELEMENT FORCES      FORCE,LENGTH UNITS= KN    METE
--------------
FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT  LOAD          QX          QY            MX           MY              MXY
                      VONT        VONB           FX           FY              FXY

 100      1         -2.58        2.10           .14          .05             -.09
                    30.28       30.28           .00          .00              .00
     TOP :  SMAX=     29.66  SMIN=     -1.21  TMAX=      15.43  ANGLE= -33.2
     BOTT:  SMAX=      1.21  SMIN=    -29.66  TMAX=      15.43  ANGLE= -33.2
          2        -11.35         .04           .43         8.56              .02
                  1252.48     1252.48           .00          .00              .00
     TOP :  SMAX=   1283.75  SMIN=     65.09  TMAX=     609.33  ANGLE=    -.2
     BOTT:  SMAX=    -65.09  SMIN=  -1283.75  TMAX=     609.33  ANGLE=    -.2
          3        -13.93        2.14           .57         8.61             -.07
                  1251.39     1251.39           .00          .00              .00
     TOP :  SMAX=   1291.90  SMIN=     85.39  TMAX=     603.25  ANGLE=     .5
     BOTT:  SMAX=    -85.39  SMIN=  -1291.90  TMAX=     603.25  ANGLE=     .5
          4          8.77        2.07          -.30        -8.50             -.12
                  1254.30     1254.30           .00          .00              .00
     TOP :  SMAX=    -44.46  SMIN=  -1275.94  TMAX=     615.74  ANGLE=    -.8
     BOTT:  SMAX=   1275.94  SMIN=     44.46  TMAX=     615.74  ANGLE=    -.8

 101      1         -6.85     -154.88           .18       -14.82             -.62
                  2242.70     2242.70           .00          .00              .00
     TOP :  SMAX=     30.14  SMIN=  -2227.48  TMAX=    1128.81  ANGLE=   -2.4
     BOTT:  SMAX=   2227.48  SMIN=    -30.14  TMAX=    1128.81  ANGLE=   -2.4
          2          6.95         .36           .08         8.31             -.02
                  1239.68     1239.68           .00          .00              .00
     TOP :  SMAX=   1245.83  SMIN=     12.39  TMAX=     616.72  ANGLE=     .1
     BOTT:  SMAX=    -12.39  SMIN=  -1245.83  TMAX=     616.72  ANGLE=     .1
          3           .10     -154.52           .26        -6.52             -.64
                  1011.31     1011.31           .00          .00              .00
     TOP :  SMAX=     47.57  SMIN=   -986.69  TMAX=     517.13  ANGLE=   -5.3
     BOTT:  SMAX=    986.69  SMIN=    -47.57  TMAX=     517.13  ANGLE=   -5.3
          4        -13.80     -155.23           .09       -23.13             -.60
                  3479.98     3479.98           .00          .00              .00
     TOP :  SMAX=     16.27  SMIN=  -3471.82  TMAX=    1744.04  ANGLE=   -1.5
     BOTT:  SMAX=   3471.82  SMIN=    -16.27  TMAX=    1744.04  ANGLE=   -1.5

 102      1        103.48      -94.23          -.43       -12.79             -.85
                  1899.59     1899.59           .00          .00              .00
     TOP :  SMAX=    -55.47  SMIN=  -1926.72  TMAX=     935.62  ANGLE=   -3.9
     BOTT:  SMAX=   1926.72  SMIN=     55.47  TMAX=     935.62  ANGLE=   -3.9
          2         16.04         .43           .41         8.40             -.02
                  1230.08     1230.08           .00          .00              .00
     TOP :  SMAX=   1259.75  SMIN=     61.66  TMAX=     599.05  ANGLE=     .2
     BOTT:  SMAX=    -61.66  SMIN=  -1259.75  TMAX=     599.05  ANGLE=     .2
          3        119.52      -93.80          -.02        -4.39             -.87
                   694.63      694.63           .00          .00              .00
     TOP :  SMAX=     22.39  SMIN=   -683.17  TMAX=     352.78  ANGLE=  -10.8
     BOTT:  SMAX=    683.17  SMIN=    -22.39  TMAX=     352.78  ANGLE=  -10.8
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ELEMENT FORCES        FORCE,LENGTH UNITS= KN    METE
--------------
FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT  LOAD          QX          QY          MX          MY            MXY
                       VONT        VONB        FX          FY            FXY

          4          87.44      -94.66        -.84      -21.19           -.82
                   3124.19     3124.19         .00         .00            .00
     TOP :  SMAX=  -120.77  SMIN= -3182.83  TMAX=  1531.03  ANGLE=   -2.3
     BOTT:  SMAX=  3182.83  SMIN=   120.77  TMAX=  1531.03  ANGLE=   -2.3

  103     1         114.86      -32.21        1.96       -8.09           -.59
                   1393.20     1393.20         .00         .00            .00
     TOP :  SMAX=   299.24  SMIN= -1219.26  TMAX=   759.25  ANGLE=   -3.3
     BOTT:  SMAX=  1219.26  SMIN=  -299.24  TMAX=   759.25  ANGLE=   -3.3
          2          24.47         .24        1.21        8.58           -.04
                   1206.37     1206.37         .00         .00            .00
     TOP :  SMAX=  1286.81  SMIN=   181.42  TMAX=   552.69  ANGLE=     .3
     BOTT:  SMAX=  -181.42  SMIN= -1286.81  TMAX=   552.69  ANGLE=     .3
          3         139.33      -31.97        3.17         .48           -.63
                    472.71      472.71         .00         .00            .00
     TOP :  SMAX=   496.46  SMIN=    51.75  TMAX=   222.35  ANGLE=  -12.5
     BOTT:  SMAX=   -51.75  SMIN=  -496.46  TMAX=   222.35  ANGLE=  -12.5
          4          90.38      -32.45         .75      -16.67           -.55
                   2563.05     2563.05         .00         .00            .00
     TOP :  SMAX=   115.24  SMIN= -2503.48  TMAX=  1309.36  ANGLE=   -1.8
     BOTT:  SMAX=  2503.48  SMIN=  -115.24  TMAX=  1309.36  ANGLE=   -1.8

  104     1          29.13       -3.85        3.64       -4.11           -.25
                   1008.89     1008.89         .00         .00            .00
     TOP :  SMAX=   547.22  SMIN=  -617.04  TMAX=   582.13  ANGLE=   -1.8
     BOTT:  SMAX=   617.04  SMIN=  -547.22  TMAX=   582.13  ANGLE=   -1.8
          2          26.15        -.16        2.51        8.84           -.05
                   1183.61     1183.61         .00         .00            .00
     TOP :  SMAX=  1325.94  SMIN=   376.03  TMAX=   474.96  ANGLE=     .4
     BOTT:  SMAX=  -376.03  SMIN= -1325.94  TMAX=   474.96  ANGLE=     .4
          3          55.29       -4.01        6.15        4.73           -.29
                    839.97      839.97         .00         .00            .00
     TOP :  SMAX=   930.92  SMIN=   701.23  TMAX=   114.85  ANGLE=  -11.3
     BOTT:  SMAX=  -701.23  SMIN=  -930.92  TMAX=   114.85  ANGLE=  -11.3
          4           2.98       -3.69        1.13      -12.95           -.20
                   2032.74     2032.74         .00         .00            .00
     TOP :  SMAX=   170.39  SMIN= -1942.18  TMAX=  1056.29  ANGLE=    -.8
     BOTT:  SMAX=  1942.18  SMIN=  -170.39  TMAX=  1056.29  ANGLE=    -.8

  105     1           -.69        -.88        3.65       -1.99           -.09
                    742.57      742.57         .00         .00            .00
     TOP :  SMAX=   546.99  SMIN=  -298.35  TMAX=   422.67  ANGLE=    -.9
     BOTT:  SMAX=   298.35  SMIN=  -546.99  TMAX=   422.67  ANGLE=    -.9
          2          16.56        -.41        3.70        9.06           -.03
                   1183.61     1183.61         .00         .00            .00
     TOP :  SMAX=  1359.15  SMIN=   555.18  TMAX=   401.98  ANGLE=     .3
     BOTT:  SMAX=  -555.18  SMIN= -1359.15  TMAX=   401.98  ANGLE=     .3
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 ELEMENT FORCES       FORCE,LENGTH UNITS= KN    METE
 --------------
 FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT   LOAD         QX          QY          MX          MY              MXY
                       VONT        VONB        FX          FY              FXY

             3       15.87       -1.28        7.35        7.07             -.12
                   1082.55     1082.55         .00         .00              .00
     TOP :  SMAX=  1109.19  SMIN=  1053.78  TMAX=    27.70  ANGLE= -21.1
     BOTT:  SMAX= -1053.78  SMIN= -1109.19  TMAX=    27.70  ANGLE= -21.1
             4      -17.25        -.47        -.06      -11.05             -.06
                   1653.11     1653.11         .00         .00              .00
     TOP :  SMAX=    -8.40  SMIN= -1657.29  TMAX=   824.45  ANGLE=    -.3
     BOTT:  SMAX=  1657.29  SMIN=     8.40  TMAX=   824.45  ANGLE=    -.3

  106        1      -10.64         .12        2.98        -.96             -.03
                    533.36      533.36         .00         .00              .00
     TOP :  SMAX=   446.62  SMIN=  -143.93  TMAX=   295.28  ANGLE=    -.4
     BOTT:  SMAX=   143.93  SMIN=  -446.62  TMAX=   295.28  ANGLE=    -.4
             2        2.44        -.51        4.23        9.16              .00
                   1190.46     1190.46         .00         .00              .00
     TOP :  SMAX=  1373.32  SMIN=   634.74  TMAX=   369.29  ANGLE=     .1
     BOTT:  SMAX=  -634.74  SMIN= -1373.32  TMAX=   369.29  ANGLE=     .1
             3       -8.21        -.39        7.21        8.20             -.03
                   1162.50     1162.50         .00         .00              .00
     TOP :  SMAX=  1229.57  SMIN=  1081.17  TMAX=    74.20  ANGLE=    1.9
     BOTT:  SMAX= -1081.17  SMIN= -1229.57  TMAX=    74.20  ANGLE=    1.9
             4      -13.08         .63       -1.25      -10.11             -.02
                   1432.46     1432.46         .00         .00              .00
     TOP :  SMAX=  -188.14  SMIN= -1517.23  TMAX=   664.55  ANGLE=    -.2
     BOTT:  SMAX=  1517.23  SMIN=   188.14  TMAX=   664.55  ANGLE=    -.2

  107        1      -13.34         .33        2.13        -.43              .00
                    356.32      356.32         .00         .00              .00
     TOP :  SMAX=   319.39  SMIN=   -64.93  TMAX=   192.16  ANGLE=    -.1
     BOTT:  SMAX=    64.93  SMIN=  -319.39  TMAX=   192.16  ANGLE=    -.1
             2      -12.24        -.46        3.96        9.11              .02
                   1186.25     1186.25         .00         .00              .00
     TOP :  SMAX=  1365.88  SMIN=   593.71  TMAX=   386.09  ANGLE=    -.3
     BOTT:  SMAX=  -593.71  SMIN= -1365.88  TMAX=   386.09  ANGLE=    -.3
             3      -25.58        -.13        6.09        8.67              .02
                   1156.86     1156.86         .00         .00              .00
     TOP :  SMAX=  1300.96  SMIN=   913.09  TMAX=   193.94  ANGLE=    -.5
     BOTT:  SMAX=  -913.09  SMIN= -1300.96  TMAX=   193.94  ANGLE=    -.5
             4       -1.10         .79       -1.83       -9.54             -.03
                   1315.28     1315.28         .00         .00              .00
     TOP :  SMAX=  -274.33  SMIN= -1430.81  TMAX=   578.24  ANGLE=    -.2
     BOTT:  SMAX=  1430.81  SMIN=   274.33  TMAX=   578.24  ANGLE=    -.2

  108        1      -12.50         .34        1.31        -.18              .01
                    210.86      210.86         .00         .00              .00
     TOP :  SMAX=   196.40  SMIN=   -26.43  TMAX=   111.41  ANGLE=     .3
     BOTT:  SMAX=    26.43  SMIN=  -196.40  TMAX=   111.41  ANGLE=     .3
```

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ELEMENT FORCES      FORCE,LENGTH UNITS= KN    METE
--------------
FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT  LOAD         QX          QY          MX          MY              MXY
                      VONT        VONB        FX          FY              FXY

         2          -23.46        -.30       2.96        8.92             .04
                   1180.64     1180.64        .00         .00             .00
  TOP :  SMAX=  1338.36   SMIN=    444.45  TMAX=   446.95   ANGLE=        -.4
  BOTT:  SMAX=  -444.45   SMIN=  -1338.36  TMAX=   446.95   ANGLE=        -.4
         3          -35.96         .04       4.27        8.75             .05
                   1136.30     1136.30        .00         .00             .00
  TOP :  SMAX=  1311.97   SMIN=    640.80  TMAX=   335.59   ANGLE=        -.7
  BOTT:  SMAX=  -640.80   SMIN=  -1311.97  TMAX=   335.59   ANGLE=        -.7
         4           10.96         .64      -1.65       -9.10            -.04
                   1259.18     1259.18        .00         .00             .00
  TOP :  SMAX=  -248.08   SMIN=  -1364.76  TMAX=   558.34   ANGLE=        -.3
  BOTT:  SMAX=  1364.76   SMIN=    248.08  TMAX=   558.34   ANGLE=        -.3

 109     1          -10.05         .16        .62        -.04             .01
                     96.81       96.81        .00         .00             .00
  TOP :  SMAX=    93.65   SMIN=     -6.03  TMAX=    49.84   ANGLE=         .5
  BOTT:  SMAX=     6.03   SMIN=    -93.65  TMAX=    49.84   ANGLE=         .5
         2          -26.68        -.11       1.57        8.65             .05
                   1197.46     1197.46        .00         .00             .00
  TOP :  SMAX=  1297.75   SMIN=    235.62  TMAX=   531.07   ANGLE=        -.4
  BOTT:  SMAX=  -235.62   SMIN=  -1297.75  TMAX=   531.07   ANGLE=        -.4
         3          -36.74         .06       2.20        8.61             .06
                   1162.64     1162.64        .00         .00             .00
  TOP :  SMAX=  1291.76   SMIN=    329.24  TMAX=   481.26   ANGLE=        -.5
  BOTT:  SMAX=  -329.24   SMIN=  -1291.76  TMAX=   481.26   ANGLE=        -.5
         4           16.63         .27       -.95       -8.69            -.05
                   1238.87     1238.87        .00         .00             .00
  TOP :  SMAX=  -141.99   SMIN=  -1303.75  TMAX=   580.88   ANGLE=        -.3
  BOTT:  SMAX=  1303.75   SMIN=    141.99  TMAX=   580.88   ANGLE=        -.3

 110     1           -5.15         .77        .14         .02             .00
                     19.24       19.24        .00         .00             .00
  TOP :  SMAX=    20.47   SMIN=      2.75  TMAX=     8.86   ANGLE=        2.0
  BOTT:  SMAX=    -2.75   SMIN=    -20.47  TMAX=     8.86   ANGLE=        2.0
         2          -16.13         .71        .40        8.37             .04
                   1227.04     1227.04        .00         .00             .00
  TOP :  SMAX=  1255.78   SMIN=     59.66  TMAX=   598.06   ANGLE=        -.3
  BOTT:  SMAX=   -59.66   SMIN=  -1255.78  TMAX=   598.06   ANGLE=        -.3
         3          -21.28        1.48        .53        8.39             .05
                   1220.49     1220.49        .00         .00             .00
  TOP :  SMAX=  1258.56   SMIN=     80.10  TMAX=   589.23   ANGLE=        -.3
  BOTT:  SMAX=   -80.10   SMIN=  -1258.56  TMAX=   589.23   ANGLE=        -.3
         4           10.98         .06       -.26       -8.35            -.04
                   1233.85     1233.85        .00         .00             .00
  TOP :  SMAX=   -39.22   SMIN=  -1253.00  TMAX=   606.89   ANGLE=        -.3
  BOTT:  SMAX=  1253.00   SMIN=     39.22  TMAX=   606.89   ANGLE=        -.3

********************END OF ELEMENT FORCES********************
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39. PRINT SUPPORT REACTION LIST 100 TO 132

SUPPORT REACTIONS -UNIT KN METE STRUCTURE TYPE = SPACE

JOINT	LOAD	FORCE-X	FORCE-Y	FORCE-Z	MOM-X	MOM-Y	MOM Z
100	1	.00	.88	.00	.00	.00	.00
	2	.00	-.61	.00	.00	.00	.00
	3	.00	.28	.00	.00	.00	.00
	4	.00	1.49	.00	.00	.00	.00
101	1	.00	1.56	.00	.00	.00	.00
	2	.00	-.81	.00	.00	.00	.00
	3	.00	.75	.00	.00	.00	.00
	4	.00	2.37	.00	.00	.00	.00
102	1	.00	1.77	.00	.00	.00	.00
	2	.00	-.61	.00	.00	.00	.00
	3	.00	1.16	.00	.00	.00	.00
	4	.00	2.39	.00	.00	.00	.00
103	1	.00	1.89	.00	.00	.00	.00
	2	.00	-.05	.00	.00	.00	.00
	3	.00	1.84	.00	.00	.00	.00
	4	.00	1.94	.00	.00	.00	.00
104	1	.00	1.36	.00	.00	.00	.00
	2	.00	.70	.00	.00	.00	.00
	3	.00	2.06	.00	.00	.00	.00
	4	.00	.67	.00	.00	.00	.00
105	1	.00	.79	.00	.00	.00	.00
	2	.00	1.07	.00	.00	.00	.00
	3	.00	1.86	.00	.00	.00	.00
	4	.00	-.28	.00	.00	.00	.00
106	1	.00	.38	.00	.00	.00	.00
	2	.00	1.12	.00	.00	.00	.00
	3	.00	1.50	.00	.00	.00	.00
	4	.00	-.74	.00	.00	.00	.00
107	1	.00	.10	.00	.00	.00	.00
	2	.00	.84	.00	.00	.00	.00
	3	.00	.94	.00	.00	.00	.00
	4	.00	-.74	.00	.00	.00	.00
108	1	.00	-.10	.00	.00	.00	.00
	2	.00	.20	.00	.00	.00	.00
	3	.00	.10	.00	.00	.00	.00
	4	.00	-.30	.00	.00	.00	.00
109	1	.00	-.24	.00	.00	.00	.00
	2	.00	-.87	.00	.00	.00	.00
	3	.00	-1.11	.00	.00	.00	.00
	4	.00	.63	.00	.00	.00	.00
110	1	.00	-.19	.00	.00	.00	.00
	2	.00	-1.22	.00	.00	.00	.00
	3	.00	-1.41	.00	.00	.00	.00
	4	.00	1.04	.00	.00	.00	.00
111	1	.00	.69	.00	.00	.00	.00
	2	.00	-.38	.00	.00	.00	.00
	3	.00	.32	.00	.00	.00	.00
	4	.00	1.07	.00	.00	.00	.00

SUPPORT REACTIONS -UNIT KN METE STRUCTURE TYPE = SPACE

JOINT	LOAD	FORCE-X	FORCE-Y	FORCE-Z	MOM-X	MOM-Y	MOM Z
112	1	.00	1.21	.00	.00	.00	.00
	2	.00	-.50	.00	.00	.00	.00
	3	.00	.70	.00	.00	.00	.00
	4	.00	1.71	.00	.00	.00	.00
113	1	.00	1.36	.00	.00	.00	.00
	2	.00	-.39	.00	.00	.00	.00
	3	.00	.97	.00	.00	.00	.00
	4	.00	1.74	.00	.00	.00	.00
114	1	.00	1.42	.00	.00	.00	.00
	2	.00	-.04	.00	.00	.00	.00
	3	.00	1.38	.00	.00	.00	.00
	4	.00	1.45	.00	.00	.00	.00
115	1	.00	1.01	.00	.00	.00	.00
	2	.00	.42	.00	.00	.00	.00
	3	.00	1.43	.00	.00	.00	.00
	4	.00	.58	.00	.00	.00	.00
116	1	.00	.58	.00	.00	.00	.00
	2	.00	.65	.00	.00	.00	.00
	3	.00	1.23	.00	.00	.00	.00
	4	.00	-.07	.00	.00	.00	.00
117	1	.00	.29	.00	.00	.00	.00
	2	.00	.68	.00	.00	.00	.00
	3	.00	.97	.00	.00	.00	.00
	4	.00	-.39	.00	.00	.00	.00
118	1	.00	.09	.00	.00	.00	.00
	2	.00	.51	.00	.00	.00	.00
	3	.00	.60	.00	.00	.00	.00
	4	.00	-.42	.00	.00	.00	.00
119	1	.00	-.05	.00	.00	.00	.00
	2	.00	.11	.00	.00	.00	.00
	3	.00	.07	.00	.00	.00	.00
	4	.00	-.16	.00	.00	.00	.00
120	1	.00	-.15	.00	.00	.00	.00
	2	.00	-.55	.00	.00	.00	.00
	3	.00	-.69	.00	.00	.00	.00
	4	.00	.40	.00	.00	.00	.00
121	1	.00	-.12	.00	.00	.00	.00
	2	.00	-.76	.00	.00	.00	.00
	3	.00	-.88	.00	.00	.00	.00
	4	.00	.64	.00	.00	.00	.00
122	1	.00	.27	.00	.00	.00	.00
	2	.00	-.14	.00	.00	.00	.00
	3	.00	.12	.00	.00	.00	.00
	4	.00	.41	.00	.00	.00	.00
123	1	.00	.47	.00	.00	.00	.00
	2	.00	-.19	.00	.00	.00	.00
	3	.00	.27	.00	.00	.00	.00
	4	.00	.66	.00	.00	.00	.00

SUPPORT REACTIONS -UNIT KN METE STRUCTURE TYPE = SPACE

JOINT	LOAD	FORCE-X	FORCE-Y	FORCE-Z	MOM-X	MOM-Y	MOM Z
124	1	.00	.52	.00	.00	.00	.00
	2	.00	-.15	.00	.00	.00	.00
	3	.00	.38	.00	.00	.00	.00
	4	.00	.67	.00	.00	.00	.00
125	1	.00	.54	.00	.00	.00	.00
	2	.00	-.02	.00	.00	.00	.00
	3	.00	.53	.00	.00	.00	.00
	4	.00	.56	.00	.00	.00	.00
126	1	.00	.39	.00	.00	.00	.00
	2	.00	.16	.00	.00	.00	.00
	3	.00	.55	.00	.00	.00	.00
	4	.00	.23	.00	.00	.00	.00
127	1	.00	.22	.00	.00	.00	.00
	2	.00	.25	.00	.00	.00	.00
	3	.00	.47	.00	.00	.00	.00
	4	.00	-.03	.00	.00	.00	.00
128	1	.00	.11	.00	.00	.00	.00
	2	.00	.26	.00	.00	.00	.00
	3	.00	.37	.00	.00	.00	.00
	4	.00	-.15	.00	.00	.00	.00
129	1	.00	.03	.00	.00	.00	.00
	2	.00	.19	.00	.00	.00	.00
	3	.00	.23	.00	.00	.00	.00
	4	.00	-.16	.00	.00	.00	.00
130	1	.00	-.02	.00	.00	.00	.00
	2	.00	.04	.00	.00	.00	.00
	3	.00	.03	.00	.00	.00	.00
	4	.00	-.06	.00	.00	.00	.00
131	1	.00	-.06	.00	.00	.00	.00
	2	.00	-.21	.00	.00	.00	.00
	3	.00	-.26	.00	.00	.00	.00
	4	.00	.15	.00	.00	.00	.00
132	1	.00	-.04	.00	.00	.00	.00
	2	.00	-.29	.00	.00	.00	.00
	3	.00	-.33	.00	.00	.00	.00
	4	.00	.24	.00	.00	.00	.00

************** END OF LATEST ANALYSIS RESULT **************

Appendix C: Use STAAD-III to solve pavement design example 2

```
 ****************************************************
 *                                                  *
 *              S T A A D - III                     *
 *              Revision 22.3b                      *
 *              Proprietary Program of              *
 *              Research Engineers, Inc.            *
 *              Date=    AUG 16, 2005               *
 *              Time=      9:16: 3                  *
 *              Build No.    2500                   *
 *         USER ID: Kawneer Company Canada Limited  *
 ****************************************************

STAAD SPACE DOWEL-1
UNIT CM
JOINT COORDINATES
1 0. 0. 0. 13 0 0 360 1
REPEAT 10 30 0 0
144 300.5 0 0 156 300.5 0 360 1
REPEAT 10 30 0 0
MEMBER INCIDE
301 131 144 313
ELEMENT INCIDENCES
1 1 2 15 14 TO 12 1 1
REPEAT 9 12 13
121 144 145 158 157 TO 132 1 1
REPEAT 9 12 13
MEMBER RELEASE
301 TO 313 START FZ FX MX MY MZ
UNIT KN CM
MEMBER PROPERTIES
* DOWEL BAR
301 TO 313 PRIS AX 4.9 IX 3.83 IY 1.92 IZ 1.92
ELEMENT PROPERTIES
1 TO 240 TH 20
UNIT KN METER
CONSTANTS
E 2E8 MEMBER 301 TO 313
E 27.6E6 ELEM 1 TO 240
POISSON 0.15 ELEM 1 TO 240
SUPPORT
1 TO 286 ELASTIC MAT DIRECTION Y SUB 80000
LOAD 1
ELEMENT LOAD
109 116 PR GY -667
PERFORM ANALYSIS
```

34. PRINT JOINT DISPL LIST 131 TO 143

JOINT DISPLACEMENT (CM RADIANS) STRUCTURE TYPE = SPACE

JOINT	LOAD	X-TRANS	Y-TRANS	Z-TRANS	X-ROTAN	Y-ROTAN	Z-ROTAN
131	1	.0000	-.0738	.0000	-.0004	.0000	-.0004
132	1	.0000	-.0613	.0000	-.0004	.0000	-.0004
133	1	.0000	-.0490	.0000	-.0004	.0000	-.0003
134	1	.0000	-.0396	.0000	-.0002	.0000	-.0003
135	1	.0000	-.0339	.0000	-.0001	.0000	-.0002
136	1	.0000	-.0316	.0000	.0000	.0000	-.0002
137	1	.0000	-.0320	.0000	.0000	.0000	-.0002
138	1	.0000	-.0332	.0000	.0000	.0000	-.0003
139	1	.0000	-.0317	.0000	-.0001	.0000	-.0003
140	1	.0000	-.0269	.0000	-.0002	.0000	-.0002
141	1	.0000	-.0219	.0000	-.0002	.0000	-.0002
142	1	.0000	-.0173	.0000	-.0001	.0000	-.0002
143	1	.0000	-.0133	.0000	-.0001	.0000	-.0002

************** END OF LATEST ANALYSIS RESULT **************

35. PRINT MEMBER FORCE LIST 301 TO 313

MEMBER END FORCES   STRUCTURE TYPE = SPACE

ALL UNITS ARE -- KN   METE

MEMBER	LOAD	JT	AXIAL	SHEAR-Y	SHEAR-Z	TORSION	MOM-Y	MOM-Z
301	1	131	.00	-13.11	.00	.00	.00	.00
		144	.00	13.11	.00	.00	.00	-.07
302	1	132	.00	-11.88	.00	.00	.00	.00
		145	.00	11.88	.00	.00	.00	-.06
303	1	133	.00	-1.05	.00	.00	.00	.00
		146	.00	1.05	.00	.00	.00	-.01
304	1	134	.00	.16	.00	.00	.00	.00
		147	.00	-.16	.00	.00	.00	.00
305	1	135	.00	.21	.00	.00	.00	.00
		148	.00	-.21	.00	.00	.00	.00
306	1	136	.00	-.10	.00	.00	.00	.00
		149	.00	.10	.00	.00	.00	.00
307	1	137	.00	-1.70	.00	.00	.00	.00
		150	.00	1.70	.00	.00	.00	-.01
308	1	138	.00	-10.78	.00	.00	.00	.00
		151	.00	10.78	.00	.00	.00	-.05
309	1	139	.00	-10.82	.00	.00	.00	.00
		152	.00	10.82	.00	.00	.00	-.05
310	1	140	.00	-1.80	.00	.00	.00	.00
		153	.00	1.80	.00	.00	.00	-.01
311	1	141	.00	-.32	.00	.00	.00	.00
		154	.00	.32	.00	.00	.00	.00
312	1	142	.00	-.02	.00	.00	.00	.00
		155	.00	.02	.00	.00	.00	.00
313	1	143	.00	.32	.00	.00	.00	.00
		156	.00	-.32	.00	.00	.00	.00

************** END OF LATEST ANALYSIS RESULT **************

```
 36. PRINT ELEMENT FORCE LIST 109 TO 120

 ELEMENT FORCES      FORCE,LENGTH UNITS= KN    METE
 --------------
 FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT   LOAD          QX          QY          MX          MY            MXY
                        VONT        VONB        FX          FY            FXY


 109        1         64.60       42.18        1.04       -1.75         -1.27
                     491.91      491.91         .00         .00           .00
    TOP :  SMAX=     228.72  SMIN=   -335.91  TMAX=   282.31  ANGLE= -21.1
    BOTT:  SMAX=     335.91  SMIN=   -228.72  TMAX=   282.31  ANGLE= -21.1

 110        1        150.34       26.91        3.40        -.79         -2.87
                     943.98      943.98         .00         .00           .00
    TOP :  SMAX=     729.48  SMIN=   -336.68  TMAX=   533.08  ANGLE= -26.9
    BOTT:  SMAX=     336.68  SMIN=   -729.48  TMAX=   533.08  ANGLE= -26.9

 111        1         70.62       -3.10        6.41         .01         -2.11
                    1106.09     1106.09         .00         .00           .00
    TOP :  SMAX=    1056.67  SMIN=    -92.97  TMAX=   574.82  ANGLE= -16.7
    BOTT:  SMAX=      92.97  SMIN=  -1056.67  TMAX=   574.82  ANGLE= -16.7

 112        1         23.69       -6.29        6.73         .35         -1.29
                    1039.51     1039.51         .00         .00           .00
    TOP :  SMAX=    1046.77  SMIN=     14.67  TMAX=   516.05  ANGLE= -11.0
    BOTT:  SMAX=     -14.67  SMIN=  -1046.77  TMAX=   516.05  ANGLE= -11.0

 113        1        -15.39       -5.55        5.69         .32          -.45
                     839.38      839.38         .00         .00           .00
    TOP :  SMAX=     859.52  SMIN=     41.83  TMAX=   408.84  ANGLE=  -4.8
    BOTT:  SMAX=     -41.83  SMIN=   -859.52  TMAX=   408.84  ANGLE=  -4.8

 114        1        -58.67        -.13        3.33        -.13           .26
                     514.01      514.01         .00         .00           .00
    TOP :  SMAX=     502.11  SMIN=    -23.04  TMAX=   262.57  ANGLE=   4.3
    BOTT:  SMAX=      23.04  SMIN=   -502.11  TMAX=   262.57  ANGLE=   4.3

 115        1       -137.99       14.35       -1.80        -.46           .74
                     310.17      310.17         .00         .00           .00
    TOP :  SMAX=     -19.61  SMIN=   -319.51  TMAX=   149.95  ANGLE= -23.9
    BOTT:  SMAX=     319.51  SMIN=     19.61  TMAX=   149.95  ANGLE= -23.9

 116        1         -3.23       25.19       -5.78        -.40          -.42
                     845.26      845.26         .00         .00           .00
    TOP :  SMAX=     -55.02  SMIN=   -871.42  TMAX=   408.20  ANGLE=   4.4
    BOTT:  SMAX=     871.42  SMIN=     55.02  TMAX=   408.20  ANGLE=   4.4

 117        1        132.97       15.72       -3.44        -.54         -1.64
                     642.48      642.48         .00         .00           .00
    TOP :  SMAX=      30.21  SMIN=   -626.84  TMAX=   328.53  ANGLE=  24.2
    BOTT:  SMAX=     626.84  SMIN=    -30.21  TMAX=   328.53  ANGLE=  24.2

 118        1         58.20        2.56         .06        -.29         -1.34
                     351.02      351.02         .00         .00           .00
```

```
    TOP :   SMAX=    185.41  SMIN=   -219.44  TMAX=    202.42  ANGLE= -41.3
    BOTT:   SMAX=    219.44  SMIN=   -185.41  TMAX=    202.42  ANGLE= -41.3


 ELEMENT FORCES     FORCE,LENGTH UNITS= KN    METE
 --------------
 FORCE OR STRESS = FORCE/UNIT WIDTH/THICK, MOMENT = FORCE-LENGTH/UNIT WIDTH

ELEMENT  LOAD        QX          QY             MX          MY           MXY
                    VONT        VONB            FX          FY           FXY


  119       1       22.13       -4.04           .82         .16          -.92
                   264.23      264.23           .00         .00           .00
     TOP :   SMAX=    219.71  SMIN=    -73.48  TMAX=    146.60  ANGLE= -35.2
     BOTT:   SMAX=     73.48  SMIN=   -219.71  TMAX=    146.60  ANGLE= -35.2

  120       1       -1.91       -2.90           .36         .36          -.37
                   109.59      109.59           .00         .00           .00
     TOP :   SMAX=    109.08  SMIN=     -1.00  TMAX=     55.04  ANGLE=  44.8
     BOTT:   SMAX=      1.00  SMIN=   -109.08  TMAX=     55.04  ANGLE=  44.8


 ********************END OF ELEMENT FORCES********************
```

37. PRINT SUPPORT REACTION LIST 131 TO 143

SUPPORT REACTIONS -UNIT KN METE STRUCTURE TYPE = SPACE

JOINT	LOAD	FORCE-X	FORCE-Y	FORCE-Z	MOM-X	MOM-Y	MOM Z
131	1	.00	1.35	.00	.00	.00	.00
132	1	.00	2.24	.00	.00	.00	.00
133	1	.00	1.79	.00	.00	.00	.00
134	1	.00	1.45	.00	.00	.00	.00
135	1	.00	1.24	.00	.00	.00	.00
136	1	.00	1.16	.00	.00	.00	.00
137	1	.00	1.17	.00	.00	.00	.00
138	1	.00	1.22	.00	.00	.00	.00
139	1	.00	1.16	.00	.00	.00	.00
140	1	.00	.99	.00	.00	.00	.00
141	1	.00	.80	.00	.00	.00	.00
142	1	.00	.63	.00	.00	.00	.00
143	1	.00	.24	.00	.00	.00	.00

************** END OF LATEST ANALYSIS RESULT **************

Appendix D: Use KENSLABS to solve pavement design example 2

1. General Information screen.

General Information of SLABSINP

TITLE: design example 2

Type of foundation (0=liquid, 1=solid, 2=layer)	(NFOUND)	0
Damage analysis (0=no, 1=PCA criteria, 2=user specified)	(NDAMA)	0
Number of periods per year	(NPY)	1
Number of load groups	(NLG)	1
Number of slab layers	(NLAYER)	1
Bond between two slab layers (0=unbonded, 1=bonded)	(NBOND)	0
Number of slabs	(NSLAB)	2
Number of joints	(NJOINT)	1
Nodal number for checking convergence	(NNCK)	1
Number of nodes for stress printout	(NPRINT)	0
Number of nodes on X axis of symmetry	(NSX)	0
Number of nodes on Y axis of symmetry	(NSY)	0
More detailed printout (0=no, 1=yes)	(MDPO)	0
Number of nodes with different thicknesses of slab layer 1	(NAT1)	0
Number of nodes with different thicknesses of slab layer 2	(NAT2)	0
System of units (0=English, 1=SI)	(NUNIT)	0

Default options are shown by black dots. If not true, please click the other button.

- with uniform load
- without uniform load

- with temperature gradient and/or checking contact
- without temperature gradient and/or checking contact

- with concentrated load
- without concentrated load

Print | *Data Set 1* 0

2. Slab Information screen.

Slab Information

Arrangement input | X-coordinate input | Y-coordinate input | Properties input

Arrangement of Slabs and Joints

Use <Ctrl>-<Del> to delete a line, <Ctrl>-<Ins> to insert a line, and <C

Slab No.	NX	NY	JONO1	JONO2	JONO3	JONO4
1	6	9	0	1	0	0
2	6	9	1	0	0	0

- JONO1/2/3/4 - joint number on four sides of each slab. Subscript '1' indicates the joint number on the left side of the slab, '2' on right side, '3' on the bottom and '4' on the top. If there is no joint on a given side, JONO = '0'.

Thickness, Poisson's Ratio, Young's Modulus, and Unit Weight of Concrete

Unit	in.		psi
Layer No.	T	PR	YM
1	7.9	.15	4000000

3. Uniform load screen.

Loaded Areas for Load Group No. 1

Use <Ctrl>-<Del> to delete a line, <Ctrl>-<Ins> to insert a line, and <Del

Unit		in.	in.	in.	in.	psi
Load Sequence	LS	XL1	XL2	YL1	YL2	QQ
1	1	145.7	157.5	0	11.8	96.7
2	1	145.7	157.5	82.7	94.2	96.7

4. Foundation Information screen.

Liquid Foundation

Double click (or press the E

Unit	pci	
	SUBMOD	NAS
	294.9	0

Solid Foundation

Young's modulus of subgrade in psi	(YMS)	38238
Poisson's ratio of subgrade	(PRS)	0.45

5. Joint Information screen.

Joint Information

Double click anywhere on a line to get auxiliary form for NNAJ.

Unit	psi	in.-lb/in.	pci	in.	in.	in.	in.	
Joint No.	SPCON1	SPCON2	SCKV	BD	BS	WJ	GDC	NNAJ
1	0	0	1500000	.98	11.8	.2	0	9

- SPCON1 - spring constant for shear transfer for each joint. SPCON1 = '0' if dowel bars are used. When grain interlock is used as loading transferring media, the stiffness of joint is represented by shear spring constant C_ω and moment spring constant C_θ as mentioned above in Chapter 2.
- SPCON2 – spring constant for moment transfer for each joint. SPCON2 = '0' if dowel bars are used. Since load is transferred across the joint mainly by shear, it is generally agreed that $C_\theta = 0$ in most occasions.
- SCKV - modulus of dowel support K, also called steel-concrete K value (pci or KN/m^3) as mentioned above in Chapter 2. Suggested K values range from 3×10^5 to 1.5×10^6 pci (81.5 to 409 GN/m^3). If SPCON1/2 ≠'0', SCKV = '0'.
- BD – dowel bar diameter (inch or cm). If SPCON1/2 ≠'0', BD = '0'.
- BS – dowel bar spacing (inch or cm). If SPCON1/2 ≠'0', BS = '0'.
- WJ – width of the joint (inch or cm). If SPCON1/2 ≠'0', WJ = '0'.
- GDC – gap between dowel bars and concrete. In the case of no gap exists between dowel bars and concrete, GDC = '0'.
- NNAJ - number of nodes at each joint, maximum 15 nodes can be defined. In the case of dowel bars are uniformly distributed along the joint, NNAJ = '0'.

INPUT FILE NAME -C:\KENPAVE\design example 2.TXT

NUMBER OF PROBLEMS TO BE SOLVED = 1

TITLE -example 2 - liquid foundation

TYPE OF FOUNDATION (NFOUND)	=	0
TYPE OF DAMAGE ANALYSIS (NDAMA)	=	0
NUMBER OF PERIODS PER YEAR (NPY)	=	1
NUMBER OF LOAD GROUPS (NLG)	=	1
TOTAL NUMBER OF SLABS (NSLAB)	=	2
TOTAL NUMBER OF JOINTS (NJOINT)	=	1

ARRANGEMENT OF SLABS

SLAB NO.	NO. NODES (NX) IN X DIRECTION	NO. NODES (NY) IN Y DIRECTION	JOINT NO. AT FOUR SIDES (JONO) LEFT	RIGHT	BOTTOM	TOP
1	6	9	0	1	0	0
2	6	9	1	0	0	0

NUMBER OF LAYERS (NLAYER)-------------------------------------	=	1
NODAL NUMBER USED TO CHECK CONVERGENCE (NNCK)-----------------	=	1
NUMBER OF NODES NOT IN CONTACT (NOTCON)-----------------------	=	0
NUMBER OF GAPS (NGAP)---	=	0
NUMBER OF POINTS FOR PRINTOUT (NPRINT)------------------------	=	0
CODE FOR INPUT OF GAPS OR PRECOMPRESSIONS (INPUT)- -----------	=	0
BOND BETWEEN TWO LAYERS (NBOND)-------------------------------	=	0
CONDITION OF WARPING (NTEMP)----------------------------------	=	0
CODE INDICATING WHETHER SLAB WEIGHT IS CONSIDERED (NWT)-------	=	0
MAX NO. OF CYCLES FOR CHECKING CONTACT (NCYCLE)---------------	=	1
NUMBER OF ADDITIONAL THICKNESSES FOR SLAB LAYER 1 (NAT1)------	=	0
NUMBER OF ADDITIONAL THICKNESSES FOR SLAB LAYER 2 (NAT2)------	=	0
NUMBER OF POINTS ON X AXIS OF SYMMETRY (NSX)------------------	=	0
NUMBER OF POINTS ON Y AXIS OF SYMMETRY (NSY)------------------	=	0
MORE DETAILED PRINTOUT FOR EACH CONTACT CYCLE (MDPO)----------	=	0
TOLERANCE FOR ITERATIONS (DEL)--------------------------------	=	0.001
MAXIMUM ALLOWABLE VERTICAL DISPLACEMENT (FMAX)----------------	=	1

SYSTEM OF UNITS (NUNIT)--------------------------------------= 0
(Length in in., force in lb, stress in psi, unit weigh in pcf
subgrade and dowel K value in pci, and temperature in F)

FOR SLAB NO. 1 COORDINATES OF FINITE ELEMENT GRID ARE:
X = 0 40 80 120 145.7 157.5
Y = 0 11.8 25 39 53 67 82.7 110 141.7

FOR SLAB NO. 2 COORDINATES OF FINITE ELEMENT GRID ARE:
X = 157.7 169.5 195.2 235.2 275.2 315.2
Y = 0 11.8 25 39 53 67 82.7 110 141.7

LAYER NO.	THICKNESS (T)	POISSON'S RATIO (PR)	YOUNG'S MODULUS (YM)
1	7.90000	0.15000	4.000E+06

No. OF LOADED AREAS (NUDL) FOR EACH LOAD GROUP ARE: 2
NO. OF NODAL FORCES (NCNF) AND MOMENTS (NCMX AND NCMY) ARE: 0 0 0

FOR LOAD GROUP NO. 1 LOADS ARE APPLIED AS FOLLOWS:

SLAB NO. (LS)	X COORDINATES (XL1)	(XL2)	Y COORDINATES (YL1)	(YL2)	INTENSITY (QQ)
1	145.70000	157.50000	0.00000	11.80000	96.70000
1	145.70000	157.50000	82.70000	94.20000	96.70000

FOUNDATION ADJUSTMENT FACTOR (FSAF) FOR EACH PERIOD ARE: 1

NUMBER OF ADDITIONAL SUBGRADE MODULI (NAS) TO BE READ IN-----= 0
SUBGRADE MODULUS (SUBMOD)------------------------------------= 294.9

NODAL COORDINATES (XN AND YN) OF INDIVIDUAL SLAB ARE:

1	0.000	0.000	2	0.000	11.800	3	0.000	25.000
4	0.000	39.000	5	0.000	53.000	6	0.000	67.000
7	0.000	82.700	8	0.000	110.000	9	0.000	141.700
10	40.000	0.000	11	40.000	11.800	12	40.000	25.000
13	40.000	39.000	14	40.000	53.000	15	40.000	67.000
16	40.000	82.700	17	40.000	110.000	18	40.000	141.700
19	80.000	0.000	20	80.000	11.800	21	80.000	25.000
22	80.000	39.000	23	80.000	53.000	24	80.000	67.000
25	80.000	82.700	26	80.000	110.000	27	80.000	141.700
28	120.000	0.000	29	120.000	11.800	30	120.000	25.000
31	120.000	39.000	32	120.000	53.000	33	120.000	67.000
34	120.000	82.700	35	120.000	110.000	36	120.000	141.700
37	145.700	0.000	38	145.700	11.800	39	145.700	25.000
40	145.700	39.000	41	145.700	53.000	42	145.700	67.000
43	145.700	82.700	44	145.700	110.000	45	145.700	141.700
46	157.500	0.000	47	157.500	11.800	48	157.500	25.000
49	157.500	39.000	50	157.500	53.000	51	157.500	67.000
52	157.500	82.700	53	157.500	110.000	54	157.500	141.700
55	157.700	0.000	56	157.700	11.800	57	157.700	25.000
58	157.700	39.000	59	157.700	53.000	60	157.700	67.000
61	157.700	82.700	62	157.700	110.000	63	157.700	141.700
64	169.500	0.000	65	169.500	11.800	66	169.500	25.000
67	169.500	39.000	68	169.500	53.000	69	169.500	67.000
70	169.500	82.700	71	169.500	110.000	72	169.500	141.700
73	195.200	0.000	74	195.200	11.800	75	195.200	25.000
76	195.200	39.000	77	195.200	53.000	78	195.200	67.000
79	195.200	82.700	80	195.200	110.000	81	195.200	141.700
82	235.200	0.000	83	235.200	11.800	84	235.200	25.000
85	235.200	39.000	86	235.200	53.000	87	235.200	67.000
88	235.200	82.700	89	235.200	110.000	90	235.200	141.700
91	275.200	0.000	92	275.200	11.800	93	275.200	25.000
94	275.200	39.000	95	275.200	53.000	96	275.200	67.000
97	275.200	82.700	98	275.200	110.000	99	275.200	141.700
100	315.200	0.000	101	315.200	11.800	102	315.200	25.000
103	315.200	39.000	104	315.200	53.000	105	315.200	67.000
106	315.200	82.700	107	315.200	110.000	108	315.200	141.700

YOUNG MODULUS OF DOWEL BAR (YMSB) = 2.900E+07
POISSON RATIO OF DOWEL BAR (PRSB) = 0.30000

JOINT NO.	SPRING CONSTANT SHEAR (SPCON1)	MOMENT (SPCON2)	MODULUS OF DOWEL SUP. (SCKV)	DOWEL DIA. (BD)	DOWEL SPACING (BS)	JOINT WIDTH (WJ)	GAP DOWEL (GDC)	NODE JOINT (NNAJ)
1	0.000E+00	0.000E+00	1.500E+06	0.980	11.800	0.200	0.00000	9

FOR JOINT NO. 1 DOWELS (BARNO) AT EACH NODE ARE: 1 1 1 1 1 1 1 1 1

JOINT NO. 1 SPRING CONSTANT OF ONE DOWEL BAR = 4.656E+05

HALF BAND WIDTH (NB) = 33

PERIOD 1 LOAD GROUP 1 AND CYCLE NO. 1

DEFLECTIONS OF SLABS (F) ARE: (DOWNWARD POSITIVE)

1	-0.00028	2	-0.00031	3	-0.00033	4	-0.00033	5	-0.00031
6	-0.00027	7	-0.00022	8	-0.00014	9	-0.00009	10	-0.00032
11	-0.00046	12	-0.00061	13	-0.00071	14	-0.00076	15	-0.00076
16	-0.00072	17	-0.00066	18	-0.00073	19	0.00196	20	0.00128
21	0.00059	22	0.00005	23	-0.00026	24	-0.00039	25	-0.00040
26	-0.00048	27	-0.00106	28	0.01197	29	0.00944	30	0.00690
31	0.00495	32	0.00396	33	0.00373	34	0.00373	35	0.00293
36	0.00063	37	0.02394	38	0.01924	39	0.01442	40	0.01085
41	0.00925	42	0.00931	43	0.00997	44	0.00780	45	0.00354
46	0.02891	47	0.02366	48	0.01795	49	0.01372	50	0.01187
51	0.01208	52	0.01315	53	0.01027	54	0.00525	55	0.02321
56	0.01984	57	0.01621	58	0.01310	59	0.01110	60	0.01001
61	0.00918	62	0.00734	63	0.00491	64	0.01694	65	0.01442
66	0.01173	67	0.00942	68	0.00788	69	0.00698	70	0.00630
71	0.00501	72	0.00316	73	0.00655	74	0.00539	75	0.00415
76	0.00310	77	0.00237	78	0.00194	79	0.00165	80	0.00119
81	0.00031	82	0.00017	83	-0.00009	84	-0.00038	85	-0.00060
86	-0.00074	87	-0.00079	88	-0.00078	89	-0.00076	90	-0.00095
91	-0.00055	92	-0.00059	93	-0.00062	94	-0.00064	95	-0.00064
96	-0.00061	97	-0.00057	98	-0.00050	99	-0.00049	100	-0.00011
101	-0.00011	102	-0.00010	103	-0.00009	104	-0.00007	105	-0.00004
106	-0.00001	107	0.00004	108	0.00008				

FOR JOINT NO. 1 SHEAR (FAJ1) AND MOMENT (FAJ2) AT THE NODES ARE:

46	-2652.8	0.0	47	-1778.6	0.0	48	-813.9	0.0
49	-288.3	0.0	50	-357.4	0.0	51	-962.3	0.0
52	-1851.8	0.0	53	-1367.7	0.0	54	-160.3	0.0

FOR JOINT NO. 1 SHEAR IN ONE DOWEL BAR (FAJPD) AT THE NODES IS:

46	-2652.8	47	-1778.6	48	-813.9	49	-288.3	50	-357.4
51	-962.3	52	-1851.8	53	-1367.7	54	-160.3		

FOR JOINT NO. 1 BEARING STRESS (BEARS) OF CONCRETE AND SHEAR STRESS (SHEARS) OF DOWELS AT THE NODES ARE:

46	-4226.2	-3516.9	47	-2833.5	-2357.9	48	-1296.6	-1079.0
49	-459.4	-382.3	50	-569.4	-473.8	51	-1533.0	-1275.7
52	-2950.2	-2455.1	53	-2179.0	-1813.2	54	-255.4	-212.5

NODAL NUMBER AND REACTIVE PRESSURE (SUBR) ARE: (COMPRESSION POSITIVE)

1	-0.082	2	-0.091	3	-0.098	4	-0.098	5	-0.092
6	-0.080	7	-0.064	8	-0.040	9	-0.027	10	-0.093
11	-0.136	12	-0.179	13	-0.210	14	-0.224	15	-0.223
16	-0.212	17	-0.195	18	-0.214	19	0.577	20	0.378
21	0.173	22	0.014	23	-0.077	24	-0.114	25	-0.119
26	-0.142	27	-0.313	28	3.529	29	2.785	30	2.036
31	1.460	32	1.168	33	1.099	34	1.101	35	0.864
36	0.187	37	7.059	38	5.674	39	4.252	40	3.198
41	2.728	42	2.746	43	2.940	44	2.299	45	1.045
46	8.526	47	6.979	48	5.295	49	4.045	50	3.499
51	3.561	52	3.879	53	3.030	54	1.549	55	6.846

56	5.852	57	4.779	58	3.862	59	3.273	60	2.951
61	2.706	62	2.163	63	1.448	64	4.997	65	4.251
66	3.459	67	2.778	68	2.323	69	2.057	70	1.858
71	1.477	72	0.932	73	1.931	74	1.590	75	1.225
76	0.914	77	0.700	78	0.572	79	0.486	80	0.352
81	0.092	82	0.049	83	-0.027	84	-0.111	85	-0.178
86	-0.217	87	-0.232	88	-0.230	89	-0.226	90	-0.279
91	-0.162	92	-0.173	93	-0.184	94	-0.190	95	-0.188
96	-0.180	97	-0.167	98	-0.148	99	-0.144	100	-0.032
101	-0.031	102	-0.029	103	-0.026	104	-0.019	105	-0.012
106	-0.002	107	0.012	108	0.024				

SUM OF FORCES (FOSUM) = 26586.7   SUM OF REACTIONS (SUBSUM) = 26586.5

NODE	LAYER	STRESS X	STRESS Y	STRESS XY	MAX.SHEAR	MAJOR	MINOR
1	1	0.000	0.000	0.000	0.000	0.000	0.000
2	1	0.000	1.153	0.000	0.577	1.153	0.000
3	1	0.000	1.991	0.000	0.995	1.991	0.000
4	1	0.000	2.080	0.000	1.040	2.080	0.000
5	1	0.000	1.575	0.000	0.788	1.575	0.000
6	1	0.000	0.726	0.000	0.363	0.726	0.000
7	1	0.000	-0.368	0.000	0.184	0.000	-0.368
8	1	0.000	-1.197	0.000	0.599	0.000	-1.197
9	1	0.000	0.000	0.000	0.000	0.000	0.000
10	1	13.867	0.000	0.000	6.934	13.867	0.000
11	1	12.338	3.496	8.187	9.304	17.222	-1.387
12	1	9.990	5.724	5.772	6.153	14.010	1.704
13	1	7.756	6.156	3.810	3.893	10.849	3.063
14	1	5.847	5.103	2.142	2.174	7.649	3.301
15	1	4.141	3.147	0.914	1.040	4.685	2.604
16	1	2.902	0.600	0.084	1.154	2.905	0.597
17	1	2.358	-2.857	1.341	2.932	2.682	-3.182
18	1	-0.190	0.000	0.000	0.095	0.000	-0.190
19	1	78.442	0.000	0.000	39.221	78.442	0.000
20	1	65.184	16.717	31.855	40.025	80.976	0.925
21	1	55.654	26.270	24.709	28.747	69.709	12.215
22	1	46.769	26.452	15.333	18.393	55.003	18.218
23	1	41.380	21.296	6.481	11.952	43.290	19.386
24	1	38.690	14.658	1.254	12.081	38.755	14.593
25	1	34.446	5.330	1.034	14.595	34.482	5.293
26	1	30.314	-6.875	8.356	20.385	32.105	-8.666
27	1	18.859	0.000	0.000	9.430	18.859	0.000
28	1	162.783	0.000	0.000	81.391	162.783	0.000
29	1	126.553	47.826	79.979	89.141	176.331	-1.952
30	1	94.411	78.372	69.461	69.922	156.314	16.469
31	1	75.590	94.649	38.570	39.730	124.850	45.390
32	1	69.531	76.414	7.866	8.586	81.558	64.387
33	1	75.765	29.285	-12.535	26.405	78.930	26.120
34	1	91.133	-11.753	-0.718	51.448	91.138	-11.758
35	1	62.221	-18.027	28.689	49.325	71.422	-27.228
36	1	38.453	0.000	0.000	19.226	38.453	0.000
37	1	-125.574	0.000	0.000	62.787	0.000	-125.574
38	1	-44.995	15.727	82.184	87.612	72.979	-102.246
39	1	8.082	144.588	75.576	101.835	178.170	-25.500
40	1	28.942	171.916	40.442	82.134	182.563	18.295
41	1	26.884	145.548	4.084	59.472	145.688	26.744
42	1	8.431	65.810	-26.607	39.128	76.249	-2.007

43	1	-24.565	-146.718	-4.103	61.214	-24.427	-146.856
44	1	1.563	-9.169	35.186	35.593	31.790	-39.396
45	1	28.830	0.000	0.000	14.415	28.830	0.000
46	1	0.000	0.000	0.000	0.000	0.000	0.000
47	1	0.000	-31.001	65.199	67.016	51.516	-82.516
48	1	0.000	183.042	71.618	116.212	207.733	-24.691
49	1	0.000	201.096	37.680	107.376	207.925	-6.828
50	1	0.000	175.154	2.159	87.604	175.181	-0.027
51	1	0.000	108.415	-34.132	64.058	118.265	-9.851
52	1	0.000	-226.214	-6.412	113.289	0.182	-226.396
53	1	0.000	8.388	33.101	33.365	37.559	-29.171
54	1	0.000	0.000	0.000	0.000	0.000	0.000

MAXIMUM STRESS (SMAX) IN LAYER 1 IS -226.396 (NODE 52)

MAXIMUM NEGATIVE STRESS IN X DIRECTION = -125.6 (NODE 37)
MAXIMUM POSITIVE STRESS IN X DIRECTION = 162.8 (NODE 28)
MAXIMUM NEGATIVE STRESS IN Y DIRECTION = -226.2 (NODE 52)
MAXIMUM POSITIVE STRESS IN Y DIRECTION = 201.1 (NODE 49)