STUDY OF CURVATURE EFFECTS IN COMPOSITE STEEL I-GIRDER BRIDGES USING THE V-LOAD METHOD

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STUDY OF CURVATURE EFFECTS IN COMPOSITE STEEL I-GIRDER BRIDGES

USING THE V-LOAD METHOD

BY Mohammad Reza Davoodi, Ryerson University - Civil Engineering
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Abstract

Horizontally curved composite I-girder bridges are being increasingly used for highway interchanges and river crossings. The V-load method is widely used as a simplified method for the analysis of horizontally curved I-girder highway bridges as a straight I-girder considering the effect of torsion due to curvature. Recently, North American bridge design codes and specifications have specified certain limitations to treat horizontally curved bridges as straight ones in structural analysis and design. The purpose of this study is to investigate the applicability of those specified limitations by the V-Load method, to compare the results from the V-Load method with those obtained from the finite element analysis and to develop empirical expressions for curvature limitation. The results of this study shows that the North American codes and specifications underestimate the response with their specified curvature limitations. Based on this study, a modified equation for the curvature limitation is proposed.

Acknowledgements

I wish to express my deep appreciation to my supervisor Dr. Khaled Sennah, Who made this work possible. His friendly guidance and expert advice have been invaluable throughout all stages of the work.
Dr. Sennah devoted his time and effort to make this study a success. His most helpful guidance is greatly appreciated.
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Thank you

TABLE OF CONTENT

LIST OF TABLES

Tables ***Page***

LIST OF FIGURES

NOTATIONS

b	Half-width of the bridge
H or d or h	Depth of the bridge
N	Number of transvers bracing
R	Radius of curvature
W or B	Width of the bridge
L	Centre line curved span length
M	Longitudinal bending moment
WS/TS	Warping Stress to Total Stress
SMF	Stress Magnification Factor in using the V-Load Method
L/R	Span-to-radius or curvature ratio

1. Introduction

As a result of complicated geometrics, limited rights of way, and traffic mitigation, horizontally curved bridges are becoming the model type of highway interchanges and urban expressways. This type of superstructure has gained popularity since the early 1960s because it addresses the needs of transportation engineering.

The design of today's roadways has placed increasing demands on the engineer. The use of horizontally curved bridges has grown out of alignment requirements and constraints. The right of way available for the construction of a new roadway may be limited because of the expansion that many cities are experiencing. It may be impossible to use straight bridge girder in curved ramps, so a curved bridge is necessary with the alignment adapted to suit the site. In addition, the girder spans can be continuous which allows shallower girders. The aesthetics of a curved continuous bridge is also an advantage. There are, however, disadvantages which the engineer should be aware of when designing curved bridges. The fabrication costs are generally higher, and the curved bridge segments are produced in smaller pieces which increases the erection and transportation costs [1]. Analysis of curved bridges is different from that for a straight bridge because of the twisting of the unit due to its curvature.

Horizontally curved steel girder bridges are often used in our modern road systems. The curve in the bridge allows for a smoother transition for traffic, which creates better road travel. The disadvantage of horizontally curved bridges is that they are more difficult to analyze and design than the conventional straight bridges. The horizontal curvature in the girders adds torsional effects, which can increase or decrease the strains in the girders based on its location in the bridge cross-section. The methods used in steel bridge analysis can generally be classified in one of two categories: hand analysis, or computer-based numerical analysis using the finite element method. The finite element method is the most common numerical method in structure analysis and design [3, 4].

The V-Load method is a widely used as an approximate method for analyzing horizontally curved I-girder bridges. The method assumes that the internal torsional load on the bridge resulting from the curvature is resisted by self-equilibrating sets of shear responses (referred to as secondary) between adjacent girders. The final response in the curved girder is the sum of the secondary response and the respective straight girder primary response.

Due to the difficulty in analyzing these bridges, a variety of approaches may be used. The appropriateness and relative accuracy of these different approaches need to be determined. In bridges with light curvature, the curvature effects on bending, shear, and torsional stresses may be ignored if they are within an acceptable range [3]. Treating horizontally curved bridges as straight ones with certain limitations is one of the methods to simplify the design procedure.

Bridge design specifications and codes have specified certain limitations to treat horizontally curved bridges as straight bridges. However, these limitations do not differentiate between bridge cross section configurations, in addition to being inaccurate in estimating the structural response. Moreover, these specifications were developed primarily for the calculation of girder bending moments [2]. The purpose of this study is to investigate the applicability of those specified limitations of bridge curvature in North American Bridge Codes to treat a horizontally curved bridge as straight ones by the V-Load method and compare the results with those obtained using the finite element method [2].

2. Overall Review, the Problem and Objectives of This Study

Horizontally curved composite I-girder bridges, shown in Figure 1, are being increasingly used for highway interchanges and river crossings. Curved bridge girders offer several inherent advantages. They are more aesthetically pleasing than a series of straight girders along the chords of a roadway curve. Curved girders allow designers to use longer spans, thus eliminating much of the substructure. Curved bridges may also result in simpler and more uniform construction details because girder spacing and concrete slab overhang are generally constant along the length of the structure. However, curved bridge design has difficulty in mathematically analyzing the curved girders; curvature causes torsional loadings that complicate the stress analysis.

Figure 1. Highfield Lane bridge [5]

Different methods have been available for the structural analysis of curved bridges, but highway engineers generally prefer simplified techniques. Most recently, curved girders are widely used in bridge superstructures. However, due to the addition of curvature, the design and construction of bridges becomes greatly more complicated than that of straight bridges and their structural behavior still not well understood. While in straight bridges, the girders, stringers, and floor beams can be designed by systematically isolating each member and applying standard loads, curved bridges must be designed with careful consideration to system-wide behavior. In essence, the addition of curvature adds torsion to the system that results in significant warping and distortional stresses within the member cross-sections. Furthermore, “secondary members” such as cross frames and diaphragms that provide stability in straight bridges become primary load carrying members in curved bridges [6].

The V-Load method is a widely used as simplified method for the analysis of horizontally curved I-girder highway bridges as straight I-girder considering the effect of torsional due to curvature . This approximate method eventually became known as the V-Load method because a large percentage of the torsional load on the girders is approximated by sets of vertical shears known as V-loads.

The V-Load method has been widely used in consulting engineering offices. According to a 1969 survey, the method was used for the design of approximately 75 percent of the curved steel I-girder bridges in the United States. Current codes pertaining to analysis and design of horizontally curved bridges are mostly based on experimental and analytical research conducted over 30 years ago as part of project CURT (Consortium of University Research Teams, 1975) [6].

Treating the horizontally curved bridges as straight one with certain limitations is one of the methods to simplify the analysis and design procedure. Recently the Canadian Highway Bridge Design Code of 2014, CHBDC [7], the AASHTO Guide Specifications for Horizontally Curved Bridges of 2003 [8] and AASHTO LRFD Bridge Design Specifications of 2017 [9] have specified certain limitations to treat horizontally curved bridges as straight one. AASHTO Guide Specifications for Horizontally Curved Bridges accepted the use of V-Load Method (VLM) as one of the methods for the analysis of horizontally curved bridges. This current study is an attempt to realize the above mentioned need in Canada. The results and the equations provided in this study will help bridge engineers in predicting new and more reliable limitations in treating the horizontally curved bridges as straight ones. Therefore, the overall objectives of the research reported herein are:

- Study the applicability of the specified limitations of bridge curvature in North American Bridge Codes to treat a horizontally curved bridge as a straight one in structural analysis and design using the V-Load method.

- Compare the results with the provided information that complements existing data and that has been done with finite element method by Khalafalla (2009) [2].

- Develop empirical expressions for stress magnification factors and use them to develop curvature limitations to treat a horizontally curved bridge as straight one.

3. Introduction of the V-Load Method

In 1984, AISC Marketing Inc. published a report explaining the "V-Load Analysis" for the analysis of curved steel I-girder bridges [6]. This report presented an approximate simplified analysis method to determine moments and shears for horizontally curved I-girder bridges. Now, this method is known as the V-Load method because a large part of the torsion load on the girders is approximated by sets of vertical shears known as "V-Loads." The V-Load method is a two-step process. First, the bridge is straightened out so that the applied vertical load is assumed to induce only flexural stresses. Second, additional fictitious forces are applied to result in final stresses similar to the ones in a curved bridge. The additional fictitious forces are determined so that they result in no net vertical, longitudinal, or transverse forces on the bridge. With this method, the girders are assumed to be straight, but point loads are placed along each span at points where the diaphragm or transversal bracing connects to the girders (see Figure 2), simulating torsional loads imposed on the girder system.

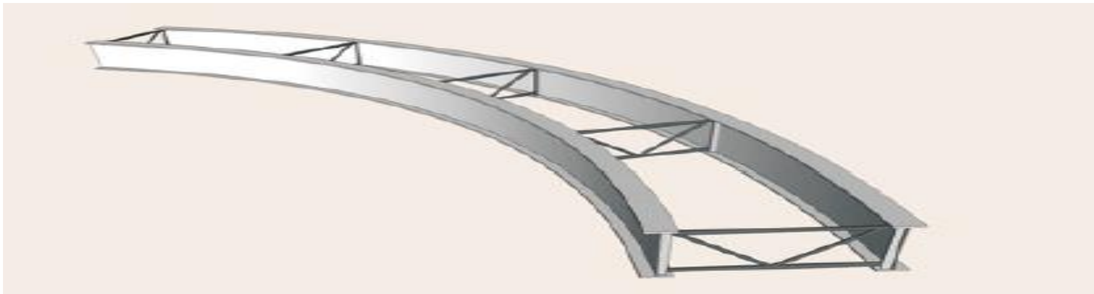


Figure 2. Braced Pair Girder [5]

In order to determine the magnitude of the extra point loads, the moments at critical sections (typically diaphragm or bracing locations) along the girders are determined, as if the bridge were a normal, straight span.

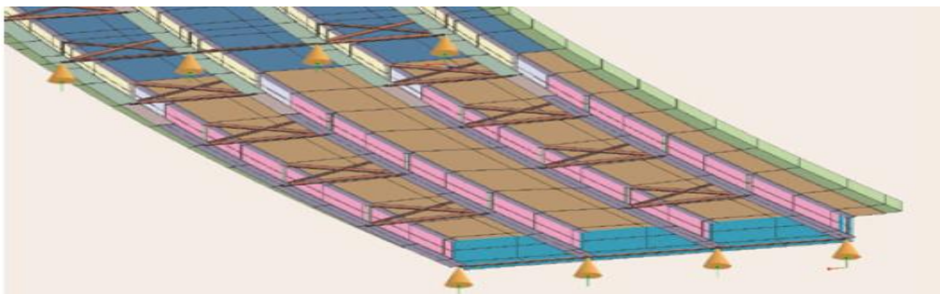


Figure 3. View of Underside of typical I-Girder Bridge [5]

Once the V-Load has been calculated, the extra loads are applied to each of the girders. The value of the loads applied depends on the position of the girder. The loads placed on the outside girder are equal to the V-Load and are acting down. The loads applied to the inside girder are also equal to the V-Load, but are acting up. The loads added to the middle girders are calculated by assuming a straight-line variation from one side of the bridge to the other. For example, if a girder is located at the center of the bridge no V-Load will be added to it. With the V-Load method, boundary conditions must be assumed which will allow evaluation by hand.

Recent efforts were made to extend the V-load method to composite curved open-framed bridges (no horizontal lateral bracing) with any general support configuration by comparing the V-load-method results to the results from several finite-element bridge models similar to that shown in Figure 3. Non-composite and composite bridges, with combinations of radial and skewed supports, were analyzed with both methods under the correct dead and live loading. The effect of horizontal lateral bracing was also studied for selected cases.

Another important consideration in the design of curved I-girder bridges is the warping stresses (lateral bending stresses) that develop in the girder flanges as depicted in Figure 4. These stresses arise from resistance to the out-of-plane warping of an I-girder cross section that is caused by the applied torsional loads. The approximate calculation of these warping stresses is also presented in this study.

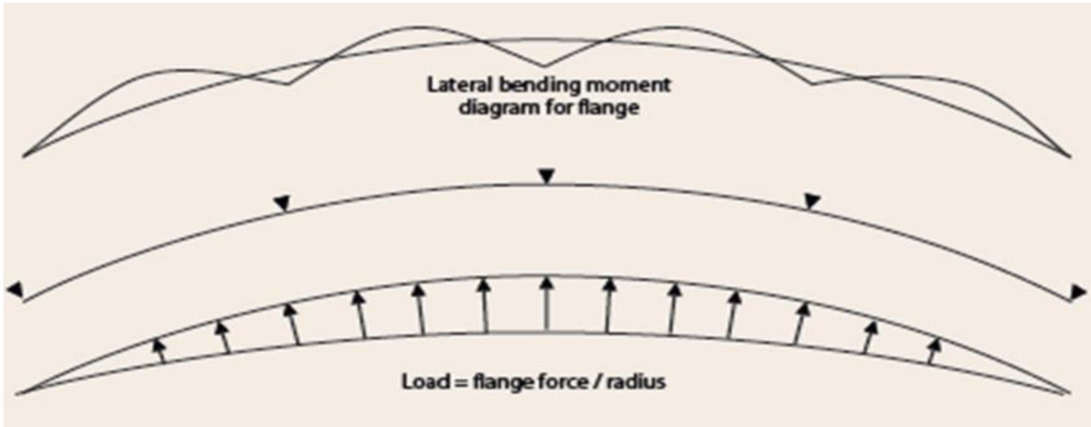


Figure 4. Lateral bending moment in girder flanges due to torsional effect [5]

For the bridge analyzed in this report there were 962 different load cases that had to be evaluated. This required repeating the V-Load calculation process 962 times. This was not practical by hand, so a set of spread sheets was developed, using Microsoft Excel, that uses the V-Load method to calculate stress at the desired points along the span.

4. V Load Method History and Review of Previous Work

One of the first presentations of an approximate analysis of horizontally curved girders was by the United States Steel Corporation in 1963. The United States Steel approximate method became known as the V-load method and was extended to analyze multi-girder bridge units in 1965. A computer program implementing the V-Load method was developed in 1966 for multi-girder bridge units with radial supports [5]. About the same time, Dabrowski developed expressions for the warping moment in a curved girder using differential equations. Developments in curved girder analysis were also made by Gillespie and Ketchek. They used an approximate analysis method where it was found that the lateral bending stress was dependent on the lateral bending moment which is related to diaphragm spacing. Another method was developed by Ketchek who, in addition to allowing for the V-Loads used in earlier reports, allowed for the direct application of uniformly distributed torsional moments to the girders. In the 1970's, CURT, Consortium of University Research Teams, was established to develop methods for curved bridge analysis and design and determine bridge requirements. Also during this time, Weissman developed a method for analyzing curved girders using statically indeterminate analysis of plane grid systems with straight elements. The slope deflection technique was used by Heins and Siminou to determine various distribution factors to relate a single straight girder to a single curved girder and then to a system of curved girders. Culver, Brogan, and Bednar utilized the flexibility method to develop an approximate analysis using equivalent straight girders. They discovered that the maximum deflection of a curved girder is much larger than that of an equivalent straight girder. For small radii of curvature, a curved beam is more flexible than the equivalent straight girder, and the ratio of deflections between a curved and a straight girder increases as the radius of curvature decreases. They also found that the diaphragm spacing influenced the maximum warping stress but not the bending stress. The approximate method predicted the outer girder stress fairly well but underestimated the stress on the inner girder.

In the 1980's, the V-Load method was revised to accommodate skewed bridges with the effort of US Steel Research and Richardson, Gordon, and Associates. Grubb found this approximate analysis method very accurate for the dead load condition. For live load, he found that the V-Load results were reasonable for the exterior girders but not for the interior girders. The accuracy was largely affected by the lateral distribution factors assumed in the V-Load method. In 1984, Heins and Jin [10] developed expressions for live load distribution factors for braced systems by the use of a three-dimensional space frame matrix formulation. Bottom bracing was added to their models to examine its effect on the load distribution. It was found that bottom bracing stiffens the system and the live load is distributed more uniformly to all the girders and the load to a given girder decreased.

5. Approximate Analysis of Horizontally Curved Bridges using the V-Load Method

Horizontally curved bridges respond to loads different from straight bridges because of the torsional forces induced by the curvature of the longitudinal axis. An approximate method of analysis for horizontally curved bridges can be developed using equivalent straight girders if the torque produced by the curvature is represented by self-equilibrating loads on the girders. These additional loads are called V-loads because they are a set of vertical shears on the equivalent straight girders. The V-loads are developed from equilibrium requirements and are primarily a function of the radius of curvature, width of the bridge unit, and spacing of diaphragms (transverse bracing) between the girders [6]. This section presents the V-Load method for approximate analysis of horizontally curved bridge units. The method was first developed for a two-girder bridge unit and then for a multi-girder bridge unit.

5.1 Two-Girder Bridge

The approximate forces on two horizontally curved girders connected with radial diaphragms (transverse bracing) can be determined from equilibrium. Figure 5 shows a schematic diagram of the analysis of the longitudinal bending moment and resolved flange forces, while Figure 6 shows a horizontally curved bridge unit with two girders spaced a distance D. The angle of curvature of the bridge is θ, the radius of the outside girder, girder 1, is R1 and the arc length is L1. The radius and arc length of girder 2 are R2 and L2, respectively. Radial diaphragms, spaced a distance D, connect girders 1 and 2. Vertical loads on the bridge produce bending moments in both girders.

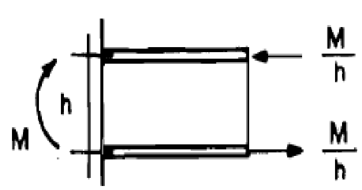


Figure 5. Longitudinal bending moment and flange forces [11]

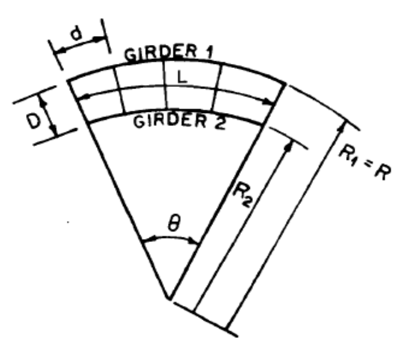


Figure 6. Horizontally curved two-girder bridge [11]

Assuming that the plate girder sections resist the bending moment entirely by longitudinal forces in the flanges, as shown in Figure 7, the force in each flange of girder 1 is M_1/h_1 where h_1 is the depth of the girder and M_1 is the bending moment. In girder 2, the bending moment is M_2 and the flange forces are M_2/h_2. However, because the flanges of the girder are horizontally curved, the longitudinal forces due to bending are not in equilibrium. To maintain radial equilibrium of the flange, the chord of the diaphragm must develop a force. Similar forces develop at the bottom cord of the diaphragm, for equilibrium of the bottom flange. Figure 7 shows a free body diagram of a diaphragm between the girders. The horizontal force, H1 and H2, and vertical shear force, V, developed in the diaphragm are found as follows by equilibrium along a radial line at the diaphragm location.

$$H_1 = \frac{M_1 \theta}{h_1} \tag{1}$$

In which θ is central angle of curvature of the girder (L_1/R_1 or L_2/R_2).

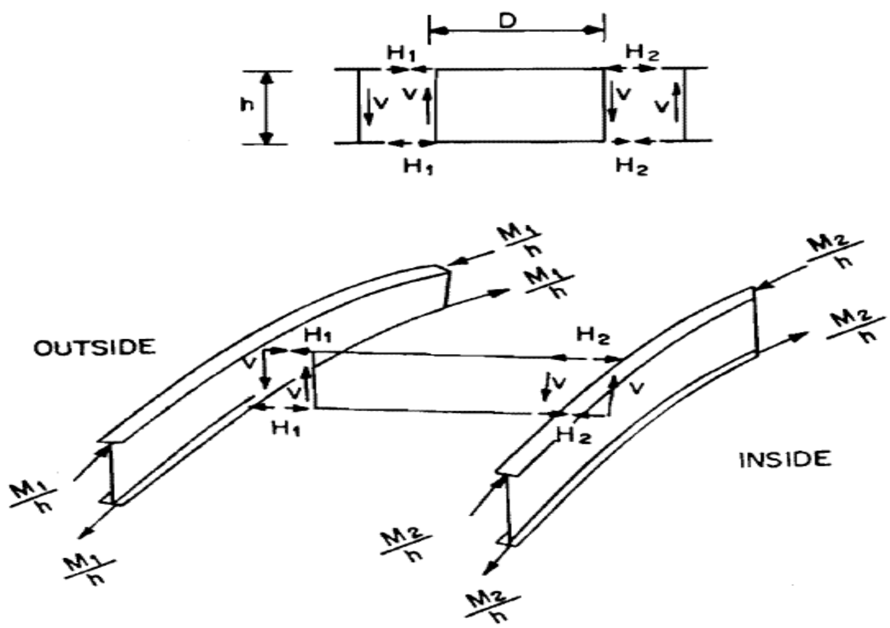


Figure 7. Free-body diagram and cross-section of bridge showing diaphragm and girders [11]

Substituting θ and H_1, repeating the same calculation for H_2 and using the equilibrium concept we have the shear force V:

$$H_1 = \frac{M_1 d_1}{h_1 R_1} \quad , \quad V = (H_1 + H_2)\frac{h}{D} \quad , \quad V = \frac{M_1 \frac{d_1}{R_1} + M_2 \frac{d_2}{R_2}}{D} \tag{2}$$

But from geometry, d1/R1 = d2/R2 = d/R and the shear force in the diaphragm (transverse bracing) is:

$$V = \frac{M_1 + M_2}{R\, D/d} \tag{3}$$

These shear forces in the diaphragm act in the opposite direction on girders 1 and 2. The shear forces, known as V-loads, are self-equilibrating forces on the bridge unit that approximate the effects of the horizontal curvature of the girders. They must be self-equilibrating forces because they are not actual loads applied to the bridge unit.

The bending moments M_1 and M_2 are the moments in girders 1 and 2 due to the applied loads and the additional forces due to curvature, as represented by the V-loads. The two contributions to the totals moment can be separated as:

$$\begin{aligned} M_1 &= M_{1p} + M_{1v} \\ M_2 &= M_{2p} + M_{2v} \end{aligned} \tag{4}$$

The subscripts p and v denote responses due to the P-Loads, which are applied loads, and V-loads respectively. In common application of the V-Load method, the bending moments produced by the concentrated V-Load forces are assumed proportional to their respective girder lengths. Equation 5 gives the magnitude of the V -loads as a function of the P-Load moments only:

$$V + \frac{M_{1p} + M_{2p}}{R\ D/d} \tag{5}$$

Where the V-Load forces act on the girders at the diaphragm location.

In summary, the V-Load method involves analyzing the girders in the bridge unit as equivalent straight girders twice as follows.

- The first analysis gives the response to P-Loads, including M_{1p} and M_{2p}.
- The second analysis gives the response to the self-equilibrating V-Load applied at the diaphragm or transverse bracing locations.

The total response on the girders is the sum of the responses to the P-Loads and V-Loads.

5.2 Multi-Girder Bridge

In a curved bridge unit with two girders, the outer girder experiences an increase in load due to the curvature while the inner one experiences a decrease in load. The same procedure can be applied in a bridge with more than two girders, but the effect of curvature must also be distributed to the inner girders. A general expression for the V-Loads acting on multiple girder units can be developed using a similar procedure as for the two-girder bridge geometry.

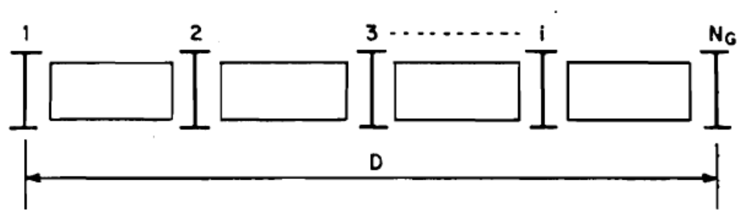


Figure 8. Cross section of a multi-girder bridge unit [11]

Figure 8 shows a cross section of a bridge unit with *N* girders, where D is the distance between outer and inner girders. Due to curvature, the section is subjected to twisting. Lateral flange forces develop and produce forces in the diaphragms as described before. The V-Loads are derived using equilibrium between the girders and the diaphragms or transverse bracing. It can be shown that equilibrium of the diaphragm panels allows summation of the lateral flange forces and with repeating same approach, the equation for V can be expressed as follows:

$$V = \frac{\sum_{i=1}^{N_g} M_{pi}}{C\,(RD/d)} \tag{6}$$

In which, Ng is the number of girders and C is equal to:

$$C = \frac{1}{6}\,\frac{N_g\,(N_g + 1)}{N_g} - 1 \tag{7}$$

The other parameters are defined earlier.

Number of Girdres, Ng	C
2	1.0000
3	1.0000
4	1.1111
5	1.2500
6	1.4000
7	1.5556
8	1.7143

Table 1. Value of C as a Function of the number of girders, N_g [5]

A check of this expression for a two-girder unit gives a value of C equal to 1.0. This is the same coefficient as found in the derivation of the two girder unit. Table 1 lists the value of C as a function of the number of girders. The summation of girder moments in V Equation can be approximately represented by the summation of primary girder moments as done for two-girder units. The V-Loads acting on the diaphragm or transverse bracing locations of the other girders are given by V-Load Equation that is based on the assumption of linearly varying diaphragm shear.

The first of the two analyses for each equivalent straight girder gives the P-Load moment, shear, and reaction responses, respectively. The second analysis gives the responses due to the V-Loads. The expression for the V-Load factor is dependent on the number of girders as derived above. The V-Loads are assumed to be distributed linearly between the outer and inner girders, Therefore, the V-Load on a girder is proportional to its distance from the bridge centerline. The magnitude of the V-Loads is observed to:

- ✓ increase with decreasing radius of curvature,
- ✓ increase with increase the curvature,
- ✓ increase with decreasing bridge unit width, and

✓ increase with increasing diaphragm (transverse bracing) spacing

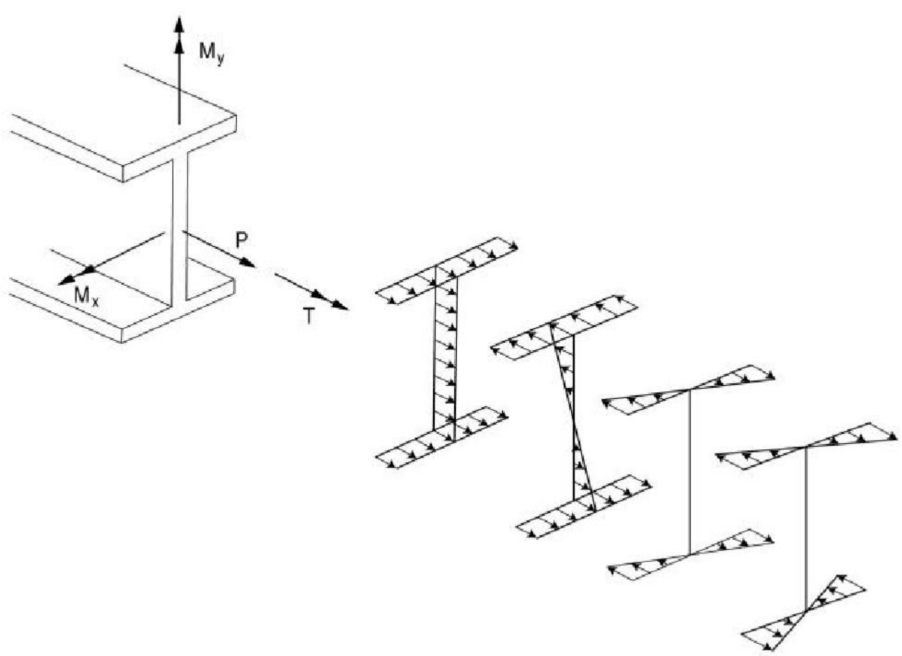
In summary, the V-Load method involves analyzing the girders in the bridge unit as equivalent straight girders twice as follows.

- The first analysis gives the response to P-Loads, including M_{pi}.
- The second analysis gives the response to the self-equilibrating V-Load applied at the diaphragm or transverse bracing locations.

The total response on the girders is the sum of the responses to the P-Loads and V-loads.

5.3 Torsional Response of Girders

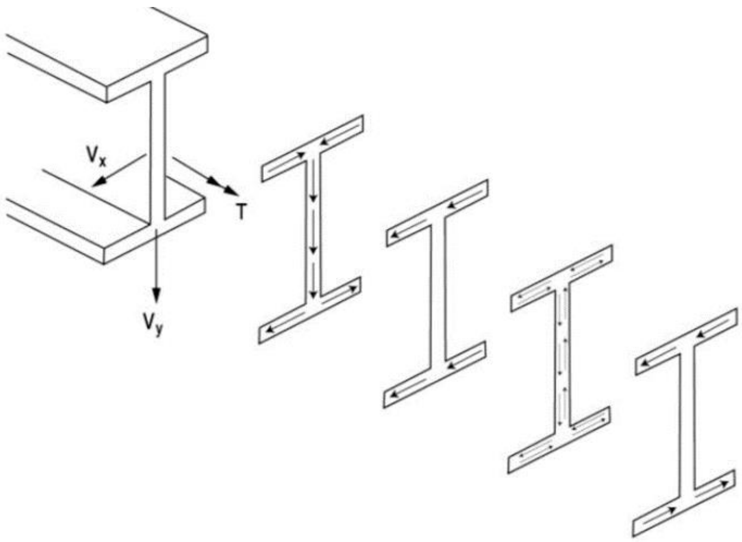
In addition to the basic vertical shear and bending effects, a curved girder is also subjected to torsional effects. Because I-shaped girders have low St. Venant torsional stiffness, they carry torsion primarily by means of warping. The total state of normal stress in an I-shaped girder is a combination of any axial stress, major axis bending stress, lateral bending stress, and warping normal stress. The total state of shear stress in an I-shaped girder is a combination of vertical shear stress, horizontal shear stress, some St. Venant torsional shear stress (typically relatively small), and warping shear stress [4].



$$\text{Total Normal Stress} = \sigma = \frac{P}{A} + \frac{M_x\,y}{I_x} + \frac{M_y\,x}{I_y} + \text{Warping Normal Stress}$$

Figure 9. The general I-girder normal stresses, which can occur in a curved I-shaped girder

The relatively low St. Venant torsional stiffness of I-shaped girders is a result of their open cross sectional geometry. The St. Venant torsional shear flow around the perimeter of the cross section can only develop relatively small force couples [12].



$$\text{Total Shear Stress} = \tau = \frac{V_y Q_x}{I_x t} + \frac{V_x Q_y}{I_y t} + \text{St. Venant Torsion} + \text{Warping Torsion}$$

Figure 10. The general I-girder shear stresses, which can occur in a curved I-shaped girder [10]

Assuming no bracing in the plate of the bottom flange, the St. Venant's stiffness for wide flange girders is much less than its warping stiffness. So, the St. Venant's torsion is neglected in an approximate analysis of curved girder bridge without bracing in the plane of the bottom flanges. All of the torsion is assumed to be resisted by warping of the girders. The approximate torsional response analysis presented as follows.

The section of a girder twisted through an angle, ϕ, and by a torque, T, is shown in Figure 11. The torque creates flange shear forces, T/h, in the direction of the torque, where h is the depth of the girder section. These flange shears cause lateral bending moments, M_f, in the flange. The effects of warping torsion can be approximated by applying lateral forces to a straight model of the bottom flange. Due to horizontal curvature, radial forces develop on the flanges to establish equilibrium.

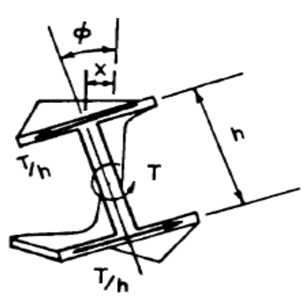


Figure 11. Girder section twisted by a torque [11]

The lateral load on the flange, W_W, varies along its length and in proportion to the bending moment as required for radial equilibrium:

$$W_W = M/Rh \tag{8}$$

Where M is the total bending moment on the girder, h is the distance between flanges, and R is the radius of the twisted by the girder. The distribution of these lateral flange loads is shown in Figure 12.

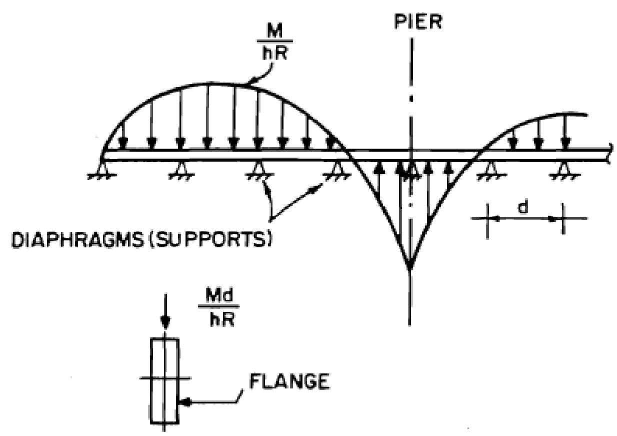


Figure 12. Distribution of lateral loads on a girder flange [11]

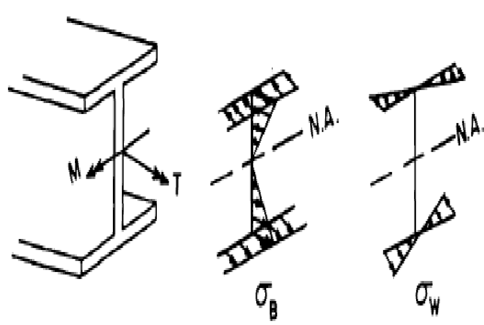


Figure 13. Bending and warping stress in girder cross section [11]

As shown in the above figure, diaphragms or transverse bracing restrain lateral bending of the girders, acting as lateral supports for the flanges. In the approximation, the diaphragms are assumed to provide rigid supports against lateral bending. The lateral bending moments in the flange resulting from this loading (flange warping), M_f, is illustrated in Figure (12). However, for simplicity, the transverse bending moment in girder flanges for cases with less than 3 intermediate cross frames (transverse bracing) or diaphragms may be determined as follows:

$$M_f = 1/8\ W_W d^2 \tag{9}$$

For other cases:

$$M_f = 1/10\ W_W d^2 \tag{10}$$

The normal warping stress at the flange tip is then given by:

$$\sigma_w = \frac{M_f}{S_f} \quad (11)$$

where S_f is the section modulus of the bottom flange for lateral bending.

The longitudinal bending stress and warping stress distributions on a girder cross section are shown in Figure 13. The summation of the stress due to bending, σ_b, and that due to warping, σ_w, gives the total stress at the tip of the flange, σ_t.

6. Analysis Procedure

The procedures relevant to the analysis of curved bridge in this study involves computing the moments, shear forces, longitudinal and warping stresses, and reactions that develop due to dead load and specified positions of live loads. A direct analysis of the structure with the prescribed loads can be performed to compute the responses.

The approximate analysis procedure, based on the V-Load method, presented in this study, computes the response of multi-girder bridge units with variable radius of curvature. The girders may include composite behavior of the steel girders and concrete slab. The analysis procedure for horizontally curved bridges is based on the V-Load method described in Section 5. Two analyses of the equivalent straight girders are performed for each load case. The applied loads on the girder are called P-Loads, and analysis of the girders subjected to these loads results in P-Load responses such as M_p ,V_p and R_p which are the bending moments, shears, and reactions, respectively. Because of the horizontal curvature of the unit, V-Loads act on the girders.

The girders are analyzed a second time with the V-Loads applied at the diaphragm or transverse bracing locations. The response due to these V-Loads result in V-Load responses, namely: *M", V"' R", representing* the bending moments, shears, and reactions, respectively. The response of the girders in the curved unit is the sum of the P-Load and V-Load responses. Each equivalent straight girder in the unit is modeled by an arbitrary number of prismatic beam elements (constant properties for each element) connected at nodes. An important requirement of the analysis is to compute the response values along the entire length of the girders, not just at the nodes. The more the locations at which the response is computed, the better the resolution of the maximum and minimum response.

7. Slab-On-Steel I-Girder Bridge Configurations

A total of 462 single span and continuous two-span straight and curved concrete slab-on-steel I-girder bridge prototypes were considered for the V-Load analysis in the parametric study.

Major parameters were considered in two steps. The first steps involves comparing the results with the available results obtained from the finite element modelling performed by Khalafalla [2]. The second step involves changing the parameters such as curvature and number of cross bracing (to limit the ratio of warping stress to total stress to a prescribed value) and to reach reliable expression for curvature limitation to treat a curved bridges as a straight one in analysis.

7.1 Parameters Considered to Compare the Result with the Finite Element Method

The parameters considered for comparison with the results from the finite-element analysis are listed as follows.

- Span length **(L)**: 15, 25, and 35 m;
- Girder spacing **(S)**: 2 m;
- Depth-to-span ratios (**D/L**): 1/25 for single span, and 1/30 for two spans;
- Number of girders **(N)**: 4 for 8 m bridge width; 6 for 12 m bridge width; 8 for 16 m bridge width;
- Span-to-radius of curvature ratio **(L/R)**: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, & 0.6;
- Number of **X**-bracing with top and bottom chord: 3 for 15 m span length; 5 for 25 m span length and 7 for 35 m span length;
- Deck slab thickness: 200 mm;
- Girder web and flange thickness: 16 mm;
- Overhang slab length: half the girder spacing;
- Bottom and top steel flange widths: 300 mm.

7.2 Parameters considered for limiting the warping stress

The parameters considered to study warping stresses include those listed in listed in Section 7.1 in addition to the following:

- adding the span-to-radius of curvature ratio (L/R) as 0.02, 0.04, 0.06 and 0.08
- considering a wide range of number of intermediate X bracing as follows:
  - For L = 15 m, 5, 4, 3, 2 for single span and 11, 9, 7, 5 for two-span (15-15) bridge
  - For L = 25 m, 7, 5, 4, 3, 2 for single span and 15, 11, 9, 7, 5 for two-span (25-25) bridge
  - For L = 35 m, 9, 7, 6, 5, 3 for single span and 19, 15, 13, 11, 7 for two-span (35-35) bridge
- Effect of depth with changing the overall depth

General cross section and geometric properties of the studied bridges are shown in Figure 14 and Table 2.

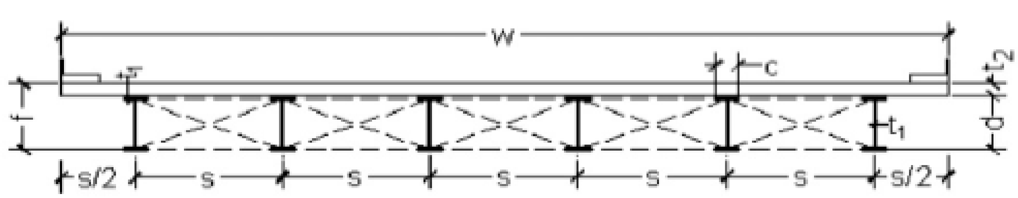


Figure 14. General cross section of the studied bridges [2]

Table 2. Geometry of the studied bridges [2]

Span L (m)	Number of girders (N)	Bridge width W (m)	Number of spans	Girder spacing S (m)	Steel bridges				
					C	d	f	t_1	t_2
15	4	8	1	2	300	700	900	16	200
15	6	12	1	2	300	700	900	16	200
15	8	16	1	2	300	700	900	16	200
15	4	8	2	2	300	700	900	16	200
15	6	12	2	2	300	700	900	16	200
15	8	16	2	2	300	700	900	16	200
25	4	8	1	2	300	1,000	1,200	16	200
25	6	12	1	2	300	1,000	1,200	16	200
25	8	16	1	2	300	1,000	1,200	16	200
25	4	8	2	2	300	900	1,100	16	200
25	6	12	2	2	300	900	1,100	16	200
25	8	16	2	2	300	900	1,100	16	200
35	4	8	1	2	300	1,400	1,600	16	200
35	6	12	1	2	300	1,400	1,600	16	200
35	8	16	1	2	300	1,400	1,600	16	200
35	4	8	2	2	300	1,200	1,400	16	200
35	6	12	2	2	300	1,200	1,400	16	200
35	8	16	2	2	300	1,200	1,400	16	200

7.3 Assumptions of this study

This study was based on the following assumptions:

1. Bridges were are analyzed as beam elements;
2. Bridges were single, and two spans;
3. Materials were elastic and homogenous; and
4. Bridges had constant radii of curvature between radial support lines.

7.4 Loads

For the purpose of comparing the results of a straight bridge with a curved one, bridges were analyzed for dead load only in this study.

7.5 Stress Magnification Factor

The stress magnification factor (SMF) for curved bridges was determined. From bridge analysis, the maximum vertical stresses were determined for straight ($\sigma_{straight}$) and curved (σ_{curve}) bridges due to a uniformly distributed dead load. Accordingly, the stress distribution factors, SMF, were calculated as follows:

$$\mathbf{SMF = \sigma_{curve} / \sigma_{straight}} \quad (12)$$

Based on research carried out using the finite element method [2], the stress magnification factors due to bending moment, including longitudinal and transverse bending moment (torsion effect in terms of warping stress) are considered for both single and two-span curved slab-on-steel I-girder bridges for the sake of comparison with the results obtained from the V-Load method. As such, results for the shear and deflection magnification factors obtained by Khalafalla [2] were not considered herein.

8. Stress Magnification Factors obtained using the V-Load Method and the Finite Element Method

A data base was generated from the parametric study using Microsoft Spread Sheet, Excel, to develop a comparison of the SMFs obtained using the finite element method and the V-Load method. Also, the results were used to examine the curvature limitation presented in CHBDC and AASHTO codes. Due to a large numbers of different bridges considered in this study, selected results are presented in this section.

Tables 3 through 11 present the stress magnification factors obtained using the finite-element method (FEM) and the V-Load method (VLM) for single span bridges with span ranging from 15 to 35 m and bridge widths ranging from 8 to 16 m. The results are also presented in a graphical form in Figures 15 through 23. One In addition, Tables 12 through 20 present the stress magnification factors obtained using the finite-element method and the V-Load method for two-equal-span continuous bridges with span ranging from 15 to 35 m and bridge widths ranging from 8 to 16 m. The results are also presented in a graphical form in Figures 24 through 32. Good correlation was observed between the results obtained from the finite-element modelling and the V-Load method. However, the results obtained from the V-Load method appear to overlap with those obtained from the finite element method in the range of span-to-radius of curvature ration, L/R, less than 0.2, with differences between the two results generally increasing with increase in L/R ratio. However, this conclusion is altered in very few bridges cases as shown in Figures 17, 19, 20, 21, 22 and 23 on which the results from the finite element method and the V-Load methods are very close to each other all over the studied range of L/R (i.e. 0 to 0.6).

Analysis based on V Load Method including flexural stress and warping stress have been performed and the following observations were made:

1. Curvature is the most important parameter that affecting the structural behavior of horizontally curved bridges. Increase in the degree of curvature leads to significant increase in the stress, deflection, and reaction distribution factors, and decrease in the frequency distribution factor.

2. The stress magnification factor increased with increase in bridge curvature.

3. The stress magnification factor for single span bridges is about double that for two-span continuous bridges at the position moment region.

4. The stress magnification factor at the negative moment region in two-span continuous bridges is significantly less than that at the position moment region. The ratio between then changes based on the studied bridge span, width and radius of curvature.

Table 3. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 8 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
15	8	0	1	0	1
15	8	0.1	1.406804044	0.2392871335	1.4439581658
15	8	0.2	1.827447166	0.3861689952	1.9498445855
15	8	0.3	2.242315563	0.4855091940	2.5176593087
15	8	0.4	2.636415213	0.5571744809	3.1474023353
15	8	0.5	2.99981764	0.6113157212	3.8390736654
15	8	0.6	3.328976962	0.6536603017	4.5926732989

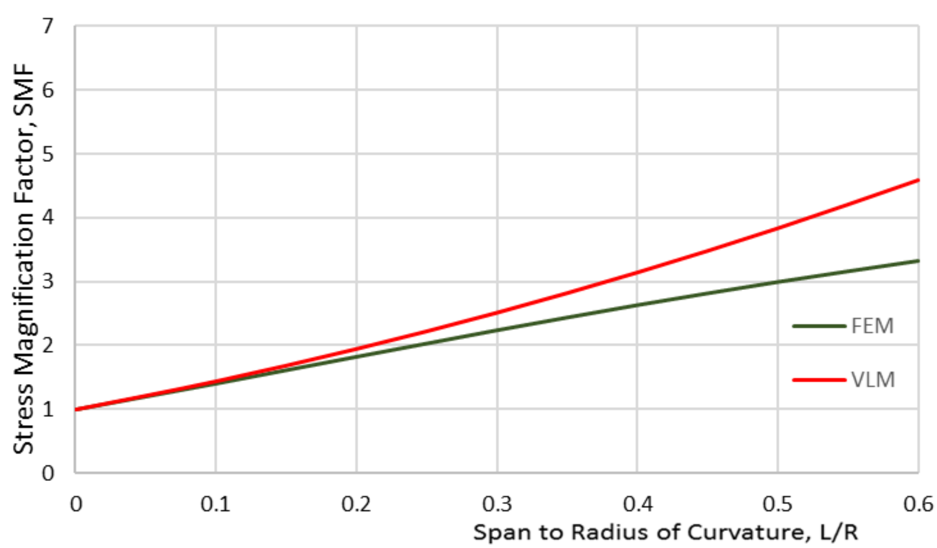


Figure 15. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 8 m width

Table 4. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
15	12	0	1	0	1
15	12	0.1	1.42174074	0.239287134	1.406986263
15	12	0.2	1.858776221	0.386168995	1.858206978
15	12	0.3	2.293010833	0.485509194	2.353662195
15	12	0.4	2.709740019	0.557174481	2.893351914
15	12	0.5	3.098060399	0.611315721	3.477276136
15	12	0.6	3.451169863	0.653660302	4.105434859

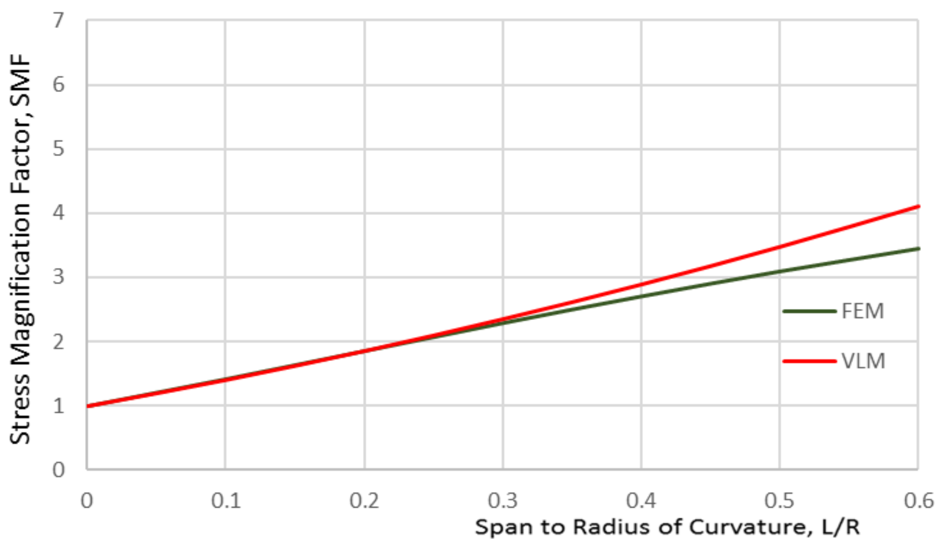


Figure 16. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 12 m width

Table 5. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 16 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
15	16	0	1	0	1
15	16	0.1	1.445873206	0.239287134	1.386445718
15	16	0.2	1.914174641	0.386168995	1.807295713
15	16	0.3	2.386164274	0.485509194	2.262550034
15	16	0.4	2.845893142	0.557174481	2.75220868
15	16	0.5	3.280602073	0.611315721	3.276271653
15	16	0.6	3.680821372	0.653660302	3.834738951

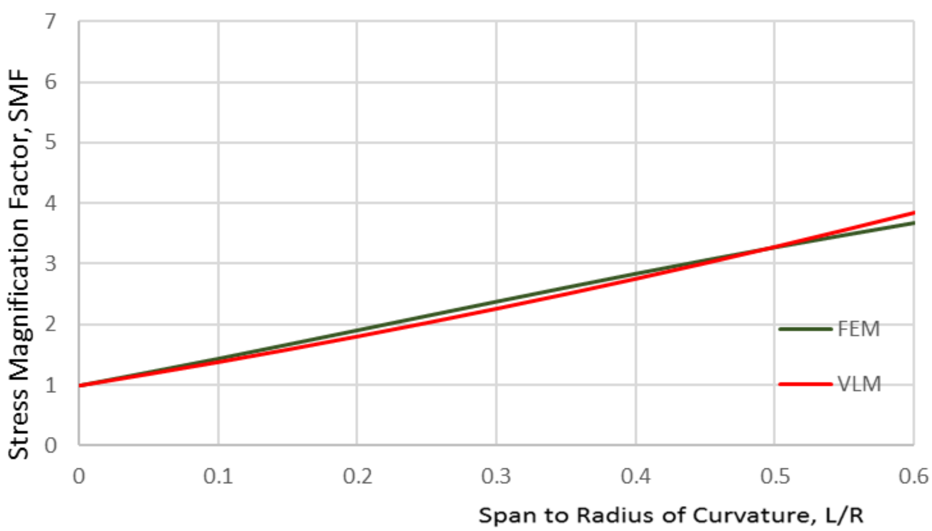


Figure 17. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 15 m span and 16 m width

Table 6. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 8 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
25	8	0	1	0	1
25	8	0.1	1.473769411	0.213543666	1.474765844
25	8	0.2	1.983512201	0.351934045	2.036332609
25	8	0.3	2.500509623	0.448908054	2.684700338
25	8	0.4	3.000539601	0.520637891	3.419869032
25	8	0.5	3.468493315	0.575845604	4.24183869
25	8	0.6	3.89771569	0.61965016	5.150609311

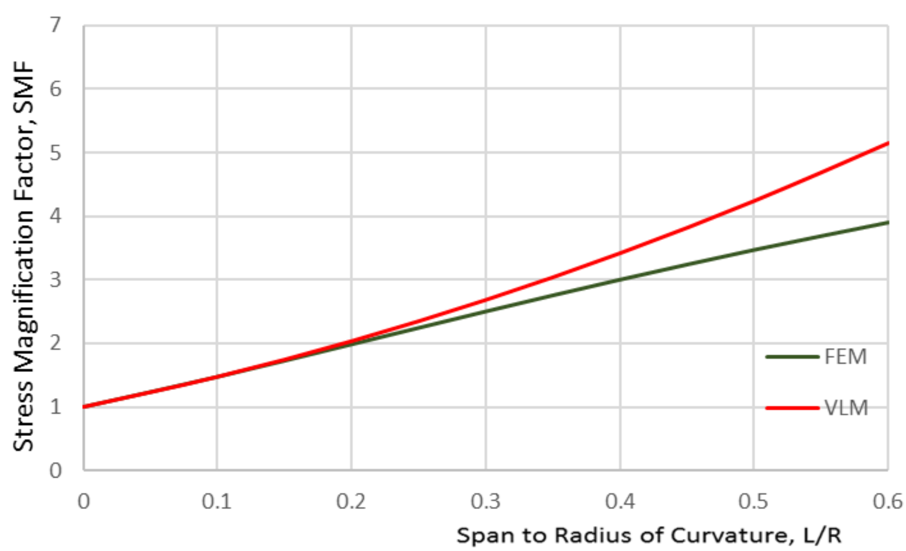


Figure 18. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 8 m width

Table 7. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
25	12	0	1	0	1
25	12	0.1	1.468007857	0.197973884	1.389195015
25	12	0.2	1.967918576	0.330514524	1.834754249
25	12	0.3	2.478126302	0.425461228	2.336677745
25	12	0.4	2.97988215	0.496822121	2.894965503
25	12	0.5	3.459198857	0.552414692	3.509617523
25	12	0.6	3.907267425	0.596945353	4.180633805

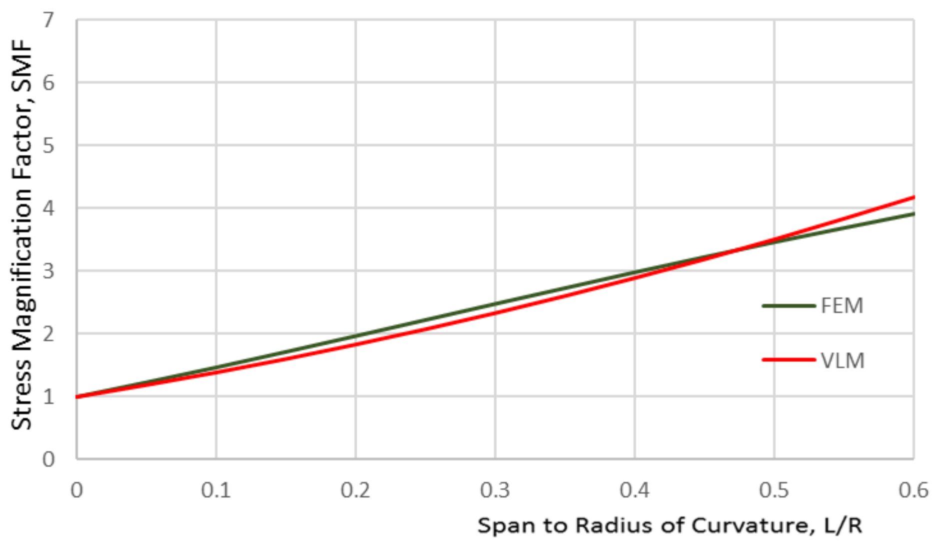


Figure 19. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 12 m width

Table 8. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 16 m width

FINITE ELEMENT METHOD (FEM) -IMAD WORK				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
25	16	0	1	0	1
25	16	0.1	1.467534004	0.213543666	1.384436289
25	16	0.2	1.963867534	0.351934045	1.81709489
25	16	0.3	2.469189829	0.448908054	2.297975848
25	16	0.4	2.966706091	0.520637891	2.82707916
25	16	0.5	3.443524542	0.575845604	3.404404828
25	16	0.6	3.890656416	0.61965016	4.029952852

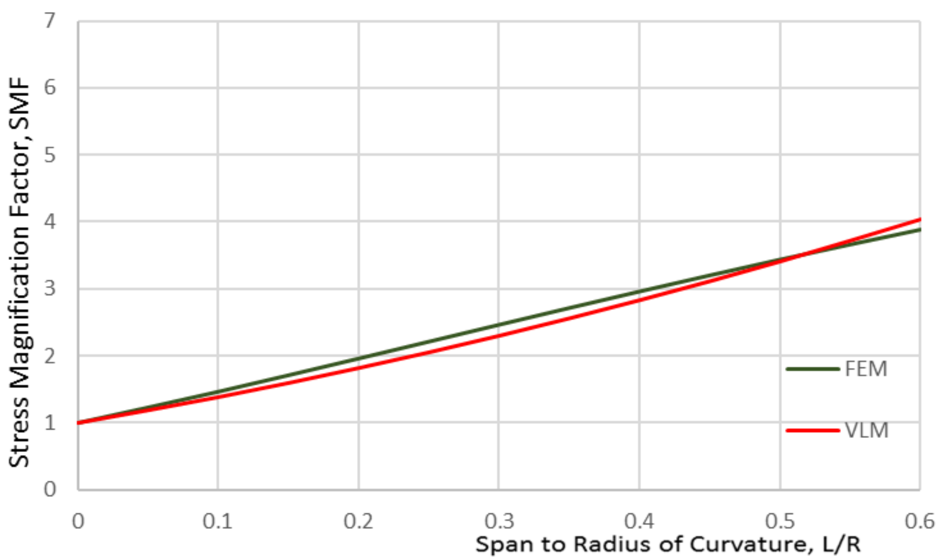


Figure 20. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 25 m span and 16 m width

Table 9. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 8 m width

FINITE ELEMENT METHOD (FEM) -IMAD WORK				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
25	16	0	1	0	1
25	16	0.1	1.467534004	0.213543666	1.384436289
25	16	0.2	1.963867534	0.351934045	1.81709489
25	16	0.3	2.469189829	0.448908054	2.297975848
25	16	0.4	2.966706091	0.520637891	2.82707916
25	16	0.5	3.443524542	0.575845604	3.404404828
25	16	0.6	3.890656416	0.61965016	4.029952852

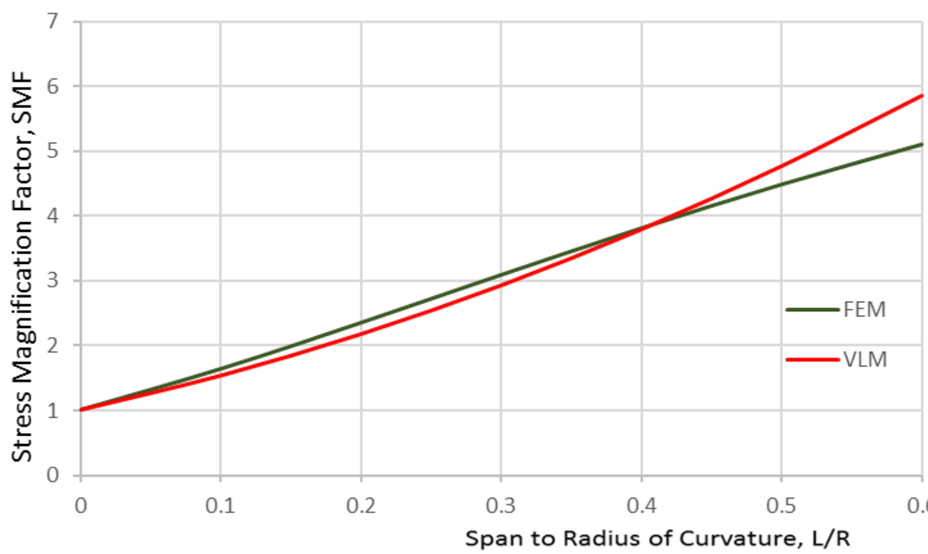


Figure 21. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 8 m width

Table 10. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
35	12	0	1	0	1
35	12	0.1	1.632154502	0.201386444	1.450267573
35	12	0.2	2.344319037	0.335256728	1.980323161
35	12	0.3	3.083019814	0.430689342	2.590166954
35	12	0.4	3.803661901	0.502160702	3.279798952
35	12	0.5	4.47830449	0.557688558	4.049219154
35	12	0.6	5.096062202	0.602072483	4.89842756

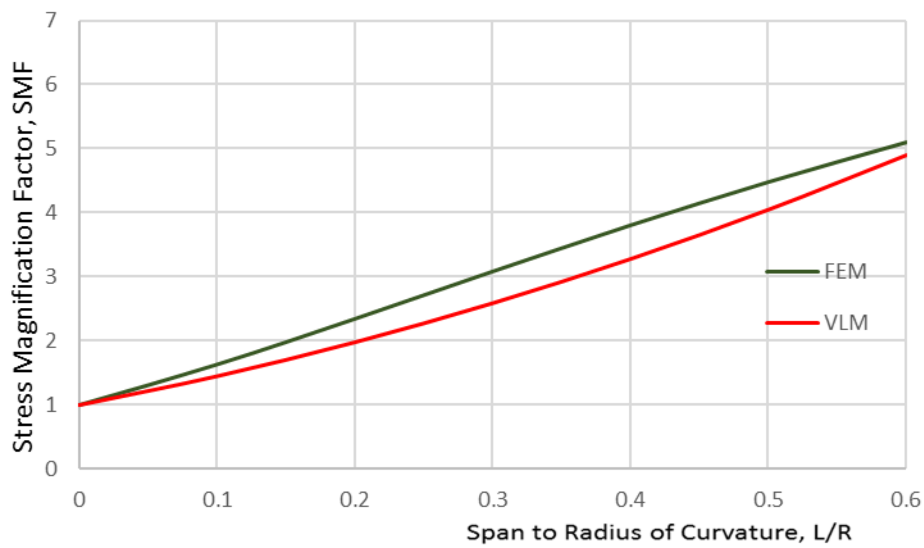


Figure 22. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 12 m width

Table 11. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 16 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009				V LOAD METHOD (VLM)	
L(m)	W(m)	Theta (L/R)	S11 Mid Span	WS/TS	SMF-Mid Span
35	16	0	1	0	1
35	16	0.1	1.593421381	0.201386444	1.406244676
35	16	0.2	2.245601794	0.335256728	1.874546139
35	16	0.3	2.926538749	0.430689342	2.404904577
35	16	0.4	3.609718415	0.502160702	2.99731999
35	16	0.5	4.274557688	0.557688558	3.651792379
35	16	0.6	4.907600299	0.602072483	4.368321742

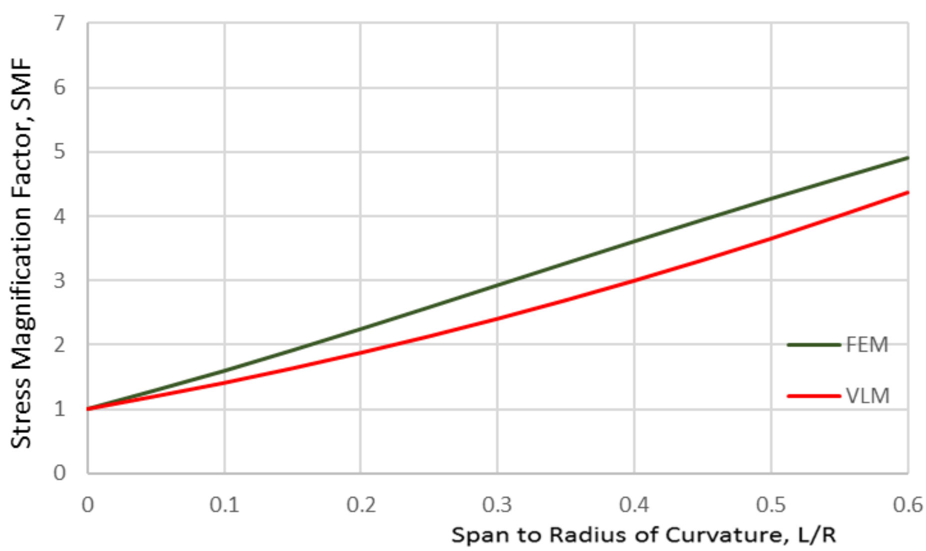


Figure 23. Comparison of the stress magnification factor obtained from the FEM and VLM for single span bridges of 35 m span and 16 m width

Table 12: Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 8 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
15	8	0	1	1	0	1	1
15	8	0.1	1.13976615	1.021919879	0.135903553	1.186266744	1.017138722
15	8	0.2	1.276094472	1.045948204	0.239287134	1.380412709	1.034277395
15	8	0.3	1.407275764	1.069698055	0.320575861	1.582437945	1.051416068
15	8	0.4	1.532241774	1.091816844	0.386168995	1.792342451	1.068554741
15	8	0.5	1.650351552	1.111349803	0.440212171	2.010126229	1.085693414
15	8	0.6	1.761216641	1.127342165	0.485509194	2.235789277	1.102832087

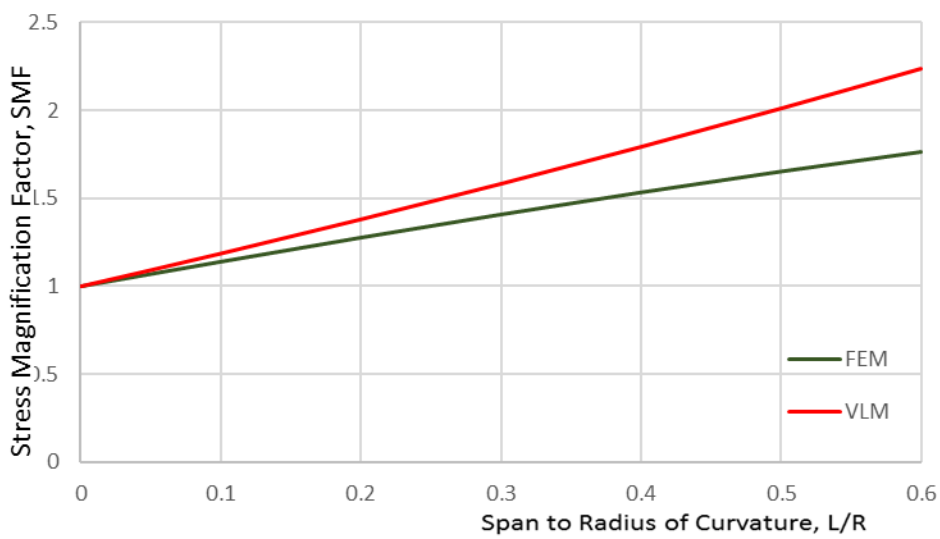


Figure 24. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 8 m width

Table 13. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
15	12	0	1	1	0	1	1
15	12	0.1	1.150165525	1.03268576	0.135903553	1.177984325	1.012241959
15	12	0.2	1.297751944	1.068215502	0.239287134	1.361596651	1.024483868
15	12	0.3	1.440873046	1.103865412	0.320575861	1.550837028	1.036725777
15	12	0.4	1.578566479	1.138513919	0.386168995	1.745705455	1.048967686
15	12	0.5	1.710043113	1.171239736	0.440212171	1.946201932	1.061209595
15	12	0.6	1.834744784	1.201361907	0.485509194	2.152326460	1.073451504

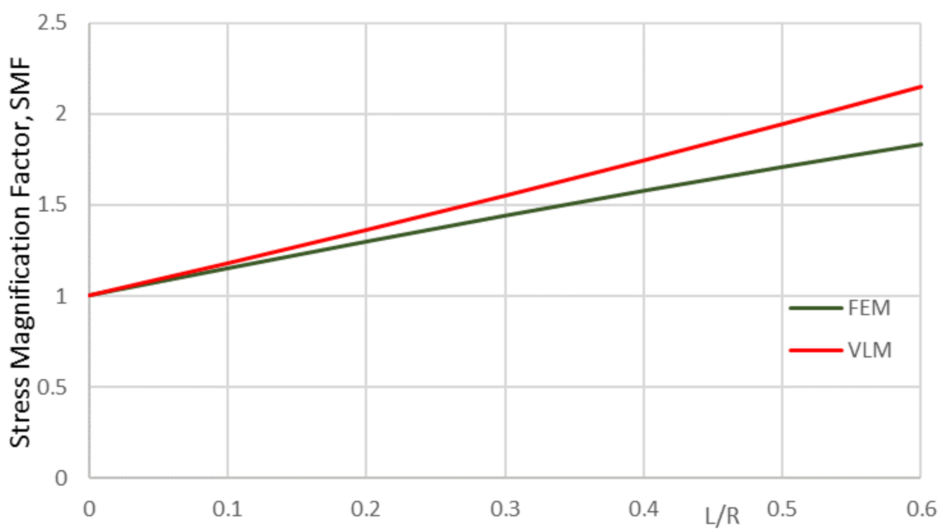


Figure 25. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 12 m width

Table 14. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 16 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
15	16	0	1	1	0	1	1
15	16	0.1	1.163541767	1.044341335	0.135903553	1.173382847	1.009521455
15	16	0.2	1.325850275	1.093120839	0.239287134	1.351142981	1.01904286
15	16	0.3	1.485171982	1.142787977	0.320575861	1.533280451	1.028564266
15	16	0.4	1.64021582	1.192334073	0.386168995	1.719795257	1.038085671
15	16	0.5	1.790326621	1.240911842	0.440212171	1.9106874	1.047607077
15	16	0.6	1.934675788	1.287875731	0.485509194	2.105956878	1.057128482

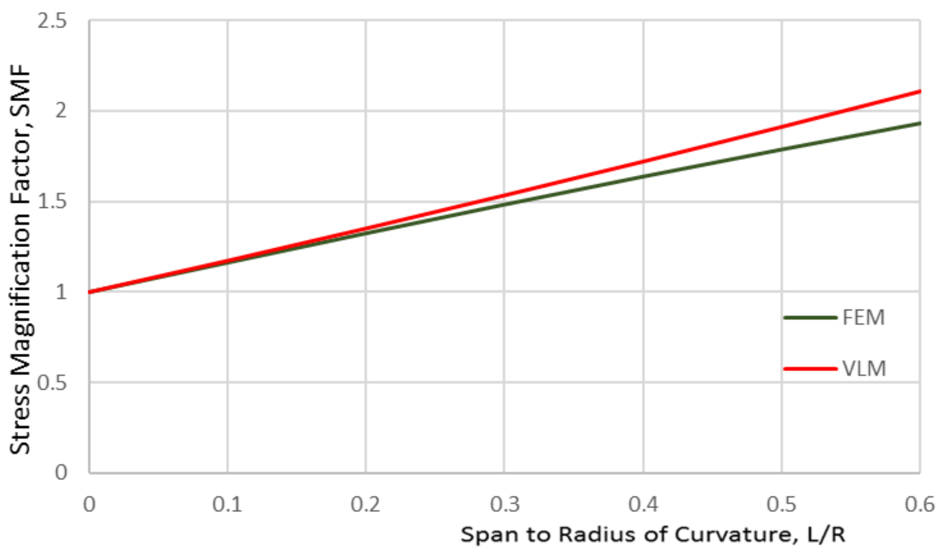


Figure 26. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 15 m span each and 16 m width

Table 15. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 8 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	**W(m)**	**Theta (L/R)**	**S11 Mid Span**	**S11 SUPPORT**	**WS/TS @MID**	**SMF Mid Span**	**SMF SUPPORT**
25	8	0	1	1	0	1	1
25	8	0.1	1.122487751	1.009258283	0.113452024	1.175768397	1.026700427
25	8	0.2	1.244475552	1.027795938	0.203784305	1.362382417	1.053400857
25	8	0.3	1.364263574	1.048337094	0.277410505	1.559841983	1.080101287
25	8	0.4	1.480551945	1.066811481	0.33857279	1.768147096	1.106801716
25	8	0.5	1.592140786	1.082143535	0.390189132	1.987297755	1.133502146
25	8	0.6	1.698130187	1.093468587	0.434332587	2.21729396	1.160202576

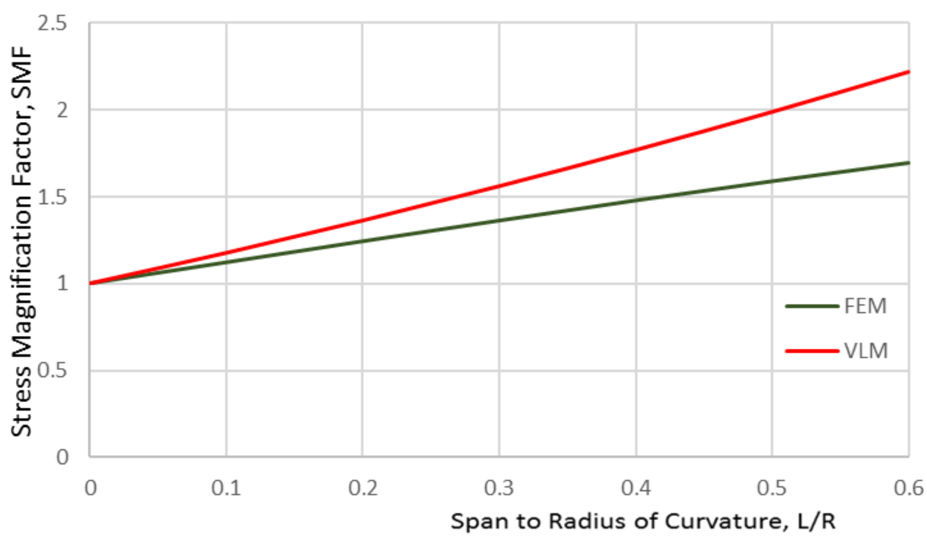


Figure 27. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 8 m width

Table 16. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
25	12	0	1	1	0	1	1
25	12	0.1	1.124170053	1.014278171	0.113452024	1.162111842	1.019071733
25	12	0.2	1.247844614	1.034047946	0.203784305	1.331970579	1.038143468
25	12	0.3	1.369140819	1.054725948	0.277410505	1.509576134	1.057215203
25	12	0.4	1.486968586	1.075615165	0.33857279	1.694928509	1.076286939
25	12	0.5	1.600436032	1.095976344	0.390189132	1.888027702	1.095358674
25	12	0.6	1.708750372	1.115196958	0.434332587	2.088873714	1.114430409

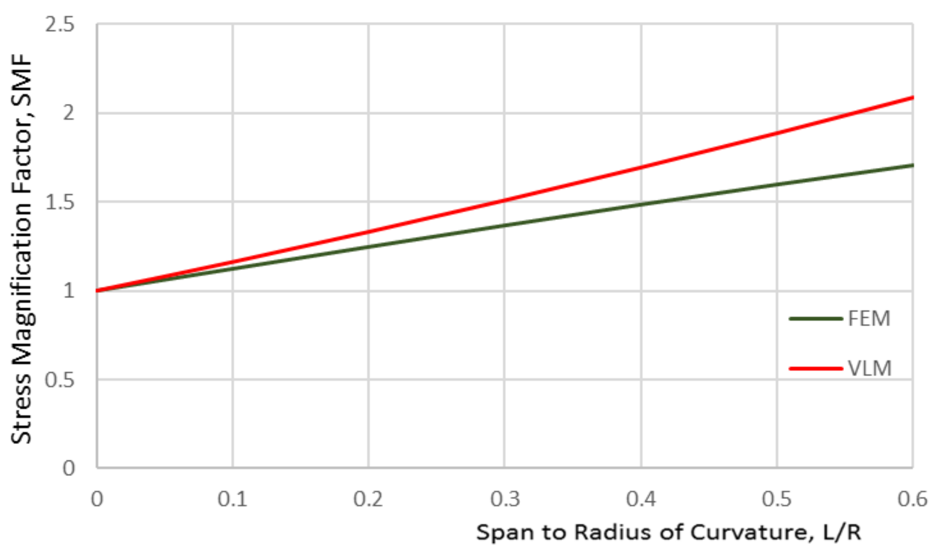


Figure 28. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 12 m width

Table 17. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 16 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
25	16	0	1	1	0	1	1
25	16	0.1	1.129267811	1.01939856	0.113452024	1.154524646	1.014833446
25	16	0.2	1.257943556	1.046272766	0.203784305	1.315074621	1.029666894
25	16	0.3	1.384448392	1.074015248	0.277410505	1.481649849	1.044500342
25	16	0.4	1.507696862	1.102011859	0.33857279	1.654250331	1.059333791
25	16	0.5	1.626702191	1.129606099	0.390189132	1.832876066	1.074167239
25	16	0.6	1.740674956	1.156247353	0.434332587	2.017527054	1.089000687

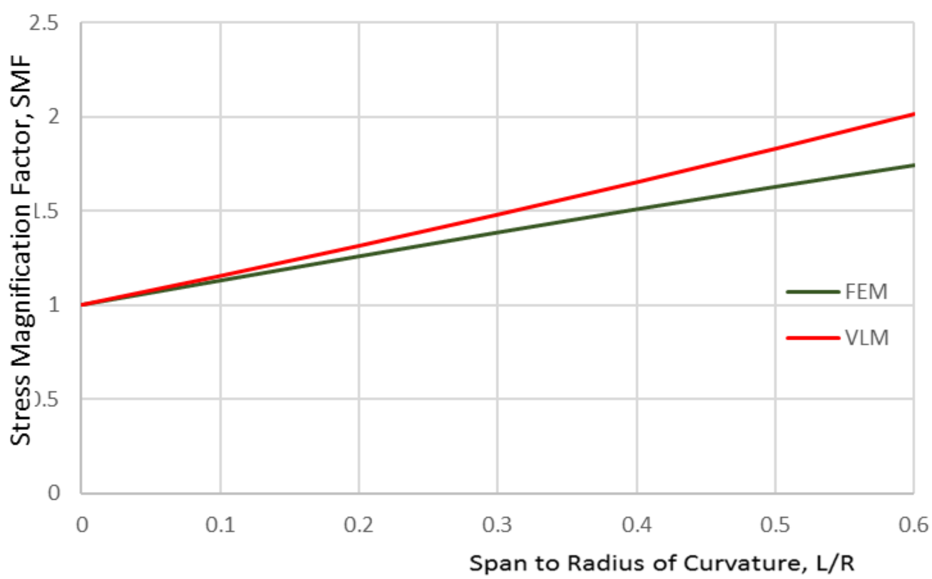


Figure 29. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 25 m span each and 16 m width

Table 18. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 8 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
35	8	0	1	1	0	1	1
35	8	0.1	1.1443283	1.011463639	0.104351033	1.181050193	1.036337261
35	8	0.2	1.289740612	1.039266078	0.188981637	1.375570311	1.072674541
35	8	0.3	1.433991483	1.067006215	0.258999391	1.583560343	1.109011821
35	8	0.4	1.574680604	1.092378861	0.317888235	1.805020289	1.145349101
35	8	0.5	1.7100271	1.114060091	0.368106141	2.039950148	1.181686381
35	8	0.6	1.838172667	1.130866159	0.411436881	2.288349921	1.218023661

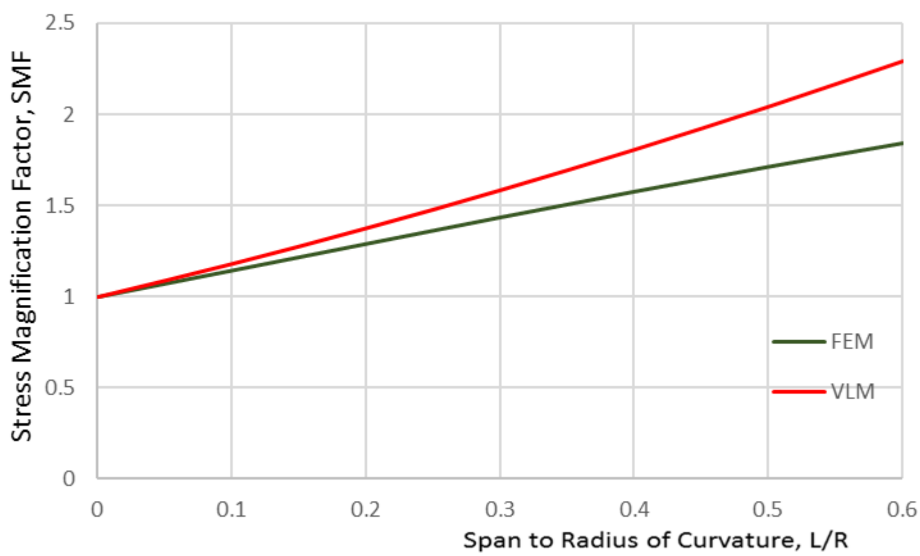


Figure 30. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 8 m width

Table 19. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 12 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
35	12	0	1	1	0	1	1
35	12	0.1	1.140778941	1.011815369	0.104351033	1.162609806	1.025955181
35	12	0.2	1.282481527	1.031948137	0.188981637	1.33484099	1.051910381
35	12	0.3	1.423106527	1.052702766	0.258999391	1.516693541	1.077865581
35	12	0.4	1.561037562	1.073177557	0.317888235	1.708167459	1.10382078
35	12	0.5	1.694735222	1.092548544	0.368106141	1.909262743	1.12977598
35	12	0.6	1.823044951	1.110007307	0.411436881	2.119979395	1.15573118

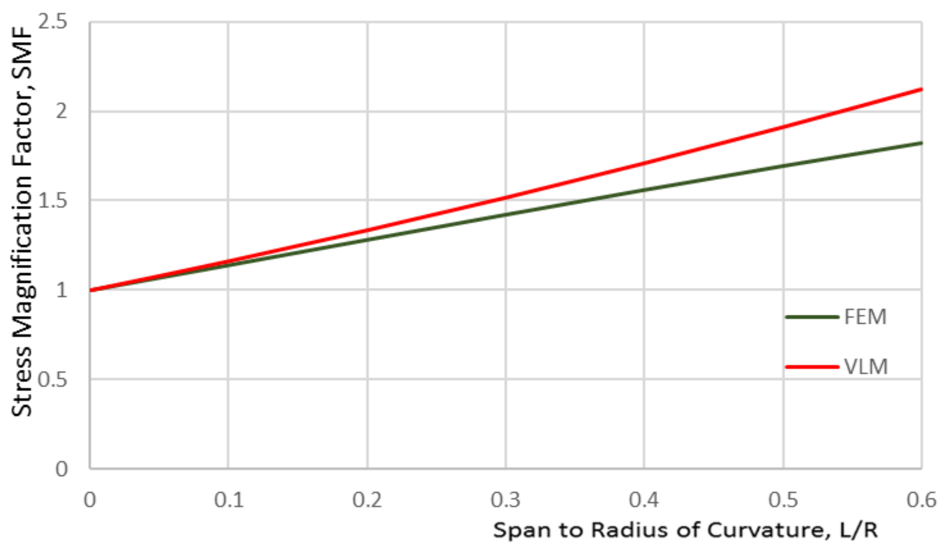


Figure 31. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 12 m width

Table 20. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 16 m width

FINITE ELEMENT METHOD (FEM) -by Khalafalla 2009					V LOAD METHOD (VLM)		
L(m)	W(m)	Theta (L/R)	S11 Mid Span	S11 SUPPORT	WS/TS @MID	SMF Mid Span	SMF SUPPORT
35	16	0	1	1	0	1	1
35	16	0.1	1.140635772	1.010933976	0.104351033	1.152364848	1.020187191
35	16	0.2	1.281731137	1.034559453	0.188981637	1.31221293	1.0403744
35	16	0.3	1.421294523	1.058635975	0.258999391	1.479544235	1.060561609
35	16	0.4	1.558023746	1.082074811	0.317888235	1.654358764	1.080748819
35	16	0.5	1.690540023	1.104051637	0.368106141	1.836656515	1.100936028
35	16	0.6	1.81769437	1.123882106	0.411436881	2.026437489	1.121123237

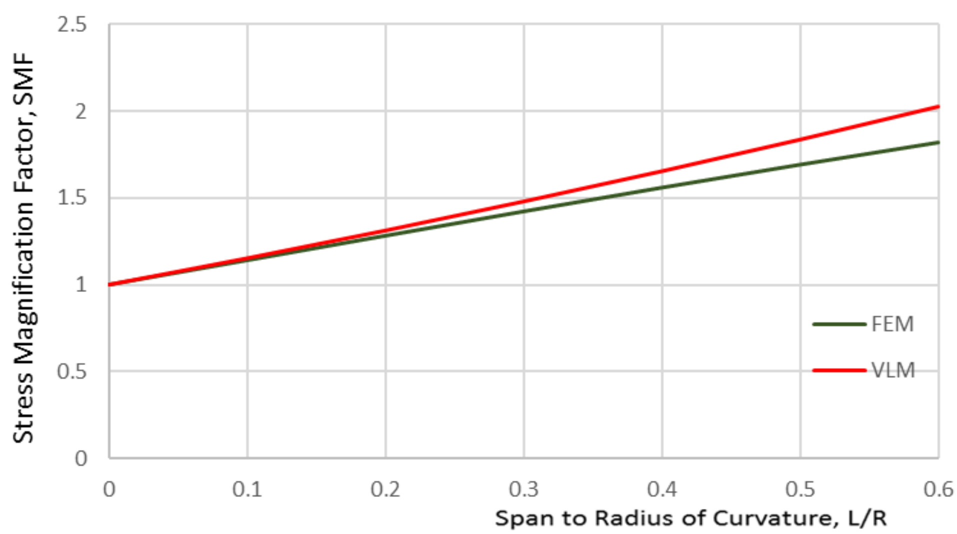


Figure 32. Comparison of the stress magnification factor obtained from the FEM and VLM for two-span continuous bridges of 35 m span each and 16 m width

9. Curvature Limitation to Treat a Horizontally Curved Bridge as a Straight Bridge

Canadian Highway Bridge Design Code CHBDC of 2014 [7] and the AASHTO Guide Specifications for Horizontally Curved Bridges of 2003 [8] and 1993 [13] specified certain limitations to treat horizontally curved bridges as straight ones. These limitations can be summarized in the following subsections.

9.1 Guide Specifications for Horizontally Curved Highway Bridges of 1993

For the Curved I-Girder bridges, the effects of curvature may be neglected in determining the primary bending moments in the longitudinal girders when the central angle subtended by each span is less than the values given in Table 21.

Table 21. Limiting central angle for neglecting curvature in determining primary bending moment in a curved bridge [13]

Numbers of Girders	Angle for one Span	Angle for two or More Spans
2	2°	3°
3 or 4	3°	4°
5 or more	4°	5°

9.2 Guide Specifications for Horizontally Curved I Steel Girder Highway Bridges of 2003

In the I-girder bridges, the effect of curvature may be ignored in the determination of vertical moment, when the following three conditions are met:

I. Girders are concentric.
II. Bearing lines are not skewed more than 10 degrees from radial, and
III. The arc span divided by the girder radius is less than 0.06 radians.

9.3 Canadian Highway Bridge Design Code of 2014

For bridges that are curved in plan and that are built with shored construction, the simplified method of analysis specified in CHBDC for straight slab-on-girder bridges can be applied to a curved bridges by treating it as a straight provided that:

a) There are at least two intermediate diaphragms per span.

b) $\mathbf{L^2/(B\ R) \leq 0.5}$ (13)

where B = bridge width, L = centerline curved span length, and R = radius of curvature.

Assuming all conditions for curvature limitations are met per CHBDC and AASHTO, the span-to-radius of curvature, L/R, limits are summarized in Tables 22 and 23 for single and two-span bridges, respectively. As shown in the tables, the L/R limit specified in CHBDC for the studied bridge geometries are higher than those specified in AASHTO Guides of 1993 and 2003. This means that CHBDC underestimates the response of curved bridges in the range of L/R between those obtained using CHBDC and AASHTO Guides.

Table 22. Curvature limitations for single span bridges

Single Span I- Girder Bridge Horizontal Curvature Limitation(L/R) in CHBDC and AASHTO					
L (m)	No. of Girder	Width B or W (m)	CHBDC	AASHTO 1993	AASHTO 2003
15	4	8	0.267	0.052	0.060
	6	12	0.400	0.070	0.060
	8	16	0.533	0.070	0.060
25	4	8	0.160	0.052	0.060
	6	12	0.240	0.070	0.060
	8	16	0.320	0.070	0.060
35	4	8	0.114	0.052	0.060
	6	12	0.171	0.070	0.060
	8	16	0.228	0.070	0.060

Table 23. Curvature limitations for two-span bridges

Two Span I- Girder Bridge Horizontal Curvature Limitation(L/R) in CHBDC and AASHTO					
L (m)	No. of Girder	Width B or W (m)	CHBDC	AASHTO 1993	AASHTO 2003
15x2	4	8	0.133	0.070	0.060
	6	12	0.200	0.087	0.060
	8	16	0.267	0.087	0.060
25x2	4	8	0.080	0.070	0.060
	6	12	0.120	0.087	0.060
	8	16	0.160	0.087	0.060
35x2	4	8	0.057	0.070	0.060
	6	12	0.086	0.087	0.060
	8	16	0.114	0.087	0.060

9.4 Curvature Limitations in North American Bridge Codes and Corresponding Stress Magnification Factor using the V-Load Method

Tables 24 and 25 presents the values of the stress magnification factors corresponding to the L/R limit to treat a curved bridges as a straight one for single span and two-span continuous bridges, respectively. As expected, the SMFs associated with the L/R limit specified per CHBDC is very much greater than those associated with L/R limits specified in AASHTO Guides of 1993 and 2003.

Table 24. Code curvature limitations and corresponding stress magnification factor for single span bridges

Single Span I - Girder Bridge (L/R) Limitation and Corresponding SMF in CHBDC and AASHTO					
L (m)	No. of Girder	Width	CHBDC	AASHTO 1993	AASHTO 2003
			L/R	L/R	L/R
		B or W (m)	SMF	SMF	SMF
15	4	8	0.267	0.052	0.060
			2.518	1.215	1.259
	6	12	0.400	0.070	0.060
			2.893	1.281	1.239
	8	16	0.533	0.070	0.060
			3.276	1.267	1.228
25	4	8	0.160	0.052	0.060
			1.812	1.227	1.274
	6	12	0.240	0.070	0.060
			2.036	1.267	1.227
	8	16	0.320	0.070	0.060
			2.404	1.265	1.225
35	4	8	0.114	0.052	0.060
			1.594	1.251	1.304
	6	12	0.171	0.070	0.060
			1.821	1.308	1.261
	8	16	0.228	0.070	0.060
			2.034	1.278	1.236

Table 25. Code curvature limitations and corresponding stress magnification factor for single span bridges

Two Span I - Girder Bridge (L/R) Limitation and Corresponding SMF in CHBDC and AASHTO					
L (m)	No. of Girder	Width	CHBDC	AASHTO 1993	AASHTO 2003
			L/R	L/R	L/R
		B or W (m)	SMF	SMF	SMF
15x2	4	8	0.133	0.070	0.060
			1.244	1.130	1.111
	6	12	0.200	0.087	0.060
			1.362	1.160	1.106
	8	16	0.267	0.087	0.060
			1.478	1.156	1.104
25x2	4	8	0.080	0.070	0.060
			1.140	1.122	1.104
	6	12	0.120	0.087	0.060
			1.196	1.113	1.096
	8	16	0.160	0.087	0.060
			1.251	1.139	1.092
35x2	4	8	0.057	0.070	0.060
			1.107	1.125	1.107
	6	12	0.086	0.087	0.060
			1.146	1.146	1.096
	8	16	0.114	0.087	0.060
			1.168	1.137	1.091

10. Examination of the Equation for Curvature Limitation proposed by Khalafalla (2009)

Recently, Khalafalla [2] investigated the curvature limitations of composite steel I-girder bridges that let to proposing the following two equations for single span and two-equal-span continuous bridges, followed by the scope of applicability of these equations with respect to bridge curved span and width. The limiting values to ignore curvature effect were determined based on 5% underestimation in design parameters.

Single Span Bridge	Two Span Bridge	
$(L/R)^{0.92} (L)^{0.5} (B)^{0.04} \leq 0.054$	$(L/R)^{0.81} (L)^{-0.06} (B)^{0.08} \leq 0.0235$	(15)
$15 \leq L \leq 35$ m	$15 \leq L \leq 35$ m (L for each span)	
$8 \leq B \leq 16$ m	$8 \leq B \leq 16$ m	

Where L is the bridge span arc length in meters, R is the radius of curvature in meters, and B is the bridge width in meters.

Results presented earlier in this project report for the V-Load method showed insignificant difference between the finite element analysis results reported by Khalafalla [2] in the range of L/R between 0 and 0.2. The above-mentioned developed equations lead to a L/R limit of less than 0.1 for bridge spans between 15 and 35 m and bridge widths between 8 and 16 m. As such, one may conclude that the V-Load method will yield similar equations to those developed earlier by Khalafalla in 2009.

11. Effect of Warping Stress

One of the most important factor that affect the total bending stress in curved girders is the contribution of warping stress. As described, the flanges of the girders are subjected to warping due to the torsion induced by the horizontal curvature of the bridge unit. In composite girders, the concrete slab acts together with the top flange to resist the warping moment. The section modulus for lateral bending of the top flange and slab is much larger than that for the bottom flange, resulting in smaller warping stresses in the former. As such, only the warping of the bottom flange is important in composite systems.

In the V–Load approximate analysis procedure, the bottom flanges of the girders are straightened and modeled as individual flange elements supported at each diaphragm (or transvers X bracing) location. The curvature of the flanges is the same as that of the girders. Using the model of the bottom flanges, an analysis of the lateral bending can be performed after the V-Loads are specified. As described earlier, the lateral bending of the flanges is caused by the radial flange forces which develop due to the horizontal curvature. As described, the warping stress combined with the longitudinal bending stress gives the total stress at the tip of the bottom flange. In the V-Load method, large warping stress exists at the diaphragm locations because the diaphragms are assumed to be infinitely stiff and provide rigid supports for the flange. This assumption is not the same in the finite element modelling, so the flanges can deflect laterally at the diaphragm locations. Based on Equations 16 and 17, the warping stress can be calculated as:

$$\sigma_w = \frac{M_f}{S_f} \tag{16}$$

$$\sigma_\omega = 6M_f/b_f^2 t_f \tag{17}$$

One may consider decreasing the warping stress in curved girders by considering the following changes:

1) Decreasing intermediate diaphragm or transverse bracing spacing.
2) Increasing girder depth.
3) Increasing flange width.

Recent study by Davidson et al. in 1996 [14] concluded that the change in warping stress with an increase in girder depth is not a significant. The reduction of warping stress again arises from the increase in the torsional warping constant, which is directly proportional to the moment of inertia of the flange, I_f, and therefore related to the width of the flange squared. Davidson et al. [14] recommended that for a preliminary design purposes, a warping-to-bending stress ratio of 0.25 should be targeted resulting in the following equation for the maximum spacing between the radial diaphragms or X bracings between the radial supporting lines:

$$S_{\max} = L\left[-\ln\left(\frac{Rb_f}{2000L^2}\right)\right]^{-1.52} \tag{18}$$

In which S_{max} is the maximum spacing of intermediate diaphragm or transverse bracing, L is the arc length of the bridge in m, R is the radius of curvature in m, and b_f is the bottom flange I-Girder in mm.

It is important to note that the magnitude of bending stress is not sensitive to the diaphragm spacing. The spacing of diaphragms has little effect on the longitudinal bending stresses but does affect the warping stresses. Furthermore, in the V-Loads analysis, the diaphragms do not contribute to the torsional stiffness of the bridge unit. The finite element analysis recognizes the contribution of the diaphragms to the torsional stiffness of the bridge unit. The larger the distance between diaphragms, the greater the warping stresses at the diaphragm locations. Decreasing the diaphragm spacing decreases the magnitudes of the warping stresses and also decreases their influence on the warping plus bending stress values.

In this study, the following parameters were considered in investigating the warping stress magnitude in curved composite I-girders as presented in the following subsections.

1) Curvature of bridge with $L/R \leq 0.1$,
2) Depth I-girders,
3) Bottom flange width, and
4) Number of X bracings.

11.1 Effect of depth of I-Girder on warping stress

Using the V-Load meth, Table 26 presents the change in the stress magnification factor, SMF, and the ratio of warping to total bending stress, WS/TS, in the outer girder of a 25 m span, 6-girder, bridge of 12 m width with the change in girder depth from 1 m to 1.2 ad 1.4 m and change in L/R ratio from 0 to 0.1. Results for SMF and WS/TS are presented in graphical form in Figures 33 and 34, respectively. In this analysis, the. It can be observed that the change in girder depth has insignificant effect on SMF of the studied bridges irrespective of the L/R value. Also, changing the depth of the girder has insignificant effect on warping stress-total stress ratio, while the warping stress increases with increase in curvature.

Table 26. Effect of girder depth on warping stress and Stress Magnification Factor

DEPTH EFFECT, L=25, No. of GIRDER 6, W=12 (m)							
L/R	L (m)	H=1 (m)		H=1.2 (m)		H=1.40 (m)	
		WS/TS	SMF	WS/TS	SMF	WS/TS	SMF
0	25	0.00	1.00	0.00	1.00	0.00	1.00
0.02	25	0.03	1.05	0.03	1.06	0.03	1.06
0.04	25	0.06	1.11	0.06	1.11	0.07	1.12
0.06	25	0.08	1.17	0.09	1.17	0.10	1.18
0.08	25	0.11	1.22	0.12	1.24	0.13	1.25
0.1	25	0.13	1.28	0.14	1.30	0.15	1.31

Note: WS/TS: warping stress/total stress, SMF: stress magnification factor, H: girder depth

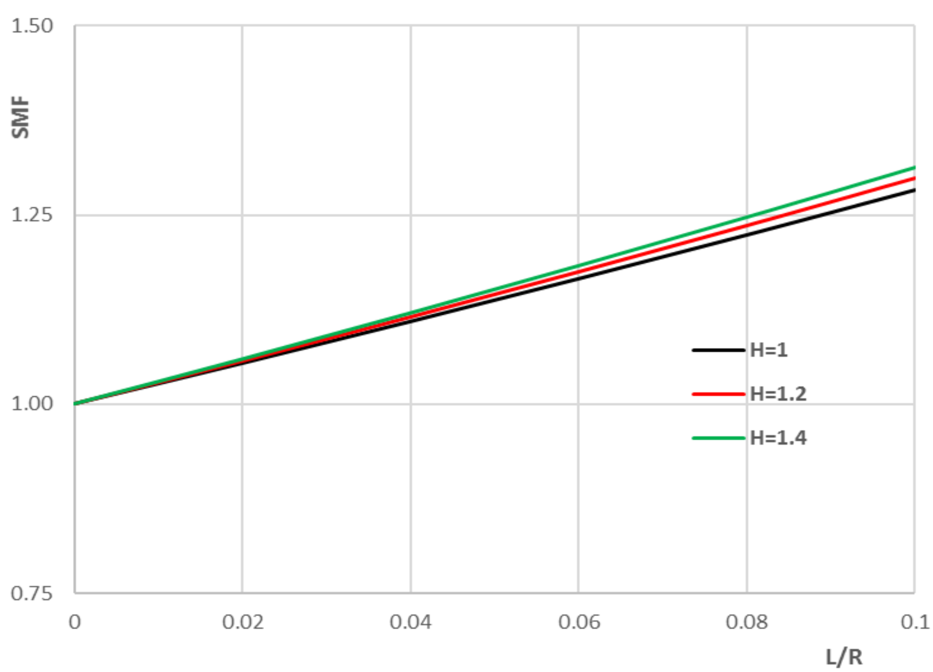


Figure 33. Variation of stress magnification factor, SMF, with the change in girder depth and curvature for 25 m span bridge of 12 m width

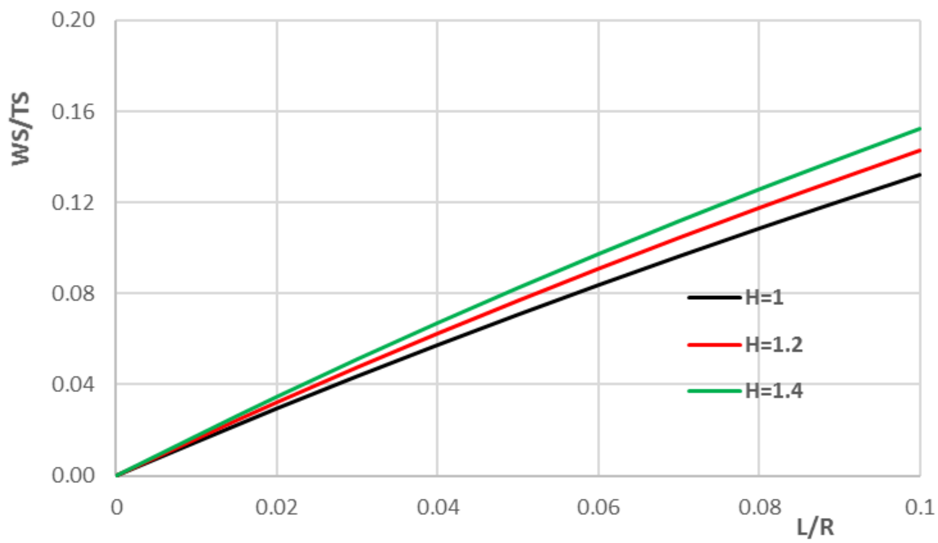


Figure 34. Variation of warping-to-total bending stress ratio, WS/TS, with the change in girder depth and curvature for 25 m span bridge of 12 m width

11.2 Bottom flange width effect on warping stress

Using the V-Load method, Table 27 presents the change in the stress magnification factor, SMF, and the ratio of warping to total bending stress, WS/TS, in the outer girder of a 25 m span, 6-girder, bridge of 12 m width With the change in bottom flange width from 200 mm to 300 and 400 mm and change in L/R ratio from 0 to 0.1. Results for SMF and WS/TS are presented in graphical form in Figures 35 and 36, respectively. In this analysis, the. It can be observed that the SMF decrease with increase in flange width, with the rate of decrease significantly increasing with increase in L/R ratio. Similar behavior to SMF is observed for the WS/TS ratios. As such, one may conclude that the flange width has an effect on the warping stress and thus the SMF of the I-girder design.

Table 27: Effect of girder flange width on warping stress and Stress Magnification Factor

I GIRDER FLANGE WIDTF EFFECT, L=25, No. of GIRDER 6, W=12 (m)							
L/R	L (m)	b=0.2 (m)		b=0.3 (m)		b=0.4 (m)	
		WS/TS	SMF	WS/TS	SMF	WS/TS	SMF
0	25	0.00	1.00	0.00	1.00	0.00	1.00
0.02	25	0.05	1.08	0.03	1.05	0.02	1.04
0.04	25	0.10	1.16	0.06	1.11	0.04	1.09
0.06	25	0.15	1.25	0.08	1.17	0.06	1.13
0.08	25	0.19	1.34	0.11	1.22	0.07	1.18
0.1	25	0.22	1.43	0.13	1.28	0.09	1.22

Note: WS/TS: Warping Stress/Total Stress, SMF: Stress Magnification Factor, b: Girder Flange Width.

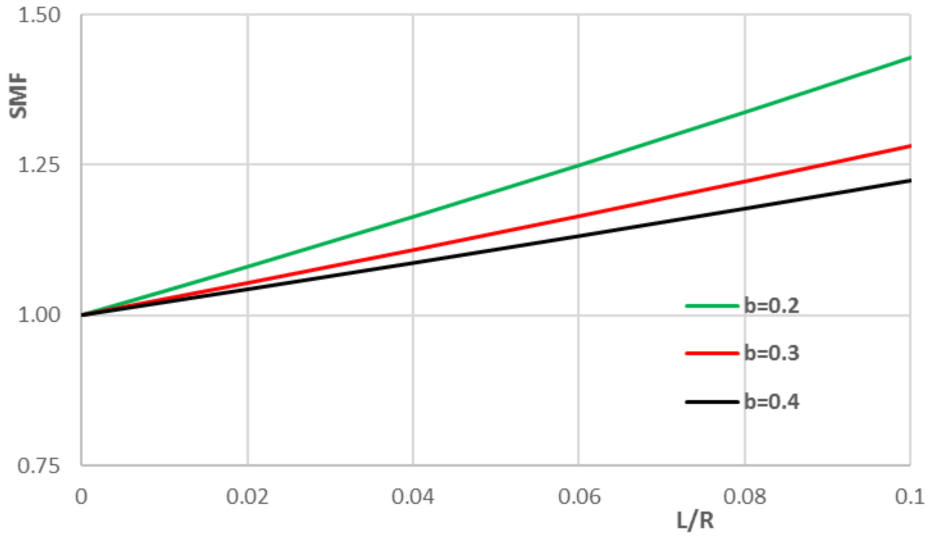


Figure 35. Variation of stress magnification factor, SMF, with the change in girder flange width and curvature for 25 m span bridge of 12 m width

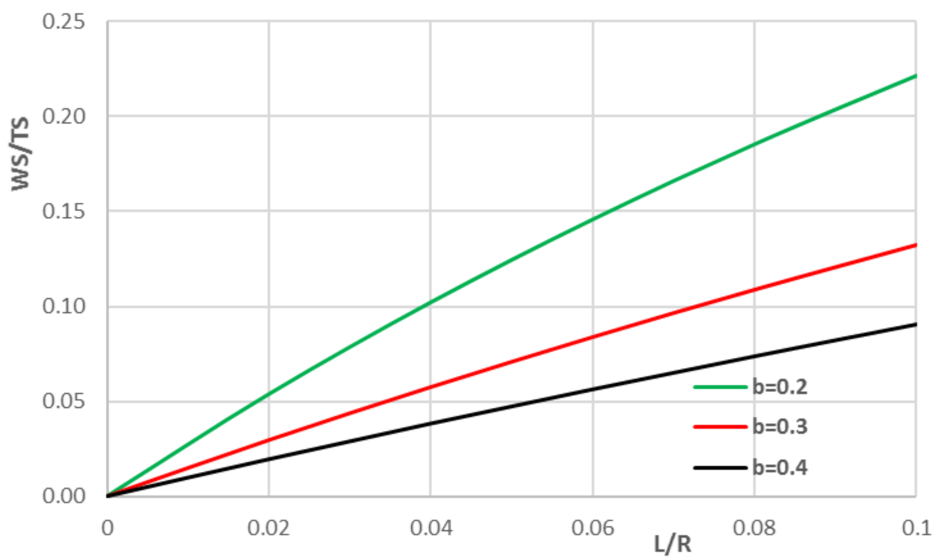


Figure 36. Variation of stress magnification factor, SMF, with the change in girder flange width and curvature for 25 m span bridge of 12 m width

11.3 The effect of Number of Transverse Bracing on Warping Stress

As mentioned earlier, Davidson et al. recommended, for a preliminary design purposes, a warping-to-bending stress ratio of 0.25 and the corresponding maximum spacing between bracings as shown in equation (18). Results using the V-Load method are summarized in Appendices 3 and 4 considering the following parameters in addition to bridge configurations in Section 7:

- L/R ratio: 0.0, 0.02, 0.04, 0.06, 0.08 and 0.1;
- Number of intermediate X bracing:
  - For L = 15 m: 5, 4, 3, 2 per span
  - For L = 25 m: 7, 5, 4, 3, 2 per span
  - For L = 35m: 9, 7, 6, 5, 3 per span

Results showed that the transverse bracing numbers or spacing has a great effect on warping stress, the warping-to-bending stress ratio and stress magnification factor (SMF). Also, it was observed that decreasing the number of transverse bracing at a certain curvature (L/R) causes an increase in warping stress. Moreover, it was observed that the effect of number of transverse bracing on warping stress in a two-span bridge is less significant than that in a single span bridge of the same width, curved span, cross section configuration and radius of curvature.

12. Correlation between the V-Load Method Results and Curvature Limitations

One the objectives of this study is to examine the available equations for a curvature limitation for curved bridge that can be treated as a straight bridge in structural design. For this task, the data generated from the parametric V-Load analysis were correlated with those presented by Khalafalla [2] and Khalafalla and Sennah [15] (i.e. equation 15). If should be noted that Khalafalla used the following general equation form in developing those in equation 15.

$$(L/R)^{\alpha}(L)^{\beta}(B)^{\gamma} \leq \phi \qquad (19)$$

In the finite element analysis conducted by Khalafalla [2], the number of bracing spacing obtained from equation 18 was considered in the studied bridges to determine curvature limitations. However, in the current study by the V-Load method, the number of cross-bracing spacing, N, was considered and the following modified form of equation 19 was considered to develop new equation for bridge curvature limitations.

$$(L/R)^{\alpha}(L)^{\beta}(B)^{\gamma}(N)^{\rho} \leq \phi' \qquad (20)$$

Following the same procedure presented by Khalafalla [2], the following two equations were developed as a modification to equation (15) for single- and two-span bridges to account the effect of number of transverse bracing on the stress magnification factor.

For single-span bridge: Modification factor = $1.124\ N^{-0.050}$ (21)

For two-span bridge: Modification factor = $1.070\ N^{-0.047}$ (22)

Equations 21 and 22 can be simplified by considering them as **1.10** $N^{-0.050}$ for the effect of number of transverse bracing to Equation 15. As such, equation 15 can be rewritten as follows, considering the same scope of applicability.

For single-span bridges:

$$(L/R)^{0.92} \times (L)^{0.5} \times (B)^{0.04} \times (N)^{-0.05} \leq 0.0491 \quad (23)$$

For two-span bridges:

$$(L/R)^{0.81} \times (L)^{-0.06} \times (B)^{0.08} \times (N)^{-0.05} \leq 0.0214 \quad (24)$$

Tables 28 and 29 summarize the curvature limitation derived from equation (15) and equations 23 and 24. Good correlations were observed.

Table 28. Single-span bridge curvature limitations as obtained from the FEA modelling and V-Load method for a specific number of transverse X bracing

Single Span Bridge				
CURVATURE (L/R) Limitation with Khalafalla, 2009 Equation and V Load Method				
L (m)	W (m)	No. of Bracing	L/R (Khalafalla, 2009)	L/R (V Load Method)
15	8	3	0.0088	0.0084
15	12	3	0.0086	0.0083
15	16	3	0.0085	0.0082
25	8	5	0.0067	0.0065
25	12	5	0.0065	0.0064
25	16	5	0.0065	0.0064
35	8	7	0.0055	0.0056
35	12	7	0.0054	0.0055
35	16	7	0.0054	0.0054

Table 29. Two-span continuous bridge curvature limitations as obtained from the FEA modelling and V-Load method for a specific number of transverse X bracing

Two Span Bridge Span				
CURVATURE (L/R) Limitation with Khalafalla, 2009 Equation and V Load Method				
L (m)	W (m)	No. of Bracing	L/R (Khalafalla, 2009)	L/R (V Load Method)
15	8	3	0.0097	0.0093
15	12	3	0.0093	0.0089
15	16	3	0.0091	0.0086
25	8	5	0.0101	0.0099
25	12	5	0.0097	0.0095
25	16	5	0.0094	0.0093
35	8	7	0.0103	0.0104
35	12	7	0.0099	0.0100
35	16	7	0.0096	0.0097

13. Conclusions

The purpose of this study is to investigate the applicability of those specified limitations of bridge curvature in North American Bridge Codes to treat a horizontally curved bridge as straight one by the V-Load method and compare the results with the results obtained from the finite element modelling by others. Based on the data generated form this study, the following conclusions are drawn.

1- The stress magnification factor is the governing factor to design on whether single- and two-span curved slab on steel I-girder bridges can be treated as a straight one per the outcome from the V-Load method.
2- In general, good correlation between the finite element results and the results from the V-Load method for bridges of shallow curvature, while the V-Load method provides conservative values for the bending moment and warping stresses for bridges of high curvature. This conclusion is dependent of the bridge configuration, including span length, bridge width and number of cross-bracing spacing.
3- The developed equations for curvature limitations by Khalafalla in 2009 correlate well with the results from the V-Load method, given the limiting values of the span-to-radius of curvature ration, L/R, are generally less than 0.1.
4- A modification to the curvature limitations by Khalafalla in 2009 was proposed using the data generated from the V-Load method to consider the effect of number of bracing spacing on the total bending moment on the girder.

14. Recommendations for Future Research

Further research is recommended in the following areas:

1) Study of curvature limitations for the truck loading conditions.
2) Study of curvature limitations for the bare steel girders for un-shored construction.
3) Study of the presence of concrete barrier wall, built integrally with the concrete deck slab, on the results from the finite element method and the V-Load method.

APPENDIX 1.

RESULTS OF STRESS MAGNIFICATION FACTOR SINGLE SPAN BRIDGES

Table A.1. Results of stress magnification factors and
Warping-bending stress ratios for single span bridges

L(m)	W(m)	Theta(L/R)	No. of Bracing	WS/TS	SMF
15	8	0	5	0	1
15	8	0.02	5	0.027200	1.047663202
15	8	0.04	5	0.052959585	1.096398177
15	8	0.06	5	0.077390103	1.146204974
15	8	0.08	5	0.100591866	1.197083593
15	8	0.1	5	0.122655301	1.249034034
15	8	0.2	5	0.218509281	1.52486357
15	8	0.3	5	0.295481226	1.827488655
15	8	0.4	5	0.358650171	2.156909291
15	8	0.5	5	0.411423442	2.513125478
15	8	0.6	5	0.456172147	2.896137215
15	8	0	4	0	1
15	8	0.02	4	0.038704843	1.059437409
15	8	0.04	4	0.0745252	1.120359032
15	8	0.06	4	0.107771937	1.182764919
15	8	0.08	4	0.1387128	1.24665507
15	8	0.1	4	0.16757964	1.312029484
15	8	0.2	4	0.287054749	1.661165508
15	8	0.3	4	0.376538536	2.047408122
15	8	0.4	4	0.44606455	2.470757327
15	8	0.5	4	0.501639759	2.931213121
15	8	0.6	4	0.547080268	3.428775506
15	8	0	3	0	1
15	8	0.02	3	0.059187713	1.083837408
15	8	0.04	3	0.111760574	1.1701519
15	8	0.06	3	0.158768814	1.258943523
15	8	0.08	3	0.201051515	1.350212278
15	8	0.1	3	0.239287134	1.443958166
15	8	0.2	3	0.386168995	1.949844586
15	8	0.3	3	0.485509194	2.517659309
15	8	0.4	3	0.557174481	3.147402335
15	8	0.5	3	0.611315721	3.839073665

15	8	0.6	3	0.653660302	4.592673299
15	8	0	2	0	1
15	8	0.02	2	0.100591866	1.141491475
15	8	0.04	2	0.182795947	1.288947824
15	8	0.06	2	0.251231839	1.442369095
15	8	0.08	2	0.309091264	1.601755289
15	8	0.1	2	0.358650171	1.767106405
15	8	0.2	2	0.527950724	2.683335823
15	8	0.3	2	0.626536015	3.748688305
15	8	0.4	2	0.691057265	4.963163851
15	8	0.5	2	0.73656878	6.326762459
15	8	0.6	2	0.770393043	7.839484132
15	12	0	5	0	1
15	12	0.02	5	0.027200044	1.042033894
15	12	0.04	5	0.052959585	1.084833326
15	12	0.06	5	0.077390103	1.128398345
15	12	0.08	5	0.100591866	1.172728951
15	12	0.1	5	0.122655301	1.217825144
15	12	0.2	5	0.218509281	1.454789917
15	12	0.3	5	0.295481226	1.710894368
15	12	0.4	5	0.358650171	1.986138499
15	12	0.5	5	0.411423442	2.280522308
15	12	0.6	5	0.456172147	2.594045796
15	12	0	4	0	1
15	12	0.02	4	0.038704843	1.056520729
15	12	0.04	4	0.0745252	1.114306356
15	12	0.06	4	0.107771937	1.17335095
15	12	0.08	4	0.1387128	1.23365451
15	12	0.1	4	0.16757964	1.295217036
15	12	0.2	4	0.287054749	1.621914157
15	12	0.3	4	0.376538536	1.980085429
15	12	0.4	4	0.44606455	2.369730852
15	12	0.5	4	0.501639759	2.790850427
15	12	0.6	4	0.547080268	3.243444152
15	12	0	3	0	1
15	12	0.02	3	0.059187713	1.077858532
15	12	0.04	3	0.111760574	1.157486395
15	12	0.06	3	0.158768814	1.238883637
15	12	0.08	3	0.201051515	1.32205026
15	12	0.1	3	0.239287134	1.406986263

15	12	0.2	3	0.386168995	1.858206978
15	12	0.3	3	0.485509194	2.353662195
15	12	0.4	3	0.557174481	2.893351914
15	12	0.5	3	0.611315721	3.477276136
15	12	0.6	3	0.653660302	4.105434859
15	12	0	2	0	1
15	12	0.02	2	0.100591866	1.125960979
15	12	0.04	2	0.182795947	1.254762348
15	12	0.06	2	0.251231839	1.386404156
15	12	0.08	2	0.309091264	1.520886403
15	12	0.1	2	0.358650171	1.658209089
15	12	0.2	2	0.527950724	2.387429111
15	12	0.3	2	0.626536015	3.187660114
15	12	0.4	2	0.691057265	4.0589021
15	12	0.5	2	0.73656878	5.001155067
15	12	0.6	2	0.770393043	6.014419016
15	16	0	5	0	1
15	16	0.02	5	0.027200044	1.038906409
15	16	0.04	5	0.052959585	1.078408221
15	16	0.06	5	0.077390103	1.118505484
15	16	0.08	5	0.100591866	1.159198199
15	16	0.1	5	0.122655301	1.200486366
15	16	0.2	5	0.218509281	1.415858975
15	16	0.3	5	0.295481226	1.646117877
15	16	0.4	5	0.358650171	1.891263072
15	16	0.5	5	0.411423442	2.15129456
15	16	0.6	5	0.456172147	2.42621234
15	16	0	4	0	1
15	16	0.02	4	0.038704843	1.050915483
15	16	0.04	4	0.0745252	1.102655501
15	16	0.06	4	0.107771937	1.155220103
15	16	0.08	4	0.1387128	1.208609289
15	16	0.1	4	0.16757964	1.262823059
15	16	0.2	4	0.287054749	1.546260669
15	16	0.3	4	0.376538536	1.850312879
15	16	0.4	4	0.44606455	2.17497969
15	16	0.5	4	0.501639759	2.5202611
15	16	0.6	4	0.547080268	2.886157112
15	16	0	3	0	1
15	16	0.02	3	0.059187713	1.074536837

15	16	0.04	3	0.111760574	1.150449798
15	16	0.06	3	0.158768814	1.227738932
15	16	0.08	3	0.201051515	1.306404238
15	16	0.1	3	0.239287134	1.386445718
15	16	0.2	3	0.386168995	1.807295713
15	16	0.3	3	0.485509194	2.262550034
15	16	0.4	3	0.557174481	2.75220868
15	16	0.5	3	0.611315721	3.276271653
15	16	0.6	3	0.653660302	3.834738951
15	16	0	2	0	1
15	16	0.02	2	0.100591866	1.122823413
15	16	0.04	2	0.182795947	1.247855989
15	16	0.06	2	0.251231839	1.375097778
15	16	0.08	2	0.309091264	1.504548778
15	16	0.1	2	0.358650171	1.63620899
15	16	0.2	2	0.527950724	2.327648235
15	16	0.3	2	0.626536015	3.074317782
15	16	0.4	2	0.691057265	3.876217634
15	16	0.5	2	0.73656878	4.733347788
15	16	0.6	2	0.770393043	5.645708246
25	8	0	7	0	1
25	8	0.02	7	0.02959531	1.063103548
25	8	0.04	7	0.057489209	1.128137
25	8	0.06	7	0.083824317	1.195100397
25	8	0.08	7	0.108727746	1.263993739
25	8	0.1	7	0.132313155	1.334817028
25	8	0.2	7	0.233704173	1.717882655
25	8	0.3	7	0.31387886	2.149196923
25	8	0.4	7	0.378865823	2.628759832
25	8	0.5	7	0.432607253	3.156571383
25	8	0.6	7	0.47778965	3.732631575
25	8	0	5	0	1
25	8	0.02	5	0.051508119	1.088009126
25	8	0.04	5	0.097969988	1.179490247
25	8	0.06	5	0.14009255	1.274443407
25	8	0.08	5	0.178456587	1.372868606
25	8	0.1	5	0.213543666	1.474765844
25	8	0.2	5	0.351934045	2.036332609
25	8	0.3	5	0.448908054	2.684700338
25	8	0.4	5	0.520637891	3.419869032

25	8	0.5	5	0.575845604	4.24183869
25	8	0.6	5	0.61965016	5.150609311
25	8	0	4	0	1
25	8	0.02	4	0.072420437	1.127364257
25	8	0.04	4	0.135059786	1.261867618
25	8	0.06	4	0.189774245	1.403510123
25	8	0.08	4	0.237978277	1.552291774
25	8	0.1	4	0.280768637	1.70821257
25	8	0.2	4	0.438437715	2.594903726
25	8	0.3	4	0.539408137	3.660073512
25	8	0.4	4	0.609602641	4.903721926
25	8	0.5	4	0.661231226	6.32584897
25	8	0.6	4	0.700799384	7.926454643
25	8	0	3	0	1
25	8	0.02	3	0.108727746	1.15880702
25	8	0.04	3	0.196130649	1.325619697
25	8	0.06	3	0.267922105	1.500438075
25	8	0.08	3	0.32794185	1.683262154
25	8	0.1	3	0.378865823	1.874091933
25	8	0.2	3	0.549532546	2.948326336
25	8	0.3	3	0.646627414	4.222703251
25	8	0.4	3	0.709288162	5.697222678
25	8	0.5	3	0.753073647	7.371884618
25	8	0.6	3	0.785396148	9.246689069
25	8	0	2	0	1
25	8	0.02	2	0.178105065	1.252714924
25	8	0.04	2	0.302358543	1.518258481
25	8	0.06	2	0.393976703	1.796630713
25	8	0.08	2	0.464324581	2.08783162
25	8	0.1	2	0.520039004	2.391861202
25	8	0.2	2	0.684244289	4.104439235
25	8	0.3	2	0.764733998	6.137734141
25	8	0.4	2	0.812523805	8.491745921
25	8	0.5	2	0.844176401	11.16647457
25	8	0.6	2	0.866684723	14.1619201
25	12	0	7	0	1
25	12	0.02	7	0.02959531	1.053787662
25	12	0.04	7	0.057489209	1.108953815
25	12	0.06	7	0.083824317	1.1654985
25	12	0.08	7	0.108727746	1.223421718

25	12	0.1	7	0.132313155	1.282723468
25	12	0.2	7	0.233704173	1.599910209
25	12	0.3	7	0.31387886	1.951560265
25	12	0.4	7	0.378865823	2.337673635
25	12	0.5	7	0.432607253	2.758250321
25	12	0.6	7	0.47778965	3.213290321
25	12	0	5	0	1
25	12	0.02	5	0.047045858	1.073329896
25	12	0.04	5	0.089863987	1.14891432
25	12	0.06	5	0.128999764	1.226753314
25	12	0.08	5	0.164908627	1.306846879
25	12	0.1	5	0.197973884	1.389195015
25	12	0.2	5	0.330514524	1.834754249
25	12	0.3	5	0.425461228	2.336677745
25	12	0.4	5	0.496822121	2.894965503
25	12	0.5	5	0.552414692	3.509617523
25	12	0.6	5	0.596945353	4.180633805
25	12	0	4	0	1
25	12	0.02	4	0.072420437	1.113281521
25	12	0.04	4	0.135059786	1.23166239
25	12	0.06	4	0.189774245	1.355142648
25	12	0.08	4	0.237978277	1.483722295
25	12	0.1	4	0.280768637	1.617401331
25	12	0.2	4	0.438437715	2.362287354
25	12	0.3	4	0.539408137	3.23465811
25	12	0.4	4	0.609602641	4.234513601
25	12	0.5	4	0.661231226	5.361853825
25	12	0.6	4	0.700799384	6.616678783
25	12	0	3	0	1
25	12	0.02	3	0.099830161	1.136938274
25	12	0.04	3	0.181537413	1.279075013
25	12	0.06	3	0.249646068	1.426410258
25	12	0.08	3	0.307290164	1.57894401
25	12	0.1	3	0.356709384	1.736676269
25	12	0.2	3	0.525844942	2.603315165
25	12	0.3	3	0.624557272	3.59991673
25	12	0.4	3	0.689250825	4.726480963
25	12	0.5	3	0.734926378	5.983007866
25	12	0.6	3	0.768895357	7.369497437
25	12	0	2	0	1

25	12	0.02	2	0.178105065	1.242425111
25	12	0.04	2	0.302358543	1.494013519
25	12	0.06	2	0.393976703	1.754765266
25	12	0.08	2	0.464324581	2.024680352
25	12	0.1	2	0.520039004	2.303758777
25	12	0.2	2	0.684244289	3.83660099
25	12	0.3	2	0.764733998	5.598526682
25	12	0.4	2	0.812523805	7.589535852
25	12	0.5	2	0.844176401	9.809628502
25	12	0.6	2	0.866684723	12.25880463
25	16	0	7	0	1
25	16	0.02	7	0.02697741	1.045790747
25	16	0.04	7	0.052537494	1.092556163
25	16	0.06	7	0.076789087	1.140296292
25	16	0.08	7	0.099830161	1.189011132
25	16	0.1	7	0.121749142	1.238700684
25	16	0.2	7	0.217070176	1.501769123
25	16	0.3	7	0.293725718	1.78920536
25	16	0.4	7	0.356709384	2.101009394
25	16	0.5	7	0.409379365	2.437181225
25	16	0.6	7	0.454077265	2.797720854
25	16	0	5	0	1
25	16	0.02	5	0.051508119	1.073029503
25	16	0.04	5	0.097969988	1.147987858
25	16	0.06	5	0.14009255	1.224875107
25	16	0.08	5	0.178456587	1.303691251
25	16	0.1	5	0.213543666	1.384436289
25	16	0.2	5	0.351934045	1.81709489
25	16	0.3	5	0.448908054	2.297975848
25	16	0.4	5	0.520637891	2.82707916
25	16	0.5	5	0.575845604	3.404404828
25	16	0.6	5	0.61965016	4.029952852
25	16	0	4	0	1
25	16	0.02	4	0.072420437	1.105457551
25	16	0.04	4	0.135059786	1.214881219
25	16	0.06	4	0.189774245	1.328271045
25	16	0.08	4	0.237978277	1.44562703
25	16	0.1	4	0.280768637	1.566949174
25	16	0.2	4	0.438437715	2.233052271
25	16	0.3	4	0.539408137	2.998309336

25	16	0.4	4	0.609602641	3.862720367
25	16	0.5	4	0.661231226	4.826285365
25	16	0.6	4	0.700799384	5.88900433
25	16	0	3	0	1
25	16	0.02	3	0.108727746	1.14244447
25	16	0.04	3	0.196130649	1.289336472
25	16	0.06	3	0.267922105	1.440676049
25	16	0.08	3	0.32794185	1.5964632
25	16	0.1	3	0.378865823	1.756697925
25	16	0.2	3	0.549532546	2.624585164
25	16	0.3	3	0.646627414	3.603661759
25	16	0.4	3	0.709288162	4.693927712
25	16	0.5	3	0.753073647	5.895383022
25	16	0.6	3	0.785396148	7.208027688
25	16	0	2	0	1
25	16	0.02	2	0.178105065	1.236708381
25	16	0.04	2	0.302358543	1.480543703
25	16	0.06	2	0.393976703	1.731506006
25	16	0.08	2	0.464324581	1.989595292
25	16	0.1	2	0.520039004	2.254811559
25	16	0.2	2	0.684244289	3.68779763
25	16	0.3	2	0.764733998	5.298958256
25	16	0.4	2	0.812523805	7.088293435
25	16	0.5	2	0.844176401	9.055803169
25	16	0.6	2	0.866684723	11.20148746
35	8	0	9	0	1
35	8	0.02	9	0.031268493	1.061152844
35	8	0.04	9	0.060640838	1.124111248
35	8	0.06	9	0.088284436	1.188875399
35	8	0.08	9	0.114347546	1.255445299
35	8	0.1	9	0.138961943	1.323820947
35	8	0.2	9	0.244015077	1.692785404
35	8	0.3	9	0.326221173	2.106893559
35	8	0.4	9	0.392302443	2.566145412
35	8	0.5	9	0.446579496	3.070540965
35	8	0.6	9	0.491955911	3.620080215
35	8	0	7	0	1
35	8	0.02	7	0.04801255	1.096965167
35	8	0.04	7	0.091625907	1.198398286
35	8	0.06	7	0.131418204	1.304299544

35	8	0.08	7	0.167870525	1.414668941
35	8	0.1	7	0.201386444	1.529506478
35	8	0.2	7	0.335256728	2.170716255
35	8	0.3	7	0.430689342	2.92362952
35	8	0.4	7	0.502160702	3.788246271
35	8	0.5	7	0.557688558	4.76456651
35	8	0.6	7	0.602072483	5.852590237
35	8	0	6	0	1
35	8	0.02	6	0.061801922	1.112112832
35	8	0.04	6	0.116409513	1.229940872
35	8	0.06	6	0.165009908	1.353484308
35	8	0.08	6	0.208542675	1.48274314
35	8	0.1	6	0.24776115	1.617717369
35	8	0.2	6	0.397129131	2.378319453
35	8	0.3	6	0.49700593	3.281806441
35	8	0.4	6	0.568493093	4.328178334
35	8	0.5	6	0.622188851	5.51743513
35	8	0.6	6	0.663999948	6.84957683
35	8	0	5	0	1
35	8	0.02	5	0.082274315	1.138379478
35	8	0.04	5	0.152039671	1.284777066
35	8	0.06	5	0.211947307	1.439192951
35	8	0.08	5	0.263948672	1.601627135
35	8	0.1	5	0.309512019	1.772079616
35	8	0.2	5	0.472713521	2.744616492
35	8	0.3	5	0.573515917	3.917610816
35	8	0.4	5	0.641962628	5.291062586
35	8	0.5	5	0.691477592	6.864971805
35	8	0.6	5	0.728961062	8.639338471
35	8	0	3	0	1
35	8	0.02	3	0.167870525	1.256941052
35	8	0.04	3	0.287481397	1.532416419
35	8	0.06	3	0.377027849	1.826426292
35	8	0.08	3	0.446579496	2.138970669
35	8	0.1	3	0.502160702	2.47004955
35	8	0.2	3	0.668584528	4.403461526
35	8	0.3	3	0.751617033	6.800236116
35	8	0.4	3	0.801379273	9.660373319
35	8	0.5	3	0.834530275	12.98387314
35	8	0.6	3	0.858197904	16.77073557

35	12	0	9	0	1
35	12	0.02	9	0.031268493	1.052902878
35	12	0.04	9	0.060640838	1.107095388
35	12	0.06	9	0.088284436	1.162577718
35	12	0.08	9	0.114347546	1.219349868
35	12	0.1	9	0.138961943	1.277411838
35	12	0.2	9	0.244015077	1.587068986
35	12	0.3	9	0.326221173	1.928971633
35	12	0.4	9	0.392302443	2.303119778
35	12	0.5	9	0.446579496	2.709513422
35	12	0.6	9	0.491955911	3.148152564
35	12	0	7	0	1
35	12	0.02	7	0.04801255	1.083670609
35	12	0.04	7	0.091625907	1.170532557
35	12	0.06	7	0.131418204	1.260586034
35	12	0.08	7	0.167870525	1.353831039
35	12	0.1	7	0.201386444	1.450267573
35	12	0.2	7	0.335256728	1.980323161
35	12	0.3	7	0.430689342	2.590166954
35	12	0.4	7	0.502160702	3.279798952
35	12	0.5	7	0.557688558	4.049219154
35	12	0.6	7	0.602072483	4.89842756
35	12	0	6	0	1
35	12	0.02	6	0.061801922	1.098901509
35	12	0.04	6	0.116409513	1.201885255
35	12	0.06	6	0.165009908	1.308951427
35	12	0.08	6	0.208542675	1.420100024
35	12	0.1	6	0.24776115	1.535331048
35	12	0.2	6	0.397129131	2.172722552
35	12	0.3	6	0.49700593	2.912174701
35	12	0.4	6	0.568493093	3.753687494
35	12	0.5	6	0.622188851	4.697260932
35	12	0.6	6	0.663999948	5.742895015
35	12	0	5	0	1
35	12	0.02	5	0.082274315	1.124456893
35	12	0.04	5	0.152039671	1.254640953
35	12	0.06	5	0.211947307	1.390552369
35	12	0.08	5	0.263948672	1.532191141
35	12	0.1	5	0.309512019	1.679557268
35	12	0.2	5	0.472713521	2.502298237

35	12	0.3	5	0.573515917	3.468223096
35	12	0.4	5	0.641962628	4.577331844
35	12	0.5	5	0.691477592	5.829624482
35	12	0.6	5	0.728961062	7.22510101
35	12	0	3	0	1
35	12	0.02	3	0.167870525	1.241168262
35	12	0.04	3	0.287481397	1.495575268
35	12	0.06	3	0.377027849	1.763221205
35	12	0.08	3	0.446579496	2.044106074
35	12	0.1	3	0.502160702	2.338229875
35	12	0.2	3	0.668584528	4.007432854
35	12	0.3	3	0.751617033	6.007609125
35	12	0.4	3	0.801379273	8.338758688
35	12	0.5	3	0.834530275	11.00088154
35	12	0.6	3	0.858197904	13.99397769
35	16	0	9	0	1
35	16	0.02	9	0.031268493	1.04831943
35	16	0.04	9	0.060640838	1.097641858
35	16	0.06	9	0.088284436	1.14796747
35	16	0.08	9	0.114347546	1.199296267
35	16	0.1	9	0.138961943	1.251628248
35	16	0.2	9	0.244015077	1.528335932
35	16	0.3	9	0.326221173	1.830123239
35	16	0.4	9	0.392302443	2.156990168
35	16	0.5	9	0.446579496	2.508936722
35	16	0.6	9	0.491955911	2.885962898
35	16	0	7	0	1
35	16	0.02	7	0.04801255	1.076284528
35	16	0.04	7	0.091625907	1.155051146
35	16	0.06	7	0.131418204	1.236300044
35	16	0.08	7	0.167870525	1.32003122
35	16	0.1	7	0.201386444	1.406244676
35	16	0.2	7	0.335256728	1.874546139
35	16	0.3	7	0.430689342	2.404904577
35	16	0.4	7	0.502160702	2.99731999
35	16	0.5	7	0.557688558	3.651792379
35	16	0.6	7	0.602072483	4.368321742
35	16	0	6	0	1
35	16	0.02	6	0.061801922	1.091561671
35	16	0.04	6	0.116409513	1.186298347

35	16	0.06	6	0.165009908	1.284210216
35	16	0.08	6	0.208542675	1.385297279
35	16	0.1	6	0.24776115	1.489559536
35	16	0.2	6	0.397129131	2.058498724
35	16	0.3	6	0.49700593	2.706817751
35	16	0.4	6	0.568493093	3.434516619
35	16	0.5	6	0.622188851	4.241595327
35	16	0.6	6	0.663999948	5.128053874
35	16	0	5	0	1
35	16	0.02	5	0.082274315	1.116721898
35	16	0.04	5	0.152039671	1.237898181
35	16	0.06	5	0.211947307	1.363529036
35	16	0.08	5	0.263948672	1.493614464
35	16	0.1	5	0.309512019	1.628154465
35	16	0.2	5	0.472713521	2.367673062
35	16	0.3	5	0.573515917	3.218555978
35	16	0.4	5	0.641962628	4.180803214
35	16	0.5	5	0.691477592	5.254414769
35	16	0.6	5	0.728961062	6.439390644
35	16	0	3	0	1
35	16	0.02	3	0.167870525	1.232405346
35	16	0.04	3	0.287481397	1.475107365
35	16	0.06	3	0.377027849	1.728106245
35	16	0.08	3	0.446579496	1.991401986
35	16	0.1	3	0.502160702	2.264994588
35	16	0.2	3	0.668584528	3.787410515
35	16	0.3	3	0.751617033	5.567247969
35	16	0.4	3	0.801379273	7.604506949
35	16	0.5	3	0.834530275	9.899187455
35	16	0.6	3	0.858197904	12.45128949

APPENDIX 2.

RESULTS OF STRESS MAGNIFICATION FACTOR FOR TWO-SPAN BRIDGES

Table A.2. Results of stress magnification factors,

and warping-bending stress ratios for two-span continuous bridges

L(m)	W(m)	Theta(L/R)	No. of Bracing	WS/TS-MID	SMF-MID SPAN	SMF-SUPPORT
15	8	0	11	0	1	1
15	8	0.02	11	0.0137875	1.01913111	1.0032003
15	8	0.04	11	0.0272	1.03840431	1.0064005
15	8	0.06	11	0.0402526	1.05781954	1.0096007
15	8	0.08	11	0.0529596	1.07737682	1.012801
15	8	0.1	11	0.0653345	1.09707612	1.0160012
15	8	0.2	11	0.1226553	1.1977032	1.0320024
15	8	0.3	11	0.1733517	1.30188117	1.0480035
15	8	0.4	11	0.2185093	1.40961004	1.0640047
15	8	0.5	11	0.2589887	1.5208898	1.0800058
15	8	0.6	11	0.2954812	1.63572046	1.096007
15	8	0	9	0	1	1
15	8	0.02	9	0.0197343	1.02534643	1.0032832
15	8	0.04	9	0.0387048	1.05089878	1.0065664
15	8	0.06	9	0.056955	1.07665696	1.0098497
15	8	0.08	9	0.0745252	1.10262096	1.0131329
15	8	0.1	9	0.0914526	1.12879078	1.0164161
15	8	0.2	9	0.1675796	1.26272729	1.0328321
15	8	0.3	9	0.2319356	1.40180944	1.0492481
15	8	0.4	9	0.2870547	1.54603723	1.0656641
15	8	0.5	9	0.3347925	1.69541066	1.0820801
15	8	0.6	9	0.3765385	1.84992973	1.0984961
15	8	0	7	0	1	1
15	8	0.02	7	0.0304964	1.03662305	1.0034278
15	8	0.04	7	0.0591877	1.07356121	1.0068555
15	8	0.06	7	0.0862297	1.11081455	1.0102833
15	8	0.08	7	0.1117606	1.14838306	1.013711
15	8	0.1	7	0.1359036	1.18626674	1.0171387
15	8	0.2	7	0.2392871	1.38041271	1.0342774
15	8	0.3	7	0.3205759	1.58243794	1.0514161
15	8	0.4	7	0.386169	1.79234245	1.0685547

15	8	0.5	7	0.4402122	2.01012623	1.0856934
15	8	0.6	7	0.4855092	2.23578928	1.1028321
15	8	0	5	0	1	1
15	8	0.02	5	0.0529596	1.06202432	1.0037038
15	8	0.04	5	0.1005919	1.12469268	1.0074075
15	8	0.06	5	0.1436622	1.18800724	1.0111112
15	8	0.08	5	0.1827959	1.251968	1.0148149
15	8	0.1	5	0.2185093	1.31657496	1.0185186
15	8	0.2	5	0.3586502	1.64930279	1.0370371
15	8	0.3	5	0.4561721	1.99818566	1.0555556
15	8	0.4	5	0.5279507	2.36322355	1.0740741
15	8	0.5	5	0.5829908	2.74441648	1.0925926
15	8	0.6	5	0.626536	3.14176445	1.1111112
15	12	0	11	0	1	1
15	12	0.02	11	0.0137875	1.01765943	1.0022859
15	12	0.04	11	0.0272	1.03542037	1.0045718
15	12	0.06	11	0.0402526	1.05328276	1.0068577
15	12	0.08	11	0.0529596	1.07124661	1.0091436
15	12	0.1	11	0.0653345	1.08931191	1.0114294
15	12	0.2	11	0.1226553	1.18116023	1.0228588
15	12	0.3	11	0.1733517	1.2755449	1.0342882
15	12	0.4	11	0.2185093	1.37246593	1.0457176
15	12	0.5	11	0.2589887	1.47192331	1.057147
15	12	0.6	11	0.2954812	1.57391704	1.0685764
15	12	0	9	0	1	1
15	12	0.02	9	0.0197343	1.02385646	1.0023452
15	12	0.04	9	0.0387048	1.04786003	1.0046903
15	12	0.06	9	0.056955	1.07201061	1.0070355
15	12	0.08	9	0.0745252	1.09630821	1.0093806
15	12	0.1	9	0.0914526	1.12075283	1.0117258
15	12	0.2	9	0.1675796	1.24518121	1.0234515
15	12	0.3	9	0.2319356	1.37328505	1.0351772
15	12	0.4	9	0.2870547	1.50506434	1.0469029
15	12	0.5	9	0.3347925	1.64051909	1.0586286
15	12	0.6	9	0.3765385	1.7796493	1.0703543
15	12	0	7	0	1	1
15	12	0.02	7	0.0304964	1.03514666	1.0024484
15	12	0.04	7	0.0591877	1.07051839	1.0048968
15	12	0.06	7	0.0862297	1.10611525	1.0073452
15	12	0.08	7	0.1117606	1.14193723	1.0097936

15	12	0.1	7	0.1359036	1.17798433	1.012242
15	12	0.2	7	0.2392871	1.36159665	1.0244839
15	12	0.3	7	0.3205759	1.55083703	1.0367258
15	12	0.4	7	0.386169	1.74570545	1.0489677
15	12	0.5	7	0.4402122	1.94620193	1.0612096
15	12	0.6	7	0.4855092	2.15232646	1.0734515
15	12	0	5	0	1	1
15	12	0.02	5	0.0529596	1.06028121	1.0026456
15	12	0.04	5	0.1005919	1.12102182	1.0052911
15	12	0.06	5	0.1436622	1.18222401	1.0079366
15	12	0.08	5	0.1827959	1.24388778	1.0105821
15	12	0.1	5	0.2185093	1.30601311	1.0132276
15	12	0.2	5	0.3586502	1.62356337	1.0264551
15	12	0.3	5	0.4561721	1.95265294	1.0396826
15	12	0.4	5	0.5279507	2.29328182	1.0529101
15	12	0.5	5	0.5829908	2.64545001	1.0661376
15	12	0.6	5	0.626536	3.0091575	1.0793651
15	16	0	11	0	1	1
15	16	0.02	11	0.0137875	1.04259598	1.017779
15	16	0.04	11	0.0272	1.08598111	1.0355579
15	16	0.06	11	0.0402526	1.13015531	1.0533368
15	16	0.08	11	0.0529596	1.17511859	1.0711157
15	16	0.1	11	0.0653345	1.22087096	1.0888946
15	16	0.2	11	0.1226553	1.461469	1.1777892
15	16	0.3	11	0.1733517	1.72179408	1.2666838
15	16	0.4	11	0.2185093	2.00184619	1.3555784
15	16	0.5	11	0.2589887	2.30162534	1.444473
15	16	0.6	11	0.2954812	2.62113151	1.5333675
15	16	0	9	0	1	1
15	16	0.02	9	0.0197343	1.02302867	1.001824
15	16	0.04	9	0.0387048	1.04617178	1.003648
15	16	0.06	9	0.056955	1.06942923	1.005472
15	16	0.08	9	0.0745252	1.09280103	1.007296
15	16	0.1	9	0.0914526	1.11628718	1.00912
15	16	0.2	9	0.1675796	1.23543311	1.0182399
15	16	0.3	9	0.2319356	1.3574377	1.0273598
15	16	0.4	9	0.2870547	1.48230096	1.0364797
15	16	0.5	9	0.3347925	1.61002289	1.0455997
15	16	0.6	9	0.3765385	1.74060348	1.0547196
15	16	0	7	0	1	1

15	16	0.02	7	0.0304964	1.03432642	1.0019043
15	16	0.04	7	0.0591877	1.06882789	1.0038086
15	16	0.06	7	0.0862297	1.10350445	1.0057129
15	16	0.08	7	0.1117606	1.1383561	1.0076172
15	16	0.1	7	0.1359036	1.17338285	1.0095215
15	16	0.2	7	0.2392871	1.35114298	1.0190429
15	16	0.3	7	0.3205759	1.53328045	1.0285643
15	16	0.4	7	0.386169	1.71979526	1.0380857
15	16	0.5	7	0.4402122	1.9106874	1.0476071
15	16	0.6	7	0.4855092	2.10595688	1.0571285
15	16	0	5	0	1	1
15	16	0.02	5	0.0529596	1.05931278	1.0020576
15	16	0.04	5	0.1005919	1.1189824	1.0041152
15	16	0.06	5	0.1436622	1.17901102	1.0061728
15	16	0.08	5	0.1827959	1.23939864	1.0082304
15	16	0.1	5	0.2185093	1.30014525	1.010288
15	16	0.2	5	0.3586502	1.60926328	1.020576
15	16	0.3	5	0.4561721	1.92735625	1.030864
15	16	0.4	5	0.5279507	2.25442417	1.041152
15	16	0.5	5	0.5829908	2.59046703	1.05144
15	16	0.6	5	0.626536	2.93548483	1.0617279
25	8	0	15	0	1	1
25	8	0.02	15	0.01417	1.02504545	1.0077068
25	8	0.04	15	0.027944	1.05039334	1.0154136
25	8	0.06	15	0.0413385	1.07604367	1.0231204
25	8	0.08	15	0.0543688	1.10199643	1.0308272
25	8	0.1	15	0.0670496	1.12825164	1.0385339
25	8	0.2	15	0.125673	1.26406423	1.0770679
25	8	0.3	15	0.1773645	1.40743777	1.1156019
25	8	0.4	15	0.223285	1.55837227	1.1541358
25	8	0.5	15	0.2643499	1.71686772	1.1926698
25	8	0.6	15	0.3012907	1.88292412	1.2312037
25	8	0	11	0	1	1
25	8	0.02	11	0.0249554	1.03428598	1.0053401
25	8	0.04	11	0.0486956	1.06900585	1.0106802
25	8	0.06	11	0.0713072	1.10415954	1.0160203
25	8	0.08	11	0.0928689	1.13974706	1.0213603
25	8	0.1	11	0.113452	1.1757684	1.0267004
25	8	0.2	11	0.2037843	1.36238242	1.0534009
25	8	0.3	11	0.2774105	1.55984198	1.0801013

25	8	0.4	11	0.3385728	1.7681471	1.1068017
25	8	0.5	11	0.3901891	1.98729775	1.1335021
25	8	0.6	11	0.4343326	2.21729396	1.1602026
25	8	0	9	0	1	1
25	8	0.02	9	0.0354907	1.04563018	1.005472
25	8	0.04	9	0.0685485	1.09188732	1.010944
25	8	0.06	9	0.0994154	1.13877147	1.016416
25	8	0.08	9	0.1283021	1.18628264	1.021888
25	8	0.1	9	0.1553933	1.23442083	1.02736
25	8	0.2	9	0.2689878	1.48451697	1.05472
25	8	0.3	9	0.355649	1.75028847	1.08208
25	8	0.4	9	0.4239407	2.03173532	1.10944
25	8	0.5	9	0.4791437	2.32885754	1.1368
25	8	0.6	9	0.5246919	2.64165512	1.16416
25	8	0	7	0	1	1
25	8	0.02	7	0.0543688	1.06686655	1.0057129
25	8	0.04	7	0.1031305	1.13475218	1.0114258
25	8	0.06	7	0.14711	1.20365688	1.0171387
25	8	0.08	7	0.1869779	1.27358065	1.0228516
25	8	0.1	7	0.223285	1.34452349	1.0285645
25	8	0.2	7	0.3650581	1.71452376	1.0571289
25	8	0.3	7	0.4630644	2.11000082	1.0856934
25	8	0.4	7	0.5348609	2.53095466	1.1142578
25	8	0.5	7	0.5897213	2.97738529	1.1428223
25	8	0.6	7	0.6330062	3.4492927	1.1713867
25	8	0	5	0	1	1
25	8	0.02	5	0.0926669	1.11273674	1.0061657
25	8	0.04	5	0.169616	1.22743903	1.0123314
25	8	0.06	5	0.2345337	1.34410691	1.018497
25	8	0.08	5	0.2900371	1.46274038	1.0246627
25	8	0.1	5	0.3380356	1.58333942	1.0308284
25	8	0.2	5	0.5052715	2.2158184	1.0616568
25	8	0.3	5	0.60505	2.89743697	1.0924852
25	8	0.4	5	0.671336	3.62819511	1.1233136
25	8	0.5	5	0.7185696	4.40809284	1.154142
25	8	0.6	5	0.7539329	5.23713015	1.1849704
25	12	0	15	0	1	1
25	12	0.02	15	0.01417	1.02199637	1.0055048
25	12	0.04	15	0.027944	1.04420877	1.0110097
25	12	0.06	15	0.0413385	1.0666372	1.0165145

25	12	0.08	15	0.0543688	1.08928165	1.0220194
25	12	0.1	15	0.0670496	1.11214214	1.0275242
25	12	0.2	15	0.125673	1.22968496	1.0550485
25	12	0.3	15	0.1773645	1.35262846	1.0825727
25	12	0.4	15	0.223285	1.48097264	1.110097
25	12	0.5	15	0.2643499	1.6147175	1.1376212
25	12	0.6	15	0.3012907	1.75386304	1.1651455
25	12	0	11	0	1	1
25	12	0.02	11	0.0249554	1.03180256	1.0038143
25	12	0.04	11	0.0486956	1.06391507	1.0076287
25	12	0.06	11	0.0713072	1.09633746	1.011443
25	12	0.08	11	0.0928689	1.12906971	1.0152574
25	12	0.1	11	0.113452	1.16211184	1.0190717
25	12	0.2	11	0.2037843	1.33197058	1.0381435
25	12	0.3	11	0.2774105	1.50957613	1.0572152
25	12	0.4	11	0.3385728	1.69492851	1.0762869
25	12	0.5	11	0.3901891	1.8880277	1.0953587
25	12	0.6	11	0.4343326	2.08887371	1.1144304
25	12	0	9	0	1	1
25	12	0.02	9	0.0354907	1.04310632	1.0039086
25	12	0.04	9	0.0685485	1.08666045	1.0078171
25	12	0.06	9	0.0994154	1.13066245	1.0117257
25	12	0.08	9	0.1283021	1.17511232	1.0156343
25	12	0.1	9	0.1553933	1.22001006	1.0195429
25	12	0.2	9	0.2689878	1.45121676	1.0390857
25	12	0.3	9	0.355649	1.69362014	1.0586286
25	12	0.4	9	0.4239407	1.94722021	1.0781714
25	12	0.5	9	0.4791437	2.21201696	1.0977143
25	12	0.6	9	0.5246919	2.48801041	1.1172571
25	12	0	7	0	1	1
25	12	0.02	7	0.0543688	1.06418888	1.0040806
25	12	0.04	7	0.1031305	1.12910568	1.0081613
25	12	0.06	7	0.14711	1.19475038	1.0122419
25	12	0.08	7	0.1869779	1.26112299	1.0163225
25	12	0.1	7	0.223285	1.32822351	1.0204032
25	12	0.2	7	0.3650581	1.67464473	1.0408064
25	12	0.3	7	0.4630644	2.03926365	1.0612095
25	12	0.4	7	0.5348609	2.42208027	1.0816127
25	12	0.5	7	0.5897213	2.82309459	1.1020159
25	12	0.6	7	0.6330062	3.24230662	1.1224191

25	12	0	5	0	1	1
25	12	0.02	5	0.0926669	1.10970656	1.0044041
25	12	0.04	5	0.169616	1.22081707	1.0088081
25	12	0.06	5	0.2345337	1.33333158	1.0132122
25	12	0.08	5	0.2900371	1.44725008	1.0176162
25	12	0.1	5	0.3380356	1.56257256	1.0220203
25	12	0.2	5	0.5052715	2.16024479	1.0440406
25	12	0.3	5	0.60505	2.79301673	1.0660609
25	12	0.4	5	0.671336	3.46088837	1.0880811
25	12	0.5	5	0.7185696	4.16385971	1.1101014
25	12	0.6	5	0.7539329	4.90193075	1.1321217
25	16	0	15	0	1	1
25	16	0.02	15	0.01417	1.02030239	1.0042815
25	16	0.04	15	0.027944	1.0407728	1.008563
25	16	0.06	15	0.0413385	1.06141123	1.0128445
25	16	0.08	15	0.0543688	1.08221768	1.0171261
25	16	0.1	15	0.0670496	1.10319215	1.0214076
25	16	0.2	15	0.125673	1.21058481	1.0428151
25	16	0.3	15	0.1773645	1.32217795	1.0642227
25	16	0.4	15	0.223285	1.43797159	1.0856303
25	16	0.5	15	0.2643499	1.55796573	1.1070379
25	16	0.6	11	0.3012907	1.68216035	1.1284454
25	16	0	11	0	1	1
25	16	0.02	11	0.0249554	1.03042285	1.0029667
25	16	0.04	11	0.0486956	1.06108678	1.0059334
25	16	0.06	11	0.0713072	1.09199173	1.0089001
25	16	0.08	11	0.0928689	1.12313768	1.0118668
25	16	0.1	11	0.113452	1.15452465	1.0148334
25	16	0.2	11	0.2037843	1.31507462	1.0296669
25	16	0.3	11	0.2774105	1.48164985	1.0445003
25	16	0.4	11	0.3385728	1.65425033	1.0593338
25	16	0.5	11	0.3901891	1.83287607	1.0741672
25	16	0.6	9	0.4343326	2.01752705	1.0890007
25	16	0	9	0	1	1
25	16	0.02	9	0.0354907	1.04170413	1.00304
25	16	0.04	9	0.0685485	1.08375655	1.0060799
25	16	0.06	9	0.0994154	1.12615731	1.0091199
25	16	0.08	9	0.1283021	1.16890641	1.0121599
25	16	0.1	9	0.1553933	1.21200385	1.0151999
25	16	0.2	9	0.2689878	1.4327161	1.0303997

25	16	0.3	9	0.355649	1.66213682	1.0455996
25	16	0.4	9	0.4239407	1.900266	1.0607995
25	16	0.5	9	0.4791437	2.14710364	1.0759994
25	16	0.6	7	0.5246919	2.40264974	1.0911992
25	16	0	7	0	1	1
25	16	0.02	7	0.0543688	1.06270125	1.0031738
25	16	0.04	7	0.1031305	1.12596864	1.0063476
25	16	0.06	7	0.14711	1.18980218	1.0095214
25	16	0.08	7	0.1869779	1.25420187	1.0126952
25	16	0.1	7	0.223285	1.3191677	1.015869
25	16	0.2	7	0.3650581	1.65248906	1.031738
25	16	0.3	7	0.4630644	1.99996407	1.047607
25	16	0.4	7	0.5348609	2.36159273	1.063476
25	16	0.5	7	0.5897213	2.73737504	1.079345
25	16	0.6	7	0.6330062	3.127311	1.095214
25	16	0	5	0	1	1
25	16	0.02	5	0.0926669	1.10802307	1.0034253
25	16	0.04	5	0.169616	1.2171381	1.0068507
25	16	0.06	5	0.2345337	1.32734511	1.010276
25	16	0.08	5	0.2900371	1.4386441	1.0137014
25	16	0.1	5	0.3380356	1.55103508	1.0171267
25	16	0.2	5	0.5052715	2.12936967	1.0342535
25	16	0.3	5	0.60505	2.7350038	1.0513802
25	16	0.4	5	0.671336	3.36793747	1.068507
25	16	0.5	5	0.7185696	4.02817068	1.0856337
25	16	0.6	5	0.7539329	4.71570343	1.1027605
35	8	0	19	0	1	1
35	8	0.02	19	0.014694	1.02645957	1.0071725
35	8	0.04	19	0.0289624	1.05325844	1.0143451
35	8	0.06	19	0.0428235	1.08039664	1.0215176
35	8	0.08	19	0.0562944	1.10787416	1.0286902
35	8	0.1	19	0.0693914	1.13569102	1.0358627
35	8	0.2	19	0.1297774	1.27986516	1.0717255
35	8	0.3	19	0.1828042	1.43252245	1.1075882
35	8	0.4	19	0.2297398	1.59366289	1.143451
35	8	0.5	19	0.2715768	1.76328649	1.1793137
35	8	0.6	19	0.3091031	1.94139323	1.2151765
35	8	0	15	0	1	1
35	8	0.02	15	0.0227712	1.03513244	1.0072674
35	8	0.04	15	0.0445284	1.07080368	1.0145349

35	8	0.06	15	0.0653378	1.10701372	1.0218023
35	8	0.08	15	0.0852602	1.14376256	1.0290698
35	8	0.1	15	0.104351	1.18105019	1.0363373
35	8	0.2	15	0.1889816	1.37557031	1.0726745
35	8	0.3	15	0.2589994	1.58356034	1.1090118
35	8	0.4	15	0.3178882	1.80502029	1.1453491
35	8	0.5	15	0.3681061	2.03995015	1.1816864
35	8	0.6	15	0.4114369	2.28834992	1.2180237
35	8	0	13	0	1	1
35	8	0.02	13	0.029536	1.04221136	1.0073469
35	8	0.04	13	0.0573774	1.08511838	1.0146939
35	8	0.06	13	0.0836658	1.12872106	1.0220408
35	8	0.08	13	0.1085277	1.17301939	1.0293877
35	8	0.1	13	0.1320762	1.21801338	1.0367347
35	8	0.2	13	0.2333344	1.45341818	1.0734694
35	8	0.3	13	0.3134341	1.70621439	1.1102041
35	8	0.4	13	0.3783798	1.976402	1.1469388
35	8	0.5	13	0.4321002	2.26398102	1.1836734
35	8	0.6	13	0.4772742	2.56895146	1.2204081
35	8	0	11	0	1	1
35	8	0.02	11	0.0397339	1.05371386	1.0074632
35	8	0.04	11	0.076431	1.10840799	1.0149264
35	8	0.06	11	0.1104264	1.16408241	1.0223895
35	8	0.08	11	0.1420081	1.22073712	1.0298527
35	8	0.1	11	0.1714242	1.27837214	1.0373159
35	8	0.2	11	0.2926766	1.5812517	1.0746318
35	8	0.3	11	0.3829715	1.9086387	1.1119478
35	8	0.4	11	0.4528226	2.26053317	1.1492637
35	8	0.5	11	0.5084669	2.63693509	1.1865797
35	8	0.6	11	0.5538386	3.03784446	1.2238956
35	8	0	7	0	1	1
35	8	0.02	7	0.0852602	1.12033448	1.007998
35	8	0.04	7	0.157124	1.24529467	1.0159961
35	8	0.06	7	0.2185188	1.37488062	1.0239941
35	8	0.08	7	0.2715768	1.50909234	1.0319922
35	8	0.1	7	0.3178882	1.64792983	1.0399902
35	8	0.2	7	0.4824206	2.41150375	1.0799804
35	8	0.3	7	0.5830043	3.29072182	1.1199707
35	8	0.4	7	0.6508553	4.28558404	1.1599609
35	8	0.5	7	0.6997157	5.39609039	1.1999512

35	8	0.6	7	0.7365795	6.6222409	1.2399414
35	12	0	19	0	1	1
35	12	0.02	19	0.014694	1.02316059	1.0051232
35	12	0.04	19	0.0289624	1.04656354	1.0102465
35	12	0.06	19	0.0428235	1.07020886	1.0153697
35	12	0.08	19	0.0562944	1.09409656	1.020493
35	12	0.1	19	0.0693914	1.11822664	1.0256162
35	12	0.2	19	0.1297774	1.24251264	1.0512325
35	12	0.3	19	0.1828042	1.37285804	1.0768487
35	12	0.4	19	0.2297398	1.50926283	1.102465
35	12	0.5	19	0.2715768	1.65172702	1.1280812
35	12	0.6	19	0.3091031	1.80025059	1.1536975
35	12	0	15	0	1	1
35	12	0.02	15	0.0227712	1.03175224	1.005191
35	12	0.04	15	0.0445284	1.06388935	1.0103821
35	12	0.06	15	0.0653378	1.09641132	1.0155731
35	12	0.08	15	0.0852602	1.12931813	1.0207641
35	12	0.1	15	0.104351	1.16260981	1.0259552
35	12	0.2	15	0.1889816	1.33484099	1.0519104
35	12	0.3	15	0.2589994	1.51669354	1.0778656
35	12	0.4	15	0.3178882	1.70816746	1.1038208
35	12	0.5	15	0.3681061	1.90926274	1.129776
35	12	0.6	15	0.4114369	2.11997939	1.1557312
35	12	0	13	0	1	1
35	12	0.02	13	0.029536	1.03884667	1.0052478
35	12	0.04	13	0.0573774	1.07819025	1.0104956
35	12	0.06	13	0.0836658	1.11803072	1.0157434
35	12	0.08	13	0.1085277	1.1583681	1.0209912
35	12	0.1	13	0.1320762	1.19920236	1.026239
35	12	0.2	13	0.2333344	1.41082717	1.0524781
35	12	0.3	13	0.3134341	1.63487441	1.0787172
35	12	0.4	13	0.3783798	1.87134408	1.1049562
35	12	0.5	13	0.4321002	2.12023619	1.1311953
35	12	0.6	13	0.4772742	2.38155074	1.1574344
35	12	0	11	0	1	1
35	12	0.02	11	0.0397339	1.05018935	1.0053308
35	12	0.04	11	0.076431	1.10107888	1.0106617
35	12	0.06	11	0.1104264	1.15266862	1.0159925
35	12	0.08	11	0.1420081	1.20495858	1.0213234
35	12	0.1	11	0.1714242	1.25794874	1.0266542

35	12	0.2	11	0.2926766	1.53340277	1.0533085
35	12	0.3	11	0.3829715	1.82636212	1.0799627
35	12	0.4	11	0.4528226	2.13682679	1.1066169
35	12	0.5	11	0.5084669	2.46479679	1.1332712
35	12	0.6	11	0.5538386	2.81027212	1.1599254
35	12	0	7	0	1	1
35	12	0.02	7	0.0852602	1.11258381	1.0057129
35	12	0.04	7	0.157124	1.22847168	1.0114258
35	12	0.06	7	0.2185188	1.34766367	1.0171387
35	12	0.08	7	0.2715768	1.47015979	1.0228515
35	12	0.1	7	0.3178882	1.59596002	1.0285644
35	12	0.2	7	0.4824206	2.27452294	1.0571289
35	12	0.3	7	0.5830043	3.03568882	1.0856933
35	12	0.4	7	0.6508553	3.87945767	1.1142578
35	12	0.5	7	0.6997157	4.80582947	1.1428222
35	12	0.6	7	0.7365795	5.81480423	1.1713867
35	16	0	19	0	1	1
35	16	0.02	19	0.014694	1.02316059	1.0051232
35	16	0.04	19	0.0289624	1.04656354	1.0102465
35	16	0.06	19	0.0428235	1.07020886	1.0153697
35	16	0.08	19	0.0562944	1.09409656	1.020493
35	16	0.1	19	0.0693914	1.11822664	1.0256162
35	16	0.2	19	0.1297774	1.24251264	1.0512325
35	16	0.3	19	0.1828042	1.37285804	1.0768487
35	16	0.4	19	0.2297398	1.50926283	1.102465
35	16	0.5	19	0.2715768	1.65172702	1.1280812
35	16	0.6	19	0.3091031	1.80025059	1.1536975
35	16	0	15	0	1	1
35	16	0.02	15	0.0227712	1.0298743	1.0040374
35	16	0.04	15	0.0445284	1.06004795	1.0080749
35	16	0.06	15	0.0653378	1.09052092	1.0121123
35	16	0.08	15	0.0852602	1.12129322	1.0161497
35	16	0.1	15	0.104351	1.15236485	1.0201872
35	16	0.2	15	0.1889816	1.31221293	1.0403744
35	16	0.3	15	0.2589994	1.47954424	1.0605616
35	16	0.4	15	0.3178882	1.65435876	1.0807488
35	16	0.5	15	0.3681061	1.83665651	1.100936
35	16	0.6	15	0.4114369	2.02643749	1.1211232
35	16	0	13	0	1	1
35	16	0.02	13	0.029536	1.03697735	1.0040816

35	16	0.04	13	0.0573774	1.07434118	1.0081632
35	16	0.06	13	0.0836658	1.11209148	1.0122448
35	16	0.08	13	0.1085277	1.15022825	1.0163264
35	16	0.1	13	0.1320762	1.18875149	1.020408
35	16	0.2	13	0.2333344	1.38716481	1.040816
35	16	0.3	13	0.3134341	1.59523993	1.061224
35	16	0.4	13	0.3783798	1.81297687	1.081632
35	16	0.5	13	0.4321002	2.04037563	1.1020399
35	16	0.6	13	0.4772742	2.27743619	1.1224479
35	16	0	11	0	1	1
35	16	0.02	11	0.0397339	1.04823124	1.0041462
35	16	0.04	11	0.076431	1.09700704	1.0082923
35	16	0.06	11	0.1104264	1.14632745	1.0124385
35	16	0.08	11	0.1420081	1.19619246	1.0165847
35	16	0.1	11	0.1714242	1.24660208	1.0207309
35	16	0.2	11	0.2926766	1.50681926	1.0414618
35	16	0.3	11	0.3829715	1.78065157	1.0621927
35	16	0.4	11	0.4528226	2.06809903	1.0829236
35	16	0.5	11	0.5084669	2.36916162	1.1036545
35	16	0.6	11	0.5538386	2.68383936	1.1243854
35	16	0	7	0	1	1
35	16	0.02	7	0.0852602	1.10827776	1.0044433
35	16	0.04	7	0.157124	1.21912531	1.0088866
35	16	0.06	7	0.2185188	1.33254271	1.0133299
35	16	0.08	7	0.2715768	1.44852996	1.0177733
35	16	0.1	7	0.3178882	1.56708706	1.0222166
35	16	0.2	7	0.4824206	2.19842027	1.0444332
35	16	0.3	7	0.5830043	2.8939997	1.0666498
35	16	0.4	7	0.6508553	3.65382533	1.0888664
35	16	0.5	7	0.6997157	4.47789718	1.111083
35	16	0.6	7	0.7365795	5.36621524	1.1332996

APPENDIX 3.

NUMBER OF BRACING EFFECT ON STRESS MAGNIFICATION FACTOR

OF SINGLE SPAN BRIDGES

The data in this Appendix is specific to a certain L/R ratio based on the curvature limit in equation (15) developed by Khalafalla [2]. For each bridge span and width, L/R limit was calculated. Then, the V-Load method was applied for such bridges (i.e. of 3 different span values of 15, 25 and 35 m and different bridge widths of 8, 12 and 16 m) and the fixed L/R ratio for each span. In each specific bridge, the number of bracing spacing changes from 2 to 5 for span length of 15 m as depicted in Table A.3.1, from 2 to 7 for span length of 25 m as depicted in Table A.3.2 and from 2 to 9 for span length of 35 m as depicted in Table A.3.3. Along with Tables A.3.1 through A.3.3, Figures A.3.1 through A.3.3 present the results in graphical form. Using statistical package of curve fit, empirical equations were developed as shown in each of these figures. These equations formed the basis of equations 21 and 22 in this report.

Table A.3.1. Effect of number of bracing spacing on the stress magnification factor for single span bridges of 15 m span based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAGNIFICATION FACTOR, L=15			
N	**W = 8**	**W = 12**	**W= 16**
2	1.067	1.057	1.054
3	1.039	1.035	1.033
4	1.027	1.025	1.023
5	1.022	1.019	1.017

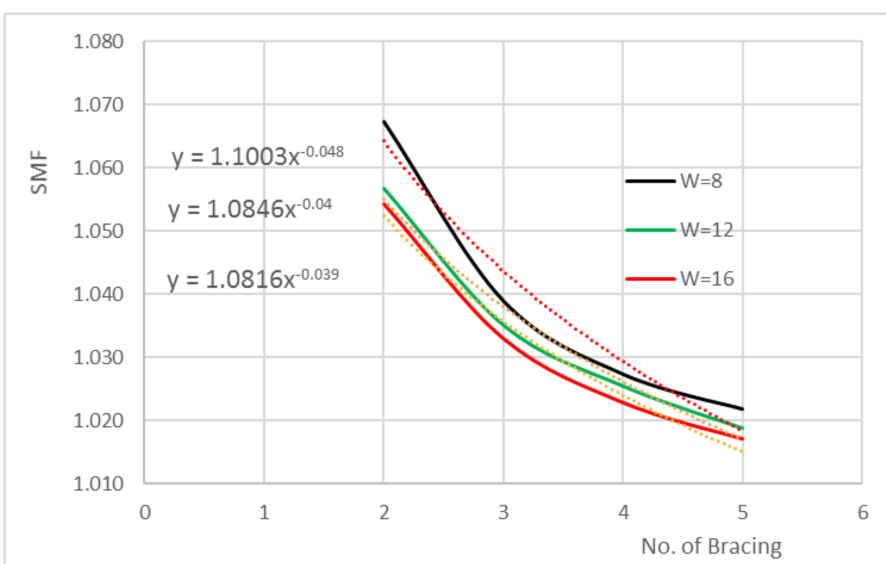


Figure A.3.1. Effect of number of bracing spacing on the stress magnification factor for 15 m single span bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

Table A.3.2. Effect of number of bracing spacing on the stress magnification factor for 25 m single span bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAGNIFICATION FACTOR, L= 25			
N	**W = 8**	**W = 12**	**W= 16**
2	1.091	1.085	1.081
3	1.058	1.048	1.049
4	1.047	1.040	1.037
5	1.032	1.025	1.025
7	1.022	1.018	1.015

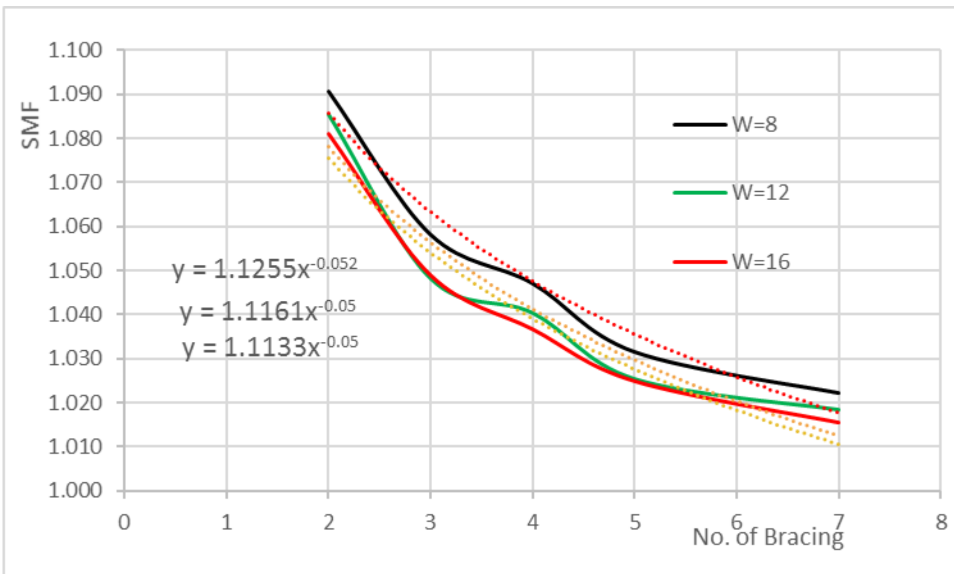


Figure A.3.2. Table A.3.1. Effect of number of bracing spacing on the stress magnification factor for 25 m single span bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

Table A.3.3. Effect of number of bracing spacing on the stress magnification factor for 35 m single span bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAGNIFICATION FACTOR, L=35			
N	**W = 8**	**W = 12**	**W= 16**
3	1.081	1.073	1.068
5	1.043	1.037	1.034
6	1.034	1.029	1.026
7	1.029	1.025	1.022
9	1.018	1.015	1.014

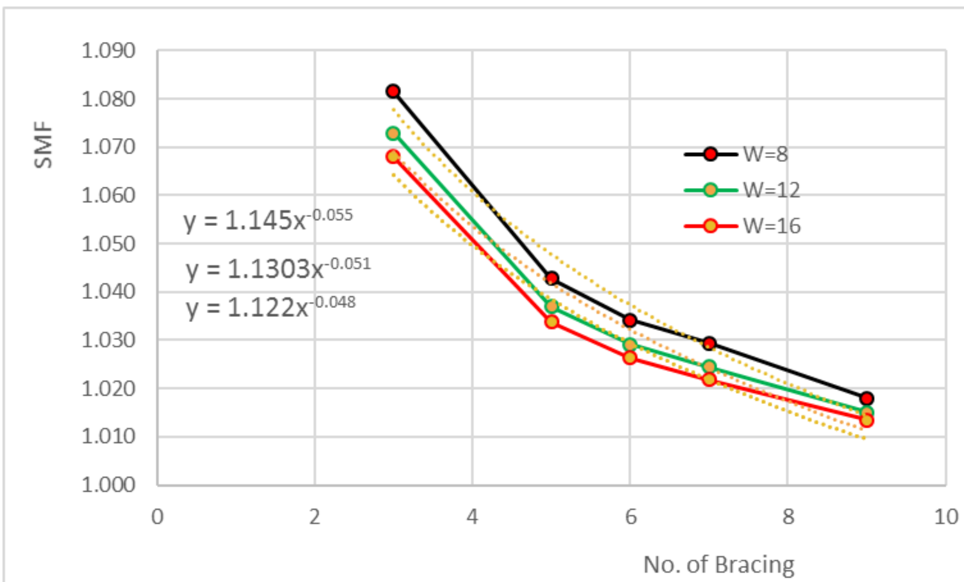


Figure A.3.3. Effect of number of bracing spacing on the stress magnification factor for 35 m single span bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

APPENDIX 4.

TWO SPAN BRIDGE, NUMBER OF BRACING EFFECT ON STRESS MAGNIFICATION FACTOR

The data in this Appendix is specific to a certain L/R ratio based on the curvature limit in equation (15) developed by Khalafalla [2]. For each bridge span and width, L/R limit was calculated. Then, the V-Load method was applied for such bridges (i.e. of 3 different span values of 15, 25 and 35 m and different bridge widths of 8, 12 and 16 m) and the fixed L/R ratio for each span. In each specific bridge, the number of bracing spacing changes from 2 to 5 for span length of 15 m as depicted in Table A.4.1, from 2 to 7 for span length of 25 m as depicted in Table A.4.2 and from 2 to 7 for span length of 35 m as depicted in Table A.4.3. Along with Tables A.4.1 through A.4.3, Figures A.4.1 through A.4.3 present the results in graphical form. Using statistical package of curve fit, empirical equations were developed as shown in each of these figures. These equations formed the basis of equations 21 and 22 in this report.

Table A.4.1. Effect of number of bracing spacing on the stress magnification factor for two-span (15-15 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAAGNIFICATION FACTOR						
L	W	L/R	N=2	N=3	N=4	N=5
15	8	0.014631	1.046318	1.027254	1.018844	1.014204
15	12	0.015229	1.046603	1.027105	1.018389	1.013601
15	16	0.015668	1.047026	1.034606	1.027165	1.01822

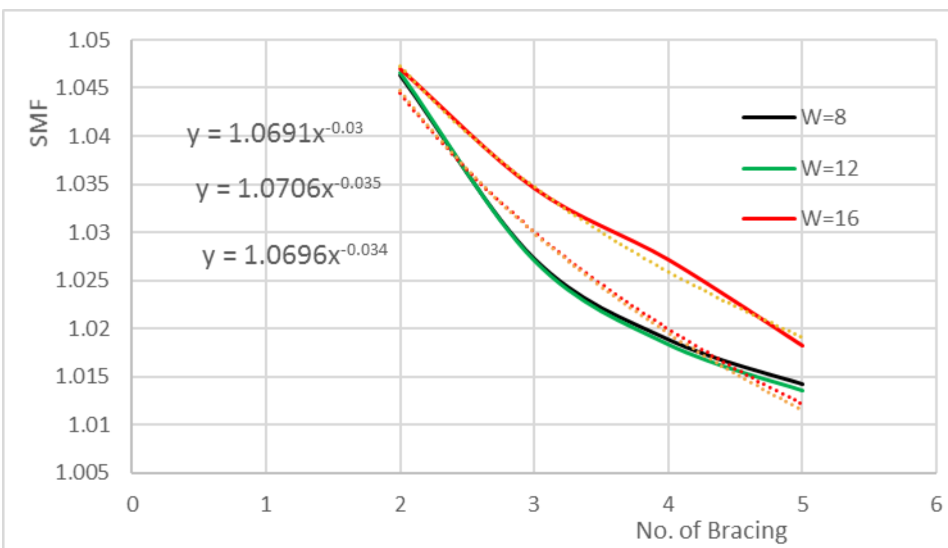


Figure A.4.1. Effect of number of bracing spacing on the stress magnification factor for two-span (15-15 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

Table A.4.2. Effect of number of bracing spacing on the stress magnification factor for two-span (25-25 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAAGNIFICATION FACTOR							
L	W	L/R	N=2	N=3	N=4	N=5	N=7
25	8	0.015196	1.088642	1.052352	1.035622	1.026709	1.019488
25	12	0.015816	1.088979	1.051913	1.034798	1.02564	1.017737
25	16	0.016272	1.089667	1.051936	1.034497	1.025144	1.016791

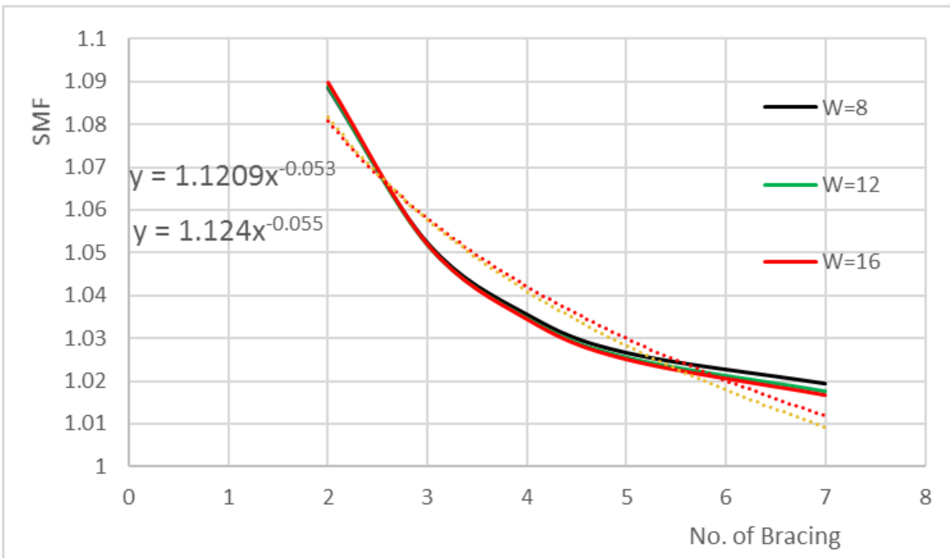


Figure A.4.2. Effect of number of bracing spacing on the stress magnification factor for two-span (25-25 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

Table A.4.3. Effect of number of bracing spacing on the stress magnification factor for two-span (35-35 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

STRESS MAAGNIFICATION FACTOR							
L	W	L/R	N=2	N=3	N=4	N=5	N=7
25	8	0.015196	1.088642	1.052352	1.035622	1.026709	1.019488
25	12	0.015816	1.088979	1.051913	1.034798	1.02564	1.017737
25	16	0.016272	1.089667	1.051936	1.034497	1.025144	1.016791

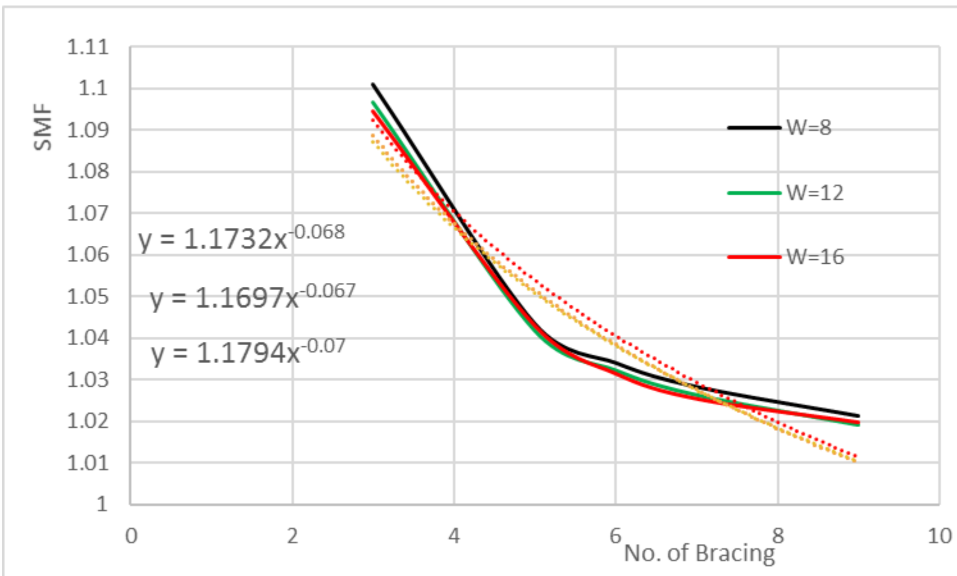


Figure A.4.3. Effect of number of bracing spacing on the stress magnification factor for two-span (35-35 m) bridges based on curvature limitation, L/R, obtained from Khalafalla's equations

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