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Reusability Evaluation Models In Reverse Logistics

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REUSABILITY EVALUATION MODELS IN REVERSE LOGISTICS

By

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DECLARATION

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ABSTRACT

This study focuses on the problem of reusability evaluation in reverse logistics. To deal with it, products are categorized into two types: well established products, and products with fast innovations. An innovative reliability based model (model 1) is suggested to evaluate reusability of returns for the first category.

For the second category, a fuzzy multiple participant-multiple criteria (MPMC) decision making model is presented, which is a modified combination of two previous researches: the disposal cause analysis matrix (Umeda et al., 2005), and the fuzzy analytical hierarchy process (AHP) method (van Laarhoven and Pedrycz, 1983).

To present the application of model 1, a green manufacturing system with an (s, Q) inventory control policy is simulated using Arena®. With the aid of it, the system is analyzed in two situations: with recovery operations, and without recovery operations to investigate the effects of both model 1 and recovery operations on the system parameters.

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ABBREVIATIONS

AFR	Annualized Failure Rate
AHP	Analytical Hierarchy Process
CDF	Cumulative Distribution Function
DCA	Disposal Cause Analysis
DM	Decision Maker
DV	Design Verification
EBO	Expected Backorder
EOL	End Of Life
EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
FMEA	Failure Mode and Effect Analysis
FTA	Fault Tree Analysis
ICT	Information and Communication Technologies
KKT	Krush-Kuhn-Tucker
K-S	Kolmogorov-Smirnov
LCA	Life Cycle Analysis
LCD	Life Cycle Design
LCOP	Life Cycle Options
MCDM	Multiple Criteria Decision Making
METRIC	Multi-Echelon Techniques for Recoverable Item Control
MPMC	Multiple Participant-Multiple Criteria
MPSC	Multiple Participant-Single Criteria
MTBF	Mean Time Between Failures

MTTF	Mean Time To Failure
OARA	Ontario Automotive Recyclers Association
OEM	Original Equipment Manufacturer
PDF	Probability Density Function
PG	Product Gain
PLCC	Product Life Cycle Cost
PVL	Product Value
QFD	Quality Function Deployment
SL	Safety Level
SPMC	Single Participant-Multiple Criteria
TFN	Triangular Fuzzy Number, usually shown by (l, m, u)
VIN	Vehicle Identity Number

NOMENCLATURE

$R\ (t)$	Reliability function of an item
R^*	Reliability threshold for deciding on reusability of returns
C_{Mx}	Maximum warranty cost per product
C_o	Average cost per repair
C_p	Product's price
t_o	Warranty period
$t_o^{\ c}$	Warranty period offered by the competitor
t	Product's life /time in service
$t_o^{\ *}$	Calculated warranty period based on the maximum warranty cost
$W(t_0)$	Expected number of renewals during warranty period
$f\,(t)$	Probability density function (pdf) of an item
$F(t)$	Cumulative density function (cdf) of an item
$h(t)$	Hazard rate
n_i	Number of recovered products sold in time period *i*
a_{ij}	Number of failures that happened in time period j to the items sold in period *i*
l	Maximum time in service
$a_{i.}$	Total number of failures among n_i recovered units
$a_{.j}$	Total number of failures in *j* periods
$a..$	Total number of failures among all sold recovered products
Level2	Level of inventory 2 (inventory for reusable items)
C_1	Total testing cost
C_2	Total recycling cost
C_3	Total holding cost for reusable items

C_4	Total restoring cost
C_5	Total holding cost for serviceable items
C_6	Total manufacturing cost
C_7	Total shortages cost
n_1	Average number of arrivals per time
n_2	Average number of recycled batches per time
n_3	Average number of recycled per time
n_4	Average number of reusable items per time
n_5	Average number of remanufactured batches per time
n_6	Average number of remanufactured items per time
n_7	Average number of refurbished batches per time
n_8	Average number of refurbished items per time
n_9	Average number of orders for remanufactured items per time
n_{10}	Average number of serviceable items per time
n_{11}	Average number of manufactured batches per time
n_{12}	Average number of manufactured items per time
n_{13}	Average number of orders for manufactured items per time
n_{14}	Average number of backorders per time
C_{Vt}	Testing Variable cost
C_{Fp}	Recycling fixed cost
C_{Vp}	Recycling variable cost
C_{hR}	Holding cost for reusable items
C_{hM}	Holding cost for serviceable items
L_R	Remanufacturing lead time
L_M	Manufacturing lead time

C_{FR}	Fixed remanufacturing cost
C_{VR}	Variable remanufacturing cost
C_{Ff}	Fixed refurbishing cost
C_{Vf}	Variable refurbishing cost
C_{SR}	Setup cost for remanufacturing operations
C_{FM}	Fixed manufacturing cost
C_{VM}	Variable manufacturing cost
C_{SM}	Setup cost for manufacturing operations
C_b	Fixed backorder cost
d_j	Disposal cause
f_j	Functions of a component/subassembly
C_k	Component k
r_j	importance of cause j for function j
M_{ik}	Cause-component matrix
M_k	Total importance of component k
I_k	Relative importance of each component
VI_k	Value importance of each component
PI_k	Physical importance of each component
W_{ij}	Disposal cause-function matrix
W_{jk}	Function-component matrix
PIP	Physical importance of the product
VIP	Value importance of the product
A_i	Attribute i
X_j	Alternative j
P_{ij}	Number of DMs in each cell of comparison matrix
W_i	Weight of attribute i
μ_M	Fuzzy membership function

Chapter 1

Introduction

This chapter provides a brief review of the fundamental subjects of this thesis, such as reliability engineering, warranty, multiple criteria decision making, reverse logistics, and simulation. The problem definition, objectives, and structure of this thesis are discussed here too.

1.1 Reliability engineering and reusing old products

Reliability has been defined as the probability that an item (component, device, or system) performs its function adequately for certain period of time under specified design conditions. *Reliability engineering* is the study of the performance of a system or a component to measure its reliability over time. It is the regulation of ensuring that a product will be reliable enough when used in a specified way.

The main function of reliability engineering is to define reliability requirements for a product, and to develop a proper reliability program. Reliability engineers are completely familiar with statistics, probability theory, and reliability theory. They conduct analyses and tasks, such as reliability prediction, reliability planning and specifications, reliability testing, accelerated life testing, allocation, failure mode and effect analysis (FMEA), stress screening, warranty analysis, and fault tree analysis (FTA) to ensure the product will meet the defined requirements. Figure 1.1 shows a sample reliability program developed by an auto part producer to meet a reliability target and customer satisfaction for a new product, e.g. an A/C Compressor (Yang, 2007). A reliability program developed for a remanufactured or refurbished product would be very similar to the one shown in Figure 1.1 and with the focus on the second life of the product.

Reliability engineers try to maximize the reliability of a product and minimize the effects of the item's failures. Generally, there are three steps to achieve this goal. First step, which is the most important one, is building maximum reliability into an item in the product's design step (in the same way, design for reuse is very important). The second step is to minimize variations of production processes. The third step is to establish a proper maintenance plan for the product. Many engineers, such as system engineers, electronics engineers, mechanical engineers, and quality engineers use the same system of methods and the tools used by reliability engineers.

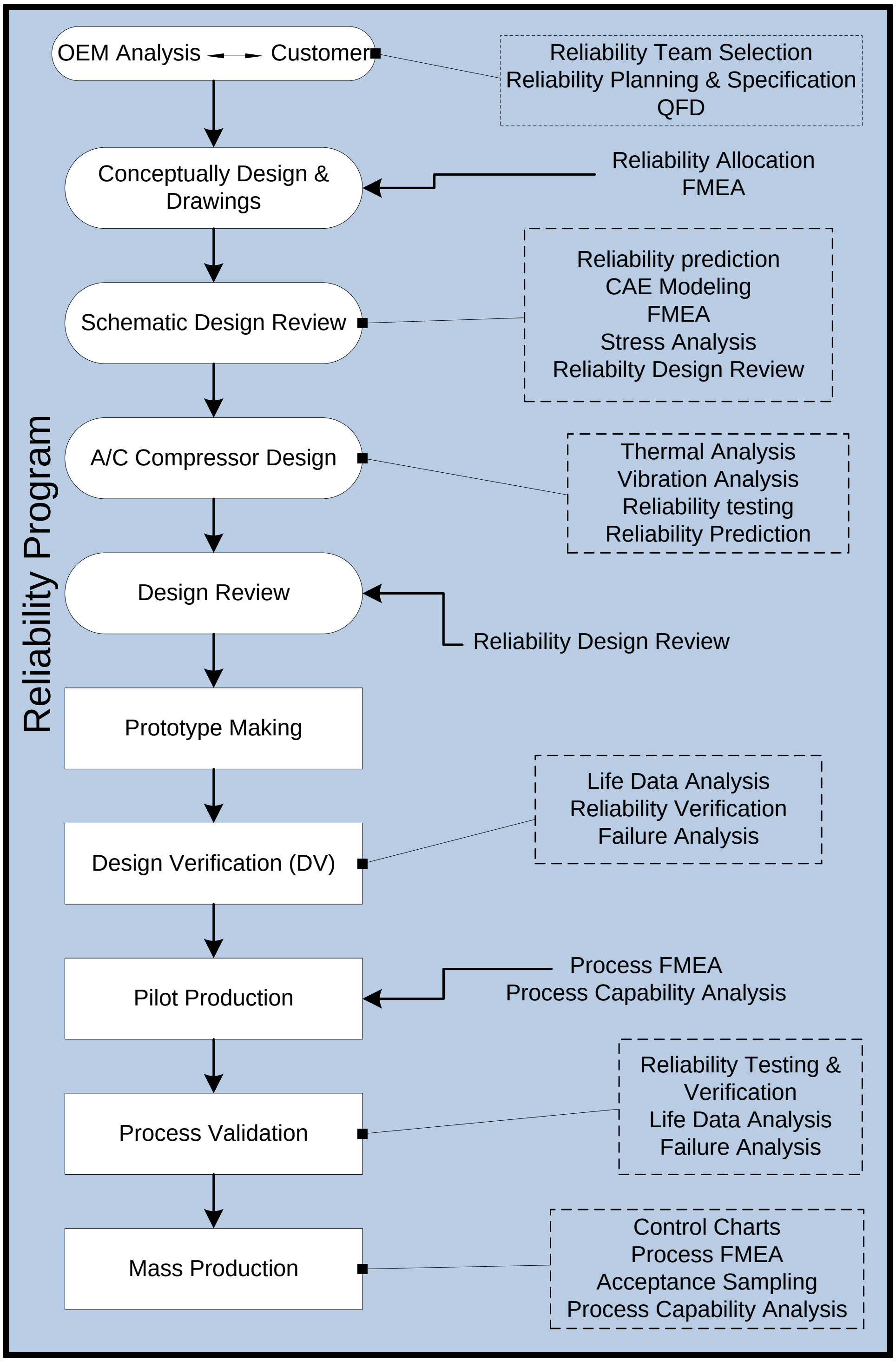


Figure 1.1: Reliability program for an automotive part

Reliability of used products is a concern that must be discussed prior to choosing a reuse alternative to avoid extra costs associated with pre-recovery operations, such as disassembling, cleaning, and testing as well as additional failure costs and poor quality of the recovered item.

Reuse means to use an item more than once. Reusing old products is not a new practice. Since ancient times, materials from used products have been reused. Metal scrap brokers and waste paper recyclers are examples that have been existed for a long time. In these cases due to the profitability recovery options are more attractive than disposal.

Generally, there are two types of reusing policies: when the item is reused for the same function and when it is reused for a new function. In contrast, recycling is converting the old item into raw materials to make new items. Reuse helps in saving time, money, energy, and resources. In a broader economic sense, reuse offers quality products to people and organizations at a better price and can create jobs and business activities that contribute to the economy.

Moreover, reusing old products is the most environmentally conscious strategy in green manufacturing systems, which in recent years has gained a lot of interest from both environmental and economic perspectives.

Recovery options include reusing options plus recycling. Recovery options are as follows:

- *Repair:* bringing back the product to a working condition
- *Refurbishing:* restoring a product to an “as-good-as-new” situation by minor operations.
- *Remanufacturing:* restoring a product to an “as-good-as-new” situation by major operations
- *Cannibalization:* restoring the subassemblies of a product
- *Recycling:* recovering the product’s materials

Some practical/in practice examples of the applications of repair, refurbishing, remanufacturing, or cannibalization for used products to prolong the useful life of them and delay the final disposal or recycling phase are:

- Packaging appliances	- Electronic devices	- Auto spare parts
- Aerospace spare parts	- Furniture	- Electrical home appliances

1.2. Warranty

Warranty is a seller's assurance to the buyer that the goods or property is as presented and, if not, will be replaced or repaired (Webster's College Dictionary, 1997). Traditionally, the *"specified design conditions"* in the definition of the reliability in section 1.1 refers to what the reliability testing methods determine. However, field data provide better information about the lifetime distribution of an item. For example, the specified conditions resulting from laboratory testing of an auto brake pad may not include the fact that some users may drive their cars with the hand brake on. Therefore, warranty data can reflect the real usage of the product better. This closely depends on the data collection system and proper analysis.

1.2.1 Warranty policies

A warranty policy consists of three elements: period of warranty (e.g. calendar time, or usage), failure coverage, and seller-buyer financial responsibilities/conditions for the provided warranty service. The most common warranty policies are as follows (Rai, 2009):

1. *Free replacement policy:* In this policy if a product fails within the warranty period and failure coverage, it is repaired/replaced by the seller free of charge to the buyer.

2. *Pro-rata replacement policy:* If a product fails within the warranty period, it is repaired or replaced by the seller at a fraction of repair/replacement cost to buyer. The amount of money payable by the buyer would be determined based on the usage of the product. The longer the product has been used, the more the buyer has to pay.

3. *Combination free and pro-rata replacement policy:* This policy specifies two warranty periods for the product, say t_1 & t_2, $(t_1 < t_2)$. If a product fails before t_1, the repair/replacement would be free of charge to the buyer. If the product fails in the interval $[t_1, t_2]$, the repair/replacement would be done by the seller at a fraction of the cost to the buyer.

All policies mentioned above are non-renewing policies, which mean that neither repair nor replacement of a failed product renews the warranty period. In other words, the repaired/replaced product would be covered for the remaining length of the original warranty period only. Usually the non-renewing warranty policies would be considered for repairable items, and the renewing warranty policies cover mostly non-repairable items.

Between the three elements of a warranty policy, the warranty period has the most effect on the warranty costs. Due to this fact, manufacturers usually decide on an optimal warranty period by considering several factors, such as item's reliability, cost per repair, sales volume, product's price, legal requirements, and market competition.

One remaining point to mention is that warranty period can be offered in two dimensions too (e.g. time and usage). In this case, when either of the dimensions reaches its limit, the warranty expires. The warranty offered by tire makers is a good example of a two-dimensional warranty with a *combination free and pro-rata policy* (third type of warranty policy), in which the arbitrary dimensions could be 24 months after the sales date and the first 2/3 inch tire wear out.

1.2.2 Warranty data analysis

When failures of products that are sold under a warranty are claimed, information related to the failed product is disclosed to the producer. That information is called warranty data. Generally, warranty data contain information, such as *product data* (e.g. serial number, production date, sales date), *failure data* (e.g. accumulated use, failed part numbers, causes), and *repair data*, which refers to information, such as labor time, cost, and date of repair (Yang, 2007).

Because warranty databases provide information on a product's performance in real field conditions, many producers have decided to collect and analyze warranty data as a way to improve the reliability of their products. Analysis using warranty data generally supports three levels of decision making in a company: Strategic, tactical, and operational (Rai, 2009). For example, increasing warranty period may impact the company for many years.

Moreover, warranty data can be used in reliability estimation, modeling, analysis, and prediction. They can also be used for detecting critical failure modes, or estimating the number of warranty claims. The main advantage of warranty data is that they can be routinely collected and updated from costumers' claims during the warranty period without any additional cost. Although warranty data are more realistic than testing data there are some limitations associated with warranty data that should be considered.

Warranty data are criticized as being "dirty" due to their limitations, such as: use conditions are not differentiable from the warranty data; different lots that are affected differently by the remanufacturing process are mixed and treated equally; failure times are not accurate due to the reporting delay caused by costumers, and some warranty claims are not real failures due to the fact that the producer tries to satisfy picky customers. Also, the accurate numbers of the items under the warranty, which have been salvaged, are not known.

Regardless of the above issues, there are four popular methods for collecting warranty data that are in practice. Those common ways for collecting sales and returns information are:

1. *Nevada Chart approach*: information about number of items produced and/or shipped in a certain time period, and the information about the number of returns for that production lot in the subsequent time periods should be available.

2. *Time-to-Failure approach*: times-to-failure for returned items should be collected.

3. *Dates of Failure approach*: exact dates of shipments and warranty claims should be recorded.

4. *Usage approach*: exact dates of production/shipment and the usage of returned items should be known.

Implementing any of the above approaches for warranty data collection depends on the producer’s policy, budget, and type of the product. *Weibell++7* software can effectively be used to analyze warranty data that are collected by using any of these approaches.

1.2.3 Warranty and remanufactured products

In the subject of offering warranty for remanufactured products, the key point is that the role of warranty in selling remanufactured products is even more important than its role in selling new products, because generally customers think that used products have a lower quality than new ones. Offering warranty for remanufactured products plays an important role in changing their idea. Obviously, offering warranty is costly for the seller, but usually those costs can be optimized by applying statistical methods.

1.3 Multiple criteria decision making (MCDM)

The concept of the MCDM is closely related to this study, and is briefly reviewed in this thesis. Because the problem of deciding on recovery options for returns, which includes reusability evaluation, is indeed a multiple criteria decision making (MCDM) problem.

Figure 1.2 shows the overall steps involved **i**n an MCDM method (Hobbs and Meier, 2000), which can be performed in different combinations depending on the problem and the analyst. One key point is that an MCDM problem can be generalized to a multiple participant-multiple criteria (MPMC) problem when there is more than one decision maker.

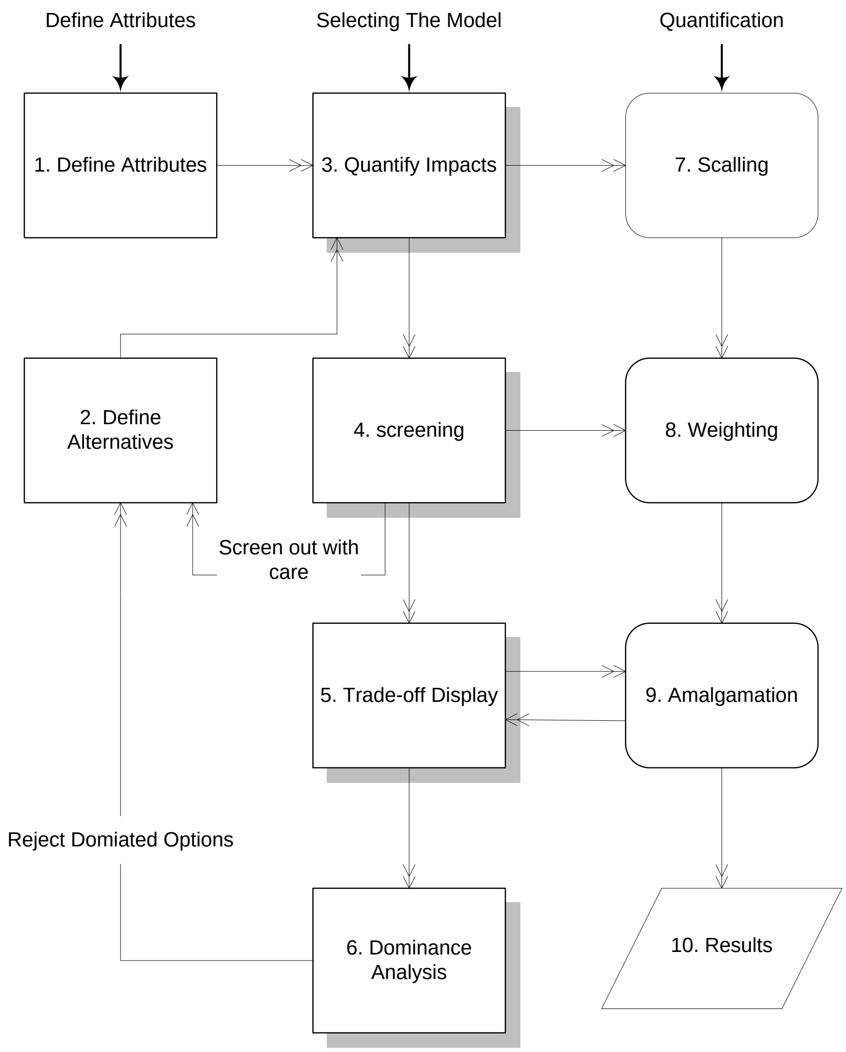


Figure 1.2: Algorithm of an MCDM method

By applying MCDM methods, a decision maker can decide on alternatives with multiple criteria in an easier, systematically, and more efficient way. MCDM methods are very effective, as they are designed based on the following fact: "Psychological research has shown that, when alternatives have many attributes (l0 or more are common in energy planning), decision makers are inconsistent in their subjective evaluations of the options. Often the mind will focus on two or three attributes, ignoring the others, or it will flit inconsistently among the attributes" (Shepard, 1964; Slovic and Lichtenstein, 1971).

An MCDM approach helps an analyst who deals with a multiple criteria problem by providing a way to structure the problem systematically, a frame work for solving the problem, tools to display tradeoffs, necessary tools to perform dominance analysis, and a method to quantify the value of judgments for him/her. Finally, in a systematic way, it makes selection among alternatives possible for the analyst. Utilizing it involves the following steps:

A) Problem definition steps:

1. Attributes selection & definition, A_i
2. Alternatives/options definition, O_j
3. Quantification of the levels of A_{ij} & A_{ij} = estimated level of attribute i for alternative j

B) Trade-off analysis steps

4. Construction of trade-off curves
5. Dominance analysis

C) Evaluation steps

6. Preliminary screening of the alternatives
7. Scaling of the attributes
8. Selection of weights for each attribute
9. Determination and application of an amalgamation rule
10. Resolution of differences between methods and among stakeholders

Some important points regarding the above algorithm can be listed as follows:

- It is suggested to define attributes before defining alternative, as thinking about attributes would result in finding new options ("value-focused thinking").

- When there is uncertainty associated with an attribute, risk can be considered as a attribute

- Generally, attributes can be quantified by four approaches: indirect valuation methods, methods that use mitigation scenarios, directly quantifying attributes as a single attribute value function, which is the most practiced approach (Proposed by Keeney and Nair, 1977), and methods applying an appropriate proxy attribute.

- Trade-off analysis is the first analytical step in MCDM, in which the decision maker (DM) graphically analyzes the situation; it can assist in defining the strategies; it can serve as a screening tool, and it highlights the trade-offs for DM.

- Different people react differently to trade-off displays. Hence, using more than one approach for the trade-off display is strongly recommended,

- The main aim of screening is to accomplish two things: eliminating options that are unlikely to be chosen, and to provide a range of options for DMs.

- Exclusionary screening is the most popular screening method, and trade-off curves, Cartesian plots, and value path graphs are examples of the tools used in the screening step.

- Value scaling is the creation of a value function, $V_{\mathrm{i}}(A_{\mathrm{ij}})$, which represents the worth of the alternative *j* for the attribute *i*; it can be a linear function, exponential function, etc.

- Usually a value of one is assigned to the best outcome, and a value of zero to the worst outcome ($0 \leq V_{\mathrm{i}}(A_{\mathrm{ij}}) \leq 1$).
- The additive value function method of amalgamation is the amalgamation method in widest use, in which alternatives would be ranked by the score shown in Equation (1-1).

  $$\textit{Maximize } TV_j = \sum w_i \times V_i(A_{ij}) \quad (1\text{-}1)$$

  Where TV_j = Total value of alternative *j* and w_i = weight of attribute *i*

- Weights represent the importance of attributes from the DM's point of view, and the analyst by applying a proper method for the weighting process tries to avoid bias or skew preferences.

- Some weighting methods are: equal weights method, observer derived weights, direct weighting, analytical hierarchy process (AHP; Saaty, 1980), swing weights, indifference trade off weights, and gamble method.

- Analytical Hierarchy Process (AHP), which is the popular version of ratio questioning, is widely used, and it consists of these steps: collecting every possible pair-wise comparisons of attributes/criteria for the DM (importance is measured on an integer-valued 1-9 scale), tabulating all DMs' stated ratios in the matrix of importance (R = [r_{ij}]), and obtaining the vector of weights (W) by solving Equation (1-2):

  $$R \times W = \lambda \times W \qquad (1\text{-}2)$$

  Where, R = matrix of ratios = [r_{ij}], W = vector of weights, and λ = eigenvalue of the matrix R, Obtaining a closer value of λ to the number of attributes indicates more consistency in the assessment. Table 1.1 shows the interpretation of each number

Table 1.1: Interpretation of entities in a pair-wise comparison

Value of r_{ij}	**Interpretation**
1	If the two attributes are judged to be equally important
3	If attribute 1 is judged to be slightly more important than attribute 2
5	If attribute 1 is judged to be moderately more important than attribute 2
7	If attribute 1 is judged to be strongly more important than attribute 2
9	If attribute 1 is judged to be extremely more important than attribute 2
2,4,6,8	If intermediate values between two adjacent judgments are needed

- A suggested estimation of AHP consists of these steps: Adding up the values in each column of matrix R, normalizing the matrix by dividing each element by corresponding value obtained in the last step, and computing the average of the elements in each row of the normalized matrix.

- To simplify calculations in AHP when the number of attributes is large, one can group the attributes; then the vector of weights can be calculated using Equation (1-3).

  $$W = W_{in-gr} \times W_{gr} \qquad (1\text{-}3)$$

Where, $W_{in\text{-}gr}$ = conditional weight assessed within its own group, and W_{gr} = weight of the group itself. Suggested approximate method for AHP can be applied for calculating both types of weights.

1.4 Reverse logistics

Traditionally, supply chains were dealing only with the one-way product flows, which are the flow from manufacturers to customers, but reverse logistics deals with flows in the opposite direction. Examples of such two-way streams of products are remanufactured auto spare parts, EOL computer devices, reusable bottles, and reusable printer cartridges. Some definitions of Reverse Logistics are as follows, which shows how the definition of it has changed over time (Dekker, 2004):

i. "going the wrong way" (Stock, 1981)

ii. "…the movement of goods from a consumer towards a producer in a channel of distribution" (Pohlen and Farris, 1992)

iii. "Reverse Logistics is a broad term referring to the logistics management and disposing of hazardous or non-hazardous waste from packaging and products. It includes reverse distribution…which cause goods and information to flow in the opposite direction of normal logistics activities" (Kopicky et al., 1993)

iv. " The process of planning, implementing and controlling backward flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal." (The European Working Group on Reverse Logistics, REVLOG, 1998)

In addition, a complete definition of reverse logistics (Wikipedia, the free encyclopedia) is: "reverse logistics stands for all operations related to the reuse of products and materials. It is the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. More precisely, reverse logistics is the process of moving goods from their typical final destination for the purpose of capturing value, or proper disposal. Remanufacturing and refurbishing activities also may be included in the definition of reverse logistics. The reverse logistics process includes the management and the sale of surplus as well as returned equipment and machines from the hardware leasing business. Normally, logistics deal with events that bring the product towards the customer. In the case of reverse, the resource goes at least one step back in the supply chain. For instance, goods move from the customer to the distributor or to the manufacturer".

Analyzing reverse logistics utilizing the "Five Ws" (and one H) concept can effectively help to get a better understanding of *reverse logistics*; some results of applying the "Five Ws" (and one H) concept, which is completing a check list consisting questions forming by putting *who, what, when, where, why, and how* in front of every important thing in the subject under study, would be as follows:

- What are returned items?

  Helps to know what the attractive characteristics of the returns are

- Why items are returned?

  Helps to understand driving forces behind the manufacturers in the *reverse logistics* (e.g. economics, and legislations, motivation, or culture)

- Who are involved in reverse logistics?

  Helps in identifying reverse chain actors.

- How reverse logistics work in the real world?

  Helps to understand recovery options or value added operations in the reverse chain.

In summary, reverse logistics includes: production planning, managing the returns, collection issues, distribution issues, coordination of supply chains with returns, and inventory control for joint manufacturing and remanufacturing systems.

1.5 Inventory control systems in the reverse logistics

Although inventory control is not within the scope of this thesis, it is briefly discussed here due to its importance for the reverse logistics. Also, Section 3.3 introduces inventory control system as the main application of the proposed models, and chapter 5 shows the models application in an inventory control system with returns by a simulation model built with *Arena®*.

Traditionally, inventory refers to raw inputs, work-in-process, and finished products, and the goal of inventory control is to keep their level at the lowest possible cost. In a closed-loop system, there are two inventories: *serviceable products inventory*, and *reusable products* *inventory*. Serviceable inventory refers to inventory of finished products, and reusable inventory refers to inventory of returned used products.

The process of control in a manufacturing system with returns or a closed-loop system is more complex than the traditional one due to the existence of the additional uncertain recovery operations such as return arrivals, testing, disassembling, and remanufacturing that need to be controlled. Usually this type of inventory system is optimized under different policies. One of the several objectives of an inventory control system with returns is to minimize stock shortage costs and to find an optimum inventory level for reusable products to start the recovery process.

1.6 Simulation

Jerry Banks et al. (2005) define simulation as:

"the imitation of the operation of a real-world process or system over time. Whether done by hand or on a computer, simulation involves the generation of an artificial history of a system and the observation of that artificial history to draw inferences concerning the operating characteristics of the real system."

Kelton et al. (2004) define simulation as:

"a broad collection of methods and applications to mimic the behavior of real systems, usually on a computer with appropriate software."

1.6.1 Different kinds of simulation

According to Kelton et al. (2004), simulation models can be classified as follows:

- Static vs. Dynamic: Time is a concern in dynamic models, but is not in static models.

- Continuous vs. Discrete: the state of the system is continuously changing in a continuous model, while in a discrete model the state of the system changes only at discrete points of time.

- Deterministic vs. Stochastic: when all inputs are known and deterministic, the model is a deterministic model, but if there are uncertainties associated with some inputs, we are dealing with a stochastic model. Probability distributions will be applied to picture the uncertainties in the model.

Nowadays, there are many simulation software packages available, which are designed for different needs. Some of them are as follows:

Simulink from the MathWorks, *Arena*- developed by Rockwell Automation, *Goldsim*-embedded in a Monte Carlo framework, and *SIMUL8* - Software for discrete event or process based simulation.

1.6.2 Simulation steps

Generally, making a simulation model of a system includes following steps:

1. Setting the objective(s) and Justification of simulation application

2. Studying the system: System behavior and its characteristics

3. Input analysis: Data collection and fitting probability distribution on them

4. Building the model: Choosing the best panels to model the system closely (using Arena)

5. Model validation or verification: compare the simulation results with results of similar situation or verify the simulation model results or behavior with what is expected

6. Output analysis: truncation if analysis calls and investigating the effect of model variables on each other and on objective

1.7 Problem definition

In this thesis, the focus is on reusability evaluation, which is one of the aspects of reverse logistics as highlighted in Figure 1.3. The figure also shows some other elements of the broad concept of the reverse logistics. This Figure also presents which elements of reverse logistics are closely related with the focus of this thesis. In addition, the important role of information and communication technologies (ICT) is highlighted by showing it as a central element.

From one point of view, reusability evaluation deals with the problem of finding a reliability threshold to help in deciding which recovery option to choose for a used product. On the other hand, one point to take in consideration is that the problem of reusability evaluation is not limited to finding a reliability threshold, and some products are discarded due to other reasons than failures. For example, some customers want to have the end of market products, and whenever a newer model of a product comes to the market, they buy it and discard the older model while it still works.

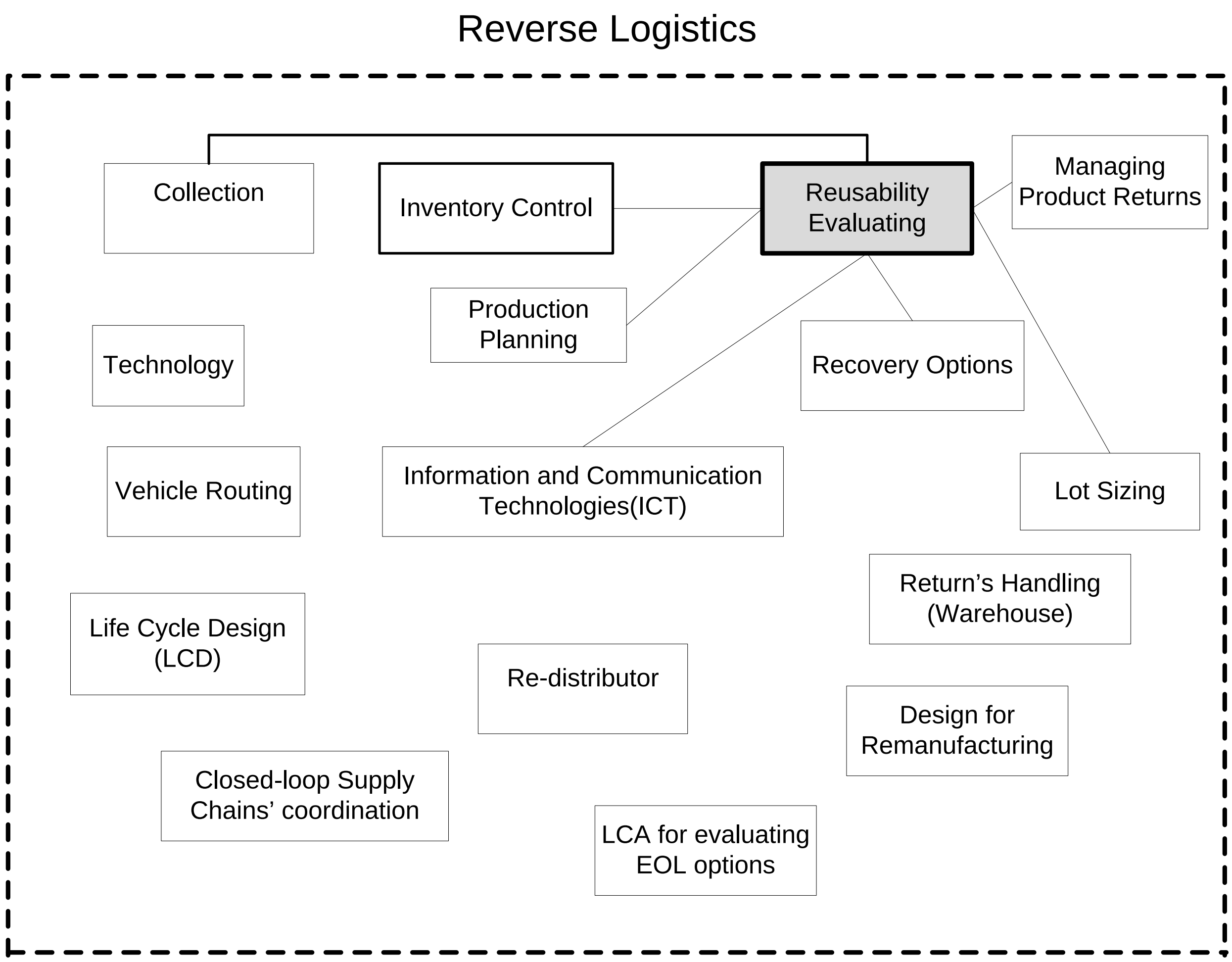


Figure 1.3: Some elements of the reverse logistics

Therefore, the problem considered in this study has two parts: reusability evaluation for products for which reliability theory can be applied to obtain a solution (well established products), and reusability evaluation for products for which reliability theory is not applicable in solving the problem (products with fast innovations).

1.8 Motivation of the research and its objectives

As it has been proved that reusing old products is the most environmentally conscious strategy in green manufacturing, the motivation for this study comes from the fact that reviewing recent research indicated that the subject of "evaluating the reusability of returns" has gained less attention than other aspects of the management of returns.

Also, the importance of the subject of saving our planet from being polluted by landfills, which many researchers are working on, and the fact that Canada is actively getting involved in research related to environment issues and effectively is implementing the results makes this study as a focused responsibility.

The objectives of this study are as follows:

- To develop a model for evaluating the reusability of used products for a joint manufacturing and remanufacturing system, with the focus on well established products.

- To develop a model to help in deciding on selection of the best recovery option for products with fast innovations.

- To present the application of a reusability evaluation model and their potential benefits in a manufacturing system with returns by a simulation model, this can be applied for analyzing the system under different scenarios.

1.9 Shortcomings of previous research

Studies related to the problem of reusability evaluation for product recovery are very limited, and a general solution is not yet proposed. Most of the previous research is confined to repair or ageing modeling. Finding a reliability based threshold to help in deciding on the selection of the best recovery alternative has not gained that much attention.

In addition, most of the previous studies assumed that the product is not usable due to a functionality reason and that it has been thrown away because it is not in a proper working condition. The fact is that the other group of products that are not being used because of other reasons than failures should be considered too. Some examples of the second group of products that are being thrown away while still are in working condition are cellular handsets, digital cameras, electronic games, and TVs.

1.10 Thesis structure

This thesis reports two models for evaluating the reusability of old products: A reliability based model, and a fuzzy MPMC model. Also, a simulation model to present the application of the first model in reverse logistics is presented. The structure of the thesis is as follows:

In chapter 1 a brief background is provided, and chapter 2 is assigned to a literature review on the subjects related to those discussed in chapter 1. In chapter 3 a reliability based method for evaluating the reusability of used products along with an illustration example is presented, which is applicable only for well established products. Chapter 4 presents the second model to help the producer in deciding on recovery options for used products, which is applicable for products with fast innovations and model presented in chapter 3 is not providing a solution for them. The proposed fuzzy MPMC model in chapter 4 is illustrated by a numerical example. In chapter 5, a simulation model of a manufacturing system with returns, which has an (s, Q) inventory control system is presented. The simulation model is used to study the system under two scenarios: the system with recovery operations, and the system without recovery operations. Chapter 6 is assigned to conclusions and future work.

This chapter discussed the basic background of this thesis; in chapter 2 a review of recent research that are related to the above subjects will be presented. It also shows that what types of topics are searched to find an answer to this study's problem.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, a review of recent research on the following subjects is summarized:

Reverse logistics, reliability based models for reuse/repair, warranty and recovery, multiple criteria decision making models, fuzzy AHP, inventory control models with returns, and simulation models of systems with recovery.

The most recent research (e.g. 2001-2009 for studies with the focus on reliability) are chosen to review with the aim of getting familiar with updated information and also to be aware of what are done by previous researchers, as usually previous research is mentioned in any new research.

2.2 Reverse logistics and recovery options

In this section, the framework of reverse logistics and the importance of recovery options are investigated by reviewing relevant literature:

Fleischmann et al. (2000) investigate the structure of suitable logistics networks to deal with the opposite flows of finished products towards the producers. They categorize general characteristics of product recovery networks for different industries.

Ferguson et al. (2001) explain the issues in the reverse logistics with the focus on information requirements. They discuss how distribution ways can be developed and how important the role of information systems is in the reverse logistics. All these issues are illustrated through a study of end-of-life vehicles (ELVs).

Beullens (2004) discuss the importance of the economic aspects of product recovery. He describes recent frameworks, models, and approaches for product recovery. He explores the issues related to the strategic objectives of the firms as well as the issues related to implementing recovery processes.

Patel et al. (2006) compare various problems, frameworks, models, and applications of the reverse logistics suggested by other researchers. More specifically, they provide a review and methodology classification for inventory models in the reverse logistics, and they suggest some solutions for selecting practical inventory models.

King et al. (2006) discuss two European Union policies along with four alternatives to deal with the waste landfill problem. The four strategies are: repairing, reconditioning, remanufacturing or recycling. They show how remanufacturing is the best alternative to choose. They argue that from thermodynamic/energy point of view, recycling option is not the best solution compared to remanufacturing.

2.3 Reliability based models for reuse and repair

In this section the application of reliability in reuse and repair is investigated by reviewing relevant literature on the subject.

Kijima et al. (1988) introduce the virtual age of a system, and a stochastic repairable model is built. The assumption is that a maintenance plan with minimum repair is developed for the system, in which a scheduled replacement brings back the system to its operating condition just before it fails. The effect of that maintenance plan is studied numerically assuming the g-renewal function for failures, and expected cost per unit time is calculated. The g-renewal function, H (t) is an element of the model's cost function, and h (t) is called a g-renewal density, which can be estimated by a *Voltera* integral equation.

Murayama et al. (2001) present an application of the reliability for reusing without repair by considering two aspects:

1. Reliability model for material-flow simulation, which by using Petri nets simulates the material flow. Two main reliability-parameters inputs are: time to failure and material quality. The first one is used to simulating of usage and end-of-life phases, and second one is considered as a base for deciding on reusability of the old item.
2. Reliability model for production management, in which a proposed production management method deals with the issues related to timings and quantities of returns. A reliability based prediction method helps in production planning.

In addition, the role of reliability in remanufacturing is discussed by them considering the following:

i. Reliability modeling for remanufacture, which describes product's repair during remanufacturing or maintenance as it is leaving the system in neither "same-as-good" situation nor "same-as-old" situation.
ii. Analyses of remanufacturer waste stream, which provides information about how top product sectors can be classified by the number of remanufacturers in each sector (e.g. automotive aftermarket parts as first sector and electrical parts as the second sector). Also, provides some references to obtain other critical information (e.g. root causes tables), which are helpful in remanufacturing design step.

iii. Modified Failure Modes and Effects Analysis (FMEA) for remanufacture, which helps a remanufacturer in detecting and repairing failed items. The application of modified FMEA method for an automotive reproducer is presented.

Murayama et al. (2004) built a reliability based model using the item's use time for evaluating the reusability of an item. The model is a function of the product's use time assuming that the product has reached its useful life, but some components in it are still usable. Their interesting claim resulting from graphs obtained from the model by choosing different values for parameters, such as the shape parameter of a Weibull distribution, is that "the reusability of a component does not always decrease as the use time is lengthened". (Murayama, 2004)

Kara et al. (2004) present a literature review on proposed methods for evaluating reliability parameters, such as lifetime, mean time to failure (MTTF), remaining life, and mean time between failures (MTBF) by reviewing 91 papers. They come to following conclusions: reusing is the most efficient and most economical end-of-life alternative; Reusing is feasible and does not reduce the item's quality. The potential of reusing can be measured and might require disassembly of the returns before testing; the product's history can be recorded using cheap electronic devices. Collected operating data can be used in reliability estimation. Weibull analysis is a good tool in forecasting the remaining life of items. Developing qualitative and quantitative decision tools for applying recovery and item take-back is strongly recommended. The current approaches in machine condition monitoring can be used for determining the reuse potential, and developing reliability methods for used products is strongly recommended.

Anityasari et al. (2005) propose a model with the aim of finding an indicator measuring the potential reusability of the product to provide a practical method. They present an application of findings in the reusing main subassemblies of a TV. They introduce three new parameters to apply in their model: Product Gain *(PG),* Product Value *(PVL),* and Product Life Cycle Cost *(PLCC),* which are related to each other *(PG = PVL - PLCC). PG* represents the financial outcome from the sales of the product. *PVL* represents the quality status of the product, and *PLCC* represents all costs that occurred during the product's life cycle. Using simulation with @RISK Software Version 4 and Excel, a model is built using probability distributions known from their experiments and assuming some random values for parameters, such as the distance from the collection spot, disassembly time, length of first life, and cleaning time. The results of statistical analysis of the outputs provide a base for decision makers to decide on which recovery option to choose. The decision making is based on the value of *Δ PG,* which is the difference between the value of *PG* for choosing to reuse an old component and the value of it for choosing the alternative of producing a new component. A positive value of ΔPG measures the potential for reuse.

Jiang et al. (2006) provide a review on several Proportional Intensity (PI) methods for repairable system reliability assessment, and discuss some engineering applications of those methods along with guidance in selecting the proper method.

Anityasari et al. (2008) propose a methodology to determine a threshold value of reliability for reuse evaluation. In their method the producer subjectively decides how many total failures would be acceptable. The threshold would be found based on that acceptable level.

Zhang et al. (2009) propose a method to find reliability indices for a k/n (F) system with repairable items using queuing theory, geometric process, and Markov process. They assume that all items' failure times are exponentially distributed.

Wilson et al. (2009) propose a model, which is based on the Bayesian theorem for estimating the life time probability density function (pdf) of an item from field return data by considering the actual installation and return times.

2.4 Warranty, reliability, and recovery

Reliability plays a vital role in warranty determination, and warranty is very important factor in selling recovered products. Some related researches to warranty and reliability/recovery are:

Zhi-Jie et al. (2006) illustrate that although reliability determination is an engineering decision and determining the price and warranty are marketing decisions, they should be determined jointly. They propose a model to help in making these decisions for new products jointly and under two pricing policies. Two pricing policies are: constant price, and dynamic price. In a dynamic price policy, price changes over the product's life cycle and decreases with the production quantity. The model is illustrated numerically too.

Anityasari et al. (2007) propose a model for reused products to investigate the effects of the reliability and related costs. They provide a methodology to assess the warranty cost reserved by manufacturers in the reuse strategy and to investigate how this additional cost influences the profitability. They show how to estimate the remaining life of a used product considering Weibull probability distribution properties. An example of renewing under free replacement warranty policy is illustrated.

Mohan et al. (2009) illustrate how a producer should select three inter-related important parameters (reliability, warranty period, and price) in the design step of a product to maximize the profit. Through several examples, they numerically show how reliability of different subassemblies of a product should be allocated to maximize total profit. Under a *free replacement warranty policy* and by assuming a discounted pricing policy, they find that the optimum reliability for each subassembly, the optimum warranty period, and the optimum price of the product. Also, in their proposed method, reasonably, it has been considered that if sales volume increases, manufacturing costs will decrease.

Saidi-Mehrabad et al. (2009) deal with the problem of minimizing the warranty costs for second-hand products by using two approaches: virtual age approach (kijima's model), and screening test approach. They consider the fact that upgrading the used products is costly, but also can reduce the warranty costs. Then they use those approaches to find the best reliability improvement (upgrading) strategy for used products sold under different warranty policies.

2.5 Multiple criteria decision making (MCDM)

The problem of selecting a proper recovery option for used products is an MCDM problem. Some related research to decision making models with multiple attributes and recovery options that are reviewed are as follows:

Hipel et al. (1993) show that Single Participant-Multiple Criteria (SPMC) decision problems and Multiple Participant-Single Criteria (MPSC) decision problems can be solved using same methods. Also, they illustrate how a decision on Multiple Participant-Multiple Criteria (MPMC) problem can be made by converting it to an MPSC decision situation, which simplifies solving the MPMC decision problem, and proper method for converting process and suitable MPSC and SPMC techniques are suggested for solving the problem. They classify all possible combinations of four main factors of a decision environment (the degree of existence of uncertainties, possibility of quantification of costs or benefits, number of objectives, type and number of decision makers) into five condensed categories from 16 possible combinations, and discuss each category with showing proper methods for solving each of them.

Fang et al. (2003, part I) propose a flexible graph model along with its formulation to study and resolve a real-world strategic conflict in part 1. They develop a comprehensive decision support system (graph model), for conflict resolution (GMCR II), which allows practitioners, researchers, and other interested parties to utilize a flexible conflict decision methodology. The proposed model, by using their illustrated method and special data, generates all possible states, removes infeasible states, and determines allowable state transitions. The model works, based on systematic approaches for ordinal favorite drawing-out. That logically draws out a DM’s relative favorite over other states.

Fang et al. (2003, part II) describe how the GMCR II's output engine works along with presentation of the output analysis of the proposed graph model. Also, the application of the proposed model in an international trade conflict (softwood lumber dispute between Canada and the U.S.A) is illustrated. The software package of the proposed system has been provided too, and is demonstrated in this research.

Umeda et al. (2005) categorize lifetime of a product to two categories: physical lifetime and value life time. Physical lifetime is the time until a product fails, and it is predictable by reliability theory. Value lifetime is the time until a product is thrown away while it might works well, but a customer decided to throw it away because he or she was not happy with its performance or its appearance. They propose using Disposal Cause Analysis matrix (DCA matrix) for estimating value lifetimes of products to help in determining life cycle options (LCOP) of a product. Some examples of LCOPs are: maintenance, upgrading, remanufacturing, reuse, recycling, and dumping.

Wadhwa et al. (2007) describe how the methodology of fuzzy reasoning can provide a framework to deal with the complexities in selecting the best reverse manufacturing alternative. They propose a fuzzy based MCDM model, which can help producers in designing a return policy.

2.6 Fuzzy AHP

In some decision problems, it is easier for decision makers to give interval judgments than fixed values. Using the concept of fuzzy set theory (Zadeh, 1965); fuzzy AHP methods are proposed to help DMs by presenting a systematic approach for selecting between options when the nature of the selection requires a fuzzy comparison. A review on researches proposing different methods of fuzzy AHP is investigated. Many fuzzy AHP methods can be found. The first one is proposed by van Laarhoven and Predrycz (1983), and to the best knowledge of the author all proposed methods are acceptable with no advantages over the others (only different calculations).

Van Laarhoven and Predrycz (1982) propose a fuzzy version of Saaty's extended popular pairwise comparison method (1980), which was proposed by deGraan (1980) and Lootsma (1981). In their method fuzzy ratios, which are presented as triangular fuzzy numbers (TFN) are applied instead of the crisp numbers, and they use their own rules for adding and multiplying TFNs. In their method first weights would be found, and then fuzzy scores of the alternatives are found, and as a result, the DM would be able to select between alternatives.

Chang (1996) presents a new method for pair-wise comparison of fuzzy AHP. He uses triangular fuzzy numbers and the extent analysis method for the synthetic extent values of the pair-wise comparison. The example that he illustrates is a modified version of the example illustrated by van Laarhoven (1982), which was a selection problem between three applicants, who have applied for a position as a professor when attributes are mathematical creativity, creativity implementation, administration capabilities, and human maturities.

Tolga et al. (2005) propose a method for solving a technology selection problem. Their goal is creating an operating system selection framework for DMs when they have to consider the economic aspect of technology selection as well as noneconomic aspects of it. They conclude that fuzzy AHP methods are more systematic and capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision-making problems are considered.

Wang et al. (2007) show that the method proposed by Chang (1996) to obtain a crisp priority vector from a triangular fuzzy comparison matrix cannot estimate the true weights. They show that by re-examining three numerical examples.

2.7 Inventory control systems with returns

In this section, a review of research proposing different inventory control models with returns is investigated. A major classification of related research can be made into deterministic models versus stochastic ones. Two models of each category are investigated:

Schardy (1967) proposed the first deterministic model, which was built for military service. He proposed a model with constant demand and return rate, and both lead times of two inventories were assumed to be fixed. In his model, costs are: fixed setup costs for manufacturing and recovery process, and linear holding costs for both inventories. To solve the model he proposed a control policy in which batch sizes are fixed and priority is given to the recovered products to satisfy the demand for; his model is a case with one recovery setup and several manufacturing setups.

Van der Laan (1997) developed a complete framework to study the situation; his stochastic model allows studying the situation applying various control policies. He also numerically compared different disposal strategies.

Kiesmüller et al. (2003) proposed a stochastic model, in which simple formulas are developed for finding the optimums. The optimums are: the produce-up-to level S and the remanufacture-up-to level M in a stochastic inventory control model with returns. Formulas can easily be implemented within spreadsheet applications. The formulas are news-vendor-type formulas that are based on underage and average cost considerations. They proposed different formulas depending on whether lead times for production and remanufacturing are identical or not. They performed a simulation study to measure the cost performance of their proposed approximate parameters with the optimal ones.

Saadany et al. (2010) extended the model by Dobos and Ritcher (2003, 2004) and built deterministic models in which two variables are considered: quality of used items, and used item's price, which were not considered in Dobos and Ritcher models. They proposed two models: (i) a single remanufacturing cycle and a single production cycle, and (ii) multiple remanufacturing and production cycles. The models' assumptions are: production and remanufacturing rates are finite; remanufacturing operations bring back the used products to a "as-good-as-new" condition; demand is known and is constant; there is no lead time and backorders are not allowed; there is a single product case; storage capacity is unlimited, and planning horizon is infinite.

2.8 Simulation models & product recovery

Simulation models have been widely used in studying systems. This section investigates the application of simulation in modeling inventory control systems and in the reverse logistics.

Kara et al. (2007) build a model of a reverse logistics networks in order to estimate the collection cost for used appliances in a specific geographic area in the USA. Also, the simulated model is used for the "what if" analysis to find important factors and to provide a better understanding of the relationship between the elements of a closed loop system, such as producer, customer, and distributer.

Jie et al. (2008) using *Arena*® simulate an inventory system, (s, S) with random lead times, in which orders can enter the system at any time. Their goal is to find out why the results of other attempts of simulating the same system do not match. They prove that optimizing using *OptQuest*® works efficiently for optimizing a stochastic constrained model by verifying the results obtained from *OptQuest* with a statistical testing of Krush-Kuhn-Tucker (KKT).

Tao et al. (2009) build a discrete-event stochastic model with *Arena*® to model a closed-loop inventory system for repairable items. Their objective is to investigate the effect of depot repair capacity on Expect Backorder (EBO) and to optimize spares locations. They verify the model by applying several examples, and show that their simulated model is more practical and that it runs better than Multi-Echelon Techniques for Recoverable Item Control (METRIC).

Chapter 3

Model 1: A reliability based method to evaluate the reusability for product recovery

3.1 Introduction

Many producers are now considering closed loop systems or reverse logistics to restructure their supply chain processes and redesign their products due to several reasons. First, in some countries laws are forcing producers to either recover their used products or recycle them properly. Second, customers are now more environmentally conscious, and they prefer to buy products with a 'green' image. Third, producers are realizing that adding a proper recovery process is profitable.

As an example, the Ontario Automotive Recyclers Association (OARA) green parts program allows customers to find used auto parts in a systematic way. To provide this service, many auto recyclers are members of this association. These modern auto recyclers in Ontario, Canada take out all reusable parts from old cars and recycle the rest of them. The process that they apply includes three major steps: any fluids remaining in the old car are drained; reusable parts are removed, tested and coded to be prepared for re-sale (they are kept in a computerized inventory, in which all related information including data related to the old car that parts came from, such as model, and VIN is recorded); finally, the old car is compressed and shredded into small handfuls of metal to be reused as raw materials (for more information go to www.gogreenparts.ca).

This program is established a recovery line with a simple process of go/no go testing, complex recycling operations, and without any remanufacturing/refurbishing process. However, the process of control in a manufacturing system with returns is more complex than what it is in the OARA green parts program. The complexity results from highly stochastic return process that makes it difficult to predict the arrivals of used items, and uncertain recovery operations, such as testing, disassembling, remanufacturing, and refurbishing that need to be controlled.

As a result, developing a method that makes an economic decision for remanufacturing, refurbishing, and recycling of returned products becomes important. The method presented in this chapter is designed with the focus on the role that reliability engineering plays in green manufacturing, and the objective is to develop a method to find a reliability threshold for evaluating the reusability of returned products that can be applied for well established products only. The used products can be selected for restoring operations only if their reliability measures greater than or equal to the reliability threshold.

The proposed method covers only well established products, as usually this type of products will be returned due to a functionality problem. In other words, people will keep them till they work properly. An example of this type of products is an electrical hair cutter. In contrast, products with fast innovations (e.g. cellular handsets) can be disposed when still are in working condition, so the reliability is not a proper parameter for evaluating them (see Chapter 4).

Applying the proposed method by the producer eliminates the costs of primary recovery operations, such as cleaning, and testing for returns, which are not reliable enough to be reused. Further, it reduces unexpected warranty costs for recovered items, as recovered items failures increases if their reliability does not measure enough at return time.

3.2 Model-description

Beside the economic aspects of the reuse processes, the reliability test will determine if the product can be restored or should be recycled. Figure 3.1 shows how the returns will be separated into two categories: those suitable for reusing and those that should be recycled.

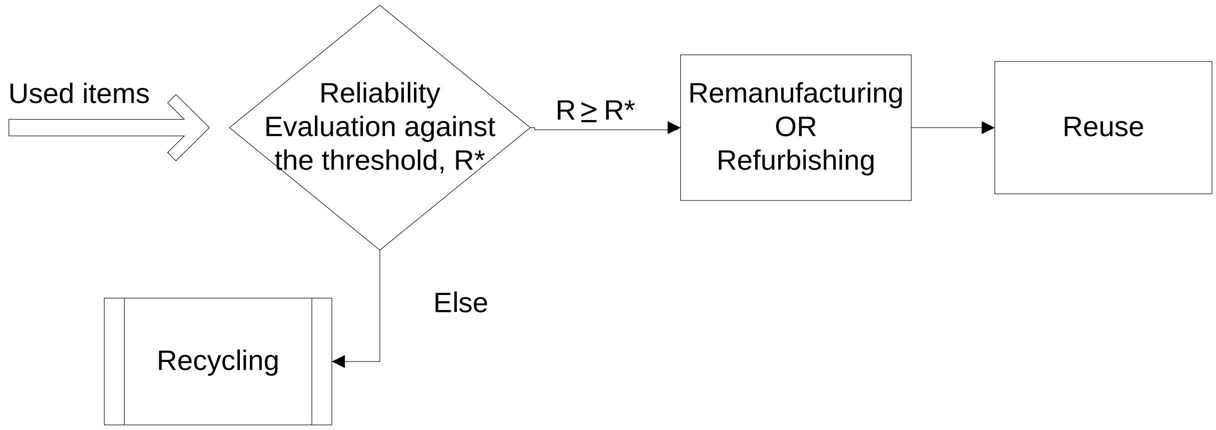


Figure 3.1: Reliability test to separate used products

The test is designed based on the fact that the reliability of all products will decrease when their service time increases and the fact that recovery is a feasible option for a used product only if it is reliable enough.

As it is shown in Figure 3.1, a decision on reusability of an item will be made based on its measured reliability (*R*) and the threshold (*R**). Products whose reliability measured greater or equal than the threshold will be selected for remanufacturing/refurbishing (restoring).

Figure 3.2 shows a general situation for a reusable item. It graphically shows how the general idea of decreasing the reliability by increasing item's service time is applied in the proposed method. A returned item, which has failed after working for t_1 units of time, can be restored, as its reliability, $R(t_1)$, measures greater than the threshold (R^*) or it can be restored because it has been used for t_1 units of time, which is less than the service time threshold (T^*).

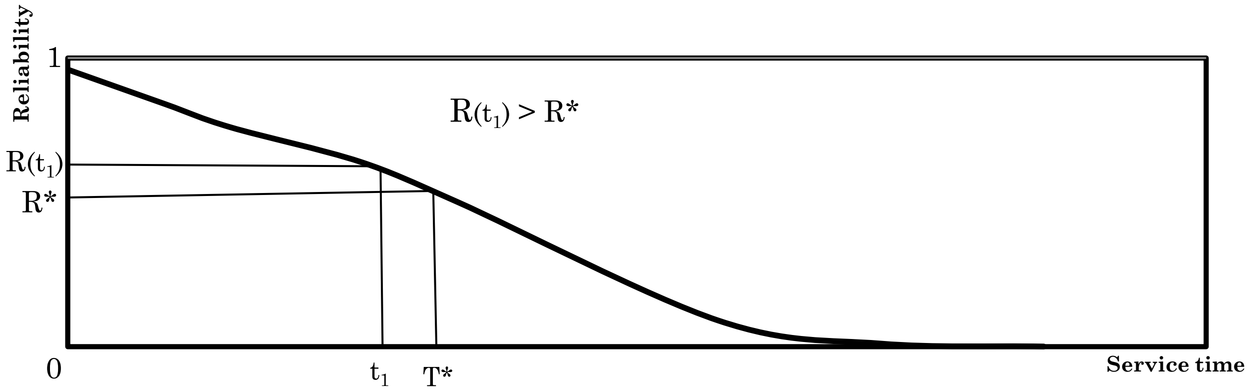


Figure 3.2: A reusable item

To get a better understanding of how the reliability measurement test works let's consider a personal computer (PC) producer with recovery operations. A reliability measurement testing procedure can help the producer to decide on which option to take for received used PCs; for instance, if some of the used PC's reliability measured higher than the threshold (say, R^*= 85%), then restoring (remanufacturing or refurbishing) would be the best option as the product has the ability to be brought back to a "as good as new" condition.

In the proposed method recovery options are: remanufacturing, refurbishing, and recycling only. The other recovery option, cannibalization, has not been considered for simplicity, which has no affect on the method. The proposed method can easily be applied for subassemblies as well.

The recovery process would be called 'refurbishing' if only minor operations are needed for restoring the product to an "as good as new" condition, and it will be designated as 'remanufacturing' if major operations are needed.

In an inventory system with returns, which consists of two stocking points: reusable inventory (for stocking reusable returns) and serviceable inventory (for stocking new products), both remanufactured and refurbished products will be stored in the serviceable inventory.

3.3 Application of the model

The main application of the model is in inventory control systems with returns (closed-loop inventory control systems). A general framework of the situation is depicted in Figure 3.3:
The used products are received by the producer. They will be grouped into two different categories based on their measured reliability: they can be restored or they have to be recycled. As the main application of the proposed method is in an inventory control of a closed-loop manufacturing system, Figure 3.3 shows how the reliability measurement fits in such system.

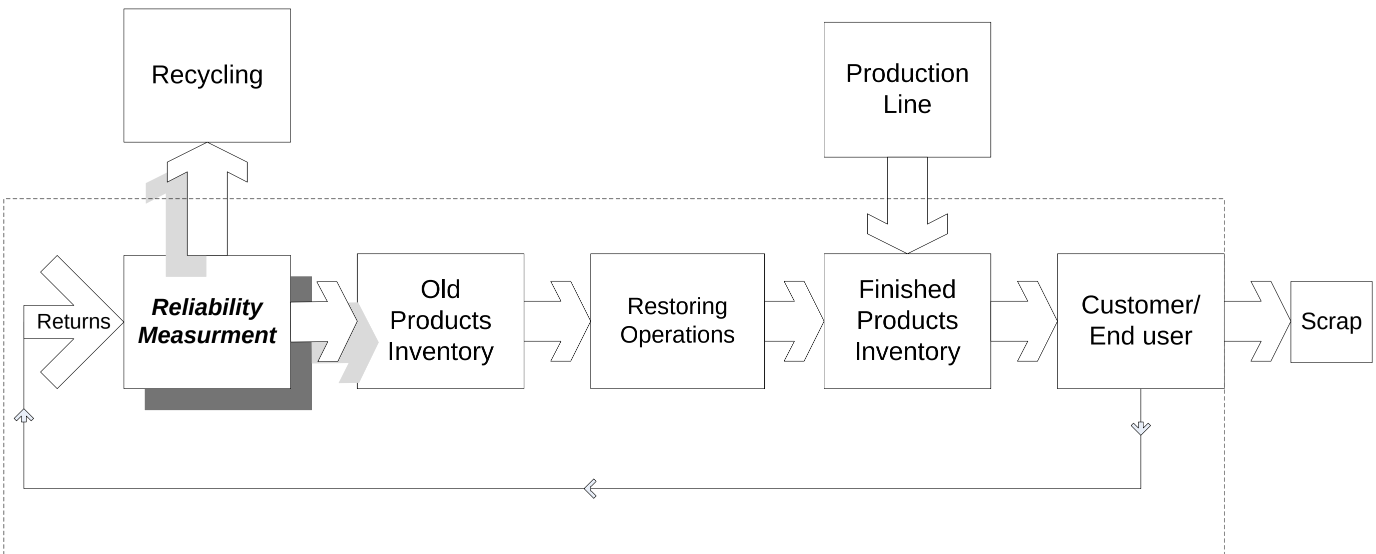


Figure 3.3: A closed-loop manufacturing system with reliability test

Therefore, by adding the reliability measurement testing procedure to a closed loop system, recovery process will be controlled and only used products that are reliable enough will be selected for restoring, otherwise their materials will be recycled. One point to take in consideration in planning for recovery operations including the new reliability testing is that they are highly stochastic in nature due to the uncertainty in timing, quantity and quality of returned products.

3.4 Warranty costs and reliability threshold determination

The threshold, R*, will be determined based on the warranty costs offered for recovered products. In other words, it will be calculated based on the warranty period that the producer is able to offer. The reason that warranty costs are chosen for justifying the model is that the role of warranty in selling recovered products is even more important than its role in selling new products.

The reason for this importance is that generally costumers think that used products have a lower quality than new ones. One way to attract those customers is to offer them a warranty, and of course recovered products are sold cheaper than new ones. Therefore, due to the importance of the warranty costs in the threshold determination, the warranty cost formulations and a brief description of how they are used in the proposed method to find the optimum warranty period are discussed first. Table 3.1 provides warranty cost formulations for different common warranty policies (Yang, 2007).

Table 3.1: Expected warranty cost per item under different warranty policies

Warranty Policy	**Expected warranty cost per item (C_w)**
Free Replacement with a 'good-as-new' repair	$C_w = C_0 * W(t_0) = C_0[F(t_0) + \int_0^{t_0} W(t_0 - x) f(x) dx]$
Free Replacement with a 'same-as-old' repair	$C_w = C_0 * W(t_0)$; $W(t_0) = \int_0^{t_0} h(t) dt = \ln \frac{1}{1-F(t_0)}$
Pro-Rata Replacement	$E[C_w(t)] = \int_0^{\infty} C_w(t) dF(t) = C_p \left[F(t_0) - \frac{\mu(t_0)}{t_0} \right]$, $\mu(t_0) = \int_0^{t_0} t dF(t)$
Combination Free and Pro-Rata Replacement	$E[C_w(t)] = \frac{C_p}{t_0 - t_1} \{ t_0 F(t_0) - t_1 F(t_1) - \mu(t_0) + \mu(t_1) \}$

The cost of warranty depends on the number of claims, warranty period, and the type of warranty policy. Note that the number of failures within the warranty period depends on the failure probability distribution of the product, and this has been considered in the warranty cost formulations.

There are different warranty policies, and warranty costs are calculated differently in each case. The most common types of warranty policies are: *Free replacement policy, Pro-rata replacement policy, and Combination free and pro-rata replacement policy*, which are introduced in Section 1.2.1. Warranty cost formulations can be applied to find the optimum warranty period for different policies to meet targeted warranty costs. Because it is reasonable to assume that the seller knows how much the maximum warranty cost per unit (C_{Mx}) is in order to reach the targeted profit. The seller can estimate it through a cost analysis.

In the newly developed method, C_{Mx} is a known input. When the maximum warranty cost per unit is given, the optimum warranty period (t_o*) can be obtained from a warranty cost formulation.

For example, in Section 3.7 (illustration), the optimum warranty period (t_o*) is found when a *free replacement with a 'good-as-new' repair* policy is selected by the producer, and the threshold would be found by considering that obtained optimum warranty period.

To show how Table 3.1 is used in section 3.7, consider a free replacement policy, if average cost per repair is denoted by C_0, the expected warranty cost per item, C_w, can be found from Equation 3-2, where $W(t_0)$ is the expected number of renewals and can be calculated from Equation 3-3 or Equation 3-4. Note that the relation between the cdf ($F(t)$), pdf ($f(t)$), and reliability function ($R(t)$) of the product is introduced in Equations 3-1 to 3-1-2.

$$F(t) = \int_{-\infty}^{t} f(x)dx \quad (3\text{-}1); \qquad f(t) = \frac{d(F(t))}{dt} \quad (3\text{-}1\text{-}1); \qquad R(t) = 1 - F(t) \quad (3\text{-}1\text{-}2)$$

Equation 3-3 applies when a good-as-new repair is performed. Equation 3-4 applies when repair is a same-as-old repair. Equation 3-3 is a *Volterra integral* equation of the second kind, and cannot be solved classically. It can be estimated using numerical methods. One of those numerical solutions for it is using Equation 3-5, which has been recognized as a good estimator for *Voltera integral* (Yang, 2007).

$$C_w = C_0 W(t_0) \tag{3-2}$$

$$W(t_0) = F(t_0) + \int_0^{t_0} W(t_0 - x) f(x)dx \tag{3-3}$$

$$W(t_0) = \int_0^{t_0} h(t)dt = \ln\frac{1}{1 - F(t_0)} \tag{3-4}$$

$$N_i = \frac{1}{1 - F_1}\left[A_i + \sum_{j=1}^{i-1}\left(N_j - N_{j-1}\right)F_{i-j+1} - N_{i-1}F_1\right], 1 \le i \le n \tag{3-5}$$

For the same policy used in Section 3.7, but with a 'same-as-old' repair, t_o* is found from Equation 3-6, which is the result of solving Equation 3-4 for finding warranty period when C_{Mx} and C_0 are given. Note that if $F(t_0)$ is very small (e.g. because of a short warranty period or high reliability), Equation 3-4 can be approximated by Equation 3-7.

$$t_o = F^{-1}\left(1 - e^{C_{Mx}/C_o}\right) \tag{3-6}$$

$$W\left(t_0\right) \approx F\left(t_0\right) \tag{3-7}$$

For the other remained two types of policies (pro-rata warranty policy and combination of free and pro-rata warranty policy), the optimum warranty period, t_o*, has to be calculated numerically with the aid of a simple computer program or a spread sheet, in which warranty costs for different warranty periods are calculated to find for which warranty period the related warranty costs start to be greater than C_{Mx}. Say, for warranty period equals a, the related warranty costs starts to appear greater than C_{Mx}, then a-1 is the answer ($t_o* = a-1$).

3.5 Model-assumptions

The main model's assumptions are as follows:

1) Reliability of any item reduces by increasing its service time.
2) The product is well established and has been in the market for a while.
3) The product is slow to moderate changes. Therefore, restoring/repair can be an economic option for used products.
4) The producer is willing to offer warranty for remanufactured/refurbished products.
5) The producer has established a warranty data collection system.
6) The producer knows how much the maximum warranty cost per item is and can estimate the failure probability distribution of the products.

3.6 Reliability threshold determination

Figure 3.4 shows how the reliability threshold (R^*) is found by following these steps:

Step 1) maximum warranty cost per remanufactured item and item's estimated reliability function should be obtained, which are the inputs of the model

Step 2) choosing the type of warranty policy, which is a tactical decision and usually should be made by the management.

Step 3) calculating the warranty period (Table 3.1 can be applied: when maximum warranty cost and cdf of the remanufactured item are given, the economic warranty period (t_o*) to offer can be calculated for a single item or for a batch of remanufactured products (see Section 3.4)).

Step 4) verifying the warranty period found in step 3

This can be done by comparing the warranty period found in step 3 with warranty period offered by the competitors (see Section 3.6.1).

Step 5) using Table 3.1, calculate the number of failures during the warranty period ($n*$). It means that $W(t_o*)$ is calculated by using proper formula in Table 3.1.

Step 6) finding the lifetime of the $n*$th failure ($T*$). This can be done using statistical methods (e.g. by using Annualized Failure Rate (AFR) formula)

Step 7) calculating the reliability measure associated to the $T*$ to obtain the threshold or $R*$

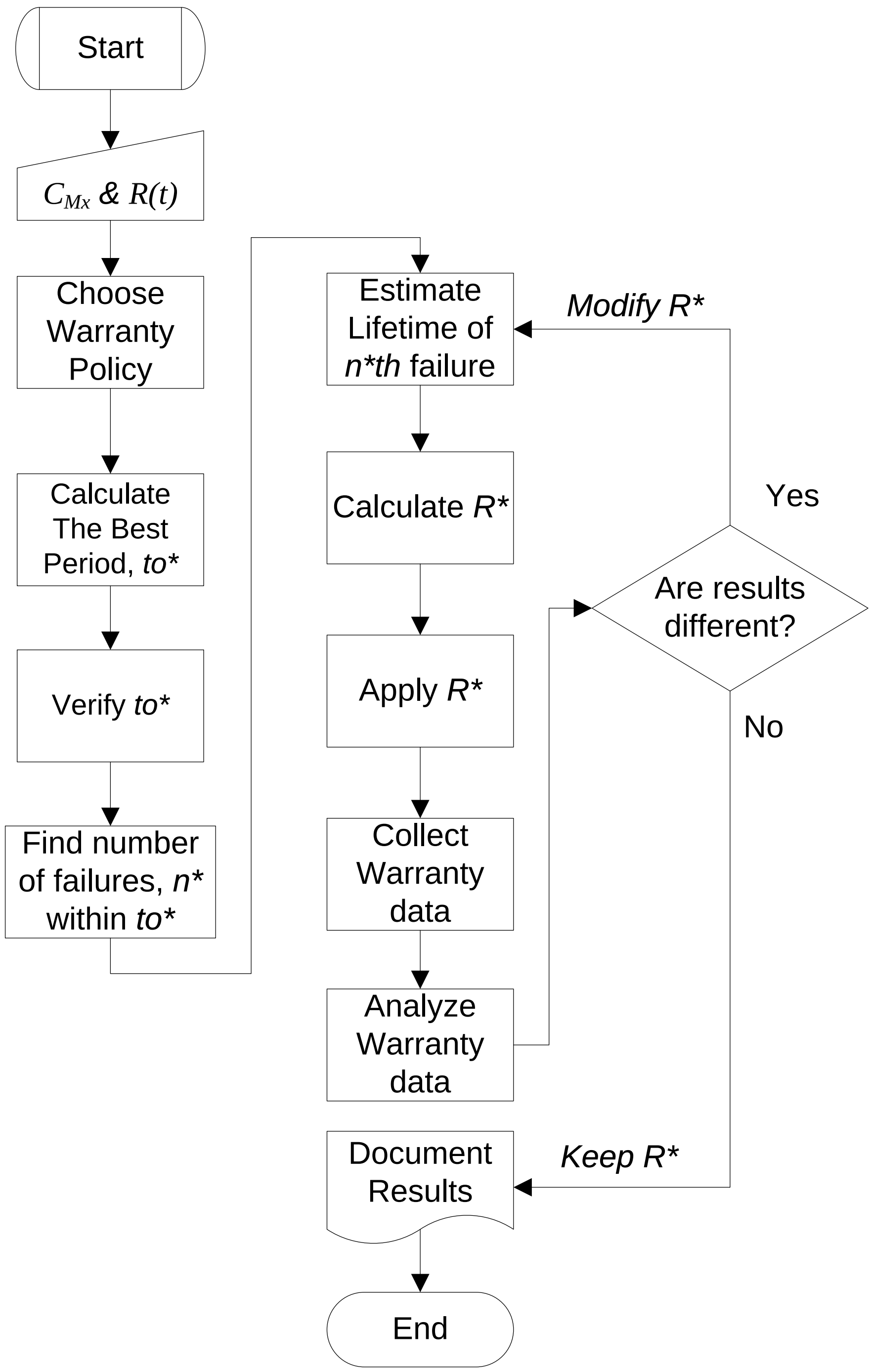


Figure 3.4: Algorithm for finding the threshold, *R**

Steps 8) using the reliability threshold, evaluate the reusability of used products. Those that are reliable enough will be restored (remanufactured or refurbished) and will be prepared for re-selling to the customers.

Step 9) collecting the warranty data related to the sold remanufactured products

Step 10) analyzing the warranty data to find out how good the threshold was selected (the threshold will be changed if the warranty analysis calls for it, and new threshold will be applied)

The method continues modifying itself until it finds the accurate threshold. Some information related to warranty data can be found in section 1.2. Section 3.6.2 describes warranty analysis in details.

The method to find the reliability threshold (R^*), which is shown in Figure 3.4, is developed based on the following facts:

- It is reasonable to assume that producer can estimate how much the maximum warranty cost per unit (C_{Mx}) is through a cost analysis.

- The probability density function (pdf) of any item's life can be estimated when failure data is available.

- Offering warranty for recovered products is strongly recommended, and in practice is widely used. Because generally costumers think that used products have a lower quality than new ones. The producer can change their idea by offering a proper warranty.

3.6.1 Warranty period verification

The procedure for verifying the obtained warranty period, t_o^* is shown in Figure 3.5: The warranty period obtained based on the C_{Mx} and the selected warranty policy will be compared with the warranty period offered by competitor(s), t_o^c for the same recovered product. If there is not any other seller for the same recovered product, then t_o^c equals to the warranty period offered for the new product or to the warranty period estimated through a survey.

If case *I)* happens (see Figure 3.5), the producer can either offer a longer warranty period than the competitor(s) or save on the warranty cost by reducing the calculated warranty period, t_o^* to t_o^c .

If case *II)* applies, obviously the best strategy is to keep t_o^*, while increasing it can be considered, although this would be costly.

If case *III)* applies, then the producer has to consider some options such as reliability improvement, or lean manufacturing. The producer would be able to offer a longer warranty period either by improving the reliability of its recovered products or by considering a bigger maximum warranty cost through some savings (e.g. Lean manufacturing process).

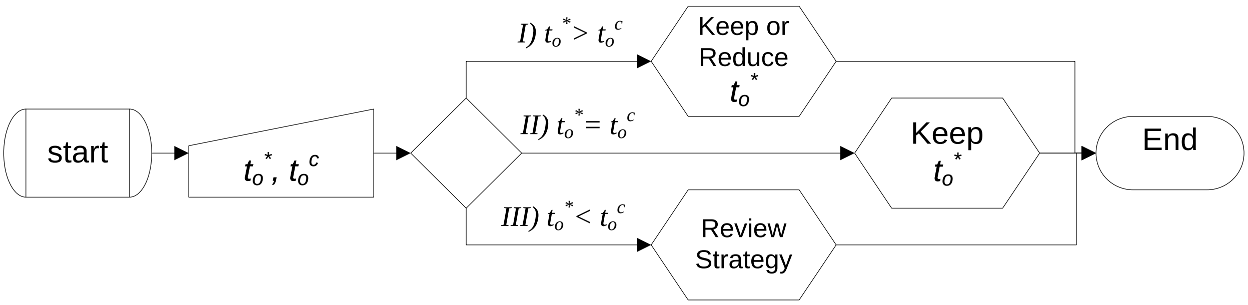


Figure 3.5: Verification of warranty period offered for restored items

3.6.2 Warranty data analysis

The objective of warranty data analysis is to find out how effective the method was in reusability evaluation and calculating the threshold, *R**. To do so, warranty data related to sold recovered items should be collected.

Warranty data usually contains information such as 1) *product data*: e.g. serial number, remanufacturing date, sales date, price, and repair history, 2) *failure data:* e.g. use condition at failure, accumulated use, and customer complaint, and 3) *repair data:* e.g. labor cost, part cost, date of repair, and others.

By statistically analyzing the warranty data, the reliability function that fits the data and the number of recovered products that would fail by the end of the warranty period can be estimated. Thus, we can measure the effectiveness of the method in selecting the threshold, and the threshold should be modified if the analysis calls for it.

Although warranty data are more realistic than testing data, there are some limitations associated with warranty data that should be considered (Yang, 2007).

Warranty data are criticized as being "dirty" due to these deficiencies: Usage ways are not differentiable from the warranty data, different lots produced under different remanufacturing conditions are mixed and treated equally, failure times are not accurate due to the reporting delay caused by the costumers, some warranty claims are not real failures due to the fact that the producer tries to satisfy picky customers, and the accurate number of the salvaged units under the warranty is not known. The following procedure is one of the suggested methods for warranty data analysis, which reduces the effect of some of those limitations.

For example, to reduce the effect of treating data equally with returns belonging to different lots, data can be collected periodically (e.g. monthly) and the products can be grouped assuming that those sold in the same time period have the same time in service, and the products that failed in the same period are assumed to have the same lifetime.

If the item's failure times are recorded, then the warranty data can be tabulated as shown in Table 3.2.

Table 3.2: Warranty data tabulated for reliability estimation

Time In Service (TIS)	Time To Failure (TTF)					Total	Sales Volume
	1	2	3	. . .	l		
1	a_{11}					$a_{1.}$	n_1
2	a_{21}	a_{22}				$a_{2.}$	n_2
3	a_{31}	a_{32}	a_{33}			$a_{3.}$	n_3
. . .	. . .	. . .	. . .	. . .		. . .	. . .
l				. . .	a_{ll}	$a_{l.}$	n_l
Total	$a_{.1}$	$a_{.2}$	$a_{.3}$	. . .	$a_{.l}$	$a_{..}$	

Where n_i = number of recovered products sold in time period i; a_{ij} = number of failures that happened in time period j to the products sold in time period i; l = maximum time in service;

$i = 1, 2, \ldots\ l$ and $j = 1,2,\ldots, l$; $a_{i.} = \sum_{j=1}^{i} a_{ij}$, $a_{.j} = \sum_{i=j}^{l} a_{ij}$, and $a_{..} = \sum_{i=1}^{l} a_{i.} = \sum_{j=1}^{l} a_{.j}$

Where $a_{i.}$= total number of failures among n_i units; $a_{.j}$ = total number of failures in j periods, and $a_{..}$ = total number of failures among all sold recovered products.

From data in Table 3.2, Table 3.3 should be tabulated, which is multiply right-censored data and the number of censored recovered products is shown in it (e.g. n_1 - $a_{1.}$ products are censored at the end of period 1).

Table 3.3: Multiply right-censored data

TTF	Number of Failures	Number of Censored Units
1	$a_{.1}$	$n_1 - a_{1.}$
2	$a_{.2}$	$n_2 - a_{2.}$
3	$a_{.3}$	$n_3 - a_{3.}$
. . .	. . .	. . .
l	$a_{.l}$	$n_l - a_{l.}$

After tabulating data in Table 3.3, they can be statistically analyzed to estimate the appropriate fitting distribution parameters. After the analysis, the number of failures should be estimated to determine whether warranty costs per product was lower than C_{Mx}.

3.7 Illustration

ABC company, which is a leading auto electrical spare part producer, has decided to establish recovery processes for one of its popular products (e.g. starter), which is in the market for many years. All related costs for recovering the starter has been estimated including the maximum warranty cost per product. ABC wants to know how to differentiate used starters to reusable and recyclable to make quality remanufactured starters. ABC has already established a failure data collecting system, which would be applied for remanufactured products as well.

The maximum warranty cost per starter, C_{Mx} is \$250/item for a contract of selling a batch of 200 remanufactured items. A *free replacement warranty policy* is chosen by ABC. The average cost per repair, C_0 is \$500/item.

To apply model 1, first, assume that an accelerated life test on 10 starters is performed. Results are shown in Table 3.4 as sorted failure data. The aim is to estimate the remanufactured starter life's probability density function (pdf). Note that for simplicity both refurbished and remanufactured starters are called remanufactured starters, as both are in the "good-as-new" condition. The pdf is considered as an input to model 1. However, it is shown how it can be estimated for illustration purpose only.

Table 3.4: Items failure times-a reliability test results

***i* (item number)**	1	2	3	4	5	6	7	8	9	10
t_i (days)	2.6	6.4	7.8	8.8	9.5	22.1	27.3	32.6	45.9	78.3

The failure data in Table 3.4 were analyzed using MATLAB. Figure 3.6 shows the exponential probability plot, which does not reject an exponential fit. Figure 3.7 shows a fitted exponential distribution as the best fit using the least squares method by the program. From the estimated parameter by program the pdf is found to be:

$$f(t) = \lambda^{\wedge} \exp(-\lambda^{\wedge} t)\,,\ \lambda^{\wedge} = 0.041 \text{ failure/unit of time} \tag{3-8}$$

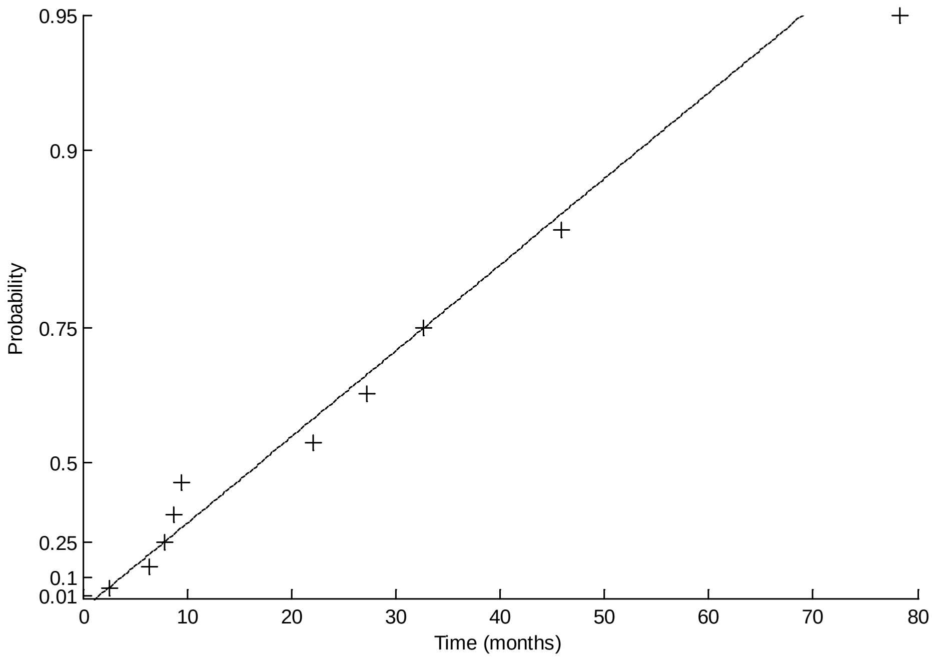


Figure 3.6: Probability plot for exponential distribution

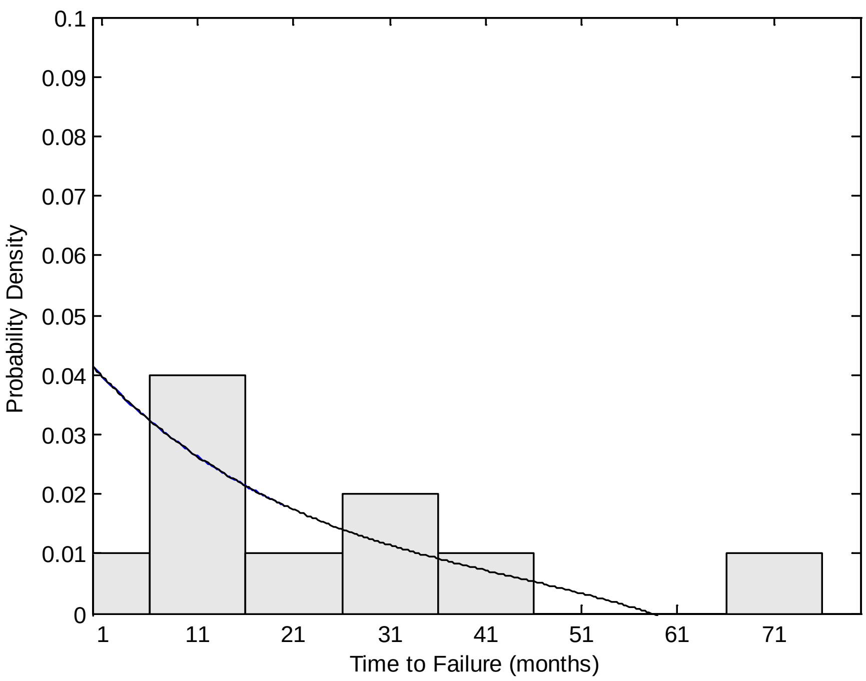


Figure 3.7: Fitting exponential distribution on the failure data

K-S test with significance level of 0.05 is chosen to test the goodness of fit; results obtained by using MATLAB are as follows:

h = 0, p = 0.8590, ksstat = 0.1774, and cv = 0.4093 ksstat is not greater than $D_{10}(0.05) = 0.410$. Thus, the hypothesized exponential distribution model for time-to-failure of item cannot be rejected.

As *free replacement warranty policy* is chosen, Equation 3-3, which is numerically estimated by Equation 3-5, can be used.

$$N_i = \frac{1}{1-F_1}\left[A_i + \sum_{j=1}^{i-1}\left(N_j - N_{j-1}\right)F_{i-j+1} - N_{i-1}F_1\right], 1 \le i \le n \quad (3\text{-}5)$$

For calculating N_i, for a fixed $t \ge 0$, the time interval [0, t] should be divided to: $0=t_0<t_1<t_2<\ldots<t_n=t$. Where $t = id$ for a given grid size $d > 0$, $A_i = F(id)$, $F_i = F\left(\left(i-0.5\right)d\right)$ and $N_0 = 0$. Note that in this illustrated case, $F_i = 1-\exp\left[-d\left(i-0.5\right)\lambda^{\wedge}\right]$ (3-9), and $A_i = 1-e^{-id\lambda^{\wedge}}$ (3-10), as found in pdf estimation step, and $d = 1$.

Table 3.5: Voltera Integral estimates for i = 1 to 24

i	***N_i***	**Estimated Warranty Cost ($)**
1	0.0410	4100
2	0.0820	8200
3	0.1230	12300
4	0.1640	16400
5	0.2050	20500
6	0.2460	24600
7	0.2870	28700
8	0.3280	32800
9	0.3690	36900
10	0.4100	41000
11	0.4510	45100
12 (t_o^*)	**0.4920**	**49200**
13	0.5330	53300
14	0.5740	57400
15	0.6150	61500
16	0.6560	65600
17	0.6970	69700
18	0.7381	73810
19	0.7791	77910
20	0.8201	82010
21	0.8611	86110
22	0.9021	90210
23	0.9431	94310
24	0.9841	98410

In searching an interval of [1, 24] months, the biggest i with an associated warranty cost lower than C_{Mx} is the desired answer. A MATLAB program is developed to calculate N_i, and in each estimation of N_i, the warranty cost related to that period should be compared with C_{Mx}. For the value of i where the related cost is greater than C_{Mx}, the "i - 1" should be selected. The results are shown in Table 3.5. The optimum warranty period, t_o^* equals 12 month.Assume that case II applies and competitor(s) are also offering a 12-months *free replacement warranty* package to their customers (see Section 3.6.1).

As a result, the t_o^* found in the last step can be kept, and the estimated total number of warranty claims (n*) would be:

$$200 \times W(12) = 200 \times \left[F(t_0) + \int_0^{t_0} W(t_0 - x) f(x) dx \right] = 98 \tag{3-11}$$

Therefore, the *98*th failure determines the threshold. Practically, it can happen within the warranty period or after it. To estimate when *98*th failure will happen, the hazard/failure rate concept is used. The lifetime of the remanufactured starter has an exponential distribution with $\lambda\hat{} = 0.0410$, so h(t) = 0.0410, and it can be estimated that in average 8.2 ≈ 8 starters will fail per month for a batch of 200 items. It can be concluded that *98*th failure can happen approximately to a product with 12 months time in service. Thus, $R^* = \exp(-12 \times 0.0410) = 61.14\%$ (3-12)

Table 3.6: Warranty data for 13 consecutive months

TIS	TTF													Sales Volume
	1	2	3	4	5	6	7	8	9	10	11	12	Total	
1	0												0	200
2	2	2											4	342
3	3	2	6										11	336
4	2	6	5	1									14	83
5	3	4	8	1	0								16	70
6	4	0	2	3	1	0							10	253
7	5	6	9	3	3	5	4						35	500
8	0	0	12	6	5	3	7	6					39	188
9	0	8	6	0	3	9	0	6	12				44	316
10	6	7	3	5	2	4	7	6	16	10			66	151
11	8	2	0	3	3	2	9	14	0	8	16		65	415
12	6	7	12	5	4	7	10	11	14	9	15	28	128	400
13	6	6	9	3	3	5	8	0	12	8	23	30	113	165
Total	45	50	72	30	24	35	45	43	54	35	54	58	545	3419

Applying the threshold that found by utilizing the proposed method, ABC rejects restoring used starters, which were in service longer than 12 months, and they can be recycled. Assume that

ABC keeps checking the reusability of returned starters based on the threshold "R^* = 61.14%", and tracks the warranty data related to the remanufactured starters. Table 3.6 shows the failure data for items in service for the past 13 months. ABC sold 3419 remanufactured products during the past 13 consecutive months.

Table 3.7: Life data of the remanufactured product

TTF(months)	1	2	3	4	5	6	7	8	9	10	11	12
Number of failures	45	50	72	30	24	35	45	43	54	35	54	58
Number of censored units	200	338	325	79	54	243	465	149	272	85	350	324

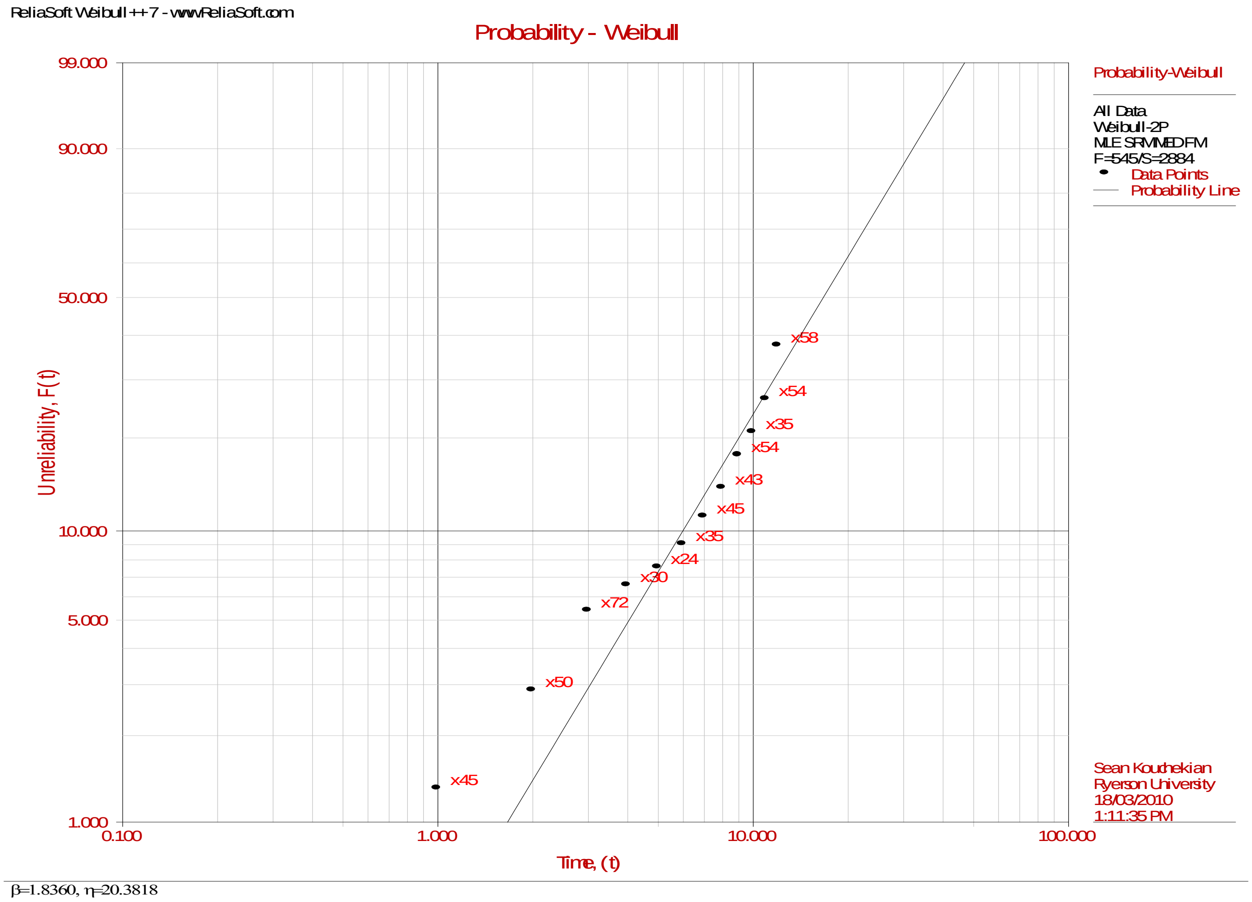


Figure 3.8: Probability plot for weibull distribution

The life data are tabulated in Table 3.7 shows the number of censored remanufactured products. Note that the failure data for the 113 items in service for the 13th month are treated as having had 12 months in service and are combined with the 128 items having had 12 months in service to calculate the number of survivals. The life data shown in Table 3.7 are analyzed using Weibull++7 software. Ranked results and graphical analysis implies that the Weibull distribution can be chosen as a good fit. Figure 3.8 Shows the Weibull probability plot. As a straight line can closely pass through the data, it is demonstrating Weibull probability distribution as a good fit.

Further analysis, using the maximum likelihood method yields estimates of the parameters as follows: α = 20.3818 and β = 1.8360. Figure 3.9 shows the estimated Weibull reliability function plot versus time for the remanufactured product. Figures C.1-C.5 in Appendix C, show more related plots.

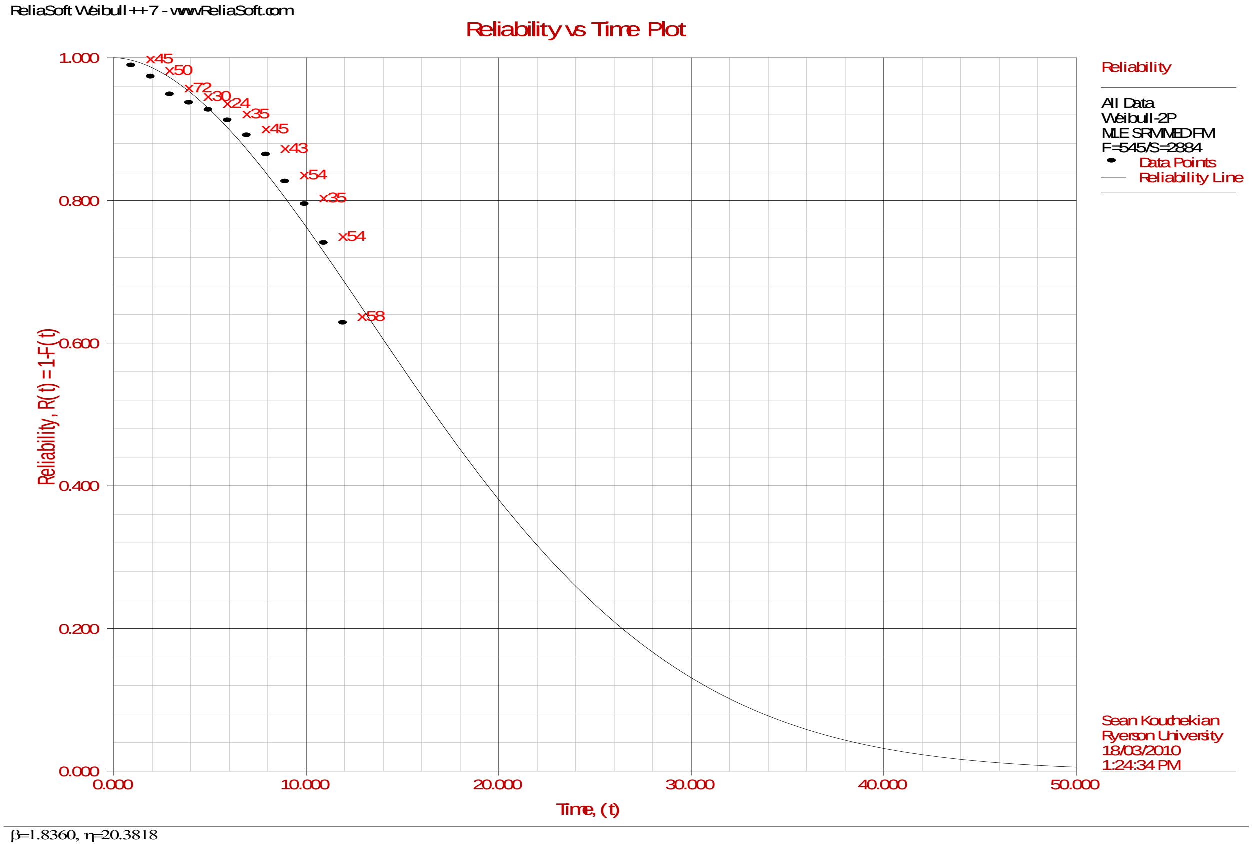


Figure 3.9: Reliability function plot vs. time for the remanufactured item

Total failures within the warranty period can be estimated from the Weibull-cdf found through warranty data analysis. Where shape parameter, α = 20.3818 and scale parameter, β =1.8360.

- The cumulative probability of failures during the warranty period is:

$$F(12) = 1 - \exp\left[-(12/20.3818)^{1.8360}\right] = 0.3148 \tag{3-13}$$

Therefore, total failures $= 3419 \times 0.3148 = 1076$.

As a result, 1076-545= 531 more remanufactured products could be expected to fail.

- Total failures accrued for the first batch of recovered products equals 63 failures $(200 \times 0.3148 = 62.96 \approx 63)$.

Thus, it can be concluded that the value of R^* should be modified. Further, the first batch of recovered products appeared more reliable than what the producer estimated. As a result, the producer has gained more profit than expected one, as the number of failed recovered products even could have been reached to 98 failures.

- To estimate when 98th failure has occurred, the concept of the Annualized Failure Rate (AFR) can be applied. Equations 3-14 to 3-16 can be used to calculate AFR, while the lifetime of products have a Wiebull distribution with shape parameter, α = 20.3818 and scale parameter, β =1.8360.

$$AFR(T_2 - T_1) = \frac{\int_{T_1}^{T_2} h(t)dt}{(T_2 - T_1)} = \frac{H(T_2 - T_1)}{(T_2 - T_1)} \tag{3-14}$$

$$H(t) = \int_0^t h(t)dt = -\ln R(t) \tag{3-15}$$

$$AFR(0,T) = \frac{H(T)}{T} = \frac{-\ln R(t)}{T} \tag{3-16}$$

Substituting the parameters, $AFR(0,12) = \frac{-\ln R(12)}{12} = \frac{-\ln(1-F(12))}{12} = \frac{-\ln(0.6852)}{12} = 0.0315$

Therefore, number of claims for first batch/month $= 200 \times 0.0315 = 6.3$ failure/month.

Also, the lifetime of the *98*th failure is 98/6.3 = 15.5556 ≈ 16 months.

Thus, new threshold is equal to: $R^* = \exp\left[-\left(16/20.3818\right)^{1.8360}\right] = 0.5267 \approx 53\%$

3.8 Discussion

To deal with the problem of reusability evaluation, products are categorized into two groups: well established products, and products with fast innovations. Model 1 presented in this chapter, is applicable to well established products, and model 2 presented in the next chapter, is applicable to the second group of the products. One possible question regarding model 1 is that how precisely the reliability threshold (R^*) should be calculated to minimize the related costs.

To answer, one important point to take into consideration is that having proper recovery operations is recognized as an economic decision that can be made by a producer regardless of the value of R^*. This is due to several reasons including followings:

First, usually used products are very cheap, but restored products can be sold at prices close to those of new ones. Second, if the used product is selected for recycling while it is reusable, still it is a profitable selection for the producer. Because recycled materials are sold at prices greater than used items' purchasing prices, and usually governments provide some incentives for the producers who establish recovery operations. Third, the green image that the producer would gain after adding recovery operations helps in increasing sales.

In addition, if the used item is selected for restoring operations while it is recyclable, the testing operations will catch its deficiencies, and only the cost of restoring may increase, which is the nature of uncertainties involved in restoring operations that can be controlled.

However, in situations where real data are accessible, the sensitivity analysis for R^* can be done to investigate the effect of different values of R^*. To get an idea of the effects of different values of R^* on cost parameters, in chapter 5 the sensitivity of R^* is investigated by using the simulation model.

The other question that might rise is that if the above reasons are satisfactory, and recovery operations are profitable regardless of how accurate the R^* is, then why the reusability evaluation model is needed.

The main benefit of model 1 is that the reusability evaluation helps in providing quality recovered products. It also, helps in saving time on unnecessary recovery operations, such as cleaning, or testing. Because the reliability threshold, R^*, helps the producer to decide on which recovery alternative to choose for a used product without spending time on testing it. Thus, model 1 serves as a reliable pre scanning process to minimize costs related to recovery operations including warranty costs and to improve the quality of restored products.

In conclusion, applying model 1 helps in providing quality restored products for the customers, and can effectively improve the recovery line's speed. It can be applied by the producers who have established a data collecting system, which records all related information of the products' failures.

Chapter 4

Model 2: A fuzzy MPMC model to evaluate the reusability for product recovery

4.1 Introduction

In chapter 3 a model was presented to deal with the problem of reusability evaluation for well established products. The assumption was that the products are returned due to the failures, so reliability is an important parameter to consider. Also, it was assumed that the change in the products is slow to moderate, so reuse can be an economic option.

In contrast, there are some products that do not satisfy the above assumptions. A product of this group is discarded although it still works well, but the end user decided to throw it away because its performance or its appearance was no longer desirable. For example sometimes a product, such as a cellular phone, computer, or electronic game is returned because the user has switched to a newer model of that type of product with a better performance. Because this group of products is subject to fast changes, restoring them is rarely feasible.

In this chapter a multiple participant-multiple criteria (MPMC) model with the application of fuzzy AHP is presented to help the producer in deciding on recovery options for products with fast innovations. The key point is: it might be true that for example, people living in North America are not going to buy a specific model of cellular phone, but maybe there is a good market for it in another specific area of the world.

In building model 2, the proposed method by Umeda et al. (2005) for estimating value lifetimes is considered to help in distinguishing targeted products from well established ones. A table of interpretations for pair-wise comparisons with fuzzy elements is defined, and then the fuzzy AHP method proposed by van Laarhoven and Pedrycz (1983) is utilized to perform evaluation steps.

In following sections, value lifetime, physical lifetime, DCA matrix, and fuzzy AHP steps are defined. Then the algorithm of model is presented, and it is illustrated by a numerical example.

4.2 Value lifetime vs. physical lifetime

Physical lifetime is the time until a product fails; it can be predicted by reliability theory. In contrast, value lifetime is the time until a product is thrown away while it might still be working well. However, a customer decided to throw it away because he or she was not happy with its performance or its appearance. Therefore, dispose causes for this group of products usually are value causes (e.g. fashion), and their return is a function of time not service time.

4.3 DCA matrix (Umeda et al., 2005)

A disposal cause analysis (DCA) matrix is a tool to collect customers' reasons for disposing of a product that is based on the quality function deployment technique (QFD). A sample of a DCA matrix is shown in Table 4.2.

The inputs of a DCA matrix include: market data, worldwide data, and customer data. Market data is made of information, such as sales, discard amount, fashion, and available technologies. Customer data are composed of: purchasing time, discard time, critical function for discarding, culture, and discard causes. Worldwide data include same information as market and customer data for other parts of the world that potentially can be considered as a good area for selling the recovered product. A DCA matrix consists of three sub matrixes with three additional rows: I_k, PI_k, and VI_k. Three sub matrixes are as follows, and Equation 4-1 shows their relation.

1. Disposal cause-function matrix denoted by W_{ij}
2. Function-component matrix denoted by W_{jk}
3. Cause-component matrix denoted by M_{ik}

$$M_{ik} = W_{ij} \times W_{jk} \tag{4-1}$$

$$\text{Total importance} = M_k = \sum_i M_{ik} \tag{4-2}$$

i = 1, 2 denote physical causes and i = 3, 4, 5 denote value causes. These five categories of disposal causes are shown in Table 4.1. Also, j = number of functions, k = 1, 2 …, n, and n = number of components / subassemblies.

Table 4.1: Categories of disposal causes

Physical causes	i = 1 : Use function	i = 2 : Failure	
Value causes	i = 3 : Appearance	i = 4 : Capacity & size	i = 5 : Value worsening

Table 4.2: Disposal cause analysis matrix (Umeda et al., 2005)

Disposal Causes (d_i)		Importance (r_i)	Function (f_j)				
			Function 1	Function 2	Function 3	Function 4	Function 5
Physical causes	Use function						
	Failure						
Value causes	Appearance						
	Capacity & Size						
	Value worsening						
Importance of functions		40.0					

Components (C_k)					
Component A					
Component B					
Component C					
Component D					
Component E					
Component F					
Product (P)					

	Cause-component matrix					Total Importance M_k	Relative Importance (I_k)	Value Importance (VI_k)	Physical Importance (PI_k)
	d_5	d_4	d_3	d_2	d_1				
Component A	0.1	0.5	0.9	4.6	0.0		15%	3.8%	11.5%
Component B									
Component C									
Component D									
Component E									
Component F									
Product (P)						40.0	100%	42.55	57.5%

Three additional rows are as follows:

1. Relative importance = $I_k = \dfrac{M_k}{\sum M_k}$ (4-3)

Relative importance (I_k) indicates the ratio of product disposal owing to component/subassembly k to the total amount of disposal.

2. Physical Importance = $PI_k = \dfrac{\sum_i M_{ik}}{\sum M_k}$ & i=1, 2 (4-4)

If important causes are from physical causes, then i = 1, 2

3. Value Importance = $VI_k = \dfrac{\sum_i M_{ik}}{\sum M_k}$ & i= 3, 4, 5 (4-5)

If important causes are from value causes, then i = 3, 4, 5.

The results of a DCA matrix are as follows:

1. PIP = Physical Importance of the whole Product = $\sum PI_k$ (4-6)
2. VIP = Value Importance of the whole Product = $\sum VI_k$ (4-7)
3. PI_k = Physical Importance of each component/subassembly
4. VI_k = Value Importance of each component/subassembly

Consider DCA matrix shown in Table 4.2, which for simplicity is partially filled out. To find out if the product is disposed due to the physical lifetime or value lifetime, all VI_k values and all PI_k values should be calculated and the total value of them will be shown in the last row.

For instance, to find values of indicators for component A (k=1), calculations are as follows:

$$I_1 = \frac{0.1 + 0.5 + 0.9 + 4.6}{40} = 15.25\% \approx 15\% \tag{4-8}$$

$$PI_1 = \frac{4.6}{40} = 11.5\% \tag{4-9}$$

$$VI_1 = \frac{0.1 + 0.5 + 0.9}{40} = 3.75\% \approx 3.8\% \tag{4-10}$$

Assume that after doing the same calculations for other components, the total value of indicators for the product is obtained as: PIP = 57.5% and VIP = 42.5%. Therefore, the product was discarded due to the physical lifetime, and model 1 can be applied to evaluate its reusability.

Note that in (Umeda et al., 2005), DCA matrix is applied to estimate a distribution function for the value lifetime, but in the presented method (model 2), the DCA matrix is used for identifying the type of product (fast innovative or well established).

In other words, in the model 2, DCA matrix, which is tabulated through surveys from the customers who have thrown away a product, only helps in identifying products that are discarded due to the value lifetimes (products with fast innovations).

4.4 Fuzzy AHP-Van laarhoven and Predrycz's approach

Figure 4.1 shows a general situation of a MPMC decision problem: each option/alternative is judged against others depending on its overall score on all attributes.

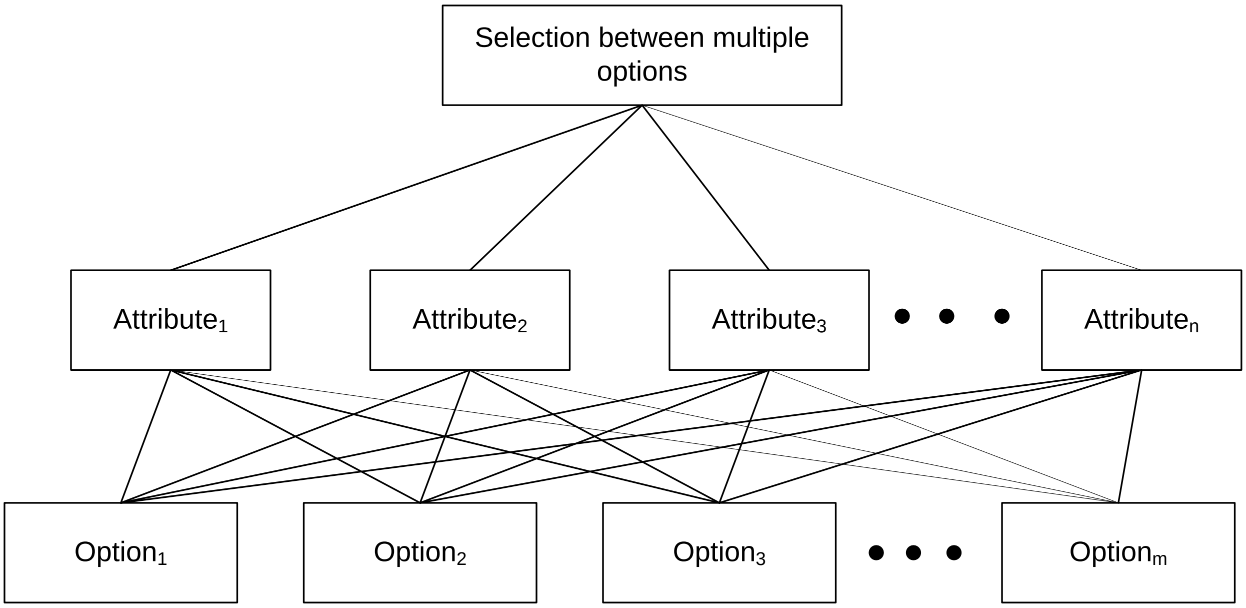


Figure 4.1: A general MPMC problem

There are many fuzzy AHP methods, which all use a systematic approach for deciding on alternatives by utilizing the concept of fuzzy set theory (Zadeh, 1965) along with the popular analytic hierarchy process (AHP). It means that in these methods, multiple DMs instead of using crisp values use a range of values to express their judgments. The first developed fuzzy AHP method (van Laarhoven and Pedrycz, 1983) is applied in model 2.

In van Laarhoven and Pedrycz's fuzzy AHP method, triangular fuzzy numbers (TFNs) are used instead of crisp numbers, and their method is an extension of popular AHP method (Saaty, 1980). Their method is made of several steps as follows:

1) Step 1: Obtain n+ 1 reciprocal matrixes of pair-wise comparisons, in which the decision makers (DMs) describe their opinions using TFNs, where n is the number of attributes.

2) Step 2: Define Z_i as a triangular fuzzy number: $Z_i = (l_i , m_i , u_i)$. Solve the following linear equations obtained from each matrix of step1.

$$l_i\left(\sum_{\substack{j=1\\j\neq i}}^{n} P_{ij}\right)-\sum_{\substack{j=1\\j\neq i}}^{n} P_{ij}u_j=\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln l_{ijk}\right),\forall i \qquad (4\text{-}11)$$

$$m_i\left(\sum_{\substack{j=1\\j\neq i}}^{n} P_{ij}\right)-\sum_{\substack{j=1\\j\neq i}}^{n} P_{ij}m_j=\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln m_{ijk}\right),\forall i \qquad (4\text{-}12)$$

$$u_i\left(\sum_{\substack{j=1\\j\neq i}}^{n}P_{ij}\right)-\sum_{\substack{j=1\\j\neq i}}^{n}P_{ij}l_j=\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln u_{ijk}\right),\forall i \qquad (4\text{-}13)$$

Where, n = number of attributes;

P_{ij} = number of DMs in each cell of matrixes;

P_{ij} = 0 OR $P_{ij} \geq 1$ (it equals zero when no DM expresses an idea)

Fuzzy membership function of Z_i , which is a TFN is shown in Equation 4-14. Note that TFNs are usually shown by (*l*, *m*, *u*), in which *l* refers to the lower bound, *m* refers to the mode, and *u* refers to the upper bound of the TFN. Also, *x* shown in Equation 4-14 refers to the crisp number, and $x \in [-\infty, +\infty]$.

$$\mu_M=\begin{cases}\dfrac{x-l_i}{m_i-l_i}, x\in[l_i,m_i]\\[2ex]\dfrac{u_i-x}{u_i-m_i}, x\in[m_i,u_i]\\[2ex]0, Otherwise\end{cases} \qquad (4\text{-}14)$$

3) Step 3: Find fuzzy weights from Equation 4-15:

$$w_i=\left(\lambda_1\exp(l_i),\lambda_2\exp(m_i),\lambda_3\exp(u_i)\right) \qquad (4\text{-}15)$$

Where $\lambda_1=\left[\sum_{i=1}^{n}\exp\left(u_i\right)\right]^{-1}$ $\lambda_2=\left[\sum_{i=1}^{n}\exp\left(m_i\right)\right]^{-1}$ $\lambda_3=\left[\sum_{i=1}^{n}\exp\left(l_i\right)\right]^{-1}$

4) Step 4: Repeat step 1 to step 3 for all reciprocal matrixes and get solutions for all related linear equations.

4.5 Algorithm of model 2

Model 2 is a modified combination of two methods proposed by other researchers (van Laarhoven and Pedrycz, 1983 and Umeda et al., 2005). The algorithm of model 2 shows how those methods should be applied along with each other to find a solution to the reusability evaluation for products with fast innovations. Model 2 is made of several steps as follows, which are shown in Figure 4.2.

Step 1: Collect required data including: product's information, such as sales, discard amount, fashion, available technologies, purchase timing, discard timing, discard causes, culture, critical function(s) for discarding, and worldwide data. Step 1 is the most important part of the algorithm, as it helps in defining both attributes and alternatives.

Step 2: Construct DCA matrix using information obtained in step 1. A sample DCA matrix and its required calculations are shown in section 4.6. The key point in its required calculations is that in calculating the product of W_{ij} and W_{jk} , the basic rule of matrix multiplication should be considered, and this is true about the result of the multiplication too. (see Appendix B.4)

Step 3: Choose model 1, if physical importance of the product (*PIP*) is greater than or equal to its value importance. Otherwise, continue with model 2.

Step 4: Define alternatives. Although there are four recovery options (see Section 1.3) they are not applicable for every product, and also thinking about alternatives may help in defining attributes better.

Step 5: Define attributes, which are the most important parameters of the model 2. DCA matrix helps in identifying products with fast innovations and gives the analysts some valuable ideas about the attributes of the product, but the analysts need more updated information to define the attributes.

Steps 6: Calculate weights utilizing fuzzy AHP method proposed by Van laarhoven and Predrycz (1983).

Step 7: Construct matrix of importance. Matrix of importance is calculated in the same way as matrix of weights. Section 4.6 provides the required formulas along with a numerical example for calculating it.

Step 8: Using Equation 4-16, the product of matrix of importance, *R*, and weights matrix, *W*, determines which alternative should be selected (see Section 4.6.6).

$$U_i = \sum_{j=1}^{m} W_j r_{ij} \text{ , m = number of alternatives} \tag{4-16}$$

Step 9: Select the best alternative among calculated U_i. The best alternative is the one with the greatest TFN. Note that when TFNs are shown by (*l*, *m*, *u*), the TFN with the biggest parameter *m*, which corresponds to the maximum grade of the TFN's membership function, 1, is the greatest TFN (see Section 4.6.6).

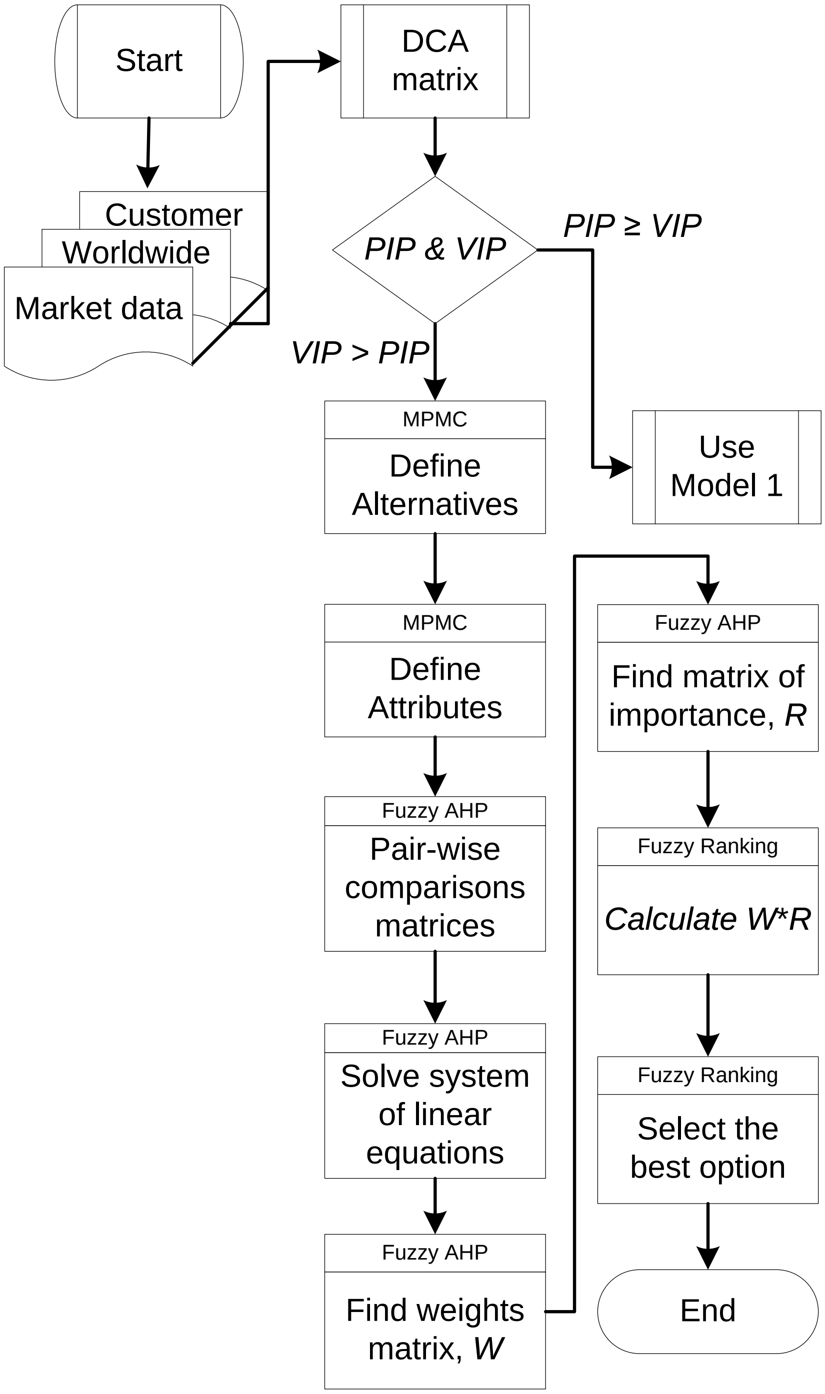


Figure 4.2: A Fuzzy MPMC algorithm for reusability evaluation

4.6 Illustration

According to new legislation, a cellular handset producer, DEF Company, has to either restore returned handsets or recycle them properly. The company has hired three experts to help in solving the problem. They use the proposed fuzzy MPMC method to decide on recovery options.

Consider that the handsets include three main subassemblies and five main functions. The following shows how DEF is utilizing model 2.

Solution steps are:

The first step is to collect return reasons to find out whether reliability is a good parameter for evaluating the reusability of returned handsets or not. Survey results from costumers, who have thrown away the handsets, are shown in Table 4.3.

Table 4.3: DCA matrix for a cellular phone

Disposal Causes (d_i)		Importance (r_i)	Function (f_j)				
			Function 1	Function 2	Function 3	Function 4	Function 5
Physical causes	Use function	**8**	**5**	**3**			
	Failure	**5**	**2**	**2**	**1**		
Value causes	Appearance	**15**		**2**	**8**	**5**	
	Capacity & Size	**12**		**2**	**5**	**5**	
	Value worsening	**10**	**2**		**4**	**2**	**2**
Importance of functions		**50**	**9**	**9**	**18**	**12**	**2**
		Subassemblies (C_k)					
		Subassembly1	**0.5**	**0.0**	**0.1**	**0.3**	**0.5**
		Subassembly2	**0.0**	**0.2**	**0.9**	**0.4**	**0.0**
		Subassembly3	**0.5**	**0.8**	**0.0**	**0.3**	**0.5**
		Product					

	Cause-component matrix					Total Importance M_k	Relative Importance (I_k)	Value Importance (VI_k)	Physical Importance (PI_k)
	d_5	d_4	d_3	d_2	d_1				
Subassembly1	2.5	1.1	2.3	2	3	10.9	21.8%	11.8%	10%
Subassembly2	0.6	1.3	9.6	6.9	4.4	22.8	45.6%	23%	22.6%
Subassembly3	4.9	2.6	3.1	3.1	2.6	16.3	32.6%	21.2%	11.4%
Product (P)						50	100%	**56%**	**44%**

Because *PIP* = 44% and *VIP* = 56%, the product is discarded due to the value lifetime, and model 2 can be applied to decide on recovery options. Appendix B.4 shows DCA calculations.

The next step is to define attributes and options. Results of this step are shown in Figure 4.3. Alternatives can be listed as follows. Note that remanufacturing and refurbishing are considered together. If the product is selected for restoring, then for each single item restoring can be done either by remanufacturing or by refurbishing, depending on the returned item's condition.

- **a)** Cannibalization, X_1
- **b)** Restoring, X_2, (remanufacturing or refurbishing)
- **c)** Recycling, X_3

Attributes are listed as follows, but the list is not exhaustive. These attributes are chosen to illustrate the algorithm only.

1) Demand (A_1)

2) Quality (A_2)
3) Cost (A_3)

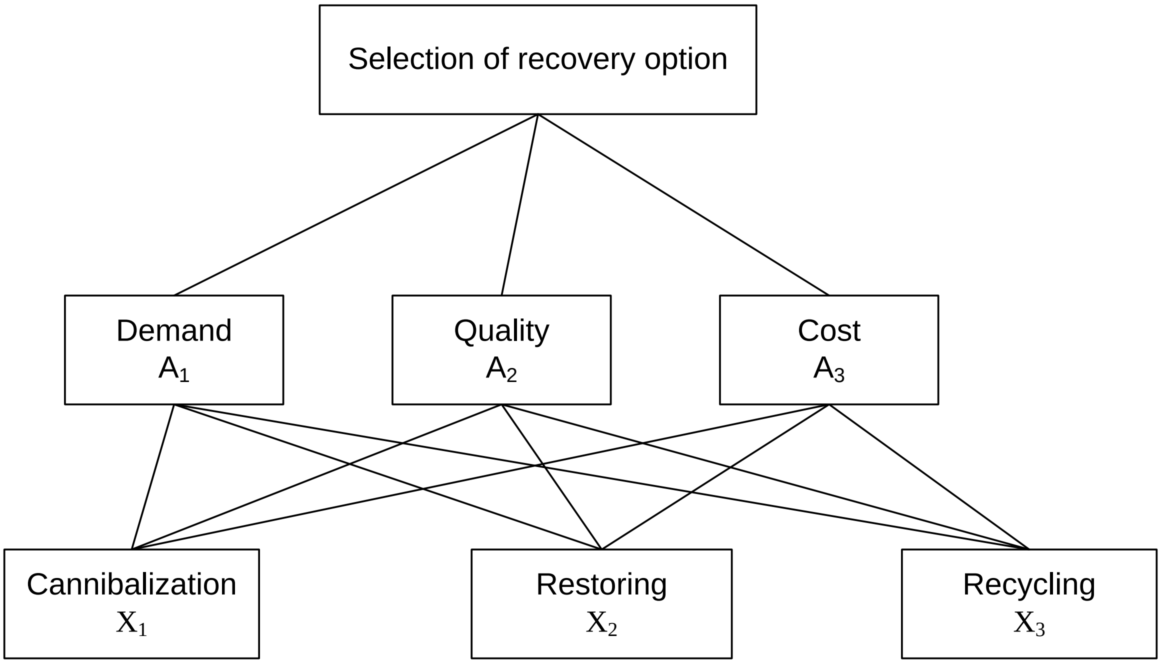


Figure 4.3: Selecting between recovery options-cellular phone case

In order to apply the discussed fuzzy AHP method, first the fuzzy interpretations for pair-wise comparisons are defined as triangular fuzzy numbers (TFNs), which are shown in Table 4.4. Figure 4.4 graphically shows the membership functions for these TFNs.

The next step is to obtain the weights matrix for the attributes by applying the fuzzy AHP method. In order to do that, the fuzzy AHP method, which is explained in section 4.4, is applied as follows:

Table 4.4: Fuzzy interpretation for pair-wise comparisons

Fuzzy value of $\tilde{a}_{ij}$	Interpretation
(1, 1, 1)	If the two attributes are equally important
(2/9, 1/4, 2/7)	If attribute 1 is not important compared to attribute 2
(2/7, 1/3, 2/5)	If attribute 1 has minor importance compared to attribute 2
(2/5, 1/2, 2/3)	If attribute 1 has low importance compared to attribute 2
(2/3, 1, 3/2)	If attribute 1 is moderately more important than attribute 2
(3/2, 2, 5/2)	If attribute 1 is more important than attribute 2
(5/2, 3, 7/2)	If attribute 1 is strongly more important than attribute 2

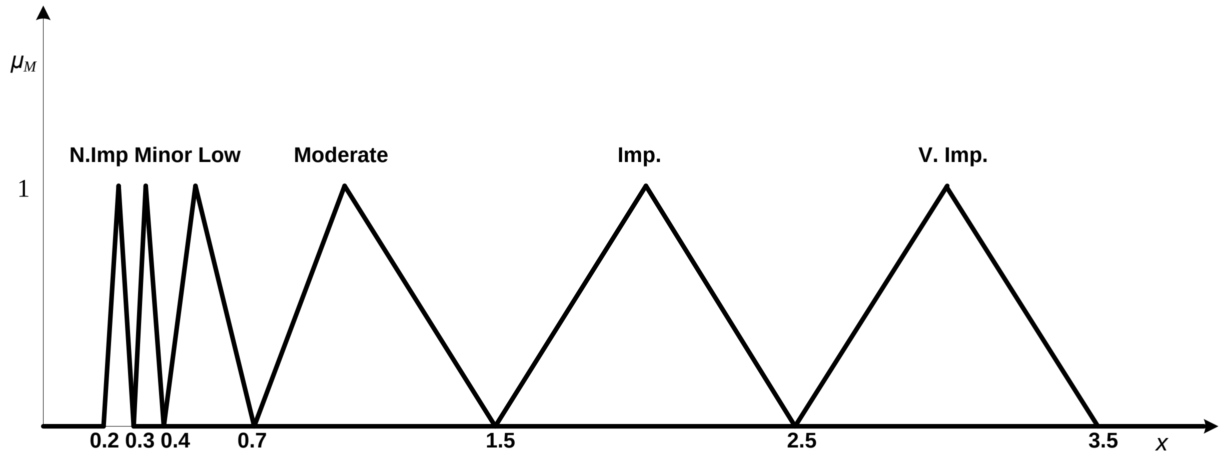


Figure 4.4: Membership functions of interpretations' TFNs

The first step of fuzzy AHP is to obtain *n* +1 reciprocal matrixes, where *n* is number of attributes. Therefore, four matrixes will be formed: three matrixes show the pair-wise comparisons of options for each attribute, and the last one shows pair-wise comparisons of attributes. There are three experts, who express their judgments in fuzzy numbers using Table 4.4. The maximum number of fuzzy numbers in each matrix cell is three when all decision makers (DMs) express their judgments, and the minimum number of fuzzy numbers in each matrix cell is zero when no DM expresses his or her idea. Tables 4.5 to 4.8 show the results of pair-wise comparisons.

Table 4.5: Pair-wise comparisons of recovery options for Demand

	X_1	X_2	X_3
X_1	(1, 1, 1)	(5/2, 3, 7/2) (5/2, 3, 7/2) (2/3, 1, 3/2)	(2/3, 1, 3/2) (2/5, 1/2, 2/3)
X_2	(2/7, 1/3, 2/5) (2/7, 1/3, 2/5) (2/3, 1, 3/2)	(1, 1, 1)	(2/3, 1, 3/2)
X_3	(2/3, 1, 3/2) (3/2, 2, 5/2))	(2/3, 1, 3/2)	(1, 1, 1)

Table 4.6: Pair-wise comparisons of recovery options for quality

	X_1	X_2	X_3
X_1	(1, 1, 1)	(2/5, 1/2, 2/3) (5/2, 3, 7/2)	(5/2, 3, 7/2)
X_2	(3/2, 2, 5/2) (2/7, 1/3, 2/5)	(1, 1, 1)	(2/7, 1/3, 2/5)
X_3	(2/7, 1/3, 2/5)	(5/2, 3, 7/2)	(1, 1, 1)

Table 4.7: Pair-wise comparisons of recovery options for Cost

	X_1	X_2	X_3
X_1	(1, 1, 1)	(5/2, 3, 7/2)	(2/3, 1, 3/2) (2/5, 1/2, 2/3)
X_2	(2/7, 1/3, 2/5)	(1, 1, 1)	(5/2, 3, 7/2)
X_3	(2/3, 1, 3/2) (3/2, 2, 5/2)	(2/7, 1/3, 2/5)	(1, 1, 1)

Table 4.8: Pair-wise comparisons of attributes

	A_1	A_2	A_3
A_1	(1, 1, 1)	(5/2, 3, 7/2) (5/2, 3, 7/2) (2/7, 1/3, 2/5)	(3/2, 2, 5/2) (2/5, 1/2, 2/3)
A_2	(2/7, 1/3, 2/5) (2/7, 1/3, 2/5) (5/2, 3, 7/2)	(1, 1, 1)	(2/7, 1/3, 2/5)
A_3	(2/5, 1/2, 2/3) (3/2, 2, 5/2)	(5/2, 3, 7/2)	(1, 1, 1)

The next step is finding linear equations from equations 4-11 to 4-13 for each matrix of pair-wise comparisons. Required calculations for Table 4.5 are as follows (note that equations 4-11 to 4-13 are shown again to show the calculations better):

$$l_i\left(\sum_{\substack{j=1\\ j\neq i}}^{n} P_{ij}\right)-\sum_{\substack{j=1\\ j\neq i}}^{n} P_{ij}u_j=\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln l_{ijk}\right), \forall i \qquad (4\text{-}11)$$

a) ***i* = 1:** $l_1\left(\sum_{j=2}^{3} P_{1j}\right)-\sum_{j=2}^{3} P_{1j}u_j=\sum_{j=2}^{3}\sum_{k=1}^{p_{1j}}\left(\ln l_{1jk}\right)$

$$l_1\left(p_{12}+p_{13}\right)-\left(p_{12}u_2+p_{13}u_3\right)=\sum_{j=2}^{3}\sum_{k=1}^{p_{1j}}\ln\left(l_{1jk}\right)$$

$$l_1\left(p_{12}+p_{13}\right)-\left(p_{12}u_2+p_{13}u_3\right)=\sum_{k=1}^{p_{12}}\ln\left(l_{12k}\right)+\sum_{k=1}^{p_{13}}\ln\left(l_{13k}\right),\ p_{12}=3,\ p_{13}=2$$

$$5l_1-\left(3u_2+2u_3\right)=\ln(5/2)+\ln(5/2)+\ln(2/3)+\ln(2/5)+\ln(2/3)$$

$$\underline{5l_1-3u_2-2u_3=0.1054}$$

b) ***i* = 2:** $l_2\left(\sum_{j=1}^{3} P_{2j}\right)-\sum_{j=1}^{3} P_{2j}u_j=\sum_{j=1}^{3}\sum_{k=1}^{p_{2j}}\left(\ln l_{2jk}\right),\ j\neq 2$

$$l_2\left(p_{21}+p_{23}\right)-\left(p_{21}u_1+p_{23}u_3\right)=\sum_{j=1}^{3}\sum_{k=1}^{p_{2j}}\ln\left(l_{2jk}\right),\ j\neq 2,\ p_{21}=3,\ p_{23}=1$$

$$l_2\left(p_{21}+p_{23}\right)-\left(p_{21}u_1+p_{23}u_3\right)=\sum_{k=1}^{p_{21}}\ln\left(l_{21k}\right)+\sum_{k=1}^{p_{23}}\ln\left(l_{23k}\right),\ p_{21}=3,\ p_{23}=1$$

$$4l_2-3u_1-u_3=2\ln(2/7)+2\ln(2/3)$$

$$\underline{4l_2-3u_1-u_3=-3.3165}$$

c) ***i* = 3:** $l_3\left(\sum_{j=1}^{2}P_{3j}\right)-\sum_{j=1}^{2}P_{3j}u_j=\sum_{j=1}^{2}\sum_{k=1}^{p_{3j}}\left(\ln l_{3jk}\right)$

$$l_3\left(p_{31}+p_{32}\right)-\left(p_{31}u_1+p_{32}u_2\right)=\sum_{k=1}^{p_{31}}\ln\left(l_{31k}\right)+\sum_{k=1}^{p_{32}}\ln\left(l_{32k}\right)$$

$$l_3\left(p_{31}+p_{32}\right)-\left(p_{31}u_1+p_{32}u_2\right)=\ln(2/3),\ p_{31}=2,\ p_{32}=1$$

$$\underline{3l_3-2u_1-u_2=-0.4055}$$

$$m_i\left(\sum_{\substack{j=1\\j\neq i}}^{n}P_{ij}\right)-\sum_{\substack{j=1\\j\neq i}}^{n}P_{ij}m_j=\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln m_{ijk}\right),\forall i \qquad (4\text{-}12)$$

d) ***i* = 1:** $m_1\left(\sum_{j=2}^{3}P_{1j}\right)-\sum_{j=2}^{3}P_{1j}m_j=\sum_{j=2}^{3}\sum_{k=1}^{p_{1j}}\left(\ln m_{1jk}\right)$

$$m_1\left(p_{12}+p_{13}\right)-\left(p_{12}m_2+p_{13}m_3\right)=\sum_{j=2}^{3}\sum_{k=1}^{p_{1j}}\ln\left(m_{1jk}\right)$$

$$m_1\left(p_{12}+p_{13}\right)-\left(p_{12}m_2+p_{13}m_3\right)=\sum_{k=1}^{p_{12}}\ln(m_{12k})+\sum_{k=1}^{p_{13}}\ln(m_{13k})$$

$$5m_1-3m_2-2m_3=2\ln(3)+\ln(1/2)$$

$$\underline{5m_1-3m_2-2m_3=1.5041}$$

e) ***i* = 2:** $m_2\left(\sum_{j=1}^{3} P_{2j}\right) - \sum_{j=1}^{3} P_{2j} m_j = \sum_{j=1}^{3}\sum_{k=1}^{p_{2j}}\left(\ln m_{2jk}\right),\ j \neq 2$

$4m_2 - 3m_1 - m_3 = 2\ln(1/3)$

$\underline{4m_2 - 3m_1 - m_3 = -2.1972}$

f) ***i* = 3:** $m_2\left(\sum_{j=1}^{2} P_{3j}\right) - \sum_{j=1}^{2} P_{3j} m_j = \sum_{j=1}^{2}\sum_{k=1}^{p_{3j}}\left(\ln m_{3jk}\right)$

$3m_3 - 2m_1 - m_2 = \ln(2)$

$\underline{3m_3 - 2m_1 - m_2 = 0.6931}$

$$u_i\left(\sum_{\substack{j=1\\ j\neq i}}^{n} P_{ij}\right) - \sum_{\substack{j=1\\ j\neq i}}^{n} P_{ij} l_j = \sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{k=1}^{p_{ij}}\left(\ln u_{ijk}\right), \forall i \qquad (4\text{-}13)$$

g) ***i* =1:** $5u_1 - 3l_2 - 2l_3 = 2\ln(7/2) + \ln(3/2)$

$\underline{5u_1 - 3l_2 - 2l_3 = 2.91110}$

h) ***i* = 2:** $\underline{4u_2 - 3l_1 - l_3 = -1.0217}$

i) ***i* = 3:** $\underline{3u_3 - 2l_1 - l_2 = 1.7272}$

4.6.1 Linear equations for matrix of pair-wise comparisons for demand

The system of linear equations calculated for Table 4.5 is as follows. Note that finding the underlined equations resulting from application of any of equations 4-11 to 4-13 to any Table (4. 5 to 4.8) simplifies the calculation for the others, as they have the same pattern. This is shown in the last calculation for finding the last underlined equations.

The system of equations is solved with MATLAB as shown in Appendix B.5, and the results are shown in Table 4.9. Note that solving this system of equations by using regular methods is not possible, as the matrix for equations is close to singular and its inverse cannot be obtained. Solving the problem is possible by using “pinv” command in MATLAB, and all other systems of equations are solved in the same way.

$5m_1 - 3m_2 - 2m_3 = 1.5041$

$4m_2 - 3m_1 - m_3 = -2.1972$

$3m_3 - 2m_1 - m_2 = 0.6931$

$5l_1 - 3u_2 - 2u_3 = 0.1054$

$4l_2 - 3u_1 - u_3 = -3.3165$

$3l_3 - 2u_1 - u_2 = -0.4055$

$5u_1 - 3l_2 - 2l_3 = 2.91110$

$4u_2 - 3l_1 - l_3 = -1.0217$

$3u_3 - 2l_1 - l_2 = 1.7272$

Table 4.9: Solutions to the system of equations for Table 4.4

i	***l_i***	***m_i***	***u_i***	**exp(*l_i*)**	**exp(*m_i*)**	**exp(*u_i*)**
1	0.0767	0.1613	0.2489	1.0797	1.1750	1.2826
2	-0.5299	-0.3749	-0.2075	0.5887	0.6874	0.8126
3	-0.0384	0.2136	0.4502	0.9623	1.2381	1.5686

4.6.2 Linear equations for matrix of pair-wise comparisons for quality

The systems of equations calculated for Table 4.6 are as follows:

$3m_1 - 2m_2 - m_3 = 1.5041$

$3m_2 - 2m_1 - m_3 = -1.5041$

$2m_3 - m_1 - m_2 = 0$

$3l_1 - 2u_2 - u_3 = 0.9163$

$3l_2 - 2u_1 - u_3 = -2.1001$

$2l_3 - u_1 - u_2 = -0.3365$

$3u_1 - 2l_2 - l_3 = 2.1001$

$3u_2 - 2l_1 - l_3 = -0.9163$

$2u_3 - l_1 - l_2 = 0.3365$

4.6.3 Linear equations for matrix of pair-wise comparisons for cost

The systems of equations calculated for Table 4.7 are as follows:

$$3m_1 - m_2 - 2m_3 = 0.4055$$

$$2m_2 - m_1 - m_3 = 0$$

$$3m_3 - 2m_1 - m_2 = -0.4055$$

$$3l_1 - u_2 - 2u_3 = -0.4055$$

$$2l_2 - u_1 - u_3 = -0.3365$$

$$3l_3 - 2u_1 - u_2 = -1.2528$$

$$3u_1 - l_2 - 2l_3 = 1.2528$$

$$2u_2 - l_1 - l_3 = 0.3365$$

$$3u_3 - 2l_1 - l_2 = 0.4055$$

4.6.4 Linear equations for matrix of pair-wise comparisons of attributes

The systems of equations calculated for Table 4.8 are as follows:

$$5m_1 - 3m_2 - 2m_3 = 1.0986$$

$$6m_2 - 3m_1 - 3m_3 = -2.1972$$

$$5m_3 - 2m_1 - 3m_2 = 1.0986$$

$$5l_1 - 3u_2 - 2u_3 = 0.0690$$

$$6l_2 - 3u_1 - 3u_3 = -2.8420$$

$$5l_3 - 2u_1 - 3u_2 = 0.4055$$

$$5u_1 - 3l_2 - 2l_3 = 2.1001$$

$$6u_2 - 3l_1 - 3l_3 = -1.4961$$

$$5u_3 - 2l_1 - 3l_2 = 1.7636$$

Table 4.10: The exponentials of all solutions

i	Table 4.5	Table 4.6	Table 4.7	Table 4.8
	exp(l_i), exp(m_i), exp(u_i)	exp(l_i), exp(m_i), exp(u_i)	exp(l_i), exp(m_i), exp(u_i)	exp(l_i), exp(m_i), exp(u_i)
1	1.0797, 1.1750, 1.2826	1.2162, 1.3509, 1.5032	0.9227, 1.0845, 1.284	0.9713, 1.1051, 1.2540
2	0.5887, 0.6874, 0.8126	0.6652, 0.7402, 0.8223	0.9970, 1.0000, 1.0030	0.6437, 0.6707, 0.7020
3	0.9623, 1.2381, 1.5686	0.9396, 1.0000, 1.0643	0.7788, 0.9221, 1.0838	1.1831, 1.3493, 1.5273

4.6.5 Calculating weights

To calculate weights, equation 4-15 is applied as follows:

$$\lambda_1 = \left[\sum_{i=1}^{3} \exp(u_i)\right]^{-1} = (3.4833)^{-1} = 0.2871$$

$$\lambda_2 = \left[\sum_{i=1}^{3} \exp(m_i)\right]^{-1} = (3.1251)^{-1} = 0.3200$$

$$\lambda_3 = \left[\sum_{i=1}^{3} \exp(l_i)\right]^{-1} = (2.8017)^{-1} = 0.3569$$

$$w_i = \left(\lambda_1 \exp(l_i), \lambda_2 \exp(m_i), \lambda_3 \exp(u_i)\right) \tag{4-15}$$

W = [(0.2789, 0.3536, 0.4476), (0.1858, 0.2146, 0.2505), (0.3397, 0.4318, 0.5451)]

4.6.6 Ranking the alternatives

To rank the alternatives, first their performance scores are calculated. After that the product of the matrix of performance scores and the weights matrix determines which alternative should be chosen.

The matrix of performance scores is calculated in the same way as the weights matrix. For example, considering Table 4.5, the performance score of each alternative for demand is calculated as follows:

$$r_{11} = \left(\lambda_1 \exp(l_1), \lambda_2 \exp(m_1), \lambda_3 \exp(u_1)\right) = (0.3100, 0.3760, 0.4578)$$

$$r_{21} = \left(\lambda_1 \exp(l_2), \lambda_2 \exp(m_2), \lambda_3 \exp(u_2)\right) = (0.1690, 0.2200, 0.2900)$$

$$r_{31} = \left(\lambda_1 \exp(l_3), \lambda_2 \exp(m_3), \lambda_3 \exp(u_3)\right) = (0.2763, 0.3962, 0.5598)$$

The matrix of performance scores of all alternatives for all attributes is shown in Table 4.11.

Table 4.11: Matrix of performance scores for recovery options

	A_1	A_2	A_3
X_1	(0.3100, 0.3760, 0.4578)	(0.3492, 0.4323, 0.5365)	(0.2649, 0.3470, 0.4583)
X_2	(0.1690, 0.2200, 0.2900)	(0.1910, 0.2369, 0.2935)	(0.2862, 0.3200, 0.3580)
X_3	(0.2763, 0.3962, 0.5598)	(0.2698, 0.3200, 0.3798)	(0.2236, 0.2951, 0.3868)

The last step is to rank alternatives by applying equation 4-16, which is the product of the matrix of performance scores and the weights matrix. Calculating alternatives' ranks is as follows:

$$U_i = \sum_{j=1}^{m} W_j r_{ij} \text{ , m = number of alternatives} \tag{4-16}$$

U_1 = ([0.31 0.3492 0.2649]$\times$[0.2789; 0.1858; 0.3397], [0.3760 0.4323 0.3470]$\times$[0.3536; 0.2146; 0.4318], [0.4578 0.5365 0.4583]$\times$[0.4476; 0.2505; 0.5451])

U_2 = ([0.169 0.191 0.2862]$\times$[0.2789; 0.1858; 0.3397], [0.22 0.2369 0.32]$\times$[0.3536; 0.2146; 0.4318], [0.29 0.2935 0.358]$\times$[0.4476; 0.2505; 0.5451])

U_3 = ([0.2763 0.2698 0.2236]$\times$[0.2789; 0.1858; 0.3397], [0.3962 0.32 0.2951]$\times$[0.3536; 0.2146; 0.4318], [0.5598 0.3798 0.3868]$\times$[0.4476; 0.2505; 0.5451])

U_1 = (0.2413, 0.3756, 0.5891)
U_2 = (0.1798, 0.2668, 0.3985)
U_3 = (0.2031, 0.3362, 0.5566)

$$\left(m_1 = 0.3756\right) > \left(m_2 = 0.2668\right) > \left(m_3 = 0.3362\right)$$

Therefore, the first alternative, cannibalization, is the best recovery option for the used cellular phone type under study.

The membership functions of ranking functions of alternatives are graphically shown in Figure 4.5. Mathematical expression of U_1 can be found in equation 4-17.

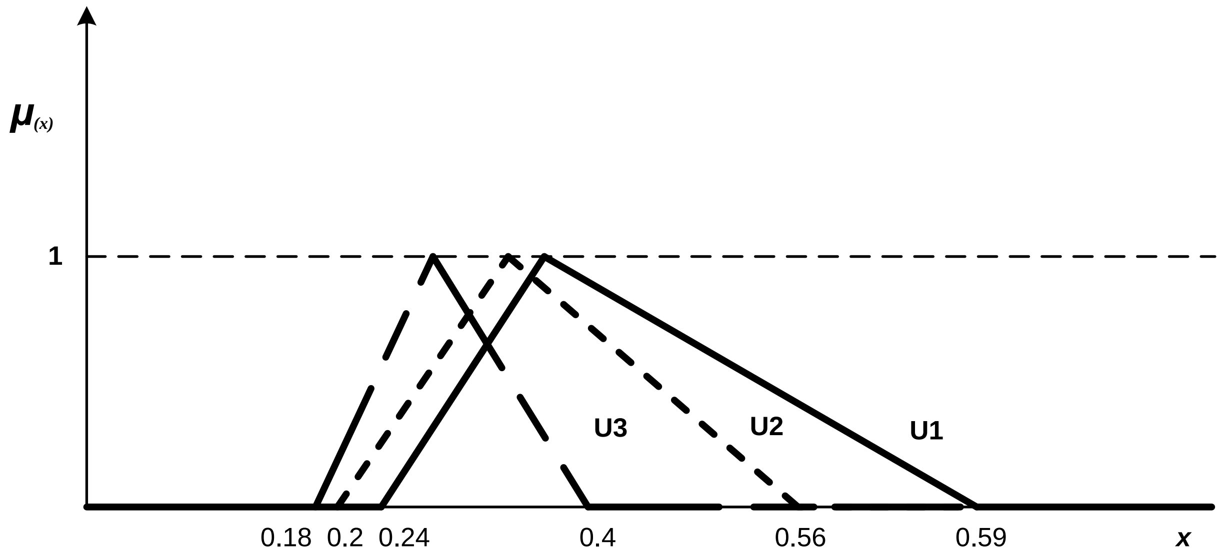


Figure 4.5: TFNs for ranked recovery options/alternatives

$$\mu_{(x)} = \begin{cases} \dfrac{x-0.24}{0.14}, x \in [0.24, 0.38] \\ \\ \dfrac{0.59-x}{0.21}, x \in [0.38, 0.59] \\ \\ 0, Otherwise \end{cases} \tag{4-17}$$

4.7 Results

In this chapter the second model for reusability evaluation is presented, which is applicable to products with fast innovations. This model along with the reliability based model presented in chapter 3 provides a solution for the problem of reusability evaluation for product recovery.

While physical lifetime of a product is a function of its service time, value lifetime is a function of time. It means that a product's value decreases from the shipment time regardless of its usage. In this chapter, the presented fuzzy model helps in evaluating the reusability for products that are discarded due to their value lifetime or products with fast innovations. The model can serve as a practical one only if it is attached to a strong and effective global information system, which can collect required data.

Information plays a vital role in model 2, as decision makers need updated and complete information to make timely deciding on recovery options for this type of product. For example, refurbishing might be a good recovery option for a digital camera today, but not next month, as remanufacturing could have been a good option for TVs in previous years, but not these days when customers are looking for the newest model.

Next chapter presents a simulation model of the main application of model 1. An (s, Q) inventory control system with returns is simulated, in which the reliability of returns are measured against the reliability threshold. Only those that pass the test will go through remanufacturing or refurbishing operations. The rest of the returned products will be selected for recycling operations.

The main goal of presenting the simulation model is to provide a reliable and powerful tool for manufacturers to investigate the effect of system parameters on total costs when recovery operations are added to the manufacturing ones.

Chapter 5

Model 3: A simulation model using Arena® to present the application of model 1

5.1 Introduction

Chapters 3 and 4 present two models to evaluate the reusability for product recovery: Model 1, and Model 2, which together provide an answer to the problem of reusability evaluation in the reverse logistics. This chapter presents the third model, a simulation model, which is built to show the application of the first model in manufacturing systems.

As the main application of the reusability evaluation models is in an inventory control system, the focus of the simulation model is on the inventory system of a manufacturer with (s, Q) inventory control policy taking into account the manufacturing and remanufacturing operations. Model 1 is chosen for simulation as it covers most cases. In other words, most of the products are returned due to failures. Also, in Model 1, different recovery options can be selected, but in Model 2, once the decision on the recovery option selection is made, all used products will be operated under that selected option; therefore, model 1 is more general and shows the recovery operations better as well.

The application of Model 1 is shown by utilizing the same hypothetical data presented in section 3.7: ABC Company wants to start recovery operations for an item (e.g. a starter); it knows the failure probability distribution of its new remanufactured item and the upper limit of warranty costs to make the contract profitable. It applies Model 1 to obtain the reliability threshold, R^*. Next, it applies R^* to evaluate the reusability of returns. Applying that threshold to select the proper recovery option along with other operations is shown in the simulation model.

The proposed simulation model is built after completely studying the system. It offers a practical and powerful tool to study the system under various scenarios. It removes many limitations, which have to be considered in analytic models. Also, it can be used to help in finding the optimums of a complex and highly variable stochastic inventory control system with returns by using *OptQuest®* software by Arena, which uses heuristic methods. Showing this benefit of the simulation model can be considered as a future work.

Therefore, several goals can be achieved by analyzing the simulation model's outputs. For instance, one possible goal is to investigate the influence of system parameters, such as lead-times, holding costs, and return rate on system performance. By reaching these goals, the simulation analyst can help ABC in establishing a profitable remanufacturing system and in selecting the best inventory control policy for used items. In addition, the simulation model will help ABC to know which information should be collected to start and maintain recovery lines.

5.2 Objective

The objective is to investigate the effects of adding recovery operations to manufacturing system parameters, such as average manufacturing cost, inventory level, holding costs, total cost, and backorder costs.

To achieve this goal, a producer with an (s, Q) inventory control policy is considered: The producer always has a safety stock on hand, whenever the inventory level reaches a quantity equal to or less than that safety level, a manufacturing order of a specified amount of product will be released to the production line (see Section 5.4.1).

The producer's inventory control system before and after adding the recovery processes is simulated and the outputs are statistically analyzed. A comparison between the results of these two scenarios is made to provide a picture for realizing what would be the effects of both using model 1 on system parameters for selecting between recovery options and of adding recovery operations.

5.3 System study

In this study, an auto spare parts producer, ABC Company, with an (s, Q) inventory control system is considered. ABC wants to start recovery operations for one of its popular products, a starter (see Section 3.7). The company wants to investigate how profitable it would be to add a recovery line and what would be the effects of adding recovery operations to its inventory control system for that specific product, the starter.

ABC is evaluating the reusability of old products by using model 1 to ensure that costs of warranty claims for its new remanufactured item will not pass the upper bound of warranty costs, which is determined by ABC. The system that ABC wants to simulate is shown in Figure 5.1. The results obtained in Section 3.7 are as follows:

- The item's failure time is exponentially distributed with the rate of $\hat{\lambda} = 0.041$.
- Applying model 1 indicates: the best warranty period = 12 months and $R^* = 61.14\%$.

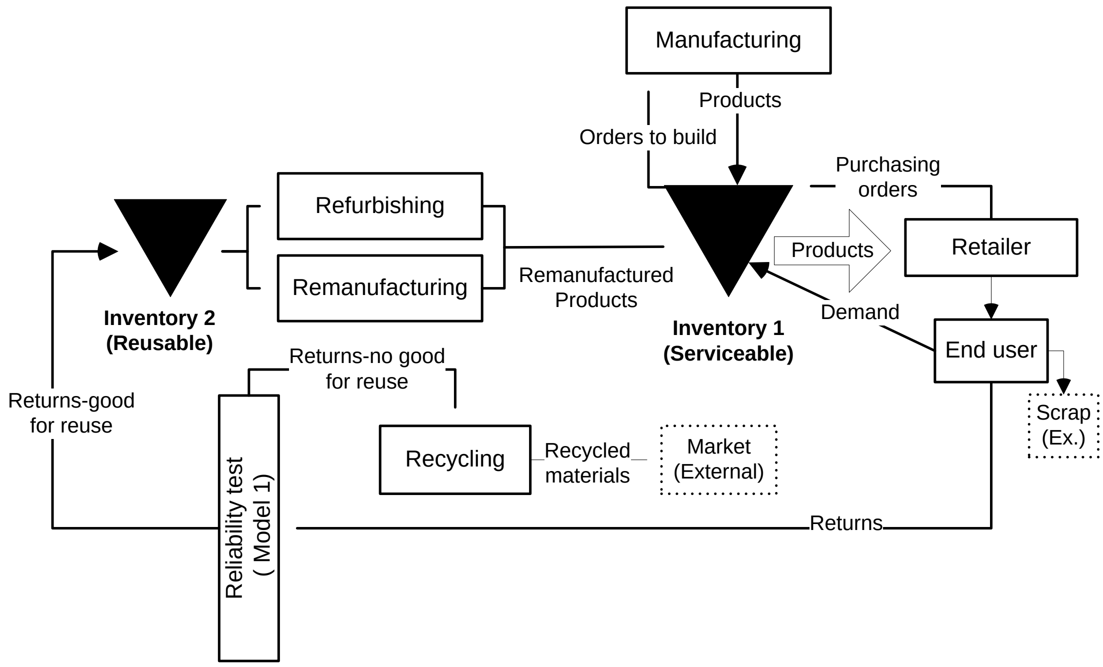


Figure 5.1: A green manufacturing system with reusability evaluation test

As is shown in Figure 5.1, the inventory control system of ABC with returns will consist of two inventories: *serviceable products inventory*, and *reusable products inventory*. Serviceable inventory refers to the inventory of finished products, and reusable inventory refers to the inventory of returned used products, which are selected to be prepared for reuse. Used products are received and those whose reliability measure is greater than the threshold, are moved into the second inventory (reusable inventory). There are several policies defined for controlling the flow of used products. Under one of those policies, used products will be kept in the second inventory until their number reaches a specified level (e.g. *Level2*), and then they will be restored by either remanufacturing or refurbishing depending on their condition. This policy has been referred to as *PUSH* policy (van der Laan, 1997). In contrast, another policy is to assume that a restoring operation will start when the serviceable inventory level is at or below a certain level, which has been referred to as *PULL* policy (van der Laan, 1997). The third policy can be defined under which every single used item that is suitable to be prepared for reuse (see Figure 5.1) will go through a restoring process immediately.

Assume that for this study, the *PUSH* policy is chosen by ABC, which seems more practical than the third policy. It is more practical due to the uncertain results of restoring operations and high setup costs that would be associated with the restoration of single items. ABC does not want to change its current (s, Q) policy, so it rejects the *PULL* policy. As a result, the parameters that should be determined to build the simulation model of the system are as follows:

- Returns rate or returns inter arrivals
- R^* (reliability threshold)
- Manufacturing, remanufacturing, refurbishing, and recycling lead times
- Manufacturing, remanufacturing, refurbishing, and recycling costs per unit.
- Holding costs associated with inventory 1, and with inventory 2
- Manufacturing batch size (see Section 5.4.1)
- Purchase orders' inter arrivals
- Manufacturing orders' timing (see Section 5.4.1)
- Demand size (or purchasing orders' quantities)
- Capacity of inventories

5.4 System behaviour and its characteristics

The focus of the study is on the inventory control system of the ABC Company. To meet the objective, two scenarios are considered: inventory control system without recovery operations (Scenario A), and inventory control system with recovery operations (Scenario B). To build the simulation models, the systems' behaviour and their characteristics should be known. Sections 5.4.1 to 5.4.2 discuss the system behaviour and its characteristics under each scenario.

5.4.1 Scenario A

Figure 5.2 graphically shows ABC's inventory system's behaviour before adding recovery operations (a traditional inventory control system with (s, Q) policy):

Attempts are always made to keep the serviceable inventory level, $I(t)$, at safety level (s). Whenever $I(t) \leq s$, a manufacturing order with quantity of ($s - I(t)+Q$) will be released, and manufacturing processes will start. The manufactured items will be added to the serviceable inventory when the manufacturing operations with a manufacturing process lead time, are finished.

The manufacturing process lead time is stochastic and is denoted by Lm. The inventory evaluation period is denoted by Δt. Therefore, the inventory system is controlled by three rigid variables: s, Q, and Δt.

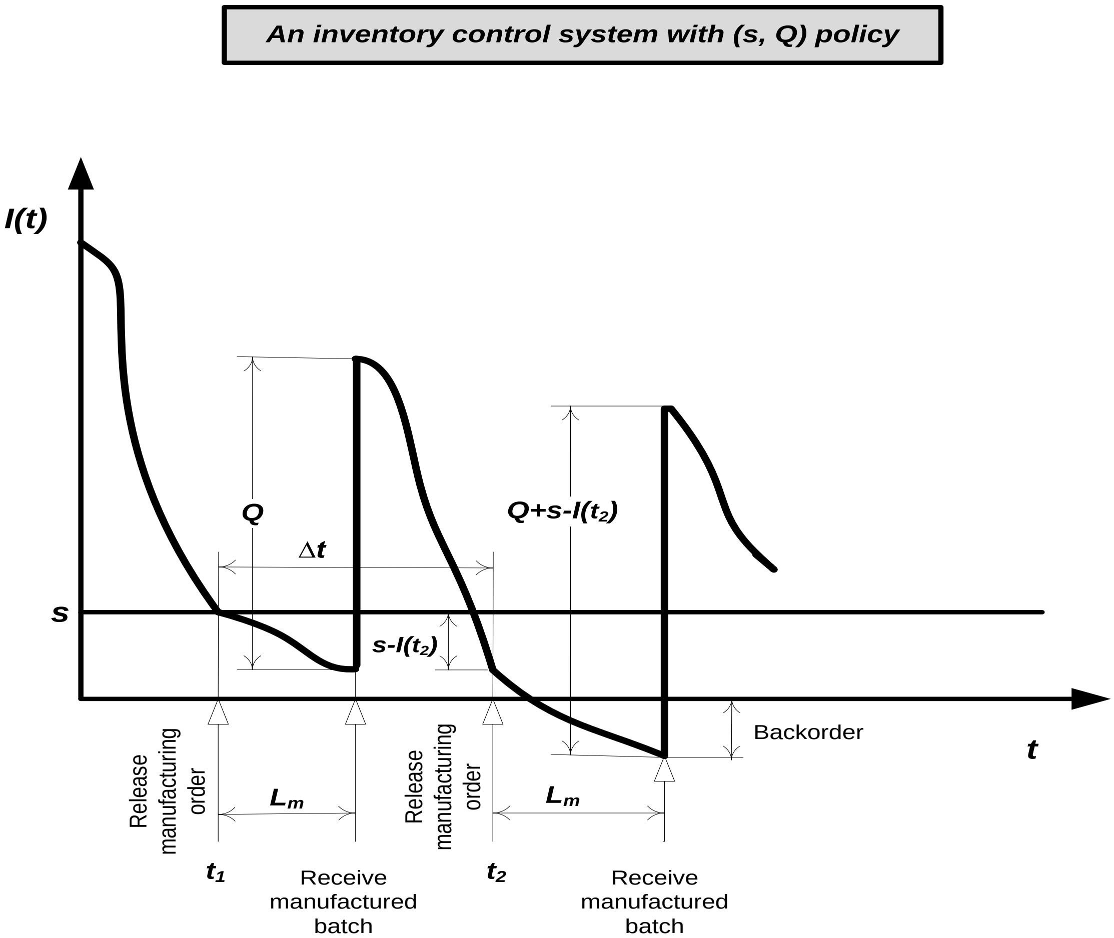


Figure 5.2: ABC's inventory control system without recovery operations

Identifying the system's characteristics will help in finding the important system's parameters. This is done by utilizing the "*five Ws & H*" concept as follows:

- What is the goal of inventory control? How can it be achieved?
  - ✓ To minimize backorders/shortages cost and it can be achieved by choosing the optimum values for *s*, *Q*, and Δ *t*. In the present case, ABC determines them.
- How many item types are kept in the inventory?
  - ✓ An inventory with a single item, a starter, is considered.
- What parameters control the inventory and how?
  - ✓ *s*, *Q*, and Δ *t; s* refers to the safety level that ABC tries to keep its inventory level at. *Q* determines the size of the manufacturing batch, and the inventory level is checked periodically at interval times of Δ *t*.
- When is a manufacturing order released? What is the order quantity?

- ✓ Whenever *inventory level* $\leq s$, a manufacturing order with quantity of (s - $I(t)+Q$) will be released where $I(t)$ refers to the inventory level at time t.
- ➢ When is the manufactured batch received?
  - ✓ It takes a stochastic lead time (*Lm*) starting right after releasing an order.
- ➢ What are the other costs associated with the system?
  - ✓ Costs associated with manufacturing are: setup cost per batch, and manufacturing cost per unit. Costs associated with the inventory are: holding cost/item/day, and shortages cost/item/day.
- ➢ How often is a purchasing order received? How many items are in the order?
  - ✓ Customers' order inter-arrival times and the number of items per order are stochastic variables, which are estimated by ABC.
- ➢ Who provides the required information to estimate system parameters?
  - ✓ Estimated system parameters, such as *Lm*, demand, customers' order inter arrivals, and all cost elements are provided by ABC, but the input analysis is performed by the analyst when needed.
- ➢ When is the inventory level checked? Is the inventory monitored automatically?
  - ✓ The inventory clerk checks it regularly within a defined period (Δt). The system is computerized, and the manufacturing order is released by the inventory clerk.

Finding system characteristics helps the analyst to know what parameters to look for. He/she can get the required information from the producer or should find them by using sampling methods.

5.4.2 Scenario B

Figure 5.3 shows ABC's inventory system's behaviour with the recovery operations:
Those used products that pass the reliability test will be kept in the inventory 2 (reusable inventory) until the number of them reaches a specified number; then restoring operations will start. When restoring operations are finished, the restored products will be added to the inventory 1 (serviceable). Note that while a general graph of two inventories (inventory 1 and inventory 2) are shown together, the graph related to inventory 1, I_1 (t), shows how products are released from the inventory 1, but the related graph to inventory 2, I_2 (t), shows how used products are collected in the inventory 2 after passing the reliability test.

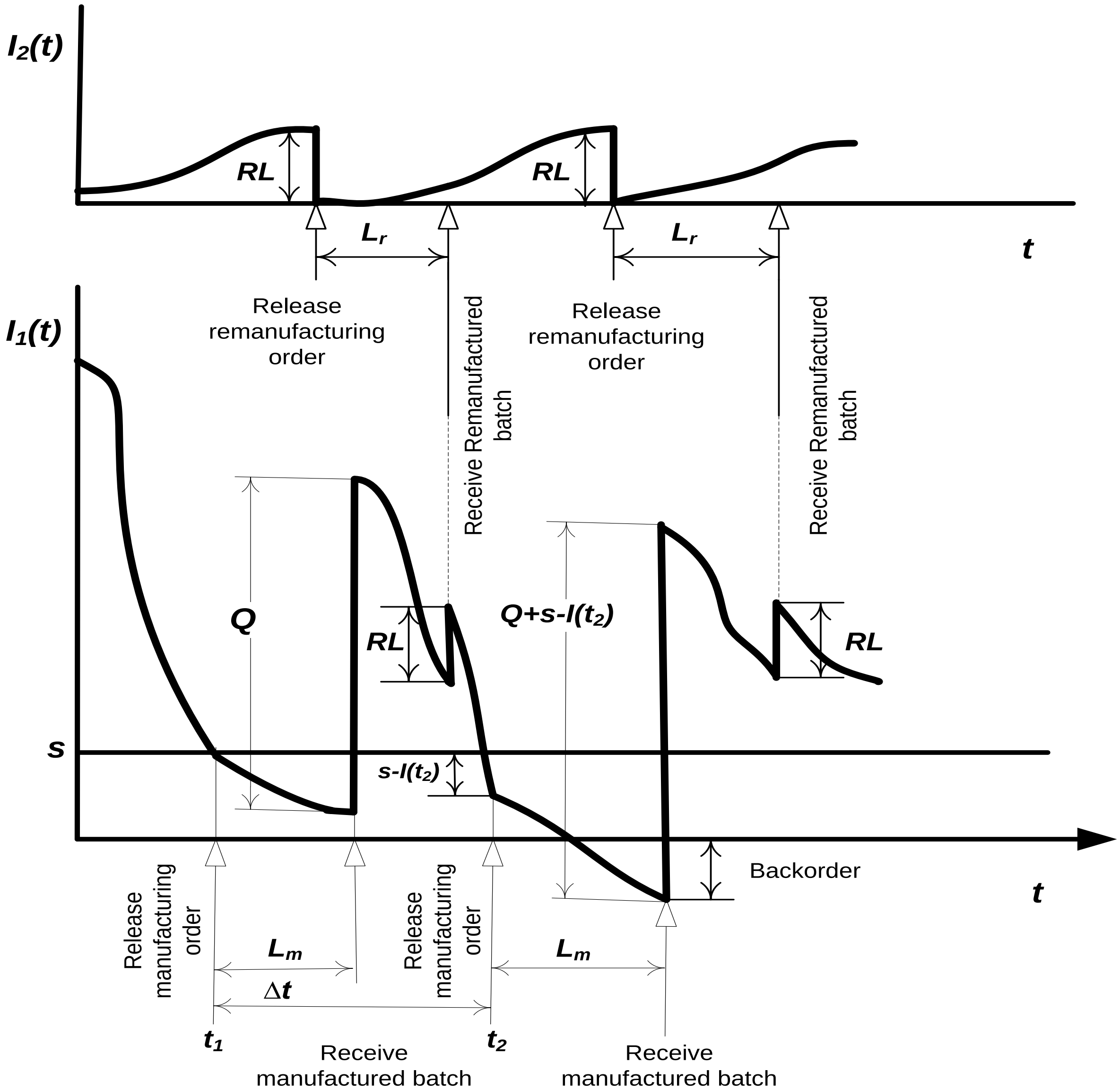


Figure 5.3: ABC's inventory control system after adding the recovery operations (an *s* and *Q* inventory control with *PUSH* policy for returns)

Note that here the remanufacturing term refers to both remanufacturing and refurbishing, and it will be considered the same in the rest of this chapter for simplicity.

As a result of studying the system, the system is highly variable, and uncertainties exist in every parameter of it, such as demand, lead times, returns, and testing procedure, so it justifies building a simulation model. The system characteristics can be summarized as follows:

i. Demand rate for the product, recovery operations, and collection rate of old products are all stochastic.

ii. There are two sources to meet the demand, i.e. *new products* and *remanufactured products*.
iii. The reliability of all returns will be measured against a threshold, R^*/T^*. Those that can be remanufactured will be kept in inventory 2 and the rest will be recycled.
iv. Remanufacturing processes will start immediately after the number of collected used products reaches a specified number (*RL* or *Level2*).
v. *Reusable* source is not always available, while it is reasonable to assume there is always a supplier for *new products* source.
vi. Remanufactured products are sold first.
vii. The new products' source is more reliable than the remanufactured products' source, due to its random availability.
viii. There is a high variation in the quality of subassemblies, so remanufacturing operations, such as cleaning, repairing, and testing are highly variable.

Note that the main difference between what this simulated stochastic inventory model offers, and what previous models offer is:

- A reliability based control system is attached to evaluate the reusability of returns, which is not considered in previous models.

5.5 Analyzing the model - cost elements

The objective of this section is to find required cost formulations for this simulation model. To analytically investigate the cost elements of the system, subsystems are numbered as nodes, which are shown in Figure 5.4. A cost is associated with each node from the inventory control point of view (except node 1). These nodes help the analyst to completely review all cost elements without missing any, and node 1 helps in estimating the costs associated with others.

One point to take into consideration is that all costs formulations are considered for a general situation to be applicable in any similar system; some of them appear differently in this simulation model due to the attempt of making formulas simpler.

For example, in a real situation, there is a fixed remanufacturing cost associated with each batch of reusable products and a variable remanufacturing cost associated with each reusable product, but for simplicity only a constant remanufacturing cost per item is considered in the simulation model. The cost formulations at each node shown in Figure 5.4 are as follows:

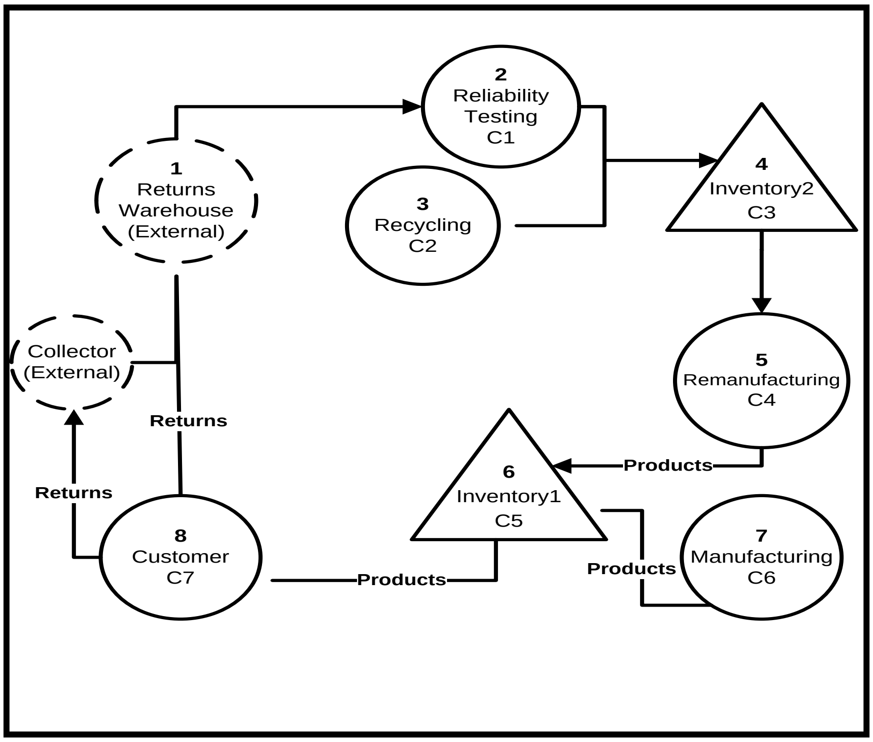


Figure 5.4: Costs after adding recovery operations

Node 1) *returns:* Stochastic returns and stochastic demands are considered. An Exponential inter-arrival for returns and a discrete probability distribution for demand are considered.

Node 2) *testing:* While in some models it has been assumed that all returns are qualified for recovery, in this model testing is a stochastic process that generally includes a reliability check, appearance check, and so on. An immediate testing process is considered, which involves a variable cost, C_{Vt}, per item. Total testing cost, C_1, can be obtained as:

C_1 = *average number of arrivals per time* (n_1) * C_{Vt}

Node 3) *recycling:* It is a variable process with variable cost, C_{Vp}, per recycled item and fixed cost, C_{Fp} per batch. Apparently, the number of recycled items is equal to the total number of returned items minus the total number of recovered items, and total recycling cost, C_2, is:

C_2 = *average number of recycled batches per time* (n_2)* C_{Fp} +*average number of recycled per time* (n_3)* C_{Vp}

Node 4) *reusable inventory:* This inventory is considered to have an unlimited capacity and a holding cost of C_{hR}. It is reasonable to assume that the holding cost for reusable items is less than the holding cost for serviceable items. The objective is to find the optimum level of reusable inventory to start the recovery processes. Total holding cost for reusable items, C_3, is:

C_3 = *average number of reusable items per time* (n_4)* C_{hR}

Node 5) *restoring or remanufacturing*: Consists of two operations: refurbishing and remanufacturing, both of which will restore the item to an 'as-good-as-new' condition; refurbishing is less costly than remanufacturing as it requires minor operations. Parameters related to remanufacturing are:

- Lead time, L_R, which is assumed as a random variable with triangular distribution.
- Fixed remanufacturing cost, C_{FR}, per batch, and variable remanufacturing cost, C_{VR}, per item.
- Fixed refurbishing cost, C_{Ff}, per batch, and variable refurbishing cost, C_{Vf}, per item.
- Fixed setup cost, C_{SR}, per order, and unlimited capacity.

Total cost of remanufacturing, C_4, is:

C_4 = average number of remanufactured batches per time (n_5) C_{FR} + average number of remanufactured items per time (n_6)* C_{VR} + average number of refurbished batches per time (n_7)* C_{Ff} + average number of refurbished items per time (n_8)* C_{Vf} + average number of orders for remanufactured items per time (n_9)* C_{SR}*

Node 6) *serviceable inventory:* This inventory has been considered with unlimited capacity, and the holding cost here, C_{hM}, is assumed greater than the holding cost for reusable inventory. Total related cost, C_5, is:

C_5 = average number of serviceable items per time (n_{10}) C_{hM}*

Node 7) *manufacturing:* A manufacturing process with unlimited capacity is assumed. Parameters related to manufacturing are:

- Lead time, L_M, which is assumed as a random variable with triangular distribution.
- Fixed manufacturing cost, C_{FM}, per batch, and variable remanufacturing cost, C_{VM}, per item.
- Fixed setup cost, C_{SM}, per order.

Total cost of manufacturing, C_6, would be:

C_6 = average number of manufactured batches per time (n_{11}) C_{FM} + average number of manufactured items per time (n_{12})* C_{VM} + average number of orders for manufactured items per time (n_{13})* C_{SM}*

Node 8) *customer:* In some models shortages were not allowed, but it is more realistic to assume that there is a fixed back order cost, C_b. Total cost associated with back orders would be:

C_7 = average number of backorders per time (n_{14})C_b*

Finally, in a general situation, the objective is to find all parameters, which would minimize total cost: Obj. Min Z = $\sum_{i=1}^{7} C_i$

5.6 Modelling Approach

In this section, the modelling process is briefly discussed, and some challenges that are overcome are described. First, the objective of building the simulation model is to investigate the effects of adding recovery operations for ABC Company. In order to do that, ABC's inventory system under two scenarios is simulated: An inventory system without recovery operations (scenario A), and an inventory system with recovery operations (scenario B). Comparing outputs of these two scenarios will help in reaching the goal. One point is that building only one simulation model for scenario B is enough , as recovery line is modeled independently (separate sub-model), so it can be stopped working at any time changing to a simulation model for scenario A.

5.6.1 Input analysis

Values defined in the model (e.g. *expressions*), usually are found through a proper input analysis. Since, there is no data available for model 3, and inventory system of ABC does not exist, Table 5.1 and 5.2 (see Section 5.6.2) are completed by relying on hypothetical data and experts' suggestions (Kelton, 2007). But, as an example, assuming that real data for manufacturing lead time is available, a sample input analysis for 100 observations is done using Arena. The results of sample input analysis is shown in Appendix A.3. Note that later, in the modelling section, manufacturing lead time is defined as the average of two expressions (e.g. *Lm* and *L2m*; see Table 5.2) assuming that the manufacturing consists of two interrelated processes. The sample input analysis is assigned to one of those processes.

As the results of input analysis show, a Uniform probability distribution with parameters: a = 0.5 and b = 1.0 is recognized as the best fit by Arena: the *mean square error* of fitting Uniform probability distribution (0.0044) is the minimum between the mean square errors of fitting all reported probability distributions. Also, both goodness of fit tests (K-S test and chi-square test) do not reject fitting the Uniform probability distribution on the observations. In addition, because P values for both goodness-of-fit hypothesis tests, K-S test and chi-square test, are greater than 0.10 (fairly high), they indicate that Uniform probability distribution is representing the data well (Kelton, 2007).

5.6.2 Modelling

Although it would have been a lot easier to build a simulation model for ABC's inventory system with recovery operations with modules from the Basic Process and Advanced Process panels, the model is built using only modules from the Blocks and Elements panels to increase simulation model's flexibility in the sense of simplicity that it provides for making changes.

Appendix A.1 shows the simulation model and its sub-models. Figure A.1 in Appendix A.1, shows the completed model, and three sub-models are shown in Figures A.2 to A.4 separately. The modules shown in Appendix A.2 are called *elements*, and they define different objects for the model. For example, the Variables element initializes variables of the model. Tables 5.1 to 5.2 provide the list of all defined variables and expressions along with their initialized values.

Table 5.1: List of variables and their initialized values[1]

Alt.	Variable	Initial value	Alt.	Variable	Initial value
V1	Total Manufacturing Cost	0	V14	Unit Refurbishing Cost	3
V2	Total Remanufacturing Cost	0	V15	Unit Recycling Cost	1.5
V3	Total Refurbishing Cost	0	V16	Used Unit Holding Cost	1
V4	Total Recycling Cost	0	V17	Level2	30
V5	Inventory Level	50	V18	Used item Reliability	0
V6	Setup cost	40	V19	Threshold	0.61
V7	Remanufacturing Setup cost	50	V20	Total Refs	0
V8	SL	20	V21	Total Rems	0
V9	Q	30	V22	Total Recs	0
V10	Unit Manufacturing Cost	10	V23	Test	0
V11	Unit Holding Cost	2	V24	Tag	0
V12	Unit Shortage Cost	5	V25	Used Inventory level	0
V13	Unit Remanufacturing Cost	6	V26	Run Period	119.999

[1] V1 to V26 are alternate names for Variables, which are used in model 3's flow chart (see Figure 5.5)

The reason for introducing variables and expressions in the beginning of explaining the modeling approach is that by looking at both them and the picture of the model, Figure A.1, one can get an idea of how the modeling is done.

In Appendix A.2, Figures A.5 and A.6, provide enough information about how the rest of the *elements* are defined (e.g. in the Replicate Element, Replication Length is defined by the variable "Run Period" instead of a rigid value of 1 to make future changes easier).

With the aid of *elements* in Arena, the model is designed flexible, and any desired changes for tuning the model or defining a new scenario can be made using them without changing whatever is inside the Blocks modules unless a major change is required.

In Table 5.1, variables with initialized value of zero are serving as counters. For example, test counts the number of times that remanufactured items are added to the inventory 1, or "Total Refs" counts the total number of refurbished items.

Table 5.2: List of expressions and their values[2]

Alt.	Expression	Value
E1	Order Size	DISC(0.167,1, 0.5,5, 0.833,7, 1.0,10)
E2	Inventory Counting Period	1
E3	Order Inter Arrival	EXPO(0.1)
E4	Lm	UNIF(0.5, 1.0)
E5	L2m	UNIF(0.07, 0.08)
E6	Return interarrival	EXPO(0.2)
E7	Total Service Time	DISC(0.1,5, 0.2,18, 0.5,8, 0.7,5, 1.0,7)
E8	Reliability	EP(-0.04* Total Service Time)
E9	L1r	UNIF(0.3, 0.45)
E10	L2r	UNIF(0.1, 0.15)
E11	Good Percent	TRIA(0.1, 0.15, 0.2)

The simulated model behaves exactly the same as is shown in Figure 5.1, and costs associated with every step, including recycling, would be calculated and reported by the model. In order to do that, all costs are defined in Outputs Element and DStats Element (see Figures A.5 and A.6).

For example, Arena formulas shown under the DStats Element are related to total holding cost, shortages cost, and holding cost related to inventory 2(reusable inventory) respectively. With the same idea, Arena formulas shown under the Outputs Element are related to average manufacturing cost, average remanufacturing cost, average recycling cost, and average total cost respectively. The function of the remaining output, Tag, is counting the number of used products that have passed the reliability test; its application will be described later.

[2] E1 to E11 are alternate names for Expressions, which are used in model 3's flow chart (see Figure 5.5)

The model consists of three sub-models, which are shown in Figures A.2 to A.4, and four types of entities with random arrivals are created:

- ✓ Purchase Order– *Entity type 1*
- ✓ Inventory Clerk- *Entity type 2*
- ✓ Used Item- *Entity type 3*
- ✓ Inventory Clerk2- *Entity type 4*

Entity type 1 in the sub model 1 (see Figure A.4), models the process of receiving customers' orders, and whenever an order is received, it will decrease the serviceable inventory level to "Inventory Level - Order Size".

In the sub-model 1 (see Figure A.4), the inventory level is measured daily by *Entity type2*; this means that $\Delta\, t = 1$. If "Inventory Level $\leq SL$", a manufacturing order of "Manufacturing Batch" is realized. Otherwise, it will be disposed, and Manufacturing Batch = SL - Inventory Level +Q.

Entity type2 also models the manufacturing process: It is modeled simply with a Delay module and a variable lead-time, which has uniform probability distribution resulting from a combination of two variables (*Lm* and *L2m)* with uniform distributions. The idea is to increase the variation of the model, and to substitute the effect of manufacturing quantity, which is not considered directly due to the fact that it will not affect the modeling objective.

In the sub model 2 (see Figure A.3), *Entity type3* models the return and reliability test processes. Also, all costs associated with recovery operations and number of all types of recovered items (recycled, remanufactured, and refurbished) are calculated in the sub model 2. One of the challenges in the modeling was defining the reliability, the solution found is: It can be defined by an expression with the value of EP (- 0.04 $\times$ *Total Service Time*) to assign a random reliability measure to each *Entity type3* with random arrival times, where the expression "Total Service Time" is a random variable with Discrete probability distribution representing failure times. Note that the reliability function used here is the same as the one found in Chapter 3 (see Section 3.7). Also, the initial value of the variable "Threshold", 0.61, is the same as the result that was found in the section 3.7 before the warranty data analysis step.

Recovery options are recycling, remanufacturing and refurbishing. Selection between refurbishing and remanufacturing was the other challenge, which is done by defining an expression "Good Percent", which is a random variable with triangular distribution (see Table 5.2). This variable is used in estimating the number of refurbished or remanufactured items: Each time that number of reusable items reaches to Level2, the variable "Good Percent$\times$Level2" determines the number of refurbished items, and consequently, "(1-Good Percent) $\times$ Level2" determines the number of remanufactured items.

As was mentioned earlier (see Section 5.3), ABC applies a *PUSH* policy to control inventory 2.

Modeling the timing of adding remanufactured items to the inventory was another challenge. This is overcome by using one of the mathematical functions provided in Arena, "Real Remainder". In sub model 2, a counter is defined (*Tag*), which counts the number of those used items that pass the reliability test; whenever the real remainder of "*Tag*" and "*Level2*" equals zero, total number of remanufactured items (*Total Rems*), total number of refurbished items (*Total Refs*), total remanufacturing cost, and total refurbishing cost are calculated.

Finally, in the sub-model 3 (see Figure A.2), *Entity type4* models remanufacturing operations similarly to what *Entity type2* does, and the inventory level will be increased as much as "*Level2*" after a lead time, if "*Used inventory Level* $\geq$ *Level2*". Note that the variable "*Used inventory Level*" refers to the level of inventory 2. Next section provides more details about how the simulation model works by showing the model's logics with a flow chart.

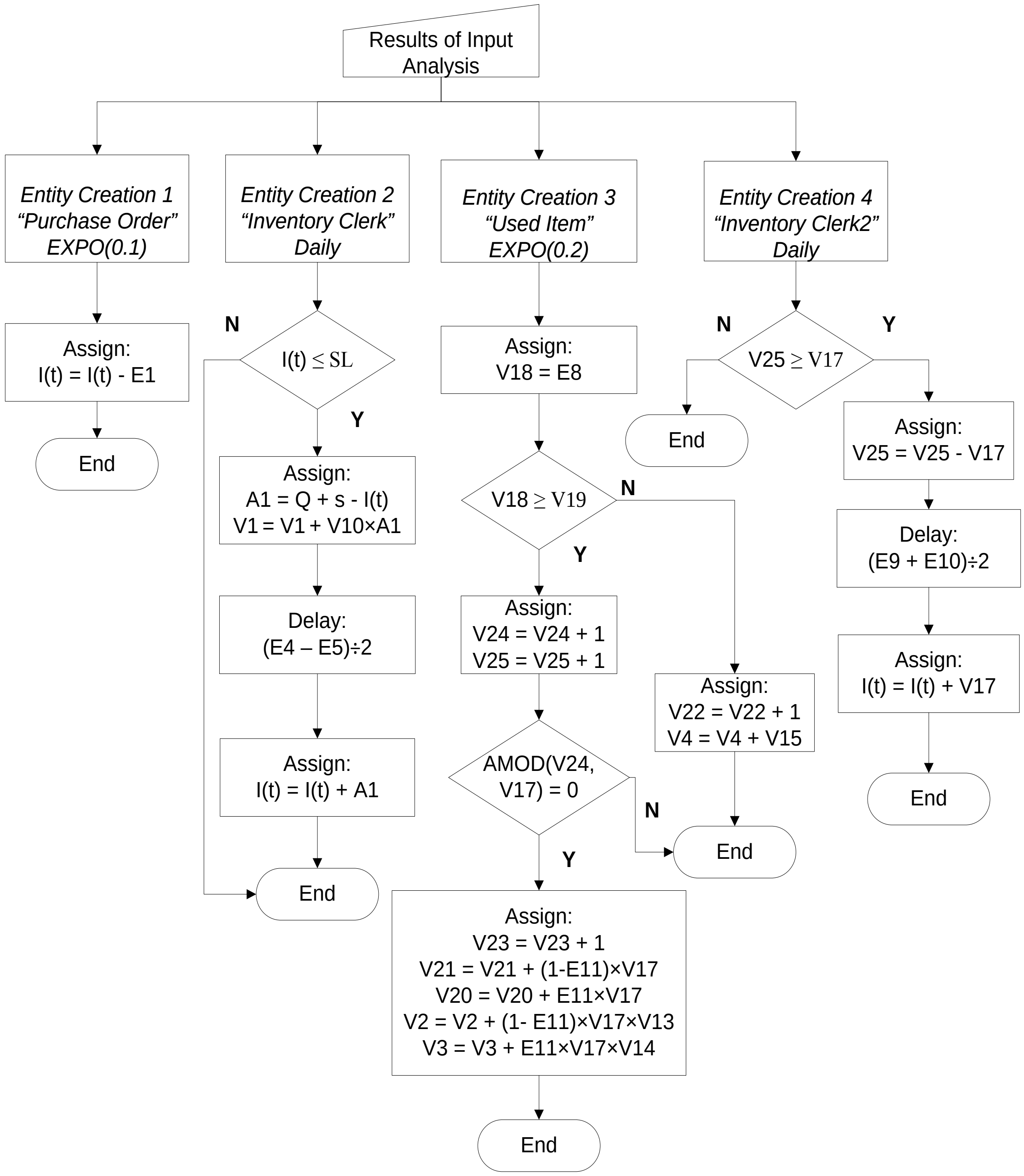
5.6.3 Flow chart of model 3

Figure 5.5 shows model3 with a flow chart. It shows in detail how the modules from the Blocks panel are applied. This figure should be applied along with Tables 5.1 and 5.2. For example, V1 refers to the first variable defined in Table 5.1, Total Manufacturing Cost. In the same way, E4 refers to the fourth expression defined in Table 5.2, Lm (manufacturing lead time), and A1 refers to the only attribute defined in the Attributes element, Manufacturing Batch.

The flow chart shows the exact algorithm explained in Section 5.4.2 with additional information about building the simulation model. For example, Figure A.1 shows that Entity type1, Purchase order, is created by Arena with an interval of "Order Inter Arrival", which changes the value of the variable shown in the Assign module (Inventory Level), but it is not shown how this variable is changed. This is shown in Figure 5.5: Each time Entity type1 passes through the Assign module, the inventory level, I(t), is decreased to a new value of "I(t) - E1". The only remaining point is that "AMOD" is one of the mathematical functions provided by Arena, and is called "Real remainder". It determines when the level of inventory 2 reaches a specified level (Level2). This is where "Tag" is used. Equation 5-1 shows how this function works:

$$\text{AMOD}(a_1, a_2) = (a_1 - (\text{AINT}(a_1/a_2) \times a_2)) \qquad (5\text{-}1)$$

In Equation 5-1, AINT is another mathematical function provided by Arena, which returns the truncated value of (a_1/a_2). For instance, AINT (63/30) = 2, and AMOD (63, 30) = 3.



Notation

V1 = Total manufacturing cost	V17 = Level2	V25 = Used inventory level
V2 = Total remanufacturing cost	V18 = Used item reliability	E1 = Order size
V3 = Total refurbishing cost	V19 = Threshold	E4 = Lm
V4 = Total recycling cost	V20 = Total Refs	E5 = L2m
V10 = Unit manufacturing cost	V21 = Total Rems	E8 = Reliability
V13 = unit remanufacturing cost	V22 = Total Recs	E9 = L1r
V14 = Unit refurbishing cost	V23 = Test	E10 = L2r
V15 = Unit recycling cost	V24 = Tag	E11 = Good percent

& A1 = Manufacturing batch

Figure 5.5: The flow chart of Model 3

5.7 Model verification & output analysis

Both scenarios A and B are run for 120 days with 100 replications using model 3, and output analysis is done. In order to verify the performance of model 3, both output graphs and numerical results are compared with expected outputs under both scenarios.

First, Figure 5.6 and Figure A.8 show sample graphs of inventory level vs. days under scenario A for the 55th replication. Figure 5.6 shows the same data shown in Figure A.8 (see Appendix A.4), but over 10 days instead of 120 days. The similarity between Figures 5.6 and A.8 on the one hand and Figure 5.2 on the other hand (see Section 5.4.1) roughly verifies model 3's performance. Note that Figure 5.2 is drawn based on classical theoretical inventory graphs, and Figure 5.6 is provided to make the comparison easier.

Next, with the same idea, inventory level vs. days, and shortages cost vs. days under scenario B, are shown in Figures A.9 and A.10 in Appendix A.4, and similar graphs over 60 days are shown in Figures 5.7 and 5.8 respectively.

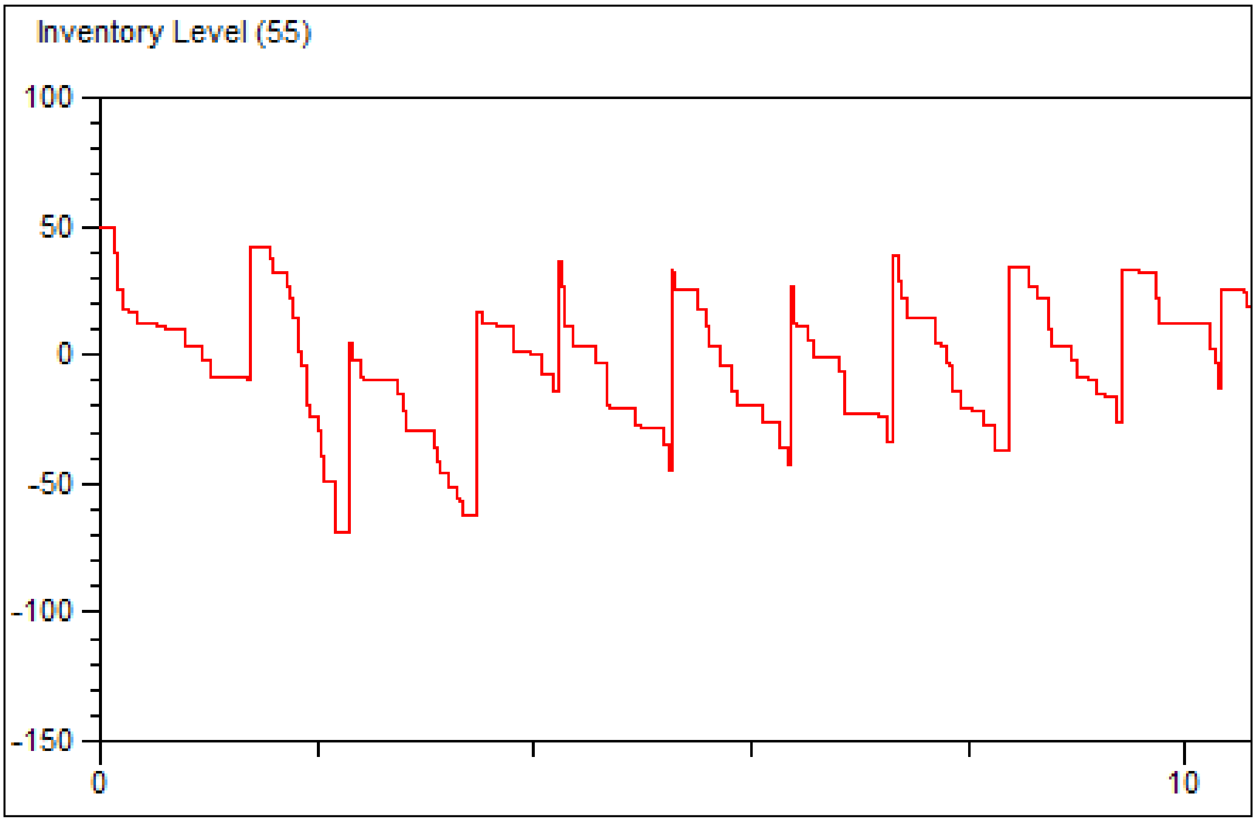


Figure 5.6: Inventory level vs. days - scenario A - replication # 55

A comparison between Figures 5.7 and 5.8 helps in verifying the performance of model 3: It shows a direct relation between inventory level and shortages cost in the sense of whenever the inventory level is zero or greater than zero, no shortages cost is calculated by the model. Also, a comparison between Figure 5.7/A.9 and Figure 5.3 (see Section 5.4.2) verifies model 3's performance, as both figures show the system's behaviour similarly. The above presented comparisons, graphically verifies Model 3. In addition to those comparisons, in the following paragraphs, model 3 is numerically verified by comparing outputs with what would be expected.

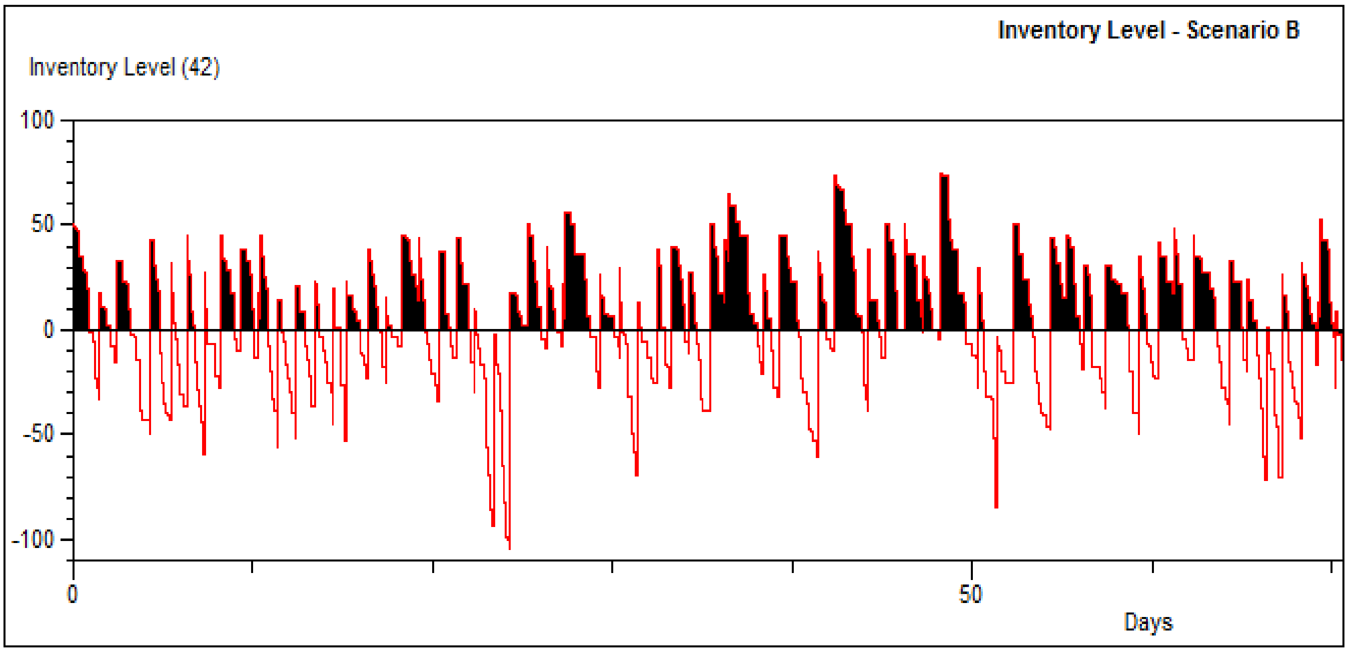


Figure 5.7: Inventory level vs. days – scenario B – replication # 42

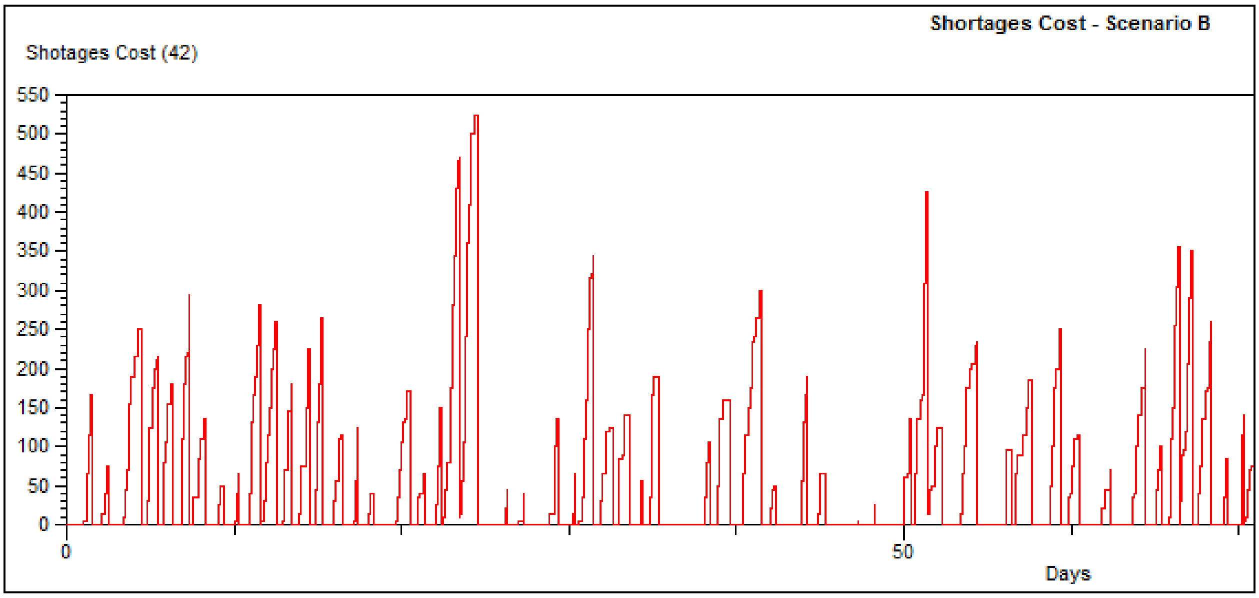


Figure 5.8: Shortages cost vs. days – scenario B – replication # 42

For instance, under Reports tab, in Category Overview, under Entity, the average Number In for *Entity type 1* (Purchase order), 1202.12 (see Appendix A.6), can be considered as an indicator showing that model 3 is working good. Because on the other hand, the process of receiving costumers' orders which is modeled with a random variable (Order Inter Arrival) that is distributed exponentially with $\lambda = 0.1$, and the expectation is that the sales department on average should receive 10 orders per day, which matches with the results provided by Arena ($120 \times 10 \approx 1202.12$). More importantly, model 3 is numerically verified by comparing the output data of the levels of the two inventories with what is expected. Tables 5.3 and 5.4 provide these comparisons under scenario B by using sample data obtained from the 42nd replication.

Table 5.3: Sample inventory 2's level for 30 days - on marked days (**) it reaches to *Level2*

Day	Level of Inv.2	Note	Day	Level of Inv.2	Note	Day	Level of Inv.2	Note	Day	Level of Inv.2	Note
0	0		7.88	26		16.4	22		23.5	18	
0	0		8.69	27		16.5	23		23.9	19	
0.206	1		8.7	28		16.8	24		23.9	20	
0.241	2		9.21	29		16.9	25		24	21	
0.655	3		9.38	30		17.4	26		24.2	22	
1.04	4		9.45	31		17.5	27		24.2	23	
1.3	5		9.79	32		18	28		25.5	24	
2.41	6		9.93	33		18.1	29		25.8	25	
2.48	7		9.94	34		18.3	30		25.9	26	
2.59	8		10	4	**	18.8	31		26.5	27	
2.68	9		10.2	5		18.8	32		26.7	28	
4.67	10		10.3	6		19	2	**	26.9	29	
4.97	11		10.4	7		19	3		27	30	
5.45	12		10.4	8		19.4	4		27	0	**
5.46	13		10.7	9		19.5	5		27.4	1	
5.52	14		12.2	10		19.9	6		27.5	2	
5.59	15		12.4	11		20.1	7		27.8	3	
5.71	16		12.7	12		20.2	8		28	4	
5.78	17		13	13		20.3	9		28.3	5	
6.19	18		13.1	14		21.1	10		28.5	6	
6.43	19		13.5	15		21.4	11		28.8	7	
6.75	20		14.3	16		21.7	12		28.8	8	
6.8	21		15.2	17		21.9	13		29.3	9	
6.85	22		15.3	18		22.1	14		30	10	
7.13	23		15.5	19		22.5	15		30	11	
7.49	24		16	20		22.7	16				
7.83	25		16.1	21		23	17				

Table 5.4: Verification of model 3 using sample inventory level for 30 days[3]

Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)
0	50		4.25	-50		7.12	-43		10.5	35		13.6	12	
0	50		4.29	43	*	7.15	-44		10.7	25		13.7	2	
0.0449	49		4.5	36		7.17	-54		10.7	20		13.7	-3	
0.0912	48		4.51	31		7.26	-59		10.8	13		13.8	-10	
0.247	47		4.61	24		7.32	27	*	10.8	12		14	-15	65
0.296	42		4.74	19		7.33	20		10.8	2		14.1	-25	
0.36	35		4.77	9		7.33	15		10.8	-3		14.3	-30	
0.507	28		4.77	4		7.34	10		10.9	-8		14.4	-35	
0.628	27		4.81	-6		7.42	3		10.9	-13		14.4	-45	
0.792	20		4.81	-11		7.45	-2		11	-20	70	14.4	20	*
0.841	13		4.89	-18		7.49	-7		11	-25		14.5	15	
0.849	6		4.95	-25	75	7.83	-17		11	-26		14.5	8	
0.909	-1	51	5.01	-35		7.92	-22	72	11.1	-33		14.5	1	
1.08	-6		5.17	-40		8.09	-27		11.1	-38		14.8	-9	
1.24	-13		5.29	-41		8.22	45	*	11.3	-39		14.9	-16	
1.25	-23		5.36	-42		8.26	40		11.4	-46		14.9	-26	76
1.32	-28		5.39	-43		8.31	35		11.4	-56		15.1	-36	
1.41	-33		5.42	32	*	8.36	34		11.4	14	*	15.1	-46	
1.45	18	*	5.5	25		8.46	33		11.5	9		15.1	-53	
1.56	11		5.52	18		8.56	28		11.6	-1		15.2	23	*
1.8	10		5.54	11		8.73	18		11.7	-6		15.2	16	
1.89	3		5.58	4		8.75	17		11.8	-16		15.5	15	
1.89	2	48	5.58	3		8.92	12		11.9	-23	73	15.5	10	
2.08	-3		5.73	-4		9	2	48	12.1	-30		15.6	9	
2.11	-8		5.77	-11		9.02	-5		12.1	-40		15.8	4	
2.33	-15		5.78	-16		9.07	-10		12.3	-45		16	-1	
2.41	33	*	5.86	-21		9.35	38	*	12.4	-52		16	-6	
2.7	23		5.87	-31	81	9.63	33		12.4	21	*	16	-11	61
2.92	22	na	6.1	-36		9.88	26		12.6	14		16.1	-12	
3.08	15		6.35	45	*	9.92	21		12.6	9		16.2	-17	
3.09	10		6.35	40		9.93	20		12.9	-1		16.3	-22	
3.12	5		6.35	33		9.95	10	40	12.9	-8		16.3	-23	
3.21	-2		6.51	26		10	0		13	-9	59	16.4	38	*
3.42	-3		6.52	19		10.1	-1		13	-14		16.6	33	
3.48	-4		6.58	9		10.1	-8		13.1	-15		16.6	26	
3.49	-9		6.72	2		10.1	-13		13.2	-22		16.7	21	
3.49	-14		6.83	-8		10.2	17	**	13.2	-29		16.8	16	
3.68	-21		6.83	-15		10.3	10		13.2	-36		16.8	11	
3.69	-31		6.9	-22		10.4	5		13.4	23	*	16.9	10	40
3.73	-38		6.9	-29		10.4	45	*	13.5	22		17	0	
3.85	-43	93	6.98	-36	86	10.5	40		13.6	17		17.1	-1	

[3] On days that are marked with "*" or "**", a manufacturing or remanufacturing batch is added to the inventory respectively. Manufacturing batch size and a check point is shown, wherever the column of "Q +s-I(t)" is listed.

Table 5.4: (*Continue*)

Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)	Day	I(t)	Q + s - I(t)
17.2	-11		21	0		23.5	-3		26.5	20				
17.2	-18		21	-7	57	23.5	-10		26.7	10				
17.3	-25		21	-8		23.5	-11		26.8	0				
17.3	15	*	21.2	-13		23.6	-16		26.9	-1	51			
17.4	8		21.3	44	*	23.7	-21		27.1	-8				
17.4	7		21.6	39		23.7	-31		27.3	22	**			
17.5	2		21.6	32		23.8	-38		27.3	12				
17.8	-3	53	21.7	22		23.8	-48		27.4	5				
18.1	-8		22	15		23.8	-55		27.4	56	*			
18.3	45	*	22	10	40	23.9	-65		27.6	51				
18.4	44		22	0		23.9	-72	122	27.9	41				
18.6	43		22.1	-5		24	-82		27.9	36				
18.8	33		22.1	-15		24	-92		28.4	29				
18.8	26	na	22.3	-20		24.1	-99		28.5	24				
19.1	21		22.3	-30		24.2	-100		28.5	17				
19.1	14		22.3	10	*	24.3	-105		28.5	7				
19.2	44	**	22.4	9		24.3	17	*	28.8	-3	53			
19.2	34		22.5	8		24.6	16		29.1	-13				
19.3	29		22.5	-2		24.7	9		29.1	-20				
19.4	24		22.5	-9		24.8	8		29.2	-27				
19.5	14		22.6	-16		24.9	7		29.3	26	*			
19.6	9		22.9	-23		25	2	48	29.3	16				
19.6	-1		22.9	-28		25.2	50	*	29.4	15				
19.7	-2		22.9	-35		25.4	45		29.5	8				
19.7	-7		22.9	-42	92	25.6	40		29.7	7	43			
19.8	-14		23	-49		25.6	33							
19.9	-21	71	23	-56		25.7	23							
20.1	-26		23.1	-61		25.8	16							
20.2	-27		23.1	-68		25.8	11							
20.2	-34		23.1	-69		26	1	49						
20.3	37	*	23.1	-74		26.1	-4							
20.6	32		23.2	-81		26.2	-9							
20.7	25		23.2	-86		26.3	40	*						
20.7	15		23.3	-93		26.3	33							
20.7	8		23.4	-94		26.4	28							
20.9	1		23.4	-2	*	26.5	21							

Table 5.3 includes data related to the inventory 2 (reusable inventory): On days marked with two asterisks, the number of reusable items is greater than or equal to *Level2*, namely 30. It is expected that, first, on these days the level of inventory 2 decreases as much as *Level2*, and the

second expectation is that, after a random time of (L1r + L2r) / 2 days, 30 remanufactured items should be added to the inventory. Table 5.3 satisfies the first expectation, and a comparison between Tables 5.3 and 5.4 satisfies the second one. These verifications are explained in detail in the following paragraphs. Note that as Table 5.2 shows $0.2 \le \frac{L1r + L2r}{2} \le 0.3$.

Table 5.4 includes data related to the serviceable inventory: On days that the size of the manufacturing batch is shown (column "Q+s-I(t)"), the inventory level, I(t), is less than or equal to *SL*, namely 20. Otherwise, the check point is marked with "na"(not applicable); means that there is no need to manufacture more items. It is expected that after a random time of (Lm - L2m) / 2 days, the inventory level increases as much as the manufacturing batch, *Q + s - I(t)*. Note that as Table 5.2 shows $0.2150 \le \frac{Lm - L2m}{2} \le 0.4650$.

Tracing the data shown in Table 5.4 indicates that the above expectation is satisfied. For instance, at the end of day 1, the inventory level reaches (-1), so a batch of 50- (-1) = 51 items should be manufactured; this batch size (51) is shown in Table 5.4 in front of the related day. The inventory level should be increased as much as 51 items after a random number of days, which has a lower and upper bound of 0.2150 and 0.4650 days respectively. This expectation is satisfied as, on day 1.45 a manufacturing batch of 51 items is received, and the inventory level is changed from (-33) items to 18 items (51+ (-33) = 18). This increase of the inventory level, which verifies model 3, is marked with an asterisk. As Table 5.4 shows, the manufacturing batch is received after 0.45 day (1.45-1.00 = 0.45), which is within the limits ($0.2150 \le 0.45 \le 0.4650$). In the same way as described above, the output data are traced for 30 days to verify model 3.

A comparison between Table 5.3 and 5.4 verifies the second expectation of model 3's behaviour for remanufacturing operations: On days that are marked with two asterisks in Table 5.4, the remanufactured items are added to the serviceable inventory. Those days are matched with data presented in Table 5.3. For instance, as Table 5.3 shows on day 9.94, the number of reusable items reaches 34 items, which is greater than *Level2*, namely 30. It is expected that the number of reusable items in inventory 2 decreases as much as *Level2* immediately and 30 remanufactured items are to be added to the serviceable inventory after a random time, which is something between 0.2 and 0.3 days. As Table 5.3 shows, the first expectation is satisfied, and on day 10, the level of inventory 2 is dropped to 4. Also, considering Table 5.4, searching among days that are close to day 10, indicates that on day 10.2, the inventory level is increased by as much as 30 items. This is marked with two asterisks in Table 5.4, and is within the limits ($0.2 \le 0.2 \le 0.3$).

In addition, Appendix A.7 provides verification of model 3 under scenario A (see Table A.1), and some additional graphs for analyzing the outputs per each replication and under each scenario are provided in Appendix A.4, Figures A.11 to A.19. For example, the average manufacturing cost per replication under the two scenarios can be compared with each other by using Figures A.11 and A.12, and Figures A.14 and A.15 do the same by showing the histograms of the inventory level under each scenario. Figure A.13 shows average remanufacturing cost per replication. Figures A.16 and A.17 show histograms, by which the comparison of the shortage costs under the two scenarios are made possible. The graphs of average total cost per replication under scenarios B and A are shown in Figures A.18 and A.19 respectively.

The next sections are assigned to investigating the effects of adding recovery operations to an (*s*, *Q*) inventory control system, and to the effects of model 1 on an (*s*, *Q*) inventory control system with returns.

5.8 Comparing scenario A with scenario B

In order to compare the two scenarios, first, Arena reports are used. Next, the two scenarios are compared using statistical methods.

Appendix A.5 shows the category overview report by Arena for scenario A, which is run for 120 days and with 100 replications. Note that as other sections of the report are not used in this thesis, only "category overview" is provided in the Appendixes section.

Appendix A.6 shows the "category overview" report by Arena for scenario B. The average values of some useful statistics found in "category overview" reports for both scenarios are shown in Table 5.5 to make the comparison between two scenarios possible. This helps in investigating the effects of adding recovery operations to the system outputs.

Table 5.5: Average statistics' values for scenarios A and B

Item	Scenario A	Scenario B
WIP-Inventory Clerk	0.3143	0.2952
Time Persistent- Shortage Cost	52.74	48.64
Inv Level	0.00	3.21
Holding Cost	21.01	42.19
Average Manufacturing Cost	614.54	576.60
Average Remanufacturing Cost	N/A	184.43
Average Recycling Cost	N/A	88.77

Note that in the reports, items “Backorder Cost” and “Shortage Cost” both refer to the same thing, shortage costs, and both could be used here. The first item, which is defined by the author, records the last value of the shortage costs, and the second item is automatically produced by Arena and collects the average values of the shortage costs.

The average WIP (Work In Process), which is an Arena term indicates how much activity, can be assigned to an entity (e.g. Inventory Clerk). That statistic for scenario B is less than for scenario A. Therefore, it can be concluded that there was less need for manufacturing items after adding recovery operations. Also, the average of shortage costs in scenario B is improved, but the average holding cost is increased after adding recovery operations. In order to compare two scenarios better, a statistical comparison of them using the Output Analyzer (a separate application provided by Arena) is done.

To do so, a mean-comparison between total costs assigned to the two scenarios is performed using the Output Analyzer, in which, with a 0.95 confidence interval, the expected differences between total costs of the two scenarios are compared. Note that scenario B (with recovery) is the base case, so the subtractions of means are in the direction B – A. Figure 5.9 shows the result of this comparison along with the result of the Hypothesis Test on the expected difference between total costs of the two scenarios. Since the interval misses zero, it is concluded that a significant difference exists, and the hypothesis test verifies it as well.

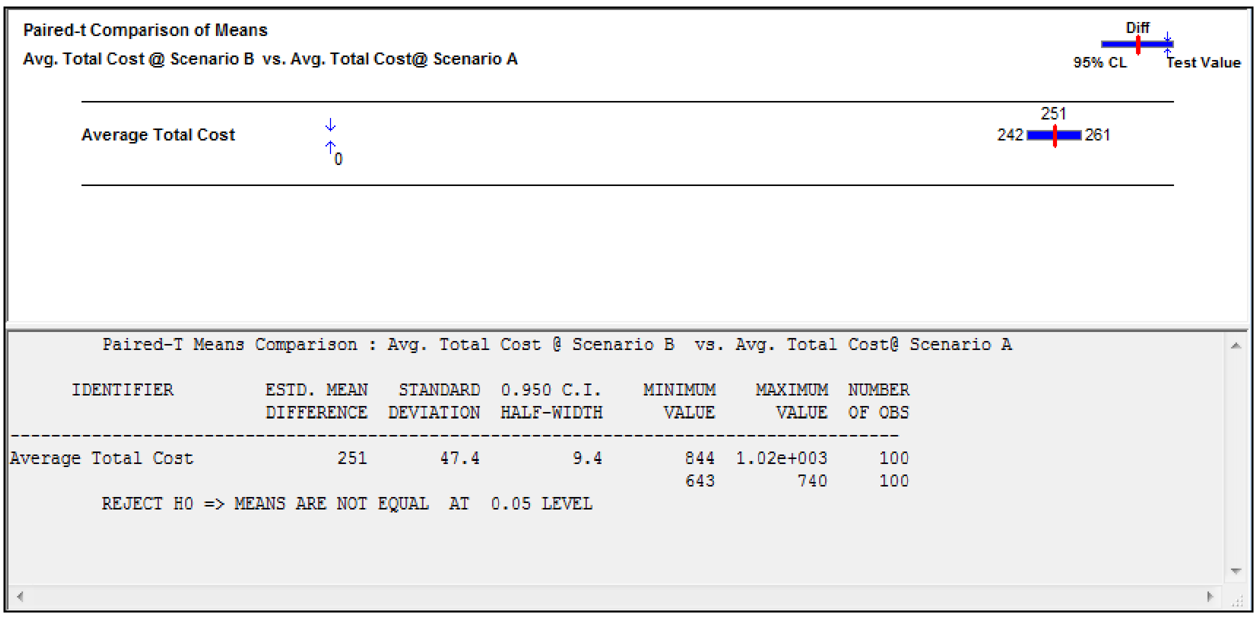


Figure 5.9: Confidence interval and hypothesis test on the expected difference between average total costs of scenarios B and A

As a result, this comparison gives the producer an idea of what would be the cost of adding the recovery operations. Note that the decision on implementing a project of adding recovery operations cannot be made based on this comparison only. There are other factors that can be considered such as the higher benefit gained from selling restored products, for which the assigned costs are lower than for new ones.

For example, one statistic that can be considered is that on average, 15.51 times remanufactured items are added to the inventory (see Appendix A.6), so on average, 465.3 products ($15.51 \times$ Level2 = 15.51×30) are remanufactured during each period (120 days), which are less costly according to the ratio of the average cost assigned to them to average manufacturing cost ($\frac{184.43}{576.60} \times 100 = 31.9858$). Another factor is the potential benefits of having recycling operations.

For example, selling recovered materials is beneficial, and usually government financial aids reduce the related costs; more importantly, the green image, which can be gained for the company, helps in increasing sales. To get a better idea, the result of performing the same statistical analysis, comparing the average manufacturing costs under the two scenarios is shown in Figure 5.10.

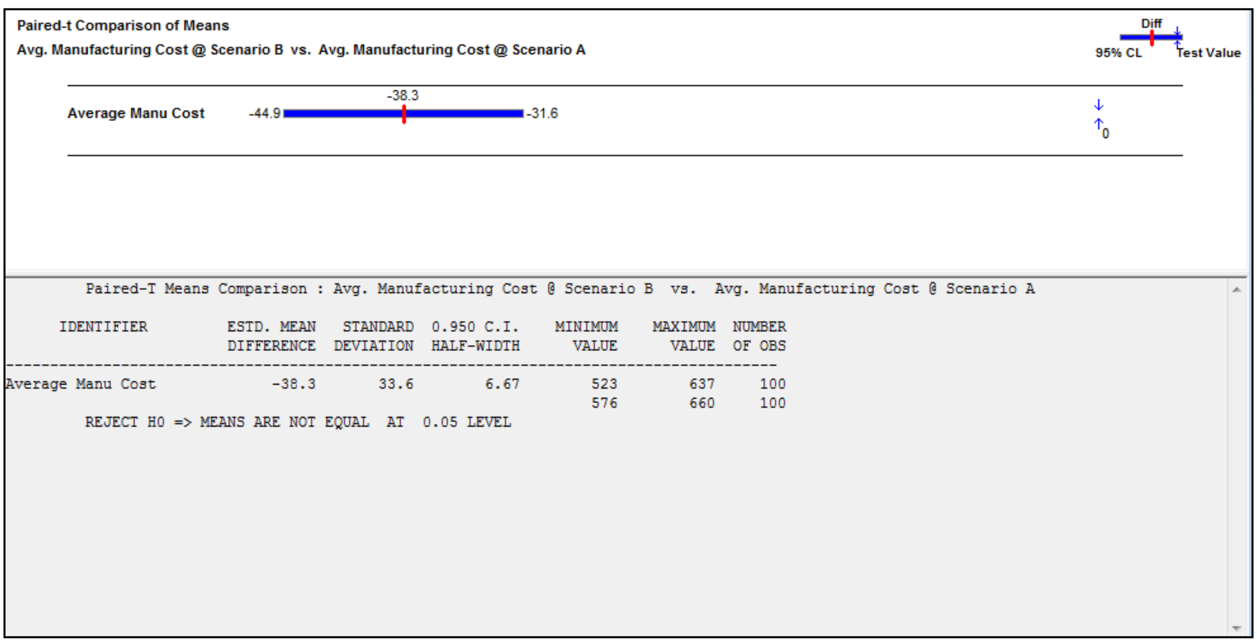


Figure 5.10: Confidence interval and hypothesis test on the expected difference between average manufacturing cost under scenarios B and A

It can be concluded that under scenario B, on average, manufacturing cost is improved. Finally, as is shown in Table 5.5, the average inventory level under scenario B, namely 3.21, is greater than the average inventory level under scenario A, namely0.00. Therefore, under scenario B, more finished products are available, and activities, such as marketing can be considered to increase the sales.

In this section, it is shown how a simulation model can help in analyzing and comparing the two scenarios. In the next section, the effect of model 1 on the system is investigated using it.

5.9 Sensitivity analysis for model 1

In this section, the effect of adding model 1 to an inventory system with (*s*, *Q*) control policy and with returns, is investigated. It is done by analyzing the sensitivity of the threshold, *R**, introduced by model 1 (see Chapter 3).

In order to do that, the outputs of the simulation model (scenario B), such as total remanufactured items (Total Rems), total refurbished items (Total Refs), total recycled items (Total Recs), average remanufacturing cost, and average total costs are obtained under different values for the threshold, *R**.

The simulation model is run for 120 days and with 100 replications. The results for some threshold values between 25% and 90% ($0.25 \leq R^* \leq 0.90$), are shown in Table 5.6. Note that in the base case simulation model, $R^* = 0.61$.

Table 5.6: Model outputs under different values for *R**

No	**Reps**	**R***	$\mathbf{n_1}$	$\mathbf{n_2}$	$\mathbf{n_3}$	$\mathbf{C_1}$	$\mathbf{C_2}$
1	100	0.25	500	88	0	289.65	945.59
2	100	0.40	500	88	0	289.65	945.59
3	100	0.45	500	88	0	289.65	945.59
4	100	0.50	449	79	60	235.82	919.19
5	100	0.55	395	69	118	184.43	940.62
6	100	**0.61**	395	69	118	184.43	940.62
7	100	0.68	395	69	118	184.43	940.62
8	100	0.75	244	43	299	74.65	1318.88
9	100	0.80	93	16	479	13.46	2152.35
10	100	0.85	0	0	600	0	2949.55
11	100	0.90	0	0	600	0	2949.55

Notation

n_1 = Total remanufactured items C_1 = Average remanufacturing cost
n_2 = Total refurbished items C_2 = Average Total cost
n_3 = Total recycled items Reps = number of replications

Figure 5.11 shows a plot of data presented in Table 5.6. As is shown in this figure, when the reliability threshold decreases from what is set in the base simulation model, 61%, the number of recycled items decreases, and the number of both remanufactured and refurbished items increases. It is reasonable that there would be even no recycled items for threshold values lower than 45%, as the used items' reliabilities are not lower than those thresholds. Also, total cost increases by setting the reliability threshold to a higher level. This is due to the fact that holding costs increase, and remanufacturing cost decreases, as the number of items that can pass the reliability test would be less than those in the base case.

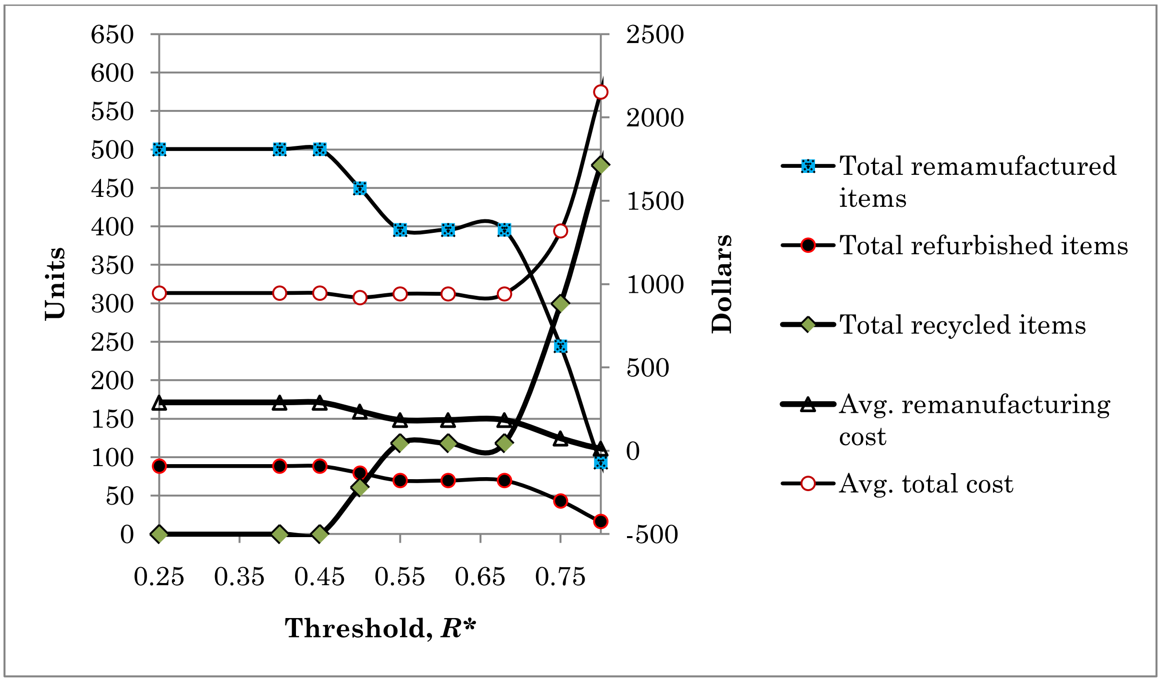


Figure 5.11: Sensitivity analysis for *R**

5.10 Conclusion

As was shown in this chapter, the simulation model can serve as a powerful tool to analyze the system statistically, and to get an idea of the system performance under various scenarios. Of course, in cases with real data it serves even better, as for example, it offers savings in costs related to complex calculations by its ability to produce and report various outputs. It also offers savings on costs related to estimating effects of changes by providing a reliable model to perform complete statistical predictions to choose the best alternative.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, one aspect of reverse logistics is studied: checking the reusability of used products. For solving the problem of reusability evaluation of returns, two models are proposed: An innovative reliability based method for well established products, model 1, and a fuzzy MPMC decision making model, model 2, for products with fast innovations. They are presented to determine if the used product or its subassemblies has to be restored or recycled.

In addition, a simulation model is built to show the application of the reliability based model for reusability evaluation of returns, by which the inventory control of a green manufacturing system can be effectively studied to forecast the system parameters, such as total cost or inventory level under different values of the reliability threshold introduced in model 1 as well as to provide answers to “what if” questions when a recovery line is added to a manufacturing system.

The main advantages of model 1 are: It modifies itself by applying a statistical analysis of warranty data and it is built with consideration of the role of warranty for restored products, as the maximum warranty cost that the producer can consider, and the estimated reliability function of the recovered product are the two inputs of model 1. Model 2 does the reusability evaluation for products with fast innovations, which is indeed a complex decision making problem. The role of an updated information collecting system in model 2 is vital to let the process of decision making find a feasible answer.

The reusability evaluation models will help producers in establishing recovery operations by realizing the importance of such subjects as service time traceability, reliability, and design for remanufacturing. Also, the simulation model helps the producer to save on related costs and in establishing recovery operations by getting an idea of the structure and the optimums of the inventory system. The simulation model can be applied as a powerful tool for studying the system's behaviour and analyzing its output under different scenarios.

6.2 Future work

There are many areas remaining for future work. For instance, the reliability based model for evaluating the reusability for product recovery, model 1, can be applied by a producer who has already established a recovery process line to investigate the effects of not having R^* on the cost elements compared to those of having it. Also, as the effectiveness of model 1 is closely related to the pattern of failures of an item or the item's failure probability distribution, finding an economic way to trace failure timing accurately is another topic for future work, and of course different types of products require different treatments. Some researches (e.g. Anityasari, 2008) suggest the idea of installing an electronic chip in every product to record failure timings. As yet there is no such device in the market, so that could be an option to take for future research.

In addition, applying model 1 in a real case, establishing the recovery operations from scratch, and evaluating its efficiency is strongly suggested, and is one of the author's future goals.

Moreover, the fuzzy MPMC decision model, model 2, will become a complete model with a suitable data collecting system designed for it. Designing such system is left as future work that can be done with the cooperation of industrial engineering and IT engineering. Also, making a computer program for model 2 can be considered as a later research topic.

Modifying the simulation model of the inventory control system of a joint manufacturing and remanufacturing system with an (s, Q) inventory control policy, model 3, which presents the application of model 1, in a way to show the end users, is another area for future work. In this way, the failures resulting from different causes, such as user's fault, environmental causes, or reliability (design or manufacturing factors) can be modeled to investigate a situation closer to a real situation. Also, model 3 can be applied to optimize a complex inventory control system with returns using *OptQuest by Arena®* to find the optimum safety stock, SL^*, optimum manufacturing batch size determiner, Q^*, optimum level of inventory 2, $Level2^*$, and optimum inventory counting period, Δt^*.

In order to do that, different policies can be tried (e.g. first optimizing the case without returns, and then optimizing the case with returns or vice versa). Also, comparing optimization results using *OptQuest* with related analytical models' results is suggested.

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Appendixes

A. Simulation

B. MATLAB Programs

C. Additional Plots Resulting From Warranty Data Analysis

Appendix A.1: Pictures of model 3 and its sub-models

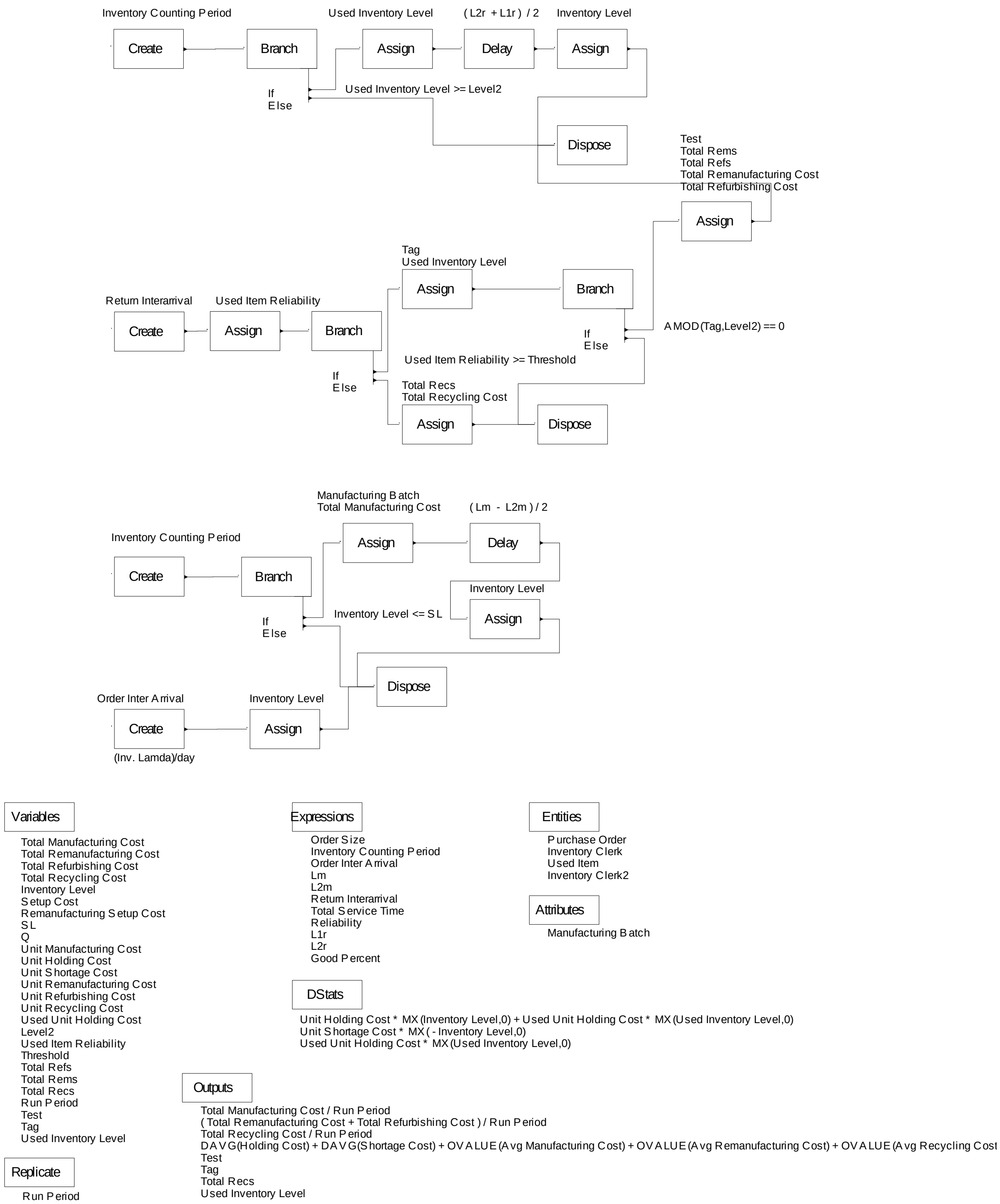


Figure A.1: Simulation model of an (*s*, *Q*) inventory control system with both returns and a reliability test

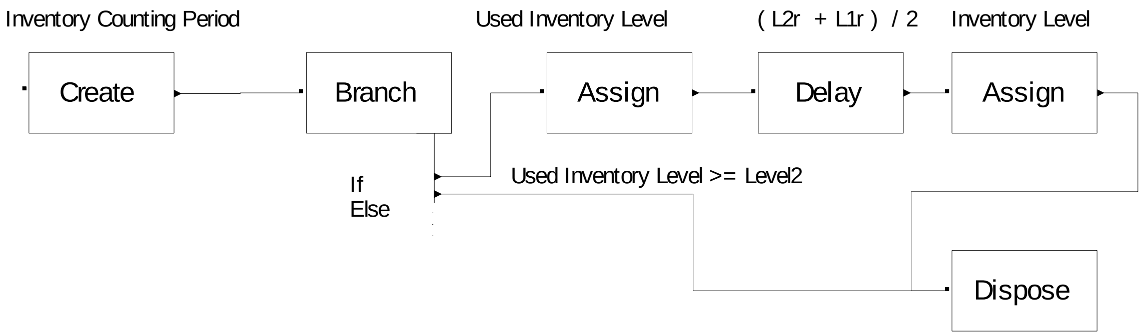


Figure A.2: Sub model 3 - recovery process

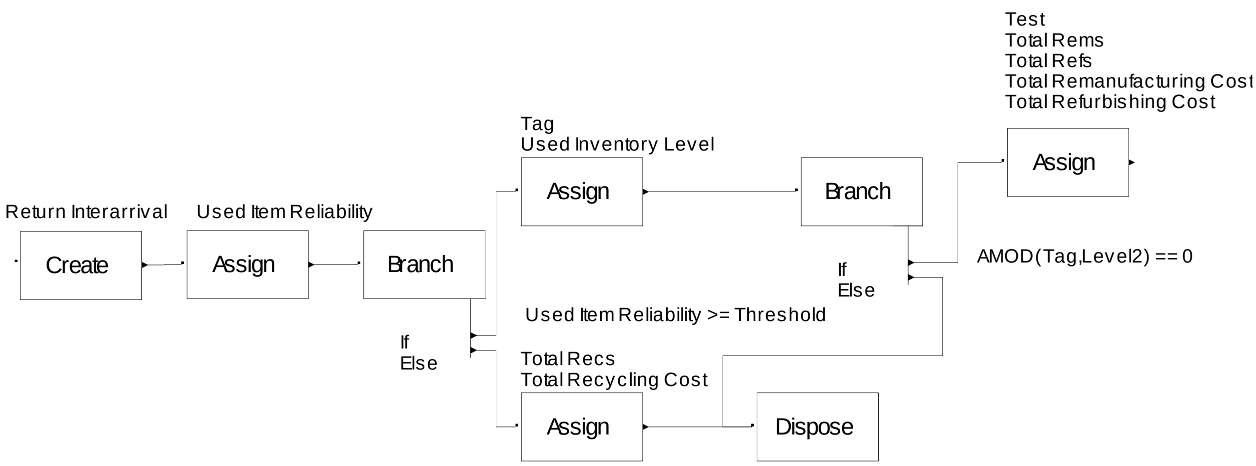


Figure A.3: Sub model 2 – reliability test and recovery costs calculations

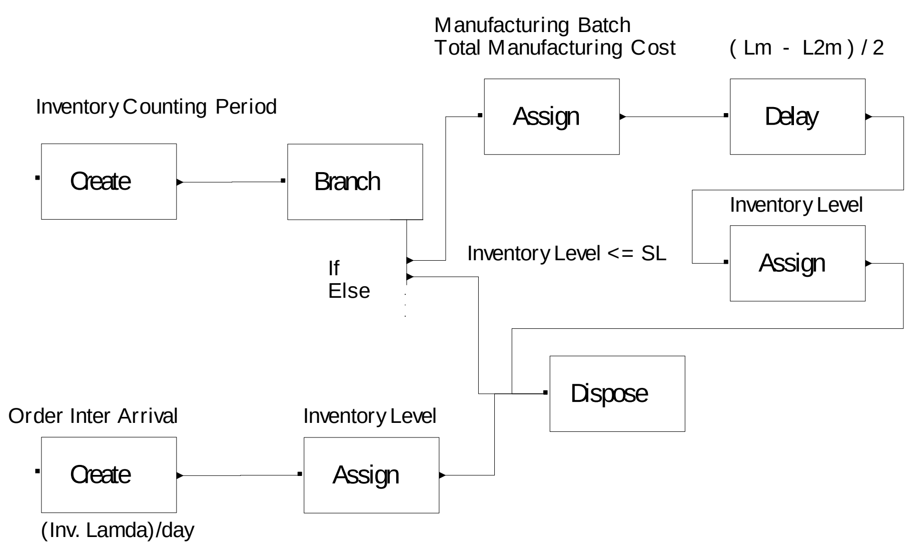


Figure A.4: Sub model 1 – sales department and production line

Appendix A.2: Pictures of Elements modules

Variables

Total Manufacturing Cost
Total Remanufacturing Cost
Total Refurbishing Cost
Total Recycling Cost
Inventory Level
Setup Cost
Remanufacturing Setup Cost
SL
Q
Unit Manufacturing Cost
Unit Holding Cost
Unit Shortage Cost
Unit Remanufacturing Cost
Unit Refurbishing Cost
Unit Recycling Cost
Used Unit Holding Cost
Level2
Used Item Reliability
Threshold
Total Refs
Total Rems
Total Recs
Run Period
Test
Tag
Used Inventory Level

Replicate

Run Period

Outputs

Total Manufacturing Cost / Run Period
(Total Remanufacturing Cost + Total Refurbishing Cost) / Run Period
Total Recycling Cost / Run Period
DAVG(Holding Cost) + DAVG(Shortage Cost) + OVALUE(Avg Manufacturing Cost) + OVALUE(Avg Remanufacturing Cost) + OVALUE(Avg Recycling Cost)
Test
Tag
Total Recs
Used Inventory Level

Figure A.5: Variables, Replicate, and Outputs modules

Expressions

Order Size
Inventory Counting Period
Order Inter Arrival
Lm
L2m
Return Interarrival
Total Service Time
Reliability
L1r
L2r
Good Percent

Entities

Purchase Order
Inventory Clerk
Used Item
Inventory Clerk2

Attributes

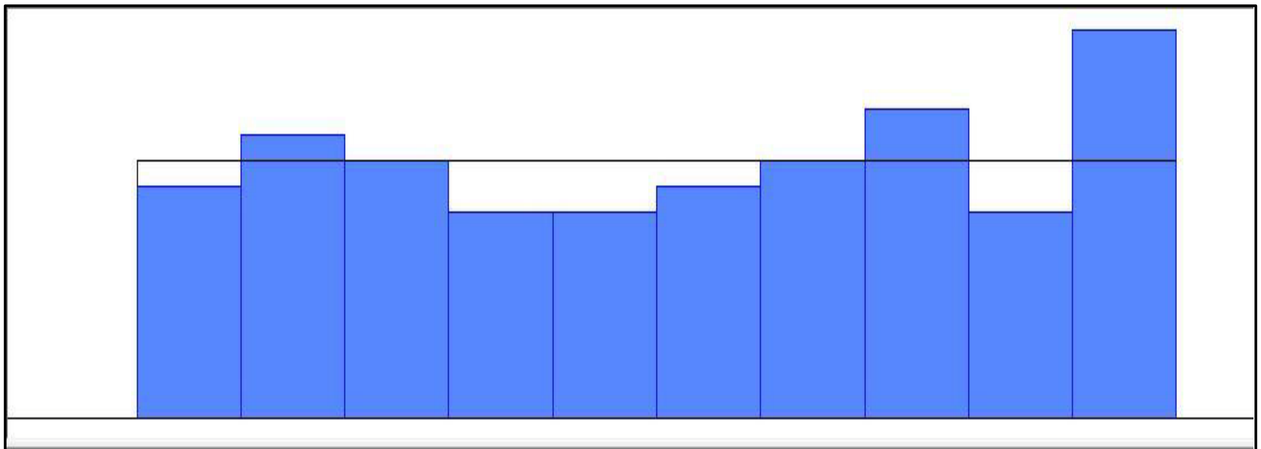
Manufacturing Batch

DStats

Unit Holding Cost * MX(Inventory Level,0) + Used Unit Holding Cost * MX(Used Inventory Level,0)
Unit Shortage Cost * MX(- Inventory Level,0)
Used Unit Holding Cost * MX(Used Inventory Level,0)

Figure A.6: Expressions, DStats, Entities, and Attributes modules

Appendix A.3: Sample input analysis using Arena®



```
      Distribution Summary

Distribution:   Uniform
Expression:     UNIF(0.5, 1)
Square Error:   0.004400

Chi Square Test
  Number of intervals   = 9
  Degrees of freedom    = 8
  Test Statistic        = 2.35
  Corresponding p-value > 0.75

Kolmogorov-Smirnov Test
  Test Statistic        = 0.0764
  Corresponding p-value > 0.15

      Data Summary

Number of Data Points   = 100
Min Data Value          = 0.506
Max Data Value          = 0.985
Sample Mean             = 0.764
Sample Std Dev          = 0.149

      Histogram Summary

Histogram Range         = 0.5 to 1
Number of Intervals     = 10
```

```
Function      Sq Error
-----------------------
Uniform       0.0044
Normal        0.0221
Weibull       0.0224
Triangular    0.0282
Lognormal     -1.#J
Beta          0.00224
Gamma         0.0241
Erlang        0.0243
Exponential   0.0324
```

Figure A.7: Fitting a uniform probability distribution on 100 observations

Appendix A.4: Output analysis

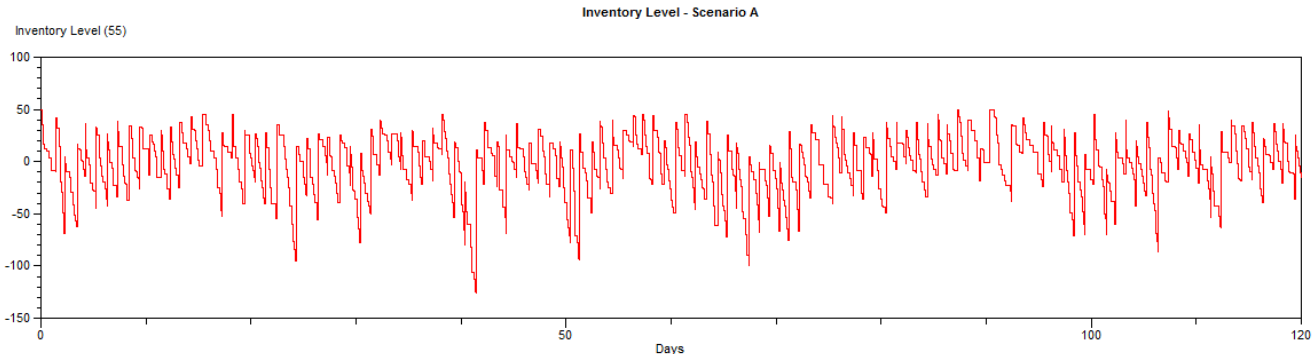


Figure A.8: Inventory level vs. days – scenario A (replication # 55)

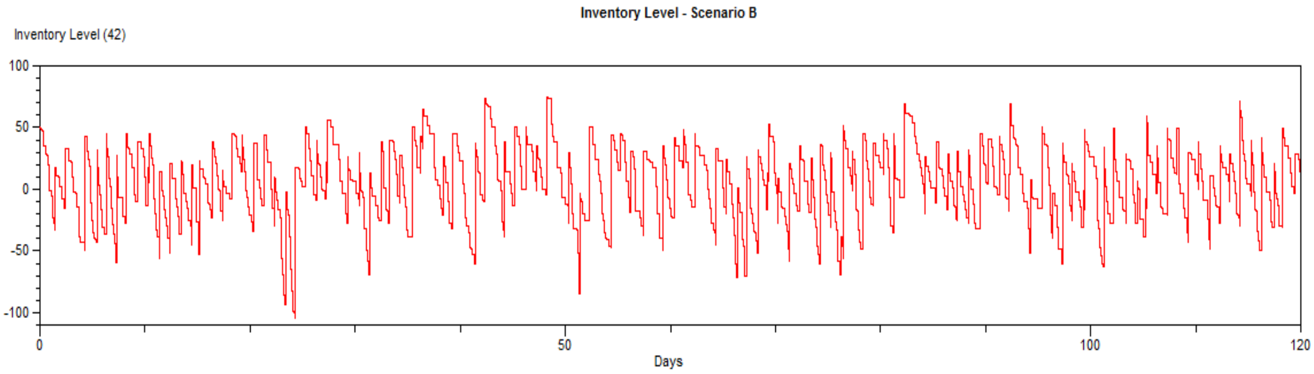


Figure A.9: Inventory level vs. days – scenario B (replication # 42)

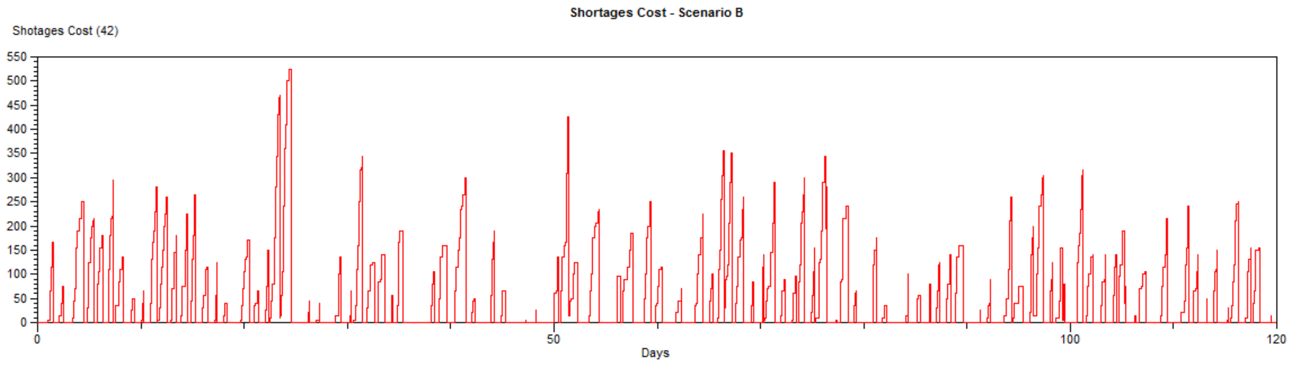


Figure A.10: shortages Cost vs. days-Scenario B (replication # 42)

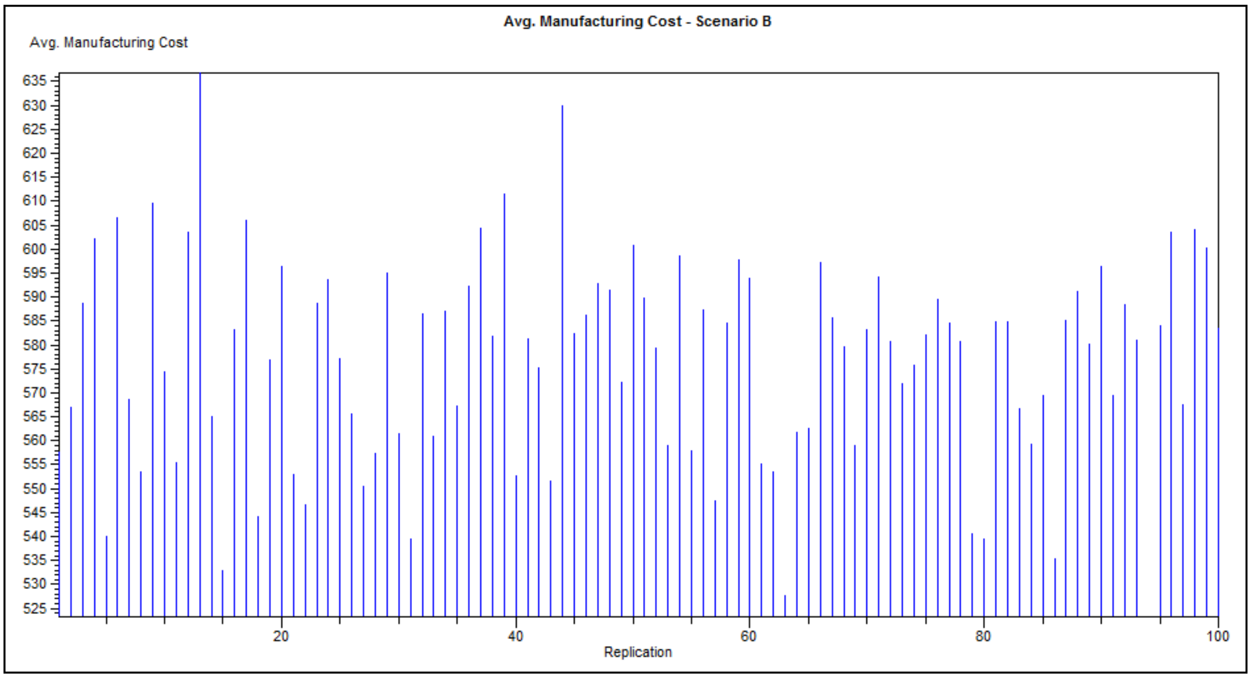


Figure A.11: Average manufacturing cost per replication under scenario B

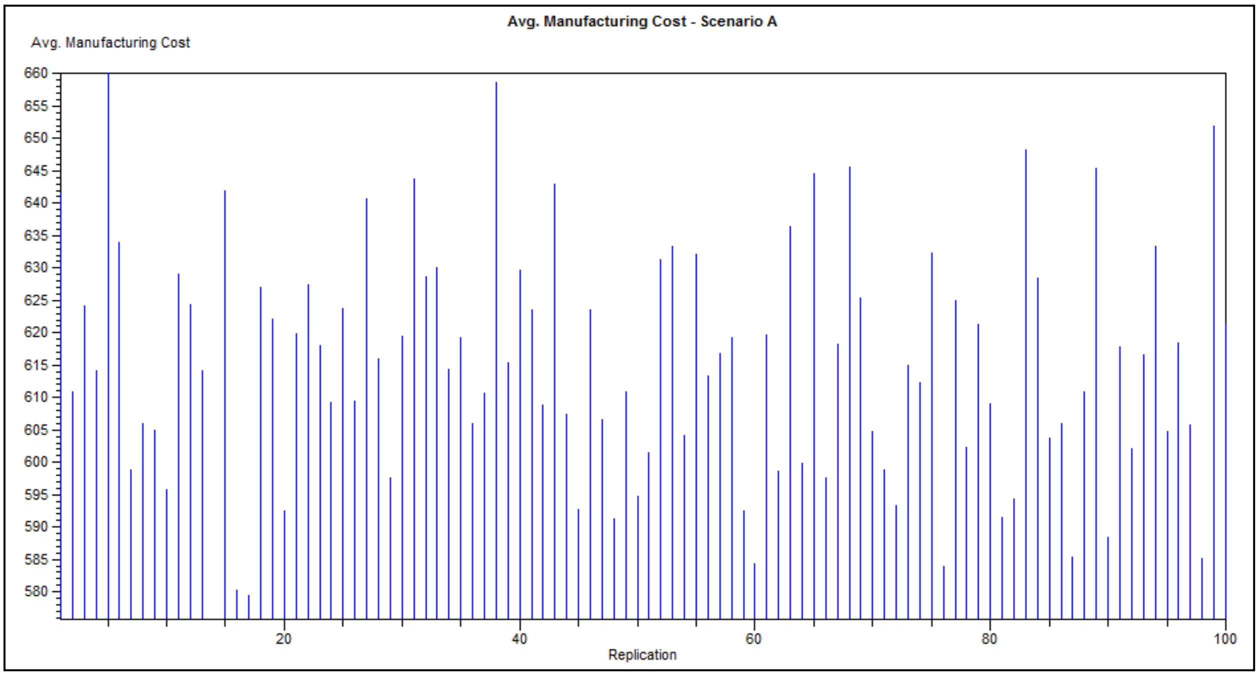


Figure A.12: Average manufacturing cost per replication under scenario A

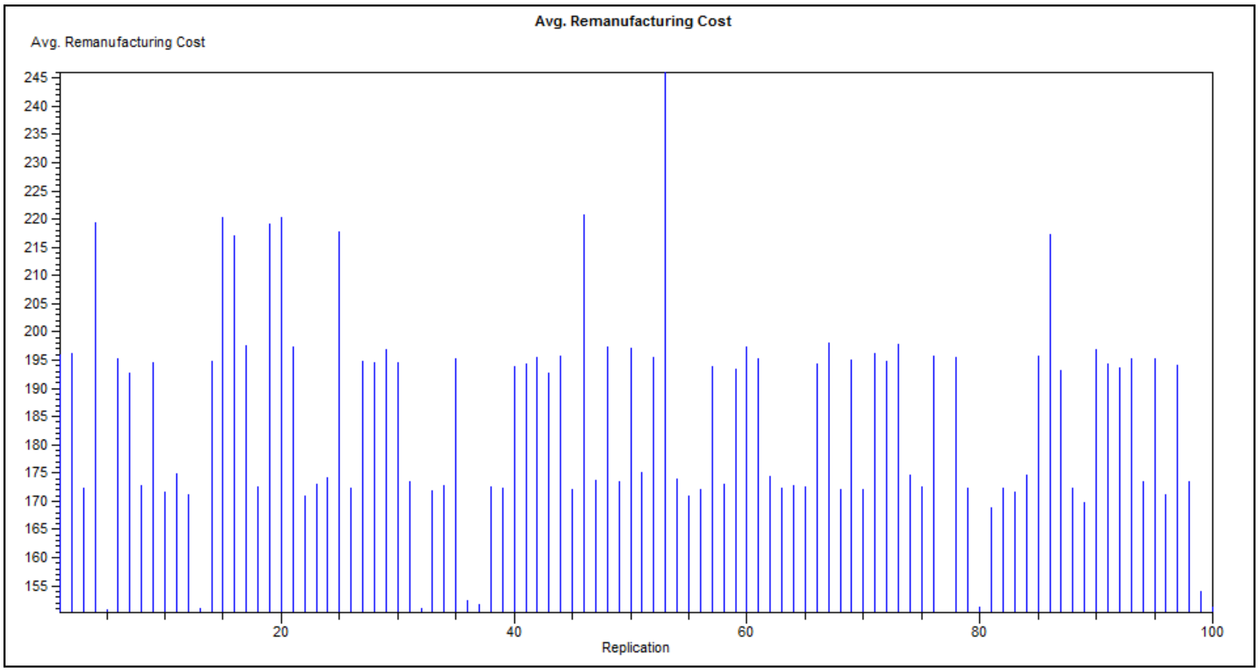
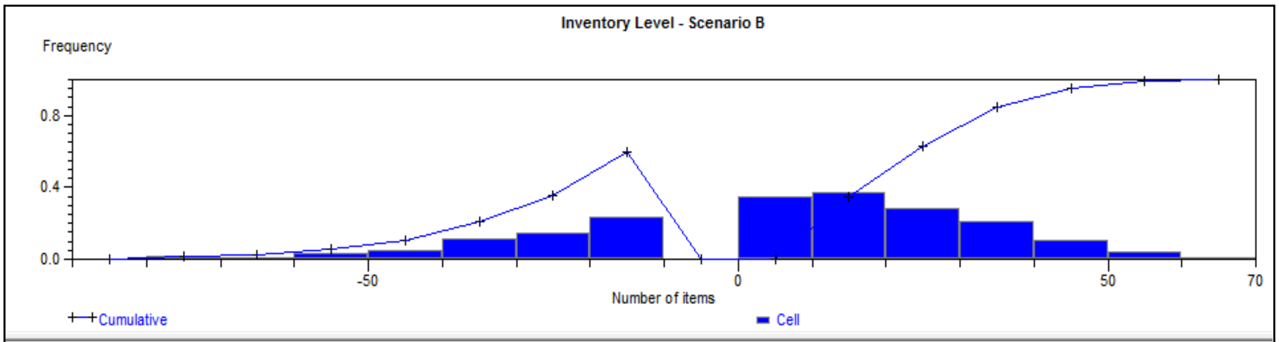


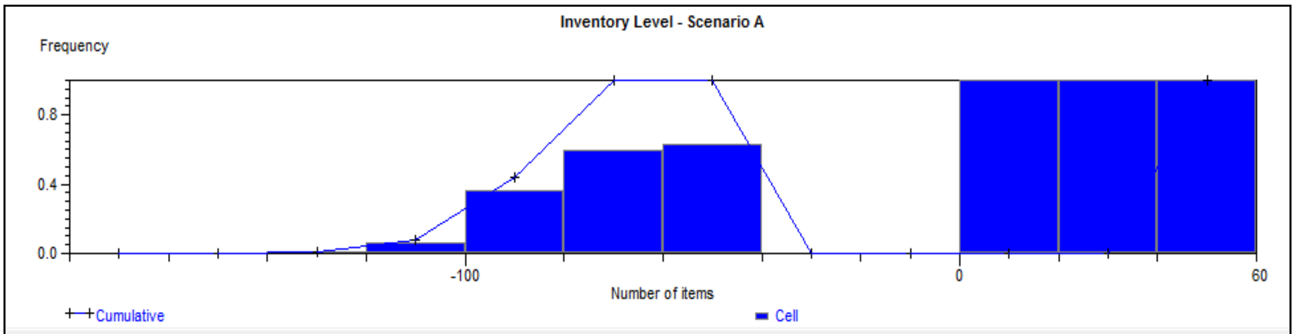
Figure A.13: Average remanufacturing cost per replication



Histogram Summary
Inventory Level - Scenario B

Cell	Cell Limits From	Cell Limits To	Abs. Freq. (Time) Cell	Abs. Freq. (Time) Cumul.	Rel. Freq. Cell	Rel. Freq. Cumul.
1	-Infinity	-80	0	0	0	0
2	-80	-70	0.9747	0.9747	0.01874	0.01874
3	-70	-60	0.2898	1.264	0.005571	0.02431
4	-60	-50	1.616	2.881	0.03108	0.05539
5	-50	-40	2.382	5.262	0.04579	0.1012
6	-40	-30	5.681	10.94	0.1092	0.2104
7	-30	-20	7.592	18.54	0.146	0.3564
8	-20	-10	12.34	30.88	0.2373	0.5937
9	-10	0	-50.11	-19.23	-0.9635	-0.3698
10	0	10	17.94	-1.295	0.3449	-0.02489
11	10	20	19.4	18.1	0.3729	0.348
12	20	30	14.68	32.78	0.2823	0.6303
13	30	40	11.1	43.88	0.2134	0.8437
14	40	50	5.497	49.38	0.1057	0.9494
15	50	60	2.12	51.5	0.04075	0.9901
16	60	+Infinity	0.5136	52.01	0.009875	1

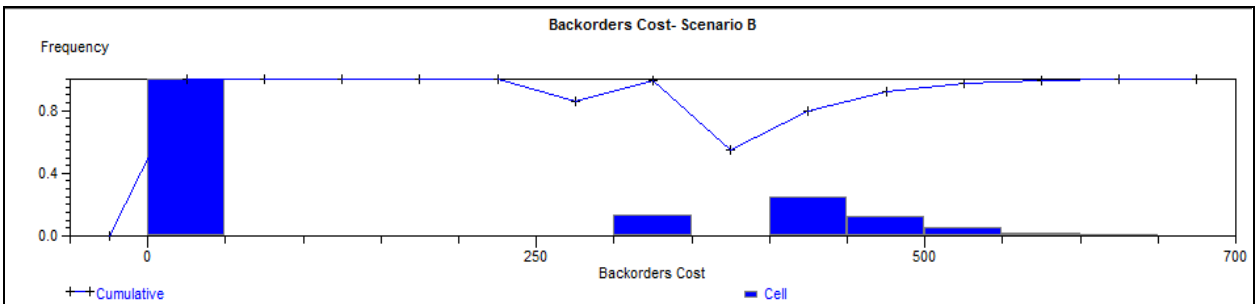
Figure A.14: Histogram of inventory level under scenario B



Histogram Summary
Inventory Level - Scenario A

Cell	Cell Limits From	To	Abs. Freq. (Time) Cell	Cumul.	Rel. Freq. Cell	Cumul.
1	-Infinity	-160	0	0	0	0
2	-160	-140	0.3744	0.3744	0.00312	0.00312
3	-140	-120	0.958	1.332	0.007983	0.0111
4	-120	-100	7.518	8.85	0.06265	0.07375
5	-100	-80	43.63	52.48	0.3636	0.4373
6	-80	-60	71.72	124.2	0.5977	1.035
7	-60	-40	75.71	199.9	0.6309	1.666
8	-40	-20	-734.4	-534.5	-6.12	-4.454
9	-20	0	-1639	-2174	-13.66	-18.11
10	0	20	123.2	-2050	1.027	-17.09
11	20	40	1698	-352	14.15	-2.933
12	40	+Infinity	472	120	3.933	1

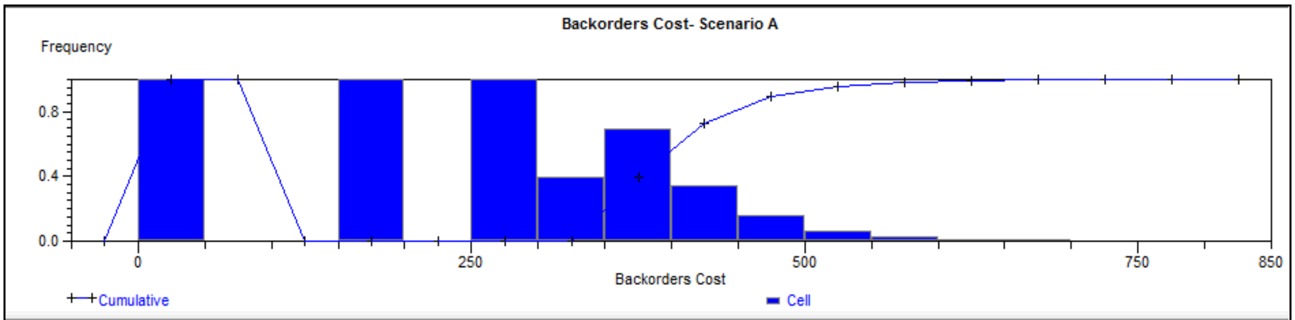
Figure A.15: Histogram of inventory level under scenario A



Histogram Summary
Backorders Cost- Scenario B

Cell	Cell Limits From	To	Abs. Freq. (Time) Cell	Cumul.	Rel. Freq. Cell	Cumul.
1	-Infinity	0	0	0	0	0
2	0	50	1998	1998	16.65	16.65
3	50	100	-685.8	1312	-5.715	10.93
4	100	150	-856	456	-7.133	3.8
5	150	200	-186.4	269.6	-1.553	2.246
6	200	250	-58.1	211.5	-0.4841	1.762
7	250	300	-108.2	103.3	-0.9017	0.8605
8	300	350	15.49	118.7	0.1291	0.9896
9	350	400	-52.73	66.02	-0.4394	0.5502
10	400	450	29.6	95.62	0.2467	0.7969
11	450	500	14.91	110.5	0.1242	0.9211
12	500	550	6.3	116.8	0.0525	0.9736
13	550	600	2.286	119.1	0.01905	0.9926
14	600	650	0.8332	119.9	0.006943	0.9996
15	650	+Infinity	0.05132	120	0.0004277	1

Figure A.16: Histogram of backorders cost under scenario B



Histogram Summary
Backorders Cost- Scenario A

Cell	Cell Limits From	To	Abs. Freq. (Time) Cell	Cumul.	Rel. Freq. Cell	Cumul.
1	-Infinity	0	0	0	0	0
2	0	50	1496	1496	12.47	12.47
3	50	100	-1028	467.8	-8.57	3.898
4	100	150	-649.5	-181.7	-5.413	-1.514
5	150	200	161.2	-20.52	1.343	-0.171
6	200	250	-231.8	-252.3	-1.932	-2.103
7	250	300	168.7	-83.64	1.406	-0.697
8	300	350	47.03	-36.61	0.3919	-0.3051
9	350	400	83.56	46.96	0.6964	0.3913
10	400	450	40.67	87.63	0.3389	0.7302
11	450	500	19.28	106.9	0.1607	0.8909
12	500	550	7.811	114.7	0.06509	0.956
13	550	600	2.663	117.4	0.0222	0.9782
14	600	650	1.228	118.6	0.01023	0.9884
15	650	700	0.9706	119.6	0.008088	0.9965
16	700	750	0.2673	119.8	0.002228	0.9987
17	750	800	0.1526	120	0.001271	1
18	800	+Infinity	0	120	0	1

Figure A.17: Histogram of backorders cost under scenario A

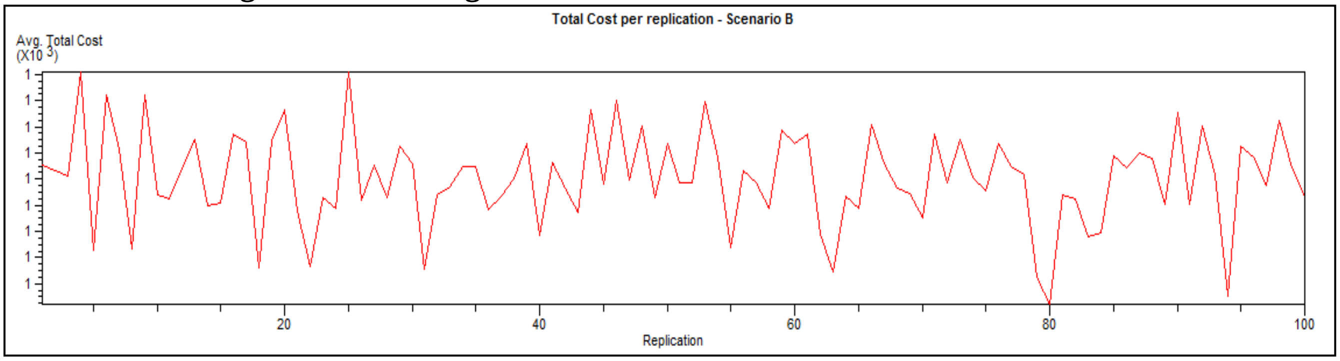


Figure A.18: Average total cost per replication - scenario B

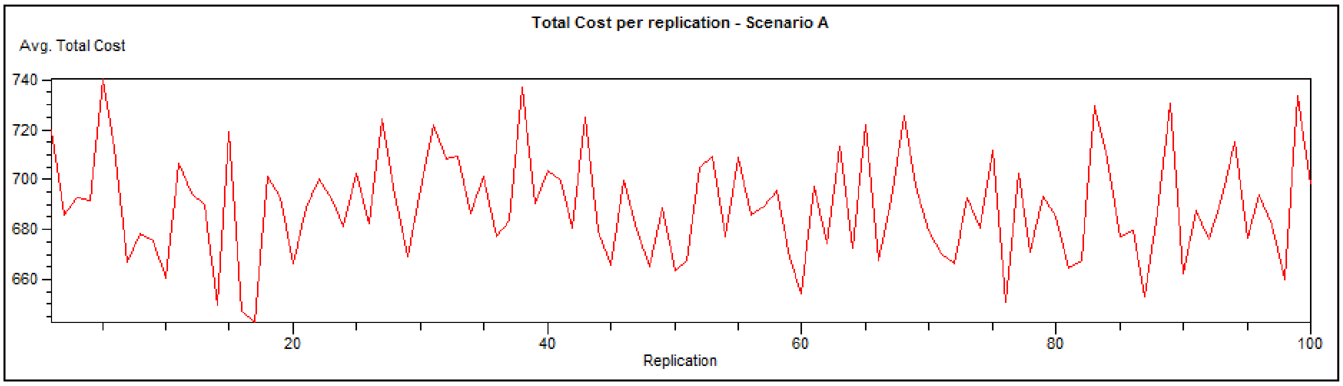


Figure A.19: Average total cost per replication - scenario A

Appendix A.5: Arena® reports (category overview) - scenario A

An s and Q inventory control model Scenario A

Replications: 100 Time Units: Days

Key Performance Indicators

All Entities

	Average
Non-Value Added Cost	0
Other Cost	0
Transfer Cost	0
Value Added Cost	0
Wait Cost	0
Total Cost	0

All Resources

	Average
Busy Cost	0
Idle Cost	0
Usage Cost	0
Total Cost	0

System

	Average
Total Cost	0
Number Out	0

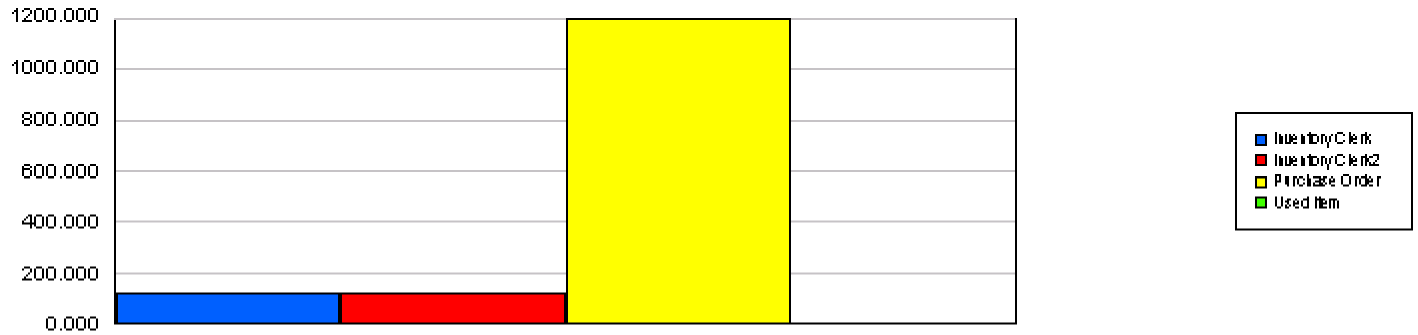
An s and Q inventory control model Scenario A

Replications: 100 Time Units: Days

Entity

Other

Number In	Average	Half Width	Minimum Average	Maximum Average
Inventory Clerk	120.00	0.00	120.00	120.00
Inventory Clerk2	120.00	0.00	120.00	120.00
Purchase Order	1197.79	6.82	1116.00	1283.00
Used Item	1.0000	0.00	1.0000	1.0000



Number Out	Average	Half Width	Minimum Average	Maximum Average
Inventory Clerk	120.00	0.00	120.00	120.00
Inventory Clerk2	120.00	0.00	120.00	120.00
Purchase Order	1197.79	6.82	1116.00	1283.00
Used Item	1.0000	0.00	1.0000	1.0000

WIP	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Inventory Clerk	0.3143	0.00	0.2728	0.3327	0.00	1.0000
Inventory Clerk2	0.00	0.00	0.00	0.00	0.00	1.0000
Purchase Order	0.00	0.00	0.00	0.00	0.00	1.0000
Used Item	0.00	0.00	0.00	0.00	0.00	1.0000

An s and Q inventory control model Scenario A

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Replications: 100 Time Units: Days

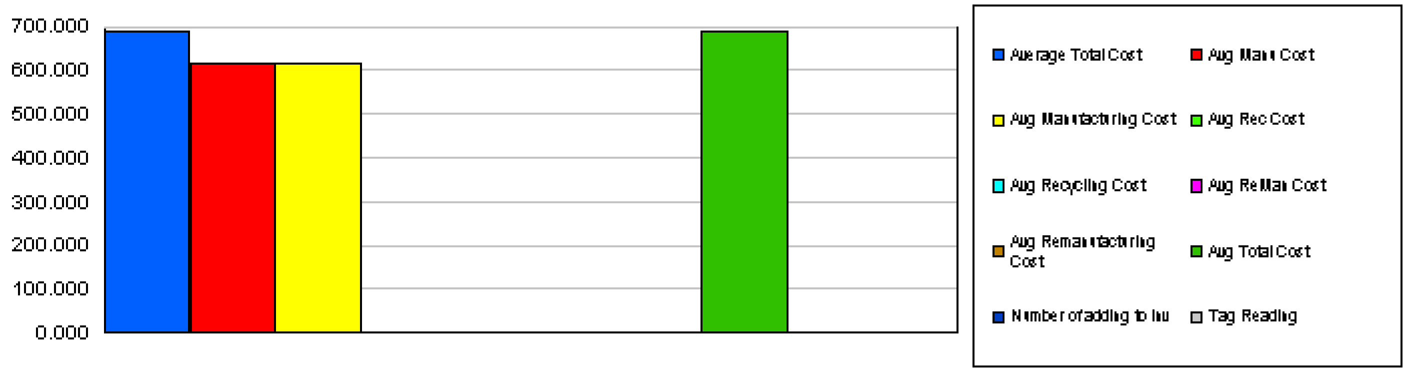
User Specified

Time Persistent

Time Persistent	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Backorder Cost	56.7385	1.11	39.5394	72.5471	0.00	795.00
Holding Cost	21.0140	0.27	17.5157	25.0440	0.00	100.00
Holding Cost Inv2	0.00	0.00	0.00	0.00	0.00	0.00
Inv Level	0	0.30	-4	3.3977	-159	50.0000
Level Of Inv2	0.00	0.00	0.00	0.00		
Shortage Cost	52.7437	0.99	39.5698	65.3616	0.00	795.00
Used Inv Level	0.00	0.00	0.00	0.00	0.00	0.00

Output

Output	Average	Half Width	Minimum Average	Maximum Average
Average Total Cost	688.29	4.33	640.00	746.88
Avg Manu Cost	614.54	3.69	572.33	664.00
Avg Manufacturing Cost	614.54	3.69	572.33	664.00
Avg Rec Cost	0.00	0.00	0.00	0.00
Avg Recycling Cost	0.00	0.00	0.00	0.00
Avg ReMan Cost	0.00	0.00	0.00	0.00
Avg Remanufacturing Cost	0.00	0.00	0.00	0.00
Avg Total Cost	688.29	4.33	640.00	746.88
Number of adding to Inv	0.00	0.00	0.00	0.00
Tag Reading	0.00	0.00	0.00	0.00



Usage

Ouput	Average	Half Width	Minimum Average	Maximum Average
Total Recycled Items	0.00	0.00	0.00	0.00

Appendix A.6: Arena® reports (category overview) - scenario B

An s and Q inventory control model Scenario B

Replications: 100 Time Units: Days

Key Performance Indicators

All Entities

	Average
Non-Value Added Cost	0
Other Cost	0
Transfer Cost	0
Value Added Cost	0
Wait Cost	0
Total Cost	0

All Resources

	Average
Busy Cost	0
Idle Cost	0
Usage Cost	0
Total Cost	0

System

	Average
Total Cost	0
Number Out	0

An s and Q inventory control model Scenario B

Replications: 100 Time Units: Days

Entity

Other

Number In	Average	Half Width	Minimum Average	Maximum Average
Inventory Clerk	120.00	0.00	120.00	120.00
Inventory Clerk2	120.00	0.00	120.00	120.00
Purchase Order	1202.12	7.82	1113.00	1305.00
Used Item	598.87	5.34	545.00	674.00

Number Out	Average	Half Width	Minimum Average	Maximum Average
Inventory Clerk	120.00	0.00	120.00	120.00
Inventory Clerk2	120.00	0.00	120.00	120.00
Purchase Order	1202.12	7.82	1113.00	1305.00
Used Item	598.87	5.34	545.00	674.00

WIP	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Inventory Clerk	0.2952	0.00	0.2665	0.3315	0.00	1.0000
Inventory Clerk2	0.03204929	0.00	0.02826651	0.03591371	0.00	2.0000
Purchase Order	0.00	0.00	0.00	0.00	0.00	1.0000
Used Item	0.00	0.00	0.00	0.00	0.00	1.0000

An s and Q inventory control model Scenario B

Replications: 100 Time Units: Days

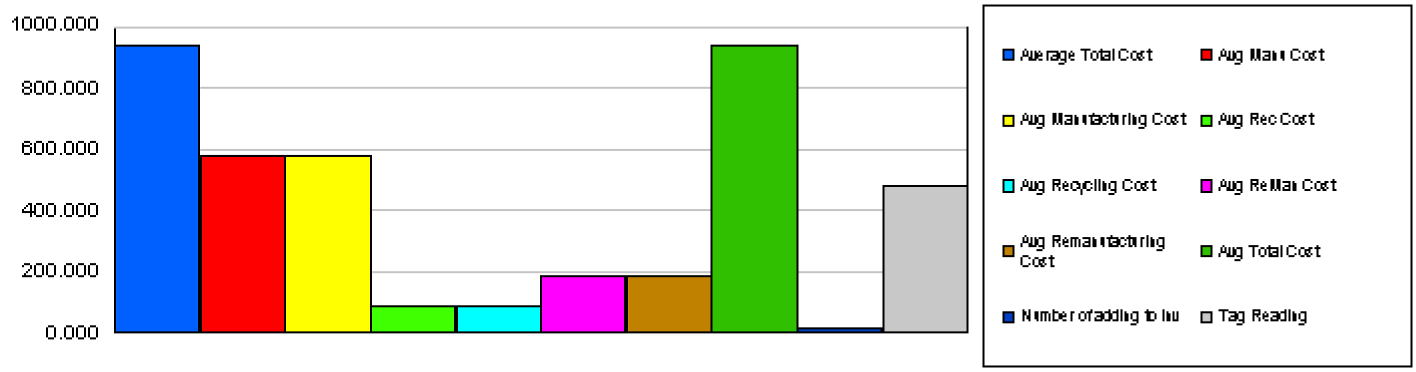
User Specified

Time Persistent

Time Persistent	Average	Half Width	Minimum Average	Maximum Average	Minimum Value	Maximum Value
Backorder Cost	50.0209	1.11	34.5939	61.1272	0.00	670.00
Holding Cost	42.1876	0.33	38.5014	45.9840	0.00	168.00
Holding Cost Inv2	16.3133	0.11	15.0117	17.6537	0.00	39.0000
Inv Level	3.2100	0.34	-1	7.4257	-134	80.0000
Level Of Inv2	18.6500	1.61	4.0000	34.0000		
Shortage Cost	48.6358	1.10	34.6435	59.7081	0.00	670.00
Used Inv Level	16.3133	0.11	15.0117	17.6537	0.00	39.0000

Output

Output	Average	Half Width	Minimum Average	Maximum Average
Average Total Cost	940.62	7.17	844.22	1021.91
Avg Manu Cost	576.60	4.37	523.25	636.75
Avg Manufacturing Cost	576.60	4.37	523.25	636.75
Avg Rec Cost	88.7671	3.18	55.8125	139.69
Avg Recycling Cost	88.7671	3.18	55.8125	139.69
Avg ReMan Cost	184.43	3.72	150.27	245.99
Avg Remanufacturing Cost	184.43	3.72	150.27	245.99
Avg Total Cost	940.62	7.17	844.22	1021.91
Number of adding to Inv	15.5100	0.16	14.0000	18.0000
Tag Reading	480.65	4.65	427.00	540.00



Usage

An s and Q inventory control model Scenario B

8

Replications: 100 Time Units: Days

User Specified

Usage

Ouput	Average	Half Width	Minimum Average	Maximum Average
Total Recycled Items	118.22	2.10	94.0000	149.00

Appendix A.7: Verification of simulation model under scenario A

Table A.1: Sample inventory level for 10 days[4]- replication # 55

Day	Inv. Level	Q+s-I(t)	Lead time	Day	Inv. Level	Q+s-I(t)	Lead time
0	50			2.79	-22		
0	50			2.82	-29	+ 79	
0.124	40			3.08	-36		
0.144	35			3.11	-41		
0.164	25			3.12	-46		
0.221	18			3.21	-51		
0.266	17			3.28	-56		
0.343	12			3.32	-57		
0.522	11			3.33	-62		
0.589	10			3.46	17	*	0.64
0.786	3			3.53	12		
0.949	-2	+ 52		3.66	11		
1.03	-9			3.81	1		
1.36	-10			3.95	0	+ 50	
1.38	42	*	0.431	4.07	-7		
1.56	37			4.18	-14		
1.58	32			4.22	36	*	0.27
1.72	27			4.25	26		
1.75	22			4.27	21		
1.77	15			4.29	11		
1.83	8			4.35	4		
1.84	1			4.56	-3		
1.85	-4			4.67	-10		
1.9	-14			4.68	-20		
1.9	-19			4.7	-21		
1.93	-24	+ 74		4.93	-22		
2.01	-29			4.93	-27		
2.03	-34			4.98	-28	+ 78	
2.04	-39			5.18	-35		
2.06	-44			5.24	-45		
2.07	-49			5.26	33	*	0.28
2.15	-59			5.26	32		
2.17	-69			5.3	25		
2.29	5	*	0.36	5.51	18		
2.32	-2			5.57	11		
2.4	-9			5.6	4		
2.41	-10			5.63	3		
2.74	-15			5.7	-4		

[4] Check points are shown with “+”, and adding to the inventory is shown with “*”.

Table A.1: (continue)

Day	Inv. Level	Q+s-I(t)	Lead time	Day	Inv. Level	Q+s-I(t)	Lead time
5.82	-14			9.25	-16		
5.88	-19	+ 69		9.37	-26		
6.11	-26			9.41	33	*	0.41
6.26	-36			9.58	32		
6.33	-43			9.72	27		
6.36	26	*	0.48	9.73	22		
6.38	19			9.76	12	+ 38	
6.4	12			10.2	7		
6.41	11			10.2	2		
6.51	6			10.3	-3		
6.58	-1			10.3	-13		
6.8	-6			10.3	25	*	0.54
6.85	-13						
6.87	-23	+ 73					
7.17	-24						
7.24	-34						
7.31	39	*	0.44				
7.36	29						
7.37	22						
7.44	15						
7.7	5						
7.76	4						
7.79	-3						
7.83	-4						
7.85	-14						
7.92	-21	+ 71					
8.02	-22						
8.14	-27						
8.23	-37						
8.38	34	*	0.46				
8.56	27						
8.64	22						
8.74	15						
8.75	10						
8.78	3						
8.95	-2						
9	-9	+ 59					
9.1	-10						
9.18	-15						

Appendix B.1: Fitting exponential distribution on failure data with MATLAB

```
%program 'DisfittingbyShahin.m'


life = [ 2.6 6.4 9.5 8.8 7.8 22.1 27.3 32.6 45.9 78.3 ];
binWidth = 10;
binCtrs = 1:binWidth:80;
hist(life,binCtrs);
xlabel('Time to Failure'); ylabel('Frequency'); ylim([0 5]);
h = get(gca,'child');
set(h,'FaceColor',[.98 .98 .98],'EdgeColor',[.94 .94 .94]);
counts = hist(life,binCtrs);
hold on
plot(binCtrs,counts,'o');
hold off
paramEsts = expfit(life);
n = length(life);
prob = counts / (n * binWidth);
bar(binCtrs,prob,'hist');
h = get(gca,'child');
set(h,'FaceColor',[.9 .9 .9]);
xlabel('Time to Failure (months)'); ylabel('Probability Density'); ylim([0 0.1]);
xgrid = linspace(0,20,100);
pdfEst = exppdf(xgrid,paramEsts);
line(xgrid,pdfEst)
```

Appendix B.2: Kolmogorov-Smirnov test (K-S test) with MATLAB

```
%program 'KStestbyShahin.m'


x=[2.6;6.4;7.8;8.8;9.5;22.1;27.3;32.6;45.9;78.3];
F=expcdf(x,1/0.041);
cdf=[x F];
[h,p,ksstat,cv]=kstest(x,cdf,0.05);
h;
p;
ksstat;
cv;
```

Appendix B.3: Estimating *Voltera integral equation* (2nd kind) with MATLAB

```
%program 'Voltera2ndbyshahin.m'

L=.041;
N(1)=(1- exp(-L))/exp(-.5*L);
F1=1-exp(-.5*L);

for i=2:1:48
  sum=0;
    for j=1:1:(i-1)
       F=1-exp(-1*(i-j+0.5)*L);
        if j==1
          sum=sum+((N(j)-0)*(F));
        else
         sum=sum+((N(j)-N(j-1))*(F));
        end
    end

N(i)=(1/(1-F1))*((1- exp(-i*L))+sum-(N(i-1)*F1));


end



N
```

Appendix B.4: Sample DCA matrix calculations using MATLAB

W_{ij} = a

W_{jk} = b   (Note that rows and columns are switched to make the multiplication possible)

$M_{ik} = W_{ij} \times W_{jk}$

```
>> a=[5 3 0 0 0;2 2 1 0 0;0 2 8 5 0;0 2 5 5 0;2 0 4 2 2];

>> b=[.5 0 .5;0 .2 .8;.1 .9 0;.3 .4 .3;.5 0 .5];

>> a*b

ans =

   2.5000    0.6000    4.9000
   1.1000    1.3000    2.6000
   2.3000    9.6000    3.1000
   2.0000    6.9000    3.1000
   3.0000    4.4000    2.6000
```

Switching rows with columns, cause -subassembly/component matrix (M_{ik}) is found to be:

[2.5 1.1 2.3 2 3; 0.6 1.3 9.6 6.9 4.4;4.9 2.6 3.1 3.1 2.6];

M_1 =2.5+1.1+2.3+2+3 = 10.9; I_1 = 10.9/50 = 21.8%

M_2 =0.6+1.3+9.6+6.9+4.4 = 22.8; I_2 = 22.8/50 = 45.6%

M_3 = 4.9+2.6+3.1+3.1+2.6 = 16.3; I_3 = 16.3/50 = 32.6%

VI_1 = (2.5+1.1+2.3)/50 = 11.8%; PI_1 = (2+3)/50 = 10%

VI_2 = (0.6+1.3+9.6)/50 = 23%; PI_2 = (6.9+4.4)/50 = 22.6%

VI_3 = (4.9+2.6+3.1)/50 = 21.2%; PI_3 = (3.1+2.6)/50 = 11.4%

Appendix B.5: Solving system of linear equations with MATLAB

```
>> a=[5 -3 -2;-3 4 -1;-2 -1 3];
>> b=[1.5041;-2.1972;0.6931];
>> pinv(a)*b

ans =
    0.1613
   -0.3749
    0.2136

>> b=[0.1054;-3.3165;-0.4055;2.9110;-1.0217;1.7272];
>> a=[5 0 0 0 -3 -2;0 4 0 -3 0 -1;0 0 3 -2 -1 0;0 -3 -2 5 0 0;-3 0 -1 0 4 0;-2 -1 0 0 0 3];
>> pinv(a)*b
ans =
    0.0767
   -0.5299
   -0.0384
    0.2489
   -0.2075
    0.4502
```

Appendix C.1: Failure/success histogram

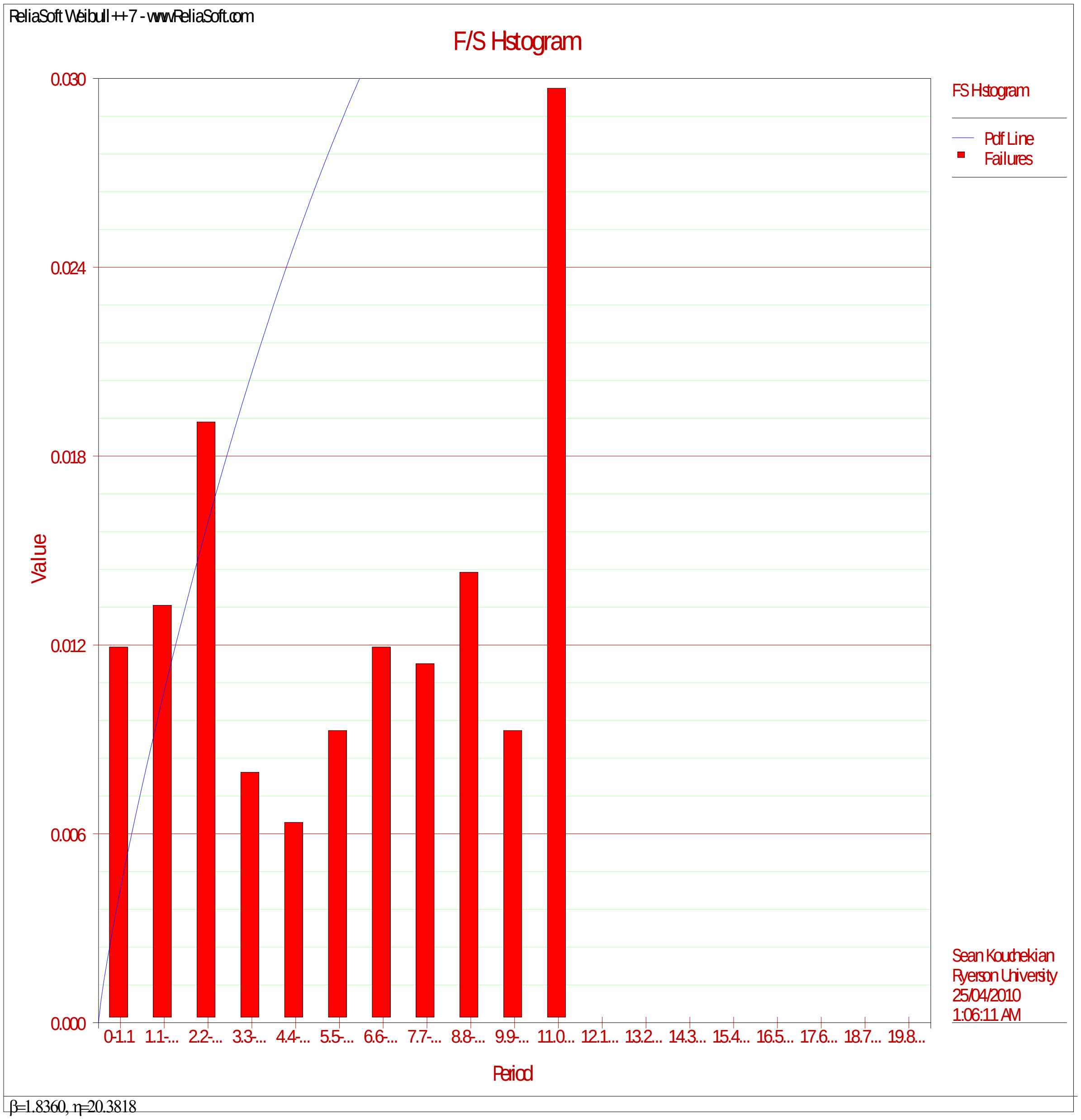


Figure C.1: Failure/Success histogram for the remanufactured item

Appendix C.2: Failure/success timeline

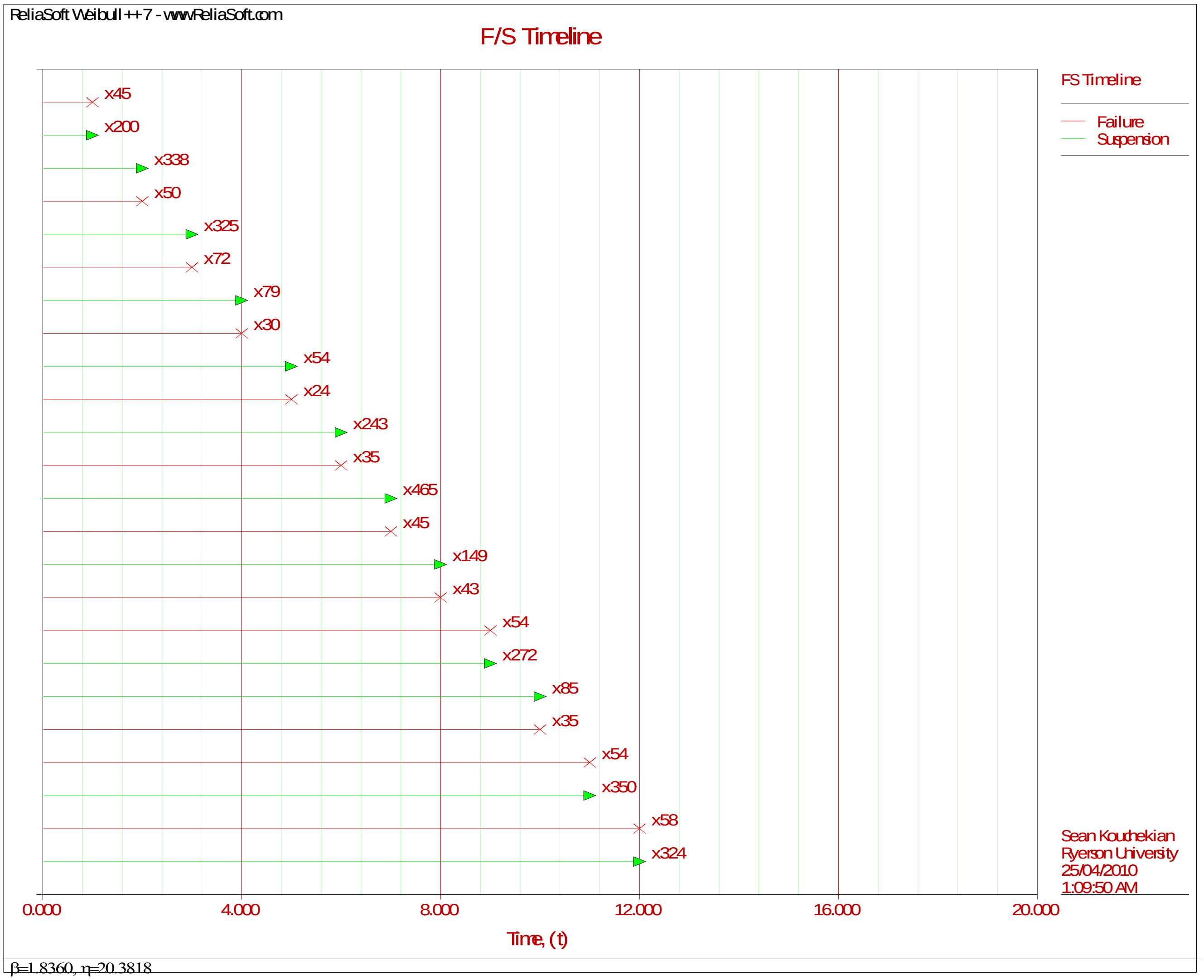


Figure C.2: Failure/Success timeline for the remanufactured item

Appendix C.3: Unreliability vs. time plot

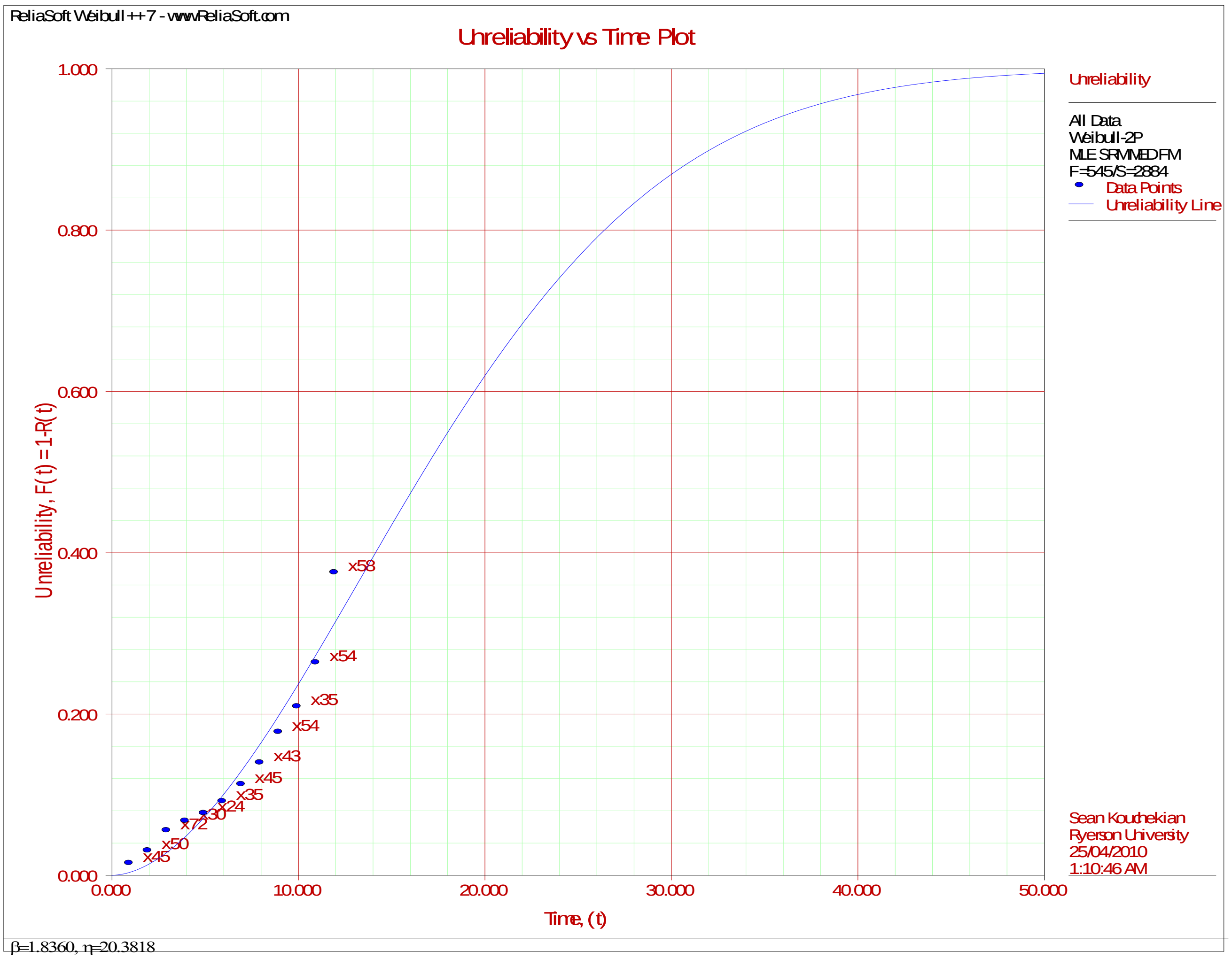


Figure C.3: Unreliability plot vs. time for the remanufactured item

Appendix C.4: Probability density function (pdf) plot vs. time

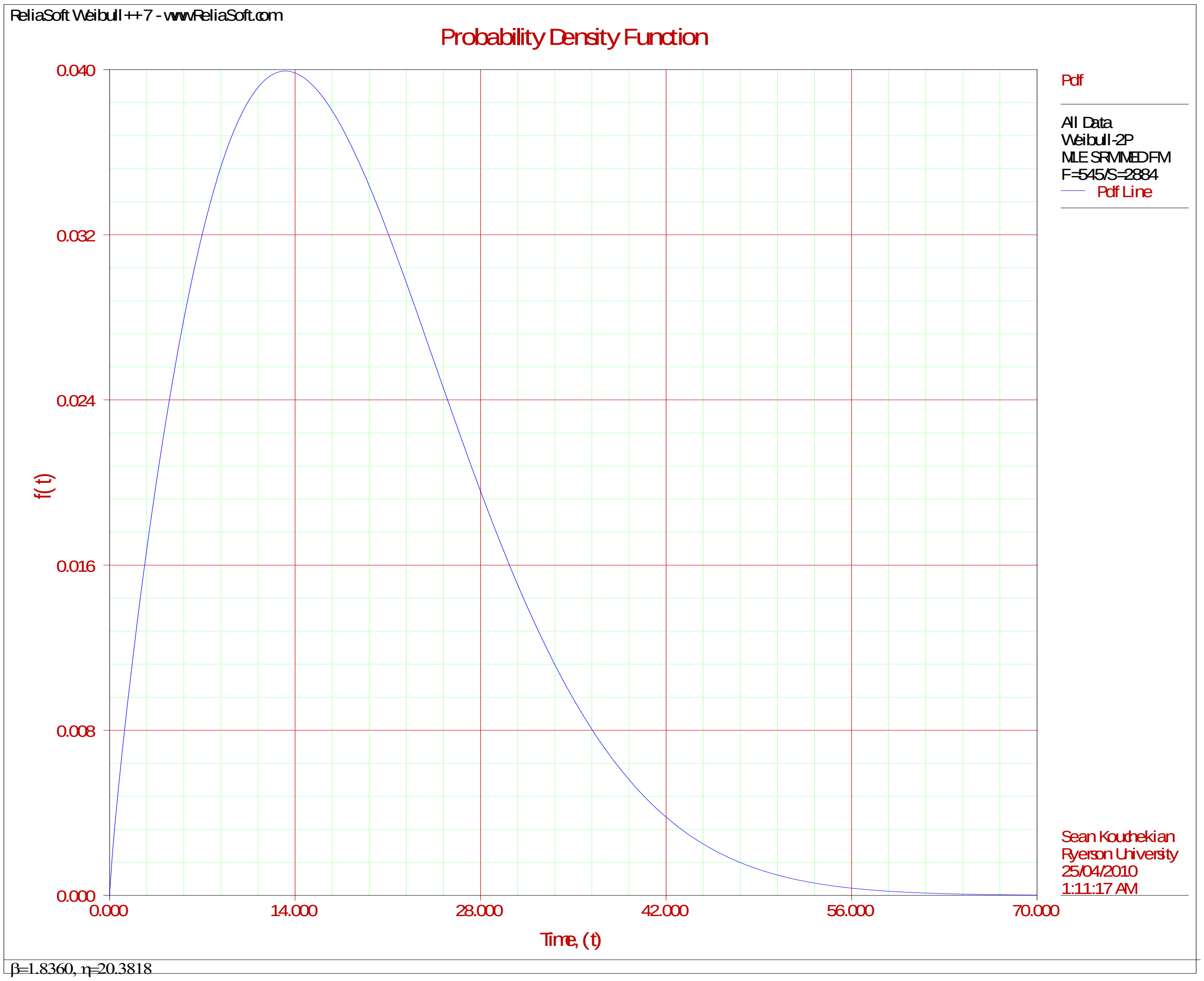


Figure C.4: Weibull pdf plot vs. time for the remanufactured item

Appendix C.5: Failure/success pie

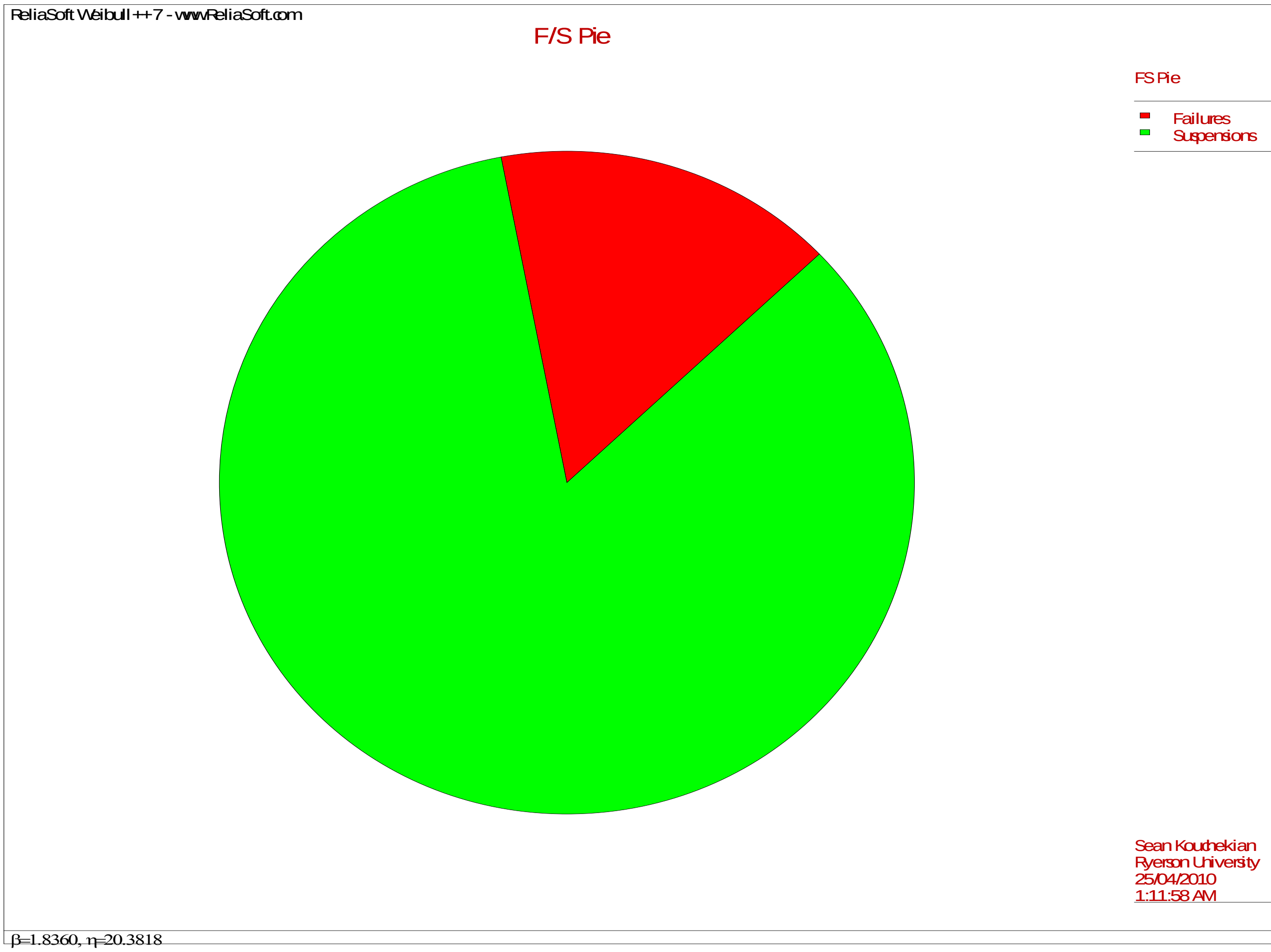


Figure C.5: Failure/Success pie for the remanufactured item