PROFIT MAXIMIZATION MODELS FOR A TWO-LEVEL DUAL-CHANNEL SUPPLY CHAIN

by

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AUTHOR'S DECLARATION

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ABSTRACT

Internet-based technologies have changed the way firms do business and manage their supply chains. They have influenced customers' purchase patterns, thereby motivating manufacturers to introduce online channels alongside traditional ones. Such structures are known as dual-channels. Nowadays, an increasing number of manufacturers offer a return policy to attract more customers and to stay competitive. Furthermore, learning-based continuous improvements help firms cope with market changes and be competitive, flexible and efficient. This thesis presents three main models:

The first model investigates the effect of adopting a dual-channel (comprised of a retail channel and an online channel) on the performance of a two-level (vendor-retailer) supply chain. The objective is to maximize the total profit of the system by finding the optimal markup margin and inventory decisions before and after adopting the dual-channel. The results show that adding an online channel would increase the profit of the system. However, it creates a conflict due to competition between the retail and online channels.

The second model studies a supply chain system, which is comprised of production, refurbishing, collection, and waste disposal processes. A return policy in which customers can return the purchased item for a refund is also considered. The purpose is to examine the effect of different return policies on the behavior of the system before and after adopting the dual-channel strategy. In both strategies, the model analyzes the change in the profit, the pricing and inventory decisions. The findings demonstrate that the more generous the return policy is, the higher the demand, the selling prices and the overall profit.

The third model investigates the effects of learning and forgetting in the vendor's production processes. It also considers single- and dual-channel strategies. Each channel structure can adopt any of six inventory policies. Learning and forgetting effects are considered in all policies except one. The objective is to maximize the profit of the system by finding the joint optimal pricing and inventory decisions. The results suggests that learning, despite being impeded by forgetting, reduces inventory-related costs thereby allowing the chain to reduce the prices of its product(s), which increases demand and subsequently sales.

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Dedication

In memory of my mother, Fatemah Batarfi, whose enduring love made this possible.

You are loved beyond words and missed beyond measure

This is for you mom

Raaid Batarfi - April 2017

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The following notations are used in Chapter CHAPTER 4:

Notation	Description
D_r, D_d	Demand of the retail and the direct online channel, respectively, (units/year);
a	Primary demand (potential demand when the items are free of charge), (units/year);
$\theta, (1-\theta)$	Percentage share of the demand going to the direct online and retail channel, respectively, (%);
α_r	Coefficient of price elasticity of D_r, (unit^2/\$/year);
α_{dk}	Coefficient of price elasticity of D_d, $k = 1, 2, \dots, N$, (unit^2/\$/year);
ρ	Cross-price sensitivity, (-);
l_d	Quoted delivery lead-time (i.e., waiting time) of customized items, (day);
β_r	Sensitivity to quoted delivery lead-time of the demand D_r, (customer/day);
β_d	Sensitivity to quoted delivery lead-time of the demand D_d, (customer/day);
η_d	Product differentiation $\eta_d = \sum_{i=1}^{I}(\lambda_i \cdot w_i)$, (-);
w_i	Weight of importance of feature i where $(0 \leq w_i \leq 1)$ and $i = 1,2, \dots, I$, (-);
λ_i	Availability of the feature i, (-);
φ_{dk}	Percentage of core items stock used for customized item k, $k = 1, 2, \dots, N$, (-);
ψ_r	Sensitivity to product differentiation of D_r, (-);
ψ_d	Sensitivity to product differentiation of D_d, (-);
P_r	Production rate for the standard item, $P_r > a$, (units/year);
P_d	Production rate for the core item for eventual customization, $P_d > a$, (units/year);
c_P	Production cost for the standard item, (\$/unit);;
c_{dk}	Production cost for the customized item k, $k = 1, 2, \dots, N$, (\$/unit);
c_r	Vendor's wholesale price of the standard item to the retailer, (\$/unit);
p_r	Retailer's selling price of the standard item; $p_r > c_r > c_P$, (\$/unit);
p_{dk}	Vendor's selling price of the customized item k, $k = 1, 2, \dots, N$, (\$/unit);
S_r	Vendor's setup cost for the standard item, (\$/setup);

S_d	Vendor's setup cost for the core item, (\$/setup);
h_1	Vendor's holding cost which includes financial cost and storage cost, (\$/unit/year);
h_2	The supply chain unit holding cost for items at the retailer side (vendor's financial unit holding cost + buyer's unit storage cost), (\$/unit/year);
O_r	Retailer's order cost, (\$/order);
T_r	Cycle time (interval length) of the standard item, where $T_r = n_r q_r / D_r$, (year);
T_d	Cycle time (interval length) of the core item, where $T_d = q_d / D_d$, (year);
V_N	Set of variant k, where $k \in V_N$, and $k = 1, 2, \dots, N$, (-);
N	Total number of variants, (-);
I	Total number of custom features, (-);
m	Markup margin, (%);
q_r	Ordered quantity of the standard item to the retail channel, (units);
q_d	Production quantity of the core item for customization, (units);
n_r	Number of shipments of the standard item to the retail channel.

Note: non-dimensional parameters are indicated by (-)

The following notations are used in Chapter 5:

Notation	Description
i	Subscript indicating the type of item; $s=$ standard, $f=$ refurbished standard, $z=$ customized, and $fz=$ refurbished customized
D_i	Demand rate for the i item, (unit/year)
a_i	Primary demand for the i item, (unit/year)
δ_i	price elasticity for the i item, (unit^2/\$/year)
γ_i	Sensitivity of demand D_i with respect to the return policy, (unit^2/\$/year)
ζ	A migration parameter, (unit^2/\$/year)
l	Lost value parameter, (\$/unit)
ρ_i	Proportion of the i items returned (repairable and non-repairable items) from the demand D_i; $0 \leq \rho_i < 1$ (%)
α_i	Proportion of non-repairable (disposed) returned i items form the demand D_i; $0 \leq \alpha_i < 1$, (%)
β_i	Proportion of repairable i items returned for refurbishing from the demand D_i; $0, \leq \beta_i < 1$, $\beta_i = (1-\alpha_i)$, (%)
P_i	Production rate for the i item; $P_i > D_i$ (unit/year),
c_i	Unit production/processing cost of the i item $i \neq z, fz$, (\$/unit)
c_{ik}	Unit production/processing cost of the i item $i \neq s, f$, and variant k, $k = 1, 2, \dots, N$, (\$/unit)
c_w	Cost of disposing a non-repairable item, (\$/unit)
S_i	Setup cost for the i item, (\$/setup)
$h_{v_1 i}$	Holding cost at vendor 1's side for a unit of item $i \neq f, fz$ (financial and physical

storage cost), ($/unit/year)

$h_{bi}^{v_1}$ Financial holding cost for a unit of item $i \neq z, fz$, at the retailer's side paid by vendor 1, ($/unit/year)

h_{bi}^{b} Physical storage holding cost for a unit of item $i \neq z, fz$, at the retailer's side paid by the retailer, ($/unit/year)

h_{bi} Total holding cost for a unit of item $i \neq z, fz$, at the retailer's side, where $h_{bi} = h_{bi}^{b} + h_{bi}^{v_1}$

$h_{v_2 i}^{v_1}$ Financial holding cost for a unit of item $i \neq s, z$ at vendor 2's side paid by vendor 1, ($/unit/year)

$h_{v_2 i}^{v_2}$ Physical storage holding cost for a unit of item $i \neq s, z$ at vendor 2's side paid by vendor 2, ($/unit/year)

$h_{v_2 i}$ Total holding cost for a unit of item $i \neq s, z$ at vendor 2's side where $h_{v_2 i} = h_{v_2 i}^{v1} + h_{v_2 i}^{v2}$

$h_{bu}^{v_1}$ Financial holding cost for a repairable item at the retailer's side paid by vendor 1, ($/unit/year)

h_{bu}^{b} Physical storage holding cost for a repairable item at the retailer's side, ($/unit/year)

h_{bu} Total holding cost for a repairable item at the retailer's side, where $h_{bu} = h_{bu}^{v_1} + h_{bu}^{b}$; $h_{bf} > h_{v_2 f} > h_{bu}$

$h_{v_2 u}^{v_1}$ Financial holding cost for a repairable item at the side of vendor 2 paid by vendor 1, ($/unit/year)

$h_{v_2 u}^{v_2}$ Physical storage holding cost for a repairable item at the side of vendor 2 paid by vendor 2, ($/unit/year)

h_{v_2u}	Total holding cost for a repairable item at vendor 2's side, where $h_{v_2u} = h_{v_2u}^{v_1} + h_{v_2u}^{v_2}$
T_i	Length of time interval for the process of i itme, (year)
p_{v_1i}	Vendor 1's selling price (wholesale price) of i item $i \neq z, fz$ to the retailer, ($/unit)
O_{bi}	Retailer's ordering cost for i items $i \neq z, fz$, ($/order)
φ_{ik}	Percentage of core item stock used for ik item, $i \neq s, f$ and $k = 1,2, \dots N$
η_i	Proportion of the selling price of the i item refunded to a customer, where $0 \leq \eta_i \leq 1$
r_i	Refunded amount per unit of i item, where $r_i = \eta_i p_i$, $0 \leq r_i \leq p_i$, ($/unit)
r_{ik}	Refunded amount per unit of ik item , $i \neq s, f$ and $k = 1,2, \dots N$, where $r_{ik} = \eta_i p_{ik}$, $0 \leq r_{ik} \leq p_{ik}$, ($/unit)
V_N	Set of variant k, where $k \in V_N$ and $k = 1, 2, \dots, N$
N	Total number of variants, (-)
I	Total number of custom features
p_i	Selling price of i item, $i \neq z, fz$ to customers , ($/unit)
p_{ik}	Selling price of ik item, $i \neq s, f$, $k = 1, 2, \dots, N$, ($/unit)
q_i	Shipment (batch) size for i item, $i \neq z, fz$ (unit)
q_{ik}	Production quantity of item ik, $i \neq s, f$ and $k = 1,2, \dots N$, (unit)
n_i	Number of shipment of i item $i \neq z, fz$ and n_i integer $n_i \geq 1$

Note: non-dimensional parameters are indicated by (-)

The following notations are used in Chapter 6:

Notation	Description
$D_{r,i}, D_{d,i}$	Demand for the retail and the direct channel for cycle i, respectively, (unit/year);
a	Primary demand (potential demand when the item is free of charge), (unit/year);
$\theta, (1-\theta)$	Percentage share of the demand going to the direct and retail channel, respectively;
α_r	Coefficient of price elasticity of the standard item, (unit^2/\$/year)
α_{dk}	Coefficient of price elasticity of the customized k item, where $k = 1, 2, \dots, N$, (unit^2/\$/year);
ρ	Cross-price sensitivity;
l_d	Quoted delivery lead-time (i.e., waiting time) of customized items, (day);
β_r	Sensitivity to quoted delivery lead-time of the demand D_r, (unit/day);
β_d	Sensitivity to quoted delivery lead-time of the demand D_d, (unit/day);
φ_{dk}	Percentage of core item stock used for customized item k, $k = 1, 2, \dots, N$, (-);
P_r	Production rate for the standard item, $P_r > a$, (unit/year);
P_d	Production rate for the core item for eventual customization, $P_d > a$, (unit/year);
c_P	Unit production/labour cost for the standard item, (\$/unit);
c_{dk}	Unit production/labour cost for the customized k item, $k = 1, 2, \dots, N$, (\$/unit);
c_r	Vendor's wholesale price of the standard item to the retailer, (\$/unit);
S_r	Vendor's setup cost for the standard item, (\$/setup);
S_d	Vendor's setup cost for the core item, (\$/setup);
O_r	Retailer's order cost for the standard item, (\$/order);

h_{v1}	Holding cost at the vendor's side, which includes the financial and physical storage holding cost, (\$/unit/year);
h_{v2}	Financial holding cost for a unit of the standard item at the retailer's side paid by the vendor, (\$/unit/year);
h_r	Physical storage holding cost for a unit of the standard item at the retailer's side paid by the retailer, (\$/unit/year);
$t_{r,i}$	The time required to produce the ith $q_{r,i}$ of the standard item in cycle i, (year);
$t_{d,i}$	The time required to produce the ith $q_{d,i}$ of the core item in cycle i, (year);
T_{r1}	The time required to process the first unit of the standard item, (year/unit);
T_{d1}	The time required to produce the first unit of the customized core item, (year/unit);
b_r	Learning curve exponent for the standard item, (-);
b_d	Learning curve exponent for the core item, (-);
B_r	The time for total forgetting of the standard item, (year);
B_d	The time for total forgetting of the standard item, (year);
V_N	Set of variant k, where $k \in V_N$ and $k = 1, 2, \ldots, N$, (-);
N	Total number of variants (-);
Z	Total number of custom features(-);
p_r	Selling price of a standard item to customers, (\$/unit);
p_{dk}	Selling price of a customized k item, $k = 1, 2, \ldots, N$, (\$/unit);
q_r	Shipment (batch) size for standard item, (unit);
q_d	Production quantity of the core item for eventual customization, (unit);
n_r	Number of shipments of the standard item, integer $n_r \geq 1$

Note: non-dimensional parameters are indicated by (-)

LIST OF ACRONYMS

3PL: Third-party logistics

4PL: Fourth-party logistics

AHP: Analytic Hierarchy Process

BTO: Build-to-order

CAD: Computer-aided design

CAM: Computer-aided manufacturing

CPFR: Collaborative planning, forecasting and replenishment

CS: Consignment stock

CSCMP: Council of Supply Chain Management Professionals

EDI: Electronic data interchange

EOL: End-of-life

EOQ: Economic order quantity

EPQ: Economic production quantity

FC: Forgetting curve

FL: Forward logistics

FMS: Flexible manufacturing system

IT: Information technologies

JELS: Joint economic lot size

LC: Learning curve

LFCM: The learn-forget curve model

LFL: Lot-for-lot

LR: Learning rate

MC Mass customization

MADM:	Multi-Attribute Decision-Making
MTS:	Make-to-stock
NB:	National brand
OEM:	Original equipment manufacturer
QR:	Quick response
RL:	Reverse logistics
SB:	Store brand
SCM:	Supply chain management
VBA:	Visual Basic for Applications
VMI:	Vendor-managed inventory
WLC:	Wright learning curve

CHAPTER 1. INTRODUCTION

This chapter provides the basis of the topics that are researched in this thesis. First, it gives a brief overview of the main concepts of supply chain management (SCM), inventory management, and supply chain coordination. Then it gives a more detailed overview of the main subjects that are covered in this thesis, namely: dual-channel supply chain, reverse logistics, and learning and forgetting effects.

1.1 Supply Chain Management (SCM)

The concept of supply chain management (SCM) has risen to prominence over the past two decades due to the substantial interest among researchers and practitioners (Cooper et al., 1997). The concept has been defined in various ways, but most researchers agree that it is the management (oversight) of the flow of materials, (i.e. information, goods, services and financial flow) between different supply chain players. These players can be suppliers, manufacturers, distributors, retailers, and customers. Effective management of these flows is a challenging task (Lambert, 2008). It requires the collaboration and the coordination of these flows both within and among the supply chain members to achieve a common global decision, that is enhancing and/or optimizing the supply chain system by maximizing (minimizing) the total profit (cost) of the system. In the literature, this type of decision is defined as "centralized decision–making" In some cases, however, this decision is not aligned with the supply chain players, in which each player attempts to optimize its own objective without considering the objective of the other players in the system. This type of decision is referred to as "decentralized decision–making". However, this type of decision often leads to insufficient sub-optimal results or conflicts of interest between the different players in the system (Thomas and Griffin, 1996).

Centralized decision–making has been extensively studied throughout the literature. The bulk of these studies discussed quantitative models to optimize the total profit (cost) of the supply chain system. Most of these models considered some of the following costs elements: inventory holding costs, ordering costs, purchasing costs, setup costs, production costs, lost sales costs and/or shortages costs. Models build based on a centralized decision–making may increase (decrease) the profit (cost) of one supply chain player over the other in the system. Therefore, coordination schemes, such as profit sharing or quantity discount can be used to compensate the losing player. The mathematical models in this thesis are built on the concept of centralized decision–making.

1.2 Inventory Management

Inventory management is considered one of the main aspects that affect the total profit (cost) of the supply chain system. It accounts for almost 50% of that total logistics costs (Jaber and Zolfaghari, 2008). Therefore, modeling inventory management in supply chains is a prime concern for managers, practitioners and researchers. One method to attain inventory management is to coordinate the ordered quantities and the number of shipments in a supply chain, which have been shown to reduce costs, increase profitability and achieve customer satisfaction. There are many models and strategies that can be applied in designing and managing the inventory of the supply chain. The most commonly-used models in this areas are the economic order quantity model (EOQ), and the economic production quantity model (EPQ). The EOQ model was developed by Harris (1915) to optimize the total cost by finding the optimal ordered quantity and holding costs of a supply chain player. This model was then extended by Taft (1918) who introduced the EPQ model (a modification of the EOQ model) in which a finite production rate was assumed. These two models are considered the foundation of more sophisticated inventory models in the literature, such as the joint economic lot size (JELS) models (Glock, 2012). The JELS models are used to determine the optimal inventory decisions between different players in the supply chain system. These coordination models are discussed in the next section, and based on their advantages and disadvantages, this thesis selects one model to be used as its focus.

1.3 Supply chain coordination

Coordination is an important element for a successful SCM. In the literature there are different forms of coordination mechanisms that help in coordinating the activities between the different players in the system. Collaborative planning, forecasting and replenishment (CPFR), quick response, (QR), vendor-managed inventory (VMI) and consignment stock (CS) policy are some of the initiatives that are used to coordinate product and information flows between the supply chain players.

The CPFR policy is a technique "designed to link consumer demand with supply chain planning and execution by promoting a single, jointly owned demand plan and forecast throughout the entire supply chain" (Min and Zhou, 2002, p. 10). The model allows information to be shared between the supply chain players in order to coordinate operations and increase trades while reducing costs and improving client services (Bowersox et al., 2000).

The QR policy, a concept derived from the military, was applied to textile, clothing, and footwear industries and is used as a supply chain strategy to improve the speed-to-market of products. It allows the production and delivery of products to move quickly from the supplier, to manufacturers, to retailers, and finally to end users through the use of information technology (IT) and computerized equipment (i.e., electronic data interchange (EDI)). The benefits of using this coordination mechanism is to allow for shorter delivery cycles, lower inventory level and higher customer service level (Perry et al., 1999).

The VMI is an initiative and effective supply chain policy used as a coordination mechanism to allow the vendor (supplier) to manage or monitor the buyer's inventory levels, and make periodic replenishment decisions (such as ordered quantities, number of shipments and replenishment timing) (Fugate et al., 2006). The model has been used by a wide range of companies to increase the efficiency of the supply chain by increasing (decreasing) the profit (cost) and improving the level of customer service.

The CS policy has been implemented in different kinds of industries such as automobile and auto parts, consumer electronics, pharmaceutical and papermaking (Liu et al., 2007; Zahran et al., 2015). The CS has also been implemented with other coordination mechanisms such as the VMI. For example, companies such as Wal-Mart, Procter and Gamble, Dell, Barilla, Costco, and Campbell's Soup have revolutionized their relationships with suppliers by using VMI and/or CS policy (Cigolini et al., 2004). Most importantly, the CS is most effective for new and unproven items where fluctuation in demand exist, when the vendor needs to free its inventory to account for other possible items or when sales is questionable such as selling expensive items (Mandal and Giri, 2015). In the CS coordination mechanism, the vendor owns the inventory of the final item and stocks it at the retailer's warehouse, who sells the item to the end users from the consigned inventory and pays the vendor the wholesale price for only the withdrawn quantities that have been sold.

1.4 Dual-channel

The last few decades witnessed remarkable changes in the business world. One of these changes is attributed to the rapid development of the internet and e-commerce technologies, which have been significantly influencing customers' purchase behavior. Such changes led to an increase in the number of manufacturers redesigning their supply chain system and restructuring their distribution channels. Many manufacturers today sell their products using a multi-channel strategy such as a traditional retail (indirect) channel, an online (direct) channel or through a dual-channel, which is a mix of both channels. It was forecast that direct online sales in the United States and Canada will reach US$500 billion by 2018 ("Retail Sales Worldwide Will Top $22 Trillion This Year - eMarketer," 2014). This growing size of the e-marketplace is promising; therefore, many manufacturers have been motivated to redesign their supply chain channel distribution through the adoption of an online channel in addition to their existing traditional retail channel (Yao et al., 2005). A dual-channel strategy has helped many manufacturers in increasing their shares of markets by reaching out to different customer segments (Hua et al., 2010). Apple, Dell, Estee Lauder, Hewlett-Packard (hp), IBM, Nike, and Pioneer Electronics are just a few examples of companies that illustrate the adoption of a dual-channel strategy (Tedeschi, 2000; Chiang et al., 2003; Tsay and Agrawal, 2000; Hua et al., 2010; Huang et al., 2012, 2013). There are several success factors of an online channel mentioned in the literature. Some of them are measuring its performance, design and implement websites that are user-friendly and accessible, and monitoring the online channels of competitors (Dubelaar et al., 2005). E-commerce technologies have also allowed many manufacturers to be more interactive with customers, understand their individual needs and preferences, and deliver products that are individually customized to their specific requirements (i.e. mass-customized items) (Mukhopadhyay and Setoputro, 2004; Liu et al., 2012).

Mass customization (MC) is an industrial system practice, triggered by advances in information technologies (IT), flexible manufacturing system (FMS) and computer-aided design/manufacturing (CAD/CAM) to produce a wide range of customized items at a cost that is close (but higher) to mass produced ones (Liu et al., 2012; Batarfi et al., 2016). In practice, successful MC companies build products from combining preassembled components, which the literature refers to as build-to-order (BTO). Most manufacturers that offer BTO customized items do not consider offering them as an isolated business strategy, but alongside an existing standardized production strategy. The motivation behind this is to enrich the capability portfolio of the manufacturer, which will increase their market share and improve their profit. For example, manufacturers such as Apple, Dell, and Nike coupled a MC strategy with a dual-channel strategy to increase their competitiveness and market share, thereby generating more profit (Salvador et al., 2009; Shao, 2013). Manufacturers adopting a dual-channel strategy and offering identical items to those of the retail channels through online channels risk losing their retail partners who feel threatened by competition (Takahashi et al., 2011).

When a dual-channel supply chain is adopted, the problem of managing the inventory of the supply chain becomes more complex, in which the online channel may cause difficulties that could cannibalize the retailer's market share and impact the inventory decisions (Takahashi et al., 2011). Manufacturers, therefore, are required to redesign their supply chain structure and, accordingly, determine their pricing strategy and their inventory decisions.

Although it would seem that the adoption of an online channel would create conflict between the retailer and the manufacturer, or that it will affect the retailer's market share, studies have shown that the introduction of an online channel will most likely result in a reduction in the wholesale price of the product sold through the retail channel, and increase the supply chain's total profit, thus benefiting both firms in the end (Hua et al., 2010; Shao, 2013). A reduction in the wholesale price may also indicate that the seller has reduced its total cost, or its markup margin, a percentage above the unit cost of a product; therefore, establishing an accurate markup is an imperative part of the pricing strategy.

1.5 Reverse logistics supply chain

The importance of managing the inventory of the supply chain has received tremendous attention from researchers and businesses. However, this attention, in most cases, is concentrated on managing the inventory of the forward logistics (FL) of an item from the point-of-origin (i.e., vendor) to the point-of-consumption (i.e., consumers) (Agrawal et al., 2015). Recently, however, the scope of the supply chain has been extended to include managing the inventory of items from consumers to the point-of-origin, known as reverse logistics (RL) (Agrawal et al., 2015). RL involves products that fall into two categories. The first deals with products that are returned by customers to the point-of-origin either because the products failed to function as designed or because the customers, for whatever other reasons, are not satisfied with them. In most cases, these returned items are sold to a secondary market as repaired or refurbished items (Cheng and Lee, 2010). The second category deals with products that are collected from customers for the purpose of recovery (remanufacturing, recycling, disposal, etc.) once they have reached the end of their useful lives (Prahinski and Kocabasoglu, 2006; Agrawal et al., 2015).

Offering a return policy, in which customers can return the purchased items for a refund, has been used as an important marketing tool and a competitive strategy to substantially increase customer satisfaction and improve product sales (Mukhopadhyay and Setoputro, 2004). A return policy can take many forms such as full or partial refund of the selling price, exchanging the item, store credit or no refund whatsoever (Mukhopadhyay and Setoputro, 2004). In practice, the type of return policy may vary from business to business or from one industry to another. It also depends on the type of item. For example, in the computer industry, Dell Computers offers a full return policy that allows its customers to return purchased items, even customized ones. Apple Computers, on the other hand, for most of its products, offers a full return policy on non-customized items, and no return for special custom orders (e.g. engraving on iPad) (Mukhopadhyay and Setoputro, 2005). Companies that establish a return policy as a competitive tool need to recognize its impact on the selling prices and the supply chain inventory decisions.

Recently, RL has received a growing importance by many researchers and firms for its economic benefits, competitive advantage and corporate social image (Agrawal et al., 2015). However, managing the RL of a supply chain is significantly more complex than managing a traditional FL supply chain (Krumwiede and Sheu, 2002). This complexity is due to the lack of resources and/or the capabilities to manage returned items from customers. It can substantially increase the cost of the supply chain. Due to this complexity, many firms (such as K-mart, Best Buy and Philips) have outsourced the RL part of their supply chains to third-party logistics (3PL) providers (such as GENCO, Ryder, UPS and OZARK) to reduce cost and ensure an effective RL process (Cheng and Lee, 2010; Krumwiede and Sheu, 2002; Olorunniwo and Li, 2011).

Firms have relied on 3PL providers for logistic operations (e.g., transportation) for many years. However, the reliance on 3PL providers for RL is fairly new (Bloomberg et al., 2001; Prahinski and Kocabasoglu, 2006). For example, in the computer and electronic industry, Thomson Consumer Electronics outsourced its RL part of the supply chain to a 3PL provider, GENCO, to facilitate returns and for the refurbishment of repairable items in Mexico; unrepairable items are disposed of in the US before they are shipped to Mexico (Dhanda and Hill, 2005). As well, Cerplex Group built a RL and repair system business to provide services for the computer and electronics industry that range from the handling of returned items to the refurbishments and repairs of these items (Dhanda and Hill, 2005). Similarly, Philips outsourced the collection, repairing, repackaging, refurbishing and disposition of returned items to a 3PL provider, Ryder Supply Chain Solutions (Sharma et al., 2012). It was proven that firms that outsourced their RL to 3PL providers reduced their annual logistics cost up to 10% (Krumwiede and Sheu, 2002), and were able to reduce inventories and improve field engineering productivity by up to 40% (Cheng and Lee, 2010). Martin, Guide, & Craighead (2010) reported that remanufacturing was traditionally performed by independent small and privately owned 3PL remanufacturers; however, some original equipment manufacturer (OEM) prefer to remanufacture in-house for several reasons. For example, they noted that Caterpillar established a remanufacturing decision for its products to gain a competitive edge in a growing lower-price market segment. They also mentioned that when the products contain high levels of proprietary technology, firms remanufacture in-house to avoid the exposure of these components; Xerox is one such company. For the purposes of this thesis, it is assumed that the 3PL remanufacturing facility is a division of the parents company, such that legal concerns about intellectual property are not a concern.

1.6 Learning and forgetting effects on supply chain

To maintain a sustainable and effective supply chain, many companies implement continuous improvement programs such as fostering organizational learning within the operations of the supply chain. Learning-based continuous improvement programs improve competitiveness by eliminating inefficiencies and thus reducing costs (Jaber et al., 2010). Learning-by-doing is fundamental to these programs and to improving the overall (including production) efficiency (Jaber, 2011), especially in labor intensive and ramp-up processes (Glock et al., 2012), in which continuous improvements are needed. Learning also improves the overall efficiency of the supply chain in areas where new products are introduced or where a new production system is implemented (Glock et al., 2012). The learning phenomenon in earlier studies focused mainly on the behavior of individual subjects (Jaber and Bonney, 1999). The phenomenon is captured by a learning curve (LC) and suggests that the time required to perform a repetitive task declines with experience (Jaber, 2011).

The application of LC in various industrial settings resulted in different LC forms (Jaber, 2011; Grosse et al., 2015). For example, some of these LC forms have been known as “progress function” (Glover, 1965), “start-up curve” (Baloff, 1970), “improvement curve” (Steedman, 1970), and “power form or aircraft learning curve” (Wright, 1936). The power-form LC, or Wright learning curve (WLC) (Wright, 1936) remains the most favored among researchers and practitioners. The WLC quantitatively formulates the relationship between learning variables. In general, the LC has been applied to a diverse set of industries and management decision areas to better make use of resources, coordinate and dispatch inventory, and manage production and distribution with the end goal being to improve the overall system productivity and reduce the costs involved therein (Jaber and El Saadany, 2011).

Although the learning phenomenon improves production efficiency and minimizes cost, its opposite phenomenon, “forgetting”, impedes it (Jaber, 2006a). Changes in products and processes, production interruptions, job rotations, knowledge interference, and labor turnover are among the reasons for loss of knowledge or forgetting, which impedes performance and increases costs (Jaber, 2011; Sikström and Jaber, 2012).

CHAPTER 2. LITERATURE REVIEW

This chapter presents an overview of the literature related to the intended thesis. The overview is classified mainly on three streams of research. The first stream of research, overviews studies that have considered investigating the effects of adopting a dual-channel strategy on the supply chain system, in different cases. The second stream of research overviews studies that have considered investigating inventory management issues in the reverse logistics (RL) supply chain system. The third stream of research overviews studies that have considered investigating the learning and forgetting effects on the supply chain system, specifically in the joint economic lot size (JELS) problem.

2.1 Dual-channel supply chain

In the last few years, research on dual-channel supply chains has gained considerable attention in the supply chain management (SCM) literature. Most of the studies have focused on pricing decisions and channel coordination. This thesis is related to this stream of research. Earlier studies that are related to this type of pricing and channel structure/coordination problem appear in the work of Chiang et al. (2003) and Tsay and Agrawal (2004) who considered pricing and channel structure as decision variables. In the first study, Chiang et al. (2003) argued that the introduction of an online channel can induce the retailer to decrease its price, which in return increases the demand for the retail channel and consequently increases the profit of the manufacturer (even if no sale occurs in the online channel). Their main finding indicated that the strategic use of the online channel increases channel efficiency. In the second study, Tsay and Agrawal (2004) extended the work of Chiang et al. (2003) by incorporating the unit cost of supplying a product while studying channel conflict and service interaction between the upstream and the downstream players in a dual-channel supply chain setting. They showed that the direct channel would benefit both the vendor and the retailer if the wholesale price were to be adjusted; however, this benefit depends on the channels' sales effort. Fruchter and Tapiero (2005) on the other hand, studied the pricing policy for a dual-channel supply chain using dynamic hierarchal game theory. Their optimal finding indicated that the vendor can charge the same price across both channels, and found that customers' heterogeneity affects the direct channel price, the retailer's markup and supply chain profitability. Cattani et al. (2006) investigated pricing strategies for a vendor who competes with a traditional retail channel by adding a direct online channel. The authors proposed a generalized equal-pricing strategy in which the selling price of an item in the online channel matches the retailer's price. Their findings indicated that the proposed equal-pricing strategy maximizes the vendor's profit and is preferred by the retailer and customers. Kurata et al. (2007) explored cross-brand and cross-pricing policies in multiple distribution channels. They considered channel competition between a national brand (NB) and store brand (SB). They assumed that the NB could be distributed through a direct online channel as well as an indirect retail channel, whereas the SB can be distributed only through the indirect retail channel. Their findings showed that building brand loyalty is profitable for both the NB and the SB, and that marketing decisions are more restrictive for the NB than they are for the SB.

The benefits of demand sharing and channel coordination have been investigating in the supply chain literature. For example, Yao and Liu (2003) and Yue and Liu (2006) studied the benefits of demand diffusion and demand sharing between an online channel and a retail channel. The first study proved how the demand between the two channels could be stable under some conditions. The second study investigated pricing and production quantity of a dual-channel supply chain and found that the online channel has a negative impact on the performance of the retailer. Mukhopadhyay et al. (2008), on the other hand, presented a game-theoretic model to investigate optimal contract design for a two-level dual-channel supply chain (vendor-retailer) under two cases: complete information and asymmetric information sharing. They studied the optimal pricing decisions and the retailer's added value to its product offering to differentiate itself from the vendor. Their findings showed when information asymmetry is considered, the selling price in the direct channel does not change while the selling price in the retail channel increases. Cai (2010) investigated channel selection and coordination contract in a two-level dual-channel supply chain. The author considered four channel structures: (1) a traditional retail channel, (2) a direct online channel, (3) a dual-channel with a direct and a retail channels and (4) a dual-channel with two retailers. One of his key findings suggested that channel structures with and without coordination depend on factors such as the base demand, the operational costs and channel substitutability. Similarly, Chen et al. (2012) implemented coordination contracts in a dual-channel supply chain using a Stackelberg scenario (where one firm is the leader, and the other is a follower). They examined pricing decisions and found that the dual-channel supply chain increases the profits of the manufacturer and the retailer while improving the overall supply chain efficiency under some circumstances. Lei et al. (2014) studied vertical and horizontal demand information sharing and channel structure in a dual-channel supply chain. They found that the retailers have the incentive to share information under horizontal integration but not under vertical integration. Chen et al. (in press) explored the impact of pricing and product quality on three channel structures: (1) a traditional retail channel, (2) a direct online channel and (3) a dual-channel. They modeled each channel structure under centralized and decentralized decision-making system. Their results showed that in the centralized scenario, the direct online channel provides the highest quality level and selling price among the three-channel structure. However, in the decentralized scenario, the price and quality level depend on factors that are related to channel relative power, quality coefficient and channel substitutability. Recently, Li et al. (2016a) investigated pricing strategies and channel coordination for a two-level supply chain (supplier-retailer) in which the supplier is risk-neutral and the retailer is risk-averse. Using a Stackelberg game model they obtained the equilibrium solutions in two scenarios (decentralized and centralized). Their findings indicated the selling price will decrease and that the supplier initial stock will increase when the retailer becomes more risk-averse.

Pricing policies in a dual-channel supply chain were also investigated under environmental factors. For example, Li et al. (2016b) examined pricing policies in a dual-channel supply chain when the vendor produces green items for the environmentally-conscious. They analyzed pricing decisions under centralized and decentralized settings. Their main findings indicated that the vendor should open the online channel when the greening costs and customers' loyalty to the retail channel satisfy certain conditions. Ji et al. (2017) investigated pricing and the carbon emission reduction decisions in a single-channel and a dual-channel supply chain system. The study considered cap-and-trade regulations and consumers' low-carbon preference. Their results suggested that the introduction of the vendor's online channel is profitable when the degree of consumers' low-carbon sensitivity satisfies certain conditions. They also showed that when consumers have low-carbon preference, the cap-and-trade mechanism is acceptable for the supply chain players.

Pricing decisions in a dual-channel supply chain have also been investigated under demand and/or product disruption, (Huang et al., 2012, 2013; Cao, 2014; Zhang et al., 2015; Matsui, 2016). For example, Huang et al. (2012, 2013) investigated pricing and production decisions in a dual-channel supply chain under centralized and decentralized settings and considered demand disruption and production cost disruption, respectively. In the first study, they found that the optimal production plan has some robustness when demand is disrupted. Production and pricing plans are adjusted only when the change in the market scale exceeds a threshold. In the second study, they found that pricing and production quantity are disrupted when production cost disruption exceeds a threshold value. Cao (2014) examined optimal decisions (pricing and production) and coordination for a dual-channel supply chain with and without demand disruption and proposed an improved revenue-sharing contract. His results recommended adjusting the prices and the production quantities. Along the same stream of research, Zhang et al. (2015) examined the coordination of dual-channel supply chains when demand and production cost are disrupted using a contract. They found that by adjusting the parameters of the contract, coordination is still possible with disruptions. Matsui (2016), on the other hand, investigated the optimal product distribution for two symmetric manufacturers facing price competition and using dual-channel supply chains. Their main findings indicated that an asymmetric distribution policy for a manufacturer encountering price competition is not necessarily optimal.

Researchers also investigated channels' competitions and the mitigation of the effect of adopting an online channel on the retail channel. In this area Chen et al. (2008), Dumrongsiri et al. (2008) Yan and Pei (2009) and Dan et al. (2012) studied the strategic role of the retail services and channel competition in a dual-channel supply chain system. Chen et al. (2008) found that the optimal channel strategy of the manufacturer depends on the channel environment and that the retail channel generates less profit than the direct channel. Dumrongsiri et al. (2008) showed that adding an online channel in a centralized case can increase the overall profit. They also showed that increasing the quality of the retailer's service can increase the vendor's profit. Similarly, Yan and Pei (2009) suggested that the retail service directly affects the overall profit of the system and the pricing strategies of the vendor and the retailer. Likewise, Dan et al. (2012) presented a model for retail service and pricing policy in centralized and decentralized dual-channel supply chains to test the effects of the retail service and the degree of customers' loyalty on the retail channel. Their findings showed that retail service has a direct effect on the overall profit of the supply chain system and on the pricing strategies of both the vendor and the retailer. Chen (2015) evaluated the effect of pricing and cooperative advertising policies on a two-level dual-channel supply chain system. Their results showed that the total profits of both channels are highly sensitive to the level of local advertising. On examining the effect of delivery lead-time in a dual-channel supply chain, Hua et al. (2010) investigated the optimal prices of a dual-channel supply chain under centralized and decentralized settings while taking into consideration the delivery lead-time. They found that the delivery lead-time affects pricing strategies and the supply chain profit. They also found that customer acceptance of the online channel has a great effect on the supply chain.

The research of this thesis is also largely related to the literature on channel coordination when standardized and customized products are sold. In this area, Xia and Rajagopalan (2009) investigated the standardization and customization of two competing firms by incorporating consumer heterogeneity, product variety, delivery lead-time and price. Their results showed that increasing product variety or decreasing delivery lead-time can increase market share and margin. Shao (2013) explored the effectiveness of a mass customization (MC) strategy and the conditions under which the manufacturing firm should offer customization. The study was analyzed under centralized and decentralized decision-making situations. His findings showed that the introduction of a MC strategy in a centralized setting increased the expected profit of the system by reducing the average costs of the standard items. Xiao et al. (2014) used a Stackelberg pricing model to investigate product variety and channel structure in which the retail channel sells standard products and the online channel offers customized products. They found that when the reservation price in the retail channel was sufficiently low, the unit wholesale and the retail prices of a standard item sold through a retail channel were increased due to the introduction of an online channel for customized items. In a study of a two-level supply chain (vendor-retailer), Li et al. (2015) investigated firms' decisions on offering customization through an online channel based on customers' acceptance level of that channel and customers' sensitivity to product differences. A key finding was that it is not necessary that the price of the standard product offered by the retailer decrease due to offering a customized product online. They also found, under some conditions, that when the manufacturer offered customized products online, the profits of the manufacturer and the retailer improved due to alleviated vertical competition and an increase in the market share.

This thesis also largely relates to the literature on inventory control and management of multi-channel supply chains. Here is a brief review of the literature. Chiang and Monahan (2005) proposed a two-echelon dual-channel supply chain model for a vendor and a retailer. They applied a lot-for-lot (LFL) replenishment policy where the stock is kept at the vendor's warehouse and sent to the buyer on an "as needed" basis. In their model, the total cost consisted of the sum of the inventory holding cost and lost sales, with no consideration for setup cost and ordering cost. They found that a dual-channel strategy outperformed a single-channel strategy. Takahashi et al. (2011) also proposed a new inventory control policy for a two-echelon dual-channel supply chain by considering production setup and delivery costs as part of the total cost of the supply chain. They calculated the total cost of the system using Markov analysis and showed that the proposed model could reduce the number of setups with no additional increase in the inventory levels of the vendor and the retailer. Ryan et al. (2013) analyzed the optimal price and order quantity for a two-level dual-channel supply chain using a newsvendor problem. They proposed two revenue-sharing contracts to enable coordination. Rodríguez and Aydin (2015) analyzed pricing and assortment decisions in a dual-channel SC. They accounted for inventory related costs, but did not specify an inventory policy. Their results showed that adopting a dual-channel increased the manufacturer's market share and profit. However, they discussed that the two channels compete with one another with conflicting objectives between the manufacturer and the retailer: the first pushing the retailer for storing items with high demand variability, with the latter pushing for low variability. Yang et al. (2016) investigated inventory completion of a perishable item in a two-level (vendor-retailer) dual-channel supply chain using a newsvendor model, and considered the delivery lead-time, the retailer's ordered quantity and the vendor's inventory level as decision variables. They compared the optimal decisions in two scenarios: centralized and decentralized. They found that at least one of the supply chain members will overstock in the decentralized scenario as compared with the centralized scenario. They also found that customers in the decentralized scenario enjoy a better service in the online channel by having a shorter delivery lead-time.

2.2 Reverse logistics (RL) supply chain

In the literature, reverse logistics (RL), which falls under the umbrella of closed loop supply chain, is considered a separate line of research. This line of research, which grew in the late twentieth century, includes all aspects of managing a business, including the inventory parts of the supply chain. The research in this thesis relates to the literature that considered a RL supply chain in either single or dual-channel structures and examined the pricing of products, return policy, and/or inventory decisions. Readers can refer to Prahinski and Kocabasoglu (2006) and Agrawal et al. (2015) for a complete literature review on the RL supply chain. It was reported that more than 56% of the RL research considered the problems of production planning and inventory management (Rubio et al., 2008). In the literature, there are numerous mathematical models that considered RL supply chains with inventory models, based on the economic order/production quantity (EOQ/EPQ) (Bazan et al., 2016). The research in this thesis is largely related to this stream of research in which it considers investigating inventory models for the forward and the reverse flow of the material. To the best of the author's knowledge, the research in this thesis did not come across a study that models inventory management in a RL for items returned by unsatisfied customers and/or using a dual-channel strategy. However, there are multiple studies that considered RL with inventory management models for end-of-life (EOL) items that are collected from customers for the purpose of recovery or recycling. Although these studies do not directly relate to the research on this thesis, the ideas brought by these studies could be useful. In this research area, it is believed that Schrady (1967) made the initial attempt to investigate a reverse logistic supply chain for a repair inventory system. The author developed a deterministic EOQ model for repaired items and assumed a fixed return rate. A flurry of articles extended the work of Schrady (1967) and accounted for other aspects of inventory problems. For example, Nahmias and Rivera (1979) extended the work of Schrady (1967) by accounting for a finite return rate and a limited storage capacity in the production and repair shops. In the late of 1990s Richter (1996a, 1996b) developed two EOQ repair and waste disposal models. In the first study, the author assumed that the demand could be satisfied from newly-produced and repaired used items. The second study included the case of variable setup numbers. Following these two studies, Richter (1997) extended his earlier work by assuming extreme waste disposal rates. His results showed that a pure (bang-bang) policy of either total repair (no waste disposal) or full waste disposal (no repair) is more beneficial than a mixed strategy. Along the same line of research, Teunter (2001, 2002, 2004), extended the work of Schrady (1967) by investigating a deterministic EOQ inventory system for items that can be recovered (repaired/refurbished/remanufactured). In the first study, the author assumed more than one manufacturing batch and different holding costs rates for manufactured and recovered items. In the second study, the author investigated a discounted cost inventory system and used a stochastic model in which the demand and returns rates follow a distribution that represents randomness. In the third study, the author developed a lot-sizing inventory system with product recovery, in which recovered items are assumed to be as-good-as-new. The developed model was valid for finite and infinite production rates. Similarly, Dobos and Richter (2003, 2004, 2006) extended the work of Richter (1997) and developed a production-recycling model with stationary demand and return rates. They assumed that the demand can be satisfied from newly produced and recycled items and that non-recycled items are disposed of. Their first study considered that in each time interval there is a single production and a single repair cycle. In the second study, they developed a more generalized model to consider multiple cycles per time interval. In their third study, quality consideration was taking into account. Later, Mitra (2009) considered a return policy for EOL items in a two-level RL supply chain to find the optimal value of the inventory variables under deterministic and stochastic settings. Through a simulation study, he showed that the model performs very well with respect to the optimal solutions. El Saadany and Jaber (2010, 2011) proposed two RL inventory models in which the demand is satisfied from newly produced items and remanufactured EOL items (where remanufactured items are assumed to be as-good-as-new). The first study considered that the return rate depends on the price and the quality of the product and showed that a mixed strategy of production and remanufacturing is optimal compared to a pure strategy. The second study considered that collected used items are disassembled into components, which are then used in the production and remanufacturing processes. In this study, they showed that a pure production or remanufacturing policy is more optimal than a mixed strategy. In the same stream of research, Jaber and El Saadany (2009) and Hasanov et al. (2012) developed a RL supply chain model, where production, remanufacturing and waste disposal processes are considered. They assumed that newly produced and remanufactured “repaired” items are of incompatible quality and hence, the demand for both items is different. In the first study, lost sales scenario for stock-out periods of manufactured and remanufactured items are considered. This results in a shortage of remanufactured (manufactured) items over the manufacturing (remanufacturing) segment of the production cycle. The second study assumed that stock-out situations are met through a full or partial backorder. The results of Jaber and El Saadany (2009) suggested that the collection of used items from the secondary market is only optimal for certain values of the input parameters (e.g., setup costs, holding costs, etc). One main finding of Hasanov et al. (2012) is that when there are several production and manufacturing batches in an interval, an optimal policy can occur. Jaber et al. (2014) extended the model of production, remanufacturing and waste disposal system by considering a two-level (vendor-retailer) RL supply chain with a consignment stock (CS) policy as a coordination mechanism. Their finding showed that both the collection and repair rates of used items have a significant impact on both the total cost and the batch size. For more literature review on the mathematical inventory models in RL, readers can refer to the work of Bazan et al. (2016).

Other researchers investigated the use of an online channel to sell returned items. In this research area, Choi et al. (2004) investigated the optimal return policy of a two-level supply chain where returned items from the retailer to the vendor could be sold on an e-marketplace. Through a simulation study, they showed this practice to be very profitable. They also showed that optimal solutions of the problem exist under some conditions. For example, the selling price of the returned items on the e-marketplace should be set when the actual amount of returned items is known. Using an online direct channel strategy, Li et al. (2013) examined the relationship between return policy, product quality and pricing strategies and its impact on customer’s purchase and return decisions.

Their main findings indicated that when product quality improves, the direct online channel should provide a lenient return policy and can increase the selling prices. Mukhopadhyay and Setoputro (2004) developed a profit maximization model for a RL supply chain in an e-business context. They studied optimal pricing decisions and return policy regarding certain market reaction parameters. From their result, a number of managerial insights were derived based on the optimum price and return policy that can maximize the total profit of the supply chain. One year later, Mukhopadhyay and Setoputro (2005) developed another RL supply chain model. However, they considered a BTO (build-to-order) product produced from pre-assembled modules and sold to customers through an online channel. They analyzed the effect of two decision variables, namely return policy and modularity level, on the demand, the refund amount returned to customers and the overall profit of the supply chain. Their results showed that offering a higher return policy and/or a higher modularity level, can increase the revenue and the total cost of the returned items. Liu et al. (2012) extended the work of Mukhopadhyay and Setoputro (2005) and developed an analytical model to investigate optimal pricing, modularity and return policy under MC environment. Through a sensitivity analysis, they showed that when the primary demand increases, the decision variables in their model decrease. Yao et al. (2005) estimated the optimal ordered quantities and return policy for a two-level supply chain (vendor-retailer) with the addition of a direct channel under two cases: information sharing and non-information sharing. They found that when the vendor shares information with the retailer, the return policy remains unchanged regardless of customer shift between the two channels. They also found that the total profit of the supply chain in the information sharing case is higher than in the non-information sharing case. Ofek et al. (2011) examined the impact of product return, pricing strategies and physical store assistance levels of two retailers under the adoption of a dual-channel strategy. They indicated that when product differentiation between the two retailers is not too high, opening an online channel could increase the investment in store assistance level and decrease profits. Using different return policies Chen and Bell (2012) showed how customer returns could enhance the profit of the firm and impact the optimal pricing and ordering decisions of the firm. Ketzenberg and Zuidwijk (2009) studied optimal pricing, ordering and return policies for consumer goods in two demand cases: deterministic and stochastic. They showed that the results of the stochastic demand case do not give further insight. They also showed that errors in the parameters related to demand and purchase cost have a great impact on the expected profit. Using a buyback policy contract, Liu et al. (2014) examined the impact of returned unsold items from the retailer and returned items from customers, on the coordination of a two-level supply chain and under demand uncertainty. They showed that the optimal refund amount, the ordered quantity and the total profit in a centralized decision are always much higher than in a decentralized decision. Environmental concerns were also examined in a RL dual-channel supply chain. In this area, He et al. (2016) investigated the impact of consumer free riding behavior on carbon emissions in a closed loop dual-channel supply chain system. Consumer free riding occur when customers enjoy the retail service but they make purchases from the online channel due to cost saving. The authors analyzed the effect of governmental e-commerce tax on carbon emissions. Their results showed that even though the vendor may gain economic advantages from consumer free riding, the supply chain's total carbon emissions increased. They suggested that imposing a governmental tax on e-commerce can reduce consumer free riding and total carbon emissions.

Some other studies in RL supply chain considered the benefits of outsourcing the RL part of the supply chain to another firm. For example, Krumwiede and Sheu (2002) proposed a model for a RL supply chain and showed how companies that would like to pursue RL could use a third party logistics (3PL) provider for all or portions of their RL. Savaskan et al. (2004) considered the problem of choosing the right channel structure (manufacturer, retailer, or 3PL) for the collection of used EOL items from customers. They analyzed the optimal decisions of the supply chain members under three channels and compared the retail price, the wholesale price, the return rate and the profit gained by selling the returned used items to evaluate the merits of the three channels. They showed that the retailer is the most effective undertaker for the collection of EOL items. Cheng and Lee (2010) presented a systematic approach to examine the importance of RL requirements and to select the appropriate 3PL provider. Through a case study, they showed that IT management is an imperative requirement that should be considered when outsourcing RL to a 3PL. In another study by Mukhopadhyay and Setaputra (2006), they examined the optimal pricing decisions (including buyback price of retuned items) and the return policy decisions when the RL part of the supply chain is outsourced to a fourth-party logistics (4PL) provider, who is responsible for refurbishing all returned items. Managerial insights were provided using marketing and operational strategy variables to show how a manager can influence some market parameters (provided in their model) to obtain the optimal prices, return policy and outsourced fees paid to the 4PL. The literature does not provide a clear distinction between 3PL and 4PL. The Council of Supply Chain Management Professionals (CSCMP) defines 3PL as the companies integrating and offering subcontracted logistics and transportation services. 4PL differs from 3PL as representing an interface between a client and multiple logistics service providers.

2.3 The learning and forgetting effects

For almost seven decades, the learning curve (LC) phenomena have been extensively discussed and used by many researchers and practitioners (Jaber et al., 2010). The phenomena provide a managerial technique and the means to model, quantify and predict the continuous improvement of products and services (Jaber and Bonney, 2003). The theory in its most popular form states that production time per unit of an item declines by a constant percentage when the total quantity of the produced item doubles (Jaber and Guiffrida, 2004). The literature discussed three main LC model types: power models, hyperbolic models and exponential models. However, a unanimous agreement among researchers stated that the power form proposed by Wright (Wright, 1936) is the first, most widely used and accepted model to formulate learning effects in a quantitative way due to its simplicity and efficiency (Yelle, 1979). Most studies confirmed that as the learning rate (LR) increases, the systems productivity improves and, therefore, reduces the systems cost. These studies also confirmed that it is beneficial for the supply chain system to produce in smaller lots more frequently (Jaber and Bonney, 1999; Jaber, 2011). However, although learning reduces costs and improves systems productivity, this improvement is impeded when forgetting is considered.

In the last decade, researchers in industrial engineering extended the LC theory by given considerable attentions to its opposite phenomenon, the forgetting curve (FC) (Jaber and Guiffrida, 2004). These extensions introduced new parameters that adjusted the prediction of Wright's LC model, which, as suggested by Globerson et al. (1989) and Jaber and Bonney (1996), may be considered as a mirror image of the LC model.

Both theories (i.e. LC and FC) have been investigated in different industrial settings such as lot-sizing problem (Jaber and Bonney, 1999), dual resource constrained system (Jaber et al., 2003), production quality (Jaber and Guiffrida, 2004), supplier quality (Jaber et al., 2008), supply chain coordination (Jaber et al., 2010) and order picking (logistics) (Grosse et al., 2013). The use of LC and FC to study its effects on the lot-sizing problems can be found as early as the 1960s. For example, Keachie and Fontana (1966) were the first to study the effects of learning and forgetting in an EPQ model where production is intermittent. They studied three different cases (full transfer, partial transfer, and no transfer) of LC. This work was followed by articles in the 70s and 80s to regain interest in the 1990s. For example, Adler and Nanda (1974a, 1974b) investigated the effect of production breaks on the optimal lot-sizing determination for a single and multi-product scenarios and proposed two refined mathematical models (equal lot sizes and equal production interval). Globerson et al. (1989), on the other hand, investigated the relation between production breaks and forgetting and found that the degree of forgetting is a function of the level of experience prior to the break and the length of the production break. Although these articles discussed different situations, they have a common finding that learning suggests producing in smaller, more frequent lots and that learning decreases costs. Forgetting as a result of production breaks does the opposite. Jaber and Bonney (1999) provided a review of inventory models with learning and forgetting effects, with Adler and Nanda (1974b) being the only study that investigated multiple products.

Furthermore, Jaber and Bonney (1996) developed a mathematical model referred to as the learn-forget curve model (LFCM). The LFCM was tested by many researchers to investigate its effects in different cases on the lot-sizing problems (Jaber and Bonney, 2007; Jaber and Guiffrida, 2007; Khan et al., 2010). It has also satisfactorily proven that learning and forgetting are mirror images of each other (Jaber, 2011). Moreover, the LFCM was shown to produce better results than other models considered in the literature (Jaber and Bonney, 1997; Jaber, 2016).

A few of the inventory models reviewed in Jaber and Bonney (1999) were extended to consider two or more players. For example, Nanda and Nam (1992, 1993) examined the JELS model for a vendor and a retailer (or multi-retailers). They considered a LFL policy with quantity discount. For simplicity, they assumed forgetting to be a constant percentage of cumulative experience. Readers may refer to Glock (2012) for a comprehensive review on JELS models. Jaber et al. (2010) studied the effects of learning-based continuous improvements in a coordinated three-level supply chain (supplier-vendor-retailer) where learning and forgetting occur in setup, production, and product quality. They showed that as improvement becomes faster, the manufacturer offers the retailer a discount to order in smaller quantities. Jaber and Bonney (2011, chap. 14) provided an updated version of (Jaber and Bonney, 1999) that covers the literature up until 2011.

Of the works that did not appear in (Jaber and Bonney, 2011) and of importance to this thesis is the work of Zanoni et al. (2012) who investigated VMI-CS (a vendor-managed inventory (VMI) with CS policy) for a two-level (vendor-retailer) supply chain with learning and forgetting in the production process of the vendor. Three different production and shipment cases were suggested. They showed that learning in production gives flexibility to the supply chain members by assigning the size and the time of the shipment. Other recent studies that considered learning and/or forgetting are those of Khan et al. (2012, 2014), who investigated the effect of different human factors (e.g. learning in quality and production) on the performance of a two-level supply chain. Their main results indicated that accounting for the learning factor brings in a substantial drop in the total cost of the supply chain.

For the convenience of the reader, Table 2.1 summarises the literature relevant to this thesis. It classifies the literature by the following sub-headings: dual-channel, the type of product (standard and/or customized), pricing decision, inventory decision, inventory policy, the use of reverse logistics, the implementation of learning and forgetting effects, and the investigation of other decision variables. A paper that addresses a specific sub-heading is indicated by a tick mark (✓). The last row of the table indicates how this thesis differs from the literature.

Table 2.1 Summary of the relevant literature

Research Paper	Dual-channel	Standard item	Customized item	Pricing decisions	Inventory decisions	Inventory policy	Reverse logistics	Learning and forgetting	Other decision
Chiang et al. (2003)	✓	✓		✓					
Tsay and Agrawal (2004)	✓	✓		✓					✓
Fruchter and Tapiero (2005)	✓	✓		✓					✓
Cattani et al. (2006)	✓	✓		✓					
Kurata et al. (2007)	✓	✓		✓					✓
Yao and Liu (2003)	✓	✓							✓
Yue and Liu (2006)	✓	✓	✓	✓					✓
Mukhopadhyay et al. (2008)	✓	✓		✓					✓
Cai (2010)	✓	✓							✓
Chen et al. (2012)	✓	✓		✓					
Lei et al. (2014)	✓	✓		✓					
Chen et al. (in press)	✓	✓		✓					✓
Li et al. (2016a)	✓	✓		✓	✓				
Li et al. (2016b)	✓	✓		✓					✓
Ji et al. (2017)	✓	✓		✓					✓
Huang et al. (2012)	✓	✓		✓	✓				
Huang et al. (2013)	✓	✓		✓	✓				
Cao (2014)	✓	✓		✓	✓				
Zhang et al. (2015)	✓	✓		✓	✓				
Matsui (2016)	✓	✓		✓					
Chen et al. (2008)	✓	✓							✓
Dumrongsiri et al. (2008)	✓	✓		✓	✓				
Yan and Pei (2009)	✓	✓		✓					✓

Research Paper	Dual-channel	Standard item	Customized item	Pricing decisions	Inventory decisions	Inventory policy	Reverse logistics	Learning and forgetting	Other decision
Dan et al. (2012)	✓	✓		✓					✓
Chen (2015)	✓	✓		✓					✓
Hua et al. (2010)	✓	✓		✓					✓
Xia and Rajagopalan (2009)	✓	✓	✓	✓					✓
Shao (2013)	✓	✓	✓	✓	✓				
Xiao et al. (2014)	✓	✓	✓	✓					✓
Li et al. (2015)	✓	✓	✓	✓	✓				✓
Chiang and Monahan (2005)	✓	✓				✓			✓
Takahashi et al. (2011)	✓	✓			✓	✓			
Ryan et al. (2013)	✓	✓		✓	✓	✓			
Rodríguez and Aydin (2015)	✓	✓	✓	✓					✓
Yang et al. (2016)	✓	✓			✓	✓			✓
Schrady (1967)		✓			✓	✓			
Nahmias and Rivera (1979)		✓			✓	✓			
Richter (1996a)		✓			✓	✓			
Richter (1996b)		✓			✓	✓			
Richter (1997)		✓			✓	✓			
Teunter (2001)		✓			✓	✓			
Teunter (2002)		✓			✓	✓			
Teunter (2004)		✓			✓	✓			
Dobos and Richter (2003)		✓			✓	✓			
Dobos and Richter (2004)		✓			✓	✓			
Dobos and Richter (2006)		✓			✓	✓			
Mitra (2009)		✓			✓	✓	✓		

Research Paper	Dual-channel	Standard item	Customized item	Pricing decisions	Inventory decisions	Inventory policy	Reverse logistics	Learning and forgetting	Other decision
El Saadany and Jaber (2011)		✓		✓	✓	✓	✓		
El Saadany and Jaber (2011)		✓			✓	✓	✓		
Jaber and El Saadany (2009)		✓			✓	✓	✓		
Hasanov et al. (2012)		✓			✓	✓	✓		
Jaber et al. (2014)		✓			✓	✓	✓		
Choi et al. (2004)	✓	✓		✓	✓	✓	✓		
Li et al. (2013)		✓		✓	✓		✓		✓
Mukhopadhyay and Setoputro (2004)		✓		✓			✓		✓
Mukhopadhyay and Setoputro (2005)			✓				✓		✓
Liu et al. (2012)			✓	✓			✓		✓
Yao et al. (2005)	✓	✓			✓		✓		✓
Ofek et al. (2011)	✓		✓				✓		✓
Chen and Bell (2012)	✓	✓		✓			✓		✓
Ketzenberg and Zuidwijk (2009)		✓		✓			✓		✓
Liu et al. (2014)		✓					✓		✓
He et al. (2016)	✓	✓		✓			✓		✓
Savaskan et al. (2004)	✓	✓		✓			✓		✓
Mukhopadhyay and Setaputra (2006)	✓	✓		✓			✓		✓
Keachie and Fontana (1966)		✓				✓		✓	✓
Adler and Nanda (1974a)		✓				✓		✓	✓
Adler and Nanda (1974b)		✓				✓		✓	✓
Globerson et al. (1989)		✓				✓		✓	✓
Jaber and Bonney (1996)		✓				✓		✓	✓
Nanda and Nam (1992)		✓				✓		✓	✓

Research Paper	Dual-channel	Standard item	Customized item	Pricing decisions	Inventory decisions	Inventory policy	Reverse logistics	Learning and forgetting	Other decision
Nanda and Nam (1993)		✓				✓		✓	✓
Jaber et al. (2010)		✓				✓		✓	✓
Zanoni et al. (2012)		✓				✓		✓	✓
Khan et al. (2012)		✓				✓		✓	✓
Khan et al. (2014)		✓				✓		✓	✓
This thesis	✓	✓	✓	✓	✓	✓	✓	✓	✓

CHAPTER 3. RESEARCH GAPS

As seen from the above-reviewed literature (Chapter 2), it is apparent that dual-channel supply chains have frequently been studied in different situations. However, most studies reported in the literature in this area are general economic models in which important cost factors such as inventory holding costs and other related operational costs (i.e. setup and ordering costs) are ignored. It was proven that when these costs are ignored, the results of these general economic models do not hold anymore (Reyniers, 2001). To fulfill the gaps lacking in this area, the research in this thesis investigates the effect of a dual-channel strategy on the profit of a two-level (vendor-retailer) supply chain system in the presence of inventory costs in both channels. The dual-channel strategy here is composed of a vendor and a retailer and deals with two channels in which the retail channel offers a standard items, and the online channel offers customized items. Three main models were developed. These models are presented in the next three chapters as follows:

Chapter 4 investigates the optimal pricing decision (by finding the optimal markup margin), and the optimal inventory decisions (optimal ordered/production quantity and the number of shipments) before and after adopting the dual-channel supply chain system. The content of this chapter is published in a peer-reviewed journal (Applied Mathematical Modelling, *40*(21), 9454-9473; doi.org/10.1016/j.apm.2016.06.008).

Chapter 5 builds on the work of the previous chapter (Batarfi et al. (2016)), by modeling a more complex supply chain structure to account for the returns of newly produced and refurbished standard and customized items (four return flows). Specifically, Chapter 5 investigates the effect of different return policies on the pricing strategy, optimal inventory decisions and the profits of a reverse logistics dual-channel supply chain and in the presence of inventory, refurbishing, outsourcing, and disposal costs. The content of this chapter is published in a peer-reviewed journal (Computers & Industrial Engineering, 106, 58–82; doi.org/10.1016/j.cie.2017.01.024).

Chapter 6 investigates the effects of learning and forgetting on the performance of a dual-channel supply chain. Two main strategies are investigated (i.e., single- and dual-channel strategies). For each strategy, six different policies are investigated. The first policy discusses the behavior of both inventories without considering learning and forgetting effects. The other policies discuss five different shipment arrangements between the vendor and the retailer when learning and forgetting effects are considered. The objective is to maximize the total profit of the supply chain system by finding the optimal pricing and inventory decisions. The content of this chapter was submitted for review in a peer-reviewed journal.

Finally, Chapter 7 outlines the final reflections of the proposed thesis and provides directions for future research work.

CHAPTER 4. DUAL-CHANNEL SUPPLY CHAIN: A STRATEGY TO MAXIMIZE PROFIT

(Elements of this chapter are taken from (Batarfi et al., 2016))

The aim of this chapter is to investigate the effect of adopting a dual-channel strategy on the performance of a two-level supply chain. This is investigated under centralized decision-making situations. The dual-channel strategy here is composed of a vendor and a retailer, and deals with two channels in which the retail channel offers a standard item and the online channel offers customized items. There could be other scenarios where a vendor offers standard items through a retailer and directly online or where a vendor offers a standard items through both channels as well as offering customization online. However, the existing literature on the dual-channel supply chain is mainly focused on optimizing the profit of a supply chain when both the vendor and the retailer offer a single item (standard) through the dual-channel (Yue and Liu, 2006; Chen et al., 2008; Hua et al., 2010; Huang et al., 2012). Few studies have investigated the effect of selling standard and customized items in a dual-channel supply chain. We limit our investigation to a supply chain structure where a standard item is distributed through a retailer and customized items are sold through the online channel. The previous studies in this area, however, assumed no price discrimination of the customized items and did not consider the effect of the dual-channel on inventory decisions (Shao, 2013; Xiao et al., 2014; Li et al., 2015). We address this gap by investigating the optimal pricing decisions by finding the optimal markup margin, order and production quantities, and the number of shipments that will maximize the total profit of the system.

The remainder of the chapter is organized as follows. Section 4.1 describes the developed models. Sections 4.2 and 4.3 present the assumptions and notations. Sections 4.4 introduces the mathematical models. Section 4.5 discusses numerical examples and the obtained results. Section 4.6 illustrates the effect of different input parameters through sensitivity analysis. Finally, the conclusion and the discussion are provided in Section 4.7.

4.1 Model Description

Consider a supply chain that consists of a vendor and a retailer. The vendor has a flexible production system in place that allows it to produce make-to-stock (MTS) and build-to-order (BTO) items. However, the vendor currently produces only one type of item based on the MTS process, which is then sold to customers through a traditional retail channel. This item is comprised of a core item (unfinished item) with some basic features added (henceforth a standard item). To meet customers' demand and their preferences, the vendor adopts a dual-channel strategy in which it opens an online channel in addition to the existing retail channel. Adopting a dual-channel strategy would allow the vendor to sell its standard items indirectly through the retail channel (as shown by the solid line in Figure 4.1) and to offer customized items, which follow a BTO process, directly to customers through an online channel (as demonstrated by the dotted line in Figure 4.1). A customer, for example, may therefore purchase a pre-configured (standard) item only from the retail channel or a customized item only from the vendor's online channel.

The objective of this chapter is to investigate the effect of a dual-channel strategy on the performance of the supply chain system when inventory costs are considered. The total profit of the supply chain is the performance measure, which is maximized for the optimal markup margin, production/ordered quantity and the number of shipments.

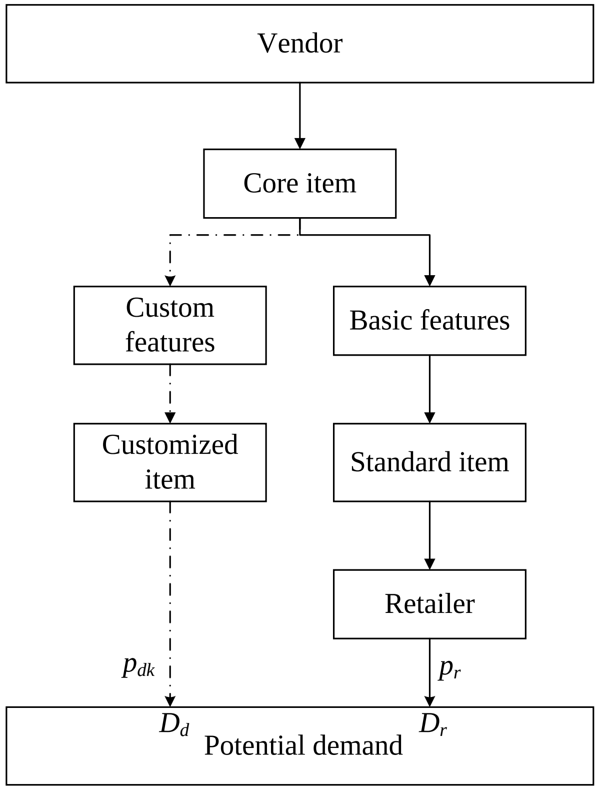

Figure 4.1 Dual-channel supply chain

The investigation starts first with a benchmark scenario, where the supply chain system is composed of a single channel (retail channel) and only one type of item is offered (the standard item). The single-channel strategy is similar to the one of Braglia and Zavanella (2003) who adopted a cost minimization approach. However, the developed model in this chapter deals with a profit maximization (Zanoni et al., 2014b; Zanoni and Jaber, 2015). Therefore, the revenue part will be implemented into their total cost equation. Moreover, Braglia and Zavanella (2003) assumed deterministic demand, whereas the demand function in this chapter is dependent on several parameters, including price. The second scenario in the chapter investigates the effect of adding an online channel to the existing traditional retail channel to form a dual-channel scenario.

The optimal values of the decision variables of the dual-channel will be analyzed, and then compared with the benchmark scenario (the single channel strategy).

4.2 Assumptions

This chapter assumes the following:

(1) The vendor has a production system in place to make standardized and customized items and does not need to invest in adopting a mass customization system.

(2) Production rates for the standard and customized items are greater than the base demand, i.e. no shortages allowed.

(3) The lead-time between the vendor and the retailer is zero (i.g., overnight deliveries (Maddah and Noueihed, 2017)).

(4) The lead-time between the supplier (who supplies the additional custom features) and the vendor is zero.

(5) A quoted delivery lead-time between the vendor and the customer is considered. It accounts for the waiting time from the point of ordering a customized item through the online channel to the point of delivering it to the consumer.

(6) The vendor and the retailer have a common cycle time for the retail channel (standard item).

(7) The vendor has a different cycle time for the production of the core item that will be used in the customization process.

(8) All input parameters are positive.

4.3 Notations

In this chapter, all the notations, parameters and decision variables are defined in situ. However, for the convenience of the reader, a nomenclature list is provided below:

Input parameter

D_r, D_d	Demand of the retail and the direct channel, respectively, (units/year);
a	Primary demand (potential demand when the items are free of charge), (units/year);
$\theta, (1-\theta)$	Percentage share of the demand going to the direct and retail channel, respectively, (%);
α_r	Coefficient of price elasticity of D_r, (unit^2/\$/year);
α_{dk}	Coefficient of price elasticity of D_d, $k = 1, 2, \dots, N$, (unit^2/\$/year);
ρ	Cross-price sensitivity, (-);
l_d	Quoted delivery lead-time (i.e., waiting time) of customized items, (day);
β_r	Sensitivity to quoted delivery lead-time of the demand D_r, (customer/day);
β_d	Sensitivity to quoted delivery lead-time of the demand D_d, (customer/day);
η_d	Product differentiation $\eta_d = \sum_{i=1}^{I}(\lambda_i \cdot w_i)$, (-);
w_i	Weight of importance of feature i where $(0 \leq w_i \leq 1)$ and $i = 1,2, \dots, I$, (-);
λ_i	Availability of the feature i, (-);
ψ_r	Sensitivity to product differentiation of D_r, (-);
ψ_d	Sensitivity to product differentiation of D_d, (-);
φ_{dk}	Percentage of core items stock used for customized item k, $k = 1, 2, \dots, N$, (-);
P_r	Production rate for the standard item, $P_r > a$, (units/year);
P_d	Production rate for the core item for eventual customization, $P_d > a$, (units/year);

c_P	Production cost for the standard item, ($/unit);;
c_{dk}	Production cost for the customized item k, $k = 1, 2, \dots, N$, ($/unit);
c_r	Vendor's wholesale price of the standard item to the retailer, ($/unit);
p_r	Retailer's selling price of the standard item; $p_r > c_r > c_P$, ($/unit);
p_{dk}	Vendor's selling price of the customized item k, $k = 1, 2, \dots, N$, ($/unit);
S_r	Vendor's setup cost for the standard item, ($/setup);
S_d	Vendor's setup cost for the core item, ($/setup);
h_1	Vendor's holding cost which includes financial cost and storage cost, ($/unit/year);
h_2	The supply chain unit holding cost for items at the retailer side (vendor's financial unit holding cost + buyer's unit storage cost), ($/unit/year);
O_r	Retailer's order cost, ($/order);
T_r	Cycle time (interval length) of the standard item, where $T_r = n_r q_r / D_r$, (year);
T_d	Cycle time (interval length) of the core item, where $T_d = q_d / D_d$, (year);
V_N	Set of variant k, where $k \in V_N$, and $k = 1, 2, \dots, N$, (-);
N	Total number of variants, (-);
I	Total number of custom features, (-);

Decision variables

m	Markup margin, (%);
q_r	Ordered quantity of the standard item to the retail channel, (units);
q_d	Production quantity of the core item for customization, (units);
n_r	Number of shipments of the standard item to the retail channel.

4.4 Mathematical models

4.4.1 Single-channel strategy

In this strategy, the vendor produces a MTS standard item only at a cost c_P, which is then sold to the retailer at the wholesale price c_r (after adding a markup percentage m), where $c_r = c_P(1 + m)$. The retailer then adds the same markup percentage m to the wholesale price and sells the item to the consumers at the retail price p_r, where $p_r > c_r > c_P$ and $p_r = c_P(1 + m)^2$. The central decision–maker seeks to maximize the total profit of the system by finding the optimal markup percentage m, order quantity q_r, and the number of shipments n_r of the two firms.

It is assumed that all orders of the standard item are filled by inventory and are replenished in a stock-driven environment. Motivated by the observation of the consignment stock (CS) policy (Battini et al., 2010; Valentini and Zavanella, 2003), the coordination mechanism between the vendor and the retailer is assumed to be under the CS policy. In the CS coordination mechanism, the vendor owns the inventory of the final item and stocks it at the retailer's warehouse, who sells the item to the end users from the consigned inventory and pays the vendor the wholesale price for only the withdrawn quantities that have been sold. Hence, the model in this chapter accounts for the production and inventory related costs of the standard item.

As shown in Figure 4.2, the vendor produces a lot size $n_r q_r$ units of the standard item every T_r units of time, where $T_r = n_r q_r / D_r$. The lot is shipped to the retailer in equal batches of q_r units each every q_r / P_r units of time. The vendor will continue the production and shipments of its inventory of the standard item until the retailer's inventory reaches the maximum limit $(n_r q_r - (n_r - 1)\, q_r D_r / P_r)$, which is determined by the number of shipments n_r and the ordered batch size

q_r. The retailer's inventory is depleted over the period $T_r = n_r q_r / D_r$. Before the retailer's inventory is completely exhausted, the vendor starts the production for the next cycle.

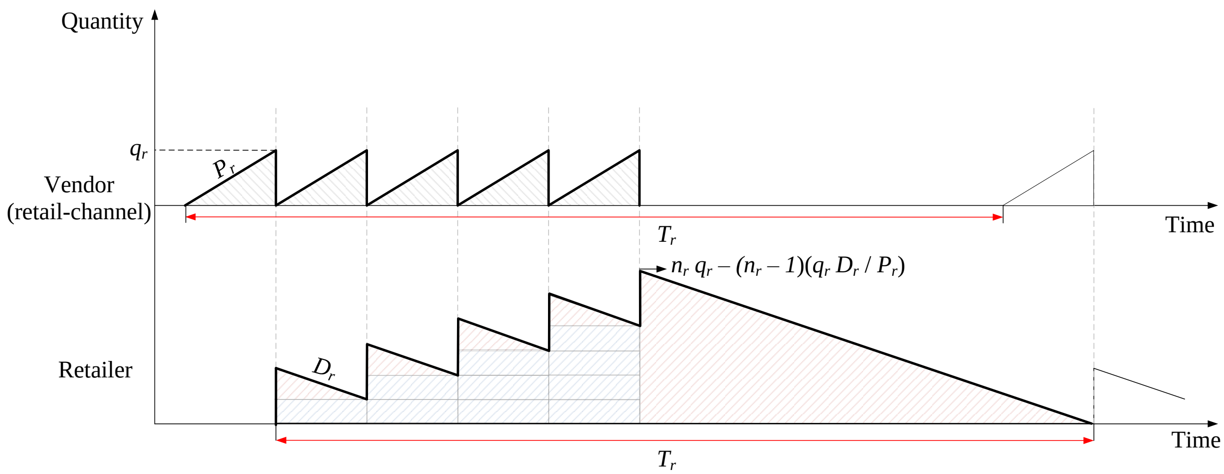

Figure 4.2: Behavior of the supply chain between the vendor and the retailer under the CS policy

Similar to Tsay and Agrawal (2000) Mukhopadhyay and Setoputro (2005), Sajadieh and Jokar (2009), and Kim et al. (2011), a linear demand function is assumed, in which the demand of the retailer D_r decreases linearly with the retail price p_r and is written as follows:

$$D_r = a - \alpha_r p_r \tag{4.1}$$

Without loss of generality, p_r could be written as $p_r(m) = c_P\,(1+m)^2$. Eq. (4.1) could then be rewritten as follows:

$$D_r = a - \ \alpha_r\, c_P (1+m)^2 \tag{4.2}$$

The next subsections, introduce the profit functions of the vendor and the retailer. The total profit of the supply chain is determined by the sum of both profits. All revenues and costs are based on per unit of time and are given as follows:

4.4.1.1 Vendor's profit

The profit of the vendor in per unit of time is given as follows:

$$\pi_{1,s} = c_p\ (1+m)\ D_r - \left(\frac{S_r\ D_r}{n_r\ q_r} + \frac{h_1\ q_r\ D_r}{2P_r} + c_P\ D_r\right) \tag{4.3}$$

where the first term represents the vendor's revenue per unit of time. The second term represents the vendor's total cost per unit of time, which has been given by Braglia and Zavanella (2003). This cost includes the setup cost, the holding cost and the production cost.

4.4.1.2 Retailer's profit

The profit of the retailer in per unit of time is given as follows:

$$\pi_{2,s} = c_P\ (1+m)^2\ D_r - \left(\frac{O_r D_r}{q_r} + \ c_p\ (1+m) D_r + \frac{h_2}{2}\left(n_r q_r - (n_r - 1)\frac{q_r D_r}{P_r}\right)\right) \tag{4.4}$$

where the first term represents the retailer's revenue per unit of time. The second term represents the retailer's total cost per unit of time, which is given by Braglia and Zavanella (2003). This cost includes the ordering cost, the purchasing cost and the retailer's inventory holding cost. The supply chain's total profit is the sum of Eqs. (4.3) and (4.4) and could be written as follows:

$$\Pi_s = \ \pi_{1,s} + \pi_{2,s} \tag{4.5}$$

4.4.1.3 Optimal decisions of the single-channel strategy

In the single-channel strategy when the vendor sells only the standard item through the retail channel, the total profit of the system Π_s is a concave function in m, q_r and n_r. The proof of concavity is shown in Appendix A.1. The optimal (indicated by an asterisk) solutions are given as follows:

$$m^{*} = -1 + \sqrt{\frac{1}{2} + \frac{a}{2\ \alpha_r\ c_P} + \frac{(S_r + n_r O_r)}{2\ c_P\ n_r q_r} + \frac{q_r\ (h_1 + h_2 - n_r h_2)}{4\ c_P P_r}} \tag{4.6}$$

$$q_r^{*} = \sqrt{\frac{2P_r(S_r + n_r O_r)D_r}{n_r((h_1 + h_2 - n_r h_2)\ D_r + P_r n_r h_2)}} \tag{4.7}$$

$$n_r^{*} = \frac{1}{q_r}\sqrt{\frac{2\ P_r\ S_r D_r}{(\ P_r\ h_2 - h_2 D_r)}} \tag{4.8}$$

One can notice that the optimal solutions $m^{*}, q_r^{*},$ and n_r^{*} are not independent of each other. Therefore, finding these optimal values require the use of a numerical procedure with nested iterations that can be easily implemented using a well-known mathematical software such as MATLAB, Maple, or Mathematica. In this chapter, an algorithm is developed in Microsoft Excel using the Solver Tool add-in, and enhanced with Visual Basic for Applications (VBA) Macros. The algorithm is developed to find the optimal solutions for these decision variables, in which the total profit of the system is maximized. The initial procedure for solving this problem is obtained by setting $q_r = 1$, and $n_r = 1$ and then finding the optimal values of m that maximize the total profit of the system Π_S. Following that the solutions are determined for $n_r = n_r + 1$ where the total system's profit Π_s is compared against previous iterations and then repeated until the maximum system profit is found for the first iteration. This iteration is repeated for $q_r > 1$ and until the maximum system profit for all iteration sets is found. A similar algorithm has been proposed in Jaber and Goyal (2008, p. 4), Zanoni and Jaber (2015, p. 6) and Bazan et al. (2015, p. 9).

4.4.2 Dual-channel strategy

This section modifies the single-channel strategy by introducing an online channel as part of a dual-channel selling strategy. The objective of the central decision-maker (i.e. the vendor) is to maximize the total profit of the supply chain when the dual-channel strategy is adopted. The vendor in this scenario produces two items: a standard item, sold through the retail channel and a customized item, sold through the online-channel. The standard item is produced using the MTS process and follows the same inventory policy as in Section 4.4.1. To supply customized items, it is assumed that the vendor offers online a set of custom features that can be added to the core item. Hence, the customized items are BTO, in which each order of a customized item is prepared once a customer's order arrives to the vendor through the online channel.

In practice there is no backlogging (zero inventory) for BTO or finished customized items (Gunasekaran and Ngai, 2009). Dell, for example, has developed a close relationship with its suppliers, which has allowed the company to operate with almost "no-work-in process inventory". The company pulls component from the supplier just as needed for producing the customized items (Gunasekaran and Ngai, 2009). Hence it is assumed that there is no inventory for the finished customized items. However, this model assumes that the vendor carries inventory of the core item that will be used in the customization process. The additional custom features that can be added to the core item are outsourced and are then supplied to the vendor once needed with zero lead-time. For example, when a customer orders a customized item through the online channel, the vendor takes one core item from the inventory and customizes it by adding as many features as requested by the customer. Once the customization is complete, the vendor delivers the item to the customer. The vendor's inventory of the core item behaves like the Economic Production Quantity (EPQ) model as shown Figure 4.3. In the EPQ model, the vendor produces q_d units of the core item for customization at a rate P_d in T_d units of time, where $T_d = q_d/D_d$ and D_d is the demand of the online channel.

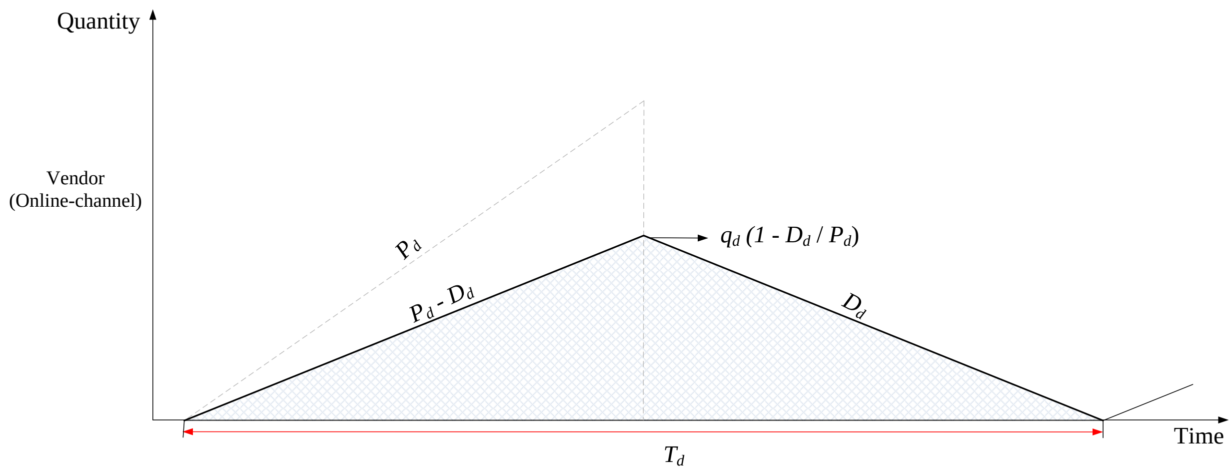

Figure 4.3: Behavior of the vendor's online-channel inventory under the EPQ model

The different combinations of features added to the core customizable item are denoted as a set of variants V_N, where each variant k ($k \in V_N$ and $k = 1,2, \dots, N$) is made up of one core item and at least one additional custom feature i, where $i = 1, 2, \dots, I$. For example, assuming that there are no dependencies between the additional features, if the vendor is offering feature X and Y, then the customer may request one feature (X or Y) or both (X and Y) to be added to the core item.

Each additional i feature is priced independently; hence, the price of each variant k is different. Therefore, the production cost and the selling price of the customized items are not fixed (it depends on the number of features added to the core item). Let c_{dk} be the production cost of a unit of variant k, which includes the cost of the core item, the cost of the additional added feature(s) and a fixed processing cost for each customized item ordered through the online channel. The vendor sells each customized item at a price p_{dk}, where $k \in V_N$ and $k = 1, 2, \dots, N$. To avoid triviality problems, it is assumed that the selling price of each customized item p_{dk} is always higher than the wholesale price of the standard item c_r ($\text{i.e.}\, p_{dk} > c_r$). This is not an arbitrary condition, because if the wholesale price is higher than the selling price of the customized item ($\text{i.e.}\, p_{dk} < c_r$), the retailer or any other arbitrator can obtain the item from the vendor's direct channel at a lower price (Hua et al., 2010; Huang et al., 2012, 2013).

Customers are heterogeneous in their preference of the standard or the customized items. Therefore, the number of customers choosing between the standard and customized items is affected by many factors. Following Hua et al. (2010), Huang et al. (2012, 2013), Yue and Liu (Yue and Liu, 2006), Zhang et al. (2015) and Raju and Roy (Raju and Roy, 2000), it is assumed that the demand functions for the standard and the customized items are linear. Specifically, the demand functions of the retail channel and the online channel are as follows:

$$D_r = (1-\theta)a - \alpha_r p_r + \rho \sum_{k=1}^{N} p_{dk} + \beta_r l_d - \psi_\mathrm{r} \eta_d \tag{4.9}$$

$$D_d = \theta a - \sum_{k=1}^{N} \alpha_{dk}\, p_{dk} + \rho\, p_r - \beta_d l_d + \psi_\mathrm{d} \eta_d \tag{4.10}$$

where $\eta_d = \sum_{i=1}^{I} (\lambda_i \cdot w_i)$

In Eqs. (4.9) and (4.10), D_r is the retailer's demand of the standard item and D_d is the vendor's direct channel demand of the customized item. The parameter a ($a > 0$) is the primary demand (potential demand when the two items are free of charge). The parameter θ $(0 < \theta < 1)$ and the term $(1-\theta)$ represent the percentage of the demand going to the direct channel and the retail channel, respectively, when p_r and p_{dk} are zero. In the literature, θ has been referred to as customer's acceptance (preference) of the online channel when the items are free of charge (Hua et al., 2010). α_r and α_{dk} are the price elasticity coefficients of p_r and p_{dk}, respectively, representing the amount of decrease/increase in market demand when both channels increase/decrease the price by one dollar. It is assumed that ρ, which represents the cross-price sensitivities, is symmetric. The cross-price sensitivities reflect the degree to which the items sold through the two channels are substitutable. It is assumed that $\alpha_r > \rho$ and $\alpha_{dk} > \rho$, meaning that the self-price effects are greater than the cross-price effects, a common expression used in economic and operation management literature (Huang et al., 2012)

The quoted delivery lead-time is incorporated into the model, since it significantly affects the customer's decision when buying the customized items online (Shao, 2013). Denote l_d as the quoted delivery lead-time, which is defined as the waiting time from the time an order is placed to the time it is delivered to the customer (Webster, 2002). β_r and β_d represent the elasticity of customer demand function, D_d , to changes in l_d of the retail and online channel, respectively, where D_d decreases as l_d becomes longer. For example, if l_d for a customized items increases by one unit, β_d units of demand will be lost, of which β_r units of that will be transferred to the retail channel and $\beta_d - \beta_r$ $(\beta_r < \beta_d)$ units will be lost to both channels (Hua et al., 2010).

Another important factor that affects customer decisions of buying a customized item from the online channel (versus a standard one from the retail channel) is the customized product differentiation, a marketing strategy that is used to distinguish one item from the other (Feitzinger and Lee, 1997). η_d is denoted as the customized product differentiation. It is assumed that η_d is a function of λ_i (denoted as the availability of the added custom features) and w_i (denoted as and the weight of importance of the added feature). The weight of importance is a measure of the preference of a customer for one feature compared to another. In practice, the weight of importance can be measured using a Multi-Attribute Decision-Making (MADM) method, such as the Analytic Hierarch Process (AHP) developed by Saaty (1980) or its extended model, fuzzy AHP developed by Van Laarhoven and Pedrycz (1983). The parameters ψ_{r} and ψ_{d} represent the product differentiation's elasticity of the demand in the retail channel and online channel, respectively, which means that if the product differentiation increases by one unit, ψ_{d} units of the demand will increase in the online channel of which ψ_{r} of the demand would be lost in the retail channel ($\psi_{\mathrm{r}} < \psi_{\mathrm{d}}$).

A company would usually generate the same return per dollar invested irrespective of the item sold; therefore, the selling price of the standard item p_r and the selling price of the customized item p_{dk}, could be written as $p_r(m) = c_p(1+m)^2$ and $p_{dk}(m) = c_{dk}(1+m)$, respectively. Considering a fixed markup margin for both standard and custom items, Eqs. (4.9) and (4.10) could be rewritten as follows:

$$D_r = (1-\theta)a - \alpha_r\, c_P(1+m)^2 + \rho \sum_{k=1}^{N} c_{dk}(1+m) + \beta_r l_d - \psi_{\mathrm{r}}\eta_d \tag{4.11}$$

$$D_d = \theta a - \sum_{k=1}^{N} \alpha_{dk}\, c_{dk}(1+m) + \rho\, c_P(1+m)^2 - \beta_d l_d + \psi_{\mathrm{d}}\eta_d \tag{4.12}$$

4.4.2.1 Vendor's profit

The profit of the vendor per unit of time from selling the standard item to the retailer, $\pi_{1,s}$, is given by Eq. (4.3).

The profit of the vendor in per unit of time from selling the finished customized items through the online channel is given by:

$$\pi_{1,c} = \sum_{k=1}^{N} c_{dk}(1+m)\ \varphi_{dk}\ D_d - \left(\frac{S_d\ D_d}{q_d} + \left(\frac{h_1\ q_d}{2}\right)\left(1-\frac{D_d}{P_d}\right) + \sum_{k=1}^{N} c_{dk}\ \varphi_{dk}\ D_d\right) \qquad (4.13)$$

where the first and second terms are, respectively, the vendor's revenue and total cost in per unit of time. The second term includes the vendor's setup cost of the core item S_d, the vendor's holding cost h_1 and the vendor's production cost c_{dk}. The parameter φ_{dk}, represents the percentage of the core item's stock that is used to customize item k.

4.4.2.2 Retailer's profit

The profit of the retailer per unit of time from selling the standard item to customers, $\pi_{2,s}$, is given by Eq. (4.4).

The dual-channel supply chain's total profit is, $\Pi_{s,c}$, is given by:

$$\Pi_{s,c} = \pi_{1,s} + \pi_{1,c} + \pi_{2,s} \qquad (4.14)$$

4.4.2.3 Optimal decisions of the dual-channel strategy

The total profit of the dual-channel supply chain system, $\Pi_{s,c}$, is a concave function in m, q_r, n_r and q_d. The proof of concavity and finding the optimal solutions are given in Appendix A.2. The optimal (indicated by an asterisk) solutions are given as follows:

$$q_r^* = \sqrt{\frac{2P_r(S_m + n_r\, O_r)\, D_r}{n_r((h_1 + h_2 - n_r h_2)\, D_r + h_2\, P_r\, n_r\,)}} \tag{4.15}$$

$$n_r^* = \frac{1}{q_r}\sqrt{\frac{2\, P_r\, S_m\, D_r}{(h_2\, P_r - h_2\, D_r)}} \tag{4.16}$$

$$q_d^* = \sqrt{\frac{S_d D_d}{\frac{h_1}{2}\left(1 - \frac{D_d}{P_d}\right)}} \tag{4.17}$$

One can notice that the optimal solution occurs for $q_r^*, n_r^*,$ and q_d^*, which are not independent of one another. Moreover, a closed-form expression for m, is complex to find. Therefore, finding these optimal values requires the use of a numerical procedure with nested iterations that can be easily implemented using mathematical software. Therefore, an algorithm procedure in Microsoft Excel using the Solver Tool add-in enhanced with VBA codes was developed. The developed algorithm was used to find the values of the decision variables that produce optimal solutions, where the total profit is the performance measure to be maximized. A similar algorithm has been proposed in Jaber and Goyal (2008, p. 4), Zanoni and Jaber (2015, p. 6) and Bazan et al. (2015, p. 9).

4.5 Numerical examples

This section presents numerical examples to illustrate the behavior of the developed models. The purpose is to analyze the vendor's decisions about the dual-channel strategy and whether adopting an online channel to sell customized items is worthwhile. The numerical analysis is applied first on the single-channel strategy, and then on the dual-channel strategy. For simplicity, in the dual-channel strategy it was assumed that the vendor offers two customizable features to customers. Three different combinations of these features can be added to the core item. Table 4.1 lists the system input parameters that were used in the numerical example. The values of the input parameters in Table 4.1, were logically chosen based on the assumptions presented in Section 4.2 and from different published studies to make the model more meaningful (Waters, 2003; Hua et al., 2010; Huang et al., 2013; Shao, 2013). For example, the vendor's holding cost was set 20% of the unit production cost and retailer's holding cost was set at 30% of the unit production cost. Moreover, some other conditions were applied such as that the base demand is less than the production rates, to insure a no shortage condition.

The numerical example is solved for both the single-channel and dual-channel strategies using similar steps to the referenced algorithms in Section 4.4.1.3. Table 4.2 describes how the search for the optimal solution using the algorithms was performed.

Table 4.1 Values of the chosen parameters of the single-channel and dual-channel strategies

Input parameters	Value	Unit	Input parameters	Value	Unit
P_r	18,000	(units/year)	λ_1	1	(-)
P_d	18,000	(units/year)	λ_2	1	(-)
a	15000	(unit/year)	λ_3	1	(-)
θ	0.3	(%)	φ_{d1}	0.25	(%)
α_r	20	(unit^2/\$/year)	φ_{d2}	0.3	(%)
α_{d1}	2	(unit^2/\$/year)	φ_{d3}	0.45	(%)
α_{d2}	2	(unit^2/\$/year)	c_P	150	(\$/unit)
α_{d3}	2	(unit^2/\$/year)	c_{d1}	400	(\$/unit)
ρ	1.8	(-)	c_{d2}	450	(\$/unit)
β_r	40	(customer/day)	c_{d3}	500	(\$/unit)
β_d	50	(customer/day)	h_1	30	(\$/unit/year)
l_d	6	(day)	h_2	45	(\$/unit/year)
ψ_r	80	(-)	S_r	1000	(\$/setup)
ψ_d	100	(-)	S_d	800	(\$/setup)
w_1	0.4	(%)	O_r	300	(\$/order)
w_2	0.6	(%)	N	3	(-)

Table 4.2 Sample search for the optimal solution of the single-channel strategy

n_r^*	m^*	q_r^*	Π_s^*				
2	0.7342	335	\$ 1,771,459.44				
n_r	m	q_r	Π_s	Value 1		Value 2	Optimal
1	0.7348	532	\$ 1,770,750.03				
2	0.7342	335	\$ 1,771,459.44	\$ 1,770,750.03	$<$	\$ 1,771,459.44	\$ 1,771,459.44
3	0.7340	257	\$ 1,770,457.28	\$ 1,771,459.44	$>$	\$ 1,770,457.28	\$ 1,771,459.44

The results of both the single-channel and the dual-channel strategies are summarized in Table 4.3. The results of the single-channel strategy showed that the profits of the vendor and the retailer are \$647,734.52 and \$1,123,724.92, respectively (column 2 and 3 of Table 4.3), corresponding to a total profit of \$1,771,459.44. For the dual-channel strategy, the results show that the profits of the vendor and the retailer are \$762,571.08 and \$1,115,635.63, respectively (columns 4 and 5 of Table 4.3), corresponding to a total profit of \$1,878,206.71. The results show that the profit of the retailer dropped in the dual-channel as compared to the single-channel strategy. This could be due to the effect of competition between the retail and the online channels. This finding corroborates to those of Shao (2013) and Li et al. (2015). The results also show that the ordered quantity of the standard item decreased from 335 for the single-channel to 325 units for the dual-channel with the number of shipments for both channels being the same. This decrease is a result of a decrease in demand. The findings also show that having a dual-channel strategy increases the markup margin from 0.73% (when only the standard item is sold) to 0.76 % (when the standard and the customized items are sold). For example, the retailer's selling price of the standard item, p_r, was found to increase from \$451.12 to \$465.44, when a dual-channel strategy is adopted.

To improve the profit of the system without a loss in the retailer's profit, the retailer must be motivated to improve its retail service (e.g. provide partnered marketing, exclusive pricing/incentives, and rebranding). The framework developed by Yan and Pei (2009) provides some evidence of how improving the retail service can effectively improve the supply chain performance and the overall profit. Moreover, offering permissible delay in payments between the vendor and the retailer could help in maximizing the profits of the two players. For instance, Aljazzar et al. (2015) showed that incorporating permissible delay in payment in different production policies has reduced the total cost (and increased the profit) of a two-level supply chain.

Table 4.3 Results of the optimal solution for the single-channel and dual-channel strategies

	Single-channel		Dual-channel	
	Vendor	Retailer	Vendor	Retailer
Revenue	$ 1,554,948.52	$ 2,697,607.06	$1,797,309.94	$2,621,201.10
Total cost	($ 907,214.00)	($ 1,572,882.14)	($1,034,738.86)	($1,505,565.46)
Profit	$ 647,734.52	$ 1,123,724.92	$762,571.08	$1,115,635.63
System's total profit	$ 1,771,459.44		$1,878,206.71	
m^*	0.74%		0.76%	
q_r^*	335		325	
n_r^*	2		2	
q_d^*	-		144	

4.6 Sensitivity analysis

This section expands on the presented numerical example by examining the effect of different input parameters on the behavior of the developed supply chain models to attain some qualitative insights. Table 4.4 summarizes the effect of the selected input parameters on the developed model. Each sensitivity analysis is tested on the optimal decisions and is discussed in the following subsections. Managerial insights are also provided after each sensitivity analysis.

Table 4.4 The effect of the selected input parameters on the optimal decisions

	Dual-channel					Single-channel			
Input parameter	$\Pi_{s,c}$	m	q_r	n_r	q_d	Π_s	m	q_r	n_r
P_r	↓	↑	↑	↓	-	↓	↑	↑	↓
P_d	↓	↑	-	-	↓	-	-	-	-
h_1/h_2	↑	-	↑	↑	-	↑	↓	↑	↑
S_r/O_r	↓	↑	↑	↓	-	↓	↑	↑	↓
θ	↑	↓	↓	↓	↑	-	-	-	-
l_d	↓	↓	↑	-	↓	-	-	-	-
ψ_d	↑	↑	↓	↓	↑	-	-	-	-

Note: ↑= increase; ↓= decrease

4.6.1 Effect of varying production rates of the standard and the core customizable items

The production rate is an important factor that affects the behavior of a supply chain system. This subsection examines the effect of varying the value of the production rate on the supply chain for the single-channel and dual-channel strategies.

4.6.1.1 Single-channel

The results showed that when the production rate of the standard item, P_r, increases from 6,500 to 22,500 units/year, the total profit of the single-channel strategy, Π_s, decreases and the markup margin, m, slightly increases (from 0.728 to 0.734, as seen in Figure 4.4). It was also observed that the ordered quantity, q_r, increases and the number of shipments, n_r, decreases with the increasing P_r (Figure 4.5). The significant increase in q_r is due to a sudden decrease in n_r, which indicates a change in the inventory policy (which is the number of shipments and the shipment size). To get a clearer picture, one can plot the lot size ($n_r \cdot q_r$) against the production rate, which will show a decreasing function. Meaning that as the production rate increases, the lot size decreases. This variation in q_r and n_r has to do with the adjustments of the cost function to the minimum cost.

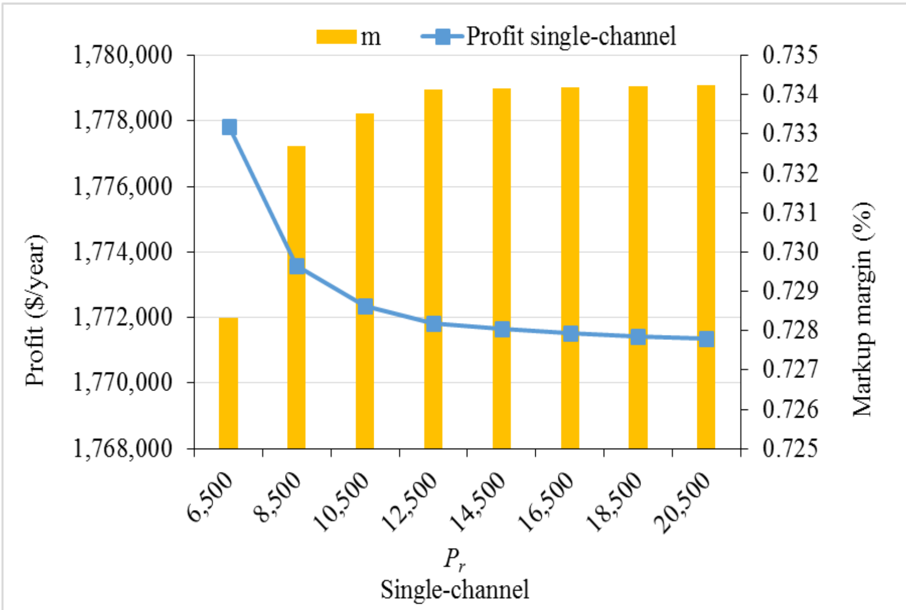

Figure 4.4 Effect of P_r on Π_s and m (Single-channel)

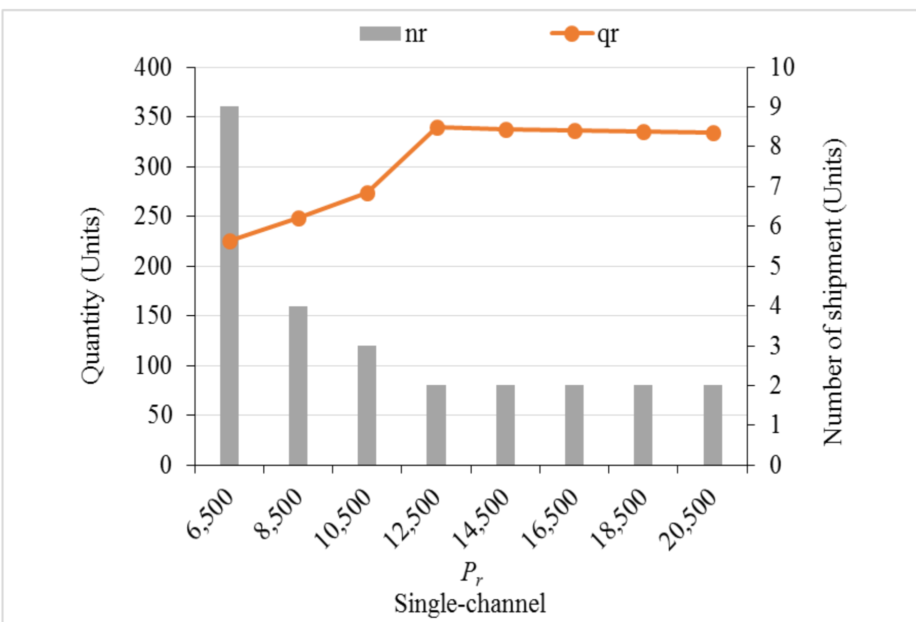

Figure 4.5 Effect of P_r on q_r and n_r (Single-channel)

4.6.1.2 Dual-channel

The results showed that as the production rate of the standard item, P_r, increases, the profit of the dual-channel strategy, $\Pi_{s,c}$, decreases and the markup margin, m, increases (Figure 4.6). Moreover, the ordered quantity, q_r, increases and the number of shipments, n_r, decreases as P_r increases (Figure 4.7). Similarly, the significant increase in q_r is due to the sudden decrease in n_r, which indicates a change in the inventory policy (which is the number of shipments and the shipment size). Which indicate that the as production rate increases, the lot size decreases. This variation in q_r and n_r has to do with the adjustments of the cost function to the minimum cost. Figure 4.7 also shows that the production quantity of the core item, q_d, is insensitive to the changes in P_r.

We also examined the effect of varying the production rate of the core customizable item, P_d, on the developed model. The results showed that as P_d increases, the profit of the dual-channel strategy, $\Pi_{s,c}$, decreases and the markup margin, m, increases (Figure 4.8). On the other side, the production quantity of the core item, q_d, decreases with increasing, P_d, (Figure 4.9). It was also noted that varying P_d has no effect on the ordered quantity, q_r, and the number of shipments, n_r (Figure 4.9).

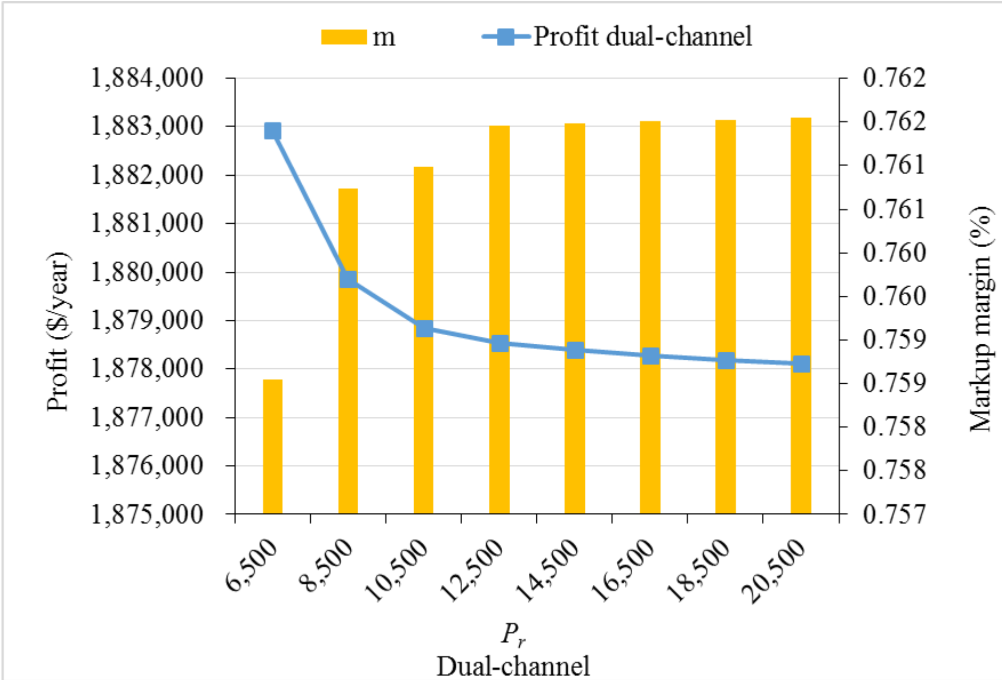

Figure 4.6 Effect of P_r on $\Pi_{s,c}$ and m (Dual-channel)

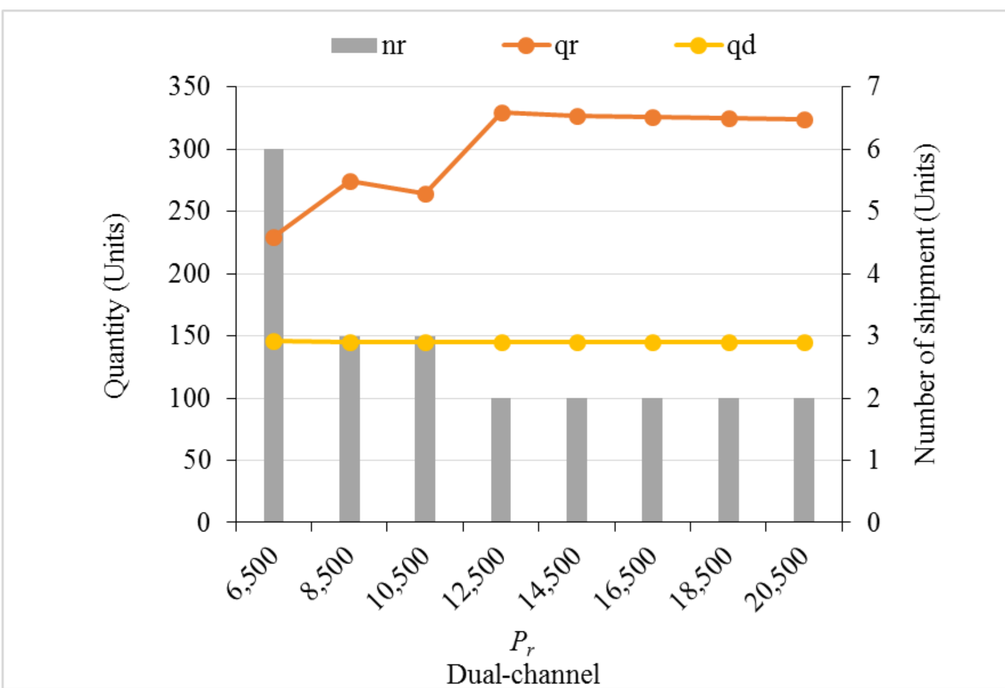

Figure 4.7 Effect of P_r on q_r, q_d and n_r (Dual-channel)

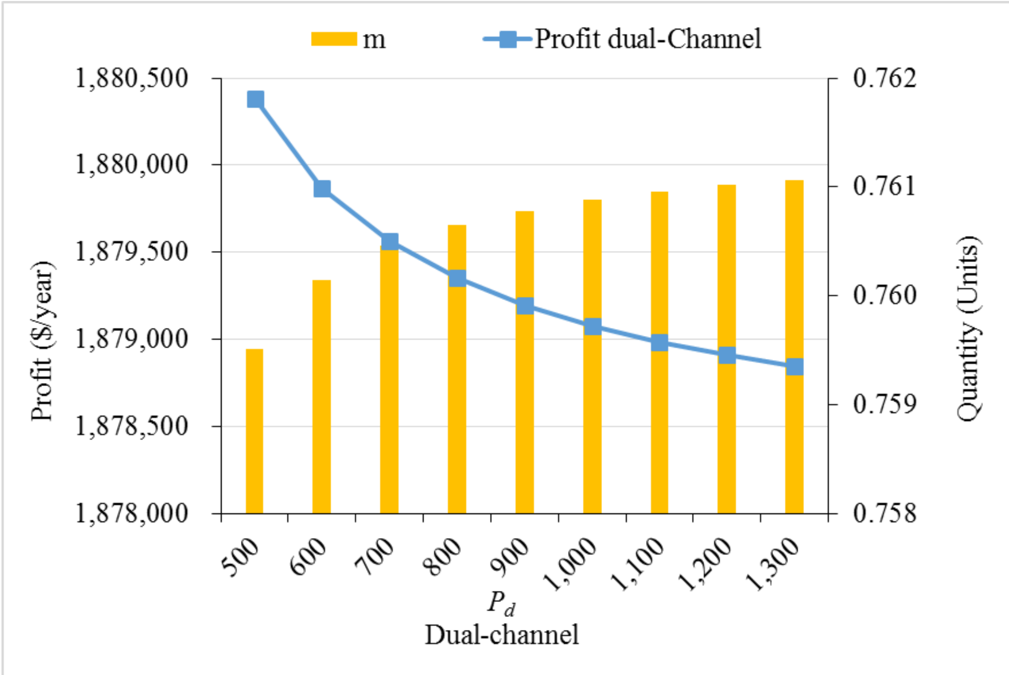

Figure 4.8 Effect of P_d on $\Pi_{s,c}$ and m (Dual-channel)

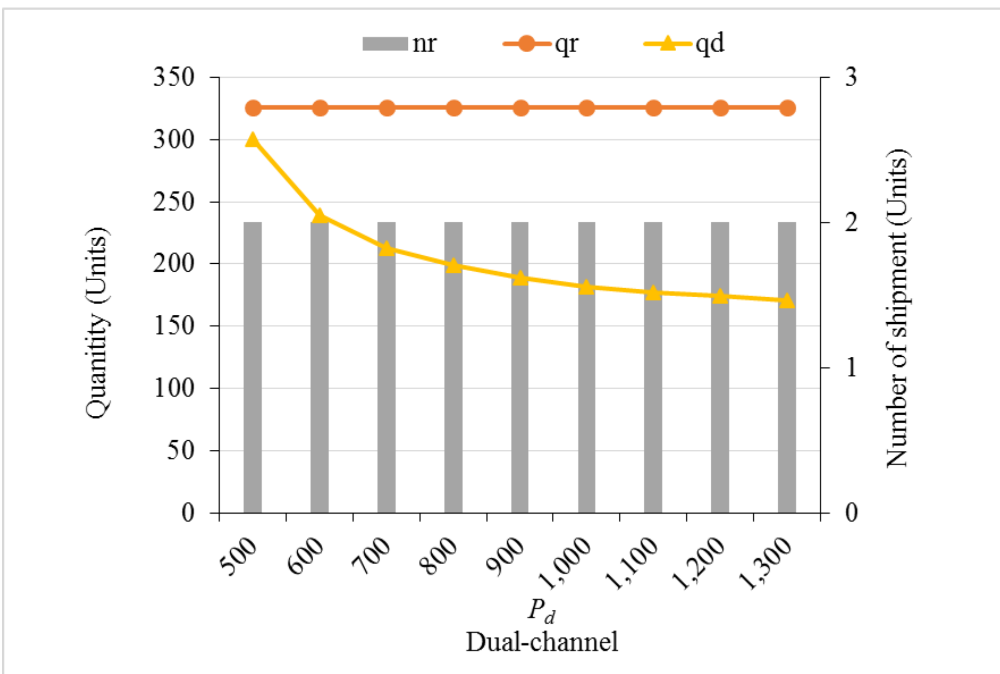

Figure 4.9 Effect of P_d on q_r, q_d and n_r (Dual-channel)

4.6.1.3 Managerial insights

The results suggest that it is not beneficial for the vendor to produce at a fast production rate for both strategies as items will remain in stock for longer periods, thus increasing holding costs and decreasing profits. It is recommended that the vendor produces at a rate that is close to the demand rate. Moreover, it was found that the dual-channel strategy is more profitable, even when production rate is fast. For example, the loss of profit between producing at 6500 (units/year) and producing at 22,500 (units/year) of standard items in the dual-channel strategy is ≅ $ 4,725 (0.25%) as compared to the single-channel strategy ≅ $ 6526 (0.37 %).

4.6.2 Effect of varying the holding cost of the vendor and the retailer

The inventory holding cost plays a major part in affecting the supply chain behavior. For instance, a low/high inventory cost may influence the vendor to produce large/small quantities of its items. In this analysis, the effect of the holding cost ratio (vendor to the retailer holding cost) on the supply chain system is examined.

4.6.2.1 Single-channel

As demonstrated in Figure 4.10 for the single-channel strategy, as the inventory unit holding of the retailer, h_2, decreases (while that of the vendor h_1 unchanged), the total profit of the single-channel strategy, Π_s, increases and the markup margin, m, decreases. On the other hand, the ordered quantity, q_r, and the number of shipments, n_r, increase with increasing the ratio of h_1/h_2 (Figure 4.11). A significant drop in q_r occurs due to the increase in n_r, which indicates a change in the inventory policy. For example, plotting the lot size $(n_r \cdot q_r)$ against (h_1/h_2) shows an increasing function. This means that as the ratio (h_1/h_2) increases, the lot size increases.

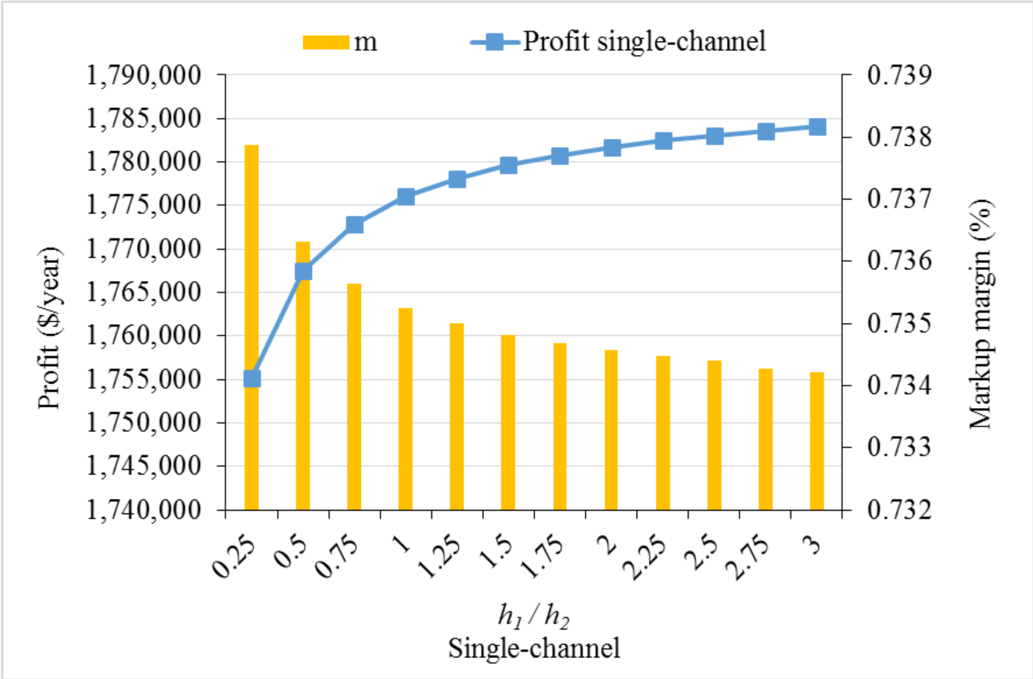

Figure 4.10 Effect of h_1/h_2 on Π_s and m (Single-channel)

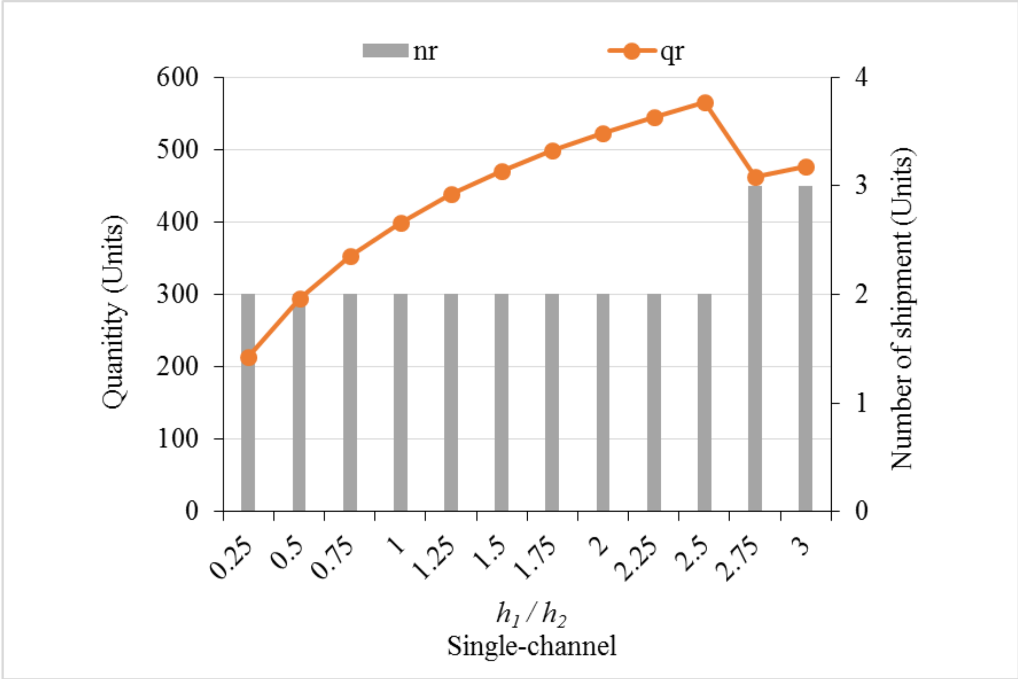

Figure 4.11 Effect of h_1/h_2 on q_r and n_r (Single-channel)

4.6.2.2 Dual-channel

The same sensitivity analysis (h_1/h_2) is investigated when adopting the dual-channel strategy. The results are demonstrated in Figure 4.12 and Figure 4.13. In Figure 4.12, an increase in h_1/h_2 results in an increase in the profit of the dual-channel strategy, $\Pi_{s,c}$, with no change in the value of the markup margin, m. However, the ordered quantity, q_r, and the number of shipments, n_r, increases with the increase of h_1/h_2. Similarly, the significant drop in q_r occurs due to the increase in n_r, which indicates a change in the inventory policy which means that as the ratio (h_1/h_2) increases, the lot size increases. The production quantity, q_d, is not affected by the change in the ratio of the holding cost (Figure 4.13).

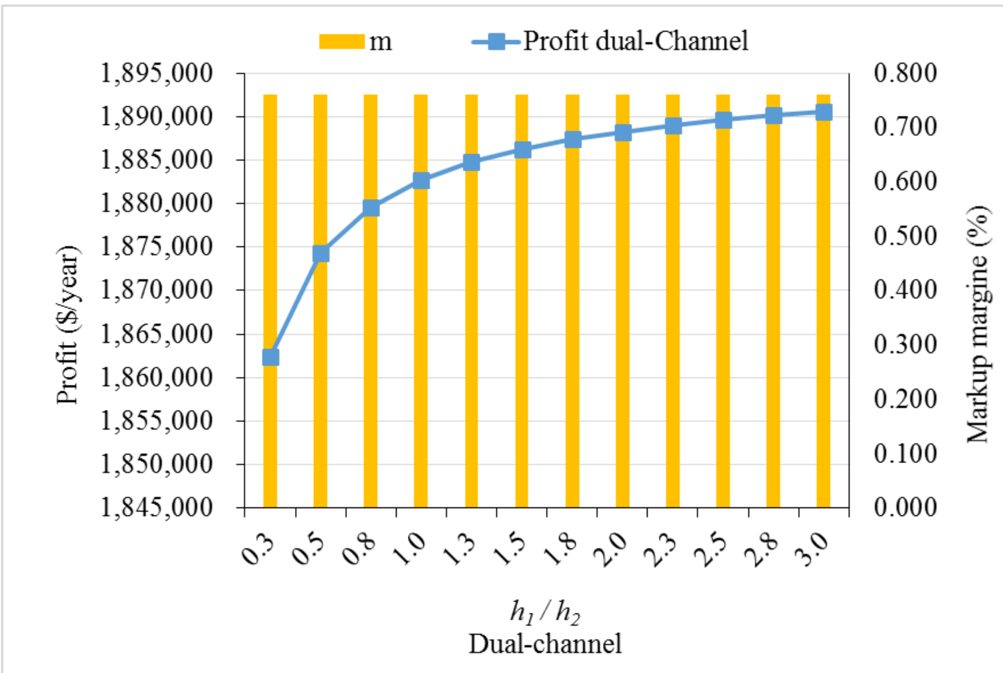

Figure 4.12 Effect of h_1/h_2 on $\Pi_{s,c}$ and m (Dual-channel)

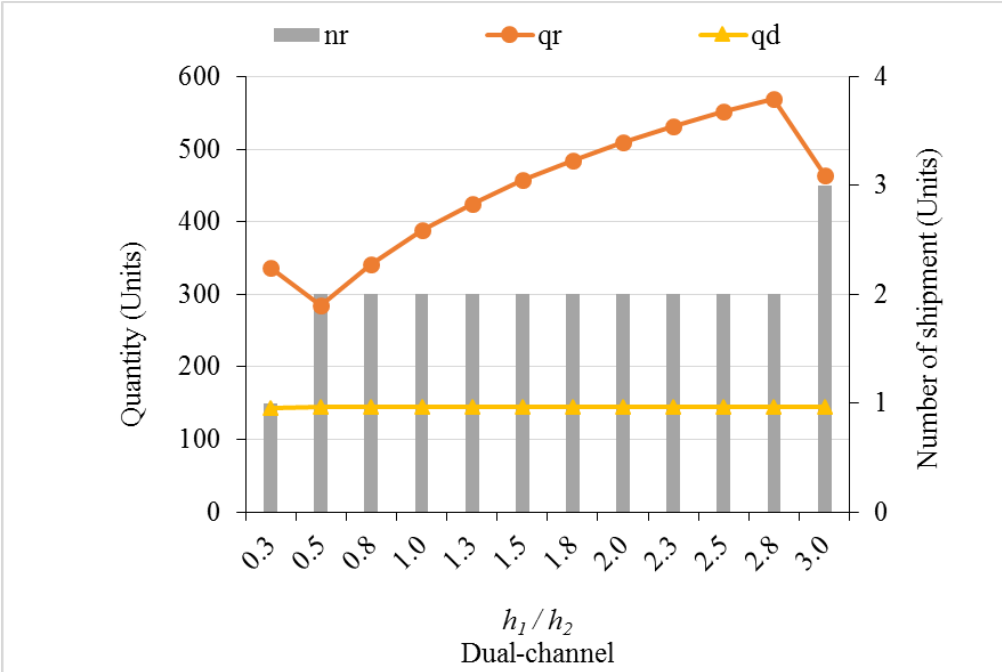

Figure 4.13 Effect of h_1/h_2 on q_r , q_d and n_r (Dual-channel)

4.6.2.3 Managerial insights

The results suggest that it is beneficial for the supply chain system to have a low inventory unit holding cost at the side of the retailer, which is in line with the literature (Zanoni et al., 2014a; Mandal and Giri, 2015; Zahran et al., 2015), as it helps the vendor in saving on its holding costs by making more frequent and larger shipments to the retailer. It helps to free space at the vendor for other items. The results also suggested that it is more profitable for the vendor to adopt a dual-channel strategy, as it increases sales while maintaining the same markup margin and lower holding costs.

4.6.3 Effect of varying the vendor's setup cost and the retailer's ordering cost

Similar to examining the effect of the inventory holding cost on the developed model, the ratio of the vendor's setup cost over the retailer's ordering cost (S_r/O_r) is investigated. To do this, the retailer's ordering cost has been varied while the vendor's setup cost has been fixed. The results are demonstrated in Figure 4.14 and Figure 4.15 for the single-channel and Figure 4.16 and Figure 4.17 for the dual-channel.

4.6.3.1 Single-channel

As shown in Figure 4.14, the total profit of single-channel strategy, Π_s, decreases and the markup margin, m, slightly increases when the ratio S_r/O_r increases. Figure 4.15 shows that increasing S_r/O_r increases the ordered quantity, q_r, and decreases the number of shipments, n_r. The significant increase in q_r is due to the sudden decrease in n_r, which indicates a change in the inventory policy (which is the number of shipments and the shipment size). To get a clearer picture, one can plot the lot size $(n_r \cdot q_r)$ against the production rate, which will show a decreasing function. Meaning that as the ratio S_r/O_r increases, the lot size decreases. This variation in q_r and n_r has to do with the adjustments of the cost function to the minimum cost.

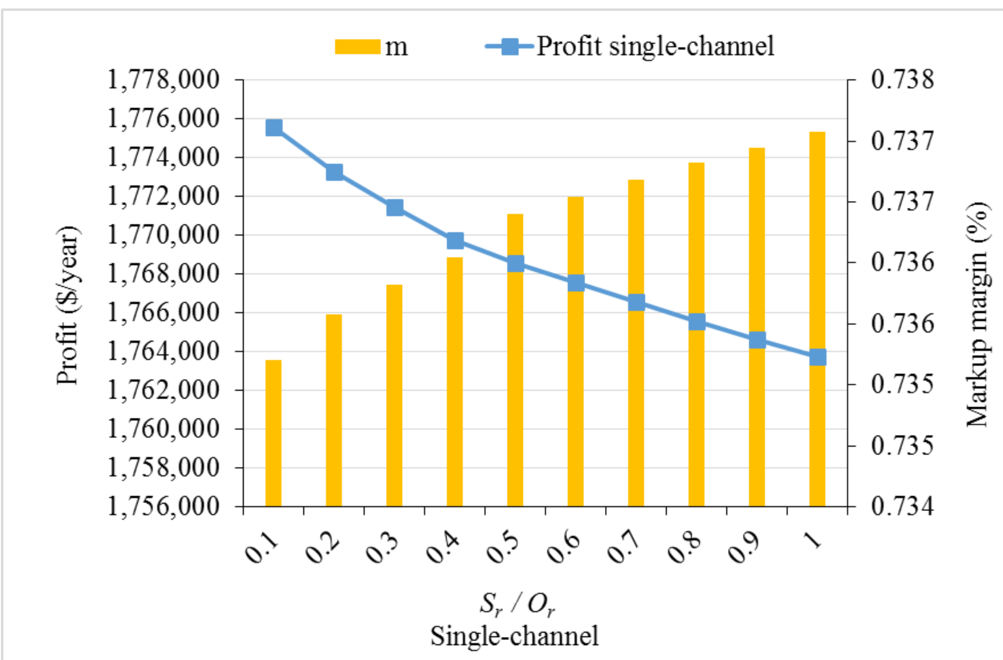

Figure 4.14 Effect of S_r/O_r on Π_s and m (Single-channel)

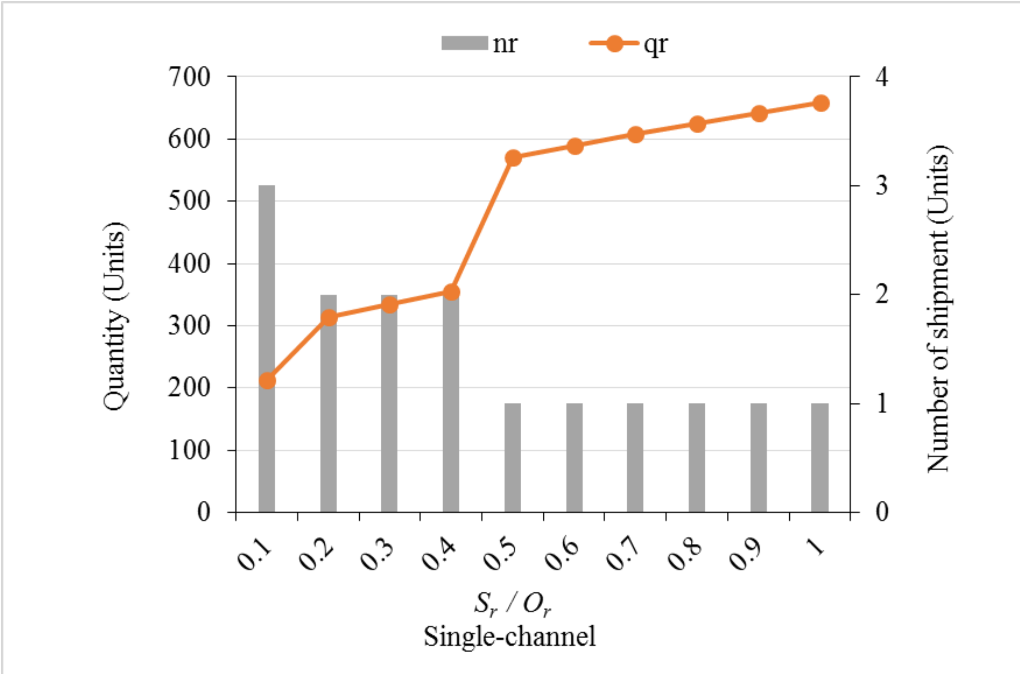

Figure 4.15 Effect of S_r/O_r on q_r and n_r (Single-channel)

4.6.3.2 Dual-channel

The ratio of the vendor's setup cost over the retailer's ordering cost (S_r/O_r) has a similar effect on the total profit of the dual-channel strategy, $\Pi_{s,c}$, and the markup margin, m, when the dual-channel strategy is implemented; the total system's profit, $\Pi_{s,c}$, decreases with a slight increase in the markup margin, m, when increasing the ratio from 0.1 to 1, (Figure 4.16). The results show that increasing S_r/O_r, increases q_r and decreases n_r, with no effect on the production quantity, q_d (Figure 4.17). Similarly, the significant drop in q_r occurs due to the decrease in n_r, meaning that as the ratio (S_r/O_r) increases, the lot size decreases.

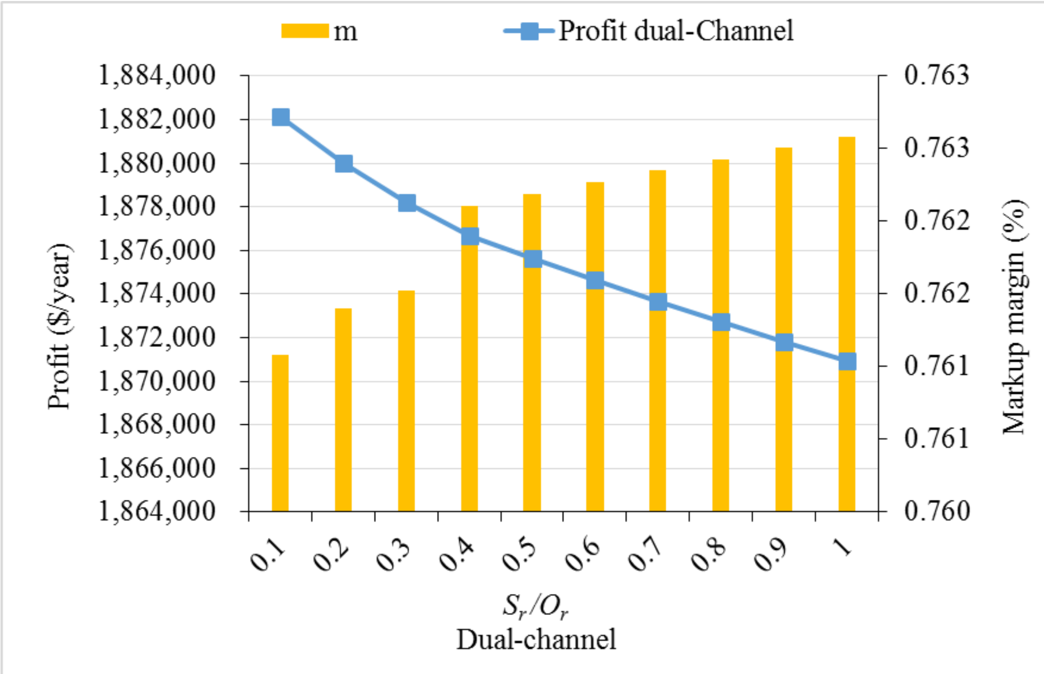

Figure 4.16 Effect of S_r/O_r on $\Pi_{s,c}$ and m (Dual-channel)

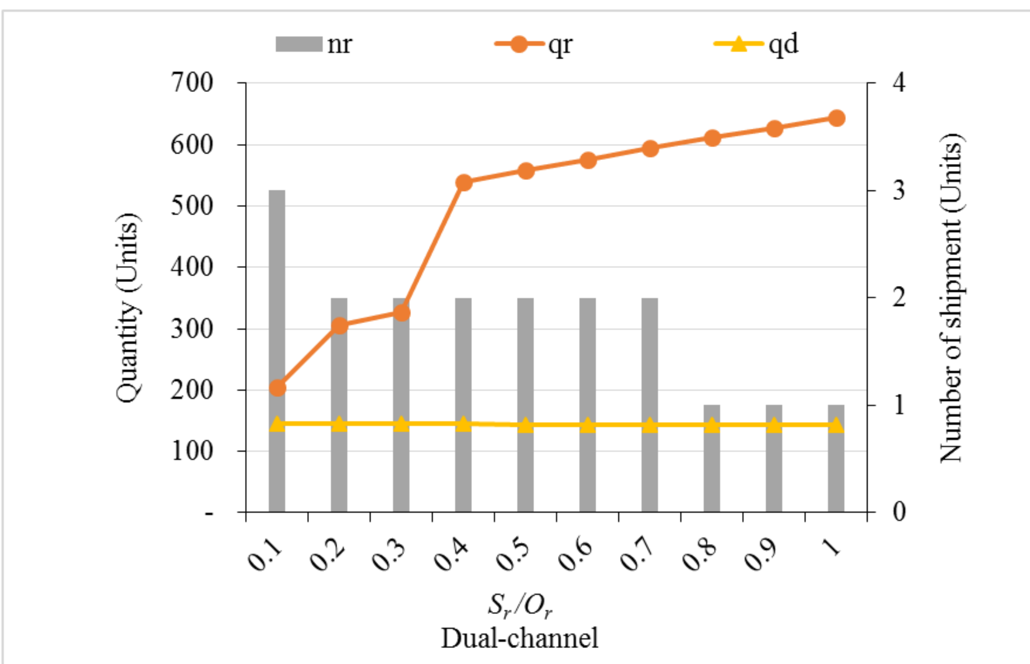

Figure 4.17 Effect of S_r/O_r on q_r, q_d and n_r (Dual-channel)

4.6.3.3 Managerial insights

The results suggest that a lower ordering cost reduces the total cost of the supply chain as frequent and larger batches of the standard item are made by the retailer whose holding cost is also lower than that of the vendor. As both strategies behave in a similar manner for the variance in the ordering cost, it can be noted that as the ordering cost is lower, the movement of items (from the vendor to the retailer) in the supply chain increases, and reduces significantly when the ordering cost increases. It is suggested that the supply chain and in particular the retailer apply new methods or implement new technologies that help reduce its order cost.

4.6.4 Effect of varying customers' acceptance of the online channel

Customers' acceptance of buying online is increasing (Li et al., 2015). The effect of varying customers' acceptance, θ, on the behavior of the centralized dual-channel supply chain is shown in Figure 4.18. The results show that the total profit of the dual-channel strategy, $\Pi_{s,c}$, increases with an increasing θ (from 0.3 to 0.65), but slightly decreases the markup margin, m, (from 0.76 to 0.73). This suggests that increasing customer loyalty does not necessarily mean that a firm has to sacrifice its profit margin. The results in Figure 4.19 show that as θ increases (from 0.3 to 0.65), q_r and n_r decrease. The sudden decrease in q_r occurs due to a sudden decrease in n_r, meaning that as the θ increases, the lot size $(q_r . n_r)$ decreases. Additionally, it was found that q_d increases with an increasing value of θ.

4.6.4.1 Managerial insights

The results suggest that as customers' acceptance of buying online increases the total profit of the dual-channel supply chain increases. Given current market trends toward higher acceptance of the online channel, vendors need to strongly consider adopting dual-channel strategies. Therefore, the vendor should also consider what type of items to be sold through the online channel and whether or not its customers will be willing to purchase it, this is in line with the recommendations of some studies (Hua et al., 2010). For example, items that require inspection before committing to purchasing them, or require after-sale service, are less amenable to be sold through the direct channel, than items that require less inspection and services.

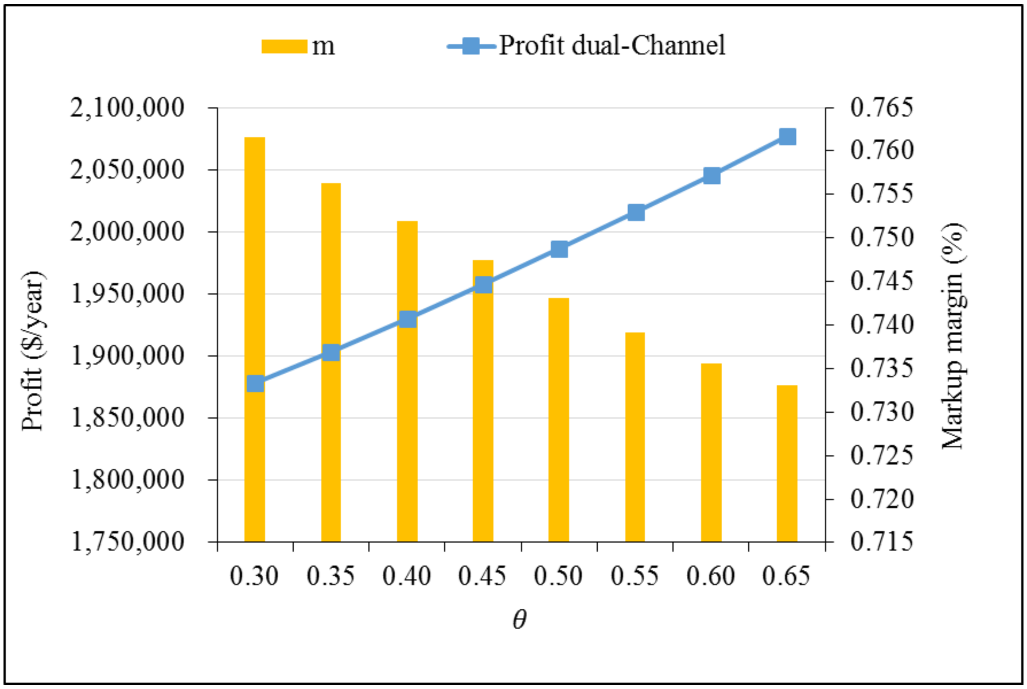

Figure 4.18 Effect of θ on $\Pi_{s,c}$ and m (Dual-channel)

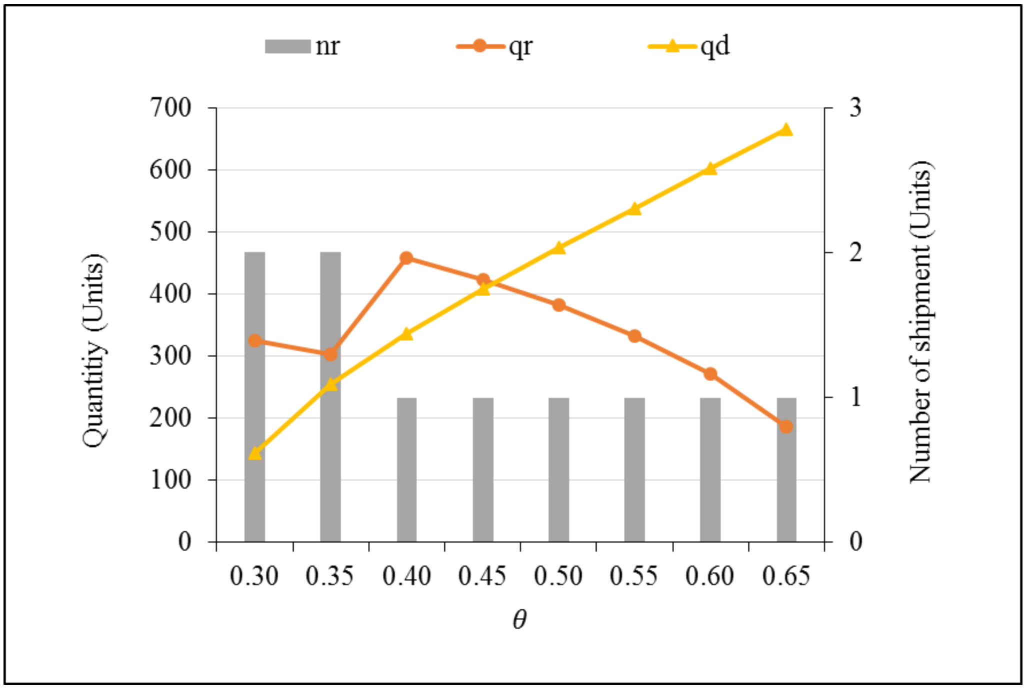

Figure 4.19 Effect of θ on q_r, q_d and n_r (Dual-channel)

4.6.5 Effect of varying the quoted delivery lead-time

The quoted delivery lead-time, l_d, of the customized items to customers is an important factor that affects the demand of the online channel. The results in Figure 4.20 show that the dual-channel strategy would be more profitable than the single-channel strategy if l_d of customized items is relatively short. Moreover, even with a longer l_d, the dual-channel strategy is much preferred to the single-channel strategy. This insight is consistent with the conclusion of Shao (2013). The effect of l_d on q_r, q_d and n_r is shown in Figure 4.21. It was found that, as l_d becomes longer, q_r slightly increases with n_r remains unchanged. The production quantity, q_d, on the other hand, decreases significantly for longer l_d.

4.6.5.1 Managerial insights

When a vendor is faced with long quoted delivery lead-times, it is recommended that it considers hiring a reliable third party logistics (3PL) firm to handle the delivery of customized items to customers quickly. If it is expensive to hire a 3PL, the vendor should consider shortening the quoted delivery-time through learning-based continuous improvement efforts (Jaber, 2016). However, such efforts incur costs such as training workers, shortening the time to processing and customize orders, and using economical and effective modes of delivery (i.e. consolidated shipments, zone skipping).

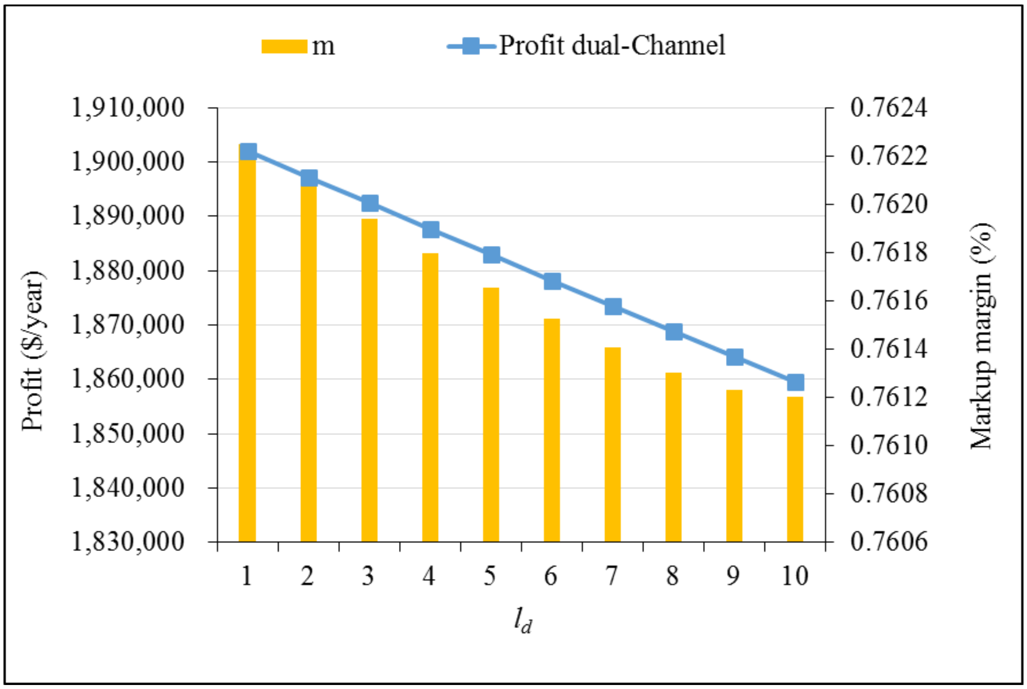

Figure 4.20 Effect of l_d on q_r, q_d and n_r (Dual-channel)

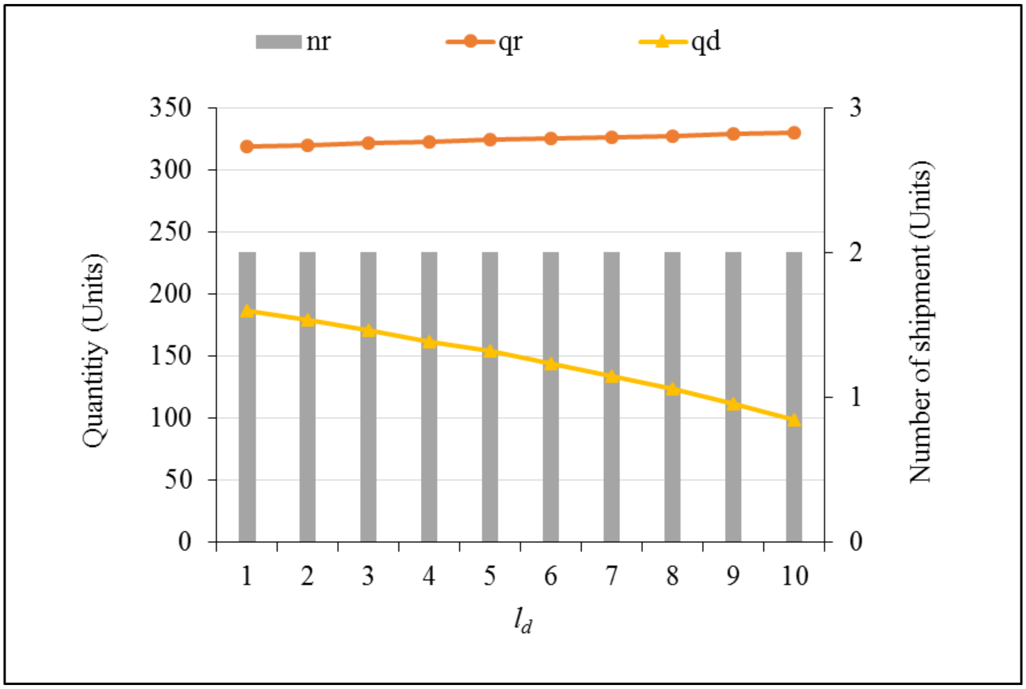

Figure 4.21 Effect of l_d on $\Pi_{s,c}$ and m (Dual-channel)

4.6.6 Effect of varying sensitivity to product differentiation

Product differentiation, ψ_d affects the profitability of a dual-channel supply chain, $\Pi_{s,c}$,as shown in Figure 4.22 and Figure 4.23. Figure 4.22 shows that an increase in ψ_d will lead to an increase in $\Pi_{s,c}$ and the markup margin m. In contrast, Figure 4.23 shows that as ψ_d increases, the lot size, $q_r \cdot n_r$, of the standard item follows a decreasing trend, despite its sudden jump. This decrease is due to a decrease in the demand of the standard item. Comparatively, the production quantity q_d steadily increases with increasing ψ_d (Figure 4.23).

4.6.6.1 Managerial insights

One key insight from the above analysis is that increasing the elasticity of product differentiation could increase the system's profit. This may indicate that offering more customized items satisfy a wider range of customer requirements, thus increasing demand and subsequently sales and profits. However, an indiscriminate increase in product customization through offering a large list of custom features could incur extra costs. One suggestion to be made here is that the vendor should involve customers at an early stage in the design process of new customized items. This could determine the desired level of product customization and therefore, have a better projection of the increase in sales (Blecker and Friedrich, 2006)

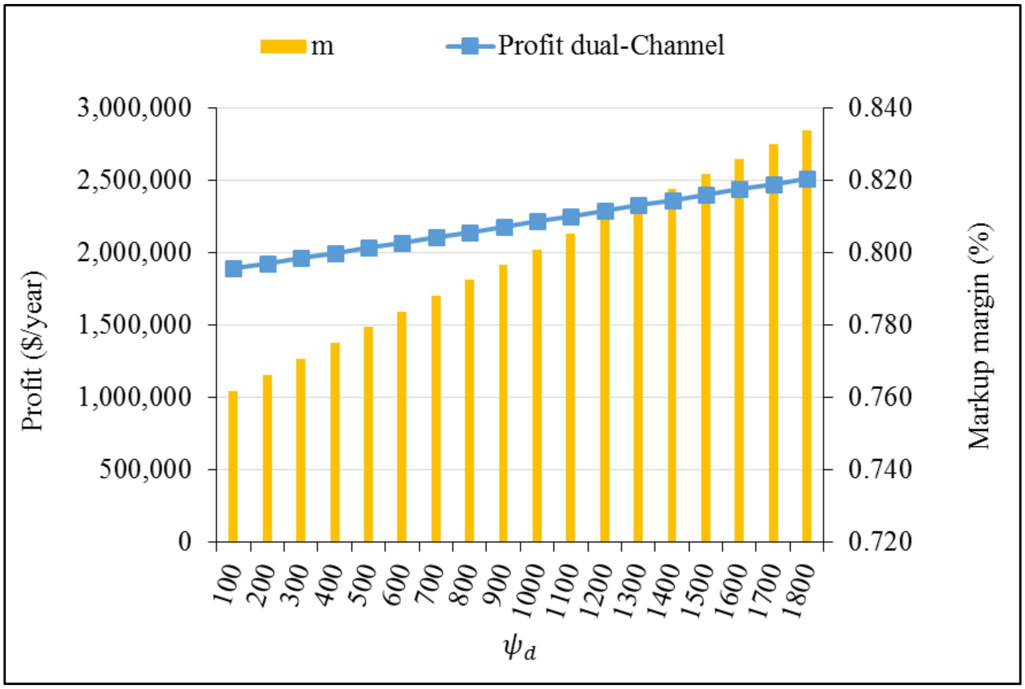

Figure 4.22 Effect of ψ_d on $\Pi_{s,c}$ and m (Dual-channel)

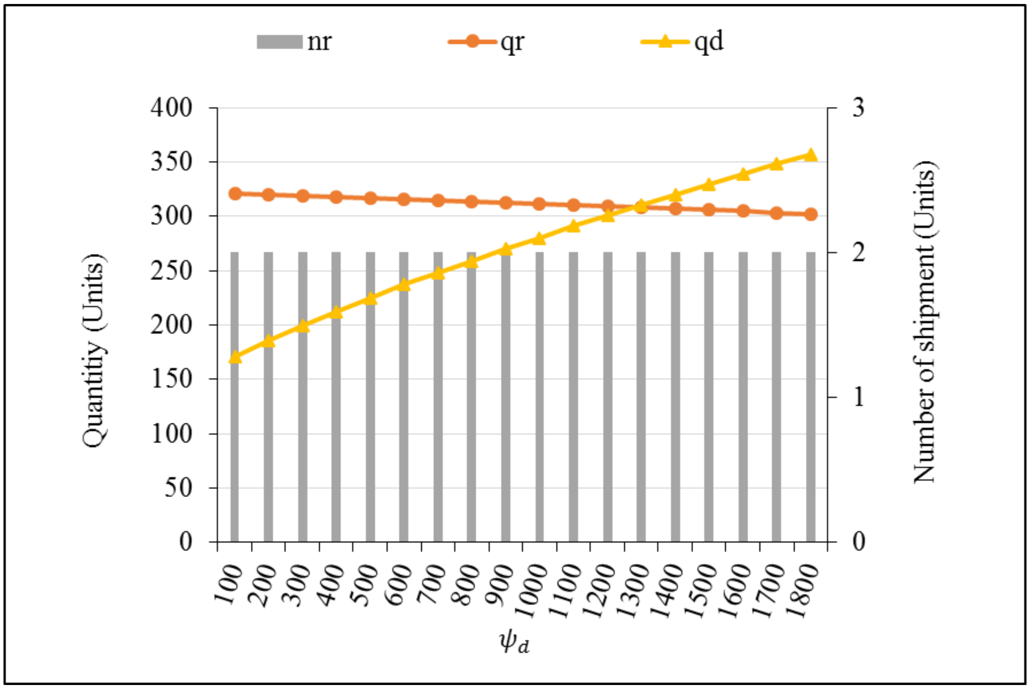

Figure 4.23 Effect of ψ_d on q_r, q_d and n_r (Dual-channel)

4.7 Summary and Conclusions

This chapter aimed at investigating the dual-channel strategy's effect on the behavior of a two-level supply chain (vendor-retailer) model, where a standard item is sold through a retail channel and customized products are sold online by the vendor. Specifically, this chapter explored how the total profit, the markup margin, and the inventory decisions of this supply chain are affected by the adoption of an online customizable-item channel. Two strategies were considered. The first strategy (a benchmark scenario) analyzed the behavior of the system when the supply chain is composed of a single channel (retail channel) in which the standard item is offered based on the MTS process. Since this chapter introduces a new channel to a two-level supply chain, where the vendor needs to account for inventory space for the new channel and where demand of the retail channel will be affected by the introduction of the online channel, the consignment stock policy was used as a coordination mechanism between the vendor and the retailer. In this scenario, the profit was maximized by finding the optimal markup margin, ordered quantity and number of shipments of the standard item. The second strategy analyzed the effect of the dual-channel (where the vendor offers standard items through the retail channel and customized items through the online channel) on the behavior of supply chain. The consignment stock policy was again used between the vendor and the retailer for the standard items. However, the production-inventory behavior of the core item followed the economic production quantity model. No inventory was assumed for finished customized items. In this scenario, the objective was to find the optimal profit of the dual-channel system by finding the optimal markup margin of the two items, the order quantity and number of shipments of the standard item, and the production quantity of the core item.

A numerical example was performed on both strategies and the results were compared. The results showed that the dual-channel strategy outperforms the single-channel strategy, and that the vendor is the one who benefits from the strategy by having a higher markup margin and profit. On the other hand, the retailer's profit was shown to decrease in the dual-channel strategy due to a decrease in the ordered quantity of the standard item per shipment. In general, the increase in the total profit of the system is typically due to two reasons. Firstly, adopting the dual-channel strategy reduces the double marginalization problem in the retail channel. Secondly, adopting the dual-channel strategy increases the market coverage.

Sensitivity analysis was also performed to examine the effect of varying some of the input parameters, first when the single-channel strategy is used, and then when the dual-channel is adopted. The results showed that changing some of the input parameters have a major impact on the optimal decisions of the supply chain system. For example, when varying the ratio of the holding cost (vendor to retailer), the total profit of the system in both strategies increased;

indicating that a lower holding cost at the retailer's side benefits the supply chain system. The results also showed that customer acceptance of the online-channel, the quoted delivery lead-time, and elasticity of product differentiation (θ, l_d and ψ_d, respectively), have a great impact on the total profit of the dual-channel strategy.

Since the adoption of the online channel can benefit the vendor by increasing its total profit, while cannibalizing the retailer's profit, improving the retail service as suggested by Yan, Z. Pei (Yan and Pei, 2009) can play a major part in improving the performance of the supply chain and the total profit of the system. The success of the retail channel has a direct impact on the customer perception and decision to adopt the online channel.

CHAPTER 5. A PROFIT MAXIMIZATION FOR A REVERSE LOGISTICS DUAL-CHANNEL SUPPLY CHAIN WITH A RETURN POLICY

(Elements of this chapter are taken from (Batarfi et al., 2017))

This chapter studies a supply chain system that comprises of production, refurbishing, collection, and waste disposal processes. An original equipment manufacturer (OEM) manages the forward logistics (FL) part of the supply chain; whereas, the reverse logistics (RL) part of the supply chain is outsourced to a third-party logistics (3PL) provider for the refurbishment of returned repairable items. The objective is to examine the effect of adopting different return policies when a dual-channel strategy is adopted on the behavior of the supply chain system while considering inventory, refurbishing, outsourcing, and disposal costs. The chapter analyzes first the behavior of the system when the supply chain system is composed of a single-channel strategy in which the retail channel offers both standard and refurbished standard items. Then, the chapter analyzes the behavior of the system when a dual-channel strategy is adopted, where the retail channel offers standard items whereas the online channel offers customized and refurbished items (refurbished standard and refurbished customized). In both strategies, the primary objective is to investigate the optimal prices and the optimal inventory decisions under different return policies (i.e., full, partial, or no refund) that maximize the total profit of the system.

The remainder of the chapter is organized as follows. Section 5.1 describes the developed models. Section 5.2 and 5.3 introduces the assumption and notations used in developing the proposed models. Sections 5.4 presents the mathematical models. Section 5.5 discusses numerical examples and the obtained results. Section 5.6 discusses sensitivity analysis of different input parameters. Section 5.7 presents the conclusion remarks and outlines future extensions.

5.1 Model Description

Consider a closed-loop supply chain system that consists of an OEM (henceforth vendor 1), a 3PL (henceforth vendor 2) and a retailer. It is assumed that the vendor 1 is capable of producing MTS (make-to-stock) and BTO (build-to-order) items. Presently, vendor 1 offers a standard item (a core item with basic features) based on the MTS process, sold to customers through the retail channel. To increase the market share and meet other customers' demand, vendor 1 adopts a dual-channel strategy in which it opens an online channel in addition to its existing retail channel. Doing this allows vendor 1 to offer MTS standard items indirectly through the retail channel and BTO customized items directly through the online channel.

Moreover, to achieve a market competitive advantage, vendor 1 offers a return policy agreement in which unsatisfied customers with their purchased item may return the item for a refund. If the returned item is repairable, it will be then refurbished and offered to customers at a lower price than the original price of a newly produced item. Returned non-repairable items are disposed of. To reduce logistics and inventory costs, the returned repairable items are refurbished through a contracted 3PL provider (vendor 2) (Cheng and Lee, 2010). The role of the 3PL in this chapter is restricted to refurbishing returned repairable items and does not include transportation. However, as it has been noted in the introduction (Section 1.5), a 3PL can perform both activities; these activities could be addressed in a future work. According to the contract agreement, vendor 1 pays vendor 2 a fee for each refurbished unit. Since vendor 2 is only responsible for the refurbishing processes, the ownership of the refurbished items belongs to vendor 1. Additionally, it is assumed that a customer unsatisfied with the purchase of a refurbished item can return the item for a refund. Returned refurbished items are repaired and then offered to customers as refurbished items.

5.2 Assumptions

This chapter assumes the following

(1) Refurbished items are not considered as-good-as-new.

(2) Lead-time between vendor 1 and the retailer in the single-channel strategy is zero.

(3) Lead-time between vendor 2 and the retailer in the single-channel strategy is zero.

(4) Vendor 2 has a production system to refurbish the returned repairable items.

(5) Collection rates for returned items are known and constant.

(6) All input parameters are positive.

(7) Infinite planning horizon.

(8) All cost input parameters do not vary over time.

(9) Shortages are not allowed.

5.3 Notations

<u>Input parameter</u>

i	Subscript indicating the type of item; $s =$ standard, $f =$ refurbished standard, $z =$ customized, and $fz =$ refurbished customized
D_i	Demand rate for the i item, (unit/year)
a_i	Primary demand for the i item, (unit/year)
δ_i	price elasticity for the i item, (unit^2/\$/year)
γ_i	Sensitivity of demand D_i with respect to the return policy, (unit^2/\$/year)

ζ A migration parameter, (unit^2/\$/year)

l Lost value parameter, (\$/unit)

ρ_i Proportion of the i items returned (repairable and non-repairable items) from the demand D_i; $0 \leq \rho_i < 1$ (%)

α_i Proportion of non-repairable (disposed) returned i items form ρ_i; $0 \leq \alpha_i < 1$, (%)

β_i Proportion of repairable i items returned for refurbishing from ρ_i; $0, \leq \beta_i < 1$, $\beta_i = (1 - \alpha_i)$, (%)

P_i Production rate for the i item; $P_i > D_i$ (unit/year),

c_i Unit production/processing cost of the i item $i \neq z, fz$, (\$/unit)

c_{ik} Unit production/processing cost of the i item $i \neq s, f$, and variant k, $k = 1, 2, \dots, N$, (\$/unit)

c_w Cost of disposing a non-repairable item, (\$/unit)

S_i Setup cost for the i item, (\$/setup)

$h_{v_1 i}$ Holding cost at vendor 1's side for a unit of item $i \neq f, fz$ (financial and physical storage cost), (\$/unit/year)

$h_{bi}^{v_1}$ Financial holding cost for a unit of item $i \neq z, fz$, at the retailer's side paid by vendor 1, (\$/unit/year)

h_{bi}^{b} Physical storage holding cost for a unit of item $i \neq z, fz$, at the retailer's side paid by the retailer, (\$/unit/year)

h_{bi} Total holding cost for a unit of item $i \neq z, fz$, at the retailer's side, where $h_{bi} = h_{bi}^{b} + h_{bi}^{v_1}$

$h_{v_2 i}^{v_1}$ Financial holding cost for a unit of item $i \neq s, z$ at vendor 2's side paid by vendor 1, (\$/unit/year)

$h_{v_2 i}^{v_2}$ Physical storage holding cost for a unit of item $i \neq s, z$ at vendor 2's side paid by vendor 2, (\$/unit/year)

$h_{v_2 i}$ Total holding cost for a unit of item $i \neq s, z$ at vendor 2's side where $h_{v_2 i} = h_{v_2 i}^{v1} + h_{v_2 i}^{v2}$

$h_{bu}^{v_1}$ Financial holding cost for a repairable item at the retailer's side paid by vendor 1, (\$/unit/year)

h_{bu}^{b} Physical storage holding cost for a repairable item at the retailer's side, (\$/unit/year)

h_{bu} Total holding cost for a repairable item at the retailer's side, where $h_{bu} = h_{bu}^{v_1} + h_{bu}^{b}$; $h_{bf} > h_{v_2 f} > h_{bu}$

$h_{v_2 u}^{v_1}$ Financial holding cost for a repairable item at the side of vendor 2 paid by vendor 1, (\$/unit/year)

$h_{v_2 u}^{v_2}$ Physical storage holding cost for a repairable item at the side of vendor 2 paid by vendor 2, (\$/unit/year)

$h_{v_2 u}$ Total holding cost for a repairable item at vendor 2's side, where $h_{v_2 u} = h_{v_2 u}^{v_1} + h_{v_2 u}^{v_2}$

T_i Length of time interval for the process of i itme, (year)

$p_{v_1 i}$ Vendor 1's selling price (wholesale price) of i item $i \neq z, fz$ to the retailer, (\$/unit)

O_{bi} Retailer's ordering cost for i items $i \neq z, fz$, (\$/order)

φ_{ik} Percentage of core item stock used for ik item, $i \neq s, f$ and $k = 1,2, \dots N$

η_i Proportion of the selling price of the i item refunded to a customer, where $0 \leq \eta_i \leq 1$

r_i Refunded amount per unit of i item, where $r_i = \eta_i p_i$, $0 \leq r_i \leq p_i$, (\$/unit)

r_{ik} Refunded amount per unit of ik item , $i \neq s, f$ and $k = 1,2, \dots N$, where $r_{ik} = \eta_i p_{ik}$, $0 \leq r_{ik} \leq p_{ik}$, (\$/unit)

V_N Set of variant k, where $k \in V_N$ and $k = 1, 2, \dots, N$

N Total number of variants

I Total number of custom features

Decision variables

p_i Selling price of i item, $i \neq z, fz$ to customers , (\$/unit)

p_{ik} Selling price of ik item, $i \neq s, f$, $k = 1, 2, \dots, N$, (\$/unit)

q_i Shipment (batch) size for i item, $i \neq z, fz$ (unit)

q_{ik} Production quantity of item ik, $i \neq s, f$ and $k = 1,2, \dots N$, (unit)

n_i Number of shipment of i item $i \neq z, fz$ and n_i integer $n_i \geq 1$

5.4 Mathematical models

5.4.1 Single-channel strategy

In the single-channel strategy, the supply chain system is comprised of a production, refurbishing, collection, and waste disposal processes. Figure 5.1 depicts the flow of the standard and refurbished items from the system to the market (forward) and the flow of the returned items from the market to the system (reverse). In Figure 5.1, there are three subsystems: vendor 1's system, vendor 2's system and the retailer's system. Vendor 1's system is responsible for the production of the standard items that are sold to the retailer at p_{v_1s}. Vendor 2's system, on the other hand, is responsible for refurbishing the returned repairable items, which are then sold back to the retailer at p_{v_1f}. The retailer's system offers both items, standard and refurbished, at p_s and p_f, respectively. With a return policy agreement provided by vendor 1, unsatisfied customers can return the standard or refurbished items to the retailer, and receive a refund of r_s or r_f, respectively. The refunded amounts r_s and r_f can be a full, a proportional or a zero (denied) refund of the selling price p_s and p_f, respectively, where $r_s = \eta_s p_s$, $r_f = \eta_f p_f$ and $0 \leq \eta_s, \eta_f \leq 1$.

Returned standard and refurbished standard items ($\rho_s D_s$ and $\rho_f D_f$, respectively) received by the retailer are screened immediately, and only the repairable-returned ones ($\beta_s \rho_s D_s$ and $\beta_s \rho_s D_s$) are kept in the retailer's inventory and then are shipped in one batch to vendor 2's facility for refurbishing. Non-repairable items ($\alpha_s \rho_s D_s$ and $\alpha_f \rho_f D_f$) are disposed at c_w per unit that is charged to vendor 1.

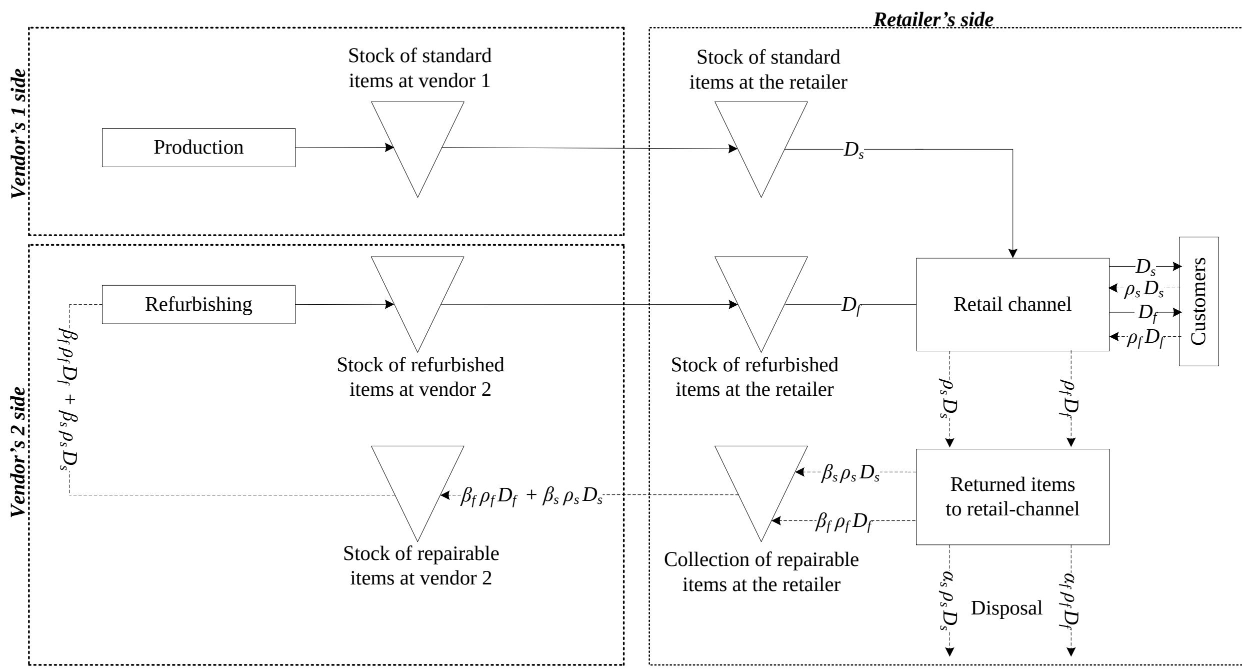

Figure 5.1 Forward and reverse material flow system for the single-channel strategy

The objective in the single channel strategy is to maximize the total profit of the system by finding the optimal selling price p_s and p_f, the optimal batch (shipment) size q_s and q_f, and the optimal number of shipments n_s and n_f.

The behavior of the inventory for vendor 1 and the retailer for the standard items is based on the consignment stock (CS) policy agreement and is illustrated in Figure 5.2. In Figure 5.2 vendor 1 produces the standard items at a production rate P_s in an interval of T_s units of time, where $T_s = (n_s q_s)/D_s$. A lot of size $n_s q_s$ units of the standard items is shipped to the retailer in equal batches of q_s units each every q_s/P_s and until the retailer's inventory reaches a maximum limit of $(n_s q_s - (n_s - 1)\ q_s D_s/P_s)$, which is determined by the number of shipments n_s and the ordered batch size q_s (Zahran et al., 2015). During the retailer's inventory depletion period, vendor 1 does not produce. Just before the retailer's inventory is completely exhausted, vendor 1 starts the production for the next cycle.

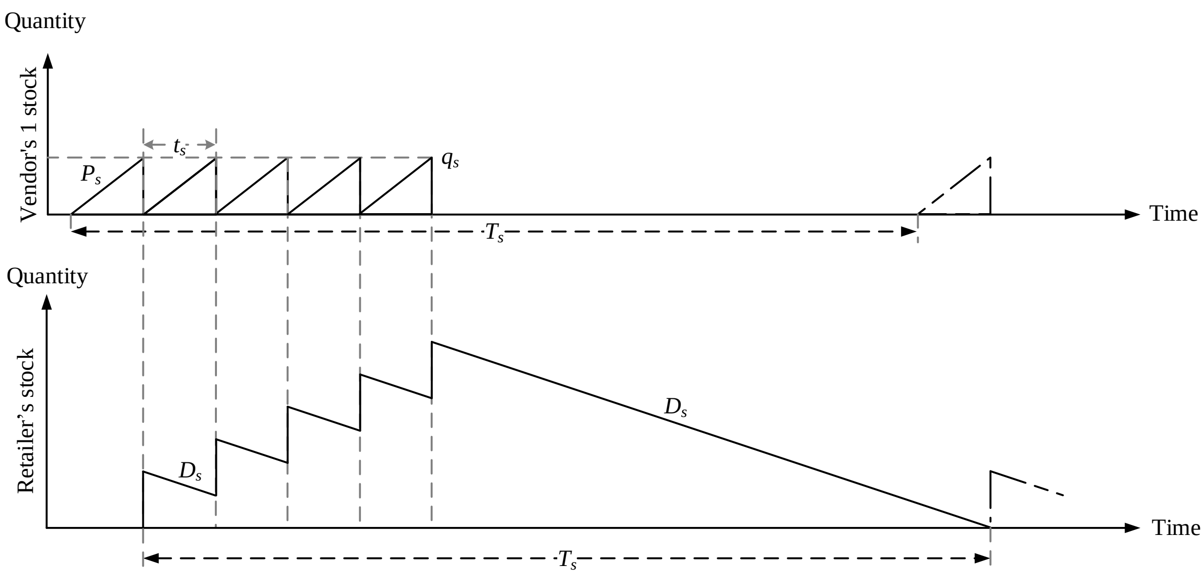

Figure 5.2: Inventory behavior at vendor 1's and the retailer's side for the standard item

On the other hand, vendor 2 refurbishes a lot of size $n_f \cdot q_f$ units of returned repairable items every T_f unit of time ($T_f = (n_f q_f)/D_f$). The processed refurbished items are shipped to the retailer's warehouse in equal batches of size q_f each at time intervals t_f, ($t_f = q_f/D_f$). Figure 5.3 shows the behavior of the inventory of the refurbished items between vendor 2 and the retailer.

To refurbish $n_f q_f$, vendor 2 receives from the retailer $\beta_s \rho_s D_s T_s$ of returned repairable standard items and $\beta_f \rho_f D_f T_f$ of returned repairable refurbished standard items $\left(\text{e.g.}\, n_f q_f = \beta_s \rho_s D_s T_s + \beta_f \rho_f D_f T_f\right)$ as shown in Figure 5.4. From this, we have $n_f q_f = \beta_s \rho_s n_s q_s + \beta_f \rho_f n_f q_f$. Hence, $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f(1-\beta_f \rho_f)}$.

There could be other scenarios where the time interval of receiving the returned repairable refurbished standard items is shorter (longer) than that for receiving the returned repairable standard items. However, this does not affect the behavior of the model. Therefore, we choose a classical scenario as considering all scenarios will proportionally expand the chapter.

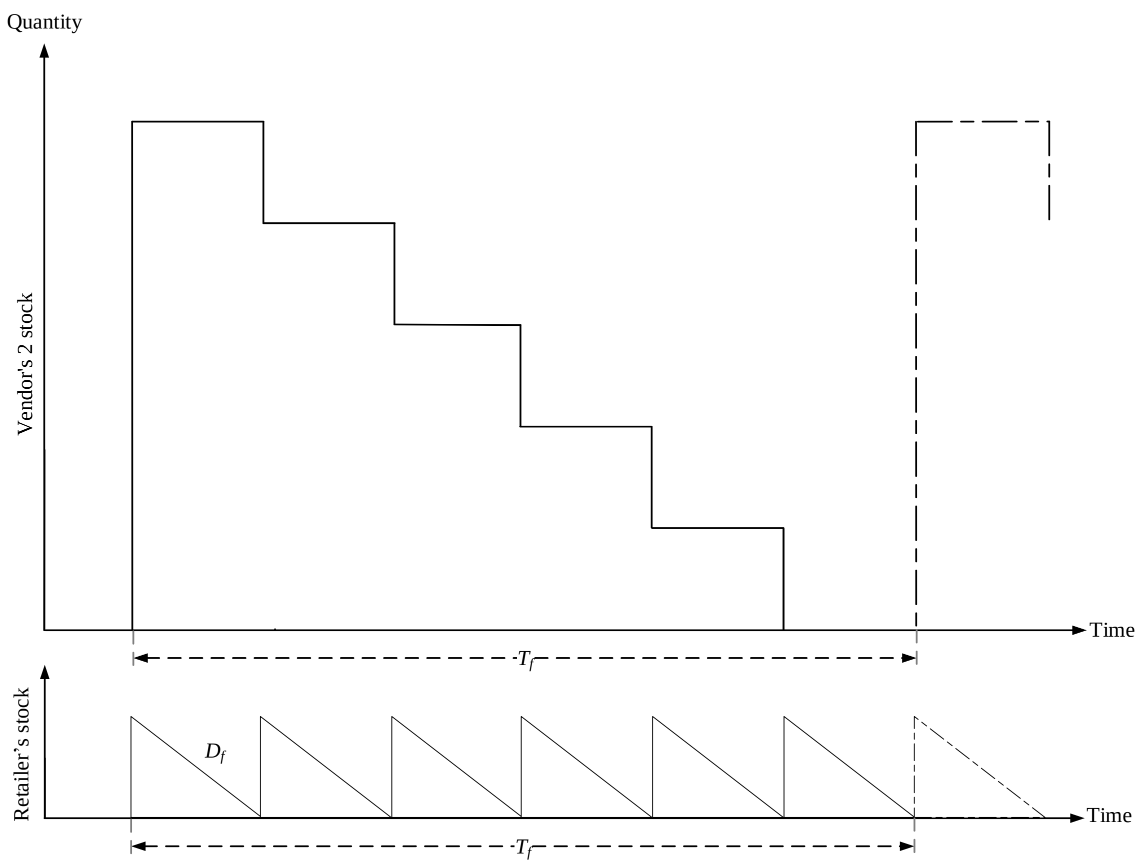

Figure 5.3: Inventory behavior at vendor 2's and the retailer's side for the refurbished items

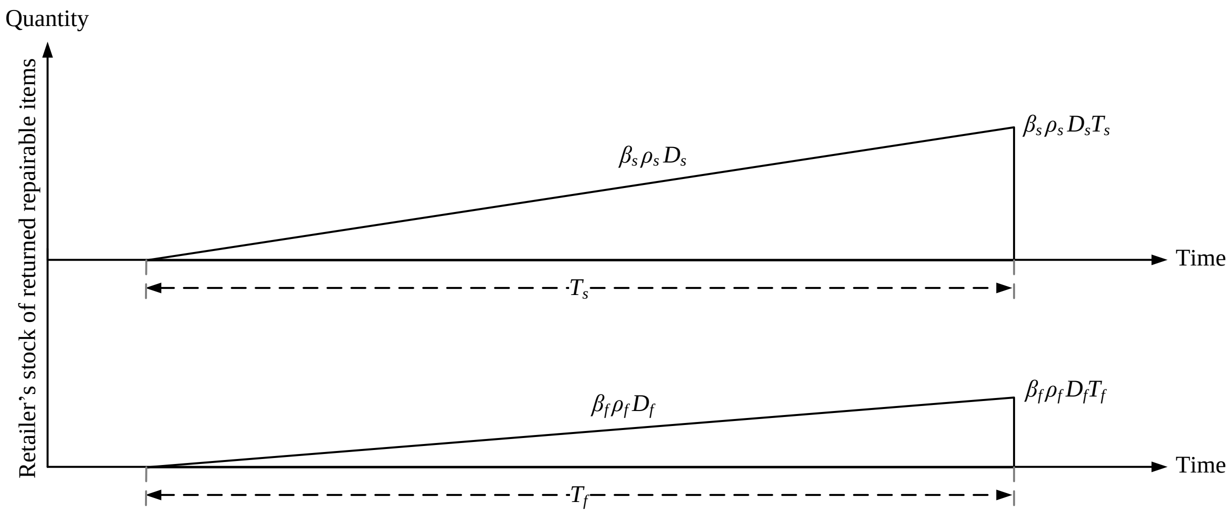

Figure 5.4: Inventory behavior of the returned repairable items at the retailer's side

5.4.1.1 Demand functions

In the single-channel strategy, there are two demand functions that are satisfied through the retail channel, which are the demand for the standard item, D_s, and the demand for the refurbished item, D_f. It is assumed that customers are sensitive to the price of the standard item, p_s, and the price of refurbished standard items, p_f, in which a higher price has a negative effect on the demand D_s and D_f. This sensitivity is moderated by the coefficients δ_s and δ_f that represent the elasticity of the two demand functions to p_s and p_f, respectively. Additionally, it is assumed that customers are sensitive to the return policy offered by vendor 1, in which a generous (tightened) return policy for the standard and refurbished items, $r_s = \eta_s p_s$ and $r_f = \eta_f p_f$ with $0 \leq \eta_s, \eta_f \leq 1$, increases (decreases) the demand, D_s and D_f. This sensitivity is moderated by the coefficients γ_s and γ_f that represent the elasticity of the two demand functions with respect to the return policy r_s and r_f. Since the two items (standard and refurbished) are in direct competition, there will be a migration factor of demand from one to the other depending on the relative price of the two items. Hence, the parameter ζ is a migration parameter, which represents the amount of demand lost by the standard item to the refurbished item. Note that $p_s - p_f$ is the price difference between the two items; this price difference is moderated by a loss of a value l for the refurbished item (Mukhopadhyay and Setaputra, 2006). Like many in the literature, this chapter assumes a linear demand function (Tsay and Agrawal, 2000; Mukhopadhyay and Setaputra, 2006; Li et al., 2013). Specifically, the demand functions for the standard and the refurbished items are similar in form to the demand functions proposed by Mukhopadhyay and Setaputra (2006) and are of the following forms:

$$D_s = a_s - \delta_s\, p_s + \gamma_s \eta_s p_s - \zeta\left(p_s - p_f - l\right) \tag{5.1}$$

$$D_f = a_f - \delta_f\, p_f + \gamma_f\, \eta_f p_f + \zeta\left(p_s - p_f - l\right) \tag{5.2}$$

where a_s and a_f represent the primary demand for the newly produced standard and the refurbished standard items, respectively. The primary demand depends on general economic factors (i.e., brand image, product quality) that are outside the scope of this thesis.

5.4.1.2 Vendor 1's profit function

The total profit of vendor 1 in per unit of time is given as:

$$\pi_{v_1} = TV_{v_1} - \left(PC_{v_1}^{s} + HC_{bs}^{v_1} + HC_{v_2 f}^{v_1} + HC_{bf}^{v_1} + HC_{bu}^{v_1} + C_{v_1}^{e} + C_W + TR\right) \tag{5.3}$$

where the equation above consists of the following components:

a) Vendor 1's total revenue

Under the single-channel strategy, vendor 1 has two revenues. The first revenue is generated from selling newly produced standard items to the retailer, which can be calculated in per unit of time by $\left(\frac{p_{v_1 s} n_s q_s}{T_s}\right)$. The second revenue is generated from selling refurbished standard items to the retailer, which is calculated in per unit of time by $\left(\frac{p_{v_1 f} n_f q_f}{T_f}\right)$. The total revenue of vendor 1 is given as:

$$TV_{v_1} = \frac{p_{v_1 s} n_s q_s}{T_s} + \frac{p_{v_1 f} n_f q_f}{T_f} = p_{v_1 s} D_s + p_{v_1 f}\, D_f \qquad (5.4)$$

where $T_s = \frac{n_s q_s}{D_s}$, $T_f = \frac{n_f q_f}{D_f}$ and $p_{v_1 s} > p_{v_1 f}$.

b) Vendor 1's production cost for the standard items

Vendor 1's production cost for the standard items is the sum of three costs: setup $\left(\frac{S_s}{T_s}\right)$, production $\left(c_s \frac{n_s q_s}{T_s}\right)$, and holding $\left(\frac{1}{T_s}\left(h_{v_1 s} \frac{n_s q_s^2}{2P_s}\right)\right)$. The holding cost was calculated by summing the areas under the curve of vendor's 1 stock in Figure 5.2. The total production cost in per unit of time is given as:

$$PC_{v_1}^{s} = \frac{S_s}{T_s} + c_s \frac{n_s q_s}{T_s} + \frac{1}{T_s}\left(h_{v_1 s} \frac{n_s q_s^2}{2P_s}\right) = \frac{S_s\, D_s}{n_s q_s} + c_s\, D_s + h_{v_1 s} \frac{q_s\, D_s}{2P_s} \qquad (5.5)$$

where $T_s = \frac{n_s q_s}{D_s}$.

c) <u>Vendor 1's financial holding cost for the standard items at the retailer's side</u>

In the CS policy, the vendor incurs the financial holding cost component for standard items stocked at the retailer's side (Jaber et al., 2014; Zanoni et al., 2014a). This holding cost is calculated by summing the areas under the curve of the retailer's stock (see Figure 5.2), and it is written in per unit of time as:

$$HC_{bs}^{v_1} = h_{bs}^{v_1}(n_s - 1)\,\frac{D_s q_P^2}{2T_s P_s^2} + h_{bs}^{v_1}\frac{n_s(n_s-1)q_s^2}{2T_s P_s}\left(1 - \frac{D_s}{P_s}\right)$$

$$+ \frac{h_{bs}^{v_1}}{2T_s}\left(\frac{n_s^2 q_s^2}{D_s} - \frac{n_s^2 D_s q_s^2}{P_s D_s} + \frac{n_s q_s^2 D_s}{P_s D_s} - \frac{n_s^2 q_s^2}{P_s} + \frac{n_s^2 D_s q_s^2}{P_s^2} - \frac{n_s q_s^2 D_s}{P_s^2}\right.$$

$$\left. + \frac{n_s q_s^2}{P_s} - \frac{n_s D_s q_s^2}{P_s^2} + \frac{D_s q_s^2}{P_s^2}\right)$$

$$= \frac{h_{bs}^{v_1}}{2}\left(n_s q_s - (n_s - 1)\,\frac{q_s D_s}{P_s}\right) \tag{5.6}$$

where $T_s = \frac{n_s q_s}{D_s}$.

d) <u>Vendor 1's financial holding cost for a refurbished item at vendor 2's side</u>

Vendor 1 also incurs the financial holding cost component for a refurbished item while it is at vendor's 2 side. This holding cost is calculating by summing the areas under the curve of vendor's 2 stock (see Figure 5.3), and it is given in per unit of time as follows:

$$HC_{v_2 f}^{v_1} = h_{v_2 f}^{v_1}\frac{q_f^2\, n_f\left(n_f - 1\right)}{2D_f T_f} = h_{v_2 f}^{v_1}\frac{q_f\left(n_f - 1\right)}{2} = h_{v_2 f}^{v_1}\frac{n_s q_s \beta_s \rho_s\left(n_f - 1\right)}{2n_f\left(1 - \beta_f \rho_f\right)} \tag{5.7}$$

where $T_f = \frac{n_f q_f}{D_f}$ and $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f(1-\beta_f \rho_f)}$.

e) Vendor 1's financial holding cost for the refurbished items at the retailer's side

After the returned repairable items are refurbished by vendor 2, they are shipped in equal batches to the retailer. During the time that the refurbished items are at the retailer's side, vendor 1 incurs the financial holding cost for these items. This holding cost is calculated by summing the areas under the curve of the retailer's stock in Figure 5.3, and it is given in per unit of time as follows:

$$HC_{bf}^{v_1} = h_{bf}^{v_1} \frac{q_f^2\, n_f}{2D_f T_f} = h_{bf}^{v_1} \frac{q_f}{2} = h_{bf}^{v_1} \frac{n_s q_s \beta_s \rho_s}{2n_f\left(1 - \beta_f \rho_f\right)} \tag{5.8}$$

where $T_f = \frac{n_f q_f}{D_f}$, $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f(1-\beta_f \rho_f)}$ and $h_{bf}^{v_1} > h_{v_2 f}^{v_1}$.

f) Vendor 1's financial holding cost for the returned repairable items at the retailer's side

Returned repairable items received by the retailer arrive at a rate of $\beta_s \rho_s D_s$ for the standard items and $\beta_f \rho_f D_f$ for the refurbished items. When the received repairable items are physically at the retailer's side, vendor 1 incurs the financial holding cost of these items until the retailer ships them to vendor's 2 facility for refurbishing. This holding cost is calculated by summing the areas under the curve of the retailer (see Figure 5.4) and is given as follows:

$$HC_{bu}^{v_1} = h_{bu}^{v_1} \frac{(\beta_s \rho_s D_s T_s)\, T_s}{2T_s} + h_{bu}^{v_1} \frac{\left(\beta_f \rho_f D_f T_{bf}\right) T_{bf}}{2T_f}$$

$$= h_{bu}^{v_1} \frac{\beta_s \rho_s n_s q_s}{2} + h_{bu}^{v_1} \frac{\beta_f\ \rho_f n_f q_f}{2}$$

$$= h_{bu}^{v_1} \frac{\beta_s \rho_s n_s q_s}{2} + h_{bu}^{v_1} \frac{\beta_f\ \rho_f\ \beta_s\ \rho_s n_s q_s}{2(1 - \beta_f\ \rho_f)} \tag{5.9}$$

where $T_s = \frac{n_s q_s}{D_s}$, $T_f = \frac{n_f q_f}{D_f}$ and $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f(1-\beta_f \rho_f)}$.

g) Vendor 1's cost for outsourcing the refurbishing processes

Returned repairable standard items $(\beta_s \rho_s D_s)$ and previously refurbished ones $\left(\beta_f \rho_f D_f\right)$ are refurbished at a cost c_e per item, which is paid as an outsourcing fee to vendor 2 for refurbishing these items. This cost could be agreed upon as a unit cost plus a markup percentage. The total cost for outsourcing the refurbishing processes in per unit of time is written as:

$$C_{v_1}^e = c_e\left(\beta_s \rho_s D_s + \beta_f \rho_f D_f\right) \tag{5.10}$$

h) Vendor 1's disposing (waste) cost for non-repairable items

Non-repairable standard items $(\alpha_s \rho_s D_s)$ and refurbished standard items $\left(\alpha_f \rho_f D_f\right)$ are disposed outside of the system at a cost of c_w. The total cost for disposing these items in per unit of time is given by:

$$C_W = c_w\left(\alpha_s\, \rho_s D_s + \alpha_f\, \rho_f D_f\right) \tag{5.11}$$

i) Vendor 1's return policy (refunded dollars) to customers

Through the return policy agreement provided by vendor 1, customers unsatisfied with their purchase can be refunded r_s for a returned standard item (where $r_s = \eta_s p_s$) and r_f for a returned refurbished item (where $r_f = \eta_f p_f$). The total refund of both items to customers in per unit of time is written as:

$$TR = r_s\, \rho_s D_s + r_f\, \rho_f D_f = \eta_s p_s \rho_s D_s + \eta_f p_f \rho_f D_f \tag{5.12}$$

5.4.1.3 Vendor 2's profit function (3PL provider)

The total profit of vendor 2 in per unit of time is given as:

$$\pi_{v_2} = TV_{v_2} - TC_{v_2}^{f} \tag{5.13}$$

where the equation above consists of the following components:

a) *Vendor 2's total revenue*

Vendor's 2 revenue is generated from the received outsourcing fee, c_e, for the refurbishing processes, which is paid by vendor 1 per returned repairable item. The total revenue of vendor 2 in per unit of time is similar to Eq. (5.10) and is given as

$$TV_{v_2} = c_e\left(\beta_s \rho_s D_s + \beta_f \rho_f D_f\right) \tag{5.14}$$

b) *Vendor 2's total refurbishing cost*

Vendor's 2 total cost for the refurbishing processes of the returned repairable items is the sum of three main costs: setup $\left(\frac{S_f}{T_f}\right)$, refurbishing $\left(c_f \frac{n_f q_f}{T_f}\right)$ and holding $\left(h_{v_2 f}^{v_2} \frac{q_f^2 n_f (n_f - 1)}{2 D_f T_f}\right)$. The total refurbishing cost in per unit of time is given as:

$$TC_{v_2}^{f} = \frac{S_f}{T_f} + c_f \frac{n_f q_f}{T_f} + \left(h_{v_2 f}^{v_2} \frac{q_f^2 n_f \left(n_f - 1\right)}{2 D_f T_f}\right)$$

$$= \frac{S_f\ D_f}{n_f\ q_f} + c_f\ D_f + h_{v_2 f}^{v_2} \frac{q_f(n_f - 1)}{2}$$

$$= \frac{S_f\ D_f(1 - \beta_f\ \rho_f)}{\beta_s\ \rho_s n_s q_s} + c_f\ D_f + h_{v_2 f}^{v_2} \frac{n_s q_s \beta_s \rho_s (n_f - 1)}{2 n_f (1 - \beta_f \rho_f)} \quad (5.15)$$

where $T_f = \frac{n_f q_f}{D_f}$ and $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f(1-\beta_f \rho_f)}$.

5.4.1.4 Retailer's profit function

The total profit of the retailer per unit of time is given as:

$$\pi_b = TV_b - \left(C_b^{G,O} + HC_{bs}^b + HC_{bf}^b + HC_{bu}^b\right) \quad (5.16)$$

where the equation above consists of the following components:

a) <u>The retailer's revenue</u>

In the single-channel strategy, the retailer generates two revenues: one is generated from selling newly produced standard items to customers, which is calculated in per unit of time by $\left(p_s \frac{n_s q_s}{T_s}\right)$, the second is generated from selling refurbished standard items to customers, which is calculated in per unit of time by $\left(p_f \frac{n_f q_f}{T_f}\right)$. The total revenue of the retailer is given as:

$$TV_b = p_s \frac{n_s q_s}{T_s} + p_f \frac{n_f q_f}{T_f} = D_s\ p_s + D_f\ p_f \quad (5.17)$$

where $T_s = \frac{n_s q_s}{D_s}$ and $T_f = \frac{n_f q_f}{D_f}$.

b) <u>The retailer's purchasing and ordering cost</u>

The retailer has two purchasing costs $\left(\frac{p_{v_1 s} n_s q_s}{T_s}\right)$ and $\left(\frac{p_{v_1 f} n_f q_f}{T_f}\right)$ for each standard and refurbished standard item. The retailer also incurs two ordering costs $\left(\frac{O_{bs}\, n_s}{T_s}\right)$ and $\left(\frac{O_{bf} n_f}{T_f}\right)$ for each standard and refurbished standard item. The sum of these costs is given in per unit of time in Eq. (5.18) as:

$$C_b^{G,O} = \frac{p_{v_1 s} n_s q_s}{T_s} + \frac{p_{v_1 f} n_f q_f}{T_f} + \frac{O_{bs}\, n_s}{T_s} + \frac{O_{bf} n_f}{T_f}$$

$$= p_{v_1 s}\, D_s + p_{v_2 f}\, D_f + \frac{O_{bs} D_s}{q_s} + \frac{O_{bf} D_f}{q_f}$$

$$= p_{v_1 s}\, D_s + p_{v_2 f}\, D_f + \frac{O_{bs} D_s}{q_s} + \frac{O_{bf} D_f n_f (1 - \beta_f\, \rho_f)}{\beta_s\, \rho_s n_s q_s} \tag{5.18}$$

where $T_s = \frac{n_s q_s}{D_s}$ and $T_f = \frac{n_f q_f}{D_f}$ and $q_f = \frac{n_s q_s \beta_s \rho_s}{n_f (1 - \beta_f \rho_f)}$.

c) <u>The retailer's physical storage holding cost for the standard items</u>

The physical storage holding cost of the retailer for a standard item is determined similarly to Eq. (5.6), where $h_{bs}^{v_1}$ is replaced by h_{bs}^{b}. This holding cost is given in per unit of time as:

$$HC_{bs}^{b} = \frac{h_{bs}^{b}}{2}\left(n_s q_s - (n_s - 1)\frac{q_s D_s}{P_s}\right) \tag{5.19}$$

d) <u>The retailer's physical storage holding cost for the refurbished items</u>

The physical storage holding cost of the retailer for a refurbished item is determined similarly to Eq. (5.8), where $h_{bf}^{v_1}$ is replaced by h_{bf}^{b}. This holding cost is given in per unit of time as:

$$HC_{bf}^{b} = h_{bf}^{b} \frac{n_s q_s \beta_s \rho_s}{2n_f(1 - \beta_f \rho_f)} \tag{5.20}$$

e) *<u>The retailer's physical storage holding cost for the returned repairable items</u>*

This holding cost is determined similarly to Eq. (5.9), where $h_{bu}^{v_1}$ is substituted by h_{bu}^{b}. The per unit of time of this holding cost is given as:

$$HC_{bu}^{b} = h_{bu}^{b} \frac{\beta_s \rho_s n_s q_s}{2} + h_{bu}^{b} \frac{\beta_f\ \rho_f\ \beta_s\ \rho_s n_s q_s}{2(1 - \beta_f\ \rho_f)} \tag{5.21}$$

5.4.1.5 Total profit of the supply chain system

The total profit of the supply chain system in the single-channel strategy is given as:

$$\Pi_{single} = \pi_{v_1} + \pi_{v_2} + \pi_b \tag{5.22}$$

5.4.1.6 Optimal decision in the single-channel strategy

Eq. (5.22) is a concave function in p_s, p_f, q_s, n_s and n_f. The proof of concavity is given in Appendix B.1. The optimal (indicated by an asterisk) solutions are given as follows:

$$p_s^* = \frac{FE}{(4AB - E^2)} + \frac{2BC}{(4AB - E^2)} \tag{5.23}$$

$$p_f^* = \frac{CE}{(4AB - E^2)} + \frac{2AF}{(4AB - E^2)} \tag{5.24}$$

where

$$A = \eta_s^2 \rho_s \gamma_s + \delta_s + \zeta - \eta_s \gamma_s - \eta_s \rho_s \delta_s - \eta_s \rho_s \zeta$$

$$B = \eta_f^2 \rho_f \gamma_f + \delta_f + \zeta - \eta_f \gamma_f - \eta_f \rho_f \delta_f - \eta_f \rho_f \zeta$$

$$E = 2\zeta - \eta_s \rho_s \zeta - \eta_f \rho_f \zeta$$

$$\begin{aligned} C = {} & a_s(1 - \eta_s \rho_s) + \zeta l(1 - \eta_s \rho_s) - c_s(\eta_s \gamma_s - \zeta - \delta_s) - c_f \zeta - c_w \alpha_s \rho_s (\eta_s \gamma_s - \zeta - \delta_s) \\ & - c_w \zeta \alpha_f \rho_f - \frac{(S_s + O_{bs} n_s)(\eta_s \gamma_s - \zeta - \delta_s)}{n_s q_s} - \frac{\zeta\left(S_f + O_{bf} n_f\right)\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} \\ & - \frac{\left(h_{v_1 s} - h_{bs} + h_{bs}\, n_s\right)(\eta_s \gamma_s - \zeta - \delta_s) q_s}{2P_s} \end{aligned}$$

$$\begin{aligned} F = {} & a_f\left(1 - \eta_f \rho_f\right) + \zeta l\left(1 - \eta_f \rho_f\right) - c_f\left(\eta_f \gamma_f - \zeta - \delta_f\right) - c_s \zeta - c_w \alpha_f \rho_f \left(\eta_f \gamma_f - \zeta - \delta_f\right) \\ & - c_w \zeta \alpha_s \rho_s - \frac{\left(S_f + O_{bf} n_f\right)\left(\eta_f \gamma_f - \zeta - \delta_f\right)\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} - \frac{\zeta(S_s + O_{bs} n_s)}{n_s q_s} \\ & - \frac{\left(h_{v_1 s} + h_{bs} - h_{bs}\, n_s\right)(\eta_s \gamma_s - \zeta - \delta_s) q_s \zeta}{2P_s} \end{aligned}$$

$$q_s^* = \sqrt{\frac{\frac{(S_s + O_{bs} n_s) D_s \beta_s \rho_s + \left(S_f + O_{bf} n_f\right)\left(1 - \beta_f \rho_f\right) D_f}{n_s \beta_s \rho_s}}{\frac{\left(h_{v_1 s} + h_{bs} - h_{bs}\, n_s\right) D_s}{2P_s} + \frac{\left(h_{v_2 f} n_f - h_{v_2 f} + h_{bf}\right) n_s \beta_s \rho_s}{n_f\left(1 - \beta_f \rho_f\right)} + \frac{h_{bu} n_s \beta_s \rho_s \beta_f \rho_f}{2\left(1 - \beta_f \rho_f\right)} + \frac{h_{bs} n_s}{2} + \frac{h_{bu} \beta_s \rho_s n_s}{2}}} \tag{5.25}$$

$$n_s^* = \frac{1}{q_s} \sqrt{\frac{\frac{S_s D_s \beta_s \rho_s + \left(S_f + O_{bf} n_f\right)\left(1 - \beta_f \rho_f\right) D_f}{\beta_s \rho_s}}{\frac{\beta_s \rho_s \left(h_{v_2 f} n_f - h_{v_2 f} + h_{bf}\right)}{2 n_f \left(1 - \beta_f \rho_f\right)} + \frac{h_{bs}}{2}\left(1 - \frac{D_s}{P_s}\right) + \frac{h_{bu} \beta_s \rho_s}{2} + \frac{h_{bu} \beta_s \rho_s \beta_f \rho_f}{2\left(1 - \beta_f \rho_f\right)}}} \tag{5.26}$$

$$n_f^* = \frac{1}{2}\frac{\sqrt{2}\sqrt{O_{bf}D_f\left(h_{bf} - h_{v_2 f}\right)}\,\beta_s \rho_s q_s n_s}{O_{bf}D_f\left(1 - \beta_f \rho_f\right)} \tag{5.27}$$

The optimal solutions of Eqs. (5.23)−(5.27) are not independent of each other. Therefore, the problem is solved using a solution procedure that is modeled in Microsoft Excel using the Solver Tool add-in and enhanced with Visual Basic Macro. This solution procedure is similar to the one found in Jaber and Goyal (2008). The initial procedure for solving this problem is obtained by setting $q_s = 1$, $n_s = 1$ and $n_f = 1$ and then finding the optimal values of p_s and p_f that maximize Π_{single}. Following that the solutions are determined for $n_s = n_s + 1$ and $n_f = n_f + 1$ where Π_{single} is compared against previous iterations and then repeated until the maximum Π_{single} is found for the first iteration. This iteration is repeated for $q_s > 1$ and until the maximum system's profit is found.

5.4.2 Dual-channel strategy

In this section, the single-channel strategy is modified by adding an online channel to form a dual-channel selling strategy. The objective is to reinvestigate managerial insight related to selling prices and inventory decisions when the dual-channel strategy is adopted. The modified forward and reverse flow of the material in the dual-channel strategy is demonstrated in Figure 5.5.

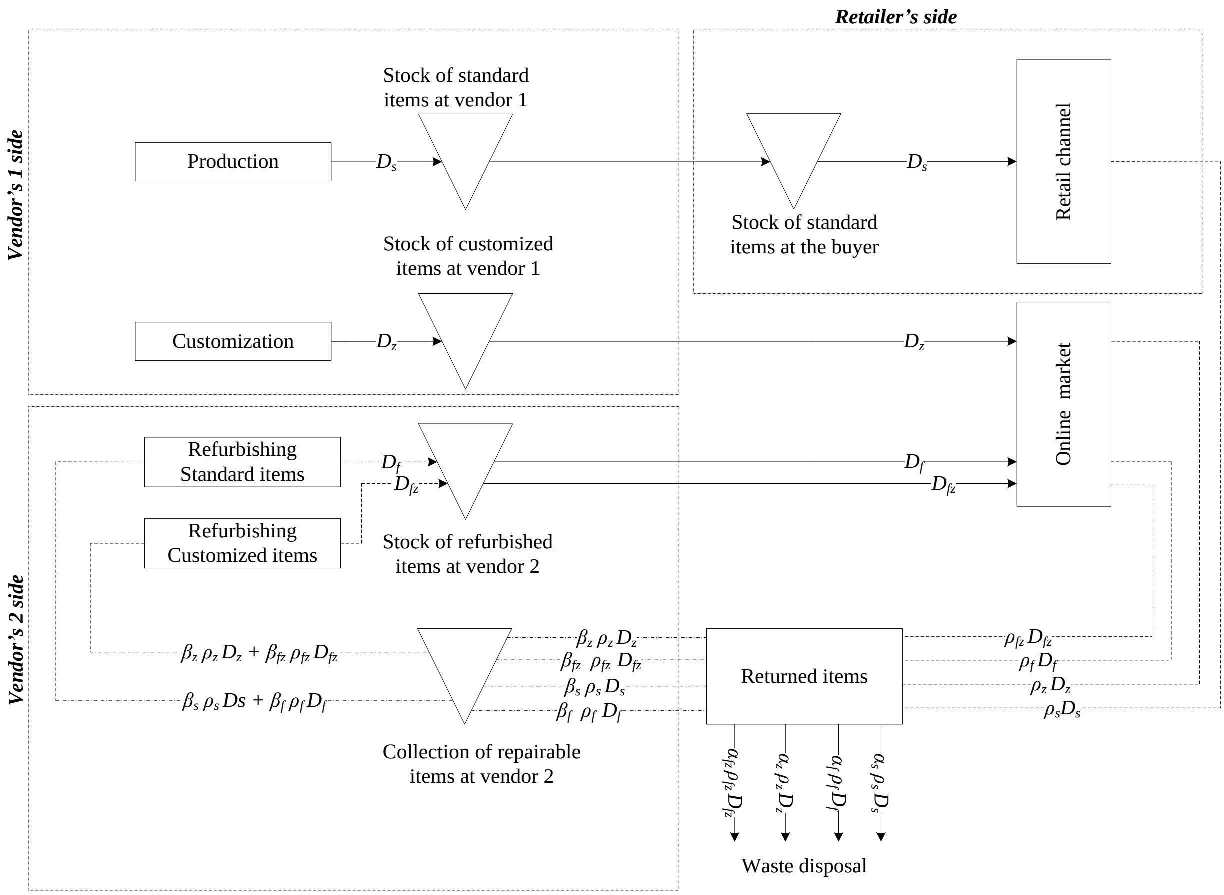

Figure 5.5 Forward and reverse material flow system for the dual-channel strategy

In the dual-channel strategy, vendor 1 produces two items: MTS standard items sold to costumers through the retail channel, and BTO customized items sold to customers directly through the online channel. The standard items follow the same inventory policy (CS policy) described in Section 5.4.1 (see Figure 5.2).

The retailer receives the returned standard items and then ships them in a single batch to the facility of vendor 2. The latter then refurbishes these items and offers them to customers directly through the online channel of vendor 1. In practice, the handling of both refurbishing and the distribution of returned items by a 3PL provider has shown to reduce transportation and other costs related to carbon footprint (Partridge, 2011; Pattinson, 2014). Therefore, it is assumed that vendor 2 in this strategy offers the refurbished standard items through vendor 1's online channel rather than through the retail channel. It is assumed that the refurbished standard items, offered through the online channel, has a similar return policy as the one applied to a newly produced standard item. A customer unsatisfied with the online purchase of a refurbished standard item, can send the item directly to vendor 2's facility through special instructions provided by vendor 1 (e.g. printing a return label addressed to vendor 2's facility and paid by vendor 1) (Mukhopadhyay and Setoputro, 2005; Aulkemeier et al., 2015). The inventory for the refurbished standard items at vendor 2's facility behaves similarly to the economic production quantity (EPQ) model (see Figure 5.6), in which vendor 2 refurbishes q_f unit at a production rate P_f at time T_f, where $T_f = q_f/D_f$ and the finished refurbished standard items consumed at a certain rate.

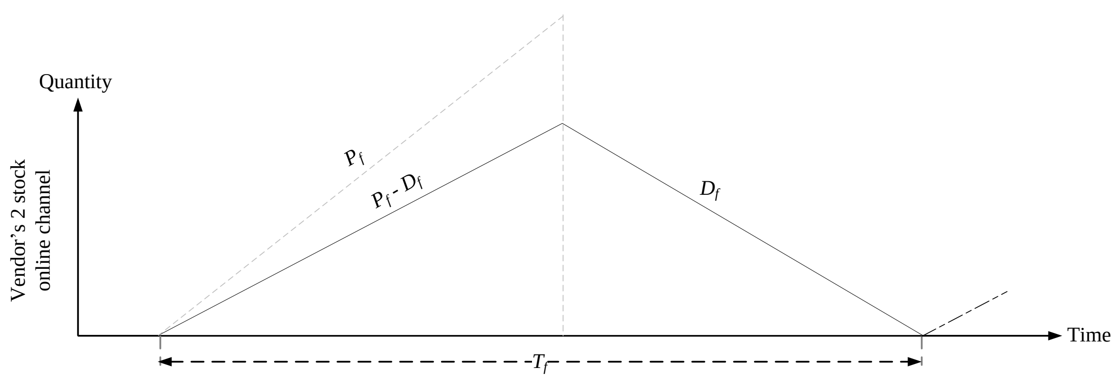

Figure 5.6 Inventory behavior at vendor's 2 side for the refurbished standard items

For the customized items, it is assumed that each customized item is built or prepared once the customer's order is received through the online channel. To do this, it is assumed that vendor 1 offers a set of custom features online that can be added to the core item. Due to the close relationship with suppliers, firms (e.g. Dell) that offer BTO customized items, usually carries zero inventory (no backlogging) of finished customized item (Gunasekaran and Ngai, 2009). However, in this chapter, it is assumed that vendor 1 carries the inventory of the core item that will be used in the eventual customization process. The additional custom features that can be added to the core item are outsourced and are supplied to vendor 1 once needed with zero lead-time; hence, we consider no inventory for the additional custom features. For the inventory of the core item at vendor 1's facility it is assumed that it behaves according to an EPQ model where the vendor produces q_z units of core items at time T_z, where $T_z = q_z/D_z$. The core items are consumed during the processing run for the customization orders. The behavior of the inventory of the core items is shown in Figure 5.7

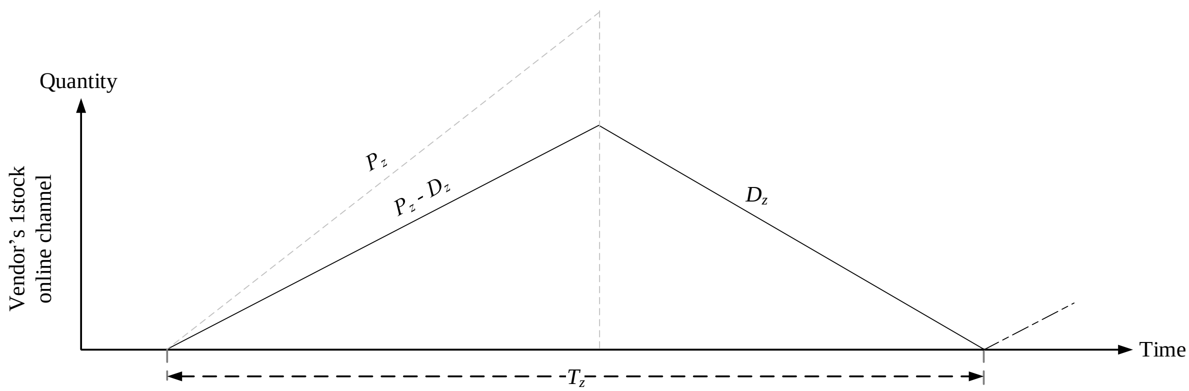

Figure 5.7 Inventory behavior at vendor's 1 side for the core customizable item

V_N is defined as the set of variant features, for each variant k ($k \in V_N$ and $k = 1,2, \dots, N$). A core item is customized by adding i (where $i = 1, 2, \dots, I$) features of the set V_N, each at a price. The production cost for a customized item is c_{zk} and its selling price is p_{zk}, and both vary from item to item (it depends on the number of features added to the core product + the core item + a processing fee) (Batarfi et al., 2016). To avoid triviality problems, it is assumed that the selling price of each customized item is always higher than the selling price of the standard item to the retailer. This is not an arbitrary condition, because if the selling price of the standard item to the retailer is higher than the selling price of the customized product to customers, then the retailer or any other arbitrator can obtain the customized item from vendor 1's direct channel at a lower price.

It is assumed that customized items purchased through the online channel can also be returned for a refund of r_{zk}. The customized items can be returned directly to vendor 2's facility in a similar manner to returning a refurbished standard item ordered online. Vendor 2 then refurbishes the customized items and reoffers them through vendor 1's online channel as a refurbished customized items at p_{fzk}which is agreed upon with vendor 1. The behavior of the inventory for the refurbished customized items at vendor 2's facility also behaves like an EPQ model (see Figure 5.8).

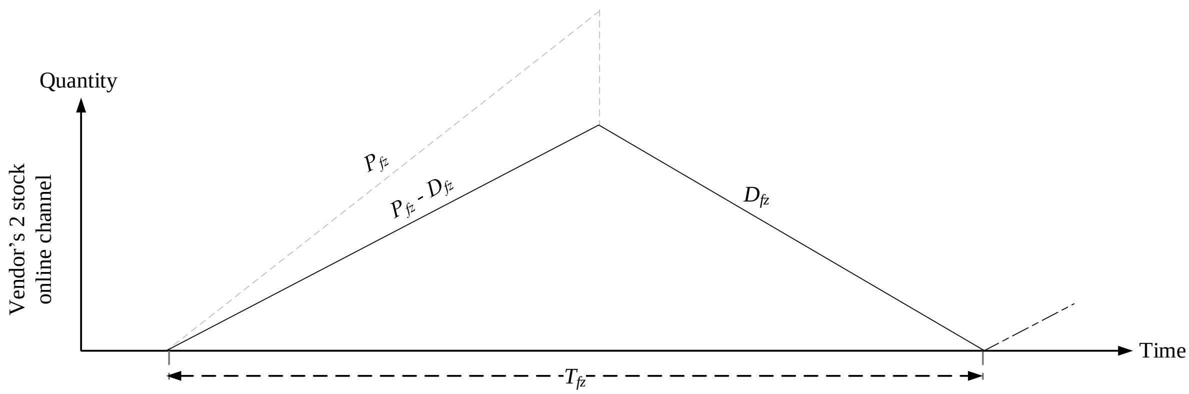

Figure 5.8 Inventory behavior at vendor's 2 side for the refurbished customized items

To refurbish q_f units, vendor 2 receives $\beta_s \rho_s D_s T_s$, where $T_s = n_s q_s / D_s$ of returned repairable standard items from the retail channel and $\beta_f \rho_f D_f T_f$, where $T_f = q_f / D_f$, of returned repairable refurbished standard items from the online channel. Accordingly we have $q_f = \beta_s \rho_s n_s q_s + \beta_f \rho_f q_f$ and hence, $q_f = \frac{n_s q_s \beta_s \rho_s}{(1-\beta_f \rho_f)}$. Both repairable inventories are replenished instantaneously and consumed at a rate P_f. Figure 5.9 shows the behavior of the returned repairable inventory of the standard and refurbished standard items.

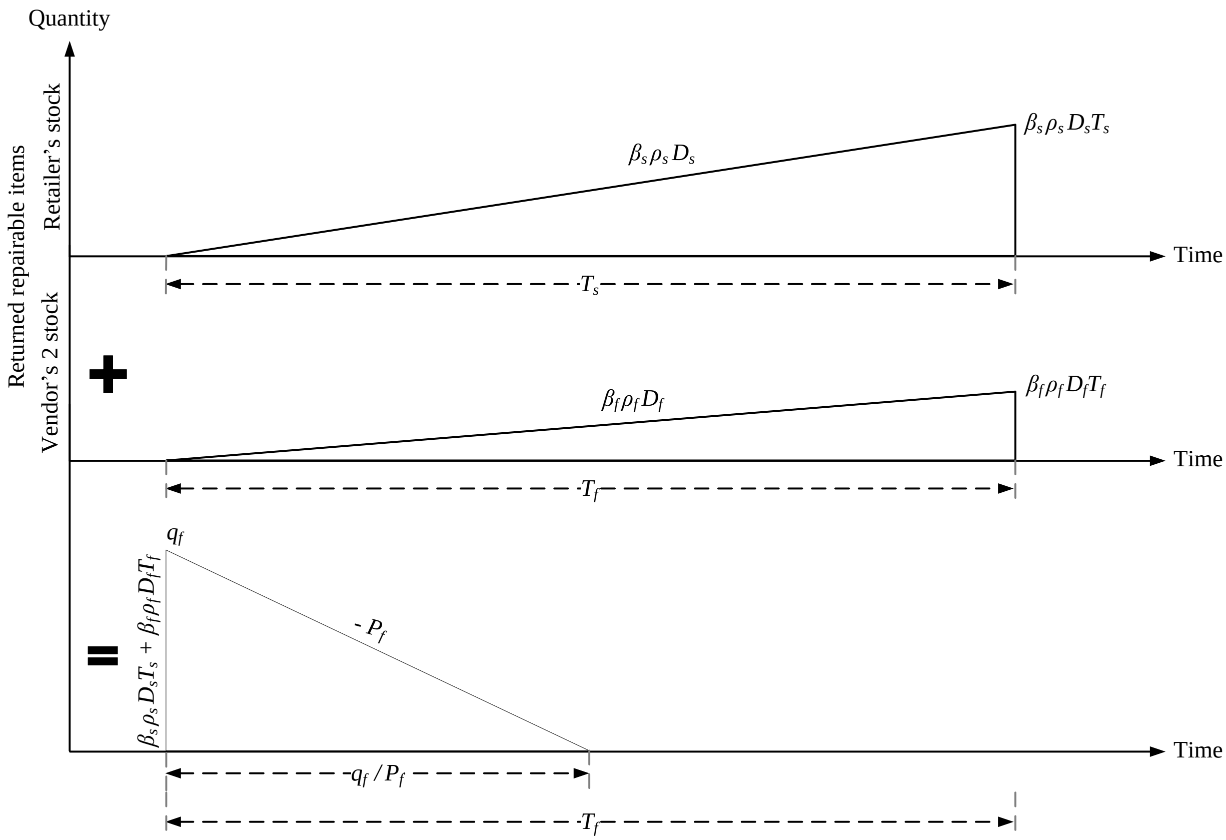

Figure 5.9 Inventory behavior for returned repairable standard and refurbished standard items

Similarly, to refurbish q_{fz} units, vendor 2 receives $\beta_z \rho_z D_z T_Z$, where $T_z = q_z / D_z$ of returned repairable customized items and $\beta_{fz} \rho_{fz} D_{fz} T_{fz}$, where $T_{fz} = q_{fz} / D_{fz}$ of returned repairable refurbished customized items from which we have $q_{fz} = \beta_z \rho_z q_z + \beta_{fz} \rho_{fz} q_{fz}$. Therefore, $q_{fz} = \frac{q_z \beta_z \rho_z}{(1 - \beta_{fz} \rho_{fz})}$. The returned inventories are replenished instantaneously and consumed at a rate of P_{fz} (see Figure 5.10).

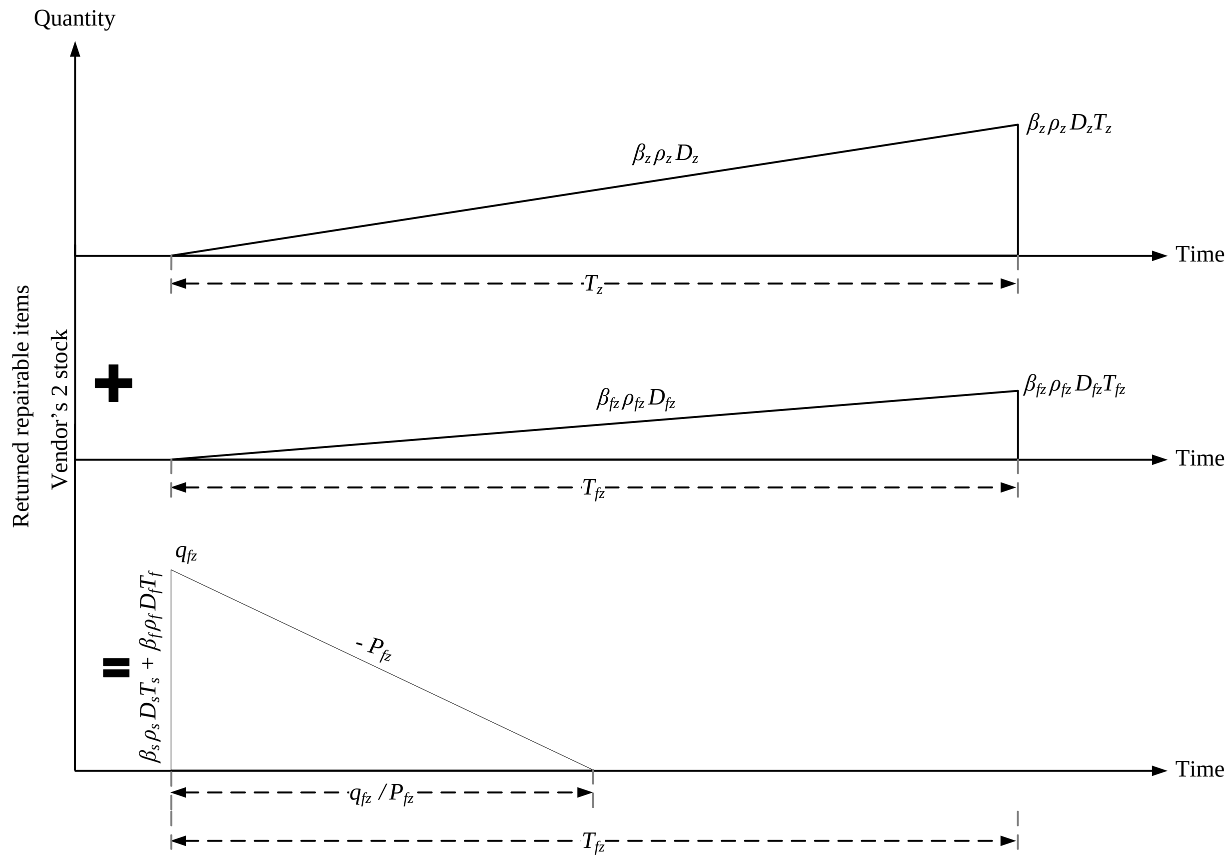

Figure 5.10 Inventory behavior for returned repairable customized and refurbished customized items

5.4.2.1 Demand functions

In the dual-channel strategy, there are four demands functions: demand for the standard items, D_s, demand for the refurbished standard items, D_f, demand for the customized items, D_z, and a demand for the refurbished customized items, D_{fz}. These demand functions are developed in a similar manner to what has been done in Section 5.4.1.1. The descriptions of the terms in these functions are the same as those in the noted Section. The four demand functions are given as:

$$D_s = a_s - \delta_s \, p_s + \gamma_s \, \eta_s p_s - \zeta\left(p_s - p_f - l\right) \tag{5.28}$$

$$D_f = a_f - \delta_f \, p_f + \gamma_f \, \eta_f p_f + \zeta(p_s - p_f - l) \tag{5.29}$$

$$D_z = a_z - \delta_z \sum_{k=1}^{N} p_{zk} + \gamma_z \, \eta_z \sum_{k=1}^{N} p_{zk} - \zeta \sum_{k=1}^{N} (p_{zk} - p_{fzk} - l) \tag{5.30}$$

$$D_{fz} = a_{fz} - \delta_{fz} \sum_{k=1}^{N} p_{fzk} + \gamma_{fz} \, \eta_{fz} \sum_{k=1}^{N} p_{fzk} + \zeta \sum_{k=1}^{N} (p_{zk} - p_{fzk} - l) \tag{5.31}$$

where a_s, a_f, a_z and a_{fz} represent the primary demand rates for the standard, refurbished standard, customized and refurbished customized items, respectively. δ_s, δ_f, δ_z and δ_{fz} are the coefficients of the price elasticity of p_s, p_f, p_{zk} and p_{fzk}, respectively. The coefficients γ_s, γ_f, γ_z and γ_{fz} are the elasticity of the demand rates D_s, D_f, D_z and D_{fz}, respectively in regards to the return policy. The parameter η_s, η_f, η_z and η_{fz} are the returned proportion of the selling price p_s, p_f, p_{zk} and p_{fzk}, respectively to customers unsatisfied with their purchase. ζ is a migration parameter, which represents the amount of demand lost by the newly produced item (standard or customized) item to the refurbished item (refurbished standard or refurbished customized). For simplicity, it is assumed that the migration parameter in all the demand functions is the same. The parameter l represents the lost-in value of the refurbished item to the newly produced item.

5.4.2.2 Vendor 1's profit function

The total profit of vendor 1 per unit of time is given as:

$$\begin{aligned} \pi_{v_1} = TV_{v1} \, - \big(& PC_{v_1}^{s} + PC_{v_1}^{z} + HC_{bs}^{v_1} + HC_{v_2 f}^{v_1} + HC_{v_2 fz}^{v_1} + HC_{v_2 u,s,f}^{v_1} + HC_{v_2 u,z,fz}^{v_1} \\ & + C_{v_1}^{e} + C_W + TR \, \big) \end{aligned} \tag{5.32}$$

where the equation above consists of the following components:

a) <u>Vendor 1's total revenue</u>

In the dual-channel strategy, vendor 1 generates four revenues. The first revenue is generated from selling newly produced standard items to the retailer, which is calculated in per unit of time by $\left(\frac{p_{v_1 s} n_s q_s}{T_s}\right)$. The second revenue is generated from the online selling of refurbished standard items, which is calculated in per unit of time by $\left(\frac{p_f q_f}{T_f}\right)$. The third revenue is generated from the online selling of newly produced customized items, which is calculated in per unit of time by $\left(\frac{q_z}{T_z} \sum_{k=1}^{N} p_{zk}\, \varphi_{zk}\right)$, where φ_{zk} represents the percentage of the core item used to customized item k. The last revenue is generated from the online selling of refurbished customized items, which is calculated in per unit of time by $\left(\frac{q_{fz}}{T_{fz}} \sum_{k=1}^{N} p_{fzk} \varphi_{fzk}\right)$. The total revenue of vendor 1 is given as:

$$TV_{v1} = \frac{p_{v_1 s} n_s q_s}{T_s} + \frac{p_f q_f}{T_f} + \frac{q_z}{T_z} \sum_{k=1}^{N} p_{zk}\, \varphi_{zk} + \frac{q_{fz}}{T_{fz}} \sum_{k=1}^{N} p_{fzk} \varphi_{fzk}$$

$$= p_{v_1 s}\, D_s + p_f\, D_f + D_z \sum_{k=1}^{N} p_{zk} \varphi_{zk} + D_{fz} \sum_{k=1}^{N} p_{fzk} \varphi_{fzk} \tag{5.33}$$

where $T_s = \frac{n_s q_s}{D_P}$, $T_z = \frac{q_z}{D_z}$, $T_f = \frac{q_f}{D_f}$ and $T_{fz} = \frac{q_{fz}}{D_{fz}}$

b) <u>Vendor 1's production cost for the standard items</u>

Vendor's 1 producing cost for the standard items in the dual-channel strategy is the same as in Eq. (5.5) and is given by:

$$PC_{v_1}^{s} = \frac{S_s\, D_s}{n_s q_s} + c_s\, D_s + h_{v_1 s}\frac{q_s\, D_s}{2P_s} \quad (5.34)$$

where $T_s = \frac{n_s q_s}{D_P}$

c) Vendor 1's production cost for the core customizable items

Vendor's 1 producing cost for the core customizable items is the sum of three costs: setup $\left(\frac{S_z}{T_z}\right)$, production $\left(\frac{q_z}{T_z}\sum_{k=1}^{N} c_{zk}\varphi_{zk}\right)$ and holding $\left(\frac{h_{v_1 z}\, q_z^2}{2D_z T_z}\left(1-\frac{D_z}{P_z}\right)\right)$. The total production cost in per unit of time is given as:

$$PC_{v_1}^{z} = \frac{S_z}{T_z} + \frac{q_z}{T_z}\sum_{k=1}^{N} c_{zk}\varphi_{zk} + \frac{h_{v_1 z}\, q_z^2}{2D_z T_z}\left(1-\frac{D_z}{P_z}\right)$$

$$= \frac{S_z D_z}{q_z} + D_z\sum_{k=1}^{N} c_{zk}\varphi_{zk} + h_{v_1 z}\left(\frac{q_z}{2}\right)\left(1-\frac{D_z}{P_z}\right) \quad (5.35)$$

where $T_z = \frac{q_z}{D_z}$

d) Vendor 1's financial holding cost for the standard items at the retailer's side

The per unit of time of this cost is the same as in Eq. (5.6)

$$HC_{bs}^{v_1} = \frac{h_{bs}^{v_1}}{2}\left(n_s q_s - (n_s - 1)\frac{q_s D_s}{P_s}\right) \quad (5.36)$$

e) Vendor 1's financial holding cost for refurbished items at vendor 2's side is

Vendor 1 also incurs the financial holding costs for the refurbished standard items and the refurbished customized items while these items at vendor's 2 stock are determined in a similar manner to that of the EPQ. The sum of the holding costs per unit of time for the standard and refurbished items is given as:

$$HC_{v_2 f}^{v_1} + HC_{v_2 fz}^{v_1} = \frac{h_{v_2 f}^{v_1}}{T_f}\left(\frac{q_f^2}{2D_f}\right)\left(1-\frac{D_f}{P_f}\right) + \frac{h_{v_2 fz}^{v_1}}{T_{fz}}\left(\frac{q_{fz}^2}{2D_{fz}}\right)\left(1-\frac{D_{fz}}{P_{fz}}\right)$$

$$= h_{v_2 f}^{v_1}\left(\frac{q_f}{2}\right)\left(1-\frac{D_f}{P_f}\right) + h_{v_2 fz}^{v_1}\left(\frac{q_{fz}}{2}\right)\left(1-\frac{D_{fz}}{P_{fz}}\right)$$

$$= h_{v_2 f}^{v_1}\left(\frac{n_s q_s \beta_s \rho_s}{2\left(1-\beta_f \rho_f\right)}\right)\left(1-\frac{D_f}{P_f}\right) + h_{v_2 fz}^{v_1}\left(\frac{q_z \beta_z \rho_z}{2\left(1-\beta_{fz} \rho_{fz}\right)}\right)\left(1-\frac{D_{fz}}{P_{fz}}\right) \qquad (5.37)$$

where $T_f = \frac{q_f}{D_f}$, $T_{fz} = \frac{q_{fz}}{D_{fz}}$, $q_f = \frac{n_s q_s \beta_s \rho_s}{(1-\beta_f \rho_f)}$ and $q_{fz} = \frac{q_z \beta_z \rho_z}{(1-\beta_{fz} \rho_{fz})}$.

f) *Vendor 1's financial holding cost for returned repairable standard and refurbished standard items*

As demonstrated in Figure 5.9, returned repairable standard items arrive at the retailer at a rate of $\beta_s \rho_s D_s$ over T_s and are then shipped to Vendor's 2 facility for refurbishing. In contrast, returned refurbished standard items arrive directly to vendor's 2 facility at a rate of $\beta_f \rho_f D_f$ over T_f. During the time that these items are physically at the retailer's, or vendor's 2 side, vendor 1 carries the financial holding costs of these items. These holding costs are calculated by summing the areas under the curves of Figure 5.9, and is written in per unit of time as:

$$HC^{v_1}_{v_2u,s,f} = h^{v_1}_{bu}\frac{(\beta_s\ \rho_s D_s T_s)\ T_s}{2T_s} + h^{v_1}_{v_2u}\frac{\left(\beta_f\ \rho_f D_f T_f\right)\ T_f}{2T_f} + h^{v_1}_{v_2u}\frac{q_f^2}{2P_f T_f}$$

$$= h^{v_1}_{bu}\frac{\beta_s \rho_s n_s q_s}{2} + h^{v_1}_{v_2u}\frac{\beta_f\ \rho_f q_f}{2} + h^{v_1}_{v_2u}\frac{q_f D_f}{2P_f}$$

$$= h^{v_1}_{bu}\frac{\beta_s \rho_s n_s q_s}{2} + h^{v_1}_{v_2u}\frac{\beta_f\ \rho_f \beta_s \rho_s n_s q_s}{2(1-\beta_f \rho_f)} + h^{v_1}_{v_2u}\frac{\beta_s \rho_s n_s q_s D_f}{2P_f\left(1-\beta_f \rho_f\right)} \tag{5.38}$$

where $T_s = \frac{n_s q_s}{D_s}$, $T_f = \frac{q_f}{D_f}$, $q_f = \frac{n_s q_s \beta_s \rho_s}{(1-\beta_f \rho_f)}$ and $q_{fz} = \frac{q_z \beta_z \rho_z}{(1-\beta_{fz}\rho_{fz})}$. The term $\left(h^{v_1}_{v_2u}\frac{\beta_s \rho_s n_s q_s D_f}{2P_f(1-\beta_f \rho_f)}\right)$ is the accumulated returned repairable inventories of both items at vendor 2's side, which are now set for the refurbishing process.

g) *Vendor 1's financial holding cost for returned repairable customized and refurbished customized items*

As demonstrated in Figure 5.10, returned repairable customized and refurnished customized items, arrive directly to vendor's 2 facility at a rate of $\beta_z \rho_z D_z$ over T_z and a rate of $\beta_{fz}\rho_{fz}D_{fz}$ over T_{fz}, respectively. During the time that these items are physically at vendor's 2 side, vendor 1 carries the financial holding costs of these items. These holding costs are calculated by summing the areas under the curves of Figure 5.10, and is written in per unit of time as:

$$HC^{v_1}_{v_2u,z,fz} = h^{v_1}_{v_2u}\frac{(\beta_z\ \rho_z D_z T_z)\ T_z}{2T_z} + h^{v_1}_{v_2u}\frac{\left(\beta_{fz}\ \rho_{fz} D_{fz} T_{fz}\right)\ T_{fz}}{2T_{fz}} + h^{v_1}_{v_2u}\frac{q_{fz}^2}{2P_{fz} T_{fz}}$$

$$= h^{v_1}_{v_2u}\frac{\beta_z\ \rho_z q_z}{2} + h^{v_1}_{v_2u}\frac{\beta_{fz}\ \rho_{fz} q_{fz}}{2} + h^{v_1}_{v_2u}\frac{q_{fz} D_{fz}}{2P_{fz}}$$

$$= h_{v_2u}^{v_1}\frac{\beta_z\,\rho_z q_z}{2} + h_{v_2u}^{v_1}\frac{\beta_{fz}\,\rho_{fz}\beta_z\rho_z q_z}{2\left(1-\beta_{fz}\rho_{fz}\right)} + h_{v_2u}^{v_1}\frac{\beta_z\rho_z q_z D_{fz}}{2P_{fz}\left(1-\beta_{fz}\rho_{fz}\right)} \tag{5.39}$$

where $T_z = \frac{q_z}{D_z}$, $T_{fz} = \frac{q_{fz}}{D_{fz}}$, $q_f = \frac{n_s q_s \beta_s \rho_s}{(1-\beta_f\rho_f)}$ and $q_{fz} = \frac{q_z\beta_z\rho_z}{(1-\beta_{fz}\rho_{fz})}$. The term $\left(h_{v_2u}^{v_1}\frac{\beta_z\rho_z q_z D_{fz}}{2P_{fz}(1-\beta_{fz}\rho_{fz})}\right)$ is the accumulated returned repairable inventories of both items at vendor 2's side, which are now set for the refurbishing process.

h) *<u>Vendor 1's cost for outsourcing the refurbishing processes</u>*

This cost is calculated similarly to Eq. (5.10). However, returned repairable customized items $(\beta_z\rho_z D_z)$ and returned repairable refurbished customized items $\left(\beta_{fz}\rho_{fz}D_{fz}\right)$ are included here. The total outsourcing fee in per unit of time is given as:

$$C_{v_1}^{e} = c_e\left(\beta_s\rho_s D_s + \beta_f\rho_f D_f + \beta_z\rho_z D_z + \beta_{fz}\rho_{fz}D_{fz}\right) \tag{5.40}$$

i) *<u>Vendor 1's disposing (waste) cost for non-repairable items</u>*

This cost is calculated similarly to Eq. (5.11). However, non-repairable customized items $(\alpha_z\,\rho_z D_z)$ and non-repairable refurbished customized items $\left(\alpha_{fz}\,\rho_{fz}D_{fz}\right)$ are included here. The total outsourcing fee in per unit of time is given as:

$$C_W = c_w\left(\alpha_s\,\rho_s D_s + \alpha_f\,\rho_f D_f + \alpha_z\,\rho_z D_z + \alpha_{fz}\,\rho_{fz}D_{fz}\right) \tag{5.41}$$

j) *<u>Vendor 1's return policy (refunded dollars) to customers</u>*

Through the return policy agreement provided by vendor 1, customers unsatisfied with their purchase can be refunded r_s for a returned standard item (where $r_s = \eta_s p_s$), r_f for a returned standard refurbished item (where $r_f = \eta_f p_f$), r_z for a returned customized item (where $r_z =$

$\eta_z \sum_{k=1}^{N} p_{zk}\varphi_{zk}$) and r_{fz} for a returned refurbished customized item (where $r_{fz} = \eta_z \sum_{k=1}^{N} p_{fzk}\varphi_{fzk}$). The total refund in per unit of time is written as:

$$R = r_s\, \rho_s D_s + r_f\, \rho_f D_f + \rho_z D_z \sum_{k=1}^{N} r_{zk}\varphi_{zk} + \rho_{fz} D_{fz} \sum_{k=1}^{N} r_{fzk}\varphi_{fzk}$$

$$= \eta_s p_s \rho_s D_s + \eta_f p_f \rho_f D_f + \eta_z \rho_z D_z \sum_{k=1}^{N} p_{zk}\varphi_{zk} + \eta_{fz}\rho_{fz} D_{fz} \sum_{k=1}^{N} p_{fzk}\varphi_{fzk} \tag{5.42}$$

5.4.2.3 Vendor 2's profit function (3PL provider)

The total profit of vendor 2 per unit of time is given as

$$\pi_{v_2} = TV_{v_2} - \left(PC_{v_2}^{f} + PC_{v_2}^{fz} + HC_{v_2 u}^{v_2}\right) \tag{5.43}$$

where the equation above consists of the following components:

a) <u>Vendor 2's total revenue</u>

Vendor's 2 revenue is generated from the received outsourcing fee, c_e, for the refurbishing processes, which is paid by vendor 1 per returned repairable item. The total revenue of vendor 2 in per unit of time is written similarly to Eq. (5.40) and is given as:

$$TV_{v_2} = c_e\left(\beta_s \rho_s D_s + \beta_f \rho_f D_f + \beta_z \rho_z D_z + \beta_{fz} \rho_{fz} D_{fz}\right) \tag{5.44}$$

b) <u>Vendor 2's refurbishing cost for the standard and the refurbished standard items</u>

Vendor's 2 total cost for the refurbishing processes of the returned repairable standard and refurbished standard items is the sum of three costs setup $\left(\frac{S_f}{T_f}\right)$, refurbishing $\left(c_f\frac{q_f}{T_f}\right)$ and the holding $\left(\frac{h_{v_2f}^{v_2}q_f^2}{2D_fT_f}\left(1-\frac{D_f}{P_f}\right)\right)$. The total refurbishing cost in per unit of time is given as:

$$PC_{v_2}^f=\frac{S_f}{T_f}+c_f\frac{q_f}{T_f}+\frac{h_{v_2f}^{v_2}q_f^2}{2D_fT_f}\left(1-\frac{D_f}{P_f}\right)$$

$$=\frac{S_f\,D_f}{q_f}+c_f\,D_f+h_{v_2f}^{v_2}\left(\frac{q_f}{2}\right)\left(1-\frac{D_f}{P_f}\right)$$

$$=\frac{S_f\,D_f(1-\beta_f\rho_f)}{n_sq_s\beta_s\rho_s}+c_f\,D_f+h_{v_2f}^{v_2}\left(\frac{n_sq_s\beta_s\rho_s}{2(1-\beta_f\rho_f)}\right)\left(1-\frac{D_f}{P_f}\right) \qquad (5.45)$$

where $T_f=\frac{q_f}{D_f}$ and $q_f=\frac{n_sq_s\beta_s\rho_s}{(1-\beta_f\rho_f)}$.

c) <u>Vendor 2's refurbishing cost for the customized and the refurbished customized items</u>

Similar to the above equation, vendor's 2 total cost for the refurbishing processes of the returned repairable customized and refurbished customized items is computed by the sum of three costs setup $\left(\frac{S_{fz}}{T_{fz}}\right)$, refurbishing $\left(\frac{q_{fz}}{T_{fz}}\sum_{k=1}^{N}c_{fzk}\varphi_{fzk}\right)$ and holding $\left(\frac{h_{v_2fz}^{v_2}q_{fz}^2}{2D_{fz}T_{fz}}\left(1-\frac{D_{fz}}{P_{fz}}\right)\right)$. The total refurbishing cost in per unit of time is given as:

$$PC_{v_2}^{fz}=\frac{S_{fz}}{T_{fz}}+\frac{q_{fz}}{T_{fz}}\sum_{k=1}^{N}c_{fzk}\varphi_{fzk}+\frac{h_{v_2fz}^{v_2}q_{fz}^2}{2D_{fz}T_{fz}}\left(1-\frac{D_{fz}}{P_{fz}}\right)$$

$$= \frac{S_{fz}\, D_{fz}}{q_{fz}} + D_{fz} \sum_{k=1}^{N} c_{fzk}\varphi_{fzk} + h_{v_2 fz}^{v_2}\left(\frac{q_{fz}}{2}\right)\left(1 - \frac{D_{fz}}{P_{fz}}\right)$$

$$= \frac{S_{fz}\, D_{fz}\left(1 - \beta_{fz}\rho_{fz}\right)}{q_z\beta_z\rho_z} + D_{fz} \sum_{k=1}^{N} c_{fzk}\varphi_{fzk} + h_{v_2 fz}^{v_2}\left(\frac{q_z\beta_z\rho_z}{2\left(1 - \beta_{fz}\rho_{fz}\right)}\right)\left(1 - \frac{D_{fz}}{P_{fz}}\right) \quad (5.46)$$

where $T_{fz} = \frac{q_{fz}}{D_{fz}}$ and $q_{fz} = \frac{q_z\beta_z\rho_z}{(1-\beta_{fz}\rho_{fz})}$.

d) *Vendor 2's physical storage holding cost for returned repairable items*

Vendor 2 pays the physical storage holding costs of all received returned repairable items. These holding costs are calculated by the sum of Eqs. (5.38) and (5.39), except that $h_{v_2 u}^{v_1}$ in Eqs. (5.38) and (5.39) is replaced by $h_{v_2 u}^{v_2}$ in Eq. (5.47), and is given as follows:

$$HC_{v_2 u}^{v_2} = h_{v_2 u}^{v_2}\frac{\beta_f\, \rho_f\beta_s\rho_s n_s q_s}{2(1 - \beta_f\rho_f)} + h_{v_2 u}^{v_2}\frac{\beta_s\rho_s n_s q_s D_f}{2P_f\left(1 - \beta_f\rho_f\right)} + h_{v_2 u}^{v_2}\frac{\beta_z\, \rho_z q_z}{2} + h_{v_2 u}^{v_2}\frac{\beta_{fz}\, \rho_{fz}\beta_z\rho_z q_z}{2\left(1 - \beta_{fz}\rho_{fz}\right)} + h_{v_2 u}^{v_2}\frac{\beta_z\rho_z q_z D_{fz}}{2P_{fz}\left(1 - \beta_{fz}\rho_{fz}\right)} \quad (5.47)$$

5.4.2.4 Retailer's profit function

The total profit of the retailer per unit of time is given as:

$$\pi_b = TV_b - TC_b \quad (5.48)$$

where the equation above consists of the following components:

a) <u>The retailer's total revenue</u>

In the dual-channel strategy, the retailer has only one revenue, which is generated from selling newly produced standard items to customers $\left(p_s \frac{n_s q_s}{T_s}\right)$. The total revenue in per unit of time is written as:

$$TV_b = \frac{p_s n_s q_s}{T_s} = p_s D_S \tag{5.49}$$

b) <u>The retailer's total cost</u>

The total cost of the retailer is comprised of the following: purchasing cost $\left(p_{v_1 s}\, D_s\right)$, ordering cost $\left(\frac{O_{bs} D_s}{q_s}\right)$, the physical storage holding cost for a new standard item at the retailer side $\left(\frac{h_{bs}^b}{2}\left(n_s q_s - (n_s - 1)\frac{q_s D_s}{P_s}\right)\right)$, and the physical storage holding cost for a returned repairable item at the retailer's side $\left(h_{bu}^b \frac{\beta_s \rho_s n_s q_s}{2}\right)$. The total cost in unit per of time is written as:

$$TC_b = p_{v_1 s}\, D_s + \frac{O_{bs} D_s}{q_s} + \frac{h_{bs}^b}{2}\left(n_s q_s - (n_s - 1)\frac{q_s D_s}{P_s}\right) + h_{bu}^b \frac{\beta_s \rho_s n_s q_s}{2} \tag{5.50}$$

5.4.2.5 Total profit of the supply chain system

The total profit of the supply chain system in the dual-channel strategy is given as:

$$\Pi_{dual} = \pi_{v_1} + \pi_{v_2} + \pi_b \tag{5.51}$$

5.4.2.6 Optimal decision in the dual-channel strategy

Eq. (5.51) is a concave function in p_s, p_f, p_{zk}, p_{fzk} q_s, q_z, and n_s. The proof of concavity is shown in Appendix B.2. The optimal (indicated by an asterisk) solutions are given as follows:

$$p_s^* = \frac{F'E'}{(4A'B' - E'^2)} + \frac{2B'C'}{(4A'B' - E'^2)} \tag{5.52}$$

$$p_f^* = \frac{C'E'}{(4A'B' - E'^2)} + \frac{2A'F'}{(4A'B' - E'^2)} \tag{5.53}$$

where

$$A' = \eta_s^2 \rho_s \gamma_s + \delta_s + \zeta - \eta_s \gamma_s - \eta_s \rho_s \delta_s - \eta_s \rho_s \zeta$$

$$B' = \eta_f^2 \rho_f \gamma_f + \delta_f + \zeta - \eta_f \gamma_f - \eta_f \rho_f \delta_f - \eta_f \rho_f \zeta$$

$$E = 2\zeta - \eta_s \rho_s \zeta - \eta_f \rho_f \zeta$$

$$\begin{aligned} C' = {} & a_s(1 - \eta_s \rho_s) + \zeta l(1 - \eta_s \rho_s) - c_s(\eta_s \gamma_s - \zeta - \delta_s) - c_f \zeta - c_w \alpha_s \rho_s (\eta_s \gamma_s - \zeta - \delta_s) \\ & - c_w \zeta \alpha_f \rho_f - \frac{(S_s + O_{bs} n_s)(\eta_s \gamma_s - \zeta - \delta_s)}{n_s q_s} - \frac{\zeta S_f \left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} \\ & - \frac{\left(h_{v_1 s} - h_{bs} + h_{bs}\, n_s\right)(\eta_s \gamma_s - \zeta - \delta_s) q_s}{2P_s} + \frac{n_s q_s \beta_s \rho_s \zeta \left(h_{v_2 f} - h_{v2u}\right)}{2P_f \left(1 - \beta_f \rho_f\right)} \end{aligned}$$

$$F' = a_f(1-\eta_f\rho_f) + \zeta l(1-\eta_f\rho_f) - c_f(\eta_f\gamma_f - \zeta - \delta_f) - c_s\zeta - c_w\alpha_f\rho_f(\eta_f\gamma_f - \zeta - \delta_f)$$

$$- c_w\zeta\alpha_s\rho_s - \frac{S_f(\eta_f\gamma_f - \zeta - \delta_f)(1-\beta_f\rho_f)}{n_s q_s \beta_s \rho_s} - \frac{\zeta(S_s + O_{bs}n_s)}{n_s q_s}$$

$$- \frac{(h_{v_1 s} + h_{bs} - h_{bs}\, n_s)q_s\zeta}{2P_s} + \frac{n_s q_s \beta_s \rho_s(\eta_f\gamma_f - \zeta - \delta_f)(h_{v_2 f} - h_{v2u})}{2P_f(1-\beta_f\rho_f)}$$

$$\sum_{k=1}^{N} p^*_{z,k} = \frac{YL}{(4MX\varphi_{zk}\varphi_{fzk} - L^2)} + \frac{2XZ\varphi_{fzk}}{(4MX\varphi_{zk}\varphi_{fzk} - L^2)} \tag{5.54}$$

$$\sum_{k=1}^{N} p^*_{fz,k} = \frac{ZL}{(4MX\varphi_{zk}\varphi_{fzk} - L^2)} + \frac{2YM\varphi_{zk}}{(4MX\varphi_{zk}\varphi_{fzk} - L^2)} \tag{5.55}$$

$$M = \eta_z^2\rho_z\gamma_z + \delta_z + \zeta - \eta_z\gamma_z - \eta_z\rho_z\delta_z - \eta_z\rho_z\zeta$$

$$X = \eta_{fz}^2\rho_{fz}\gamma_{fz} + \delta_{fz} + \zeta - \eta_{fz}\gamma_{fz} - \eta_{fz}\rho_{fz}\delta_{fz} - \eta_{fz}\rho_{fz}\zeta$$

$$L = \zeta\varphi_{zk} + \zeta\varphi_{fzk} - \eta_s\rho_s\zeta\varphi_{zk} - \eta_{fz}\rho_{fz}\zeta\varphi_{fzk}$$

$$Z = a_z\varphi_{zk}(1-\eta_z\rho_z) + \zeta l\varphi_{zk}(1-\eta_z\rho_z) - c_{zk}\varphi_{zk}(\eta_z\gamma_z - \zeta - \delta_z) - \zeta c_{fzk}\varphi_{fzk}$$

$$- c_w\alpha_z\rho_z(N\eta_z\gamma_z - N\zeta - N\delta_z) - c_w\zeta\alpha_{fz}\rho_{fz}N - \frac{S_z(N\eta_z\gamma_z - N\zeta - N\delta_z)}{q_z}$$

$$- \frac{S_{fz}\zeta N(1-\beta_{fz}\rho_{fz})}{q_z\beta_z\rho_z} + \frac{h_{v_1 z}q_z(N\eta_z\gamma_z - N\zeta - N\delta_z)}{2P_z} + \frac{q_z\beta_z\rho_Z\zeta N(h_{v_2 fz} - h_{v_2 u})}{2P_{fz}(1-\beta_{fz}\rho_{fz})}$$

$$
\begin{aligned}
Y = {} & a_{fz}\varphi_{fzk}(1-\eta_{fz}\rho_{fz}) + \zeta l\varphi_{fzk}(1-\eta_{fz}\rho_{fz}) - c_{fzk}\varphi_{fzk}(\eta_{fz}\gamma_{fz}-\zeta-\delta_{fz}) - \zeta c_{zk}\varphi_{zk} \\
& - c_w\alpha_{fz}\rho_{fz}(N\eta_{fz}\gamma_{fz}-N\zeta-N\delta_{fz}) - c_w\zeta\alpha_z\rho_z N \\
& - \frac{S_{fz}(N\eta_{fz}\gamma_{fz}-N\zeta-N\delta_{fz})(1-\beta_{fz}\rho_{fz})}{q_z\beta_z\rho_z} - \frac{S_z\zeta N}{q_z} + \frac{h_{v_1 z}q_z\zeta N}{2P_z} \\
& + \frac{q_z\beta_z\rho_Z(N\eta_{fz}\gamma_{fz}-N\zeta-N\delta_{fz})(h_{v_2 fz}-h_{v_2 u})}{2P_{fz}(1-\beta_{fz}\rho_{fz})}
\end{aligned}
$$

$$
q_s^* = \sqrt{\frac{\dfrac{(S_s+O_{bs}n_s)D_s\beta_s\rho_s + S_f D_f(1-\beta_f\rho_f)}{n_s\beta_s\rho_s}}{\dfrac{(h_{v_1 s}+h_{bs}-h_{bs}\,n_s)D_s}{2P_s} + \dfrac{h_{bs}n_s}{2} + \dfrac{h_{v_2 f}n_s\beta_s\rho_s}{2(1-\beta_f\rho_f)}\left(1-\dfrac{D_f}{P_f}\right) + \dfrac{h_{v_2 u}n_s\beta_s\rho_s\beta_f\rho_f}{2(1-\beta_f\rho_f)} + \dfrac{h_{v_2 u}n_s\beta_s\rho_s D_f}{2P_f(1-\beta_f\rho_f)} + \dfrac{h_{bu}\beta_s\rho_s n_s}{2}}} \tag{5.56}
$$

$$
n_s^* = \frac{1}{q_s}\sqrt{\frac{\dfrac{S_s D_s\beta_s\rho_s + S_f D_f(1-\beta_f\rho_f)}{\beta_s\rho_s}}{\dfrac{h_{v_2 f}\beta_s\rho_s}{2(1-\beta_f\rho_f)}\left(1-\dfrac{D_f}{P_f}\right) + \dfrac{h_{v_2 u}\beta_s\rho_s\beta_f\rho_f}{2(1-\beta_f\rho_f)} + \dfrac{h_{v_2 u}\beta_s\rho_s D_f}{2P_f(1-\beta_f\rho_f)} + \dfrac{h_{bs}}{2}\left(1-\dfrac{D_s}{P_s}\right) + \dfrac{h_{ub}\beta_s\rho_s}{2}}} \tag{5.57}
$$

$$
q_z^* = \sqrt{\frac{\dfrac{S_z D_s\beta_z\rho_z + S_{fz}D_{fz}(1-\beta_{fz}\rho_{fz})}{\beta_z\rho_z}}{\dfrac{h_{v_1 z}}{2}\left(1-\dfrac{D_z}{P_z}\right) + \dfrac{h_{v_2 fz}\beta_z\rho_z}{2(1-\beta_{fz}\rho_{fz})}\left(1-\dfrac{D_{fz}}{P_{fz}}\right) + \dfrac{h_{v_2 u}\beta_z\,\rho_z q_z}{2} + \dfrac{h_{v_2 u}\beta_{fz}\rho_{fz}\beta_z\rho_z}{2(1-\beta_{fz}\rho_{fz})} + \dfrac{h_{v_2 u}\beta_z\rho_z D_{fz}}{2P_{fz}(1-\beta_{fz}\rho_{fz})}}} \tag{5.58}
$$

5.5 Numerical example

In this section, a simulation study using nested loop search coupled with the Microsoft Excel Solver tool written in Visual Basic for Applications (VBA) codes similar to the one provided in Jaber and Goyal (2008) is conducted to solve more than one thousand numerical examples for both strategies. The objective is to maximize the total profit of the system by setting q_s, n_s and n_f as decision variables in the single channel strategy and setting q_s, q_z, and n_s as decision variables in the dual-channel strategy. In these numerical examples, the values of the input parameters were generated randomly from a uniform distribution each over its range of minimum and maximum values. The minimum and maximum values of some of the input parameters were determined from published studies such as Jaber et al. (2014) and Mukhopadhyay and Setaputra (2006). Other input parameters were logically estimated to meet the considered assumption and conditions in this chapter, such as the production rates are greater than the demand rates. Moreover, the percentage values of the financial and physical holding costs were based on the suggestions of Waters (2003). For example, the physical storage holding cost was set at 9% of the unit cost; on the other hand, the financial holding cost was set at 10% of the unit cost. Table 5.1 shows the ranges of the values of the input parameters and the values of a one randomly selected numerical example that was used for illustration purposes.

Table 5.1 Ranges of input parameters for the 1000 numerical examples and values of the selected random example

Parameter	Max.	Avg.	Min.	*Example	Parameter	Max.	Avg.	Min.	*Example
a_s	10,000	8,037.86	5,000	6,433	P_s	15,000	11,444.14	6,000	1,0499
a_f	5,000	2,780.11	1,000	2,267	P_f	15,000	10,153.77	6,000	9,843
a_z	10,000	8,827.12	5,000	9,960	P_z	15,000	11,620.28	6,000	11,001
a_{fz}	5,000	2,812.55	1,000	2,951	P_{fz}	15,000	10,083.41	6,000	8,505
δ_s	10	7.35	5	5	S_s	250	198.01	150	226
δ_f	15	12.41	10	11	S_f	150	97.12	50	67
δ_z	10	6.27	5	6	S_z	250	201.39	150	211
δ_{fz}	15	11.90	10	13	S_{fz}	150	100.89	50	103
γ_s	5	3.01	1	3	O_{bs}	50	40.32	30	49
γ_f	5	2.88	1	5	O_{bf}	25	21.08	15	18
γ_z	5	3.95	1	4	$h_{v_1 s}$	74.10	54.82	45.90	46.8
γ_{fz}	5	3.47	1	4	$h_{v_1 z}$	105	89.18	78	96
ζ	30	17.43	10	10	h_{bs}	68.05	50.34	42.15	42.98
l	20	12.66	5	17	h_{bf}	158.22	95.42	52.02	77.42
ρ_s	15%	10%	5%	5%	$h_{v_2 f}$	56.32	41.66	34.88	35.57
ρ_f	30%	24%	15%	25%	$h_{v_2 fz}$	79.80	67.78	59.28	72.96
ρ_z	15%	8%	5%	12%	h_{bu}	68.05	50.34	42.15	42.98
ρ_{fz}	30%	25%	15%	25%	$h_{v_2 u}$	48.63	39.36	34.03	39.03
α_s	25%	13%	1%	18%	c_s	250	182.73	150	156
α_f	50%	40%	30%	46%	c_f	50	36.55	30	31.2
α_z	25%	13%	1%	2%	c_z	350	297.26	260	320
α_{fz}	50%	40%	30%	34%	c_{fz}	70	59.45	50	64
					c_w	5	3.65	3	3.12

* Values of selected numerical example that is used for illustration purposes

The selected random example was solved for both the single-channel and the dual-channel strategies and the results are shown in Table 5.2 and Table 5.3, respectively. In Table 5.2, the first two columns show an analysis of the proportion of the selling price that is refunded to customers. In the interest of creating a return policy that is beneficial for vendor 1, it is assumed that when vendor 1 offers a full or a partial refund on the standard items, the refund on the refurbished items should be equal to or lesser than the refund on the standard items. For instance, in the single channel strategy, when vendor 1 offers a full refund (100%) for both standard and refurbished items ($\eta_s = \eta_f = 1$), the optimal selling prices for the standard and refurbished items were p_s =\$763.71 and p_f =\$566.07, respectively, and the optimal inventory decisions were q_s =148, n_s =2 and n_f =1 with an optimal profit of \$2,015,989. In another scenario, when vendor 1 offers a 75% refund on the standard item, the refund on the refurbished item should be less than or equal 75%. Assuming $\left(\eta_s = \eta_f = 0.75\right)$, the optimal selling prices were found to be p_s =\$637.68 and p_f =\$431.29 with optimal inventory decisions of q_s =164, n_s =2 and n_f =1, and a total profit of \$1,587,885.03. Correspondingly, in Table 5.3 when offering a full refund in the dual-channel strategy the optimal decisions were as follows: the selling prices are $p_s = \$763.46, p_f = \$564.93, p_z = \$1{,}032.00$, and $p_{fz} = \$596.08$, optimal inventory decisions are $q_s = 105, n_s = 3$ and $q_z = 230$ with a maximum profit of \$4,859,322.90.

From Table 5.2 and Table 5.3, it was observed that the selling prices increase as the refund proportion increases, indicating that, the more generous the return policy is, the higher the demands, the selling prices and the total profits. This is logical since, if vendor 1 offers a higher refund, it will face a risk of losing the value of the product where it will be considered as a returned item rather than a new one. Davis et al. (1995, p. 9) provided a rational explanation for higher price-higher refund policy. They mentioned that a full refund policy (money-back guarantee) "may allow the retailer to charge higher prices because the reduction in the risk from the product's being a poor match with customers' tastes may increase consumers' willingness to pay." Additionally, the strategy of offering a lower refund and decreasing the prices could be used as a discount selling strategy. For example, if vendor 1 would like to offer a 50% discount on the standard product, then the vendor could use the no-refund policy. Many companies nowadays use "final sale" "no-return" policy on sale or clearance items; Sears, for example, offers a full refund policy to its customers on regular priced items, whereas, marked-down priced items are offered with no return policy (Chen and Bell, 2012).

Comparing the results of the single-channel strategy in Table 5.2 with the dual-channel strategy in Table 5.3 it was found the dual-channel strategy outperformed the single channel strategy. Moreover, the results also revealed that the adoption of the dual-channel strategy has no significant effect on the prices of the standard and refurbished standard items. For example, under the full refund policy, the price of the standard and refurbished standard items item in the single-channel strategy were p_s = 763.71 and p_f = 566.07, respectively. Under the dual-channel strategy the two prices were p_s = 763.46 and p_f = 564.93, respectively.

Table 5.2 Optimal results of the single-channel strategy for the selected random example

Single channel							
η_s	η_f	p_s^*	p_f^*	q_s^*	n_s^*	n_f^*	Π_{single}
1	1	$763.71	$566.07	148	2	1	$2,015,989.00
1	0.75	$706.51	$471.69	164	2	1	$1,807,737.59
1	0.5	$665.73	$404.23	121	3	1	$1,658,625.96
1	0.25	$634.97	$353.60	126	3	1	$1,546,586.70
1	0	$611.02	$314.19	129	3	1	$1,459,311.29
0.75	0.75	$637.68	$431.29	164	2	1	$1,587,885.03
0.75	0.5	$604.54	$372.20	173	2	1	$1,466,162.58
0.75	0.25	$579.49	$327.33	125	3	1	$1,373,635.23
0.75	0	$559.69	$292.08	128	3	1	$1,300,916.55
0.5	0.5	$554.65	$346.07	172	2	1	$1,309,971.11
0.5	0.25	$533.61	$305.69	179	2	1	$1,232,062.18
0.5	0	$516.94	$273.75	183	2	1	$1,170,381.55
0.25	0.25	$495.21	$287.58	177	2	1	$1,114,151.61
0.25	0	$480.96	$258.30	181	2	1	$1,061,062.34
0	0	$450.16	$245.11	180	2	1	$968,252.52

Table 5.3 Optimal results of the dual-channel strategy for the selected random example

Dual-channel											
η_s	η_f	η_z	η_{fz}	p_s^*	p_f^*	p_{zk}^*	p_{fzk}^*	q_s^*	n_s^*	q_z^*	Π_{dual}
1	1	1	1	\$763.46	\$564.93	\$1,032.00	\$596.08	105	3	230	\$4,859,322.90
1	0.75	1	0.75	\$706.41	\$470.96	\$998.83	\$532.66	114	3	238	\$4,510,541.22
1	0.5	1	0.5	\$665.57	\$403.71	\$972.51	\$482.01	120	3	244	\$4,250,795.99
1	0.25	1	0.25	\$635.00	\$353.22	\$951.11	\$440.56	97	4	248	\$4,049,512.25
1	0	1	0	\$611.10	\$313.92	\$933.34	\$405.95	99	4	252	\$3,888,792.00
0.75	0.75	0.75	0.75	\$637.54	\$430.50	\$889.12	\$483.77	114	3	232	\$3,895,194.11
0.75	0.5	0.75	0.5	\$604.48	\$371.63	\$868.46	\$439.45	119	3	237	\$3,686,719.89
0.75	0.25	0.75	0.25	\$579.46	\$326.88	\$851.55	\$402.92	96	4	241	\$3,523,843.45
0.75	0	0.75	0	\$559.72	\$291.75	\$837.44	\$372.25	98	4	244	\$3,392,994.29
0.5	0.5	0.5	0.5	\$554.56	\$345.45	\$786.34	\$406.01	118	3	232	\$3,243,086.46
0.5	0.25	0.5	0.25	\$533.67	\$305.21	\$772.57	\$373.19	95	4	235	\$3,108,132.43
0.5	0	0.5	0	\$517.05	\$273.37	\$761.04	\$345.52	97	4	238	\$2,999,179.78
0.25	0.25	0.25	0.25	\$495.14	\$287.06	\$708.25	\$349.10	120	3	230	\$2,771,583.17
0.25	0	0.25	0	\$481.03	\$257.88	\$698.61	\$323.77	96	4	233	\$2,679,209.02
0	0	0	0	\$450.20	\$244.64	\$646.54	\$305.71	95	4	228	\$2,414,425.39

5.6 Sensitivity analysis

This section expands on the selected numerical example presented above, by performing a number of sensitivity analysis to examine the effect of changing the value of different input parameters on the behavior of the developed supply chain models. When changing the value of one parameter, (in the selected numerical example) the values of the remaining parameters are kept fixed at their initial values. Each sensitivity analysis is tested on the optimal profit, and pricing decisions for both the single-channel and the dual-channel strategies and are discussed in detail in the following subsections, with graphical representations.

5.6.1 Effect of varying the coefficient of the price elasticity

The effect of varying the value of the coefficient of the price elasticity of the standard and the refurbished standard items (δ_s and δ_f, respectively) on the supply chain's behavior is examined in this subsection. The results as demonstrated in Figure 5.11 showed that as δ_s increases (from 3 to 15), the total profit of the single-channel strategy (Π_{single}) decreases by about 85%, whereas the total profit of the dual-channel strategy (Π_{dual}) decreases by about 46%. The price of the standard and the refurbished standard items (p_s and p_f, respectively) reduced significantly. Similarly, as δ_f increases (from 10 to 20), the total profit Π_{single} decreases by about 51%, the total profit Π_{dual} decreased by about 22%, and both prices p_s and p_f decrease (see Figure 5.12).

5.6.1.1 Managerial insight

One can notice that as customers are more sensitive to the price of the standard item (i.e., δ_s increases), the decision maker (i.e., the vendor) needs to decrease p_s in order to stem the decline in demand and the overall profit of both the single channel, Π_{single}, and the dual-channel, Π_{dual}. Moreover, the manufacturer needs to lower the price of the refurbished (p_f) items to accommodate the decrease in p_s. Therefore, the decision maker needs to ensure a lower δ_s to achieve a beneficial and profitable supply chain and to increase the differences between the two prices p_s and p_f. One possible way of making customers less price-sensitive is by differentiating the standard item from the competition (i.e., improving the quality of the item) and thereby reducing customer's sensitivity to higher prices (Mukhopadhyay and Setaputra, 2006). A similar managerial guideline is also gained when δ_f is varied (the decision maker needs to ensure a lower δ_f). This can be possibly done by providing the same warranty given on new items, on the refurbished ones. Another interesting managerial guideline that is gained from this analysis is that the dual-channel strategy is more profitable than the single channel strategy even with higher sensitivity to prices. Therefore, adopting a dual-channel strategy is the profitable decision to make.

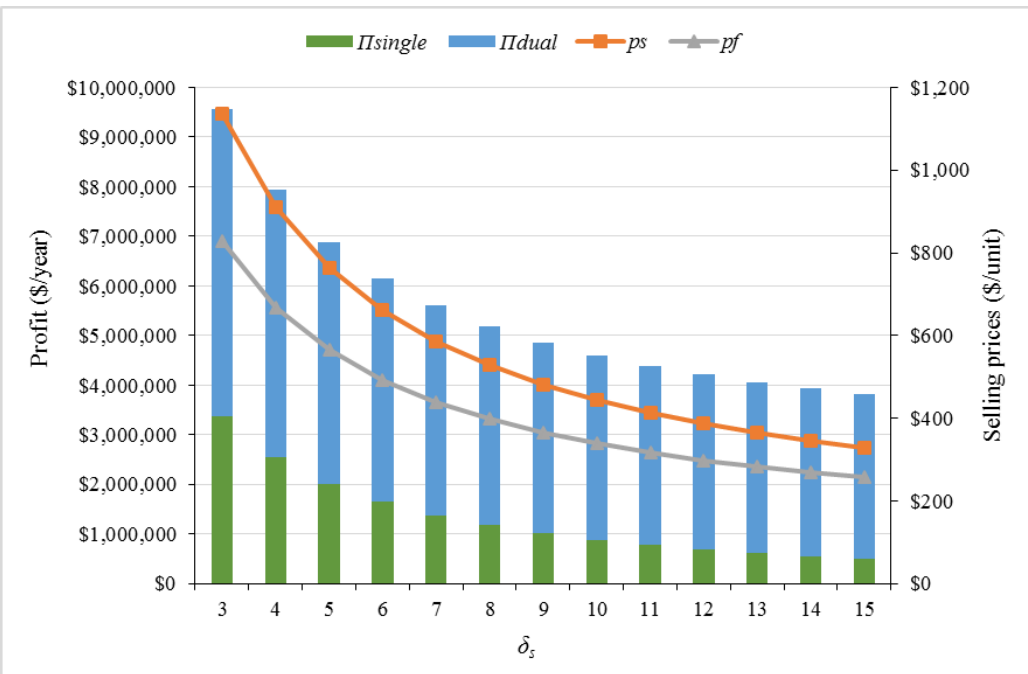

Figure 5.11 Effect of varying δ_s on Π_{single}, Π_{dual}, p_s and p_f

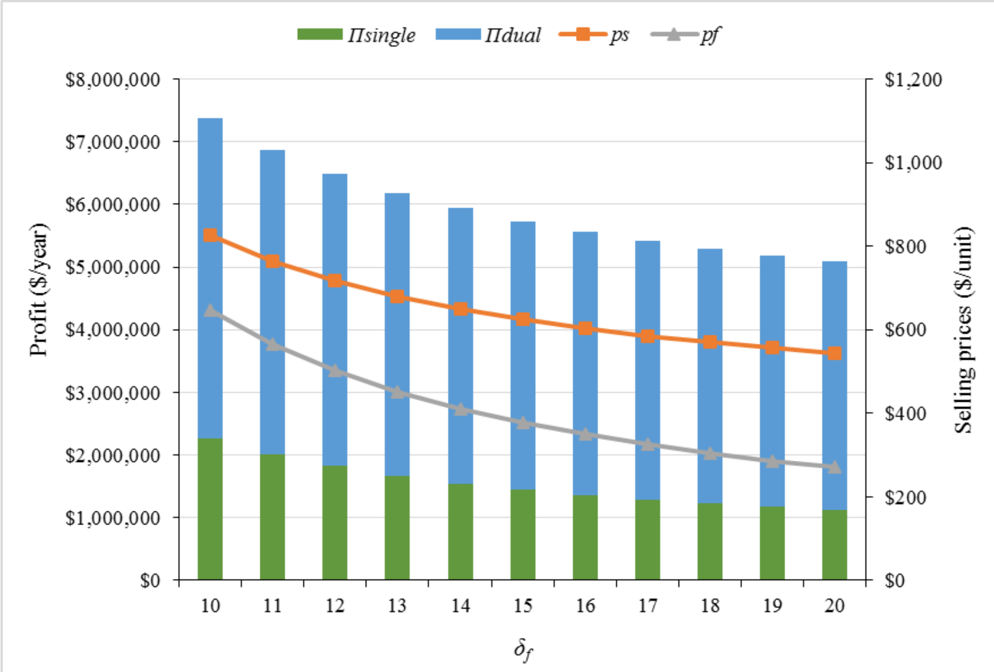

Figure 5.12 Effect of varying δ_f on Π_{single}, Π_{dual}, p_s and p_f

5.6.2 Effect of varying the sensitivity of the demand with respect to the return policy

In this sensitivity analysis, when the sensitivity of the demand with respect to the return policy of the standard and the refurbished standard item (γ_s) increases (from 1 to 5) as shown in Figure 5.13, the total profit of the single-channel strategy (Π_{single}), increases by about 59%. However, the total profit of the dual-channel strategy (Π_{dual}) increases by about 32%. Moreover, it was observed that the two prices of the standard and refurbished standard items (p_s and p_f, respectively) have significant increase as γ_s increases. We also found that the difference between the two prices ($p_s - p_f$) increase as γ_s increases.

Likewise, when γ_f (the sensitivity of the demand with respect to the return of the refurbished item) increases, the total profit Π_{single} increases by about 63%, the total profit Π_{dual} increases by about

47%, and the two prices p_s and p_f significantly increase (see Figure 5.14). Interestingly, it was found that the difference between the two prices ($p_s - p_f$) decrease as γ_f increases.

5.6.2.1 Managerial insight

The results suggest that the higher the customer's sensitivity to the return policy is, the higher the demand, the prices and the overall profit. These positive results can be achieved by offering a generous return policy that is beneficial to the supply chain. Increasing customer's sensitivity to the return policy can be achieved by increasing the customers' awareness and benefits of the offered return policy (e.g. effective advertisement) at the retail channel and/or at the online channel, for both new and refurbished products (Mukhopadhyay and Setoputro, 2005; Mukhopadhyay and Setaputra, 2006). Moreover, the decision maker needs to be aware that when the elasticity of customers to the refurbished item increases in which the difference in prices becomes smaller, customers may prefer the newly standard item to the refurbished standard item.

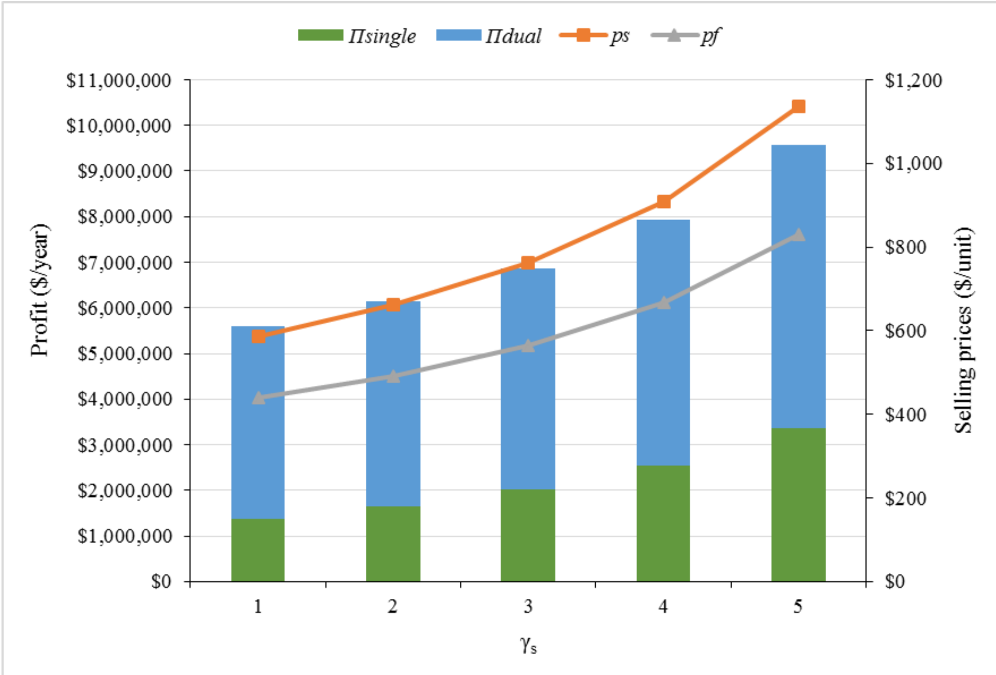

Figure 5.13 Effect of varying γ_s on Π_{singel}, Π_{dual}, p_s and p_f

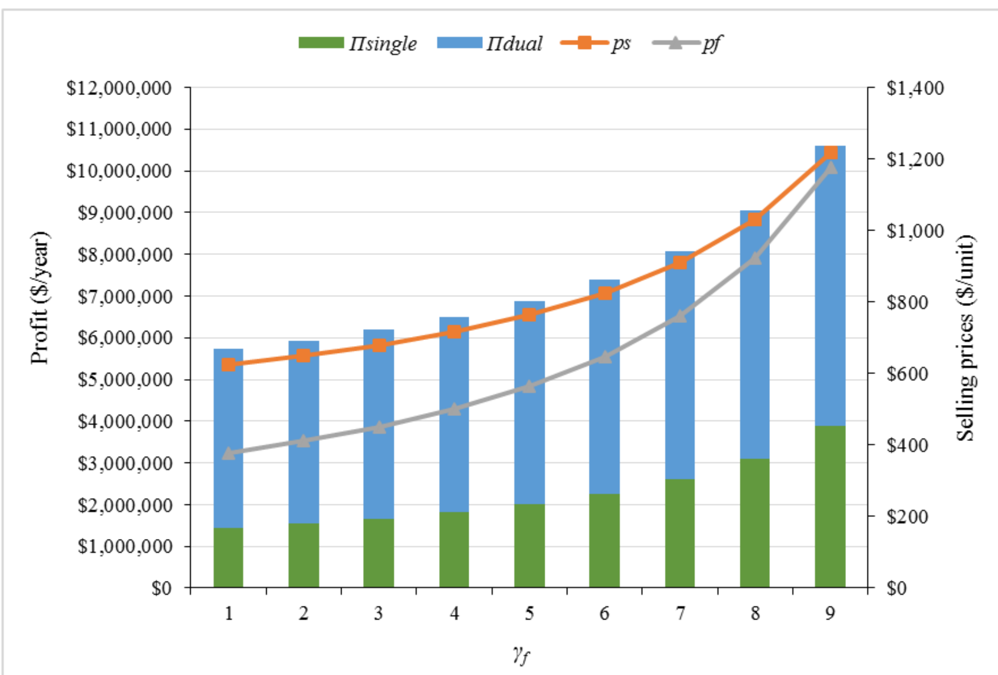

Figure 5.14 Effect of varying γ_f on Π_{singel}, Π_{dual}, p_s and p_f

5.6.3 Effect of varying migration parameter

The results of varying the migration parameter (ζ), as demonstrated in Figure 5.15, showed that as ζ increases, the total profit single-channel (Π_{single}) decreases by about 36% and the price of the standard item (p_s) decreases, whereas the price of the refurbished item (p_f) increases.

A similar analysis was performed for the dual-channel strategy and the results are shown in Figure 5.16. As ζ increases, the profit of the dual-channel (Π_{dual}) decreases by about 47%. The price of the new standard and new customized items (p_s and p_{zk}, respectively) decrease. However, the price of the refurbished standard and refurbished customized items (p_{zk} and p_{fzk}, respectively), increase.

5.6.3.1 Managerial insight

From the above sensitivity analysis, when applied to both strategies, it is clear that a lower ζ is beneficial for the supply chain. The decision maker can achieve a lower ζ through effective advertisements (for example, by showing the benefits of owning a brand-new item compared to a refurbished one). This result is consistent with the findings of Mukhopadhyay and Setaputra (2006). Furthermore, the marketing strategy could focus solely on advertising their new items rather than placing any focus at all on the refurbished items. For example, many companies such as Apple focus their advertisement on newly produced items rather than on refurbished items. With the rapid growth of secondary markets for refurbished products, companies may have to direct some of their commercial ads to those customers on budgets. Having a secondary market is beyond the scope of this chapter; however, it would indeed be an interesting idea to consider in another work.

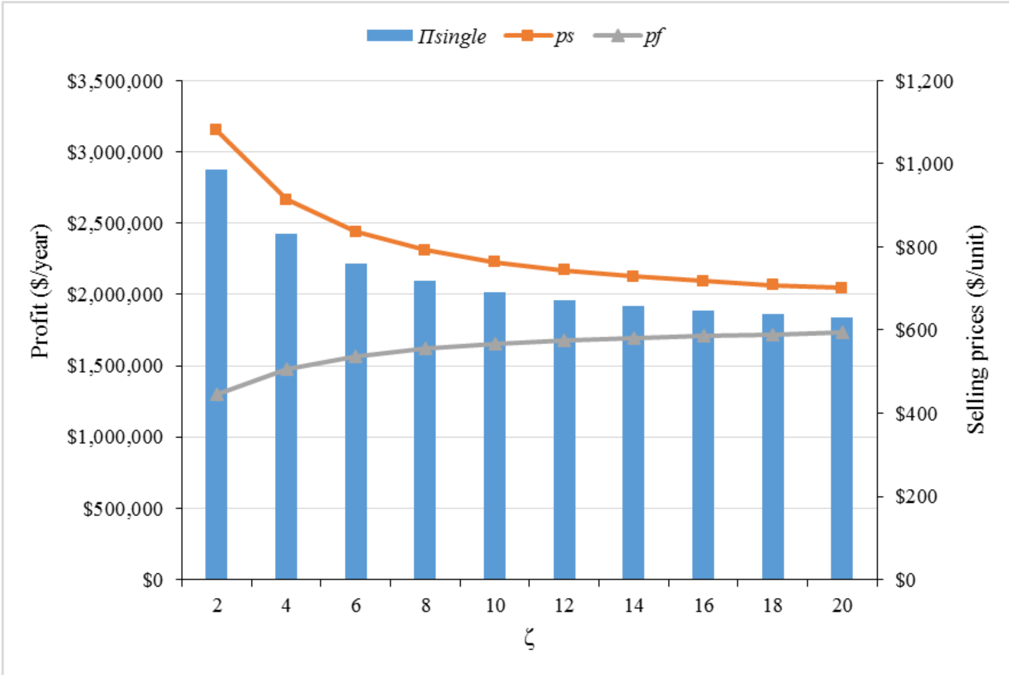

Figure 5.15 Effect of varying ζ on Π_{single}, p_s and p_f (Single-channel)

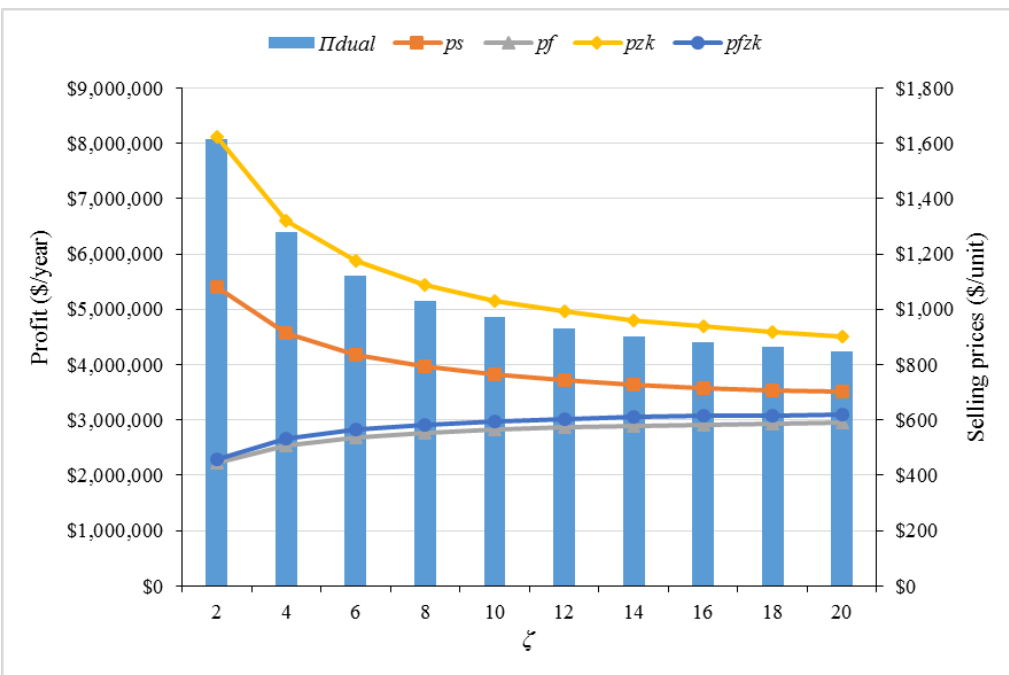

Figure 5.16 Effect of varying ζ on Π_{dual}, p_s, p_f, p_{zk} and p_{fzk} (Dual-channel)

5.6.4 Effect of varying the proportion of returned items

The changes in the optimal profit and pricing decisions of the two strategies with respect to varying the proportion of the returned items are examined here. In the single channel strategy, as the proportion of returned standard item (ρ_s) increases (from 5% to 50%), the total profit of the single-channel strategy Π_{single} is reduced by about 36%, the price of the standard item (p_s) increases, and the price of the refurbished standard item (p_f) decreases, as shown in Figure 5.17. We also investigated the effect of varying the proportion of returned refurbished standard items (ρ_f) on Π_{single}, p_s, and p_f and similar results were obtained.

The effect of varying ρ_s when the dual-channel strategy is adopted was also investigated and the results are shown in Figure 5.18. The results showed that as ρ_s increases (from 5% to 50%), the total profit of the dual-channel strategy (Π_{dual}) is reduced by 15%, the price p_s increases, and the price p_f decreases. Moreover, no effect was observed on the price of the customized and refurbished customized items (p_{zk} and p_{fzk}, respectively). Similar results were found when ρ_f is varied when the dual-channel strategy is adopted.

5.6.4.1 Managerial insight

From the above sensitivity analysis, it is clear that adopting the dual-channel strategy is more profitable, even when the proportion of the returned items is higher. One key insight that is gained from this sensitivity analysis is that as a lower ρ_s or ρ_f is always beneficial for the supply chain in both the single-channel and the dual-channel strategies. The decision maker of the supply chain can achieve a lower return rate by offering restrictions or a limited-time return policy to return the purchased item. This insight is consistent with what is being practiced by many firms nowadays. For example, Best Buy offers between 14-30 days return policy on some of its offered products and depending on the type of the item, it should be returned in its original packaging and/or unopened ("Returns and exchanges policy - Best Buy Canada," 2016). Other companies also restrict returns by using the "no receipt, no return" policy. Reducing return rate can also be achieved by encouraging customer review. Customer review plays an important factor in providing visual information that gives customers confidence in purchasing the desired item (Pfister, 2014).

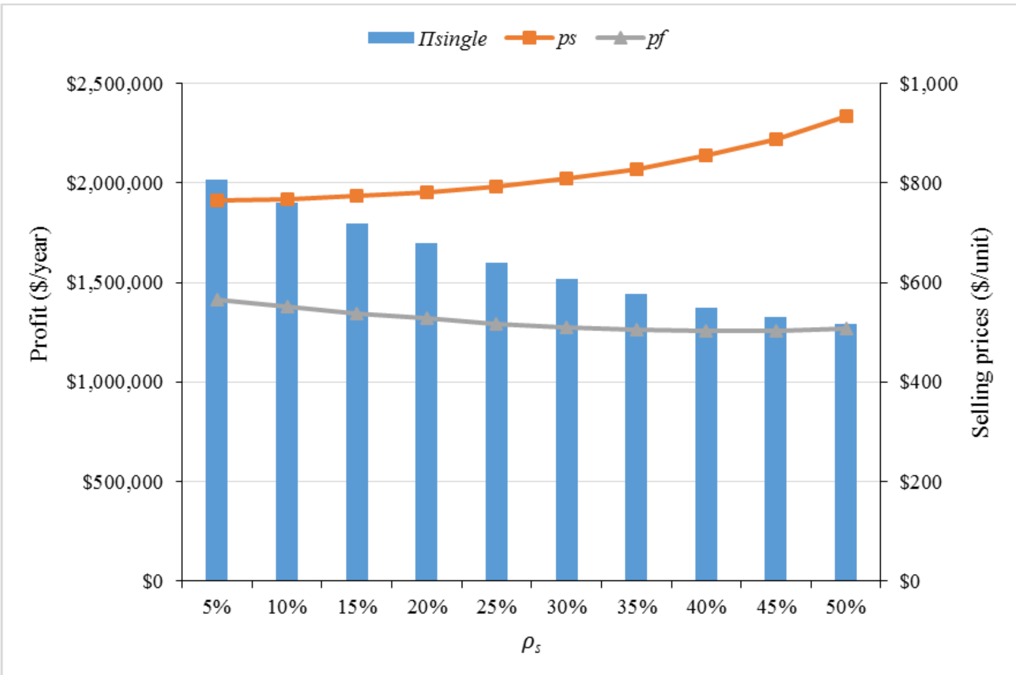

Figure 5.17 Effect of varying ρ_s on Π_{single}, p_s and p_f (Single-channel)

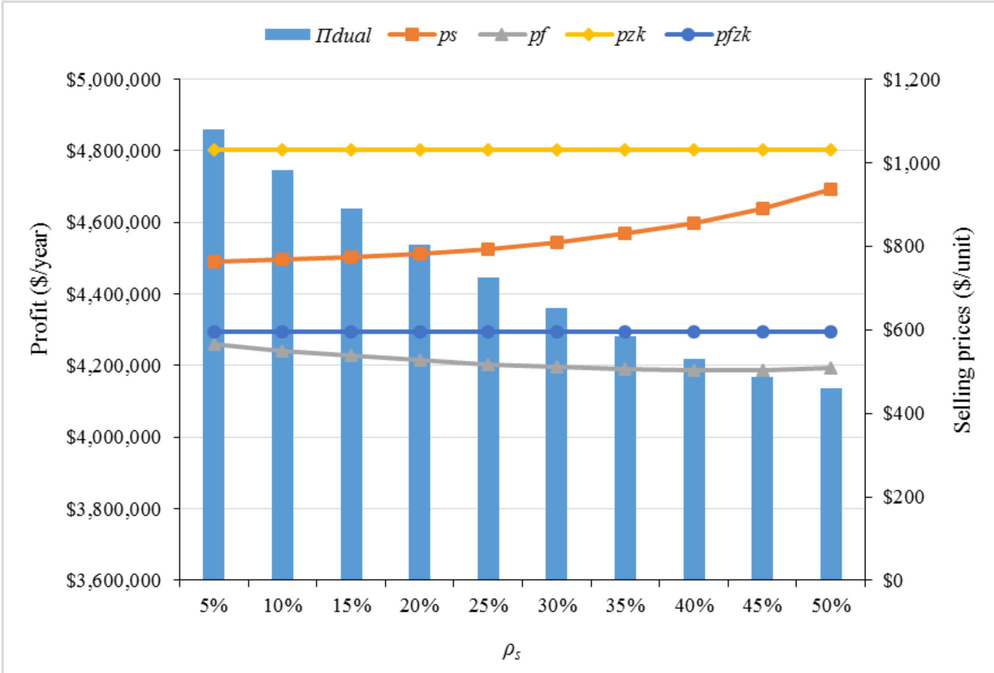

Figure 5.18 Effect of varying ρ_s on Π_{dual}, p_s and p_f (Dual-channel)

5.6.5 Effect of varying the proportion of returned repairable items

In this sensitivity, the proportion of the returned repairable items is varied to investigate its effect on the supply chain of both strategies. The results as shown in Figure 5.19, showed that as the proportion of the returned repairable standard items (β_s) increases (from 10% to 90%), the single-channel profit (Π_{single}) increases by about 0.7% only, and both the price of the standard and refurbished standard items (p_s and p_f, respectively), decrease minimally. Similar results were also obtained when the proportion of the returned repairable refurbished standard item (β_f) is varied.

Comparatively, when the dual-channel strategy is adopted, the results as demonstrated in Figure 5.20 showed that as β_s increases (from 10% to 90%), the dual-channel's profit (Π_{dual}), increases by about 0.25% and both p_s and p_f decreases minimally. No change in the price of the customized and refurbished customized items (p_{zk} and p_{fzk}, respectively), was observed when β_s or β_f was varied.

5.6.5.1 Managerial insight

From the above sensitivity analysis, it is clear that the manufacturer will be making some extra profit if the proportion of the return items is more repairable than unrepairable. This is because the returned items that are suited for repair provide the manufacturer with additional means of profit on a previously sold item even though the production cost is now higher. In contrast, if the returned items are unsuitable for repair, this presents a total loss to the manufacturer as the product is now sent for disposal. Therefore, the decision maker can seek to achieve a higher proportion of repairable items through the investment in quality improvement actions or through the implementation of an inspection and testing procedure into the production processes with the aim of reducing and/or recovering defective items (Hsieh and Liu, 2010).

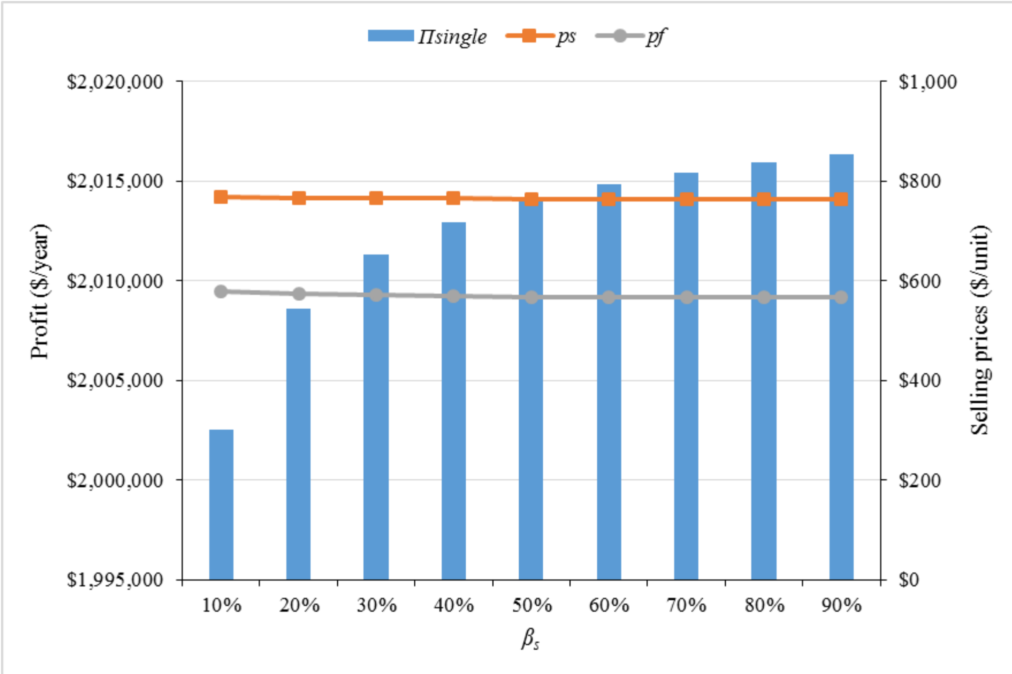

Figure 5.19 Effect of varying β_s on Π_{single}, p_s and p_f (Single-channel)

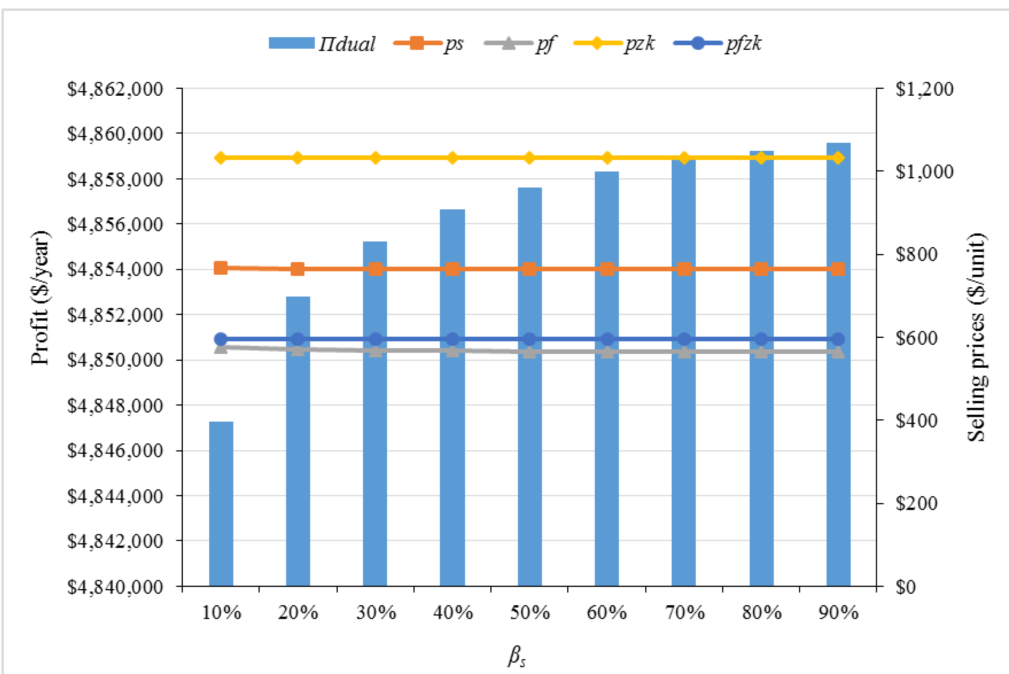

Figure 5.20 Effect of varying β_s on Π_{dual}, p_s, p_f, p_{zk} and p_{fzk} (Dual-channel)

5.7 Summary and Conclusions

This chapter considers a forward and a reverse supply chain system consisting of an original equipment manufacturer and a retailer. A return policy agreement is also considered in which unsatisfied customers with the purchase of a new item may return the item for a refund. The returned items are collected and only the repairable ones are refurbished through a contracted 3PL provider and then offered to customers at a lower price than the new items. A return policy agreement applied on the refurbished items is also considered. A linear dependent demand function, in which customers are sensitive to the prices and the return policy of the sold items (new and refurbished), is used.

The chapter investigated the effect of adopting a dual-channel strategy (retail and online channels) on the behavior of the supply chain system while taking inventory decisions, return policy and refurbishing costs into consideration. Two strategies were analyze. The first strategy analyzed the behavior of the system when the supply chain is composed of a single-channel and in which standard and refurbished standard items are offered through the retailer. The second strategy analyzed the behavior of the system when the dual-channel strategy is adopted where standard items are offered through the retail channel whereas customized and refurbished items (including refurbished standard and refurbished customized) are offered directly through the online channel. In both strategies, the objective was to maximize the total profit of the system by finding the optimal pricing strategy and inventory decisions under different return policies.

One thousand numerical example were solved from which one numerical example was randomly selected. The results showed that the dual-channel strategy is more profitable than the single-channel strategy and that the optimal prices in the single channel strategy are not affected by the adoption of the dual-channel strategy. The results also indicated that the more generous the return policy is, the higher the selling prices and the profits. Sensitivity analysis was also conducted (by varying the values of different input parameters) to determine how the optimal pricing decisions and the optimal profits are affected. From these sensitivity analysis, specific managerial insights using marketing and operational management strategies were provided.

CHAPTER 6. DUAL-CHANNEL SUPPLY CHAIN WITH LEARNING AND FORGETTING EFFECTS

This chapter considers a dual-channel supply chain where learning and forgetting occur in the production of the standard and core items. The objective is to investigate the effects of learning and forgetting on pricing and inventory decisions in a dual-cannel supply chain. The performance measure is total profit.

This chapter has seven sections. Section 6.1 briefly presents the learning and forgetting process. Section 6.2 describes the problem. Section 6.3 and 6.4 presents the assumptions and notations. Section 6.5 presents the developed models. Sections 6.6 and 6.7 present numerical examples and the sensitivity analysis, respectively, and discuss the results. Finally, Section 6.8 summarizes and concludes the chapter.

6.1 The learning and forgetting process

The literature discussed several learning and forgetting models in the past (Jaber, 2006a, 2011). The learn-forget curve model (LFCM) is one of these models that has solid theoretical and empirical evidence (Jaber and Bonney, 2003). The learning process of the LFCM follows the Wright learning curve (WLC) model, while the forgetting process is a mirror image of it (Globerson et al., 1989). The WLC is expressed as:

$$T_x = T_1\, x^{-b} \tag{6.1}$$

where T_x is the time (cost) to produce the xth unit, T_1 is the time (cost) to produce the first unit, x is the cumulative quantity produced (production count), and b is the learning curve (LC) exponent; where $0 < b < 1$ and $b = -\log(LR)/\log(2)$, and LR is the learning rate expressed as a percentage. The forgetting curve (FC) is expressed as (Globerson et al., 1989; Jaber and Bonney, 1996, 2003):

$$\hat{T}_x = \hat{T}_1\, x^f \tag{6.2}$$

where $\hat{T}_x$ is the time (cost) for the xth unit of lost experience of the FC, $\hat{T}_1$ is the intercept, and f is the forgetting exponent.

In the LFCM, unlike in Eq. (6.2), the forgetting exponent, *f*, varies from one production cycle to another, i.e., $0 \le f_i \le 1$where *i* = 1, 2, …, and is expressed as:

$$f_i = \frac{b(1-b)\log(u_i + x_i)}{\log\left(1 + B/t(u_i + x_i)\right)} \tag{6.3}$$

where B is the time for total forgetting to occur, x_i is the production quantity in cycle i up to the point of interruption, and u_i is the equivalent number of units remembered at the beginning of the ith cycle (cumulative experience). The term $t(u_i + x_i)$ is the equivalent units of cumulative production by the end of the ith cycle, and is computed from Eq. (6.1) as follows:

$$t(u_i + x_i) = \sum_{x=1}^{u_i + x_i} T_1 (x)^{-b} \cong \int_0^{u_i + x_i} T_1 x^{-b} dx = \frac{T_1}{1-b}(u_i + x_i)^{1-b} \tag{6.4}$$

The number of units remembered at the beginning of the cycle $(i+1)$ is given from Jaber and Bonney (1996) as follows:

$$u_{i+1} = (u_i + x_i)^{(b+f_i)/b}\, y_i^{-(f_i/b)} \tag{6.5}$$

where $u_1 = 0$, $u_i \leq \sum_{j=1}^{i-1} x_i$ and y_i is the number of units that would have been accumulated if production had not ceased for e_i units of time and can be computed from Eq. (6.4) as follows:

$$y_i = \left\{\frac{1-b}{T_1}\left[t(u_i + x_i)\right] + e_i\right\}^{1/(1-b)} \tag{6.6}$$

When a full transfer of learning occours, we have $u_i = \sum_{j=1}^{i-1} x_j$ and when partial or total forgetting occurs, we have $u_i \geq 0$.

6.2 Model Description

Consider a dual-channel supply chain consisting of a vendor and a retailer (see Fig. 6.1). In the first channel, the vendor produces a standard item from an unfinished core item with some added basic features. The item then enters a make-to-stock (MTS) process and is then sold to customers through a retail channel. In this channel, the vendor and the retailer use a vendor managed inventory with a consignment stock (VMI-CS) agreement. In the second channel, the vendor builds product to order (BTO) from the core items, customizes them based on the end customer's order and sells them directly to end customers online (thereby bypassing the retailer). The production of standard and core items is intermittent, improves with learning and deteriorates with production breaks (Jaber and Bonney, 1999; Jaber, 2011). The learning and forgetting process follows the LFCM (introduced in Section 6.1). Transfer of learning between cycles could be full (no forgetting) or partial (some forgetting). No transfer of learning means total forgetting of knowledge. Customized items may significantly differ from one another, suggesting that the knowledge a worker gains in producing one product is not transferable to another. Therefore, for simplicity and to keep the chapter concise, it is assumed that learning and forgetting do not occur in the BTO process. This will be dealt with in future work.

The objective of this chapter is to maximize the profit of the supply chain (both for the single- and the dual-channel strategy) by optimizing prices, production and order quantities, and the number of shipments. The chapter starts by analyzing the single-channel (retail channel), followed by the dual-channel supply chain. The results of the analyses are compared and discussed to draw insights on the relative performance of both channels.

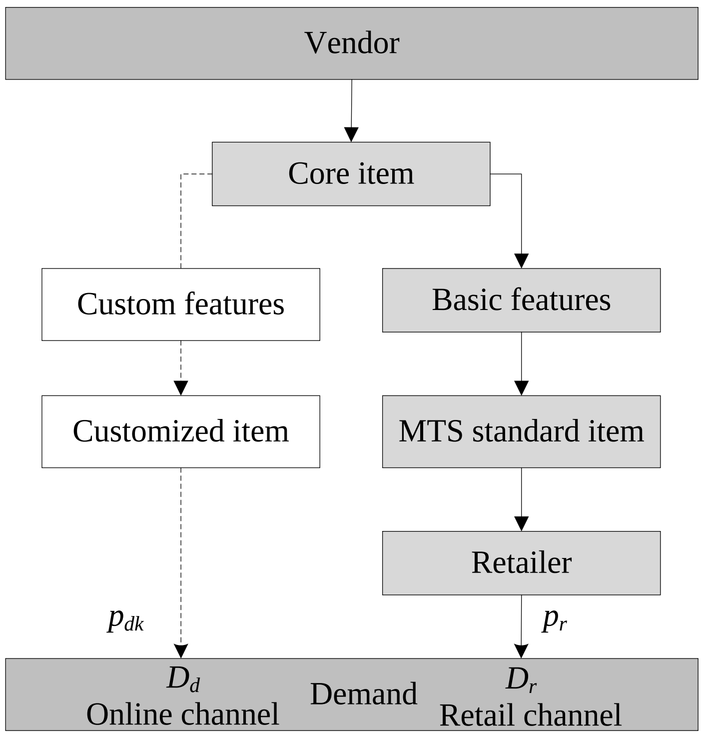

Figure 6.1 Dual-channel supply chain

6.3 Assumptions

The following additional assumptions are made:

(1) All input parameters are known and constant in time.

(2) Shortages are not allowed.

(3) Lead-time between the vendor and the retailer is zero.

(4) A quoted delivery lead-time (customer's waiting time for a customized item) is considered.

(5) The vendor's setup cost and the retailer's ordering cost are fixed and independent of the order/production quantities.

(6) The online and the retailer channel compete for customers. The share of customers preferring the online (retailer) channel is given.

(7) An infinite planning horizon is considered.

6.4 Notations

Input parameters

$D_{r,i}, D_{d,i}$	Demand for the retail and the direct channel for cycle i, respectively, (unit/year);
a	Primary demand (potential demand when the item is free of charge), (unit/year);
$\theta, (1-\theta)$	Percentage share of the demand going to the direct and retail channel, respectively;
α_r	Coefficient of price elasticity of the standard item, (unit^2/\$/year)
α_{dk}	Coefficient of price elasticity of the customized k item, where $k = 1, 2, \ldots, N$, (unit^2/\$/year);
ρ	Cross-price sensitivity;
l_d	Quoted delivery lead-time (i.e., waiting time) of customized items, (day);
β_r	Sensitivity to quoted delivery lead-time of the demand D_r, (unit/day);
β_d	Sensitivity to quoted delivery lead-time of the demand D_d, (unit/day);
φ_{dk}	Percentage of core item stock used for customized item k, $k = 1, 2, \ldots, N$, (-);
P_r	Production rate for the standard item, $P_r > a$, (unit/year);
P_d	Production rate for the core item for eventual customization, $P_d > a$, (unit/year);

c_P Unit production/labour cost for the standard item, ($/unit);

c_{dk} Unit production/labour cost for the customized k item, $k = 1, 2, \dots, N$, ($/unit);

c_r Vendor's wholesale price of the standard item to the retailer, ($/unit);

S_r Vendor's setup cost for the standard item, ($/setup);

S_d Vendor's setup cost for the core item, ($/setup);

O_r Retailer's order cost for the standard item, ($/order);

h_{v1} Holding cost at the vendor's side, which includes the financial and physical storage holding cost, ($/unit/year);

h_{v2} Financial holding cost for a unit of the standard item at the retailer's side paid by the vendor , ($/unit/year);

h_r Physical storage holding cost for a unit of the standard item at the retailer's side paid by the retailer, ($/unit/year);

$t_{r,i}$ The time required to produce the ith unit of the standard item in cycle i, (year) ;

$t_{d,i}$ The time required to produce the ith unit of the core item in cycle i, (year);

T_{r1} The time required to process the first unit of the standard item, (year/unit);

T_{d1} The time required to produce the first unit of the customized core item, (year/unit);

b_r Learning curve exponent for the standard item, (-);

b_d Learning curve exponent for the core item, (-);

B_r The time for total forgetting of the standard item, (year);

B_d The time for total forgetting of the standard item, (year);

V_N Set of variant k, where $k \in V_N$ and $k = 1, 2, \dots, N$, (-);

N Total number of variants (-);

Z Total number of custom features(-);

Decision variables

p_r	Selling price of a standard item to customers, (\$/unit);
p_{dk}	Selling price of the customized kth item, $k = 1, 2, \dots, N$, (\$/unit);
q_r	Shipment (batch) size for standard item, (unit);
q_d	Production quantity of the core item for eventual customization, (unit);
n_r	Number of shipments of the standard item, integer $n_r \geq 1$

6.5 Mathematical models

6.5.1 Single-channel strategy

Although the single-channel model considered in this chapter is similar to the one developed by Zanoni et al. (2012), it differs from it in three distinct ways. First, Zanoni et al. (2012) assumed a constant demand, whereas this model assumes a price-dependent demand. Second, this model adopts a profit maximization whereas Zanoni et al. (2012) aimed for a cost minimization. This assumption affects pricing and inventory decisions because of the price dependency of demand. Third, this model corrects the estimation of the production time in Zanoni et al. (2012) when the process is subjected to learning and forgetting effects.

The VMI-CS has been used by different industries (e.g. the automobile/auto part industry, the consumer electronics industry, the pharmaceutical industry and the papermaking industry) because of its economic benefits (Zanoni et al., 2012; Batarfi et al., 2016, 2017). Wal-Mart, Procter and Gamble, Dell, Barilla, Costco, and Campbell's Soup use VMI-CS (Cigolini et al., 2004).

In the single-channel scenario, the vendor sells the standard item to the retailer at a wholesale price c_r. The vendor ships the standard item in equal batches of size q_r to the retailer's facility every t_r units of time, where $t_r = q_r/P_r$. The retailer sells the standard item to customers at a retail price p_r, where $p_r > c_r > c_P$, and pays the vendor only when the items are withdrawn from inventory. The vendor continues producing and shipping the standard item until the retailer's inventory reaches a maximum. Figure 6.2 illustrates the behavior of the VMI-CS inventory policy at the vendor's and the retailer's side when learning and forgetting are not considered (henceforth policy 0).

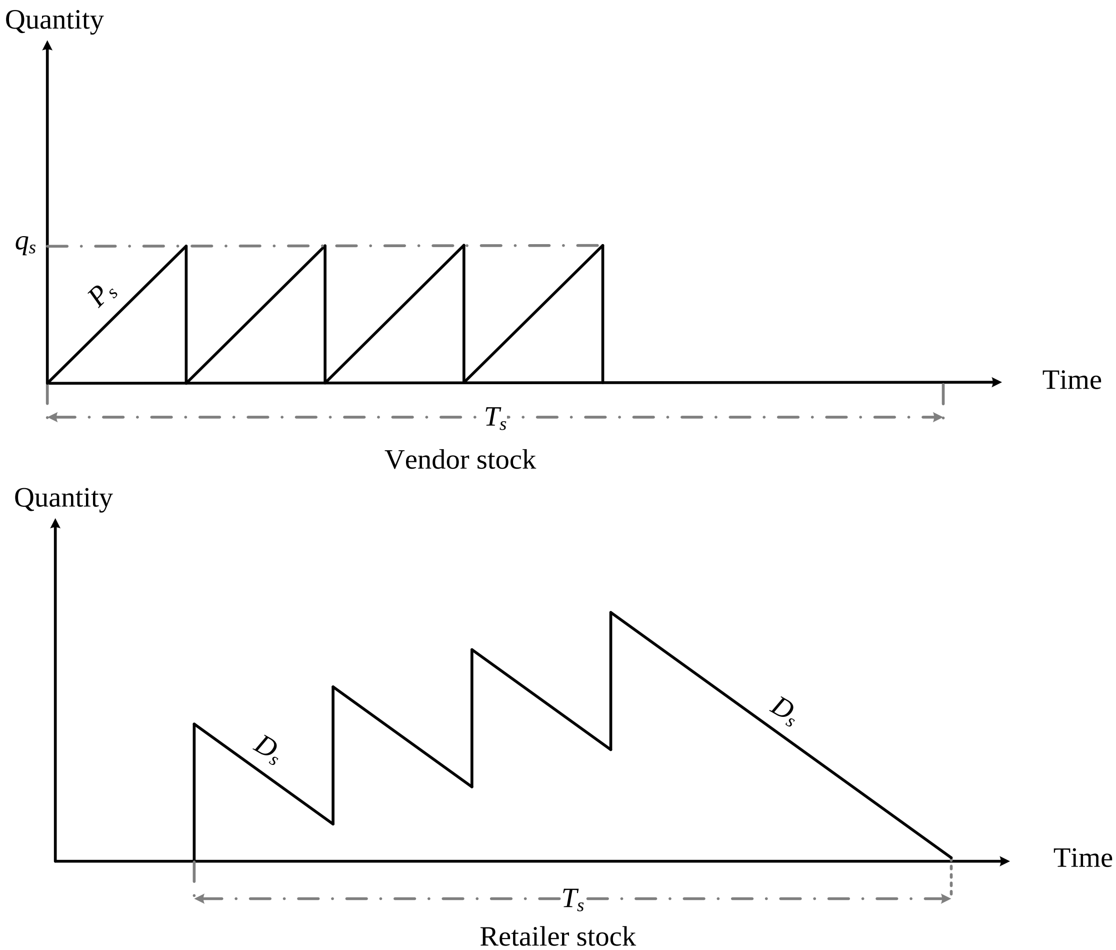

Figure 6.2 The inventory behavior of the vendor and the retailer under policy 0

Learning in production shortens the time to produce a batch and accumulates inventory at a faster rate. The vendor produces $n_{r,i}$ batches of size $q_{r,i}$ in $t_{r,i}$ units of time and delivers them to the retailer in cycle i (where $i = 1, 2, 3, ...$). Zanoni et al. (2012) used the method of Salameh et al. (1993) to calculate $t_{r,i}$, which is given by integrating Eq. (6.1) over the proper limits as:

$$t_{r,i} = \int_0^{n_{r,i}q_{r,i}} T_{r1,i}\, x^{-b_r} dx = \frac{T_{r1,i}}{1-b}\left(n_{r,i}q_{r,i}\right)^{1-b_r} \tag{6.7}$$

where $T_{r1,i} = T_{r_1}\left(u_{r,i}+1\right)^{-b_r}$. Eq. (6.7) underestimates the value of $t_{r,i}$ (Jaber and Bonney, 1998) and, therefore, has to be modified. To improve the estimation ability of (6.7), the exponent b_r has to change with every ith cycle (i.e. assuming $b_{r,i}$ instead of b_r). Jaber and Bonney (1998) provided the exact approximation of $t_{r,i}$. To find an expression for $b_{r,i}$, set Eq. (6.7), after replacing b_r with $b_{r,i}$, equal to the exact term from Jaber and Bonney (1998) as:

$$\frac{T_{r1,i}}{1-b_{r,i}}\left(n_{r,i}q_{r,i}\right)^{1-b_{r,i}} = \frac{T_{r1}}{1-b_r}\left(n_{r,i}q_{r,i}+u_{r,i}\right)^{1-b_r} - u_{r,i}^{1-b_r} \tag{6.8}$$

where $T_{r1,i} = T_{r_1}(u_i+1)^{-b_r}$. Then, solving for $b_{r,i}$ gives (refer to Appendix C.1 for details):

$$b_{r,i} = \left(\frac{-\delta \pm \sqrt{\delta^2 - 4\gamma\varepsilon}}{2\gamma}\right) \tag{6.9}$$

where,

$\gamma = 0.6443$

$\delta_i = 0.26166 - \log\left(n_{r,i}q_{r,i}\right)$

$$\varepsilon_i = 0.0143 - \left(b_r \log\left(1 + u_{r,i}\right) \log\left[\left(n_{r,i} q_{r,i} + u_{r,i}\right)^{1-b_r} - u_{r,i}^{1-b_r}\right] - \log(1 - b_r) - \log\left(n_{r,i} q_{r,i}\right)\right)$$

The parameters, γ, δ_i and ε_i are used in approximating Eq. (6.8) (refer to Appendix A for details).

Zanoni et al. (2012) suggested three production and shipment policies for the vendor. These policies differ from one another with respect to the times of shipments and batch sizes. The sequence of production and interruption periods, in turn, affect learning and forgetting. The three policies are provided below and summarized in Table 6.1.

1. Policy I: This policy releases shipments of equal sizes in equal time intervals.
2. Policy II: This policy releases shipments of unequal sizes in equal time intervals.
3. Policy III: This policy releases shipments of equal sizes in unequal time intervals.

Zanoni et al. (2012) divided policy I into three sub-policies that affect the behavior of inventory at the side of the vendor, but not the retailer except for one policy. The chapter also considers a base case policy, policy 0, with no learning and forgetting (Batarfi et al., 2016).

Table 6.1 The different policies arrangement when learning and forgetting is considered for a VMI with CS policy

Policy	Shipment batch size (q)	Shipment interval time (t)	Effect on inventory behavior		Description
			Vendor	Retailer	
I.1	Equal	Equal	✓	×	Equal shipments at equal intervals. Inventory is held for a period. Then equal lots are shipped to the retailer at the no production period. Inventory behavior of the retailer is the same as policy 0.
I.2	Equal	Equal	✓	✓	Equal shipments at equal intervals. Accumulated units are shipped at the end of the production period. Inventory behavior of the retailer is affected.
I.3	Equal	Equal	✓	×	Equal shipments at equal intervals. No accumulated inventory at the side of the vendor. Interruptions between shipments occur Inventory behavior of the retailer is the same as policy 0.
II	Different	Equal	✓	✓	Different shipments at equal intervals. Whatever the vendor accumulate, she ships. Inventory behavior of the retailer is affected.
III	Equal	Different	✓	✓	Equal shipments at different intervals. The shipments will be made more frequent. No accumulated inventory at the side of the vendor. Inventory behavior of the retailer is affected.

This chapter assumes, a price-dependent linear demand function at the single-channel strategy (retailer's side), which is expressed as (Hua et al., 2010; Huang et al., 2012, 2013; Batarfi et al., 2016):

$$D_r = a - \alpha_r p_r \tag{6.10}$$

where a ($a > 0$) represents the primary demand or the potential demand when the product is free of charge, α_r, represents the price elasticity (sensitivity) of D_r, and p_r is the retail price.

6.5.1.1 Policy 0

The behavior of inventory for policy 0 for a VMI-CS is illustrated in Figure 6.2. The total profits of the vendor, π_v^0, and the retailer, π_r^0, for policy 0 is (Batarfi et al., 2016):

$$\pi_v^0 = c_r D_r - \left(\frac{S_r D_r}{n_r q_r} + \frac{h_{v1} q_r D_r}{2P_r} + \frac{h_{v2}}{2}\left(n_r q_r - (n_r - 1)\frac{q_r D_r}{P_r}\right) + \frac{c_P D_r}{P_r}\right) \tag{6.11}$$

$$\pi_r^0 = p_r D_r - \left(\frac{O_r D_r}{q_r} + c_r D_r + \frac{h_r}{2}\left(n_r q_r - (n_r - 1)\frac{q_r D_r}{P_r}\right)\right) \tag{6.12}$$

The supply chain total profit is $\Pi_{single}^0 = \pi_v^0 + \pi_r^0$. The optimal p_r^0 that maximizes Π_{single}^0 is given by:

$$p_r^0 = \frac{A^0}{2\alpha_r} + \frac{B^0}{2\alpha_r} \tag{6.13}$$

The expression of A^0 and B^0 and the proof of concavity are reported in Appendix C.2.

6.5.1.2 Policy I.1

In this policy, the vendor produces and ships to the retailer the first $m_{r,i}$ batches. The vendor stores the remaining $n_{r,i} - m_{r,i}$ batches $(1 < m_{r,i} < n_{r,i})$ and ships them at later dates. Figure 6.3 illustrates the behavior of inventory for policy I.1, which shows the effects of learning on inventory buildup for the vendor. The behavior of inventory for the retailer is the same as for policy 0.

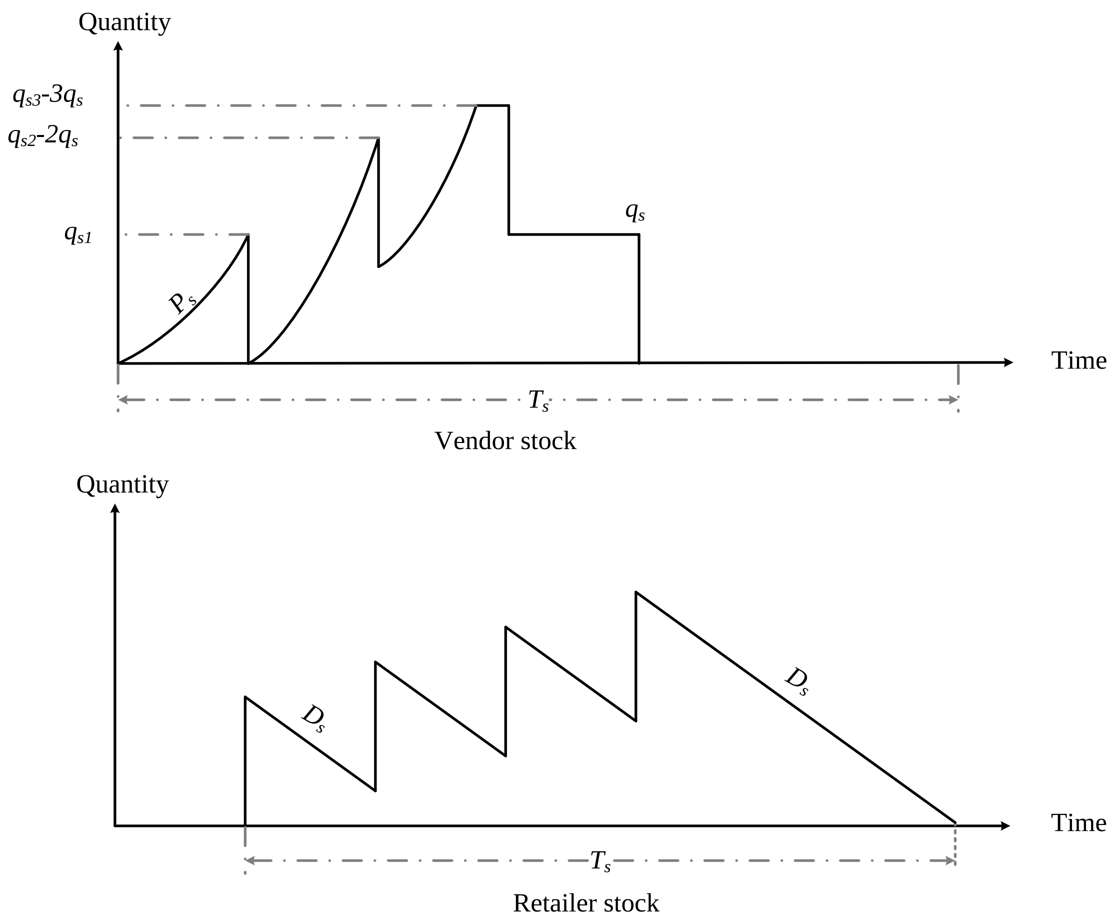

Figure 6.3 The inventory behavior of the vendor and the retailer under policy I.1

The vendor's and retailer's total profit are, $\pi_{v,i}^{\text{I.1}}$ and $\pi_{r,i}^{\text{I.1}}$, respectively, and given by:

$$\pi_{v,i}^{\mathrm{I.1}} = c_r D_{r,i} - \left(\frac{S_r D_{r,i}}{n_{r,i} q_{r,i}} + \frac{D_r}{n_{r,i} q_{r,i}} H_{v1}^{\mathrm{I.1}} + \frac{h_{v2}}{2}\left(n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right) t_{r,i} D_{r,i}\right)\right.$$
$$\left. + c_P \frac{D_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{\left(1 - b_{r,i}\right)} \left(n_{r,i} q_{r,i}\right)^{1-b_{r,i}}\right) \tag{6.14}$$

$$\pi_{r,i}^{\mathrm{I.1}} = p_{r,i} D_{r,i} - \left(\frac{O_r D_{r,i}}{q_{r,i}} + c_r D_{r,i} + \frac{h_r}{2}\left(n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right) t_{r,i} D_{r,i}\right)\right) \tag{6.15}$$

The expression of $H_{v1,i}^{\mathrm{I.1}}$ is provided in Appendix C.2. The length of the production break is: $e_{r,i}^{\mathrm{I.1}} = \frac{n_{r,i} q_{r,i}}{D_{r,i}} - \frac{T_{r1,i}}{1-b_{r,i}}\left(n_{r,i} q_{r,i}\right)^{1-b_{r,i}}$. The supply chain total profit is $\Pi_{single,i}^{\mathrm{I.1}} = \pi_{v,i}^{\mathrm{I.1}} + \pi_{r,i}^{\mathrm{I.1}}$. The optimal $p_{r,i}^{\mathrm{I.1}}$ that maximizes $\Pi_{single,i}^{\mathrm{I.1}}$ is given by:

$$p_{r,i}^{\mathrm{I.1}} = \frac{A_i^{\mathrm{I.1}}}{2\alpha_r} + \frac{B_i^{\mathrm{I.1}}}{2\alpha_r} + \frac{H_{v1,i}^{\mathrm{I.1}}}{2 n_{r,i} q_{r,i}} \tag{6.16}$$

where $A_i^{\mathrm{I.1}} = A^0$. The expression $B_i^{\mathrm{I.1}}$ and the proof of concavity are reported in Appendix C.2.

6.5.1.3 Policy I.2

This policy is similar to policy I.1 with one difference that the vendor ships the accumulated inventory as one unequal shipment by the end of the production period. Figure 6.4 illustrates the behavior of the inventory for policy I.2.

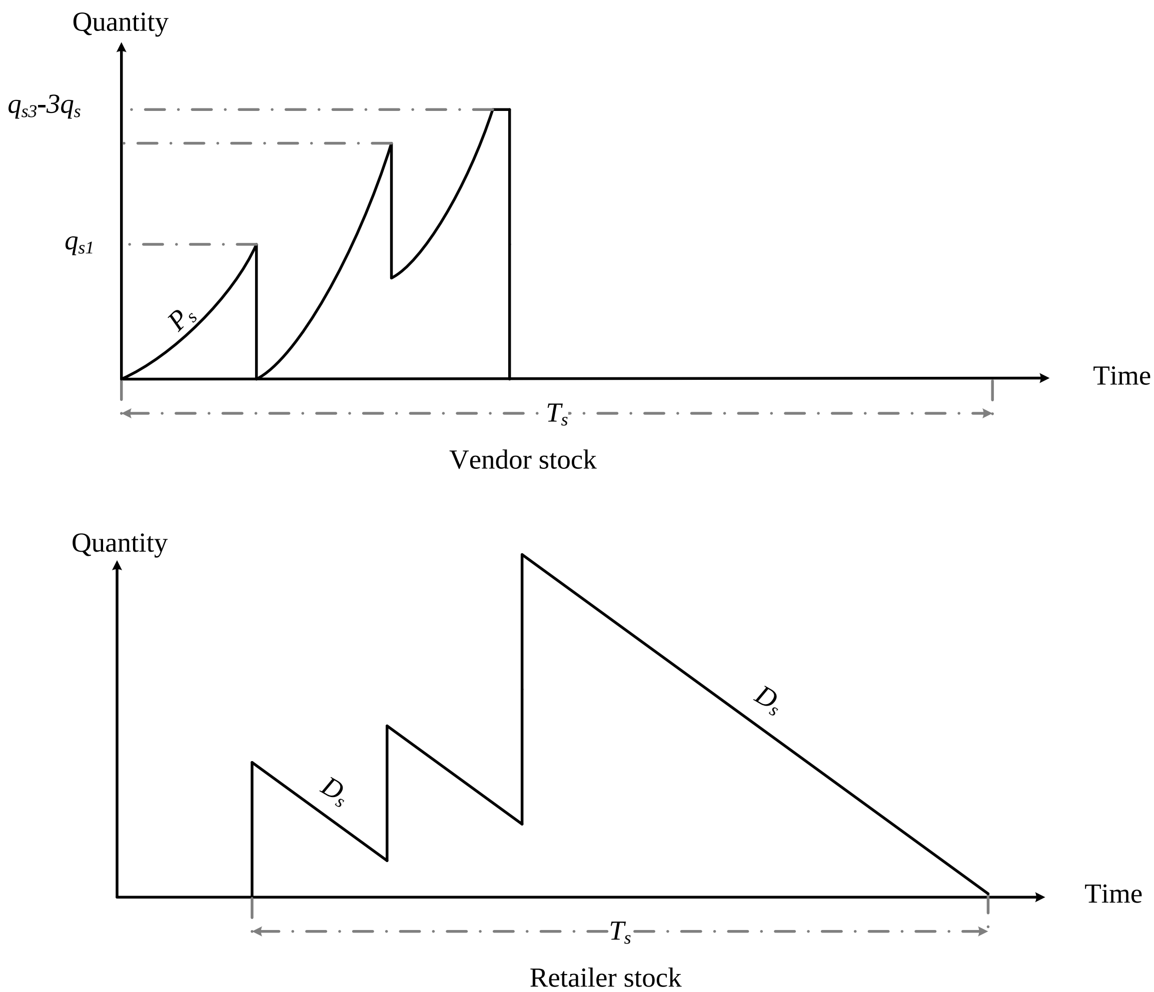

Figure 6.4 The inventory behavior of the vendor and the retailer under policy I.2

The vendor's and retailer's total profits are $\pi_{v,i}^{\mathrm{I.2}}$ and $\pi_{r,i}^{\mathrm{I.2}}$, respectively, and given by:

$$\pi_{v,i}^{\mathrm{I.2}} = c_r D_{r,i} - \left(\frac{S_r D_{r,i}}{n_{r,i} q_{r,i}} + \frac{D_{r,i}}{n_{r,i} q_{r,i}} \left(H_{v1,i}^{\mathrm{I.2}} + H_{v2,i}^{\mathrm{I.2}} \right) \right. \\ \left. + c_P \frac{D_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{\left(1 - b_{r,i}\right)} \left(n_{r,i} q_{r,i} \right)^{1 - b_{r,i}} \right) \tag{6.17}$$

$$\pi_{r,i}^{\mathrm{I.2}} = p_{r,i} D_{r,i} - \left(\frac{O_r D_{r,i}}{q_{r,i}} + c_r D_{r,i} + \frac{D_{r,i} h_r}{n_{r,i} q_{r,i} h_{v2}} H_{v2,i}^{\mathrm{I.2}} \right) \tag{6.18}$$

The expressions $H_{v1,i}^{\mathrm{I.2}}$ and $H_{v2,i}^{\mathrm{I.2}}$ are provided in Appendix C.2. The length of the production break is: $e_{r,i}^{\mathrm{I.2}} = \frac{n_{r,i} q_{r,i}}{D_{r,i}} - \frac{T_{r1,i}}{1-b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}}$. The supply chain total profit is $\Pi_{single,i}^{\mathrm{I.2}} = \pi_{v,i}^{\mathrm{I.2}} + \pi_{r,i}^{\mathrm{I.2}}$. The optimal $p_{r,i}^{\mathrm{I.2}}$ that maximizes $\Pi_{single,i}^{\mathrm{I.2}}$ is given by:

$$p_{r,i}^{\mathrm{I.2}} = \frac{A_i^{\mathrm{I.2}}}{2\alpha_r} + \frac{B_i^{\mathrm{I.2}}}{2\alpha_r} + \frac{H_{v1,i}^{\mathrm{I.2}}}{2 n_{r,i} q_{r,i}} \tag{6.19}$$

where $A_i^{\mathrm{I.2}} = A^0$. The expression $B_i^{\mathrm{I.2}}$ and the proof of concavity are reported in Appendix C.2.

6.5.1.4 Policy I.3

In this policy, the quantity produced by the vendor in $t_{r,i}$ may be large than or equal to the quantity shipped to the retailer. The vendor may choose delaying the production of subsequent shipments by τ_{m_r}, where $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_{m_r} \geq \cdots \geq \tau_{n_r}$, over which no forgetting occurs (Zanoni et al., 2012). Figure 6.5 illustrates the behavior of inventory for policy I.3, where the inventory of the retailer behaves the same as in policy 0.

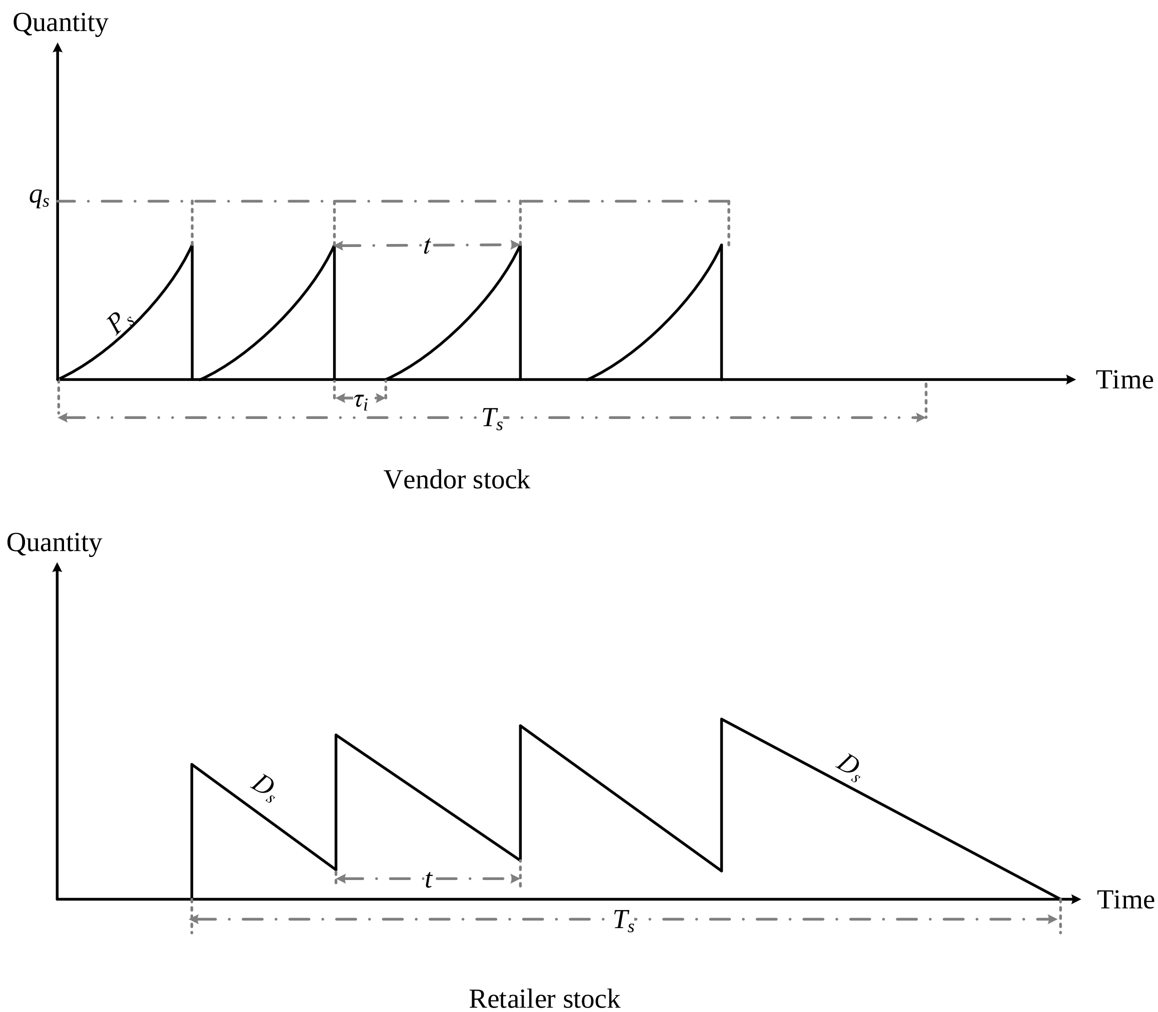

Figure 6.5 The inventory behavior of the vendor and the retailer under policy I.3

The vendor's total profit, $\pi_{v,i}^{\mathrm{I.3}}$, is given by:

$$\pi_{v,i}^{\mathrm{I.3}} = c_r D_{r,i} - \left(\frac{S_r D_{r,i}}{n_{r,i} q_{r,i}} + \frac{D_{r,i}}{n_{r,i} q_{r,i}} H_{v1,i}^{\mathrm{I.3}} + \frac{h_{v2}}{2} \left(n_{r,i} q_{r,i} - \left(n_{r,i} - 1 \right) t_{r,i} D_{r,i} \right) \right.$$
$$\left. + c_P \frac{D_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1 - b_{r,i}} \right) \tag{6.20}$$

The retailer's total profit, $\pi_{r,i}^{\mathrm{I.3}}$, is the same as in Eq. (6.15). The expression of $H_{v1,i}^{\mathrm{I.3}}$ is provided in Appendix C.2. The length of the production break is $e_{r,i}^{\mathrm{I.3}} = \frac{n_{r,i}q_{r,i}}{D_{r,i}} - n_{r,i}t_{r,i}$. The total profit of the supply chain is given by $\Pi_{single,i}^{\mathrm{I.3}} = \pi_{v,i}^{\mathrm{I.3}} + \pi_{r,i}^{\mathrm{I.3}}$. The optimal $p_{r,i}^{\mathrm{I.3}}$ that maximizes $\Pi_{single,i}^{\mathrm{I.3}}$ is given by:

$$p_{r,i}^{\mathrm{I.3}} = \frac{A_i^{\mathrm{I.3}}}{2\alpha_r} + \frac{B_i^{\mathrm{I.3}}}{2\alpha_r} + \frac{H_{v1,i}^{\mathrm{I.3}}}{2n_{r,i}q_{r,i}} \tag{6.21}$$

where $A_i^{\mathrm{I.3}} = A^0$. The expression $B_i^{\mathrm{I.3}}$ and the proof of concavity are reported in Appendix C.2.

6.5.1.5 Policy II

Due to the effect of learning, this policy suggests that the vendor ships batches of unequal sizes at equal intervals of length t_r. Compared to the traditional VMI-CS policy when learning is not considered (policy 0), this policy will affect the vendor's and the retailer's inventory behavior (see Figure 6.6).

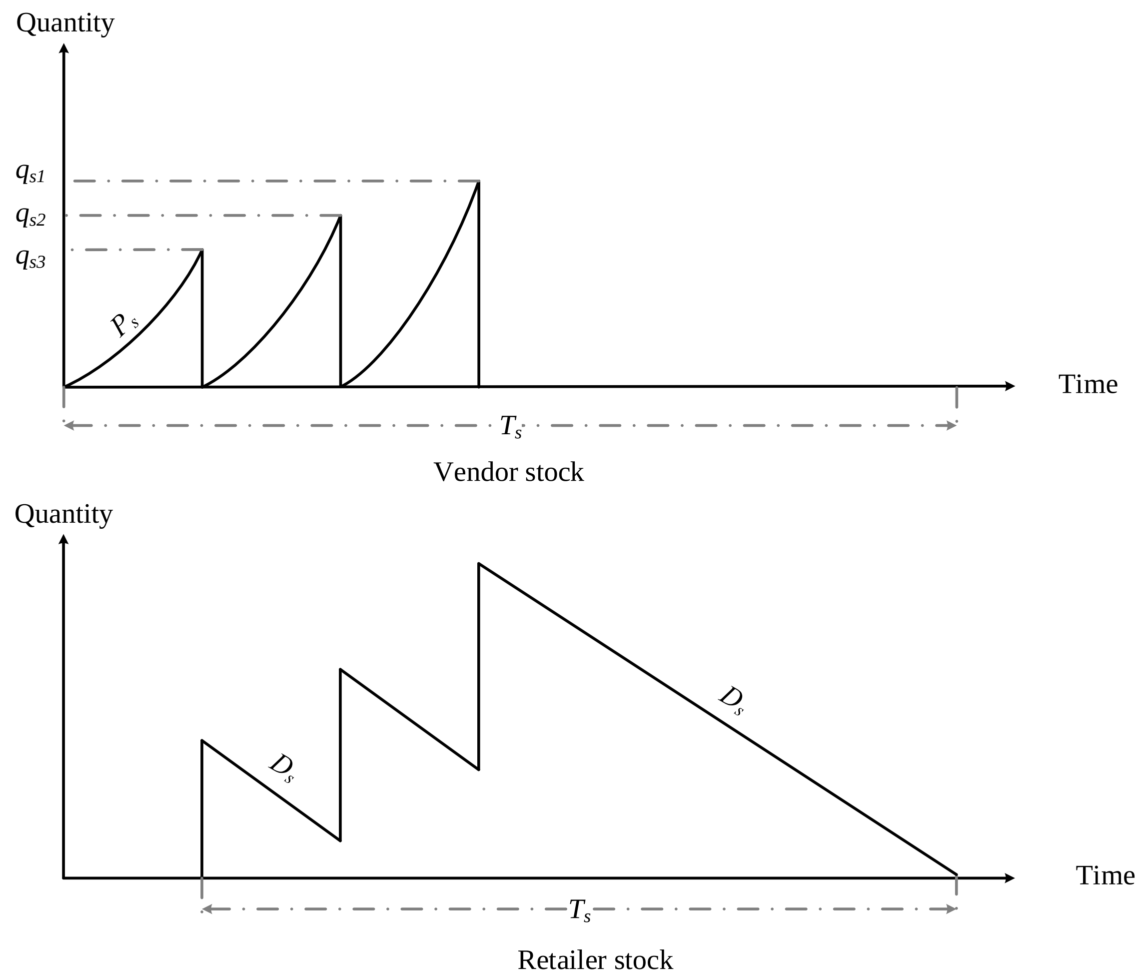

Figure 6.6 The inventory behavior of the vendor and the retailer under policy II

The vendor's and retailer's total profits are, $\pi_{v,i}^{\mathrm{II}}$, and $\pi_{r,i}^{\mathrm{II}}$, respectively, and given by:

$$\pi_{v,i}^{\mathrm{II}} = c_r D_{r,i} - \left(\frac{S_r D_{r,i}}{n_{r,i} q_{r,i}} + \frac{D_{r,i}}{n_{r,i} q_{r,i}} \left(H_{v1,i}^{\mathrm{II}} + H_{v2,i}^{\mathrm{II}} \right) + c_P \frac{D_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{\left(1 - b_{r,i}\right)} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}} \right) \tag{6.22}$$

$$\pi_{r,i}^{\mathrm{II}} = p_{r,i} D_{r,i} - \left(\frac{O_r D_{r,i}}{q_{r,i}} + c_r D_{r,i} + \frac{D_{r,i} h_r}{n_{r,i} q_{r,i} h_{v2}} H_{v2,i}^{\mathrm{II}} \right) \tag{6.23}$$

The expressions $H^{\text{II}}_{v1,i}$ and $H^{\text{II}}_{v2,i}$ are provided in Appendix C.2. The length of the production break is: $e^{\text{II}}_{r,i} = \frac{n_{r,i}q_{r,i}}{D_{r,i}} - \frac{T_{r1,i}}{1-b_{r,i}}\left(n_{r,i}q_{r,i}\right)^{1-b_{r,i}}$. The supply chain total profit is $\Pi^{\text{II}}_{single,i} = \pi^{\text{II}}_{v,i} + \pi^{\text{II}}_{r,i}$. The optimal $p^{\text{II}}_{r,i}$ that maximizes $\Pi^{\text{II}}_{single,i}$ is given by:

$$p^{\text{II}}_{r,i} = \frac{A^{\text{II}}_i}{G^{\text{II}}_i} + \frac{B^{\text{II}}_i}{G^{\text{II}}_i} + \frac{\alpha_r H^{\text{II}}_{v1,i}}{G^{\text{II}}_i n_{r,i}q_{r,i}} + \frac{\alpha_r h_{v2} F^{\text{II}}_i}{G^{\text{II}}_i n_{r,i}q_{r,i}} + \frac{\alpha_r h_r F^{\text{II}}_i}{G^{\text{II}}_i n_{r,i}q_{r,i}} \tag{6.24}$$

where $A^{\text{II}}_i = A^0$. The expressions B^{II}_i, F^{II}_i and G^{II}_i and the proof of concavity are reported in Appendix C.2.

6.5.1.6 Policy III

This policy suggests that the vendor ships batches of equal sizes in an unequal time basis (decreasing times interval due to learning effects). As cab be seen from Figure 6.7, the shipment mechanism of this policy affects the behavior of the vendor and the retailer.

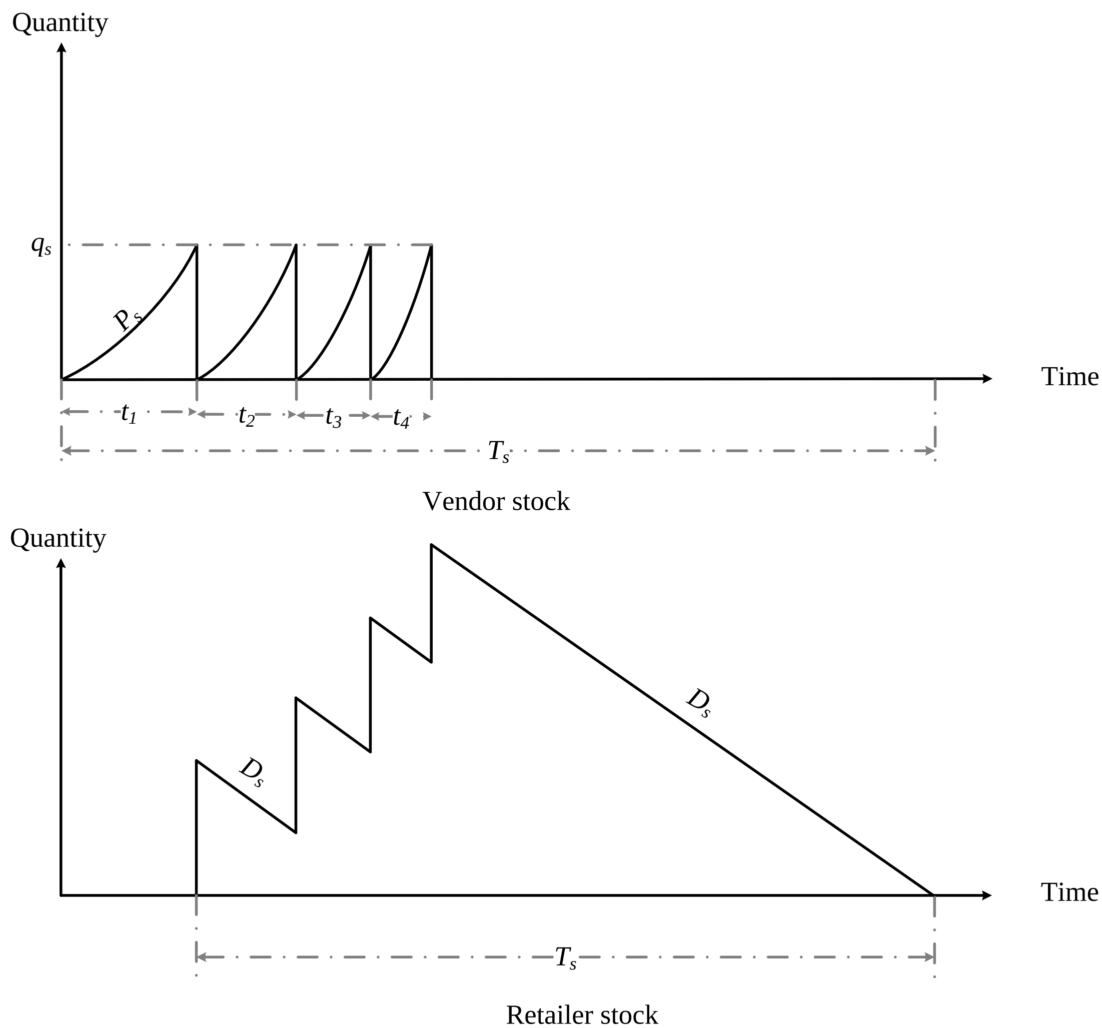

Figure 6.7 The inventory behavior of the vendor and the retailer under policy III

The vendor's and retailer's total profits are $\pi_{v,i}^{\mathrm{III}}$ and $\pi_{r,i}^{\mathrm{III}}$, respectively, and given by:

$$\pi_{v,i}^{\mathrm{III}} = c_r D_{r,i} - \left(\frac{S_r D_{r,i}}{n_{r,i} q_{r,i}} + \frac{D_{r,i}}{n_{r,i} q_{r,i}} \left(H_{v1,i}^{\mathrm{III}} + H_{v2,i}^{\mathrm{III}} \right) + c_P \frac{D_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}} \right) \tag{6.25}$$

$$\pi_{r,i}^{\mathrm{III}} = p_{r,i}D_{r,i} - \left(\frac{O_r D_{r,i}}{q_{r,i}} + c_r D_{r,i} + \frac{D_{r,i}h_r}{n_{r,i}q_{r,i}h_{v2}} H_{v2,i}^{\mathrm{III}}\right) \tag{6.26}$$

The expressions $H_{v1,i}^{\mathrm{III}}$ and $H_{v2,i}^{\mathrm{III}}$ are provided in Appendix C.2. The length of the production break is $e_{r,i}^{\mathrm{III}} = \frac{n_{r,i}q_{r,i}}{D_{r,i}} - \frac{T_{r1,i}}{1-b_{r,i}}(n_r q_r)^{1-b_{r,i}}$. The total supply chain profit is $\Pi_{single,i}^{\mathrm{III}} = \pi_{v,i}^{\mathrm{III}} + \pi_{r,i}^{\mathrm{III}}$. The optimal $p_{r,i}^{\mathrm{III}}$ that maximizes $\Pi_{single,i}^{\mathrm{III}}$ is given by:

$$p_{r,i}^{\mathrm{III}} = \frac{A_i^{\mathrm{III}}}{2\alpha_r(1-G_i^{\mathrm{III}})} + \frac{B_i^{\mathrm{III}}}{2\alpha_r(1-G_i^{\mathrm{III}})} + \frac{H_{v1,i}^{\mathrm{III}}}{2n_{r,i}q_{r,i}(1-G_i^{\mathrm{III}})} + \frac{F_i^{\mathrm{III}}(h_{v2}+h_r)}{2n_{r,i}q_{r,i}(1-G_i^{\mathrm{III}})} - \frac{aG_i^{\mathrm{III}}}{\alpha_r(1-G_i^{\mathrm{III}})} \tag{6.27}$$

where $A_i^{\mathrm{III}} = A^0$. The expressions B_i^{III}, F_i^{III} and G_i^{III} and the proof of concavity are reported in Appendix C.2.

6.5.2 Dual-channel strategy

Now, an online channel is added to the retail channel to form a dual-channel. The objective is to see how adding an online channel affect the pricing and inventory decisions and subsequently supply chain profitability. The vendor sells a standard item (retail channel) and a customized item (online channel). The standard item follows the inventory policies of Section 6.5.1. The customized item follows BTO. Customers select, from an online list, features (grouped in sets) to add to the core item. Once an online order is received by the vendor, the core item is instantly customized and shipped to the customer. No back-ordering or work-in-progress inventory situations are assumed. This is common MC industries (e.g., Dell) with BTO (Mukhopadhyay and Setoputro, 2005). Core items follow the economic production quantity (EPQ) model.

Custom features are shipped to the vendor on as-needed basis indicating zero inventory (Batarfi et al., 2016). V_N represent a set of variants, where $k \in V_N$ and $k = 1,2,\dots,N$. Each variant k represents one core item and at least one additional custom feature, z, where $z = 1,2,\dots,Z$. The production cost, c_{dk}, and the selling price, p_{dk}, of a unit of variant k depends on the number of z feature(s) added to the core item. It is assumed that c_{dk} includes the cost of one core item, the cost of the custom feature(s) and a fixed processing cost for each customized item. To avoid trivial problems, assuming $p_{dk} > c_r$, the wholesale price of the standard item is less than the selling price of a customized item. This is not an arbitrary condition, because if $p_{dk} < c_r$, the retailer or any other arbitrator can obtain the item from the vendor's online channel at a lower price (Hua et al., 2010; Huang et al., 2012, 2013).

In a traditional EPQ model (no learning and forgetting effects) the vendor produces q_d units of the core item at a production rate P_d in T_d units of time, where $T_d = q_d/D_d$ and D_d is the demand of the online channel. Therefore, the level of inventory of the core item will increase at a rate of $P_d - D_d$. The vendor will continue production until the inventory level reaches a maximum of $q_d(1 - D_d/P_d)$. The core item is consumed during the production run, and the vendor will start production in the next cycle after the inventory level has been depleted.

When learning and forgetting are considered in the production run of the core items, the EPQ inventory behavior is affected (see Figure 6.8). At the beginning of each interval T_d, the inventory of the core item builds up at a rate P_d while depleting at a constant rate of $D_{d,i}$. $t_{d,i}$ is defined as the time required to produce $q_{d,i}$ units of core items until the inventory reaches a maximum level of X_i, and $t_{c,i}$ as the time required to deplete X_i, where $T_d = t_{d,i} + t_{c,i}$. The cumulative time to produce $q_{d,i}$ is computed from Jaber and Bonney (1998) as:

$$t = t_{d,i} = \int_0^{q_{d,i}+u_{d,i}} T_{d1}\, x^{-b} dx = \frac{T_{d1}}{1-b_d}\left[\left(q_{r,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}\right] \tag{6.28}$$

Solving for q_{di} in Eq. (6.28), we get

$$q_{d,i}(t_{d,i}) = \left(\frac{1-b_d}{T_{d1}} t_{d,i} + u_{d,i}\right)^{\frac{1}{1-b_d}} - u_{d,i} \tag{6.29}$$

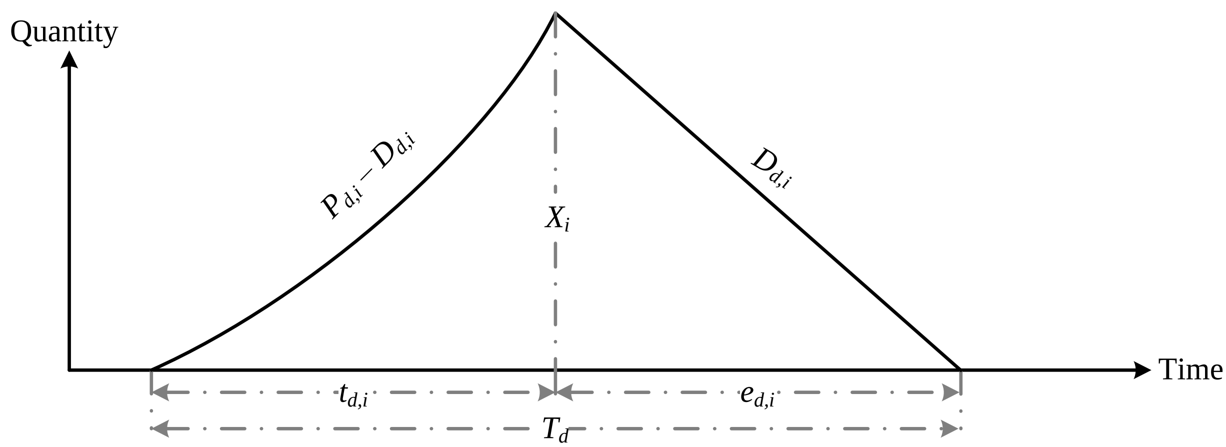

Figure 6.8 The inventory behavior of the vendor for the core customizable item with learning and forgetting effects

Due to customers' heterogeneity in their preferences between the standard and the customized items, it is assumed that there are two major factors affecting the demands for the retail channel and the online channel: the first is the selling prices of the two items (i.e. standard and customized), and the second is the quoted delivery lead-time of the customized items, l_d. Following Hua et al. (2010), Huang et al. (2013, 2012), and Batarfi et al. (2016), a linear demand function is assumed. Specifically, the demand function of the two channels take the following forms:

$$\overline{D}_r = (1-\theta)a - \alpha_r p_r + \rho \sum_{k=1}^{N} p_{dk} + \beta_r l_d \tag{6.30}$$

$$D_d = \theta a - \sum_{k=1}^{N} \alpha_{dk}\, p_{dk} + \rho\, p_r - \beta_d l_d \tag{6.31}$$

where $\overline{D}_r$ and D_d represent the demands of the retail channel and the online channel, respectively. The term θ $(0 < \theta < 1)$, represents the share of the primary demand, a, going to the online channel, which is referred to in the literature as customer's acceptance (preference) of the online channel, whereas the term $(1-\theta)$ is the share of the primary demand going to the retail channel. The two parameters α_r and α_{dk} represent the price elasticity (sensitivity) of p_r and p_{dk}, respectively. The price elasticity here is defined as the amount of decrease (increase) in the market demand when the price increases (decreases) by one dollar. The cross price sensitivity, ρ, which reflects the degree to which the two items sold through the two channels are substitutable, is assumed to be symmetric. It has to be noted that $\alpha_r > \rho$ and $\alpha_{dk} > \rho$, indicating that the self-price effects are greater than the cross-price effects, a very common assumption used in the economic and operations management literature (Huang et al., 2013).

The quoted delivery lead-time of the online channel, l_d, is defined as the time that a customer needs to wait from the time an order is placed through the online channel to the time that the item is delivered to the customer (Webster, 2002). The parameters β_r and β_d are the elasticity of the quoted delivery lead-time to changes in l_d of the two demands (retail channel and the online channel), respectively. For example, when l_d increases by one unit, β_d units of D_d will be lost, of which β_r units of that will be transferred to $\overline{D}_r$ and $\beta_d - \beta_r$ $(\beta_r < \beta_d)$ units will be lost to both channels (Hua et al., 2010).

In the next subsections, we reinvestigate the total profits of both firms (the vendor and the retailer) when the dual-channel strategy is adopted. The investigation is carried out for all policies discussed in Section 6.5.1. It has to be noted that the vendor's profit in all reinvestigated policies is majorly affected due to the adoption of the dual-channel strategy, whereas the only difference between the retailer's profit in the single channel strategy and the dual-channel strategy is just in the demand function (i.e. D_r in the single-channel strategy is replace with $\overline{D}_r$ of the dual-channel strategy).

6.5.2.1 Policy 0

The vendor's adjusted total profit for policy 0 is, $\bar{\pi}_v^0$, (i.e. no learning and forgetting effects) and is given by

$$\bar{\pi}_v^0 = \sum_{k=1}^{N} p_{dk}\varphi_{dk}D_d + c_r\overline{D}_r - \left(\frac{S_r\overline{D}_r}{n_r q_r} + \frac{S_d D_d}{q_d} + \frac{h_{v1}q_r\,\overline{D}_r}{2P_r} + \frac{h_{v2}}{2}\left(n_r q_r - (n_r-1)\frac{q_r\overline{D}_r}{P_r}\right) + \left(\frac{h_{v1}\,q_d}{2}\right)\left(1-\frac{D_d}{P_d}\right) + \frac{c_P\overline{D}_r}{P_r} + \sum_{k=1}^{N} c_{dk}\,\varphi_{dk}\,\frac{D_d}{P_d}\right) \tag{6.32}$$

The retailer's adjusted total profit, $\bar{\pi}_r^0$, in this policy is the same as in Eq. (6.12), except for replacing D_r with $\overline{D}_r$. The supply chain total profit is $\Pi_{dual}^0 = \bar{\pi}_v^0 + \bar{\pi}_r^0$, which is maximized for prices $\bar{p}_r^0$ and p_{dk}^0 given, respectively, as:

$$\bar{p}_r^0 = \sum_{k=1}^{N}\left(\frac{\bar{A}^0}{2\alpha_r\bar{G}^0} + \frac{\rho\bar{B}^0}{4\alpha_r a_{dk}\bar{G}^0} + \frac{\rho\bar{B}^0}{4\alpha_r a_{dk}\varphi_{dk}\bar{G}^0}\right) \tag{6.33}$$

$$\sum_{k=1}^{N} p_{dk}^{0} = \sum_{k=1}^{N} \left(\frac{\bar{B}^0}{2a_{dk}\varphi_{dk}\bar{G}^0} + \frac{\rho\bar{A}^0}{4\alpha_r a_{dk}\bar{G}^0} + \frac{\rho N\bar{A}^0}{4\alpha_r a_{dk}\varphi_{dk}\bar{G}^0} \right) \tag{6.34}$$

The expression $\overline{A}^0$, $\bar{B}^0$ and $\bar{G}^0$ and the proof of concavity are reported in Appendix C.3

6.5.2.2 Policy I.1

The vendor's adjusted total profit is, $\bar{\pi}_{v,i}^{\mathrm{I.1}}$, and is given by:

$$\begin{aligned} \bar{\pi}_{v,i}^{\mathrm{I.1}} = \sum_{k=1}^{N} p_{dk,i}\varphi_{dk}D_{d,i} + c_r\overline{D}_{r,i} \\ - \Bigg(\frac{S_r\overline{D}_{r,i}}{n_{r,i}q_{r,i}} + \frac{S_d D_{d,i}}{q_{d,i}} + \frac{\overline{D}_{r,i}}{n_{r,i}q_{r,i}} H_{v1,i}^{\mathrm{I.1}} \\ + \frac{h_{v2}}{2}\left(n_{r,i}q_{r,i} - (n_{r,i}-1)t_{r,i}\overline{D}_{r,i}\right) + \frac{D_{d,i}}{q_{d,i}} H_{vd,i} \\ + c_P \frac{\overline{D}_{r,i}}{n_{r,i}q_{r,i}} \frac{T_{r1,i}}{1-b_{r,i}} \left(n_{r,i}q_{r,i}\right)^{1-b_{r,i}} \\ + \sum_{k=1}^{N} c_{dk}\,\varphi_{dk}D_{d,i}\,\frac{T_{d1}}{1-b_d}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right]\Bigg) \end{aligned} \tag{6.35}$$

The calculation of $H_{vd,i}$ is given in Appendix C.3. The retailer's adjusted total profit, $\bar{\pi}_{r,i}^{\mathrm{I.1}}$, is given in Eq. (6.15), where $D_{r,i}$ is replaced by $\overline{D}_{r,i}$. The supply chain total profit is $\Pi_{dual,i}^{\mathrm{I.1}} = \bar{\pi}_{v,i}^{\mathrm{I.1}} + \bar{\pi}_{r,i}^{\mathrm{I.1}}$, which is maximized for prices $\bar{p}_{r,i}^{\mathrm{I.1}}$ and $p_{dk,i}^{\mathrm{I.1}}$ given, respectively, as:

$$\bar{p}_{r,i}^{\mathrm{I.1}} = \sum_{k=1}^{N} \left(\frac{\bar{A}_i^{\mathrm{I.1}}}{2\alpha_r \bar{G}_i^{\mathrm{I.1}}} + \frac{\rho \bar{B}_i^{\mathrm{I.1}}}{4\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.1}}} + \frac{\rho \bar{B}_i^{\mathrm{I.1}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.1}}} + \frac{H_{v1,i}^{\mathrm{I.1}}}{2n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.1}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.1}}}{4\alpha_r a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.1}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.1}}}{4\alpha_r a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.1}}} \right) \tag{6.36}$$

$$\sum_{k=1}^{N} p_{dk,i}^{\mathrm{I.1}} = \sum_{k=1}^{N} \left(\frac{\bar{B}_i^{\mathrm{I.1}}}{2a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.1}}} + \frac{\rho \bar{A}_i^{\mathrm{I.1}}}{4\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.1}}} + \frac{\rho N \bar{A}_i^{\mathrm{I.1}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.1}}} + \frac{\rho H_{v1,i}^{\mathrm{I.1}}}{4a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.1}}} - \frac{\rho N H_{v1,i}^{\mathrm{I.1}}}{4a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.1}}} \right) \tag{6.37}$$

The formulas for $\bar{A}_i^{\mathrm{I.1}}$, $\bar{B}_i^{\mathrm{I.1}}$ and $\bar{G}_i^{\mathrm{I.1}}$ and the proof of concavity are reported in Appendix C.3.

6.5.2.3 Policy I.2

The vendor's adjusted total profit is $\bar{\pi}_{v,i}^{\mathrm{I.2}}$, and is given by:

$$\begin{aligned} \bar{\pi}_{v,i}^{\mathrm{I.2}} = \sum_{k=1}^{N} p_{dk,i}\, \varphi_{dk} D_{d,i} + c_r \bar{D}_{r,i} & \\ - \Bigg(\frac{S_r \bar{D}_{r,i}}{n_{r,i} q_{r,i}} + \frac{S_d D_{d,i}}{q_{d,i}} + \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} \left(H_{v1,i}^{\mathrm{I.2}} + H_{v2,i}^{\mathrm{I.2}} \right) + \frac{D_{d,i}}{q_{d,i}} H_{vd,i} & \\ + c_P \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1 - b_{r,i}} & \\ + \sum_{k=1}^{N} c_{dk}\, \varphi_{dk} D_{d,i}\, \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i} \right)^{1 - b_d} - u_{d,i}^{1 - b_d}}{q_{d,i}} \right] \Bigg) & \end{aligned} \tag{6.38}$$

The retailer's adjusted total profit, $\bar{\pi}_{r,i}^{\mathrm{I.2}}$, is given by Eq. (6.18), where $D_{r,i}$ is replaced by $\bar{D}_{r,i}$. The supply chain total profit is $\Pi_{dual,i}^{\mathrm{I.2}} = \bar{\pi}_{v,i}^{\mathrm{I.2}} + \bar{\pi}_{r,i}^{\mathrm{I.2}}$, which is maximized for prices $\bar{p}_{r,i}^{\mathrm{I.2}}$ and $p_{dk,i}^{\mathrm{I.2}}$ given, respectively, as:

$$\bar{p}_{r,i}^{\mathrm{I.2}} = \sum_{k=1}^{N} \left(\frac{\bar{A}_{\mathrm{i}}^{\mathrm{I.2}}}{2\alpha_r \bar{G}_i^{\mathrm{I.2}}} + \frac{\rho \bar{B}_i^{\mathrm{I.2}}}{4\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.2}}} + \frac{\rho \bar{B}_i^{\mathrm{I.2}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.2}}} + \frac{H_{v1,i}^{\mathrm{I.2}}}{2n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.2}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.2}}}{4\alpha_r a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.2}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.2}}}{4\alpha_r a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.2}}} \right) \tag{6.39}$$

$$\sum_{k=1}^{N} p_{dk,i}^{\mathrm{I.2}} = \sum_{k=1}^{N} \left(\frac{\bar{B}_i^{\mathrm{I.2}}}{2a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.2}}} + \frac{\rho \bar{A}_i^{\mathrm{I.2}}}{2\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.2}}} + \frac{\rho N \bar{A}_i^{\mathrm{I.2}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.2}}} + \frac{\rho H_{v1,i}^{\mathrm{I.2}}}{4a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.2}}} - \frac{\rho N H_{v1,i}^{\mathrm{I.2}}}{4a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.2}}} \right) \tag{6.40}$$

The formulas for $\bar{A}_i^{\mathrm{I.2}}$, $\bar{B}_i^{\mathrm{I.2}}$ and $\bar{G}_i^{\mathrm{I.2}}$ and the proof of concavity are reported in Appendix C.3.

6.5.2.4 Policy I.3

The vendor's adjusted total profit is $\bar{\pi}_{v,i}^{\mathrm{I.3}}$, and is given by:

$$\begin{aligned}\bar{\pi}_{v,i}^{\mathrm{I.3}} = \sum_{k=1}^{N} p_{dk,i}\, \varphi_{dk} D_{d,i} + c_r \bar{D}_{r,i} &- \Bigg(\frac{S_r \bar{D}_{r,i}}{n_{r,i} q_{r,i}} + \frac{S_d D_{d,i}}{q_{d,i}} + \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} H_{v1,i}^{\mathrm{I.3}} + \frac{h_{v2}}{2} \left(n_{r,i} q_{r,i} - \left(n_{r,i} - 1 \right) t_r \bar{D}_r \right) \\ &+ \frac{D_{d,i}}{q_{d,i}} H_{vd,i} + c_P \frac{\bar{D}_r}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}} \\ &+ \sum_{k=1}^{N} c_{dk}\, \varphi_{dk} D_{d,i}\, \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i} \right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right] \Bigg)\end{aligned} \tag{6.41}$$

The retailer's adjusted total profit, $\bar{\pi}_{r,i}^{\mathrm{I.3}}$, is given by Eq. (6.15), where $D_{r,i}$ is replaced by $\bar{D}_{r,i}$. The supply chain total profit is $\Pi_{dual,i}^{\mathrm{I.3}} = \bar{\pi}_{v,i}^{\mathrm{I.3}} + \bar{\pi}_{r,i}^{\mathrm{I.3}}$, which is maximized for prices $\bar{p}_{r,i}^{\mathrm{I.3}}$ and $p_{dk,i}^{\mathrm{I.3}}$, given, respectively, as:

$$\bar{p}_{r,i}^{\mathrm{I.3}} = \sum_{k=1}^{N} \left(\frac{\bar{A}_i^{\mathrm{I.3}}}{2\alpha_r \bar{G}_i^{\mathrm{I.3}}} + \frac{\rho \bar{B}_i^{\mathrm{I.3}}}{4\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.3}}} + \frac{\rho \bar{B}_i^{\mathrm{I.3}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.3}}} + \frac{H_{v1,i}^{\mathrm{I.3}}}{2n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.3}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.3}}}{4\alpha_r a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.3}}} - \frac{\rho^2 N H_{v1,i}^{\mathrm{I.3}}}{4\alpha_r a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.3}}} \right) \tag{6.42}$$

$$\sum_{k=1}^{N} p_{dk,i}^{\mathrm{I.3}} = \sum_{k=1}^{N} \left(\frac{\bar{B}_i^{\mathrm{I.3}}}{2a_k \varphi_{dk} \bar{G}_i^{\mathrm{I.3}}} + \frac{\rho \bar{A}_i^{\mathrm{I.3}}}{4\alpha_r a_{dk} \bar{G}_i^{\mathrm{I.3}}} + \frac{\rho N \bar{A}_i^{\mathrm{I.3}}}{4\alpha_r a_{dk} \varphi_{dk} \bar{G}_i^{\mathrm{I.3}}} + \frac{\rho H_{v1,i}^{\mathrm{I.3}}}{4a_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.3}}} - \frac{\rho N H_{v1,i}^{\mathrm{I.3}}}{4a_{dk} \varphi_{dk} n_{r,i} q_{r,i} \bar{G}_i^{\mathrm{I.3}}} \right) \tag{6.43}$$

The formula for $\bar{A}_i^{\mathrm{I.3}}$, $\bar{B}_i^{\mathrm{I.3}}$ and $\bar{G}_i^{\mathrm{I.3}}$ and the proof of concavity are reported in Appendix C.3.

6.5.2.5 Policy II

The vendor's adjusted total profit, is $\bar{\pi}_{v,i}^{\mathrm{II}}$, and is given by:

$$\begin{aligned} \bar{\pi}_v^{\mathrm{II}} = \sum_{k=1}^{N} p_{dk,i}\, \varphi_{dk} D_{d,i} + c_r \bar{D}_{r,i} & - \left(\frac{S_r \bar{D}_{r,i}}{n_{r,i} q_{r,i}} + \frac{S_d D_{d,i}}{q_{d,i}} + \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} \left(H_{v1,i}^{\mathrm{II}} + H_{v2,i}^{\mathrm{II}} \right) + \frac{D_{d,i}}{q_{d,i}} H_{vd,i} \right. \\ & + c_P \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}} \\ & \left. + \sum_{k=1}^{N} c_{dk}\, \varphi_{dk} D_{d,i}\, \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i} \right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right] \right) \end{aligned} \tag{6.44}$$

The retailer's adjusted total profit, $\bar{\pi}_{r,i}^{\mathrm{II}}$, is given by Eq. (6.23), where $D_{r,i}$ is replaced by $\bar{D}_{r,i}$. The supply chain total profit is $\Pi_{dual,i}^{\mathrm{II}} = \bar{\pi}_{v,i}^{\mathrm{II}} + \bar{\pi}_{r,i}^{\mathrm{II}}$, which is maximized for prices $\bar{p}_{r,i}^{\mathrm{II}}$ and $p_{dk,i}^{\mathrm{II}}$ given, respectively, as:

$$\bar{p}^{\mathrm{II}}_{r,i} = \sum_{k=1}^{N}\left(\frac{n_{r,i}^2 q_{r,i}^2 \bar{A}_i^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} + n_{r,i} q_{r,i} \bar{B}_i^{\mathrm{II}} \bar{E}_{r,i}^{\mathrm{II}}}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} + \frac{H_{v1,i}^{\mathrm{II}}\left(\alpha_r n_{r,i} q_{r,i} \bar{G}_{d,i}^{\mathrm{II}} - \rho N \bar{E}_{r,i}^{\mathrm{II}}\right)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} - \frac{\rho N \bar{F}_i^{\mathrm{II}} \bar{E}_{r,i}^{\mathrm{II}}(h_{v2} + h_r)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} + \frac{\alpha_r n_{r,i} q_{r,i} \bar{F}_i^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}}\,(h_{v2} + h_r)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}}\right) \tag{6.45}$$

$$\sum_{k=1}^{N} p_{dk,i}^{\mathrm{II}} = \sum_{k=1}^{N}\left(\frac{n_{r,i} q_{r,i} \bar{A}_i^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}} + n_{r,i}^2 q_{r,i}^2 \bar{B}_i^{\mathrm{II}} \bar{G}_{r,i}^{\mathrm{II}}}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} + \frac{H_{v1,i}^{\mathrm{II}}\left(\alpha_r \bar{E}_{d,i}^{\mathrm{II}} - \rho N n_{r,i} q_{r,i} \bar{G}_{r,i}^{\mathrm{II}}\right)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} + \frac{\alpha_r \bar{F}_i^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}(h_{v2} + h_r)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}} - \frac{\rho N n_{r,i} q_{r,i} \bar{F}_i^{\mathrm{II}} \bar{G}_{r,i}^{\mathrm{II}}(h_{v2} + h_r)}{n_{r,i}^2 q_{r,i}^2 \bar{G}_{r,i}^{\mathrm{II}} \bar{G}_{d,i}^{\mathrm{II}} - \bar{E}_{r,i}^{\mathrm{II}} \bar{E}_{d,i}^{\mathrm{II}}}\right) \tag{6.46}$$

The formulas for $\bar{A}_i^{\mathrm{II}}$, $\bar{B}_i^{\mathrm{II}}$, $\bar{E}_{r,i}^{\mathrm{II}}$, $\bar{E}_{d,i}^{\mathrm{II}}$, $\bar{F}_i^{\mathrm{II}}$, $\bar{G}_{r,i}^{\mathrm{II}}$ and $\bar{G}_{d,i}^{\mathrm{II}}$ and the proof of concavity are reported in Appendix C.3.

6.5.2.6 Policy III

The vendor's adjusted total profit is $\bar{\pi}_{v,i}^{\mathrm{III}}$, and is given by:

$$\bar{\pi}_{v,i}^{\mathrm{III}} = \sum_{k=1}^{N} p_{dk,i}\,\varphi_{dk} D_{d,i} + c_r \bar{D}_{r,i} - \left(\frac{S_r \bar{D}_{r,i}}{n_{r,i} q_{r,i}} + \frac{S_d D_{d,i}}{q_{d,i}} + \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}}\left(H_{v1,i}^{\mathrm{II}} + H_{v2,i}^{\mathrm{II}}\right) + \frac{D_{d,i}}{q_{d,i}} H_{vd,i} + c_P \frac{\bar{D}_{r,i}}{n_{r,i} q_{r,i}} \frac{T_{r1,i}}{1 - b_{r,i}} \left(n_{r,i} q_{r,i}\right)^{1-b_{r,i}} + \sum_{k=1}^{N} c_{dk}\,\varphi_{dk} D_{d,i}\,\frac{T_{d1}}{1 - b_d}\left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right]\right) \tag{6.47}$$

The retailer's adjusted total profit, $\bar{\pi}_{r,i}^{\mathrm{III}}$, is given by Eq. (6.26), where $D_{r,i}$ is replaced by $\bar{D}_{r,i}$. The supply chain total profit is $\Pi_{dual,i}^{\mathrm{III}} = \bar{\pi}_{v,i}^{\mathrm{III}} + \bar{\pi}_{r,i}^{\mathrm{III}}$, which is maximized for prices $\bar{p}_{r,i}^{\mathrm{III}}$ and $p_{dk,i}^{\mathrm{III}}$ given, respectively, as:

$$
\begin{aligned}
\bar{p}_{r,i}^{\mathrm{III}} = \sum_{k=1}^{N} \frac{1}{\left(1-\bar{G}_{r,i}^{\mathrm{III}}\bar{G}_{d,i}^{\mathrm{III}}\right)} & \left(\frac{\bar{A}_i^{\mathrm{III}}}{2\alpha_r\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} - \frac{2\left((1-\theta)a+\beta_r l_d\right)\bar{E}_{r,i}^{\mathrm{III}}}{2\alpha_r\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} \right. \\
& \left. + \frac{H_{v1,i}^{\mathrm{III}}}{2n_{r,i}q_{r,i}\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} + \frac{\bar{F}_i^{\mathrm{III}}(h_{v2}+h_r)}{2n_{r,i}q_{r,i}\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} \right) \\
& + \frac{\bar{G}_{r,i}^{\mathrm{III}}}{\left(1-\bar{G}_{r,i}^{\mathrm{III}}\bar{G}_{d,i}^{\mathrm{III}}\right)} \left(\frac{\bar{B}_i^{\mathrm{III}}}{2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}} + \frac{2\left((1-\theta)a+\beta_r l_d\right)\bar{E}_{d,i}^{\mathrm{III}}}{2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}} \right. \\
& \left. - \frac{\rho N H_{v1,i}^{\mathrm{III}}}{n_{r,i}q_{r,i}\left(2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}\right)} - \frac{\rho N \bar{F}_i^{\mathrm{III}}(h_{v2}+h_r)}{n_{r,i}q_{r,i}\left(2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}\right)} \right)
\end{aligned}
\tag{6.48}
$$

$$
\begin{aligned}
\sum_{k=1}^{N} p_{dk,i}^{\mathrm{III}} = \sum_{k=1}^{N} \frac{\bar{G}_{d,i}^{\mathrm{III}}}{(1-\bar{G}_r^{\mathrm{III}}\bar{G}_d^{\mathrm{III}})} & \left(\frac{\bar{A}_i^{\mathrm{III}}}{2\alpha_r\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} - \frac{2\left((1-\theta)a+\beta_r l_d\right)\bar{E}_{r,i}^{\mathrm{III}}}{2\alpha_r\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} \right. \\
& \left. + \frac{H_{v1,i}^{\mathrm{III}}}{n_{r,i}q_{r,i}\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} + \frac{\bar{F}_i^{\mathrm{III}}(h_{v2}+h_r)}{2n_{r,i}q_{r,i}\left(1-\bar{E}_{r,i}^{\mathrm{III}}\right)} \right) \\
& + \frac{1}{\left(1-\bar{G}_{r,i}^{\mathrm{III}}\bar{G}_{d,i}^{\mathrm{III}}\right)} \left(\frac{\bar{B}_i^{\mathrm{III}}}{2a_{dk}\varphi_{dk}-\rho\bar{E}_{dk}^{\mathrm{III}}} + \frac{2\left((1-\theta)a+\beta_r l_d\right)\bar{E}_{d,i}^{\mathrm{III}}}{2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}} \right. \\
& \left. - \frac{\rho N H_{v1,i}^{\mathrm{III}}}{n_{r,i}q_{r,i}\left(2a_{dk}\varphi_{k}-\rho\bar{E}_{d,i}^{\mathrm{III}}\right)} - \frac{\rho N \bar{F}_i^{\mathrm{III}}(h_{v2}+h_r)}{n_{r,i}q_{r,i}\left(2a_{dk}\varphi_{dk}-\rho\bar{E}_{d,i}^{\mathrm{III}}\right)} \right)
\end{aligned}
\tag{6.49}
$$

The formula for $\bar{A}_i^{\mathrm{III}}$, $\bar{B}_i^{\mathrm{III}}$, $\bar{E}_{r,i}^{\mathrm{III}}$, $\bar{E}_{d,i}^{\mathrm{III}}$, $\bar{F}_i^{\mathrm{III}}$, $\bar{G}_{r,i}^{\mathrm{III}}$ and $\bar{G}_{d,i}^{\mathrm{III}}$ and the proof of concavity are reported in Appendix C.3.

6.6 Numerical example

Numerical examples to illustrate the behavior of the developed models for the single- and dual-channel strategies and their inventory policies, with and without learning and forgetting effects, are provided in this section. The results of both strategies are compared and analyzed to determine when it is profitable for the vendor to introduce an online channel, and which inventory policy is most profitable for the supply chain. For simplicity and conciseness, it is assumed in this section that the vendor offers one customizable item. However, the mathematics of the dual-channel strategy consider multiple customizable products. The values of some input parameters were taken from Jaber and Bonny (1998), Zanoni et al. (2012) and Batarfi et al. (2016), while the values of the other parameters were assumed within reason and in accordance with the assumptions made and conditions presented in this chapter. The breakdown of the unit holding cost into financial and physical storage is as suggested by Waters (2003). VMI-CS policy works when it is cheaper for the vendor to store items at the retailer's workhouse; i.e. $h_{v1} > h_{v2} + h_r$. The learning rate (LR_r and LR_d) is 87%, corresponding to $b_r = b_d = -\log(LR)/\log(2) = 0.2009$. The time to produce the first unit of the standard item is $T_{r1} = 1/P_r$ and the time to produce the first unit of the core item is $T_{d1} = 1/P_d$. Table 6.2 lists the values of the input parameters used in the numerical examples.

Table 6.2 Values of the input parameters of the numerical example

Input parameters	Value	Unit	Input parameters	Value	Unit
P_r	18,000	(unit/year)	c_P	500,000	($/year)
P_d	18,000	(unit/year)	c_{d1}	750,000	($/unit)
a	15,000	(unit/year)	c_r	200	($/setup)
θ	0.3	(%)	S_r	1000	($/setup)
α_r	20	(unit^2/$/year)	S_d	800	($/setup)
α_{d1}	2	(unit^2/$/year)	O_r	300	($/order)
ρ	1.8	(-)	h_{v1}	30	($/unit/year)
β_r	40	(customer/day)	h_{v2}	20	($/unit/year)
β_d	50	(customer/day)	h_r	10	($/unit/year)
l_d	6	(day)	b	0.2009	(-)
φ_{d1}	1	(%)	N	1	(-)

The optimal solutions were found using a similar solution procedure to the one developed by Jaber and Goyal (2008). The decision variables for the single-channel strategy are n_r and q_r and for the dual-channel strategy $\bar{n}_r$, $\bar{q}_r$ and q_d. A subscript i ($i = 1, 2, \dots, 10$) is added to each decision variable as their values change with every cycle because of learning and forgetting effects. Each cycle i for every policy (policies I.1, I.2, I.3, II and III) is optimized independently from the previous cycle and the values of the optimal solutions are averaged over the ten consecutive cycles. The optimal results for all policies of the single-channel and the dual-channel are summarized in Table 6.3.

In Table 6.3, the optimal price of the standard item is $p_r^0 = \$389.80$ and the optimal lot size is $n_r^0 q_r^0 = 2 \times 438 = 876$ units. The corresponding supply chain total profit is $\Pi_{single}^0 =$ $2,581,710.10. Comparing the results of the single-channel strategy shows that with learning and forgetting the optimal price of the standard item drops to around $p_r^{\mathrm{I.1}} = p_r^{\mathrm{I.2}} = p_r^{\mathrm{I.3}} = p_r^{\mathrm{II}} = p_r^{\mathrm{III}} \approx$

$379 and the optimal lot size for all policies, expect policy III, increase. Table 6.3 shows that learning in production increases the supply chain total profit. For example, comparing policy 0 to policy I.1, the price of the standard item in policy I.1 decreases by 3% ($p_r^{\mathrm{I.1}} = 378.98$), the lot size increases by 4% ($n_{r,i}^{\mathrm{I.1}} q_{r,i}^{\mathrm{I.1}} = 908$ units) and the profit increases by 6% ($\Pi_{single}^{\mathrm{I.1}} = \$2{,}739{,}445.46$).

Table 6.3 also shows that for policy 0 in the dual-channel strategy the optimal prices of the standard item and the customized items are $\bar{p}_r^0 = \$409$ and $p_{dk}^0 = \$1427.3$, respectively, the optimal lot size of the standard item is $\bar{n}_r^0 \bar{q}_r^0 = 2 \times 369 = 738$ units and the optimal production quantity of the core customizable item is $q_d^0 = 354$ units. The corresponding supply chain total profit is $\Pi_{dual}^0 = \$4{,}808{,}104.99$. With learning and forgetting effects, the optimal average prices of both the standard and the customized items reduce to $\bar{p}_r^{\mathrm{I.1}} = \bar{p}_r^{\mathrm{I.2}} = \bar{p}_r^{\mathrm{I.3}} = \bar{p}_r^{\mathrm{II}} = \bar{p}_r^{\mathrm{III}} \approx \$399$ and $p_{dk}^{\mathrm{I.1}} = p_{dk}^{\mathrm{I.2}} = p_{dk}^{\mathrm{I.3}} = p_{dk}^{\mathrm{II}} = p_{dk}^{\mathrm{III}} \approx \$1413$, respectively. The optimal lot sizes of standard increase for all policies except for policy III. The supply chain total profit also increases for all policies.

Comparing the results of the two strategies (single-channel and dual-channel), it was found that adopting the dual-channel strategy increases the total profit of the supply chain. The findings also show that in both strategies, both players (the vendor and the retailer) benefit from learning in production. Moreover, the results indicate that in both strategies, policy II generates the highest profit and policy III generated the least profit compared to all other policies with learning and forgetting effects. Furthermore, the results indicate that customers benefit from the coordination policies proposed in this chapter (as the prices of the products decrease, customers are able to consume more, which increases their benefit/utility).

Table 6.3 Optimal results of the single-channel and the dual-channel strategies for all policies

Single-channel						
	Policy 0	Policy I.1	Policy I.2	Policy I.3	Policy II	Policy III
p_r	\$389.80	\$378.98	\$378.79	\$378.84	\$378.86	\$379.31
q_r	438	908	1,003	953	963	722
n_r	2	1	1	1	1	1
t_r	-	0.0118	0.0129	0.0129	0.0125	0.0111
u_r	-	1,645.86	1,708.113	1,690.4	1,700.2	1,536.2
TC_v	\$217,971.44	\$65,680.43	\$63,987.74	\$64,179.33	\$63,852.14	\$67,915.03
TC_r	\$1,449,230.74	\$1,491,115.42	\$1,492,165.63	\$1,491,791.86	\$1,491,765.63	\$1,490,941.33
π_v	\$1,222,822.54	\$1,418,383.56	\$1,420,851.06	\$1,420,447.30	\$1,420,722.54	\$1,414,844.53
π_r	\$1,358,887.57	\$1,321,061.90	\$1,320,043.38	\$1,320,408.23	\$1,320,432.36	\$1,321,181.93
Π_{single}	\$2,581,710.10	\$2,739,445.46	\$2,740,894.44	\$2,740,855.53	\$2,741,154.90	\$2,736,026.46
Dual-channel						
	Policy 0	Policy I.1	Policy I.2	Policy I.3	Policy II	Policy III
p_r	\$409.97	\$399.29	\$399.17	\$398.96	\$399.14	\$399.36
p_{dk}	\$1,427.31	\$1,413.20	\$1,413.20	\$1,413.08	\$1,413.20	\$1,413.14
q_r	369	770	797	767	803	623
n_r	2	1	1	1	1	1
q_d	354	588	587	547	587	587
t_r	-	0.0106	0.0109	0.0098	0.0110	0.0097
t_d	-	0.0112	0.0111	0.0102	0.0111	0.0112
u_r	-	1,174.88	1,196.21	1,958.98	1,200.87	1,687.78
u_d	-	5.72	5.72	28.23	5.72	5.73
TC_v	\$252,975.89	\$92,150.21	\$91,224.90	\$91,225.90	\$91,028.86	\$91,969.05
TC_r	\$1,029,283.27	\$1,065,562.09	\$1,066,114.66	\$1,066,114.30	\$1,066,236.84	\$1,066,003.54
π_v	\$3,742,532.30	\$3,924,317.33	\$3,925,418.33	\$3,925,417.29	\$3,925,652.69	\$3,924,410.15
π_r	\$1,065,572.70	\$1,049,862.93	\$1,049,862.93	\$1,049,638.89	\$1,049,587.76	\$1,049,178.77
Π_{dual}	\$4,808,104.99	\$4,974,180.26	\$4,974,180.32	\$4,975,055.32	\$4,975,239.45	\$4,973,588.92

Note: TC_v is the total is cost of the vendor and TC_r is the total cost of the retailer.

6.7 Sensitivity analysis

This section investigates the effects of varying the values of the learning and forgetting parameters, B and LR, on the optimal decisions and the behavior of the single- and dual-channel models developed earlier. The results of the sensitivity analysis are then discussed to draw some managerial insights.

6.7.1 Effects of varying the time for total forgetting

In this subsection, the time for total forgetting is varied from 0.08 year (30 days) to 10 years while fixing LR at 87%. The results of the single channel strategy in Table 6.4 show that when the value of the time for total forgetting of the standard item, B_r, increases from 0.08 to 10 years, the average total profit of the supply chain, Π_{single}, for each policy increases, while the average lot size of the standard item ($n_r q_r$) decreases. Noticeably, the average price of the standard item, p_r, is insensitive to changes in B_r, and so are demand and revenues. Policy III is the least profitable, while policy I.2 is the most profitable. These findings suggest that learning positively affects competitiveness and efficiency of the supply chain, whereas forgetting impedes these attributes. The results in Table 6.4 are different from those of Zanoni et al. (2012) who showed that policy I.1 is the most sensitive, while policy I.3 is the least sensitive to the changes in B_r. The difference in the results can be attributed to two main factors. First, the proposed models in this chapter adopt a profit maximization – and not a cost minimization – approach with a price-dependent (and not a constant) demand. Second, Zanoni et al. (2012) used the model of Salameh et al. (1993) that underestimates the effects of learning and forgetting on the production and holding costs.

Table 6.4 Optimal results of the single-channel strategy for all policies with learning and forgetting effects when varying the value of B_r

Single-channel							
Policy	B_r	p_r	n_r	q_r	π_v	π_r	Π_{single}
I.1	0.08	\$379.86	1	1,239	\$1,398,015.41	\$1,323,480.81	\$2,721,496.22
	0.5	\$379.11	1	933	\$1,415,655.97	\$1,321,496.07	\$2,737,152.04
	1	\$378.98	1	908	\$1,418,383.56	\$1,321,061.90	\$2,739,445.46
	5	\$378.84	1	883	\$1,421,388.74	\$1,320,565.14	\$2,741,953.88
	10	\$378.82	1	879	\$1,421,716.84	\$1,320,518.31	\$2,742,235.16
I.2	0.08	\$378.86	1	1,785	\$1,410,638.38	\$1,317,461.08	\$2,728,099.46
	0.5	\$378.91	1	1,066	\$1,417,904.45	\$1,320,300.42	\$2,738,204.87
	1	\$378.79	1	1,003	\$1,420,851.06	\$1,320,043.38	\$2,740,894.44
	5	\$378.66	1	952	\$1,423,825.55	\$1,319,706.52	\$2,743,532.07
	10	\$378.64	1	942	\$1,424,478.13	\$1,319,627.14	\$2,744,105.27
I.3	0.08	\$379.64	1	1,311	\$1,401,194.26	\$1,322,395.79	\$2,723,590.05
	0.5	\$378.97	1	983	\$1,417,713.10	\$1,320,813.39	\$2,738,526.49
	1	\$378.84	1	953	\$1,420,447.30	\$1,320,408.23	\$2,740,855.53
	5	\$378.70	1	924	\$1,423,420.26	\$1,319,944.95	\$2,743,365.21
	10	\$378.67	1	918	\$1,424,097.01	\$1,319,838.24	\$2,743,935.25
II	0.08	\$379.67	1	1,325	\$1,401,546.61	\$1,322,427.69	\$2,723,974.30
	0.5	\$378.98	1	994	\$1,417,974.78	\$1,320,842.64	\$2,738,817.42
	1	\$378.86	1	963	\$1,420,722.54	\$1,320,432.36	\$2,741,154.89
	5	\$378.71	1	933	\$1,423,700.72	\$1,319,964.52	\$2,743,665.24
	10	\$378.68	1	928	\$1,424,390.45	\$1,319,848.63	\$2,744,239.08
III	0.08	\$380.11	2	606	\$1,394,944.89	\$1,321,680.68	\$2,716,625.57
	0.5	\$379.42	1	700	\$1,412,377.67	\$1,321,326.36	\$2,733,704.03
	1	\$379.31	1	722	\$1,414,844.53	\$1,321,181.93	\$2,736,026.46
	5	\$379.14	1	709	\$1,418,134.38	\$1,320,614.08	\$2,738,748.46
	10	\$379.10	1	706	\$1,418,898.93	\$1,320,480.79	\$2,739,379.71

A sensitivity analysis of the policies of the dual-channel strategy was performed for changes in B_r (standard) and B_d (core) from 0.08 to 10 years. The results for policy I.1, I.2, I.3, II and III are summarized in Tables 6.5, 6.6, 6.7, 6.8 and 6.9, respectively. The results show that the supply chain average total profit, Π_{dual}, increased for all policies. The average lot size of the standard item ($\bar{n}_r \bar{q}_r$) and the average production quantity of the core item, q_d, decreased. Prices of the standard and the customized items, $\bar{p}_r$ and p_{dk}, were shown to be insensitive to B_r and B_d. The results also indicate that the supply chain system performed better when $B_r < B_d$. For example, when $B_r = 0.08$ and $B_d = 0.5$, the total profit of the system, $\Pi_{dual} = \$4{,}961{,}203.63$ in policy I.1. Whereas, when $B_r > B_d$ ($B_r = 0.5$ and $B_d = 0.08$), the supply chain total profit performed better, $\Pi_{dual} = \$4{,}971{,}724.48$.

Comparing the results of the single- and dual-channel, the results show that the dual-channel strategy outperforms the single-channel strategy. The results also show that policy III in both strategies generates the lowest average profit, whereas policy II in the dual-channel strategy generates the highest average profit. Comparing the results of the two strategies with those of policy 0 show that learning improves competitiveness by lower the process and increasing demand and reducing inventory-related costs. The effects of learning are more prevalent when forgetting effects are insignificant (large values of B_r and B_d).

The findings in Tables 6.4, 6.5, 6.6, 6.7, 6.8, and 6.9 show that the profit of the retailer for all policies reduces considerably, compared to policy 0, when the vendor undergoes learning-based improvements, whose profit increases significantly. The vendor clusters the additional profits and savings from learning. To sustain collaboration, the vendor has to compensate the retailer for its losses. Mitigating the effects of forgetting could be achieved through training and worker transfer policies in a manufacturing environment where workers perform functionally different tasks, and possibly through increasing the factor of similarity between the tasks (Jaber et al., 2003). Moreover, studies has shown that when workers are exposed to shorter spaced learning (training) sessions the results of retaining the information is by far better than massing the learning (training) in one whole session (Sikström and Jaber, 2012).

Table 6.5 Optimal results of the dual-channel strategy for policy I.I when varying the values of B_r and B_d

Dual-channel										
Policy	B_r	B_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
I.1	0.08	0.08	$400.13	$1,413.26	1	1007	605	$3,910,866.84	$1,050,336.79	$4,961,203.63
		0.5	$400.13	$1,413.23	1	1006	595	$3,911,128.84	$1,050,327.56	$4,961,456.40
		1	$400.13	$1,413.18	1	1006	593	$3,911,364.58	$1,050,308.32	$4,961,672.90
		5	$400.13	$1,413.12	1	1006	561	$3,912,063.75	$1,050,283.95	$4,962,347.70
		10	$400.13	$1,413.09	1	1007	546	$3,912,391.04	$1,050,271.97	$4,962,663.01
	0.5	0.08	$399.44	$1,413.25	1	792	608	$3,921,689.27	$1,050,035.21	$4,971,724.48
		0.5	$399.44	$1,413.20	1	792	604	$3,921,953.83	$1,050,018.34	$4,971,972.18
		1	$399.44	$1,413.20	1	792	587	$3,922,179.35	$1,050,017.74	$4,972,197.10
		5	$399.44	$1,413.12	1	792	560	$3,922,879.58	$1,049,988.17	$4,972,867.75
		10	$399.44	$1,413.09	1	791	546	$3,923,201.50	$1,049,979.05	$4,973,180.56
	1	0.08	$399.29	$1,413.25	1	770	606	$3,923,832.28	$1,049,884.65	$4,973,716.92
		0.5	$399.29	$1,413.20	1	768	602	$3,924,085.60	$1,049,873.44	$4,973,959.04
		1	$399.29	$1,413.20	1	770	588	$3,924,317.33	$1,049,862.93	$4,974,180.26
		5	$399.29	$1,413.12	1	768	559	$3,925,010.71	$1,049,843.07	$4,974,853.78
		10	$399.29	$1,413.09	1	768	547	$3,925,341.09	$1,049,827.57	$4,975,168.66
	5	0.08	$399.12	$1,413.25	1	746	607	$3,926,156.52	$1,049,712.53	$4,975,869.05
		0.5	$399.12	$1,413.23	1	747	595	$3,926,423.00	$1,049,701.30	$4,976,124.30
		1	$399.11	$1,413.20	1	748	586	$3,926,647.36	$1,049,691.82	$4,976,339.17
		5	$399.11	$1,413.12	1	746	559	$3,927,347.44	$1,049,664.80	$4,977,012.25
		10	$399.11	$1,413.09	1	746	546	$3,927,674.91	$1,049,651.89	$4,977,326.79
	10	0.08	$399.08	$1,413.25	1	742	608	$3,926,696.55	$1,049,669.89	$4,976,366.45
		0.5	$399.08	$1,413.23	1	742	596	$3,926,953.85	$1,049,662.53	$4,976,616.38
		1	$399.08	$1,413.20	1	740	586	$3,927,181.12	$1,049,657.15	$4,976,838.27
		5	$399.07	$1,413.13	1	742	558	$3,927,885.69	$1,049,624.05	$4,977,509.74
		10	$399.07	$1,413.09	1	742	547	$3,928,215.87	$1,049,608.22	$4,977,824.09

Table 6.6 Optimal results of the dual-channel strategy for policy I.2 when varying the values of B_r and B_d

					Dual-channel					
Policy	B_r	B_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
I.2	0.08	0.08	$399.95	$1,413.25	1	1050	608	$3,912,515.46	$1,049,948.86	$4,962,464.32
		0.5	$399.95	$1,413.22	1	1050	597	$3,912,776.13	$1,049,939.77	$4,962,715.90
		1	$399.95	$1,413.20	1	1050	587	$3,913,005.18	$1,049,931.60	$4,962,936.78
		5	$399.95	$1,413.12	1	1050	560	$3,913,706.48	$1,049,901.59	$4,963,608.07
		10	$399.95	$1,413.08	1	1050	548	$3,914,036.48	$1,049,886.44	$4,963,922.92
	0.5	0.08	$399.32	$1,413.25	1	824	608	$3,922,791.67	$1,049,785.91	$4,972,577.59
		0.5	$399.32	$1,413.22	1	824	597	$3,923,052.12	$1,049,776.90	$4,972,829.02
		1	$399.32	$1,413.20	1	824	587	$3,923,281.58	$1,049,768.12	$4,973,049.70
		5	$399.31	$1,413.12	1	824	559	$3,923,981.84	$1,049,738.73	$4,973,720.57
		10	$399.31	$1,413.08	1	824	547	$3,924,311.59	$1,049,723.57	$4,974,035.17
	1	0.08	$399.17	$1,413.25	1	797	608	$3,924,928.57	$1,049,655.68	$4,974,584.25
		0.5	$399.17	$1,413.22	1	797	596	$3,925,188.96	$1,049,646.67	$4,974,835.63
		1	$399.17	$1,413.20	1	797	587	$3,925,418.42	$1,049,637.89	$4,975,056.31
		5	$399.16	$1,413.12	1	797	559	$3,926,118.39	$1,049,608.63	$4,975,727.02
		10	$399.16	$1,413.08	1	797	547	$3,926,448.34	$1,049,593.35	$4,976,041.68
	5	0.08	$399.00	$1,413.25	1	772	608	$3,927,241.79	$1,049,495.82	$4,976,737.61
		0.5	$399.00	$1,413.22	1	772	596	$3,927,502.22	$1,049,486.76	$4,976,988.98
		1	$399.00	$1,413.20	1	772	587	$3,927,731.50	$1,049,478.07	$4,977,209.56
		5	$399.00	$1,413.12	1	772	559	$3,928,431.64	$1,049,448.65	$4,977,880.29
		10	$399.00	$1,413.08	1	772	547	$3,928,761.30	$1,049,433.54	$4,978,194.84
	10	0.08	$398.96	$1,413.25	1	767	608	$3,927,773.15	$1,049,457.32	$4,977,230.47
		0.5	$398.96	$1,413.22	1	767	596	$3,928,033.48	$1,049,448.30	$4,977,481.78
		1	$398.96	$1,413.20	1	767	587	$3,928,262.90	$1,049,439.55	$4,977,702.44
		5	$398.96	$1,413.12	1	767	559	$3,928,962.97	$1,049,410.15	$4,978,373.12
		10	$398.96	$1,413.08	1	767	547	$3,929,292.60	$1,049,395.05	$4,978,687.65

Table 6.7 Optimal results of the dual-channel strategy for policy I.3 when varying the values of B_r and B_d

Dual-channel										
Policy	B_r	B_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
I.3	0.08	0.08	$399.95	$1,413.25	1	1050	608	$3,912,514.13	$1,049,948.96	$4,962,463.09
		0.5	$399.95	$1,413.22	1	1050	597	$3,912,774.81	$1,049,939.87	$4,962,714.67
		1	$399.95	$1,413.20	1	1050	587	$3,913,004.38	$1,049,931.13	$4,962,935.52
		5	$399.95	$1,413.12	1	1050	560	$3,913,705.16	$1,049,901.69	$4,963,606.85
		10	$399.95	$1,413.08	1	1050	548	$3,914,035.15	$1,049,886.54	$4,963,921.70
	0.5	0.08	$399.32	$1,413.25	1	824	608	$3,922,790.46	$1,049,786.03	$4,972,576.49
		0.5	$399.32	$1,413.22	1	824	597	$3,923,050.96	$1,049,776.96	$4,972,827.92
		1	$399.32	$1,413.20	1	824	587	$3,923,280.38	$1,049,768.23	$4,973,048.61
		5	$399.31	$1,413.12	1	824	559	$3,923,980.65	$1,049,738.81	$4,973,719.46
		10	$399.31	$1,413.08	1	824	547	$3,924,310.39	$1,049,723.70	$4,974,034.09
	1	0.08	$399.17	$1,413.25	1	797	608	$3,924,927.42	$1,049,655.80	$4,974,583.22
		0.5	$399.17	$1,413.22	1	797	596	$3,925,187.88	$1,049,646.74	$4,974,834.62
		1	$399.17	$1,413.20	1	797	587	$3,925,417.29	$1,049,638.00	$4,975,055.29
		5	$399.16	$1,413.12	1	797	559	$3,926,117.47	$1,049,608.59	$4,975,726.06
		10	$399.16	$1,413.08	1	797	547	$3,926,447.16	$1,049,593.49	$4,976,040.65
	5	0.08	$399.00	$1,413.25	1	772	608	$3,927,240.71	$1,049,495.93	$4,976,736.64
		0.5	$399.00	$1,413.22	1	772	596	$3,927,501.16	$1,049,486.86	$4,976,988.02
		1	$399.00	$1,413.20	1	772	587	$3,927,730.50	$1,049,478.14	$4,977,208.64
		5	$399.00	$1,413.12	1	772	559	$3,928,430.61	$1,049,448.73	$4,977,879.34
		10	$399.00	$1,413.08	1	772	547	$3,928,760.24	$1,049,433.63	$4,978,193.87
	10	0.08	$398.96	$1,413.25	1	767	608	$3,927,772.08	$1,049,457.43	$4,977,229.52
		0.5	$398.96	$1,413.22	1	767	596	$3,928,032.50	$1,049,448.37	$4,977,480.88
		1	$398.96	$1,413.20	1	767	587	$3,928,261.86	$1,049,439.65	$4,977,701.51
		5	$398.96	$1,413.12	1	767	559	$3,928,961.93	$1,049,410.25	$4,978,372.18
		10	$398.96	$1,413.08	1	767	547	$3,929,291.59	$1,049,395.12	$4,978,686.71

Table 6.8 Optimal results of the dual-channel strategy for policy II when varying the values of B_r and B_d

								Dual-channel		
Policy	B_r	B_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
II	0.08	0.08	\$399.92	\$1,413.25	1	1058	608	\$3,912,815.52	\$1,049,877.13	\$4,962,692.65
		0.5	\$399.92	\$1,413.22	1	1058	597	\$3,913,076.17	\$1,049,868.05	\$4,962,944.23
		1	\$399.92	\$1,413.20	1	1058	587	\$3,913,305.73	\$1,049,859.33	\$4,963,165.06
		5	\$399.92	\$1,413.12	1	1058	560	\$3,914,006.51	\$1,049,829.86	\$4,963,836.37
		10	\$399.92	\$1,413.08	1	1058	547	\$3,914,336.50	\$1,049,814.71	\$4,964,151.21
	0.5	0.08	\$399.29	\$1,413.25	1	831	608	\$3,923,023.89	\$1,049,732.39	\$4,972,756.28
		0.5	\$399.29	\$1,413.22	1	831	597	\$3,923,284.36	\$1,049,723.33	\$4,973,007.68
		1	\$399.29	\$1,413.20	1	831	587	\$3,923,513.88	\$1,049,714.53	\$4,973,228.42
		5	\$399.29	\$1,413.12	1	831	559	\$3,924,214.17	\$1,049,685.09	\$4,973,899.27
		10	\$399.29	\$1,413.08	1	831	547	\$3,924,543.90	\$1,049,669.97	\$4,974,213.87
	1	0.08	\$399.14	\$1,413.25	1	803	608	\$3,925,161.97	\$1,049,605.64	\$4,974,767.62
		0.5	\$399.14	\$1,413.22	1	803	596	\$3,925,422.55	\$1,049,596.52	\$4,975,019.07
		1	\$399.14	\$1,413.20	1	804	587	\$3,925,652.69	\$1,049,586.76	\$4,975,239.45
		5	\$399.14	\$1,413.12	1	804	559	\$3,926,352.91	\$1,049,557.25	\$4,975,910.16
		10	\$399.14	\$1,413.08	1	804	547	\$3,926,682.94	\$1,049,541.69	\$4,976,224.63
	5	0.08	\$398.97	\$1,413.25	1	778	608	\$3,927,470.85	\$1,049,449.38	\$4,976,920.23
		0.5	\$398.97	\$1,413.22	1	778	596	\$3,927,731.25	\$1,049,440.33	\$4,977,171.58
		1	\$398.97	\$1,413.20	1	778	587	\$3,927,960.84	\$1,049,431.50	\$4,977,392.34
		5	\$398.97	\$1,413.12	1	778	559	\$3,928,660.85	\$1,049,402.14	\$4,978,062.99
		10	\$398.97	\$1,413.08	1	778	547	\$3,928,990.55	\$1,049,387.00	\$4,978,377.55
	10	0.08	\$398.93	\$1,413.25	1	772	608	\$3,928,000.50	\$1,049,411.70	\$4,977,412.20
		0.5	\$398.93	\$1,413.22	1	772	596	\$3,928,260.89	\$1,049,402.65	\$4,977,663.53
		1	\$398.93	\$1,413.20	1	772	587	\$3,928,490.44	\$1,049,393.85	\$4,977,884.29
		5	\$398.93	\$1,413.12	1	772	559	\$3,929,190.52	\$1,049,364.44	\$4,978,554.97
		10	\$398.93	\$1,413.08	1	772	547	\$3,929,520.19	\$1,049,349.32	\$4,978,869.51

Table 6.9 Optimal results of the dual-channel strategy for policy III when varying the values of B_r and B_d

Dual-channel										
Policy	B_r	B_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
III	0.08	0.08	\$400.47	\$1,413.33	2	505	608	\$3,909,244.18	\$1,048,623.74	\$4,957,867.92
		0.5	\$400.47	\$1,413.30	2	505	597	\$3,909,504.92	\$1,048,614.67	\$4,958,119.59
		1	\$400.47	\$1,413.28	2	505	587	\$3,909,734.61	\$1,048,605.92	\$4,958,340.52
		5	\$400.47	\$1,413.20	2	505	560	\$3,910,435.70	\$1,048,576.44	\$4,959,012.14
		10	\$400.47	\$1,413.16	2	505	548	\$3,910,765.84	\$1,048,561.29	\$4,959,327.13
	0.5	0.08	\$399.47	\$1,413.18	1	633	608	\$3,922,431.99	\$1,049,276.62	\$4,971,708.61
		0.5	\$399.47	\$1,413.16	1	633	597	\$3,922,692.53	\$1,049,267.55	\$4,971,960.08
		1	\$399.47	\$1,413.13	1	633	587	\$3,922,921.99	\$1,049,258.83	\$4,972,180.81
		5	\$399.47	\$1,413.06	1	633	560	\$3,923,622.37	\$1,049,229.43	\$4,972,851.79
		10	\$399.47	\$1,413.02	1	633	547	\$3,923,952.20	\$1,049,214.28	\$4,973,166.47
	1	0.08	\$399.36	\$1,413.19	1	623	608	\$3,923,920.19	\$1,049,196.58	\$4,973,116.77
		0.5	\$399.36	\$1,413.17	1	623	597	\$3,924,180.72	\$1,049,187.50	\$4,973,368.22
		1	\$399.36	\$1,413.14	1	623	587	\$3,924,410.15	\$1,049,178.77	\$4,973,588.92
		5	\$399.36	\$1,413.07	1	623	560	\$3,925,110.46	\$1,049,149.38	\$4,974,259.85
		10	\$399.36	\$1,413.03	1	623	547	\$3,925,440.24	\$1,049,134.26	\$4,974,574.50
	5	0.08	\$399.24	\$1,413.17	1	647	608	\$3,925,379.93	\$1,049,230.88	\$4,974,610.81
		0.5	\$399.24	\$1,413.14	1	647	597	\$3,925,640.43	\$1,049,221.81	\$4,974,862.24
		1	\$399.24	\$1,413.12	1	647	587	\$3,925,869.84	\$1,049,213.10	\$4,975,082.93
		5	\$399.24	\$1,413.04	1	647	559	\$3,926,570.10	\$1,049,183.71	\$4,975,753.81
		10	\$399.23	\$1,413.00	1	647	547	\$3,926,899.87	\$1,049,168.57	\$4,976,068.44
	10	0.08	\$399.21	\$1,413.17	1	644	608	\$3,925,774.43	\$1,049,209.73	\$4,974,984.16
		0.5	\$399.21	\$1,413.15	1	644	596	\$3,926,034.93	\$1,049,200.66	\$4,975,235.59
		1	\$399.21	\$1,413.12	1	644	587	\$3,926,264.33	\$1,049,191.95	\$4,975,456.28
		5	\$399.21	\$1,413.05	1	644	559	\$3,926,964.57	\$1,049,162.56	\$4,976,127.13
		10	\$399.21	\$1,413.01	1	644	547	\$3,927,294.33	\$1,049,147.43	\$4,976,441.76

6.7.2 Effects of varying the learning rate

This subsection investigates the effect of the learning rate, LR, on the behavior of the developed models. Dar-El et al. (1995, p. 273) categorize LR into three different groups: LR = 70-75% (fast), LR = 75-80% (moderate), LR = 85-90% (slow); also refer to Jaber (2006b). Accordingly, three different values of LR are considered, 70% (b_r = 0.5146), 80% (b_r = 0.3219), and 90% (b_r = 0.1520). Table 6.10 shows that as the learning rate of producing the standard item, LR_r, becomes faster, the performance of the supply chain improves, i.e., Π_{single} increases, p_r decreases, demand increases and $n_r q_r$, decreases. For example, policy I.1 in Table 6.10, when LR = 90%, $\Pi_{single}^{\mathrm{I.1}} = \$2{,}719{,}422.22$, $p_r^{\mathrm{I.1}} = \$380.35$, and $n_r^{\mathrm{I.1}} q_r^{\mathrm{I.1}} = 889$ units, in contrast, when, when LR = 70%, $\Pi_{single}^{\mathrm{I.1}} = \$2{,}782{,}028.43$, $p_r^{\mathrm{I.1}} = \$376.17$, and $n_r^{\mathrm{I.1}} q_r^{\mathrm{I.1}} = 864$ units.

Table 6.11 shows how the performance of the dual-channel model varies for every inventory policy and different values of LR_r and LR_d. The results indicate that fast learnings rates increase Π_{dual} and decrease $\bar{p}_r$, p_{dk}, $\bar{n}_r\bar{q}_r$, and q_d. For example, in policy I.1 when $LR_r = LR_d = 90\%$, $\Pi_{dual}^{\text{I.1}} = \$4{,}951{,}104.17$, $\bar{p}_r^{\text{I.1}} = \$400.67$, $p_{dk}^{\text{I.1}} = \$1{,}415.27$, $\bar{n}_r^{\text{I.1}}\bar{q}_r^{\text{I.1}} = 756$ units and $q_d^{\text{I.1}} = 585$ units. However, when $LR_r = LR_d = 70\%$, $\Pi_{dual}^{\text{I.1}} = \$5{,}029{,}706.82$, $\bar{p}_r^{\text{I.1}} = \$396.36$, $p_{dk}^{\text{I.1}} = \$1{,}407.99$, $\bar{n}_r^{\text{I.1}}\bar{q}_r^{\text{I.1}} = 730$ units and $q_d^{\text{I.1}} = 462$ units. The results also indicate that supply chain performs better when the learning rate of the standard item, LR_r, is faster than the learning rate of the core customizable item, LR_d (e.g. $b_r > b_d$).

The results suggest that faster learning shortens production time and reduces the lot size and subsequently production and inventory costs. Lowering costs mean better competitiveness and profitability. Managers can achieve a fast learning rate through training and better worker-task assignment. Having a flexible and cross-trained workforce increases responsiveness to changes in product or market requirements. Schilling et al. (2003), for example, noted that learning rates are faster for workers who perform variety of tasks than for those performing a repetitive task or a groups of unrelated tasks. This notion has its parallel in the psychology literature where shifting learning context (novelty) improves performance and mitigates forgetting effects (Sikström and Jaber, 2012).

Table 6.10 Optimal results of the single-channel strategy for all policies with learning and forgetting effects when varying the value of LR_r

Single-channel							
Policy	LR_r	p_r	n_r	q_r	π_v	π_r	Π_{single}
I.1	90%	\$380.35	1	889	\$1,393,078.19	\$1,326,344.03	\$2,719,422.22
	80%	\$377.14	1	904	\$1,452,913.72	\$1,313,942.41	\$2,766,856.13
	70%	\$376.17	1	864	\$1,471,814.69	\$1,310,213.74	\$2,782,028.43
I.2	90%	\$380.10	1	1005	\$1,396,468.49	\$1,325,043.28	\$2,721,511.78
	80%	\$377.05	1	958	\$1,453,987.30	\$1,313,428.00	\$2,767,415.30
	70%	\$376.14	1	886	\$1,472,106.10	\$1,310,036.89	\$2,782,143.00
I.3	90%	\$380.15	1	953	\$1,396,008.71	\$1,325,423.97	\$2,721,432.67
	80%	\$377.09	1	922	\$1,453,745.84	\$1,313,682.93	\$2,767,428.78
	70%	\$376.16	1	868	\$1,472,020.05	\$1,310,152.70	\$2,782,172.75
II	90%	\$380.17	1	968	\$1,396,418.50	\$1,325,451.93	\$2,721,870.43
	80%	\$377.10	1	925	\$1,453,845.49	\$1,313,698.05	\$2,767,543.54
	70%	\$376.16	1	868	\$1,472,040.62	\$1,310,157.91	\$2,782,198.53
III	90%	\$380.76	1	650	\$1,388,708.77	\$1,326,257.02	\$2,714,965.79
	80%	\$377.29	1	810	\$1,451,249.11	\$1,313,965.60	\$2,765,214.72
	70%	\$376.21	1	807	\$1,471,357.26	\$1,310,026.17	\$2,781,383.43

Table 6.11 Optimal results of the dual-channel strategy for all policies with learning and forgetting effects when varying the values of LR_r and LR_d

Dual-channel										
Policy	LR_r	LR_d	p_r	p_{dk}	n_r	q_r	q_d	π_v	π_r	Π_{dual}
I.1	90%	90%	\$400.68	\$1,415.27	1	756	585	\$3,898,666.80	\$1,052,437.37	\$4,951,104.17
		80%	\$400.68	\$1,410.08	1	755	541	\$3,922,883.35	\$1,050,576.49	\$4,973,459.83
		70%	\$400.68	\$1,407.99	1	755	464	\$3,933,680.71	\$1,049,816.83	\$4,983,497.54
	80%	90%	\$397.39	\$1,415.27	1	764	582	\$3,937,751.92	\$1,048,055.25	\$4,985,807.18
		80%	\$397.39	\$1,410.08	1	763	543	\$3,961,888.27	\$1,046,216.99	\$5,008,105.26
		70%	\$397.39	\$1,407.99	1	763	461	\$3,972,646.39	\$1,045,476.25	\$5,018,122.65
	70%	90%	\$396.35	\$1,415.27	1	731	584	\$3,950,746.49	\$1,046,666.93	\$4,997,413.42
		80%	\$396.35	\$1,410.08	1	730	543	\$3,974,855.04	\$1,044,840.50	\$5,019,695.54
		70%	\$396.35	\$1,407.99	1	730	462	\$3,985,603.93	\$1,044,102.89	\$5,029,706.82
I.2	90%	90%	\$400.50	\$1,415.27	1	797	585	\$3,900,211.36	\$1,052,117.34	\$4,952,328.70
		80%	\$400.50	\$1,410.08	1	796	544	\$3,924,426.97	\$1,050,254.06	\$4,974,681.03
		70%	\$400.50	\$1,407.99	1	796	462	\$3,935,218.59	\$1,049,501.52	\$4,984,720.11
	80%	90%	\$397.34	\$1,415.27	1	775	584	\$3,938,221.27	\$1,047,955.34	\$4,986,176.61
		80%	\$397.34	\$1,410.08	1	774	543	\$3,962,353.62	\$1,046,120.71	\$5,008,474.33
		70%	\$397.34	\$1,407.99	1	774	461	\$3,973,111.95	\$1,045,379.57	\$5,018,491.52
	70%	90%	\$396.34	\$1,415.27	1	733	584	\$3,950,867.66	\$1,046,644.02	\$4,997,511.68
		80%	\$396.34	\$1,410.08	1	733	543	\$3,974,975.96	\$1,044,817.78	\$5,019,793.74
		70%	\$396.34	\$1,407.99	1	732	461	\$3,985,724.51	\$1,044,080.07	\$5,029,804.58
I.3	90%	90%	\$399.86	\$1,415.27	1	793	585	\$3,907,913.76	\$1,051,290.25	\$4,959,204.01
		80%	\$399.86	\$1,410.08	1	792	544	\$3,932,112.25	\$1,049,432.79	\$4,981,545.04
		70%	\$399.86	\$1,407.99	1	792	462	\$3,942,897.06	\$1,048,682.56	\$4,991,579.63
	80%	90%	\$397.34	\$1,415.27	1	775	584	\$3,938,220.77	\$1,047,955.38	\$4,986,176.16
		80%	\$397.34	\$1,410.08	1	774	543	\$3,962,353.17	\$1,046,120.71	\$5,008,473.89
		70%	\$397.34	\$1,407.99	1	774	461	\$3,973,111.45	\$1,045,379.56	\$5,018,491.01
	70%	90%	\$396.34	\$1,415.27	1	733	584	\$3,950,867.66	\$1,046,643.83	\$4,997,511.49
		80%	\$396.34	\$1,410.08	1	733	543	\$3,974,975.82	\$1,044,817.79	\$5,019,793.61
		70%	\$396.34	\$1,407.99	1	732	461	\$3,985,724.37	\$1,044,080.08	\$5,029,804.45
II	90%	90%	\$400.46	\$1,415.27	1	806	585	\$3,900,546.73	\$1,052,045.19	\$4,952,591.91
		80%	\$400.46	\$1,410.08	1	805	544	\$3,924,761.16	\$1,050,182.56	\$4,974,943.72
		70%	\$400.46	\$1,407.99	1	805	462	\$3,935,551.99	\$1,049,430.48	\$4,984,982.48
	80%	90%	\$397.33	\$1,415.27	1	777	584	\$3,938,313.99	\$1,047,936.14	\$4,986,250.13
		80%	\$397.33	\$1,410.08	1	776	543	\$3,962,446.23	\$1,046,101.51	\$5,008,547.74
		70%	\$397.33	\$1,407.99	1	776	461	\$3,973,204.40	\$1,045,360.41	\$5,018,564.81
	70%	90%	\$396.34	\$1,415.27	1	734	584	\$3,950,889.26	\$1,046,639.75	\$4,997,529.01
		80%	\$396.34	\$1,410.08	1	733	543	\$3,974,997.40	\$1,044,813.71	\$5,019,811.11
		70%	\$396.34	\$1,407.99	1	733	461	\$3,985,746.02	\$1,044,075.91	\$5,029,821.93
III	90%	90%	\$400.80	\$1,415.27	1	596	586	\$3,898,157.33	\$1,051,591.06	\$4,949,748.39
		80%	\$400.80	\$1,410.08	1	592	544	\$3,923,539.49	\$1,049,642.76	\$4,973,182.25
		70%	\$400.80	\$1,407.99	1	591	462	\$3,934,866.16	\$1,048,852.41	\$4,983,718.57
	80%	90%	\$397.46	\$1,415.27	1	692	584	\$3,937,437.28	\$1,047,737.14	\$4,985,174.42
		80%	\$397.46	\$1,410.08	1	686	543	\$3,962,338.64	\$1,045,848.44	\$5,008,187.08
		70%	\$397.46	\$1,407.99	1	683	461	\$3,973,487.61	\$1,045,080.36	\$5,018,567.96
	70%	90%	\$396.40	\$1,415.27	1	723	584	\$3,950,309.13	\$1,046,625.35	\$4,996,934.48
		80%	\$396.40	\$1,410.08	1	717	543	\$3,974,852.92	\$1,044,772.86	\$5,019,625.78
		70%	\$396.40	\$1,407.99	1	714	461	\$3,985,820.91	\$1,044,021.65	\$5,029,842.56

6.8 Summary and Conclusions

This chapter investigated the effects of learning and forgetting in a two-level single-and dual-channel strategy supply chain. The vendor in the single-channel produces and sells a standard item through a retail channel. The vendor in the dual-channel, and in parallel to the standard item channel, produces a core item, customizes it and sells directly to customers through an online channel. The vendor and the retailer follow the vendor-managed inventory with consignment stock agreement. The inventory behavior of the core items follows the economic production quantity model. No inventory was considered for customized items. Each strategy was investigated for six different inventory policies. Policy 0, a base case, does not consider learning and forgetting effects. The remaining five policies (policy I.1, I.2, I.3, II, and III) do. The profits of the centralized single- and dual-channel supply chain models were maximized by optimizing the prices of the standard and/or the customized item, the lot sizes of the standard item and/or the core item and the number of shipments.

The numerical results showed that the dual-channel strategy outperformed the single channel strategy with the profit being the performance measure. The results suggested that learning, despite being impeded by forgetting, reduces inventory-related costs thereby allowing the chain to reduce the prices of its product(s), which increases demand and subsequently sales. Although the five policies (I.1, I.2, I.3, II, and III) performed better than policy 0, policy II in both strategies performed the best and policy III the worst among the five policies.

The sensitivity of the single- and dual-channel supply chains were investigated for different values of the time for total forgetting, B, and the learning rate, LR. The results showed that faster forgetting, small values of B, and slower learning, and higher values of LR, all reduce profits.

However, *B* was found to have very little effect on prices. On the other hand, lower forgetting intensity, a very large *B*, and faster learning improve competitiveness and profitability.

The results showed that when the vendor considers the learning and forgetting effects savings and additional profits reaped from improvements shift to its side. The vendor has to compensate the retailer for its loss so as to foster collaboration. Compensation may take the form of profit sharing or price/quantity discounts.

CHAPTER 7. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

This chapter summaries the research presented in this thesis and discusses its main contributions. It also presents the limitation of this thesis and suggests future research directions.

7.1 Thesis summary

Significant changes have affected the modern business world and the management of supply chains. Some of these changes are attributed to the recent technological developments of the internet and e-commerce technologies. These have influenced customers' purchase patterns, thereby motivating manufacturers to adopt a dual-channel strategy (a mixture of a traditional retail channel and an online channel). The research presented in this thesis investigates the effect of adopting a dual-channel strategy on the performance of a two-level (vendor-retailer) supply chain. The performance measure is a profit maximization. The following chapters summarizes the models investigated in this thesis:

Chapter 4 investigated two scenarios. The first scenario discussed a single-channel strategy in which a standard item is produced using make-to-stock processes and is sold to customers through a retail channel. The second scenario discussed the effect of adopting a dual-channel strategy on the supply chain, in which a standard item is sold through a retail channel and customized items are sold to customers through an online channel. The customized items, which are produced based on build-to-order processes, are made of a core item and custom added features. It was assumed that the inventory behavior between the vendor and the retailer follows the consignment stock agreement. No inventory was assumed for the finished customized items and the custom added features. However, it was assumed that the production-inventory behavior of the core item follows the economic production quantity model. A numerical analysis was performed on both strategies, and the results were compared. The results showed that the dual-channel strategy outperforms the single-channel strategy and that the vendor is the one who benefits from the strategy by having a higher markup margin and profit. On the other hand, the retailer's profit was shown to decrease due to a decrease in demand for the standard items. The developed models were investigated for different varying conditions (production rate, holding cost, ordering cost, customer acceptance, quoted delivery lead-time, and product differentiation) to determine how inventory policies, price markups, and the supply chain total profit are affected. The results showed that changing some of the input parameters have a major impact on the optimal decisions of the supply chain system.

Chapter 5 developed a closed-loop (forward and reverse logistics) dual-channel supply chain system. The system is comprised of production, refurbishing, collection, and waste disposal processes. A return policy in which unsatisfied customers can return the purchased item for a refund was also considered. The objective was to examine the effect of different return policies on the behavior of supply chain systems before and after adopting the dual-channel. In both strategies, the chapter analyzed the change in the profit, the pricing and inventory decisions. Numerical examples and sensitivity analysis were provided and their results discussed. The results demonstrated that in both strategies, the more generous the return policy is, the higher the demand, the selling prices and the overall profit. The results also indicated that adopting a dual-channel strategy is more profitable to the supply chain.

Chapter 6 investigated the effects of learning and forgetting modeling on the adoption of a two-level dual-channel supply chain system. Two strategies were considered: a single-channel strategy and a dual-channel strategy. Each strategy, was investigated for six different policies. The first policy (policy 0) discussed the behavior of both inventories without considering learning and forgetting effects. The rest of the other policies (I.1, I.2, I.3, II, and III) discussed five different shipment arrangements between the vendor and the retailer when learning and forgetting effects are considered. The objective was to maximize the total profit of the supply chain system by finding the optimal pricing and inventory decisions. A numerical example was used to test the developed models and to obtain results. Sensitivity analysis were also used to build further upon the numerical results and to draw specific managerial decisions. The results suggested that learning, despite being impeded by forgetting, reduces inventory-related costs thereby allowing the chain to reduce the prices of its product(s), this increases demand and subsequently sales. Although the five policies (I.1, I.2, I.3, II, and III) performed better than policy 0, policy II in both strategies performed the best and policy III the worst among the five policies. Moreover, the results showed that when the vendor undergoes learning and forgetting effects, savings and additional profits reaped from improvements shift to its side. The vendor has to compensate the retailer for its loss so as to foster collaboration. Compensation may take the form of profit sharing or price/quantity discounts. The results of the sensitivity analysis indicated that faster forgetting and slower learning all reduce profits.

7.2 Thesis contributions

The main research contributions of this thesis can be summarized as follows:

The first contribution of this thesis is presented in Chapter 4. The presented research in Chapter 4 is the first to address a dual-channel supply chain in which two type of items are sold and in which inventory policies are considered. It is believed that the chapter contributed to the growing research on the dual-channel supply chain and the inventory management of the supply chain literature. Specifically, the contribution of Chapter 4 is two-fold. First, the chapter investigated the effect of adopting a dual-channel system where standardized and customized items are sold, on the total profit of a supply chain with price discrimination for customized items. This marketing strategy has been receiving increasing attention by business and researchers. Moreover, Chapter 4 contributed to the literature by studying the inventory management policy for standardized, customized and core items, where a unit of a core item is the basis for producing a standardized and/or customized items. The content of this chapter is published in a peer-reviewed journal (Applied Mathematical Modelling, *40*(21), 9454-9473; doi.org/10.1016/j.apm.2016.06.008).

The second contribution of this thesis is presented in Chapter 5. To the best of the author's knowledge, the mathematical models in this chapter are the first to address reverse logistics for a two-level dual-channel supply chain system. To be specific, the mathematical models of Chapter 5 contributed to the growing literature of the reverse logistics supply chain by investigating the effects of different return policies on a single-channel and a dual-channel supply chain system. Moreover, Chapter 5 studied the optimal pricing decisions (of both newly produced and refurbished items), the optimal inventory decisions (including the production/ordered quantity) and the number of shipments in a reverse logistics supply chain and when a return policy is considered. The content of this chapter is published in a peer-reviewed journal (Computers & Industrial Engineering, 106, 58–82; doi.org/10.1016/j.cie.2017.01.024).

The third and final contribution of this thesis is presented in Chapter 6. It is believed that the research in this chapter made an original contribution to the supply chain literature by investigating the effect of adopting a dual-channel strategy on the performance of a two-level supply chain system (in which prices and inventory decisions are decision variables) and when learning and forgetting effects are considered. Considering learning and forgetting effects in such systems increases competitiveness by reducing costs and increasing sales. Although it is important, there is no paper in the literature that combines the two streams. Furthermore, studying learning and forgetting in a dual-channel supply chain may lead to interesting insights due to the following reasons. First, moving from a situation where the company produces only a single standard item to a scenario where it offers a customized item in addition ultimately takes away scale effects from the standard item, which may imply lower learning and higher forgetting. This aspect may significantly influence the relative efficiency of the dual channel supply chain (as compared to a traditional setup), and therefore it has to be considered when deciding about which structure to adopt. Secondly, neglecting learning and forgetting in a decision support model may lead to wrong decision support and the selection of a wrong channel strategy or production policy, which may lead to unnecessarily high costs for the supply chain. As a result, it is important to study learning and forgetting in the described context, which is the subject of the chapter at hand. The content of this chapter was submitted for review in a peer-reviewed journal.

7.3 Future research directions

The assumptions made in this thesis were necessary to make the developed mathematical models possible to track and solve. The models developed may be viewed by some as representing an ideal situation or an over-abstraction of the problems, which may affect their solutions. However, despite the limiting assumptions made, it is worth noting that the problems addressed in this thesis represent what has been in practice by some businesses, to some extent. Therefore, these models have been developed such that they may be applied to real-world scenarios with some modifications, and with similar solutions.

The limitations of the presented research in this thesis are as follows: This thesis assumed a deterministic dependent demand function, which can become random by adding noise to it. A stochastic component can either be of an additive or multiplicative form (Petruzzi and Dada, 1999). A time-dependent demand function could also be used to test the dynamicity of the models and its parameters. This thesis uses sensitivity analysis to capture this. However, having the parameter as a function of time in the developed models would make the models more realistic and dynamic but complex. Finding closed-form solutions could be challenging and worthy of another doctoral thesis. Moreover, the research in this thesis assumed that the customized items are sold only by the vendor; however, it would be interesting to offer standard and customized items in both channels. This thesis also assumed that the custom features are independent of each other. In other cases, however, these features could be dependent on each other. If the custom features are dependent, then maybe introducing a dependent feature and/or its family would affect the entire mathematics of this thesis. In addition, selecting a specific custom feature may eliminate the selection of other features to be included with the customization. This will most probably require the introduction of binary variables “yes” or “no” choices. Furthermore, the models proposed in this thesis considered a centralized scenario. Developing a decentralized scenario of the proposed models using a game theoretic (i.e., the vendor is the leader and the retailer is the follower) and comparing them with the centralized scenario could represent a valuable contribution to the supply chain literature. The absence of real-world data is also a limitation in this thesis. However, if the data becomes available, then the models of this thesis could be tested. If errors are significant, then the models should be revisited and modified. Perhaps using a control theory approach with feedback loops could result in having the models move closer to reality. Additionally, this thesis did not account for economic and strategic aspects. This could be an interesting aspect to investigate and develop.

The mathematical models presented in Chapter 4 assumed that the markup margin between the vendor and the retailer is the same as between the vendor and customers. This limitation could be addressed in future works. Another limitation in this thesis is that the custom features are assumed to be outsourced; it will be interesting to see the effect of moving some or all of the production of these features in-house at the manufacturer's facility. This problem would be interesting, and perhaps challenging, to model, solve and analyze.

The mathematical models in Chapter 5 assumed that returned unrepairable items cannot be recovered and are disposed of. Future research may consider that unrepairable items can be disassembled to recover useful components and parts (Tahirov et al., 2016) and that an item can be refurbished for a limited number of times (El Saadany et al., 2013). The quality and acquisition price of returned items, learning-by-doing of remanufacturing activities, and frequency of remanufacturing need also to be thought about in a future work (El Saadany and Jaber, 2010; Martin et al., 2010; Jaber, 2011). Future research may also consider zone-based or clustering based return approach where the return policy depends on the item and the culture of customers.

The mathematical models in Chapter 6 considered no learning and forgetting for the customized items. Addressing this limitation requires accounting for the inter-arrival time between orders of the same type of customized items and the patterns of arrivals. It also requires developing a stochastic learning and forgetting model to investigate their effects in such an environment. Workforce flexibility and cross-training could also be investigated to address this limitation. Another limitation in is the assumption of perfect production; i.e., no defective units are generated that require rework. This also could be addressed in a future work by considering improvements in quality and setups. Finally, the models of Chapter 6 considered the forward flow. It is common that there are unsatisfied customers who would return their products to the retailer/vendor. This would be a challenging problem, especially when considering refurbishing or remanufacturing returns and selling them in a secondary market. Adding a reverse channel affects the pricing and inventory decisions of the forward channel.

APPENDICES

Appendix A.1 (Chapter 4)

Single-channel strategy

To determine if the total profit of the single-channel supply chain strategy, Π_s, is a concave function in m, q_r, and n_r, one need to examine all its second derivatives with respect to m, q_r, and n_r, as follows:

$$\frac{\partial^2 \Pi_s}{\partial m^2} = -2\alpha_r c_P^2 < 0; \qquad \text{(A.1.1)}$$

$$\frac{\partial^2 \Pi_s}{\partial q_r^2} = -\frac{2\,(S_r + n_r O_r) D_r}{n_r q_r^3} < 0; \qquad \text{(A.1.2)}$$

$$\frac{\partial^2 \Pi_s}{\partial n_r^2} = -\frac{2\,S_r D_r}{n_r^3 q_r} < 0 \qquad \text{(A.1.3)}$$

From the above, since all the parameters in this chapter are positive and a should be large enough to insure that the demand function is positive, the total profit Π_s is a concave function in m, q_r and n_r which indicate that an optimal solution exist.

The optimal solution of m, q_r and n_r can be found by setting the first-order partial derivative of Π_s to zero and solving for m, q_r and n_r as follows:

Finding the optimal solution for m

$$\frac{\partial \Pi_s}{\partial m} = -2\,A\,\alpha_r\,c_P^2 + a\,c_P + \alpha_r\,c_P^2 + \frac{\alpha_r(S_r + n_r O_r)}{n_r q_r} + \frac{\alpha_r\,q_r\,c_P(h_1 + h_2 - n_r h_2)}{2P_r} = 0 \qquad \text{(A.1.4)}$$

Therefore, m^* equals to

$$m = -1 + \sqrt{\frac{1}{2} + \frac{a}{2\,\alpha_r\,c_P} + \frac{(S_r + n_r O_r)}{2\,c_P\,n_r q_r} + \frac{q_r\,(h_1 + h_2 - n_r h_2)}{4\,c_P P_r}}$$

Finding the optimal solution for q_r

$$\frac{\partial \Pi_s}{\partial q_r} = \frac{(S_r + n_r O_r)\,D_r}{n_r q_r^2} - \frac{(h_1 + h_2 - n_r h_2) D_r}{2P_r} - \frac{h_2 n_r}{2} = 0 \qquad \text{(A.1.5)}$$

Therefore, q_r^* equals to

$$q_r^* = \sqrt{\frac{2P_r(S_r + n_r O_r) D_r}{n_r\big((h_1 + h_2 - n_r h_2) D_r + P_r n_r h_2\big)}}$$

Finding the optimal solution for n_r

$$\frac{\partial \Pi_s}{\partial n_r} = \frac{S_r D_r}{n_r^2 q_r} + \frac{h_2\,q_r D_r}{2P_r} - \frac{h_2 q_r}{2} = 0 \qquad \text{(A.1.6)}$$

Therefore, n_r^* equals to

$$n_r^* = \frac{1}{q_r}\sqrt{\frac{2\,P_r\,S_r D_r}{(\,P_r\,h_2 - h_2 D_r)}}$$

Appendix A.2 (Chapter 4)

Dual-channel strategy

To determine if the total profit of the dual-channel supply chain strategy, $\Pi_{s,c}$, is a concave function, one must examine all its second derivative with respect to m, q_r, q_d and n_r as follows:

$$\begin{aligned}\frac{\partial^2 \Pi_{s,c}}{\partial m^2} &= -12\,(1+m)^2\,\alpha_r\,c_P^2 + 6\,(1+m)\rho\,c_P\,\sum_{k=1}^{N} c_{dk}\,\varphi_{dk} \\ &\quad + 6\,(1+m)\rho\,c_P\sum_{k=1}^{N} c_{dk}\; - 2\sum_{k=1}^{N}\,(\alpha_{dk}c_{dk}^2\,\varphi_{dk}) \\ &\quad - 2\,\rho\,c_P\,\sum_{k=1}^{N} c_{dk}\,\varphi_{dk} + 2\,c_P(1-\theta)a + 2\,c_P\,\beta_r\,l_d - 2\,c_P\,\psi_r\,\eta_d \\ &\quad + 2\,\alpha_r\,c_P^2 - \frac{2\,S_d\,\rho\,c_P}{q_d} + \frac{h_1\,q_d\,\rho\,c_P}{P_d} + \frac{2(S_r + O_r\,n_r)\,\alpha_r\,c_P}{n_r\,q_r} \\ &\quad + \frac{(h_1 + h_2 - n_r\,h_2)\,q_r\,\alpha_r\,c_P}{P_r};\end{aligned} \tag{A.2.1}$$

Due to the complexity of proving that the second derivative of $\Pi_{s,c}$ is a concave function in $\left(\frac{\partial^2 \Pi_{s,c}}{\partial m^2} < 0\right)$, a simulation with random number is done and the solution of which have shown that its negative for all values $\left(\frac{\partial^2 \Pi_{s,c}}{\partial m^2} < 0\right)$. Therefore $\Pi_{s,c}$ is concave in m

The proof of concavity of $\Pi_{s,c}$ with respect to the other variables (q_r, n_r *and* q_d) is as follows:

$$\frac{\partial^2 \Pi_{s,c}}{\partial q_r^2} = -\frac{2\,(S_r + n_r O_r)D_r}{n_r q_r^3} < 0; \tag{A.2.2}$$

$$\frac{\partial^2 \Pi_{s,c}}{\partial n_r^2} = -\frac{2\, S_r D_r}{n_r^3 q_r} < 0; \tag{A.2.3}$$

$$\frac{\partial^2 \Pi_{s,c}}{\partial q_d^2} = -\frac{2\, S_d D_d}{q_d^3} < 0 \tag{A.2.4}$$

The optimal solution of q_r, n_r, and q_d can be found by setting the first-order partial derivative of $\Pi_{s,c}$ to zero and solving q_r, n_r, and q_d as follows:

Finding the optimal solution for q_r

$$\frac{\partial \Pi_{s,c}}{\partial q_r} = \frac{(S_r + n_r\, O_r)\, D_r}{n_r q_r^2} - \frac{(h_1 + h_2 - n_r h_2) D_r}{2 P_r} - \frac{n_r\, h_2}{2} = 0 \tag{A.2.5}$$

Therefore q_r^* equals to

$$q_r^* = \sqrt{\frac{2 P_r (S_r + n_r\, O_r) D_r}{n_r \big((h_1 + h_2 - n_r h_2) D_r + P_r\, n_r\, h_2\big)}}$$

Finding the optimal solution for n_r

$$\frac{\partial \Pi_{s,c}}{\partial n_r} = \frac{S_r D_r}{n_r^2 q_r} + \frac{h_2\, q_r D_r}{2 P_r} - \frac{h_2\, q_r}{2} = 0 \tag{A.2.6}$$

Therefore n_r^* equals to

$$n_r^* = \frac{1}{q_r} \sqrt{\frac{2\, P_r\, S_r D_r}{(\, P_r\, h_2 - h_2\, D_r)}}$$

Finding the optimal solution for $\boldsymbol{q_d}$

$$\frac{\partial \Pi_{s,c}}{\partial q_d} = \frac{S_d\, D_d}{q_d^2} - \frac{h_1}{2}\left(1 - \frac{D_d}{P_d}\right) = 0 \qquad \text{(A.2.7)}$$

Therefore q_d^* equals to

$$q_d^* = \sqrt{\frac{S_d D_d}{\frac{h_1}{2}\left(1 - \frac{D_d}{P_d}\right)}}$$

Appendix B.1 (Chapter 5)

Single-channel strategy

To determine if the total profit of the system Π_{single} is a concave function in p_s, p_f, q_s, n_s, and n_f, one need to examine all its second derivatives with respect to p_s, p_f, q_s, n_s, and n_f as follows:

$$\frac{\partial^2 \Pi_{single}}{\partial p_s^2} = -2\eta_s^2\gamma_s\rho_s - 2\zeta - 2\delta_s + 2\zeta\eta_s\rho_s + 2\delta_s\eta_s\rho_s + 2\eta_s\gamma_s < 0; \tag{B.1.1}$$

$$\frac{\partial^2 \Pi_{single}}{\partial p_f^2} = -2\eta_f^2\gamma_f\rho_f - 2\zeta - 2\delta_f + 2\zeta\eta_f\rho_f + 2\delta_f\eta_f\rho_f + 2\eta_f\gamma_f < 0; \tag{B.1.2}$$

$$\frac{\partial^2 \Pi_{single}}{\partial q_s^2} = -\frac{2\,(S_s + n_s O_{bs})D_s}{n_s q_s^3} - \frac{2\,S_f\left(1-\beta_f\rho_f\right)D_f}{n_s q_s^3\beta_s\rho_s} - \frac{2\,O_{bf}\left(1-\beta_f\rho_f\right)n_f D_f}{n_s q_s^3\beta_s\rho_s} < 0; \tag{B.1.3}$$

$$\frac{\partial^2 \Pi_{single}}{\partial n_s^2} = -\frac{2(S_s + O_{bs})D_s}{n_s^3 q_s} - \frac{2S_f\left(1-\beta_f\rho_f\right)D_f}{n_s^3 q_s\beta_s\rho_s} < 0; \tag{B.1.4}$$

$$\frac{\partial^2 \Pi_{single}}{\partial n_f^2} = \frac{h_{v_2f} n_s q_s\beta_s\rho_s}{n_f^2\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2f} n_s q_s\beta_s\rho_s(n_f-1)}{n_f^3\left(1-\beta_f\rho_f\right)} - \frac{h_{bf} n_s q_s\beta_s\rho_s}{n_f^3\left(1-\beta_f\rho_f\right)} < 0; \tag{B.1.5}$$

From the above, since all the parameters in this chapter (Chapter 5) are positive, $D_s > 0$, $D_f > 0$, and $h_{bf} > h_{v_2f}$, the total profit of the system Π_{single} is concave in p_s, p_f, q_s, n_s, and n_f which indicate that an optimal solution exist.

The optimal solution of p_s, p_f, q_s, n_s, and n_f can be found by taking the first-order partial derivative of Π_{singel} with respect to p_s, p_f, q_s, n_s, and n_f and setting to zero as follows:

Finding the optimal solution for $\boldsymbol{p_s}$

$$\frac{\partial \Pi_{single}}{\partial p_s} = a_s - 2\delta_s p_s + 2\gamma_s \eta_s p_s - 2\zeta p_s + 2\zeta p_f + \zeta l - \frac{S_s(\eta_s\gamma_s - \zeta - \delta_s)}{n_s q_s}$$

$$- \frac{S_f \zeta\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} - c_s(\eta_s\gamma_s - \zeta - \delta_s) - c_f \zeta$$

$$- \frac{h_{v_1 s}\, q_s(\eta_s\gamma_s - \zeta - \delta_s)}{2P_s} + \frac{h_{bs}\, q_s(n_s - 1)(\eta_s\gamma_s - \zeta - \delta_s)}{2P_s} \tag{B.1.6}$$

$$- \frac{O_{bs}(\eta_s\gamma_s - \zeta - \delta_s)}{q_s} - \frac{O_{bf}\zeta n_f\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} - c_w \alpha_s \rho_s(\eta_s\gamma_s - \zeta - \delta_s)$$

$$- c_w \alpha_f \rho_f \zeta - a_s \eta_s \rho_s + 2\eta_s p_s \rho_s \delta_s - 2\eta_s^2 \rho_s \gamma_s p_s + 2\eta_s p_s \rho_s \zeta$$

$$- \eta_s \rho_s \zeta p_f - \eta_s \rho_s l \zeta - \eta_f p_f \rho_f \zeta = 0$$

Finding the optimal solution for $\boldsymbol{p_f}$

$$\frac{\partial \Pi_{single}}{\partial p_f} = a_f - 2\delta_f p_f + 2\gamma_f \eta_f p_f - 2\zeta p_f + 2\zeta p_s - \zeta l - \frac{S_s \zeta}{n_s q_s}$$

$$- \frac{S_f\left(\eta_f\gamma_f - \zeta - \delta_f\right)\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} - c_s \zeta - c_f\left(\eta_f\gamma_f - \zeta - \delta_f\right)$$

$$- \frac{h_{v_1 s}\, q_s \zeta}{2P_s} + \frac{h_{bs}\, q_s \zeta(n_s - 1)}{2P_s} - \frac{O_{bs}\zeta}{q_s}$$

$$- \frac{O_{bf} n_f\left(\eta_f\gamma_f - \zeta - \delta_f\right)\left(1 - \beta_f \rho_f\right)}{n_s q_s \beta_s \rho_s} - c_w \zeta \alpha_s \rho_s \tag{B.1.7}$$

$$- c_w \alpha_f \rho_f\left(\eta_f\gamma_f - \zeta - \delta_f\right) - a_f \eta_f \rho_f + 2\eta_f p_f \rho_f \delta_f - 2\eta_f^2 \rho_f \gamma_f p_f$$

$$+ 2\eta_f p_f \rho_f \zeta - \eta_f \rho_f \zeta p_s + \eta_f \rho_f l \zeta - \eta_s p_s \rho_s \zeta = 0$$

Solving Eqs. (B.1.7) and (B.1.7) simultaneously yields Eqs. (5.23) and (5.24).

Finding the optimal solution for q_s

$$\frac{\partial \Pi_{single}}{\partial q_s} = \frac{S_s D_s}{n_s q_s^2} + \frac{S_f D_f\left(1-\beta_f\rho_f\right)}{n_s^2 q_s \beta_s \rho_s} - \frac{h_{v_{1s}} D_s}{2P_s} - \frac{h_{bs}}{2}\left(n_s - \frac{(n_s-1)D_s}{P_s}\right)$$
$$- \frac{h_{v_2 f} n_s \beta_s \rho_s \left(n_f - 1\right)}{n_f\left(1-\beta_f\rho_f\right)} - \frac{h_{bf} n_s \beta_s \rho_s}{2n_f\left(1-\beta_f\rho_f\right)} - \frac{h_{ub}\beta_s\rho_s n_s}{2} \quad \text{(B.1.8)}$$
$$- \frac{h_{ub}\beta_f\rho_f\beta_s\rho_s n_s}{2\left(1-\beta_f\rho_f\right)} + \frac{O_{bs} D_s}{q_s^2} + \frac{O_{bf} D_f n_f\left(1-\beta_f\rho_f\right)}{n_s q_s^2 \beta_s \rho_s} = 0$$

Solving Eq. (B.1.8) gives Eq. (5.25)

Finding the optimal solution for n_s

$$\frac{\partial \Pi_{single}}{\partial n_s} = \frac{S_s D_s}{n_s^2 q_s} + \frac{S_f D_f\left(1-\beta_f\rho_f\right)}{n_s^2 q_s \beta_s \rho_s} - \frac{h_{bs}}{2}\left(q_s - \frac{q_s D_S}{P_s}\right) - \frac{h_{v_2 f} q_s \beta_s \rho_s\left(n_f - 1\right)}{2n_f\left(1-\beta_f\rho_f\right)}$$
$$- \frac{h_{bf} q_s \beta_s \rho_s}{2n_f\left(1-\beta_f\rho_f\right)} - \frac{h_{ub}\beta_s\rho_s q_s}{2} - \frac{h_{ub}\beta_f\rho_f q_s\beta_s\rho_s}{2\left(1-\beta_f\rho_f\right)} \quad \text{(B.1.9)}$$
$$+ \frac{O_{bf} D_f n_f\left(1-\beta_f\rho_f\right)}{n_s^2 q_s \beta_s \rho_s} = 0$$

Solving Eq. (A.9) gives Eq. (26)

Finding the optimal solution for n_f

$$\frac{\partial \Pi_{single}}{\partial n_f} = -\frac{h_{v_2 f} n_s q_s \beta_s \rho_s}{2n_f\left(1-\beta_f\rho_f\right)} + \frac{h_{v_2 f} n_s q_s \beta_s \rho_s\left(n_f - 1\right)}{2n_f^2\left(1-\beta_f\rho_f\right)} + \frac{h_{bf} n_s q_s \beta_s \rho_s}{2n_f^2\left(1-\beta_f\rho_f\right)}$$
$$- \frac{O_{bf} D_f\left(1-\beta_f\rho_f\right)}{n_s q_s \beta_s \rho_s} = 0 \quad \text{(B.1.10)}$$

Solving Eq. (B.1.10) gives Eq. (5.27)

Appendix B.2 (Chapter 5)

Dual-channel strategy

To determine if the total profit of the system Π_{dual} is a concave function in p_s, p_f, $\sum_{k=1}^{N} p_{zk}$, $\sum_{k=1}^{N} p_{fzk}$, q_s, n_s, and q_z, one need to examine all its second derivatives with respect to p_s, p_f, $\sum_{k=1}^{N} p_{zk}$, $\sum_{k=1}^{N} p_{fzk}$, q_s, n_s, and q_z as follows:

$$\frac{\partial^2 \Pi_{dual}}{\partial p_s^2} = -2\eta_s^2\gamma_s\rho_s - 2\zeta - 2\delta_s + 2\zeta\eta_s\rho_s + 2\delta_s\eta_s\rho_s + 2\eta_s\gamma_s < 0; \tag{B.2.1}$$

$$\frac{\partial^2 \Pi_{dual}}{\partial p_f^2} = -2\eta_f^2\gamma_f\rho_f - 2\zeta - 2\delta_f + 2\zeta\eta_f\rho_f + 2\delta_f\eta_f\rho_f + 2\eta_f\gamma_f < 0; \tag{B.2.2}$$

$$\begin{aligned}\frac{\partial^2 \Pi_{dual}}{\partial \sum_{k=1}^{N} p_{zk}^2} = &-\sum_{k=1}^{N}(2\eta_z^2\gamma_z\rho_z\varphi_{zk} - 2\zeta\eta_z\rho_z\varphi_{zk} - 2\delta_z\eta_z\rho_z\varphi_{zk}) \\ &+\sum_{k=1}^{N}(2\eta_z\gamma_z\varphi_{zk} - 2\zeta\varphi_{zk} - 2\delta_z\varphi_{zk}) < 0;\end{aligned} \tag{B.2.3}$$

$$\begin{aligned}\frac{\partial^2 \Pi_{dual}}{\partial \sum_{k=1}^{N} p_{fzk}^2} = &-\sum_{k=1}^{N}\left(2\eta_{fz}^2\gamma_{fz}\rho_{fz}\varphi_{fzk} - 2\zeta\eta_{fz}\rho_{fz}\varphi_{fzk} - 2\delta_{fz}\eta_{fz}\rho_{fz}\varphi_{fzk}\right) \\ &+\sum_{k=1}^{N}\left(2\eta_{fz}\gamma_{fz}\varphi_{fzk} - 2\zeta\varphi_{fzk} - 2\delta_{fz}\varphi_{fzk}\right) < 0;\end{aligned} \tag{B.2.4}$$

$$\frac{\partial^2 \Pi_{dual}}{\partial q_s^2} = -\frac{2\,(S_s + n_s O_{bs})D_s}{n_s q_s^3} - \frac{2\,S_f\left(1-\beta_f\rho_f\right)D_f}{n_s q_s^3\beta_s\rho_s} < 0; \tag{B.2.5}$$

$$\frac{\partial^2 \Pi_{dual}}{\partial n_s^2} = -\frac{2S_sD_s}{n_s^3 q_s} - \frac{2S_fD_f\left(1-\beta_f\rho_f\right)}{n_s^3 q_s\beta_s\rho_s} < 0; \tag{B.2.6}$$

$$\frac{\partial^2 \Pi_{dual}}{\partial q_z^2} = -\frac{2S_z D_s}{q_z^3} + \frac{2S_{fz} D_{fz}\left(1-\beta_{fz}\rho_{fz}\right)}{q_z^3 \beta_z \rho_z} < 0; \tag{B.2.7}$$

The optimal solution of p_s, p_f, $\sum_{k=1}^{N} p_{zk}$, $\sum_{k=1}^{N} p_{fzk}$, q_s, q_z, and n_s can be found by taking the first-order partial derivative of Π_{dual} with respect to p_s, p_f, $\sum_{k=1}^{N} p_{zk}$, $\sum_{k=1}^{N} p_{fzk}$, q_s, q_z, and n_s and setting to zero as follows:

Finding the optimal solution for $\boldsymbol{p_s}$

$$\begin{aligned}
\frac{\partial \Pi_{dual}}{\partial p_s} = {} & a_s - 2\delta_s p_s + 2\gamma_s \eta_s p_s - 2\zeta p_s + 2\zeta p_f + \zeta l - \frac{S_s(\eta_s\gamma_s - \zeta - \delta_s)}{n_s q_s} \\
& - \frac{S_f \zeta\left(1-\beta_f\rho_f\right)}{n_s q_s \beta_s \rho_s} - c_s(\eta_s\gamma_s - \zeta - \delta_s) - c_f\zeta \\
& - \frac{h_{v_1 s}\, q_s(\eta_s\gamma_s - \zeta - \delta_s)}{2P_s} + \frac{h_{bs}\, q_s(n_s - 1)(\eta_s\gamma_s - \zeta - \delta_s)}{2P_s} \\
& + \frac{h_{v_2 f} n_s q_s \beta_s \rho_s \zeta}{2P_f\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2 u}\beta_s\rho_s n_s q_s \zeta}{2P_f\left(1-\beta_f\rho_f\right)} - \frac{O_{bs}(\eta_s\gamma_s - \zeta - \delta_s)}{q_s} \\
& - c_w\alpha_s\rho_s(\eta_s\gamma_s - \zeta - \delta_s) - c_w\alpha_f\rho_f\zeta - a_s\eta_s\rho_s + 2\eta_s p_s\rho_s\delta_s \\
& - 2\eta_s^2\rho_s\gamma_s p_s + 2\eta_s p_s \rho_s \zeta - \eta_s\rho_s\zeta p_f - \eta_s\rho_s l\zeta - \eta_f p_f \rho_f \zeta = 0
\end{aligned} \tag{B.2.8}$$

Finding the optimal solution for $\boldsymbol{p_f}$

$$\frac{\partial \Pi_{dual}}{\partial p_f} = a_f - 2\delta_f p_f + 2\gamma_f \eta_f p_f - 2\zeta p_f + 2\zeta p_s - \zeta l - \frac{S_s \zeta}{n_s q_s} - \frac{S_f(\eta_f\gamma_f - \zeta - \delta_f)(1-\beta_f\rho_f)}{n_s q_s \beta_s \rho_s} - c_s\zeta - c_f(\eta_f\gamma_f - \zeta - \delta_f) - \frac{h_{v_1 s}\, q_s \zeta}{2P_s} + \frac{h_{bs}\, q_s\zeta(n_s - 1)}{2P_s} + \frac{h_{v_2 f} n_s q_s \beta_s \rho_s (\eta_f\gamma_f - \zeta - \delta_f)}{2P_f(1-\beta_f\rho_f)} - \frac{h_{v_2 u} n_s q_s \beta_s \rho_s(\eta_f\gamma_f - \zeta - \delta_f)}{2P_f(1-\beta_f\rho_f)} - \frac{O_{bs}\zeta}{q_s} - c_w\zeta\alpha_s\rho_s - c_w\alpha_f\rho_f(\eta_f\gamma_f - \zeta - \delta_f) - a_f\eta_f\rho_f + 2\eta_f p_f \rho_f \delta_f - 2\eta_f^2\rho_f\gamma_f p_f + 2\eta_f p_f \rho_f \zeta - \eta_f\rho_f\zeta p_s + \eta_f\rho_f l\zeta - \eta_s p_s \rho_s \zeta = 0 \quad \text{(B.2.9)}$$

Solving Eqs. (B.2.8) and (B.2.9) simultaneously yields Eqs. (5.52) and (5.53).

Finding the optimal solution for $\boldsymbol{\sum_{k=1}^{N} p_{zk}}$

$$\frac{\partial \Pi_{dual}}{\partial \sum_{k=1}^{N} p_{zk}} = \sum_{k=1}^{N}\big(a_z\varphi_{zk} - 2\delta_z p_{zk}\varphi_{zk} + 2\gamma_z\eta_z p_{zk}\varphi_{zk} - 2\zeta p_{zk}\varphi_{zk} + \zeta p_{fzk}\varphi_{zk} + \zeta l\varphi_{zk}\big) + \sum_{k=1}^{N}\zeta p_{fzk}\varphi_{fzk} - \frac{S_z(N\eta_z\gamma_z - N\zeta - N\delta_z)}{q_z} - \frac{S_{fz}\zeta N(1-\beta_{fz}\rho_{fz})}{q_z\beta_z\rho_z} - (\eta_z\gamma_z - \zeta - \delta_z)\sum_{k=1}^{N} c_{zk}\varphi_{zk} - \zeta\sum_{k=1}^{N} c_{fzk}\varphi_{fzk} + \frac{h_{v_1 z}\, q_z(N\eta_z\gamma_z - N\zeta - N\delta_z)}{2P_z} + \frac{h_{v_2 fz}\beta_z\rho_z q_z\zeta N}{2P_{fz}(1-\beta_{fz}\rho_{fz})} - \frac{h_{v_2 u}\beta_z\rho_z q_z \zeta N}{2P_{fz}(1-\beta_{fz}\rho_{fz})} - c_w\alpha_z\rho_z(N\eta_z\gamma_z - N\zeta - \delta_z) - c_w\alpha_{fz}\rho_{fz}\zeta N - \sum_{k=1}^{N}\big(a_z\eta_z\rho_{zk}\varphi_{zk} - 2\eta_z p_{zk}\rho_z\delta_z\varphi_{zk} + 2\eta_z^2\rho_z\gamma_z p_{zk}\varphi_{zk} - 2\eta_z p_{zk}\varphi_{zk}\rho_z\zeta + \eta_z\rho_z\zeta p_{fzk}\varphi_{zk} + \eta_z\rho_z\varphi_{zk} l\zeta\big) - \sum_{k=1}^{N}\eta_{fz} p_{fzk}\varphi_{fzk}\rho_{fz}\zeta = 0 \quad \text{(B.2.10)}$$

Finding the optimal solution for $\sum_{k=1}^{N} p_{fzk}$

$$\begin{aligned}
\frac{\partial \Pi_{dual}}{\partial \sum_{k=1}^{N} p_{fzk}} = \sum_{k=1}^{N} \big(& a_{fz}\varphi_{fzk} - 2\delta_{fz}p_{fzk}\varphi_{fzk} + 2\gamma_{fz}\eta_{fz}p_{fzk}\varphi_{fzk} + \zeta p_{zk}\varphi_{fzk} \\
& - 2\zeta p_{fzk}\varphi_{fzk} - \zeta l\varphi_{zk}\big) + \sum_{k=1}^{N} \zeta p_{zk}\varphi_{zk} - \frac{S_z\zeta N}{q_z} \\
& - \frac{S_{fz}\left(N\eta_{fz}\gamma_{fz} - N\zeta - N\delta_{fz}\right)\left(1 - \beta_{fz}\rho_{fz}\right)}{q_z\beta_z\rho_z} - \zeta \sum_{k=1}^{N} c_{zk}\varphi_{zk} \\
& - \left(\eta_{fz}\gamma_{fz} - \zeta - \delta_{fz}\right) \sum_{k=1}^{N} c_{fzk}\varphi_{fzk} + \frac{h_{v_1 z}\, q_z N\zeta}{2P_z} \\
& + \frac{h_{v_2 fz}\beta_z\rho_z q_z\left(N\eta_{fz}\gamma_{fz} - N\zeta - N\delta_{fz}\right)}{2P_{fz}\left(1 - \beta_{fz}\rho_{fz}\right)} \\
& - \frac{h_{v_2 u}\beta_z\rho_z q_z\left(N\eta_{fz}\gamma_{fz} - N\zeta - N\delta_{fz}\right)}{2P_{fz}\left(1 - \beta_{fz}\rho_{fz}\right)} - c_w\alpha_z\rho_z N\zeta \\
& - c_w\alpha_{fz}\rho_{fz}\left(N\eta_{fz}\gamma_{fz} - N\zeta - N\delta_{fz}\right) \\
& - \sum_{k=1}^{N} \big(a_{fz}\eta_{fz}\rho_{fzk}\varphi_{fzk} - 2\eta_{fz}p_{fzk}\rho_{fz}\delta_{fz}\varphi_{fzk} \\
& + 2\eta_{fz}^2\rho_{fz}\gamma_{fz}p_{fzk}\varphi_{fzk} - 2\eta_{fz}p_{fzk}\varphi_{fzk}\rho_{fz}\zeta + \eta_{fz}\rho_{fz}\zeta p_{zk}\varphi_{fzk} \\
& - \eta_{fz}\rho_{fz}\varphi_{fzk} l\zeta\big) - \sum_{k=1}^{N} \eta_z p_{zk}\varphi_{zk}\rho_z\zeta = 0
\end{aligned} \tag{B.2.11}$$

Solving Eqs. (B.2.10) and (B.2.11) simultaneously yields Eqs. (5.54) and (5.55).

Finding the optimal solution for q_s

$$\frac{\partial \Pi_{dual}}{\partial q_s} = -\frac{h_{bu}\beta_s\rho_s n_s}{2} + \frac{S_f D_f\left(1-\beta_f\rho_f\right)}{n_s q_s^2 \beta_s \rho_s} - \frac{h_{v_1 s} D_s}{2P_s} - h_{bs}\left(n_s - \frac{(n_s-1)D_s}{P_s}\right) + \frac{S_s D_s}{n_s q_s^2}$$
$$- \frac{h_{v_2 f} n_s \beta_s \rho_s \left(1-\frac{D_f}{P_f}\right)}{2\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2 u}\beta_f\rho_f\beta_s\rho_s n_s}{2\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2 u}\beta_s\rho_s D_s}{P_f\left(1-\beta_f\rho_f\right)} + \frac{O_{bs}D_s}{q_s^2} = 0 \quad \text{(B.2.12)}$$

Solving Eq. (B.2.12) gives Eq. (5.56)

Finding the optimal solution for n_s

$$\frac{\partial \Pi_{dual}}{\partial n_s} = -\frac{h_{v_2 u}\beta_s\rho_s\beta_f\rho_f q_s}{2\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2 u}\beta_s\rho_s q_s D_f}{2P_f\left(1-\beta_f\rho_f\right)} - \frac{h_{v_2 f} q_s\beta_s\rho_s\left(1-\frac{D_f}{P_f}\right)}{2\left(1-\beta_f\rho_f\right)} - \frac{h_{bs}}{2}\left(q_s - \frac{q_s D_s}{P_s}\right)$$
$$+ \frac{S_s D_s}{n_s^2 q_s} + \frac{S_f D_f\left(1-\beta_f\rho_f\right)}{n_s^2 q_s \beta_s\rho_s} - \frac{h_{bu}\beta_s\rho_s q_s}{2} = 0 \quad \text{(B.2.13)}$$

Solving Eq. (B.2.13) gives Eq. (5.57)

Finding the optimal solution for q_z

$$\frac{\partial \Pi_{dual}}{\partial q_z} = -\frac{h_{v1z}}{2}\left(1-\frac{D_z}{P_z}\right) - \frac{h_{v_2 fz}\beta_z\rho_z}{2(1-\beta_{fz}\rho_{fz})}\left(1-\frac{D_{fz}}{P_{fz}}\right) - \frac{h_{v_2 u}\beta_z\,\rho_z q_z}{2}$$
$$- \frac{h_{v_2 u}\beta_{fz}\rho_{fz}\beta_z\rho_z}{2(1-\beta_{fz}\rho_{fz})} + \frac{h_{v_2 u}\beta_z\rho_z D_{fz}}{2P_{fz}(1-\beta_{fz}\rho_{fz})} + \frac{S_z D_s}{q_z^2} \quad \text{(B.2.14)}$$
$$+ \frac{S_{fz}D_{fz}(1-\beta_{fz}\rho_{fz})}{q_z^2\beta_z\rho_z} = 0$$

Solving Eq. (B.2.14) gives Eq. (5.58)

Appendix C.1 (Chapter 6)

To formulate a closed from for $b_{r,i}$, Eq. (6.8) was simplified by taking the log of both side as follows:

$$\begin{aligned} b_r \log(1+u_{r,i}) \log\left[\left(n_{r,i}q_{r,i}+u_{r,i}\right)^{1-b_r} - u_{r,i}^{1-b_r}\right] \\ - \log(1-b_r) - \log\left(n_{r,i}q_{r,i}\right) \\ = -b_{r,i}\log\left(n_{r,i}q_{r,i}\right) - \log\left(1-b_{r,i}\right) \end{aligned} \tag{C.1.1}$$

The term $\left[-\log\left(1-b_{r,i}\right)\right]$ was plotted for different range of b_r (i.e. $b_r = 0.05$ to 0.7). The range of b_r was chosen based on the suggestion of Dar-El et al. (1995, p. 273). Figure C.1 depict the plotting of $\left[-\log\left(1-b_{r,i}\right)\right]$.

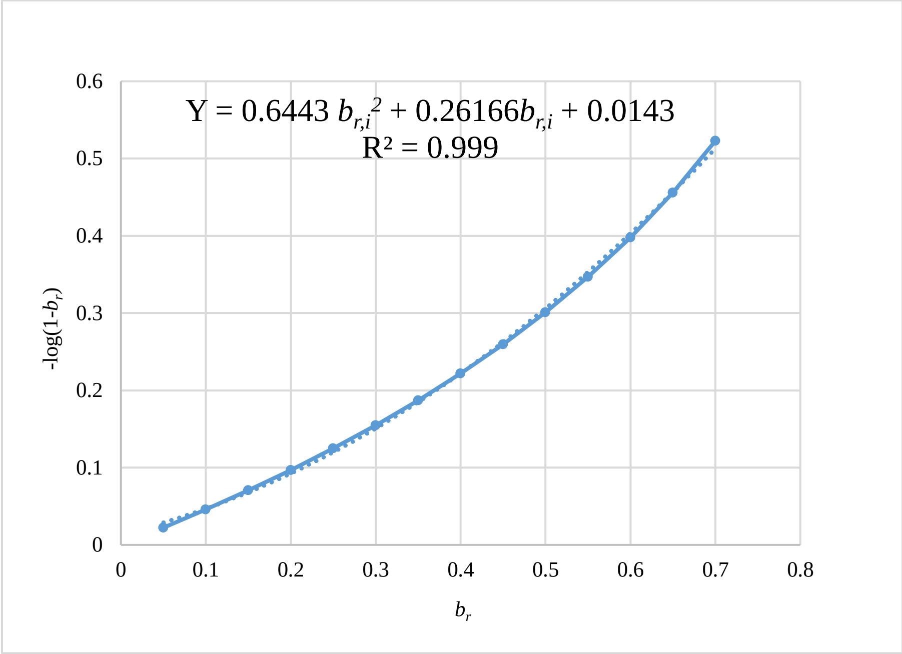

Figure. C.1: Approximation of -log(1-$b_{r,i}$)

Let:

$$\gamma = 0.6443$$

$$\delta_i = 0.26166 - \log\left(n_{r,i}q_{r,i}\right)$$

$$\varepsilon_i = 0.0143 - \left(b_r \log\left(1 + u_{r,i}\right) \log\left[\left(n_{r,i}q_{r,i} + u_{r,i}\right)^{1-b_r} - u_{r,i}^{1-b_r}\right] - \log(1 - b_r) - \log\left(n_{r,i}q_{r,i}\right)\right)$$

Then $b_{r,i}$ can be found using the expression $b_{r,i} = \left(\frac{-\delta_i \pm \sqrt{\delta_i^2 - 4\gamma\varepsilon_i}}{2\gamma}\right)$

Appendix C.2 (Chapter 6)

Single-channel strategy-policy 0

It was not possible to find a globally optimal solution for the concave function Π^0_{single} in p^0_r, q^0_r, and n^0_r, due to the complexity of the hessian matrix. However, alternatively, a local optimal solution with respect to p^0_r was obtained as follows:

$$\frac{\partial^2 \Pi^0_{single}}{\partial (p^0_r)^2} = -2\alpha_r < 0$$

The expression of A^0 and B^0 in Eq. (6.13) are as follows:

$$A^0 = a + \frac{S_r \alpha_r}{n_r q_r} + \frac{O_r \alpha_r}{q_r}$$

$$B^0 = \frac{c_P \alpha_r}{P_r} + \frac{h_{v1} q_r \alpha_r}{2P_r} - \frac{h_{v2}(n_r - 1) q_r \alpha_r}{2P_r} - \frac{h_r (n_r - 1) q_r \alpha_r}{2P_r}$$

Single-channel strategy-policy I.1

The holding cost of the vendor at the vendor's side for policy I.1 is calculated by first finding the production quantity, q_r, as a function of time, which is found from Eq. (6.7) as follows:

$$q_{r,i}(t_{r,i}) = \left(\frac{1 - b_{r,i}}{T_{r1,i}} t_{r,i}\right)^{1/(1-b_{r,i})} \quad \text{(C.2.1)}$$

The number of shipped batches from the vendor to the retailer, $m_{r,i}$, is calculated as follows:

$$m_{r,i} = \left\lceil \frac{1/P_r(1 - b_{r,i}) \left[n_{r,i} \left(q_{r,i}^{1-b_{r,i}} \right) \right]}{1/P_r(1 - b_{r,i}) \left(q_{r,i}^{1-b_{r,i}} \right)} \right\rceil = \left\lceil n_{r,i}^{1-b_{r,i}} \right\rceil \quad \text{(C.2.2)}$$

From Fig. 6.3, the total inventory holding cost for the vendor at its side, H_{v1}^{I1}, is given by:

$$H_{v1,i}^{\mathrm{I.1}} = \sum_{j=1}^{m_{r,i}} H_j + H_{m_{r,i}} + H_{n_{r,i}} \tag{C.2.3}$$

where $j = 1, 2, 3, \dots m_{r,i}$, and the parameters H_j, $H_{m_{r,i}}$ and $H_{n_{r,i}}$are calculated as follows:

$$\sum_{j=1}^{m_{r,i}} H_j = h_{v1} \frac{T_{r1,i}}{2-b_{r,i}} \left(\frac{1-b_{r,i}}{T_{r1,i}} m_{r,i} t_{r,i}\right)^{\frac{2-b_{r,i}}{1-b_{r,i}}} - h_{v1} \frac{m_{r,i}(m_{r,i}-1)}{2} q_{r,i} t_{r,i} \tag{C.2.3a}$$

$$H_{m_{r,i}} = h_{v1} q_{r,i} \left(n_{r,i} - (m_{r,i}-1)\right)\left(m_{r,i} t_{r,i} - \frac{T_{r1,i}(n_{r,i} q_{r,i})^{1-b_{r,i}}}{(1-b_{r,i})}\right) \tag{C.2.3b}$$

$$H_{n_{r,i}} = h_{v1} q_{r,i} t_{r,i} \frac{(n_{r,i}-m_{r,i})(n_{r,i}-m_{r,i}+1)}{2} \tag{C.2.3c}$$

The proof of concavity for $\Pi_{single,i}^{\mathrm{I.1}}$ was shown to be similar to the one provided in "Single-channel strategy-policy 0" and is given as:

$$\frac{\partial^2 \Pi_{single,i}^{\mathrm{I.1}}}{\partial\left(p_{r,i}^{\mathrm{I.1}}\right)^2} = -2\alpha_r < 0$$

The expression $B_i^{\mathrm{I.1}}$ in Eq. (6.16) is given by:

$$B_i^{\mathrm{I.1}} = \frac{c_P \alpha_r T_{r1,i}(n_{r,i} q_{r,i})^{-b_{r,i}}}{1-b_{r,i}} - \frac{(h_{v2}+h_r)(n_{r,i}-1) t_{r,i} \alpha_r}{2}$$

Single-channel strategy-policy I.2

In policy I.2 and looking at Fig. 6.4, one can notice that $m_{r,i} = n_{r,i}$, therefore, $H_{n_{r,i}} = 0$. The holding cost for the vendor at its side, $H_{v1,i}^{\mathrm{I.2}}$, can be calculated as follows.

$$H_{v1,i}^{\mathrm{I.2}} = \sum_{j=1}^{m_{r,i}} H_j + H_{m_{r,i}} = h_{v1} q_{r,i} \left(n_{r,i} t_{r,i} - \frac{T_{r1,i} \left(n_{r,i} q_{r,i} \right)^{1-b_{r,i}}}{\left(1 - b_{r,i} \right)} \right) \tag{C.2.4}$$

Also from Fig.6.4, the total holding cost for the vendor at the retailer's side, $H_{v2,i}^{\mathrm{I.2}}$, can be calculated as follows:

$$\begin{aligned} H_{v2,i}^{\mathrm{I.2}} = h_{v2} \frac{n_{r,i} - 1}{2} \left[q_{r,i} t_{r,i} n_{r,i} \left(n_{r,i} - 1 \right) D_{r,i} t_{r,i}^2 \right] \\ + h_{v2} \frac{\left(n_{r,i} q_{r,i} - \left(n_{r,i} - 1 \right) D_{r,i} t_{r,i} \right)^2}{2 D_{r,i}} \end{aligned} \tag{C.2.5}$$

The proof of concavity for $\Pi_{single,i}^{\mathrm{I.2}}$ was shown to be similar to the one provided in "Single-channel strategy-policy 0" and is given as:

$$\frac{\partial^2 \Pi_{single,i}^{\mathrm{I.2}}}{\partial \left(p_{r,i}^{\mathrm{I.2}} \right)^2} = -2\alpha_r < 0$$

The expression $B_i^{\mathrm{I.2}}$ in Eq. (6.19) is given by:

$$B_i^{\mathrm{I.2}} = \frac{c_P \alpha_r T_{r1,i} \left(n_{r,i} q_{r,i} \right)^{-b_{r,i}}}{1 - b_{r,i}} - \left(h_{v2} + h_r \right) \frac{\alpha_r n_{r,i} t_{r,i}}{2} + \left(h_{v2} + h_r \right) \frac{\alpha_r t_{r,i}}{2}$$

Single-channel strategy-policy I.3

For policy I.3 and from Fig 6.5, the total holding cost for items at the vendor's side is given by:

$$H_{v1,i}^{1.3} = h_{v1}\frac{T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{1-b_{r,i}}}{2-b_{r,i}}j \tag{C.2.6}$$

The proof of concavity for $\Pi_{single,i}^{\mathrm{I}.3}$ was shown to be similar to the one provided in "Single-channel strategy-policy 0" and is given as:

$$\frac{\partial^2 \Pi_{single,i}^{\mathrm{I}.3}}{\partial\left(p_{r,i}^{\mathrm{I}.3}\right)^2} = -2\alpha_r < 0$$

The formula for $B_i^{\mathrm{I}.1}$ in Eq. (6.21) is as follows:

$$B_i^{\mathrm{I}.3} = \frac{c_P\alpha_r T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)} - \frac{(h_{v2}+h_r)\left(n_{r,i}-1\right)t_{r,i}\alpha_r}{2}$$

Single-channel strategy-policy II

In policy II, the time to produce all the $n_{r,i}q_{r,i}$ batches in a production cycle is determined as:

$$t_{total} = \frac{T_{1i}}{1-b_{r,i}}\left(n_{r,i}q_{r,i}\right)^{1-b_{r,i}} \tag{C.2.7}$$

The time between shipments from the vendor to the retailer is given by:

$$t_{r,i} = \frac{t_{tot}}{n_{r,i}} \tag{C.2.8}$$

The total inventory holding cost for the vendor at its side is given by

$$H_{v1,i}^{\mathrm{II}} = \sum_{j=1}^{n_{r,i}} H_j = \frac{T_{r1,i}}{2-b_{r,i}}\left(\frac{1-b_{r,i}}{T_{r1,i}} n_{r,i} t_{r,i}\right)^{\frac{2-b_{r,i}}{1-b_{r,i}}}$$

$$- h_{v1}\left(\frac{1-b_{r,i}}{2-b_{r,i}}\right)\left(\frac{1-b_{r,i}}{T_{r1,i}}\right)^{1/(1-b_{r,i})}\left[\left(n_{r,i}-0.5\right)t_{r,i}\right]^{\frac{2-b_{r,i}}{1-b_{r,i}}} \tag{C.2.9}$$

The total inventory holding cost for the vendor at the retailer's side is as follows:

$$H_{v2,i}^{\mathrm{II}} = h_{v2}\left(\frac{1-b_{r,i}}{T_{r1,i}} t_{r,i}\right)^{\frac{1}{1-b_{r,i}}}\left(\frac{1-b_{r,i}}{2-b_{r,i}}\right)\left[\left(n_{r,i}-0.5\right)^{\frac{2-b_{r,i}}{1-b_{r,i}}} - 0.5^{\frac{2-b_{r,i}}{1-b_{r,i}}}\right]$$

$$- h_{v2,i}\left(n_{r,i}-1\right)^2 D_{r,i} t_{r,i}^2 \tag{C.2.10}$$

$$+ \frac{h_{v2}}{2D_{r,i}}\left(\left(\frac{1-b_{r,i}}{T_{r1,i}} n_{r,i} t_{r,i}\right)^{\frac{1}{1-b_{r,i}}} - \left(n_{r,i}-1\right)D_{r,i} t_{r,i}\right)^2$$

The proof of concavity for $\Pi_{single,i}^{\mathrm{II}}$ was shown to be similar to the one provided in "Single-channel strategy-policy 0" and is given as:

$$\frac{\partial^2 \Pi_{single,i}^{\mathrm{II}}}{\partial\left(p_{r,i}^{\mathrm{II}}\right)^2} = -2\alpha_r + \frac{\alpha_r^2 t_{r,i}^2 (h_{v2}+h_r)\left(n_{r,i}-1\right)^2}{n_{r,i} q_{r,i}} < 0$$

The expressions B_i^{II}, F_i^{II} and G_i^{II} in Eq. (6.24) are as follows:

$$B_i^{\mathrm{II}} = \frac{c_P \alpha_r T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{-b_{r,i}}}{1-b_{r,i}} - \frac{a h_{v2}\left(n_{r,i}-1\right)^2 t_{r,i}^2 \alpha_r}{n_{r,i}q_{r,i}} - \frac{a h_r\left(n_{r,i}-1\right)^2 t_{r,i}^2 \alpha_r}{n_{r,i}q_{r,i}}$$

$$- \frac{h_{v2}\left(n_{r,i}-1\right)\alpha_r t_{r,i}}{n_{r,i}q_{r,i}}\left(\frac{\left(1-b_{r,i}\right)n_{r,i}t_{r,i}}{T_{r1,i}}\right)^{\frac{1}{1-b_{r,i}}}$$

$$- \frac{h_r\left(n_{r,i}-1\right)\alpha_r t_{r,i}}{n_{r,i}q_{r,i}}\left(\frac{\left(1-b_{r,i}\right)n_{r,i}t_{r,i}}{T_{r1,i}}\right)^{\frac{1}{1-b_{r,i}}}$$

$$G_i^{\mathrm{II}} = \frac{2\alpha_r n_{r,i}q_{r,i} - \alpha_r^2 h_{v2}\left(n_{r,i}-1\right)^2 t_{r,i}^2 - \alpha_{r,i}^2 h_{r,i}\left(n_{r,i}-1\right)^2 t_{r,i}^2}{n_{r,i}q_{r,i}}$$

$$F^{\mathrm{II}} = \left(\frac{\left(1-b_{r,i}\right)t_{r,i}}{T_{r1,i}}\right)^{\frac{1}{1-b_{r,i}}}\left(\frac{1-b_{r,i}}{2-b_{r,i}}\right)\left[\left(n_{r,i}-0.5\right)^{\frac{2-b_{r,i}}{1-b_{r,i}}} - 0.5^{\frac{2-b_{r,i}}{1-b_{r,i}}}\right]$$

Single-channel strategy-policy III

The total holding cost of the vendor at its side is given by:

$$H_{v1,i}^{\mathrm{III}} = h_{v1}\frac{T_{r1,i}}{2-b_{r,i}}\left(q_{r,i}n_{r,i}\right)^{2-b_{r,i}} \tag{C.2.11}$$

The total holding cost of the vendor at the retailer’s side is as follows:

$$
\begin{aligned}
H_{v2,i}^{\mathrm{III}} = h_{v2}\frac{T_{r1,i}}{1-b_{r,i}}q_{r,i}^{2-b_{r,i}}&\left[\frac{\left(n_{r,i}+0.5\right)^{3-b_{r,i}}-\left(n_{r,i}-0.5\right)^{3-b_{r,i}}}{3-b_{r,i}}\right.\\
&\left.-\frac{\left(n_{r,i}+0.5\right)^{2-b_{r,i}}}{2-b_{r,i}}+\frac{1.5^{2-b_{r,i}}}{2-b_{r,i}}\right]\\
&-h_{v2}\frac{T_{r1,i}}{1-b_{r,i}}q_{r,i}^{2-b_{r,i}}\left[\frac{1.5^{3-b_{r,i}}-0.5^{3-b_{r,i}}}{3-b_{r,i}}-\frac{1.5^{2-b_{r,i}}}{2-b_{r,i}}\right]\\
&-h_{v2}D_{r,i}\left(\frac{T_{r1,i}}{1-b_{r,i}}q_{r,i}^{1-b_{r,i}}\right)^2\left[\frac{\left(n_{r,i}+0.5\right)^{2-2b_{r,i}}-1.5^{2-2b_{r,i}}}{2}\right.\\
&\left.-\left(n_{r,i}+0.5\right)^{1-b_{r,i}}+0.5^{1-b_{r,i}}\right]\\
&+h_{v2}D_{r,i}\left(T_{r1,i}q_{r,i}^{1-b_{r,i}}\right)^2\frac{n_{r,i}^{1-2b_{r,i}}}{2\left(1-2b_{r,i}\right)}\\
&+\frac{h_{v2}}{2D_{r,i}}\left[n_{r,i}q_{r,i}-\frac{T_{r1,i}}{1-b_{r,i}}q_{r,i}^{1-b_{r,i}}\left(n_{r,i}^{1-b_{r,i}}-1\right)\right]^2
\end{aligned}
\tag{C.2.12}
$$

The proof of concavity for $\Pi_{single,i}^{\mathrm{III}}$ was shown to be similar to the one provided in "Single-channel strategy-policy 0" and is given as:

$$\frac{\partial^2 \Pi^{\mathrm{III}}_{single,i}}{\partial (p^{\mathrm{III}}_{r.i})^2}$$

$$= -2\alpha_r - \frac{2\alpha_r^2(h_{v2}+h_r)T_{r1,i}^2\left(q_{r,i}^{1-b_{r,i}}\right)^2 n_{r,i}^{1-2b_{r,i}}}{n_{r,i}q_{r,i}\left(2-4b_{r,i}\right)}$$

$$+ \frac{2\alpha_r^2(h_{v2}+h_r)T_{r1,i}^2\left(q_{r,i}^{1-b_{r,i}}\right)^2\left(\frac{1}{2}\left(n_{r,i}+0.5\right)^{2-2b_{r,i}} - \frac{1}{2}(1.5^{2-2b_{r,i}}) - \left(n_{r,i}+0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}}\right)}{n_{r,i}q_{r,i}\left(1-b_{r,i}\right)^2}$$

$$< 0$$

The expression B_i^{III} and G_i^{III} in Eq. (6.27) are as follows:

$$B_i^{\mathrm{III}} = \frac{c_P\alpha_r T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)}$$

$$F_i^{\mathrm{III}} = \frac{T_{r1,i}}{1-b_{r,i}}q_{r,i}^{2-b_{r,i}}\left[\frac{\left(n_{r,i}+0.5\right)^{3-b_{r,i}} - \left(n_{r,i}-0.5\right)^{3-b_{r,i}}}{3-b_{r,i}} - \frac{\left(n_{r,i}+0.5\right)^{2-b_{r,i}}}{2-b_{r,i}} + \frac{1.5^{2-b_{r,i}}}{2-b_{r,i}}\right]$$

$$- h_{v2}\frac{T_{r1,i}}{1-b_{r,i}}q_r^{2-b_{r,i}}\left[\frac{1.5^{3-b_{r,i}} - 0.5^{3-b_{r,i}}}{3-b_{r,i}} - \frac{1.5^{2-b_{r,i}}}{2-b_{r,i}}\right]$$

$$G_i^{\mathrm{III}} = \left(\frac{\alpha_r T_{r1,i}^2\left(q_{r,i}^{1-b_{r,i}}\right)^2\left(\frac{1}{2}\left(n_{r,i}+0.5\right)^{2-2b_{r,i}} - \frac{1}{2}1.5^{2-2b_{r,i}} - \left(n_{r,i}+0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}}\right)}{n_{r,i}q_{r,i}\left(1-b_{r,i}\right)^2}\right.$$

$$\left. - \frac{\alpha_r T_{r1,i}^2\left(q_{r,i}^{1-b_{r,i}}\right)^2 n_{r,i}^{1-2b_{r,i}}}{n_{r,i}q_{r,i}\left(2-4b_{r,i}\right)}\right)(h_{v2}+h_r)$$

Appendix C.3 (Chapter 6)

Dual-channel strategy-policy 0

In the dual-channel strategy the holding cost of the vendor for the core items is given by Jaber and Bonney (1998) as follows:

$$H_{vd,i} = h_{v1}\left(\frac{q_{d,i}^2}{2D_{d,i}} - \frac{T_{d1}\left[\left(q_{d,i}+u_{d,i}\right)^{2-b_d} - u_i^{2-b_d}\right]}{(1-b_d)(2-b_d)} + \frac{T_{d1}q_{d,i}u_{d,i}^{1-b_d}}{(1-b_d)}\right) \tag{C.3.1}$$

A global optimal solution to determine that Π_{dual}^0 is a concave in $\bar{p}_r^0$, p_{dk}^0, q_r^0, q_d and n_r^0 was not possible, due to the complexity of the hessian matrix. However, a local optimal solution with respect to $\bar{p}_r^0$ and p_{dk}^0 was obtained as follows:

$$H^0 = \begin{pmatrix} \frac{\partial^2 \Pi_{dual}^0}{\partial(\bar{p}_r^0)^2} & \frac{\partial^2 \Pi_{dual}^0}{\partial^2 \bar{p}_r^0 p_{dk}^0} \\ \frac{\partial^2 \Pi_{dual}^0}{\partial^2 p_{dk}^0 \bar{p}_r^0} & \frac{\partial^2 \Pi_{dual}^0}{\partial(p_{dk}^0)^2} \end{pmatrix} = \begin{pmatrix} -2\alpha_r & \varphi_{dk}\rho + \rho N \\ \varphi_{dk}\rho + \rho N & -2\varphi_{dk}\alpha_{dk} \end{pmatrix}$$

$$|H^0| = (-2\alpha_r)(-2\varphi_{dk}\alpha_{dk}) - (\varphi_{dk}\rho + \rho N)(\varphi_{dk}\rho + \rho N) = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2$$

Since $\frac{\partial^2 \Pi_{dual}^0}{\partial(\bar{p}_r^0)^2} < 0$, $\frac{\partial^2 \Pi_{dual}^0}{\partial(p_{dk}^0)^2} < 0$, and $|H^0| = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2 > 0$, therefore, Π_{dual}^0 is concave in $\bar{p}_r^0$ and p_{dk}^0 for a given value of the other decision variables. Since local optimal was shown, one can conjuncture that a global optimal exists.

The expression of A^0 and B^0 in Eq. (13) are as follows:

Formulas for calculating $\overline{A}^0$, $\bar{B}^0$ and $\bar{G}^0$ in Eqs. (33) and (34) are as follow:

$$\bar{A}^0 = (1-\theta)a + \beta_r l_d + \frac{S_r\alpha_r}{n_r q_r} - \frac{S_d\rho}{q_d} + \frac{O_r\alpha_r}{q_r} + \frac{c_P\alpha_r}{P_r} - \sum_{k=1}^{N}\frac{c_{dk}\varphi_{dk}\rho}{P_d} + \frac{h_{v1}q_r\alpha_r}{2P_r} + \frac{h_{v1}q_d\rho}{2P_d} - \frac{h_{v2}(n_r-1)q_r\alpha_r}{2P_r} - \frac{h_r(n_r-1)q_r\alpha_r}{2P_r}$$

$$\bar{B}^0 = \theta a\varphi_{dk} - \beta_d l_d\varphi_{dk} - \frac{S_r\rho N}{n_r q_r} + \frac{S_d\alpha_{dk}}{q_d} - \frac{O_r\rho N}{q_r} - \frac{c_P\rho N}{P_r} + \sum_{k=1}^{N}\frac{c_{dk}\varphi_{dk}\alpha_{dk}}{P_d} - \frac{h_{v1}q_r\rho N}{2P_r} - \frac{h_{v1}q_d\alpha_{dk}}{2P_d} + \frac{h_{v2}(n_r-1)q_r\rho N}{2P_r} + \frac{h_r(n_r-1)q_r\rho N}{2P_r}$$

$$\bar{G}^0 = \frac{4\alpha_r\alpha_{dk}\varphi_{dk} - \varphi_{dk}^2\rho^2 - \varphi_{dk}\rho^2 - \varphi_{dk}\rho^2 N - \rho^2 N}{4\alpha_r\alpha_{dk}\varphi_{dk}}$$

Dual-channel strategy-policy I.1

The proof of concavity for $\Pi_{dual,i}^{\mathrm{I.1}}$ was shown to be similar to the one provided in "Dual-channel strategy-policy 0" and is given as:

$$H^{\mathrm{I.1}} = \begin{pmatrix} \frac{\partial^2 \Pi_{dual}^{\mathrm{I.1}}}{\partial(\bar{p}_r^{\mathrm{I.1}})^2} & \frac{\partial^2 \Pi_{dual}^{\mathrm{I.1}}}{\partial^2 \bar{p}_r^{\mathrm{I.1}} p_{dk}^{\mathrm{I.1}}} \\ \frac{\partial^2 \Pi_{dual}^{\mathrm{I.1}}}{\partial^2 p_{dk}^{\mathrm{I.1}} \bar{p}_r^{\mathrm{I.1}}} & \frac{\partial^2 \Pi_{dual}^{\mathrm{I.1}}}{\partial(p_{dk}^{\mathrm{I.1}})^2} \end{pmatrix} = \begin{pmatrix} -2\alpha_r & \varphi_{dk}\rho + \rho N \\ \varphi_{dk}\rho + \rho N & -2\varphi_{dk}\alpha_{dk} \end{pmatrix}$$

$$|H^{\mathrm{I.1}}| = (-2\alpha_r)(-2\varphi_{dk}\alpha_{dk}) - (\varphi_{dk}\rho + \rho N)(\varphi_{dk}\rho + \rho N) = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2$$

As $\frac{\partial^2 \Pi_{dual}^{\mathrm{I.1}}}{\partial(\bar{p}_r^{\mathrm{I.1}})^2} < 0$, $\frac{\partial^2 \Pi_{dual}^{0}}{\partial(p_{dk}^{\mathrm{I.1}})^2} < 0$, and $|H^0| = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2 > 0$, therefore, $\Pi_{dual}^{\mathrm{I.1}}$ is concave with respect to $\bar{p}_r^{\mathrm{I.1}}$ and $p_{dk}^{\mathrm{I.1}}$.

Formulas for calculating $\bar{A}_i^{\mathrm{I.1}}$, $\bar{B}_i^{\mathrm{I.1}}$ and $\bar{G}^{\mathrm{I.1}}$ in Eqs. (36) and (37) are as follow:

$$\bar{A}_i^{\mathrm{I.1}} = (1-\theta)a + \beta_r l_d + \frac{S_r \alpha_r}{n_{r,i} q_{r,i}} - \frac{S_d \rho}{q_{d,i}} + \frac{O_r \alpha_r}{q_{r,i}} + \frac{c_P \alpha_r T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)}$$

$$- c_{dk} \varphi_{dk} \rho \frac{T_{d1}}{1-b_d} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right] - \frac{h_{v2}\left(n_{r,i} - 1\right) t_{r,i} \alpha_r}{2}$$

$$- \frac{h_r\left(n_{r,i} - 1\right) t_{r,i} \alpha_r}{2} + \rho h_{v1} T_{d1} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1-b_d)(2-b_d)} \right] - \frac{\rho h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1-b_d}$$

$$\bar{B}_i^{\mathrm{I.1}} = \theta a \varphi_{dk} - \beta_d l_d \varphi_{dk} - \frac{S_r \rho N}{n_{r,i} q_{r,i}} + \frac{S_d \alpha_{dk}}{q_{d,i}} - \frac{O_r \rho N}{q_{r,i}} - \frac{c_P \rho N T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{b_{r,i}}}{\left(1-b_{r,i}\right)}$$

$$+ c_{dk} \varphi_{dk} \alpha_{dk} \frac{T_{d1}}{1-b_d} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right] + \frac{h_{v2}\left(n_{r,i} - 1\right) t_{r,i} \rho N}{2}$$

$$+ \frac{h_r\left(n_{r,i} - 1\right) t_{r,i} \rho N}{2} - \alpha_{dk} h_{v1} T_{d1} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1-b_d)(2-b_d)} \right]$$

$$+ \frac{\alpha_{dk} h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1-b_d}$$

$$\bar{G}^{\mathrm{I.1}} = \frac{4\alpha_r \alpha_{dk} \varphi_{dk} - \varphi_{dk}^2 \rho^2 - \varphi_{dk} \rho^2 - \varphi_{dk} \rho^2 N - \rho^2 N}{4\alpha_r \alpha_{dk} \varphi_{dk}}$$

Dual-channel strategy-policy I.2

The proof of concavity for $\Pi_{dual,i}^{\mathrm{I.2}}$ was shown to be similar to the one provided in “Dual-channel strategy-policy 0” and is given as:

$$H^{I.2} = \begin{pmatrix} \frac{\partial^2 \Pi_{dual}^{I.2}}{\partial (\bar{p}_r^{I.2})^2} & \frac{\partial^2 \Pi_{dual}^{I.2}}{\partial^2 \bar{p}_r^{I.2} p_{dk}^{I.2}} \\ \frac{\partial^2 \Pi_{dual}^{I.2}}{\partial^2 p_{dk}^{I.2} \bar{p}_r^{I.2}} & \frac{\partial^2 \Pi_{dual}^{I.2}}{\partial (p_{dk}^{I.2})^2} \end{pmatrix} = \begin{pmatrix} -2\alpha_r & \varphi_{dk}\rho + \rho N \\ \varphi_{dk}\rho + \rho N & -2\varphi_{dk}\alpha_{dk} \end{pmatrix}$$

$$|H^{I.2}| = (-2\alpha_r)(-2\varphi_{dk}\alpha_{dk}) - (\varphi_{dk}\rho + \rho N)(\varphi_{dk}\rho + \rho N) = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2$$

As $\frac{\partial^2 \Pi_{dual}^{\mathrm{I.2}}}{\partial(\bar{p}_r^{\mathrm{I.2}})^2} < 0$, $\frac{\partial^2 \Pi_{dual}^{\mathrm{I.2}}}{\partial(p_{dk}^{\mathrm{I.2}})^2} < 0$, and $|H^{\mathrm{I.2}}| = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2 > 0$, therefore, $\Pi_{dual}^{\mathrm{I.2}}$ is concave with respect to $\bar{p}_r^{\mathrm{I.2}}$ and $p_{dk}^{\mathrm{I.2}}$.

Formulas for calculating $\bar{A}_i^{\mathrm{I.2}}$, $\bar{B}_i^{\mathrm{I.2}}$ and $\bar{G}^{\mathrm{I.2}}$ in Eqs. (39) and (40) are as follow:

$$\bar{A}_i^{\mathrm{I.2}} = (1-\theta)a + \beta_r l_d + \frac{S_r\alpha_r}{n_{r,i}q_{r,i}} - \frac{S_d\rho}{q_{d,i}} + \frac{O_r\alpha_r}{q_{r,i}} + \frac{c_P\alpha_r T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)}$$
$$- c_{dk}\varphi_{dk}\rho\frac{T_{d1}}{1-b_d}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] - \frac{h_{v2}\left(n_{r,i}-1\right)t_{r,i}\alpha_r}{2}$$
$$- \frac{h_r\left(n_{r,i}-1\right)t_{r,i}\alpha_r}{2} + \rho h_{v1}T_{d1}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1-b_d)(2-b_d)}\right] - \frac{\rho h_{v1}T_{d1}u_{d,i}^{(1-b_d)}}{1-b_d}$$

$$\bar{B}_i^{\mathrm{I.2}} = \theta a\varphi_{dk} - \beta_d l_d\varphi_{dk} - \frac{S_r\rho N}{n_{r,i}q_{r,i}} + \frac{S_d\alpha_{dk}}{q_{d,i}} - \frac{O_r\rho N}{q_{r,i}} - \frac{c_P\rho N T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{b_{r,i}}}{\left(1-b_{r,i}\right)}$$
$$+ c_{dk}\varphi_{dk}\alpha_{dk}\frac{T_{d1}}{1-b_d}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] + \frac{h_{v2}\left(n_{r,i}-1\right)t_{r,i}\rho N}{2}$$
$$+ \frac{h_r\left(n_{r,i}-1\right)t_{r,i}\rho N}{2} - \alpha_{dk}h_{v1}T_{d1}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1-b_d)(2-b_d)}\right]$$
$$+ \frac{\alpha_{dk}h_{v1}T_{d1}u_{d,i}^{(1-b_d)}}{1-b_d}$$

$$\bar{G}^{\mathrm{I.2}} = \frac{4\alpha_r\alpha_{dk}\varphi_{dk} - \varphi_{dk}^2\rho^2 - \varphi_{dk}\rho^2 - \varphi_{dk}\rho^2 N - \rho^2 N}{4\alpha_r\alpha_{dk}\varphi_{dk}}$$

Dual-channel strategy-policy I.3

The proof of concavity for $\Pi_{dual,i}^{\mathrm{I.3}}$ was shown to be similar to the one provided in "Dual-channel strategy-policy 0" and is given as:

$$H^{I.3} = \begin{pmatrix} \frac{\partial^2 \Pi_{dual}^{I.3}}{\partial(\bar{p}_r^{I.3})^2} & \frac{\partial^2 \Pi_{dual}^{I.3}}{\partial^2 \bar{p}_r^{I.3} p_{dk}^{I.3}} \\ \frac{\partial^2 \Pi_{dual}^{I.3}}{\partial^2 p_{dk}^{I.3} \bar{p}_r^{I.3}} & \frac{\partial^2 \Pi_{dual}^{I.3}}{\partial(p_{dk}^{I.3})^2} \end{pmatrix} = \begin{pmatrix} -2\alpha_r & \varphi_{dk}\rho + \rho N \\ \varphi_{dk}\rho + \rho N & -2\varphi_{dk}\alpha_{dk} \end{pmatrix}$$

$$|H^{I.3}| = (-2\alpha_r)(-2\varphi_{dk}\alpha_{dk}) - (\varphi_{dk}\rho + \rho N)(\varphi_{dk}\rho + \rho N) = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2$$

As $\frac{\partial^2 \Pi_{dual}^{\mathrm{I.3}}}{\partial\left(\bar{p}_r^{\mathrm{I.3}}\right)^2} < 0$, $\frac{\partial^2 \Pi_{dual}^{\mathrm{I.3}}}{\partial\left(p_{dk}^{\mathrm{I.3}}\right)^2} < 0$, and $|H^{\mathrm{I.3}}| = 4\varphi_{dk}\alpha_r\alpha_{dk} - (\varphi_{dk}\rho + \rho N)^2 > 0$, therefore, $\Pi_{dual}^{\mathrm{I.3}}$ is concave with respect to $\bar{p}_r^{\mathrm{I.3}}$ and $p_{dk}^{\mathrm{I.3}}$.

Formulas for calculating $\bar{A}_i^{\mathrm{I.3}}$, $\bar{B}_i^{\mathrm{I.3}}$ and $\bar{G}_i^{\mathrm{I.3}}$ in Eqs. (42) and (43) are as follow:

$$\begin{aligned} \bar{A}_i^{\mathrm{I.3}} = {} & (1-\theta)a + \beta_r l_d + \frac{S_r\alpha_r}{n_{r,i}q_{r,i}} - \frac{S_d\rho}{q_{d,i}} + \frac{O_r\alpha_r}{q_{r,i}} + \frac{c_P\alpha_r T_{r1,i}\left(n_{r,i}q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)} \\ & - c_{dk}\varphi_{dk}\rho\frac{T_{d1}}{1-b_d}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] - \frac{h_{v2}\left(n_{r,i}-1\right)t_{r,i}\alpha_r}{2} \\ & - \frac{h_r\left(n_{r,i}-1\right)t_{r,i}\alpha_r}{2} + \rho h_{v1}T_{d1}\left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] - \frac{\rho h_{v1}T_{d1}u_{d,i}^{(1-b_d)}}{1-b_d} \end{aligned}$$

$$\bar{B}_i^{\mathrm{I}.3} = \theta a\varphi_{dk} - \beta_d l_d \varphi_{dk} - \frac{S_r \rho N}{n_{r,i} q_{r,i}} + \frac{S_d \alpha_{dk}}{q_{d,i}} - \frac{O_r \rho N}{q_{r,i}} - \frac{c_P \rho N T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{b_{r,i}}}{\left(1 - b_{r,i}\right)}$$

$$+ c_{dk} \varphi_{dk} \alpha_{dk} \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] + \frac{h_{v2}\left(n_{r,i} - 1\right) t_{r,i} \rho N}{2}$$

$$+ \frac{h_r\left(n_{r,i} - 1\right) t_{r,i} \rho N}{2} - \alpha_{dk} h_{v1} T_{d1} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1 - b_d)(2 - b_d)}\right]$$

$$+ \frac{\alpha_{dk} h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1 - b_d}$$

$$\bar{G}^{\mathrm{I}.3} = \frac{4\alpha_r \alpha_{dk} \varphi_{dk} - \varphi_{dk}^2 \rho^2 - \varphi_{dk} \rho^2 - \varphi_{dk} \rho^2 N - \rho^2 N}{4\alpha_r \alpha_{dk} \varphi_{dk}}$$

Dual-channel strategy-policy II

The proof of concavity for $\Pi_{dual,i}^{\mathrm{II}}$ was shown to be similar to the one provided in “Dual-channel strategy-policy 0” and is given as:

$$H^{II} = \begin{pmatrix} \dfrac{\partial^2 \Pi_{dual}^{II}}{\partial (\bar{p}_r^{II})^2} & \dfrac{\partial^2 \Pi_{dual}^{II}}{\partial^2 \bar{p}_r^{II} p_{dk}^{II}} \\ \dfrac{\partial^2 \Pi_{dual}^{II}}{\partial^2 p_{dk}^{II} \bar{p}_r^{II}} & \dfrac{\partial^2 \Pi_{dual}^{II}}{\partial (p_{dk}^{II})^2} \end{pmatrix}$$

$$= \begin{pmatrix} -2\alpha_r + \dfrac{\alpha_r^2 t_{r,i}^2 (h_{v2} + h_r)\left(n_{r,i} - 1\right)^2}{n_{r,i} q_{r,i}} & \varphi_{dk}\rho + \rho N - \dfrac{\rho N \alpha_r t_{r,i}^2 (h_{v2} + h_r)\left(n_{r,i} - 1\right)^2}{n_{r,i} q_{r,i}} \\ \varphi_{dk}\rho + \rho N - \dfrac{\rho N \alpha_r t_{r,i}^2 (h_{v2} + h_r)\left(n_{r,i} - 1\right)^2}{n_{r,i} q_{r,i}} & -2\varphi_{dk}\alpha_{dk} + \dfrac{\rho^2 N^2 t_{r,i}^2 (h_{v2} + h_r)\left(n_{r,i} - 1\right)^2}{n_{r,i} q_{r,i}} \end{pmatrix}$$

Due to the complexity of proving that the second derivatives of Π_{dual}^{II} is a concave function, a simulation with more than 20,000 random examples is done and the solution of which have shown that $\frac{\partial^2 \Pi_{dual}^{\mathrm{II}}}{\partial(\bar{p}_r^{\mathrm{II}})^2} < 0$, $\frac{\partial^2 \Pi_{dual}^{\mathrm{II}}}{\partial(p_{dk}^{\mathrm{II}})^2} < 0$, and $|H^{\mathrm{II}}| > 0$, therefore, Π_{dual}^{II} is concave with respect to $\bar{p}_r^{\mathrm{II}}$ and p_{dk}^{II}.

Formulas for calculating $\bar{A}_i^{\mathrm{II}}$, $\bar{B}_i^{\mathrm{II}}$, $\bar{E}_{r,i}^{\mathrm{II}}$, $\bar{E}_{d,i}^{\mathrm{II}}$, $\bar{F}_i^{\mathrm{II}}$, $\bar{G}_{r,i}^{\mathrm{II}}$ and $\bar{G}_{d,i}^{\mathrm{II}}$ in Eqs. (45) and (46) are as follow:

$$
\begin{aligned}
\bar{A}_i^{\mathrm{II}} = (1-\theta)a + \beta_r l_d + \frac{S_r \alpha_r}{n_{r,i} q_{r,i}} - \frac{S_d \rho}{q_{d,i}} + \frac{O_r \alpha_r}{q_{r,i}} + \frac{c_P \alpha_r T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{-b_{r,i}}}{\left(1-b_{r,i}\right)} \\
- c_{dk} \varphi_{dk} \rho \frac{T_{d1}}{1-b_d} \left[\frac{\left(q_{d,i}+u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right] \\
- \frac{2(h_{v2}+h_r)\left(n_{r,i}-1\right)^2 t_{r,i}^2 \alpha_r \left((1-\theta)a + \beta_r l_d\right)}{n_{r,i} q_{r,i}} \\
+ \frac{(h_{v2}+h_r)\left(n_{r,i}-1\right)^2 t_{r,i}^2 \alpha_r \left((1-\theta)a + \beta_r l_d\right)}{n_{r,i} q_{r,i}} \\
- \frac{(h_{v2}+h_r)\left(n_{r,i}-1\right) t_{r,i} \alpha_r}{2 n_{r,i} q_{r,i}} \left(\frac{(1-b_i) n_{r,i} t_{r,i}}{T_{r1,i}}\right)^{\frac{1}{1-b_{r,i}}} \\
+ \rho h_{v1} T_{d1} \left[\frac{\left(q_{d,i}+u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1-b_d)(2-b_d)}\right] - \frac{\rho h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1-b_d}
\end{aligned}
$$

$$\bar{B}^{\mathrm{II}} = \theta a\varphi_{dk} - \beta_d l_d \varphi_{dk} - \frac{S_r \rho N}{n_{r,i} q_{r,i}} + \frac{S_d \alpha_{dk}}{q_d} - \frac{O_r \rho N}{q_{r,i}} - \frac{c_P \rho N T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{b_{r,i}}}{\left(1 - b_{r,i}\right)}$$

$$+ c_{dk} \varphi_{dk} \alpha_{dk} \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right]$$

$$+ \frac{2(h_{v2} + h_r)\left(n_{r,i} - 1\right)^2 t_{r,i}^2 \rho N \left((1-\theta)a + \beta_r l_d\right)}{n_{r,i} q_{r,i}}$$

$$- \frac{(h_{v2} + h_r)\left(n_{r,i} - 1\right)^2 t_{r,i}^2 \rho N \left((1-\theta)a + \beta_r l_d\right)}{n_{r,i} q_{r,i}}$$

$$+ \frac{(h_{v2} + h_r)\left(n_{r,i} - 1\right) t_{r,i} \rho N}{2 n_{r,i} q_{r,i}} \left(\frac{\left(1 - b_{r,i}\right) n_{r,i} t_{r,i}}{T_{r1,i}} \right)^{\frac{1}{1-b_{r,i}}}$$

$$- \alpha_{dk} h_{v1} T_{d1} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1 - b_d)(2 - b_d)} \right] + \frac{\alpha_{dk} h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1 - b_d}$$

$$\bar{E}_{r,i}^{\mathrm{II}} = \varphi_{dk} \rho n_{r,i} q_{r,i} + \rho n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right)^2 t_{r,i}^2 \alpha_r \rho (h_{v2} + h_r)$$

$$\bar{E}_{d,i}^{\mathrm{II}} = \varphi_{dk} \rho n_{r,i} q_{r,i} + \rho N n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right)^2 t_{r,i}^2 \alpha_r \rho N (h_{v2} + h_r)$$

$$\bar{F}^{\mathrm{II}} = \left(\frac{1 - b_{r,i}}{T_{r1,i}} t_{r,i} \right)^{\frac{1}{1-b_{r,i}}} \left(\frac{1 - b_{r,i}}{2 - b_{r,i}} \right) \left[\left(n_{r,i} - 0.5\right)^{\frac{2-b_{r,i}}{1-b_{r,i}}} - 0.5^{\frac{2-b_{r,i}}{1-b_{r,i}}} \right]$$

$$\bar{G}_{r,i}^{\mathrm{II}} = \frac{2\alpha_r n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right)^2 t_{r,i}^2 \alpha_r^2 (h_{v2} + h_r)}{n_{r,i} q_{r,i}}$$

$$\bar{G}_{d,i}^{\mathrm{II}} = \frac{2\alpha_{dk} \varphi_{dk} n_{r,i} q_{r,i} - \left(n_{r,i} - 1\right)^2 t_{r,i}^2 \rho^2 N (h_{v2} + h_r)}{n_{r,i} q_{r,i}}$$

Dual-channel strategy-policy III

The proof of concavity for $\Pi_{dual,i}^{\mathrm{III}}$ was shown to be similar to the one provided in "Dual-channel strategy-policy 0" and is given as:

$$H^{\mathrm{III}} = \begin{pmatrix} \dfrac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial (\bar{p}_r^{\mathrm{III}})^2} & \dfrac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial^2 \bar{p}_r^{\mathrm{III}} p_{dk}^{\mathrm{III}}} \\ \dfrac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial^2 p_{dk}^{\mathrm{III}} \bar{p}_r^{\mathrm{III}}} & \dfrac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial (p_{dk}^{\mathrm{III}})^2} \end{pmatrix}$$

$$\frac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial (\bar{p}_r^{\mathrm{III}})^2}$$

$$= -2\alpha_r - \frac{2\alpha_r^2 (h_{v2} + h_r) T_{r1,i}^2 \left(q_{r,i}^{1-b_{r,i}}\right)^2 n_{r,i}^{1-2b_{r,i}}}{n_{r,i} q_{r,i} (2 - 4b_{r,i})}$$

$$+ \frac{2\alpha_r^2 (h_{v2} + h_r) T_{r1,i}^2 \left(q_{r,i}^{1-b_{r,i}}\right)^2 \left[\frac{1}{2}\left(n_{r,i} + 0.5\right)^{2-2b_{r,i}} - \frac{1}{2}(1.5^{2-2b_{r,i}}) - \left(n_{r,i} + 0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}}\right]}{n_{r,i} q_{r,i} \left(1 - b_{r,i}\right)^2}$$

$$\frac{\partial^2 \Pi_{dual}^{\mathrm{III}}}{\partial (p_{dk}^{\mathrm{III}})^2}$$

$$= -2\varphi_{dk}\alpha_{dk} - \frac{2\rho^2 N^2 T_{r1,i}^2 (h_{v2} + h_r) \left(q_{r,i}^{1-b_{r,i}}\right)^2 n_{r,i}^{1-2b_{r,i}}}{n_{r,i} q_{r,i} (2 - 4b_{r,i})}$$

$$+ \frac{2\rho^2 N^2 (h_{v2} + h_r) T_{r1,i}^2 \left(q_{r,i}^{1-b_{r,i}}\right)^2 \left[\frac{1}{2}\left(n_{r,i} + 0.5\right)^{2-2b_{r,i}} - \frac{1}{2}(1.5^{2-2b_{r,i}}) - \left(n_{r,i} + 0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}}\right]}{n_{r,i} q_{r,i} \left(1 - b_{r,i}\right)^2}$$

$$\frac{\partial^2 \Pi_{dual}^{\text{III}}}{\partial^2 \bar{p}_r^{\text{III}} p_{dk}^{\text{III}}} = \frac{\partial^2 \Pi_{dual}^{\text{III}}}{\partial^2 p_{dk}^{\text{III}} \bar{p}_r^{\text{III}}}$$

$$= \varphi_{dk}\rho + \rho N + \frac{2\alpha_r \rho N T_{r1,i}^2 (h_{v2} + h_r)\left(q_{r,i}^{1-b_{r,i}}\right)^2 n_{r,i}^{1-2b_{r,i}}}{n_{r,i} q_{r,i}}$$

$$- \frac{2\alpha_r \rho N (h_{v2} + h_r) T_{r1,i}^2 \left(q_{r,i}^{1-b_{r,i}}\right)^2 \left[\frac{1}{2}\left(n_{r,i} + 0.5\right)^{2-2b_{r,i}} - \frac{1}{2}(1.5^{2-2b_{r,i}}) - \left(n_{r,i} + 0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}}\right]}{n_{r,i} q_{r,i} \left(1 - b_{r,i}\right)^2}$$

Due to the complexity of proving that the second derivatives of Π_{dual}^{III} is a concave function, a simulation with more than 20,000 random examples is done and the solution of which have shown that $\frac{\partial^2 \Pi_{dual}^{\text{III}}}{\partial (\bar{p}_r^{\text{III}})^2} < 0$, $\frac{\partial^2 \Pi_{dual}^{\text{III}}}{\partial (p_{dk}^{\text{III}})^2} < 0$, and $|H^{\text{III}}| > 0$, therefore, Π_{dual}^{III} is concave with respect to $\bar{p}_r^{\text{III}}$ and p_{dk}^{III}.

Formulas for calculating $\bar{A}_i^{\text{II}}$, $\bar{B}_i^{\text{II}}$, $\bar{E}_{r\ i}^{\text{II}}$, $\bar{E}_{d\ i}^{\text{II}}$, $\bar{F}_i^{\text{II}}$, $\bar{G}_{r\ i}^{\text{II}}$ and $\bar{G}_{d\ i}^{\text{II}}$ in Eqs. (48) and (49) are as follow:

$$\bar{A}_i^{\text{III}} = (1-\theta)a + \beta_r l_d + \frac{S_r \alpha_r}{n_{r,i} q_{r,i}} - \frac{S_d \rho}{q_{d,i}} + \frac{O_r \alpha_r}{q_{r,i}} + \frac{c_P \alpha_r T_{r1,i}\left(n_{r,i} q_{r,i}\right)^{b_{r,i}}}{\left(1 - b_{r,i}\right)}$$

$$- c_{dk}\varphi_{dk}\rho \frac{T_{d1}}{1 - b_d}\left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}}\right]$$

$$+ \rho h_{v1} T_{d1}\left[\frac{\left(q_{d,i} + u_i\right)^{2-b} - u_{d,i}^{2-b_d}}{q_{d,i}(1 - b_d)(2 - b_d)}\right] - \frac{\rho h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1 - b_d}$$

$$\bar{B}_i^{\mathrm{III}} = \theta a \varphi_{dk} - \beta_d l_d \varphi_{dk} - \frac{S_r \rho N}{n_{r,i} q_{r,i}} + \frac{S_d \alpha_{dk}}{q_{d,i}} - \frac{O_r \rho N}{q_{r,i}} - \frac{c_P \rho N T_{r1,i} \left(n_{r,i} q_{r,i}\right)^{b_{r,i}}}{\left(1 - b_{r,i}\right)}$$

$$+ c_{dk} \varphi_{dk} \alpha_{dk} \frac{T_{d1}}{1 - b_d} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{1-b_d} - u_{d,i}^{1-b_d}}{q_{d,i}} \right]$$

$$- \alpha_{dk} h_{v1} T_{d1} \left[\frac{\left(q_{d,i} + u_{d,i}\right)^{2-b_d} - u_{d,i}^{2-b_d}}{q_{d,i}(1 - b_d)(2 - b_d)} \right] + \frac{\alpha_{dk} h_{v1} T_{d1} u_{d,i}^{(1-b_d)}}{1 - b_d}$$

$$\bar{F}_i^{\mathrm{III}} = \frac{T_{r1,i}}{1 - b_{r,i}} q_{r,i}^{2-b_{r,i}} \left(\frac{\left(n_{r,i} + 0.5\right)^{3-b_{r,i}} - \left(n_{r,i} - 0.5\right)^{3-b_{r,i}}}{3 - b_{r,i}} - \frac{\left(n_{r,i} + 0.5\right)^{2-b_{r,i}}}{2 - b_{r,i}} + \frac{1.5^{2-b_{r,i}}}{2 - b_{r,i}} \right)$$

$$- h_{v2} \frac{T_{r1,i}}{1 - b_{r,i}} q_{r,i}^{2-b_{r,i}} \left[\frac{1.5^{3-b_{r,i}} - 0.5^{3-b_{r,i}}}{3 - b_{r,i}} - \frac{1.5^{2-b_{r,i}}}{2 - b_{r,i}} \right]$$

$$\bar{E}_{r,i}^{\mathrm{III}}$$

$$= (h_{v2}$$

$$+ h_r) \left(\frac{\alpha_r T_{r1,i}^2 \left(q_{r,i}^{(1-b_{r,i})} \right)^2 \left(\frac{1}{2} \left(n_{r,i} + 0.5\right)^{2-2b_{r,i}} - \frac{1}{2} 1.5^{2-2b_{r,i}} - \frac{1}{2} \left(n_{r,i} + 0.5\right)^{1-b_{r,i}} + 0.5^{1-b_{r,i}} \right)}{n_{r,i} q_{r,i} \left(1 - b_{r,i}\right)^2} \right.$$

$$\left. - \frac{\alpha_r T_{r1,i}^2 \left(q_{r,i}^{(1-b_{r,i})} \right)^2 n_{r,i}^{1-2b_{r,i}}}{2 n_{r,i} q_{r,i} \left(1 - 2b_{r,i}\right)} \right)$$

$$
\bar{E}_{d,i}^{\mathrm{III}} = \left(h_{v2} + h_r\right)\left(\frac{\rho N T_{r1,i}^{2}\ \left(q_{r,i}^{(1-b_{r,i})}\right)^{2}\left(\frac{1}{2}\left(n_{r,i}+0.5\right)^{2-2b_{r,i}}-\frac{1}{2}1.5^{2-2b_{r,i}}-\frac{1}{2}\left(n_{r,i}+0.5\right)^{1-b_{r,i}}+0.5^{1-b_{r,i}}\right)}{n_{r,i}q_{r,i}\left(1-b_{r,i}\right)^{2}} - \frac{\rho N\, T_{r1,i}^{2}\ \left(q_{r,i}^{(1-b_{r,i})}\right)^{2} n_{r,i}^{1-2b_{r,i}}}{2n_{r,i}q_{r,i}\left(1-2b_{r,i}\right)}\right)
$$

$$
\bar{G}_{r,i}^{\mathrm{III}} = \frac{\rho\varphi_{dk}+\rho-2\rho E_{r,i}^{\mathrm{III}}}{2\alpha_r\left(1-E_{r,i}^{\mathrm{III}}\right)}
$$

$$
\bar{G}_{d,i}^{\mathrm{III}} = \frac{\rho\varphi_{dk}+\rho N-2\alpha_r E_{d,i}^{\mathrm{III}}}{\left(2\alpha_{dk}-\rho E_{d,i}^{\mathrm{III}}\right)}
$$

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