TWO-PRICE POLICY FOR A NEWSVENDOR PRODUCT SUPPLY CHAIN WITH TIME AND PRICE SENSITIVE DEMAND

by
Omid Jadidi

Bachelor of Science in Industrial Engineering, Iran University of Science and Technology, Iran, 2002
Master of Science in Industrial and System Engineering, University Putra Malaysia, 2009

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ABSTRACT

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Omid Jadidi
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Ryerson University

In this study, a dominant manufacturer wholesales a technological product to a retailer. In technology-related industries, the obsolescence of an existing product and/or the appearance of a new product decrease the attractiveness of the existing product. This study also assumes that the market demand is stochastic and price-sensitive, where this price-sensitivity increases by time. Hence, retailers need to decline the retail price during the product life cycle to alleviate the effect of time on the demand. Here, two models/cases are considered. In the first model, the retailer decreases the retail price at midlife without any compensation from the manufacturer. In the second model, the manufacturer gives rebate to the retailer when the retailer declines the retail price at midlife. In addition, the performance of the proposed models is numerically compared with wholesale-price-only and the buyback policies.

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CHAPTER 1 - INTRODUCTION

A supply chain involves a group of independent firms that add value on products/services in order to satisfy the end customers. It links the raw materials and components to the finished goods. Supply chain can be called “value chain” if it is perceived as a chain of value adding activities. Recognizing supply chain as a separate discipline traced back to just before the 1960s (Upadhyay, 2012).

Supply chain management (SCM) attempts to manage the flows of material, information, and finance throughout the supply chain from the upstream suppliers to the downstream customers (see Figure 1).

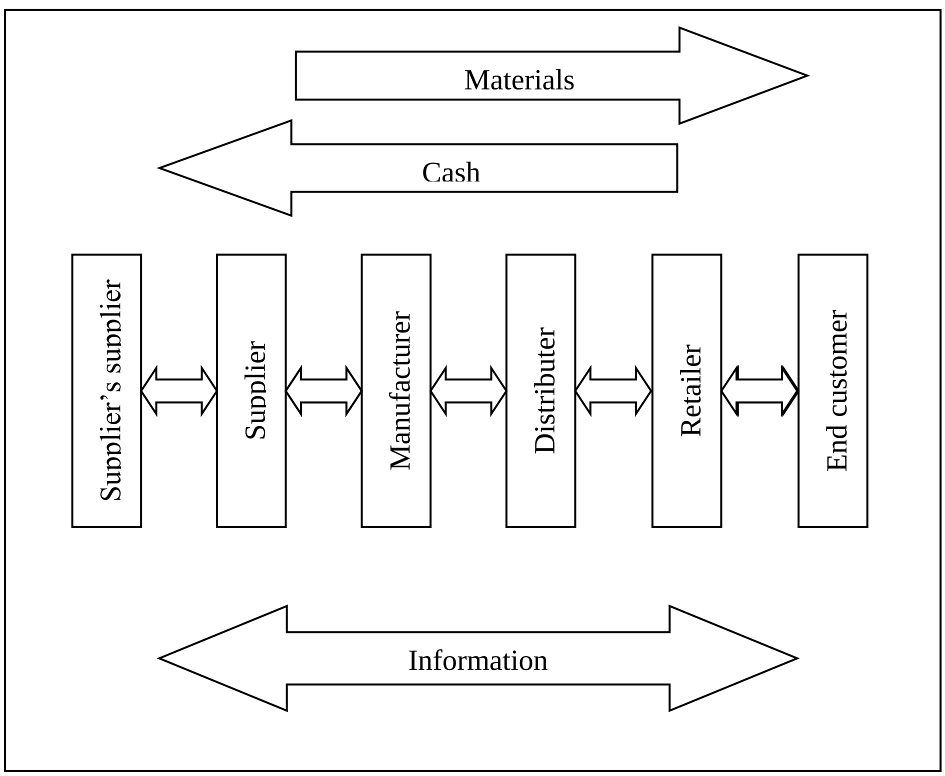

Figure 1.1. The supply chain Model

SCM is one of the new and well growing management approaches that because of some factors, such as globalization, high customer expectation and severe competition, its importance has increased to the managers. It is recognized from the literature that different factors make SCM a very difficult approach. The factors can be:

- Demand, supply and lead time uncertainty,
- Conflicting objectives of the members,
- The dynamic nature of the supply chain.

SCM consists of the four elements as follows (Linton, 2014):

- *Demand Management*

Demand management is an important element in SCM that cause companies and their partners to focus on meeting customers' needs and requirements. In a supply chain, companies collaboratively maximize the quality of supplies and add value to the finished products in order to satisfy their customers. This can improve the ability of the entire supply chain to maintain competitiveness in the market that increases opportunities for the entire supply chain.

- *Communication*

With effective communication in a supply chain, all members will share the demand and operational information that enables the whole supply chain to rapidly match their operations with demand changes, to take quick action on new business opportunities, to rapidly introduce new products to market, etc. In other words, effective communication helps the whole supply chain to enhance its operations' efficiency and productivity.

- *Integration*

Integrating supply chain processes requires the members to develop a single information network, making them able to access and share demand and supply information. This will decrease the inventory costs of each member.

- *Collaboration*

Collaboration in supply chain means that all members work together towards a common purpose while enhancing the whole supply chain performance. This fortifies the relationships between the supply chain members. Despite the importance of supply chain collaboration, the managers usually seek to optimize their own objectives (Cachon, 2003).

Supply chain managers take into account different criteria for the evaluation of the supply chain performance, such as customer satisfaction, cost of operations, innovativeness, price and quality of the product/service, flexibility/adaptability, and ability to collaborate (Vaidya and Hudnurkar, 2013).

The supply chain managers should understand that their success and survival strongly depend on their ability to cooperate with the other members of the supply chain. Supply chain coordination intends to optimize the performance of the entire supply chain by contracting for a set of transfer payments that align all members' objectives with the supply chain objective (Cachon, 2003). However, the managers usually seek to optimize their own objectives that results in a poor supply chain performance (Cachon, 2003). Now the obstacles to supply chain coordination are briefly reviewed (Makajić-Nikolić et al., 2014):

- *Incentive obstacles*

If incentives concentrate only on the local impact of an action, decisions made by the members attempt to maximize their individual profit rather than the whole supply chain.

- *Information processing obstacles*

If each member within a supply chain gets the demand information from its previous stage, the information may be distorted, which results in increased variability in orders.

- *Operational obstacles*

If members for some reason (such as high fixed cost) order large lot sizes rather than actual demand, variability of orders increases in the supply chain.

- *Pricing obstacles*

Quantity discounts will encourage the members (buyer) to magnify the lot sizes that cause distortions in orders.

- *Behavioral obstacles*

The members perceive their decisions and actions locally and are unable to see the effects of those decisions and actions on the entire supply chain.

However, there may be an incentive for all members to implement a coordinating contract, if it makes each member’s profit no worse off. Academics and practitioners have identified a number of different contracts, and analyzed their advantages and disadvantages on the supply chain. They attempt to optimize the total cost or profit of the whole supply chain by implementing some contracts such as the wholesale-price-only contract, the buyback contract, the revenue sharing contract, the quantity flexibility contract, the sales rebate contract, and the quantity-discount contract. These contracts have been extensively implemented for technology products that are the main focus of this study.

Due to the rapid innovation in Technology-related industries, such as personal computers (PCs) and notebooks, the products are quickly outmoded. The introduction of a new product decreases the attractiveness of existing products such that some customers prefer to purchase them at a discount and others opt to get the new product (Lee et al., 2000; Norton and Bass, 1987). The obsolescence of the existing product is also another reason for the product attractiveness reduction. Consequently, in such industries, products have short life cycles, uncertain demand and decreasing prices due to intense competitiveness (Lee et al., 2000; Chen and Xiao, 2011). The price of a PC in average decreases by 50–58% within the first year of its life cycle (Lee et al., 2000; Chen and Xiao, 2011). Similarly, the average prices of Apple PCs drops as much as 20% in the first quarter (Philip, 2009; Chen and Xiao, 2011). It is also reported that in the second quarter of 2007 in China, the prices of notebooks and PCs, digital cameras, and mobile phones dropped monthly by 0.63%, 2.27%, and 7.05%, respectively (Chen and Xiao, 2011). Thus, retailers prefer to reduce their stock levels for products due to the uncertainty of demand, and to postpone ordering the products because of a possible price decline in the future (Lee et al., 2000). Consequently, the manufacturers employ approaches to induce the retailers to opt out these preferences.

Studies, such as Lee et al. (2000) and Taylor (2001), have modeled such problem where the product demand is uncertain and the retail price drops in the midlife due to the appearance of a new product. They also considered different channel coordination policies which aim to improve supply chain performance by synchronizing both the manufacturer and the retailer’s objectives.

The aforementioned studies assumed that the retail prices are exogenously determined. Independent variables that have influence on other variables in a model without being influenced by them are exogenous variables. However, in some situations, especially when monopolies and oligopolies are common, the supply chain members should consider the retail price as an endogenous decision variable to absorb the maximum profit earned from the market (Liu, 2005). The reason is that in such markets, manufacturers have the degree of freedom to determine their products' prices in order to keep both demand and price high (Wilcox 1999). For example, Dell, Compaq and HP occupy a great portion (52%) of the PC market in the United States (Gray, 2002). Endogenous variables are those affected by other variables in a model. They may also influence other endogenous variables.

As discussed, when a new product is introduced to the market some customers prefer to purchase the existing product at a discount. It indicates that the demand is highly price sensitive. There are studies that have investigated the behavior of the channel players under price-sensitive demand. Emmons and Gilbert (1998), Granot and Yin (2002), Ha (2001), Lau et al. (2007) and Arcelus et al. (2012a) have studied newsvendor problems with price-sensitive demand in which different channel coordination policies were considered. More studies will be considered in the literature review section.

The studies on price-sensitive demand have not considered the impact of the appearance of a new product on the existing product's price and demand. Generally, apart from uncertainty, demand for technological products can have the following characteristics:

(1) It is decreasing over the product life cycle due to the introduction of a new product or the obsolescence of the existing product.

(2) It is price sensitive: A higher price results in a lower demand. Therefore, retailers have to decrease the retail price of the existing products, particularly when a new product is introduced.

According to the literature, no study has simultaneously considered these two characteristics of the demand for technology-related products.

This study considers a two-echelon supply chain with a dominant manufacturer and a follower retailer. It considers the two aforementioned characteristics of the demand in order to develop a more realistic model for technological products. The demand of this model is price-sensitive, but the price-sensitivity is constant before a certain time, such as when a new product is introduced, and increases afterward, i.e., when it approaches its end-of-life. This type of demand can be called a price-and-time sensitive or dependent demand. In order to alleviate the effect of price on the demand, the retailer will re-determine the retail price after the certain point. In other words, the problem has two retail prices: the first one is determined at the beginning of a product's life cycle, and the other is re-determined after the certain time (or at a change point). Due to having two retail prices, the problem needs to be considered as a two-period model. The first period is from the beginning of product life cycle until the change point, and the second period is afterwards. Two different models or cases are considered for the retail price decline. In one case, it is the retailer's commitment to decrease the retail price without any compensation from the manufacturer. This case is called the two-price policy that helps the retailers keep the demand as high as possible without any effort from the manufacturer. In another case, the manufacturer gives rebates to the retailer for any product sold after the retail price declines. It is called the two-price policy with rebate. In this problem, the wholesale price and the rebate set by the manufacturer, the two retail prices and the order volume set by the retailer are the decision variables. The wholesale-price-only and the buyback policies are also designed for the problem considered in this study to investigate and compare their performances from the perspective of manufacturer and retailer's profits.

To solve the problem, it is modeled mathematically in which a game theoretic approach is used. Since the objective is to obtain the optimal decision variables at the beginning of the first period for the two-period problem, backward induction is also used. The complexity of the problem does not allow us to achieve analytical results. Therefore, the problem is programed by visual basic and FORTRAN to reach the optimal solutions.

Chapter 2 is for the literature review. Chapter 3 first describes the problem and then develops the two-price policy models along with the wholesale-price-only and the buyback policies. In Chapter 4, numerical analysis is performed that compares the performance of the policies. Concluding remarks and future research are discussed in Chapter 5.

CHAPTER 2 – LITERATURE REVIEW

In this chapter, three different areas related to this work are considered. The first subsection reviews studies that have worked on different contracts for single period problems. The second subsection considers two-period models related to technological products and the contracts that have been used. The last subsection is on price sensitive demand models and the implemented contracts.

2.1. Single period newsvendor models

Every day, the news vendors attempt to order the best number of newspapers, in which its demand is not known precisely. If the order is large, the vendors may not sell all the newspapers at the end of the day, and thus, they have to either throw the extra newspapers away or keep them in stock and pay for holding cost. If the order is small, the vendors might not be able to meet all the customers' needs, and thus, they lose the sales and the profit. Generally, in the newsvendor model a buyer purchases a single product from a manufacturer (or supplier) for a selling season in which the product demand is assumed to be uncertain. If the buyer cannot sell all products at the end of the season, he/she either keeps them in stock and pays for the holding cost or return them back to the manufacturer with a return price. On the other hand, if the retailer's stock is less than the demand, he/she incurs a shortage cost. The newsvendor model, known also as newsboy model, is a mathematical model in operations management and applied economics used to find the optimal level of inventory (newspapers) to maximize the expected profit of the buyer (news vendor).

The newsvendor model can be applied in the following situations in which the managers need to make a one-time decision on inventory level (Hill, 2011):

Products with seasonal demand: For example, a Christmas tree is only used for the celebration of Christmas. If the buyer orders too many or too few, he/she will have an over- or under-stock inventory at the end of the season, respectively.

Products with short life cycle: For the products with short life cycle, such as personal computers (PCs) and notebooks, the manufacturers need to produce the products in an adequate amount in order to prevent an over- or under-stock inventory.

Safety stock level: If the firms do not skillfully determine the safety stock level of the products, the probability of facing shortage or overage will increase.

Products with fixed capacity: In airline or hotel industries where the number of seats or rooms is fixed the managers should take advantage of a very useful pricing policy. If the price does not satisfy the customers, the companies will have empty seats or rooms that incur an opportunity cost of lost revenue.

Different coordinating contracts have been employed by manufacturers and retailers in a supply chain. In this part, wholesale-price-only, buyback and rebate policies are reviewed in detail, since they are used in this study.

Wholesale-price-only policy

Wholesale-price-only (WPO) policy is a simple and the most common policy (or contract) in practice which is presented in Figure 2.1. W is the wholesale price per unit incurred to the retailer by the manufacturer, P is the retail price per unit incurred to the end customers by the retailer, and Q is the order volume that the retailer orders from the manufacturer.

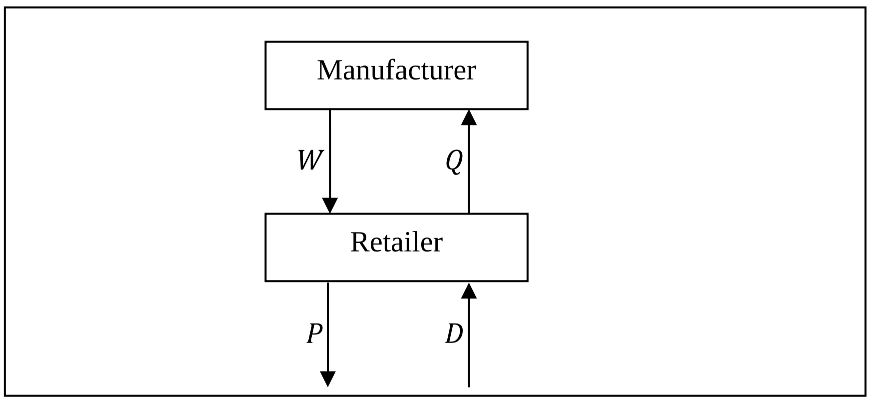

Figure 2.1. The wholesale-price-only policy

There are two types of WPO contracts in the standard newsvendor setting: (1) the push WPO contract in which the retailer places the order before observing the uncertain demand and the supplier accordingly supplies the products; (2) the pull WPO contract where the retailer places the order after realizing the actual demand whereas, the supplier should be ready to meet the order in advance (Cachon, 2004). Lariviere and Porteus (2001) analyzed the properties of push WPO contracts in which the supplier, who plays the role of Stackelberg leader, can meet all the buyer's orders. Cachon and Netessine (2004) considered the pull WPO policy where the supplier has to determine its capacity level before the buyer places the order.

In this chapter, the models that are very close to the models proposed in this study are mathematically described for the purpose of comparative analysis.

Now, the push WPO newsvendor problem is mathematically modeled where a buyer purchases a newsvendor type product from a manufacturer. Let D denote the random demand of a product, and $f(\cdot)$ and $F(\cdot)$ signify its probability density and cumulative distribution functions, respectively. Also, let μ_D and σ_D present the mean and standard deviation of D. s_l is the shortage cost per each unit demand lost and h is the overage or holding cost per each unit unsold that the retailer incurs. The retailer's expected profit π_{Re} (Re stands for the Retailer) when he/she purchases Q units from the manufacture is formulated as follows:

$$\pi_{Re}(Q) = P\ E[\min(Q\ , D)] - s_l\ E[(D - Q)^+] - h\ E[(Q - D)^+] - Q\ W. \tag{2.1}$$

The first term in Eq. (2.1) is the expected revenue of selling the product to the end customers, the second term is the expected stock-out/shortage cost at the end of the season, the third term represents the expected holding cost, and the last term is the purchasing cost. In the third term, when h is negative (e.g. $h = -2$), it will be $-(-2)E[(Q - D)^+] = +\,2\ E[(Q - D)^+]$. This means that the retailer obtains a profit from the left over. Therefore, the third term can represent the expected salvage value if h is negative.

By taking the first derivative of Eq. (2.1) with respect to Q and setting it to zero, the optimal order volume is obtained as

$$Q^* = F^{-1}[(P^* + s_l - W)/(P^* + s_l + h)]. \tag{2.2}$$

If the production cost per unit is C_m, then the manufacturer's profit π_M (*M* stands for the Manufacturer) is formulated as follows:

$$\pi_M(W) = (W - C_m)\,Q. \tag{2.3}$$

Buyback or return policy

Under a buyback policy, the manufacturer allows the retailer to return unsold products at the end of the season with a buyback price R. Since the retailer does not generate any profit from the left over inventory, it has been assumed that $R \leq W$. Figure 2.2 presents a buyback policy.

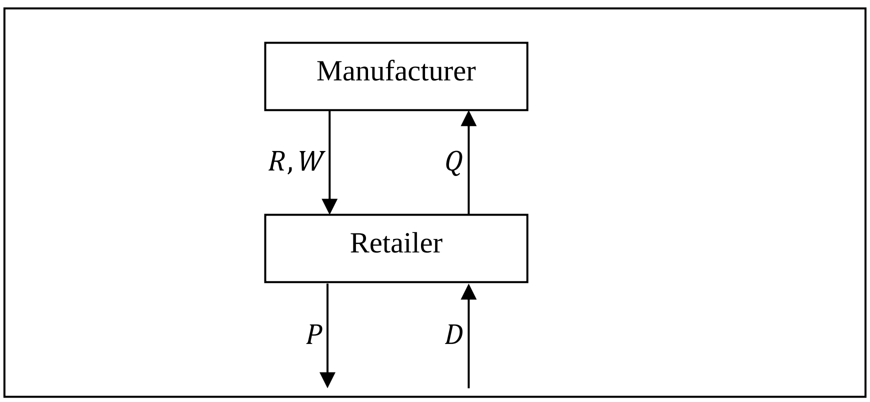

Figure 2.2. The buyback policy

When the demand is uncertain, there is always the risk of having unsold products at the end of the season. Buyback policies help the retailers to share this risk with the manufacturers. Pasternack (1985) demonstrated that supply chain coordination for risk-neutral firms can be achieved under a full leftover return and partial compensation. In addition, he proved that supply chain coordination cannot be obtained if the retailer can return all unsold products with full compensation (i.e., $R = W$). In another study, Cachon

(2003) showed that coordination is attainable under partial return and partial compensation, but not under partial return and full compensation. Su (2009) considered the effect of full and partial return policies on supply chain performance.

By this policy the retailer's expected profit π_{Re} can be expressed as follows:

$$\pi_{Re}(Q) = P\,E[\min(Q\,,D)] - s_l\,E[(D-Q)^+] + R\,E[(Q-D)^+] - Q\,W. \tag{2.4}$$

The third term is the revenue obtained from returning the unsold products to the manufacturer at the end of the season. In newsvendor models, if the retailer keeps unsold items at the end of the season, it then incurs a holding cost as considered in the third term of Eq. (2.1). However, the retailer can sometimes sell them to the market at a reduced price (known as salvage value) or can return them to the manufacturer for a buyback price R. In this case, there is no holding costs for the retailer.

By taking the first derivative of Eq. (2.4) with respect to Q and setting it to zero, the optimal order volume is obtained as

$$Q^* = F^{-1}[(P^* + s_l - W)/(P^* + s_l - R)]. \tag{2.5}$$

It is perceived that the retailer is not charged any cost for the left over stock as he/she can return them to the manufacturer. Therefore, the manufacturer's expected profit π_M is formulated as follows:

$$\pi_M(W,R) = (W - C_m)\,Q - R\,E[(Q-D)^+]. \tag{2.6}$$

Manufacturers can salvage the return products by recycling program, which is the reuse of technological devices, or by reusing their components.

Rebate policy

By this contract the manufacturer charges the retailer $\$W$/per unit purchased, but then gives the retailer a rebate ρ per unit sold. In the buyback policy, the manufacturer compensates the buyer for unsold products, while in the rebate policy he/she gives rebate for sold products. In literature, there are two types of rebate contract between manufacturers and buyers (Hezarkhani and Kubiak, 2010): (1) a contract that rebates for all sold products, and (2) a contract that rebates for sold products above a threshold τ (see Figure 2.3). Taylor (2002) proved that a supply chain can achieve coordination under the second type.

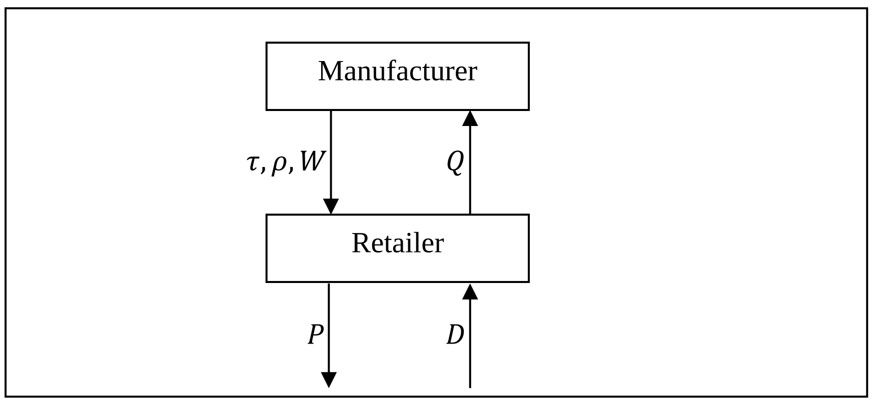

Figure 2.3. The rebate policy

Using the second type policy, the retailer's expected profit π_{Re} can be formulated as follows:

$$\pi_{Re}(Q) = P\,E[\min(Q\,,D)] + \rho\,E[(\min(Q\,,D) - \tau)^+] - s_l\,E[(D-Q)^+] - h\,E[(Q-D)^+] - Q\,W. \quad (2.7)$$

If the retailer sells less than τ, it receives no rebate from the manufacturer. Therefore, the model is converted to the WPO contract and its optimal order volume is obtained as $Q_1 = F^{-1}[(P^* + s_l - W)/(P^* + s_l + h)]$. If the retailer can sell more than τ, then he/she receives rebate and the optimal order volume is obtain as $Q_2 = F^{-1}[(P^* + \rho + s_l - W)/(P^* + \rho + s_l + h)]$. Q_2 is obtained by taking the first derivative of Eq. (2.7) and setting it to zero.

For this model, Taylor (2002) proved that there exists a unique $\tau_0 \in (Q_1, Q_2)$ such that if $\tau < \tau_0$, then Q^* (the optimal order volume of Eq.(2.7)) $= Q_2$; if $\tau > \tau_0$, then $Q^* = Q_1$; if $\tau = \tau_0$, then $Q^* = \{Q_1, Q_2\}$ (see Figure 2.4). In general, $Q^* =$ arg $max_{Q=\{Q_1,Q_2\}}\, \pi_{Re}(Q)$.

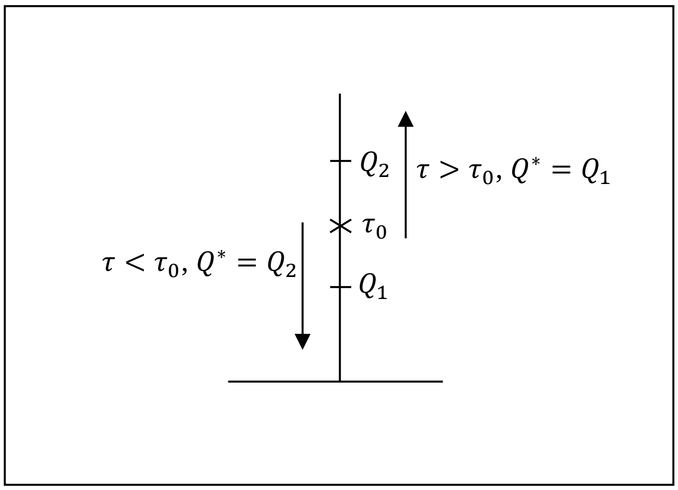

Figure 2.4. The optimal order volume Q^* in rebate policy

One can see that the manufacturer pays the retailer for every product sold after the threshold τ. Therefore, the manufacturer's expected profit π_M is expressed as follows:

$$\pi_M(W, \rho, \tau) = (W - C_m)\, Q - \rho\, E[(\min(Q\,, D) - \tau)^+]. \qquad (2.8)$$

There is another type of rebate policy where the manufacturer or the retailer pays the end customer via a coupon (Chen et al., 2007). Chen et al. (2007) investigated the impact of manufacturer's rebate on both the manufacturer and retailer's expected profits, where the demand was price-sensitive. They considered two types of rebate, the mail-in and the instant rebates, and showed that the former may benefit the manufacturer while the latter may not. In the mail-in rebate, a customer, after purchasing a product, can mail in its receipt and/or barcode to the manufacturer in order to receive a cheque. In instant rebate, the customer receives the discount immediately.

The revenue sharing policy

With this policy the retailer shares a percentage of his/her revenue with the manufacturer. Also, the retailer incurs $\$W$/unit for the products purchased from the manufacturer. It has been greatly useful and successful in the video cassette rental industry (Hezarkhani and Kubiak, 2010).

2.2. Two-period newsvendor models for technology-related industries

For the technology-related industries, it is known that the products' value drops drastically over the product life cycle (Lee et al., 2000). This means that the products have two retail prices. P_1 is the product's retail price at the beginning of the period, and P_2 is the retail price at its midlife ($P_2 < P_1$). Figure 2.5 depicts that the inventory level at the retailer's side is Q that can be sold at $\$P_1$ per unit. At a change point, where the inventory level is x, the retail price declines to $\$P_2$ per unit.

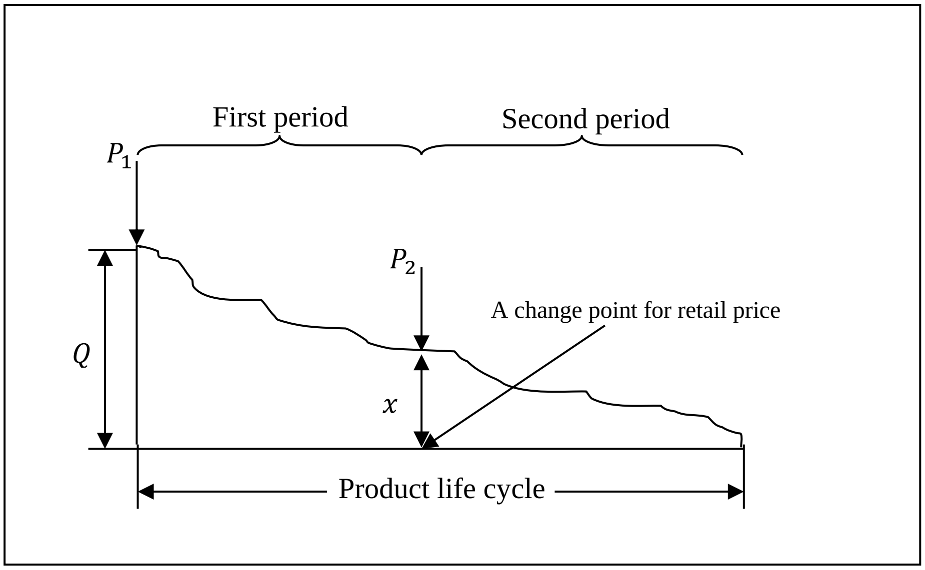

Figure 2.5. A product life cycle is presented by two periods

The product life cycle can be divided into two periods. First period starts from the beginning of the cycle and ends at the change point with the random demand D_{P_1}. $f_{P_1}(\cdot)$ and $F_{P_1}(\cdot)$ indicate the probability density and cumulative distribution functions of D_{P_1}, respectively. The second period starts afterwards with the random demand D_{P_2}. $f_{P_2}(\cdot)$ and $F_{P_2}(\cdot)$ specify the probability density and cumulative distribution functions of D_{P_2}, respectively. This decision problem is dealt with recursively: first, the retailer's expected profit in the second period is modeled, and then, the retailer's expected profit in the first period is formulated.

If the leftover stock of the first period is *x*, the retailer's expected profit in period 2 π_{Re2} is formulated as

$$\pi_{Re2}(x) = P_2\, E\left[\min\left(x\,, D_{P_2}\right)\right] - sl_2\, E\left[\left(D_{P_2} - x\right)^+\right] - h_2\, E\left[\left(x - D_{P_2}\right)^+\right]. \qquad (2.9)$$

Moving back to the first period, the retailer's expected profit, π_{Re1} with order volume Q is given by

$$\pi_{Re1}(Q) = P_1\, E\left[\min\left(Q\,, D_{P_1}\right)\right] - sl_1\, E\left[\left(D_{P_1} - Q\right)^{+}\right] - h_1\, E\left[\left(Q - D_{P_1}\right)^{+}\right] - Q\, W + \alpha\, E\left[\pi_{Re2}\left(Q - D_{P_1}\right)^{+}\right], \quad (2.10)$$

where α is the discount factor for cost in the second period, $0 < \alpha \leq 1$.

By taking the first derivative of π_{Re1} with respect to Q and setting it to zero, the following equation is obtained

$$P_1 + sl_1 - W - \left[P_1 + s_{l_1} + h_1 - \alpha\left(P_2 + s_{l_2}\right)\right]F_{P_1}(Q) - \alpha(P_2 + s_{l_2} + h_2)]F_3(Q) = 0, \quad (2.11)$$

where $F_3(\cdot)$ is the convolution of $f_{P_1}(\cdot)$ and $f_{P_2}(\cdot)$, i.e., the cumulative distribution of $D_3 = D_{P_1} + D_{P_2}$; s_{l_1} and s_{l_2} are the shortage costs per each unit demand lost at the first and second periods, respectively; h_1 and h_2 are the holding cost per each unit unsold at the first and second periods, respectively. As there are two different distribution functions, $F_{P_1}(Q)$ and $F_3(Q)$ in Eq. (2.11), Lee et al. (2000) could not find a close form solution for the optimal order volume Q. Therefore, the optimal order volume is the one that satisfies Eq. (2.11). For more details, the readers are refered to Lee et al. (2000).

Since the manufacturer provides the retailer with Q at the beginning of the first period, the manufacturer's profit π_M can be formulated as

$$\pi_M(W) = (W - C_m)\, Q. \quad (2.12)$$

Different policies have been considered for such situation.

Price protection (PP) policy

Extreme reduction in product price over the product life cycle makes the price protection a widely used policy that both the manufacturers and the retailers utilize in the PC industry. Under this policy, the manufacturer gives rebate ρ_p to the retailer for unsold units at the end of period 1 according to the decline in the retail price, i.e., ρ_p is a function of the retail price decline.

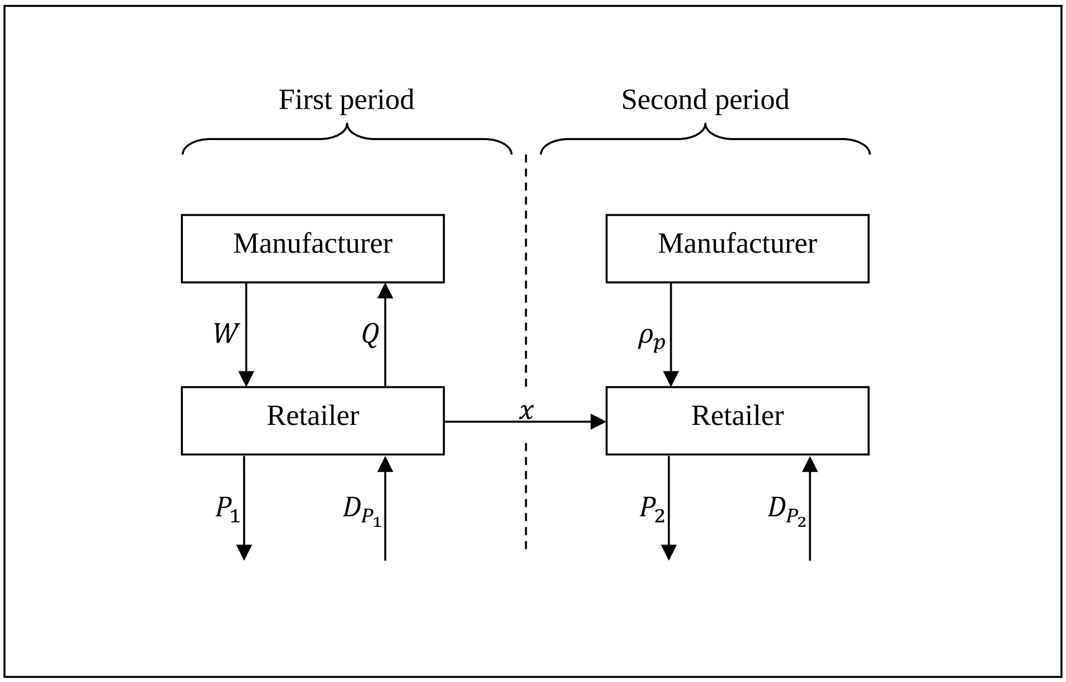

Figure 2.6. The price protection policy in a two-period setting with a single buying opportunity

Based on the literature, the study of Lee et al. (2000) was the first one who considered price protection policy in a dynamic (two periods) model. They investigated two models: (1) the retailer has a single buying opportunity, and (2) the retailer has two buying opportunities one at the beginning of the life cycle and another at the change point of retail price (midlife).

They solved the decision problem recursively. For a single buying opportunity, if the leftover stock of the first period is x, the retailer's expected profit in period 2 π_{Re2} is formulated as

$$\pi_{Re2}(x) = x\rho_p + P_2\, E\left[\min\left(x\,, D_{P_2}\right)\right] - s_{l_2}\, E\left[\left(D_{P_2} - x\right)^+\right] - h\, E\left[\left(x - D_{P_2}\right)^+\right]. \qquad (2.13)$$

It can be seen that the only difference between Eq. (2.9) and Eq. (2.13) is that the retailer receives a rebate per each unit left unsold at the end of period 1, i.e., $x\,\rho_p$. The retailer's expected profit in the first period is expressed as

$$\pi_{Re1}(Q) = P_1\, E\left[\min\left(Q\,, D_{P_1}\right)\right] - s_{l_1}\, E\left[\left(D_{P_1} - Q\right)^+\right] - h\, E\left[\left(Q - D_{P_1}\right)^+\right] - Q\, W + \alpha\, E\left[\pi_{Re2}\left(Q - D_{P_1}\right)^+\right]. \qquad (2.14)$$

Here, no one can see any visible difference between Eq. (2.14) and Eq. (2.10), because the retailer receives nothing from the manufacturer in period 1. However, $\pi_{Re2}\left(Q - \widetilde{D}_{P_1}\right)^+$ in Eq. (2.14) defers from its counterpart in Eq. (2.10) due to $x\,\rho_p$. As before, by taking the first derivative of π_{Re1} with respect to Q and setting it to zero, the following equation is obtained:

$$P_1 + s_{l_1} - W - \left[P_1 + s_{l_1} + h_1 - \alpha\left(P_2 + s_{l_2} + \rho_p\right)\right]F_{P_1}(Q) - \alpha(P_2 + s_{l_2} + h_2)]F_3(Q) = 0. \qquad (2.15)$$

For the price protection policy model, the optimal order volume Q satisfies Eq. (2.15).

Lee et al., (2000) showed that if the price protection credit is chosen properly, it can coordinate the supply chain. Sourirajan et al. (2008) examined price-protection contracts where the manufacturer offers full price protection for a limited time. Zhang (2008) also considered price-protection contracts for a two-period model where the retailer has only one buying opportunity.

The two buying opportunities models are briefly described below. It is assumed that both the manufacturing cost and wholesale price decreases over time. W_1 and W_2 are the wholesale prices at period 1 and 2, respectively, such that $W_1 > W_2$. In comparison with a single buying opportunity model where the manufacturer’s rebate was a function of the retail price decline, in two buying opportunities, the manufacturer’s rebate is a function of the wholesale price decline. Since the wholesale price decreases in the second period, the manufacturer compensates the retailer for a portion of the wholesale price decline. Figure 2.7 shows how the mechanism of the price protection policy is applicable for two buying opportunity models. In Figure 2.7, Q_1 is the optimal order volume in period 1, and Q_2 is the optimal inventory level at period 2. If $Q_2 > x$, then the retailer increases his/her inventory level up to Q_2 by purchasing $(Q_2 - x)$ units from the manufacture, and does nothing otherwise.

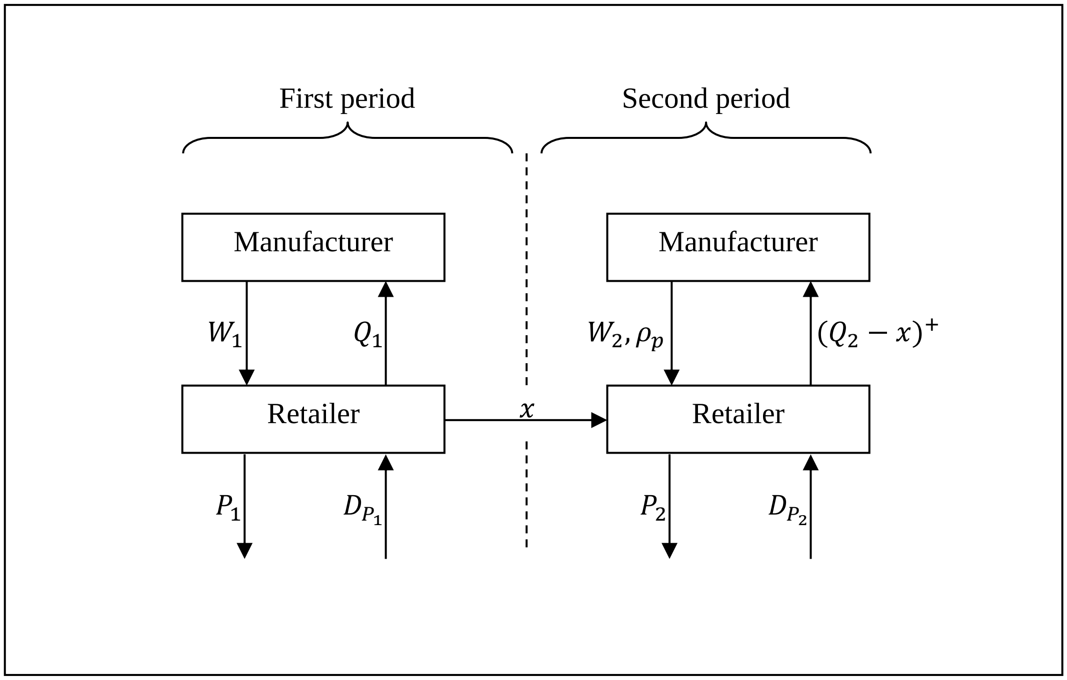

Figure 2.7. The price protection policy in a two-period setting with two buying opportunities

For the model in which the retailer has two buying opportunities, Lee et al., (2000) also showed that channel coordination is achieved if both the price protection credit and wholesale prices are set endogenously. However, when the wholesale prices are exogenous the policy may not guarantee channel coordination.

Return policy

Taylor (2001) considered tree channel policies that are used when price declines over the product life cycle: (1) price protection where a manufacturer pays the retailer a credit for each unit unsold at the end of period 1 when $W_1 > W_2$; (2) Midlife return, in which at the end of period 1 the manufacturer permits the retailer to either keep the unsold products and receive the price protection rebate or return them and receive the buyback rebate; (3) End-of-life returns where the manufacturer allows the retailer to return unsold products at the end of period 2 at some rebate. He proved that the integration of these three policies can attain both coordination and a win-win outcome. In addition, if the retail price is constant over the product life cycle, then a combination of the two return policies can guarantee both coordination and a win-win outcome.

In another study, Chen and Xiao (2011) examined two channel coordination policies for a single buying opportunity model: PME that is the combination of price protection, Midlife return, and End-of-life return; ME that is the combination of Midlife return and End-of-life return. They demonstrated that both policies may cause a win–win outcome under some conditions.

Rebate policy

Under this policy the manufacturer gives a credit to the retailer for each unit sold at period 2 (i.e., after the retail price decline). Lu et al. (2007) examined price protection, rebate, Midlife return, End-of-life return as well as their combinations for both the single- and the two-buying opportunity models. They showed that for the single buying opportunity model, both PME (the combination of price protection, midlife return and end-of-life return) and RME (the combination of rebate plus midlife return and end-of-life returns) can achieve a win-win outcome. However, both price protection and ME (midlife returns plus end-of-life returns) fail to achieve a win-win outcome.

Revenue sharing policy

Linh and Hong (2009) examined the performance of a revenue sharing contract on the supply chain coordination with both the single- and the two-buying opportunity models.

They demonstrated that this policy achieves channel coordination between a manufacturer and a retailer. Wang et al. (2013a) also considered the revenue sharing policy for both the single- and the two-buying opportunity models. Zhou and Wang (2012) proposed an improved revenue sharing contract for a two-echelon supply chain with two ordering opportunities. They showed that this contract can achieve the perfect supply chain coordination. They also demonstrated that channel coordination can be achieved under this policy. Hematyar and Chaharsooghi (2014) considered a two-period two-echelon supply chain where the retailer has one buying opportunity. They performed a comparison between revenue sharing contract and the insurance contract. In addition, Zhang et al. (2014) introduced two revenue sharing contracts when the retailer has two buying opportunities.

2.3. Newsvendor model with price-sensitive demand

Considering both retail price and order quantity as decision variables has become a widely used approach in operations research studies (Liu, 2005). These studies have assumed that demand is a function of price in two different forms (Liu, 2005; Lau et al., 2007):

(1) Linear form: $D(P) = A - bP$,

(2) Iso-elastic form with constant elasticity: $D(P) = K\,P^{-\theta}$,

where A, b, K and θ are given parameters.

Figure 2.8 shows how the demand is decreasing on price.

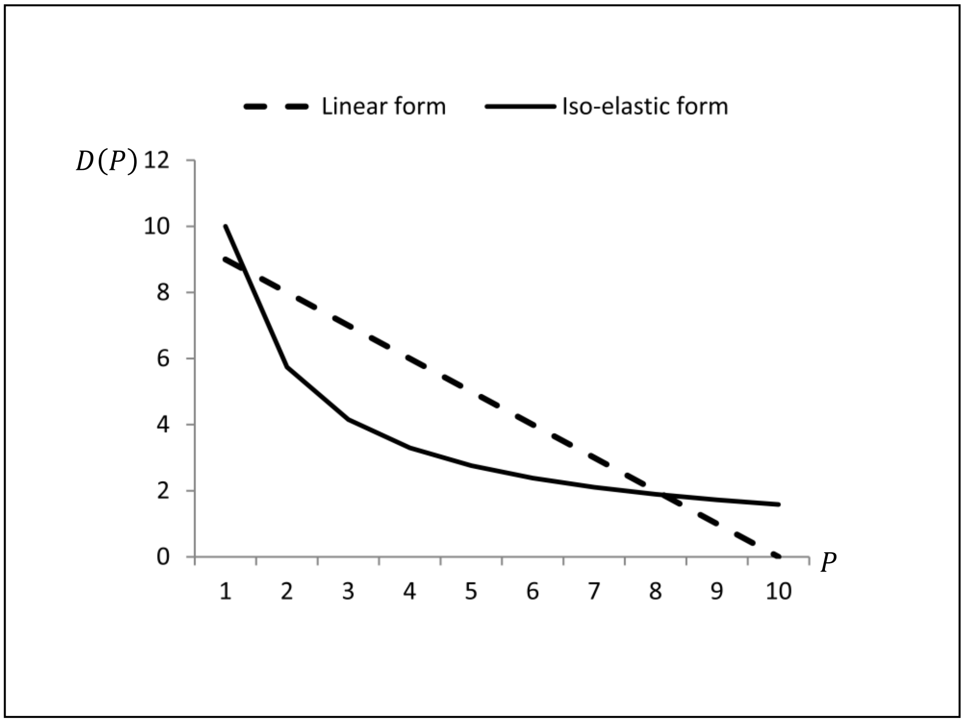

Figure 2.8. The effect of price on demand with the two different forms when $A = K = 10$, $b = 1$, and $\theta = 0.8$.

In addition, the randomness of price-sensitive demand is modeled by using two common forms (Liu, 2005; Lau et al., 2007):

(1) the "multiplicative" form: $D = D(P) \times \tilde{\varepsilon}$, in which $\mu_\varepsilon = 1$ and $\sigma_\varepsilon \geq 0$,

(2) the "additive" form: $D = D(P) + \tilde{\varepsilon}$, where $\mu_\varepsilon = 0$ and $\sigma_\varepsilon \geq 0$,

where $\tilde{\varepsilon}$ is a random variable and D is the random demand. As can be seen, D consists of a deterministic portion $D(P)$ and a stochastic portion $\tilde{\varepsilon}$.

Similar to other situations, some of which discussed in two previous subsections, different coordinating contracts such as buyback, rebate, etc., have been used for price-sensitive demand problems.

Wholesale-price-only policy

One of the early studies for price-sensitive demand can be Whitin (1955) who considered a linear additive form for D as $D = (A - bP) + \tilde{\varepsilon}$. Then, for the same demand form, Mills (1959) extended the work of Whitin (1955) and demonstrated that the optimal price for a problem with uncertain demand is lower than a problem with deterministic demand. Other related studies on linear additive form of demand are Thowsen (1975) and Lau and Lau (1988). In contrast with Mills (1959), Karlin and Carr (1962) demonstrated that for a multiplicative form of demand, the optimal price for uncertain demands is higher than for deterministic demands. For this type of demand (the multiplicative form), Zabel (1970) examined a problem in which the stochastic portion of the demand follows uniform and exponential distributions. Zabel (1972) considered the multiplicative form of demand for a multi-period model. Federgruen and Heching (1999) also considered a multi-period model with price-sensitive demand. Dana and Petruzzi (2001) investigated a model in which the demand was price and inventory-level dependent. Zhao et al. (2012) considered a wholesale-price-only contract for a two-echelon supply chain consisting of one retailer and two competitive manufacturers who produce two substitutable products, respectively. In their model, the customers' demand for each product is price-sensitive and characterized as fuzzy variables. Huang et al. (2013) took into consideration a pricing and production decision problem for a dual-channel supply chain in which production costs are disrupted and demand is deterministic and price-sensitive. Figure 2.9 depicts a two-echelon supply chain in which the manufacturer utilizes two ways for selling the products to the market. In one way, the manufacturer sells its products through a retailer, while in the other way, he/she is directly connected to the final customers through the internet.

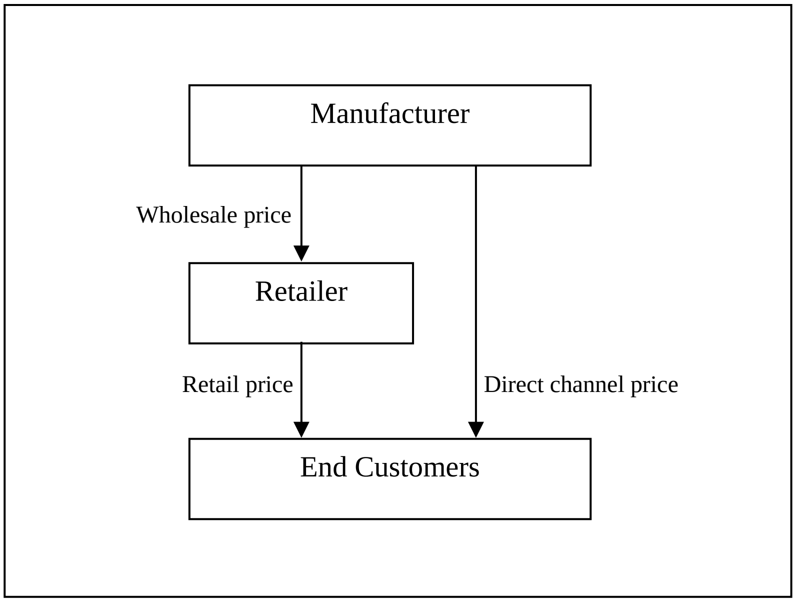

Figure 2.9. A dual-channel supply chain

Lu and Liu (2013) studied a situation in which a supplier sells a product through a dual-channel distribution system, i.e., physical retailers and e-tailers. In their model, the demand was price and service level sensitive.

Two-part tariff policy

In a two-part tariff policy, the company sets its product or service price into two parts: the entry fee and the usage fee. One of the first studies that considered channel coordination under price-sensitive demand was Weng (1997). Weng (1997) proposed a two-part tariff for a situation in which the supply chain members do not have the same level of information (asymmetric information). Ha (2001) proposed three different policies for channel coordination: The first one is a quantity fixing policy in which the manufacturer determines the quantity sent to the retailer; the second one is a two-part linear pricing policy that contains a fixed fee and a variable price for the supplied part; the third one is a buyback policy. In another study, Wang et al. (2012) compared the performance of several promising contracts, a retailer-implemented two-part tariffs and a retailer-implemented the volume discount scheme, for a two-echelon supply chain with one manufacturer and one dominant retailer for a price-sensitive demand.

Buyback or return policy

For price-sensitive demand, Yao et al. (2004) showed that if the wholesale and the buyback prices are a function of the retail price, channel coordination can be achieved. For a multi-retailer setting, Bernstein and Federgruen (2005) showed that by using a price discount sharing policy where the buyback price depends on the retail price, coordination can be achieved. Lau et al. (2002) investigated the effect of demand uncertainty on the manufacturer’s wholesale and buyback prices, as well as the retailer’s retail price and order volume. Lau et al. (2007) considered a dominant manufacturer newsvendor problem where the demand was price-sensitive and distributed uniformly. They examined the performance of wholesale-price-only, buyback and manufacturer-imposed retail price policies on channel coordination. In the manufacturer-imposed retail price policy the manufacturer stipulates the retail price in addition to the wholesale and buyback prices. Lau et al. (2008) studied the performance of different contracts, such as buyback and revenue sharing, on a dominant retailer’s profit.

Rebate policy

Arcelus et al. (2012a) considered a price-sensitive demand problem and investigated the effect of direct rebate to the end customers from the dominant manufacturer and/or the follower retailer. Arcelus et al. (2012b) studied a situation in which the retailer may have an opportunity to backlog a portion of lost sales by giving a rebate as an incentive for waiting, i.e., customers whose demand was not satisfied will receive a rebate if they wait. In addition, Arcelus et al. (2012c) assessed the pricing and ordering policies of a retailer under different degrees of risk tolerance.

Markup pricing policy

Markup pricing refers to a strategy where a company first determines its product's actual cost, and then applies a fixed percentage to that cost which gives the product's retail price. Markup pricing contract guarantees the retailer's financial prudence, as he/she can charge a retail margin over the wholesale price. Wang et al. (2013b) employed two different types of markup pricing contract for price-dependent demand situations in which a dominant retailer purchases two substitutable products from two competitive manufacturers. In addition, Wang et al. (2013c) also considered markup pricing contracts for a two-echelon supply chain with one manufacturer and one dominant retailer.

Lau et al. (2012) and Wang et al. (2013d) considered situations in which the stochastic demand was the price and effort dependent.

2.4. The gap in the literature

As discussed in this chapter, no study has considered a model in which the demand is price-sensitive and this price sensitivity increases over the product life cycle. This study develops a two-period newsvendor model with price-sensitive demand that allows the parties in a supply chain to reduce the retail price during the product life cycle. Table 2.1 shows the contribution on the related literature.

Table 2.1. The gap in the literature

Article	Stochastic demand	Price-sensitive demand	Single period model	Multi-period model
Lee et al., (2000), Taylor (2001), Chen and Xiao (2011), Lu et al. (2007), Linh and Hong (2009), Wang et al. (2013 a), Sourirajan et al. (2008), Zhou and Wang (2012), Wang et al. (2013), Zhang et al. (2014), Hematyar and Chaharsooghi (2014), Zhang (2008).	Yes	No	----	Yes
Whitin (1955), Mills (1959), Thowsen (1975), Lau and Lau (1988), Karlin and Carr (1962), Zabel (1970), Zabel (1972), Dana and Petruzzi (2001), Weng (1997), Ha (2001), Lau et al. (2002), Lau et al. (2007), Arcelus et al. (2012a), Arcelus et al. (2012b), Arcelus et al. (2012c), Wang et al. (2012), Huang et al. (2013), Zhao et al. (2012), Wang et al. (2013b), Wang et al. (2013c), Lu and Liu (2013), Lau et al. (2012), Wang et al. (2013d)	Yes	Yes	Yes	----
Liu (2005)	No	Yes	----	Yes
This study	Yes	Yes	----	Yes (two period)

CHAPTER 3 – THE PROPOSED MODELS

In this chapter, Subsection 3.1 describes the problem under consideration. Then, Subsection 3.2 develops the two-price policy models in which the two following assumptions are considered: (1) the retailer is responsible for the retail price decline, and (2) the manufacturer is responsible for the retail price decline (manufacturer pays rebate to the retailer). Finally, Subsection 3.3 develops the wholesale-price-only and buyback polices as benchmark.

3.1. Problem description

This study considers a two-echelon supply chain model with a manufacturer wholesaling a product to a retailer. In this problem, the product life cycle consists of T time-units, where each time-unit has a stochastic and price-sensitive demand, independent of the other time-units. Any time-unit can be considered such as an hour, a day, a week, or a month. Let D_t denote the random demand per each time-unit, where $t = 1, 2, \ldots, T$. The probability density and the cumulative distribution functions of D_t are denoted by $f(\cdot)$ and $F(\cdot)$, respectively, and μ_{D_t} and σ_{D_t} are its mean and standard deviation. For price sensitivity of the demand, the linear demand-curve function is employed that indicates how μ_{D_t} varies with P: $D(P) = \mu_{D_t} = A - b_t P$, where $(A/b_t) > C_m$ and C_m is the manufacturing cost per unit. The linear demand-curve function is the popular one in the literature (Lau and Lau, 2002). To model the randomness (or σ_{D_t}) of price-sensitive demand, the 'additive' form is used, under which μ_{D_t} is randomized by the additive relationship, $D_t = \mu_{D_t} + \tilde{\varepsilon}$, where $\mu_\varepsilon = 0$ and $\sigma_\varepsilon \geq 0$. Since many regression models assume that the standard deviations of the error terms are constant and not dependent on the x-value, an empirically estimated demand curve matches closer to the additive form than the multiplicative form (Lau et al., 2007). This means that the 'additive' form provides a closer fit to real situations (Lau et al., 2007).

In this case, $\sigma_{D_t} = \sigma_\varepsilon \; \forall t$ remains constant as P varies. μ_{D_t} and $\tilde{\varepsilon}$ are interpreted as the deterministic and stochastic portions of the demand, respectively. Following the literature, it is assumed that $\tilde{\varepsilon}$ follows a uniform distribution, and hence, the finite ranges of D_t are obtained as $D_{t,min} = \mu_{D_t} - \sigma_{D_t}\sqrt{3}$ and $D_{t,max} = \mu_{D_t} + \sigma_{D_t}\sqrt{3}$ (Lau et al., 2007; Arcelus et al., 2012c) by solving the following equations

$$\begin{cases} \mu_{D_t} = \frac{D_{t,max}+D_{t,min}}{2} \\ \sigma_{D_t} = \frac{D_{t,max}-D_{t,min}}{2\sqrt{3}} \end{cases}.$$

Since in this study it is assumed that the attractiveness of a product may decrease due to its obsolescence or due to the introduction of a new product, there should be a point $t_{st} \in [1, T]$ from which the price-sensitivity of the demand increases. In other words, it is assumed that b_t remains constant up to t_{st}, and increases after t_{st} (i.e., $b_t < b_{t+1}$). As a result, b_t aggregates with a constant variable, β, after t_{st}, i.e.,

$$b_t = \begin{cases} b &, \; 1 \le t \le t_{st} \\ b_{t-1} + \beta, & t_{st} < t \le T \end{cases}.$$

It is assumed that the product's quality and functional features cannot compete with the new product. As a result, the retailer follows the price differentiation policy (pricing strategy) and decides to decrease (re-determine) the retail price during the product life cycle at $t_r \in [t_{st}, T]$ to keep the demand as high as possible. In this case, there are two retail prices, \$$P_1$/unit which is determined at the beginning of the product life cycle, and \$$P_2$/unit which is re-determined at midlife, i.e., t_r. Since the retailer is able to re-determine the retail price in the product midlife, the model is called a two-price policy.

It is assumed that the retailer has one opportunity to purchase Q units from the manufacturer at the beginning of the product life cycle. Therefore, during the life cycle the quality and other features of the product remains constant.

This model implements a manufacturer Stackelberg game where the manufacturer first decides on his/her wholesale price \$$W$/unit, and subsequently, the retailer responds to the wholesale price with an optimal order volume Q and the optimal retail prices P_1 and P_2. Always, $W < P_2 \leq P_1$. The manufacturer incurs the manufacturing cost of \$$C_m$/unit ($C_m < W$). Without loss of generality, it is assumed C_m = 1 (see for example Lau et al., 2007). In this case, if the actual C_m is 9, as an example, then the achieved decision variables, such as W, P_2, P_1, are multiplied by 9. The retailer incurs the goodwill cost of \$$s_l$/unit when a customer's demand is not satisfied through the product life cycle. At the end of the product life cycle, the buyer also incurs a holding cost of \$$h$/unit for the unsold products.

3.2. Two-price policy models

Since there is a flexibility to reduce the retail price at time-unit t_r, the model is considered as a two-period newsvendor problem. The first period is $[1, t_r)$ with the retail price P_1 and the random demand D_{P_1}. Due to the independence of the demands in the time-units, the finite range of D_{P_1} can be obtained as $D_{P_1,min} = \sum_{t=1}^{t_r-1} D_{t,min}$ and $D_{P_1,max} = \sum_{t=1}^{t_r-1} D_{t,max}$. The second period is $[t_r, T]$ with the retail price P_2 and the random demand

D_{P_2} with $D_{P_2,min} = \sum_{t=t_r}^{T} D_{t,min}$ and $D_{P_2,max} = \sum_{t=t_r}^{T} D_{t,max}$. α , where $0 < \alpha \leq 1$, is considered as the discount factor for the costs in the second period.

3.2.1. The integrated channel for the two-price policy models

A vertically integrated channel is first considered for the channel efficiency analysis. An integrated channel means that there is a single manufacturing/retailing entity. The integrated firm's expected profit for the entire life cycle π_{I1} (I stands for the Integrated) is presented in Eq. (3.1):

$$\pi_{I1}(Q, P_1, P_2) = P_1\, E\left[\min\left(Q\, , D_{P_1}\right)\right] - s_l\, E\left[\left(D_{P_1} - Q\right)^+\right] - h\, E\left[\left(Q - D_{P_1}\right)^+\right] - Q\, C_m + \alpha\, E\left[\pi_{I2}\left(\left(Q - D_{P_1}\right)^+, P_2\right)\right]. \quad (3.1)$$

The first three terms in Eq. (3.1) signify the expected revenue, the expected stock-out/shortage cost, and the expected holding cost at the first period, respectively. The fourth term represents the manufacturing cost for the integrated channel, and the last term is the expected profit at period 2, i.e., $\pi_{I2}\left(\left(Q - D_{P_1}\right)^+, P_2\right)$ is the integrated firm's expected profit in the second period if the leftover stock from the first period is positive with the value of $Q - D_{P_1}$. The decision problem has been dealt with recursively, which means the expected profit in the second period is first obtained, and then, from which the expected profit in the first period is computed.

If the leftover stock of the first period is x, the integrated firm's expected profit in period 2 π_{I2} is formulated as

$$\pi_{I2}(\, P_2) = P_2\, E\left[\min\left(x\, , D_{P_2}\right)\right] - s_l\, E\left[\left(D_{P_2} - x\right)^+\right] - h\, E\left[\left(x - D_{P_2}\right)^+\right]. \quad (3.2)$$

The first term in Eq. (3.2) is the expected revenue of period 2, the second term is the expected stock-out/shortage cost at the end of period 2, and the third term represents the expected holding cost.

3.2.2. Two-price policy model when the retailer is committed for the price decline $[P_1, P_2]$

Under the manufacturer-Stackelberg game, the manufacturer first announces a unit wholesale price W. Subsequently for a given W, the retailer responds with the optimal unit retail prices P_1^* and P_2^* and the optimal order volume Q^*, which maximize the retailer's expected profit for the entire life cycle π_{Re1} (*Re* stands for the Retailer) in Eq.(3.3):

$$\pi_{Re1}(Q, P_1, P_2) = P_1\, E\left[\min\left(Q\,, D_{P_1}\right)\right] - s_l\, E\left[\left(D_{P_1} - Q\right)^+\right] - h\, E\left[\left(Q - D_{P_1}\right)^+\right] - Q\, W + \alpha\, E\left[\pi_{Re2}\left(\left(Q - D_{P_1}\right)^+, P_2\right)\right]. \tag{3.3}$$

The first three terms in Eq. (3.3) signify the expected revenue, the expected stock-out/shortage cost, and the expected holding cost at the first period, respectively. The fourth term represents the purchasing cost from the manufacturer, and the last term is the expected profit at period 2 if the leftover stock from the first period is positive with the value of $Q - D_{P_1}$. Similar to the integrated case, the decision problem is solved using the recursive method.

If the leftover stock of the first period is x, the retailer's expected profit in period 2 π_{Re2} is expressed as

$$\pi_{Re2}(P_2) = P_2\, E\left[\min\left(x\,, D_{P_2}\right)\right] - s_l\, E\left[\left(D_{P_2} - x\right)^+\right] - h\, E\left[\left(x - D_{P_2}\right)^+\right]. \tag{3.4}$$

The first term in Eq. (3.4) is the expected revenue of period 2, the second term is the expected stock-out/shortage cost at the end of period 2, and the third term represents the expected holding cost.

Since the manufacturer provides the retailer with Q at the beginning of the first period, the manufacturer's profit π_M (M stands for the Manufacturer) is presented as

$$\pi_M(W) = (W - C_m)\, Q\ . \qquad (3.5)$$

The manufacturer also tries all possible wholesale prices to find the optimal W^* that maximizes her/his profit π_M in Eq. (3.5).

To calculate the channel efficiency (CE), the following formula is used:

$$CE = (\pi_M + \pi_{Re1})/\pi_{I1},$$

where π_M, π_{Re1} and π_{I1} are obtained by Eqs (3.5), (3.3) and (3.1), respectively.

It is recognized from the literature that such problem cannot be solved analytically (Lau et al., 2007). In addition, a close form formulation cannot be obtained for Q^*. Therefore, a Sudoku algorithm is developed for solving the problem presented in Appendix A.

3.2.3. Two-price policy model when the manufacturer is committed for the price decline [P_1,P_2,Δ]

For this model, it is assumed that the manufacturer pays $\Delta = P_1 - P_2$ as a rebate to the retailer for each unit sold at the time of selling to the customers in period 2. Since the retailer declines the retail price at time-unit t_r, the model is again considered as a two-period newsvendor problem. The first period is $[1, t_r)$ with the retail price P_1 and the random demand D_{P_1} whose finite ranges are $D_{P_1,min} = \sum_{t=1}^{t_r-1} D_{t,min}$ and $D_{P_1,max} = \sum_{t=1}^{t_r-1} D_{t,max}$. The second period is $[t_r, T]$ with the retail price $P_2 = P_1 - \Delta$ and the random demand D_{P_2} whose finite ranges are $D_{P_2,min} = \sum_{t=t_r}^{T} D_{t,min}$ and $D_{P_2,max} = \sum_{t=t_r}^{T} D_{t,max}$.

Under the manufacturer-Stackelberg game, the manufacturer first announces the wholesale price \$$W$/unit and the rebate \$$\Delta$/unit. Then, the retailer responds with the optimal unit retail price P_1^* and the optimal order volume Q^*, which maximize the retailer's expected profit for the entire life cycle π_{Re1} in Eq.(3.6):

$$\pi_{Re1}(Q, P_1) = P_1\, E\left[\min\left(Q\,, D_{P_1}\right)\right] - s_l\, E\left[\left(D_{P_1} - Q\right)^+\right] - h\, E\left[\left(Q - D_{P_1}\right)^+\right] - Q\, W + \alpha\, E\left[\pi_{Re2}\left(\left(Q - D_{P_1}\right)^+, P_1\right)\right], \tag{3.6}$$

where $\pi_{Re2}\left(\left(Q - D_{P_1}\right)^+, P_1\right)$ is the retailer's expected profit in the second period if the leftover stock from the first period is positive with the value of $Q - D_{P_1}$.

The retailer does not lose any revenue for the products sold in the second period as the manufacturer pays the Δ. Therefore, if the leftover stock of the first period is x, the retailer's expected profit in period 2 π_{Re2} is expressed as

$$\pi_{Re2}(P_1) = P_1\, E\left[\min\left(x\,, D_{P_2}\right)\right] - s_l\, E\left[\left(D_{P_2} - x\right)^+\right] - h\, E\left[\left(x - D_{P_2}\right)^+\right]. \tag{3.7}$$

The first term in Eq. (3.7) can be rewritten as $(P_2 + \Delta)E\left[\min\left(x\,, D_{P_2}\right)\right]$ where the manufacturer compensates the retailer for $\Delta\, E\left[\min\left(x\,, D_{P_2}\right)\right]$. Therefore, since the manufacturer provides the retailer with Q at the beginning of the first period, the manufacturer's expected profit π_M can be formulated as

$$\pi_M(W, \Delta) = (W - C_m)\, Q - \alpha\, \Delta E\left[\min\left(x\,, D_{P_2}\right)\right]. \tag{3.8}$$

The manufacturer also tries all possible W and Δ to find the optimal W^* and Δ^* that maximize her/his profit in Eq. (3.8). To find the optimal solution, a Sudoku algorithm is developed similar to the one developed in Appendix A. Please see Appendix B for the algorithm.

The following formula is used to compute the channel efficiency: $CE = (\pi_M + \pi_{Re1})/\pi_{I1}$, where π_M, π_{Re1} and π_{I1} are obtained by Eqs (3.8), (3.6) and (3.1), respectively.

3.3. Benchmark models

To better investigate the performance of the two different two-price policy models, two well-known policies are considered in this study: the wholesale-price-only and the buyback. In either policy the retailer does not reduce the retail price over the product life cycle; therefore, the problem is a single period model. In the buyback policy, the manufacturer allows the retailer to return unsold products at the end of the product life cycle, while in the wholesale-price-only there is no incentive from the manufacturer.

3.3.1. The integrated channel for a single period model

For the channel efficiency analysis, a vertically integrated channel is considered. An integrated channel means that there is a single manufacturing/retailing entity in which the manufacturer itself produces and sells the products to the end customers. The integrated firm's expected profit for the entire life cycle π_I is presented in Eq. (3.9):

$$\pi_I(Q, P) = P\, E[\min(Q\ , D)] - s_l\, E[(D - Q)^+] - h\, E[(Q - D)^+] - Q\, C_m. \quad (3.9)$$

The first three terms of Eq. (3.9) are the expected revenue, the expected stock-out/shortage cost, and the expected holding cost, respectively. The fourth term is the manufacturing cost.

By taking the first derivative of Eq. (3.9) with respect to Q and setting it to zero, the optimal order volume is obtained as $Q^* = F^{-1}[(P^* + s_l - C_m)/(P^* + s_l + h)]$.

3.3.2. The wholesale-price-only policy $[W]$.

If the retailer only determines one retail price, instead of determining two retail prices, the two-price policy is transferred to the wholesale-price-only policy. Therefore, the model is a single period newsvendor model, and thus, the random demand of the entire single period is denoted by D whose finite ranges are $D_{min} = \sum_{t=1}^{T} D_{t,min}$ and $D_{max} = \sum_{t=1}^{T} D_{t,max}$. The dominant manufacturer first announces a unit wholesale price W and then the retailer responds with the optimal unit retail price P^* and the optimal order volume Q^* to maximize the retailer's expected profit as

$$\pi_{Re}(Q, P) = P\, E[\min(Q\ , D)] - s_l\, E[(D - Q)^+] - h\, E[(Q - D)^+] - Q\, W. \quad (3.10)$$

The first three terms in Eq. (3.10) are the expected revenue, the expected stock-out/shortage cost, and the expected holding cost, respectively. The fourth term is the purchasing cost from the manufacturer.

Within a possible range for the wholesale price, the manufacturer finds W^* that maximizes his/her profit as

$$\pi_M(W) = (W - C_m)\,Q. \tag{3.11}$$

Typically, $C_m \leq W \leq P$.

By taking the first derivative of Eq. (3.10) with respect to Q and setting it to zero, the optimal order volume is obtained by $Q^* = F^{-1}[(P^* + s_l - W)/(P^* + s_l + h)]$ (see Lau et al., 2007).

3.3.3. The buyback policy [W,R].

If in the wholesale-price-only policy the manufacturer allows the retailer to return unsold products at the end of the period, the wholesale-price-only policy is transferred to the buyback policy. In this policy, the dominant manufacturer simultaneously announces a unit buyback price R and a unit wholesale price W and then the retailer responds with the optimal unit retail price P^* and the optimal order volume Q^* that maximize the retailer's expected profit by

$$\pi_{Re}(Q,P) = P\,E[\min(Q\,,D)] - s_l E[(D-Q)^+] + R\,E[(Q-D)^+] - Q\,W. \tag{3.12}$$

The third term is the expected revenue resulted from returning the unsold products to the manufacturer at the end of the product life cycle. The first, second and the last terms are the same as those in Eq. (3.10).

Within a possible range for both R and W, the manufacturer finds the optimal wholesale and buyback prices that maximize her/his expected profit as

$$\pi_M(W) = (W - C_m)\, Q - R\, E[(Q - D)^+]. \quad (3.13)$$

It is assumed that $0 \leq R \leq W \leq P$.

By taking the first derivative of Eq. (3.12) with respect to Q and setting it to zero, the optimal order volume is obtained by $Q^* = F^{-1}[(P^* + s_l - W)/(P^* + s_l - R)]$ (see also Lau et al., 2007).

CHAPTER 4 – NUMERICAL ANALYSIS

This numerical example illustrates the performance of the two-price policy models by comparing them with the wholesale-price-only and buyback policies. The demand is price sensitive and its sensitivity increases over time. It is assumed that the product's life cycle consists of 2 time-units, $T = 2$, and the product attractiveness decreases after the first time-unit, i.e., $t_{st} = 1$. This means that b_t aggregates with β in the second time-unit. By the definition of $b_t = \begin{cases} b & , \ 1 \leq t \leq t_{st} \\ b_{t-1} + \beta, & t_{st} < t \leq T \end{cases}$, $b_1 = b$ and $b_2 = b_1 + \beta$. Recall that it is assumed that $C_m = 1$ and $\tilde{\varepsilon}$ follows a uniform distribution. In order to study the behavior of the two-price policy models, they are solved for different combinations of parameter values. In addition, in this numerical example $\alpha = 1$, since the proposed models are going to be compared with other policies.

This numerical analysis is divided into two subsections. In subsection 4.1, the solutions of the four policies are presented and analyzed. A comparison of the results obtained in subsection 4.1 is discussed in Subsection 4.2. For the proposed two-price policy models, the computations were performed on the gpc supercomputer at the SciNet HPC Consortium (Chris Loken et al. 2010). SciNet is funded by: the Canada Foundation for Innovation under the auspices of Compute Canada; the Government of Ontario; Ontario Research Fund - Research Excellence; and the University of Toronto.

4.1. The numerical example

<u>*The solutions of the two-price policy*</u> $[P_1, P_2]$ <u>*without rebate*</u>.

Since the channel efficiency of the policies is going to be evaluated, the optimal solution of the integrated firm formulated by Eq. (3.1) is presented in Table 4.1. The solutions are categorized in four different cases. For example, in Case 1 there are three sub-cases where the difference among them is in the value of β.

Table 4.1. Optimal solutions for the integrated firm of the two-price policy

Parameter values						*Integrated firm optimal solution*			
s_l	h	A	b	β	σ_ε	P_1	P_2	Q	π_{I1}
Case 1									
0	0	10	0.5	0.10	0.1	10.50	8.83	9.59	81.79
0	0	10	0.5	0.15	0.1	10.50	8.19	9.56	78.60
0	0	10	0.5	0.20	0.1	10.50	7.64	9.53	75.86
Case 2									
0	0	10	0.5	0.10	0.5	10.50	8.70	9.66	77.94
0	0	10	0.5	0.15	0.5	10.50	8.06	9.60	74.85
0	0	10	0.5	0.20	0.5	10.50	7.50	9.55	72.22
Case 3									
0	0	10	0.5	0.10	0.9	10.50	8.59	9.82	74.75
0	0	10	0.5	0.15	0.9	10.50	7.94	9.73	71.75
0	0	10	0.5	0.20	0.9	10.50	7.39	9.64	69.20
Case 4									
9	0	10	0.5	0.10	0.1	10.50	8.84	9.65	81.59
9	0	10	0.5	0.15	0.1	10.50	8.20	9.63	78.40
9	0	10	0.5	0.20	0.1	10.50	7.64	9.61	75.67
Case 5									
0	5	10	0.5	0.10	0.1	10.50	8.81	9.50	81.13
0	5	10	0.5	0.15	0.1	10.50	8.16	9.46	77.97
0	5	10	0.5	0.20	0.1	10.50	7.62	9.43	75.25

For the integrated case, since the manufacturer produces and then sells the product to the market, without the retailer, there is no wholesale price. If one compares the result of the integrated case with other cases presented in Tables 4.2 and 4.3, in can be seen that the retail prices are less and the order quantities are more. The reason is due to the fact that in the integrated case, since the manufacturer sells the products directly to the customers, he/she can set a lower retail price. Therefore, both the demand and order quantity increase.

Table 4.2 illustrates the optimal solutions of the two-price policy with different parameter values where the manufacturer gives no rebate to the retailer. In Table 4.2, the retailer’s and manufacturer’s profits are obtained using Eqs. (3.3) and (3.5), respectively.

Table 4.2. Optimal solutions for the two-price policy with no rebate

Parameter values						*Two-price policy optimal solution*						
s_l	h	A	b	β	σ_ε	P_1	P_2	W	Q	π_M	π_{Re1}	$\frac{\pi_M}{\pi_{Re1}}$
Case 1												
0	0	10	0.5	0.10	0.1	14.72	12.98	9.44	4.77	40.30	21.41	1.882
0	0	10	0.5	0.15	0.1	14.53	12.16	9.08	4.74	38.32	20.97	1.827
0	0	10	0.5	0.20	0.1	14.40	11.47	8.80	4.67	36.50	20.54	1.777
Case 2												
0	0	10	0.5	0.10	0.5	14.55	12.50	9.10	4.83	39.12	21.20	1.845
0	0	10	0.5	0.15	0.5	14.40	11.71	8.80	4.75	37.05	20.70	1.790
0	0	10	0.5	0.20	0.5	14.25	11.02	8.50	4.70	35.19	20.45	1.721
Case 3												
0	0	10	0.5	0.10	0.9	14.22	11.58	8.45	4.80	35.76	20.77	1.722
0	0	10	0.5	0.15	0.9	14.09	11.12	8.19	4.71	34.11	20.01	1.705
0	0	10	0.5	0.20	0.9	13.87	10.56	7.75	4.62	32.84	19.30	1.702
Case 4												
9	0	10	0.5	0.10	0.1	14.76	13.07	9.53	4.80	40.78	20.49	1.990
9	0	10	0.5	0.15	0.1	14.58	12.25	9.17	4.78	39.10	19.71	1.984
9	0	10	0.5	0.20	0.1	14.38	11.50	8.79	4.75	37.45	19.08	1.963
Case 5												
0	5	10	0.5	0.10	0.1	14.75	13.00	9.50	4.70	40.08	20.86	1.921
0	5	10	0.5	0.15	0.1	14.62	12.22	9.26	4.61	38.37	20.10	1.909
0	5	10	0.5	0.20	0.1	14.35	11.40	8.74	4.56	36.72	19.64	1.870

In this policy, CE obtained for all cases varies from 0.750 to 0.774, which shows the channel is not coordinated since $CE < 1$. This phenomenon has been emphasized by the earlier studies for the price sensitive demand. Furthermore, since the manufacturer leads the retailer, one can see that $\pi_M > \pi_{Re1}$, where $1.7 < \pi_M/\pi_{Re1} < 2$. It can be seen that when β increases, the two retail prices, P_1 and P_2, and the wholesale price, W, decrease. The interpretation is when the demand is more sensitive to price, the retailer is not able to keep the retail price high. The manufacturer also needs to decrease the wholesale price to support the retailer for this retail price decline. However, the decline of the prices cannot mitigate the effect of the increase in β, and as one sees when β increases the optimal order quantity decreases. All these effects cause the expected profits of both parties to decrease since the retailer will face with a higher drop in demand due to the increase of β. Other cases do not provide additional insights.

The first three cases show that when σ_ε increases, the two parties decrease their prices, since they face with a higher risk. This results in a decreased expected profit for both the manufacturer and the retailer.

Case 4 states when the shortage cost increases, the retailer needs to increase the inventory level so that he/she does not face stock-out. This means that the manufacturer can sell more products, and thus, compared to Case 1, the manufacturer's expected profit grows while the retailer's expected profit decreases.

The results presented in Case 5 indicate that when the holding cost grows, the retailer has to decrease the inventory level. Therefore, unlike Case 4, both parties lose their expected profits compared to Case 1.

Figures 4.1–4.3 are depicted based on the first row of Case 1 in Table 4.2. In Figure 4.1, W gradually increases and one can see its effect on the manufacturer's and retailer's expected profits. Figure 4.1 shows that the manufacturer's expected profit is concave, and the retailer's expected profit decreases with respect to W.

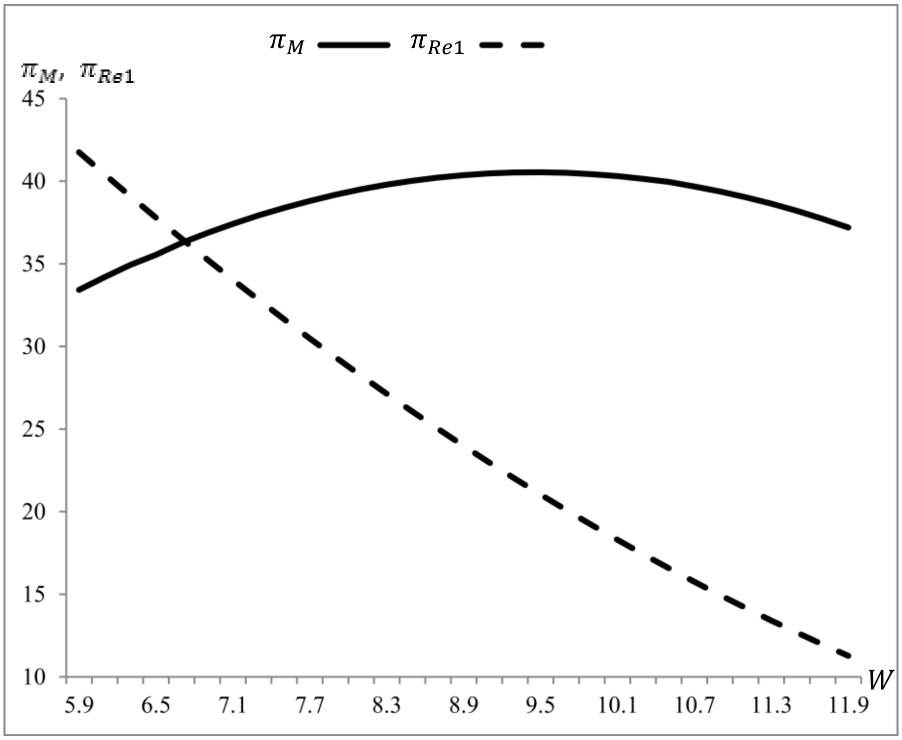

Figure 4.1. The effect of the wholesale price on the parties' expected profits.

Figure 4.2 shows how the three decision variables, P_1, P_2, Q, change as W increases steadily. P_1 and P_2 are increasing and Q is decreasing in W. Another point that one can get from Figure 4.2 is P_1 and P_2 move parallely (i.e., $P_1 - P_2 = 1.7$ for all the values of W).

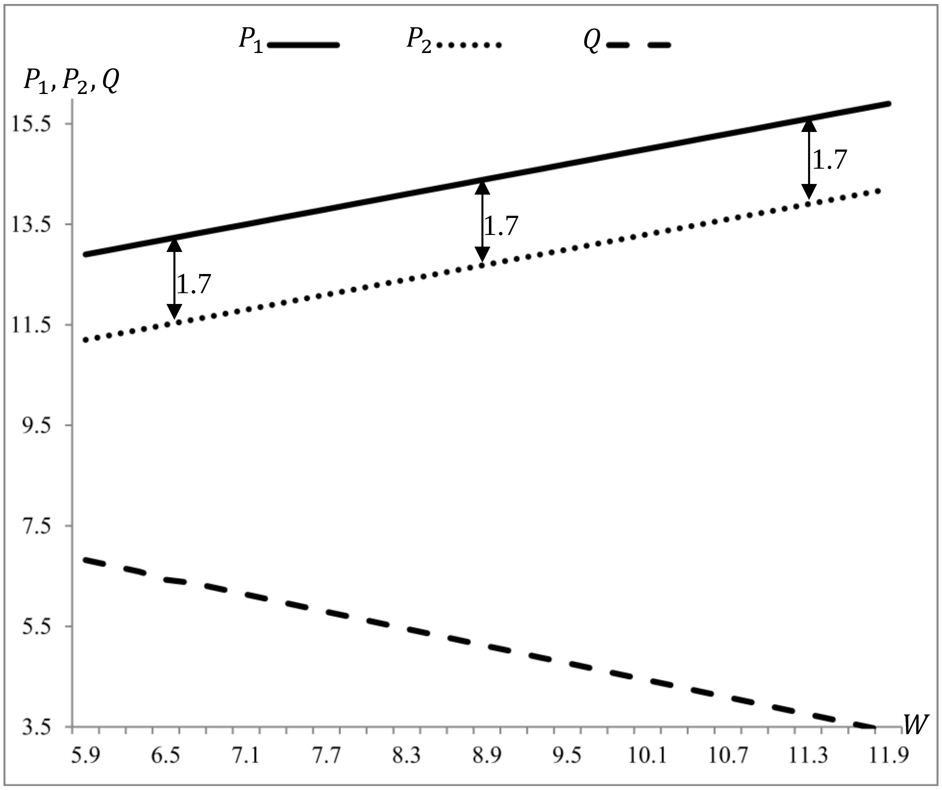

Figure 4.2. The effect of the wholesale price on P_1, P_2, Q.

In Figure 4.3, W is kept in its optimal value, i.e., 9.44, but P_1 gradually increases. Figure 4.3 shows the effect of P_1 changes on the manufacturer's and retailer's expected profits. Now, one can see that the manufacturer's expected profit is decreasing and the retailer's expected profit is concave with respect to P_1.

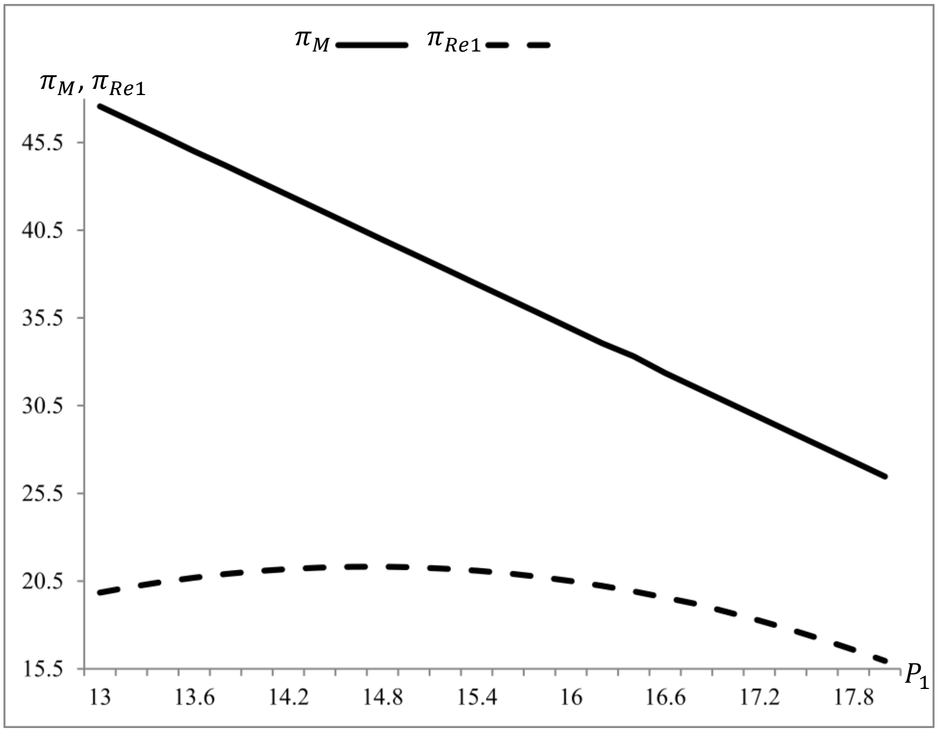

Figure 4.3. The effect of P_1 on the parties' expected profits.

<u>*The solutions of the two-price policy with rebate*</u> [P_1,P_2, Δ].

In this policy, the manufacturer compensates the retailer for the retail price decline ($\Delta = P_1 - P_2$) at the second period. The performance of this policy is evaluated by different combinations of parameter values that are illustrated in Table 4.3. In Table 4.3, the retailer's and manufacturer's profits are obtained using Eqs. (3.6) and (3.8), respectively.

Table 4.3. Optimal solutions for the two-price policy with rebate

Parameter values						*Two-price with rebate optimal solution*							
s_l	h	A	b	β	σ_ε	P_1	P_2	W	Δ	Q	π_M	π_{Re1}	$\frac{\pi_M}{\pi_{Re1}}$
Case 1													
0	0	10	0.5	0.10	0.1	14.73	13.01	10.42	1.72	4.76	41.19	20.28	2.031
0	0	10	0.5	0.15	0.1	14.54	12.18	10.43	2.36	4.74	39.99	19.26	2.076
0	0	10	0.5	0.20	0.1	14.36	11.46	10.44	2.90	4.72	39.08	18.30	2.136
Case 2													
0	0	10	0.5	0.10	0.5	14.49	12.58	10.14	1.90	4.86	40.57	19.99	2.030
0	0	10	0.5	0.15	0.5	14.32	11.77	10.19	2.55	4.82	39.45	18.89	2.088
0	0	10	0.5	0.20	0.5	14.16	11.06	10.21	3.10	4.80	38.6	17.96	2.149
Case 3													
0	0	10	0.5	0.10	0.5	14.29	12.16	9.93	2.13	4.95	40.06	19.51	2.053
0	0	10	0.5	0.15	0.5	14.10	11.37	9.95	2.73	4.92	38.16	18.68	2.043
0	0	10	0.5	0.20	0.5	13.97	10.63	10.00	3.34	4.90	37.07	17.91	2.070
Case 4													
9	0	10	0.5	0.10	0.1	14.77	13.05	10.45	1.71	4.81	41.80	19.25	2.171
9	0	10	0.5	0.15	0.1	14.57	12.24	10.46	2.32	4.78	40.59	18.15	2.236
9	0	10	0.5	0.20	0.1	14.38	11.55	10.48	2.83	4.78	39.68	17.04	2.328
Case 5													
0	5	10	0.5	0.10	0.1	14.74	12.99	10.46	1.75	4.72	40.99	19.94	2.056
0	5	10	0.5	0.15	0.1	14.55	12.17	10.47	2.39	4.70	39.81	18.94	2.102
0	5	10	0.5	0.20	0.1	14.41	11.40	10.49	3.00	4.69	39.75	18.01	2.207

In this policy, CE obtained for all cases varies from 0.751 to 0.796, which shows the channel is not coordinated since $CE < 1$. Also, $\pi_M > \pi_{Re1}$, in which $\pi_M/\pi_{Re1} > 2$. In addition, as β increases, the manufacturer pays a higher rebate that makes him/her able to set a higher wholesale price. Except Δ and W, one can see that the retail prices, the order quantity and the expected profits decrease in β. Other cases offer no additional insights beyond Case 1.

Figures 4.4–4.6 are drawn based on the date in the first row of Case 1 in Table 4.3. Similar to Figure 4.1, Figure 4.4 illustrates that the manufacturer's expected profit is concave, and the retailer's expected profit is decreasing with respect to W.

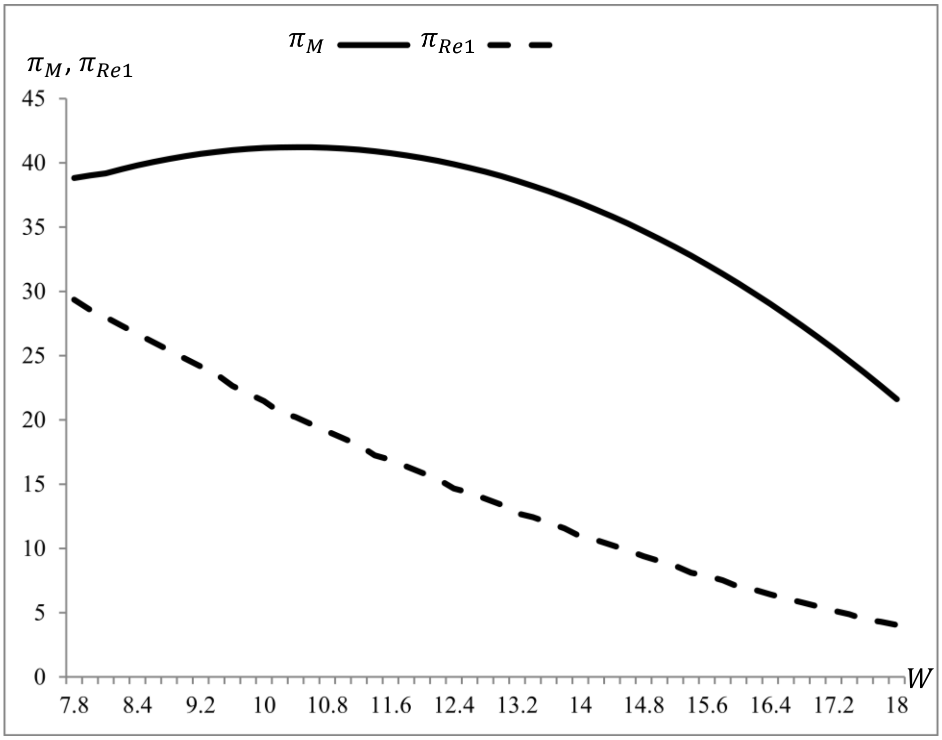

Figure 4.4. The effect of the wholesale price on the parties' expected profits.

Figure 4.5 presents the effect of the wholesale price on P_1, P_2, Δ and Q. It can be seen when the manufacturer increases the wholesale price, he/she should pay a higher Δ. The retailer also responses to the increase of wholesale price with a higher retail price P_1. The point is that P_1 and Δ increase in parallel. Therefore, P_2 remains approximately constant.

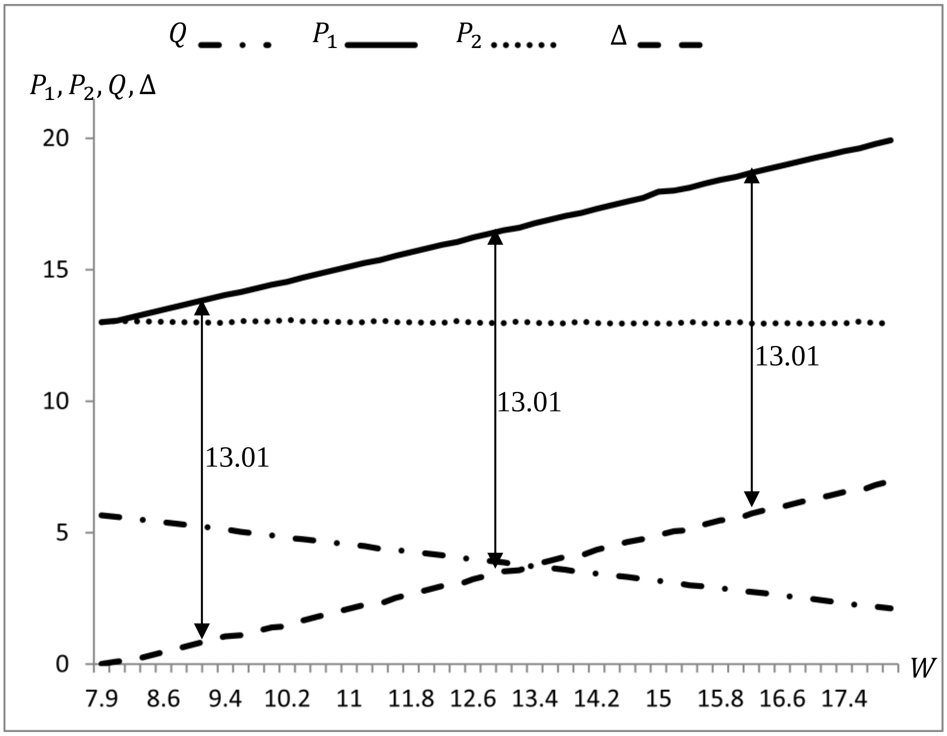

Figure 4.5. The effect of the wholesale price on P_1, P_2, Δ, Q.

In Figure 4.6, W is kept in its optimal value, i.e., 10.42, but Δ steadily increases to see its effect on the manufacturer's and retailer's expected profits. Now, one can see that the manufacturer's expected profit is concave and the retailer's expected profit is increasing with respect to Δ.

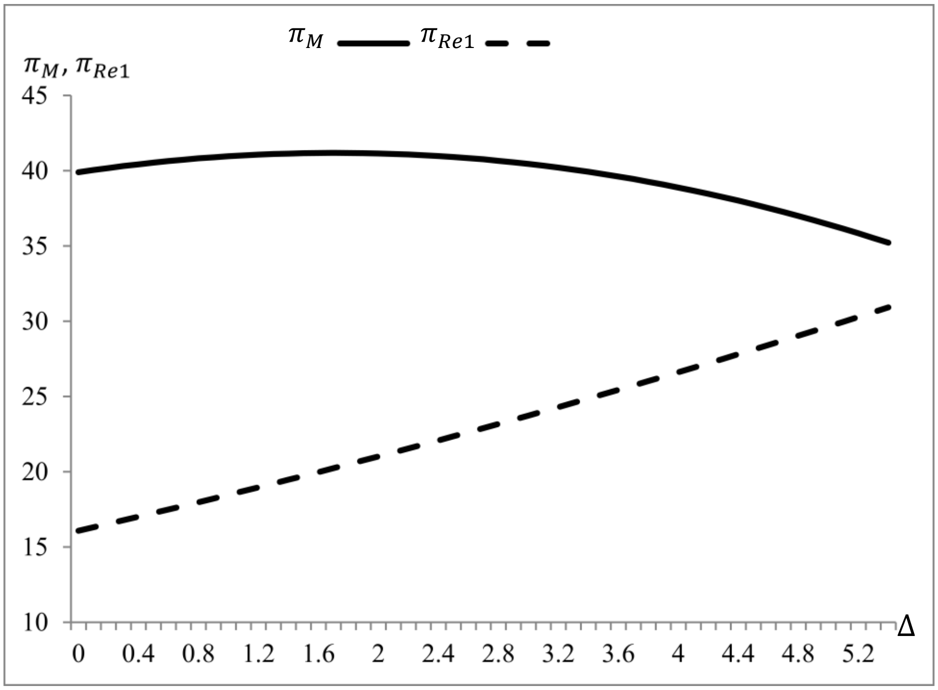

Figure 4.6. The effect of Δ on the parties' expected profits.

<u>*The solutions of the wholesale-price-only scheme*</u> [*W*].

Here, only Case 1 is considered for the analysis since it is already demonstrated that other cases do not provide additional insights. The optimal solutions of the integrated firm formulated by Eq. (3.9) are presented in Table 4.4.

Table 4.4. Optimal solutions for the integrated firm of benchmark models

Parameter values						*Integrated firm optimal solution*		
s_l	h	A	b	β	σ_ε	P	Q	π_I
0	0	10	0.5	0.10	0.1	9.27	9.75	80.77
0	0	10	0.5	0.15	0.1	9.176	9.72	76.84
0	0	10	0.5	0.20	0.1	8.81	9.69	73.25

Table 4.5 presents the solutions of the wholesale-price-only policy with different β. In Table 4.5, the retailer's and manufacturer's profits are obtained using Eqs.(3.10) and (3.11), respectively.

Table 4.5. Optimal solutions for the wholesale-price-only policy

Parameter values						*wholesale-price-only optimal solution*					
s_l	h	A	b	β	σ_ε	P	W	Q	π_M	π_{Re}	$\frac{\pi_M}{\pi_{Re}}$
0	0	10	0.5	0.10	0.1	13.70	9.28	4.82	39.71	21.35	1.859
0	0	10	0.5	0.15	0.1	13.11	8.87	4.80	37.78	20.33	1.858
0	0	10	0.5	0.20	0.1	12.59	8.55	4.77	36.01	19.22	1.874

For wholesale-price-only policy, CE varies 0.754 to 0.756. Similar to the previous policies, β has a negative impact on all the decision variables as well as the expected profits. Coordination cannot be achieved in this policy. Also, $\pi_M/\pi_{Re} < 2$.

<u>*The solutions of the buyback scheme*</u> [*W*,*R*].

Table 4.6 shows the solutions of the buyback policy. In Table 4.6, the retailer's and manufacturer's profits are obtained using Eqs. (3.12) and (3.13), respectively.

Table 4.6. Optimal solutions for the buyback policy

Parameter values						*buyback optimal solution*							
s_l	h	A	b	β	σ_ε	P	W	R	Q	π_M	π_{Re}	$\frac{\pi_M}{\pi_{Re}}$	CE
0	0	10	0.5	0.10	0.1	13.88	9.68	8.59	4.94	40.97	19.78	2.071	0.752
0	0	10	0.5	0.15	0.1	13.28	9.26	8.23	4.93	38.97	18.87	2.065	0.753
0	0	10	0.5	0.20	0.1	12.74	8.91	7.89	4.91	37.15	17.93	2.072	0.752

For buyback policy, CE varies from 0.752 to 0.753. Here, it is perceived that a higher β results in a lower buyback price. This can be interpreted as when β increases, order volume decreases, and therefore, the manufacturer can offer a lower buyback price. Similar to other policies, one can see that $CE < 1$ and $\pi_M > \pi_{Re}$, where $\pi_M/\pi_{Re} > 2$.

4.2. A comparative analysis

For comparative analysis, Figures 4.7–4.9 are used that were extracted from Case 1 of Tables 4.1–4.6. Since the manufacturer is a dominant player, his/her expected profit is first analyzed for different policies depicted in Figure 4.7.

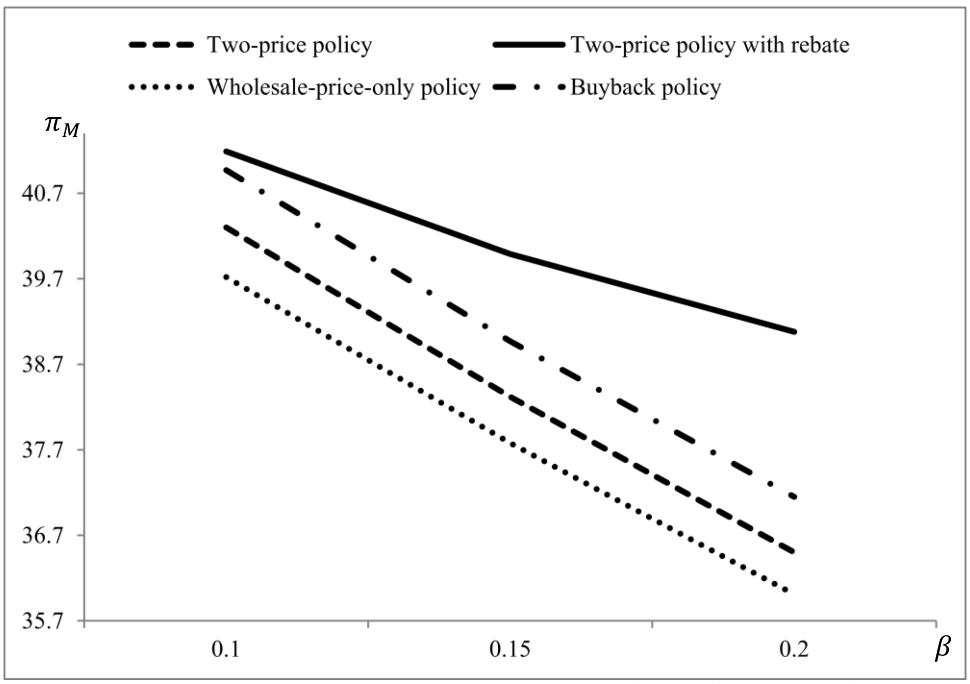

Figure 4.7. The manufacturer's expected profit for different policies.

It is interesting to see that the two-price policy with rebate $[P_1,P_2,\Delta]$ gives a higher expected profit to the manufacturer than the two-price policy without rebate $[P_1,P_2]$. This is a counter-intuitive phenomenon since one can expect that when the manufacturer compensates the retail price decline, he/she should obtain less. However, in the $[P_1,P_2,\Delta]$ policy, the manufacturer has the power of setting two manipulating tools, the wholesale price W and the rebate Δ, to increase his/her expected profit. The same result can be perceived for the buyback policy [W,R]. Since the manufacturer takes the advantage of setting W and R in [W,R], his/her expected profit is greater than its counterpart in [W] (see also Lau and Lau, 2002 and Lau et al., 2007). This numerical analysis also shows that the two-price policy with rebate [P_1,P_2, Δ] may be more interesting for the manufacturer in comparison with the buyback policy [W,R]. In addition, the two-price policy without rebate [P_1,P_2] may be better than the wholesale-price-only policy [W] for the manufacturer as [P_1,P_2] gives a higher expected profit.

Figure 4.8 illustrates the retailer's expected profits obtained by the four different policies.

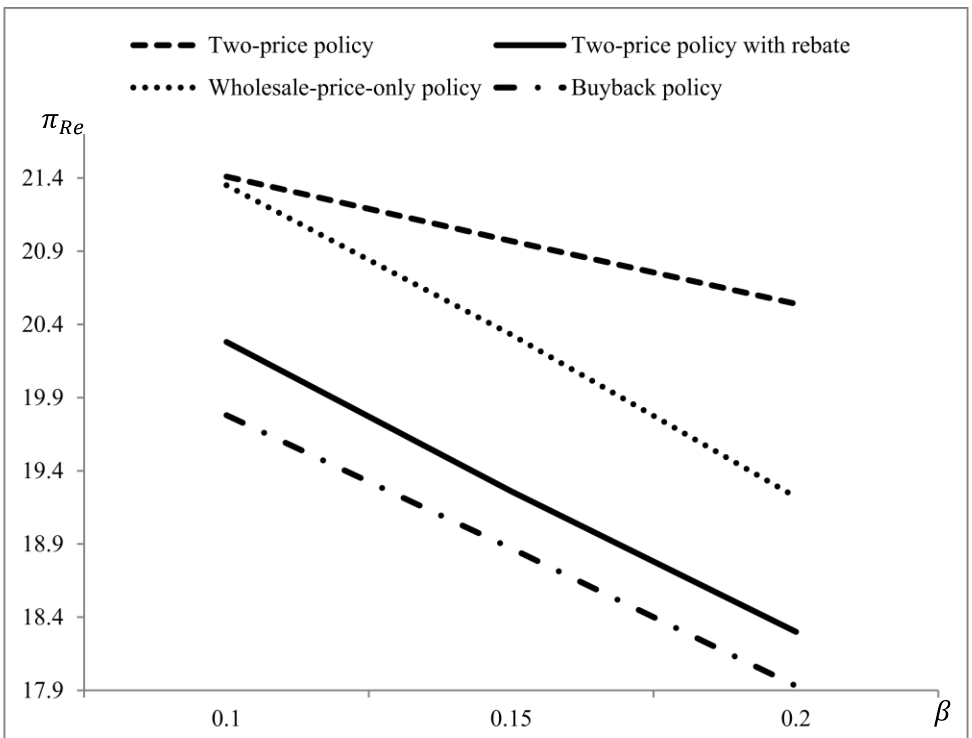

Figure 4.8. The retailer's expected profit for different policies.

This comparative analysis shows that the two-price policy without rebate [P_1,P_2] is the most beneficial policy for the retailer because he/she uses this price decline to increase his/her own expected profit. In addition, Figure 4.1 and Figure 4.2 show that [P_1,P_2, Δ] may be more interesting than [W,R] to both the manufacturer and the retailer since they are better off by the [P_1,P_2, Δ] policy. It can be also concluded that the retailer is not willing the manufacturer offers any incentive that decreases the retailer's expected profit compared to [W].

Figure 4.9 shows that the two-price policies give the highest $(\pi_M + \pi_{Re})$ and the buyback policy gives the least.

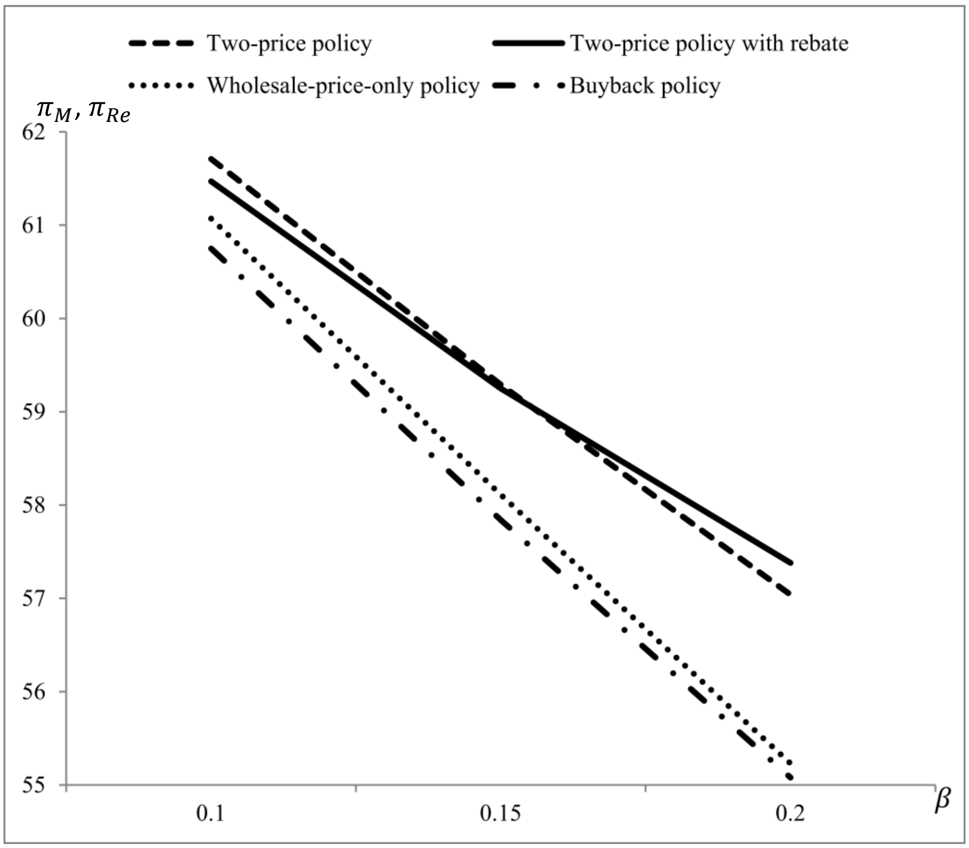

Figure 4.9. $(\pi_M + \pi_{Re})$ for different policies.

The integrated firm of the two-price policy has the higher expected profit compared to the benchmark (wholesale-price-only and buyback policies).

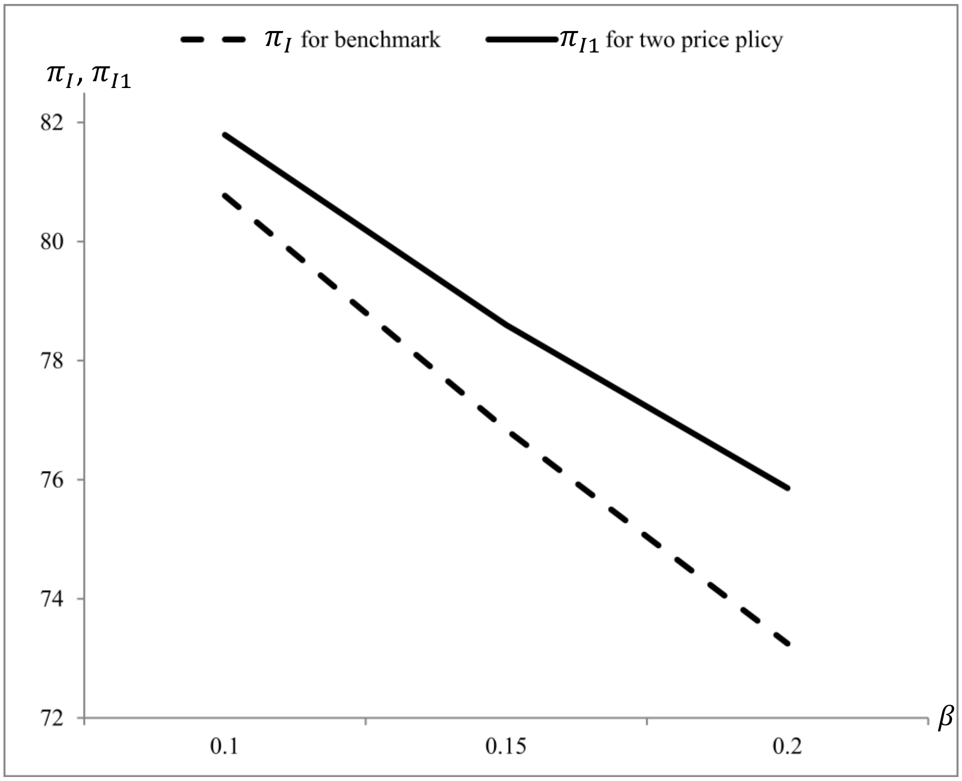

Figure 4.10. The integrated firms' expected profit.

CHAPTER 5 – CONCLUDING REMARKS AND FUTURE STUDIES

This study considered a two-echelon supply chain (newsvendor) models consisting of a dominant manufacture and a follower retailer. The manufacturer wholesales a single product, such as personal computers or notebooks, to the retailer in which the product's demand is stochastic and price-sensitive. This sensitivity to price may increase over the product's life cycle. In other words, demand decreases during the product's life cycle either because a new product is introduced or the existing product becomes obsolete. Therefore, the retailer needs to decrease the retail price in midlife to keep the demand as high as possible.

In this study, a new policy was proposed: the two-price policy [P_1,P_2] that enables the retailer to set the retail price twice during the product's life cycle: once at the beginning of the product life cycle and the other at the midlife so that $P_1 - P_2 \geq 0$. In this new policy, two different situations or models were considered. In the first situation, which is called the two-price policy [P_1,P_2], it was assumed that the retailer is committed to decrease the retail price without any compensation from the manufacturer. In the second model, the manufacturer gives rebates to the retailer for the retail price decline, i.e., the manufacturer pays the $\Delta = P_1 - P_2$ for the products sold after the decline of the retail price. Hence, it was called the two-price policy with rebate [P_1,P_2, Δ]. The two policies were modeled as a two-period problem in which the first period was from the first-of-life to the midlife, and the second period was afterwards. To investigate the performance of these two proposed policies, the wholesale-price-only [W] and the buyback [W, R] policies were also considered. The numerical analysis demonstrated that: (a) although the manufacturer pays Δ and R in the [P_1,P_2,Δ] and [W,R] policies, respectively, these two policies give the manufacturer a higher expected profit than the [P_1,P_2] and [W]; (b) although under the [P_1,P_2] policy the retailer is solely committed for the retail price decline, the retailer obtains the most benefit; (c) the [W, R] and [P_1, P_2, Δ] policies give the retailer the lowest expected profit, respectively. As a result of this numerical analysis, the [P_1, P_2] policy may interest the retailer and the [W, R] and [P_1, P_2, Δ] may interest the manufacturer.

It can be seen that the results of [W, R] and [W] policies are in agreement with the literature that verifies the accuracy of this study's results for these two policies (see Lau et al., 2007). Therefore, one may be able to trust the accuracy of the results obtained by the [P_1, P_2] and [P_1, P_2, Δ] policies. However, it is suggested the supply chain managers carefully model their problems and analyze the solutions for making an appropriate pricing decision.

In reality, the attractiveness of a product decreases over the product life cycle, especially when a new product is introduced to the market. Existing products cannot compete with the new product if the price does not drop off (i.e., for those customers who are concerned with the price). For the first time in the literature, this study considered such real and complicated situation, and proposed the two-price policy models. This study enables the supply chains' managers to design their own optimal pricing schemes.

In this study, the holding cost of the products over the product life cycle was not considered. Therefore, it may be interesting for future studies to incorporate the products' holding cost during the product life cycle to their formulation, similar to the economic order quantity models. In reality, as a result of economic conditions or for other reasons, the demand of the product may fluctuate over the product life cycle. This neglected phenomenon can be considered as a direction for future. The proposed models were compared with the wholesale-price-only and the buyback policies. Therefore, considering other policies, such as two-part tariffs, revenue sharing, and etcetera can be considered as future studies. In addition, this study took into account a situation where the retailer has only one buying opportunity over the product life cycle, while in practice they may have more. It was assumed that the demand is stochastic, which is predicted by historical data. When the historical data is not available, fuzzy set theory is usually used to model the demand. For future study, researchers can consider a fuzzy price-sensitive demand which is more realistic when incomplete data and information is available.

APPENDIX A

Now a Sudoku algorithm is introduced for finding the optimal decision variables (W^*, P_1^*, P_2^* and Q^*) for the two-price policy without rebate. For this algorithm, first a suitable range is defined for all decision variables: $W \in (C_m, \overline{W})$, where $\overline{W}$ is the first W-value that makes $\pi_M \leq 0$ (as the wholesale price increases, the retailer's order volume decreases which consequently decreases the manufacturer's profit); $Q \in [D_{min}, D_{max}]$ where $D_{min} = \sum_{t=1}^{T} D_{t,min}$ and $D_{max} = \sum_{t=1}^{T} D_{t,max}$ are the finite ranges of the demand for the entire product life cycle D; $W < P_2 \leq P_1 < \bar{P}$ where $\bar{P}$ is the first P_1-value that makes $\pi_{Re1} \leq 0$ (when the retail price increases, the demand decreases which consequently decreases both the retailer and the manufacturer's profit).

The Algorithm:

Set Π_{R1}, Π_{R2}, Π_{R3} and Π_M to $-\infty$,

For $W = C_m + \varepsilon$ to $\overline{W}$,

    For $P_1 = W$ to $\bar{P}$,

        For $P_2 = W$ to P_1,

            For $Q = \sum_{t=1}^{T} D_{t,min}$ to $\sum_{t=1}^{T} D_{t,max}$,

                Calculate π_{Re1},

                If $\pi_{Re1} > \Pi_{R1}$ then

                    $\Pi_{R1} = \pi_{Re1}$, $Q^* = Q$, $W^* = W$, $P_1^* = P_1$, $P_2^* = P_2$,

                End

            End

            If $\Pi_{R1} > \Pi_{R2}$ then

                $\Pi_{R2} = \Pi_{R1}$, $Q^{**} = Q^*$, $W^{**} = W^*$, $P_1^{**} = P_1^*$, $P_2^{**} = P_2^*$,

            End

        End

        If $\Pi_{R2} > \Pi_{R3}$ then

$\Pi_{R3} = \Pi_{R2}$, $Q^{***} = Q^{**}$, $W^{***} = W^{**}$, $P_1^{***} = P_1^{**}$, and $P_2^{***} = P_2^{**}$,

End

End

Compute π_M,

If $\pi_M > \Pi_M$ then

$\Pi_M = \pi_M$, $Q^{****} = Q^{***}$, $W^{****} = W^{***}$, $P_1^{****} = P_1^{***}$, and $P_2^{****} = P_2^{***}$,

End

End

where W^{****}, Q^{****}, P_1^{****}, and P_2^{****} are the optimal wholesale price, the optimal order volume, and the optimal two retail prices, respectively.

APPENDIX B

Now a Sudoku algorithm is introduced for finding the optimal decision variables (W^*, Δ^*, P_1^* and Q^*) for the two-price policy with rebate. It is first needed to define a suitable range for all decision variables: $\Delta \in (0, W)$, and the range for other variables are the same as what was defined in Appendix A.

The Algorithm:

Set Π_{R1}, Π_{R2}, Π_{R3} and Π_M to $-\infty$,

For $W = C_m + \varepsilon$ to $\bar{W}$,

For $\Delta = 0$ to W,

For $P_1 = W$ to $\bar{P}$,

$P_2 = P_1 - \Delta$,

For $Q = \sum_{t=1}^{T} D_{t,min}$ to $\sum_{t=1}^{T} D_{t,max}$,

Calculate π_{Re1},

If $\pi_{Re1} > \Pi_{R1}$ then

$\Pi_{R1} = \pi_{Re1}$, $Q^* = Q$, $W^* = W$, $P_1^* = P_1$, $\Delta^* = \Delta$,

End

End

If $\Pi_{R1} > \Pi_{R2}$ then

$\Pi_{R2} = \Pi_{R1}$, $Q^{**} = Q^{*}$, $W^{**} = W^{*}$, $P_1^{**} = P_1^{*}$, and $\Delta^{**} = \Delta^{*}$,

End

End

If $\Pi_{R2} > \Pi_{R3}$ then

$\Pi_{R3} = \Pi_{R2}$, $Q^{***} = Q^{**}$, $W^{***} = W^{**}$, $P_1^{***} = P_1^{**}$, and $\Delta^{***} = \Delta^{**}$,

End

End

Compute π_M,

If $\pi_M > \Pi_M$ then

$\Pi_M = \pi_M$, $W^{****} = W^{***}$, $Q^{****} = Q^{***}$, $P_1^{****} = P_1^{***}$, and $\Delta^{****} = \Delta^{***}$,

End

End

where W^{****}, Δ^{****}, Q^{****} and P_1^{****} are the optimal wholesale price, the optimal rebate, the optimal order volume, and the optimal retail prices, respectively.

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NOTATIONS

Parameters

T product life cycle. The product life cycle consists of T time-units ($t = 1,2, \dots, T$).

D_t random demand per each time-unit

$f(\cdot)$ probability density function of D_t

$F(\cdot)$ cumulative distribution function of D_t

μ_{D_t} mean of D_t

σ_{D_t} standard deviation of D_t

b_t price-sensitivity coefficient of D_t in time-unit *t*

$D_{t,min}$ & $D_{t,max}$ the finite ranges of D_t

t_{st} a time-unit from which the price-sensitivity of the demand increases, $t_{st} \in [1, T]$

t_r a time-unit when the retailer decides to decrease the retail price, $t_r \in [t_{st}, T]$

x inventory level at time-unit t_r

C_m manufacturing cost per unit incurred to the manufacturer

s_l goodwill cost per unit incurred to the retailer for the demand is not satisfied

h holding cost per unit incurred to the retailer for the unsold products

α discount factor for the costs in the second period, $0 < \alpha \leq 1$

Decision variables

P_1 retail price per unit determined at the beginning of the product life cycle

P_2 retail price per unit determined at t_r

$\Delta = P_1 - P_2$ rebate for each unit sold after t_r paid by the manufacturer to the retailer

Q order volume that the retailer orders from the manufacturer

W wholesale price per unit charged by the manufacturer to the retailer

R buyback price per unit paid by the manufacturer to the retailer for units unsold at the end of the product life cycle

Decision variables used in the literature review

ρ rebate paid by the manufacturer to the retailer for each unit sold in rebate policy

τ threshold in which if the retailer sells more than it, the manufacturer pays the ρ

ρ_p rebate paied by the manufacturer to the retailer for unsold units after the retail price declines in price protection policy