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Michael Hoy
University of Guelph

Brennan Scott Thompson
Ryerson University

Buhong Zheng
University of Colorado

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Michael Hoy,[1] Brennan Scott Thompson,[2] and Buhong Zheng[3]

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Abstract

This paper demonstrates the implications of adopting an approach to measuring poverty that takes into account the lifetime experience of individuals rather than simply taking a static or cross-sectional perspective. Our approach follows the theoretical innovations in Hoy and Zheng (2008) which address various aspects of the specific pattern of any poverty spells experienced by an individual as well as a possible retrospective consideration that an individual might have concerning his life experience as a whole. For an individual, our perspective of lifetime poverty is influenced by both the snapshot poverty of each period and the poverty level of the permanent lifetime consumption; it is also influenced by how poverty spells are distributed over the lifetime. Using PSID data for the US, we demonstrate empirically the power of alternative axioms concerning how lifetime poverty should be measured when making pairwise comparisons of individual lifetime profiles of consumption (income) experiences. We also demonstrate the importance of taking a lifetime view of poverty in comparing poverty between groups by use of the classic FGT 'snapshot' poverty index in conjunction with period weighting functions that explicitly reflect concerns about the pattern of poverty spells over individuals' lifetimes.

Keywords: Lifetime poverty, snapshot poverty, chronic poverty, early poverty, poverty measurement

JEL classification: I32

[1] University of Guelph, E-mail: mhoy@uoguelph.ca; [2] Ryerson University, E-mail: brennan@ryerson.ca; [3] University of Colorado, Denver, E-mail: buhong.zheng@cudenver.edu

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UNU World Institute for Development Economics Research (UNU-WIDER)
Katajanokanlaituri 6 B, 00160 Helsinki, Finland

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I. Introduction

It has long been recognized that the impact of poverty experienced by an individual for a long and sustained period of time is very different than poverty experienced for one or more relatively short, intermittent, periods within a person's lifetime. These two temporal characterizations of poverty have come to be known as chronic and transient poverty, respectively. Clark and Hume (2010) provide a very useful background to the historical development of concern about, and progress with, the conceptualization and measurement of individual poverty over time. They note (Clark and Hume, 2010, p. 351) that "breakthroughs in terms of conceptualizing the depth and breadth of poverty were not generally matched by equivalent, systematic advances with regard to time prior to the late 1980s."[1] However, even the terminology of chronic poverty is long-standing. For example, Godley (1847, p. 2) describes his concern with the persistent class differences between the "English Protestant aristocracy and middle class, and the Irish Roman Catholic peasantry, ... (being) two nations within Ireland." He describes the result of this social relationship (p. 10) as creating "political and religious anomalies .... (that have) produced chronic anarchy, with its necessary concomitant, chronic poverty."

As a result of increased availability of panel data, there has recently also been significant empirical research performed regarding the transitions of individuals into and out of poverty and comparing duration of poverty for different individuals within and between countries or various population subgroups. Much of the focus of this research has been on the identification of chronic and transient poverty and the measurement of their relative importance. These approaches typically start by aggregating all poverty spells of all individuals in order to obtain a measure of "total poverty." One can then establish an estimate of each individual's permanent income and use this as the basis for determining the level of chronic poverty for each person in the population. The difference (residual) between total poverty and chronic poverty is then frequently referred to as transient poverty. Methods for creating these types of distinctive measures can include a concern with intensity of poverty in both dimensions of chronic and transient poverty by adopting an appropriate poverty index just as is generally done in static poverty measurement exercises.[2] Since policy prescriptions that deal with these two types of poverty are likely to differ, these exercises are both useful and important.

Although important and insightful research on poverty dynamics of both a theoretical and empirical nature has been gaining in quantity and sophistication, development of complete axiomatic characterizations for measuring poverty through time

[1] See also Addison, Hulme, and Kanbur (2008, p. 8) who note that "The introduction of time into the economic theory of poverty measurement is relatively recent."

[2] Good examples of this approach are Jalan and Ravallion (1998, 2000), Ravalliaon (1998), Baulch and Hoddinott (2000), and Duclos, Araar, and Giles (2010). See also the seminal paper of Rodgers and Rodgers (1993).

is an area of research in its infancy. The axiomatic foundations of static poverty analysis has been well established for some time.[3] Developing a complementary approach that incorporates sensitivity to the pattern of poverty spells through time can borrow from that literature. However, there are many new challenges in developing an axiomatic characterization that incorporates a temporal perspective. In order to reflect chronic poverty concerns, one must address the manner and extent to which the clumping of poverty spells together or in "nearby periods" should matter. The permanent income approach, although useful empirically, does not distinguish, for example, the difference between someone who experiences poverty in every second period of her lifetime and someone who spends the first half of her life constantly in poverty and the second half out of poverty. Both individuals could have the same permanent income and hence be assigned the same measure of chronic poverty even though one could well argue that the second person suffered more chronic poverty over her lifetime. Other questions that arise when considering how to measure chronic poverty include the extent to which chronic poverty relief results from an interruption of several consecutive periods of poverty with one or two periods of non-poverty.

Besides addressing the property of chronic poverty over a person's lifetime, another consideration that has been deemed important in comparing the temporal pattern of poverty is that of the importance of poverty experienced early in life. There is substantial empirical evidence that poverty in earlier stages of life not only affects consumption in later periods but also leaves an inherently deeper mark on lifetime deprivation. Recent research in neuroscience (*e.g.,* see Farah, et al., 2006) suggests that children growing up in poor families with low social status not only suffer from inadequate nutrition and exposure to environmental toxins but also suffer from elevated stress hormones that generally impair neural development, including effects on language and memory.[4] This suggests at least two channels for adverse effects on the individual later in life. First is a direct lasting impact from poverty early in life in that, due to the vulnerability of children, the physiological and psychological effects noted above reduce their future enjoyment from life for any given future (continuing) stream of consumption levels. Secondly, early poverty worsens an individual's capability to generate higher consumption later in life due to a compromised ability to accumulate human capital and obtain favorable employment opportunities. This second effect is captured implicitly if one includes all future levels of consumption in the measurement of lifetime poverty. However, to account for the first (direct) effect, one may require that any lifetime measure of poverty place greater weight on poverty experienced earlier in life.

We believe that developing an axiomatic approach to measuring poverty in a way that captures the subtle but important nature of the temporal pattern in which an

[3]See for example the surveys of Zheng (1997, 2000a).

[4]See also the reports on this line of research from the 2008 meetings of the American Association for the Advancement of Science (Boston) described in an article by Clive Cookson (*The Financial Times*, February 16, 2008) and also by Paul Krugman (*New York Times*, February 18, 2008).

individual experiences poverty during her lifetime can advance the way in which we conceptualize and measure lifetime poverty. Examples of papers that take this approach and investigate the implications of precisely stated axioms relating to chronic poverty include Calvo and Dercon (2007), Foster (2007), Bossert, Chakravarty, and D'Ambrosio (2007), and Hoy and Zheng (2008).[5] In the early stages of this research program we believe it is important to (i) demonstrate the normative appeal of axioms adopted through use of simple hypothetical examples, (ii) illustrate the structure imposed by axioms on the implied measurement approach, (iii) explain how different poverty measures imply different effects from changes in the pattern of lifetime poverty experiences, and (iv) demonstrate through examples the empirical value of the measurement approach. In this paper we develop these themes for the approach of Hoy and Zheng (2008). We review and expand on the normative implications of the axioms adopted in their approach and provide empirical applications using the PSID data set for the US.

In this paper we focus on the axioms from Hoy and Zheng (2008) that bring out a concern with measuring lifetime poverty in a way that emphasizes the presence of chronic poverty and poverty experienced early in life for an individual. In conjunction with other axioms that reflect standard concerns or properties of poverty from the literature on measurement of poverty in a static setting, one obtains a simple structure for a lifetime poverty measurement approach. Specifically, the axioms together imply that lifetime poverty is measured as a weighted sum of all spells or "snapshot" experiences of poverty over the individual's lifetime in conjunction with a retrospective property of lifetime poverty based on average lifetime or permanent consumption. We also show how pairs of individuals' lifetime poverty experiences can be compared on the basis of these axioms by use of relevant dominance conditions that are related to the familiar "concentration dominances" but based on vectors constructed from the time profiles of consumptions of pairs of individuals.

We demonstrate our approach using PSID data for the US from which we are able to create income profiles over twenty-six years for 1,494 individuals. Without any structure on the weights, a pairwise lifetime poverty ordering requires vector dominance of lifetime poverty profiles. We find that few pairwise comparisons are unambiguous. The structure imposed by both the chronic and early poverty axioms significantly extends the fraction of unambiguous pairwise comparisons that can be made. Further, by adopting specific poverty measures that are consistent with our axioms, we show how the comparisons of poverty between subpopulations can be influenced by a concern with the temporal pattern of poverty experiences. The implied cost of poverty differs from that implied by an approach that simply averages each person's poverty experiences over time.

In Section II we lay out the structure of our lifetime poverty measurement ap-

[5]See also Cruces (2005), Grab and Grimm (2007) and Carter and Ikegami (2007). Bossert, D'Ambrosio, and Peragine (2008) and Hoy and Zheng (2008) are, to our knowledge, the only papers that provide a complete axiomatic characterization of lifetime poverty measurement.

proach, explain its normative features, and demonstrate how to implement it. Section III provides an elaboration of the important temporal aspects of chronic and early poverty effects. Empirical applications are explored and reported on in Section IV, followed by a section with remarks and conclusions.

II. The Measurement of Individual Lifetime Poverty

In this section we explain the properties of the lifetime poverty measure used in our empirical analysis. This lifetime poverty approach is developed formally in Hoy and Zheng (2008).[6] The measurement for individual lifetime poverty consists of three steps: (1) the measurement of each individual's "snapshot poverty" at each time period in life, (2) the aggregation of these snapshot poverty spells across all periods, and (3) the inclusion of a retrospective view of poverty over the lifetime as a whole. Here a period is interpreted as the basic unit of time that poverty is measured; income is collected at the beginning of each period to enable consumption in that period and in the subsequent periods.[7] A person is poor in a period if and only if his consumption level in that period falls short of the poverty line. The measurement of snapshot poverty at each period is straightforward; each individual's poverty is measured as his consumption deprivation from the poverty line. The conventional literature on poverty measurement provides ample guidelines for this stage of the measurement.

It is the second stage of lifetime poverty measurement that expands the literature on poverty measurement. When viewed from a lifetime perspective, the suffering and deprivation of an individual in each period transmits into the lifetime evaluation of poverty. All other things equal, the more deprivation a person endures in a given period, the more lifetime poverty is created for or experienced by the individual. This "experience axiom" is akin to the monotonicity axiom or the subgroup consistency axiom typically used in the measurement of snapshot poverty for a single period of time. All these snapshot poverty experiences considered in isolation of each other, however, may not suffice to determine lifetime poverty. One should also account for periods in which an individual lives out of poverty. Over a lifetime an individual, in retrospect, may well benefit from high consumption experienced in non-poverty spells. Given that the deprivation in one period can be offset at least in part by the experience of affluent living in another period, lifetime poverty is also influenced by consumption over one's lifetime as a whole when it is compared with a sort of "lifetime poverty line." The essence of this argument is reflected in our "retrospective axiom" which stipulates that lifetime poverty is a function of the lifetime permanent

[6] The axiomatic characterization, as well as proofs, are provided in Hoy and Zheng (2008).

[7] Consumption is generally thought to be a better measure of the standard of living than is income. However, in many data sets, as in our's, information on consumption is not available. We do not try to distinguish between income and consumption in this paper, although we recognize the importance of doing so.

consumption poverty (in this paper permanent consumption is approximated by the average lifetime consumption in the absence of discounting). This reflects a sense of how an individual has experienced lifetime poverty from a retrospective view of her consumption profile or a sense of poverty over the lifetime considered as a whole. The particular pattern or timing of poverty spells also may affect the overall lifetime poverty of an individual. We explore these further axioms on the timing of poverty spells (see below), along with other regularity assumptions, to derive a general class of lifetime poverty indices that are the weighted sum between the (weighted) average snapshot poverty level across all periods and the poverty of average lifetime consumption.

Consider an individual who lives through T periods. Each period can be interpreted as a year or as a phase of life such as youth, middle age and old age. In each period t, $t = 1, 2, ..., T$, the individual has a non-negative level of consumption x_t. In each period, the individual's poverty status is determined by comparing his consumption level with the poverty line $0 < z < \infty$ which is exogenously given and remains constant throughout the T periods. The individual is poor in period t if and only if his consumption level x_t is strictly less than z. Denote $\mathbf{x} = (x_1, x_2, ..., x_T)'$ the profile of the individual's lifetime consumptions, his lifetime poverty level is a function $P(\mathbf{x}; z)$ which maps each consumption profile $\mathbf{x}$ into $[0, \infty)$. The average consumption of the individual over the T periods is $\bar{x}$. For each consumption variable, we also define its censored consumption as $\tilde{x}_t = \min\{x_t, z\}$.

At the beginning of each period, the individual collects income and allocates it to the consumption of that period and the periods to come; at the end of each period, the individual compares his consumption level x_t with the poverty line z. If $x_t < z$, he has poverty deprivation which is measured by $p(x_t; z)$: $p(x_t; z) > 0$ if $x_t < z$; otherwise he lives out of poverty: $p(x_t; z) = 0$ if $x_t \geq z$. We refer to $p(x_t; z)$ as the "snapshot poverty" of period t. The measurement of poverty deprivation has been well studied in the literature. In general, we assume that $p(x_t; z)$ is continuous, $\frac{\partial p(x_t;z)}{\partial x_t} < 0$ (denoted as $p' < 0$ hereafter) and also in some cases we also impose $\frac{\partial^2 p(x_t;z)}{\partial x_t^2} > 0$ (denoted $p'' > 0$ hereafter) for all $x_t \in [0, z)$.[8] That is, poverty deprivation decreases as consumption increases ($p' < 0$); it is often presumed to decrease, however, at a slower pace as consumption increases ($p'' > 0$) – in part this is to reflect the poverty aversion consideration (Zheng, 2000). Although higher-order conditions can be entertained, in this paper we limit our investigation to only the first two orders.[9] Accordingly,

[8] Other axioms in the literature such as the increasing poverty line axiom – $p(x_t; z)$ is increasing in z – and the unit-consistency axiom (Zheng, 2007) – which implies that $p(x_t; z)$ is a homogeneous function of x_t and z - may also be considered to further specify the functional form of $p(x_t; z)$.

[9] Our choice also reflects the fact that poverty orderings at third and above orders may collapse to second-order if the poverty line is uncertain and expands over a large interval (Zheng, 1999). With uncertain poverty lines, the consideration of higher-than-second-order conditions may introduce little additional insights on poverty orderings. Shorrocks and Foster (1987) and Davies and Hoy (1994, 1995) explore the implications of third-order stochastic dominance in making inequality comparisons.

we also assume the lifetime poverty measure $P(\mathbf{x}; z)$ to exhibit similar properties as $p(x; z)$, *i.e.*, $\frac{\partial P(\mathbf{x};z)}{\partial x_t} \leq 0$ and possibly $\frac{\partial^2 P(\mathbf{x};z)}{\partial x_t^2} \geq 0$ for all $x_t \in [0, \infty)$. Note that here we require only weak inequalities and the range for x_t is over $[0, \infty)$ rather than $[0, z)$. This is because, unlike in the measurement of snapshot poverty where any change in x_t above z has no effect on the poverty level, here such a change may or may not affect the lifetime poverty – as we will see below.

In Hoy and Zheng (2008) it is shown that the axioms we impose imply a specific functional form of $P(\mathbf{x}; z)$; namely one that is additive in the snapshot poverty experiences from each period, $p(x_t; z)$, as well as the amount of poverty from a retrospective view, measured by $p(\overline{x}; z)$, where $\overline{x}$ is average lifetime income. The resulting functional form for $P(\mathbf{x}; z)$ turns out to be:

$$P(\mathbf{x}; z) = \beta(T) \left\{ \sum_{t=1}^{T} \alpha(t, T) p(x_t; z) \right\} + [1 - \beta(T)] p(\bar{x}; z). \tag{1}$$

The weights $\alpha(t, T)$ applied to each poverty spell have mathematical properties that reflect the axioms relating to the way in which we account for our emphasis on chronic and early aspects of the pattern of lifetime poverty experiences. The early poverty axiom implies that $\alpha(t, T)$ is falling in t while the chronic poverty axiom implies that $\alpha(t, T)$ is concave in t. The weight $\beta(T) \in [0, 1]$ reflects how much emphasis is placed on these individual spells of poverty incurred by an individual while the residual weight, $[1 - \beta(T)]$, is placed on the overall lifetime (permanent) consumption $\bar{x}$ of the individual, where $p(\bar{x}; z) > 0$ if $\bar{x} < z$ and $p(\bar{x}; z) = 0$ if $\bar{x} \geq z$. This implied structure allows for a straightforward approach to generating lifetime poverty indices. One can simply choose any of the standard static poverty measures for $p(x_t; z)$ and $p(\bar{x}; z)$, such as the classic FGT measures, which embody important properties of static poverty measurement. This structure also allows for the creation of dominance conditions to test for potential unambiguous poverty companions between pairs of lifetime income profiles. We use empirical examples to demonstrate the usefulness of both of these results in Section IV. Here we elaborate on the axioms that give rise to this structure for lifetime poverty measurement and then provide implementation results.

In an ideal empirical application one can imagine the subscript t representing the age in years (or set of years such as early childhood, middle childhood, teenage years, young adulthood, etc.) and using data to follow a cohort of individuals born (approximately) at the same time and living the same number of years T.[10] Our approach, however, could also be used to study some subset of individuals' lifetimes, say the young childhood years up to age 12 ($t = 1, 2, ..., 12$) or the teenage/young adult years ($t = 1$ to 11, where $t = 1$ refers to age 13 and $t = 11$ refers to age 23). However, real world data provides challenges to empirical applications of lifetime

[10] One important issue that we do not address is the complication associated with individuals living different lengthed lives. For important work on this topic, see Kanbur and Mukherjee (2007).

poverty measurement. Panel data often covers only five to ten years of lifetimes and it isn't clear how accurate a picture of lifetime experiences one obtains from such a limited stretch of time. Also, most panel data sets don't have sufficient numbers to allow for tracking and comparison of specific cohorts. We are able to extract 26 consecutive years of income experience using PSID data and follow a mixture of cohorts based on the entire sample. We also identify subgroups based on ethnicity and region. For our regional analysis, we also split the sample into two equal length periods of 13 years to address the question of how representative a limited stretch of time may be as an approximation to lifetime experience. We address these issues, and others, later in the paper.

In equation (1) it is clear that any poverty experienced in a given period t contributes to lifetime poverty through the "snapshot poverty" index $p(x_t; z)$. But the individual's picture of lifetime living is not entirely dictated by the poverty spells that he has experienced in the various periods. He might view that "even though I experienced some poverty spells at various points in time, life as a whole has been very good to me since I had an affluent living later in my life." This means that the individual registers all poverty deprivations but we also allow periods of rich living in the rest of his lifetime to offset some of the bad experiences or memories. In other words, consider a person who has experienced some periods of poverty in his lifetime while in other periods had consumption levels "just sufficient" to be considered non-poor. Another individual who experienced the same number and pattern of spells of poverty, but who also enjoyed periods of relatively high consumption when not in poverty, would from a whole lifetime or retrospective manner presumably feel he has experienced less poverty than the first person. Should we treat such individuals differently in measuring their lifetime poverty? A natural and manageable way to consider poverty for the entire lifetime in a retrospective manner is to model consumption over the lifetime as if it were completely smoothed out. The poverty level is then computed by comparing lifetime (permanent) consumption with the lifetime (permanent) poverty line. Since we assume the poverty line remains the same throughout all T periods, all consumption levels will be in real terms and will not be discounted either (or consider they are already discounted). Therefore, we proxy permanent consumption with a simple average consumption over the lifetime.[11] Noting that $p(\bar{x}; z)$ is the poverty of lifetime average consumption, our second axiom summarizes the afore-discussed influence of lifetime smoothed consumption on lifetime poverty. We refer to this aspect as a retrospective property or view of poverty over the individual's lifetime taken as whole.

With the retrospective property, the standard focus axiom in the literature (*i.e.,* that any change made in any above-the-poverty-line consumption has no affect on the poverty level) needs to be modified. Since now a change in a consumption may affect

[11] If a more suitable representation of permanent consumption is deemed necessary, we can replace $\bar{x}$ with such a permanent-consumption function $\mu(x_1, x_2, ...x_T)$ in the rest of the paper. All results involving $\bar{x}$ will also hold with some appropriate modifications.

lifetime poverty through two routes: through the snapshot poverty in each period and through the poverty of average-lifetime-consumption. It follows that the new focus axiom should be reformulated as: a change in a period's consumption has no effect on lifetime poverty if and only if both the consumption level in that period and the average-consumption level of the entire lifetime are above the poverty line. This is to say: (i) if $x_t < z$ then $\frac{\partial P(\mathbf{x};z)}{\partial x_t} < 0$, (ii) if $\bar{x} < z$ then $\frac{\partial P(\mathbf{x};z)}{\partial x_t} < 0, \forall t$ (since raising any x_t will raise $\bar{x}$ and hence lower $P(\mathbf{x};z)$), while (iii) if $x_t \geq z$, $\forall t$, then $\frac{\partial P(\mathbf{x};z)}{\partial x_t} = 0$. These results are all clear in equation (1).

But this leaves us with the question of how the combination of snapshot poverty spells may interact with each other to jointly determine lifetime poverty. In order to maintain the simple additive form of $P(\mathbf{x}, z)$ in equation (1), we assume that the impact of each period's snapshot poverty $p(x_t; z)$ on lifetime poverty $P(\mathbf{x}; z)$ is independent of the level of any other period's snapshot poverty $p(x_s; z)$ for $s \neq t$ and of the poverty level of the average lifetime consumption $p(\bar{x}; z)$.[12] This property is akin to the decomposability axiom used in the snapshot poverty measurement; it enables researchers to compute the contribution of each year's consumption and the smoothed consumption to total poverty and allows policy-makers to identify the specific factors that are responsible for changes in the overall poverty value. This requirement amounts to saying that there is no interaction among the poverty levels of the various periods and that of the average consumption. More formally, this property can be stated as $\frac{\partial^2 P(\mathbf{x};z)}{\partial p(x_t;z)\partial p(x_s;z)} = 0$ for all $s \neq t$ and $\frac{\partial^2 P(\mathbf{x};z)}{\partial p(x_t;z)\partial p(\bar{x};z)} = 0$ for all $t = 1, 2, ..., T$.

The property of independence does not rule out the possibility of interesting temporal relationships between spells of poverty. In fact, it is the salience of the temporal pattern of snapshot poverty experiences that we believe lies at the heart of the difference between measuring poverty at a given point in time and an individual's lifetime poverty. In our approach this is established through the properties of the weighting function $\alpha(t, T)$ as will become evident below. However, before addressing those properties of the poverty measure $P(x; z)$, we note two technical or normalization assumptions that we make. First, we assume that $P(\mathbf{x}; z) = p(x; z)$ if $x = x_1 = x_2 = ... = x_T$. Secondly, without loss of generality, we choose the weights $\alpha(t, T)$ and $\beta(T)$ such that $0 < \alpha(t, T) < 1$, $0 < \beta(T) < 1$ and $\sum_{t=1}^{T} \alpha(t, T) = 1$. The result is that we can use a straightforward additive functional form to provide a useful characterization of how poverty over the lifetime can be conceptualized as a weighted average of snapshot poverty levels and a level of poverty associated with average lifetime consumption.

Consider the balance between that part of lifetime poverty that depends on the weighted sum of snapshot poverty levels, $\sum_{t=1}^{T} \alpha(t, T)p(x_t; z)$, and that part that

[12] Note, however, that we later introduce an axiom (chronic poverty axiom) that allows for the "closeness" in time of any two poverty spells to influence the lifetime poverty measure. So in terms of the timing of poverty spells there will be a relationship between any pair $x_s < z$ and $x_t < z$ to lifetime poverty.

depends on permanent consumption level, $p(\bar{x}; z)$. The coefficient $\beta(T)$ plays the role of balancing between these two aspects (i.e., the weighted average "snapshot poverty" and the average-lifetime-consumption or retrospective poverty). A larger value of $\beta(T)$ means that the individual is concerned more about the poverty incidences he has experienced on a period by period basis and less about when his life as a whole is evaluated. In the limiting case of $\beta(T) = 1$, the individual gives no consideration to the average or smoothed lifetime consumption; the individual's lifetime poverty in this case is determined exclusively by the poverty deprivations he has had in his life no matter how affluent he may be when life as a whole is judged. On the other hand, $\beta(T) = 0$ means that the individual cares about only the lifetime aggregate or average consumption and the poverty deprivation in any period matters not at all in the evaluation of lifetime poverty. In this sense, we may label $\beta(T)$ as a "memory parameter" – $\beta(T) = 1$ is the polar case of "perfect memory" and $\beta(T) = 0$ is the other polar case of "no memory," respectively.[13] To compute the individual's lifetime poverty index using (1) – which will be used in the rest of the paper, the memory parameter must be specified. If all possible values of $\beta(T)$ are considered, then we have

Result 1: For two lifetime consumption profiles $\mathbf{x}$ and $\mathbf{y}$, $P(\mathbf{x}; z) \geq P(\mathbf{y}; z)$ for any poverty measure of the form (1) and for all $\beta(T) \in (0, 1)$ if and only if

$$\sum_{t=1}^{T} \alpha(t, T) p(x_t; z) \geq \sum_{t=1}^{T} \alpha(t, T) p(y_t; z) \quad \text{and} \quad p(\bar{x}; z) \geq p(\bar{y}; z). \tag{2}$$

This result can be regarded as our first dominance condition. The result is reasonable and intuitive. To characterize further the lifetime poverty index and establish additional dominance conditions, we need to take a closer look at the weighting function $\alpha(t, T)$. Suppose the researcher wishes to remain entirely agnostic about how to place relative importance of poverty spells experienced at different points in time in an individual's life. Then this will mean that to say lifetime poverty for one profile of consumption values $\mathbf{x}$ is unambiguously higher than for some other profile $\mathbf{y}$ it must follow that the conclusion (equation (1)) must hold for any set of weights $\alpha(t, T)$. For this most general perspective, then, we have:

Result 2. For two lifetime consumption profiles $\mathbf{x}$ and $\mathbf{y}$, $P(\mathbf{x}; z) > P(\mathbf{y}; z)$ for all $\beta(T) \in [0, 1]$ and all possible values of $\alpha(t, T)$s if and only if

$$\tilde{x}_t \leq \tilde{y}_t \text{ and } \min\{\bar{x}, z\} \leq \min\{\bar{y}, z\} \tag{3}$$

for $t = 1, 2, ..., T$ and the strict inequality holds at least once, where $\tilde{x}_t = \min\{x_t, z\}$.

[13] By "perfect memory" we mean that the individual (or ethical observer) takes into account – and only takes into account – all the details involved with the pattern of lifetime consumptions while by "no memory" only the average lifetime consumption is used as a sufficient statistic to evaluate lifetime poverty.

Condition (3) states that for profile $\mathbf{x}$ to have more lifetime poverty than $\mathbf{y}$ unambiguously (*i.e.*, for all possible weights), it must be the case that the consumption in each period of $\mathbf{x}$, if it is below the poverty line, is no greater than that in $\mathbf{y}$ (i.e., vector dominance is required). Moreover, it has also to be the case that the average lifetime consumption, if it is below the poverty line, is no greater in $\mathbf{x}$ than that in $\mathbf{y}$.

By contrast, suppose one considers that all periods of poverty experiences are equally important; *i.e.*, $\alpha(s,T) = \alpha(t,T)$ for all s and t. This corresponds to treating lifetime poverty as the simple arithmetic average of all spells of poverty. This leads to a rather simplistic and strong view of lifetime poverty in that it ignores any possible importance of the pattern of poverty spells such as a concern with early or chronic poverty experiences. The following result can be easily verified using standard results from the literature of majorization.

Result 3. For two lifetime consumption profiles $\mathbf{x}$ and $\mathbf{y}$, if the $\alpha(t,T)$s are the same, then $P(\mathbf{x};z) \geq P(\mathbf{y};z)$ for all $\beta(T) \in [0,1]$ if and only if

(a) $\sum_{t=1}^{T} p(x_t;z) \geq \sum_{t=1}^{T} p(y_t;z)$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$ for a given deprivation function p; or

(b) also for all deprivation functions p such that $p^{'} < 0$, vector $(\tilde{x}_1, ..., \tilde{x}_T)$ is rank dominated by vector $(\tilde{y}_1, ..., \tilde{y}_T)$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$; or

(c) also for all deprivation functions p such that $p^{'} < 0$ and $p^{''} > 0$, vector $(\tilde{x}_1, ..., \tilde{x}_T)$ is generalized Lorenz dominated by vector $(\tilde{y}_1, ..., \tilde{y}_T)$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$.

There is an advantage in empirical implementations of this strict and simplistic view as it ties down the set of weight function that one can use. That is, in comparison to Result 2, the implications of requiring weights $\alpha(t,T)$ to be equal for all t rather than allowing for any set of weights leads to a larger fraction of cases that can be ranked as demonstrated in the section presenting our empirical results.

One can think of the assumptions made in Results 2 and 3 as extreme cases of weakness and strength regarding permissible patterns of the weight functions $\alpha(t,T)$ for comparing the importance of the timing of poverty in terms of its impact on lifetime poverty. There is clearly a dramatic loss of power in making pairwise rankings by moving from the strongest to weakest set of restrictions. In what follows we consider intermediate positions which we argue reflect received concerns about relative timing of poverty in an individual's life. As noted in the introduction, it seems that there is a consensus that early stages of life such as childhood matter more than later-life periods in shaping the individual's lifetime well-being/poverty. Translated in terms of the weighting function $\alpha(t,T)$, this notion can be formally stated as an axiom.

The early-poverty axiom. The weighting function $\alpha(t,T)$ is nonincreasing in time t.

Note that what the early-poverty axiom states is different from the discounting concern that is usually imposed on aggregation over time, although the discounting weight function $\alpha(t,T) = \rho^t$ with $0 < \rho < 1$ does happen to satisfy the axiom. Here we do not discount over time per se; our concern is purely about the size of impact of each period's poverty deprivation on the aggregate lifetime poverty.[14] With this additional requirement, we have

Result 4. For two lifetime consumption profiles $\mathbf{x}$ and $\mathbf{y}$, $P(\mathbf{x}; z) \geq P(\mathbf{y}; z)$ for all $\beta(T) \in [0,1]$ and all possible values of $\alpha(t,T)$s satisfying the early-poverty axiom if and only if

(a) $\sum_{t=1}^{l} p(x_t; z) \geq \sum_{t=1}^{l} p(y_t; z)$ for $l = 1, 2, ..., T$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$ for a given deprivation function p; or

(b) also for all deprivation functions p such that $p^{'} < 0$, vector $(\tilde{x}_1, ..., \tilde{x}_l)$ is rank dominated by vector $(\tilde{y}_1, ..., \tilde{y}_l)$ for $l = 1, 2, ..., T$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$; or

(c) also for all deprivation functions p such that $p^{'} < 0$ and $p^{''} > 0$, vector $(\tilde{x}_1, ..., \tilde{x}_l)$ is generalized Lorenz dominated by vector $(\tilde{y}_1, ..., \tilde{y}_l)$ for $l = 1, 2, ..., T$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$.

A necessary condition for parts (b) and (c) in the above proposition is

$$\sum_{t=1}^{l} \tilde{x}_t \leq \sum_{t=1}^{l} \tilde{y}_t \text{ for } l = 1, 2, ..., T \tag{4}$$

which is the concentration curve dominance between (censored) lifetime consumption profiles of $\mathbf{x}$ and $\mathbf{y}$. Concentration curve dominance is constructed similarly to the generalized Lorenz curve dominance with the exception that the values of $\{x_t\}$ and $\{y_t\}$ are not sorted before the construction. The concentration curve dominance condition can be handily used to screen out consumption profiles in (lifetime) poverty orderings.

Finally, we introduce the axiom that characterizes the chronic aspect of lifetime poverty. Suppose an individual has to endure two poverty spells within certain periods of time. Chronic poverty is generally defined as living in poverty continuously for an extended period of time. In terms of our general setup, experiencing two consecutive periods of poverty would have a greater (negative) impact on an individual's life than if these two spells were separated by a period in which poverty was not experienced. We generalize this notion by suggesting that any two spells of poverty, say even if separated by one or more periods of non-poverty, would have less impact if separated by more periods of non-poverty. This idea is formalized below.

The chronic-poverty axiom. For a given consumption $a < z$ that occurs in two spells, the closer the two spells together, the greater is the resulting lifetime poverty, *i.e.*,

$$\alpha(s,T)p(a;z) + \alpha(u,T)p(a;z) \geq \alpha(r,T)p(a;z) + \alpha(v,T)p(a;z) \tag{5}$$

for all $1 \leq r < s \leq u < v \leq T$ such that $s - r = v - u$.

[14] In fact, as we will see later, the usual discount-weighting scheme is ruled out when the further axiom reflecting a concern with chronic poverty is introduced.

Notice that the chronic-poverty axiom specifies that two equivalent spells of poverty occurring in periods (s,u) have greater impact than if the same spells had occurred in periods spread out symmetrically by $k>0$ periods in both directions; *i.e.*, in periods $(r,v)=(s-k,u+k)$. So, for example, spreading out equivalent spells of poverty in periods $(s,u)=(17,19)$ to periods $(r,v)=(16,20)$ implies a reduction in lifetime poverty due to "added relief" from the chronic poverty that arises from only one period of prosperity separating the poverty spells compared to three periods of separation. Note that there appears to be an implicit conflict with the early poverty axiom in that the movement of a spell of poverty from period 17 to period 16 implies an increase in concern from earlier poverty being experienced. However, this is countered by the pushing apart of the two spells of poverty as the second spell is delayed from period 19 to 20. Without the implicit symmetry requirement, pushing the time period of poverty experienced in period 17 to the first period of life (e.g., choosing $(r,v)=(1,20)$) would imply that the earlier poverty effect from experiencing the spell in period $s=17$ to occur 16 periods earlier in life $(r=1)$ is countered by the pushing of poverty from period $u=19$ just one period later $(v=20)$.[15]

The chronic-poverty axiom (*including* the requirement of $s-r=v-u$) implies that the weighting function $\alpha(t,T)$ is concave in t.[16] To see this simply divide equation (5) by $p(a;z)$ and rearrange to obtain

$$\alpha(s-k,T)-\alpha(s,T)\leq\alpha(u,T)-\alpha(u+k,T),\ u\geq s \tag{6}$$

which is satisfied if and only if $\alpha(t,T)$ is concave in t. This implies:

$$0<\alpha(1,T)-\alpha(2,T)\leq...\leq\alpha(T-1,T)-\alpha(T,T)\leq\alpha(T,T) \tag{7}$$

Examples of satisfactory poverty indices are $\alpha(t,T)=(1-\frac{t}{T+1})^{\gamma}$ with $0<\gamma<1$ and $\alpha(t,T)=c_0-c_1t-c_2(t-1)^2$, $c_0,c_1,c_2\geq 0$ (with $c_0=c_1T+c_2(T-1)^2+1$ to ensure $\alpha_t>0$ for all t). The discount-rate coefficient $\alpha(t,T)=\rho^t$ with $0<\rho<1$ does not satisfy the chronic poverty axiom since it is convex in t.

Result 5. For two lifetime consumption profiles $\mathbf{x}$ and $\mathbf{y}$, $P(\mathbf{x};z)\geq P(\mathbf{y};z)$ for all $\beta(T)\in[0,1]$ and all possible values of $\alpha(t,T)$s satisfying the early-poverty axiom and the chronic-poverty axiom if and only if

(a) $l\sum_{t=1}^{T-l+1}p(x_l,z)+\sum_{m=1}^{l-1}(l-m)p(x_{T-l+m+1},z)\geq l\sum_{t=1}^{T-l+1}p(y_l,z)+\sum_{m=1}^{l-1}(l-m)p(y_{T-l+m+1},z)$ for $l=1,2,...,T$ and $\min\{\bar{x},z\}\leq\min\{\bar{y},z\}$ for a given deprivation function p; or

(b) also for all deprivation functions p such that $p^{'}<0$, vector

$$\tilde{\mathbf{x}}_l=(\tilde{x}_1,..,\tilde{x}_1,...,\tilde{x}_{T-l+1},..,\tilde{x}_{T-l+1},\tilde{x}_{T-l},..,\tilde{x}_{T-l},\tilde{x}_{T-l},..,\tilde{x}_{T-l},...,\tilde{x}_T)$$

[15] This issue is considered more formally in Hoy and Zheng (2008) where it is shown that absence of a symmetry requirment or some other similar restriction leads to the result that the early poverty axiom becomes impotent.

[16] Recall that the early poverty axiom implies that the weights $\alpha(t,T)$ are non-increasing in t.

is rank dominated by a similarly defined vector $\tilde{\mathbf{y}}_l$ for $l = 1, 2, ..., T$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$; or

(c) also for all deprivation functions p such that $p^{'} < 0$ and $p^{''} > 0$, vector $\tilde{\mathbf{x}}_l$ is generalized Lorenz dominated by vector $\tilde{\mathbf{y}}_l$ for $l = 1, 2, ..., T$ and $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$.

Note that a dominance between $\tilde{\mathbf{x}}_l$ and $\tilde{\mathbf{y}}_l$ does not imply nor is implied by the dominance between $(\tilde{x}_1, .., \tilde{x}_T)$ and $(\tilde{y}_1, .., \tilde{y}_T)$ because none of the vectors is ordered (if they are increasingly ordered then the two types of dominance would be equivalent). To see this, consider $\mathbf{x} = (3, 1)$ and $\mathbf{y} = (2, 4)$ with $z = 5$. Clearly $\mathbf{x}$ is rank ordered by $\mathbf{y}$ but $\tilde{\mathbf{x}}_2 = (3, 3, 1)$ is not rank ordered by $\tilde{\mathbf{y}}_2 = (2, 2, 4)$. The method for implementing results 5 (b) and (c) are explained in the appendix.

III Elaboration of Early and Chronic Poverty Concerns

In this section we elaborate through use of examples of alternative weighting functions, $\alpha(t, T)$, the way in which one can incorporate a concern with chronic and early poverty in a lifetime poverty measure. As noted in the discussion above, one needs to be careful in considering how the two axioms reflecting chronic and early poverty concerns interact. In order to bring out these issues as clearly as possible, throughout we assume zero weight placed on poverty related to average lifetime consumption (i.e., $p(\overline{x}; z)$) and so $[1 - B(T)] = 0$. This allows us to focus only on that part of the lifetime poverty measure that aggregates the snapshot poverty experiences. Incorporating the term involving $p(\overline{x}; z))$ is straightforward.

Our chronic poverty axiom implies that the weighting function $\alpha(t, T)$ must be concave in t. Moreover, since any pair of weights generated by two different weighting functions are normalized by dividing each α_t by the sum of weights in order to have the normalized weights sum to unity, any measure of concavity of $\alpha(t, T)$ should be preserved by any positive linear transformation. Thus, it is natural to compare the properties of this function to those of the von Neumann-Morgenstern elementary utility index used in risk theory; i.e., $u(x)$, and its standard measure of curvature or degree of concavity $R_A(x) = -\frac{u''(x)}{u'(x)}$ (i.e., the Arrow-Pratt measure of absolute risk aversion). In the case of our lifetime poverty measure, however, income appears in the snapshot poverty measure $(p(x_t; z))$ while it is a spread in the time dimension, something we will call a "temporal spread", that is the relevant exercise in our case rather than a mean-preserving spread of incomes as used in analyzing risk theory. Thus, there isn't a direct analogy between the cost of poverty and the cost of risk (risk premium). Nonetheless, a similar measure of curvature or concavity of $\alpha(t, T)$ is apporpriate. Thus, we use $C_A(t) = \frac{\alpha''(t,T)}{\alpha'(t,T)}$ as a measure of concavity or curvature of the weighting function.[17]

[17] Since $\alpha(t, T)$ has both negative first and second derivatives, we omit the negative sign in our measure of curvature. Also, in poverty measurement concavity of $\alpha(t, T)$ means a temporal spread reduces lifetime poverty, which is a "good thing".

Due to the requirement that weights be normalized in order to sum to unity, it is not possible to move from one weighting function to another which has a greater or lesser degree of curvature without also having an effect on the extent to which the lifetime poverty measure is sensitive to early poverty. This is easily seen by considering the family of weighting functions $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$ with $0 < \gamma < 1$ and associated index of concavity $C_A(t) = \frac{\alpha''(t,T)}{\alpha'(t,T)} = \frac{1-\gamma}{T+1}\left(1 - \frac{t}{T+1}\right)^{-1}$. In the case of $\gamma = 0$ the normalized weights are equal ($\alpha_t = 1/T$) and so the lifetime poverty measure would be completely insensitive to the temporal pattern of snapshot poverty experiences (i.e., to either early or chronic poverty concerns). In the case of $\gamma = 1$, the weighting function will be linear and decreasing and so the associated lifetime poverty measure would be sensitive to early poverty but not chronic poverty. As one reduces the value of γ from 1 to 0, the degree of curvature of the weights increases but there is also a change in the sensitivity of the implied lifetime poverty measure to early poverty. As seen in the figure below, a lower value of γ implies less concern with early poverty (i.e., with lower γ the normalized weights become "less steep in t" along with having a higher degree of curvature). As an example, the funtions $\alpha 1(t)$, $\alpha 2(t)$ in Figure 1 below are generated from $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$ for the weights $\gamma = 0.2, 0.5$ respectively, $T = 4$.

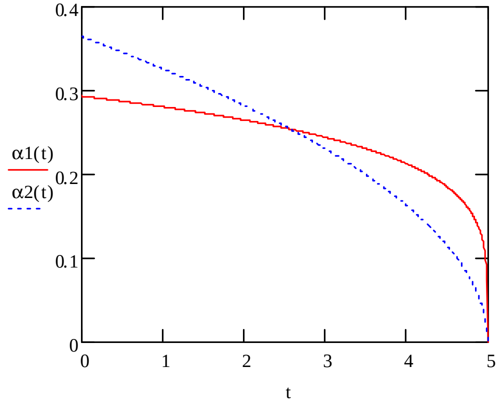

Figure 1: Weights for $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$
with $\gamma = 0.2$ (i.e., $\alpha1(t)$) and $\gamma = 0.5$ (i.e., $\alpha2(t)$)

If one used only linear, decreasing weights that sum to unity, one could simply use the absolute value of the first derivative to measure the sensitivity of the implied lifetime poverty measure to early poverty concerns. However, once one accepts the possibility of weights that are nonlinear in time, as one must to reflect sensitivity to chronic poverty, one cannot create a simple global measure reflecting the degree of sensitivity to early poverty. However, a crude reflection of concern to early poverty over the entire range of $t = 1, 2, ..., T$ is the overall difference in weights from beginning to end; i.e., $\alpha(1,T) - \alpha(T,T)$. We will call this the (overall) *drop in weights* and use it as our measure of sensitivity to early poverty conditions. It is clear that for the above example, this value is less for the set of weights that has a higher degree of curvature. However, one can also choose a family of weights that allow for a movement in the direction of a lifetime poverty measure that becomes more sensitive to early poverty through parameter changes that imply either a higher or a lower degree of curvature. A simple example of such a family of weighting functions is the quadratic function mentioned earlier (i.e., $\alpha(t,T) = c_0 - c_1 t - c_2(t-1)^2$, $c_0, c_1, c_2 \geq 0$, and $c_0 = c_1 T + c_2(T-1)^2 + 1$ to ensure $\alpha_t > 0$ for all t).

The measure of curvature for this weighting function is $C_A(t) = \frac{2c_2}{c_1 + 2c_2(t-1)}$. This function is strictly concave for $c_2 > 0$ and the measure of curvature is increasing in c_2 and decreasing in c_1. For the non-normalized weights, $\alpha(1,T) = c_0 - c_1$ and $\alpha(T,T) = 1$ and so the difference $\alpha(1,T) - \alpha(T,T) = c_1(T-1) + c_2(T-1)$. Thus, the overall drop in the set of non-normalized weights is increasing in both c_1 and c_2. The anchor for any set of non-normalized weights is $\alpha(T,T) = 1$. Thus, increasing either c_1 or c_2 will also increase the drop in the set of normalized weights. Increasing c_1 when $c_2 = 0$ leads to a greater sensitivity to early poverty concerns while retaining linearity of the weighting function and hence no sensitivity to chronic poverty. Suppose instead that one begins with an initial value of $c_2 > 0$. In this case one can create a set of weights with more sensitivity to early poverty by increasing the parameter c_1 (c_2) and in doing so one obtains a set of weights with a lower (higher) degree of curvature.

In developing explanatory examples and empirical illustrations it is useful to have a method of reporting lifetime poverty that allows for intuitive comparisons. In this regard we follow Duclos, et al. (2010) in defining a money metric cost of poverty as the "equally-distributed equivalent" (EDE) poverty gap; that is, the level of poverty gap if distributed equally to all persons in all periods of life that would produce the same measure of poverty as for the actual lifetime profiles of poverty experienced across the population. Naturally the EDE poverty gap will depend on both the properties of the snapshot poverty index, $p(x_t, z)$ and the weights $a(t, T)$. We investigate these effects by first considering the weighting function $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$ with $0 < \gamma < 1$ and its associated index of concavity $C_A(t) = \frac{\alpha''(t,T)}{\alpha'(t,T)} = \frac{1-\gamma}{T+1}\left(1 - \frac{t}{T+1}\right)^{-1}$ which is decreasing in γ. Thus, in comparing any pair of such functions, the one with the smaller value of γ has a higher degree of concavity.[18] Besides changing the way in which chronic poverty contributes to an overall poverty measure, we also show how different values of γ imply different costs of poverty arising from earlier versus later poverty experiences. As shown above, in general it is not possible to perfectly isolate these different implications.

Consider the weight function values for $\gamma = 0.5$ and 0.2, with the latter having the higher degree of concavity as measured by $C_A(t)$ and also less sensitivity in early poverty as measured by the drop $\alpha(1,T) - \alpha(T,T)$. As always the weights are normalized by dividing by the sum $\sum_{t=1}^{T} \alpha(t,T)$ to create a sum of 1 for the final weights. We use the FGT poverty index $p(x_t; z) = (1 - x_t/z)^{\varepsilon}$, with $\varepsilon = 1$ unless otherwise stated. The poverty line is always $z = 5$.

Consider the income vector $\mathbf{x} = (5, 1, 1, 5)$. Upon spreading the two poverty spells by $k = 1$ period in each direction we obtain $\mathbf{y} = (1, 5, 5, 1)$. Using α_t for $\alpha(t,T)$, we obtain the following sets of weights for $\gamma = 0.5$, 0.2:

$$\gamma = 0.5: \;\; \alpha_1 = 0.325, \;\; \alpha_2 = 0.282, \;\; \alpha_3 = 0.230, \;\; \alpha_4 = 0.163 \tag{8}$$

$$\gamma = 0.2: \;\; \widetilde{\alpha}_1 = 0.280, \;\; \widetilde{\alpha}_2 = 0.264, \;\; \widetilde{\alpha}_3 = 0.244, \;\; \widetilde{\alpha}_4 = 0.212 \tag{9}$$

Both sets of weights come from a "concave function" and so spreading out the two periods of poverty in a manner described by the chronic poverty axiom will lead to an increase in poverty. This can be illustrated by the results that, for $\gamma = 0.5$, $P(\mathbf{x}, z) = 0.40952 > P(\mathbf{y}, z) = 0.39048$ and for $\gamma = 0.2$, $P(\mathbf{x}, z) = 0.40636 > P(\mathbf{y}, z) = 0.39364$.

We now explain how the "equally-distributed equivalent" (EDE) poverty gap can be interpreted as a cost of lifetime poverty and how to associate various aspects of

[18] It is clear that the weight function with the smaller value of γ can be written as a strictly concave function of the other.

the temporal pattern of spells on the "overall" cost of poverty.[19] Let $\mathbf{g_x}$ represent the vector of poverty gaps associated with the income vector (profile) $\mathbf{x}$. For the above example $\mathbf{g_x} = (0, 4, 4, 0)$ and $\mathbf{g_y} = (4, 0, 0, 4)$. Let $\overline{\mathbf{g}}_\mathbf{x}$ be the vector formed by inserting the average poverty gap in each period, and so $\overline{\mathbf{g}}_\mathbf{x} = \overline{\mathbf{g}}_\mathbf{y} = (2, 2, 2, 2)$. We use the same symbol, but not in bold (i.e., $\overline{g}_\mathbf{x}$) to refer to the average value of this poverty gap (i.e., $\overline{g}_\mathbf{x} = 2$ for the above example). The EDE is the *equivalent constant poverty gap* for a consumption profile is that (single valued) poverty gap which, if incurred in each period of life, would generate the same poverty level as that for some existing lifetime profile of poverty spells. Let this value be represented by $\widehat{g}_\mathbf{x}$. Clearly, this value will depend both on the properties of the snapshot poverty function $p(x_t; z)$ and the choice of weighting function. In the context of our above choices, we write this as a function of the relevant parameters of these functions; that is, $\widehat{g}_\mathbf{x}(\gamma, \varepsilon)$. The extent to which this value differs from the average poverty gap reflects several aspects of the cost of poverty, including the intensity of poverty across poverty experiences (as measured by $p(x_t, z)$) and the temporal pattern (i.e., early and chronic poverty properties).

To isolate that aspect due to the temporal pattern of spells, we select $\varepsilon = 1$ and begin by selecting the equal weights case ($\alpha_t = \frac{1}{T}$, $\forall t = 1, 2, ..., T$) which eliminates chronic poverty and early poverty considerations. We then introduce a weighting scheme with sensitivity to timing of poverty spells. In terms of our weighting function above this would correspond to initial selection of $\gamma = 0$ to reflect equal weights. For the poverty gap measure, i.e., the FGT measure with $\varepsilon = 1$, in conjunction with $\gamma = 0$, we obtain $\widehat{g}_\mathbf{x}(0, 1) = \widehat{g}_\mathbf{y}(0, 1) = \overline{g}_\mathbf{x} = \overline{g}_\mathbf{y} = 2$. Taking into account the temporal aspects through choice of $\gamma = 0.5$ we obtain $\widehat{g}_\mathbf{x}(0.5, 1) = 2.048$ and $\widehat{g}_\mathbf{y}(0.5, 1) = 1.952$. Compared to equal weighting across time periods, the cost of poverty in profile $\mathbf{x}$ rises while that for $\mathbf{y}$ falls. This highlights an important point. In comparison to equal weighting, introducing a concern for time sensitivity can of course lead to a reduction in the measured value of lifetime poverty. Thus, accounting for the temporal aspect of poverty can lead either to an increase or decrease in poverty compared to an approach that weights periods equally. In conjunction with the use of $\varepsilon = 1$ for the FGT measure, which is neutral in regards to intensity of poverty, note that equal weighting across time periods implies vectors of poverty gaps $(0, 4, 4, 0)$, $(4, 0, 0, 4)$, and $(2, 2, 2, 2)$ all generate the same measure of lifetime poverty. But once we introduce a concern with chronic poverty - and early poverty - by using a weighting function that is both decreasing and concave in t - the cost of poverty rises above 2 for the distribution with more chronic poverty and falls below 2 for the distribution with poverty spells (more) spread out over time. In one sense profile $\mathbf{y} = (1, 5, 5, 1)$ may seem worse than $(2, 2, 2, 2)$ in that it has an earlier spell of poverty which is deeper, but it also has less poverty in the middle periods of life - with some poverty "pushed"

[19] We follow quite closely the approach developed in Duclos, et al. (2010), which in turn ins based on the idea of the cost of inequality measurement in Kolm (1969) and Atkinsons (1970). However, we take a very different approach in how we take into account chronic poverty measurement.

to a later period. Moreover, the two spells of poverty in $\mathbf{y}$ are separated by two time periods while for $(2,2,2,2)$ poverty is experienced in any pair of successive years and so no relief from consecutive poverty experiences occurs. Thus, it is not unrealistic to assign a lower value of poverty to profile $(1,5,5,1)$ than to $(2,2,2,2)$, especially given the presumption that intensity of spells in a snapshot sense is irrelevant (i.e., through use of $\varepsilon = 1$ for the FGT measure).

Similarly, using $\gamma = 0.2$, implying a greater measure of concavity of weights, the cost of poverty for profiles $\mathbf{x}$ and $\mathbf{y}$ become $\widehat{g}_{\mathbf{x}}(0.5,1) = 2.032$ and $\widehat{g}_{\mathbf{y}}(0.5,1) = 1.968$. Note that although there is more chronic poverty present in profile $\mathbf{x}$ than in $\mathbf{y}$, the increased cost of poverty associated with the temporal pattern in $\mathbf{x}$ compared to that for $\mathbf{y}$ is less when the weighting function displays a greater degree of concavity (i.e., for $\gamma = 0.2$ compared to than $\gamma = 0.5$). The reason is that the importance of early poverty is also different for the different values of γ and it is not really possible to separate these two influences. Overall, one can see that, despite the simplicity of the example above, comparisons are indeed rather complicated. We would argue that in fact this should be the case.[20] It is not and should not generally be possible to separate in a simplistic manner concerns with early poverty, chronic poverty, and different intensities of poverty across different time periods. Spreading out poverty spells to earlier and later periods in life create less chronic poverty but also different patterns of early/late poverty experiences. Despite the complications, however, one can obtain some insights into actual lifetime poverty comparisons between individuals and groups by taking care to use alternative weighting functions and alternative snapshot poverty measures to "tease out" different properties of lifetime poverty and comparisons between groups.

This can be seen by comparing two income profiles with one identical spell of poverty but in one case, $\mathbf{x}$, the spell occurs in the first period of life, with $g_{\mathbf{x}} = (4,0,0,0)$, while for the other, $\mathbf{y}$, it appears in the last period of life, with $g_{\mathbf{y}} = (0,0,0,4)$. The average poverty gap in each case is $\overline{g}_{\mathbf{x}} = \overline{g}_{\mathbf{y}} = 1$. Under equal weighting (and $\varepsilon = 1$) we of course obtain $\widehat{g}_{\mathbf{x}}(0,1) = \widehat{g}_{\mathbf{y}}(0,1) = \overline{g}_{\mathbf{x}} = \overline{g}_{\mathbf{y}} = 1$. Taking into account the temporal aspects through choice of $\gamma = 0.5$, we obtain $\widehat{g}_{\mathbf{x}}(0.5,1) = 1.30$ and $\widehat{g}_{\mathbf{y}}(0.5,1) = 0.65$. Compared to equal weighting, the cost of inequality in profile $\mathbf{x}$ rises by 0.3 while that for y falls by 0.35 due to the earlier versus later experience of poverty. Using $\gamma = 0.2$, the cost of inequality in profile $\mathbf{x}$ rises by only 0.12 while that for $\mathbf{y}$ falls by only 0.15. Thus, although the smaller value of γ implies a greater measure of concavity of weights, it also clearly has an impact on the cost of inequality due to an earlier versus later spell of poverty. This can be seen by comparing the overall drop in the weights, which for $\gamma = 0.5$ is $\alpha_1 - \alpha_4 =$

[20] If one begins with a concave weighting function and wants to obtain a "more concave" function through a concave transformation of the first function, then the resulting two functions will not generally have the same range $\alpha(1,T) - \alpha(T,T)$ and certainly can't have the same first derivative throughout domain $t \in [1,T]$ which reflects the relative sensitivity of the lifetime measure to an earlier poverty concern at each point in time t.

$0.325 - 0.163 = 0.162$ while for $\gamma = 0.2$ we have $\widetilde{\alpha}_1 - \widetilde{\alpha}_4 = 0.0.280 - 0.212 = 0.068$. Again, we repeat that these two aspects of chronic and early poverty cannot be easily disentangled and nor *should* they. However, we can apply weighting functions such as from the quadratic family ($\alpha(t,T) = c_0 - c_1 t - c_2(t-1)^2$) to do robustness checks when comparing the implications of accounting for temporal patterns on lifetime poverty for individuals or for group comparisons. The following example shows the importance of doing so.

Comparing the above two weighting functions, based on the family $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$ with $\gamma = 0.2$ and 0.5, we can see, roughly speaking, that the weighting function that has a higher degree of curvature also has less sensitivity to early poverty. As shown earlier, one can easily generate weighting functions for which this is not the case. Consider the two weighting functions based on the family $\alpha(t,T) = c_0 - c_1 t - c_2(t-1)^2$:

$$\alpha(t,T) = 14 - t - 0.5(t-1)^2 \tag{10}$$

$$\widetilde{\alpha}(t,T) = 14 - t - (t-1)^2 \tag{11}$$

The function $\widetilde{\alpha}(t,T)$ has the greater degree of curvature for all $t \in [1,4]$ *and* a greater overall drop with $\widetilde{\alpha}_1 - \widetilde{\alpha}_4 = 0.406 - 0.031 = 0.375$ compared to $\alpha_1 - \alpha_4 = 0.333 - 0.141 = 0.192$. Thus, $\widetilde{\alpha}(t,T)$ displays both more sensitivity to chronic and early poverty. Using these weighting functions, we find for the above case that for $\mathbf{g_x} = (0,4,4,0)$ and $\mathbf{g_y} = (4,0,0,4)$, and $\varepsilon = 1$ for the FGT snapshot poverty index, that using $\alpha(t,T)$ above we obtain $\widehat{g}_{\mathbf{x}} = 1.333$ and $\widehat{g}_{\mathbf{y}} = 0.564$ while using $\widetilde{\alpha}(t,T)$ above we obtain $\widehat{g}_{\mathbf{x}} = 1.625$ and $\widehat{g}_{\mathbf{y}} = 0.125$. Thus, contrary to the comparison using $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$, the cost of poverty for distribution $\mathbf{x}$, which displays more chronic poverty, is greater when the more concave weighting function is applied, while that for $\mathbf{y}$ is less. The cost of early poverty is also greater for weighting function $\widetilde{\alpha}(t,T)$ than for $\alpha(t,T)$ from equations (10) and (11).

IV. Empirical Results

The data we use is taken from the PSID.[21] We use information only on those individuals who were in the first year of the panel (1968) and for whom we have continued data throughout to year 1993 (T=26 consecutive years, with reports being for 1967 to 1993, and a total of 1,494 individuals). Although some data is sporadically available for later years, we do not use it as we require an unbroken sequence of time periods. Unfortunately, information on consumption is not available so we use income as a proxy. In each year we know the family size of the unit in which the individual lived and we use this information to generate a sequence of 26 consecutive years of income information for each individual. We need comparable incomes across time that take into account changes in the CPI and family size. We also need a consistent

[21] This data set is publicly available (see http://psidonline.isr.umich.edu/).

poverty line (threshold) for the entire period. We accomplish this by first converting all incomes into real (CPI-adjusted) values, using 1983 as our base year.

The poverty thresholds for various family sizes were obtained from the U.S. Census Bureau. These are reported in 1983 constant dollars and vary slightly from year to year since they are not based on the all-goods CPI. This variation, however, is small. For a single person household this poverty threshold varies from a minium of \$4,993.57 to a maximum of \$5,070.87 in 1983 dollars over the 1967-1992. To achieve a single poverty line that does not vary by year and is the same for average lifetime income we take the average of the values over the 26 years which is \$5,047.69. Also for the purpose of consistency, we choose equivalence scales based on the ratio of the poverty line used in the PSID for each family size relative to an individual in a single household. For example, since the poverty line for a 2-person family is 6483.79, the equivalence scale for a two-person family is $6483.79/5047.69 = 1.28$. The full set of poverty lines is reported in Table A1 in the appendix.

Using time consistent values as described above leads to almost identical annual poverty rates as from the use of year-specific poverty lines used in the PSID. The average annual poverty rate for our subset of individuals is 6.87% while lifetime poverty (based on average lifetime income) is 3.28%.[22]

Pairwise Comparisons

The fraction of those in the population who are never poor (*i.e.,* in any year) is 59.6%. We are interested in seeing how different criteria (axioms) used to assess lifetime poverty, including those axioms that highlight the importance of chronic and early poverty, lead to different conclusions regarding how many pairs of individuals can be ordered in regard to their lifetime poverty status. Therefore, although we realize our data is not representative of the US population[23] this is not particularly worrisome since it is only the relative power of the axioms that interest us and this is simply a demonstration of that. In this analysis we first concern ourselves only with comparisons based on general classes of poverty indices (*i.e.,* for all deprivation functions p such that $p' < 0$ or $p' < 0$ and $p'' > 0$).

Since 59.6% of the sample never experience poverty, a large fraction of possible pairwise comparisons are of no interest (i.e., those in which neither individual in such pairs ever experienced poverty or if one person experienced no poverty in any given period while the other did). Thus, there are 603 individuals of interest to attempt to compare lifetime poverty, leading to $C^2_{603} = 181,503$ pairwise comparisons.

We first focus on the possibility of making general pairwise comparisons; that is, when can one say that person A suffers more or less lifetime poverty than person

[22] That is, based on average lifetime equivalized income, 3.28% of the individuals in our sample had average income over the 26 years of recorded income below the poverty line of \$5,047.69 in constant 1983 dollars.

[23] Perhaps the most important reason our sample is not representative is that we only choose individuals who have lived throughout the 33 year period and were traced and agreed to be interviewed in each of those years.

B for given (standard) restrictions on the snapshot poverty index, $p(x_t; z)$, as well as the retrospective poverty index $p(\overline{x}; z)$, and "new" restrictions on the weighting function $\alpha(t, T)$.[24] We consider two usual possibilities for the properties of $p(x_t; z)$ and $p(\overline{x}; z)$; namely, $p' < 0$ for $x_t < z$ - the notion that poverty deprivation decreases as consumption increases - and $p'' > 0$ - the notion that poverty deprivation decreases at a lower rate as consumption increases. If one considers only one period in which to make pairwise poverty comparisons, then such comparisons are straightforward. However, in comparing lifetime profiles of consumption of two persons, it is not so clear what additional restrictions, if any, to place on the lifetime poverty measure. The most general perspective would presumably be to say person A is more poor than person B if, for every period of life $t = 1, 2, ..., T$, person A incurs at least as much poverty as person B and strictly more poverty in at least one period. Not surprisingly, adopting such a general perspective does not allow for much power in making pairwise comparisons since the comparison of lifetime poverty must hold for every possible set of non-negative weights $\alpha(t, T)$. We find that among those persons who experience at least one spell of poverty, only 11% of pairwise comparisons are ranked (Result 2).

At the other extreme (Results 3 (b) and (c)), suppose we consider weighting all poverty spells equally regardless of the stage of life in which a poverty spell is experienced or how close poverty spells occur within one's lifetime. This would be equivalent to using weights $\alpha(t, T) = 1/T$ for all t, while allowing for any value $\beta(T) \in [0, 1]$. Such a strong assumption does indeed improve the power of the poverty ordering induced by $P(\mathbf{x}; z)$ with 64% ranked when only $p' < 0$ is assumed and 77% ranked when $p'' > 0$ is also assumed. Although this is a very significant improvement in the ranking ability of the lifetime poverty measure, the assumption of equal weights is rather like ignoring any possible importance in the timing of poverty spells. On the other hand, if one adopts the rather weak requirement of the early poverty axiom (Results 4(b) and (c)); that is, that weights $\alpha(t, T)$ be nonincreasing in t, then we are able to rank 38% when only $p' < 0$ is assumed and 43% when $p'' > 0$ is also assumed. The ordering power improves further to 49% and 66% of cases, respectively, when one also adopts the chronic poverty axiom (Results 5 (b) and (c)) which restricts the set of weight functions $\alpha(t, T)$ to be nonincreasing and concave.

Our results above suggest that moving to a lifetime poverty measurement approach in which one needs to compare many time periods of possible poverty experiences between individuals rather than simply a single period, cross-sectional application, may reduce one's ability to say when one person has (unambiguously) experienced more lifetime poverty than another. However, adoption of rather general and, we believe, normatively pleasing axioms about how the temporal pattern of poverty spells affects a person's lifetime poverty experience as a whole, improves the possibility of ranking individuals who have experienced at least one spell of poverty in terms of their

[24] We explain in the appendix how to make these comparisons using the various dominance criteria associated with each set of requirements.

overall lifetime poverty. Making pairwise rankings within populations may seem of secondary importance to overall population comparisons. However, when considering a policy change which may affect transitory poverty experiences as well as have a specific temporal effect in terms of alleviation of chronic and/or early poverty, it may be very useful to be able to make pairwise comparisons for each individual before and after a (possibly hypothetical) policy change. Doing so may allow one to estimate what fraction of individuals will experience more or less lifetime poverty as a result of such a policy for a broad range of lifetime poverty measures in much the same spirit as stochastic dominance has allowed researchers a way to determine when one distribution of income generates more or less inequality or welfare than another for a wide class of relevant measures rather than one specific one.[25]

Group Comparisons

We now move on to a set of applications involving comparisons of lifetime poverty between groups or subpopulations. Our data allow us to identify ethnic groups "white" and "non-white". Since it is well established from cross-sectional studies that there is more poverty among non-whites than whites, it is interesting here to determine whether recognizing temporal aspects of poverty in a lifetime poverty measurement exercise changes the extent to which these sub-groups experience poverty. In terms of numbers of individuals who experience some poverty in their lifetime, the percentage for whites is 27.4% while that for non-whites is 70.7%. The cumulative frequency for those who do experience at least one spell of poverty is provided in the appendix (Figure A1). We see that, amongst those who experience some poverty, 88% of non-whites experience more than one spell of poverty while only 65.5% of whites experience multiple periods of poverty. Moreover, among those experiencing any poverty, the percentage incurring five or more spells of poverty is 28.6% for whites and 63% for non-whites Thus, one would expect much more potential for chronic poverty to have a large influence on the lifetime poverty measure for non-whites. However, if multiple spells are sufficiently spread out over time then departing from equal (or linear) weights to a concave set of weights will not necessarily inflate the lifetime poverty measure for non-whites more than it does for whites. The extent to which poverty spells are experienced early in life, in addition to the extent to which they are "clumped together" will of course also have a bearing on how accounting for the temporal pattern of poverty experiences impacts the lifetime poverty measure for non-whites compared to whites.

We begin our formal analysis of these issues with weights based on the function $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$, $0 \leq \gamma \leq 1$, and explore how changing the parameter γ affects the cost of poverty for the two groups as reflected by the EDE poverty gap. We present results using the FGT family of poverty measures for our snapshot poverty index, $p(x_t; z) = (1 - x_t/z)^{\varepsilon}$, for both cases of $\varepsilon = 1$,2. Note that when choosing

[25]For example, see Shorrocks and Foster (1987), Davies and Hoy (1994, 1995) for general applications of stochastic dominance and Davies and Hoy (2002) for a specific policy example involving hypothetical changes to income tax schedules.

$\gamma = 0$ we are adopting equal weights $\alpha_t = 1/T$. At the other extreme, $\gamma = 1$ implies a concern with early poverty but no concern with chronic poverty. Moving from $\gamma = 1$ to $\gamma = 0$ represents a move to a more concave weighting function but also to one with a lesser degree of emphasis on early poverty. Results are summarized in the following two tables.

Table 1: EDE poverty gap

$\widehat{g}(\gamma, \varepsilon)$, $\varepsilon = 1$

γ	White	Non-white
0.00	92.17	437.27
0.25	88.73	438.89
0.50	86.06	442.41
0.75	83.99	446.77
1.00	82.38	452.03

Table 2: EDE poverty gap

$\widehat{g}(\gamma, \varepsilon)$, $\varepsilon = 2$

γ	White	Non-white
0.00	500.48	1020.77
0.25	486.77	1019.38
0.50	475.48	1021.98
0.75	466.21	1026.83
1.00	458.58	1032.81

In the case of $\varepsilon = 1$ (Table 1), we see that as γ rises, the cost of aggregate lifetime poverty for whites falls by over 10% while that for non-whites rises by about 3%. In particular, by comparing the case of $\gamma = 0$ (equal weights) to $\gamma = 1$ (decreasing weights in t) we can focus on the implications of a sensitivity to early poverty since in neither case is the weighting function strictly concave. With no temporal sensitivity in measuring lifetime poverty ($\gamma = 0$) the ratio of the cost of poverty of non-whites to whites is $437.27/92.17 = 4.74$ while introducing a concern for early poverty this ratio becomes $452.03/82.38 = 5.49$. Thus, the relative cost of lifetime poverty between non-whites and whites is sensitive to how one accounts for temporal properties of the timing of poverty spells, and in particular a concern with early poverty.

Considering the case of $\varepsilon = 2$ (Table 2) allows us to investigate the implication of accounting for the intensity of poverty experiences across poverty spells. This has a greater effect on the measurement of poverty for whites, with an increase in measured cost of poverty of a factor of approximately 5 (for each value of γ) than for non-whites, with an increase in the cost of poverty of a factor of approximately 2.3. The direction of the effect of changes in γ on the cost of poverty for whites and non-whites is similar to the case of $\varepsilon = 1$, although the magnitude of the effect on the cost of poverty for non-whites is substantially reduced. This demonstrates that one must be aware of possible interactions of assumptions about the temporal properties of lifetime poverty as well as about the importance of the intensity of poverty reflected in the snapshot index.

Roughly speaking, using the weighting function $\alpha(t,T) = (1-\frac{t}{T+1})^{\gamma}$ and snapshot poverty index $p(x_t;z) = (1 - x_t/z)^{\varepsilon}$ suggest that measures of the cost of lifetime poverty for whites is relatively more sensitive to the assumption about the intensity of poverty than for non-whites. Lifetime poverty measurement for whites falls as one increases concern with early poverty experiences while it rises for non-whites. As mentioned earlier, one cannot necessarily disentangle how changing a parameter in a weighting function affects a concern with early poverty from the extent of its implied concern with chronic poverty. Therefore we also use a weighting function based on the quadratic function described earlier (i.e., $\alpha(t,T) = c_0 - c_1 t - c_2(t-1)^2$, $c_0, c_1, c_2 > 0$, and $c_0 = c_1 T + c_2(T-1)^2 + 1$ to ensure $\alpha_t > 0$ for all t). In this case, by fixing c_1 and increasing c_2 one can generate a new set of weights that is more sensitive to early poverty and displays a higher degree of curvature. Fixing c_2 and increasing c_1 also creates a set of weights that is more sensitive to early poverty but (for $c_2 > 0$) creates a set of weights that display less curvature. The results for various values of c_1 and c_2 are summarized in Tables 3 through 6 below.

When $c_1 = c_2 = 0$ the weighting function implies equal weights for each t; i.e., $\alpha_t = 1/T$. Moving in the tables either by increasing c_1 or c_2, while holding the other at zero, creates a set of weight that are decreasing in t. However, in the case of increasing c_1 the weight function is linear and so the lifetime poverty measure does not become sensitive to chronic poverty, while it does in the case of increasing c_2. Increasing only c_1 while holding $c_2 = 0$ leads to a modest increase in lifetime poverty for non-whites. On the other hand, increasing c_2 while holding $c_1 = 0$ leads to a somewhat larger increase in the measure of lifetime poverty for nonwhites (3.4% vs. 1.0% when $\varepsilon = 1$). There is even less of an effect when using $\varepsilon = 2$ where we see lifetime poverty for non-whites even falls marginally when increasing c_1. The effect of creating decreasing weights has roughly the same type of effect on the measure of lifetime poverty for whites as was induced by changing the parameter γ for the other weighting function.

In both cases of $\varepsilon = 1, 2$ we find that, having chosen a positive value for c_2, moving from $c_1 = 0$ leads to a "clawing back" of the effect on poverty from having increased c_2 for non-whites. Thus, when moving to a weighting function that is more sensitive to early poverty, the impact on lifetime poverty measurement depends on whether one does this using a function with a greater or lesser degree of curvature. In this case, using a lesser degree of curvature reduces the implied increase in poverty. This suggests that chronic poverty is also an important property of the lifetime income profiles for non-whites in terms of contributing to the measured cost of lifetime poverty.

Table 3: EDE poverty gap for whites

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

		c_1		
		0	0.5	1
c_2	0	92.17	83.32	83.30
	0.5	83.06	83.27	83.27
	1	82.38	83.22	83.25

Table 4: EDE poverty gap for non-whites

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

		c_1		
		0	0.5	1
c_2	0	437.27	441.66	441.67
	0.5	451.01	442.00	441.84
	1	452.03	442.32	442.01

Table 5: EDE poverty gap for whites

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 2$

		c_1		
		0	0.5	1
c_2	0	500.48	464.56	464.47
	0.5	461.60	464.29	464.33
	1	458.58	464.03	464.20

Table 6: EDE poverty gap for non-whites

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 2$

		c_1		
		0	0.5	1
c_2	0	1020.77	1018.89	1018.89
	0.5	1031.98	1019.33	1019.11
	1	1032.81	1019.74	1019.33

We now turn our attention to regional comparisons.[26] Bishop, Formby, and Thistle (1992, 1994) used cross-section data from the Census of Population and Housing public-use tapes to compare regional income distributions for the years 1969 and 1979 (i.e., using the 1970 and 1980 survey years). Their goal was to establish the extent to which regional income distributions and welfare had converged or diverged over this period. Following their lead, we consider the cost of poverty for the same regions over the longer time period of 1967 to 1992. Using PSID data allows us to

[26]Individuals are assigned to a region based on where they lived the majority of their lives.

incorporate a concern for the temporal pattern of lifetime poverty experiences (i.e., use of lifetime poverty measures) for these regions. Our main goal is not so much as to establish convergence/divergence trends but rather to see how incorporating a sensitivity for the temporal pattern of incomes may affect regional comparisons. Will the cost of poverty over individuals lifetimes by region be closer or further apart according to alternative assumptions about sensitivity to the temporal pattern of poverty experiences in these regions? In particular, compared to equal weighting of snapshot poverty experiences, we want to determine if an emphasis on early and chronic poverty characteristics of individuals' poverty experiences reduces or increases the wedge in the cost of poverty between regions.

Twenty-six years of panel data of course does not represent the entire lifetime experience of an individual. However, it is a longer span of time than most panel data sets allow and certainly gives a potentially enhanced view of poverty than one can obtain by comparing two years of snapshot poverty experiences. One interesting finding of the analysis performed by Bishop, Formby, and Thistle (1992,1994) is that the South's income distribution either converged or moved significantly closer to the income distribution of the rest of the country between 1969 and 1979. However, over the same time interval the income distributions that make up the Non-South were diverging from each other.[27] We add to this analysis by comparing the poverty experienced by individuals over the subinterval 1967-1979 to that of 1980-1992, each subinterval being composed of 13 years. Although by splitting our data into two subintervals we lose some of the advantage of taking a lifetime measurement approach, this exercise allows for some interesting comparisons over two fairly long stretches of time used to construct poverty measures with at least a dynamic, if not lifetime, approach. Thus, we enhance the knowledge obtained through the cross-sectional approach of Bishop, Formby, and Thistle (1992, 1994).

The table below indicates the percentage of individuals in each region and for each time interval who have experienced at least one spell of poverty.

Table 7: Percentage experiencing at least one spell of poverty

	West	Northeast	North Central	South
1967-1992	29.8%	29.8%	34.4%	54.6%
1967-1979	21.4%	18.2%	25.5%	44.7%
1980-1992	21.4%	21.6%	27.6%	41.5%

Graphs illustrating the cumulative frequency curves for those who do experience at least one spell of poverty within each region (and each time period considered) are provided in the appendix (see Figures A2 to A4). It is clear that a greater fraction in the South experience a greater number of spells of poverty than for any of the other regions, while those in the West experience the least, regardless of what cutoff number

[27] Roughly speaking, our results are similar to their's.

one considers. In fact, our measures of lifetime poverty measurement developed and reported on below indicate the South as having the most and the West the least amount of poverty in all comparisons. Over the entire interval (1967-1992), the cumulative frequency curves of the Northeast and North Central intersect a number of times and so no clear or unambiguous ranking would seem apparent on this basis. However, the cumulative frequency curve for the Northeast is mostly above that for the North Central region and the fraction experiencing at least one spell of poverty is higher for the North Central region. Thus, it is not suprising that in all of our lifetime poverty comparisons the Northeast has a lower cost of poverty than the North Central.

There are some interesting differences between the two subintervals. The cumulative frequency curve for the South seems somewhat "worse" in the latter interval of 1980-1992 (i.e., mostly lower which suggests more people with many spells of poverty), compared to the earlier interval of 1967-1979. However, the fraction of people who experience no spells of poverty is higher in the later interval. Our lifetime poverty measures suggest the second subinterval for the South has less lifetime poverty despite the apparent potential for more chronic poverty. The cumulative frequency curve for the West is mostly higher in the latter interval suggesting a lesser degree of chronic poverty, while the percentage incurring any poverty is the same in both subintervals. However, our lifetime poverty measures suggest the opposite conclusion with poverty in the West higher in the latter subinterval.[28] Finally, although roughly speaking the cumulative frequency curve for the Northeast is higher than that for the North Central region in both subintervals, the difference between the two curves seems greater in the latter interval. Indeed, all of our measures of lifetime poverty imply a bigger difference in the cost of poverty between the two regions (higher for the North Central region) and this difference is greater in the latter subinterval. The North Central also has higher average poverty rates in both subintervals.

We now turn to our formal measures of lifetime poverty. Due to the large number of results we restrict our attention to a single application of the FGT parameter $\varepsilon = 1$. The implications of also considering $\varepsilon = 2$ are very similar; that is, the overall pattern of poverty comparisons is the same although the size of the cost of poverty is always higher when $\varepsilon = 2$ is used and the biggest relative difference is for those cases where poverty as measured by the application of $\varepsilon = 1$ is lower.[29]

We find that regardless of the weighting function and choice of parameter ε for the FGT measure[30], and also which time period, the relative comparisons of lifetime poverty between regions remains the same. The cost of poverty for the time

[28] However, the increase in cost of poverty in the West for the second subinterval is relatively small, with the greatest increase for the lifetime poverty measures occurring when weighting each period equally. This last result indicates a higher average poverty gap in the second subinterval.

[29] The set of results for $\varepsilon = 2$ is available upon request. We also use a finer grid for the c_1, c_2 values and find no important effects upon doing so.

[30] Here we only report the results for $\varepsilon = 1$. The qualitative nature of comparisons, however, is the same for $\varepsilon = 2$. These results are available upon request.

period 1967-1992 is always highest for the South, followed by the North Central, Northeast, and finally the West with the lowest cost of poverty. However, departing from weighting each period equally, adopting a decreasing (and sometimes concave) weighting function leads to interesting and different implications for the different regions.

Consider first the weighting function $\alpha(t,T) = (1 - \frac{t}{T+1})^{\gamma}$. Recall, as we increase γ from zero (equal weighting of each time period) to $\gamma = 1$ we generate a lifetime poverty measure that is increasingly more sensitive to early poverty, while at the same time the degree of concavity of the measure (for $0 < \gamma < 1$) falls. Results for this weighting function are reported in tables 8 through 10 below. We see that increasing the degree of sensitivity for early poverty when using the entire period (1967-1992) leads to a lower measured cost of poverty compared to equal weighting for the Northeast (8.0%), North Central (13%), and the West (7.4%) but leads to a higher measured cost of poverty for the South (4%). Note that the number in brackets in each case is the maximum possible effect resulting from choosing a value of γ different from zero and this maximal effect occurs for choice of $\gamma = 1$ in each case.

There are a number of reasons for our decision to break the sample into two parts of equal length. Firstly, panel data is often only available for relatively short lengths of time, such as 5 to 10 years. Thus, if we find we obtain quite different results for our two time periods, this suggests a high degree of caution should be exercised when treating results from such studies as indicative of *lifetime* poverty measures. Also, from the cross-sectional studies mentioned above, we have reason to believe that the extent of poverty in the regions has changed somewhat over our time period, and especially for the South. We wish to see whether and, if so, how such changes are realized in terms of "partial" lifetime poverty measurement. Thirdly, although in general we would ideally like to think of our time parameter t reflecting more or less an actual age of each individual (i.e., the person's actual age), even with the PSID data set it is not possible to create sufficiently large groups of separate cohorts that we can trace over time. However, there is a sense in which breaking up the period into two subintervals indirectly allows for an approximate comparison of the two cohorts of families and their children of the two subintervals. So, for example, if in the first subinterval (1967-79) early poverty is more of a concern than in the second subinterval (1980-1992), then we can expect there was more child poverty for children living in the relevant families in that earlier period. A case in point is the South. By breaking up the time period we find that in the first subinterval (1967-79) incorporating the early poverty concern through increasing γ from zero to one leads to an increase in measured lifetime poverty of 12.2% while the effect is an increase of only 4.9% in the second subinterval (1980-92).[31]

There are other interesting results arising from the subinterval analysis. For North

[31] Admittedly, given the mix of cohorts and family composition types in each subinterval, these results are only suggestive.

Central, we had noted that taking account of early poverty concerns actually decreased the measure of lifetime poverty by up to 13%. However, in each of the two subintervals this effect was not more than 2%. Thus, using either of the shorter intervals of time leads to quite a different conclusion on the effect of temporal considerations on the measurement of lifetime poverty than one obtains by using the entire period. It is also interesting to note the implications for the West. Using the entire interval, the effect of increasing γ from zero to one leads to a reduction in lifetime poverty of 7.4% while in the two subintervals the amounts are a 5.0% *increase* for 1967-79 and a decrease of 11.7% for 1980-92. Thus, to the extent that these reflect implications for two different cohorts of (young) children in the two subintervals, this points to the need to be careful in selecting time frames or making conclusions using relatively short panels.

It is also interesting to note that the degree of sensitivity to timing of poverty implied by a set of weights used in a lifetime poverty measure affects substantially the size of the measured increase in "lifetime" poverty in the West between the two time periods. Using $\gamma = 0$ the implied increase in poverty in the West in the second subinterval is 23.7% (64.94 to 80.92). Due to the importance of the temporal pattern, however, if one adopts $\gamma = 1$, the increase is only 3.9% (68.20 to 70.87).

Tables 11 through 18 provide results from the same analysis as above except for the weighting function $\alpha(t, T) = c_0 - c_1 t - c_2(t-1)^2$. The qualitative nature, and even the quantitative nature, of the results are quite similar.

Table 8: EDE poverty gap for regions (1967-1992)

$\widehat{g}(\gamma, \varepsilon)$, $\varepsilon = 1$

γ	Northeast	North Central	South	West
0	93.28	155.18	316.95	72.61
0.25	91.15	149.13	318.08	70.02
0.50	89.15	143.76	321.05	68.51
0.75	87.37	139.02	325.05	67.70
1	85.83	134.89	329.57	67.32

Table 9: EDE poverty gap for regions (1967-1979)

$\widehat{g}(\gamma, \varepsilon)$, $\varepsilon = 1$

γ	Northeast	North Central	South	West
0	79.21	113.00	329.55	64.94
0.25	79.31	111.65	341.09	65.71
0.50	79.22	110.82	351.61	66.52
0.75	79.04	110.38	361.11	67.35
1	78.85	110.22	369.67	68.20

Table 10: EDE poverty gap for regions (1980-1992)

$\widehat{g}(\gamma, \varepsilon)$, $\varepsilon = 1$

γ	Northeast	North Central	South	West
0	107.35	197.36	304.35	80.29
0.25	107.35	198.09	297.95	76.50
0.50	106.78	198.88	293.70	73.84
0.75	106.03	199.61	291.04	72.04
1	105.11	200.19	289.49	70.87

Table 11: Northeast EDE poverty gap (1967-1992)

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

		c_1		
		0	0.5	1
c_2	0	93.28	87.24	87.23
	0.5	86.35	87.18	87.20
	1	85.83	87.13	87.17

Table 12: Northeast EDE poverty gap

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

		FIRST HALF (1967-1979)			SECOND HALF (1980-1992)		
		c_1			c_1		
c_2		0	0.5	1	0	0.5	1
	0	79.21	78.80	78.80	107.35	107.49	107.49
	0.5	78.90	78.80	78.80	105.39	107.33	107.41
	1	78.85	78.80	78.80	105.11	107.18	107.33

Table 13: North Central EDE poverty gap (1967-1992)

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

		c_1		
		0	0.5	1
c_2	0	155.18	139.20	139.16
	0.5	136.29	139.03	139.08
	1	134.89	138.86	138.99

Table 14: North Central EDE poverty gap

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

	FIRST HALF (1967-1979)			SECOND HALF (1980-1992)		
	c_1			c_1		
c_2	0	0.5	1	0	0.5	1
0	113.00	109.14	109.10	197.36	200.66	200.69
0.5	110.57	109.18	109.12	199.84	200.66	200.69
1	110.22	109.21	109.14	200.19	200.66	200.69

Table 15: South EDE poverty gap (1967-1992)

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

	c_1		
c_2	0	0.5	1
0	316.95	320.32	320.32
0.5	328.70	320.62	320.48
1	329.57	320.90	320.62

Table 16: South EDE poverty gap

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

	FIRST HALF (1967-1979)			SECOND HALF (1980-1992)		
	c_1			c_1		
c_2	0	0.5	1	0	0.5	1
0	329.55	361.67	362.01	304.35	286.80	286.61
0.5	364.66	362.53	362.45	291.34	286.81	286.62
1	369.67	363.30	362,86	289.49	286.82	286.64

Table 17: West EDE poverty gap (1967-1992)

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

	c_1		
c_2	0	0.5	1
0	72.61	66.52	66.50
0.5	67.69	66.53	66.51
1	67.32	66.54	66.51

Table 18: West EDE poverty gap

$\widehat{g}(c_1, c_2, \varepsilon)$, $\varepsilon = 1$

	FIRST HALF (1967-1979) c_1			SECOND HALF (1980-1992) c_1		
c_2	0	0.5	1	0	0.5	1
0	64.94	66.76	66.78	80.29	69.66	69.54
0.5	67.79	66.88	66.84	72.05	69.64	69.54
1	68.20	66.98	66.90	70.87	69.62	69.53

V. Some Remarks and Conclusion

In this paper we have used data from the PSID that provides information on incomes of 1,494 individuals over 26 consecutive years in order to illustrate the implications of measuring poverty from a lifetime perspective. Our formulation of lifetime poverty measurement recognizes that an individual's lifetime poverty should be reflected both by poverty experiences of each period (*i.e.,* "snapshot" poverty) and the poverty level of "permanent" or retrospective lifetime consumption. By using a weighted sum of the individual snapshot poverty experiences, we are able to reflect the sensitivity of how any poverty spells are distributed over a person's lifetime on lifetime poverty through the pattern of weights. In particular, we investigate the implications of adopting an "early poverty axiom" and a "chronic poverty axiom." The early poverty axiom reflects the well-established argument that poverty early in life is more critical than poverty later in life while the chronic poverty axiom reflects the idea that, for example, two spells of poverty of a given intensity are more harmful to an individual's well-being the closer in time that these spells occur. This is a more general view of chronic poverty than many others in the literature.

The property of early poverty sensitivity, if it holds globally over a person's lifetime, means the weights must be nonincreasing over the time that an individual lives, while sensitivity to chronic poverty means the weights must be concave in time. In our empirical applications we have used specific algebraic functions, $\alpha(t, T)$, to generate appropriate weights (i.e., that are nonincreasing and satisfy the concavity property as described by conditions of equation (7)). However, one may well wish to select weights based on explicity empirical evidence of the impact of early versus late poverty, including in particular childhood poverty, and the increased impact of spells of poverty that occur close in time. One can also, of course, depart from our axioms. In fact, the appropriate set of weights may depend on country-specific matters such as availability of age-related public goods and services. For example, suppose the researcher has evidence suggesting that in a given context one should be especially sensitive to poverty late in life due to fragility of the elderly. Thus, with continued acceptance of the early poverty concern for (say) the childhood years, one might wish to use weights $\alpha(t, T)$ that are U-shaped in t. One must recognize, however, that to do so will mean the chronic poverty axiom cannot be satisfied.

In fact, in general we have demonstrated that one must recognize a potential relationship or even conflict between the concern of sensitivity to early/late timing of poverty spells and a concern about chronic poverty. It would be possible, for example, to have a set of weights that are decreasing in t for a set of consecutive periods early in life (say $t = 1, 2, ..., T_1$) and increasing in t for a set of consecutive periods late in life (say $t = T_2, T_2 + 1, ..., T$, with $T_2 \geq T_1$). Further, one could impose concavity on each of these two sets of weights in order to ensure that spreading out of poverty spells that occur strictly within the first (early) set of periods or strictly within the second (late) set of periods would lessen the impact of lifetime poverty; that is, allow for the chronic poverty axiom to be reflected within the early and late time periods. However, in doing so one must recognize that the property of sensitivity to chronic poverty could not be satisfied globally.[32] In the current example this means that spreading of a pair of spells of equivalent poverty, with one period from the early set and the other from the late set, must lead to an increase in lifetime poverty. The increased emphasis on the earlier and later poverty experiences form each set trumps the possibility that relief from chronic poverty should imply less lifetime poverty. Our approach explicitly recognizes such conflicts and, we believe, emphasizes the need to consider such matters about the temporal pattern of poverty spells in the conceptualization of lifetime poverty.

We have used the PSID data set and explored the power of orderings for pairwise comparisons as implied by alternative combinations of properties that can be made for any set of well-behaved snapshot poverty indices in conjunction with our axioms on how the temporal pattern of poverty spells may influence lifetime poverty. Placing no structure on how to compare poverty at different periods of life leads to a very weak general ordering principle. Adding the early poverty axiom and the chronic poverty axiom sequentially markedly improves the power of the ordering procedure. We also used specific weighting functions and snapshot poverty indices (based on the FGT measures) to illustrate how assumptions made about the temporal pattern of poverty spells may affect lifetime poverty comparisons between groups. In particular, we compared lifetime poverty measures for whites and non-whites as well as a set of regional comparisons.

We found that within the set of weights that we chose, and in conjunction with the FGT snapshot poverty gap index, the comparison of the EDE cost of poverty for non-whites and whites implied a ratio of 4.74 (non-white:white) upon equal weighting of poverty spells across time compared to 5.49 (see tables 3 and 4) with weights reflecting both a concern for early and/or chronic poverty. Our regional results contributed to the literature on convergence and divergence of regional living standards from the perspective of poverty experiences. We found that for all measurement assumptions,

[32] Mathematically, one cannot have a function that is first decreasing in t, later increasing in t, yet has a negative second derivative thoughout.

a clear poverty ranking persists whether one takes the entire time interval of our available data (1967-1992) or split the data into two equal-length subintervals (1967-1979 and 1980-1992). The South sustains the greatest cost of poverty, followed by the North Central, Northeastern, and Western regions, respectively. However, upon separate computations based on the time periods 1967-1979 and 1980-1992, the South converged relative to all the other regions while the Northeast and North Central regions diverged from each other and from the West. Our results demonstrate that the extent to which this has occurred, as well as comparisons overall between regions, depends on the assumed sensitivity to the temporal pattern of poverty spells used in constructing lifetime poverty measures.

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Appendix

Table A1: Poverty lines (in 1983 dollars) and implied equivalence scales

Family size	Poverty line	Implied equivalence scale
1	5,047.59	1.00
2	6,483.79	1.28
3	7,942.48	1.57
4	10,176.61	2.02
5	12,030.04	2.38
6	13,568.20	2.69
7	16,775.38	3.32

Implementation of Results for Pairwise Comparisons

Implementing the requirements for Results 1 and 2 (b, c) is straightforward, so we only explain the cases for Results 4 (b,c) and 5 (b, c). In result 4 we consider the implication of adopting the early poverty axiom that restricts the $\alpha(t,T)$s to be nonincreasing in time t. The implication for parts (b) and (c) is that rather than checking for rank dominance and generalized Lorenz dominance, respectively, for the complete vectors $\mathbf{x}$ and $\mathbf{y}$, we must check this for all subvectors of length 1, 2, ..., T. This is to ensure that poverty earlier in life is given prominence in making the ordering as required by the early poverty axiom. To implement comparisons, let ℓ represent the first ℓ years of a person's life. Then the subvector $\widetilde{\mathbf{x}}^{\ell} = (\widetilde{x}_1, \widetilde{x}_2, ..., \widetilde{x}_{\ell})$ provides the censored incomes of the first ℓ years, in chronological order. In line with our previous notation, this vector when sorted in ascending order is $\widetilde{\mathbf{x}}^{\ell} = (\widetilde{x}_{(1)}, \widetilde{x}_{(2)}, ..., \widetilde{x}_{(\ell)})$. To determine if income profile $\mathbf{x}$ displays more lifetime poverty than income profile $\mathbf{y}$ under the early poverty axiom for all deprivation functions p such that $p^{'} < 0$ one must check if vector $(\widetilde{x}_1, \widetilde{x}_2, ..., \widetilde{x}_{\ell})$ rank dominates vector $(\widetilde{y}_1, \widetilde{y}_2, ..., \widetilde{y}_{\ell})$ for all $\ell = 1, ..., T$. This requires $\widetilde{x}^{\ell}_{(t)} \leq \widetilde{y}^{\ell}_{(t)}\ \forall\ t = 1, 2, ..., \ell$ AND all $\ell = 1, 2, ..., T$. For the case of all deprivation functions p such that $p^{'} < 0$ and $p^{''} > 0$, we require vector $(\widetilde{x}_1, \widetilde{x}_2, ..., \widetilde{x}_{\ell})$ generalize Lorenz dominate vector $(\widetilde{y}_1, \widetilde{y}_2, ..., \widetilde{y}_{\ell})$ for all $\ell = 1, ..., T$. This requires checking $\sum_{t=1}^{k} \tilde{x}_{(t)} \leq \sum_{t=1}^{k} \tilde{y}_{(t)}$ for $k = 1, 2, ..., \ell$ AND all $\ell = 1, 2, ..., T$. As in all cases one must also check that $\min\{\bar{x}, z\} \leq \min\{\bar{y}, z\}$.

In order to determine dominance conditions that allow for implementation of Results 5 (b) and (c), rewrite equation (1) as

$$\begin{aligned} P(\mathbf{x}; z) \quad &= \quad \beta(T)\left\{\left[\sum_{t=1}^{T-1}[\alpha(t,T) - \alpha(t+1,T)]\sum_{s=1}^{t} p(x_s; z)\right] + \alpha(T,T)\sum_{s=1}^{T} p(x_s; z)\right\} \\ &\quad +[1-\beta(T)]p(\bar{x}; z). \end{aligned} \tag{12}$$

and use Abel's partial summation formula one more time (but in reserve order). Now construct the following vectors x_l, as defined below, and then check for rank dominance (case b) and generalized Lorenz dominance (case c) based on the sorted vectors x_l (in nondescending order):

$$\tilde{\mathbf{x}}_l = (\underbrace{\tilde{x}_1, .., \tilde{x}_1}_{l \text{ times}}, ..., \underbrace{\tilde{x}_{T-l+1}, .., \tilde{x}_{T-l+1}}_{l \text{ times}}, \underbrace{\tilde{x}_{T-l+2}, .., \tilde{x}_{T-l+2}}_{l-1 \text{ times}}, \underbrace{\tilde{x}_{T-l+3}, .., \tilde{x}_{T-l+3}}_{l-2 \text{ times}}, ..., \underbrace{\tilde{x}_T}_{1 \text{ time}})$$

Thus, for $l = 1$ we get just the entire vector.

$$\tilde{\mathbf{x}}_1 = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_T)$$

For $l = 2$ we get:

$$\tilde{\mathbf{x}}_2 = (\underbrace{\tilde{x}_1, \tilde{x}_1}_{l=2 \text{ times}}, \underbrace{\tilde{x}_2, \tilde{x}_2}_{l=2 \text{ times}}, ..., \underbrace{\tilde{x}_{T-1}, \tilde{x}_{T-1}}_{l=2 \text{ times}}, \underbrace{\tilde{x}_T}_{l-1=1 \text{ times}})$$

For $l = 3$ we get:

$$\tilde{\mathbf{x}}_l = (\underbrace{\tilde{x}_1, \tilde{x}_1, \tilde{x}_1}_{l=3 \text{ times}}, \underbrace{\tilde{x}_2, \tilde{x}_2, \tilde{x}_2}_{l=3 \text{ times}}, ..., \underbrace{\tilde{x}_{T-2}, \tilde{x}_{T-2}, \tilde{x}_{T-2}}_{l=3 \text{ times}}, \underbrace{\tilde{x}_{T-1}, \tilde{x}_{T-1}}_{l-1=2 \text{ times}}, \underbrace{\tilde{x}_T}_{l-2=1 \text{ times}})$$

et cetera.

When the last step, $l = T$, is reached, $\tilde{x}_1$, appears T times, and then successive elements appear $T-1$, $T-2$, ..., times, with $\tilde{x}_T$ appearing once.

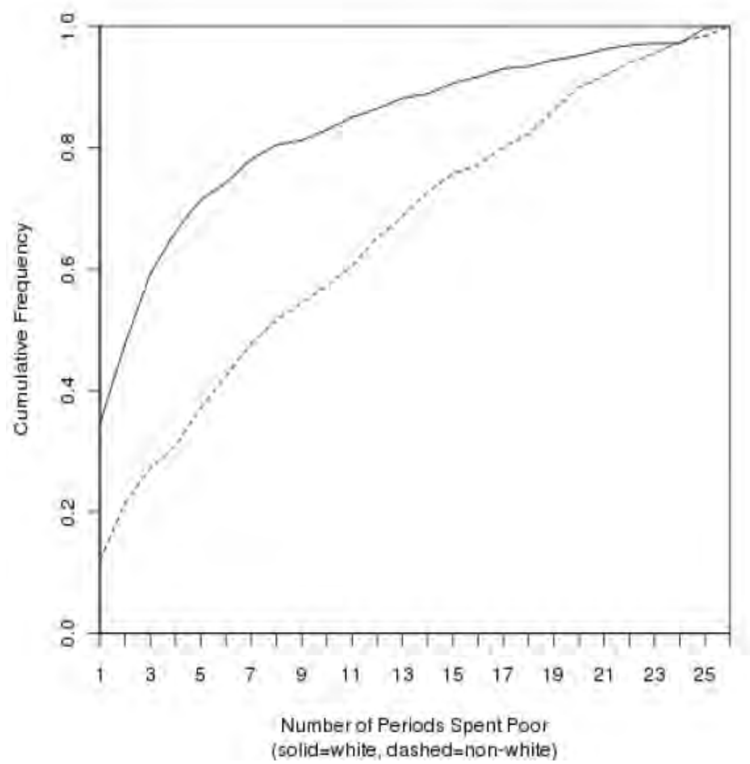

Figure A1: Cumulative frequency of number of poverty spells
Whites compared to non-whites

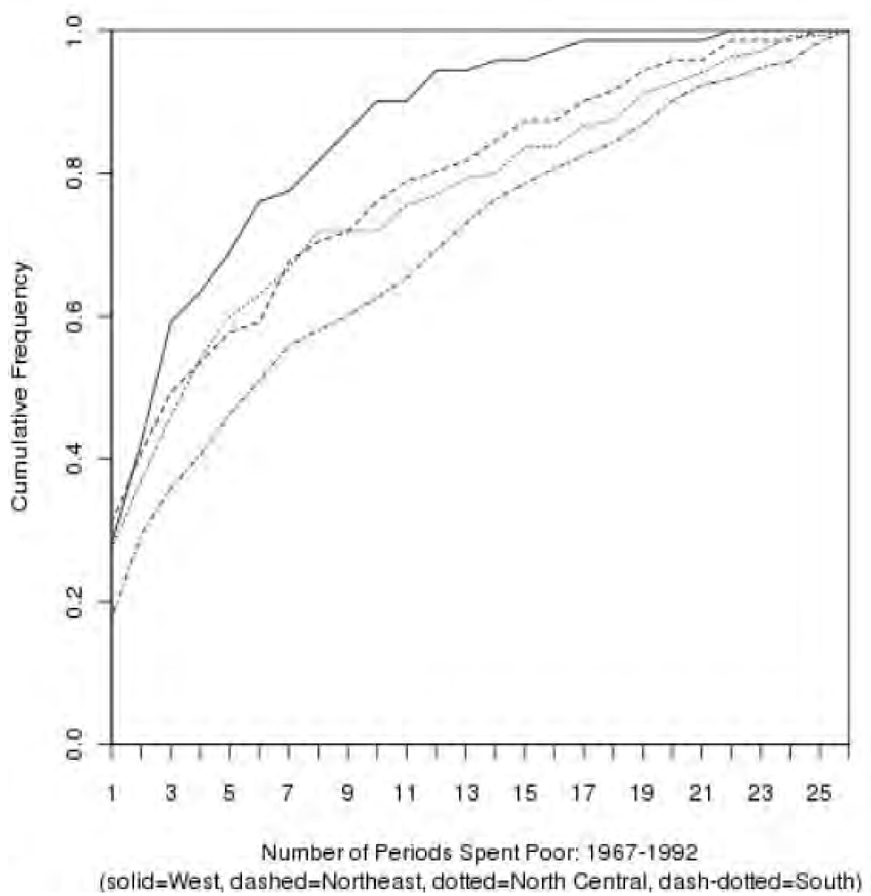

Figure A2: Cumulative frequency of number of poverty spells
Regional comparison for 1967-1992

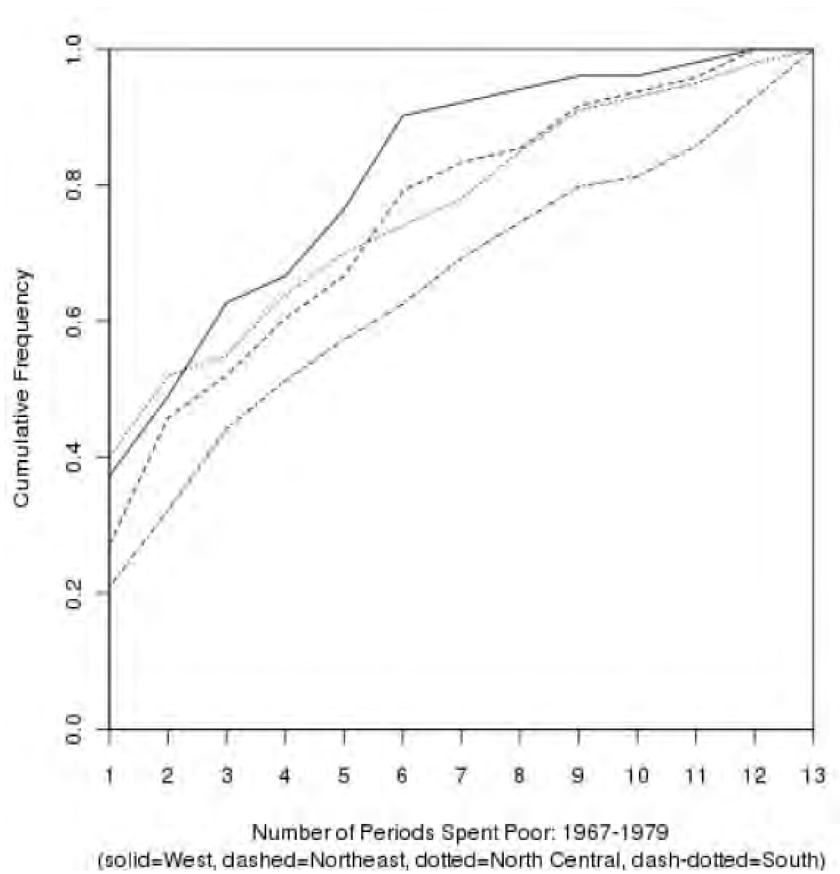

Figure A3: Cumulative frequency of number of poverty spells
Regional comparison for 1967-1979

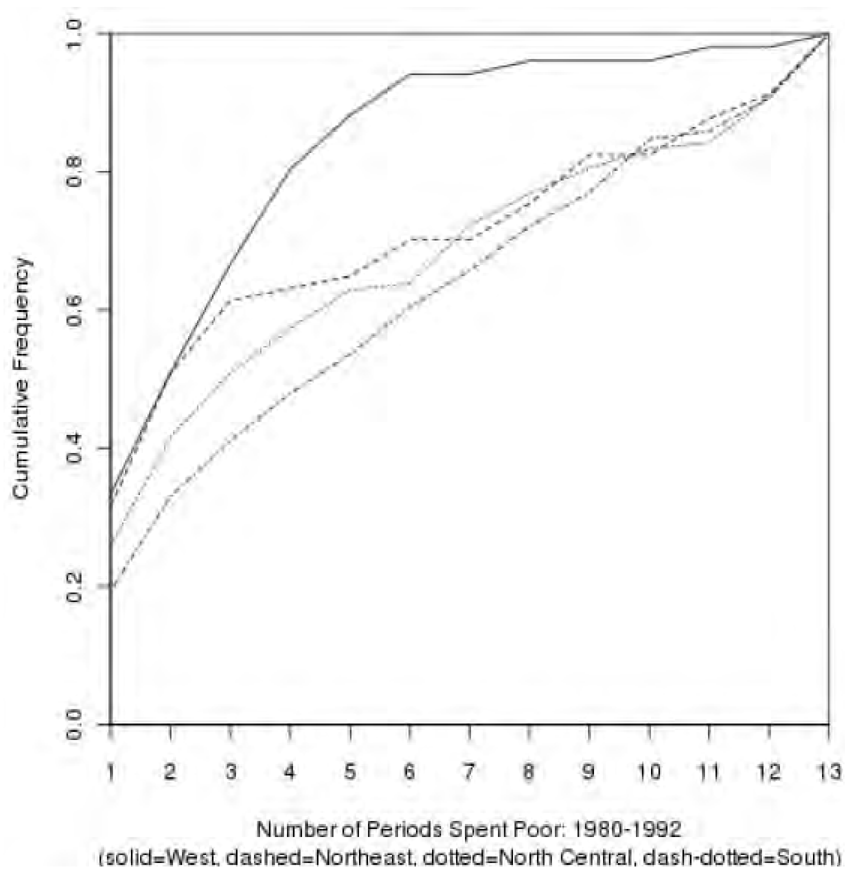

Figure A4: Cumulative frequency of number of poverty spells
Regional comparison for 1980-1992