

REAL OPTIONS VALUATION WITH DUOPOLY GAME-THEORETIC APPROACH AND REGIME  
SWITCHING

by

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A thesis  
presented to Ryerson University  
in partial fulfillment of  
the requirements for the degree of  
Master of Applied Science (MAsc)  
in the program of  
Mechanical and Industrial Engineering

Toronto, ON, Canada, 2020

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**Abstract**

New product demand continuously fluctuates through the life of the product. As a result, markets usually experience high volatility due to fluctuating demand. Hence, organizations must rapidly adapt and be flexible with such volatility. Management decisions cannot rely on traditional models that assume economic variables to be constant over time. Therefore, companies must advance and implement adequate strategies in resource allocation that help them survive in uncertain markets, and mitigate the associated risk from market uncertainty. This research aims to investigate product introduction strategies where two micro-economic theories can be applied: game-theoretic approach and a real options valuation. Specifically, a real options valuation framework with a flexible capacity that can be used to calculate the Net Present Value (NPV) and the optimal initial capacity to invest in a product, which is being introduced in a duopoly environment, is proposed. This framework is based on game theory applied to a duopoly market. A stochastic product life cycle characterized by a regime-switching approach and the consideration of two different game-theoretic models for the cases where competitors have perfect or imperfect information in the market are the main focal points of our study. Subsequently, different numerical examples comparing the results of NPV and optimal initial capacities from both game-theoretic models are presented. It is observed that the leader in the model with perfect information has an important advantage over its opponent because this company makes the first move to establish quantity to supply. Also, the leader in perfect information game has an advantage over the game with imperfect information since decisions with imperfect information have to be made simultaneously, and players need to balance all possible outcomes when making a choice. Consequently, in both models, one of the competitors obtains its maximum NPV when the other invests in the lowest

level of capacity. Lastly, expansion and contraction costs play a critical role in the strategy of resource allocation as these costs are involved in capacity flexibility. Therefore, firms can adjust their production when product demand varies to maximize profits.

## **Acknowledgments**

I would like to thank my supervisors at Ryerson University, Dr. Mohamed Wahab Mohamed Ismail and Dr. Mucahit Cevik for the opportunity to do this research. Their guidance, motivation, and enthusiasm throughout my master's studies is greatly appreciated.

My gratitude to each of the committee members, Dr. Saman Hassanzadeh Amin, Dr. Ayse Bener and Dr. Der Chyan Lin, for their helpful feedback and suggestions that improved and enriched the content of my thesis.

My thanks also goes to all of my professors, teachers, and mentors, who have provided me with extensive personal support and professional advice. You have taught me a great deal about scientific research.

Thank you to all of my friends and family. You have played a very important role in my academic accomplishments. In the most challenging of times, you have guided and inspired me.

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# Nomenclature

## Dynamic Programming

$K_0^*$	Optimal capacity
$\ell_1$	Level of the capacity for Company 1
$\ell_2$	Level of the capacity for Company 2
$\mathcal{L}(\ell_1)$	Vector that contains the capacity for Company 1 that maximizes expected present value
$r_f$	Risk-free rate
NPV	Net present value
$\nu$	Number of node in the lattice
$\psi_{dd}$	Conditional probability of going downward in decay regime
$\psi_{dm}$	Conditional probability of going middle in decay regime
$\psi_{du}$	Conditional probability of going upward in decay regime
$\psi_{gd}$	Conditional probability of going downward in growth regime
$\psi_{gm}$	Conditional probability of going middle in growth regime
$\psi_{gu}$	Conditional probability of going upward in growth regime
$p(s)$	Probability of switching during the course of the upcoming period
$P(s)$	Probability of switching during the next period
$s$	Time step each year
$t$	Period of time

$E(PV)$  Expected present value

### **Flexible Capacity**

$c_\kappa$  Installation cost per unit of  $K$

$\delta$  Percentage of expansion cost

$v$  Percentage of contraction cost

$s_f$  Fixed switching cost

$S$  Switching capacity function

### **Game Theory**

$a_i^*$  Action selected by company  $i$

$K_{i\ell}$  Capacity for company  $i$  at level  $\ell$

$c_i$  Variable cost for company  $i$

$o_i$  Overhead cost for company  $i$

$C_i$  Total cost function of company  $i$

$w_i$  Marginal cost for company  $i$

$\lambda_i$  Demand-Price elasticity for company  $i$

$R_i(Q_j)$  Reaction function of company  $i$  to company  $j$

$P_i$  Price for company  $i$

$\Pi_i$  Profit for company  $i$

$Q_i$  Production quantity for company  $i$

$u_i$  Utility function for company  $i$

### **Lattice and Regime Switching**

$\theta$	Stochastic demand parameter
$D_0$	Initial demand
$\mu_d$	Mean of the decay regime
$\mu_g$	Mean of the growth regime
$\mu_S$	Mean of regime switching
$h$	Period of time between two layers
$\sigma_d$	Standard deviation of the regime decay
$\sigma_g$	Standard deviation of the growth regime
$\sigma_S$	Standard deviation of regime switching
$\phi$	Maximum step size in the lattice
$\phi_\alpha$	Lattice step size in regime $\alpha$

### **Real Options**

$B$	Amount to borrow
$\varepsilon$	Exercise price
$\hat{I}_t$	Income certainty equivalent at period $t$
$I_0$	Investment at period $t = 0$
$I_t$	Cash inflow in period $t$
$d$	Percentage of stock price going downwards
$u$	Percentage of stock price going upwards
$N$	Number of shares to buy
$O_t$	Future investments at period $t$

$\psi$	Conditional probability
$r_a$	Risk-adjusted discount rate
$\varsigma$	Strike price
$\tau$	Time to expiration
$C$	American call option
$c$	European call option
$P$	American put option
$p$	European put option

# Acronyms

**AP** Application Provider. *Glossary:* [AP](#)

# Chapter 1

## Introduction

The world economy has continuously advanced technologically. Therefore, decisions to invest in new product expansions have been the main focus of many companies to survive in the market. Introducing new products involve decisions and activities to present them to their target markets. This is often the most expensive and risky, yet least managed part of product development ([Govil and Proth, 2002](#)). Often companies are too enthusiastic about new product ideas that they might fail to research on their feasibility before launching. They may also ignore what the research tells regarding the design and implementation as well as the competition in the market or the life cycle of the new product.

Successful new product introductions result from an integrated process. This process relies heavily on research and solving potential issues such as timing, investment, initial capacity, expected profits, and the product life spans ([Saravanan and Vikkraman, 2013](#)). In particular, the demand for trendy goods might be unpredictable and can be highly influenced by consumption tendencies and product market circumstances ([Chiu et al., 2017](#)). Most companies, such as the ones in manufacturing, frequently faces high demand volatility that comes from volume, product assortment, and customized requirements. To manage these challenges, companies need to acquire some degree of flexibility to remain competitive and profitable in the market ([Bengtsson, 2001](#)).

For the products that heavily rely on new technologies, product lifetime expectancy tends to decrease. Further, shorter lifetimes intensify companies' pressure to develop products with novel innovations that stay in the market for a shorter period of time to avoid losses ([Lukas et al., 2017](#)). Additionally, companies that are manufacturing products with short life cycles compete intensely in a market where consumers look for very specific product characteristics; therefore, they experience difficulties in predicting the demand ([Bayus, 1998](#)).



In assessing the product introduction strategies, companies take into account the Product Life Cycle (PLC), which represents the unit sales curve for a product that ranges from the time it is introduced to the market until it is removed (Rink and Swan, 1979). PLC provides a flexible framework to plan and deliver a proper strategy that guides the company through the anticipation of unexpected results from the competing market's underlying dynamics (Day, 1981). Figure 1.1 illustrates product life cycles for Apple iPhone models indicating that different products go through similar introduction, growth, maturity, and decay patterns.

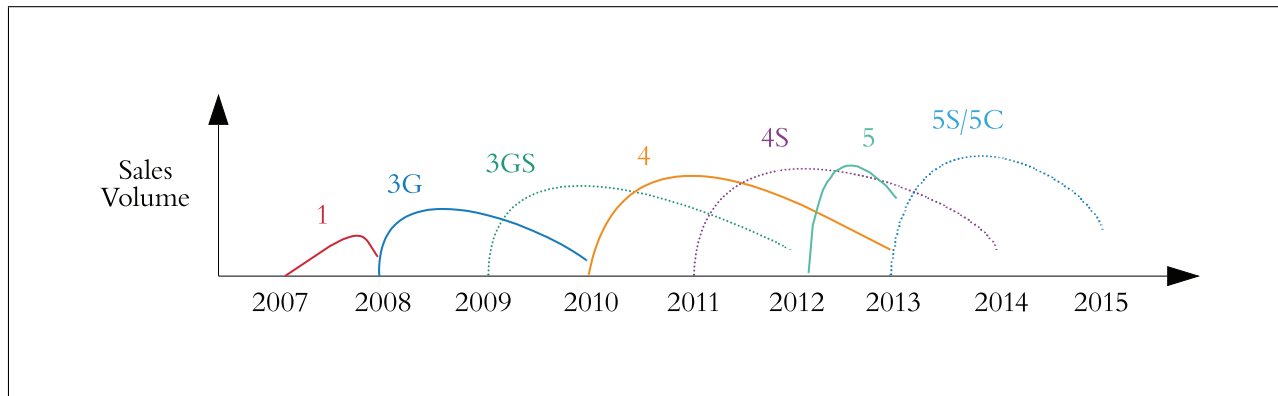


Figure 1.1: The life cycle of all the iPhone models (Ewa et al., 2017)

PLC also affects the investment opportunities of the company since it directly alters the company's cash flow, and it is highly correlated with the state of the market. Thus, ignoring cycles can be misleading in terms of calculation of required capacity needed to start production (Jeon and Nishihara, 2015; Lukas et al., 2017). Capacity is defined as the volume of goods or services that can be produced by an enterprise using current resources (He and Pindyck, 1989). Additionally, companies continuously face the dilemma of when to introduce a product and also when to retire the existing product. Product lines competition causes extra challenges, and for instance, some new goods may take over the sales of other goods in the same line and decreases profitability of the whole line in the market (Dash et al., 2016).

New products are more likely to fail than to succeed in today's rapidly changing markets. However, competitiveness requires companies to keep investing in innovative projects, even when they are not certain about their new products' commercial feasibility (Hauser et al., 2006). An unsuccessful introduction can be attributed to the insufficient resources from unknown innovation ventures that results in wrong investment amounts for many decision-makers (Klingebiel and

[Rammer, 2014](#)). For a new product or service to be successful, a manufacturer must have the personnel and capacity to deal with the success as well. For instance, extended lead times can be highly detrimental for a well-performing product. Further, as capacity decisions have a large influence on all issues relating to manufacturing competitiveness, many factors must be taken into account while making such decisions. Production costs, diversity of products, manufacturing lead times, introduction of new products, and market share are some examples of these factors. Note that the resource allocation strategies of a company are highly impacted by these factors, though they have barely been presented in research on innovation execution. Current models in the literature indicate that resource allocation behavior has been described by low initial commitment, sequencing, and reallocation ([Chi et al., 2019](#)).

Direct cash flow is one of the most frequently used methods to evaluate the investment projects in a company. This method uses actual cash inflows and out flows from the company's operations and only measures cash that has been received. Typically this cash is from customers and the payments or outflows to suppliers. The inflows and outflows are netted to arrive at the cash flow ([Bradbury, 2011](#)). Therefore, some companies must frequently correct the project investments to adjust operational flexibility and other strategic plans that they believe to be as important as direct cash flow ([Schachter and Mancarella, 2016](#)). Specifically, flexibility is defined as the faculty to operate with a profit at different production levels considering the possibility of changing volume capacity at anytime during the project lifetime ([Fontes, 2008](#)). In this regard, strategy in resource allocation and managerial flexibility must be included at the time of evaluating a product's innovation project.

To decide between different capital investment alternatives, companies have frequently relied on metrics such as the payback period, Internal Rate of Return (IRR), and Net Present Value (NPV). However, decision-makers in companies do not consider that those metrics require the assumption of certainty for project cash flow calculations, despite the fact that this is infrequently the case in real-world situations. For instance, NPV measure operates better when future cash flows are reasonably certain. The Discounted Cash Flow (DCF) methodology is not accurate when used to assess strategic investments where the payoff is uncertain. That is, the product market may become volatile and consequently cause big fluctuations in the demand due to high uncertainty and very competitive interactions. Therefore, the real cash flow diverges from what the company

expects ([Gaspars-Wieloch, 2019](#); [Miller and Park, 2002](#); [Trigeorgis, 1996](#)). Most results based on these metrics usually does not achieve companies' expectations. Real observations frequently deviates from the current cash flow estimations, and as a result, there exists discrepancies between traditional finance methodology and business reality. For instance, it is shown that operating cash flows consistently outperformed earnings in predicting future cash flows ([Nallareddy et al., 2020](#)).

Another fallacy in those financial metrics is reflected in the NPV evaluation with the assumption of a constant initial capacity for the operational system of the project. Such capacity runs immediately and uninterruptedly, and produces the same amount of product during the expected project life ([Smit and Trigeorgis, 2006](#)). Companies prefer to value NPV incorporating flexible capacity when there is a large market with potential uncertainty in the demand. As information in the product market is updated, future cash flows are adjusted by modifying the original operational strategy with flexible capacity. As a consequence, companies can capitalize on their investments based on promising future opportunities and to mitigate losses ([Wei et al., 2019](#)).

It is also important to note that the companies operate in highly competitive industries. They must carefully monitor the signals sent out by the other competitors and incorporate this information when planning their own strategies to defend their positions. This has lead researchers to identify factors related to a firms knowledge, initiative, and efficacy. These factors are used in predicting its competitive behaviour. Empirical studies have shown evidence that factors such as opponents' action features include awareness, action volume, visibility and potential impact ([Chen et al., 2017](#)). One of the strategies that competitors use is an advance notice. This pre-announcement may discourage competitors from entering the market. It may also impact financial markets, discourage potential customers from buying existing products, and to test a new concept. Many researchers have noted that advance notice are often made to provide knowledge to competitors. Although many view these knowledge as risks to the innovator company ([Klastorin et al., 2016](#)). The real options approach has been proven as a valuable tool to resolve these problems in the fields of finance, economics and other academic disciplines.

[Bayus \(1992\)](#) provides an insight into how competitiveness and micro-economic variables may affect product introduction strategies. This study presents a model predicting an association between product development time and cost as product incorporation in the market. The model includes variables related to different markets, product demand, costs that maximize profit, time

to market and decisions based on product execution. Bayus (1992) focuses on two premises: First, a competitive product is introduced to the market and the choice to improve development focuses on enhancing the opponents' innovations. Second, the competitive product is projected to be introduced in the market in the future. The alternative is to release the product in the market before the competition. In both scenarios, Bayus (1992) analyses variables such as product lifetime, time to peak sales, product margins decreasing over time, opponent product performance, marketing advantage, minimum and maximum costs.

## 1.1 Aim and Research Contributions

In this research, we develop a real options valuation model in a duopoly environment that integrates stochastic product life cycle and flexible capacity considerations to devise strategies in introducing a new product to the market. We assess the option by switching capacities during the project's lifetime. The idea of switching capacities is to expand the scale of production if market conditions improve more than expected, or to contract the scale of production if market conditions get worse (Pindyck, 1988). The two companies participating in the market compete on the quantity to produce; and to calculate such quantity, we use game-theoretic models for a duopoly market. In particular, we consider two different game-theoretic approaches, which are based on the concept of Nash Equilibrium (NE). In the first approach, competitors have imperfect information (i.e. Cournot's model) and in the second approach competitors have perfect information (i.e. Stackelberg's model). Further, we represent the stochastic product life cycle using a lattice approach for two regime-switching processes: growth and decay. We assume that the cycle starts in the growth regime where demand generally increases and then, the regime switches to decay at some point during the project life-time, where demand generally decreases.

The main criteria to evaluate our models is to select a capacity level at the beginning of the project that maximizes the NPV. This capacity value corresponds to the optimal capacity and represents the investment amounts by the company. NPV calculations in our model are based on the expected discounted cash flows that are obtained through dynamic programming. More specifically, the cash flows represent the profit functions of the demand dynamics, and a set of the different capacity values and the production quantities are obtained from each game-theoretic

models. By comparing the outcomes of the game theoretic models, we aim to identify the best game-theoretic strategy to follow in a duopoly market, which maximizes the overall NPV.

It is important to note that our analysis is based on the observation that real options assessment can be complemented with game-theoretic approaches in a competitive environment to develop a strategic investment model that includes endogenous competitive reaction. The assumption of contrarian quantity competition establishes a reaction function that helps calculate quantity, profits, and project values under equilibrium actions. The methodology to assess the real option is similar to a binomial option-valuation tree influenced by different market structures ([Smit and Trigeorgis, 1995](#)). [Wu and Chen \(2015\)](#), [Elliott and Siu \(2015\)](#) and [Thijssen \(2013\)](#) investigated financial options in portfolio valuation with sub-games NE, preemption, Markov perfect equilibrium and mixed strategy game-theoretic models where the underlying assets are an interest rate or price of the stock to calculate efficient frontiers. Their approaches are similar to the one employed in this research. However, they do not incorporate all the elements described in this research. They use multi-period and regime-switching lattice in a continuous time with finite states and a hidden Markov-modulate Poisson process to solve their models. The model evaluated in this research employs discrete-time, real options valuation with capacity flexibility found on the underlying product demand follows a geometric Brownian motion and the game-theoretic strategy is pure based on the concept of NE. Furthermore, these models are not used to assess the feasibility of introducing a product to the market. Some other studies focused on real options with regime-switching processes only for mixed strategies in a Stackelberg game based on a continuous-time and infinite-horizon analytic model with stochastic demand behavior, and the addition of other production variables in their models other than operational costs ([Dias and Teixeira, 2003](#); [Li and Sethi, 2016](#)). Although these studies applied a Brownian motion theory and a strategy of a leader and follower in a duopoly market, they differ from the model proposed in this study since they do not use discrete-time, finite lifetime of the project and pure strategy in their game-theoretic methodology. In addition, several studies proposed game-theoretic models for two-period problems with a binomial lattice or the comparison with a multi-period problem with also a binomial lattice. They only use real options with empirical information to make decisions about timing, expanding, delaying or abandoning the project in a discrete-time environment ([Ashuri and Kashani, 2011](#); [Charalampopoulos et al., 2011](#); [Pendharkar, 2010](#)). Their valuation ignores some elements of our theoretical model such as: the

effects of the competition, the product life cycle and the flexibility in capacity when a contraction must be applied due to an unfavourable change in the demand.

The main contribution this thesis makes to literature is the application of real options valuation in a duopoly market where participants compete for quantity to produce. Such quantity is established by a game-theoretic model equilibrium, defined as a pure strategy where none of the competitors have any incentive to deviate. A pure strategy allows a complete description of how every competitor will play the game. Particularly, such strategy determines the move a competitor will make for any state it could possibly face. Contrastingly, a mixed strategy assigns a probability to every pure strategy. The probability assignment permits a competitor to randomly choose a pure strategy. Because probabilities are continuous, an infinite number of mixed strategies are available to each competitor. Therefore, using a mixed strategy, the model would have a countless number of outcomes, which makes it more complex and difficult to analyze ([Rubinstein, 1991](#)). The goal of this study is to identify the game strategy that provides the maximum NPV given an optimal capacity to invest in at the beginning of the project in a multi-period framework. Additional contributions are the incorporation of the following methodologies : First, we integrate a regime-switching lattice approach to the model in order to estimate the demand for the product and calculate cash flows derived by the profits obtained in the game theory methodology ([Bollen, 1999](#); [Wahab and Lee, 2011](#)). Second, our model accounts for the managerial flexibility, which is associated with either expansion or contraction in the capacity as needed due to changes in the demand. The flexible capacity adds or subtracts value to the expected discounted present value. Third, we evaluate the model using dynamic programming considering a project lifetime. As the demand varies over time, a range of different capacities is considered to meet such demand with the objective of maximizing the profits defined as cash flows. Last, we carry out an extensive numerical study and assess the sensitivity of the outcomes and the strategies to the model parameters, which provides valuable insights in implementation of the proposed models in practice.

## 1.2 Organization of the Thesis

The remainder of this thesis is organized as follows: Chapter [2](#) reviews the related literature on product introduction, options valuation, game theory, lattice and regime-switching and dynamic

programming, and also provides background information about real options valuation and game-theoretic methodology. Chapter 3 presents a general model that incorporates calculations of profits based on two different games, flexible capacity, demand dynamics, dynamic programming, and lattice construction. Chapter 4 contains numerical examples and sensitivity analysis that illustrate what capacity value maximizes the NPV of the project under the variation of inputs and game-theoretic models. Chapter 5 concludes this research.

## Chapter 2

### Literature Review

In this section, we review the existing literature on the real options valuation, capacity flexibility, game theory, regime-switching lattice and dynamic programming approaches and discuss the research gaps. This literature review particularly focuses on the application of game theory to options valuation and most common methodologies to evaluate the viability of introducing a product to the market.

Our study is an extension of two important papers in the field of financial engineering and option valuations. First, [Bollen \(1999\)](#) has created a framework of real options valuation for expansion and contraction in capacity separately. He assumes that demand follows a stochastic product life cycle represented by a regime-switching lattice in a monopolist market evaluation. Monopolist refers to only one competitor or company in the market. Second, [Wahab and Lee \(2011\)](#) presented a lattice approach to evaluate gasoline price swing option where the underlying variable is represented by an  $n$  regime-switching model. Both propose a framework to value options where the behaviour of the underlying asset is uncertain and the option is estimated by using lattice approach in a regime-switching process. However, these two studies do not consider the reaction of the competition at the moment of assessing the option. Additionally, [Bayus \(1992\)](#) argues that one of the reasons that prices of new long-lasting goods decrease with time is the method in which demand is being modeled. Furthermore, [Bayus \(1992\)](#) points out that competitive effects such as new products' introduction to the market may explain why profits decrease over time.



## 2.1 Product Introduction

Every time a product is launched, companies might need to redesign their supply chains in order to efficiently and effectively present a new product to its target market. The characteristics of a product influence the supply chain performance during the process of the new product development. The impacts of the new product characteristics is defined by decisions that involve supply chain structure, strategy and the interaction among all the participants in the supply chain (Crippa et al., 2010). Kanno and Shibata (2019) studies how a company's strategic behaviors on product design is affected by the Product Life Cycle (PLC) during the stages of growth and maturity stages. Kanno and Shibata (2019) also emphasizes that the performance of the company's strategy in the design of new products is highly influenced by the behavior of the growth, uncertainty, and the level of competition in the market. Moreover, this study states that the level and reaction of customer orientation vary with stages of the PLC.

Dawid and Gezer (2019) presents a study that evaluates the optimal timing to introduce a new product in a duopoly market, which is impacted by the initial capacities for the new product and the costs associated to such capacities. Seidl et al. (2019) focuses on the relation between new product pricing, capacity investments and the timing. The main result of this study points out that a company should invest most of the capacity just after the new product has been introduced in the market. Budler and Trkman (2017) notes that companies should examine their investment projects through game-theoretic reasoning. The study states that game-theoretic approach is useful in different strategic circumstances where different companies interact in the market in order to obtain competition advantages. The real options method has obtained massive popularity among researchers as a methodology that extends the alternative of traditional financial evaluation for projects on product innovation. This method assesses the evaluation of innovative projects and incorporate managerial flexibility in the implementation of projects with a high degree of uncertainty (Evseeva, 2019).

## 2.2 Real Options

Real Options is the implementation of option pricing methods to evaluate non-financial investments with flexibility such as manufacturing plant contraction or expansion (Myers, 1977). One of the most conventional considerations in real options is capacity flexibility. Previous literature has focused on combining operations research methods with game theory applied to the real options to improve decision-making in project investments. Such interdisciplinary research have been useful in capturing and evaluating the implicit flexibility in many operating decisions that are faced by the decision makers (Trigeorgis and Tsekrekos, 2018). The output of a real option valuation is represented by a strategic NPV that effectively integrates the interaction between the sources that add value to the project from making a strategic commitment as the optimal flexibility approach and game theoretical approaches in a production environment. By properly combining real options and game theory, the NPV provides companies a better interpretation of the numerous important components and compensation from a strong competing action plan (Dixit et al., 1994; Smit and Trigeorgis, 2017).

Real options differs from regular financial options in various ways. They are not traded as securities, and the decisions made do not rely on a traded underlying asset but mostly on a project. Also, real option holders can have an influence on the value of the project, which does not happen with a financial option where the price of the underlying asset are established by the market (Amram and Howe, 2003). Besides, financial portfolio administrators are not able to quantify the volatility associated to the project, and alternatively must depend on their own uncertainty appreciations. In contrast to the financial options, organizations must create or find real options, and such process requires effort and entrepreneurial tasks. The main advantages of the real options are that they are more valuable as volatility gets higher, and the project evaluator has the proper flexibility to modify the sequence of the project in a favorable direction, exercising the options when needed. Table 2.1 presents some business initiatives that companies can undertake to exercise the most conventional real options.

One of the problems of traditional budgeting is the ongoing inconsistency between traditional finance methodology and business validity. That is, a company's directors must constantly fix conventional investment criteria to satisfy operating flexibility and to prevent losses. Some other

Table 2.1: The most conventional real options (Trigeorgis, 1996).

Category	Description	Fields of application
Defer	The organization has a lease on a valuable land with the option to buy, then they wait a certain time to evaluate if the upcoming prices justify to build a facility.	<ul style="list-style-type: none"> <li>- Farming</li> <li>- Extraction of natural resources</li> </ul>
Time-to-build (staged investment)	Preparing investment as a sequence of payments gives the option to abandon the project halfway through if the new information is not satisfactory. Every stage is considered as an option on the value of consecutive stages and evaluated as a compound option.	<ul style="list-style-type: none"> <li>- Pharmaceutical companies</li> <li>- R &amp; D businesses</li> </ul>
Alter operating scale	When market circumstances become more beneficial than expected, the company can increase the scale of production. When the opposite, it can decrease the scale of operations.	<ul style="list-style-type: none"> <li>- Mining</li> <li>- Commercial real estate</li> </ul>
Abandon	If market conditions reduce harshly, company can abandon existing operations perpetually and sell capital equipment and the rest of assets on a secondhand market.	<ul style="list-style-type: none"> <li>- New product introduction</li> <li>- Airlines</li> </ul>
Switch	when prices or demand fluctuate, the production combination of the plant can be changed, called product flexibility. On the other hand, the same production can be used utilizing different kinds of raw material, called process flexibility.	<ul style="list-style-type: none"> <li>- Toys</li> <li>- Autos</li> <li>- Electrical power</li> </ul>
Growth	The case of a primary investment is a condition in a series of interconnected projects to create future growth openings. It is also related to an interproject compound option.	<ul style="list-style-type: none"> <li>- Multinational operations</li> <li>- Strategic acquisitions</li> </ul>
Multiple interacting	Projects in real life frequently include a set of several options. To improve when conditions get better or to protect when conditions get worse. The aggregated value may diverge from the summation of all the values separately.	<ul style="list-style-type: none"> <li>- Most of the industries in the economy</li> </ul>

failures associated with traditional capital budgeting are

- centering on quantifiable cash flow rather than on intangible strategic advantages that derive from developing competitive benefit
- inability to seek to properly adjust the timing and risk of the cash flows

Subsequently, we review three important approaches in this practice: traditional capital budgeting, option-pricing theory and discrete-time analysis, and the multiplicative binomial process.

### **2.2.1 Traditional Capital Budgeting**

Capital budgeting studies the resource allocation within long-termed investment projects. The main concept is to sacrifice current utilization of money, making an immediate investment outlay to accomplish utilization of money in the future. The interchange between utilization today and in the future is a very important choice that a person or company must directly face every day. The person or company's financial target is to choose amongst all the different patterns of consumption and investment prospects to reach the highest satisfaction that maximizes their satisfaction or consumption utility function during the time. Instead of maximizing the utilities of each stockholder that may have different levels of wealth, preferences for current versus future consumption, and attitudes toward risk; companies usually avoid such conflicts of interests by assuming all owners' wealth as maximization utility goal. One of the biggest assumptions in the capital budget model is that there is a perfect and broad capital market, where individuals can modify their income flows and investments by borrowing or lending; selling or buying in the market in the sums that maximizes each person's specific utility.

NPV is the most commonly used consistent calculation to assess the company's objective, which is to maximize its shareholder's wealth. Consequently, it enables every single shareholder to maximize his or her own utility throughout a linked financial measure in the capital markets. Other commonly employed performance measures include the IRR, book rate of return and payback period ([Brealey et al., 2014](#)).

### 2.2.2 NPV Under Certainty

NPV under certainty is considered under an investment scenario of at least two periods, and in a competitive market with perfect information where individuals borrow or lend in an unlimited way at an equal constant interest rate  $r_f$ . Therefore, a person can lend one dollar of his or her present income  $I_0$  considered as initial investment, in a trade for  $(1 + r_f)$  dollars of  $I_1$  next period as compensation, or that person can use an initial income  $I_0$  to buy a capital asset which provides the buyer a future income of  $I_1$ . This last scenario creates a present value  $PV = I_0 + \frac{I_1}{(1+r_f)}$ . In a multi-period setting, for a given project life period  $T$  and an initial investment of  $I_0$ , NPV can be obtained as

$$NPV = \sum_{t=1}^T \frac{I_t}{(1 + r_f)^t} - I_0 \quad (2.1)$$

where  $r_f$  is the risk-free rate or opportunity capital cost and  $I_t$  is the certain net cash inflow in year  $t$ . Additionally, if a set of future investments  $O_t$  are required, instead of only one investment; the present value of cash outflows is subtracted from the present value of cash inflows to obtain the NPV. The present value of cash outflows is represented by,

$$I_0 = \sum_{t=0}^T \frac{O_t}{(1 + r_f)^t} \quad (2.2)$$

Besides, if the discount rate changes from period to period, the NPV is calculated as

$$NPV = \sum_{t=1}^T \frac{I_t}{(1 + r_{f1})(1 + r_{f2}) \cdots (1 + r_{ft})} - I_0 \quad (2.3)$$

where  $r_{ft}$  is the risk-free interest rate at period  $t$ .

### 2.2.3 NPV Under Uncertainty

Uncertainty and risk are unavoidable in a volatile market where the corporate decisions must be made. Volatility is fixed progressively, and the cash flows' estimation is usually faulty and conditioned to an error. Hence, stakeholders' attitude toward risk must be considered in the development of capital budgeting. When volatility and uncertainty exist, variables are random and follow a probability distribution of their likely results. Their dispersion of likely outcomes

becomes an indicator of how risky this variability is, therefore NPV is estimated under a expected value of cash flows instead of constant ones, that is,

$$\text{NPV} = \sum_{t=1}^T \frac{\hat{I}_t}{(1 + r_f)^t} - I_0 \quad (2.4)$$

Note that  $\hat{I}_t$  is substituted by its certainty equivalent quantity in this formula. Such amount represents a determined cash flow in period  $t$  that equals the *present value* to the uncertain one in the same period of time denoted by

$$\text{PV} = \frac{\hat{I}_t}{(1 + r_f)^t} = \frac{\text{E}(I_t)}{(1 + r_a)^t} \quad (2.5)$$

where  $\text{E}(I_t)$  is the expected cash flow value in period  $t$  and  $r_a$  is the risk-adjusted discount rate that is explained in the next subsection. Subsequently, NPV under uncertainty can be estimated by combining the Equations (2.4) and (2.5) as follows:

$$\text{NPV} = \sum_{t=1}^T \frac{\hat{I}_t}{(1 + r_f)^t} - I_0 = \sum_{t=1}^T \frac{\text{E}(I_t)}{(1 + r_a)^t} - I_0 \quad (2.6)$$

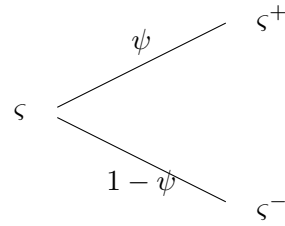
#### 2.2.4 Option-Pricing Theory and Discrete-Time Analysis.

In this section, we summarize and explain fundamental definitions and formulas of financial options pricing applied to real options valuation in a discrete time. Options are a very important and useful tool to mitigate the risk in financial instruments; therefore, the option pricing theory is established to evaluate real options. There are four types of financial options which are presented below and all of them depend on the underlying asset (in this case of a stock price) at a moment  $t$ , the strike price,  $\varsigma$ , the maturity or expiration date,  $T$ , and the exercise price,  $\varepsilon$ . The notation below represents the values of the different options and is used for further analysis in this subsection (Trigeorgis, 1996):

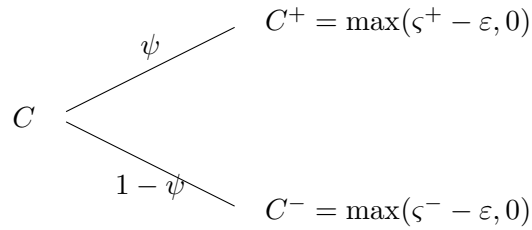
- $C = C(\varsigma, \tau, \varepsilon)$ , American call option
- $c = c(\varsigma, \tau, \varepsilon)$ , European call option
- $P = P(\varsigma, \tau, \varepsilon)$ , American put option

- $p = p(\varsigma, \tau, \varepsilon)$ , European put option

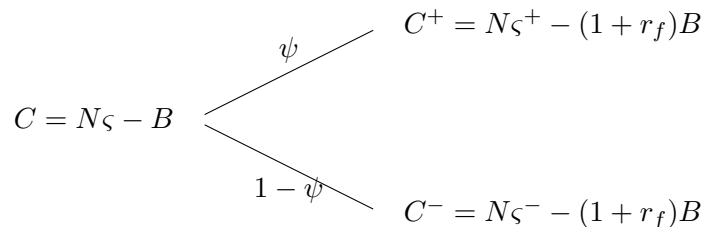
where  $\tau$  is the option time to expiration ( $T - t$ ). The difference between American and European options is that the first one can be exercised at any time before maturity, while the second can only be exercised at maturity. The goal of this evaluation is to build a portfolio that involves buying  $N$  shares of the underlying stock, and borrowing an amount  $B$  to purchase them at the risk-free interest rate  $r_f$ . It would precisely imitate the upcoming option's returns in different states of nature. One of the biggest assumptions in this technique is that the stocks must be sold for the current price to avoid free-of-risk arbitrage. Specifically, for the purpose of this study the only valid type of trade is hedging. The price of the stock may move up ( $u$ ) with a price of  $\varsigma^+ = u\varsigma$  or down ( $d$ ) with  $\varsigma^- = d\varsigma$ .  $u$  and  $d$  are called multiplicative parameters and there is a probability ( $\psi$ ) attached to go up and  $(1 - \psi)$  to go down. These parameters are used all the way along this section.



The option price attached to the underlying stock ( $\varsigma$ ) during the same period is:



Building the portfolio explained above, it is for a Call option = (Buy  $N$  shares at  $\varsigma$  and borrow  $B$  at  $r_f$ ). Consequently, it represents  $C = N\varsigma - B$ . Substituting those values in the previous decision tree,



The values of  $N$  and  $B$  need to be calculated; therefore, solving  $C^+$  and  $C^-$ ,

$$N = \frac{C^+ - C^-}{\varsigma^+ - \varsigma^-} \quad (2.7)$$

and

$$B = \frac{N\varsigma^- - C^-}{1 + r_f} \quad (2.8)$$

$N$  is known as hedge ratio and is acquired as the difference between option prices divided by the difference of stock prices in the discrete event.

Substituting Equations (2.7) and (2.8) in  $C = N\varsigma - B$ , and in

$$C = \frac{\psi C^+ - (1 - \psi)C^-}{1 + r_f} \quad (2.9)$$

The risk-neutral probability is obtained by two different formulas. This probability is the one that overcomes in a setting where investors are indifferent to risk,

$$\psi = \frac{(1 + r_f)\varsigma - \varsigma^-}{\varsigma^+ - \varsigma^-} \quad (2.10)$$

A put option can be assessed in a same way, the only difference is that the shares must be sold, instead of being bought; and lending, instead of borrowing, at the risk-free rate. Hence, Put option = (Sell  $N$  shares at  $\varsigma$  and lend  $B$  at  $r_f$ ).  $N$  is equal to value of the similar call option minus 1. The negative sign specifies selling instead of buying.

$$B = \frac{N\varsigma^- - P^-}{1 + r_f} \quad (2.11)$$

where  $P = N\varsigma - B$

The list of fundamental assumptions that are used for option valuation which permit continuous trading is provided below.

- Markets with no friction which implies that there are no transactions costs and taxes, no limits for margin requirement, all shares of the stock are infinitely divisible; and borrowing and



lending are not delimited.

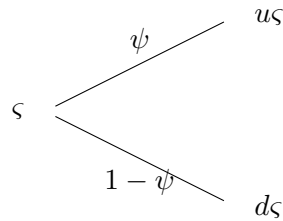
- The short-term risk-free rate is considered estimated and constant during the option's lifetime.
- The stock or asset does not pay dividends during the option's lifetime.
- Stock prices have a stochastic diffusion Wiener process expressed in the equation below. This process is substituted by a multiplicative binomial process or random walk in the discrete-time situation represented by

$$\frac{d\varsigma}{\varsigma} = \mu dt + \sigma dz \quad (2.12)$$

where  $\mu$  is the stock's sudden expected return,  $\sigma$  is the stock's sudden standard deviation that is considered constant, and  $dz$  is the differential of a standard Wiener process, with mean 0 and variance  $dt$ .

### 2.2.5 The Multiplicative Binomial Process: Discrete Time.

This process assumes that the stock price's moves is distributed by a multiplicative binomial function over consecutive periods defined by



where  $u$  and  $d$  characterize the continuously compounded rate of return when  $\varsigma$  moves either up or down, respectively, assuming  $d = 1/u$ ; and to stop traders from risk-free arbitrage profit chances,

$u > (1 + r_f) > d$ . We obtain following formulas from this process:

$$u = \frac{\varsigma^+}{\varsigma} \quad (2.13)$$

$$d = \frac{\varsigma^-}{\varsigma} \quad (2.14)$$

$$N = \frac{C^+ - C^-}{(u - d)\varsigma} \quad (2.15)$$

$$B = \frac{dC^+ - uC^-}{(u - d)(1 + r_f)} \quad (2.16)$$

$$\psi = \frac{(1 + r_f) - d}{u - d} \quad (2.17)$$

This procedure can be prolonged to multiple periods. When  $\tau$ , the option time to expiration, is divided into  $n$  equivalent intervals,  $h = \tau/n$ , and similar estimation is recurrent at the expiration date beginning and moving backwards recursively; therefore, the official multiplicative binomial option-pricing formula for  $n$  can be denoted as

$$C = \frac{\sum_{j=0}^n \binom{n}{j} \psi^j (1 - \psi)^{n-j} \max[u^j d^{n-j}(\varsigma - \varepsilon), 0]}{(1 + r_f)^n} \quad (2.18)$$

where  $\max[u^j d^{n-j}(\varsigma - \varepsilon), 0]$  is the result of the call option value at maturity given that the stock goes up  $j$  times, each one by  $u\%$ , and down  $n - j$  times, each one by  $d\%$  during  $n$  periods. The sum of all the likely  $j$  option values at maturity, times the probability that each happens gives the expected final option value. This result is discounted at the risk-free rate during the total  $n$  periods.

In addition to the financial option pricing theory explained previously, which provides the fundamentals to assess real options valuation, some researchers have added more variable to strength such valuation. They have studied volume flexibility in manufacturing, that is defined as a system that is able to operate making profits at a different overall output levels. Such flexibility allows the plant to expand or contract production within certain ranges (Bengtsson, 2001). Tannous (1996) created a model to estimate volume flexible for equipment when demand is uncertain, which is subsequently used to find the optimal level of investment. They find that manufacturing systems that incorporate a high level of flexibility can be quickly changed to a low expenditure in accordance with the properties of the specific production task (Abele et al., 2006).

[Bengtsson \(1999\)](#) formulates a real industry situation where the company has the value of an option to hire personnel on short contracts when demand of a product is uncertain. Nonetheless, in our study, we follow this real options valuation theory with capacity flexibility by adding the effect of two companies competing for production in the market on a perfect equilibrium game theory and a multi-period regime-switching approach to estimate a value of an optimal capacity that gives us the value to invest in to obtain the maximum NPV.

## 2.3 Game Theory

Nash equilibrium (NE) is defined as a result of a game that involves two or more players, where each player assumes or knows the strategies of his or her opponents, and no player has the incentive to deviate by changing only his or her own strategy unilaterally ([Tang and Zhang, 2016](#)).

[Stamatopoulos \(2016\)](#) explains NE with the interaction of two firms. His paper describes the theory of strategic competitiveness to compare between the models of simultaneous and sequential quantity competition in a duopoly scenario. Cournot and Stackelberg's models of competition are two important strategies in industrial economics. The first model assumes that companies compete simultaneously in the market. On the other hand, the second is established on the assumption that they compete sequentially. The outcomes of the two models generally differ regarding equilibrium prices, quantities and profits. Duopoly is the definition of a market influenced by two companies, where the action of one company affects the other; and consequently, the price of the good ([Ibrahim, 2019](#)).

Game theory is an examination of strategies in which decision makers interact. It applies to competitive scenarios where the result of a participant's possible course of action relies judgmentally on other participants' actions. Players illustrates an idea into a model, including characteristics of the situation with possible likely logical scenarios that seem to be relevant. This theory primarily focuses on illustrating political, economic, and biological phenomena. For the purpose of this paper, game theoretic models discuss are only applied to economics; specifically highlighting companies competing for business. For ease of presentation, this theme is explained by some basic concepts, namely, Nash Equilibrium theory, Cournot's model of oligopoly and Stackelberg's model of duopoly ([Osborne, 2004](#)).

These models are based on the theory of rational choice, which is defined as decision-makers select the best action (chosen by a set  $A$  containing all actions accessible at certain moment) consistent with their preferences (according to their utility function) amongst all other preferences available to them. It is important to point out that these decisions are not to be made based on their likes or dislikes, but on the diverse sets of available actions. In the preference passage, it is presumed that decision-makers recognize what pair of actions they prefer when any pair are accessible, or whether both actions are equally desirable, so they are indifferent between them. One clear example is if the player prefers the action  $a$  to the action  $b$ , and the action  $b$  over the action  $c$ , that implies that he or she prefers action  $a$  to action  $c$ . As an alternative method, a player's preference can be denoted by a payoff function, that assigns a number to every action inferring that greater numbers are preferred. To clarify this point, the payoff or utility function  $u$  denotes a decision-maker preference if for any actions  $a$  and  $b$  in the set  $A$ ,  $u(a) > u(b)$  if and only if the player prefers  $a$  over  $b$ . In this study, the utility function (or payoff) theory is applied for all the assumptions. Fundamentals of this theory is out of scope of this research, and more details about this theory can be found in [Winston and Goldberg \(2004\)](#)'s study.

A strategic game contains three basic elements: (1) a set of players that make decisions, (2) a set of probable actions associated to each decision-maker that allow them to interact and also be influenced by the actions of all the decision-makers, and (3) the player's preferences across the set of actions quantified by the utility function. For this study, the players are companies or firms, the actions are quantities to produce and the preferences are the result of the company's profits further represented in a NPV as an outcome to compare between each competitor in a different game-theoretic model ([Watson, 2002](#)).

### 2.3.1 Nash Equilibrium

NE aims to answer the question of what actions should be taken by the players based on their preferences. This is when the theory of rational choice play an important role. The leading action for each player is generally affected by the other players' actions in a specific game. Consequently, one decision-maker must consider what actions other players select. Habitually, these decisions are influenced by what the players believe their opponents have chosen in previous experiences playing the same game. It provides the player a pattern of how the adversary might behave. There are two

strong assumptions in this model: First, the player does not know, or nobody tells him or her how the other player(s) might behave. All is derived by prior involvement in this game. And secondly, the decision-maker has prior experience playing this game and observes each play in the game separately. The player does not condition the actions on the type of rival he or she faces because the decision-maker is not familiar with a specific opponent's performance. The player assumes his or her actions have certain effects on opponent's forthcoming behavior.

Nash equilibrium refers to an action  $a^*$  with the property that any other player  $i$  cannot do better by selecting a different action  $a_i^*$  when the opponent  $j$  adheres to  $a_j^*$  (Osborne, 2004).

$$u_i(a_i^*) \geq u_i(a_i, a_{-i}^*) \text{ for each } a_i \text{ decision-maker } i, \quad (2.19)$$

where  $u_i$  is the utility function for preference player's  $i$  and  $a_{-i}^*$  is the action selected  $a_i^*$  excluded from the set  $A$ .

### 2.3.2 Cournot's Model of Oligopoly

This model studies the results of the competition amongst firms in an industry that relies on the demand characteristics, the type of the company's cost functions, and the quantity of firms. In the field of economics, oligopoly refers to a market where a small number of suppliers control a product; and each of these participants can influence prices and affect other competitors. The general assumptions for this model are

- Only one product manufactured by  $n$  firms.
- The company  $i$ 's cost of making  $Q_i$  units of the product is given by  $C_i(Q_i)$ .
- $C_i(Q_i)$  is an increasing function, which means the bigger the production, the more costly it is to manufacture.
- The product is sold at a unique price  $P(Q)$  established by the company's total production  $Q$  and the product's demand.
- $P$  is called the inverse demand function, which means when the company's total production rises, the price declines, and  $P$  is always a positive value.

- If production of each company  $i$  is  $Q_i$ , the price function is  $P(Q_1 + Q_2 + \dots + Q_n)$ .

Accordingly, the company  $i$ 's profit can be obtained as:

$$\Pi_i(Q_1, \dots, Q_n) = P(Q_1 + \dots + Q_n)Q_i - C_i(Q_i) \quad (2.20)$$

Cournot's model is proposed to be the next strategic game: The companies are the players, the production is the company's set of actions, and each firm's profit is the utility of its preferences. To explain how this model works, an example with only two firms is presented. A market where only two companies are the suppliers of a specific product is called duopoly. In addition to this example, the unit costs are constant ( $c$ ); and the price follows a linear inverse demand function. The values of  $C_i = c$  and  $P$  are used to obtain a Cournot's Nash equilibrium point in this game. Let the price function be (Pindyck, 1988),

$$P(Q) = \begin{cases} \theta - Q, & \text{if } Q \leq \theta \\ 0, & \text{if } Q > \theta \end{cases} \quad (2.21)$$

where  $\theta$ , the product's demand in the market, and  $c$  are constants greater than zero. We set  $c < \theta$  to assure that for some value of  $Q$ ,  $P(Q) > c$ , then the company can make any profit. Figure 2.1 shows the inverse demand function.

To calculate the Nash equilibria, the company's best response function is used and explained afterwards. Assuming that companies' production equals to  $Q_1$  and  $Q_2$ , respectively, and based on Equations (2.20) and (2.21).

$$\Pi(Q_1, Q_2) = \begin{cases} Q_1(\theta - c - Q_1 - Q_2), & \text{if } Q_1 + Q_2 \leq \theta \\ -cQ_1, & \text{if } Q_1 + Q_2 > \theta \end{cases} \quad (2.22)$$

The next step is to infer about Company 1's best response, then this company's profit must be represented by a function of  $Q_1$  with  $Q_2$ . Let  $Q_2 = 0$ , then  $\Pi(Q_1, 0) = Q_1(\theta - c - Q_1)$  only if  $Q_1 < \theta$ . As shown in the darkest curve in Figure 2.2, it is a quadratic function that equals zero for  $Q_1 = 0$  and  $Q_1 = \theta - c$ . The value that maximizes the profit according to the quadratic symmetry, deriving and equaling the derivative to zero is  $Q_1 = \frac{1}{2}(\theta - c)$ . Consequently, the Company 1's best response

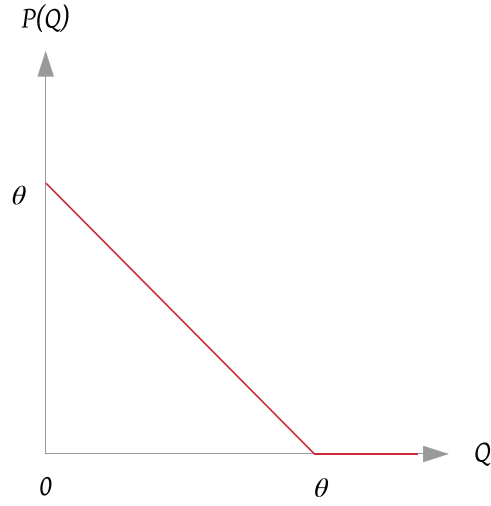


Figure 2.1: The inverse price-demand function

to a production of zero for Company 2 is  $R_1(Q_2 = 0) = \frac{1}{2}(\theta - c)$ .

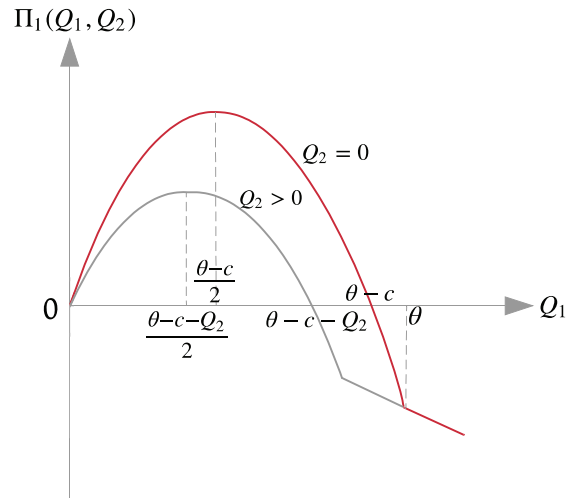


Figure 2.2: Company 1's profit as a function of its production given Company 2's production

The light curve in Figure 2.2 shows the behavior of  $\Pi(Q_1, Q_2)$  for  $Q_2 > 0$  and  $Q_2 < \theta - c$ . Following the same methodology for  $Q_2 = 0$ , the next formula represents a general solution of the

Company 1's best response,

$$R_1(Q_2) = \begin{cases} \frac{1}{2}(\theta - c - Q_2), & \text{if } Q_2 \leq \theta - c \\ 0, & \text{if } Q_2 > \theta - c \end{cases} \quad (2.23)$$

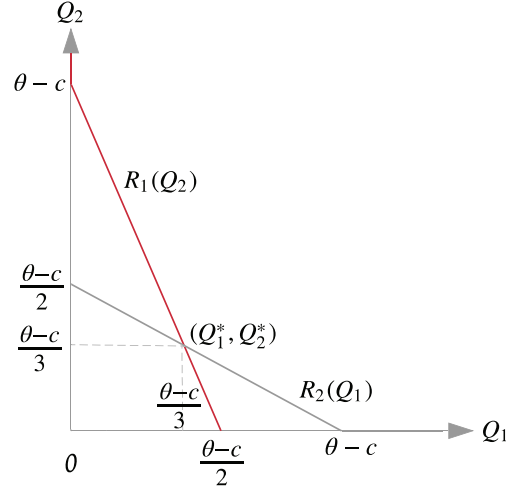


Figure 2.3: The unique Nash equilibrium for the best response functions

For the cost function for companies 1 and 2 are identical, the best response function for  $b_2$  is the same, and we obtain  $R_1(Q_2) = R_2(Q_1)$ . Figure 2.3 illustrates how the Nash equilibrium point  $(Q_1^*, Q_2^*)$  is acquired from the intersection of the best response curves  $b_1$  (values plotted on the vertical axis depending on  $Q_2$ ) and  $b_2$  (values plotted on the horizontal axis depending on  $Q_1$ ). Therefore, we observe that  $Q_1^* = R_1(Q_2)$  and  $Q_2^* = R_2(Q_1)$ . As detailed in the graphic and solving the below two linear equations:

$$\begin{aligned} Q_1 &= \frac{1}{2}(\theta - c - Q_2) \\ Q_2 &= \frac{1}{2}(\theta - c - Q_1) \end{aligned}$$

which leads to the optimal production points for each firm as

$$Q_1^* = Q_2^* = \frac{1}{3}(\theta - c) \quad (2.24)$$



The total production in the equilibrium point is  $Q_1^* + Q_2^* = \frac{2}{3}(\theta - c)$  and the price at this quantity is  $P[\frac{2}{3}(\theta - c)] = \frac{1}{3}(\theta + 2c)$ .

### 2.3.3 Stackelberg's Model of Duopoly

This methodology is built on the same assumptions, players, actions, and preferences as Cournot's model of oligopoly with two players. However, the companies make decisions sequentially instead of concurrently. This means that one company selects its production level, and consequently the other carries out knowing the level chosen by the first one.

This game is described as follows: Company 1 proceeds at the beginning, then the Company 1's strategy is merely a production level. Company 2 moves next after seeing the history where Company 1 picks a production level. Therefore, Company 2's strategy becomes a function that links a Company 2's production with every likely production level of Company 1. Stackelberg's model uses backward induction to calculate the sub-game perfect equilibrium point, which is explained below:

- Initially, for any production level of Company 1 ( $Q_1$ ), the Company 2's production level that maximizes its profit is calculated  $R_2(Q_1)$ . Subsequently, the strategy for Company 2's perfect equilibrium in the sub-game is  $R_2$ .
- Then, Company 1's level of production which maximizes its profit given Company 2's strategy is calculated. Company 1 chooses production level  $Q_1$ , then Company 2 selects the production level  $R_2(Q_1)$ , obtaining a total production level of  $Q_1 + R_2(Q_1)$ , and therefore  $P(Q_1 + R_2(Q_1))$ . As such, the next formula denotes firm 1's profit equation:

$$\Pi(Q_1, Q_2) = Q_1 P(Q_1 + R_2(Q_1)) - C_1(Q_1) \quad (2.25)$$

There is an unique value for  $Q_1$  which is  $Q_1^*$ .

As it can be seen in this model, Company 2 possesses only best response  $R_2(Q_1)$  to every  $Q_1$ , and Company 1 has a unique best action  $Q_1^*$  based on the best response of Company 2. Therefore, the perfect equilibrium of the game is  $(Q_1^*, R_2)$ . The Company 2's production given Company 1's equilibrium is  $R_2(Q_1^*) = Q_2^*$ .

For this example, unit cost and price function are the same as Cournot's example. Under the previous assumptions, Company 2 has only best response to every production level  $Q_1$ , denoted by

$$R_2(Q_1) = \begin{cases} \frac{1}{2}(\theta - c - Q_1), & \text{if } Q_1 \leq \theta - c \\ 0, & \text{if } Q_1 > \theta - c \end{cases}$$

In this sub-game the function to maximize is

$$Q_1(\theta - c - (Q_1 + \frac{1}{2}(\theta - c - Q_1))) = \frac{1}{2}Q_1(\theta - c - Q_1)$$

Note that, this is a quadratic function that becomes zero when  $Q_1 = 0$  and  $Q_1 = \theta - c$ . The maximum point is  $Q_1^* = \frac{1}{2}(\theta - c)$  and Company 2's production level  $Q_2^* = R_2(Q_1^*) = \frac{1}{4}(\theta - c)$ . The respective profits are  $\Pi_1(Q_1^*, Q_2^*) = Q_1^*(P(Q_1^* + Q_2^*) - c) = \frac{1}{8}(\theta - c)^2$  and Company 2's profit is  $\Pi_2(Q_1^*, Q_2^*) = Q_2^*(P(Q_1^* + Q_2^*) - c) = \frac{1}{16}(\theta - c)^2$ .

[Azevedo and Paxson \(2014\)](#) introduce one of the most common methodology in the Real Option Games (ROG) literature that is called pre-emption game. It establishes a two-player game in a continuous and infinite time horizon which strategy is that competitors, for example in R&D, enhance their innovations over time and keep these innovations secret before they introduce them into the market. The final goal is to establish the proper time to invest or defer in the project. However, our model looks for a different approach where the life time of the project becomes discrete and defined by an integer, and the strategy of the real option is flexibility in capacity instead of the combination of models with invest-defer criteria. Moreover, the same study concludes that many other sources of uncertainty must be considered in this analysis. However, this is outside the scope of our research since we aim to study only one variable of uncertainty, which is the demand.

Recent literature has addressed solutions to assess the impact of an optimal investment in a duopoly market based on a Stackelberg model strategy in an imperfectly competitive market. Such market imperfect competitiveness is reflected by the asymmetry between companies, that occurs when one of the competitors in an economic transaction has much more knowledge about components of the market than the other. As a result, it provides the first mover a bigger advantage over the second mover; and finds that higher uncertainty delays investment ([Pawlina and Kort,](#)

2006). In Cumbul (2019), an oligopoly market of  $n$  firms is designed by a sequential interaction of a Cournot model sub-game with a Stackelberg model competing with a product with that possesses a stochastic demand behaviour in a market with private or asymmetric information. In contrast to the previous studies, in our study, we consider a Cournot duopoly where companies make their moves at the same time while in Stackelberg duopoly one firm becomes the leader and makes the first move followed by the other firm that becomes the follower. Moreover, our model is based on the assumption that participants have access to complete information that is an economic situation or game in which knowledge about other market participants or players is equally available to all competitors (Bajari et al., 2010).

Luckraz (2011) presents the stability of properties of a duopoly Cournot model when the price-demand function elasticity is isoelastic to prove of existence and uniqueness of a Nash Equilibrium even in cases where price-demand function is not linear. Specifically, an isoelastic function is defined as a function that exhibits a constant elasticity coefficient (Simon and Blume, 1994). In another research, Ibrahim (2019) employs the isoelastic demand function and linear production costs to compare profits obtained from companies competing using the Cournot and Stackelberg duopoly approaches. As a result, the study shows that regardless the game companies are playing, the company with the lower production cost would make a greater profit, and also, such company with bigger production costs is better off making the first move instead of moving simultaneously. On the other hand, the company with lower production is better off in becoming a follower rather than making the first move or moving simultaneously. Similar to the two studies presented previously, our study considers an isoelastic linear price-demand function. However, our approach assumes that vendors and suppliers in the market are the same; therefore, the variable and marginal that represent our quadratic cost function of production for both competitors are identical and those costs only vary for the sensitive analysis of the model to observe how they affect the NPV and optimal capacity in each participant within the two games.

Empirical data models in option valuation with game theory has shown, for instance, that in a liner shipping industry that competes in non-cooperative game-theoretic oligopoly, the investment decisions may be drawn by real options theory with the assumption of cooperative or pure strategy game theory in a perfect competition market (Rau and Spinler, 2016). Additionally, the competitive interaction within players can be modeled by random Poisson arrival. The same study suggests

that investment models in continuous-time and discrete-time should incorporate more stochastic variables as cost and fuel prices. Another empirical research in a competitive electricity market compares a mixed strategy of Nash Cournot Equilibrium with the Stackelberg equilibrium having as output strategic production quantities and then analyses the results from a social benefit point of view ([Lee, 2014](#)). In another empirical research paper, in a port industry, two ports compete on future quantity to produce for a flexible investment decision that provides leader-follower timing game and the option to delay such investment in a capacity level that has not been forecasted previously. The model is based on a Cournot competition and uncertain demand defined by a geometric Brownian motion. This game-theoretic model includes additional macro-economic variables such as public money involvement, congestion costs, and the cost advantage of one port. The results show that competition and slight cost differences drive the leader's investment threshold and capacity decrement; and subsequently, it allows the follower to invest later in more capacity ([Balliauw et al., 2019](#)). Investment threshold is defined as the minimum amount of costs that a company must incur in to qualify for a project's eligibility ([Lawinsider, 2020](#)).

Although these empirical cases bring insights of the mix of real options and game theory, our study focuses on the construction of a theoretical model. We aim to analyse the feasibility of introducing a product in the market considering a flexible option in expansion and contraction of capacity affected by a drift in product demand, which is the only stochastic variable. Additionally to what is presented in the previous empirical researches, the underlying variable in our study is assumed to follow a normal distribution and the interaction of the competitors are not defined by any probability distribution. Competitors' interactions are established by a NE in a cooperative market with pure strategy. We discard a mixed strategy since it may need a new algorithm based on heuristics to find such mixed strategy NE as presented by [Lee \(2013\)](#) and this topic is out of the scope of this study. On the other hand, the players decide to invest at the same time in the optimal capacity that provides the maximum NPV and the project is not deferred.

Some studies have found it difficult to obtain an optimal solution applying existing methodologies in a game theoretic environment. For instance, [Frantsev et al. \(2012\)](#) applied a combination of evolutionary bi-level programming structure and heuristic approach to solve a multi-period Stackelberg competition model. Bi-level programming involves mathematical algorithms with optimization problems in their constraints, and the main problem is called the upper-level or the

leader and the nested problem is called the lower-level or the follower (Colson et al., 2007). This study is later on extended by Sinha et al. (2014) using a multi-period multi-leader–follower in a oligopoly Stackelberg competition where cases were solved by different number of leaders and followers showing how the entrance or exit of a player affects the profits of the other players. Furthermore, some researchers use an approach that imposes mixed strategies for identical companies in a duopoly sequential competition shows that the utility of the leader and the follower are identical when both utility functions intersect (Huisman et al., 2003).

Our study focuses on a duopoly market where competitors make their decisions based on either perfect or imperfect information and subsequently compare the investment decisions and payoff based on the maximum NPV. To evaluate such payoffs in the game theoretic models, we use a multi-period lattice approach with regime switching and a dynamic programming method. Contrary to the three presented previous studies, we solve our models using dynamic programming, not based on heuristic approaches. Moreover, we highlight different outputs in our numerical study to measure the effects of only two firms choosing to produce either simultaneously or subsequently.

## 2.4 Lattice and Regime Switching

We assume that a product life cycle are formulated by regime-switching models that represent a simple method to include the stochastic volatility pattern for the demand. In each regime, the underlying asset dynamics is discretized by a lattice obtained by a simple modification of the parameters. For these transformed parameters, the one with greatest volatility tree is identified to allow a simultaneous representation of the asset demand in all the regimes (Costabile et al., 2014). Specifically, a Markov regime-switching model is one of the most popular nonlinear time series models in the literature. It involves multiple structures that identify the time series behaviors in various regimes (Kuan, 2002).

A Markovian regime-switching geometric Brownian motion and a Markovian regime-switching jump-diffusion model are considered as methods to assess the option value for a financial asset in a game theoretic approach (Siu, 2008). An option pricing jump-diffusion model is a form of mixture model of jump process and a diffusion process (Kou, 2002). Two players in a Stackelberg game use regime switching to determine their respective optimal market entry and establish their

irreversible investments. They integrate timing flexibility, competition, and changes in the market environment that further translate in cash flows represented by operational profits (Bensoussan et al., 2017). Liu and Zhao (2013) studies options with two underlying assets whose prices are governed by the regime-switching geometric Brownian motion, where a lattice is constructed with a jump step size for the two variables. The lattice nodes recombines along each variable and the result is that the two-dimensional lattice increases quadratically as the number of time steps grows. In their paper, Yuen and Yang (2010) suggest a trinomial tree approach to price options in a regime-switching environment. Such tree is combined in a lattice with  $2t-1$  nodes for a give period  $t$ , and instead of changing the volatility if the regime changes, they change the probability of going to one regime to the other. Additionally, recombining trinomial tree as a lattice for real option valuation with changing volatility has been done by simultaneously choosing a parameterization that set an equidistant regime space and transition probabilities between the nodes. The volatility is expressed in the transition probabilities. (Haahtela, 2010). Pentanomial lattices are used to evaluate different regime-switching models capturing the temperature dynamics in weather derivatives and compare their performance with a single-regime model. Moreover, some other studies show the importance of using pentanomial regime-switching lattice in option pricing to create new, sophisticated and accurate option evaluation models (Christoforidou, 2015; Elias et al., 2014).

## 2.5 Dynamic Programming

Dynamic programming principle is one of the most important tools in solving Markov regime-switching stochastic optimal control problems. Currently, numerous researches involve in this field and outstanding results have been reached; see e.g., Insley and Wirjanto (2010), Blanco et al. (2011), Pringles et al. (2015), Bender et al. (2018), Lai et al. (2010), Kraft and Steffensen (2013), Nadarajah (2014), Wahab et al. (2005), and Shen and Siu (2012), among many others. Moreover, this approach has been applied to a recursive utility portfolio optimization problem in financial markets (Sun et al., 2018).

Optimal solutions and timing for the investment in an uncertain environment can be determined by the combination of dynamic programming with other methods. Such combination is useful to design methodologies in options valuation to calculate the optimal investment in a project

(Kozlova, 2017). Dynamic programming algorithms are used in a oligopoly Stackelberg game under a structure modelled by a discrete time dynamic bilevel optimization problem. The upper level is assigned to the decisions of the leader, and the lower level to the decisions of the leader and the followers (Nie et al., 2006). Furthermore, Dynamic programming method has been used with an novel algorithm to model projects' stochastic processes and real options with decision trees to improve the current real options valuation methodology, and propose a framework that allow the problem to be solved with a decision analysis software (Brandão and Dyer, 2005).

Lattice approach and dynamic programming have been two very useful, accurate and fast techniques to solve financial and real option valuation. In our study, such techniques are implemented since they help us combine capacity flexibility, stochastic demand and product life cycle to calculate NPV.

Table 2.2 lists and summarizes the most important articles reviewed for this study. It is categorized in the first column by the topic of studies according to: Real Options (RO), Game Theory (GT) and the Combination of real options and game theory (C). The second column indicates the name of the author(s) of the article. The third column explains the main focus on the research of each paper. The last column, called “Addressed gap in this thesis”, identifies the approaches that are incorporated to our models and are not considered in the corresponding study.

Table 2.2: Literature review summary

Topic	Author	Focus	Addressed gap in this thesis
RO	Bollen (1999)	Real options monopolist valuation - Regime-switching lattice - Expansion and contraction in capacity studied separately	- Competition - Study of flexible capacity combined (expansion and contraction)
RO	Wahab and Lee (2011)	Gasoline price swing option - Financial options pricing - n regime-switching model	- Competition
RO	Elliott and Siu (2015)	Asset pricing - Multi-period regime-switching lattice in a continuous time - Finite states - Hidden Markov-modulate Poisson process	- Competition - Discrete time - Dynamic programming - Geometric Brownian Motion
RO	Charalampopoulos et al. (2011)	Empirical model - Option to expand - Discrete time	- Capacity flexibility - Competition - Theoretical model
RO	Bengtsson (1999) Bengtsson (2001) Tannous (1996) Abele et al. (2006)	Volume flexible for equipment with demand uncertain - Expand or contract production within certain ranges - Optimal level of investment - High level of flexibility changed to a low expenditure - Option to hire personnel on short contracts	- Competition - Dynamic programming - Multi-period analysis - Regime switching
GT	Cumbul (2019) Li and Sethi (2016) Pawlina and Kort (2006)	Oligopoly in games and sub-games models for time to invest with incomplete information - Optimal investment in a Stackelberg's model - Cournot and Stackelberg with private information - Continuous-time and infinite-horizon - Mixed strategies	- Complete information - Pure strategy - Real options - Regime switching
GT	Ibrahim (2019) Luckraz (2011)	Duopoly model with variation of price-demand function elasticity and cost function - Prove of uniqueness of a NE - Price-demand not linear function - Linear production costs	- Real options - Dynamic programming - Flexible capacity - Quadratic cost function - Real options - Regime switching
GT	Colson et al. (2007) Frantsev et al. (2012) Sinha et al. (2014)	Oligopoly competition and optimization constrains - Bi-level programming - Multi-level programming - Heuristic approach - Multi-period multi-leader follower game	- Dynamic programming - Real options - Regime switching
C	Smit and Trigeorgis (2017)	Combining real options and game theory - Defer and abandon project - Two periods of study	- Capacity flexibility - Multi-period analysis - Regime switching
C	Azevedo and Paxson (2014)	Two-player game in a continuous and infinite time horizon - Time to invest or defer - Pre-emption game - Product innovation	- Capacity flexibility - Discrete time - Dynamic programming - Regime switching
C	Ashuri and Kashani (2011) Pendharkar (2010)	Empirical model in two stages - Comparison multi-stage and two-stage - Timing, expanding, abandoning and delaying	- Capacity flexibility - Regime switching - Theoretical model
C	Balliauw et al. (2019) Lee (2013) Lee (2014) Rau and Spinler (2016)	Empirical models in some industries to measure competitiveness - Cooperative game-theoretic oligopoly - Random Poisson arrival - Social benefit point of view - Timing and option to delay - Macro-economic variables - Mixed strategy NE	- Micro-economic variables - Normal distribution - Product introduction - Pure strategy - Theoretical model
C	Thijssen (2013) Wu and Chen (2015)	Financial options valuation in portfolio - NE subgames - Efficient frontier - Pre-emption games - Markov perfect equilibrium	- Capacity flexibility - Nash equilibrium - Product introduction - Real options
C	Dias and Teixeira (2003)	Mixed strategy in a follower-leader environment	- Capacity flexibility - Pure strategy



## Chapter 3

### The General Model

Companies tend not to consider the following premises, upon which this model is based:

- Typical product life cycle behaviour
- Variable production volume throughout the lifetime of the project
- Competitor rates of production
- Demand volatility; fluctuations in the market through the lifetime of the product

Figure 3.1 exhibits graphically the previous statements. The typical product life cycle is represented by a bell curve. The part of the curve to the left of the apex is called the growth stage, and the part to the right of the apex is the decay stage. Companies that don't consider variable production volumes throughout the lifetime of the project may face periods where they're unable to meet expanding demands during the growth phase or over-produce during the decay stage. Volume production of competitors must also be considered, because consumers prefer companies whose supply meets their demand and ignore ones that do not. Finally, the volatility in demand from forecast values must be considered to avoid over or under producing.

For these reasons, the model employs a theoretical real options valuation model in a duopoly environment for product introduction that integrates:

- Stochastic product life cycle that is represented by a regime-switching lattice explained by a geometric Brownian motion.
- Flexible capacity that contains the option of expansion and contraction due to changes in demand.

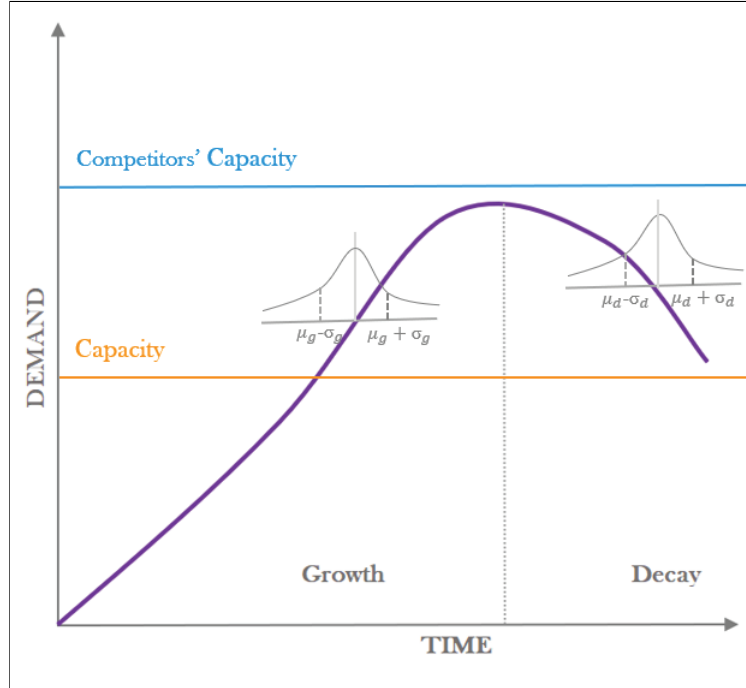


Figure 3.1: Product life cycle, capacity, competition and demand volatility

- Game theory using NE pure strategy to estimate the optimal quantity to produce for both companies based on two different approaches: Cournot and Stackelberg's models.
- Dynamic programming to calculate the models' evaluation criteria: the optimal initial capacity and the maximum NPV.

### 3.1 Model Assumptions

This model presents two companies that are planning the production to introduce strategically one product in the market. The companies seek to maximize the project value by selecting an optimal capacity at the beginning of the project. Additionally, the companies seek to evaluate the option to switch the capacity in the future during project lifetime. The idea of flexible capacity is to provide company  $i = 1, 2$  the option to shift a discrete number of capacity levels,  $K_{i\ell}$ , at any given time during the project's lifetime. It motivates the firm to either contract or expand capacity for the production due to demand fluctuations.

For the purpose of this model, it is assumed that only two firms are competing in the market for the same product (Duopoly). The future market demand shift for the product is represented by

the parameter  $\theta$  at a time  $t$ , which is identical for both competing companies (Osborne, 2004). The profit function defines the cash flow or payoff in this model. To calculate profits at anytime during the product lifetime, a price and a cost function are determined. The price follows an inverse positive demand function for each company  $i = 1, 2$  (Watson, 2002), that is,

$$P_i(\theta, Q_1 + Q_2) = \begin{cases} \theta - \lambda_i(Q_1 + Q_2), & \text{if } Q_1 + Q_2 \leq \frac{\theta}{\lambda_i} \\ 0, & \text{if } Q_1 + Q_2 > \frac{\theta}{\lambda_i} \end{cases} \quad (3.1)$$

where  $\lambda_i$  is a constant associated to each company's price function, called price elasticity of demand. This value is defined as an indicator (percentage) of the responsiveness of consumers to a change in a product's price. The greater the price elasticity of demand the more responsive consumers are to a price change (Pindyck, 1988). This inverse price-demand function is appropriate for this model since it shows the relationship between the demand and supply for a product and its price. The economic principle behind this formula establishes that, assuming all other variables are equal, the supply for a product drops as its price increases and vice versa. On the other hand, all other variables remaining the same, the price of a product rises when the demand goes up. Additionally, the elasticity price-demand constant reflects exogenous factors such as: variation in consumer incomes, changes in product taste preferences, variation in prices of substitute goods, and consumers' brand loyalty. Furthermore, prices can be instantly adjusted with changes in demand and supply, and consequently, manifested in the profits during the product life cycle. The cost function for both companies follows a variable ( $c_i$ ) or linear, and marginal ( $w_i$ ) or quadratic component derived from Trigeorgis (1996):

$$C_i(Q_i) = c_i Q_i + \frac{1}{2} w_i Q_i^2 \quad (3.2)$$

Specifically,  $w_i$  grows with quantity to produce. A marginal cost describes the incremental cost due to an additional unit of a product is made. A way to estimate it is by dividing the total change in the cost of making more units over the change in the number of units made.

Similar to Bollen (1999), we consider flexible capacity plan. That is the optimal capacity for each participating company  $i$  at a certain level  $\ell$ ,  $K_{i\ell}$ , is added to the cost function, and an overhead

cost,  $o_i$ , that is the coefficient for this capacity value. This cost incentives companies to reduce operations when excess in capacity is not likely to be utilized and to retard expansion until demand justifies acquiring additional capacity. Moreover,  $w_i$  is divided by  $K_{i\ell}$  since the marginal cost is a declining function of the capacity.

$$C_i(Q_i, K_{i\ell}) = c_i Q_i + \frac{1}{2} \frac{w_i}{K_{i\ell}} Q_i^2 + o_i K_{i\ell} \quad (3.3)$$

The profit has the form  $\Pi_i = P_i Q_i - C_i$ , which can be expanded as

$$\Pi_i(\theta, Q_i, K_{i\ell}) = [\theta - \lambda_i(Q_1 + Q_2)]Q_i - (c_i Q_i + \frac{1}{2} \frac{w_i}{K_{i\ell}} Q_i^2 + o_i K_{i\ell}) \quad (3.4)$$

To ease calculations and as it was explained in the literature review, we assume for our model that costs are the same for both companies since they interact in a complete market information. Furthermore, because this model is based on two competitors producing the same item, game theory is used to find out the quantities each company must produce to achieve the Nash equilibrium. Specifically, the Cournot and Stackelberg's models are considered to calculate  $Q_i^*$ . This assumption allows an easy profit maximization case with quantity as the prime variable.

Since flexibility considers a production's capacity, then it restricts production every period and can never be negative. Therefore, the formula for the production in each term can be obtained as (Bollen, 1999):

$$Q_i = \max[0, \min(Q_i^*, K_{i\ell})] \quad (3.5)$$

The model aims to maximize the NPV that measures the feasibility of a project where the same product is introduced to the market by two companies. The decision-making criteria is to select the initial capacity that makes such NPV the highest of all. This optimal capacity is determined from a set of fixed capacities levels established as input of the model for both companies. NPV represents the individual utility function value or payoff for the participants within either Duopoly Cournot or Stackelberg's games.

The variations in demand at every node in each period of time is estimated at the beginning, according to the mean and standard deviation assigned for each regime. This model provides a

product's life cycle that is represented by two regimes: growth and decay. One assumption is that the price is adjusted immediately in each period evaluated to balance quantities supplied and demanded. This product is sold at a price shown by Equation (3.1). The cost composition and demand-price elasticity do not change during the periods of the project evaluation.

The model follows the next steps to obtain the maximum NPV and initial optimal capacity in each game-theoretic model. First, the product life cycle is incorporated into the analysis by employing the methodology of regime-switching lattice presented in Bollen (1999) and Wahab and Lee (2011) to estimate the drift on demand. Second, a probability of switching from regime growth to decay is estimated. Third, the analysis is expanded to estimate the cash flows in every node, represented by the profits obtained by Cournot and Stackelberg's game-theoretic concepts. Fourth, the values of capacity flexibility are determined by computing the incremental increase in project NPV when capacity is allowed to change. Fifth, a dynamic programming approach is applied to calculate the maximum NPV and optimal initial capacity for each participant in the two games. Lastly, we compare the results obtained of NPV and optimal capacity between the two game theory perspectives. The results are presented as an individual utility function for each player, not as a joint utility function. Cournot's model is represented by two firms (or participants), Cournot 1 and Cournot 2 ( $C_1$  and  $C_2$ ). Similarly, Stackelberg's game is based on two participants, namely, Stackelberg Leader ( $S_L$ ) and Stackelberg Follower ( $S_F$ ). The outcomes for two competitors in these two different models are assessed using NPV (utility function or payoff) and the associated optimal initial capacity values. Our detailed numerical analysis provide insights on product introduction strategies integrating resource allocation in a multi-period framework when two suppliers dominate the market for a commodity or service.

### 3.2 Lattice Generation and Demand Dynamics

The first step in this model is to create a lattice which specifies the evolution of the drift on demand during the multiple periods of time of the project. such drift on demand is used to compute the expected future profits derived from game-theoretic calculations. Additional, the profits in each node of the lattice are also a function of the different set of capacity levels assigned to each company. In this model the range of capacities for both companies are the same and vary from

1 to  $l$  and is represented in by a  $l \times l$  square matrix. Moreover, this lattice connects the cash flow of switching capacity during one period to another.

Binomial options pricing model methodology delivers an induced numerical approach for options valuation. This method is used in variables that follow a geometric Brownian motion. Basically, the algorithm adopts a discrete-time lattice-based pattern that varies the underlying financial instrument price over time dealing with nodes where the mathematical expression of Black–Scholes formula is necessary (Cox et al., 1979). But, these lattice approaches only takes into account one regime for the underlying stochastic variable. Unlike the binomial lattice approach proposed by Hull (2018), Wahab and Lee (2011) suggests a pentanomial lattice method to price options with the underlying asset adopting a two regime-switching pattern. In this approach, a trinomial lattice represents every regime with one of its branches used by both. To reduce the number of nodes in this lattice, nodes from both regimes are combined by applying the jump sizes of the regime in a 1:2 ratio. Zhao and Tseng (2003) employs trinomial lattice to model the fluctuation of a parking lot demand to valuate flexibility in infrastructure expansion. Liu and Zhao (2013) expands the previous method of regime-switching recombining tree to estimate option pricing for two underlying assets whose prices are influenced by a Geometric Brownian Motion regime-switching model and the lattice increases in a quadratic manner. This study suggests a lattice methodology to assess real options when the underlying asset adopts a regime-switching process with  $n$  regimes and each of them is explained by a geometric Brownian motion. The lattice approach described in this study has the flexibility to estimate an option with several regimes for an underlying asset and discretized demand.

Every regime displays a trinomial lattice that shares the middle branch with the other regime. To minimize the number of nodes in the lattice and ease the model's calculations, every node is consolidated by modifying the size of every step for the  $n - 1$  remaining regimes. The idea is that all the nodes for the two regimes are uniformly separated. Such steps are individually estimated following Equation (3.6) expressed in Wahab and Lee (2011), to build a pentanomial lattice for a two-regime Markov regime-switching process.

$$\phi_\alpha = \sqrt{\sigma_\alpha^2 h + \mu_\alpha^2 h^2}, \quad \alpha = 1, 2 \quad (3.6)$$

where  $\mu_\alpha$  represents the mean and  $\sigma_\alpha$  the standard deviation of the  $\alpha$ th regime. In addition,  $h$  indicates the period of time between two layers in the lattice. This parameter  $h$  has to be very small in order to satisfy that all probabilities in the branches are positive.

$\phi_1$  and  $\phi_2$  are defined as the respective step sizes in every regime  $\alpha = 1, 2$ . The first step is to organize and re-index them in ascendant order.  $\phi_1$  is the step size with the minimum value and  $\phi_2$  is the one with the maximum value. Then, the value of the step size is defined as  $\phi = \max(\phi_1, \frac{\phi_2}{2})$ , for instance,  $\phi = \frac{\phi_\alpha}{\alpha}$ , for  $\frac{\phi_\alpha}{\alpha} \geq \frac{\phi_\beta}{\beta}$ ,  $\forall \beta \neq \alpha$ . Note that  $\alpha$  represents the regime with the highest volatility and  $\beta$  represents the other regimes. The last stage is to adjust each regime step size according to the formula

$$\phi_\beta = \begin{cases} \phi_\alpha & \text{if } \alpha = \beta, \\ \beta \frac{\phi_\alpha}{\alpha}, & \text{if } \alpha \neq \beta \end{cases} \quad (3.7)$$

The approach to calculate the conditional probabilities in each branch coming from a node is obtained by equating the first and second moments of the branches for a continuous process of the underlying instrument or asset. The formula for the trinomial conditional branch probabilities for every single regime  $\alpha$  is acquired by

$$\psi_{\phi_\beta u} = \frac{e^{\mu_\beta h} - e^{(-\beta \frac{\phi_\alpha}{\alpha})} - \psi_{\phi_\beta m}[1 - e^{(-\beta \frac{\phi_\alpha}{\alpha})}]}{e^{(\beta \frac{\phi_\alpha}{\alpha})} - e^{(-\beta \frac{\phi_\alpha}{\alpha})}} \quad (3.8)$$

$$\psi_{\phi_\beta m} = 1 - \frac{\phi_\beta^2}{(-\beta \frac{\phi_\alpha}{\alpha})^2} \quad (3.9)$$

$$\psi_{\phi_\beta d} = 1 - \psi_{\phi_\beta u} - \psi_{\phi_\beta m} \quad (3.10)$$

where  $\psi_{\phi_\beta u}$ ,  $\psi_{\phi_\beta m}$ , and  $\psi_{\phi_\beta d}$  are the respective conditional probabilities of going upward, middle and downward in the branches of the trinomial lattice. It can be observed that when  $\beta = \alpha$ , the middle branch conditional probability is zero; therefore, one of the two regimes is considered a binomial lattice or a trinomial one with middle probability of zero. Equations (3.7) to (3.10) are also obtained from [Wahab and Lee \(2011\)](#).

Additionally, the model is founded on a linear demand behaviour where the demand continuously compounded with growth rate  $\theta$ , which is considered to be normally distributed and also an

exponential function of  $\phi$ , that is,

$$\theta = D_0 e^{\{2(t-1) - (\nu-1)\}\phi} \quad (3.11)$$

where  $t$  is the period of time of evaluation,  $D_0$  is the initial demand, and  $\nu$  is a corresponding node in the lattice in such period.  $\nu$  is an integer that varies from 1 to  $4t - 3$ , which is the maximum number of branches of the pentanomial lattice in a specific period of time. For instance, at  $t = 3$ , the lattice has 9 branches; therefore,  $\nu = 1, 2, 3, \dots, 9$ . Specifically,  $\nu = 1$  correspond to the node where  $\theta$  has its highest value and  $\nu = 9$  correspond to the one with the lowest value. Equation (3.11), which defines the drift of demand in any node of the lattice at any period, is determined by our model in this study. Another assumption is that the project has a finite lifetime defined by  $T$ . Figure 3.2 shows an example of pentanomial lattice connection between demand, profits and switching costs in a 3-period lifetime project.

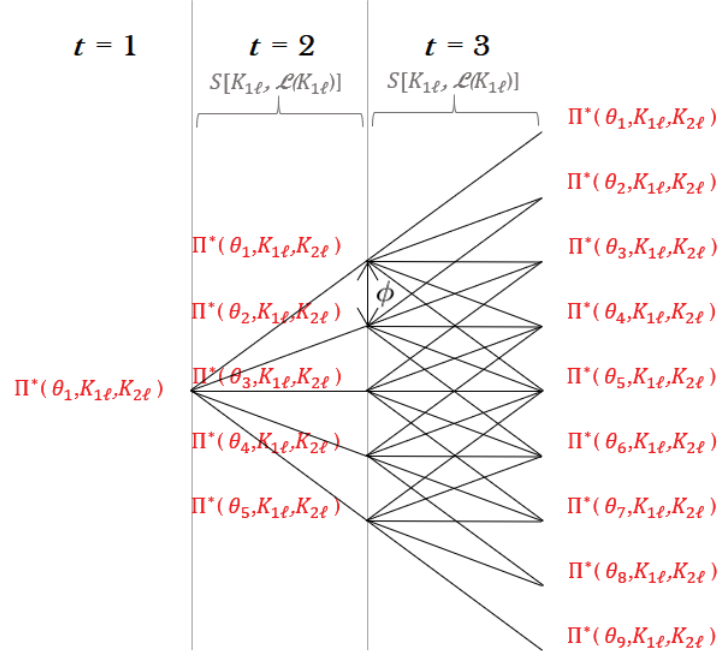


Figure 3.2: Example of a pentanomial lattice

Similar to life cycles of most products, the demand starts in the growth stage and shifts at most once to the decay stage during the product life cycle. This consideration is appropriate to demonstrate how the path of the optimal solution may vary if the cycle switches from one regime to



the other. Furthermore, the initial parameters of mean for growth and decay, with their respective standard deviations, are considered to be risk-adjusted with the purpose of discounting the cash flows at a risk-free rate as explained in Section 2.2.1.

For the model's valuation, the drift in the demand must be known and established by the parameter  $\theta$ . This variable was previously shown as a mean to calculate future profits for each of the different game theoretic models. A generalized assumption in the real options research is that the underlying stochastic variable is ruled by a geometric diffusion. we consider the likelihood that the demand dynamics varies in an fundamental direction through the path of the project's duration. Additionally, this model is based on the assumption that product life cycle may decay at some time due to introduction of contending products, market saturation, improvement of technology, or substituting taste among some other reasons (Bollen, 1999). Those cycles in the product life are considered as regime-switching models, where different phases of the product's life are identified by basically various stochastic demand schedules. Besides, the length of each phase or stage is unknown.

A project value is calculated at each node in the lattice. The nodes are conditioned to the respective level of demand and the value of the capacity. The present value at each node is maximized by exploring over an array of capacities per period, which are increased at a constant unit.

### 3.3 Probability of Switching

The second step is to calculate the probability of switching from one regime to another using the formulas and methodology established by Bollen (1999) and Hamilton (1990). The probability must be estimated in order to adjust the NPV to changes in the regime behavior. The variation of such probability, at any moment during the next year, is represented by a normally distributed cumulative function of elapsed time. Specifically, product life cycle goes from growth to decay, but the reverse cannot be true. That is if a product enters in the decay stage, it remains in that cycle until the end of the project valuation. The first step for this calculation is to find the probability of switching over the upcoming period step. One assumption is that the previously mentioned probability is constant during the period of evaluation, assuming that  $s$  time steps exist

each year. The probability of switching from one regime to the other over the upcoming time step is established as a constant value over  $s$  time steps. Therefore, the cumulative probability during the year equals the initial probability of switching.  $p(s)$  represents the switching probability during the course of the upcoming period.  $P(s)$  is chosen as the switching probability during the next period step, such that

$$p(s) = 1 - [1 - P(s)]^s \quad (3.12)$$

subsequently, we obtain

$$P(s) = 1 - [1 - p(s)]^{\frac{1}{s}} \quad (3.13)$$

The switching probability distribution that rules the product cycle by jumping from growth to decay is important to estimate PV. Such probability influences  $\theta$ ; therefore,  $Q$  and  $\Pi$  associated to the respective period.

### 3.4 Game Theory Approach

The third step in the model is to estimate the profits that are considered as the cash flows in each node of the lattice. We employ two game theory strategies that are frequently used in economic models to describe an industry structure in which companies compete on the amount of output they will produce. Those theories are Cournot and Stackelberg's. There is another game theoretic approach suitable for oligopoly markets, which is called Bertrand's model of oligopoly. It was studied during this research, however, this strategy is to compete for price in the market, not for the quantity to produce. A brief explanation of its methodology is presented in Appendix A.1. As mentioned at the beginning of this chapter, the price-demand function is obtained from [Watson \(2002\)](#), the cost function is acquired from [Bollen \(1999\)](#), and the profit function is expressed in Equation (3.4). The methodology to calculate the optimal quantity to produce,  $Q_i^*$ , in both Cournot and Stackelberg's models, is derived from [Osborne \(2004\)](#) and their respective formulas for both competitors are obtained as part of the model in this research.

### 3.4.1 Cournot's Model of Oligopoly

In this competition structure, the firms  $i = 1, 2$  compete on the total amount of output,  $Q_i$  that is produced. Since, this is an extensive game with imperfect information, both participants choose to produce simultaneously and obtain identical results under an equal pair of capacities. The biggest assumption is that such companies decide on independently of each other, and their strategy is based on either previous games or statistical data available. Oligopoly is defined as a competitiveness between a small number of suppliers (Osborne, 2004). To simplify calculations in this model, we assume that there are two companies in the market (Duopoly), and any equation for Company 1 is also applied to Company 2.

$$\Pi_1(\theta, Q_1, Q_2, K_{1\ell}) = [\theta - \lambda_1(Q_1 + Q_2)]Q_1 - (c_1Q_1 + \frac{1}{2}\frac{w_1}{K_{1\ell}}Q_1^2 + o_1K_{1\ell}) \quad (3.14)$$

$$\Pi_2(\theta, Q_1, Q_2, K_{2\ell}) = [\theta - \lambda_2(Q_1 + Q_2)]Q_2 - (c_2Q_2 + \frac{1}{2}\frac{w_2}{K_{2\ell}}Q_2^2 + o_2K_{2\ell}) \quad (3.15)$$

The next step is to find for Company 1, the  $Q_1^*$  that maximizes  $\Pi_1$  given  $Q_2$  (i.e. the amount produced by the competitor) using partial derivatives  $\frac{\partial \Pi_1}{\partial Q_1} = \frac{\partial \Pi_2}{\partial Q_2} = 0$ .

$$\frac{\partial \Pi_1}{\partial Q_1} = \theta - c_1 - \lambda_1Q_2 - (2\lambda_1 + \frac{w_1}{K_{1\ell}})Q_1 \quad (3.16)$$

$$\frac{\partial \Pi_2}{\partial Q_2} = \theta - c_2 - \lambda_2Q_1 - (2\lambda_2 + \frac{w_2}{K_{2\ell}})Q_2 \quad (3.17)$$

We solve Equations (3.16) and (3.17) for  $Q_1$  and  $Q_2$  as follows:

$$\theta - c_1 - \lambda_1Q_2 - (2\lambda_1 + \frac{w_1}{K_{1\ell}})Q_1 = 0$$

$$\theta - c_2 - \lambda_2Q_1 - (2\lambda_2 + \frac{w_2}{K_{2\ell}})Q_2 = 0$$

$$(2\lambda_1 + \frac{w_1}{K_{1\ell}})Q_1 = \theta - c_1 - \lambda_1Q_2$$

$$(2\lambda_2 + \frac{w_2}{K_{2\ell}})Q_2 = \theta - c_2 - \lambda_2Q_1$$

We obtain the Company 1 and 2 reaction functions as,

$$R_1(Q_2) = \frac{\theta - c_1 - \lambda_1 Q_2}{\left(\frac{w_1}{K_{1\ell}} + 2\lambda_1\right)} \quad (3.18)$$

$$R_2(Q_1) = \frac{\theta - c_2 - \lambda_2 Q_1}{\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right)} \quad (3.19)$$

where  $R_1(Q_2) = Q_1$  and  $R_2(Q_1) = Q_2$ . Solving the two linear Equations (3.18) and (3.19):

$$\begin{aligned} R_2(Q_1) &= R_1(Q_2) \\ \frac{\theta - c_1 - \lambda_1 Q_2}{\left(2\lambda_1 + \frac{w_1}{K_{1\ell}}\right)} &= \frac{\theta - c_2 - \lambda_2 Q_1}{\left(2\lambda_2 + \frac{w_2}{K_{2\ell}}\right)} \\ Q_1 &= \frac{(\theta - c_1 - \lambda_1 Q_2)(2\lambda_2 + \frac{w_2}{K_{2\ell}})}{\left(2\lambda_1 + \frac{w_1}{K_{1\ell}}\right)} \\ Q_1 &= \frac{\left\{\theta - c_1 - \lambda_1 \left[\frac{\theta - c_2 - \lambda_2 Q_1}{\left(2\lambda_2 + \frac{w_2}{K_{2\ell}}\right)}\right]\right\}(2\lambda_2 + \frac{w_2}{K_{2\ell}})}{\left(2\lambda_1 + \frac{w_1}{K_{1\ell}}\right)} \end{aligned}$$

We obtain the Cournot's Nash equilibrium  $Q_1$  as

$$Q_1^* = \frac{(\theta - c_1)\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right) - \lambda_1(\theta - c_2)}{\left(\frac{w_1}{K_{1\ell}} + 2\lambda_1\right)\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right) - \lambda_1\lambda_2} \quad (3.20)$$

Substituting  $Q_1^*$  in  $R_2(Q_1)$ ,

$$Q_2 = \frac{\theta - c_2 - \lambda_2 \left[ \frac{(\theta - c_1)\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right) - \lambda_1(\theta - c_2)}{\left(\frac{w_1}{K_{1\ell}} + 2\lambda_1\right)\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right) - \lambda_1\lambda_2} \right]}{\left(2\lambda_2 + \frac{w_2}{K_{2\ell}}\right)}$$

The optimal value of  $Q_2^*$  is

$$Q_2^* = \frac{(\theta - c_2)\left(\frac{w_1}{K_{1\ell}} + 2\lambda_1\right) - \lambda_2(\theta - c_1)}{\left(\frac{w_2}{K_{2\ell}} + 2\lambda_2\right)\left(\frac{w_1}{K_{1\ell}} + 2\lambda_1\right) - \lambda_1\lambda_2} \quad (3.21)$$

Therefore, the optimal profit function for Company 1 and Company 2 for  $Q_1, Q_2 \geq 0$  can be obtained by replacing the values of Equations (3.20) and (3.21) in Equations (3.14) and (3.15),

that is,

$$\Pi_1^* = -o_1 K_{1\ell} + (\theta - c_1 - \lambda_1 Q_2) Q_1 - (\lambda_1 + \frac{1}{2} \frac{w_1}{K_{1\ell}}) Q_1^2 \quad (3.22)$$

$$\Pi_2^* = -o_2 K_{2\ell} + (\theta - c_2 - \lambda_2 Q_1) Q_2 - (\lambda_2 + \frac{1}{2} \frac{w_2}{K_{2\ell}}) Q_2^2 \quad (3.23)$$

### 3.4.2 Stackelberg's Model of Duopoly

This case shows the effect when a company invests first (leader), and the other defers its investment until next period (follower). The follower (in this case Company 2) bases its decision on competitor's  $Q_1$  production and its own reaction function  $R_2(Q_1)$ , presented in Equation (3.19). Different than Cournot's, this model is an extensive game with perfect information, therefore one produce subsequently to the other's decision. Thus, under an equal pair of capacities, results differ within competitors. Additionally, this model maximizes the profits, applying partial derivatives to the following profit equation:

$$\Pi_1(\theta, Q_1, R_2(Q_1), K_{1\ell}) = [\theta - \lambda_1(Q_1 + R_2(Q_1))] Q_1 - (c_1 Q_1 + \frac{1}{2} \frac{w_1}{K_{1\ell}} Q_1^2 + o_1 K_{1\ell}) \quad (3.24)$$

Substituting the value obtained in Equation (3.18),

$$\Pi_1 = \left\{ \theta - \lambda_1 \left[ Q_1 + \frac{\theta - c_2 - \lambda_2 Q_1}{(2\lambda_2 + \frac{w_2}{K_{2\ell}})} \right] \right\} Q_1 - (c_1 Q_1 + \frac{1}{2} \frac{w_1}{K_{1\ell}} Q_1^2 + o_1 K_{1\ell})$$

and applying partial derivatives  $\frac{\partial \Pi_1}{\partial Q_1} = 0$

$$\frac{\partial \Pi_1}{\partial Q_1} = \theta - c_1 - \lambda_1 \left( \frac{\theta - c_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} \right) + [2\lambda_1 \left( \frac{\lambda_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} - 1 \right) - \frac{w_1}{K_{1\ell}}] Q_1 \quad (3.25)$$

We solve Equation (3.25) for  $Q_1$  as follows:

$$\begin{aligned} \theta - c_1 - \lambda_1 \left( \frac{\theta - c_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} \right) + [2\lambda_1 \left( \frac{\lambda_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} - 1 \right) - \frac{w_1}{K_{1\ell}}] Q_1 &= 0 \\ [2\lambda_1 \left( \frac{\lambda_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} - 1 \right) - \frac{w_1}{K_{1\ell}}] Q_1 &= -[\theta - c_1 - \lambda_1 \left( \frac{\theta - c_2}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} \right)] \end{aligned}$$

which leads to the optimal quantity  $Q_1^*$  in equilibrium for company 1 or the leader as

$$Q_1^* = \frac{(\theta - c_1)(\frac{w_2}{K_{2\ell}} + 2\lambda_2) - \lambda_1(\theta - c_2)}{(\frac{w_1}{K_{1\ell}} + 2\lambda_1)(\frac{w_2}{K_{2\ell}} + 2\lambda_2) - 2\lambda_1\lambda_2} \quad (3.26)$$

and replacing  $Q_1^*$  in  $R_2(Q_1)$ ,

$$Q_2 = \frac{\theta - c_2 - \lambda_2 \left[ \frac{(\theta - c_1)(\frac{w_2}{K_{2\ell}} + 2\lambda_2) - \lambda_1(\theta - c_2)}{(\frac{w_1}{K_{1\ell}} + 2\lambda_1)(\frac{w_2}{K_{2\ell}} + 2\lambda_2) - 2\lambda_1\lambda_2} \right]}{(2\lambda_2 + \frac{w_2}{K_{2\ell}})}$$

Then, the Company 2 (or follower) optimal quantity is

$$Q_2^* = \frac{(\theta - c_2)(\frac{w_1}{K_{1\ell}} + 2\lambda_1) - \lambda_2(\theta - c_1)}{(\frac{w_2}{K_{2\ell}} + 2\lambda_2)(\frac{w_1}{K_{1\ell}} + 2\lambda_1)} \quad (3.27)$$

Due to the flexibility and production capacity, the optimal production for each company follows Equation (2.24). Then, the optimal profit functions for Company 1 (leader) and Company 2 (follower) for  $Q_1, Q_2 \geq 0$ , which is obtained by replacing the values of Equations (3.26) and (3.27) in Equation (3.24) are

$$\Pi_1^* = -o_1 K_{1\ell} + (\theta - c_1 - \lambda_1 Q_2)Q_1 - \left( \frac{\lambda_2 \lambda_1}{\frac{w_2}{K_{2\ell}} + 2\lambda_2} + \frac{1}{2} \frac{w_1}{K_{1\ell}} \right) Q_1^2 \quad (3.28)$$

$$\Pi_2^* = -o_2 K_{2\ell} + (\theta - c_2 - \lambda_2 Q_1)Q_2 - \left( \frac{\lambda_1 \lambda_2}{\frac{w_1}{K_{1\ell}} + 2\lambda_1} + \frac{1}{2} \frac{w_2}{K_{2\ell}} \right) Q_2^2 \quad (3.29)$$

### 3.4.3 Flexible Capacity

The fourth step is to compute the cash flow corresponding to exchanging capacity from  $K_1$  to  $K_2$  in a specific time can be obtained as

$$S(K_1, K_2) = \begin{cases} vc_\kappa(K_2 - K_1) + s_f, & \text{if } K_2 > K_1 \\ \delta c_\kappa(K_1 - K_2) + s_f, & \text{if } K_1 > K_2 \\ 0, & \text{if } K_1 = K_2 \end{cases} \quad (3.30)$$

The parameters  $v$  and  $\delta$  can take positive or negative values and constitute a percentage of the total initial installment cost. Capacity expansion normally demands cash outflows which typically makes  $v$  negative. Contraction in capacity may produce cash outflows when the company incurs in clean up costs, or inflows in case the firm obtains a salvage value of machinery. Such previous situation leads  $\delta$  to take either positive or negative values; however, we only contemplate positive amounts.  $s_f$  represents fixed switching costs. Equation (3.30) is a cost function that could be expanded to let switching costs vary by time index.

The capacity flexibility value is calculated by estimating the cumulative increase in the NPV of the project when capacity is permitted to be adjusted. NPV by capacity level can be obtained as

$$\text{NPV}_i(K_0) = -c_K K_0 + \sum_{t=1}^T e^{-r_f t} \{E[\Pi_t^*(\theta_t, K_{t-1}) + S(K_{t-1}, K_t)]\} \quad (3.31)$$

Specifically, the capacity in time  $t$  is defined by the capacity assigned in time  $t - 1$ , and the changes in such capacities in period  $t$  takes effect in the next period. Equations (3.30) and (3.31) are acquired from Bollen (1999).

### 3.5 Dynamic Programming

The fifth step is to implement dynamic programming in our model, which is one of the most commonly used analytical approaches to evaluate real options due to the multi-period form of evaluation. It is based on creating a discrete-valued lattice to estimate the future outputs of the fundamental stochastic variable, in this study is  $\theta$ , in each node. Profits are computed in each node of the lattice according to the equation derived from each game-theoretic model, and subject to the respective stages of demand and established set of capacities. The estimation process starts at the culmination of the project cycle and turns back recurrently to the present to contemplate promptly option exercise. The nodes belonged to period  $T$  are called terminal nodes and project value is purely the ultimate cash flow. The other nodes are called intermediate nodes and the project value rests on the summation of the present cash flow in each period and the discounted expected value of the project in the succeeding period. A stochastic process that rules the stochastic demand progression governs the expected values calculations. Particularly, the current project's present

value is computed at the seed node, then, the option amount is either the addition or subtraction of managerial flexibility of significance.

Next, we sketch the game-theoretic models enunciated previously in a lattice for finite lifetime project established by  $T$  in years. This lattice is built on  $n$  time steps, which is integer. It makes the span of every step in years,  $h$ , equals to  $T/n$ . The quantity demanded and, as a consequence, the capacity  $K$  are escalated by  $h$  when the latter diminishes. Let us assume the parameter for continuous rate of demand drift  $\theta$  is distributed Normal with mean  $\mu$  and standard deviation  $\sigma$ . Thus, the equivalent demand parameter  $\theta$  over  $h$  is also distributed Normal with a new mean  $\mu h$  and standard deviation  $\sigma\sqrt{h}$ .

Subsequently, another characteristic of a project valuation is to decide what the proper discount rate is. This model creates two origins of risk. First, uncertainty associated to regime switch. Such risk can be mitigated by investing in other projects. Diversification theory is suitable for companies that are regularly substituting current for upgraded products. The effect of this risk is not appraised in the market due to its diversifiable attributes. On the other hand, a secondary origin of risk is the demand, inasmuch as  $\theta$  follows a stochastic process in every node of the product life cycle. A substantial assumption for this model is that every time an asset is traded with a fluctuation in time for demand, the project applies a risk-neutral valuation where the cash flows are discounted at the market  $r_f$ , and respective mean for growth ( $\mu_g$ ) and decay ( $\mu_d$ ) are adjusted to contemplate a risk premium. The most common method to evaluate risk market premium is the Capital Asset Pricing Model (CAPM) explained by [Fama and French \(2004\)](#). Another method is to deduce the risk-adjusted rates straight from derivatives prices, as presented in the Section [2.2.3](#).

### 3.6 Project Estimation and Option Values

In the final step, after establishing the lattice construction, we evaluate the project and respective options. Using dynamic programming, calculations start at the end of the lattice and operate backwards to the present. The project value is estimated at each node, based on the value of  $\theta$  and  $K_{i\ell}$  of the preceding node. The entire set of possible previous capacity levels are evaluated since the optimal capacity level is unknown from the prior node. The methodology utilized in this step is derived from [Bollen \(1999\)](#) and [Wahab and Lee \(2011\)](#). However, the final equations and figures



are the result of our model.

For terminal nodes, the project value is equal to the last period's profit given the terminal value of  $\theta$  and the capacity level of previous time step. For intermediate nodes, project value is calculated as the sum of the present period's profits and the expected discounted project values presuming an optimal production plan. Capacity is entitled to change at each node and such changes are supposed to happen in the next node. To create capacity selection, given each capacity level of the previous node, present value (PV) is maximized by exploring over changes to all the set of possible capacity levels. Furthermore, the expected discounted forthcoming PV at a possible choice capacity level includes the probability of switching from one regime to the other at some point over the next period.

Let  $PV(\nu, \alpha, \ell_1, \ell_2, t)$  express the PV conditioned in regime  $\alpha$ , where  $\nu$  illustrates the node where  $\theta$  is presented,  $\ell_1$  indicates the capacity level in previous node for Company 1,  $\ell_2$  represents a given capacity level for Company 2 that is used for profit calculations. Specifically, the switch in capacity level for Company 1 is evaluated given a capacity level for Company 2 that varies in the same set of values as Company 1's. Therefore, the results of PV are presented in a square matrix  $l \times l$ , where  $l$  is the number of capacity levels that are being considered to assess the model for both companies.  $t$  represents the period of evaluation. Let  $E[PV(\nu, \alpha, \mathcal{L}(\ell_1), \ell_2, t)]$  indicate the expected discounted forthcoming PV given a change to the possible choice of capacity level  $\mathcal{L}(\ell_1)$ . For each level of capacity  $\ell_1$ , the PV is maximized by looking through all the changes in all other possible levels of capacity as follows:

$$PV(\nu, \alpha, \ell_1, \ell_2, t) = \max_{\mathcal{L}(\ell_1)} \{ \Pi^*(\nu, \ell_1, \ell_2, t) + S[\ell_1, \mathcal{L}(\ell_1)] + E[PV(\nu, \alpha, \mathcal{L}(\ell_1), \ell_2, t + 1)] \} \quad (3.32)$$

The calculation of  $E[PV]$  varies across the two regimes in the pentanomial lattice because each regime is described by a different branches as explained in Section 3.2. Suppose that growth is represented by the trinomial and the outer branches since the decay regime has the smallest  $\phi$ . At period  $t$ , the expected discounted future PV conditional to still being in growth at the same period,

with  $\theta$  at a  $\nu$  level and possible choice of capacity level  $\mathcal{L}(\ell_1)$  is calculated as

$$E[PV(\nu, g, \mathcal{L}(\ell_1), \ell_2, t)] = e^{-r_f h} \left\{ (1 - P(t)) [\psi_{gu} PV(\nu, g, \mathcal{L}(\ell_1), \ell_2, t + 1) + \psi_{gm} PV(\nu + 2, g, \mathcal{L}(\ell_1), \ell_2, t + 1) + \psi_{gd} PV(\nu + 4, g, \mathcal{L}(\ell_1), \ell_2, t + 1)] + P(t) [\psi_{du} PV(\nu + 1, d, \mathcal{L}(\ell_1), \ell_2, t + 1) + \psi_{dd} PV(\nu + 3, d, \mathcal{L}(\ell_1), \ell_2, t + 1)] \right\} \quad (3.33)$$

where  $P(t)$  is the probability of switching from growth to decay and  $\psi$  is the conditional branch probability. The first subfix of  $\psi$  represents the regime and the second one indicates the up, middle or down branches.

If demand switches to decay at  $t$ , then the expected discounted future PV is equal to:

$$E[PV(\nu, d, \mathcal{L}(\ell_1), \ell_2, t)] = e^{r_f h} [\psi_{du} PV(\nu + 1, d, \mathcal{L}(\ell_1), \ell_2, t + 1) + \psi_{dd} PV(\nu + 3, d, \mathcal{L}(\ell_1), \ell_2, t + 1)] \quad (3.34)$$

The induction goes backwards to the present. The first node in the lattice provides the NPV of the project including capacity options for each initial capacity level. The initial capacity level that maximizes NPV is the optimal. Figure 3.3 illustrates an example of previous definitions.

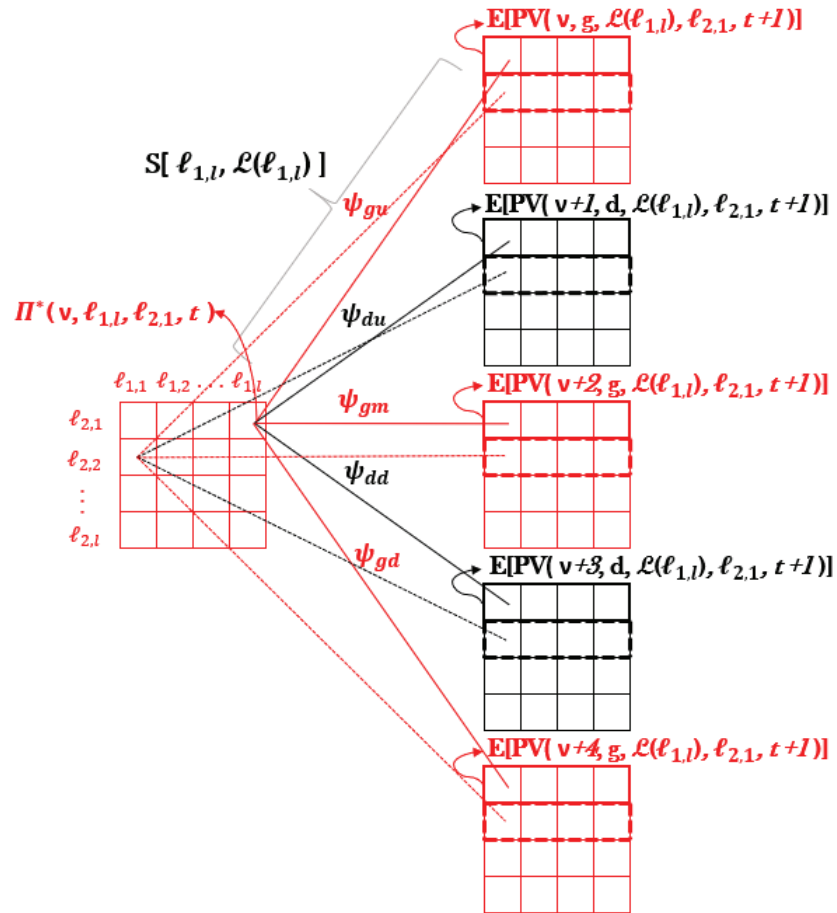


Figure 3.3: Example of dynamic programming

## Chapter 4

### Numerical Examples and Sensitivity Analysis

We conduct a detailed numerical study in MATLAB to illustrate how the companies behave under different game mechanisms in a regime-switching environment. Table 4.1 provides the values of the important model parameters below:

Table 4.1: Model input

(a) Demand		(b) Profits		(c) Switching Costs	
Parameter	Value	Parameter	Value	Parameter	Value
$\mu_g$	7.00%	$\lambda_1 = \lambda_2$	1.00	$c_\kappa$	3.00
$\sigma_g$	4.50%	$c_1 = c_2$	0.25	$\delta$	0.75
$\mu_d$	-2.50%	$w_1 = w_2$	1.50	$v$	-0.90
$\sigma_d$	3.00%	$o_1 = o_2$	0.15	$s_f$	0.25
$\mu_S$	30			$r_f$	8.00%
$\sigma_S$	3				
$h$	0.08333				
$T$	60				
$D_0$	100				

The parameters for mean and standard deviation for each regime are given in years; however, every period of evaluation is established monthly, which makes  $h = 1/12 = 0.0833$ . Using prior input for demand,  $\phi = 0.0089$ , and the trinomial part belongs to the growth regime linked to the outer branches with conditional probabilities governing the upward, middle and downward branches of 0.4808, 0.3611, and 0.1581 respectively. On the other hand, decay is represented by the binomial part with conditional probabilities of upward, middle and downward branches of 0.3810, 0.0000, and 0.6190. The mean and standard deviation to calculate the switching probability are established in months. The initial capacity and increments in capacity are 5 units until we reach a value of 300 units, in this case, we set a range of 60 capacity levels. Some experiments require a different range of capacities and project lifetime; therefore, those values are specified at the beginning of the respective experiment.

Experiments in Section 4.7 are run with a project lifetime of 5 and 10 months, but their mean and standard deviation of switching are specified in the respective experiment. Additionally, with the assumption of similar input for Companies 1 and 2, and according to Equation (3.20) that establishes an identical formula for  $C_1$  and  $C_2$ ; the results in Cournot's model is presented as one.

## 4.1 Valuation of the Model for Incremental Changes in Capacity

We first investigate how the maximum NPV varies in relation to changes on different levels of capacities. Specifically, we search over a range of capacities from 0.01 of the initial demand to a maximum value that calculation is explained next, in increments,  $\Delta K$ , of 0.01 units of the initial demand (1 unit). To set the maximum level of capacity, we calculate the highest value that demand rate,  $\theta$ , can take if it always grows in the outer branch of the pentanomial lattice for the growth regime during the project lifetime. Applying Equation (3.11), the biggest value of the drift of demand is when life time period,  $T$ , is equal to 60 and the node,  $\nu$ , equals to 1. Therefore,  $\theta_{max} = 1 \cdot e^{\{[2(60-1)-(1-1)]0.0089\}} = 2.861$ . The estimation of the maximum demand, previously calculated, shows the highest demand is approximately 3. Therefore, we consider the maximum capacity is set as 3 times the initial demand. Additionally, we vary the initial capacity and its increments in sets of 0.02, 0.03, 0.04 and 0.05 of the initial demand, respectively, until all of them reach 300 units.

This exercise is repeated by using a  $T = 10$ , where  $\theta_{max} = 1.174$ . Therefore, for this project lifetime the maximum capacity established is 1.2 units. Specifically, if capacity starts in 0.01, this is assigned to the first capacity level ( $\ell = 1$ ), then 0.02 is assigned to the second capacity level ( $\ell = 2$ ), and we continue until we reach 3 units of capacity that is capacity level 300 ( $\ell = 300$ ). Table 4.2 shows the range of capacity levels needed in every period according to the increments to reach the maximum capacity. For instance, in a 60-month lifetime project with capacity increments of 0.05 units, 60 levels are needed to look through all ranges of capacity values to find the one that maximizes the NPV. On the other hand, for a 10-month lifetime project, only 24 levels are required when capacity increments are 0.05 units.

Table 4.3 compares the maximum NPV and optimal initial capacity for the two participants in the two game-theoretic approaches with a project life time of 60 months. It has been observed that for all the three scenarios, the maximum NPV for player 1 is always obtained when player 2

Table 4.2: Range of capacity levels using different increments to achieve maximum capacity in each period

$\Delta K$	$T = 10$	$T = 60$
0.01	120	300
0.02	60	150
0.03	40	100
0.04	30	75
0.05	24	60

operates with the first capacity level; accordingly, player 1's optimal initial capacity is shown in column  $K_0^*$ . On the other hand, with one player at the lowest capacity and the other at the optimal capacity, Cournot's model presents a higher total NPV adding the two values of the participants. Appendix A.2 exhibits payoff matrices with increments of capacity in 5 units for each participant in Cournot and Stackelberg's models. We observe that  $C_1$  a maximum NPV of 137,981.30, given an optimal initial capacity of 60 units and 5 units for  $C_2$ .  $S_L$  obtains a maximum NPV of 146,722.53, given an optimal initial capacity of 60 units and 5 units for the  $S_F$ . Adding up the two values of NPV in the cells of optimal capacity for Company 1 equals 60 units and Company 2 equals 5 units, we obtain a total NPV of 208,918.03 for Cournot's model, and for Stackelberg leader equals 60 units and for Stackelberg follower 5 units, the total NPV is 189,262.47 for Stackelberg's model.

In this numerical example, the broader the increments in capacity, the lower the NPV. Moreover, when all the increments get bigger, the capacity level that makes the highest NPV gets lower. For example, for increments of 1 unit in Cournot's model, the biggest NPV comes from the sixty fourth level of capacity ( $1 \cdot 64 = 64$ ), for increments of 2 units is the thirty second level ( $2 \cdot 32 = 64$ ), for increments of 3 units is the twenty first level ( $3 \cdot 21 = 63$ ), for increments of 4 units is the sixteenth level ( $4 \cdot 16 = 64$ ) and for increments of 5 units is the twelfth level ( $5 \cdot 12 = 60$ ).

Table 4.3: Comparisons of different outcomes for Cournot and Stackelberg's models due to changes in increments in capacity

(a) $C_1$ and $C_2$			(b) $S_L$			(c) $S_F$		
$\Delta K$	NPV	$K_0^*$	$\Delta K$	NPV	$K_0^*$	$\Delta K$	NPV	$K_0^*$
1	153,827.03	64	1	157,897.08	63	1	145,502.24	60
2	148,275.53	64	2	155,099.70	62	2	135,753.08	58
3	144,188.51	63	3	152,305.49	60	3	129,783.13	57
4	140,868.10	64	4	149,513.28	60	4	125,540.53	56
5	137,981.30	60	5	146,722.54	60	5	122,208.24	55

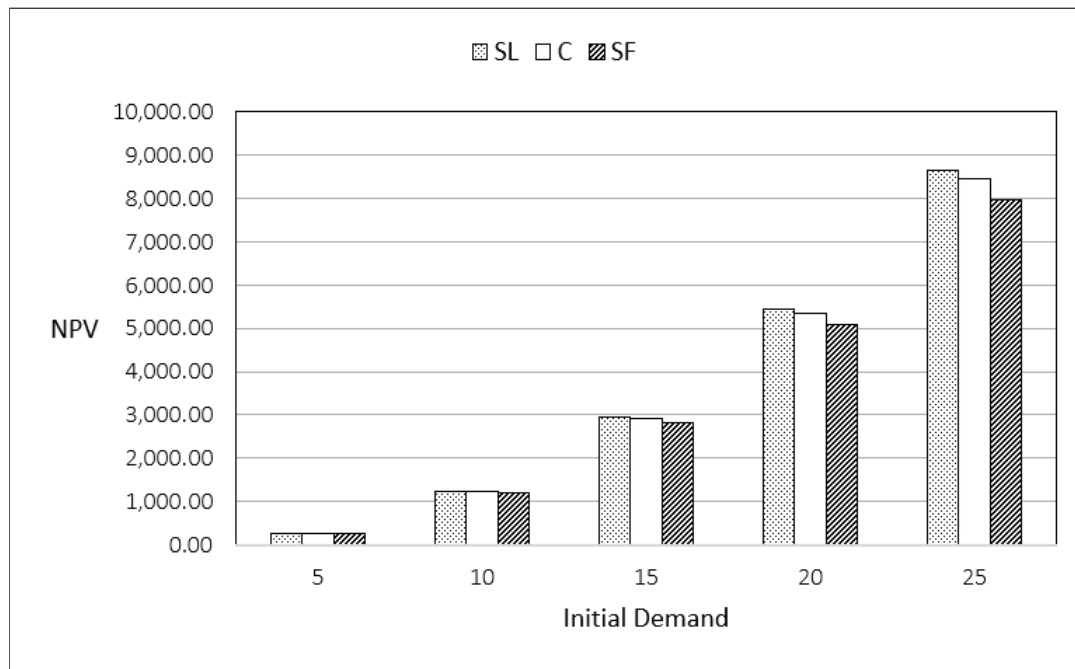
The findings show that an increase in capacity increments results in a smaller NPV. A change in increments of capacity from 1 to 2 units decreases the NPV 3.60% for Cournot, 1.77% for

Stackelberg leader and 6.70% for Stackelberg follower. Additionally, the optimal capacity remains the same for Cournot but decreases 1 unit in Stackelberg leader and 2 units in Stackelberg follower. It can be deduced because with small increments in capacity, the company can adjust easily and less expensive to an addition in quantity to produce. Additionally, the company incurs in less overhead cost that increases profitability. The results show that Cournot's model needs an higher initial capacity to operate over participants in the Stackelberg's model under the same increments of capacity. However, Stackelberg leader obtains the maximum NPV under the same scenarios.

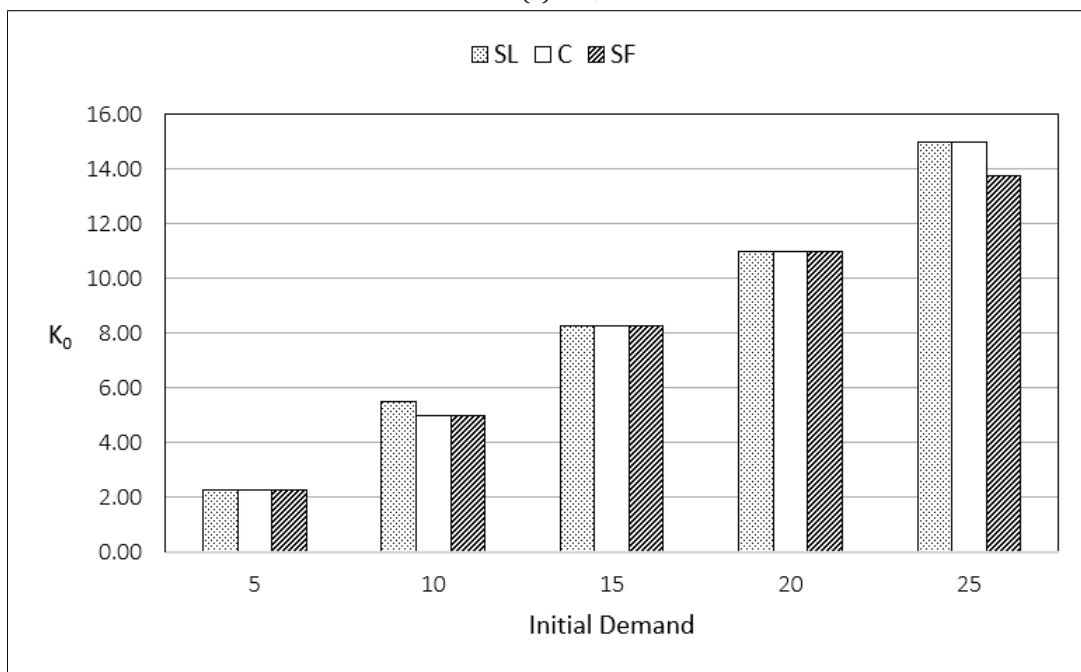
## **4.2 Valuation of the Model for Incremental Changes in Initial Demand**

Next, we examined how NPV and optimal initial capacity fluctuates with modifications in the initial demand. The NPV is maximized by exploring over an interval of capacities from 0.05 to 3 times the initial demand in each period by increments of 0.05. We establish our first value of initial demand as 5 units, then increase it every 5 units up to a value of 25 units is reached. Figure 4.1 displays that NPV and optimal initial capacity grows in an exponential mode as initial demand expands. An increment from 15 to 20 units of initial demand improves the NPV 82.53% for Cournot, 83.61% for Stackelberg Leader and 80.23% for Stackelberg Follower. On the other hand, the optimal capacity increases from 8.25 to 11.00 unit for all the three scenarios studied. In summary, these results indicate that increase in the initial demand leads to a higher NPV and initial capacity values.

Expanding initial demand makes the drift in demand become larger. This is explained because more units of the product can be supplied; therefore, more capacity is required. Additionally, given a cost structure that remains the same, more profits are generated due to increments in demand. Further, Stackelberg Leader presents a pattern for the NPV that grows faster than its opponent, Stackelberg follower, and also better than Cournot's approach.



(a) NPV



(b) Optimal Capacity

Figure 4.1: Variability in NPV and optimal capacity due to changes in initial demand



### 4.3 Valuation of the Model for Incremental Changes in Expansion and Contraction Costs

In this section, we assess how the maximum NPV and the optimal initial capacity differ by changing the percentage of contraction ( $\delta$ ) and expansion ( $v$ ) applied to installation costs for capacity flexibility calculations. We set back the initial demand as 100 units, initial and incremental units of capacity as 0.05 of the initial demand. It makes the first level of capacity 5 units and the last 300 units. These set of capacity levels are kept for the rest of the experiments with a lifetime project of 60, unless a variation is specified. Furthermore, we range values in contraction from 0.75 to 1.00, in increments of 0.05; values in expansion from -0.75 to -1.00, in increments of -0.05 as well. Table 4.4 presents the maximum NPV and its respective optimal initial capacity in every game.

We recognize that additions in the contraction option expands the NPV but the optimal capacity remains the same for the studied models. In the case of Stackelberg follower, for a given expansion value of 80%, the NPV grows 0.68% for a contraction value that goes from 85% to 90%. Additionally, if contraction value keeps expanding another extra 5.00%, it makes the NPV 0.71% higher. However, the optimal capacity stays in 45 units for the additional 10% in contraction value. Contraction benefits the NPV since it generates a profit from either obtaining a salvage value or using such extra capacity in other products manufacture.

On the other hand, the NPV is a decreasing function and optimal capacity an increasing function of expansion costs for both game theoretic models. We observe that for Stackelberg follower, given a contraction value of 80.00%, a variation of expansion costs from 85.00% to 90.00% lowers the NPV in 0.0069%, and also switches optimal capacity from 45 to 50 units. Furthermore, another increment of 5.00% in the expansion option reduces the NPV in 0.01% and increases optimal capacity in another 5 units. Reduction in the NPV is explained by an addition of switching costs at the moment the company must acquire more capacity in order to increase production to supply positive drifts in the expected demand. Lower initial capacity, as result of increments in expansion value, provides management the choice of avoiding to incur in future costs; therefore, they buy more capacity when it is cheaper. Besides, less capacity reduces overhead costs that decrease the expected profits of the project.

Table 4.4: Comparisons of NPV and optimal capacity for Cournot and Stackelberg's models due to changes in  $\delta$  and  $v$

(a)  $C_1$  and  $C_2$

Contraction Cost	Expansion Cost					
	-0.75	-0.80	-0.85	-0.90	-0.95	-1.00
0.75	138,013.31	138,001.42	137,990.72	137,981.30	137,973.74	137,969.76
	45	50	55	60	75	105
0.80	138,293.94	138,009.89	137,998.89	137,989.23	137,981.48	137,977.31
	45	50	55	60	75	105
0.85	139,102.26	138,286.89	138,007.33	137,997.36	137,989.38	137,985.02
	45	50	55	60	75	105
0.90	139,952.04	139,095.59	138,280.69	138,005.78	137,997.49	137,992.90
	45	50	55	60	75	105
0.95	140,828.62	139,945.58	139,089.74	138,275.47	138,005.87	138,000.98
	45	50	55	60	75	105
1.00	141,724.43	140,822.32	139,939.95	139,084.89	138,271.90	138,009.34
	45	50	55	60	75	105

(b)  $S_L$

Contraction Cost	Expansion Cost					
	-0.75	-0.80	-0.85	-0.90	-0.95	-1.00
0.75	146,753.65	146,742.67	146,732.03	146,722.54	146,715.31	146,711.45
	45	45	45	60	70	100
0.80	147,019.57	146,749.39	146,738.45	146,728.72	146,721.31	146,717.29
	45	45	45	60	70	100
0.85	147,856.07	147,011.50	146,745.15	146,735.11	146,727.48	146,723.27
	45	45	45	60	70	100
0.90	148,734.74	147,848.49	147,003.44	146,741.79	146,733.83	146,729.42
	45	45	45	60	70	100
0.95	149,619.70	148,727.29	147,840.92	146,996.27	146,740.48	146,735.75
	45	45	45	60	70	100
1.00	150,505.77	149,612.27	148,719.85	147,834.23	146,991.17	146,742.38
	45	45	45	60	70	100

(c)  $S_F$

Contraction Cost	Expansion Cost					
	-0.75	-0.80	-0.85	-0.90	-0.95	-1.00
0.75	122,236.79	122,226.14	122,216.60	122,208.24	122,201.47	122,197.98
	40	45	50	55	70	95
0.80	122,493.07	122,234.13	122,224.29	122,215.74	122,208.79	122,205.15
	40	45	50	55	70	95
0.85	123,331.04	122,486.41	122,232.24	122,223.41	122,216.26	122,212.45
	40	45	50	55	70	95
0.90	124,205.98	123,324.70	122,480.54	122,231.35	122,223.92	122,219.90
	40	45	50	55	70	95
0.95	125,104.90	124,199.84	123,319.11	122,475.68	122,231.82	122,227.52
	40	45	50	55	70	95
1.00	126,021.06	125,098.89	124,194.45	123,314.49	122,472.24	122,235.40
	40	45	50	55	70	95

The models are more sensitive to fluctuations in expansion costs than in contraction ones when it comes to obtain optimal initial capacity. The reason is that contraction aligns with the company's objective of maximizing profits. As a result, for a given expansion cost, extra capacity that is not necessary in the future when demand decays provides an extra positive cash flow. Results also show that  $S_L$  always presents the highest NPV for the all the combinations of expansion and contraction costs in the models. Explanation of this findings are further discussed in Section 4.7.4

## 4.4 Valuation of the Model for Incremental Changes in Installation Costs

In this experiment, we evaluate how the maximum NPV and optimal initial capacity vary with increments in installation costs ( $c_\kappa$ ) for the two different participants in the game-theoretic approach. We increase installation costs in one unit, starting from 1.00 and ending at 5.00. Table 4.5 exhibits that, for all of the models, the larger the installation costs, the lower the maximum NPV and optimal capacity. For instance, costs from 1.00 to 2.00 in Stackelberg leader reduces the NPV in 0.06% and optimal capacity in 15 units which represents 18.75% less. This results are expected since the more expensive the installation costs, the less capacity is purchased leading to a higher NPV. On the other hand, Stackelberg leader reaches the biggest NPV compared to follower and among games.

Table 4.5: Comparisons of NPV and optimal capacity in Cournot and Stackelberg's models due to changes in installation costs

$c_\kappa$	Game		
	$C_1$ and $C_2$	$S_L$	$S_F$
1.00	138,135.13	146,881.40	122,342.63
	85	80	75
2.00	138,056.64	146,800.78	122,273.38
	70	65	65
3.00	137,981.30	146,722.54	122,208.24
	60	60	55
4.00	137,907.93	146,646.52	122,144.84
	55	50	50
5.00	137,836.30	146,572.57	122,083.00
	50	45	45

Installation costs play an important role in decision making for this models in the flexibility

section because they define the initial investment given the initial optimal capacity. Additionally, such costs modify the contraction and expansion costs that are essential for dynamic decision making every period in every node. In summary, we note that the higher the installation costs, the lower the NPV and the optimal capacity.

## 4.5 Valuation of the Model for Incremental Changes in Variable and Marginal Costs

Working with the cost function, we analyze how operational costs, variable and marginal, affect the maximum NPV and optimal initial capacity for each model. Furthermore, every time one of the costs is changed, the identical value is applied for the two firms competing. We range variable costs from 0.10 to 0.25, in increments of 0.05; and marginal costs from 1.00 to 2.50, in increments of 0.5. Table 4.6 shows that expansion in variable costs make the NPV shrink but optimal capacity stays in the same level. This trend happens in both models with the participants. For instance, for a given marginal cost of 2.50 in Cournot, a shift of variable costs from 0.15 to 0.20 reduces the NPV in 0.09% and the optimal capacity remains in 80 units. These results are consistent with an inverse relationship between costs and profits. For a given volume of production, the higher the variable costs, the lower the profit: therefore, NPV decreases. In addition, since Variable costs are dependent on production output and is a constant amount per unit produced, such value does not affect the capacity required.

Furthermore, increments in marginal costs increases the NPV and optimal capacity in Cournot and Stackelberg Follower's models. We demonstrate that for a given variable cost of 0.15 in Cournot, a switch between 2.00 and 2.50 in marginal costs improves the NPV in 0.46% and adds up 10 extra units of optimal capacity. For the same given variable costs and switch in marginal costs shown in the previous Cournot's example, the NPV in Stackelberg Follower increases 1.37% and requires 5 more units of optimal capacity. However, in Stackelberg Leader, for the same variable costs and changes in marginal cost, the NPV drops 0.18% and 10 additional units of optimal capacity are needed. Consistent with the inverse relation between marginal cost and capacity, explained in the methodology section, increments in marginal costs make capacity increase to smooth the impact in the profits. Additionally, the more capacity purchased at the beginning, the more quantity to

Table 4.6: Comparisons of NPV due to changes in variable and marginal costs for both game-theoretic models

(a)  $C_1$  and  $C_2$

Variable Costs	Marginal Costs			
	1.00	1.50	2.00	2.50
0.10	137,437.97	138,356.89	139,124.60	139,762.02
	50	60	70	80
0.15	137,313.77	138,231.64	138,998.47	139,635.14
	50	60	70	80
0.20	137,189.62	138,106.44	138,872.39	139,508.32
	50	60	70	80
0.25	137,065.52	137,981.30	138,746.38	139,381.55
	50	60	70	80

(b)  $S_L$

Variable Costs	Marginal Costs			
	1.00	1.50	2.00	2.50
0.10	147,484.63	147,125.75	146,820.77	146,550.21
	45	60	65	75
0.15	147,350.10	146,991.28	146,686.45	146,416.02
	45	60	65	75
0.20	147,215.63	146,856.88	146,552.19	146,281.89
	45	60	65	75
0.25	147,081.22	146,722.54	146,417.99	146,147.82
	45	60	65	75

(c)  $S_F$

Variable Costs	Marginal Costs			
	1.00	1.50	2.00	2.50
0.10	120,304.04	122,539.57	124,495.73	126,207.77
	45	55	65	70
0.15	120,195.75	122,429.08	124,383.31	126,093.65
	45	55	65	70
0.20	120,087.52	122,318.64	124,270.93	125,979.58
	45	55	65	70
0.25	119,979.33	122,208.24	124,158.61	125,865.56
	45	55	65	70

supply that can be produced. Such effect creates a positive response in the profits and therefore, in the NPV for Cournot and Stackelberg Follower. In the case of Stackelberg Follower, the decline in NPV is caused because of the quadratic effect of quantity to product,  $Q$ , which increases the value of the total costs function, and reduces the expected profits. Additionally, Stackelberg Leader is the player that has the biggest optimal amount to produce among its opponent and the other game, as shown in (3.26).

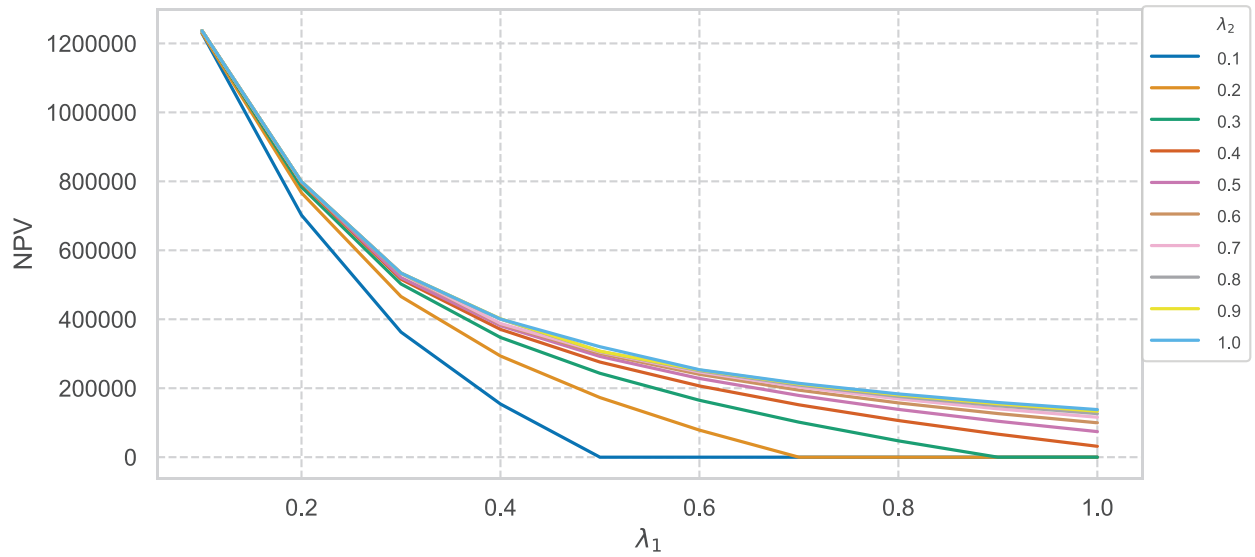
In summary, variable costs increase has a negative impact in the NPV and does not affect the optimal capacity. On the other hand, marginal costs increments improve the NPV for Cournot and Stackelberg Follower, reduce the NPV for Stackelberg Leader and expands the optimal capacity for the three scenarios. Furthermore, we again observe that Stackelberg leader provides the highest NPV compared to its opponent and the other model for all the combinations presented in this experiment.

## 4.6 Valuation of the Model for Incremental Changes in

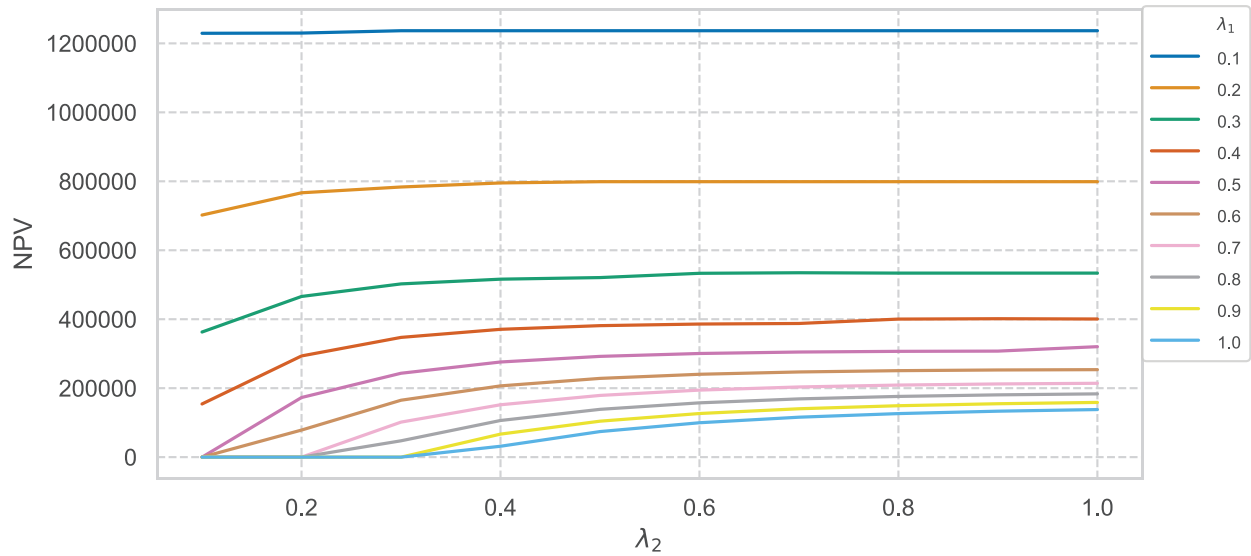
### Price-Demand Elasticity

In this experiment, we plot the response of the maximum NPV and optimal initial capacity to variations in the price-demand elasticity for both participants in the Cournot's model. The price-demand elasticity for player 1,  $\lambda_1$ , and for player 2,  $\lambda_2$ , start from 0.1 to 1.00, in increments of 0.1. Specifically, these values must be between 0.00 and 1.00 to satisfy the condition that prices cannot be negative; and we study the outcome through the effect of such elasticity in one player. Figure 4.2a shows that the NPV falls in a convex shape, identical to an exponential function, for changes in  $\lambda_1$ . In addition, the graphic displays that increases in  $\lambda_2$  make such curve decays slower. Furthermore, Figure 4.2b shows that  $\lambda_2$  has a concave shape similar to a logarithmic function. The curves go lower when the value of  $\lambda_1$  increases. For instance, given  $\lambda_1 = 0.20$  and  $\lambda_2 = 0.30$ , the NPV = 783,439.88. If  $\lambda_1$  gets increased in 0.1 units, the NPV declines 35.87% but if  $\lambda_1$  remains the same and  $\lambda_2$  increases in 0.1 units, the NPV improves by 1.50%. As such, the effect of  $\lambda_1$  is stronger than the one of  $\lambda_2$ .  $\lambda_1$  decreases the NPV in a higher percentage than  $\lambda_2$  increases the same outcome. As elasticity in player 1 increases, for a given quantity supplied, the price of the item increases; therefore, the profit grows. On the other hand, an increment in the elasticity of the

player 2 makes the price higher which creates a reaction in the consumers to buy the cheapest one. Such reaction explains why a bigger value of elasticity in player 2 generates an increase in NPV for player 1.



(a)  $\lambda_1$  contribution to participant



(b)  $\lambda_2$  contribution to participant

Figure 4.2: Variability in NPV due to changes in  $\lambda$  for Cournot's model

Figure 4.3 exhibits similar effect in the graphic trends for optimal capacity as presented for NPV. we found three outliers for the points  $(\lambda_1, \lambda_2) = \{(0.30, 0.60), (0.40, 0.80), \text{ and } (0.50, 1.00)\}$ .

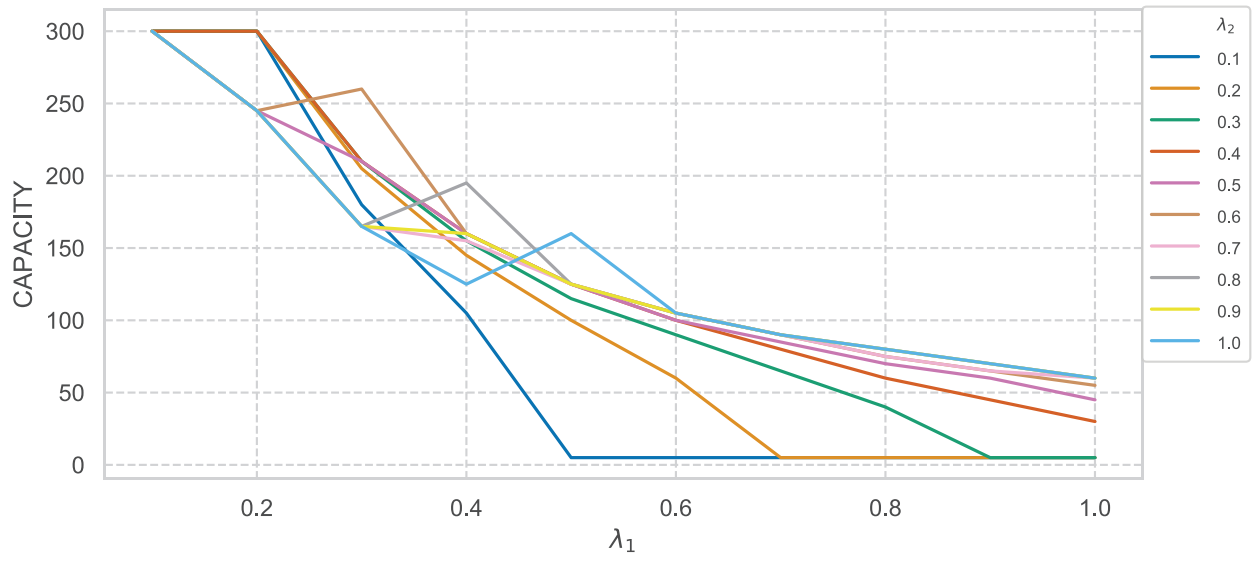
The respective values of maximum capacity are: 260, 195, and 160 units. This happens when  $\lambda_2 = 2\lambda_1$ . Such points expand the capacity to a maximum point and then contract it to a level that makes it stable. For the point (0.30, 0.60), the optimal capacity increases 6.12% from  $\lambda_1 = 0.2$  to  $\lambda_1 = 0.3$  and then, decreases 38.46% when  $\lambda_1$  jumps to 0.4. Similarly, for (0.40, 0.80) the capacity switches 18.18%, then decays 35.90%; and (0.50, 1.00) the capacity increases 28.00% and then decreases 34.38%. We note that the greater the values of  $\lambda_1$  and  $\lambda_2$  in the pair of points, the higher the growth and the smaller the decay.

In summary, the greater the  $\lambda_1$ , the lower the NPV and optimal capacity. Contrary, the bigger the  $\lambda_2$ , the higher the NPV and the optimal capacity. Such outcome is explained by the inverse price-demand relation. An increment in  $\lambda_1$  can have two effects considering the drift on demand,  $\theta$  and the quantity produced by the other company,  $Q_2$ , constant: First, the price of the good decreases given a constant quantity to produce by this company,  $Q_1$ . Therefore, the profits reduces and consequently, the NPV. Second, the amount to produce by the company has to be reduced for a given price. Hence, the lower the quantity to produce, the less capacity level needed. For instance, an elasticity close to the value of 0 makes consumers inelastic to the price. Therefore, the responsiveness to the price make customers indifferent to buy the item. Consequently, the company can produce more and acquire more capacity. The opposite is expected when the elasticity is close to 1. On the other hand, when Company 2's elasticity,  $\lambda_2$ , is increased, the same two effects as described previously for Company 1 happen to Company 2. Therefore, Company 2 produces less and Company 1 has the opportunity to produce more to supply demand. Thus, for a given  $Q_1$  and  $\lambda_1$ , if  $Q_2$  is smaller, the price for Company 1 increases and consequently, their respective profits.

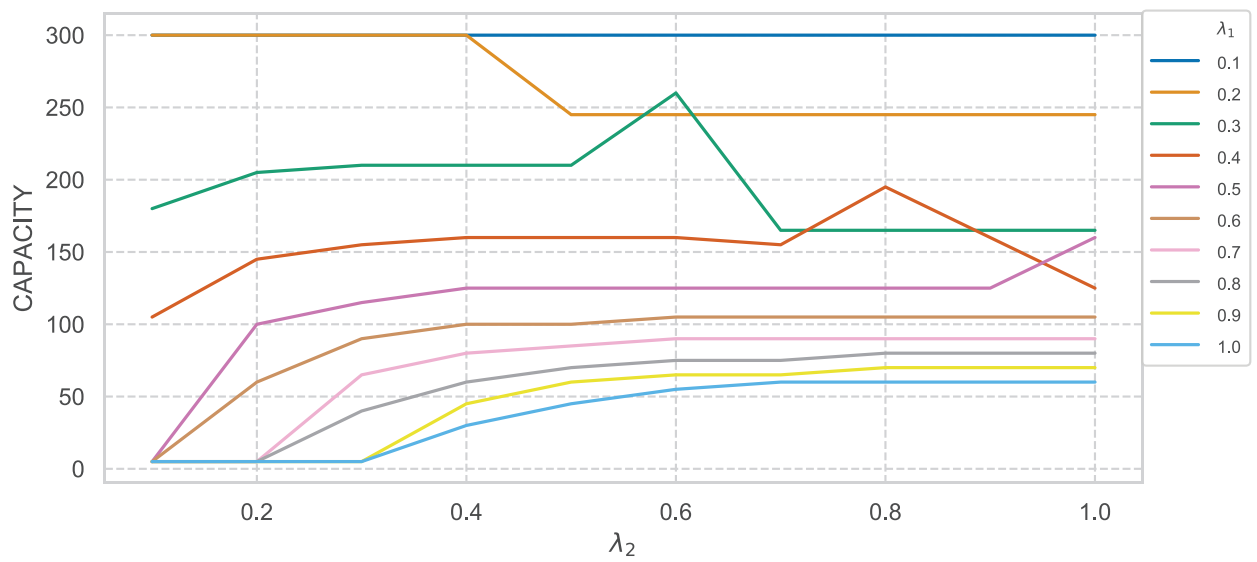
## 4.7 Example of Optimal Path for the Model

We create a graphical representations and decide to evaluate three different scenarios by standing out the most likely path and its behaviour in a lattice for a 10-period life cycle project for a Cournot's model. In these scenarios, all the input remain the same except that for a 10-month lifetime, the maximum capacity to be considered is 1.2 times initial demand as shown in Table 4.2 (120 units of production equal to 24 levels). The switching mean,  $\mu_S$ , equals to 3, 5 and 7 months, and the switching standard deviation,  $\sigma_S$ , is equal 0.1 for all the scenarios.





(a)  $\lambda_1$  contribution to participant



(b)  $\lambda_2$  contribution to participant

Figure 4.3: Variability in initial capacity due to changes in  $\lambda$  for Cournot's model

As we explained at the beginning of Section 4, the regime growth is a trinomial lattice represented by the extreme branches in the pentanomial lattice, and decay is a binomial lattice constituted by the middle branches. For every node involved in the most likely path, branches that connect such nodes are highlighted in red according to the highest criteria of selection. The criteria of selection is defined by the multiplication of each conditional probability in the respective branch times one minus switching probability,  $1 - P(s)$ , in the period of evaluation in growth regime; and times switching probability,  $P(s)$ , in the same period for the decay regime. Additionally, values of the expected NPV, optimal capacity, profit, quantity to produce and demand drift are shown in the tables related to each node involved in the such path. Every node displays five branches corresponding to the events (a) growth upward, (b) decay upward, (c) growth middle, (d) decay downward, and (e) growth downward. Figure 4.4 shows graphically these concept.

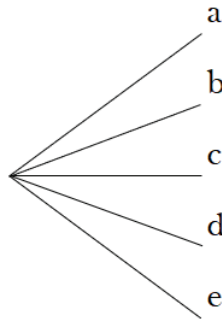


Figure 4.4: Nodes

#### 4.7.1 Scenario 1: $\mu_S = 3$ periods and $\sigma_S = 0.1$ periods

Figure 4.5 presents an ascent from period 1 to 4 by remaining in node (a) permanently. Suddenly, there is a break point in period 4 that makes the path descend continuously over node (d) up to the end of the cycle in period equals 10.

Table 4.7 summarizes the outcomes described in the previous paragraph. Findings highlighted in red belongs to the path when lattice is most likely to stay in growth upward and bold black results are the ones when the path is most likely to stay in decay downward. For example, the break point in period 4 is obtained by the maximum criteria value  $0.2063 \cdot 0.4808 = 0.3816$ , in period 4. Then, in period 5, the maximum criteria is  $1.000 \cdot 0.6190 = 0.6190$ . As a result, demand decreases 0.89% jumping from 105.49 in node a to 104.55 in node d for period 4 to 5. Such

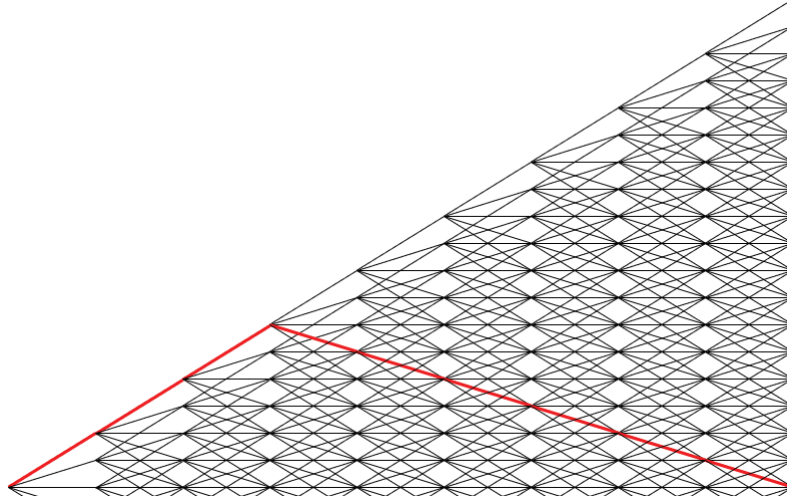


Figure 4.5: Optimal path scenario 1

drift in demand decreases 0.89% of quantity to produce, 1.85% profits, 15.32% the expected NPV. However, optimal capacity remains constant but explanation of this effect is explained in section 4.7.4.

#### 4.7.2 Scenario 2: $\mu_S = 5$ periods and $\sigma_S = 0.1$ periods

Similar to Section 4.7.1, Figure 4.6 presents an constant ascent from period 1 to 6 by staying in node (a). Suddenly, there is a break point in period 6 that makes the path descend continuously over node (d) up to the end of the cycle in period equals 10.

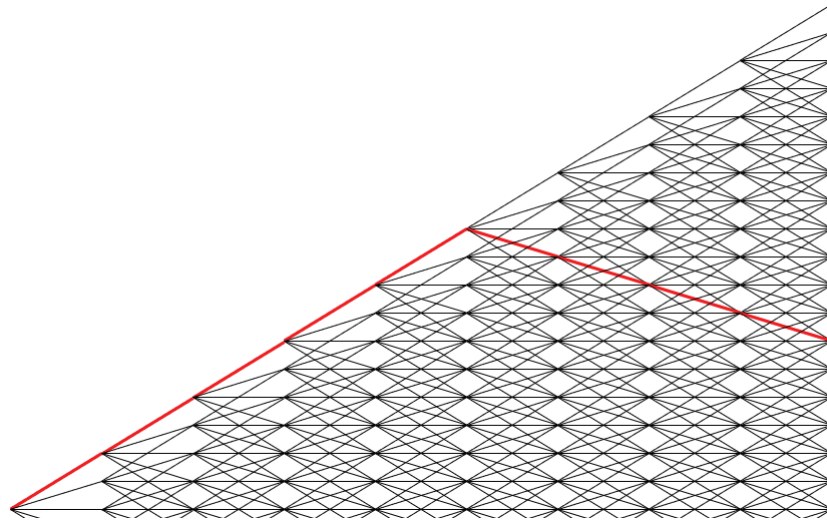


Figure 4.6: Optimal path scenario 2

Table 4.7: Values in the nodes of the most likely path for Figure 4.5

(a) Criteria

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	1.0000	0.4808	0.4808	0.3816	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
b		0.0000	0.0000	0.0786	0.3810	0.3810	0.3810	0.3810	0.3810	0.3810
c		0.3611	0.3611	0.2866	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
d		0.0000	0.0000	0.1277	0.6190	0.6190	0.6190	0.6190	0.6190	0.6190
e		0.1581	0.1581	0.1255	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(b) Net Present Value, NPV

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	20,599.19	19,197.37	17,579.12	15,877.72	14,213.26	11,705.53	9,261.42	6,879.68	4,559.08	2,185.39
b		18,844.22	17,256.05	15,586.07	13,952.48	11,491.06	9,092.13	6,754.45	4,476.81	2,145.01
c		18,497.45	16,938.90	15,299.71	13,696.41	11,280.47	8,925.89	6,631.47	4,396.02	2,105.36
d		18,157.03	16,627.72	15,019.03	13,444.95	11,073.66	8,762.65	6,510.72	4,316.69	2,066.42
e		17,822.69	16,322.13	14,743.45	13,198.40	10,871.21	8,603.36	6,393.78	4,241.46	2,028.19

(c) Profit,  $\Pi$

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	2,077.58	2,162.12	2,244.55	2,330.04	2,418.70	2,373.97	2,330.04	2,286.91	2,244.55	2,185.39
b		2,122.01	2,202.96	2,286.91	2,373.97	2,330.04	2,286.91	2,244.55	2,202.96	2,145.01
c		2,082.63	2,162.12	2,244.55	2,330.04	2,286.91	2,244.55	2,202.96	2,162.12	2,105.36
d		2,043.95	2,122.01	2,202.96	2,286.91	2,244.55	2,202.96	2,162.12	2,122.01	2,066.42
e		2,005.98	2,082.63	2,162.12	2,244.55	2,202.96	2,162.12	2,122.01	2,082.63	2,028.19

(d) Quantity,  $Q$

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	35.45	36.28	36.94	37.60	38.28	37.94	37.60	37.27	36.94	36.13
b		35.96	36.61	37.27	37.94	37.60	37.27	36.94	36.61	35.81
c		35.64	36.28	36.94	37.60	37.27	36.94	36.61	36.28	35.49
d		35.32	35.96	36.61	37.27	36.94	36.61	36.28	35.96	35.18
e		35.01	35.64	36.28	36.94	36.61	36.28	35.96	35.64	34.87

(e) Demand,  $\theta$

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	100.00	101.80	103.63	105.49	107.39	106.43	105.49	104.55	103.63	102.71
b		100.89	102.71	104.55	106.43	105.49	104.55	103.63	102.71	101.80
c		100.00	101.80	103.63	105.49	104.55	103.63	102.71	101.80	100.89
d		99.11	100.89	102.71	104.55	103.63	102.71	101.80	100.89	100.00
e		98.23	100.00	101.80	103.63	102.71	101.80	100.89	100.00	99.11

(f) Optimal capacity,  $K_0^*$

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	60	90	90	90	90	90	90	90	90	40
b		90	90	90	90	90	90	90	90	40
c		90	90	90	90	90	90	90	90	40
d		90	90	90	90	90	90	90	90	40
e		90	90	90	90	90	90	90	90	40

Table 4.8 displays the results of this case. The break point in period 6 is calculated by the maximum criteria value of  $0.1294 \cdot 0.4808 = 0.4186$ ; subsequently, in period 7, the maximum criteria is  $1.000 \cdot 0.6190$ . As a result, demand decreases 3.50% jumping from 106.43 in node (a) to 102.71 in node (d) in the period 6 to 7. Such decline in demand decreases 0.89% of quantity to produce, 1.85% of profits, and 20.89% of the expected NPV. we notice that, although demand has a higher value at the break point than the one from scenario 1, the quantity to produce and profits slightly change. However, the expected NPV at the break point compared with scenario 1 increases 5.57%. Such result is due to the longer time the optimal path maintains in growth upward node.

#### 4.7.3 Scenario 3: $\mu_S = 7$ periods and $\sigma_S = 0.1$ periods

Figure 4.7 illustrates an increase from period 1 to 8 by staying up in node (a) for such span. Suddenly, there is a break point in period 8 that makes the path descend continuously over node (d) up to the end of the cycle in period 10.

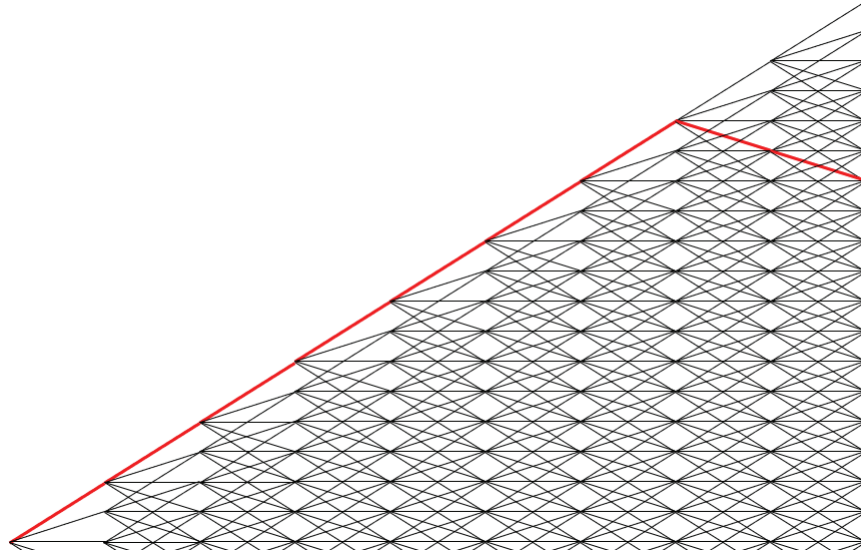


Figure 4.7: Optimal path scenario 3

Table 4.9 shows the findings this case. The calculation of the break point in period 8 is obtained by the maximum criteria  $0.0943 \cdot 0.4808 = 0.4355$  in the respective period. Afterwards, in period 9, the maximum criteria is  $1.000 \cdot 0.6190$ . As a result, demand decreases 3.50% changing from 104.55 in node (a) to 100.90 in node (d) for period 8 to 9. Such decay in demand decreases 0.89% of quantity to produce and 1.85% profits. The expected NPV goes down 33.80% . Although

Table 4.8: Values in the nodes of the most likely path for Figure 4.6

(a) Criteria

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	1.0000	0.4808	0.4808	0.4808	0.4808	0.4186	0.0000	0.0000	0.0000	0.0000
b		0.0000	0.0000	0.0000	0.0000	0.0493	0.3810	0.3810	0.3810	0.3810
c		0.3611	0.3611	0.3611	0.3611	0.3143	0.0000	0.0000	0.0000	0.0000
d		0.0000	0.0000	0.0000	0.0000	0.0801	0.6190	0.6190	0.6190	0.6190
e		0.1581	0.1581	0.1581	0.1581	0.1377	0.0000	0.0000	0.0000	0.0000

(b) Net Present Value, NPV

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	21,004.71	19,615.97	18,011.31	16,269.95	14,384.36	12,372.81	10,344.07	7,680.83	5,085.38	2,443.70
b		19,254.99	17,680.16	15,971.10	14,120.50	12,146.33	10,155.61	7,541.17	4,993.63	2,398.67
c		18,900.55	17,355.02	15,677.67	13,861.40	11,923.92	9,970.21	7,404.02	4,903.53	2,354.45
d		18,552.70	17,035.99	15,389.73	13,607.05	11,705.53	9,788.15	7,269.34	4,815.06	2,311.02
e		18,211.12	16,722.75	15,107.06	13,357.35	11,491.06	9,609.38	7,137.09	4,728.17	2,268.38

(c) Profit,  $\Pi$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	2,077.58	2,162.12	2,244.55	2,330.04	2,418.70	2,510.64	2,605.98	2,557.88	2,510.64	2,443.70
b		2,122.01	2,202.96	2,286.91	2,373.97	2,464.25	2,557.88	2,510.64	2,464.25	2,398.67
c		2,082.63	2,162.12	2,244.55	2,330.04	2,418.70	2,510.64	2,464.25	2,418.70	2,354.45
d		2,043.95	2,122.01	2,202.96	2,286.91	2,373.97	2,464.25	2,418.70	2,373.97	2,311.02
e		2,005.98	2,082.63	2,162.12	2,244.55	2,330.04	2,418.70	2,373.97	2,330.04	2,268.38

(d) Quantity,  $Q$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	35.45	36.28	36.94	37.60	38.28	38.97	39.67	39.32	38.97	38.12
b		35.96	36.61	37.27	37.94	38.62	39.32	38.97	38.62	37.78
c		35.64	36.28	36.94	37.60	38.28	38.97	38.62	38.28	37.45
d		35.32	35.96	36.61	37.27	37.94	38.62	38.28	37.94	37.11
e		35.01	35.64	36.28	36.94	37.60	38.28	37.94	37.60	36.78

(e) Demand,  $\theta$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	100.00	101.80	103.63	105.49	107.39	106.43	105.49	104.55	103.63	102.71
b		100.89	102.71	104.55	106.43	105.49	104.55	103.63	102.71	101.80
c		100.00	101.80	103.63	105.49	104.55	103.63	102.71	101.80	100.89
d		99.11	100.89	102.71	104.55	103.63	102.71	101.80	100.89	100.00
e		98.23	100.00	101.80	103.63	102.71	101.80	100.89	100.00	99.11

(f) Optimal capacity,  $K_0^*$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	60	90	90	90	90	90	90	90	90	40
b		90	90	90	90	90	90	90	90	40
c		90	90	90	90	90	90	90	90	40
d		90	90	90	90	90	90	90	90	40
e		90	90	90	90	90	90	90	90	40

demand has a higher switch from scenario 2, the quantity to produce and profits slightly change but NPV grows 12.91% compared to the one in scenario 2.

In summary, given a switching standard deviation, the bigger the switching mean, the greater the NPV. The increase in NPV is the result of the longer the optimal path remains in the outer upward branch, which is the one with the highest drift in demand. Such demand provides more opportunity to produce and increase capacity levels.

#### 4.7.4 Variation in Optimal Path for Changes in Contraction and Expansion Costs

To evaluate the behaviour of resource allocations, we conduct an experiment that shows how the dynamic of flexibility works. We define three pairs of distinct contraction and expansion options, and the maximum NPV is evaluated with a sample of the most likely path in the lattice for a Cournot's model. The lattice contains a project lifetime,  $T$ , of 5, a switching mean of 2.50; and a standard deviation of 0.1. Figure 4.8 exhibits the optimal path for such input. This path illustrates the pattern of going through the cycle of growth upward from period 1 to 3, suddenly, it switches to decay from period 4 to 5. We notice that this is the regular behaviour we have observed in previous scenarios.

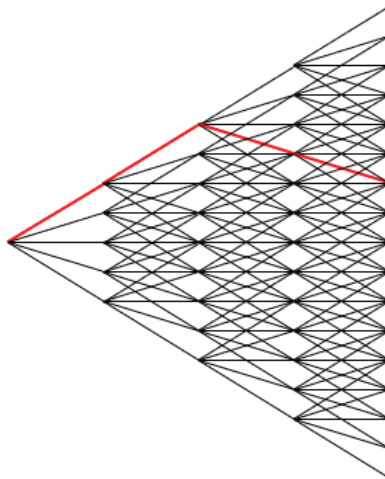


Figure 4.8: Optimal path scenario 4

The three pairs are the combinations of the values of contraction and expansion costs  $(v, \delta) = \{(-0.85, 1.00), (-0.95, 0.90) \text{ and } (-0.99, 0.85)\}$ . Installment and fixed switching costs remain the

Table 4.9: Values in the nodes of the most likely path for Figure 4.7

(a) Criteria

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	1.0000	0.4808	0.4808	0.4808	0.4808	0.4808	0.4808	0.4355	0.0000	0.0000
b		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0359	0.3810	0.3810
c		0.3611	0.3611	0.3611	0.3611	0.3611	0.3611	0.3270	0.0000	0.0000
d		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0584	0.6190	0.6190
e		0.1581	0.1581	0.1581	0.1581	0.1581	0.1581	0.1432	0.0000	0.0000

(b) Net Present Value, NPV

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	21,263.21	19,882.86	18,286.80	16,554.16	14,677.40	12,648.57	10,459.33	8,113.15	5,666.57	2,725.79
b		19,516.89	17,950.57	16,250.21	14,408.41	12,417.41	10,269.07	7,967.02	5,564.14	2,681.56
c		19,157.54	17,620.42	15,951.72	14,144.21	12,190.30	10,082.00	7,823.05	5,467.25	2,632.24
d		18,804.84	17,296.36	15,658.67	13,884.73	11,967.11	9,897.93	7,680.83	5,370.85	2,583.82
e		18,458.52	16,978.18	15,370.94	13,629.94	11,747.93	9,717.18	7,541.17	5,273.96	2,536.26

(c) Profit,  $\Pi$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	2,077.58	2,162.12	2,244.55	2,330.04	2,418.70	2,510.64	2,605.98	2,704.84	2,807.37	2,725.79
b		2,122.01	2,202.96	2,286.91	2,373.97	2,464.25	2,557.88	2,654.96	2,755.64	2,681.56
c		2,082.63	2,162.12	2,244.55	2,330.04	2,418.70	2,510.64	2,605.98	2,704.84	2,632.24
d		2,043.95	2,122.01	2,202.96	2,286.91	2,373.97	2,464.25	2,557.88	2,654.96	2,583.82
e		2,005.98	2,082.63	2,162.12	2,244.55	2,330.04	2,418.70	2,510.64	2,605.98	2,536.26

(d) Quantity,  $Q$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	35.45	36.28	36.94	37.60	38.28	38.97	39.67	40.39	41.11	40.00
b		35.96	36.61	37.27	37.94	38.62	39.32	40.03	40.75	39.86
c		35.64	36.28	36.94	37.60	38.28	38.97	39.67	40.39	39.51
d		35.32	35.96	36.61	37.27	37.94	38.62	39.32	40.03	39.16
e		35.01	35.64	36.28	36.94	37.60	38.28	38.97	39.67	38.81

(e) Demand,  $\theta$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	100.00	101.80	103.63	105.49	107.39	106.43	105.49	104.55	103.63	102.71
b		100.89	102.71	104.55	106.43	105.49	104.55	103.63	102.71	101.80
c		100.00	101.80	103.63	105.49	104.55	103.63	102.71	101.80	100.89
d		99.11	100.89	102.71	104.55	103.63	102.71	101.80	100.89	100.00
e		98.23	100.00	101.80	103.63	102.71	101.80	100.89	100.00	99.11

(f) Optimal capacity,  $K_0^*$ 

Node	Period									
	1	2	3	4	5	6	7	8	9	10
a	60	90	90	90	90	90	90	90	90	40
b		90	90	90	90	90	90	90	90	40
c		90	90	90	90	90	90	90	90	40
d		90	90	90	90	90	90	90	90	40
e		90	90	90	90	90	90	90	90	40



same. Table 4.10 shows the results of the 3 different scenarios.

Table 4.10: Combinations of contraction and expansion costs for an optimal path

(a)  $v = 0.85$  and  $\delta = 1.00$

Period	0	1	2	3	4	5
Node		a	a	a	d	d
$\Pi^*$		2,081.73	2,161.13	2,243.47	2,201.93	2,145.01
Switching Cost		0.00	0.00	0.00	101.75	
E(PV)		8,430.21	6,535.80	4,498.89	2,161.60	
Installment Costs	-240.00					
NPV	10,271.94	10,511.94	8,696.93	6,742.37	4,465.27	
$\theta$		100.00	101.80	103.63	102.71	101.80
$Q^*$		35.59	36.24	36.89	36.56	35.81
$K_t^*$	80	80	80	80	80	40

(b)  $v = 0.95$  and  $\delta = 0.90$

Period	0	1	2	3	4	5
Node		a	a	a	d	d
$\Pi^*$		2,077.58	2,162.74	2,242.68	2,203.65	2,145.01
Switching Cost		-135.25	99.50	-94.75	199.25	
E(PV)		8,534.28	6,539.34	4,597.55	2,161.60	
Installment Costs	-180.00					
NPV	10,296.61	10,476.61	8,801.58	6,745.48	4,564.50	
$\theta$		100.00	101.80	103.63	102.71	101.80
$Q^*$		35.46	36.35	36.86	36.68	35.81
$K_t^*$	60	60	110	75	110	40

(c)  $v = 0.99$  and  $\delta = 0.85$

$t$	0	1	2	3	4	5
Node		a	a	a	d	d
$\Pi^*$		2,075.74	2,162.74	2,236.73	2,203.65	2,145.01
Switching Cost		-140.50	163.10	-140.50	207.65	
E(PV)		8,555.48	6,496.72	4,605.89	2,161.60	
Installment Costs	-165.00					
NPV	10,325.72	10,490.72	8,822.56	6,702.12	4,572.90	
$\theta$		100.00	101.80	103.63	102.71	101.80
$Q$		35.40	36.35	36.69	36.68	35.81
$K_t^*$	55	55	110	55	110	40

In the first event, we assess a combination of low contraction with a high expansion option. Table 4.10a shows that the optimal capacity to invest in is 80 units. Those units of capacity stay in this level from period 1 to 4, even though the optimal capacity needed to supply 35.59 is not bigger than 40 units. Although, demand increases every node of the optimal path in the first 3 periods, no more than 36.89 units are required to be produced. However, it is seen in the very last period, capacity is reduced to 40 units (the maximum required) obtaining an extra profit of \$101.75 for employing the salvage value. Besides, due to the high value of expansion costs, it is always better

off to buy all the required capacity at the beginning. That is the maximum cost and there is not an expected cheaper cost.

In the second event, we aim to mix a higher contraction with a lower expansion option. Table 4.10b illustrates the trend of this combination. We observe that the optimal capacity to invest in is 60 units. Once more, the optimal capacity required to produce 35.46 units is 40 units. As it can be seen, the highest quantity to supply in all the evaluated periods is 36.86 units. Nevertheless, in period 1, after starting production, the firm buys 50 extra units of capacity reaching 110 units of capacity. In period 2, by contracting 35 units ( $110 - 75$ ), the firm obtains a positive switching cost of 99.50. Different than period 2, 35 units of capacity are expanded in period 3, and again 70 units of capacity are reduced in period 4 acquiring a positive switching cost of 199.25. In period 5, there are only 40 units of capacity, which are the ones needed to produce 35.81 units. The very last scenario maintains the previous dynamic of percentage combination ( $\delta > v$ ), but contraction cost is increased more and expansion cost is decreased more, all the opposite to the first experiment. Table 4.10c provides such outcome. Initially, the optimal capacity to acquire at the initial period is 55 units, which is used in period 1. In the following period, the capacity is expanded in 55 extra units, and then, in period 3 is contracted to 55 units again. Subsequently, as done in the second period, 55 extra units of capacity are expanded in period 4. In the very last period, those 110 units are reduced to 40 units that are maximum needed to produce 35.81 units. This experiment and the preceding one experience an identical dynamic.

We find that the higher the contraction and the lower the expansion option, the greater the NPV. NPV increases 0.24% from scenario 1 to 2 in this section, and 0.28% from scenario 2 to 3 in this section. Furthermore, the optimal capacity to purchase in the initial period declines when the expansion cost does. The reason that companies prefer to buy more capacity whenever it becomes cheaper. At initial time, installation costs is 3.00 for all the scenarios. However, it is subsequently 5% and 1% more economical for scenario 2 and 3, respectively.

## 4.8 Valuation of the Model for Incremental Changes in Mean and Standard Deviation for Each Regime

We analyze how the values of mean growth,  $\mu_g$ , standard deviation growth,  $\sigma_g$ , mean decay,  $\mu_d$ , and standard deviation decay,  $\sigma_d$ , affect the outcome of the maximum NPV and optimal initial capacity. We first present variations in growth regime followed by the decay regime. For growth, the mean varies from 6.00% to 9.00%, in increments of 1.00%; and the standard deviation goes from 3.50% to 6.50%, in increments of 1.00%. For decay, the mean varies from -1.50% to -4.50%, in increments of -1.00%; and the standard deviation goes from 2.00% to 5.00%, in increments of 1.00%. We observe that for this experiment the optimal capacity for Cournot Model's 1 and 2 and Stackelberg Leader is always 60 units and for Stackelberg follower is 55 units in all the combinations of mean and standard deviation. Once more, we outcomes in Stackelberg Leader are greater than Cournot's, and Cournot's are higher than Stackelberg Follower.

Table 4.11 shows that for an increasing growth mean, with a given standard deviation in Stackelberg Follower, the NPV increases. On the other hand, increases in the standard deviation in growth, for a given mean, also improves NPV. For example, when  $\mu_g = 8.00\%$  and  $\sigma_g = 4.50\%$ , the NPV is worth 4.63% more with an addition of 1.00% to the mean. Such gain is due to a change in the set of probabilities in this regime, going from  $\{0.5206, 0.3282, 0.1512\}$  to  $\{0.5626, 0.2910, 0.1464\}$ , and the decay set remains the same. The up and middle side associated with unusually high growth rates outweighs the downside associated with unusually low growth rates; therefore, the project value is increasing in growth mean. Comparably, an increase of an identical amount in the standard deviation, for a specified mean, just adds 0.24% to the NPV. The drift is also expected for a variation due to a new set of probabilities  $\{0.6507, 0.0656, 0.2836\}$  with similar decay set.

Table 4.12 reveals that the NPV deteriorates when decay mean increases for a given standard deviation. However, an increment of the standard deviation in this regime, for a given mean, enhances the NPV. When the mean goes from -2.50% to -3.50%, with a standard deviation of 2.00%, NPV declines 6.84% in the Stackelberg Follower's case. This alteration is caused by a group of probabilities in this regime, that starts at  $\{0.2241, 0.2569, 0.5190\}$  and ends at  $\{0.2067, 0.1747, 0.6186\}$ . The downside associated with low decay rates outweighs the upside associated

Table 4.11: Changes in NPV due to variations in mean and volatility in growth regimen for both models

(a)  $C_1$  and  $C_2$ 

$\sigma_g$	$\mu_g$			
	0.060	0.070	0.080	0.090
0.035	131,737.10	137,727.42	144,051.70	150,730.03
0.045	131,978.20	137,981.30	144,319.08	151,011.67
0.055	132,280.27	138,299.35	144,654.04	151,364.51
0.065	132,644.93	138,683.64	145,059.27	151,792.02

(b)  $S_L$ 

$\sigma_g$	$\mu_g$			
	0.060	0.070	0.080	0.090
0.035	140,014.82	146,447.42	153,240.37	160,415.35
0.045	140,276.10	146,722.54	153,530.10	160,720.54
0.055	140,603.52	147,067.21	153,893.09	161,102.89
0.065	140,998.73	147,483.73	154,332.24	161,566.15

(c)  $S_F$ 

$\sigma_g$	$\mu_g$			
	0.060	0.070	0.080	0.090
0.035	116,701.26	121,985.16	127,563.06	133,452.62
0.045	116,913.12	122,208.24	127,798.01	133,700.10
0.055	117,178.56	122,487.71	128,092.34	134,010.14
0.065	117,499.02	122,825.41	128,448.40	134,385.77

with high decay rates. Similarly, for a designated mean, a standard deviation from 2.00% to 3.00% increases the NPV in marginally 0.20%. This effect is generated by a move of decay from trinomial to binomial in the lattice. It implies that growth step size is more volatile; therefore decay is adjusted, and as a consequence, a new distribution in decay conditional probabilities probabilities of  $\{0.3810, 0.000, 0.6190\}$  where the upside increases associated with high decay rates.

In summary, increases in the growth cycle parameters benefit the optimal payoff of this model making NPV grows. On the other hand, in the decay cycle, increases in the mean lowers the NPV and increases in the standard deviation improves NPV.

Table 4.12: Changes in NPV due to variations in mean and volatility in decay regimen for both models

(a)  $C_1$  and  $C_2$

$\sigma_d$	$\mu_d$			
	-0.015	-0.025	-0.035	-0.045
<b>0.020</b>	138,926.93	137,953.80	137,006.34	136,083.76
<b>0.030</b>	138,954.82	137,981.30	137,033.57	136,110.83
<b>0.040</b>	138,996.02	138,021.52	137,072.87	136,149.24
<b>0.050</b>	139,034.02	138,057.99	137,107.69	136,182.36

(b)  $S_L$

$\sigma_d$	$\mu_d$			
	-0.015	-0.025	-0.035	-0.045
<b>0.020</b>	147,737.77	146,692.69	145,675.29	144,684.72
<b>0.030</b>	147,768.04	146,722.54	145,704.84	144,714.13
<b>0.040</b>	147,812.66	146,766.13	145,747.46	144,755.78
<b>0.050</b>	147,853.87	146,805.59	145,785.10	144,791.53

(c)  $S_F$

$\sigma_d$	$\mu_d$			
	-0.015	-0.025	-0.035	-0.045
<b>0.020</b>	123,042.36	122,184.04	121,348.31	120,534.47
<b>0.030</b>	123,066.94	122,208.24	121,372.28	120,558.31
<b>0.040</b>	123,103.09	122,243.61	121,406.84	120,592.08
<b>0.050</b>	123,136.45	122,275.56	121,437.35	120,621.11

## Chapter 5

### Conclusions and Discussion

This research presents a model that integrates demand uncertainty, product life cycle, competition and operational capacity to evaluate the feasibility of a project to introduce a new product that competes in a Duopoly market. The evaluation is based on the real options approach with capacity flexibility. Capacity flexibility is achieved by optimally expanding or contracting during project lifetime. The product life cycle, growth and decay, and the product demand uncertainty are represented by a two regime-switching lattice approach where the stochastic demand is considered to follow a Geometric Brownian Motion. The competition is added into the model with the Nash Equilibrium game-theoretic approach using Cournot and Stackelberg duopoly models. The two companies compete in the market for quantity to supply and competitors either have imperfect information (Cournot's model) or perfect information (Stackelberg's model). The maximum NPV and the optimal initial capacity are the output of this model. The NPV is calculated by using dynamic programming and is the criteria to select the initial capacity to invest in that it maximizes the discounted expected profits during the project lifetime. Such outputs are compared for two different game theoretic approaches.

This research contributes to the field of operations research and financial engineering in various ways. Our detailed numerical analysis points to several significant findings. We have observed that the Stackelberg and Cournot games behave very much alike since competition is on production quantity. Hence, changes in the costs and demand parameters affect in both models in a similar way. However, the first move in the game theory strategy gives the leader in Stackelberg a crucial advantage over the follower. Moreover, the leader in the Stackelberg game has an advantage over Cournot's model. This is based on the important assumption of perfect information when NPV for each company is analyzed individually. Nonetheless, when the two companies add up their

NPV, the joint effect is bigger with Cournot's model than with Stackelberg's. The summation of both companies' NPV in Cournot makes the product more profitable in the market. Even though, when competitors have the option to play a perfect information game, synergy between those two companies to play an imperfect information game is desirable to get the maximum benefit and return on investment in the market. Another important result observed is that for two competitors, one of the companies always get the maximum NPV, for all the possible pairs of combinations of capacities, when the other company operates at the smallest capacity level. This is established by the inverse price-demand relation in the model. Changes in the value of the price-elasticity constant have a significantly effect in this model valuation. We observe that for an elasticity close to 0 increases price the price and the amount to supply; therefore, increases the profits. This effect is also reflected in a greater NPV and initial capacity to invest in. However, when the elasticity is close to 1, price and quantity to produce trend to decrease and profits are lower, and as a consequence NPV and initial optimal capacity decrease. The variations of the inputs in the mean and volatility of each regime show that they define the distribution in the lattice. Such parameters establish what regime functions a binomial or trinomial lattice. If the mean and volatility of the growth regime is bigger than decay's, the NPV increases in the project. However, if decay mean is bigger, the NPV trends to decrease since probability of going on decay upward branch increases, therefore less demand and profits are associated to this effect. The volatility on decay produces the contrary effect which makes NPV increases. This effect is due to the dispersion may be associated to have a higher demand and a higher probability of moving upwards or downwards on the branches of the lattice. Additionally, costs of expansion and contraction play a critical role in the resource allocation strategy. Companies can take a great advantage to invest in initial capacity when the contraction costs are higher, and expansion costs are lower. This is measured by the additional profit that is estimated at every node when capacity is contracted.

## 5.1 Future Research

This research offers several future research directions, such as other game-theoretic models with a pure strategy, e.g., Bertrand model, where the two players compete for price instead of quantity, centipede game, that as Stackelberg's highlights the concept of backward induction adding an iter-

ated elimination of dominated strategies to provide a solution to the game. [Harsanyi et al. \(1988\)](#) also incorporate NE but using a different approach. The one-shot deviation principle in game theory can also be implemented for future research. This principle also uses dynamic programming in a strategy where players can deviate only once in any period of the game to increase their profits, and their strategies are divided in subgames called Subgame Perfect Equilibrium (SPE). Additionally, the combination of competition in an oligopoly market, more than one product to introduce may also be investigated in the future. This research may provide a better estimation of the NPV when using empirical data to evaluate the feasibility of product introduction strategies. It can also be expanded with a model where costs are considered variable during the project life time and methodology, such linear programming, can be integrated to minimized them in every node. A simulation could be another tool to assess this model and obtain results and compare them between different game-theoretic approaches.

## 5.2 Critiques

One of the weaknesses of this model is its complexity; many parameters are used as inputs, which can make it difficult to apply in practice. However, companies that operate in highly competitive industries which invest heavily in innovation (i.e. technology and automotive) can approximate the product life cycle mean and volatility from historic variations in demand or volume of sales. Another weakness is that the model only uses micro-economic variables. Macro-economic variables such inflation, unemployment, interest rates and gross domestic product may be considered for the model's accuracy since they directly affect the demand, costs, and responsiveness of consumers to the product.



# Appendices

## Other Game-Theoretic Approaches and Results

### A.1 Bertrand's Model of Oligopoly

The previous two NE models showed that price is established as a function of the demand and quantity produced by each firm involved in the competition. Different than previous models, companies may choose both quantities and prices at the same time [Watson \(2002\)](#). In some cases, firms select one variable first (either quantity or price) and then set the other to whatever the market allows. For instance, companies with extensive production cycles, such pharmaceutical and automobiles, must determine production targets in advance, and subsequently adapt the price to make supply meet the demand (Cournot's model). On the other hand, companies that produce in short notice, such a software design, set the price first and then produce to supply demand. In this model, firms assign a price to its product and then produce as much output as necessary to make a profit and supply the demand ([Osborne, 2004](#)). Bertrand's model assumes that competitors decide at the beginning how much to produce (similar to Cournot's) but the outcomes are different. Each company (two in this case) produces  $Q_i$  units, with a cost structure of  $c_i(Q_i)$  and only one product is created. The assumptions of this microeconomics model are:

- The demand function depends on the price selected by each firm  $\theta(P_i)$ . Opposite to Cournot's model where the price is a negative function of supply.
- Consumers buy the product with the lowest price when all the firms set different prices and all produce enough to supply such demand. For this study,  $Q_i$  must be less than or equal to  $K_{i\ell}$ .
- When two or more companies set the same price and such price is the lowest, the demand is

shared equally among the firms.

- If a company price is not the lowest, this company does not have a demand and the quantity produce becomes zero.
- Companies do not choose the amount to produce strategically, they make enough product to satisfy the demand given a price. It also works for prices that are lower than the unit cost. However, it must be one of the conditions to modify to make sure that profit becomes greater than zero.

The quantity  $Q_i$  is calculated by the expression:

$$Q_i = \frac{\theta(P_i)}{m}$$

Then, the formula for Company 1's profit for a duopoly can be obtained as:

$$\Pi_1(P_1, P_2) = \begin{cases} -o_1 K_{1\ell} - \frac{c_1 \theta}{\lambda_1} - \frac{w \theta_t^2}{2K_{1\ell} \lambda_1^2} + [\frac{\theta + c_1}{\lambda_t} + \frac{w_1 \theta}{K_{1\ell} \lambda_1}] P_i - [\frac{1}{\lambda_1} + \frac{w_1}{2K_{1\ell} \lambda_1^2}] P_1^2, & \text{if } P_1 < P_2 \\ -o_1 K_{1\ell} - \frac{c_1 \theta}{2\lambda_1} - \frac{w \theta_t^2}{8K_{1\ell} \lambda_1^2} + [\frac{\theta + c_1}{2\lambda_t} + \frac{w_1 \theta}{2K_{1\ell} \lambda_1}] P_1 - [\frac{1}{2\lambda_1} + \frac{w_1}{8K_{1\ell} \lambda_1^2}] P_1^2, & \text{if } P_1 = P_2 \end{cases}$$

If  $P_1 > P_2$ , the Company 1 produces zero and the profit becomes negative with a loss of  $o_1 K_{1,\ell}$ .

In this specific model, as a game situation, the price of firm 1 depends on the behaviour of  $P_2$  with respect to  $c_1$ .

- If  $P_2 > c_1$ , the best response is  $P_1 > P_2$  since any  $P_1$  lower than  $P_2$  results in a negative profit, and on the other hand,  $P_1$  greater than  $P_2$  leads to zero production.
- If  $P_2 = c_1$ , the best response is similar to the prior one, the only difference is that no matter what  $P_1$  is selected, the profit is always zero.
- If  $c_1 < P_2 \leq P_1^*$ , there is not a best response.  $P_1$  increases the profits as  $P_2$  increases. However, there might be a point where  $P_1$  falls to the point  $P_1$ . In this case, firm 1 is better off if the price is close to the price in 2 since there is always a price 1 smaller than price 2.
- If  $P_2 > P_1^*$ , then  $P_1^*$  is the best response. In this range, if  $P_1 = P_2$ , the equation for the optimal price is:

$$P_1^* = \frac{2\lambda_1(\theta c_1 K_{1\ell} + w_1 \theta)}{4K_{1\ell} \lambda_1 + w_1}$$

Otherwise, if  $P_1 > P_2$ :

$$P_1^* = \frac{\lambda_1(\theta c_1 K_{1\ell} + w_1 \theta)}{2K_{1\ell} \lambda_1 + w_1}$$

## A.2 Payoff Tables

This section presents the results of the numerical analysis when the model is calculated with all the initial parameters and capacity initial capacity and increments of 5 units until a value of 300 units in capacity is reached.

### A.2.1 Cournot's Payoff

Table A1 exhibits the results of NPV for each capacity combination of  $C_1$  (columns) and  $C_2$  (rows). The first value within the parenthesis shows the NPV of  $C_1$  and the second value the NPV of  $C_2$ .

### A.2.2 Stackelberg's Payoff

Table A2 exhibits the results of NPV for each capacity combination of  $S_L$  (columns) and  $S_F$  (rows). The first value within the parenthesis shows the NPV of  $S_L$  and the second value the NPV of  $S_F$ .

Table A1: Matrix of results of NPV and optimal capacity for  $C_1$  and  $C_2$

$K_d$	$C_1$										$C_2$									
5	10	15	20	25	30	35	40	45	50	5	10	15	20	25	30	35	40	45	50	5
(136,364,46;136,364,46)	(136,757,21;124,735,72)	(137,099,96;114,572,43)	(137,392,71;104,859,57)	(137,635,46;93,345,81)	(137,828,21;85,941,01)	(137,970,96;77,385,96)	(137,976,13;72,802,14)	(137,978,58;71,457,71)	(137,980,11;71,173,81)	(124,735,21;124,735,72)	(125,103,47;125,103,47)	(125,421,22;114,915,18)	(125,688,97;105,177,32)	(125,906,72;93,638,56)	(126,074,47;86,208,76)	(126,169,20;77,846,20)	(126,172,41;73,056,46)	(126,174,42;71,916,24)	(126,175,59;71,631,33)	(126,176,71;68,971,43)
10	(124,735,21;124,735,72)	(125,421,22;114,915,18)	(125,688,97;105,177,32)	(125,906,72;93,638,56)	(126,074,47;86,208,76)	(126,169,20;77,846,20)	(126,172,41;73,056,46)	(126,174,42;71,916,24)	(126,175,59;71,631,33)	(114,572,43;124,735,21)	(115,181,15;125,421,22)	(115,448,90;115,448,90)	(115,716,65;105,177,32)	(115,934,40;93,638,56)	(116,101,15;86,208,76)	(116,268,40;77,846,20)	(116,435,65;73,056,46)	(116,602,90;71,916,24)	(116,769,15;68,971,43)	(116,936,40;65,781,15)
15	(114,572,43;124,735,21)	(115,181,15;125,421,22)	(115,448,90;115,448,90)	(115,716,65;105,177,32)	(115,934,40;93,638,56)	(116,101,15;86,208,76)	(116,268,40;77,846,20)	(116,435,65;73,056,46)	(116,602,90;71,916,24)	(104,859,57;137,392,71)	(105,177,32;125,688,97)	(105,495,27;125,688,97)	(105,813,02;125,688,97)	(106,130,77;125,688,97)	(106,448,52;125,688,97)	(106,766,27;125,688,97)	(107,084,02;125,688,97)	(107,401,77;125,688,97)	(107,719,52;125,688,97)	(108,037,27;125,688,97)
20	(104,859,57;137,392,71)	(105,177,32;125,688,97)	(105,495,27;125,688,97)	(105,813,02;125,688,97)	(106,130,77;125,688,97)	(106,448,52;125,688,97)	(106,766,27;125,688,97)	(107,084,02;125,688,97)	(107,401,77;125,688,97)	(93,345,81;137,828,21)	(93,638,56;126,074,47)	(93,956,31;126,074,47)	(94,274,06;126,074,47)	(94,591,81;126,074,47)	(94,909,56;126,074,47)	(95,227,31;126,074,47)	(95,545,06;126,074,47)	(95,862,81;126,074,47)	(96,180,56;126,074,47)	(96,498,31;126,074,47)
25	(93,345,81;137,828,21)	(93,638,56;126,074,47)	(93,956,31;126,074,47)	(94,274,06;126,074,47)	(94,591,81;126,074,47)	(94,909,56;126,074,47)	(95,227,31;126,074,47)	(95,545,06;126,074,47)	(95,862,81;126,074,47)	(86,208,76;126,169,20)	(86,526,51;126,169,20)	(86,844,26;126,169,20)	(87,162,01;126,169,20)	(87,479,76;126,169,20)	(87,797,51;126,169,20)	(88,115,26;126,169,20)	(88,433,01;126,169,20)	(88,750,76;126,169,20)	(89,068,51;126,169,20)	(89,386,26;126,169,20)
30	(86,208,76;126,169,20)	(86,526,51;126,169,20)	(86,844,26;126,169,20)	(87,162,01;126,169,20)	(87,479,76;126,169,20)	(87,797,51;126,169,20)	(88,115,26;126,169,20)	(88,433,01;126,169,20)	(88,750,76;126,169,20)	(77,846,20;126,172,41)	(78,164,95;126,172,41)	(78,482,70;126,172,41)	(78,800,45;126,172,41)	(79,118,20;126,172,41)	(79,435,95;126,172,41)	(79,753,70;126,172,41)	(80,071,45;126,172,41)	(80,389,20;126,172,41)	(80,706,95;126,172,41)	(81,024,70;126,172,41)
35	(77,846,20;126,172,41)	(78,164,95;126,172,41)	(78,482,70;126,172,41)	(78,800,45;126,172,41)	(79,118,20;126,172,41)	(79,435,95;126,172,41)	(79,753,70;126,172,41)	(80,071,45;126,172,41)	(80,389,20;126,172,41)	(72,802,14;137,978,58)	(73,056,46;137,978,58)	(73,300,78;137,978,58)	(73,545,10;137,978,58)	(73,789,42;137,978,58)	(74,033,74;137,978,58)	(74,278,06;137,978,58)	(74,522,38;137,978,58)	(74,766,70;137,978,58)	(75,011,02;137,978,58)	(75,255,34;137,978,58)
40	(72,802,14;137,978,58)	(73,056,46;137,978,58)	(73,300,78;137,978,58)	(73,545,10;137,978,58)	(73,789,42;137,978,58)	(74,033,74;137,978,58)	(74,278,06;137,978,58)	(74,522,38;137,978,58)	(74,766,70;137,978,58)	(71,457,71;137,978,58)	(71,702,03;137,978,58)	(71,946,35;137,978,58)	(72,190,67;137,978,58)	(72,435,00;137,978,58)	(72,679,32;137,978,58)	(72,923,64;137,978,58)	(73,168,96;137,978,58)	(73,413,28;137,978,58)	(73,657,60;137,978,58)	(73,902,92;137,978,58)
45	(71,457,71;137,978,58)	(71,702,03;137,978,58)	(71,946,35;137,978,58)	(72,190,67;137,978,58)	(72,435,00;137,978,58)	(72,679,32;137,978,58)	(72,923,64;137,978,58)	(73,168,96;137,978,58)	(73,413,28;137,978,58)	(70,916,24;137,980,11)	(71,160,56;137,980,11)	(71,404,88;137,980,11)	(71,649,20;137,980,11)	(71,893,52;137,980,11)	(72,137,84;137,980,11)	(72,382,16;137,980,11)	(72,626,48;137,980,11)	(72,870,80;137,980,11)	(73,115,12;137,980,11)	(73,359,44;137,980,11)
50	(71,160,56;137,980,11)	(71,404,88;137,980,11)	(71,649,20;137,980,11)	(71,893,52;137,980,11)	(72,137,84;137,980,11)	(72,382,16;137,980,11)	(72,626,48;137,980,11)	(72,870,80;137,980,11)	(73,115,12;137,980,11)	(70,916,24;137,980,11)	(71,160,56;137,980,11)	(71,404,88;137,980,11)	(71,649,20;137,980,11)	(71,893,52;137,980,11)	(72,137,84;137,980,11)	(72,382,16;137,980,11)	(72,626,48;137,980,11)	(72,870,80;137,980,11)	(73,115,12;137,980,11)	(73,359,44;137,980,11)
55	(71,041,61;137,980,96)	(71,294,83;126,176,15)	(71,548,05;115,859,60)	(71,801,27;105,999,38)	(72,053,69;105,999,38)	(72,306,11;105,999,38)	(72,558,53;105,999,38)	(72,810,95;105,999,38)	(73,063,37;105,999,38)	(71,980,96;137,980,96)	(72,233,28;137,980,96)	(72,485,60;137,980,96)	(72,737,92;137,980,96)	(72,990,24;137,980,96)	(73,242,56;137,980,96)	(73,494,88;137,980,96)	(73,747,20;137,980,96)	(73,999,52;137,980,96)	(74,251,84;137,980,96)	(74,504,16;137,980,96)
60	(70,936,74;137,980,96)	(71,189,72;126,176,25)	(71,442,94;115,859,60)	(71,695,16;105,999,38)	(71,947,38;105,999,38)	(72,199,60;105,999,38)	(72,451,82;105,999,38)	(72,704,04;105,999,38)	(72,956,26;105,999,38)	(70,841,98;137,980,96)	(71,094,20;137,980,96)	(71,346,42;137,980,96)	(71,598,64;137,980,96)	(71,850,86;137,980,96)	(72,103,08;137,980,96)	(72,355,30;137,980,96)	(72,607,52;137,980,96)	(72,859,74;137,980,96)	(73,111,96;137,980,96)	(73,364,18;137,980,96)
65	(70,841,98;137,980,96)	(71,094,20;137,980,96)	(71,346,42;137,980,96)	(71,598,64;137,980,96)	(71,850,86;137,980,96)	(72,103,08;137,980,96)	(72,355,30;137,980,96)	(72,607,52;137,980,96)	(72,859,74;137,980,96)	(70,771,77;137,980,87)	(71,024,36;126,175,45)	(71,276,95;126,175,45)	(71,529,54;126,175,45)	(71,782,13;126,175,45)	(72,034,72;126,175,45)	(72,287,31;126,175,45)	(72,539,90;126,175,45)	(72,792,49;126,175,45)	(73,045,08;126,175,45)	(73,297,67;126,175,45)
70	(70,771,77;137,980,87)	(71,024,36;126,175,45)	(71,276,95;126,175,45)	(71,529,54;126,175,45)	(71,782,13;126,175,45)	(72,034,72;126,175,45)	(72,287,31;126,175,45)	(72,539,90;126,175,45)	(72,792,49;126,175,45)	(70,647,66;137,979,44)	(70,899,97;126,173,73)	(71,152,30;126,173,73)	(71,404,61;126,173,73)	(71,656,92;126,173,73)	(71,909,23;126,173,73)	(72,161,54;126,173,73)	(72,413,85;126,173,73)	(72,666,16;126,173,73)	(72,918,47;126,173,73)	(73,170,78;126,173,73)
75	(70,647,66;137,979,44)	(70,899,97;126,173,73)	(71,152,30;126,173,73)	(71,404,61;126,173,73)	(71,656,92;126,173,73)	(71,909,23;126,173,73)	(72,161,54;126,173,73)	(72,413,85;126,173,73)	(72,666,16;126,173,73)	(70,596,97;137,978,46)	(70,849,28;126,171,40)	(71,101,59;126,171,40)	(71,353,90;126,171,40)	(71,606,21;126,171,40)	(71,858,52;126,171,40)	(72,110,83;126,171,40)	(72,363,14;126,171,40)	(72,615,45;126,171,40)	(72,867,76;126,171,40)	(73,120,07;126,171,40)
80	(70,596,97;137,978,46)	(70,849,28;126,171,40)	(71,101,59;126,171,40)	(71,353,90;126,171,40)	(71,606,21;126,171,40)	(71,858,52;126,171,40)	(72,110,83;126,171,40)	(72,363,14;126,171,40)	(72,615,45;126,171,40)	(70,550,91;137,977,34)	(70,803,22;126,169,10)	(71,055,53;126,169,10)	(71,307,84;126,169,10)	(71,560,15;126,169,10)	(71,812,46;126,169,10)	(72,064,77;126,169,10)	(72,317,08;126,169,10)	(72,569,39;126,169,10)	(72,821,70;126,169,10)	(73,074,01;126,169,10)
85	(70,550,91;137,977,34)	(70,803,22;126,169,10)	(71,055,53;126,169,10)	(71,307,84;126,169,10)	(71,560,15;126,169,10)	(71,812,46;126,169,10)	(72,064,77;126,169,10)	(72,317,08;126,169,10)	(72,569,39;126,169,10)	(70,510,11;137,976,10)	(70,762,42;126,168,90)	(71,014,73;126,168,90)	(71,267,04;126,168,90)	(71,519,35;126,168,90)	(71,771,66;126,168,90)	(72,023,97;126,168,90)	(72,276,28;126,168,90)	(72,528,59;126,168,90)	(72,780,90;126,168,90)	(73,033,21;126,168,90)
90	(70,510,11;137,976,10)	(70,762,42;126,168,90)	(71,014,73;126,168,90)	(71,267,04;126,168,90)	(71,519,35;126,168,90)	(71,771,66;126,168,90)	(72,023,97;126,168,90)	(72,276,28;126,168,90)	(72,528,59;126,168,90)	(70,460,09;137,975,58)	(70,712,40;126,168,70)	(70,964,71;126,168,70)	(71,217,02;126,168,70)	(71,469,33;126,168,70)	(71,721,64;126,168,70)	(71,973,95;126,168,70)	(72,226,26;126,168,70)	(72,478,57;126,168,70)	(72,730,88;126,168,70)	(72,983,19;126,168,70)
100	(70,460,09;137,975,58)	(70,712,40;126,168,70)	(70,964,71;126,168,70)	(71,217,02;126,168,70)	(71,469,33;126,168,70)	(71,721,64;126,168,70)	(71,973,95;126,168,70)	(72,226,26;126,168,70)	(72,478,57;126,168,70)	(70,409,82;137,974,62)	(70,662,13;126,163,84)	(70,914,44;126,163,84)	(71,166,75;126,163,84)	(71,419,06;126,163,84)	(71,671,37;126,163,84)	(71,923,68;126,163,84)	(72,175,99;126,163,84)	(72,428,30;126,163,84)	(72,680,61;126,163,84)	(72,932,92;126,163,84)
105	(70,409,82;137,974,62)	(70,662,13;126,163,84)	(70,914,44;126,163,84)	(71,166,75;126,163,84)	(71,419,06;126,163,84)	(71,671,37;126,163,84)	(71,923,68;126,163,84)	(72,175,99;126,163,84)	(72,428,30;126,163,84)	(70,358,06;137,964,52)	(70,610,37;126,155,78)	(70,862,68;126,155,78)	(71,114,99;126,155,78)	(71,367,30;126,155,78)	(71,619,61;126,155,78)	(71,871,92;126,155,78)	(72,124,23;126,155,78)	(72,376,54;126,155,78)	(72,628,85;126,155,78)	(72,881,16;126,155,78)
110	(70,358,06;137,964,52)	(70,610,37;126,155,78)	(70,862,68;126,155,78)	(71,114,99;126,155,78)	(71,367,30;126,155,78)	(71,619,61;126,155,78)	(71,871,92;126,155,78)	(72,124,23;126,155,78)	(72,376,54;126,155,78)	(70,307,05;137,960,50)	(70,559,36;126,151,80)	(70,811,67;126,151,80)	(71,063,98;126,151,80)	(71,316,29;126,151,80)	(71,568,60;126,151,80)	(71,820,91;126,151,80)	(72,073,22;126,151,80)	(72,325,53;126,151,80)	(72,577,84;126,151,80)	(72,830,15;126,151,80)
115	(70,307,05;137,960,50)	(70,559,36;126,151,80)	(70,811,67;126,151,80)	(71,063,98;126,151,80)	(71,316,29;126,151,80)	(71,568,60;126,151,80)	(71,820,91;126,151,80)	(72,073,22;126,151,80)	(72,325,53;126,151,80)	(70,256,08;137,956,54)	(70,508,39;126,147,79)	(70,760,70;126,147,79)	(71,012,99;126,147,79)	(71,265,30;126,147,79)	(71,517,61;126,147,79)	(71,769,92;126,147,79)	(72,022,23;126,147,79)	(72,274,54;126,147,79)	(72,526,85;126,147,79)	(72,779,16;126,147,79)
120	(70,307,05;137,960,50)	(70,559,36;126,151,80)	(70,811,67;126,151,80)	(71,063,98;126,151,80)	(71,316,29;126,151,80)	(71,568,60;126,151,80)	(71,820,91;126,151,80)	(72,073,22;126,151,80)	(72,325,53;126,151,80)	(70,256,08;137,956,54)	(70,508,39;126,147,79)	(70,760,70;126,147,79)	(71,012,99;126,147,79)	(71,265,30;126,147,79)	(71,517,61;126,147,79)	(71,769,92;126,147,79)	(72,022,23;126,147,79)	(72,274,54;126,147,79)	(72,526,85;126,147,79)	(72,779,16;126,147,79)
125	(70,307,05;137,960,50)	(70,559,36;126,151,80)	(70,811,67;																	

Table A1: Matrix of results of NPV and optimal capacity for  $C_1$  and  $C_2$  (continued)

$K_d$	$C_1$	$C_2$									
		55	60	65	70	75	80	85	90	95	100
5	(137,980.96;71,041.61)	(137,981.29;70,936.74)	(137,980.92;70,771.77)	(137,980.26;70,705.62)	(137,979.44;70,647.66)	(137,978.46;70,596.47)	(137,977.34;70,550.91)	(137,976.10;70,510.11)	(137,974.85;70,473.36)	(137,973.60;70,436.61)	(137,972.35;70,399.86)
10	(126,176.25;71,189.72)	(126,175.95;71,100.75)	(126,175.45;71,027.46)	(126,174.66;70,958.06)	(126,173.73;70,899.97)	(126,172.67;70,848.65)	(126,171.40;70,802.99)	(126,169.70;70,762.09)	(126,167.50;70,722.96)	(126,164.89;70,683.83)	(126,161.88;70,644.70)
15	(115,859.60;71,493.84)	(115,859.46;71,393.47)	(115,858.99;71,304.31)	(115,858.27;71,227.34)	(115,856.27;71,161.28)	(115,853.05;71,095.61)	(115,848.57;71,030.05)	(115,843.84;70,964.85)	(115,838.84;70,900.26)	(115,833.54;70,836.11)	(115,827.94;70,771.86)
20	(106,000.44;71,693.86)	(106,000.05;71,597.46)	(106,000.05;71,508.08)	(106,000.05;71,418.72)	(106,000.05;71,329.36)	(106,000.05;71,240.00)	(106,000.05;71,150.64)	(106,000.05;71,061.28)	(106,000.05;70,971.92)	(106,000.05;70,882.56)	(106,000.05;70,793.20)
25	(96,343.40;71,757.38)	(96,342.77;71,651.54)	(96,341.89;71,561.95)	(96,340.72;71,472.36)	(96,339.30;71,383.34)	(96,337.64;71,294.38)	(96,335.76;71,205.42)	(96,333.67;71,116.46)	(96,331.37;71,027.50)	(96,328.87;70,938.54)	(96,326.17;70,849.51)
30	(86,796.91;71,811.75)	(86,796.04;71,705.66)	(86,794.95;71,615.87)	(86,793.72;71,526.82)	(86,792.35;71,437.84)	(86,790.84;71,348.83)	(86,789.19;71,259.82)	(86,787.41;71,170.80)	(86,785.50;71,081.78)	(86,783.47;70,992.75)	(86,781.32;70,903.62)
35	(78,179.95;71,821.93)	(78,179.47;71,715.72)	(78,178.47;71,626.82)	(78,177.51;71,537.69)	(78,176.40;71,448.63)	(78,175.14;71,359.56)	(78,173.74;71,270.49)	(78,172.21;71,181.42)	(78,170.55;71,092.35)	(78,168.77;70,999.28)	(78,166.88;70,906.21)
40	(73,593.27;71,824.37)	(73,593.06;71,718.15)	(73,592.54;71,628.25)	(73,591.78;71,541.05)	(73,590.82;71,451.94)	(73,589.70;71,362.83)	(73,588.44;71,273.72)	(73,587.05;71,184.61)	(73,585.54;71,095.50)	(73,583.92;71,006.39)	(73,582.19;70,917.28)
45	(72,246.38;71,825.79)	(72,246.16;71,719.57)	(72,245.64;71,629.46)	(72,244.88;71,542.36)	(72,243.92;71,453.25)	(72,242.75;71,364.14)	(72,241.48;71,275.03)	(72,240.09;71,185.92)	(72,238.58;71,096.81)	(72,236.96;71,007.70)	(72,235.24;70,918.59)
50	(71,960.50;71,826.49)	(71,960.28;71,720.27)	(71,959.76;71,630.36)	(71,958.99;71,553.16)	(71,958.03;71,464.05)	(71,956.96;71,374.94)	(71,955.79;71,285.83)	(71,954.52;71,196.72)	(71,953.15;71,107.61)	(71,951.68;71,018.50)	(71,950.11;70,929.39)
55	(71,826.67;71,826.67)	(71,826.45;71,720.44)	(71,825.92;71,630.33)	(71,825.15;71,553.13)	(71,824.19;71,464.02)	(71,823.03;71,374.91)	(71,821.67;71,285.80)	(71,820.21;71,196.69)	(71,818.65;71,107.58)	(71,816.99;71,018.47)	(71,815.24;70,929.36)
60	(71,720.44;71,826.45)	(71,720.22;71,720.22)	(71,719.70;71,630.33)	(71,718.93;71,553.11)	(71,717.97;71,464.00)	(71,716.81;71,374.89)	(71,715.46;71,285.78)	(71,713.91;71,196.67)	(71,712.26;71,107.56)	(71,710.51;71,018.45)	(71,708.66;70,929.34)
65	(71,630.33;71,825.92)	(71,630.11;71,719.70)	(71,629.78;71,629.78)	(71,629.01;71,552.88)	(71,627.95;71,463.77)	(71,626.69;71,374.66)	(71,625.24;71,285.55)	(71,623.69;71,196.44)	(71,622.04;71,107.33)	(71,620.29;71,018.22)	(71,618.44;70,929.11)
70	(71,553.11;71,825.15)	(71,553.11;71,718.93)	(71,552.58;71,629.01)	(71,551.81;71,551.81)	(71,550.84;71,463.70)	(71,549.72;71,374.59)	(71,548.46;71,285.48)	(71,547.05;71,196.37)	(71,545.50;71,107.26)	(71,543.85;71,018.15)	(71,542.10;70,929.04)
75	(71,486.32;71,824.19)	(71,486.10;71,717.96)	(71,485.57;71,628.05)	(71,484.80;71,550.84)	(71,483.83;71,463.63)	(71,482.71;71,374.52)	(71,481.45;71,285.41)	(71,479.95;71,196.30)	(71,478.30;71,107.19)	(71,476.55;71,018.08)	(71,474.70;70,928.97)
80	(71,427.62;71,823.07)	(71,427.39;71,716.84)	(71,426.86;71,626.92)	(71,426.09;71,549.72)	(71,425.12;71,462.52)	(71,423.99;71,374.41)	(71,422.72;71,285.30)	(71,421.31;71,196.19)	(71,419.76;71,107.08)	(71,418.01;70,928.97)	(71,416.16;70,839.86)
85	(71,375.75;71,821.84)	(71,375.53;71,715.62)	(71,374.99;71,625.70)	(71,374.22;71,548.50)	(71,373.25;71,461.34)	(71,372.13;71,374.17)	(71,370.91;71,285.06)	(71,369.60;71,195.95)	(71,368.19;71,106.84)	(71,366.68;71,017.73)	(71,365.07;70,928.62)
90	(71,329.61;71,820.69)	(71,329.38;71,714.47)	(71,328.85;71,624.55)	(71,328.07;71,546.35)	(71,327.10;71,460.19)	(71,325.98;71,374.02)	(71,324.71;71,284.91)	(71,323.31;71,195.80)	(71,321.80;71,106.69)	(71,320.19;71,017.58)	(71,318.48;70,928.41)
95	(71,288.28;71,819.25)	(71,288.05;71,713.02)	(71,287.52;71,623.41)	(71,286.74;71,545.89)	(71,285.77;71,460.73)	(71,284.65;71,374.56)	(71,283.38;71,285.45)	(71,282.00;71,196.34)	(71,280.51;71,107.23)	(71,278.91;71,018.12)	(71,277.20;70,929.01)
100	(71,251.05;71,817.18)	(71,250.82;71,710.53)	(71,250.29;71,620.40)	(71,249.52;71,543.38)	(71,248.55;71,458.22)	(71,247.42;71,373.05)	(71,246.15;71,283.94)	(71,244.76;71,194.83)	(71,243.27;71,105.72)	(71,241.67;71,016.61)	(71,240.06;70,927.50)
105	(71,217.35;71,815.12)	(71,217.12;71,708.66)	(71,216.58;71,616.72)	(71,215.81;71,539.49)	(71,214.84;71,454.33)	(71,213.71;71,365.16)	(71,212.44;71,276.05)	(71,211.05;71,186.94)	(71,209.56;71,097.83)	(71,207.96;71,008.72)	(71,206.35;70,919.61)
110	(71,186.69;71,808.77)	(71,186.46;71,702.52)	(71,185.92;71,612.99)	(71,185.14;71,533.37)	(71,184.17;71,448.20)	(71,183.04;71,359.03)	(71,181.76;71,270.92)	(71,179.37;71,181.81)	(71,177.88;71,092.70)	(71,176.28;71,003.59)	(71,174.67;70,914.48)
115	(71,159.33;71,806.80)	(71,159.10;71,696.51)	(71,158.56;71,606.38)	(71,157.91;71,517.13)	(71,156.94;71,432.06)	(71,155.81;71,342.95)	(71,154.54;71,253.84)	(71,153.15;71,164.73)	(71,151.66;71,075.62)	(71,150.06;71,000.51)	(71,148.45;70,911.40)
120	(71,132.98;71,800.80)	(71,132.75;71,694.56)	(71,132.21;71,604.63)	(71,131.44;71,524.36)	(71,130.47;71,439.29)	(71,129.34;71,349.78)	(71,128.06;71,260.67)	(71,126.67;71,171.56)	(71,125.18;71,082.45)	(71,123.58;71,000.51)	(71,121.97;70,911.40)
125	(71,109.33;71,796.76)	(71,109.10;71,690.51)	(71,108.56;71,600.38)	(71,107.91;71,512.36)	(71,107.04;71,427.29)	(71,106.07;71,338.22)	(71,105.00;71,249.11)	(71,103.83;71,160.00)	(71,102.55;71,070.89)	(71,101.16;71,000.51)	(71,099.77;70,911.40)
130	(71,087.49;71,792.68)	(71,087.26;71,686.44)	(71,086.72;71,596.30)	(71,085.95;71,512.18)	(71,084.98;71,427.11)	(71,083.85;71,338.04)	(71,082.67;71,249.03)	(71,081.40;71,160.92)	(71,079.99;71,071.81)	(71,078.50;71,000.51)	(71,076.91;70,911.40)
135	(71,067.26;71,788.44)	(71,067.03;71,682.33)	(71,066.49;71,592.30)	(71,065.71;71,515.17)	(71,064.74;71,426.10)	(71,063.62;71,337.03)	(71,062.35;71,248.95)	(71,061.08;71,160.84)	(71,059.75;71,071.74)	(71,058.26;71,000.51)	(71,056.67;70,911.40)
140	(71,048.47;71,784.44)	(71,048.23;71,678.19)	(71,047.70;71,588.26)	(71,046.92;71,511.04)	(71,045.95;71,421.97)	(71,044.82;71,332.90)	(71,043.55;71,244.82)	(71,042.28;71,156.75)	(71,040.96;71,067.64)	(71,039.39;71,000.51)	(71,037.80;70,911.40)
145	(71,030.96;71,780.27)	(71,030.73;71,674.03)	(71,030.19;71,584.09)	(71,029.41;71,506.87)	(71,028.44;71,417.80)	(71,027.31;71,328.73)	(71,026.04;71,240.66)	(71,024.77;71,152.59)	(71,023.45;71,064.52)	(71,022.08;71,000.51)	(71,020.49;70,911.40)
150	(71,014.39;71,669.84)	(71,014.16;71,563.84)	(71,013.85;71,579.91)	(71,013.07;71,492.69)	(71,012.10;71,407.62)	(71,011.03;71,322.55)	(71,010.06;71,237.48)	(71,009.44;71,152.41)	(71,008.59;71,067.34)	(71,007.52;71,000.51)	(71,006.03;70,911.40)
155	(70,999.32;71,771.89)	(70,999.09;71,665.84)	(70,998.55;71,575.91)	(70,997.78;71,490.48)	(70,996.80;71,405.41)	(70,995.63;71,320.34)	(70,994.44;71,235.27)	(70,993.17;71,150.20)	(70,991.81;71,065.13)	(70,990.34;71,000.51)	(70,988.75;70,911.40)
160	(70,984.98;71,767.66)	(70,984.75;71,661.42)	(70,984.21;71,571.48)	(70,983.43;71,494.26)	(70,982.45;71,409.19)	(70,981.38;71,324.12)	(70,980.20;71,239.05)	(70,978.92;71,153.98)	(70,977.46;71,068.91)	(70,976.00;71,000.51)	(70,974.31;70,911.40)
165	(70,971.50;71,763.42)	(70,971.27;71,657.18)	(70,970.73;71,567.24)	(70,969.95;71,490.02)	(70,968.98;71,404.95)	(70,967.81;71,319.88)	(70,966.54;71,234.81)	(70,965.17;71,149.74)	(70,963.71;71,064.67)	(70,962.25;71,000.51)	(70,960.56;70,911.40)
170	(70,958.82;71,759.17)	(70,958.59;71,652.92)	(70,958.04;71,562.99)	(70,957.27;71,485.77)	(70,956.29;71,400.70)	(70,955.12;71,315.63)	(70,953.85;71,230.56)	(70,952.48;71,145.49)	(70,951.02;71,060.42)	(70,949.56;71,000.51)	(70,947.87;70,911.40)
175	(70,946.85;71,754.90)	(70,946.62;71,648.65)	(70,946.08;71,558.72)	(70,945.30;71,481.50)	(70,944.32;71,400.43)	(70,943.15;71,315.36)	(70,941.88;71,230.29)	(70,940.51;71,145.22)	(70,939.05;71,060.15)	(70,937.59;71,000.51)	(70,935.90;70,911.40)
180	(70,935.55;71,750.62)	(70,935.31;71,644.37)	(70,934.77;71,554.44)	(70,933.99;71,477.21)	(70,932.99;71,400.14)	(70,931.82;71,315.07)	(70,930.55;71,230.00)	(70,929.18;71,144.93)	(70,927.72;71,059.86)	(70,926.26;71,000.51)	(70,924.57;70,911.40)
185	(70,924.85;71,746.33)	(70,924.62;71,640.08)	(70,924.08;71,550.14)	(70,923.30;71,472.92)	(70,922.30;71,400.85)	(70,921.13;71,315.78)	(70,919.86;71,230.71)	(70,918.49;71,145.64)	(70,917.03;71,060.57)	(70,915.57;71,000.51)	(70,913.88;70,911.40)
190	(70,914.72;71,742.02)	(70,914.48;71,635.77)	(70,913.94;71,545.84)	(70,913.16;71,468.62)	(70,912.16;71,391.55)	(70,911.00;71,306.48)	(70,909.73;71,221.41)	(70,908.36;71,136.34)	(70,906.90;71,051.27)	(70,905.44;71,000.51)	(70,903.75;70,911.40)
195	(70,905.10;71,737.71)	(70,904.87;71,631.46)	(70,904.33;71,541.52)	(70,903.55;71,464.30)	(70,902.55;71,387.23)	(70,901.39;71,302.16)	(70,900.12;71,217.09)	(70,898.75;71,132.02)	(70,897.29;71,046.95)	(70,895.83;71,000.51)	(70,894.14;70,911.40)
200	(70,895.96;71,733.39)	(70,895.73;71,627.14)	(70,895.19;71,537.20)	(70,894.41;71,459.98)	(70,893.44;71,382.91)	(70,892.31;71,307.84)	(70,891.04;71,222.77)	(70,889.67;71,137.70)	(70,888.21;71,052.63)	(70,886.75;71,000.51)	(70,885.06;70,911.40)
205	(70,887.27;71,729.05)	(70,887.03;71,622.81)	(70,886.49;71,532.87)	(70,885.71;71,455.65)	(70,884.74;71,380.62)	(70,883.61;71,305.55)	(70,882.34;71,220.48)	(70,881.07;71,135.41)	(70,879.70;71,049.34)	(70,878.24;71,000.51)	(70,876.55;70,911.40)
210	(70,878.99;71,724.71)	(70,878.75;71,618.47)	(70,878.21;71,528.53)	(70,877.43;71,451.31)	(70,876.46;71,376.08)	(70,875.33;71,300.99)	(70,874.06;71,225.92)	(70,872.69;71,140.85)	(70,871.23;71,055.78)	(70,869.77;71,000.51)	(70,868.08;70,911.40)
215	(70,871.09;71,720.37)	(70,870.85;71,614.12)	(70,870.31;71,524.18)	(70,869.53;71,446.96)	(70,868.56;71,371.73)	(70,867.43;71,301.64)	(70,866.16;71,226.57)	(70,864.79;71,141.50)	(70,863.33;71,056.43)	(70,861.87;71,000.51)	(70,860.18;70,911.40)
220	(70,863.55;71,716.01)	(70,863.31;71,609.76)	(70,862.77;71,519.83)	(70,861.99;71,442.61)	(70,861.0						



Table A1: Matrix of results of NPV and optimal capacity for  $C_1$  and  $C_2$  (continued)

$K_d$	$C_1$																				$C_2$																			
	155	160	165	170	175	180	185	190	195	200	155	160	165	170	175	180	185	190	195	200	155	160	165	170	175	180	185	190	195	200										
5	(137,936;19,70,224.85)	(137,932.04;70,210.69)	(137,927.87;70,197.39)	(137,923.68;70,184.86)	(137,919.47;70,173.05)	(137,915.25;70,161.89)	(137,911.01;70,151.33)	(137,906.76;70,141.33)	(137,902.50;70,131.83)	(137,898.22;70,122.81)	(137,936;19,70,224.85)	(137,932.04;70,210.69)	(137,927.87;70,197.39)	(137,923.68;70,184.86)	(137,919.47;70,173.05)	(137,915.25;70,161.89)	(137,911.01;70,151.33)	(137,906.76;70,141.33)	(137,902.50;70,131.83)	(137,898.22;70,122.81)	(137,893.97;70,113.81)	(137,936;19,70,224.85)	(137,932.04;70,210.69)	(137,927.87;70,197.39)	(137,923.68;70,184.86)	(137,919.47;70,173.05)	(137,915.25;70,161.89)	(137,911.01;70,151.33)	(137,906.76;70,141.33)	(137,902.50;70,131.83)	(137,898.22;70,122.81)									
10	(126,112.06;70,476.16)	(126,112.06;70,476.16)	(126,118.86;70,448.63)	(126,114.64;70,436.08)	(126,110.42;70,424.24)	(126,106.15;70,413.05)	(126,101.88;70,401.86)	(126,097.61;70,390.67)	(126,093.34;70,379.48)	(126,089.07;70,368.29)	(126,112.06;70,476.16)	(126,112.06;70,476.16)	(126,118.86;70,448.63)	(126,114.64;70,436.08)	(126,110.42;70,424.24)	(126,106.15;70,413.05)	(126,101.88;70,401.86)	(126,097.61;70,390.67)	(126,093.34;70,379.48)	(126,089.07;70,368.29)	(126,084.80;70,357.10)	(126,112.06;70,476.16)	(126,112.06;70,476.16)	(126,118.86;70,448.63)	(126,114.64;70,436.08)	(126,110.42;70,424.24)	(126,106.15;70,413.05)	(126,101.88;70,401.86)	(126,097.61;70,390.67)	(126,093.34;70,379.48)	(126,089.07;70,368.29)									
15	(115,806.39;70,678.26)	(115,802.18;70,664.03)	(115,797.95;70,650.67)	(115,793.72;70,637.31)	(115,789.49;70,623.95)	(115,785.26;70,610.59)	(115,781.03;70,597.23)	(115,776.80;70,583.87)	(115,772.57;70,570.51)	(115,768.34;70,557.15)	(115,806.39;70,678.26)	(115,802.18;70,664.03)	(115,797.95;70,650.67)	(115,793.72;70,637.31)	(115,789.49;70,623.95)	(115,785.26;70,610.59)	(115,781.03;70,597.23)	(115,776.80;70,583.87)	(115,772.57;70,570.51)	(115,768.34;70,557.15)	(115,764.11;70,543.79)	(115,806.39;70,678.26)	(115,802.18;70,664.03)	(115,797.95;70,650.67)	(115,793.72;70,637.31)	(115,789.49;70,623.95)	(115,785.26;70,610.59)	(115,781.03;70,597.23)	(115,776.80;70,583.87)	(115,772.57;70,570.51)	(115,768.34;70,557.15)									
20	(96,281.08;70,932.99)	(96,276.80;70,918.70)	(96,272.51;70,905.27)	(96,268.23;70,891.93)	(96,263.95;70,878.59)	(96,259.67;70,865.25)	(96,255.39;70,851.91)	(96,251.11;70,838.57)	(96,246.83;70,825.23)	(96,242.55;70,811.89)	(96,281.08;70,932.99)	(96,276.80;70,918.70)	(96,272.51;70,905.27)	(96,268.23;70,891.93)	(96,263.95;70,878.59)	(96,259.67;70,865.25)	(96,255.39;70,851.91)	(96,251.11;70,838.57)	(96,246.83;70,825.23)	(96,242.55;70,811.89)	(96,238.27;70,798.55)	(96,281.08;70,932.99)	(96,276.80;70,918.70)	(96,272.51;70,905.27)	(96,268.23;70,891.93)	(96,263.95;70,878.59)	(96,259.67;70,865.25)	(96,255.39;70,851.91)	(96,251.11;70,838.57)	(96,246.83;70,825.23)	(96,242.55;70,811.89)									
25	(86,729.72;70,995.46)	(86,725.41;70,981.13)	(86,721.10;70,966.87)	(86,716.79;70,953.54)	(86,712.48;70,940.21)	(86,708.17;70,926.88)	(86,703.86;70,913.55)	(86,699.55;70,900.22)	(86,695.24;70,886.89)	(86,690.93;70,873.56)	(86,729.72;70,995.46)	(86,725.41;70,981.13)	(86,721.10;70,966.87)	(86,716.79;70,953.54)	(86,712.48;70,940.21)	(86,708.17;70,926.88)	(86,703.86;70,913.55)	(86,699.55;70,900.22)	(86,695.24;70,886.89)	(86,690.93;70,873.56)	(86,686.62;70,860.23)	(86,729.72;70,995.46)	(86,725.41;70,981.13)	(86,721.10;70,966.87)	(86,716.79;70,953.54)	(86,712.48;70,940.21)	(86,708.17;70,926.88)	(86,703.86;70,913.55)	(86,699.55;70,900.22)	(86,695.24;70,886.89)										
30	(78,121.70;70,994.69)	(78,117.48;70,980.35)	(78,113.25;70,966.02)	(78,109.02;70,951.69)	(78,104.79;70,937.36)	(78,100.56;70,923.03)	(78,096.33;70,908.70)	(78,092.10;70,894.37)	(78,087.87;70,880.04)	(78,083.64;70,865.71)	(78,121.70;70,994.69)	(78,117.48;70,980.35)	(78,113.25;70,966.02)	(78,109.02;70,951.69)	(78,104.79;70,937.36)	(78,100.56;70,923.03)	(78,096.33;70,908.70)	(78,092.10;70,894.37)	(78,087.87;70,880.04)	(78,083.64;70,865.71)	(78,079.41;70,851.38)	(78,121.70;70,994.69)	(78,117.48;70,980.35)	(78,113.25;70,966.02)	(78,109.02;70,951.69)	(78,104.79;70,937.36)	(78,100.56;70,923.03)	(78,096.33;70,908.70)	(78,092.10;70,894.37)	(78,087.87;70,880.04)										
35	(73,538.59;70,997.09)	(73,534.37;70,982.75)	(73,530.15;70,968.41)	(73,525.93;70,954.07)	(73,521.71;70,940.73)	(73,517.49;70,926.39)	(73,513.27;70,912.05)	(73,509.05;70,897.71)	(73,504.83;70,883.37)	(73,500.61;70,869.03)	(73,538.59;70,997.09)	(73,534.37;70,982.75)	(73,530.15;70,968.41)	(73,525.93;70,954.07)	(73,521.71;70,940.73)	(73,517.49;70,926.39)	(73,513.27;70,912.05)	(73,509.05;70,897.71)	(73,504.83;70,883.37)	(73,500.61;70,869.03)	(73,496.39;70,854.69)	(73,538.59;70,997.09)	(73,534.37;70,982.75)	(73,530.15;70,968.41)	(73,525.93;70,954.07)	(73,521.71;70,940.73)	(73,517.49;70,926.39)	(73,513.27;70,912.05)	(73,509.05;70,897.71)	(73,504.83;70,883.37)										
40	(72,191.68;70,998.48)	(72,187.45;70,984.14)	(72,183.22;70,969.80)	(72,178.99;70,955.46)	(72,174.76;70,941.12)	(72,170.53;70,926.78)	(72,166.30;70,912.44)	(72,162.07;70,898.10)	(72,157.84;70,883.76)	(72,153.61;70,869.42)	(72,191.68;70,998.48)	(72,187.45;70,984.14)	(72,183.22;70,969.80)	(72,178.99;70,955.46)	(72,174.76;70,941.12)	(72,170.53;70,926.78)	(72,166.30;70,912.44)	(72,162.07;70,898.10)	(72,157.84;70,883.76)	(72,153.61;70,869.42)	(72,149.38;70,855.08)	(72,191.68;70,998.48)	(72,187.45;70,984.14)	(72,183.22;70,969.80)	(72,178.99;70,955.46)	(72,174.76;70,941.12)	(72,170.53;70,926.78)	(72,166.30;70,912.44)	(72,162.07;70,898.10)	(72,157.84;70,883.76)										
45	(71,905.75;70,999.16)	(71,901.52;70,984.82)	(71,897.29;70,970.47)	(71,893.06;70,956.13)	(71,888.83;70,941.79)	(71,884.60;70,927.45)	(71,880.37;70,913.11)	(71,876.14;70,898.77)	(71,871.91;70,884.43)	(71,867.68;70,870.09)	(71,905.75;70,999.16)	(71,901.52;70,984.82)	(71,897.29;70,970.47)	(71,893.06;70,956.13)	(71,888.83;70,941.79)	(71,884.60;70,927.45)	(71,880.37;70,913.11)	(71,876.14;70,898.77)	(71,871.91;70,884.43)	(71,867.68;70,870.09)	(71,863.45;70,855.75)	(71,905.75;70,999.16)	(71,901.52;70,984.82)	(71,897.29;70,970.47)	(71,893.06;70,956.13)	(71,888.83;70,941.79)	(71,884.60;70,927.45)	(71,880.37;70,913.11)	(71,876.14;70,898.77)	(71,871.91;70,884.43)										
50	(71,771.89;70,999.32)	(71,767.66;70,984.98)	(71,763.43;70,970.64)	(71,759.20;70,956.30)	(71,754.97;70,941.96)	(71,750.74;70,927.62)	(71,746.51;70,913.30)	(71,742.28;70,898.96)	(71,738.05;70,884.62)	(71,733.82;70,870.28)	(71,771.89;70,999.32)	(71,767.66;70,984.98)	(71,763.43;70,970.64)	(71,759.20;70,956.30)	(71,754.97;70,941.96)	(71,750.74;70,927.62)	(71,746.51;70,913.30)	(71,742.28;70,898.96)	(71,738.05;70,884.62)	(71,733.82;70,870.28)	(71,729.59;70,855.94)	(71,771.89;70,999.32)	(71,767.66;70,984.98)	(71,763.43;70,970.64)	(71,759.20;70,956.30)	(71,754.97;70,941.96)	(71,750.74;70,927.62)	(71,746.51;70,913.30)	(71,742.28;70,898.96)	(71,738.05;70,884.62)										
55	(71,665.64;70,999.09)	(71,661.41;70,984.75)	(71,657.18;70,970.41)	(71,652.95;70,956.07)	(71,648.72;70,941.73)	(71,644.49;70,927.39)	(71,640.26;70,913.05)	(71,636.03;70,898.71)	(71,631.80;70,884.37)	(71,627.57;70,870.03)	(71,665.64;70,999.09)	(71,661.41;70,984.75)	(71,657.18;70,970.41)	(71,652.95;70,956.07)	(71,648.72;70,941.73)	(71,644.49;70,927.39)	(71,640.26;70,913.05)	(71,636.03;70,898.71)	(71,631.80;70,884.37)	(71,627.57;70,870.03)	(71,623.34;70,855.69)	(71,665.64;70,999.09)	(71,661.41;70,984.75)	(71,657.18;70,970.41)	(71,652.95;70,956.07)	(71,648.72;70,941.73)	(71,644.49;70,927.39)	(71,640.26;70,913.05)	(71,636.03;70,898.71)	(71,631.80;70,884.37)										
60	(71,575.70;70,998.55)	(71,571.47;70,984.21)	(71,567.24;70,969.87)	(71,563.01;70,955.53)	(71,558.78;70,941.19)	(71,554.55;70,926.85)	(71,550.32;70,912.51)	(71,546.09;70,898.17)	(71,541.86;70,883.83)	(71,537.63;70,869.49)	(71,575.70;70,998.55)	(71,571.47;70,984.21)	(71,567.24;70,969.87)	(71,563.01;70,955.53)	(71,558.78;70,941.19)	(71,554.55;70,926.85)	(71,550.32;70,912.51)	(71,546.09;70,898.17)	(71,541.86;70,883.83)	(71,537.63;70,869.49)	(71,533.40;70,855.15)	(71,575.70;70,998.55)	(71,571.47;70,984.21)	(71,567.24;70,969.87)	(71,563.01;70,955.53)	(71,558.78;70,941.19)	(71,554.55;70,926.85)	(71,550.32;70,912.51)	(71,546.09;70,898.17)	(71,541.86;70,883.83)										
65	(71,498.48;70,997.78)	(71,494.25;70,983.43)	(71,490.02;70,969.09)	(71,485.79;70,954.75)	(71,481.56;70,940.41)	(71,477.33;70,926.07)	(71,473.10;70,912.73)	(71,468.87;70,898.39)	(71,464.64;70,884.05)	(71,460.41;70,869.71)	(71,498.48;70,997.78)	(71,494.25;70,983.43)	(71,490.02;70,969.09)	(71,485.79;70,954.75)	(71,481.56;70,940.41)	(71,477.33;70,926.07)	(71,473.10;70,912.73)	(71,468.87;70,898.39)	(71,464.64;70,884.05)	(71,460.41;70,869.71)	(71,456.18;70,855.37)	(71,498.48;70,997.78)	(71,494.25;70,983.43)	(71,490.02;70,969.09)	(71,485.79;70,954.75)	(71,481.56;70,940.41)	(71,477.33;70,926.07)	(71,473.10;70,912.73)	(71,468.87;70,898.39)	(71,464.64;70,884.05)										
70	(71,431.46;70,996.87)	(71,427.23;70,982.46)	(71,423.00;70,968.09)	(71,418.77;70,953.72)	(71,414.54;70,939.35)	(71,410.31;70,924.98)	(71,406.08;70,910.61)	(71,401.85;70,896.24)	(71,397.62;70,881.87)	(71,393.39;70,867.50)	(71,431.46;70,996.87)	(71,427.23;70,982.46)	(71,423.00;70,968.09)	(71,418.77;70,953.72)	(71,414.54;70,939.35)	(71,410.31;70,924.98)	(71,406.08;70,910.61)	(71,401.85;70,896.24)	(71,397.62;70,881.87)	(71,393.39;70,867.50)	(71,389.16;70,853.13)	(71,431.46;70,996.87)	(71,427.23;70,982.46)	(71,423.00;70,968.09)	(71,418.77;70,953.72)	(71,414.54;70,939.35)	(71,4													





Table A1: Matrix of results of NPV and optimal capacity for  $C_1$  and  $C_2$  (continued)

$K_{it}$	$C_1$	$C_2$											
		255	260	265	270	275	280	285	290	295	300		
5	(137,850.60;70,046.86)	(137,846.23;70,041.54)	(137,841.86;70,036.43)	(137,833.10;70,026.75)	(137,828.71;70,022.17)	(137,824.32;70,017.76)	(137,819.93;70,013.49)	(137,815.53;70,009.37)	(137,811.13;70,005.38)	(137,806.73;70,001.30)	(137,802.33;70,000.00)	(137,797.93;70,000.00)	(137,793.53;70,000.00)
10	(126,041.24;70,297.45)	(126,036.86;70,292.42)	(126,032.47;70,287.30)	(126,028.08;70,282.36)	(126,023.69;70,277.60)	(126,019.29;70,273.03)	(126,014.89;70,268.58)	(126,010.49;70,264.13)	(126,006.09;70,259.68)	(126,001.69;70,255.18)	(125,997.29;70,250.63)	(125,992.89;70,246.08)	(125,988.49;70,241.53)
15	(115,719.58;70,499.93)	(115,715.38;70,494.99)	(115,711.17;70,489.95)	(115,706.97;70,484.90)	(115,702.76;70,479.86)	(115,698.56;70,474.82)	(115,694.35;70,469.78)	(115,690.15;70,464.74)	(115,685.94;70,459.70)	(115,681.74;70,454.66)	(115,677.53;70,449.62)	(115,673.33;70,444.58)	(115,669.12;70,439.54)
20	(105,855.94;70,651.33)	(105,851.53;70,646.38)	(105,847.12;70,641.43)	(105,842.71;70,636.48)	(105,838.30;70,631.53)	(105,833.89;70,626.58)	(105,829.48;70,621.63)	(105,825.07;70,616.68)	(105,820.66;70,611.73)	(105,816.25;70,606.78)	(105,811.84;70,601.83)	(105,807.43;70,596.88)	(105,803.02;70,591.93)
25	(96,193.93;70,753.33)	(96,189.42;70,747.97)	(96,184.91;70,742.80)	(96,180.40;70,737.63)	(96,175.89;70,732.46)	(96,171.38;70,727.29)	(96,166.87;70,722.12)	(96,162.36;70,716.95)	(96,157.85;70,711.78)	(96,153.34;70,710.61)	(96,148.83;70,709.44)	(96,144.32;70,708.27)	(96,139.81;70,707.10)
30	(86,642.06;70,805.38)	(86,637.55;70,800.00)	(86,633.04;70,794.82)	(86,628.53;70,789.84)	(86,624.02;70,784.66)	(86,619.51;70,779.49)	(86,615.00;70,774.32)	(86,610.49;70,769.15)	(86,605.98;70,763.98)	(86,601.47;70,758.81)	(86,596.96;70,753.64)	(86,592.45;70,748.47)	(86,587.94;70,743.30)
35	(78,035.22;70,814.41)	(78,030.82;70,809.03)	(78,026.42;70,804.64)	(78,022.02;70,799.85)	(78,017.61;70,795.04)	(78,013.21;70,790.23)	(78,008.81;70,785.42)	(78,004.40;70,780.61)	(77,999.99;70,775.80)	(77,995.59;70,770.99)	(77,991.18;70,766.18)	(77,986.78;70,761.37)	(77,982.37;70,756.56)
40	(73,452.10;70,816.80)	(73,447.70;70,811.42)	(73,443.30;70,806.04)	(73,438.90;70,801.25)	(73,434.49;70,796.44)	(73,430.09;70,791.63)	(73,425.68;70,786.82)	(73,421.28;70,781.99)	(73,416.87;70,777.16)	(73,412.47;70,772.33)	(73,408.06;70,767.50)	(73,403.66;70,762.67)	(73,400.00;70,757.84)
45	(72,025.18;70,818.93)	(72,020.78;70,813.49)	(72,016.38;70,808.04)	(72,011.97;70,803.47)	(72,007.57;70,797.83)	(72,003.16;70,792.19)	(71,998.76;70,786.52)	(71,994.35;70,780.88)	(71,989.95;70,775.24)	(71,985.54;70,769.60)	(71,981.14;70,763.96)	(71,976.73;70,758.32)	(71,972.33;70,752.68)
50	(71,819.24;70,818.97)	(71,814.85;70,813.49)	(71,810.45;70,808.04)	(71,806.04;70,803.47)	(71,801.64;70,797.83)	(71,797.23;70,792.19)	(71,792.83;70,786.52)	(71,788.42;70,780.88)	(71,784.02;70,775.24)	(71,779.61;70,769.60)	(71,775.21;70,763.96)	(71,770.80;70,758.32)	(71,766.40;70,752.68)
55	(71,685.38;70,819.03)	(71,680.98;70,813.64)	(71,676.58;70,808.04)	(71,672.18;70,803.47)	(71,667.77;70,797.83)	(71,663.37;70,792.19)	(71,658.96;70,786.52)	(71,654.56;70,780.88)	(71,650.15;70,775.24)	(71,645.75;70,769.60)	(71,641.34;70,763.96)	(71,636.94;70,758.32)	(71,632.53;70,752.68)
60	(71,579.13;70,818.79)	(71,574.73;70,813.41)	(71,570.33;70,808.02)	(71,565.93;70,803.23)	(71,561.52;70,797.88)	(71,557.12;70,792.46)	(71,552.71;70,787.04)	(71,548.31;70,781.62)	(71,543.90;70,776.20)	(71,539.50;70,770.78)	(71,535.09;70,765.36)	(71,530.69;70,760.00)	(71,526.28;70,754.64)
65	(71,489.19;70,818.25)	(71,484.79;70,812.87)	(71,480.38;70,807.68)	(71,475.97;70,802.69)	(71,471.56;70,797.10)	(71,467.15;70,791.51)	(71,462.74;70,785.92)	(71,458.33;70,779.93)	(71,453.92;70,773.94)	(71,449.51;70,767.95)	(71,445.10;70,761.96)	(71,440.69;70,755.97)	(71,436.28;70,749.98)
70	(71,411.97;70,817.47)	(71,407.57;70,812.08)	(71,403.17;70,806.90)	(71,398.77;70,801.91)	(71,394.36;70,796.42)	(71,389.95;70,791.43)	(71,385.54;70,785.94)	(71,381.13;70,779.95)	(71,376.72;70,773.96)	(71,372.31;70,767.97)	(71,367.90;70,761.98)	(71,363.49;70,755.99)	(71,359.08;70,750.00)
75	(71,344.94;70,816.50)	(71,340.54;70,811.11)	(71,336.14;70,805.93)	(71,331.74;70,800.94)	(71,327.33;70,795.13)	(71,322.92;70,789.32)	(71,318.51;70,783.51)	(71,314.10;70,777.52)	(71,309.69;70,771.53)	(71,305.28;70,765.54)	(71,300.87;70,759.55)	(71,296.46;70,753.56)	(71,292.05;70,747.57)
80	(71,286.21;70,815.37)	(71,281.81;70,809.98)	(71,277.41;70,804.80)	(71,273.01;70,799.81)	(71,268.60;70,794.32)	(71,264.19;70,788.33)	(71,259.78;70,782.34)	(71,255.37;70,776.35)	(71,250.96;70,770.36)	(71,246.55;70,764.37)	(71,242.14;70,758.38)	(71,237.73;70,752.39)	(71,233.32;70,746.40)
85	(71,234.33;70,814.13)	(71,229.94;70,808.75)	(71,225.54;70,803.56)	(71,221.13;70,798.57)	(71,216.73;70,793.76)	(71,212.32;70,787.95)	(71,207.91;70,782.14)	(71,203.50;70,776.33)	(71,199.09;70,770.52)	(71,194.68;70,764.71)	(71,190.27;70,758.90)	(71,185.86;70,752.89)	(71,181.45;70,746.88)
90	(71,188.17;70,812.99)	(71,183.77;70,807.60)	(71,179.37;70,802.42)	(71,174.97;70,797.43)	(71,170.56;70,792.62)	(71,166.15;70,787.81)	(71,161.74;70,782.99)	(71,157.33;70,778.18)	(71,152.92;70,773.37)	(71,148.51;70,768.56)	(71,144.10;70,762.75)	(71,139.69;70,756.94)	(71,135.28;70,750.95)
95	(71,146.83;70,811.48)	(71,142.43;70,806.10)	(71,138.03;70,800.92)	(71,133.63;70,795.92)	(71,129.22;70,791.11)	(71,124.81;70,786.48)	(71,120.40;70,781.67)	(71,116.00;70,776.86)	(71,111.59;70,772.05)	(71,107.18;70,767.24)	(71,102.77;70,762.43)	(71,098.36;70,757.62)	(71,093.95;70,752.81)
100	(71,109.59;70,808.89)	(71,105.20;70,803.51)	(71,100.80;70,798.32)	(71,096.40;70,793.33)	(71,092.00;70,788.34)	(71,087.59;70,778.85)	(71,083.19;70,773.86)	(71,078.78;70,768.87)	(71,074.37;70,763.88)	(71,069.96;70,758.89)	(71,065.55;70,753.90)	(71,061.14;70,748.91)	(71,056.73;70,743.92)
105	(71,075.88;70,804.93)	(71,071.48;70,799.54)	(71,067.08;70,794.36)	(71,062.68;70,789.36)	(71,058.27;70,784.35)	(71,053.87;70,779.34)	(71,049.46;70,774.33)	(71,045.05;70,769.32)	(71,040.64;70,764.31)	(71,036.23;70,759.30)	(71,031.82;70,754.29)	(71,027.41;70,749.28)	(71,023.00;70,744.27)
110	(71,045.20;70,800.88)	(71,040.81;70,795.49)	(71,036.41;70,790.31)	(71,032.00;70,785.31)	(71,027.60;70,780.30)	(71,023.19;70,775.29)	(71,018.78;70,770.28)	(71,014.37;70,765.27)	(71,009.96;70,760.26)	(71,005.55;70,755.25)	(71,001.14;70,750.24)	(70,996.73;70,745.23)	(70,992.32;70,740.22)
115	(71,017.18;70,796.91)	(71,012.79;70,791.53)	(71,008.38;70,786.34)	(71,003.98;70,781.35)	(70,999.57;70,776.34)	(70,995.16;70,771.33)	(70,990.75;70,766.32)	(70,986.34;70,761.31)	(70,981.93;70,756.30)	(70,977.52;70,751.29)	(70,973.11;70,746.28)	(70,968.70;70,741.27)	(70,964.29;70,736.26)
120	(70,991.48;70,792.90)	(70,987.09;70,787.52)	(70,982.69;70,782.33)	(70,978.28;70,777.34)	(70,973.87;70,772.35)	(70,969.46;70,767.36)	(70,965.05;70,762.37)	(70,960.64;70,757.38)	(70,956.23;70,752.39)	(70,951.82;70,747.40)	(70,947.41;70,739.41)	(70,942.99;70,734.42)	(70,938.58;70,729.43)
125	(70,967.82;70,788.86)	(70,963.43;70,783.47)	(70,959.03;70,778.29)	(70,954.62;70,773.29)	(70,950.21;70,768.30)	(70,945.80;70,763.31)	(70,941.39;70,758.32)	(70,936.98;70,753.33)	(70,932.57;70,748.34)	(70,928.16;70,743.35)	(70,923.75;70,738.36)	(70,919.34;70,733.37)	(70,914.93;70,728.38)
130	(70,945.98;70,784.77)	(70,941.58;70,779.39)	(70,937.18;70,774.20)	(70,932.78;70,769.21)	(70,928.37;70,764.22)	(70,923.96;70,759.23)	(70,919.55;70,754.24)	(70,915.14;70,749.25)	(70,910.73;70,744.26)	(70,906.32;70,739.27)	(70,901.91;70,734.28)	(70,897.50;70,729.29)	(70,893.09;70,724.30)
135	(70,925.74;70,780.66)	(70,921.34;70,775.28)	(70,916.94;70,770.09)	(70,912.54;70,765.10)	(70,908.13;70,760.11)	(70,903.72;70,755.12)	(70,899.31;70,750.13)	(70,894.90;70,745.14)	(70,890.49;70,740.15)	(70,886.08;70,735.16)	(70,881.67;70,730.17)	(70,877.26;70,725.18)	(70,872.85;70,720.19)
140	(70,906.94;70,776.52)	(70,902.54;70,771.14)	(70,898.13;70,766.95)	(70,893.73;70,761.96)	(70,889.32;70,756.97)	(70,884.91;70,751.98)	(70,880.50;70,746.99)	(70,876.09;70,742.00)	(70,871.68;70,737.01)	(70,867.27;70,732.02)	(70,862.86;70,727.03)	(70,858.45;70,722.04)	(70,854.04;70,717.05)
145	(70,889.43;70,772.36)	(70,885.03;70,766.97)	(70,880.62;70,761.79)	(70,876.21;70,756.80)	(70,871.80;70,751.81)	(70,867.39;70,746.82)	(70,862.98;70,741.83)	(70,858.57;70,736.84)	(70,854.16;70,731.85)	(70,849.75;70,726.86)	(70,845.34;70,721.87)	(70,840.93;70,716.88)	(70,836.52;70,711.89)
150	(70,873.08;70,768.17)	(70,868.68;70,762.78)	(70,864.28;70,757.39)	(70,859.87;70,752.40)	(70,855.46;70,747.41)	(70,851.05;70,739.42)	(70,846.64;70,734.43)	(70,842.23;70,729.44)	(70,837.82;70,724.45)	(70,833.41;70,719.46)	(70,829.00;70,714.47)	(70,824.59;70,709.48)	(70,820.18;70,704.49)
155	(70,857.78;70,763.96)	(70,853.38;70,758.58)	(70,848.98;70,753.60)	(70,844.57;70,748.61)	(70,840.16;70,743.62)	(70,835.75;70,738.63)	(70,831.34;70,733.64)	(70,826.93;70,728.65)	(70,822.52;70,723.66)	(70,818.11;70,718.67)	(70,813.70;70,713.68)	(70,809.29;70,708.69)	(70,804.88;70,703.70)
160	(70,843.43;70,759.74)	(70,839.03;70,754.35)	(70,834.62;70,749.17)	(70,830.21;70,744.18)	(70,825.80;70,739.19)	(70,821.39;70,734.20)	(70,816.98;70,729.21)	(70,812.57;70,724.22)	(70,808.16;70,719.23)	(70,803.75;70,714.24)	(70,799.34;70,709.25)	(70,794.93;70,704.26)	(70,790.52;70,699.27)
165	(70,829.95;70,755.50)	(70,825.56;70,750.11)	(70,821.16;70,744.93)	(70,816.75;70,739.94)	(70,812.34;70,734.95)	(70,807.93;70,729.96)	(70,803.52;70,724.97)	(70,799.11;70,719.98)	(70,794.70;70,714.99)	(70,790.29;70,710.00)	(70,785.88;70,705.01)	(70,781.47;70,699.99)	(70,777.06;70,694.99)
170	(70,817.26;70,751.24)	(70,812.86;70,745.85)	(70,808.46;70,740.67)	(70,804.05;70,735.68)	(70,799.64;70,730.69)	(70,795.23;70,725.70)	(70,790.82;70,720.71)	(70,786.41;70,715.72)	(70,782.00;70,710.73)	(70,777.59;70,705.74)	(70,773.18;70,700.75)	(70,768.77;70,695.76)	(70,764.36;70,690.77)
175	(70,805.29;70,746.97)	(70,800.89;70,741.58)	(70,796.48;70,736.40)	(70,792.07;70,731.41)	(70,787.66;70,726.42)	(70,783.25;70,721.43)	(70,778.84;70,716.44)	(70,774.43;70,711.45)	(70,770.02;70,706.46)	(70,765.61;70,701.47)	(70,761.20;70,696.48)	(70,756.79;70,691.49)	(70,752.38;70,686.50)
180	(70,793.98;70,742.69)	(70,789.59;70,737.30)	(70,785.19;70,732.12)	(70,780.78;70,727.13)	(70,776.37;70,722.14)	(70,771.96;70,717.15)	(70,767.55;70,712.16)	(70,763.14;70,710.17)	(70,758.73;70,705.18)	(70,754.32;70,700.19)	(70,749.91;70,695.20)	(70,745.50;70,689.21)	(70,741.09;70,684.22)
185	(70,783.28;70,738.39)	(70,778.89;70,733.01)	(70,774.49;70,727.82)	(70,770.08;70,722.83)	(70,765.67;70,717.84)	(70,761.26;70,712.85)	(70,756.85;70,710.86)	(70,752.44;70,705.87)	(70,748.03;70,700.88)	(70,743.62;70,695.89)	(70,739.21;70,690.90		



Table A2: Matrix of results of NPV and optimal capacity for  $S_L$  and  $S_F$  (continued)

$K_{it}$	55	60	65	70	75	80	85	90	95	100
5	(146,722.47;45,318.70)	(146,722.53;42,539.94)	(146,722.24;41,369.99)	(146,721.67;40,975.75)	(146,720.88;40,818.04)	(146,719.90;40,716.11)	(146,718.78;40,630.93)	(146,717.53;40,555.60)	(146,716.37;40,488.17)	(146,714.98;40,427.42)
10	(133,191.74;42,714.31)	(133,188.75;41,543.91)	(133,185.35;41,409.91)	(133,182.18;41,286.80)	(133,178.66;41,171.23)	(133,175.66;40,889.01)	(133,173.03;40,803.56)	(133,171.30;40,728.00)	(133,167.49;40,660.36)	(133,163.61;40,599.43)
15	(120,680.25;45,618.55)	(120,677.01;42,838.73)	(120,673.58;41,667.88)	(120,669.95;41,272.86)	(120,666.28;41,114.95)	(120,663.47;40,926.24)	(120,660.59;40,782.59)	(120,657.61;40,648.54)	(120,654.61;40,514.54)	(120,651.61;40,427.47)
20	(108,791.49;45,693.53)	(108,787.72;42,913.17)	(108,783.84;41,741.86)	(108,779.86;41,346.45)	(108,775.84;41,187.71)	(108,771.69;41,084.90)	(108,767.54;40,998.92)	(108,763.34;40,922.89)	(108,759.11;40,854.83)	(108,754.85;40,793.51)
25	(97,597.45;47,118.45)	(97,594.50;42,937.58)	(97,591.31;41,765.85)	(97,587.93;41,370.08)	(97,584.41;41,211.03)	(97,580.76;41,091.72)	(97,577.01;40,991.72)	(97,573.17;40,945.48)	(97,569.27;40,877.22)	(97,565.30;40,815.74)
30	(91,070.32;45,719.49)	(91,067.36;42,938.55)	(91,064.15;41,766.85)	(91,060.76;41,371.05)	(91,057.22;41,211.94)	(91,053.56;41,108.84)	(91,049.81;41,022.61)	(91,045.96;40,946.36)	(91,042.05;40,878.09)	(91,038.08;40,816.60)
35	(89,630.23;45,719.53)	(89,627.25;42,938.62)	(89,624.04;41,766.85)	(89,620.64;41,371.05)	(89,617.09;41,211.97)	(89,613.43;41,108.86)	(89,609.66;41,022.62)	(89,605.81;40,946.36)	(89,601.90;40,878.09)	(89,597.92;40,816.59)
40	(89,140.62;45,719.04)	(89,137.64;42,938.12)	(89,134.41;41,766.43)	(89,131.01;41,369.63)	(89,127.54;41,211.45)	(89,124.01;41,108.33)	(89,120.48;41,022.08)	(89,116.94;40,945.81)	(89,113.35;40,875.61)	(89,109.74;40,815.03)
45	(89,284.95;47,118.17)	(89,281.95;42,937.24)	(89,278.73;41,765.45)	(89,275.31;41,369.63)	(89,271.76;41,210.54)	(89,268.18;41,107.42)	(89,264.61;41,022.08)	(89,261.03;40,945.81)	(89,257.45;40,875.61)	(89,253.87;40,815.03)
50	(89,184.96;45,717.03)	(89,181.96;42,936.09)	(89,178.73;41,764.39)	(89,175.31;41,368.47)	(89,171.75;41,209.37)	(89,168.17;41,106.34)	(89,164.59;41,019.89)	(89,161.01;40,944.81)	(89,157.43;40,874.81)	(89,153.85;40,813.92)
55	(89,102.97;45,715.94)	(89,099.96;42,934.99)	(89,096.72;41,763.19)	(89,093.30;41,367.36)	(89,089.74;41,208.26)	(89,086.16;41,105.12)	(89,082.58;41,018.87)	(89,079.00;40,943.87)	(89,075.42;40,873.87)	(89,071.84;40,812.79)
60	(89,024.54;45,714.13)	(89,021.53;42,933.11)	(89,018.29;41,761.23)	(89,014.87;41,365.34)	(89,011.30;41,206.19)	(89,007.72;41,103.01)	(89,004.14;40,941.81)	(89,000.56;40,872.06)	(89,000.00;40,810.51)	(89,000.00;40,810.51)
65	(88,976.57;45,706.08)	(88,973.56;42,929.05)	(88,970.31;41,757.12)	(88,966.89;41,361.23)	(88,963.32;41,202.07)	(88,959.74;41,098.88)	(88,956.16;41,012.58)	(88,952.58;40,936.26)	(88,948.00;40,867.93)	(88,943.42;40,806.38)
70	(88,929.79;45,706.08)	(88,926.78;42,925.04)	(88,923.53;41,753.16)	(88,920.11;41,357.27)	(88,916.54;41,198.11)	(88,912.96;41,094.92)	(88,909.38;41,008.62)	(88,905.80;40,928.25)	(88,902.22;40,859.92)	(88,898.64;40,802.41)
75	(88,884.59;45,702.06)	(88,881.57;42,921.02)	(88,877.33;41,749.14)	(88,873.90;41,353.24)	(88,870.33;41,194.08)	(88,866.75;41,090.89)	(88,863.17;41,004.58)	(88,859.59;40,928.25)	(88,856.01;40,859.92)	(88,852.43;40,798.37)
80	(88,845.74;45,697.98)	(88,842.72;42,916.94)	(88,839.47;41,745.05)	(88,836.04;41,349.15)	(88,832.47;41,189.99)	(88,828.89;41,086.80)	(88,825.31;41,000.49)	(88,821.73;40,924.16)	(88,818.15;40,855.83)	(88,814.57;40,790.13)
85	(88,812.31;45,693.85)	(88,809.28;42,912.81)	(88,806.03;41,740.92)	(88,802.60;41,345.02)	(88,799.02;41,185.85)	(88,795.44;41,082.66)	(88,791.86;40,996.35)	(88,788.28;40,920.02)	(88,784.70;40,851.68)	(88,781.12;40,790.13)
90	(88,782.55;45,689.68)	(88,779.53;42,908.64)	(88,776.28;41,736.75)	(88,772.84;41,293.44)	(88,769.27;41,181.67)	(88,765.69;41,078.48)	(88,762.11;40,992.17)	(88,758.53;40,915.84)	(88,754.95;40,847.50)	(88,751.37;40,785.94)
95	(88,755.91;45,685.48)	(88,752.88;42,904.43)	(88,749.63;41,732.54)	(88,746.19;41,336.63)	(88,742.62;41,177.46)	(88,739.04;41,074.27)	(88,735.46;40,987.95)	(88,731.88;40,911.62)	(88,728.30;40,843.29)	(88,724.72;40,781.73)
100	(88,730.91;45,681.24)	(88,727.88;42,900.20)	(88,724.63;41,728.30)	(88,721.19;41,332.39)	(88,717.62;41,173.22)	(88,714.04;41,070.03)	(88,710.46;40,983.71)	(88,706.88;40,907.38)	(88,703.30;40,839.04)	(88,699.72;40,777.48)
105	(88,706.91;45,677.00)	(88,703.88;42,896.94)	(88,700.63;41,724.04)	(88,697.19;41,328.13)	(88,693.62;41,168.96)	(88,690.04;41,065.76)	(88,686.46;40,979.45)	(88,682.88;40,903.11)	(88,679.30;40,834.78)	(88,675.72;40,776.32)
110	(88,683.91;45,672.76)	(88,680.88;42,892.69)	(88,677.63;41,719.76)	(88,674.19;41,323.85)	(88,670.62;41,164.67)	(88,667.04;41,061.48)	(88,663.46;40,975.16)	(88,659.88;40,898.82)	(88,656.30;40,830.49)	(88,652.72;40,776.93)
115	(88,661.91;45,668.41)	(88,658.88;42,888.43)	(88,655.63;41,715.43)	(88,652.19;41,318.94)	(88,648.62;41,160.37)	(88,645.04;41,057.17)	(88,641.46;40,972.74)	(88,637.88;40,896.33)	(88,634.30;40,832.00)	(88,630.72;40,769.62)
120	(88,640.91;45,664.09)	(88,637.88;42,884.04)	(88,634.63;41,711.14)	(88,631.19;41,313.25)	(88,627.62;41,158.05)	(88,624.04;41,054.85)	(88,620.46;40,970.42)	(88,616.88;40,894.01)	(88,613.30;40,830.68)	(88,609.72;40,767.35)
125	(88,620.91;45,659.76)	(88,617.88;42,879.71)	(88,614.63;41,706.80)	(88,611.19;41,308.89)	(88,607.62;41,151.72)	(88,604.04;41,048.51)	(88,599.46;40,964.08)	(88,595.88;40,887.27)	(88,592.30;40,823.94)	(88,588.72;40,764.61)
130	(88,601.91;45,655.42)	(88,598.88;42,875.36)	(88,595.63;41,702.89)	(88,592.19;41,304.98)	(88,588.62;41,147.37)	(88,585.04;41,044.17)	(88,581.46;40,959.75)	(88,577.88;40,882.44)	(88,574.30;40,815.11)	(88,570.72;40,751.61)
135	(88,583.91;45,651.06)	(88,580.88;42,871.00)	(88,577.63;41,698.91)	(88,574.19;41,297.82)	(88,570.62;41,143.01)	(88,567.04;41,039.81)	(88,563.46;40,954.39)	(88,559.88;40,877.08)	(88,556.30;40,809.75)	(88,552.72;40,742.28)
140	(88,566.91;45,646.72)	(88,563.88;42,866.64)	(88,560.63;41,696.80)	(88,557.19;41,293.44)	(88,553.62;41,138.64)	(88,550.04;41,035.44)	(88,546.46;40,950.02)	(88,542.88;40,872.70)	(88,539.30;40,804.37)	(88,535.72;40,738.50)
145	(88,550.91;45,642.32)	(88,547.88;42,862.26)	(88,544.63;41,689.36)	(88,541.19;41,289.66)	(88,537.62;41,134.26)	(88,534.04;41,031.06)	(88,530.46;40,946.64)	(88,526.88;40,869.31)	(88,523.30;40,792.98)	(88,519.72;40,727.11)
150	(88,535.91;45,637.97)	(88,532.88;42,858.88)	(88,529.63;41,684.97)	(88,526.19;41,284.66)	(88,522.62;41,129.88)	(88,519.04;41,026.67)	(88,515.46;40,941.25)	(88,511.88;40,863.92)	(88,508.30;40,786.59)	(88,504.72;40,711.71)
155	(88,520.91;45,633.55)	(88,517.88;42,854.49)	(88,514.63;41,680.58)	(88,511.19;41,280.26)	(88,507.62;41,125.49)	(88,504.04;41,022.28)	(88,499.46;40,937.86)	(88,495.88;40,860.53)	(88,492.30;40,783.20)	(88,488.72;40,708.33)
160	(88,506.91;45,629.15)	(88,503.88;42,849.09)	(88,500.63;41,676.18)	(88,497.19;41,276.80)	(88,493.62;41,121.09)	(88,490.04;41,017.88)	(88,486.46;40,932.46)	(88,482.88;40,855.13)	(88,479.30;40,777.80)	(88,475.72;40,702.93)
165	(88,492.91;45,624.73)	(88,489.88;42,844.61)	(88,486.63;41,671.78)	(88,483.19;41,271.80)	(88,479.62;41,116.68)	(88,476.04;41,013.47)	(88,472.46;40,928.05)	(88,468.88;40,850.72)	(88,465.30;40,773.60)	(88,461.72;40,698.73)
170	(88,478.91;45,620.34)	(88,475.88;42,839.28)	(88,472.63;41,667.37)	(88,469.19;41,267.03)	(88,465.62;41,111.27)	(88,462.04;41,008.06)	(88,458.46;40,922.74)	(88,454.88;40,845.41)	(88,451.30;40,768.08)	(88,447.72;40,693.19)
175	(88,465.91;45,615.93)	(88,462.88;42,834.86)	(88,459.63;41,662.95)	(88,456.19;41,262.61)	(88,452.62;41,106.85)	(88,449.04;41,003.64)	(88,445.46;40,918.22)	(88,441.88;40,840.89)	(88,438.30;40,763.56)	(88,434.72;40,686.27)
180	(88,452.91;45,611.51)	(88,449.88;42,830.44)	(88,446.63;41,658.53)	(88,443.19;41,257.16)	(88,439.62;41,101.01)	(88,436.04;40,997.80)	(88,432.46;40,912.38)	(88,428.88;40,835.03)	(88,425.30;40,757.70)	(88,421.72;40,680.31)
185	(88,440.91;45,607.08)	(88,437.88;42,825.02)	(88,434.63;41,654.51)	(88,431.19;41,252.76)	(88,427.62;41,095.01)	(88,424.04;40,991.80)	(88,420.46;40,906.38)	(88,416.88;40,829.03)	(88,413.30;40,752.70)	(88,409.72;40,675.21)
190	(88,428.91;45,602.66)	(88,425.88;42,820.59)	(88,422.63;41,649.68)	(88,419.19;41,248.31)	(88,415.62;41,090.14)	(88,412.04;40,986.94)	(88,408.46;40,901.52)	(88,404.88;40,824.17)	(88,401.30;40,747.84)	(88,397.72;40,669.35)
195	(88,417.91;45,598.22)	(88,414.88;42,817.16)	(88,411.63;41,645.25)	(88,408.19;41,244.89)	(88,404.62;41,085.71)	(88,401.04;40,982.50)	(88,397.46;40,897.08)	(88,393.88;40,820.73)	(88,390.30;40,743.40)	(88,386.72;40,666.91)
200	(88,406.91;45,593.79)	(88,403.88;42,812.72)	(88,400.63;41,640.81)	(88,397.19;41,240.45)	(88,393.62;41,081.27)	(88,390.04;40,978.06)	(88,386.46;40,892.64)	(88,382.88;40,815.39)	(88,379.30;40,738.06)	(88,375.72;40,661.28)
205	(88,395.91;45,589.35)	(88,392.88;42,808.29)	(88,389.63;41,636.37)	(88,386.19;41,236.01)	(88,382.62;41,076.83)	(88,379.04;40,973.62)	(88,375.46;40,888.20)	(88,371.88;40,810.95)	(88,368.30;40,733.61)	(88,364.72;40,656.48)
210	(88,384.91;45,584.91)	(88,381.88;42,803.84)	(88,378.63;41,631.93)	(88,375.19;41,231.56)	(88,371.62;41,071.65)	(88,368.04;40,968.41)	(88,364.46;40,883.85)	(88,360.88;40,806.51)	(88,357.30;40,728.16)	(88,353.72;40,651.33)
215	(88,373.91;45,580.47)	(88,370.88;42,799.40)	(88,367.63;41,627.49)	(88,364.19;41,226.00)	(88,360.62;41,066.77)	(88,357.04;40,963.56)	(88,353.46;40,878.00)	(88,349.88;40,800.66)	(88,346.30;40,723.31)	(88,342.72;40,646.08)
220	(88,362.91;45,576.02)	(88,359.88;42,794.95)	(88,356.63;41,623.04)	(88,353.19;41,221.51)	(88,349.62;41,061.49)	(88,346.04;40,958.28)	(88,342.46;40,872.72)	(88,338.88;40,795.37)	(88,335.30;40,717.92)	(88,331.72;40,640.39)
225	(88,352.91;45,571.57)	(88,349.88;42,790.51)	(88,346.63;41,618.59)	(88,343.19;41,216.02)	(88,339.62;41,056.47)	(88,336.04;40,953.26)	(88,332.46;40,867.70)	(88,328.88;40,790.25)	(88,325.30;40,712.80)	(88,321.72;40,635.31)
230	(88,342.91;45,567.12)	(88,339.88;42,786.06)	(88,336.63;41,614.14)	(88,333.19;41,211.53)	(88,329.62;41,051.45)	(88,326.04;40,948.24)	(88,322.46;40,862.68)	(88,318.88;40,785.23)	(88,315.30;40,707.78)	(88,311.72;40,630.29)
235	(88,332.91;45,562.67)	(88,329.88;42,781.60)	(88,326.63;41,609.69)	(88,323.19;41,206.01)	(88,319.62;41,046.47)	(88,316.04;40,943.26)	(88,312.46;40,857.70)	(88,308.88;40,780.25)	(88,305.30;40,702.80)	(88,301.72;40,625.31)
240	(88,322.91;45,558.22)	(88,319.88;42,777.15)	(88,316.63;41,604.78)	(88,313.19;41,201.51)	(88,309.62;41,041.41)	(88,306.04;40,938.20)	(88,302.46;40,852.64)	(88,298.88;40,775.19)	(88,295.30;40,697.74)	(88,291.72;40



Table A2: Matrix of results of NPV and optimal capacity for  $S_L$  and  $S_F$  (continued)

$K_{UL}$	$S_L$	155	160	165	170	175	180	185	190	195	200
		5 (146,672.64;40,016.38) (133,114.02;40,163.60) (120,595.97;40,284.28) (108,706.95;40,378.62) (97,519.41;40,399.69) (86,992.15;40,400.49) (76,531.96;40,400.44) (66,332.28;40,399.85) (56,206.55;40,398.90) (46,106.51;40,397.69) (36,024.48;40,396.55) (26,089.03;40,389.88) (16,848.23;40,381.85) (7,588.05;40,377.74) (-88,773.68;40,373.59) (-98,733.92;40,369.41) (-108,653.25;40,360.93) (-118,631.51;40,356.66) (-128,661.73;40,352.37) (-138,757.09;40,343.73) (-148,917.84;40,339.39) (-159,144.40;40,331.29) (-169,437.70;40,323.67) (-179,798.39;40,316.13) (-189,122.25;40,308.57) (-198,419.93;40,301.99) (-207,690.46;40,295.32) (-216,944.40;40,288.63) (-226,182.40;40,281.91) (-235,404.40;40,275.16) (-244,611.40;40,268.38) (-253,803.40;40,261.57) (-262,980.40;40,254.73) (-272,142.40;40,247.86) (-281,290.40;40,240.96) (-290,424.40;40,234.02) (-299,544.40;40,227.06) (-308,650.40;40,220.08) (-317,742.40;40,213.07) (-326,821.40;40,206.03) (-335,888.40;40,198.95) (-344,942.40;40,191.84) (-353,983.40;40,184.69) (-362,111.40;40,177.50) (-370,226.40;40,170.27) (-378,329.40;40,163.00) (-386,420.40;40,155.70) (-394,499.40;40,148.37) (-402,566.40;40,141.00) (-410,621.40;40,133.60) (-418,664.40;40,126.18) (-426,695.40;40,118.72) (-434,714.40;40,111.22) (-442,721.40;40,103.68) (-450,716.40;40,96.10) (-458,700.40;40,88.50) (-466,672.40;40,80.88) (-474,632.40;40,73.15) (-482,579.40;40,65.40) (-490,513.40;40,57.55) (-498,434.40;40,49.60) (-506,342.40;40,41.55) (-514,238.40;40,33.40) (-522,121.40;40,25.15) (-529,992.40;40,16.80) (-537,851.40;40,8.45) (-545,697.40;40,0.00) (-553,530.40;40,0.00) (-561,350.40;40,0.00) (-569,157.40;40,0.00) (-576,952.40;40,0.00) (-584,734.40;40,0.00) (-592,504.40;40,0.00) (-600,262.40;40,0.00) (-608,008.40;40,0.00) (-615,742.40;40,0.00) (-623,464.40;40,0.00) (-631,174.40;40,0.00) (-638,873.40;40,0.00) (-646,560.40;40,0.00) (-654,235.40;40,0.00) (-661,900.40;40,0.00) (-669,554.40;40,0.00) (-677,197.40;40,0.00) (-684,829.40;40,0.00) (-692,449.40;40,0.00) (-700,058.40;40,0.00) (-707,656.40;40,0.00) (-715,243.40;40,0.00) (-722,819.40;40,0.00) (-730,384.40;40,0.00) (-737,938.40;40,0.00) (-745,481.40;40,0.00) (-752,913.40;40,0.00) (-760,334.40;40,0.00) (-767,744.40;40,0.00) (-775,143.40;40,0.00) (-782,531.40;40,0.00) (-789,908.40;40,0.00) (-797,274.40;40,0.00) (-804,629.40;40,0.00) (-811,974.40;40,0.00) (-819,308.40;40,0.00) (-826,631.40;40,0.00) (-833,944.40;40,0.00) (-841,246.40;40,0.00) (-848,537.40;40,0.00) (-855,818.40;40,0.00) (-863,089.40;40,0.00) (-870,350.40;40,0.00) (-877,591.40;40,0.00) (-884,812.40;40,0.00) (-891,923.40;40,0.00) (-899,024.40;40,0.00) (-906,115.40;40,0.00) (-913,196.40;40,0.00) (-920,267.40;40,0.00) (-927,328.40;40,0.00) (-934,379.40;40,0.00) (-941,419.40;40,0.00) (-948,449.40;40,0.00) (-955,469.40;40,0.00) (-962,479.40;40,0.00) (-969,479.40;40,0.00) (-976,469.40;40,0.00) (-983,449.40;40,0.00) (-990,419.40;40,0.00) (-997,379.40;40,0.00) (-1004,329.40;40,0.00) (-1011,269.40;40,0.00) (-1018,199.40;40,0.00) (-1025,119.40;40,0.00) (-1032,3.40;40,0.00) (-1039,103.40;40,0.00) (-1046,223.40;40,0.00) (-1053,333.40;40,0.00) (-1060,433.40;40,0.00) (-1067,523.40;40,0.00) (-1074,603.40;40,0.00) (-1081,673.40;40,0.00) (-1088,733.40;40,0.00) (-1095,783.40;40,0.00) (-1102,823.40;40,0.00) (-1109,853.40;40,0.00) (-1116,873.40;40,0.00) (-1123,893.40;40,0.00) (-1130,903.40;40,0.00) (-1137,913.40;40,0.00) (-1144,923.40;40,0.00) (-1151,933.40;40,0.00) (-1158,943.40;40,0.00) (-1165,953.40;40,0.00) (-1172,963.40;40,0.00) (-1179,973.40;40,0.00) (-1186,983.40;40,0.00) (-1193,993.40;40,0.00) (-1200,1003.40;40,0.00) (-1207,1013.40;40,0.00) (-1214,1023.40;40,0.00) (-1221,1033.40;40,0.00) (-1228,1043.40;40,0.00) (-1235,1053.40;40,0.00) (-1242,1063.40;40,0.00) (-1249,1073.40;40,0.00) (-1256,1083.40;40,0.00) (-1263,1093.40;40,0.00) (-1270,1103.40;40,0.00) (-1277,1113.40;40,0.00) (-1284,1123.40;40,0.00) (-1291,1133.40;40,0.00) (-1298,1143.40;40,0.00) (-1305,1153.40;40,0.00) (-1312,1163.40;40,0.00) (-1319,1173.40;40,0.00) (-1326,1183.40;40,0.00) (-1333,1193.40;40,0.00) (-1340,1203.40;40,0.00) (-1347,1213.40;40,0.00) (-1354,1223.40;40,0.00) (-1361,1233.40;40,0.00) (-1368,1243.40;40,0.00) (-1375,1253.40;40,0.00) (-1382,1263.40;40,0.00) (-1389,1273.40;40,0.00) (-1396,1283.40;40,0.00) (-1403,1293.40;40,0.00) (-1410,1303.40;40,0.00) (-1417,1313.40;40,0.00) (-1424,1323.40;40,0.00) (-1431,1333.40;40,0.00) (-1438,1343.40;40,0.00) (-1445,1353.40;40,0.00) (-1452,1363.40;40,0.00) (-1459,1373.40;40,0.00) (-1466,1383.40;40,0.00) (-1473,1393.40;40,0.00) (-1480,1403.40;40,0.00) (-1487,1413.40;40,0.00) (-1494,1423.40;40,0.00) (-1501,1433.40;40,0.00) (-1508,1443.40;40,0.00) (-1515,1453.40;40,0.00) (-1522,1463.40;40,0.00) (-1529,1473.40;40,0.00) (-1536,1483.40;40,0.00) (-1543,1493.40;40,0.00) (-1550,1503.40;40,0.00) (-1557,1513.40;40,0.00) (-1564,1523.40;40,0.00) (-1571,1533.40;40,0.00) (-1578,1543.40;40,0.00) (-1585,1553.40;40,0.00) (-1592,1563.40;40,0.00) (-1599,1573.40;40,0.00) (-1606,1583.40;40,0.00) (-1613,1593.40;40,0.00) (-1620,1603.40;40,0.00) (-1627,1613.40;40,0.00) (-1634,1623.40;40,0.00) (-1641,1633.40;40,0.00) (-1648,1643.40;40,0.00) (-1655,1653.40;40,0.00) (-1662,1663.40;40,0.00) (-1669,1673.40;40,0.00) (-1676,1683.40;40,0.00) (-1683,1693.40;40,0.00) (-1690,1703.40;40,0.00) (-1697,1713.40;40,0.00) (-1704,1723.40;40,0.00) (-1711,1733.40;40,0.00) (-1718,1743.40;40,0.00) (-1725,1753.40;40,0.00) (-1732,1763.40;40,0.00) (-1739,1773.40;40,0.00) (-1746,1783.40;40,0.00) (-1753,1793.40;40,0.00) (-1760,1803.40;40,0.00) (-1767,1813.40;40,0.00) (-1774,1823.40;40,0.00) (-1781,1833.40;40,0.00) (-1788,1843.40;40,0.00) (-1795,1853.40;40,0.00) (-1802,1863.40;40,0.00) (-1809,1873.40;40,0.00) (-1816,1883.40;40,0.00) (-1823,1893.40;40,0.00) (-1830,1903.40;40,0.00) (-1837,1913.40;40,0.00) (-1844,1923.40;40,0.00) (-1851,1933.40;40,0.00) (-1858,1943.40;40,0.00) (-1865,1953.40;40,0.00) (-1872,1963.40;40,0.00) (-1879,1973.40;40,0.00) (-1886,1983.40;40,0.00) (-1893,1993.40;40,0.00) (-1900,2003.40;40,0.00) (-1907,2013.40;40,0.00) (-1914,2023.40;40,0.00) (-1921,2033.40;40,0.00) (-1928,2043.40;40,0.00) (-1935,2053.40;40,0.00) (-1942,2063.40;40,0.00) (-1949,2073.40;40,0.00) (-1956,2083.40;40,0.00) (-1963,2093.40;40,0.00) (-1970,2103.40;40,0.00) (-1977,2113.40;40,0.00) (-1984,2123.40;40,0.00) (-1991,2133.40;40,0.00) (-1998,2143.40;40,0.00) (-2005,2153.40;40,0.00) (-2012,2163.40;40,0.00) (-2019,2173.40;40,0.00) (-2026,2183.40;40,0.00) (-2033,2193.40;40,0.00) (-2040,2203.40;40,0.00) (-2047,2213.40;40,0.00) (-2054,2223.40;40,0.00) (-2061,2233.40;40,0.00) (-2068,2243.40;40,0.00) (-2075,2253.40;40,0.00) (-2082,2263.40;40,0.00) (-2089,2273.40;40,0.00) (-2096,2283.40;40,0.00) (-2103,2293.40;40,0.00) (-2110,2303.40;40,0.00) (-2117,2313.40;40,0.00) (-2124,2323.40;40,0.00) (-2131,2333.40;40,0.00) (-2138,2343.40;40,0.00) (-2145,2353.40;40,0.00) (-2152,2363.40;40,0.00) (-2159,2373.40;40,0.00) (-2166,2383.40;40,0.00) (-2173,2393.40;40,0.00) (-2180,2403.40;40,0.00) (-2187,2413.40;40,0.00) (-2194,2423.40;40,0.00) (-2201,2433.40;40,0.00) (-2208,2443.40;40,0.00) (-2215,2453.40;40,0.00) (-2222,2463.40;40,0.00) (-2229,2473.40;40,0.00) (-2236,2483.40;40,0.00) (-2243,2493.40;40,0.00) (-2250,2503.40;40,0.00) (-2257,2513.40;40,0.00) (-2264,2523.40;40,0.00) (-2271,2533.40;40,0.00) (-2278,2543.40;40,0.00) (-2285,2553.40;40,0.00) (-2292,2563.40;40,0.00) (-2299,2573.40;40,0.00) (-2306,2583.40;40,0.00) (-2313,2593.40;40,0.00) (-2320,2603.40;40,0.00) (-2327,2613.40;40,0.00) (-2334,2623.40;40,0.00) (-2341,2633.40;40,0.00) (-2348,2643.40;40,0.00) (-2355,2653.40;40,0.00) (-2362,2663.40;40,0.00) (-2369,2673.40;40,0.00) (-2376,2683.40;40,0.00) (-2383,2693.40;40,0.00) (-2390,2703.40;40,0.00) (-2397,2713.40;40,0.00) (-2404,2723.40;40,0.00) (-2411,2733.40;40,0.00) (-2418,2743.40;40,0.00) (-2425,2753.40;40,0.00) (-2432,2763.40;40,0.00) (-2439,2773.40;40,0.00) (-2446,2783.40;40,0.00) (-2453,2793.40;40,0.00) (-2460,2803.40;40,0.00) (-2467,2813.40;40,0.00) (-2474,2823.40;40,0.00) (-2481,2833.40;40,0.00) (-2488,2843.40;40,0.00) (-2495,2853.40;40,0.00) (-2502,2863.40;40,0.00) (-2509,2873.40;40,0.00) (-2516,2883.40;40,0.00) (-2523,2893.40;40,0.00) (-2530,2903.40;40,0.00) (-2537,2913.40;40,0.00) (-2544,2923.40;40,0.00) (-2551,2933.40;40,0.00) (-2558,2943.40;40,0.00) (-2565,2953.40;40,0.00) (-2572,2963.40;40,0.00) (-2579,2973.40;40,0.00) (-2586,2983.40;40,0.00) (-2593,2993.40;40,0.00) (-2600,3003.40;40,0.00) (-2607,3013.40;40,0.00) (-2614,3023.40;40,0.00) (-2621,3033.40;40,0.00) (-2628,3043.40;40,0.00) (-2635,3053.40;40,0.00) (-2642,3063.40;40,0.00) (-2649,3073.40;40,0.00) (-2656,3083.40;40,0.00) (-2663,3093.40;40,0.00) (-2670,3103.40;40,0.00) (-2677,3113.40;40,0.00) (-2684,3123.40;40,0.00) (-2691,3133.40;40,0.00) (-2698,3143.40;40,0.00) (-2705,3153.40;40,0.00) (-2712,3163.40;40,0.00) (-2719,3173.40;40,0.00) (-2726,3183.40;40,0.00) (-2733,3193.40;40,0.00) (-2740,3203.40;40,0.00) (-2747,3213.40;40,0.00) (-2754,3223.40;40,0.00) (-2761,3233.40;40,0.00) (-2768,3243.40;40,0.00) (-2775,3253.40;40,0.00) (-2782,3263.40;40,0.00) (-2789,3273.40;40,0.00) (-2796,3283.40;40,0.00) (-2803,3293.40;40,0.00) (-2810,3303.40;40,0.00) (-2817,3313.40;40,0.00) (-2824,3323.40;40,0.00) (-2831,3333.40;40,0.00) (-2838,3343.40;40,0.00) (-2845,3353.40;40,0.00) (-2852,3363.40;40,0.00) (-2859,3373.40;40,0.00) (-2866,3383.40;40,0.00) (-2873,3393.40;40,0.00) (-2880,3403.40;40,0.00) (-2887,3413.40;40,0.00) (-2894,3423.40;40,0.00) (-2901,3433.40;40,0.00) (-2908,3443.40;40,0.00) (-2915,3453.40;40,0.00) (-2922,3463.40;40,0.00) (-2929,3473.40;40,0.00) (-2936,3483.40;40,0.00) (-2943,3493.40;40,0.00) (-2950,3503.40;40,0.00) (-2957,3513.40;40,0.00) (-2964,3523.40;40,0.00) (-2971,3533.40;40,0.00) (-2978,3543.40;40,0.00) (-2985,3553.40;40,0.00) (-2992,3563.40;40,0.00) (-2999,3573.40;40,0.00) (-3006,3583.40;40,0.00) (-3013,3593.40;40,0.00) (-3020,3603.40;40,0.00) (-3027,3613.40;40,0.00) (-3034,3623.40;40,0.00) (-3041,3633.40;40,0.00) (-3048,3643.40;40,0.00) (-3055,3653.40;40,0.00) (-3062,3663.40;40,0.00) (-3069,3673.40;40,0.00) (-3076,3683.40;40,0.00) (-3083,3693.40;40,0.00) (-3090,3703.40;40,0.00) (-3097,3713.40;40,0.00) (-3104,3723.40;40,0.00) (-3111,3733.40;40,0.00) (-3118,3743.40;40,0.00) (-3125,3753.40;40,0.00) (-3132,3763.40;40,0.00) (-3139,3773.40;40,0.00) (-3146,3783.40;40,0.00) (-3153,3793.40;40,0.00) (-3160,3803.40;40,0.00) (-3167,3813.40;40,0.00) (-3174,3823.40;40,0.00) (-3181,3833.40;40,0.00) (-3188,3843.40;40,0.00) (-3195,3853.40;40,0.00) (-3202,3863.40;40,0.00) (-3209,3873.40;40,0.00) (-3216,3883.40;40,0.00) (-3223,3893.40;40,0.00) (-3230,3903.40;40,0.00) (-3237,3913.40;40,0.00) (-3244,3923.40;40,0.00) (-3251,3933.40;40,0.00) (-3258,3943.40;40,0.00) (-3265,3953.40;40,0.00) (-3272,3963.40;40,0.00) (-3279,3973.40;40,0.00) (-3286,3983.40;40,0.00) (-3293,3993.40;40,0.00) (-3300,4003.40;40,0.00) (-3307,4013.40;40,0.00) (-3314,4023.40;40,0.00) (-3321,4033.40;40,0.00) (-3328,4043.40;40,0.00) (-3335,4053.40;40,0.00) (-3342,4063.40;40,0.00) (-3349,4073.40;40,0.00) (-3356,4083.40;40,0.00) (-3363,4093.40;40,0.00) (-3370,4103.40;40,0.00) (-3377,4113.40;40,0.00) (-3384,4123.40;40,0.00) (-3391,4133.40;40,0.00) (-3398,4143.40;40,0.00) (-3405,4153.40;40,0.00) (-3412,4163.40;40,0.00) (-3419,4173.40;40,0.00) (-3426,4183.40;40,0.00) (-3433,4193.40;40,0.00) (-3440,4203.40;40,0.00) (-3447,4213.40;40,0.00) (-3454,4223.40;40,0.00) (-3461,4233.40;40,0.00) (-3468,4243.40;40,0.00) (-3475,4253.40;40,0.00) (-3482,4263.40;40,0.00) (-3489,4273.40;40,0.00) (-3496,4283.40;40,0.00) (-3503,4293.40;40,0.00) (-3510,4303.40;40,0.00) (-3517,4313.40;40,0.00) (-3524,4323.40;40,0.00) (-3531,4333.40;40,0.00) (-3538,4343.40;40,0.00) (-3545,4353.40;40,0.00) (-3552,4363.40;40,0.00) (-3559,									

Table A2: Matrix of results of NPV and optimal capacity for  $S_L$  and  $S_F$  (continued)

$K_{it}$	$S_L$	205	210	215	220	225	230	235	240	245	250
		5 (146,630.05;39,833.26)	(146,625.73;39,819.73)	(146,621.40;39,806.82)	(146,617.06;39,794.49)	(146,612.72;39,782.72)	(146,608.37;39,771.45)	(146,604.01;39,760.66)	(146,599.65;39,750.32)	(146,595.28;39,740.39)	(146,590.90;39,730.87)
10	10 (130,075.13;40,003.42)	(130,070.77;39,989.85)	(130,066.40;39,976.90)	(130,062.02;39,964.53)	(130,057.64;39,952.52)	(130,053.25;39,940.42)	(130,048.86;39,928.59)	(130,044.46;39,916.21)	(130,040.06;39,903.87)	(130,035.66;39,891.51)	(130,031.25;39,879.14)
15	15 (120,551.78;40,109.98)	(120,547.37;40,096.99)	(120,542.95;40,084.59)	(120,538.53;40,072.19)	(120,534.11;40,059.79)	(120,529.69;40,047.39)	(120,525.26;40,034.99)	(120,520.84;40,022.59)	(120,516.42;40,010.19)	(120,511.99;39,997.79)	(120,507.57;39,985.39)
20	20 (108,662.65;40,193.78)	(108,658.20;40,180.12)	(108,653.75;40,167.09)	(108,649.29;40,154.65)	(108,644.84;40,142.76)	(108,640.38;40,130.38)	(108,635.92;40,117.98)	(108,631.46;40,105.58)	(108,627.00;40,093.18)	(108,622.54;40,080.78)	(108,618.08;40,068.38)
25	25 (97,476.01;40,214.35)	(97,471.63;40,200.65)	(97,467.24;40,187.58)	(97,462.85;40,174.61)	(97,458.46;40,161.91)	(97,454.07;40,149.12)	(97,449.68;40,136.72)	(97,445.29;40,124.27)	(97,440.89;40,111.82)	(97,436.50;40,099.36)	(97,432.11;40,086.90)
30	30 (90,948.73;40,215.06)	(90,944.35;40,201.42)	(90,939.96;40,188.36)	(90,935.58;40,175.88)	(90,931.20;40,163.96)	(90,926.82;40,151.78)	(90,922.44;40,140.00)	(90,918.06;40,128.22)	(90,913.68;40,116.44)	(90,909.30;40,104.66)	(90,904.92;40,092.88)
35	35 (89,580.53;40,215.06)	(89,576.15;40,201.35)	(89,571.77;40,188.28)	(89,567.39;40,175.81)	(89,562.99;40,163.88)	(89,558.60;40,151.95)	(89,554.21;40,139.67)	(89,549.82;40,127.40)	(89,545.43;40,115.13)	(89,541.04;40,102.86)	(89,536.65;40,090.59)
40	40 (89,288.83;40,214.43)	(89,284.45;40,200.75)	(89,280.06;40,187.68)	(89,275.67;40,174.20)	(89,271.28;40,161.73)	(89,266.89;40,149.26)	(89,262.50;40,136.79)	(89,258.11;40,124.32)	(89,253.72;40,111.85)	(89,249.33;40,100.56)	(89,244.94;40,089.28)
45	45 (89,163.10;40,213.49)	(89,158.71;40,199.78)	(89,154.32;40,186.81)	(89,149.93;40,173.84)	(89,145.54;40,160.87)	(89,141.15;40,147.90)	(89,136.76;40,134.93)	(89,132.37;40,121.96)	(89,127.98;40,109.49)	(89,123.59;40,097.02)	(89,119.20;40,084.55)
50	50 (89,063.05;40,212.27)	(89,058.67;40,198.56)	(89,054.28;40,185.49)	(89,049.89;40,173.02)	(89,045.50;40,159.95)	(89,041.11;40,146.88)	(89,036.72;40,133.81)	(89,032.33;40,120.74)	(89,027.94;40,107.67)	(89,023.55;40,094.60)	(89,019.16;40,081.53)
55	55 (88,981.02;40,211.12)	(88,976.63;40,197.41)	(88,972.24;40,184.34)	(88,967.84;40,171.86)	(88,963.45;40,158.79)	(88,959.06;40,145.72)	(88,954.67;40,132.65)	(88,950.28;40,119.58)	(88,945.89;40,106.51)	(88,941.50;40,093.44)	(88,937.11;40,080.37)
60	60 (88,912.35;40,208.47)	(88,907.96;40,194.76)	(88,903.57;40,181.68)	(88,899.18;40,168.61)	(88,894.79;40,155.54)	(88,890.40;40,142.47)	(88,886.01;40,129.40)	(88,881.62;40,116.33)	(88,877.23;40,103.26)	(88,872.84;40,090.19)	(88,868.45;40,079.12)
65	65 (88,854.55;40,204.32)	(88,850.16;40,190.61)	(88,845.77;40,177.53)	(88,841.38;40,164.46)	(88,836.99;40,151.39)	(88,832.60;40,138.32)	(88,828.21;40,125.25)	(88,823.82;40,112.18)	(88,819.43;40,099.11)	(88,815.04;40,086.04)	(88,810.65;40,072.97)
70	70 (88,804.75;40,200.34)	(88,800.36;40,186.62)	(88,795.97;40,173.54)	(88,791.58;40,160.47)	(88,787.19;40,147.40)	(88,782.80;40,134.33)	(88,778.41;40,121.26)	(88,774.02;40,108.19)	(88,769.63;40,095.12)	(88,765.24;40,082.05)	(88,760.85;40,068.98)
75	75 (88,761.53;40,196.29)	(88,757.14;40,182.57)	(88,752.75;40,169.49)	(88,748.36;40,156.42)	(88,743.97;40,143.35)	(88,739.58;40,130.28)	(88,735.19;40,117.21)	(88,730.80;40,104.14)	(88,726.41;40,091.07)	(88,722.02;40,078.00)	(88,717.63;40,064.93)
80	80 (88,723.66;40,192.18)	(88,719.27;40,178.46)	(88,714.87;40,165.38)	(88,710.48;40,152.31)	(88,706.09;40,139.24)	(88,701.70;40,126.17)	(88,697.31;40,113.10)	(88,692.92;40,099.03)	(88,688.53;40,085.96)	(88,684.14;40,072.89)	(88,679.75;40,060.82)
85	85 (88,690.20;40,183.83)	(88,685.81;40,174.31)	(88,681.42;40,161.23)	(88,677.02;40,148.16)	(88,672.63;40,135.09)	(88,668.24;40,122.02)	(88,663.85;40,108.95)	(88,659.46;40,095.88)	(88,655.07;40,082.81)	(88,650.68;40,069.74)	(88,646.29;40,056.67)
90	90 (88,660.43;40,183.83)	(88,656.04;40,170.12)	(88,651.65;40,157.03)	(88,647.25;40,144.54)	(88,642.86;40,131.47)	(88,638.47;40,120.40)	(88,634.08;40,107.33)	(88,629.69;40,094.26)	(88,625.30;40,081.19)	(88,620.91;40,068.12)	(88,616.52;40,055.05)
95	95 (88,633.77;40,179.61)	(88,629.38;40,165.89)	(88,624.99;40,152.81)	(88,620.59;40,140.32)	(88,616.20;40,127.25)	(88,611.81;40,114.18)	(88,607.42;40,101.11)	(88,603.03;40,088.04)	(88,598.64;40,074.97)	(88,594.25;40,061.90)	(88,589.86;40,048.83)
100	100 (88,609.76;40,175.36)	(88,605.37;40,161.64)	(88,600.98;40,148.56)	(88,596.58;40,136.07)	(88,592.19;40,123.00)	(88,587.79;40,110.93)	(88,583.40;40,097.86)	(88,579.01;40,084.79)	(88,574.62;40,071.72)	(88,570.23;40,058.65)	(88,565.84;40,045.58)
105	105 (88,588.02;40,171.08)	(88,583.63;40,157.36)	(88,579.24;40,144.28)	(88,574.84;40,131.20)	(88,570.45;40,118.13)	(88,566.06;40,105.06)	(88,561.67;40,091.99)	(88,557.28;40,078.92)	(88,552.89;40,065.85)	(88,548.50;40,052.78)	(88,544.11;40,040.71)
110	110 (88,568.24;40,166.79)	(88,563.85;40,153.07)	(88,559.46;40,139.99)	(88,555.06;40,126.92)	(88,550.67;40,113.85)	(88,546.28;40,100.78)	(88,541.89;40,087.71)	(88,537.50;40,074.64)	(88,533.11;40,061.57)	(88,528.72;40,048.50)	(88,524.33;40,035.43)
115	115 (88,548.47;40,162.48)	(88,544.08;40,149.40)	(88,539.69;40,136.32)	(88,535.29;40,123.25)	(88,530.90;40,110.18)	(88,526.51;40,097.11)	(88,522.12;40,084.04)	(88,517.73;40,070.97)	(88,513.34;40,057.90)	(88,508.95;40,044.83)	(88,504.56;40,031.76)
120	120 (88,528.69;40,158.15)	(88,524.30;40,145.07)	(88,519.91;40,132.00)	(88,515.52;40,118.93)	(88,511.13;40,105.86)	(88,506.74;40,092.79)	(88,502.35;40,079.72)	(88,497.96;40,066.65)	(88,493.57;40,053.58)	(88,489.18;40,040.51)	(88,484.79;40,027.44)
125	125 (88,508.91;40,153.80)	(88,504.52;40,140.72)	(88,500.13;40,127.65)	(88,495.74;40,114.58)	(88,491.35;40,101.51)	(88,486.96;40,088.44)	(88,482.57;40,075.37)	(88,478.18;40,062.30)	(88,473.79;40,049.23)	(88,469.40;40,036.16)	(88,465.01;40,023.09)
130	130 (88,490.25;40,149.45)	(88,485.86;40,136.37)	(88,481.47;40,123.30)	(88,477.08;40,110.23)	(88,472.69;40,097.16)	(88,468.30;40,084.09)	(88,463.91;40,071.02)	(88,459.52;40,057.95)	(88,455.13;40,044.88)	(88,450.74;40,031.81)	(88,446.35;40,018.74)
135	135 (88,471.20;40,145.09)	(88,466.81;40,131.37)	(88,462.42;40,118.30)	(88,458.03;40,105.23)	(88,453.64;40,089.16)	(88,449.25;40,076.09)	(88,444.86;40,063.02)	(88,440.47;40,049.95)	(88,436.08;40,036.88)	(88,431.69;40,023.81)	(88,427.30;40,010.74)
140	140 (88,479.08;40,140.71)	(88,474.69;40,126.99)	(88,470.30;40,113.91)	(88,465.91;40,100.84)	(88,461.52;40,087.77)	(88,457.13;40,074.70)	(88,452.74;40,061.63)	(88,448.35;40,048.56)	(88,443.96;40,035.49)	(88,439.57;40,022.42)	(88,435.18;40,009.35)
145	145 (88,467.79;40,136.33)	(88,463.40;40,122.61)	(88,459.01;40,109.53)	(88,454.62;40,096.46)	(88,450.23;40,083.39)	(88,445.84;40,070.32)	(88,441.45;40,057.25)	(88,437.06;40,044.18)	(88,432.67;40,031.11)	(88,428.28;40,018.04)	(88,423.89;40,004.97)
150	150 (88,457.38;40,131.94)	(88,452.99;40,118.22)	(88,448.60;40,105.13)	(88,444.21;40,092.06)	(88,439.82;40,078.99)	(88,435.43;40,065.92)	(88,431.04;40,052.85)	(88,426.65;40,040.78)	(88,422.26;40,027.71)	(88,417.87;40,014.64)	(88,413.48;40,001.57)
155	155 (88,447.38;40,127.54)	(88,442.99;40,113.82)	(88,438.60;40,100.74)	(88,434.21;40,087.67)	(88,429.82;40,074.60)	(88,425.43;40,061.53)	(88,421.04;40,048.46)	(88,416.65;40,035.39)	(88,412.26;40,022.32)	(88,407.87;40,009.25)	(88,403.48;40,000.18)
160	160 (88,438.14;40,123.14)	(88,433.75;40,109.42)	(88,429.36;40,096.33)	(88,424.97;40,083.26)	(88,420.58;40,070.19)	(88,416.19;40,057.12)	(88,411.80;40,044.05)	(88,407.41;40,030.98)	(88,403.02;40,017.91)	(88,398.63;40,004.84)	(88,394.24;40,000.77)
165	165 (88,429.45;40,118.73)	(88,425.06;40,105.01)	(88,420.67;40,091.92)	(88,416.28;40,078.85)	(88,411.89;40,065.78)	(88,407.50;40,052.71)	(88,403.11;40,039.64)	(88,398.72;40,026.57)	(88,394.33;40,013.50)	(88,389.94;40,000.43)	(88,385.55;40,000.36)
170	170 (88,421.26;40,114.31)	(88,416.87;40,100.59)	(88,412.48;40,087.51)	(88,408.09;40,074.44)	(88,403.70;40,061.37)	(88,399.31;40,048.30)	(88,394.92;40,035.23)	(88,390.53;40,022.16)	(88,386.14;40,009.09)	(88,381.75;40,000.02)	(88,377.36;40,000.95)
175	175 (88,413.55;40,109.89)	(88,409.16;40,096.17)	(88,404.77;40,083.09)	(88,400.38;40,069.17)	(88,395.99;40,056.10)	(88,391.60;40,043.03)	(88,387.21;40,030.96)	(88,382.82;40,017.89)	(88,378.43;40,004.82)	(88,374.04;40,000.75)	(88,369.65;40,000.68)
180	180 (88,406.26;40,105.47)	(88,401.87;40,091.75)	(88,397.47;40,078.66)	(88,393.08;40,064.74)	(88,388.69;40,051.67)	(88,384.30;40,040.60)	(88,379.91;40,029.53)	(88,375.52;40,018.46)	(88,371.13;40,007.39)	(88,366.74;40,000.32)	(88,362.35;40,000.25)
185	185 (88,399.36;40,101.04)	(88,394.97;40,087.32)	(88,390.58;40,074.23)	(88,386.19;40,061.16)	(88,381.79;40,048.09)	(88,377.40;40,035.02)	(88,373.01;40,021.95)	(88,368.62;40,010.88)	(88,364.23;40,000.81)	(88,359.84;40,000.74)	(88,355.45;40,000.67)
190	190 (88,392.83;40,096.61)	(88,388.44;40,082.89)	(88,384.04;40,069.80)	(88,379.64;40,056.73)	(88,375.24;40,043.66)	(88,370.85;40,030.59)	(88,366.46;40,017.52)	(88,362.07;40,004.45)	(88,357.68;40,000.38)	(88,353.29;40,000.31)	(88,348.90;40,000.24)
195	195 (88,386.63;40,092.17)	(88,382.24;40,078.45)	(88,377.84;40,065.37)	(88,373.44;40,052.30)	(88,369.04;40,039.23)	(88,364.65;40,026.16)	(88,360.26;40,013.09)	(88,355.87;40,000.02)	(88,351.48;40,000.95)	(88,347.09;40,000.88)	(88,342.70;40,000.81)
200	200 (88,380.73;40,087.73)	(88,376.34;40,074.01)	(88,371.95;40,060.93)	(88,367.55;40,047.86)	(88,363.15;40,034.79)	(88,358.76;40,021.72)	(88,354.37;40,008.65)	(88,349.98;40,000.58)	(88,345.59;40,000.51)	(88,341.20;40,000.44)	(88,336.81;40,000.37)
205	205 (88,375.13;40,083.29)	(88,370.74;40,069.57)	(88,366.34;40,056.48)	(88,361.94;40,043.41)	(88,357.54;40,030.34)	(88,353.15;40,017.27)	(88,348.76;40,004.20)	(88,344.37;40,000.13)	(88,339.98;40,000.06)	(88,335.59;40,000.99)	(88,331.20;40,000.92)
210	210 (88,369.79;40,078.84)	(88,365.40;40,065.12)	(88,361.00;40,052.04)	(88,356.60;40,039.55)	(88,352.20;40,026.48)	(88,347.81;40,013.41)	(88,343.42;40,000.34)	(88,339.03;40,000.27)	(88,334.64;40,000.20)	(88,330.25;40,000.13)	(88,325.86;40,000.06)
215	215 (88,364.10;40,074.40)	(88,359.71;40,060.68)	(88,355.31;40,047.59)	(88,350.92;40,034.52)	(88,346.53;40,021.45)	(88,342.14;40,008.38)	(88,337.75;40,000.31)	(88,333.36;40,000.24)	(88,328.97;40,000.17)	(88,324.58;40,000.10)	(88,320.19;40,000.03)
220	220 (88,359.83;40,069.95)	(88,3									

Table A2: Matrix of results of NPV and optimal capacity for  $S_L$  and  $S_F$  (continued)

$K_{it}$	$S_L$	$S_F$									
		255	260	265	270	275	280	285	290	295	300
5	(146,586.52;39,721.71)	(146,582.14;39,712.91)	(146,577.75;39,704.44)	(146,573.36;39,696.28)	(146,568.96;39,688.42)	(146,564.56;39,680.83)	(146,560.15;39,673.51)	(146,555.75;39,666.45)	(146,551.34;39,659.62)	(146,546.92;39,653.02)	
10	(133,031.25;39,891.53)	(133,026.81;39,882.69)	(133,022.42;39,874.20)	(133,018.01;39,866.01)	(133,013.58;39,858.12)	(133,009.16;39,850.52)	(133,004.73;39,843.17)	(132,999.87;39,836.09)	(132,995.47;39,829.24)	(132,991.44;39,822.61)	
15	(120,476.38;39,993.97)	(120,471.93;39,985.09)	(120,467.48;39,976.21)	(120,463.03;39,967.33)	(120,458.58;39,958.45)	(120,454.13;39,949.57)	(120,449.68;39,940.69)	(120,445.23;39,931.81)	(120,440.78;39,922.93)	(120,436.33;39,914.05)	
20	(108,618.07;40,081.18)	(108,613.62;40,072.29)	(108,609.17;40,063.41)	(108,604.72;40,054.52)	(108,600.27;40,045.64)	(108,595.82;40,036.75)	(108,591.37;40,027.87)	(108,586.92;40,018.98)	(108,582.47;40,010.10)	(108,578.02;40,001.22)	
25	(97,431.98;40,101.44)	(97,427.53;40,092.55)	(97,423.08;40,083.66)	(97,418.63;40,074.78)	(97,414.18;40,065.89)	(97,409.73;40,057.01)	(97,405.28;40,048.13)	(97,400.83;40,039.25)	(97,396.38;40,030.37)	(97,391.93;40,021.48)	
30	(90,904.69;40,102.20)	(90,900.24;40,093.29)	(90,895.79;40,084.38)	(90,891.34;40,075.47)	(90,886.89;40,066.56)	(90,882.44;40,057.64)	(90,877.99;40,048.73)	(90,873.54;40,039.81)	(90,869.09;40,030.90)	(90,864.64;40,021.98)	
35	(89,464.48;40,102.12)	(89,460.03;40,093.21)	(89,455.58;40,084.33)	(89,451.13;40,075.42)	(89,446.68;40,066.51)	(89,442.23;40,057.60)	(89,437.78;40,048.69)	(89,433.33;40,039.78)	(89,428.88;40,030.87)	(89,424.43;40,021.95)	
40	(89,244.78;40,101.51)	(89,240.33;40,092.59)	(89,235.88;40,084.02)	(89,231.43;40,075.15)	(89,226.98;40,066.28)	(89,222.53;40,057.37)	(89,218.08;40,048.46)	(89,213.63;40,039.55)	(89,209.18;40,030.64)	(89,204.73;40,021.72)	
45	(89,114.61;40,091.62)	(89,110.16;40,082.74)	(89,105.71;40,073.86)	(89,101.26;40,064.98)	(89,096.81;40,056.10)	(89,092.36;40,047.22)	(89,087.91;40,038.33)	(89,083.46;40,029.45)	(89,079.01;40,020.57)	(89,074.56;40,011.69)	
50	(89,018.99;40,099.33)	(89,014.54;40,090.41)	(89,010.09;40,081.49)	(89,005.64;40,072.57)	(89,001.19;40,063.69)	(89,000.27;40,054.81)	(89,000.35;40,045.93)	(89,000.43;40,037.05)	(89,000.51;40,028.17)	(89,000.59;40,019.29)	
55	(88,936.95;40,098.16)	(88,932.50;40,089.24)	(88,928.05;40,080.36)	(88,923.60;40,071.48)	(88,919.15;40,062.60)	(88,914.70;40,053.72)	(88,910.25;40,044.84)	(88,905.80;40,035.96)	(88,901.35;40,027.08)	(88,896.90;40,018.20)	
60	(88,868.48;40,095.44)	(88,864.03;40,086.52)	(88,859.58;40,077.64)	(88,855.13;40,068.76)	(88,850.68;40,059.88)	(88,846.23;40,051.00)	(88,841.78;40,042.12)	(88,837.33;40,033.24)	(88,832.88;40,024.36)	(88,828.43;40,015.48)	
65	(88,810.48;40,091.28)	(88,806.03;40,082.36)	(88,801.58;40,073.48)	(88,797.13;40,064.60)	(88,792.68;40,055.72)	(88,788.23;40,046.84)	(88,783.78;40,037.96)	(88,779.33;40,029.08)	(88,774.88;40,020.20)	(88,770.43;40,011.32)	
70	(88,760.68;40,087.30)	(88,756.23;40,078.38)	(88,751.78;40,069.50)	(88,747.33;40,060.62)	(88,742.88;40,051.74)	(88,738.43;40,042.86)	(88,733.98;40,033.98)	(88,729.53;40,025.10)	(88,725.08;40,016.22)	(88,720.63;40,007.34)	
75	(88,717.45;40,083.24)	(88,713.00;40,074.32)	(88,708.55;40,065.44)	(88,704.10;40,056.56)	(88,699.65;40,047.68)	(88,695.20;40,038.80)	(88,690.75;40,029.92)	(88,686.30;40,021.04)	(88,681.85;40,012.16)	(88,677.40;40,003.28)	
80	(88,679.58;40,079.13)	(88,675.13;40,070.21)	(88,670.68;40,061.33)	(88,666.23;40,052.45)	(88,661.78;40,043.57)	(88,657.33;40,034.69)	(88,652.88;40,025.81)	(88,648.43;40,016.93)	(88,643.98;40,008.05)	(88,639.53;40,000.17)	
85	(88,646.12;40,074.98)	(88,641.67;40,066.06)	(88,637.22;40,057.18)	(88,632.77;40,048.30)	(88,628.32;40,039.42)	(88,623.87;40,030.54)	(88,619.42;40,021.66)	(88,614.97;40,012.78)	(88,610.52;40,003.90)	(88,606.07;40,000.02)	
90	(88,616.35;40,070.78)	(88,611.90;40,061.86)	(88,607.45;40,052.97)	(88,603.00;40,044.09)	(88,598.55;40,035.21)	(88,594.10;40,026.33)	(88,589.65;40,017.45)	(88,585.20;40,008.57)	(88,580.75;40,000.00)	(88,576.30;40,000.00)	
95	(88,589.69;40,066.56)	(88,585.24;40,057.64)	(88,580.79;40,048.76)	(88,576.34;40,039.88)	(88,571.89;40,031.00)	(88,567.44;40,022.12)	(88,562.99;40,013.24)	(88,558.54;40,004.36)	(88,554.09;40,000.00)	(88,549.64;40,000.00)	
100	(88,568.68;40,068.03)	(88,564.23;40,059.15)	(88,559.78;40,050.27)	(88,555.33;40,041.39)	(88,550.88;40,032.51)	(88,546.43;40,023.63)	(88,541.98;40,014.75)	(88,537.53;40,005.87)	(88,533.08;40,000.00)	(88,528.63;40,000.00)	
105	(88,543.93;40,058.03)	(88,539.48;40,049.11)	(88,535.03;40,040.23)	(88,530.58;40,031.35)	(88,526.13;40,022.47)	(88,521.68;40,013.59)	(88,517.23;40,004.71)	(88,512.78;40,000.00)	(88,508.33;40,000.00)	(88,503.88;40,000.00)	
110	(88,524.15;40,053.73)	(88,519.70;40,044.81)	(88,515.25;40,035.93)	(88,510.80;40,027.05)	(88,506.35;40,018.17)	(88,501.90;40,009.29)	(88,497.45;40,000.41)	(88,493.00;40,000.00)	(88,488.55;40,000.00)	(88,484.10;40,000.00)	
115	(88,506.08;40,049.42)	(88,501.63;40,040.54)	(88,497.18;40,031.66)	(88,492.73;40,022.78)	(88,488.28;40,013.90)	(88,483.83;40,005.02)	(88,479.38;40,000.00)	(88,474.93;40,000.00)	(88,470.48;40,000.00)	(88,466.03;40,000.00)	
120	(88,489.51;40,045.09)	(88,485.06;40,036.21)	(88,480.61;40,027.33)	(88,476.16;40,018.45)	(88,471.71;40,009.57)	(88,467.26;40,000.69)	(88,462.81;40,000.00)	(88,458.36;40,000.00)	(88,453.91;40,000.00)	(88,449.46;40,000.00)	
125	(88,474.25;40,040.75)	(88,469.80;40,031.87)	(88,465.35;40,022.99)	(88,460.90;40,014.11)	(88,456.45;40,005.23)	(88,452.00;40,000.00)	(88,447.55;40,000.00)	(88,443.10;40,000.00)	(88,438.65;40,000.00)	(88,434.20;40,000.00)	
130	(88,460.16;40,036.39)	(88,455.71;40,027.51)	(88,451.26;40,018.63)	(88,446.81;40,009.75)	(88,442.36;40,000.87)	(88,437.91;40,000.00)	(88,433.46;40,000.00)	(88,429.01;40,000.00)	(88,424.56;40,000.00)	(88,420.11;40,000.00)	
135	(88,447.11;40,032.03)	(88,442.66;40,023.15)	(88,438.21;40,014.27)	(88,433.76;40,005.39)	(88,429.31;40,000.51)	(88,424.86;40,000.00)	(88,420.41;40,000.00)	(88,415.96;40,000.00)	(88,411.51;40,000.00)	(88,407.06;40,000.00)	
140	(88,434.99;40,027.65)	(88,430.54;40,018.77)	(88,426.09;40,009.89)	(88,421.64;40,001.01)	(88,417.19;40,000.13)	(88,412.74;40,000.00)	(88,408.29;40,000.00)	(88,403.84;40,000.00)	(88,399.39;40,000.00)	(88,394.94;40,000.00)	
145	(88,423.70;40,023.27)	(88,419.25;40,014.39)	(88,414.80;40,005.51)	(88,410.35;40,000.63)	(88,405.90;40,000.00)	(88,401.45;40,000.00)	(88,397.00;40,000.00)	(88,392.55;40,000.00)	(88,388.10;40,000.00)	(88,383.65;40,000.00)	
150	(88,413.16;40,018.88)	(88,408.71;40,009.95)	(88,404.26;40,001.07)	(88,399.81;40,000.19)	(88,395.36;40,000.00)	(88,390.91;40,000.00)	(88,386.46;40,000.00)	(88,382.01;40,000.00)	(88,377.56;40,000.00)	(88,373.11;40,000.00)	
155	(88,403.29;40,014.48)	(88,398.84;40,005.55)	(88,394.39;40,000.67)	(88,389.94;40,000.00)	(88,385.49;40,000.00)	(88,381.04;40,000.00)	(88,376.59;40,000.00)	(88,372.14;40,000.00)	(88,367.69;40,000.00)	(88,363.24;40,000.00)	
160	(88,394.05;40,010.07)	(88,389.60;40,001.15)	(88,385.15;40,000.00)	(88,380.70;40,000.00)	(88,376.25;40,000.00)	(88,371.80;40,000.00)	(88,367.35;40,000.00)	(88,362.90;40,000.00)	(88,358.45;40,000.00)	(88,354.00;40,000.00)	
165	(88,385.35;40,005.66)	(88,380.90;40,000.74)	(88,376.45;40,000.00)	(88,372.00;40,000.00)	(88,367.55;40,000.00)	(88,363.10;40,000.00)	(88,358.65;40,000.00)	(88,354.20;40,000.00)	(88,349.75;40,000.00)	(88,345.30;40,000.00)	
170	(88,377.17;40,001.25)	(88,372.72;40,000.33)	(88,368.27;40,000.00)	(88,363.82;40,000.00)	(88,359.37;40,000.00)	(88,354.92;40,000.00)	(88,350.47;40,000.00)	(88,346.02;40,000.00)	(88,341.57;40,000.00)	(88,337.12;40,000.00)	
175	(88,369.46;39,996.83)	(88,365.01;39,987.90)	(88,360.56;39,979.02)	(88,356.11;39,970.14)	(88,351.66;39,961.26)	(88,347.21;39,952.38)	(88,342.76;39,943.50)	(88,338.31;39,934.62)	(88,333.86;39,925.74)	(88,329.41;39,916.86)	
180	(88,362.17;39,992.40)	(88,357.72;39,983.48)	(88,353.27;39,974.59)	(88,348.82;39,965.71)	(88,344.37;39,956.83)	(88,339.92;39,947.95)	(88,335.47;39,939.07)	(88,331.02;39,930.19)	(88,326.57;39,921.31)	(88,322.12;39,912.43)	
185	(88,355.27;39,987.97)	(88,350.82;39,979.05)	(88,346.37;39,970.17)	(88,341.92;39,961.29)	(88,337.47;39,952.41)	(88,333.02;39,943.53)	(88,328.57;39,934.65)	(88,324.12;39,925.77)	(88,319.67;39,916.89)	(88,315.22;39,908.01)	
190	(88,348.74;39,983.54)	(88,344.31;39,974.62)	(88,339.86;39,965.74)	(88,335.41;39,956.86)	(88,330.96;39,947.98)	(88,326.51;39,939.10)	(88,322.06;39,930.22)	(88,317.61;39,921.34)	(88,313.16;39,912.46)	(88,308.71;39,903.58)	
195	(88,342.53;39,979.10)	(88,338.11;39,970.18)	(88,333.67;39,961.31)	(88,329.22;39,952.43)	(88,324.77;39,943.55)	(88,320.32;39,934.67)	(88,315.87;39,925.79)	(88,311.42;39,916.91)	(88,306.97;39,908.03)	(88,302.52;39,899.15)	
200	(88,336.64;39,974.66)	(88,332.22;39,965.74)	(88,327.78;39,956.86)	(88,323.33;39,947.98)	(88,318.88;39,939.10)	(88,314.43;39,930.22)	(88,309.98;39,921.34)	(88,305.53;39,912.46)	(88,301.08;39,903.58)	(88,296.63;39,894.70)	
205	(88,331.04;39,970.22)	(88,326.62;39,961.30)	(88,322.18;39,952.42)	(88,317.73;39,943.54)	(88,313.28;39,934.66)	(88,308.83;39,925.78)	(88,304.38;39,916.90)	(88,299.93;39,908.02)	(88,295.48;39,899.14)	(88,291.03;39,890.26)	
210	(88,325.70;39,965.78)	(88,321.27;39,956.85)	(88,316.84;39,948.00)	(88,312.40;39,939.12)	(88,307.97;39,930.24)	(88,303.54;39,921.36)	(88,299.11;39,912.48)	(88,294.68;39,903.60)	(88,290.25;39,894.72)	(88,285.82;39,885.84)	
215	(88,320.30;39,961.33)	(88,315.87;39,952.41)	(88,311.44;39,943.53)	(88,307.01;39,934.65)	(88,302.58;39,925.77)	(88,298.15;39,916.89)	(88,293.72;39,908.01)	(88,289.29;39,899.13)	(88,284.86;39,890.25)	(88,280.43;39,881.37)	
220	(88,315.74;39,956.88)	(88,311.31;39,947.96)	(88,306.88;39,939.07)	(88,302.45;39,930.19)	(88,298.02;39,921.31)	(88,293.59;39,912.43)	(88,289.16;39,903.55)	(88,284.73;39,894.67)	(88,280.30;39,885.79)	(88,275.87;39,876.91)	
225	(88,310.39;39,952.43)	(88,305.96;39,943.55)	(88,301.53;39,934.67)	(88,297.10;39,925.79)	(88,292.67;39,916.91)	(88,288.24;39,908.03)	(88,283.81;39,899.15)	(88,279.38;39,890.27)	(88,274.95;39,881.39)	(88,270.52;39,872.51)	
230	(88,306.65;39,947.97)	(88,302.22;39,939.09)	(88,297.79;39,930.21)	(88,293.36;39,921.33)	(88,288.93;39,912.45)	(88,284.50;39,903.57)	(88,280.07;39,894.69)	(88,275.64;39,885.81)	(88,271.21;39,876.93)	(88,266.78;39,868.05)	
235	(88,302.39;39,943.52)	(88,297.96;39,934.64)	(88,293.53;39,925.76)	(88,289.10;39,916.88)	(88,284.67;39,908.00)	(88,280.24;39,899.12)	(88,275.81;39,890.24)	(88,271.38;39,881.36)			

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